

Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.1-Rational-functions

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June 29, 2021

Compiled on June 29, 2021 at 12:02 Noon

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3.215	$\int (a + bx + cx^2)\left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx$	884
3.216	$\int (a + cx^2)\left(1 + \left(ax + \frac{cx^3}{3}\right)^n\right) dx$	889
3.217	$\int (bx + cx^2)\left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx$	892
3.218	$\int (a + bx + cx^2)\left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx$	894
3.219	$\int (-4 + 4x + x^2)(5 - 12x + 6x^2 + x^3) dx$	896
3.220	$\int (2x + x^3)(1 + 4x^2 + x^4) dx$	898
3.221	$\int (1 + 2x)(x + x^2)^3(-18 + 7(x + x^2)^3)^2 dx$	900
3.222	$\int x^3(1 + x)^3(1 + 2x)(-18 + 7x^3(1 + x)^3)^2 dx$	903
3.223	$\int \frac{2-x^2}{(1-6x+x^3)^5} dx$	905

3.224	$\int \frac{2x+x^2}{4+3x^2+x^3} dx$	907
3.225	$\int \frac{1+x+x^3}{4x+2x^2+x^4} dx$	909
3.226	$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bf x^3}{(c+dx+ex^2+fx^3)^2} dx$	911
3.227	$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4} dx$	914
3.228	$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx$	918
3.229	$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx$	921
3.230	$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx$	924
3.231	$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3} dx$	926
3.232	$\int \frac{-1+4x^5}{(1+x+x^5)^2} dx$	929
3.233	$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2} dx$	931
3.234	$\int x^m (a+bx+cx^2+dx^3)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x))) dx$	935
3.235	$\int x^2 (a+bx+cx^2+dx^3)^p (3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3) dx$	937
3.236	$\int x (a+bx+cx^2+dx^3)^p (2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3) dx$	939
3.237	$\int (a+bx+cx^2+dx^3)^p (a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3) dx$	941
3.238	$\int \frac{(a+bx+cx^2+dx^3)^p (b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x} dx$	943
3.239	$\int \frac{(a+bx+cx^2+dx^3)^p (-a+bp x+c(1+2p)x^2+d(2+3p)x^3)}{x^2} dx$	945
3.240	$\int \frac{(a+bx+cx^2+dx^3)^p (-2a+b(-1+p)x+2cp x^2+d(1+3p)x^3)}{x^3} dx$	947
3.241	$\int \frac{(a+bx+cx^2+dx^3)^p (-3a+b(-2+p)x+c(-1+2p)x^2+3dp x^3)}{x^4} dx$	949
3.242	$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$	951
3.243	$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$	954
3.244	$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$	957
3.245	$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$	960
3.246	$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} dx$	963
3.247	$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} dx$	966
3.248	$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)} dx$	969
3.249	$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} dx$	972
3.250	$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$	975
3.251	$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$	980
3.252	$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$	986
3.253	$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} dx$	990
3.254	$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)} dx$	994
3.255	$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)} dx$	999
3.256	$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)} dx$	1004
3.257	$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6} dx$	1009

3.258	$\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} dx$	1012
3.259	$\int \frac{-9-9x+2x^2}{-9x+x^3} dx$	1015
3.260	$\int \frac{1+2x^2+x^5}{-x+x^3} dx$	1017
3.261	$\int \frac{3+2x^2}{(-1+x)^2x} dx$	1019
3.262	$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)} dx$	1021
3.263	$\int \frac{-3+2x-3x^2+x^3}{1+x^2} dx$	1024
3.264	$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} dx$	1026
3.265	$\int \frac{1}{-18+27x-7x^2-3x^3+x^4} dx$	1029
3.266	$\int \frac{1+x^3}{-2+x} dx$	1031
3.267	$\int \frac{3x-4x^2+3x^3}{1+x^2} dx$	1033
3.268	$\int \frac{5+3x}{1-x-x^2+x^3} dx$	1035
3.269	$\int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx$	1037
3.270	$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx$	1039
3.271	$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx$	1042
3.272	$\int \frac{-1-3x+x^2}{-2x+x^2+x^3} dx$	1045
3.273	$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx$	1047
3.274	$\int \frac{-1+x+x^3}{(1+x^2)^2} dx$	1049
3.275	$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} dx$	1052
3.276	$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$	1055
3.277	$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2} dx$	1058
3.278	$\int \frac{1}{(1+x^2)(4+x^2)} dx$	1061
3.279	$\int \frac{a+bx^3}{1+x^2} dx$	1063
3.280	$\int \frac{x+x^2}{(4+x)(-4+x^2)} dx$	1066
3.281	$\int \frac{4+x^2}{(1+x^2)(2+x^2)} dx$	1068
3.282	$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)} dx$	1070
3.283	$\int \frac{1+x^4}{2+x^2} dx$	1073
3.284	$\int \frac{2+2x+x^4}{x^4+x^5} dx$	1075
3.285	$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} dx$	1077
3.286	$\int \frac{2+x+x^3}{1+2x^2+x^4} dx$	1079
3.287	$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} dx$	1082
3.288	$\int \frac{3+4x}{(1+x^2)(2+x^2)} dx$	1085
3.289	$\int \frac{2+x}{(1+x^2)(4+x^2)} dx$	1088
3.290	$\int \frac{2-x+x^3}{-7-6x+x^2} dx$	1091
3.291	$\int \frac{-1+x^5}{-1+x^2} dx$	1093
3.292	$\int \frac{5+2x-x^2+x^3}{1+x+x^2} dx$	1095

3.293	$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} dx$	1098
3.294	$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)} dx$	1101
3.295	$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx$	1103
3.296	$\int \frac{2+x^2}{(-1+x)^2x(1+x)} dx$	1105
3.297	$\int \frac{3+x^2+x^3}{(2+x^2)^2} dx$	1107
3.298	$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)} dx$	1110
3.299	$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} dx$	1113
3.300	$\int \frac{x^4}{(-1+x)(2+x^2)} dx$	1115
3.301	$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx$	1118
3.302	$\int \frac{1+2x}{-1+3x-3x^2+x^3} dx$	1120
3.303	$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} dx$	1122
3.304	$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx$	1124
3.305	$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$	1127
3.306	$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} dx$	1129
3.307	$\int \frac{4-x+2x^2}{4x+x^3} dx$	1131
3.308	$\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)} dx$	1134
3.309	$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} dx$	1138
3.310	$\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx$	1141
3.311	$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$	1144
3.312	$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)} dx$	1146
3.313	$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx$	1149
3.314	$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx$	1152
3.315	$\int \frac{1-12x+x^2+x^3}{-12+x+x^2} dx$	1155
3.316	$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx$	1157
3.317	$\int \frac{-2+3x+5x^2}{2x^2+x^3} dx$	1159
3.318	$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx$	1161
3.319	$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx$	1163
3.320	$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx$	1166
3.321	$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} dx$	1169
3.322	$\int \frac{9+x^4}{x^2(9+x^2)} dx$	1172
3.323	$\int \frac{2x+x^4}{1+x^2} dx$	1174
3.324	$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)} dx$	1177
3.325	$\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx$	1180
3.326	$\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)} dx$	1182
3.327	$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx$	1185

3.328	$\int \frac{(-1+x)^4 x^4}{1+x^2} dx$	1188
3.329	$\int \frac{-20x+4x^2}{9-10x^2+x^4} dx$	1190
3.330	$\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)} dx$	1193
3.331	$\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx$	1196
3.332	$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} dx$	1199
3.333	$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx$	1202
3.334	$\int \frac{x^2(c+dx)^2}{a+bx^3} dx$	1204
3.335	$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2} dx$	1210
3.336	$\int \frac{x+x^5}{(1+2x^2+2x^4)^3} dx$	1213
3.337	$\int \frac{a+bx+cx^2}{d+ex^2+fx^4} dx$	1216
3.338	$\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$	1222
3.339	$\int \frac{x^2}{(a+bx)(c+dx)} dx$	1228
3.340	$\int \frac{x^2}{(c+dx)(a+bx^2)} dx$	1230
3.341	$\int \frac{x^2}{(c+dx)(a+bx^3)} dx$	1233
3.342	$\int \frac{x^2}{(c+dx)(a+bx^4)} dx$	1240
3.343	$\int \frac{x}{(1-x)(1+x)^2} dx$	1245
3.344	$\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx$	1247
3.345	$\int \frac{x^3}{(1-x^3)(1+x^3)^2} dx$	1249
3.346	$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} dx$	1253
3.347	$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx$	1255
3.348	$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)} dx$	1257
3.349	$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx$	1260
3.350	$\int \frac{-3+x+x^2}{(-3+x)x^2} dx$	1263
3.351	$\int \frac{1+x+4x^2}{x+4x^3} dx$	1265
3.352	$\int \frac{1-x+3x^2}{-x^2+x^3} dx$	1267
3.353	$\int \frac{4+3x+x^2}{x+x^2} dx$	1269
3.354	$\int \frac{4+x+3x^2}{x+x^3} dx$	1271
3.355	$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx$	1274
3.356	$\int \frac{x^2}{(-1+x)(1+2x+x^2)} dx$	1276
3.357	$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)} dx$	1278
3.358	$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx$	1280
3.359	$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)} dx$	1283
3.360	$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} dx$	1286

3.361	$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} dx$	1289
3.362	$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)} dx$	1291
3.363	$\int \frac{-1+x^3}{1+x+x^2} dx$	1294
3.364	$\int \frac{-3+x^3}{-7-6x+x^2} dx$	1296
3.365	$\int \frac{1+x^3}{(13+4x+x^2)^2} dx$	1298
3.366	$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx$	1301
3.367	$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8} dx$	1304
3.368	$\int \frac{1+x^3+x^6}{x+x^5} dx$	1309
3.369	$\int \frac{1+x^2}{-x+x^2} dx$	1314
3.370	$\int \frac{1+x^3}{-x+x^3} dx$	1316
3.371	$\int \frac{1+x^3}{-x^2+x^3} dx$	1318
3.372	$\int \frac{-1+x^5}{-x+x^3} dx$	1320
3.373	$\int \frac{1+x^4}{x^3+x^5} dx$	1322
3.374	$\int \frac{1+x^2}{x+2x^2+x^3} dx$	1324
3.375	$\int \frac{1+x^5}{-10x-3x^2+x^3} dx$	1326
3.376	$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$	1328
3.377	$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$	1331
3.378	$\int \frac{x^3}{13+\frac{2}{x}+15x} dx$	1333
3.379	$\int \frac{x^2}{13+\frac{2}{x}+15x} dx$	1336
3.380	$\int \frac{x}{13+\frac{2}{x}+15x} dx$	1339
3.381	$\int \frac{1}{13+\frac{2}{x}+15x} dx$	1342
3.382	$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)} dx$	1344
3.383	$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)} dx$	1346
3.384	$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)} dx$	1349
3.385	$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)} dx$	1352
3.386	$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)} dx$	1355
3.387	$\int \frac{x^2}{2-(1+x^2)^4} dx$	1358
3.388	$\int \frac{x^2}{2-(1-x^2)^4} dx$	1362
3.389	$\int \frac{x^2}{2+(1+x^2)^4} dx$	1366
3.390	$\int \frac{x^2}{2+(1-x^2)^4} dx$	1371
3.391	$\int \frac{1-x^2}{a+b(1-x^2)^4} dx$	1376
3.392	$\int \frac{1-x^2}{a+b(-1+x^2)^4} dx$	1381

3.393	$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx$	1386
3.394	$\int \frac{(d+ex)^3}{a+cx^4} dx$	1391
3.395	$\int \frac{(d+ex)^2}{a+cx^4} dx$	1396
3.396	$\int \frac{d+ex}{a+cx^4} dx$	1401
3.397	$\int \frac{1}{a+cx^4} dx$	1405
3.398	$\int \frac{1}{(d+ex)(a+cx^4)} dx$	1409
3.399	$\int \frac{1}{(d+ex)^2(a+cx^4)} dx$	1414
3.400	$\int \frac{1}{(d+ex)^3(a+cx^4)} dx$	1420
3.401	$\int \frac{(d+ex)^3}{(a+cx^4)^2} dx$	1427
3.402	$\int \frac{(d+ex)^2}{(a+cx^4)^2} dx$	1432
3.403	$\int \frac{d+ex}{(a+cx^4)^2} dx$	1437
3.404	$\int \frac{1}{(a+cx^4)^2} dx$	1442
3.405	$\int \frac{1}{(d+ex)(a+cx^4)^2} dx$	1446
3.406	$\int \frac{1}{(d+ex)^2(a+cx^4)^2} dx$	1453
3.407	$\int \frac{1}{(d+ex)^3(a+cx^4)^2} dx$	1461
3.408	$\int \frac{(d+ex)^3}{(a+cx^4)^3} dx$	1470
3.409	$\int \frac{(d+ex)^2}{(a+cx^4)^3} dx$	1476
3.410	$\int \frac{d+ex}{(a+cx^4)^3} dx$	1482
3.411	$\int \frac{1}{(a+cx^4)^3} dx$	1487
3.412	$\int \frac{1}{(d+ex)(a+cx^4)^3} dx$	1491
3.413	$\int \frac{1}{(d+ex)^2(a+cx^4)^3} dx$	1499
3.414	$\int \frac{1}{(d+ex)^3(a+cx^4)^3} dx$	1508
3.415	$\int \frac{-1+x}{1-x+x^2} dx$	1521
3.416	$\int \frac{-1+x^2}{1+x^3} dx$	1524
3.417	$\int \frac{-4+3x}{4-2x+x^2} dx$	1527
3.418	$\int \frac{-8+2x+3x^2}{8+x^3} dx$	1530
3.419	$\int \frac{2+x}{-1+2x+x^2} dx$	1533
3.420	$\int \frac{-4+x^2}{2-5x+x^3} dx$	1535
3.421	$\int \frac{2}{-1+4x^2} dx$	1538
3.422	$\int \left(\frac{1}{-1+2x} - \frac{1}{1+2x} \right) dx$	1540
3.423	$\int \frac{x}{(1-x^2)^5} dx$	1542
3.424	$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} - \frac{5}{256(1+x)^2} \right) dx$	1544
3.425	$\int \frac{1+x^6}{-1+x^6} dx$	1546

3.426	$\int \frac{\frac{1}{x^3} + x^3}{-\frac{1}{x^3} + x^3} dx$	1549
3.427	$\int \frac{-x+x^3}{6+2x} dx$	1553
3.428	$\int \frac{x+x^3}{-1+x} dx$	1555
3.429	$\int (ac + (bc + d)x) dx$	1557
3.430	$\int (dx + c(a + bx)) dx$	1559
3.431	$\int \frac{4+4x}{x^2(1+x^2)} dx$	1561
3.432	$\int \frac{24+8x}{x(-4+x^2)} dx$	1564
3.433	$\int \frac{-1+x^2}{-2x+x^3} dx$	1566
3.434	$\int \frac{1+x^2}{3x+x^3} dx$	1568
3.435	$\int \frac{a+3bx^2}{ax+bx^3} dx$	1570
3.436	$\int \frac{-2+4x}{-x+x^3} dx$	1572
3.437	$\int \frac{4+x}{4x+x^3} dx$	1574
3.438	$\int \frac{-x+2x^3}{1-x^2+x^4} dx$	1577
3.439	$\int \frac{-3+x}{2x+3x^2+x^3} dx$	1579
3.440	$\int \frac{2+4x}{x^2+2x^3+x^4} dx$	1581
3.441	$\int \frac{1+x}{-6x+x^2+x^3} dx$	1583
3.442	$\int \frac{4x^2+x^3}{x+x^3} dx$	1585
3.443	$\int \frac{x+2x^3}{(x^2+x^4)^3} dx$	1588
3.444	$\int \frac{ax^2+bx^3}{cx^2+dx^3} dx$	1590
3.445	$\int \frac{x+x^2}{-2x-x^2+x^3} dx$	1592
3.446	$\int \frac{1-5x^2}{x^3(1+x^2)} dx$	1594
3.447	$\int \frac{2x}{(-1+x)(5+x^2)} dx$	1596
3.448	$\int \frac{2+x^2}{2+x} dx$	1599
3.449	$\int \frac{1}{(-3+x)(4+x^2)} dx$	1601
3.450	$\int \frac{-2+3x^6}{x(5+2x^6)} dx$	1604
3.451	$\int \frac{3+2x}{(-2+x)(5+x)} dx$	1606
3.452	$\int \frac{x^4}{4+5x^2+x^4} dx$	1608
3.453	$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$	1611
3.454	$\int \frac{x}{-1+x^2} dx$	1613
3.455	$\int \frac{1}{(-1+x^2)^2} dx$	1615
3.456	$\int \frac{x^2}{(1+x^2)^2} dx$	1617
3.457	$\int \frac{1}{2+3x} dx$	1619
3.458	$\int \frac{1}{a^2+x^2} dx$	1621
3.459	$\int \frac{1}{a+bx^2} dx$	1623
3.460	$\int \frac{1}{2-x+x^2} dx$	1625
3.461	$\int x^2(4-x^2)^2 dx$	1627
3.462	$\int x(1-x^3)^2 dx$	1629

3.463	$\int \frac{-4+5x^2+x^3}{x^2} dx$	1631
3.464	$\int \frac{-1+x}{3-4x+3x^2} dx$	1633
3.465	$\int (2+x^3)^2 dx$	1636
3.466	$\int \frac{-4+x^2}{2+x} dx$	1638
3.467	$\int \frac{1}{(2+x)(1+x^2)} dx$	1640
3.468	$\int \frac{1}{(1+x)(1+x^2)} dx$	1643
3.469	$\int \frac{x}{(1+x)(1+x^2)} dx$	1646
3.470	$\int \frac{2x+x^2}{(1+x)^2} dx$	1649
3.471	$\int \frac{-10+x^2}{4+9x^2+2x^4} dx$	1651
3.472	$\int \frac{31+5x}{11-4x+3x^2} dx$	1653
3.473	$\int \frac{-2+x^2+x^3}{x^4} dx$	1656
3.474	$\int \frac{1+x+x^3}{x^2} dx$	1658
3.475	$\int \frac{-2+x^2}{x(2+x^2)} dx$	1660
3.476	$\int (-3+x)(-7+4x^2) dx$	1662
3.477	$\int (-2+7x)^3 dx$	1664
3.478	$\int \frac{-7+4x^2}{3+2x} dx$	1666
3.479	$\int \frac{1+x}{(-1+x)x^2} dx$	1668
3.480	$\int \frac{1}{4x^2+4x^3+x^4} dx$	1670
3.481	$\int \frac{1+x^2}{1+x} dx$	1673
3.482	$\int \frac{-1+3x-3x^2+x^3}{x^2} dx$	1675
3.483	$\int \left(\frac{1}{2}(3-\sqrt{37})+x \right) \left(\frac{1}{2}(3+\sqrt{37})+x \right) dx$	1677
3.484	$\int \frac{4+3x^2+2x^3}{(1+x)^4} dx$	1679
3.485	$\int \frac{x}{(1+x)^2(1+x^2)} dx$	1681
3.486	$\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx$	1683
3.487	$\int \frac{-1+x^3}{-1+x} dx$	1685
3.488	$\int \frac{2+2x}{(-1+x)^3(1+x^2)} dx$	1687
3.489	$\int \frac{1}{bx+c(d+ex)^2} dx$	1689
3.490	$\int \frac{1}{a+bx+c(d+ex)^2} dx$	1692
3.491	$\int \frac{x^2}{1+(-1+x^2)^2} dx$	1695
3.492	$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4} dx$	1699
3.493	$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4} + \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2} \right) dx$	1702
3.494	$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4} \right) dx$	1705

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [494]. This is test number [51].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.39 (491)	% 0.61 (3)
Mathematica	% 100.00 (494)	% 0.00 (0)
Maple	% 98.99 (489)	% 1.01 (5)
Maxima	% 82.79 (409)	% 17.21 (85)
Fricas	% 87.25 (431)	% 12.75 (63)
Sympy	% 87.25 (431)	% 12.75 (63)
Giac	% 85.22 (421)	% 14.78 (73)
Mupad	% 98.18 (485)	% 1.82 (9)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

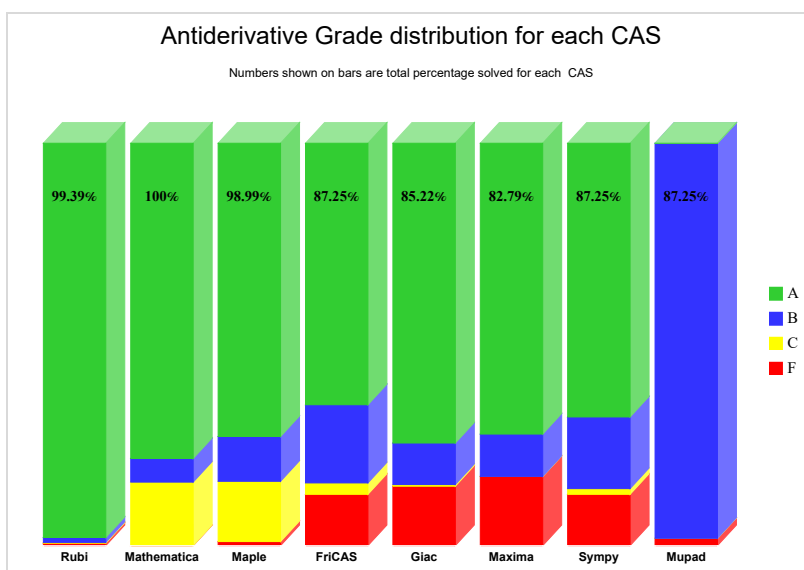
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

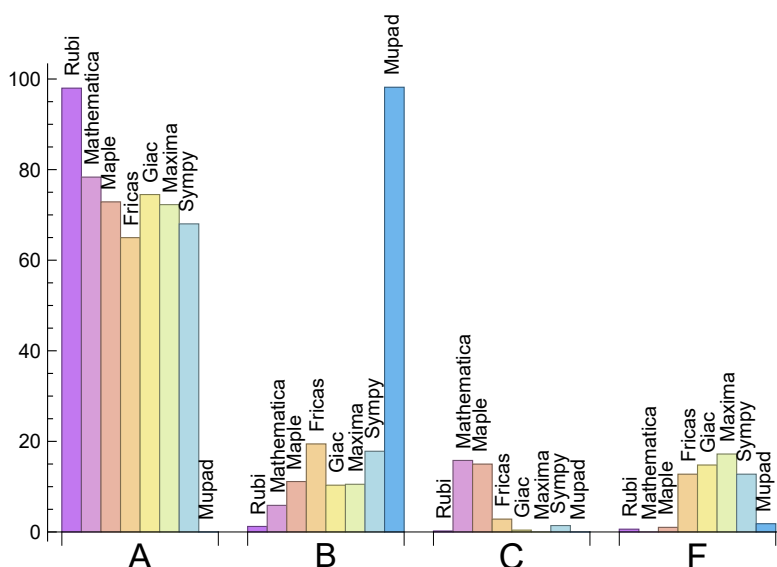
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.98	1.21	0.20	0.61
Mathematica	78.34	5.87	15.79	0.00
Maple	72.87	11.13	14.98	1.01
Maxima	72.27	10.53	0.00	17.21
Fricas	64.98	19.43	2.83	12.75
Sympy	68.02	17.81	1.42	12.75
Giac	74.49	10.32	0.40	14.78
Mupad	0.00	98.18	0.00	1.82

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	5	100.00 %	0.00 %	0.00 %
Maxima	85	94.12 %	3.53 %	2.35 %
Fricas	63	6.35 %	90.48 %	3.17 %
Sympy	63	12.70 %	87.30 %	0.00 %
Giac	73	91.78 %	2.74 %	5.48 %
Mupad	9	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

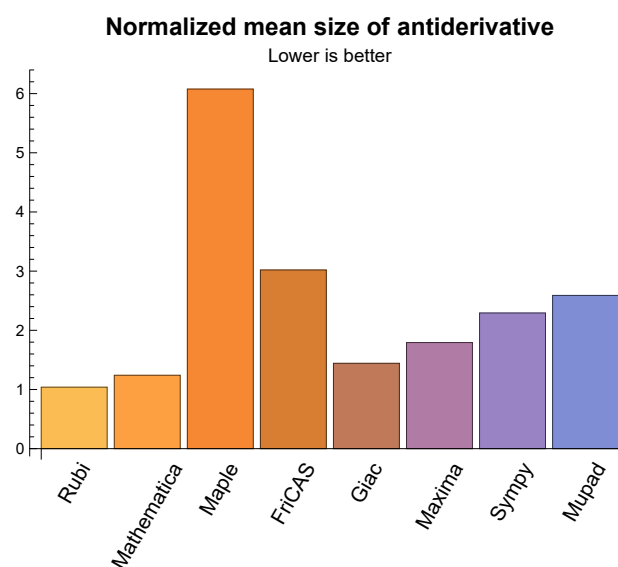
1.3 Performance

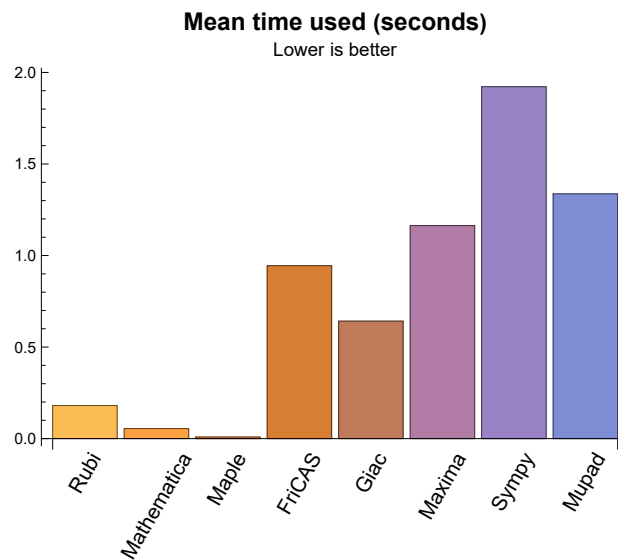
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	119.38	1.04	33.00	1.00
Mathematica	0.05	80.70	1.24	37.00	1.00
Maple	0.01	208.48	6.08	34.00	0.91
Maxima	1.16	115.91	1.79	28.00	0.88
Fricas	0.94	297.22	3.02	36.00	1.10
Sympy	1.92	141.91	2.29	39.00	0.91
Giac	0.64	110.16	1.44	28.00	0.92
Mupad	1.34	450.36	2.59	41.00	0.95

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {31, 32, 151, 152, 153, 154, 155, 156, 157}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```


1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

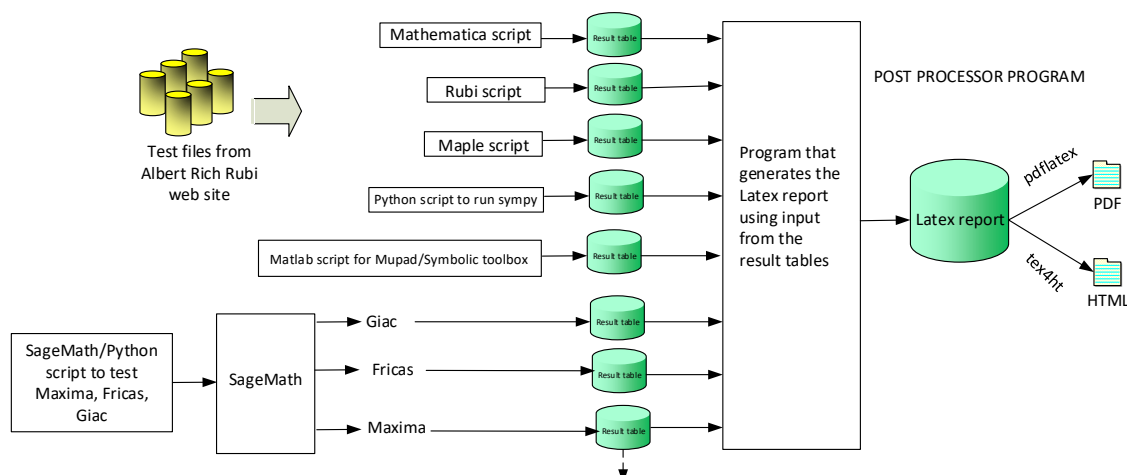
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 65, 77, 221, 222, 233, 424 }

C grade: { 174 }

F grade: { 393, 493, 494 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 193, 194, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328,

329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494 }

B grade: { 65, 92, 95, 160, 161, 162, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421 }

C grade: { 12, 13, 14, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 173, 174, 175, 176, 184, 185, 227, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 491 }

F grade: { }

2.1.3 Maple

A grade: { 2, 5, 6, 7, 8, 10, 11, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492 }

B grade: { 3, 4, 9, 15, 20, 63, 64, 65, 92, 95, 101, 102, 116, 160, 161, 162, 163, 168, 169, 170, 171, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 227, 232, 337, 338, 400, 421, 424, 443, 491, 494 }

C grade: { 1, 12, 13, 14, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 174, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 493 }

F grade: { 29, 30, 31, 32, 176 }

2.1.4 Maxima

A grade: { 2, 5, 6, 10, 11, 15, 16, 17, 18, 21, 22, 23, 24, 25, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 93, 96, 97, 98, 99, 100, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 164, 165, 166, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320,

321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492 }

B grade: { 3, 4, 7, 8, 9, 19, 20, 63, 64, 65, 66, 67, 68, 74, 87, 88, 91, 92, 94, 95, 101, 102, 161, 162, 163, 167, 169, 170, 171, 194, 197, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421, 424, 477, 493, 494 }

C grade: { }

F grade: { 1, 12, 13, 14, 26, 29, 30, 31, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 227, 228, 250, 251, 252, 253, 254, 255, 256, 257, 335, 336, 337, 338, 387, 388, 389, 390, 391, 392, 393, 490, 491 }

2.1.5 FriCAS

A grade: { 1, 2, 5, 6, 10, 11, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 96, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 224, 225, 226, 228, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 397, 404, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 3, 4, 7, 8, 9, 12, 13, 14, 15, 37, 38, 43, 44, 63, 64, 65, 66, 67, 68, 74, 75, 76, 77, 87, 91, 92, 93, 94, 95, 97, 116, 120, 121, 122, 146, 154, 160, 161, 162, 163, 168, 169, 170, 171, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 223, 229, 233, 250, 251, 252, 253, 254, 255, 256, 257, 268, 277, 343, 344, 387, 388, 389, 390, 421, 423, 424, 453, 455, 477, 484, 493, 494 }

C grade: { 49, 50, 55, 56, 103, 104, 105, 107, 108, 109, 334, 341, 368, 393 }

F grade: { 19, 20, 29, 30, 31, 32, 61, 62, 110, 111, 112, 114, 115, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 227, 337, 338, 342, 391, 392, 394, 395, 396, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 412, 413, 414, 483 }

2.1.6 SymPy

A grade: { 1, 5, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 27, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 57, 58, 59, 60, 61, 69, 70, 71, 72, 73, 75, 76, 93, 96, 97, 98, 99, 103, 104, 105, 106, 110, 111, 112, 113, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 130, 131, 132, 133, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 178, 187, 188, 189, 191, 208, 219, 220, 224, 225, 226, 228, 229, 230, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311,

312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 403, 404, 408, 409, 410, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492 }

B grade: { 3, 4, 6, 7, 8, 9, 15, 26, 50, 55, 56, 62, 63, 64, 65, 66, 67, 68, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 94, 95, 121, 122, 128, 129, 134, 135, 160, 161, 162, 168, 169, 170, 179, 184, 185, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 233, 251, 268, 339, 421, 423, 424, 459, 477, 489, 490, 493, 494 }

C grade: { 32, 89, 90, 91, 257, 279, 458 }

F grade: { 2, 18, 19, 20, 28, 29, 30, 31, 44, 100, 101, 102, 107, 108, 109, 114, 115, 136, 137, 138, 139, 140, 141, 142, 163, 171, 173, 174, 175, 176, 180, 181, 182, 183, 186, 190, 209, 217, 218, 227, 234, 235, 236, 237, 238, 239, 240, 241, 255, 337, 338, 340, 341, 342, 398, 399, 400, 405, 406, 407, 412, 413, 414 }

2.1.7 Giac

A grade: { 1, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 160, 164, 165, 166, 168, 169, 170, 172, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 189, 190, 191, 193, 194, 196, 197, 199, 200, 201, 206, 208, 209, 210, 212, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 2, 3, 4, 9, 15, 19, 20, 63, 64, 65, 74, 92, 95, 116, 159, 161, 162, 163, 173, 174, 175, 182, 188, 192, 195, 198, 202, 203, 204, 205, 207, 211, 213, 215, 234, 235, 236, 237, 238, 257, 268, 282, 324, 337, 338, 359, 421, 424, 485, 493, 494 }

C grade: { 55, 56 }

F grade: { 29, 30, 31, 32, 43, 44, 49, 50, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 171, 176, 227, 239, 240, 241, 250, 251, 252, 253, 254, 255, 256, 387, 388, 389, 390, 391, 392, 393, 413 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154,

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C grade: { }

F grade: { 31, 100, 101, 102, 167, 171, 174, 176, 227 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	143	43	0	76	60	55	51
normalized size	1	1.00	1.86	0.56	0.00	0.99	0.78	0.71	0.66
time (sec)	N/A	0.134	0.074	0.054	0.000	0.586	0.334	0.221	0.196
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	23	46	25	43	0	73	52
normalized size	1	1.00	0.77	1.53	0.83	1.43	0.00	2.43	1.73
time (sec)	N/A	0.018	0.105	0.012	0.489	0.864	0.000	0.196	2.132
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	98	216	97	107	97	97
normalized size	1	1.00	1.00	7.00	15.43	6.93	7.64	6.93	6.93
time (sec)	N/A	0.007	0.001	0.003	0.686	0.458	0.088	0.254	2.073
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	99	64	66	64	64
normalized size	1	1.00	1.00	4.64	7.07	4.57	4.71	4.57	4.57
time (sec)	N/A	0.007	0.001	0.001	0.657	0.425	0.079	0.375	0.028
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
normalized size	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.007	0.000	0.001	0.735	0.518	0.066	0.283	0.037

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	24	24	26	12	26
normalized size	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.86
time (sec)	N/A	0.009	0.005	0.023	0.649	0.590	0.183	0.264	0.034
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	57	57	61	12	59
normalized size	1	1.00	1.00	0.93	4.07	4.07	4.36	0.86	4.21
time (sec)	N/A	0.009	0.004	0.003	0.714	0.733	0.349	0.295	2.048
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	90	90	97	12	92
normalized size	1	1.00	1.00	0.93	6.43	6.43	6.93	0.86	6.57
time (sec)	N/A	0.008	0.003	0.005	0.538	0.830	0.543	0.392	2.069
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	159	295	204	166	175	166	149
normalized size	1	1.00	1.89	3.51	2.43	1.98	2.08	1.98	1.77
time (sec)	N/A	0.125	0.025	0.002	0.676	0.516	0.101	0.273	2.077
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	82	84	93	83	87	83	79
normalized size	1	1.00	1.46	1.50	1.66	1.48	1.55	1.48	1.41
time (sec)	N/A	0.068	0.010	0.002	0.636	0.693	0.083	0.344	0.039
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
normalized size	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.006	0.000	0.001	0.550	0.601	0.064	0.270	0.038

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	63	57	0	387	53	212	174
normalized size	1	1.00	0.34	0.30	0.00	2.06	0.28	1.13	0.93
time (sec)	N/A	0.312	0.021	0.005	0.000	0.860	0.406	0.358	0.494
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	112	136	0	704	192	289	247
normalized size	1	1.00	0.46	0.56	0.00	2.87	0.78	1.18	1.01
time (sec)	N/A	0.249	0.066	0.020	0.000	0.821	1.225	0.357	2.645
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	149	276	0	1268	474	366	483
normalized size	1	1.00	0.49	0.90	0.00	4.16	1.55	1.20	1.58
time (sec)	N/A	0.302	0.090	0.018	0.000	0.779	2.542	0.397	2.983
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	653	861	461	987	1018	971	787
normalized size	1	1.00	1.81	2.39	1.28	2.73	2.82	2.69	2.18
time (sec)	N/A	0.658	0.224	0.002	0.680	0.668	0.246	0.268	2.226
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	241	188	180	346	345	346	270
normalized size	1	1.00	1.25	0.97	0.93	1.79	1.79	1.79	1.40
time (sec)	N/A	0.231	0.086	0.002	0.625	0.562	0.135	0.415	0.084
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	76	51	50	62	63	54	54
normalized size	1	1.00	1.36	0.91	0.89	1.11	1.12	0.96	0.96
time (sec)	N/A	0.016	0.000	0.000	0.827	0.696	0.073	0.232	0.045

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	87	112	112	0	137	106
normalized size	1	1.00	0.93	1.01	1.30	1.30	0.00	1.59	1.23
time (sec)	N/A	0.073	0.057	0.011	0.647	10.239	0.000	0.307	2.335
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	232	398	2096	0	0	1414	1940
normalized size	1	1.00	0.99	1.70	8.96	0.00	0.00	6.04	8.29
time (sec)	N/A	0.405	0.552	0.026	1.539	0.000	0.000	0.392	8.176
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	490	1076	11005	0	0	6908	82532
normalized size	1	1.00	0.99	2.17	22.23	0.00	0.00	13.96	166.73
time (sec)	N/A	1.462	1.192	0.034	5.723	0.000	0.000	1.103	20.456
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.015	0.009	0.013	1.453	0.615	0.131	0.305	2.201
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	24	26	25
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.77	0.84	0.81
time (sec)	N/A	0.020	0.007	0.007	1.280	0.753	0.146	0.352	0.053
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.001	0.000	0.001	0.626	0.698	0.064	0.274	0.034

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
normalized size	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.015	0.006	0.009	0.587	0.890	0.180	0.362	0.060
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	18	15	24	18
normalized size	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.012	0.007	0.005	0.793	0.776	0.205	0.296	2.134
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	211	564	62	213
normalized size	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.055	0.082	0.009	0.000	0.864	4.193	0.294	0.465
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	98	299	20	112	99
normalized size	1	1.00	0.77	0.79	0.85	2.60	0.17	0.97	0.86
time (sec)	N/A	0.063	0.036	0.008	1.281	0.832	0.157	0.264	0.233
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	16	0	16	16
normalized size	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00
time (sec)	N/A	0.004	0.002	0.002	0.574	0.846	0.000	0.309	2.500
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0	56
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.02
time (sec)	N/A	0.019	0.012	0.089	0.000	0.461	0.000	0.000	2.206

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	61	0	0	0	0	0	56
normalized size	1	1.11	1.15	0.00	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.023	0.016	0.092	0.000	0.454	0.000	0.000	2.217
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	157	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.360	0.024	0.000	0.453	0.000	0.000	0.000
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	196	0	0	0	34	0	41
normalized size	1	1.26	5.60	0.00	0.00	0.00	0.97	0.00	1.17
time (sec)	N/A	0.009	0.195	0.085	0.000	0.445	10.544	0.000	2.176
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	285	392	372	277	299	277	261
normalized size	1	1.00	1.06	1.45	1.38	1.03	1.11	1.03	0.97
time (sec)	N/A	0.539	0.040	0.002	0.628	0.386	0.127	0.235	2.298
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	231	205	166	180	166	160
normalized size	1	1.00	1.00	1.35	1.20	0.97	1.05	0.97	0.94
time (sec)	N/A	0.092	0.028	0.002	0.802	0.548	0.102	0.243	2.160
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	84	94	83	95	83	82
normalized size	1	1.00	1.00	0.91	1.02	0.90	1.03	0.90	0.89
time (sec)	N/A	0.043	0.021	0.001	0.570	0.399	0.085	0.252	0.036

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
normalized size	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.006	0.000	0.002	0.651	0.385	0.067	0.268	0.037
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	71	64	0	905	88	603	1551
normalized size	1	1.00	0.13	0.12	0.00	1.71	0.17	1.14	2.93
time (sec)	N/A	0.896	0.027	0.059	0.000	0.491	1.149	0.336	4.541
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	182	232	0	3222	427	1057	5844
normalized size	1	1.00	0.24	0.31	0.00	4.32	0.57	1.42	7.83
time (sec)	N/A	1.328	0.116	0.018	0.000	0.545	109.971	0.367	4.304
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	345	500	383	353	366	323	331
normalized size	1	1.00	1.17	1.69	1.30	1.20	1.24	1.09	1.12
time (sec)	N/A	0.532	0.055	0.002	0.745	0.387	0.137	0.308	0.268
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	207	288	214	205	218	187	201
normalized size	1	1.00	1.02	1.42	1.05	1.01	1.07	0.92	0.99
time (sec)	N/A	0.123	0.029	0.001	0.598	0.377	0.109	0.238	2.243
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	109	100	101	99	112	90	98
normalized size	1	1.00	1.02	0.93	0.94	0.93	1.05	0.84	0.92
time (sec)	N/A	0.051	0.015	0.000	0.607	0.391	0.087	0.293	0.044

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	33	33	36	30	33
normalized size	1	1.00	1.00	0.92	0.89	0.89	0.97	0.81	0.89
time (sec)	N/A	0.007	0.000	0.000	0.667	0.377	0.068	0.380	0.043
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	71	67	0	1115	122	0	1264
normalized size	1	1.00	0.46	0.44	0.00	7.29	0.80	0.00	8.26
time (sec)	N/A	0.254	0.033	0.066	0.000	0.481	1.835	0.000	3.731
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	234	288	0	4285	0	0	10351
normalized size	1	1.00	0.68	0.84	0.00	12.53	0.00	0.00	30.27
time (sec)	N/A	0.532	0.193	0.023	0.000	0.734	0.000	0.000	7.109
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	85	84	84	94	84	84
normalized size	1	1.00	1.00	0.89	0.88	0.88	0.98	0.88	0.88
time (sec)	N/A	0.030	0.003	0.002	1.248	0.349	0.075	0.307	0.185
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	65	64	64	71	64	64
normalized size	1	1.00	1.00	0.88	0.86	0.86	0.96	0.86	0.86
time (sec)	N/A	0.023	0.001	0.001	0.939	0.347	0.070	0.334	0.076
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	44	44
normalized size	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.017	0.001	0.000	0.884	0.377	0.062	0.332	0.032

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.87	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.003	0.000	0.000	0.911	0.346	0.057	0.315	0.031
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	45	41	0	1015	41	0	123
normalized size	1	1.00	0.17	0.15	0.00	3.79	0.15	0.00	0.46
time (sec)	N/A	0.396	0.010	0.015	0.000	1.731	0.955	0.000	2.445
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	113	83	0	1201	3834	0	176
normalized size	1	1.00	0.32	0.23	0.00	3.36	10.74	0.00	0.49
time (sec)	N/A	0.397	0.018	0.013	0.000	2.233	3.224	0.000	0.207
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	78	77	77	94	77	77
normalized size	1	1.00	1.00	0.80	0.79	0.79	0.97	0.79	0.79
time (sec)	N/A	0.028	0.002	0.003	0.570	0.343	0.071	0.364	0.150
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.020	0.001	0.001	0.455	0.357	0.066	0.362	0.063
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	37	37	42	37	37
normalized size	1	1.00	1.00	0.84	0.82	0.82	0.93	0.82	0.82
time (sec)	N/A	0.015	0.001	0.001	0.568	0.337	0.062	0.255	0.026

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	19	17	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.90	0.81	0.81
time (sec)	N/A	0.003	0.000	0.000	0.569	0.344	0.056	0.361	0.028
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	47	41	0	499	3432	265	87
normalized size	1	1.00	0.20	0.18	0.00	2.13	14.67	1.13	0.37
time (sec)	N/A	0.315	0.016	0.007	0.000	1.330	2.572	0.520	2.359
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	108	79	0	704	3834	315	174
normalized size	1	1.00	0.34	0.25	0.00	2.22	12.09	0.99	0.55
time (sec)	N/A	0.335	0.024	0.013	0.000	1.345	3.665	0.724	2.207
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	85	84	84	100	84	84
normalized size	1	1.00	1.00	0.82	0.81	0.81	0.96	0.81	0.81
time (sec)	N/A	0.034	0.002	0.002	0.661	0.368	0.081	0.364	2.230
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	65	64	64	73	64	64
normalized size	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84
time (sec)	N/A	0.024	0.001	0.000	0.646	0.344	0.076	0.384	0.078
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	44	44	49	44	44
normalized size	1	1.00	1.00	0.87	0.85	0.85	0.94	0.85	0.85
time (sec)	N/A	0.016	0.001	0.002	0.610	0.343	0.066	0.357	0.033

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	27	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.90	0.80	0.80
time (sec)	N/A	0.004	0.000	0.001	0.462	0.342	0.058	0.300	0.019
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	55	49	0	0	41	0	123
normalized size	1	1.00	0.21	0.19	0.00	0.00	0.16	0.00	0.47
time (sec)	N/A	0.493	0.015	0.008	0.000	0.000	2.372	0.000	0.411
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	128	96	0	0	3839	0	181
normalized size	1	1.00	0.35	0.26	0.00	0.00	10.49	0.00	0.49
time (sec)	N/A	0.508	0.029	0.011	0.000	0.000	3.946	0.000	0.205
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	164	592	163	185	163	163
normalized size	1	1.00	1.00	11.71	42.29	11.64	13.21	11.64	11.64
time (sec)	N/A	0.017	0.002	0.003	0.683	0.707	0.114	0.283	0.167
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	109	228	108	114	108	108
normalized size	1	1.00	1.00	7.79	16.29	7.71	8.14	7.71	7.71
time (sec)	N/A	0.018	0.001	0.002	0.491	0.684	0.101	0.242	0.061
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	61	61	54	53	53	60	53	53
normalized size	1	4.36	4.36	3.86	3.79	3.79	4.29	3.79	3.79
time (sec)	N/A	0.014	0.000	0.001	0.458	0.563	0.074	0.292	0.024

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	46	46	49	12	48
normalized size	1	1.00	1.00	0.93	3.29	3.29	3.50	0.86	3.43
time (sec)	N/A	0.018	0.004	0.006	0.690	0.831	0.288	0.354	0.047
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	101	101	109	12	103
normalized size	1	1.00	1.00	0.93	7.21	7.21	7.79	0.86	7.36
time (sec)	N/A	0.017	0.004	0.005	0.602	0.616	0.593	0.276	2.105
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	156	156	168	12	158
normalized size	1	1.00	1.00	0.93	11.14	11.14	12.00	0.86	11.29
time (sec)	N/A	0.018	0.005	0.005	0.544	0.786	0.936	0.355	3.033
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	30	30	29	31	36
normalized size	1	1.00	1.00	0.82	0.79	0.79	0.76	0.82	0.95
time (sec)	N/A	0.025	0.008	0.009	1.113	0.793	0.154	0.295	2.165
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	65	64	64	80	64	64
normalized size	1	1.00	1.00	0.77	0.76	0.76	0.95	0.76	0.76
time (sec)	N/A	0.086	0.002	0.000	0.488	0.488	0.077	0.380	2.166
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	50	49	49	60	49	49
normalized size	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.076	0.002	0.002	0.727	0.776	0.070	0.365	0.053

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.069	0.001	0.003	0.749	0.650	0.064	0.362	0.024
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	22	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.003	0.000	0.002	0.639	0.716	0.057	0.279	0.032
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	62	42	54	56	63	62	27
normalized size	1	1.00	2.00	1.35	1.74	1.81	2.03	2.00	0.87
time (sec)	N/A	0.023	0.018	0.011	1.055	0.878	0.154	0.282	0.066
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	103	84	89	177	104	97	64
normalized size	1	1.00	1.16	0.94	1.00	1.99	1.17	1.09	0.72
time (sec)	N/A	0.061	0.063	0.023	1.289	0.819	1.359	0.278	0.084
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	137	126	119	282	134	112	93
normalized size	1	1.00	0.85	0.78	0.74	1.75	0.83	0.70	0.58
time (sec)	N/A	0.119	0.092	0.023	1.320	0.884	1.453	0.308	0.089
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	114	223	296	134	126
normalized size	1	2.25	1.45	1.27	1.25	2.45	3.25	1.47	1.38
time (sec)	N/A	0.130	0.103	0.022	1.271	0.872	1.426	0.435	2.184

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	127	81	198	209	77	87
normalized size	1	1.00	0.94	1.63	1.04	2.54	2.68	0.99	1.12
time (sec)	N/A	0.063	0.067	0.012	1.458	0.869	0.694	0.341	2.269
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	54	89	61	157	153	54	206
normalized size	1	1.00	1.08	1.78	1.22	3.14	3.06	1.08	4.12
time (sec)	N/A	0.039	0.033	0.007	1.648	0.803	0.462	0.308	0.090
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	54	50	136	124	43	46
normalized size	1	1.00	0.93	1.32	1.22	3.32	3.02	1.05	1.12
time (sec)	N/A	0.021	0.016	0.003	1.362	0.822	0.250	0.356	2.076
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	24	83	54	17	17
normalized size	1	1.00	1.00	1.33	1.14	3.95	2.57	0.81	0.81
time (sec)	N/A	0.008	0.004	0.003	1.520	0.659	0.186	0.377	0.043
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	72	68	154	738	62	173
normalized size	1	1.00	0.81	1.22	1.15	2.61	12.51	1.05	2.93
time (sec)	N/A	0.035	0.041	0.007	1.518	0.841	3.403	0.377	2.588
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	81	123	123	229	1620	117	425
normalized size	1	1.00	1.03	1.56	1.56	2.90	20.51	1.48	5.38
time (sec)	N/A	0.086	0.053	0.011	1.591	0.920	11.116	0.367	2.583

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	198	197	371	3284	195	573
normalized size	1	1.00	0.88	1.64	1.63	3.07	27.14	1.61	4.74
time (sec)	N/A	0.125	0.166	0.010	1.612	0.846	38.258	0.415	2.771
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	30	109	61	24	27
normalized size	1	1.00	1.00	1.10	0.97	3.52	1.97	0.77	0.87
time (sec)	N/A	0.024	0.012	0.006	1.561	0.723	0.207	0.453	0.057
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	86	75	253	117	65	76
normalized size	1	1.00	0.95	1.37	1.19	4.02	1.86	1.03	1.21
time (sec)	N/A	0.033	0.035	0.004	1.560	0.562	0.580	0.359	0.098
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	147	184	595	257	103	181
normalized size	1	1.00	0.82	1.62	2.02	6.54	2.82	1.13	1.99
time (sec)	N/A	0.048	0.071	0.005	1.585	1.008	1.248	0.409	2.221
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	42	66	279	92	30	31
normalized size	1	1.00	1.00	1.20	1.89	7.97	2.63	0.86	0.89
time (sec)	N/A	0.035	0.019	0.010	1.404	0.873	0.218	0.446	0.103
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	18	10	24	10	10
normalized size	1	1.00	1.00	1.10	1.80	1.00	2.40	1.00	1.00
time (sec)	N/A	0.003	0.005	0.003	1.569	0.850	0.168	0.318	0.040

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	59	51	55	56	41	42
normalized size	1	1.00	0.84	1.59	1.38	1.49	1.51	1.11	1.14
time (sec)	N/A	0.010	0.014	0.006	1.315	0.815	0.437	0.377	2.066
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	94	115	153	146	73	111
normalized size	1	1.00	0.87	1.57	1.92	2.55	2.43	1.22	1.85
time (sec)	N/A	0.016	0.020	0.006	1.505	0.839	0.931	0.304	0.121
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	32	26	25	22	22	27	10
normalized size	1	1.00	3.20	2.60	2.50	2.20	2.20	2.70	1.00
time (sec)	N/A	0.003	0.006	0.010	0.490	0.792	0.177	0.384	2.050
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	45	52	56	85	54	56	43
normalized size	1	1.00	1.15	1.33	1.44	2.18	1.38	1.44	1.10
time (sec)	N/A	0.013	0.026	0.010	0.624	0.831	0.469	0.351	2.064
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	78	122	220	141	88	114
normalized size	1	1.00	1.02	1.22	1.91	3.44	2.20	1.38	1.78
time (sec)	N/A	0.022	0.032	0.010	0.641	0.845	1.030	0.357	2.118
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	15	12	11	11	10	13	4
normalized size	1	1.00	3.75	3.00	2.75	2.75	2.50	3.25	1.00
time (sec)	N/A	0.002	0.002	0.005	0.571	0.888	0.099	0.355	0.149

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
normalized size	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.007	0.018	0.008	0.649	0.792	0.117	0.325	0.066
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	37	36	44	71	44	39	36
normalized size	1	1.00	0.82	0.80	0.98	1.58	0.98	0.87	0.80
time (sec)	N/A	0.012	0.018	0.010	0.644	0.828	0.144	0.339	2.090
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	64	64	54	54	58	62	55
normalized size	1	1.00	1.08	1.08	0.92	0.92	0.98	1.05	0.93
time (sec)	N/A	0.055	0.028	0.004	0.542	0.844	0.171	0.310	0.050
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	12	11	11	10	11	11
normalized size	1	1.00	1.10	1.20	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.011	0.007	0.001	0.678	0.677	0.086	0.353	0.032
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	35	36	35	0	23	-1
normalized size	1	1.00	1.16	0.80	0.82	0.80	0.00	0.52	-0.02
time (sec)	N/A	0.026	0.032	0.006	1.289	0.869	0.000	0.386	0.000
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	152	139	92	0	55	-1
normalized size	1	1.00	0.82	2.27	2.07	1.37	0.00	0.82	-0.01
time (sec)	N/A	0.049	0.086	0.020	1.462	0.934	0.000	0.486	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	146	135	70	0	70	-1
normalized size	1	1.00	0.81	2.32	2.14	1.11	0.00	1.11	-0.02
time (sec)	N/A	0.040	0.088	0.011	0.587	0.849	0.000	0.417	0.000
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	132	108	0	6315	238	0	374
normalized size	1	1.00	0.56	0.46	0.00	26.99	1.02	0.00	1.60
time (sec)	N/A	0.372	0.055	0.006	0.000	30.343	2.901	0.000	2.460
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	81	74	0	4759	158	0	437
normalized size	1	1.00	0.39	0.35	0.00	22.66	0.75	0.00	2.08
time (sec)	N/A	0.228	0.035	0.004	0.000	2.967	0.992	0.000	2.300
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	79	72	0	1950	83	0	145
normalized size	1	1.00	0.44	0.40	0.00	10.83	0.46	0.00	0.81
time (sec)	N/A	0.156	0.022	0.003	0.000	2.568	0.700	0.000	0.257
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	116	71	0	442	26	160	144
normalized size	1	1.00	0.83	0.51	0.00	3.16	0.19	1.14	1.03
time (sec)	N/A	0.107	0.033	0.002	0.000	0.924	0.261	0.452	2.311
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	238	119	105	0	4370	0	0	553
normalized size	1	1.06	0.53	0.47	0.00	19.51	0.00	0.00	2.47
time (sec)	N/A	0.482	0.055	0.010	0.000	2.819	0.000	0.000	0.123

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	312	173	144	0	8919	0	0	1588
normalized size	1	0.99	0.55	0.46	0.00	28.40	0.00	0.00	5.06
time (sec)	N/A	0.545	0.105	0.010	0.000	4.139	0.000	0.000	2.329
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	244	217	0	14765	0	0	1328
normalized size	1	1.00	0.62	0.55	0.00	37.57	0.00	0.00	3.38
time (sec)	N/A	0.604	0.170	0.012	0.000	11.496	0.000	0.000	2.489
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	106	97	0	0	374	0	1003
normalized size	1	1.00	0.30	0.27	0.00	0.00	1.05	0.00	2.82
time (sec)	N/A	0.426	0.055	0.016	0.000	0.000	3.736	0.000	2.692
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	106	97	0	0	274	0	625
normalized size	1	1.00	0.33	0.31	0.00	0.00	0.86	0.00	1.97
time (sec)	N/A	0.312	0.036	0.005	0.000	0.000	2.671	0.000	2.588
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	104	95	0	0	131	0	205
normalized size	1	1.00	0.40	0.36	0.00	0.00	0.50	0.00	0.79
time (sec)	N/A	0.264	0.034	0.007	0.000	0.000	0.879	0.000	2.357
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	161	94	0	189	26	103	60
normalized size	1	1.00	0.73	0.43	0.00	0.86	0.12	0.47	0.27
time (sec)	N/A	0.185	0.099	0.005	0.000	0.492	0.301	0.501	0.116

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	163	139	0	0	0	0	882
normalized size	1	1.00	0.41	0.35	0.00	0.00	0.00	0.00	2.24
time (sec)	N/A	0.466	0.073	0.008	0.000	0.000	0.000	0.000	2.183
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	238	188	0	0	0	0	2440
normalized size	1	1.00	0.48	0.38	0.00	0.00	0.00	0.00	4.92
time (sec)	N/A	0.894	0.132	0.013	0.000	0.000	0.000	0.000	2.479
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	195	264	192	219	199	219	175
normalized size	1	1.00	1.59	2.15	1.56	1.78	1.62	1.78	1.42
time (sec)	N/A	0.240	0.037	0.003	0.692	0.365	0.118	0.328	0.215
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	114	138	119	128	114	128	108
normalized size	1	1.00	0.95	1.15	0.99	1.07	0.95	1.07	0.90
time (sec)	N/A	0.063	0.020	0.000	0.667	0.364	0.090	0.296	0.095
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	63	65	65	65	65	61
normalized size	1	1.00	0.92	0.88	0.90	0.90	0.90	0.90	0.85
time (sec)	N/A	0.030	0.008	0.000	0.707	0.371	0.076	0.313	0.039
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	22	22	22
normalized size	1	1.00	1.00	0.88	0.85	0.85	0.85	0.85	0.85
time (sec)	N/A	0.004	0.000	0.000	0.627	0.354	0.061	0.303	0.018

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	49	0	457	66	0	571
normalized size	1	1.00	0.64	0.55	0.00	5.13	0.74	0.00	6.42
time (sec)	N/A	0.087	0.023	0.016	0.000	0.452	0.933	0.000	2.576
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	158	0	1948	294	0	4591
normalized size	1	1.00	0.89	0.93	0.00	11.53	1.74	0.00	27.17
time (sec)	N/A	0.291	0.075	0.013	0.000	0.437	6.317	0.000	5.352
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	254	398	0	3971	697	0	8242
normalized size	1	1.00	1.01	1.58	0.00	15.76	2.77	0.00	32.71
time (sec)	N/A	0.532	0.150	0.023	0.000	0.517	15.616	0.000	6.405
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	182	222	212	222	178
normalized size	1	1.00	0.97	1.27	0.87	1.06	1.01	1.06	0.85
time (sec)	N/A	0.226	0.035	0.003	0.621	0.360	0.123	0.390	0.217
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	130	143	113	133	128	133	113
normalized size	1	1.00	0.97	1.07	0.84	0.99	0.96	0.99	0.84
time (sec)	N/A	0.140	0.020	0.002	0.624	0.348	0.098	0.351	2.120
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	66	59	68	70	68	64
normalized size	1	1.00	0.95	0.84	0.75	0.86	0.89	0.86	0.81
time (sec)	N/A	0.076	0.008	0.000	0.616	0.386	0.077	0.348	0.038

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.010	0.002	0.000	0.611	0.397	0.061	0.360	0.022
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	59	50	0	0	155	0	275
normalized size	1	1.00	0.51	0.43	0.00	0.00	1.34	0.00	2.37
time (sec)	N/A	0.083	0.017	0.004	0.000	0.000	4.430	0.000	2.582
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	166	162	0	0	539	0	1167
normalized size	1	1.00	0.72	0.70	0.00	0.00	2.33	0.00	5.05
time (sec)	N/A	0.241	0.072	0.012	0.000	0.000	31.420	0.000	2.817
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	284	405	0	0	1102	0	2200
normalized size	1	1.00	0.81	1.16	0.00	0.00	3.16	0.00	6.30
time (sec)	N/A	0.369	0.164	0.023	0.000	0.000	88.090	0.000	3.394
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	182	222	219	222	178
normalized size	1	1.00	0.97	1.27	0.87	1.06	1.04	1.06	0.85
time (sec)	N/A	0.164	0.036	0.001	0.639	0.357	0.145	0.243	2.294
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	132	143	113	133	134	133	113
normalized size	1	1.00	0.96	1.04	0.82	0.96	0.97	0.96	0.82
time (sec)	N/A	0.121	0.019	0.003	0.563	0.342	0.097	0.285	0.092

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	66	59	68	73	68	64
normalized size	1	1.00	0.92	0.84	0.75	0.86	0.92	0.86	0.81
time (sec)	N/A	0.077	0.010	0.001	0.546	0.357	0.077	0.287	0.042
Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.009	0.004	0.000	0.609	0.345	0.062	0.313	0.021
Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	61	52	0	0	172	0	878
normalized size	1	1.00	0.62	0.53	0.00	0.00	1.74	0.00	8.87
time (sec)	N/A	0.087	0.016	0.004	0.000	0.000	7.607	0.000	2.783
Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	182	160	0	0	561	0	1218
normalized size	1	1.00	0.81	0.71	0.00	0.00	2.49	0.00	5.41
time (sec)	N/A	0.213	0.076	0.014	0.000	0.000	43.733	0.000	2.846
Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	99	93	0	0	0	0	1563
normalized size	1	1.00	0.18	0.17	0.00	0.00	0.00	0.00	2.87
time (sec)	N/A	1.476	0.069	0.015	0.000	0.000	0.000	0.000	3.175
Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	99	93	0	0	0	0	1354
normalized size	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.78
time (sec)	N/A	0.756	0.049	0.003	0.000	0.000	0.000	0.000	3.104

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	97	93	0	0	0	0	825
normalized size	1	1.00	0.29	0.28	0.00	0.00	0.00	0.00	2.47
time (sec)	N/A	0.470	0.059	0.005	0.000	0.000	0.000	0.000	3.342
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	95	91	0	0	0	0	1057
normalized size	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.25
time (sec)	N/A	0.684	0.047	0.005	0.000	0.000	0.000	0.000	2.903
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	99	90	0	0	0	0	1394
normalized size	1	1.00	0.19	0.17	0.00	0.00	0.00	0.00	2.67
time (sec)	N/A	0.861	0.066	0.005	0.000	0.000	0.000	0.000	0.711
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	157	134	0	0	0	0	4002
normalized size	1	1.00	0.28	0.24	0.00	0.00	0.00	0.00	7.11
time (sec)	N/A	1.157	0.115	0.009	0.000	0.000	0.000	0.000	2.546
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	640	163	133	0	0	0	0	2663
normalized size	1	0.99	0.25	0.21	0.00	0.00	0.00	0.00	4.13
time (sec)	N/A	1.380	0.139	0.008	0.000	0.000	0.000	0.000	2.721
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	61	56	0	0	70	0	427
normalized size	1	1.00	0.15	0.14	0.00	0.00	0.18	0.00	1.08
time (sec)	N/A	1.438	0.023	0.008	0.000	0.000	0.257	0.000	0.651

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	61	56	0	0	65	0	390
normalized size	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	1.03
time (sec)	N/A	0.910	0.014	0.009	0.000	0.000	0.275	0.000	2.704
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	61	56	0	0	61	0	276
normalized size	1	1.00	0.17	0.16	0.00	0.00	0.17	0.00	0.76
time (sec)	N/A	0.515	0.015	0.009	0.000	0.000	0.247	0.000	0.522
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	59	56	0	1277	48	0	247
normalized size	1	1.00	0.24	0.23	0.00	5.15	0.19	0.00	1.00
time (sec)	N/A	0.323	0.013	0.008	0.000	2.004	0.205	0.000	2.679
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	57	54	0	0	61	0	176
normalized size	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	0.49
time (sec)	N/A	0.548	0.016	0.008	0.000	0.000	0.257	0.000	2.423
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	62	53	0	0	65	0	306
normalized size	1	1.00	0.16	0.14	0.00	0.00	0.17	0.00	0.81
time (sec)	N/A	0.721	0.013	0.007	0.000	0.000	0.274	0.000	2.675
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	103	75	0	0	82	0	432
normalized size	1	1.00	0.25	0.18	0.00	0.00	0.20	0.00	1.04
time (sec)	N/A	0.902	0.019	0.013	0.000	0.000	0.417	0.000	2.328

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	109	74	0	0	70	0	340
normalized size	1	1.00	0.24	0.17	0.00	0.00	0.16	0.00	0.76
time (sec)	N/A	1.102	0.021	0.011	0.000	0.000	0.319	0.000	0.292
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	167	122	0	0	112	0	388
normalized size	1	1.00	0.16	0.11	0.00	0.00	0.11	0.00	0.36
time (sec)	N/A	2.504	0.040	0.015	0.000	0.000	0.397	0.000	2.341
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1005	1005	167	122	0	0	112	0	387
normalized size	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	2.403	0.030	0.016	0.000	0.000	0.403	0.000	2.304
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	677	677	167	122	0	0	112	0	388
normalized size	1	1.00	0.25	0.18	0.00	0.00	0.17	0.00	0.57
time (sec)	N/A	1.553	0.045	0.015	0.000	0.000	0.378	0.000	2.328
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	682	682	167	122	0	1445	104	0	299
normalized size	1	1.00	0.24	0.18	0.00	2.12	0.15	0.00	0.44
time (sec)	N/A	1.235	0.026	0.012	0.000	2.972	0.307	0.000	0.314
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	850	850	167	122	0	0	112	0	388
normalized size	1	1.00	0.20	0.14	0.00	0.00	0.13	0.00	0.46
time (sec)	N/A	1.470	0.036	0.013	0.000	0.000	0.388	0.000	2.420

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	873	873	167	122	0	0	112	0	387
normalized size	1	1.00	0.19	0.14	0.00	0.00	0.13	0.00	0.44
time (sec)	N/A	1.916	0.028	0.013	0.000	0.000	0.397	0.000	2.416
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	986	986	167	122	0	0	112	0	388
normalized size	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	1.927	0.035	0.014	0.000	0.000	0.387	0.000	2.478
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.055	0.002	0.000	0.443	0.708	0.094	0.370	0.037
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	114	105	105	88	365	106
normalized size	1	1.00	0.84	1.21	1.12	1.12	0.94	3.88	1.13
time (sec)	N/A	0.127	0.037	0.005	0.439	0.963	0.308	0.263	0.059
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	155	13	154	175	13	154
normalized size	1	1.00	11.47	10.33	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.018	0.006	0.003	0.436	0.655	0.131	0.300	2.224
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	156	156
normalized size	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.055	0.006	0.002	0.608	0.816	0.136	0.313	0.143

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	156	156
normalized size	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.050	0.006	0.002	0.552	0.684	0.139	0.332	2.171
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	262	0	189	229
normalized size	1	1.00	1.00	10.95	10.90	12.48	0.00	9.00	10.90
time (sec)	N/A	0.031	0.118	0.052	0.645	1.042	0.000	1.857	4.019
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
normalized size	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.004	0.004	0.000	0.555	0.776	0.127	0.230	0.050
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	18	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.87
time (sec)	N/A	0.026	0.006	0.004	0.644	0.854	0.179	0.334	2.081
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.027	0.005	0.006	0.525	1.003	0.197	0.314	0.060
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	47	23	29	0	-1
normalized size	1	1.00	1.00	1.20	3.13	1.53	1.93	0.00	-0.07
time (sec)	N/A	0.025	0.011	0.018	0.577	0.801	1.483	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	13	81	87	13	12
normalized size	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.004	0.021	0.018	0.528	0.743	0.933	0.270	4.377
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.023	0.029	0.019	0.670	0.821	1.428	0.406	2.225
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.023	0.036	0.016	0.800	0.815	2.061	0.304	7.220
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	143	0	0	-1
normalized size	1	1.00	1.00	9.67	29.14	6.81	0.00	0.00	-0.05
time (sec)	N/A	0.032	0.178	0.064	1.034	1.011	0.000	0.000	0.000
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	24	19	26	46	19	23
normalized size	1	1.00	0.89	1.26	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.005	0.009	0.005	0.559	0.785	0.710	0.374	2.095
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	97	31	35	32	0	54	45
normalized size	1	1.00	3.59	1.15	1.30	1.19	0.00	2.00	1.67
time (sec)	N/A	0.023	0.075	0.004	0.990	1.213	0.000	0.334	2.213

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	116	97	142	35	33	0	54	-1
normalized size	1	4.30	3.59	5.26	1.30	1.22	0.00	2.00	-0.04
time (sec)	N/A	0.102	0.027	0.259	1.073	0.748	0.000	0.494	0.000
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	99	33	35	34	0	58	49
normalized size	1	1.00	3.41	1.14	1.21	1.17	0.00	2.00	1.69
time (sec)	N/A	0.024	0.077	0.005	1.013	1.007	0.000	0.346	2.205
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	108	0	39	42	0	0	-1
normalized size	1	1.00	3.00	0.00	1.08	1.17	0.00	0.00	-0.03
time (sec)	N/A	0.086	0.166	0.135	1.072	0.872	0.000	0.000	0.000
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.034	0.002	0.001	0.781	0.845	0.095	0.313	2.142
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.049	0.001	0.001	0.472	0.820	0.095	0.396	0.019
Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	32
normalized size	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.76
time (sec)	N/A	0.057	0.015	0.005	1.489	1.012	0.297	0.404	2.130

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	25	38	0	25	54
normalized size	1	1.00	0.92	1.04	1.00	1.52	0.00	1.00	2.16
time (sec)	N/A	0.024	0.015	0.003	0.672	0.836	0.000	0.294	2.191
Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	21	34	24	36	0	24	46
normalized size	1	1.00	0.88	1.42	1.00	1.50	0.00	1.00	1.92
time (sec)	N/A	0.018	0.047	0.005	0.629	0.868	0.000	0.307	2.124
Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	26	39	35	0	65	51
normalized size	1	1.00	0.96	1.04	1.56	1.40	0.00	2.60	2.04
time (sec)	N/A	0.021	0.022	0.006	0.862	1.016	0.000	0.395	2.155
Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	28	0	20	39
normalized size	1	1.00	1.00	1.05	1.00	1.40	0.00	1.00	1.95
time (sec)	N/A	0.010	0.008	0.004	0.651	0.831	0.000	0.282	2.143
Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	106	26	19	26	73	19	25
normalized size	1	1.00	5.58	1.37	1.00	1.37	3.84	1.00	1.32
time (sec)	N/A	0.009	0.066	0.005	0.534	0.913	11.344	0.341	2.127
Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	108	23	31	27	76	39	26
normalized size	1	1.00	4.91	1.05	1.41	1.23	3.45	1.77	1.18
time (sec)	N/A	0.008	0.036	0.003	1.209	1.072	52.838	0.338	2.161

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	0	22	43
normalized size	1	1.00	0.95	1.05	1.00	1.45	0.00	1.00	1.95
time (sec)	N/A	0.018	0.012	0.004	0.661	0.767	0.000	0.345	2.138
Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	28	21	30	53	21	27
normalized size	1	1.00	0.90	1.33	1.00	1.43	2.52	1.00	1.29
time (sec)	N/A	0.012	0.011	0.004	0.594	0.805	1.160	0.310	2.162
Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	28	32	31	56	51	28
normalized size	1	1.00	0.92	1.17	1.33	1.29	2.33	2.12	1.17
time (sec)	N/A	0.023	0.014	0.003	0.826	0.914	6.170	0.460	2.199
Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	32	29	53	41	26
normalized size	1	1.00	1.00	1.05	1.45	1.32	2.41	1.86	1.18
time (sec)	N/A	0.009	0.010	0.005	0.895	0.981	6.154	0.314	2.188
Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	32	32	0	22	43
normalized size	1	1.00	0.95	1.05	1.45	1.45	0.00	1.00	1.95
time (sec)	N/A	0.013	0.006	0.004	0.751	1.228	0.000	0.323	2.142
Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	28	32	30	53	21	27
normalized size	1	1.00	0.90	1.33	1.52	1.43	2.52	1.00	1.29
time (sec)	N/A	0.012	0.005	0.003	0.930	0.757	1.142	0.398	2.148

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	143	25686	19	1956	1771	160	1576
normalized size	1	1.00	6.81	1223.14	0.90	93.14	84.33	7.62	75.05
time (sec)	N/A	0.125	0.151	0.004	0.740	0.558	0.420	0.299	2.979
Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	5596	18	496	469	18	418
normalized size	1	1.00	0.90	279.80	0.90	24.80	23.45	0.90	20.90
time (sec)	N/A	0.052	0.036	0.002	0.633	0.498	0.182	0.314	2.320
Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	5596	441	496	469	18	418
normalized size	1	1.00	0.95	294.53	23.21	26.11	24.68	0.95	22.00
time (sec)	N/A	0.069	0.017	0.002	0.601	0.524	0.170	0.297	2.267
Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	127	2185	14	486	483	120	438
normalized size	1	1.00	7.94	136.56	0.88	30.38	30.19	7.50	27.38
time (sec)	N/A	0.024	0.056	0.001	0.549	0.529	0.170	0.413	2.628
Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	98	89	13	88	97	13	88
normalized size	1	1.00	6.53	5.93	0.87	5.87	6.47	0.87	5.87
time (sec)	N/A	0.013	0.003	0.000	0.511	0.647	0.096	0.301	0.050
Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	98	89	88	88	97	13	88
normalized size	1	1.00	6.12	5.56	5.50	5.50	6.06	0.81	5.50
time (sec)	N/A	0.028	0.003	0.000	0.645	0.774	0.088	0.268	0.040

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	115	2205	16	488	484	136	440
normalized size	1	1.00	6.39	122.50	0.89	27.11	26.89	7.56	24.44
time (sec)	N/A	0.044	0.053	0.003	0.623	0.725	0.173	0.430	2.632
Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	98	89	15	88	97	15	88
normalized size	1	1.00	5.76	5.24	0.88	5.18	5.71	0.88	5.18
time (sec)	N/A	0.025	0.003	0.001	0.661	0.571	0.098	0.373	2.072
Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	15	88
normalized size	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.227	0.003	0.001	0.525	0.728	0.095	0.234	0.047
Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	15	88
normalized size	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.011	0.003	0.002	0.659	0.625	0.088	0.256	0.041
Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	115	2205	458	488	484	488	440
normalized size	1	1.00	6.39	122.50	25.44	27.11	26.89	27.11	24.44
time (sec)	N/A	0.060	0.011	0.002	0.595	1.061	0.169	0.317	0.568
Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	88	88
normalized size	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.022	0.003	0.001	0.598	0.711	0.094	0.306	0.040

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	98	89	88	88	97	88	88
normalized size	1	1.00	5.44	4.94	4.89	4.89	5.39	4.89	4.89
time (sec)	N/A	0.019	0.003	0.002	0.570	0.559	0.096	0.294	0.040
Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	88	88
normalized size	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.002	0.002	0.000	0.670	0.794	0.088	0.243	0.039
Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	80	67	66	66	70	24	66
normalized size	1	1.00	2.86	2.39	2.36	2.36	2.50	0.86	2.36
time (sec)	N/A	0.027	0.005	0.002	0.610	0.713	0.086	0.366	0.051
Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	108	325	187	208	194	88	180
normalized size	1	1.00	3.48	10.48	6.03	6.71	6.26	2.84	5.81
time (sec)	N/A	0.031	0.035	0.001	0.438	0.554	0.114	0.366	0.105
Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	52	48	230	30	31
normalized size	1	1.00	1.00	0.91	1.53	1.41	6.76	0.88	0.91
time (sec)	N/A	0.008	0.059	0.004	1.133	0.793	50.746	0.403	2.121
Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	54	52	0	32	58
normalized size	1	1.00	1.00	0.94	1.54	1.49	0.00	0.91	1.66
time (sec)	N/A	0.009	0.056	0.003	1.159	0.947	0.000	0.251	2.110

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	93	78	77	77	87	24	77
normalized size	1	1.00	3.10	2.60	2.57	2.57	2.90	0.80	2.57
time (sec)	N/A	0.020	0.006	0.002	0.438	0.759	0.095	0.220	0.055
Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	140	618	280	291	314	105	266
normalized size	1	1.00	4.52	19.94	9.03	9.39	10.13	3.39	8.58
time (sec)	N/A	0.038	0.046	0.002	0.455	0.993	0.143	0.336	2.266
Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	98	81	80	80	90	30	80
normalized size	1	1.00	2.88	2.38	2.35	2.35	2.65	0.88	2.35
time (sec)	N/A	0.033	0.008	0.002	0.439	0.682	0.098	0.294	0.065
Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	146	646	289	298	321	126	273
normalized size	1	1.00	3.56	15.76	7.05	7.27	7.83	3.07	6.66
time (sec)	N/A	0.045	0.051	0.001	0.446	0.931	0.150	0.312	2.283
Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	244	1523	289	309	323	37	270
normalized size	1	1.00	5.30	33.11	6.28	6.72	7.02	0.80	5.87
time (sec)	N/A	0.047	0.063	0.002	0.453	0.682	0.157	0.307	2.237
Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	248	4284	773	928	930	153	753
normalized size	1	1.00	5.28	91.15	16.45	19.74	19.79	3.26	16.02
time (sec)	N/A	0.091	0.123	0.002	0.480	0.627	0.269	0.442	2.450

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	31	54	48	201	30	33
normalized size	1	1.00	1.06	0.91	1.59	1.41	5.91	0.88	0.97
time (sec)	N/A	0.009	0.068	0.004	1.143	1.017	112.002	0.417	2.111
Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	71	57	0	36	37
normalized size	1	1.00	0.95	0.84	1.61	1.30	0.00	0.82	0.84
time (sec)	N/A	0.010	0.086	0.003	1.173	0.643	0.000	0.392	2.210
Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	43	83	72	0	42	73
normalized size	1	1.00	0.98	0.86	1.66	1.44	0.00	0.84	1.46
time (sec)	N/A	0.009	0.193	0.003	1.218	0.831	0.000	0.391	2.170
Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	30	17	29	29	30	29
normalized size	1	1.00	1.74	1.58	0.89	1.53	1.53	1.58	1.53
time (sec)	N/A	0.009	0.002	0.001	0.434	0.773	0.061	0.346	0.025
Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	18	14	17	17	22	17
normalized size	1	1.00	1.31	1.12	0.88	1.06	1.06	1.38	1.06
time (sec)	N/A	0.007	0.002	0.000	0.439	0.816	0.057	0.233	0.034
Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	86	86	94	28	86
normalized size	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.198	0.006	0.001	0.437	0.718	0.082	0.288	0.218

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	86	86	94	28	86
normalized size	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.154	0.005	0.001	0.449	0.723	0.091	0.360	0.191
Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	57	56	12	12
normalized size	1	1.00	1.00	0.93	0.86	4.07	4.00	0.86	0.86
time (sec)	N/A	0.008	0.006	0.001	0.436	0.829	0.189	0.316	2.100
Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87
time (sec)	N/A	0.009	0.004	0.002	0.442	0.786	0.089	0.371	0.049
Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	15	15	14	18	13
normalized size	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76
time (sec)	N/A	0.010	0.006	0.003	0.448	0.799	0.099	0.270	0.066
Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	28	23	23	22	24	23
normalized size	1	1.00	0.58	0.70	0.58	0.58	0.55	0.60	0.58
time (sec)	N/A	0.097	0.059	0.013	0.503	0.856	32.564	0.536	0.108
Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	98	2105	0	0	0	0	-1
normalized size	1	1.00	0.16	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.535	0.069	0.088	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	82	0	83	58	58	75
normalized size	1	1.00	0.87	1.30	0.00	1.32	0.92	0.92	1.19
time (sec)	N/A	0.066	0.029	0.049	0.000	0.756	0.126	0.274	0.185
Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	38	20	13	12
normalized size	1	1.00	1.00	0.93	1.57	2.71	1.43	0.93	0.86
time (sec)	N/A	0.050	0.008	0.007	0.436	0.892	0.096	0.289	0.040
Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	27	32	46	29	23	21
normalized size	1	1.00	0.86	0.96	1.14	1.64	1.04	0.82	0.75
time (sec)	N/A	0.028	0.013	0.006	0.446	1.000	0.109	0.392	0.037
Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	28	30	38	38	36	28	28
normalized size	1	1.00	0.47	0.51	0.64	0.64	0.61	0.47	0.47
time (sec)	N/A	0.091	0.012	0.006	0.463	0.838	0.181	1.436	0.054
Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	11	11	8	11	11
normalized size	1	1.00	1.00	3.73	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.007	0.007	0.009	0.438	0.733	0.117	0.331	2.301
Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	114	223	272	134	124
normalized size	1	2.25	1.45	1.27	1.25	2.45	2.99	1.47	1.36
time (sec)	N/A	0.149	0.090	0.019	0.987	0.941	1.362	0.372	2.369

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	44	40	0	99	49
normalized size	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.96
time (sec)	N/A	0.025	0.375	0.010	0.699	1.247	0.000	2.090	2.656
Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
normalized size	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.082	0.233	0.010	0.656	0.707	0.000	1.887	2.549
Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
normalized size	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.059	0.177	0.008	0.629	0.892	0.000	0.728	2.319
Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	37	37	0	87	37
normalized size	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.76
time (sec)	N/A	0.044	0.122	0.005	0.608	0.871	0.000	1.083	2.273
Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	33	33	0	52	19
normalized size	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.00
time (sec)	N/A	0.043	0.010	0.006	0.621	0.898	0.000	0.296	2.193
Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
normalized size	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.034	0.176	0.009	0.619	0.811	0.000	0.000	3.202

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
normalized size	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.035	0.199	0.008	0.636	1.041	0.000	0.000	3.319
Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
normalized size	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.036	0.188	0.008	0.642	1.035	0.000	0.000	3.355
Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	73	79	97	73	97
normalized size	1	1.00	0.86	0.76	0.75	0.81	1.00	0.75	1.00
time (sec)	N/A	0.138	0.036	0.007	1.520	0.687	0.245	0.309	0.194
Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	69	68	74	92	68	92
normalized size	1	1.00	0.87	0.77	0.76	0.82	1.02	0.76	1.02
time (sec)	N/A	0.123	0.023	0.006	1.880	0.800	0.247	0.378	0.178
Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	62	61	67	78	61	85
normalized size	1	1.00	0.94	0.81	0.79	0.87	1.01	0.79	1.10
time (sec)	N/A	0.121	0.031	0.005	1.441	0.545	0.232	0.391	2.290
Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	57	56	62	75	56	80
normalized size	1	1.00	0.96	0.79	0.78	0.86	1.04	0.78	1.11
time (sec)	N/A	0.093	0.023	0.005	1.586	1.037	0.232	0.307	2.292

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	56	55	61	75	55	79
normalized size	1	1.00	0.92	0.79	0.77	0.86	1.06	0.77	1.11
time (sec)	N/A	0.078	0.017	0.006	1.316	0.800	0.225	0.287	0.147
Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	60	59	65	78	60	83
normalized size	1	1.00	0.92	0.80	0.79	0.87	1.04	0.80	1.11
time (sec)	N/A	0.135	0.019	0.007	1.571	0.965	0.288	0.307	0.149
Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	65	64	76	87	65	88
normalized size	1	1.00	0.93	0.77	0.76	0.90	1.04	0.77	1.05
time (sec)	N/A	0.152	0.033	0.009	1.441	0.885	0.315	0.257	2.280
Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	69	89	94	70	92
normalized size	1	1.00	0.90	0.77	0.76	0.98	1.03	0.77	1.01
time (sec)	N/A	0.157	0.055	0.010	1.264	0.966	0.321	0.286	0.152
Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	109	74	0	1202	61	0	128
normalized size	1	1.00	0.36	0.24	0.00	3.92	0.20	0.00	0.42
time (sec)	N/A	0.578	0.021	0.008	0.000	2.942	0.985	0.000	2.169
Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	101	67	0	1145	3662	0	188
normalized size	1	1.00	0.38	0.25	0.00	4.26	13.61	0.00	0.70
time (sec)	N/A	0.392	0.016	0.007	0.000	3.031	2.732	0.000	0.134

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	94	62	0	1190	48	0	183
normalized size	1	1.00	0.41	0.27	0.00	5.17	0.21	0.00	0.80
time (sec)	N/A	0.357	0.015	0.008	0.000	3.070	0.950	0.000	0.193
Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	90	58	0	1189	46	0	181
normalized size	1	1.00	0.45	0.29	0.00	6.01	0.23	0.00	0.91
time (sec)	N/A	0.194	0.013	0.007	0.000	3.226	0.914	0.000	2.342
Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	101	67	0	1143	60	0	237
normalized size	1	1.00	0.41	0.27	0.00	4.67	0.24	0.00	0.97
time (sec)	N/A	0.473	0.020	0.011	0.000	3.241	12.657	0.000	2.342
Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	109	72	0	1245	0	0	242
normalized size	1	1.00	0.39	0.26	0.00	4.43	0.00	0.00	0.86
time (sec)	N/A	0.467	0.018	0.013	0.000	3.268	0.000	0.000	2.302
Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	116	77	0	1274	70	0	246
normalized size	1	1.00	0.37	0.24	0.00	4.02	0.22	0.00	0.78
time (sec)	N/A	0.539	0.019	0.012	0.000	3.434	2.705	0.000	2.248
Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	87	75	0	83	44	87	252
normalized size	1	1.00	4.58	3.95	0.00	4.37	2.32	4.58	13.26
time (sec)	N/A	0.105	0.047	0.095	0.000	0.756	1.045	4.080	2.270

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	34	33	47	37	34	38
normalized size	1	1.00	1.33	0.79	0.77	1.09	0.86	0.79	0.88
time (sec)	N/A	0.067	0.023	0.009	2.104	0.808	0.170	0.254	0.052
Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	18	21
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	1.06	1.24
time (sec)	N/A	0.037	0.005	0.008	1.116	0.936	0.131	0.311	2.200
Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	20	24	30
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.80	0.96	1.20
time (sec)	N/A	0.037	0.006	0.006	1.147	0.761	0.132	0.254	0.054
Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	19	18	24	14	28	18
normalized size	1	1.00	0.91	0.86	0.82	1.09	0.64	1.27	0.82
time (sec)	N/A	0.013	0.010	0.007	1.095	0.567	0.111	0.366	0.036
Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	38	22	21	21	26	22	25
normalized size	1	1.00	1.41	0.81	0.78	0.78	0.96	0.81	0.93
time (sec)	N/A	0.039	0.011	0.007	2.309	0.949	0.144	0.286	2.260
Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.015	0.004	0.001	2.281	0.819	0.082	0.270	0.031

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	22	23	23
normalized size	1	1.00	1.00	0.89	0.85	0.85	0.81	0.85	0.85
time (sec)	N/A	0.026	0.007	0.005	2.077	1.073	0.109	0.278	2.140
Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	26	25	25	26	29	25
normalized size	1	1.00	1.00	0.67	0.64	0.64	0.67	0.74	0.64
time (sec)	N/A	0.020	0.018	0.009	1.273	0.781	0.241	0.317	2.141
Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	19	18	18	17	19	18
normalized size	1	1.00	1.05	0.86	0.82	0.82	0.77	0.86	0.82
time (sec)	N/A	0.015	0.004	0.002	1.133	0.916	0.075	0.281	0.027
Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.029	0.005	0.003	2.301	0.823	0.091	0.341	2.131
Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	24	21	20	26	17	22	10
normalized size	1	1.00	2.00	1.75	1.67	2.17	1.42	1.83	0.83
time (sec)	N/A	0.021	0.011	0.008	1.078	0.726	0.098	0.297	0.072
Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	22	19	23	19
normalized size	1	1.00	1.00	0.88	0.84	0.88	0.76	0.92	0.76
time (sec)	N/A	0.036	0.006	0.007	1.129	0.872	0.102	0.269	2.139

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.028	0.006	0.005	2.157	0.828	0.114	0.309	0.038
Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	31	35	55	36	30	35
normalized size	1	1.00	1.00	0.89	1.00	1.57	1.03	0.86	1.00
time (sec)	N/A	0.034	0.017	0.007	2.223	0.778	0.147	0.329	2.125
Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	17
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.74
time (sec)	N/A	0.037	0.005	0.009	1.405	0.688	0.138	0.239	2.188
Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	20	19
normalized size	1	1.00	1.00	0.87	0.83	0.83	0.83	0.87	0.83
time (sec)	N/A	0.054	0.007	0.006	1.115	0.922	0.109	0.295	0.055
Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	23	32	20	23	25
normalized size	1	1.00	0.86	0.83	0.79	1.10	0.69	0.79	0.86
time (sec)	N/A	0.014	0.010	0.006	2.335	0.893	0.119	0.374	2.125
Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	42	41	58	49	43	55
normalized size	1	1.00	1.00	0.95	0.93	1.32	1.11	0.98	1.25
time (sec)	N/A	0.243	0.024	0.010	2.248	0.875	0.212	0.363	0.129

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	38	38	51	38	88
normalized size	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.133	0.020	0.005	2.287	0.883	0.213	0.304	0.160
Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	44	72	41	44	56
normalized size	1	1.00	1.00	1.03	1.33	2.18	1.24	1.33	1.70
time (sec)	N/A	0.166	0.022	0.012	1.156	0.852	0.186	0.301	2.162
Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	10	11	11
normalized size	1	1.00	1.00	0.71	0.65	0.65	0.59	0.65	0.65
time (sec)	N/A	0.007	0.006	0.007	2.200	0.643	0.141	0.288	0.032
Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	20	34	20	20
normalized size	1	1.00	0.92	0.88	0.83	0.83	1.42	0.83	0.83
time (sec)	N/A	0.018	0.009	0.003	2.255	0.757	0.166	0.288	2.134
Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	23	18	17	17	17	20	19
normalized size	1	1.00	1.53	1.20	1.13	1.13	1.13	1.33	1.27
time (sec)	N/A	0.060	0.006	0.008	2.313	0.959	0.137	0.319	0.056
Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	17	17	19	17	17
normalized size	1	1.00	1.00	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.012	0.009	0.007	2.398	0.872	0.148	0.375	0.050

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	27	44	29	60	35
normalized size	1	1.00	0.89	0.76	0.73	1.19	0.78	1.62	0.95
time (sec)	N/A	0.040	0.023	0.007	2.224	0.507	0.164	0.287	2.144
Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	21	21	26	21	21
normalized size	1	1.00	1.00	0.85	0.81	0.81	1.00	0.81	0.81
time (sec)	N/A	0.011	0.008	0.002	2.216	0.595	0.094	0.309	0.030
Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	16	10	11	10
normalized size	1	1.00	1.00	0.92	0.83	1.33	0.83	0.92	0.83
time (sec)	N/A	0.032	0.004	0.006	1.105	0.700	0.095	0.287	2.125
Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	20	21
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.95	1.00
time (sec)	N/A	0.030	0.007	0.008	1.092	0.793	0.132	0.282	2.144
Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	34	17	20	20
normalized size	1	1.00	1.00	0.95	0.91	1.55	0.77	0.91	0.91
time (sec)	N/A	0.015	0.008	0.005	2.231	0.722	0.112	0.282	2.124
Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	32	19	20	22
normalized size	1	1.00	1.00	0.88	0.83	1.33	0.79	0.83	0.92
time (sec)	N/A	0.016	0.011	0.007	2.176	0.789	0.116	0.405	0.029

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	33	33	39	33	56
normalized size	1	1.00	1.00	0.94	0.92	0.92	1.08	0.92	1.56
time (sec)	N/A	0.026	0.015	0.005	2.057	0.802	0.188	0.245	0.103
Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	28	27	27	29	27	37
normalized size	1	1.00	1.00	0.76	0.73	0.73	0.78	0.73	1.00
time (sec)	N/A	0.024	0.008	0.003	2.003	0.752	0.174	0.285	2.136
Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
normalized size	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.017	0.006	0.006	1.083	0.612	0.109	0.367	2.214
Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	16	15
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.74	0.84	0.79
time (sec)	N/A	0.013	0.004	0.002	1.066	0.803	0.081	0.266	0.034
Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	34	34	46	34	36
normalized size	1	1.00	1.00	0.85	0.83	0.83	1.12	0.83	0.88
time (sec)	N/A	0.028	0.014	0.005	2.178	0.916	0.118	0.288	0.043
Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	32	31	31	34	31	31
normalized size	1	1.00	0.95	0.78	0.76	0.76	0.83	0.76	0.76
time (sec)	N/A	0.028	0.007	0.004	2.074	1.114	0.118	0.278	2.116

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	24	21	20	20	24	23	20
normalized size	1	1.00	0.80	0.70	0.67	0.67	0.80	0.77	0.67
time (sec)	N/A	0.058	0.011	0.007	1.038	0.962	0.152	0.250	2.125
Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	31	30	27
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.89	0.86	0.77
time (sec)	N/A	0.040	0.008	0.008	1.118	0.746	0.149	0.324	0.044
Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	25	24	34	27	34	26
normalized size	1	1.00	0.94	0.74	0.71	1.00	0.79	1.00	0.76
time (sec)	N/A	0.057	0.014	0.008	1.133	0.602	0.145	0.387	2.109
Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	33	44	36	33	39
normalized size	1	1.00	1.00	0.83	0.79	1.05	0.86	0.79	0.93
time (sec)	N/A	0.021	0.021	0.006	2.355	0.792	0.131	0.297	2.187
Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	38	37	37	46	37	41
normalized size	1	1.00	1.00	0.78	0.76	0.76	0.94	0.76	0.84
time (sec)	N/A	0.157	0.013	0.006	2.803	0.624	0.227	0.315	0.072
Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	19	19	24	22	19
normalized size	1	1.00	1.00	0.69	0.66	0.66	0.83	0.76	0.66
time (sec)	N/A	0.053	0.007	0.008	1.166	0.582	0.142	0.275	2.182

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	34	33	33	41	34	50
normalized size	1	1.00	0.93	0.74	0.72	0.72	0.89	0.74	1.09
time (sec)	N/A	0.044	0.016	0.006	2.161	0.992	0.144	0.317	0.086
Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	17	10	15	14
normalized size	1	1.00	0.88	0.94	0.88	1.06	0.62	0.94	0.88
time (sec)	N/A	0.025	0.007	0.004	1.085	0.800	0.093	0.397	0.043
Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	16	17	17	14	12	12
normalized size	1	1.00	0.67	0.76	0.81	0.81	0.67	0.57	0.57
time (sec)	N/A	0.019	0.003	0.004	1.088	0.882	0.087	0.301	2.085
Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	23	23	17	30	15
normalized size	1	1.00	1.00	1.07	1.53	1.53	1.13	2.00	1.00
time (sec)	N/A	0.027	0.011	0.005	1.268	0.881	0.114	0.262	2.089
Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	28	28	3	29	57
normalized size	1	1.00	1.00	0.94	0.90	0.90	0.10	0.94	1.84
time (sec)	N/A	0.042	0.012	0.005	2.357	0.889	0.130	0.278	0.112
Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	22	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.042	0.006	0.007	1.088	0.804	0.141	0.287	0.060

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	24	36	20	26	22
normalized size	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73
time (sec)	N/A	0.030	0.015	0.006	1.212	0.874	0.098	0.276	2.108
Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.038	0.005	0.005	2.075	0.733	0.137	0.331	2.121
Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	73	77	136	88	74	96
normalized size	1	1.00	0.90	0.71	0.75	1.32	0.85	0.72	0.93
time (sec)	N/A	0.476	0.043	0.013	2.248	0.702	0.515	0.344	2.200
Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	25	36	24	25	32
normalized size	1	1.00	0.91	0.85	0.76	1.09	0.73	0.76	0.97
time (sec)	N/A	0.017	0.011	0.005	2.149	0.916	0.127	0.317	0.035
Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	29	44	27	30	33
normalized size	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00
time (sec)	N/A	0.041	0.019	0.008	2.254	0.778	0.149	0.379	2.113
Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	19	17	26	19
normalized size	1	1.00	1.00	0.96	0.92	0.76	0.68	1.04	0.76
time (sec)	N/A	0.041	0.005	0.007	1.046	0.564	0.096	0.291	0.043

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	31	36	31	56
normalized size	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56
time (sec)	N/A	0.115	0.015	0.004	2.096	0.842	0.202	0.336	0.106
Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	33	22	21	23
normalized size	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79
time (sec)	N/A	0.107	0.017	0.009	2.089	0.990	0.174	0.227	0.042
Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	39	46	36	49
normalized size	1	1.00	1.00	0.78	0.76	0.85	1.00	0.78	1.07
time (sec)	N/A	0.058	0.026	0.008	2.093	0.806	0.162	0.296	2.169
Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	26	19	18	18	17	20	14
normalized size	1	1.18	1.18	0.86	0.82	0.82	0.77	0.91	0.64
time (sec)	N/A	0.015	0.006	0.005	0.974	0.853	0.098	0.284	0.039
Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.039	0.006	0.007	0.850	1.135	0.136	0.381	0.065
Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	14	16	14
normalized size	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00
time (sec)	N/A	0.025	0.004	0.007	0.999	0.773	0.115	0.373	2.120

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	17	17	17	20	17
normalized size	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89
time (sec)	N/A	0.027	0.007	0.007	1.029	1.139	0.131	0.292	2.116
Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	19	17	33
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43
time (sec)	N/A	0.029	0.009	0.006	2.122	1.002	0.183	0.364	0.054
Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	50	50	68	53	58
normalized size	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92
time (sec)	N/A	0.088	0.027	0.010	2.112	0.980	0.346	0.306	2.213
Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	86	67	54	59	103	65	59	71
normalized size	1	1.25	0.97	0.78	0.86	1.49	0.94	0.86	1.03
time (sec)	N/A	0.099	0.047	0.014	2.070	0.786	0.213	0.236	2.177
Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	19	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.12	0.71	0.76	0.76
time (sec)	N/A	0.013	0.006	0.006	2.043	0.889	0.106	0.302	0.033
Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	15	17	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	0.79	0.89	0.89
time (sec)	N/A	0.027	0.005	0.004	2.030	0.908	0.101	0.269	2.098

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	28	19
normalized size	1	1.00	1.00	0.89	0.78	0.78	0.78	3.11	2.11
time (sec)	N/A	0.075	0.006	0.005	2.166	0.888	0.138	0.292	2.104
Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	12	12
normalized size	1	1.00	1.00	1.08	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.016	0.005	0.002	1.375	0.635	0.087	0.295	0.028
Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	41	51	66	99	55	48
normalized size	1	1.00	0.89	0.63	0.78	1.02	1.52	0.85	0.74
time (sec)	N/A	0.062	0.036	0.008	1.771	0.601	0.448	0.295	0.099
Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	46	24	28	28
normalized size	1	1.00	1.00	1.04	1.00	1.64	0.86	1.00	1.00
time (sec)	N/A	0.022	0.011	0.006	1.656	0.625	0.131	0.285	0.038
Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.040	0.023	0.004	1.515	0.809	0.103	0.374	0.025
Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	41	39	24	23	23	26	27	23
normalized size	1	1.32	1.26	0.77	0.74	0.74	0.84	0.87	0.74
time (sec)	N/A	0.049	0.007	0.008	0.797	0.845	0.194	0.308	0.050

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	26	19	23	30
normalized size	1	1.00	1.00	0.96	0.92	1.08	0.79	0.96	1.25
time (sec)	N/A	0.167	0.009	0.006	1.223	0.761	0.155	0.292	2.129
Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
normalized size	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.025	0.011	0.006	1.431	0.845	0.130	0.391	0.033
Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
normalized size	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.039	0.006	0.005	1.416	1.244	0.127	0.274	0.030
Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	10	13	13
normalized size	1	1.00	1.00	1.08	1.00	1.15	0.77	1.00	1.00
time (sec)	N/A	0.049	0.005	0.005	0.708	0.811	0.100	0.283	2.144
Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	193	236	199	4545	138	208	357
normalized size	1	1.00	0.94	1.15	0.97	22.06	0.67	1.01	1.73
time (sec)	N/A	0.274	0.107	0.008	1.652	2.993	1.238	0.442	0.231
Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	0	47	44	38	42
normalized size	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.93
time (sec)	N/A	0.067	0.025	0.009	0.000	0.931	0.154	1.109	2.191

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	0	75	46	42	47
normalized size	1	1.00	0.75	0.69	0.00	1.27	0.78	0.71	0.80
time (sec)	N/A	0.064	0.021	0.010	0.000	0.776	0.184	1.803	0.049
Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	234	616	0	0	0	1587	3942
normalized size	1	1.00	1.12	2.95	0.00	0.00	0.00	7.59	18.86
time (sec)	N/A	0.371	0.255	0.061	0.000	0.000	0.000	4.697	3.437
Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	245	633	0	0	0	1625	3046
normalized size	1	1.00	1.09	2.83	0.00	0.00	0.00	7.25	13.60
time (sec)	N/A	0.389	0.248	0.060	0.000	0.000	0.000	4.563	3.220
Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	60	65	190	62	61
normalized size	1	1.00	1.00	1.02	1.07	1.16	3.39	1.11	1.09
time (sec)	N/A	0.047	0.027	0.007	0.685	0.709	1.065	0.313	0.213
Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	73	87	84	162	0	85	347
normalized size	1	1.00	0.76	0.91	0.88	1.69	0.00	0.89	3.61
time (sec)	N/A	0.108	0.035	0.008	1.222	0.735	0.000	0.257	1.131
Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	228	336	279	5975	0	320	570
normalized size	1	1.00	0.86	1.27	1.06	22.63	0.00	1.21	2.16
time (sec)	N/A	0.472	0.090	0.007	1.706	2.990	0.000	0.302	2.496

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	370	422	349	0	0	401	823
normalized size	1	1.00	0.89	1.01	0.84	0.00	0.00	0.96	1.97
time (sec)	N/A	0.547	0.230	0.014	1.481	0.000	0.000	0.567	2.449
Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	24	21	20	26	19	21	12
normalized size	1	1.00	1.50	1.31	1.25	1.62	1.19	1.31	0.75
time (sec)	N/A	0.008	0.009	0.007	0.753	0.674	0.105	0.239	0.039
Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	23	34	20	30	17
normalized size	1	1.00	1.42	1.26	1.21	1.79	1.05	1.58	0.89
time (sec)	N/A	0.013	0.011	0.009	0.651	0.484	0.118	0.303	2.138
Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	90	75	106	92	77	103
normalized size	1	1.00	0.88	0.93	0.77	1.09	0.95	0.79	1.06
time (sec)	N/A	0.071	0.050	0.012	1.934	0.697	0.361	0.314	0.177
Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.106	0.008	0.005	1.400	0.633	0.120	0.289	2.120
Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.094	0.007	0.004	1.327	0.547	0.117	0.298	0.035

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	24	24	29	24	51
normalized size	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76
time (sec)	N/A	0.118	0.014	0.004	1.479	0.574	0.193	0.259	2.153
Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	14	14
normalized size	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.011	0.008	0.004	2.074	0.571	0.138	0.253	0.042
Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	7	11	10
normalized size	1	1.00	1.00	0.92	0.83	1.00	0.58	0.92	0.83
time (sec)	N/A	0.011	0.003	0.004	0.752	0.597	0.091	0.418	0.037
Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	17
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	1.55
time (sec)	N/A	0.034	0.005	0.006	1.608	0.646	0.131	0.292	2.166
Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	13	8	11	10
normalized size	1	1.00	1.00	0.92	0.83	1.08	0.67	0.92	0.83
time (sec)	N/A	0.021	0.003	0.005	0.714	0.516	0.091	0.384	0.039
Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	14	12
normalized size	1	1.00	1.00	1.08	1.00	1.00	1.00	1.17	1.00
time (sec)	N/A	0.018	0.003	0.005	0.596	0.564	0.105	0.229	0.047

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	16	23
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	1.35
time (sec)	N/A	0.037	0.005	0.005	1.532	0.570	0.133	0.243	2.284
Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	14	19
normalized size	1	1.00	1.00	1.08	1.00	1.00	0.77	1.08	1.46
time (sec)	N/A	0.035	0.008	0.006	1.410	0.631	0.132	0.343	0.060
Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	21	20	26	20	22	20
normalized size	1	1.00	0.79	0.75	0.71	0.93	0.71	0.79	0.71
time (sec)	N/A	0.011	0.015	0.007	0.662	0.593	0.112	0.297	0.069
Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	37	26	43	22
normalized size	1	1.00	1.00	0.84	0.81	1.16	0.81	1.34	0.69
time (sec)	N/A	0.026	0.017	0.009	0.746	0.530	0.134	0.298	2.226
Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	19	19	19	20	25
normalized size	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09
time (sec)	N/A	0.034	0.008	0.005	1.558	0.712	0.136	0.310	0.049
Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	36	20	47	28
normalized size	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17
time (sec)	N/A	0.035	0.014	0.007	1.600	0.566	0.141	0.303	2.111

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	37	44	39	41
normalized size	1	1.00	1.00	0.82	0.80	0.76	0.90	0.80	0.84
time (sec)	N/A	0.142	0.014	0.007	1.708	0.657	0.224	0.283	2.156
Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	19	20	19	19	19	22	19
normalized size	1	1.00	0.76	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.055	0.008	0.006	1.067	0.520	0.145	0.392	2.135
Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	47	60	63	60	61
normalized size	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02
time (sec)	N/A	0.246	0.053	0.009	2.798	0.732	0.252	0.343	0.130
Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	5	9	6
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.45	0.82	0.55
time (sec)	N/A	0.007	0.000	0.000	0.845	0.731	0.060	0.227	0.016
Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
normalized size	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.016	0.005	0.005	1.096	0.615	0.108	0.355	2.124
Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	37	37	52	37	37	49
normalized size	1	1.00	1.00	0.82	0.82	1.16	0.82	0.82	1.09
time (sec)	N/A	0.025	0.013	0.007	1.839	0.709	0.137	0.299	0.041

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	52	29	29	44
normalized size	1	1.00	1.00	0.91	0.88	1.62	0.91	0.91	1.38
time (sec)	N/A	0.254	0.019	0.010	2.025	0.626	0.263	0.325	0.069
Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	159	110	132	178	146	122	124
normalized size	1	1.00	1.07	0.74	0.89	1.20	0.99	0.82	0.84
time (sec)	N/A	0.136	0.068	0.011	2.009	0.758	0.429	0.388	2.189
Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	101	79	99	515	61	92	170
normalized size	1	1.00	0.90	0.71	0.88	4.60	0.54	0.82	1.52
time (sec)	N/A	0.114	0.049	0.007	2.058	2.414	0.965	0.390	2.228
Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	14	12
normalized size	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.016	0.003	0.005	0.907	0.751	0.103	0.262	0.044
Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	12	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83
time (sec)	N/A	0.029	0.005	0.004	1.005	0.628	0.098	0.372	2.138
Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	21	14	17	15
normalized size	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88
time (sec)	N/A	0.030	0.004	0.007	0.892	0.697	0.108	0.319	0.027

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	17	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	1.00	0.88
time (sec)	N/A	0.029	0.004	0.005	0.900	0.620	0.100	0.260	0.033
Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	25	15	23	16
normalized size	1	1.00	1.00	0.94	0.89	1.39	0.83	1.28	0.89
time (sec)	N/A	0.030	0.004	0.006	1.921	0.666	0.105	0.267	0.050
Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	14	7	11	10
normalized size	1	1.00	1.00	1.10	1.00	1.40	0.70	1.10	1.00
time (sec)	N/A	0.019	0.005	0.005	0.892	0.784	0.086	0.361	0.034
Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	36	33	30
normalized size	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71
time (sec)	N/A	0.040	0.007	0.008	0.878	0.592	0.151	0.287	0.053
Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	38	38	51	38	88
normalized size	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.125	0.017	0.002	1.972	0.769	0.212	0.355	0.002
Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	17	8
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.74	0.89	0.42
time (sec)	N/A	0.062	0.003	0.007	1.026	0.522	0.109	0.281	0.084

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	30	30	34	32	26
normalized size	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.021	0.005	0.007	0.995	0.627	0.119	0.284	2.093
Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	25	27	27	21
normalized size	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.020	0.004	0.006	1.020	0.646	0.117	0.343	0.033
Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	20	22	16
normalized size	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.015	0.005	0.006	1.027	0.832	0.117	0.270	2.117
Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	19	13
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.007	0.003	0.003	1.173	0.599	0.109	0.286	2.110
Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	19	8
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.38
time (sec)	N/A	0.010	0.003	0.005	0.926	0.731	0.110	0.275	0.078
Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	24	24	17
normalized size	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.016	0.004	0.008	0.916	0.732	0.148	0.373	0.095

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	26	30	31	29	22
normalized size	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.030	0.004	0.008	1.062	0.537	0.162	0.266	0.035
Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	39	36	34	26
normalized size	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.034	0.005	0.008	1.015	0.591	0.170	0.276	0.036
Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	36	44	41	39	32
normalized size	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.041	0.005	0.009	1.071	0.883	0.182	0.238	0.037
Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	61	54	0	1506	41	0	144
normalized size	1	1.00	0.39	0.34	0.00	9.59	0.26	0.00	0.92
time (sec)	N/A	0.168	0.016	0.014	0.000	2.684	0.225	0.000	2.745
Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	61	56	0	1546	41	0	142
normalized size	1	1.00	0.39	0.36	0.00	9.85	0.26	0.00	0.90
time (sec)	N/A	0.112	0.017	0.015	0.000	2.520	0.227	0.000	2.805
Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	61	54	0	2271	39	0	142
normalized size	1	1.00	0.32	0.29	0.00	12.08	0.21	0.00	0.76
time (sec)	N/A	0.276	0.013	0.008	0.000	2.944	0.211	0.000	2.780

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	61	56	0	2259	39	0	142
normalized size	1	1.00	0.32	0.30	0.00	12.02	0.21	0.00	0.76
time (sec)	N/A	0.156	0.013	0.010	0.000	2.704	0.212	0.000	2.740
Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	0	133	0	328
normalized size	1	1.00	0.10	0.10	0.00	0.00	0.20	0.00	0.49
time (sec)	N/A	1.108	0.037	0.074	0.000	0.000	3.924	0.000	2.699
Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	0	133	0	328
normalized size	1	1.00	0.10	0.10	0.00	0.00	0.20	0.00	0.49
time (sec)	N/A	0.648	0.031	0.002	0.000	0.000	3.920	0.000	0.002
Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	C	A	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	95	67	0	5653	42	0	504
normalized size	1	0.00	0.57	0.40	0.00	33.65	0.25	0.00	3.00
time (sec)	N/A	0.380	0.061	0.278	0.000	2.699	1.866	0.000	3.082
Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	322	314	310	0	384	311	894
normalized size	1	1.00	1.01	0.98	0.97	0.00	1.20	0.97	2.79
time (sec)	N/A	0.256	0.249	0.011	1.956	0.000	5.289	0.361	2.843
Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	243	292	275	0	277	285	556
normalized size	1	1.00	0.84	1.00	0.95	0.00	0.95	0.98	1.91
time (sec)	N/A	0.198	0.100	0.003	2.047	0.000	2.678	0.336	2.657

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	184	151	207	0	124	215	160
normalized size	1	1.00	0.84	0.69	0.95	0.00	0.57	0.98	0.73
time (sec)	N/A	0.174	0.059	0.002	2.666	0.000	0.823	0.316	2.323
Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	169	121	20	179	33
normalized size	1	1.00	0.72	0.69	0.91	0.65	0.11	0.97	0.18
time (sec)	N/A	0.098	0.018	0.003	2.367	0.421	0.161	0.325	0.083
Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	404	433	345	0	0	371	874
normalized size	1	1.00	0.97	1.04	0.83	0.00	0.00	0.89	2.10
time (sec)	N/A	0.428	0.149	0.007	2.727	0.000	0.000	0.422	0.418
Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	524	866	561	0	0	646	2436
normalized size	1	1.00	0.95	1.57	1.02	0.00	0.00	1.17	4.41
time (sec)	N/A	0.808	0.606	0.010	2.109	0.000	0.000	2.780	2.779
Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	738	1201	817	0	0	901	1955
normalized size	1	1.00	1.09	1.77	1.20	0.00	0.00	1.32	2.88
time (sec)	N/A	0.950	0.929	0.012	2.342	0.000	0.000	0.892	3.674
Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	347	390	332	0	350	342	670
normalized size	1	1.00	0.99	1.12	0.95	0.00	1.00	0.98	1.92
time (sec)	N/A	0.298	0.374	0.004	2.200	0.000	8.334	0.342	0.433

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	321	362	318	0	318	323	391
normalized size	1	1.00	1.00	1.12	0.99	0.00	0.99	1.00	1.21
time (sec)	N/A	0.269	0.316	0.007	2.630	0.000	3.521	0.364	2.483
Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	224	188	238	0	155	241	282
normalized size	1	1.00	0.93	0.78	0.99	0.00	0.64	1.00	1.17
time (sec)	N/A	0.189	0.189	0.005	2.433	0.000	1.092	0.339	0.274
Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	183	143	189	173	39	194	58
normalized size	1	1.00	0.91	0.71	0.94	0.86	0.19	0.96	0.29
time (sec)	N/A	0.116	0.107	0.005	2.108	0.632	0.297	0.246	0.089
Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	855	855	558	1122	601	0	0	771	1591
normalized size	1	1.00	0.65	1.31	0.70	0.00	0.00	0.90	1.86
time (sec)	N/A	0.849	0.391	0.019	2.291	0.000	0.000	0.504	2.995
Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1141	807	1636	961	0	0	1104	2246
normalized size	1	1.00	0.71	1.43	0.84	0.00	0.00	0.97	1.97
time (sec)	N/A	1.659	0.865	0.023	2.541	0.000	0.000	103.636	4.104
Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1384	1384	996	2121	1394	0	0	1488	3256
normalized size	1	1.00	0.72	1.53	1.01	0.00	0.00	1.08	2.35
time (sec)	N/A	1.963	1.357	0.026	2.602	0.000	0.000	1.148	5.038

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	388	470	392	0	413	389	721
normalized size	1	1.00	0.98	1.19	0.99	0.00	1.05	0.99	1.83
time (sec)	N/A	0.346	0.351	0.007	2.373	0.000	8.013	0.354	0.478
Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	358	419	364	0	374	356	676
normalized size	1	1.00	0.99	1.16	1.01	0.00	1.04	0.99	1.88
time (sec)	N/A	0.326	0.294	0.007	2.181	0.000	4.878	0.433	0.471
Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	249	222	269	0	192	260	315
normalized size	1	1.00	0.94	0.83	1.01	0.00	0.72	0.98	1.18
time (sec)	N/A	0.249	0.206	0.007	2.204	0.000	1.483	0.320	0.303
Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	200	158	212	232	63	204	80
normalized size	1	1.00	0.91	0.72	0.97	1.06	0.29	0.93	0.37
time (sec)	N/A	0.142	0.083	0.005	2.704	0.641	0.461	0.358	0.096
Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1352	1352	835	2098	1015	0	0	1259	2720
normalized size	1	1.00	0.62	1.55	0.75	0.00	0.00	0.93	2.01
time (sec)	N/A	1.412	0.738	0.025	2.408	0.000	0.000	0.910	4.406
Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1830	1830	1115	2769	1564	0	0	0	3572
normalized size	1	1.00	0.61	1.51	0.85	0.00	0.00	0.00	1.95
time (sec)	N/A	2.781	1.551	0.033	3.097	0.000	0.000	0.000	5.754

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2204	2204	1338	3334	2198	0	0	2119	6280
normalized size	1	1.00	0.61	1.51	1.00	0.00	0.00	0.96	2.85
time (sec)	N/A	3.165	2.873	0.038	3.988	0.000	0.000	1.534	7.785
Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	28	28	34	28	30
normalized size	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.019	0.010	0.002	2.532	0.980	0.112	0.295	0.041
Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	28	28	34	28	30
normalized size	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.029	0.006	0.002	2.530	1.088	0.115	0.380	0.035
Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	26	26	36	26	30
normalized size	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.017	0.009	0.003	2.156	0.800	0.116	0.388	0.043
Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	26	26	36	26	30
normalized size	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.035	0.005	0.003	2.329	0.877	0.120	0.335	0.032
Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
normalized size	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.012	0.022	0.003	2.621	0.901	0.111	0.415	2.319

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
normalized size	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.021	0.005	0.003	2.332	0.755	0.118	0.315	0.047
Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	23	18	17	17	15	15	6
normalized size	1	1.00	3.83	3.00	2.83	2.83	2.50	2.50	1.00
time (sec)	N/A	0.002	0.003	0.005	1.061	0.673	0.094	0.310	2.270
Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	18	17	17	15	19	6
normalized size	1	1.00	1.10	0.86	0.81	0.81	0.71	0.90	0.29
time (sec)	N/A	0.004	0.003	0.002	0.942	0.516	0.098	0.416	0.146
Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	24	22	9	9
normalized size	1	1.00	0.85	0.77	0.69	1.85	1.69	0.69	0.69
time (sec)	N/A	0.002	0.003	0.001	0.882	0.593	0.125	0.335	2.319
Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	81	11	58	57	24	22	57	9
normalized size	1	6.23	0.85	4.46	4.38	1.85	1.69	4.38	0.69
time (sec)	N/A	0.012	0.002	0.002	0.937	0.641	0.314	0.312	0.025
Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	66	66	85	68	94
normalized size	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.111	0.014	0.009	2.014	0.593	0.250	0.293	0.102

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	66	66	85	68	94
normalized size	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.124	0.006	0.003	1.990	0.614	0.254	0.307	0.039
Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	31	21	20	20	20	21	20
normalized size	1	1.00	1.29	0.88	0.83	0.83	0.83	0.88	0.83
time (sec)	N/A	0.019	0.006	0.003	0.631	0.546	0.078	0.307	0.037
Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	20	20	19	21	20
normalized size	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77
time (sec)	N/A	0.016	0.004	0.002	0.686	0.634	0.073	0.295	0.033
Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	22	16	15	18	15	15	17
normalized size	1	1.00	1.29	0.94	0.88	1.06	0.88	0.88	1.00
time (sec)	N/A	0.006	0.000	0.000	0.664	0.530	0.057	0.388	0.024
Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	20	20	18	15	20	17
normalized size	1	1.00	0.92	0.83	0.83	0.75	0.62	0.83	0.71
time (sec)	N/A	0.005	0.001	0.001	0.568	0.450	0.058	0.368	0.021
Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	23	22	25	20	23	28
normalized size	1	1.00	1.09	1.05	1.00	1.14	0.91	1.05	1.27
time (sec)	N/A	0.019	0.006	0.007	1.378	0.624	0.132	0.393	0.045

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	16	15	15	15	18	15
normalized size	1	1.00	1.59	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.013	0.004	0.007	0.755	0.710	0.130	0.327	0.057
Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	16	13
normalized size	1	1.00	1.00	0.74	0.68	0.68	0.63	0.84	0.68
time (sec)	N/A	0.021	0.004	0.006	0.659	0.632	0.096	0.305	2.336
Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	17	11	10	10	8	13	10
normalized size	1	1.00	1.42	0.92	0.83	0.83	0.67	1.08	0.83
time (sec)	N/A	0.006	0.004	0.001	0.621	0.639	0.089	0.398	2.269
Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	11	10	10	8	11	10
normalized size	1	1.00	1.10	1.10	1.00	1.00	0.80	1.10	1.00
time (sec)	N/A	0.009	0.006	0.002	0.610	0.711	0.132	0.332	0.056
Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.018	0.006	0.005	0.614	0.693	0.125	0.356	0.056
Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.021	0.004	0.005	1.640	0.637	0.128	0.376	0.049

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.009	0.004	0.000	0.655	0.467	0.092	0.278	0.042
Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.021	0.005	0.006	0.586	0.610	0.131	0.369	0.067
Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	14	9	9	7	9	10
normalized size	1	1.00	0.90	1.40	0.90	0.90	0.70	0.90	1.00
time (sec)	N/A	0.010	0.005	0.003	0.781	0.700	0.083	0.261	2.204
Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	20	20	17
normalized size	1	1.00	1.00	0.72	0.68	0.68	0.80	0.80	0.68
time (sec)	N/A	0.021	0.005	0.006	0.646	0.764	0.132	0.362	0.107
Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	14	14
normalized size	1	1.00	1.00	1.07	1.00	1.00	0.86	1.00	1.00
time (sec)	N/A	0.025	0.004	0.001	1.557	0.525	0.105	0.252	2.219
Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	14	30	11	16	17	11	20
normalized size	1	1.00	1.08	2.31	0.85	1.23	1.31	0.85	1.54
time (sec)	N/A	0.007	0.007	0.010	0.582	0.604	0.130	0.371	2.245

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	32	26	25	20	27	25
normalized size	1	1.00	0.96	1.23	1.00	0.96	0.77	1.04	0.96
time (sec)	N/A	0.045	0.008	0.003	0.717	0.611	0.154	0.283	2.234
Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	4	5	4	4	3	5	4
normalized size	1	1.00	0.67	0.83	0.67	0.67	0.50	0.83	0.67
time (sec)	N/A	0.008	0.001	0.001	0.570	0.665	0.064	0.280	0.018
Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	20	25	19	27	18
normalized size	1	1.00	1.00	0.95	1.00	1.25	0.95	1.35	0.90
time (sec)	N/A	0.015	0.005	0.007	0.718	0.671	0.105	0.381	0.039
Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	28	27	27	31	28	44
normalized size	1	1.00	1.05	0.74	0.71	0.71	0.82	0.74	1.16
time (sec)	N/A	0.025	0.010	0.005	1.973	0.655	0.141	0.389	0.158
Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	14	16	15
normalized size	1	1.00	1.06	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.008	0.004	0.002	0.440	0.579	0.073	0.367	0.031
Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	36	22	21	21	22	22	25
normalized size	1	1.00	1.16	0.71	0.68	0.68	0.71	0.71	0.81
time (sec)	N/A	0.012	0.005	0.005	0.995	0.466	0.141	0.274	0.053

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	17	15	17	17	13
normalized size	1	1.00	1.00	0.84	0.89	0.79	0.89	0.89	0.68
time (sec)	N/A	0.017	0.006	0.006	0.474	0.482	0.114	0.408	0.090
Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
normalized size	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.005	0.004	0.000	0.447	0.583	0.088	0.287	2.217
Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
normalized size	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.014	0.009	0.006	1.008	0.642	0.143	0.371	2.216
Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
normalized size	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.023	0.019	0.010	0.448	0.631	0.195	0.363	0.042
Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	14	8	8	7	9	8
normalized size	1	1.00	0.83	1.17	0.67	0.67	0.58	0.75	0.67
time (sec)	N/A	0.002	0.001	0.002	0.438	0.650	0.076	0.275	0.042
Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	23	34	20	25	17
normalized size	1	1.00	1.29	1.33	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.003	0.008	0.008	0.451	0.721	0.108	0.271	2.216

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
normalized size	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.004	0.008	0.006	0.996	0.551	0.101	0.290	0.027
Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	9	6
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.001	0.001	0.001	0.453	0.495	0.058	0.382	0.068
Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.003	0.002	0.003	0.995	0.579	0.112	0.261	2.245
Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
normalized size	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.007	0.004	0.002	0.982	0.780	0.134	0.379	2.243
Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
normalized size	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.011	0.006	0.002	0.955	0.598	0.108	0.380	0.032
Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	16	17
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.77
time (sec)	N/A	0.007	0.001	0.000	0.460	0.409	0.057	0.310	0.024

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	17
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.77
time (sec)	N/A	0.005	0.001	0.000	0.456	0.514	0.056	0.287	0.029
Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	10	14	15
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.62	0.88	0.94
time (sec)	N/A	0.005	0.001	0.003	0.456	0.720	0.069	0.306	0.028
Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	39	30	30
normalized size	1	1.00	1.00	0.84	0.81	0.81	1.05	0.81	0.81
time (sec)	N/A	0.021	0.010	0.005	0.976	0.599	0.118	0.266	2.219
Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	13
normalized size	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.93
time (sec)	N/A	0.004	0.000	0.001	0.466	0.494	0.054	0.364	0.024
Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	7	9	6
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.64	0.82	0.55
time (sec)	N/A	0.004	0.000	0.001	0.450	0.603	0.058	0.286	0.016
Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	20	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.80	0.80	1.00
time (sec)	N/A	0.010	0.006	0.005	0.994	0.690	0.136	0.365	0.049

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.010	0.005	0.003	1.359	0.630	0.129	0.292	2.228
Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.017	0.006	0.005	1.432	0.801	0.123	0.249	2.209
Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	7	7	8	7	12	5	8	7
normalized size	1	0.78	0.78	0.89	0.78	1.33	0.56	0.89	0.78
time (sec)	N/A	0.006	0.003	0.003	0.951	0.543	0.073	0.298	0.023
Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.011	0.013	0.007	1.579	0.482	0.149	0.292	0.055
Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	44	30	30
normalized size	1	1.00	1.00	0.84	0.81	0.81	1.19	0.81	0.81
time (sec)	N/A	0.022	0.013	0.003	1.393	0.681	0.122	0.384	0.045
Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	14	16	13
normalized size	1	1.00	1.00	0.93	1.00	1.27	0.93	1.07	0.87
time (sec)	N/A	0.004	0.002	0.004	0.969	0.495	0.086	0.269	0.026

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	15	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	1.00	0.67	0.93	0.87
time (sec)	N/A	0.004	0.001	0.005	0.948	0.646	0.076	0.280	0.023
Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	13	11	8	13	11
normalized size	1	1.00	1.00	1.09	1.18	1.00	0.73	1.18	1.00
time (sec)	N/A	0.011	0.003	0.005	0.692	0.455	0.095	0.287	0.056
Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	18	17	17	17	17	17
normalized size	1	1.00	0.86	0.82	0.77	0.77	0.77	0.77	0.77
time (sec)	N/A	0.004	0.001	0.000	0.876	0.491	0.056	0.356	0.030
Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	9
normalized size	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	0.82
time (sec)	N/A	0.001	0.001	0.000	0.862	0.602	0.057	0.216	0.142
Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	16	14	13	13	12	14	11
normalized size	1	1.00	1.23	1.08	1.00	1.00	0.92	1.08	0.85
time (sec)	N/A	0.010	0.004	0.002	0.643	0.843	0.076	0.361	2.211
Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	14	16	12
normalized size	1	1.00	1.00	0.94	0.88	1.12	0.88	1.00	0.75
time (sec)	N/A	0.006	0.003	0.007	0.588	0.620	0.100	0.360	0.034

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
normalized size	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.011	0.016	0.008	0.815	0.566	0.111	0.379	2.228
Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	12	16	15
normalized size	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88
time (sec)	N/A	0.008	0.003	0.002	0.740	0.585	0.073	0.357	0.026
Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	15	17	16
normalized size	1	1.00	1.00	0.94	0.89	1.11	0.83	0.94	0.89
time (sec)	N/A	0.006	0.001	0.005	0.728	0.619	0.078	0.371	0.029
Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	14	0	14	14	13
normalized size	1	1.00	1.00	1.56	0.78	0.00	0.78	0.78	0.72
time (sec)	N/A	0.010	0.001	0.001	1.708	0.000	0.057	0.351	0.024
Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	34	46	31	25	23
normalized size	1	1.00	1.00	0.96	1.48	2.00	1.35	1.09	1.00
time (sec)	N/A	0.018	0.010	0.006	0.738	0.556	0.109	0.369	0.032
Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	15	10	32	12
normalized size	1	1.00	0.75	0.81	0.75	0.94	0.62	2.00	0.75
time (sec)	N/A	0.012	0.006	0.004	1.789	0.734	0.107	0.262	2.219

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	26	25	25	24	26	25
normalized size	1	1.00	1.03	0.90	0.86	0.86	0.83	0.90	0.86
time (sec)	N/A	0.020	0.008	0.001	0.765	0.667	0.080	0.283	0.026
Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81
time (sec)	N/A	0.006	0.000	0.000	0.891	0.628	0.059	0.348	0.021
Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	17	25	14	12	17
normalized size	1	1.00	1.00	0.94	1.00	1.47	0.82	0.71	1.00
time (sec)	N/A	0.015	0.012	0.006	1.745	0.688	0.124	0.237	0.031
Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	68	190	151	48	42
normalized size	1	1.00	1.00	0.91	1.45	4.04	3.21	1.02	0.89
time (sec)	N/A	0.066	0.025	0.006	1.021	0.753	0.296	0.281	0.097
Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	61	0	240	294	60	82
normalized size	1	1.00	1.07	1.07	0.00	4.21	5.16	1.05	1.44
time (sec)	N/A	0.082	0.027	0.004	0.000	0.630	0.350	0.358	2.227
Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	39	308	0	247	24	147	101
normalized size	1	1.00	0.21	1.64	0.00	1.31	0.13	0.78	0.54
time (sec)	N/A	0.189	0.031	0.102	0.000	0.521	0.517	1.488	2.300

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	27	28	65	65	60	30	27
normalized size	1	1.00	0.45	0.47	1.08	1.08	1.00	0.50	0.45
time (sec)	N/A	0.136	0.015	0.007	1.249	0.459	0.229	0.397	2.337
Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	B	B	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	250	65	65	60	197	27
normalized size	1	0.00	1.00	9.26	2.41	2.41	2.22	7.30	1.00
time (sec)	N/A	0.311	0.010	0.030	1.199	0.679	0.324	0.397	0.045
Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	B	B	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	112	65	65	60	111	27
normalized size	1	0.00	1.00	4.15	2.41	2.41	2.22	4.11	1.00
time (sec)	N/A	0.433	0.010	0.019	0.975	0.660	0.288	0.447	0.043

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [412] had the largest ratio of [.8824]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	24	0.083
2	A	3	3	1.00	29	0.103
3	A	2	2	1.00	29	0.069
4	A	2	2	1.00	29	0.069
5	A	1	0	1.00	27	0.000
6	A	2	2	1.00	29	0.069
7	A	2	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	2	2	1.00	29	0.069
9	A	3	2	1.00	27	0.074
10	A	3	2	1.00	27	0.074
11	A	1	0	1.00	25	0.000
12	A	7	7	1.00	27	0.259
13	A	8	8	1.00	27	0.296
14	A	9	8	1.00	27	0.296
15	A	3	2	1.00	46	0.043
16	A	3	2	1.00	46	0.043
17	A	1	0	1.00	44	0.000
18	A	2	1	1.00	46	0.022
19	A	2	1	1.00	46	0.022
20	A	2	1	1.00	46	0.022
21	A	5	4	1.00	11	0.364
22	A	5	4	1.00	17	0.235
23	A	2	2	1.00	7	0.286
24	A	3	2	1.00	13	0.154
25	A	5	5	1.00	11	0.454
26	A	7	7	1.00	16	0.438
27	A	6	6	1.00	9	0.667
28	A	2	2	1.00	7	0.286
29	A	3	3	1.00	13	0.231
30	A	3	3	1.11	11	0.273
31	A	3	3	1.00	16	0.188
32	A	2	2	1.26	9	0.222
33	A	3	2	1.00	29	0.069
34	A	2	1	1.00	29	0.034
35	A	2	1	1.00	29	0.034
36	A	1	0	1.00	27	0.000
37	A	10	6	1.00	29	0.207
38	A	11	7	1.00	29	0.241
39	A	3	2	1.00	32	0.062
40	A	2	1	1.00	32	0.031
41	A	2	1	1.00	32	0.031
42	A	1	0	1.00	30	0.000
43	A	4	3	1.00	32	0.094

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	5	4	1.00	32	0.125
45	A	2	1	1.00	17	0.059
46	A	2	1	1.00	17	0.059
47	A	2	1	1.00	17	0.059
48	A	1	0	1.00	15	0.000
49	A	16	9	1.00	17	0.529
50	A	18	11	1.00	17	0.647
51	A	2	1	1.00	17	0.059
52	A	2	1	1.00	17	0.059
53	A	2	1	1.00	17	0.059
54	A	1	0	1.00	15	0.000
55	A	15	9	1.00	17	0.529
56	A	17	11	1.00	17	0.647
57	A	2	1	1.00	22	0.045
58	A	2	1	1.00	22	0.045
59	A	2	1	1.00	22	0.045
60	A	1	0	1.00	20	0.000
61	A	16	9	1.00	22	0.409
62	A	18	11	1.00	22	0.500
63	A	2	2	1.00	51	0.039
64	A	2	2	1.00	51	0.039
65	B	1	0	4.36	49	0.000
66	A	2	2	1.00	51	0.039
67	A	2	2	1.00	51	0.039
68	A	2	2	1.00	51	0.039
69	A	6	5	1.00	13	0.385
70	A	5	3	1.00	19	0.158
71	A	5	3	1.00	19	0.158
72	A	5	3	1.00	19	0.158
73	A	1	0	1.00	17	0.000
74	A	5	2	1.00	19	0.105
75	A	7	3	1.00	19	0.158
76	A	10	3	1.00	19	0.158
77	B	15	7	2.25	17	0.412
78	A	6	5	1.00	15	0.333
79	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	4	4	1.00	13	0.308
81	A	2	2	1.00	11	0.182
82	A	6	6	1.00	15	0.400
83	A	7	6	1.00	15	0.400
84	A	7	6	1.00	15	0.400
85	A	2	2	1.00	13	0.154
86	A	3	3	1.00	13	0.231
87	A	4	3	1.00	13	0.231
88	A	2	2	1.00	19	0.105
89	A	2	2	1.00	11	0.182
90	A	3	3	1.00	11	0.273
91	A	4	3	1.00	11	0.273
92	A	2	2	1.00	13	0.154
93	A	3	3	1.00	13	0.231
94	A	4	3	1.00	13	0.231
95	A	2	2	1.00	11	0.182
96	A	3	3	1.00	11	0.273
97	A	4	3	1.00	11	0.273
98	A	3	2	1.00	15	0.133
99	A	4	3	1.00	13	0.231
100	A	4	4	1.00	17	0.235
101	A	4	4	1.00	19	0.210
102	A	4	4	1.00	17	0.235
103	A	11	10	1.00	17	0.588
104	A	9	9	1.00	17	0.529
105	A	7	7	1.00	15	0.467
106	A	7	7	1.00	13	0.538
107	A	11	10	1.06	17	0.588
108	A	11	10	0.99	17	0.588
109	A	11	10	1.00	17	0.588
110	A	16	12	1.00	17	0.706
111	A	14	10	1.00	17	0.588
112	A	14	10	1.00	15	0.667
113	A	10	7	1.00	13	0.538
114	A	18	13	1.00	17	0.765
115	A	18	13	1.00	17	0.765

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	3	2	1.00	22	0.091
117	A	2	1	1.00	22	0.045
118	A	2	1	1.00	22	0.045
119	A	1	0	1.00	20	0.000
120	A	4	3	1.00	22	0.136
121	A	5	4	1.00	22	0.182
122	A	6	5	1.00	22	0.227
123	A	2	1	1.00	24	0.042
124	A	2	1	1.00	24	0.042
125	A	2	1	1.00	24	0.042
126	A	2	1	1.00	22	0.045
127	A	8	7	1.00	24	0.292
128	A	10	9	1.00	24	0.375
129	A	12	10	1.00	24	0.417
130	A	2	1	1.00	26	0.038
131	A	2	1	1.00	26	0.038
132	A	2	1	1.00	26	0.038
133	A	2	1	1.00	24	0.042
134	A	9	8	1.00	26	0.308
135	A	11	10	1.00	26	0.385
136	A	14	5	1.00	46	0.109
137	A	14	5	1.00	46	0.109
138	A	8	3	1.00	46	0.065
139	A	14	5	1.00	44	0.114
140	A	14	5	1.00	42	0.119
141	A	14	5	1.00	46	0.109
142	A	14	5	0.99	46	0.109
143	A	14	6	1.00	26	0.231
144	A	14	6	1.00	26	0.231
145	A	14	6	1.00	26	0.231
146	A	8	4	1.00	26	0.154
147	A	14	6	1.00	24	0.250
148	A	14	6	1.00	22	0.273
149	A	14	6	1.00	26	0.231
150	A	14	6	1.00	26	0.231
151	A	23	7	1.00	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	23	7	1.00	26	0.269
153	A	14	6	1.00	26	0.231
154	A	17	5	1.00	26	0.192
155	A	23	7	1.00	26	0.269
156	A	23	7	1.00	26	0.269
157	A	23	7	1.00	26	0.269
158	A	2	1	1.00	52	0.019
159	A	4	3	1.00	52	0.058
160	A	1	1	1.00	18	0.056
161	A	3	3	1.00	23	0.130
162	A	3	3	1.00	23	0.130
163	A	3	3	1.00	29	0.103
164	A	1	1	1.00	18	0.056
165	A	4	3	1.00	20	0.150
166	A	4	3	1.00	20	0.150
167	A	4	3	1.00	22	0.136
168	A	1	1	1.00	18	0.056
169	A	3	3	1.00	23	0.130
170	A	3	3	1.00	23	0.130
171	A	3	3	1.00	29	0.103
172	A	1	1	1.00	18	0.056
173	A	1	1	1.00	25	0.040
174	C	7	3	4.30	38	0.079
175	A	1	1	1.00	27	0.037
176	A	1	1	1.00	31	0.032
177	A	2	1	1.00	54	0.019
178	A	3	1	1.00	54	0.019
179	A	5	4	1.00	54	0.074
180	A	1	1	1.00	30	0.033
181	A	1	1	1.00	29	0.034
182	A	1	1	1.00	28	0.036
183	A	1	1	1.00	21	0.048
184	A	1	1	1.00	20	0.050
185	A	1	1	1.00	21	0.048
186	A	1	1	1.00	26	0.038
187	A	1	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	2	2	1.00	26	0.077
189	A	1	1	1.00	24	0.042
190	A	1	1	1.00	24	0.042
191	A	1	1	1.00	23	0.043
192	A	1	1	1.00	30	0.033
193	A	1	1	1.00	29	0.034
194	A	1	1	1.00	28	0.036
195	A	1	1	1.00	21	0.048
196	A	1	1	1.00	20	0.050
197	A	2	2	1.00	21	0.095
198	A	1	1	1.00	26	0.038
199	A	1	1	1.00	25	0.040
200	A	2	2	1.00	26	0.077
201	A	1	1	1.00	22	0.045
202	A	1	1	1.00	24	0.042
203	A	2	2	1.00	23	0.087
204	A	1	1	1.00	23	0.043
205	A	1	1	1.00	19	0.053
206	A	2	1	1.00	22	0.045
207	A	2	1	1.00	23	0.043
208	A	2	1	1.00	22	0.045
209	A	2	1	1.00	23	0.043
210	A	2	1	1.00	24	0.042
211	A	2	1	1.00	25	0.040
212	A	2	1	1.00	31	0.032
213	A	2	1	1.00	32	0.031
214	A	2	1	1.00	35	0.029
215	A	2	1	1.00	36	0.028
216	A	2	1	1.00	24	0.042
217	A	2	1	1.00	31	0.032
218	A	2	1	1.00	35	0.029
219	A	1	1	1.00	22	0.045
220	A	1	1	1.00	18	0.056
221	B	3	2	2.91	26	0.077
222	B	2	1	2.91	28	0.036
223	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	1	1	1.00	20	0.050
225	A	1	1	1.00	21	0.048
226	A	3	3	1.00	52	0.058
227	A	9	5	1.00	38	0.132
228	A	3	2	1.00	32	0.062
229	A	4	3	1.00	33	0.091
230	A	3	2	1.00	34	0.059
231	A	6	6	1.00	43	0.140
232	A	1	1	1.00	16	0.062
233	B	15	7	2.25	25	0.280
234	A	1	1	1.00	56	0.018
235	A	1	1	1.00	51	0.020
236	A	1	1	1.00	49	0.020
237	A	1	1	1.00	46	0.022
238	A	2	2	1.00	48	0.042
239	A	1	1	1.00	49	0.020
240	A	1	1	1.00	48	0.021
241	A	1	1	1.00	48	0.021
242	A	10	5	1.00	35	0.143
243	A	10	5	1.00	35	0.143
244	A	10	5	1.00	35	0.143
245	A	10	5	1.00	33	0.152
246	A	10	5	1.00	32	0.156
247	A	13	6	1.00	35	0.171
248	A	13	6	1.00	35	0.171
249	A	13	6	1.00	35	0.171
250	A	13	6	1.00	35	0.171
251	A	13	6	1.00	35	0.171
252	A	11	6	1.00	33	0.182
253	A	9	5	1.00	32	0.156
254	A	13	6	1.00	35	0.171
255	A	13	6	1.00	35	0.171
256	A	13	6	1.00	35	0.171
257	A	2	2	1.00	40	0.050
258	A	6	5	1.00	20	0.250
259	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	3	2	1.00	20	0.100
261	A	2	1	1.00	16	0.062
262	A	5	4	1.00	22	0.182
263	A	3	2	1.00	21	0.095
264	A	6	5	1.00	26	0.192
265	A	2	1	1.00	20	0.050
266	A	2	1	1.00	11	0.091
267	A	4	3	1.00	22	0.136
268	A	3	2	1.00	21	0.095
269	A	3	2	1.00	25	0.080
270	A	6	6	1.00	22	0.273
271	A	5	5	1.00	31	0.161
272	A	3	2	1.00	21	0.095
273	A	4	3	1.00	33	0.091
274	A	4	4	1.00	14	0.286
275	A	7	6	1.00	33	0.182
276	A	7	6	1.00	29	0.207
277	A	6	5	1.00	44	0.114
278	A	3	2	1.00	15	0.133
279	A	5	4	1.00	15	0.267
280	A	4	3	1.00	18	0.167
281	A	3	2	1.00	20	0.100
282	A	5	4	1.00	26	0.154
283	A	3	2	1.00	13	0.154
284	A	3	2	1.00	18	0.111
285	A	2	1	1.00	26	0.038
286	A	5	5	1.00	19	0.263
287	A	5	5	1.00	24	0.208
288	A	8	6	1.00	20	0.300
289	A	8	6	1.00	18	0.333
290	A	5	3	1.00	19	0.158
291	A	4	3	1.00	13	0.231
292	A	6	5	1.00	22	0.227
293	A	6	5	1.00	24	0.208
294	A	2	1	1.00	29	0.034
295	A	2	1	1.00	30	0.033

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
296	A	2	1	1.00	19	0.053
297	A	4	4	1.00	16	0.250
298	A	10	5	1.00	36	0.139
299	A	2	1	1.00	21	0.048
300	A	5	4	1.00	16	0.250
301	A	2	1	1.00	24	0.042
302	A	2	1	1.00	21	0.048
303	A	2	1	1.00	24	0.042
304	A	6	5	1.00	26	0.192
305	A	3	2	1.00	25	0.080
306	A	2	1	1.00	29	0.034
307	A	6	5	1.00	20	0.250
308	A	14	10	1.00	32	0.312
309	A	4	4	1.00	23	0.174
310	A	6	5	1.00	26	0.192
311	A	4	3	1.00	26	0.115
312	A	8	4	1.00	25	0.160
313	A	6	3	1.00	23	0.130
314	A	7	6	1.00	23	0.261
315	A	5	3	1.18	20	0.150
316	A	3	2	1.00	25	0.080
317	A	3	2	1.00	22	0.091
318	A	2	1	1.00	24	0.042
319	A	7	6	1.00	24	0.250
320	A	6	5	1.00	43	0.116
321	A	7	5	1.25	50	0.100
322	A	3	2	1.00	16	0.125
323	A	6	5	1.00	15	0.333
324	A	5	4	1.00	20	0.200
325	A	3	2	1.00	24	0.083
326	A	5	3	1.00	27	0.111
327	A	5	5	1.00	26	0.192
328	A	3	2	1.00	16	0.125
329	A	11	8	1.32	22	0.364
330	A	5	4	1.00	24	0.167
331	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
332	A	5	3	1.00	36	0.083
333	A	4	3	1.00	26	0.115
334	A	10	9	1.00	20	0.450
335	A	6	6	1.00	27	0.222
336	A	7	7	1.00	20	0.350
337	A	8	7	1.00	25	0.280
338	A	8	7	1.00	22	0.318
339	A	2	1	1.00	18	0.056
340	A	5	4	1.00	20	0.200
341	A	10	9	1.00	20	0.450
342	A	16	12	1.00	20	0.600
343	A	3	2	1.00	14	0.143
344	A	2	2	1.00	20	0.100
345	A	14	8	1.00	20	0.400
346	A	4	3	1.00	26	0.115
347	A	4	3	1.00	24	0.125
348	A	6	4	1.00	30	0.133
349	A	4	4	1.00	21	0.190
350	A	2	1	1.00	15	0.067
351	A	4	3	1.00	18	0.167
352	A	3	2	1.00	22	0.091
353	A	3	2	1.00	16	0.125
354	A	6	5	1.00	16	0.312
355	A	3	2	1.00	25	0.080
356	A	3	2	1.00	19	0.105
357	A	2	1	1.00	23	0.043
358	A	5	4	1.00	23	0.174
359	A	5	4	1.00	21	0.190
360	A	10	6	1.00	28	0.214
361	A	2	1	1.00	24	0.042
362	A	6	5	1.00	26	0.192
363	A	2	1	1.00	14	0.071
364	A	5	3	1.00	16	0.188
365	A	5	5	1.00	16	0.312
366	A	7	5	1.00	43	0.116
367	A	17	13	1.00	26	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	18	13	1.00	16	0.812
369	A	3	2	1.00	15	0.133
370	A	3	2	1.00	15	0.133
371	A	3	2	1.00	17	0.118
372	A	3	2	1.00	15	0.133
373	A	4	3	1.00	15	0.200
374	A	4	3	1.00	18	0.167
375	A	3	2	1.00	20	0.100
376	A	7	6	1.00	29	0.207
377	A	4	3	1.00	22	0.136
378	A	6	4	1.00	16	0.250
379	A	6	4	1.00	16	0.250
380	A	5	4	1.00	14	0.286
381	A	4	3	1.00	12	0.250
382	A	4	3	1.00	16	0.188
383	A	6	5	1.00	16	0.312
384	A	4	3	1.00	16	0.188
385	A	4	3	1.00	16	0.188
386	A	4	3	1.00	16	0.188
387	A	8	5	1.00	17	0.294
388	A	8	5	1.00	19	0.263
389	A	8	5	1.00	15	0.333
390	A	8	5	1.00	17	0.294
391	A	16	10	1.00	23	0.435
392	A	17	10	1.00	21	0.476
393	F	0	0	N/A	0	N/A
394	A	15	11	1.00	17	0.647
395	A	13	9	1.00	17	0.529
396	A	13	9	1.00	15	0.600
397	A	9	6	1.00	9	0.667
398	A	17	12	1.00	17	0.706
399	A	17	12	1.00	17	0.706
400	A	17	12	1.00	17	0.706
401	A	16	12	1.00	17	0.706
402	A	14	10	1.00	17	0.588
403	A	14	10	1.00	15	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
404	A	10	7	1.00	9	0.778
405	A	31	14	1.00	17	0.824
406	A	31	14	1.00	17	0.824
407	A	31	14	1.00	17	0.824
408	A	15	11	1.00	17	0.647
409	A	15	10	1.00	17	0.588
410	A	15	10	1.00	15	0.667
411	A	11	7	1.00	9	0.778
412	A	46	15	1.00	17	0.882
413	A	46	15	1.00	17	0.882
414	A	46	15	1.00	17	0.882
415	A	4	4	1.00	14	0.286
416	A	5	5	1.00	13	0.385
417	A	4	4	1.00	16	0.250
418	A	5	5	1.00	18	0.278
419	A	3	2	1.00	14	0.143
420	A	4	3	1.00	16	0.188
421	A	2	2	1.00	11	0.182
422	A	1	0	1.00	17	0.000
423	A	1	1	1.00	11	0.091
424	B	1	0	6.23	73	0.000
425	A	11	7	1.00	13	0.538
426	A	13	9	1.00	19	0.474
427	A	3	2	1.00	15	0.133
428	A	3	2	1.00	11	0.182
429	A	1	0	1.00	11	0.000
430	A	1	0	1.00	11	0.000
431	A	5	4	1.00	16	0.250
432	A	2	1	1.00	16	0.062
433	A	4	3	1.00	15	0.200
434	A	1	1	1.00	15	0.067
435	A	1	1	1.00	20	0.050
436	A	3	2	1.00	15	0.133
437	A	6	5	1.00	13	0.385
438	A	1	1	1.00	22	0.045
439	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	3	3	1.00	20	0.150
441	A	3	2	1.00	16	0.125
442	A	6	6	1.00	17	0.353
443	A	1	1	1.00	17	0.059
444	A	4	3	1.00	25	0.120
445	A	2	2	1.00	20	0.100
446	A	3	2	1.00	18	0.111
447	A	6	5	1.00	15	0.333
448	A	2	1	1.00	11	0.091
449	A	5	5	1.00	13	0.385
450	A	3	2	1.00	20	0.100
451	A	2	1	1.00	16	0.062
452	A	4	3	1.00	16	0.188
453	A	2	1	1.00	16	0.062
454	A	1	1	1.00	9	0.111
455	A	2	2	1.00	7	0.286
456	A	2	2	1.00	11	0.182
457	A	1	1	1.00	7	0.143
458	A	1	1	1.00	9	0.111
459	A	1	1	1.00	9	0.111
460	A	2	2	1.00	10	0.200
461	A	2	1	1.00	13	0.077
462	A	2	1	1.00	11	0.091
463	A	2	1	1.00	14	0.071
464	A	4	4	1.00	16	0.250
465	A	2	1	1.00	7	0.143
466	A	2	1	1.00	11	0.091
467	A	5	5	1.00	13	0.385
468	A	5	5	1.00	13	0.385
469	A	5	4	1.00	14	0.286
470	A	2	1	0.78	13	0.077
471	A	3	2	1.00	20	0.100
472	A	4	4	1.00	18	0.222
473	A	2	1	1.00	12	0.083
474	A	2	1	1.00	10	0.100
475	A	3	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
476	A	2	1	1.00	11	0.091
477	A	1	1	1.00	7	0.143
478	A	2	1	1.00	15	0.067
479	A	2	1	1.00	12	0.083
480	A	3	3	1.00	16	0.188
481	A	2	1	1.00	11	0.091
482	A	2	1	1.00	17	0.059
483	A	2	1	1.00	29	0.034
484	A	2	1	1.00	18	0.056
485	A	3	2	1.00	14	0.143
486	A	2	1	1.00	24	0.042
487	A	2	1	1.00	11	0.091
488	A	3	2	1.00	18	0.111
489	A	3	3	1.00	15	0.200
490	A	3	3	1.00	16	0.188
491	A	10	7	1.00	15	0.467
492	A	5	2	1.00	50	0.040
493	F	0	0	N/A	0	N/A
494	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

$$3.1 \quad \int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3} dx$$

Optimal. Leaf size=77

$$\frac{1}{3\sqrt{3}\sqrt{b}(\sqrt{3}\sqrt{b}-3x)} - \frac{\log(\sqrt{b}-\sqrt{3}x)}{27b} + \frac{\log(2\sqrt{b}+\sqrt{3}x)}{27b}$$

[Out] $-1/27*\ln(-x*3^{(1/2)}+b^{(1/2)})/b+1/27*\ln(x*3^{(1/2)}+2*b^{(1/2)})/b+1/9*3^{(1/2)}/b^{(1/2)}/(-3*x+3^{(1/2)}*b^{(1/2)})$

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2063, 44}

$$\frac{1}{3\sqrt{3}\sqrt{b}(\sqrt{3}\sqrt{b}-3x)} - \frac{\log(\sqrt{b}-\sqrt{3}x)}{27b} + \frac{\log(2\sqrt{b}+\sqrt{3}x)}{27b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*\text{Sqrt}[3]*b^{(3/2)} - 9*b*x + 9*x^3)^{-1}, x]$

[Out] $1/(3*\text{Sqrt}[3]*\text{Sqrt}[b]*(\text{Sqrt}[3]*\text{Sqrt}[b] - 3*x)) - \text{Log}[\text{Sqrt}[b] - \text{Sqrt}[3]*x]/(27*b) + \text{Log}[2*\text{Sqrt}[b] + \text{Sqrt}[3]*x]/(27*b)$

Rule 44

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d}, x] & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 2063

$\text{Int}[(a_ + (b_)*(x_ + (d_)*(x_)^3)^{(p_)}), x_Symbol] \rightarrow \text{Dist}[1/(3^{(3*p)}*a^{(2*p)}), \text{Int}[(3*a - b*x)^p*(3*a + 2*b*x)^{(2*p)}, x], x] /;$ FreeQ[{a, b, d}, x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3} dx &= (324b^3) \int \frac{1}{(6\sqrt{3}b^{3/2} - 18bx)^2 (6\sqrt{3}b^{3/2} + 9bx)} dx \\ &= (324b^3) \int \left(\frac{1}{324\sqrt{3}b^{7/2}(\sqrt{3}\sqrt{b} - 3x)^2} + \frac{1}{2916b^4(\sqrt{3}\sqrt{b} - 3x)} + \frac{1}{2916b^4(2\sqrt{3}\sqrt{b} + 3x)} \right) dx \\ &= \frac{1}{3\sqrt{3}\sqrt{b}(\sqrt{3}\sqrt{b} - 3x)} - \frac{\log(\sqrt{b} - \sqrt{3}x)}{27b} + \frac{\log(2\sqrt{b} + \sqrt{3}x)}{27b} \end{aligned}$$

Mathematica [A] time = 0.07, size = 143, normalized size = 1.86

$$\frac{(3x - \sqrt{3}\sqrt{b})(2\sqrt{3}\sqrt{b} + 3x)((3x - \sqrt{3}\sqrt{b})\log(3x - \sqrt{3}\sqrt{b}) + (\sqrt{3}\sqrt{b} - 3x)\log(2\sqrt{3}\sqrt{b} + 3x) + 3\sqrt{3}\sqrt{b}\log(3x - \sqrt{3}\sqrt{b}))}{81b(2\sqrt{3}b^{3/2} - 9bx + 9x^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1), x]

[Out] -1/81*((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]*Sqrt[b] + (-Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (Sqrt[3]*Sqrt[b] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x))/(b*(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3))

fricas [A] time = 0.59, size = 76, normalized size = 0.99

$$\frac{3\sqrt{3}\sqrt{b}x - (3x^2 - b)\log(2\sqrt{3}\sqrt{b} + 3x) + (3x^2 - b)\log(-\sqrt{3}\sqrt{b} + 3x) + 3b}{27(3bx^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)), x, algorithm="fricas")

[Out] -1/27*(3*sqrt(3)*sqrt(b)*x - (3*x^2 - b)*log(2*sqrt(3)*sqrt(b) + 3*x) + (3*x^2 - b)*log(-sqrt(3)*sqrt(b) + 3*x) + 3*b)/(3*b*x^2 - b^2)

giac [A] time = 0.22, size = 55, normalized size = 0.71

$$\frac{\log(|9\sqrt{3}x + 18\sqrt{b}|)}{27b} - \frac{\log(|-\sqrt{3}x + \sqrt{b}|)}{27b} - \frac{1}{9(\sqrt{3}x - \sqrt{b})\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)), x, algorithm="giac")

[Out] 1/27*log(abs(9*sqrt(3)*x + 18*sqrt(b)))/b - 1/27*log(abs(-sqrt(3)*x + sqrt(b)))/b - 1/9/((sqrt(3)*x - sqrt(b))*sqrt(b))

maple [C] time = 0.05, size = 43, normalized size = 0.56

$$\frac{\ln\left(-\text{RootOf}\left(9_Z^3 - 9_Zb + 2\sqrt{3}b^{\frac{3}{2}}\right) + x\right)}{27\text{RootOf}\left(9_Z^3 - 9_Zb + 2\sqrt{3}b^{\frac{3}{2}}\right) - 9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x)

[Out] 1/9*sum(1/(3*_R^2-b)*ln(x-_R),_R=RootOf(-9*b*_Z+9*_Z^3+2*b^(3/2)*3^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{9x^3 + 2\sqrt{3}b^{\frac{3}{2}} - 9bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)

mupad [B] time = 0.20, size = 51, normalized size = 0.66

$$\frac{2\sqrt{3}\sqrt{27}\operatorname{atanh}\left(\frac{\sqrt{3}\sqrt{27}}{27} + \frac{2\sqrt{27}x}{9\sqrt{b}}\right)}{243b} - \frac{\sqrt{3}}{27\sqrt{b}\left(x - \frac{\sqrt{3}\sqrt{b}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*3^(1/2)*b^(3/2) - 9*b*x + 9*x^3),x)

[Out] (2*3^(1/2)*27^(1/2)*atanh((3^(1/2)*27^(1/2))/27 + (2*27^(1/2)*x)/(9*b^(1/2))) / (243*b) - 3^(1/2)/(27*b^(1/2)*(x - (3^(1/2)*b^(1/2))/3))

sympy [A] time = 0.33, size = 60, normalized size = 0.78

$$-\frac{3\sqrt{3}}{81\sqrt{b}x - 27\sqrt{3}b} + \frac{-\frac{\log\left(-\frac{\sqrt{3}\sqrt{b}}{3} + x\right)}{27}}{b} + \frac{\log\left(\frac{2\sqrt{3}\sqrt{b}}{3} + x\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)

[Out] -3*sqrt(3)/(81*sqrt(b)*x - 27*sqrt(3)*b) + (-log(-sqrt(3)*sqrt(b)/3 + x)/27 + log(2*sqrt(3)*sqrt(b)/3 + x)/27)/b

3.2 $\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p dx$

Optimal. Leaf size=30

$$\frac{\left(\frac{a}{b} + x\right) \left(b^3 \left(\frac{a}{b} + x\right)^3\right)^p}{3p + 1}$$

[Out] (a/b+x)*(b^3*(a/b+x)^3)^p/(1+3*p)

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2067, 15, 30}

$$\frac{\left(\frac{a}{b} + x\right) \left(b^3 \left(\frac{a}{b} + x\right)^3\right)^p}{3p + 1}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]

[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2067

Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]

Rubi steps

$$\begin{aligned} \int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p dx &= \text{Subst} \left(\int (b^3x^3)^p dx, x, \frac{a}{b} + x \right) \\ &= \left(\left(\frac{a}{b} + x \right)^{-3p} \left(b^3 \left(\frac{a}{b} + x \right)^3 \right)^p \right) \text{Subst} \left(\int x^{3p} dx, x, \frac{a}{b} + x \right) \\ &= \frac{(a + bx) \left((a + bx)^3 \right)^p}{b(1 + 3p)} \end{aligned}$$

Mathematica [A] time = 0.11, size = 23, normalized size = 0.77

$$\frac{(a + bx) \left((a + bx)^3 \right)^p}{3bp + b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]

[Out] ((a + b*x)*(a + b*x)^3)^p/(b + 3*b*p)

fricas [A] time = 0.86, size = 43, normalized size = 1.43

$$\frac{(bx + a)(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)^p}{3bp + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="fricas")

[Out] (b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)

giac [B] time = 0.20, size = 73, normalized size = 2.43

$$\frac{(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)^p bx + (b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)^p a}{3bp + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="giac")

[Out] ((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*b*x + (b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*a)/(3*b*p + b)

maple [A] time = 0.01, size = 46, normalized size = 1.53

$$\frac{(bx + a)(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)^p}{(3p + 1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x)

[Out] (b*x+a)/b/(1+3*p)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p

maxima [A] time = 0.49, size = 25, normalized size = 0.83

$$\frac{(bx + a)(bx + a)^{3p}}{b(3p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="maxima")

[Out] (b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))

mupad [B] time = 2.13, size = 52, normalized size = 1.73

$$\left(\frac{x}{3p+1} + \frac{a}{b(3p+1)} \right) (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p,x)

[Out] (x/(3*p + 1) + a/(b*(3*p + 1)))*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\left\{ \begin{array}{ll} \frac{x}{\sqrt[3]{a^3}} & \text{for } b = 0 \wedge p = -\frac{1}{3} \\ x(a^3)^p & \text{for } b = 0 \\ \int \frac{1}{\sqrt[3]{a^3+3a^2bx+3ab^2x^2+b^3x^3}} dx & \text{for } p = -\frac{1}{3} \\ \frac{a(a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3bp+b} + \frac{bx(a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3bp+b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)

[Out] Piecewise((x/(a**3)**(1/3), Eq(b, 0) & Eq(p, -1/3)), (x*(a**3)**p, Eq(b, 0)), (Integral((a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**(-1/3), x), Eq(p, -1/3)), (a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b) + b*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b), True))

3.3 $\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3 dx$

Optimal. Leaf size=14

$$\frac{(a + bx)^{10}}{10b}$$

[Out] 1/10*(b*x+a)^10/b

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2059, 32}

$$\frac{(a + bx)^{10}}{10b}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]

[Out] (a + b*x)^10/(10*b)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2059

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[Non freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3 dx &= \int (a + bx)^9 dx \\ &= \frac{(a + bx)^{10}}{10b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^{10}}{10b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]

[Out] (a + b*x)^10/(10*b)

fricas [B] time = 0.46, size = 97, normalized size = 6.93

$$\frac{1}{10}x^{10}b^9 + x^9b^8a + \frac{9}{2}x^8b^7a^2 + 12x^7b^6a^3 + 21x^6b^5a^4 + \frac{126}{5}x^5b^4a^5 + 21x^4b^3a^6 + 12x^3b^2a^7 + \frac{9}{2}x^2ba^8 + xa^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")

[Out] $\frac{1}{10}x^{10}b^9 + x^9b^8a + \frac{9}{2}x^8b^7a^2 + 12x^7b^6a^3 + 21x^6b^5a^4 + \frac{126}{5}x^5b^4a^5 + 21x^4b^3a^6 + 12x^3b^2a^7 + \frac{9}{2}x^2b^1a^8 + xa^9$

giac [B] time = 0.25, size = 97, normalized size = 6.93

$$\frac{1}{10}b^9x^{10}+ab^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a^8bx^2+a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")`

[Out] $\frac{1}{10}b^9x^{10} + ab^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8bx^2 + a^9x$

maple [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{1}{10}b^9x^{10}+ab^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a^8bx^2+a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)`

[Out] $\frac{1}{10}b^9x^{10}+ab^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a^8bx^2+a^9x$

maxima [B] time = 0.69, size = 216, normalized size = 15.43

$$\frac{1}{10}b^9x^{10}+ab^8x^9+\frac{27}{8}a^2b^7x^8+\frac{27}{7}a^3b^6x^7+\frac{27}{4}a^6b^3x^4+a^9x+\frac{3}{4}(b^3x^4+4ab^2x^3+6a^2bx^2)a^6+\frac{9}{10}(5b^3x^6+18ab^2x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")`

[Out] $\frac{1}{10}b^9x^{10} + ab^8x^9 + \frac{27}{8}a^2b^7x^8 + \frac{27}{7}a^3b^6x^7 + \frac{27}{4}a^6b^3x^4 + a^9x + \frac{3}{4}(b^3x^4 + 4ab^2x^3 + 6a^2bx^2)a^6 + \frac{9}{10}(5b^3x^6 + 18ab^2x^5)a^4b^2 + \frac{3}{70}(10b^6x^7 + 70ab^5x^6 + 126a^2b^4x^5 + 210a^4b^2x^3 + 21(4b^3x^5 + 15ab^2x^4)a^2b)a^3 + \frac{9}{56}(7b^6x^8 + 48ab^5x^7 + 84a^2b^4x^6)a^2b$

mupad [B] time = 2.07, size = 97, normalized size = 6.93

$$a^9x+\frac{9a^8bx^2}{2}+12a^7b^2x^3+21a^6b^3x^4+\frac{126a^5b^4x^5}{5}+21a^4b^5x^6+12a^3b^6x^7+\frac{9a^2b^7x^8}{2}+ab^8x^9+\frac{b^9x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)`

[Out] $a^9x + \frac{b^9x^{10}}{10} + \frac{(9a^8bx^2)}{2} + ab^8x^9 + 12a^7b^2x^3 + 21a^6b^3x^4 + \frac{(126a^5b^4x^5)}{5} + 21a^4b^5x^6 + 12a^3b^6x^7 + \frac{(9a^2b^7x^8)}{2}$

sympy [B] time = 0.09, size = 107, normalized size = 7.64

$$a^9x+\frac{9a^8bx^2}{2}+12a^7b^2x^3+21a^6b^3x^4+\frac{126a^5b^4x^5}{5}+21a^4b^5x^6+12a^3b^6x^7+\frac{9a^2b^7x^8}{2}+ab^8x^9+\frac{b^9x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)

[Out] a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4 + 126*a**5
*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 + 9*a**2*b**7*x**8/2 +
a*b**8*x**9 + b**9*x**10/10

3.4 $\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2 dx$

Optimal. Leaf size=14

$$\frac{(a + bx)^7}{7b}$$

[Out] 1/7*(b*x+a)^7/b

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2059, 32}

$$\frac{(a + bx)^7}{7b}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]

[Out] (a + b*x)^7/(7*b)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2059

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[Non freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2 dx &= \int (a + bx)^6 dx \\ &= \frac{(a + bx)^7}{7b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^7}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]

[Out] (a + b*x)^7/(7*b)

fricas [B] time = 0.43, size = 64, normalized size = 4.57

$$\frac{1}{7}x^7b^6 + x^6b^5a + 3x^5b^4a^2 + 5x^4b^3a^3 + 5x^3b^2a^4 + 3x^2ba^5 + xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")

[Out] 1/7*x^7*b^6 + x^6*b^5*a + 3*x^5*b^4*a^2 + 5*x^4*b^3*a^3 + 5*x^3*b^2*a^4 + 3*x^2*b*a^5 + x*a^6

giac [B] time = 0.37, size = 64, normalized size = 4.57

$$\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")

[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3*a^5*b*x^2 + a^6*x

maple [B] time = 0.00, size = 65, normalized size = 4.64

$$\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)

[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^2+a^6*x

maxima [B] time = 0.66, size = 99, normalized size = 7.07

$$\frac{1}{7}b^6x^7+ab^5x^6+\frac{9}{5}a^2b^4x^5+3a^4b^2x^3+a^6x+\frac{1}{2}(b^3x^4+4ab^2x^3+6a^2bx^2)a^3+\frac{3}{10}(4b^3x^5+15ab^2x^4)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")

[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x + 1/2*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b

mupad [B] time = 0.03, size = 64, normalized size = 4.57

$$a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)

[Out] a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*x^4 + 3*a^2*b^4*x^5

sympy [B] time = 0.08, size = 66, normalized size = 4.71

$$a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)

[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7

3.5 $\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3) dx$

Optimal. Leaf size=35

$$a^3x + \frac{3}{2}a^2bx^2 + ab^2x^3 + \frac{b^3x^4}{4}$$

[Out] $a^3x + 3/2a^2bx^2 + ab^2x^3 + 1/4b^3x^4$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{3}{2}a^2bx^2 + a^3x + ab^2x^3 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3, x]

[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4

Rubi steps

$$\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3) dx = a^3x + \frac{3}{2}a^2bx^2 + ab^2x^3 + \frac{b^3x^4}{4}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3x + \frac{3}{2}a^2bx^2 + ab^2x^3 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3, x]

[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4

fricas [A] time = 0.52, size = 31, normalized size = 0.89

$$\frac{1}{4}x^4b^3 + x^3b^2a + \frac{3}{2}x^2ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="fricas")

[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3

giac [A] time = 0.28, size = 31, normalized size = 0.89

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="giac")

[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x)`

[Out] `a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4`

maxima [A] time = 0.74, size = 31, normalized size = 0.89

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="maxima")`

[Out] `1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x`

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x,x)`

[Out] `a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3`

sympy [A] time = 0.07, size = 32, normalized size = 0.91

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)`

[Out] `a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4`

$$3.6 \quad \int \frac{1}{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3} dx$$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

[Out] -1/2/b/(b*x+a)^2

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2058, 32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1), x]

[Out] -1/(2*b*(a + b*x)^2)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3} dx &= \int \frac{1}{(a + bx)^3} dx \\ &= -\frac{1}{2b(a + bx)^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1), x]

[Out] -1/2*1/(b*(a + b*x)^2)

fricas [A] time = 0.59, size = 24, normalized size = 1.71

$$-\frac{1}{2(b^3x^2 + 2ab^2x + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3), x, algorithm="fricas")

[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)

giac [A] time = 0.26, size = 12, normalized size = 0.86

$$-\frac{1}{2(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="giac")

[Out] -1/2/((b*x + a)^2*b)

maple [A] time = 0.02, size = 13, normalized size = 0.93

$$-\frac{1}{2(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)

[Out] -1/2/b/(b*x+a)^2

maxima [A] time = 0.65, size = 24, normalized size = 1.71

$$-\frac{1}{2(b^3x^2 + 2ab^2x + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")

[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)

mupad [B] time = 0.03, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)

[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)

sympy [B] time = 0.18, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)

[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)

$$3.7 \quad \int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2} dx$$

Optimal. Leaf size=14

$$-\frac{1}{5b(a+bx)^5}$$

[Out] -1/5/b/(b*x+a)^5

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2058, 32}

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2), x]

[Out] -1/(5*b*(a + b*x)^5)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2} dx &= \int \frac{1}{(a + bx)^6} dx \\ &= -\frac{1}{5b(a + bx)^5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2), x]

[Out] -1/5*1/(b*(a + b*x)^5)

fricas [B] time = 0.73, size = 57, normalized size = 4.07

$$-\frac{1}{5(b^6x^5 + 5ab^5x^4 + 10a^2b^4x^3 + 10a^3b^3x^2 + 5a^4b^2x + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")

[Out] $-1/5/(b^6x^5 + 5ab^5x^4 + 10a^2b^4x^3 + 10a^3b^3x^2 + 5a^4b^2x + a^5b)$

giac [A] time = 0.29, size = 12, normalized size = 0.86

$$-\frac{1}{5(bx+a)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")`

[Out] $-1/5/((b*x + a)^5*b)$

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{5(bx+a)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)`

[Out] $-1/5/b/(b*x+a)^5$

maxima [B] time = 0.71, size = 57, normalized size = 4.07

$$-\frac{1}{5(b^6x^5 + 5ab^5x^4 + 10a^2b^4x^3 + 10a^3b^3x^2 + 5a^4b^2x + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")`

[Out] $-1/5/(b^6x^5 + 5ab^5x^4 + 10a^2b^4x^3 + 10a^3b^3x^2 + 5a^4b^2x + a^5b)$

mupad [B] time = 2.05, size = 59, normalized size = 4.21

$$-\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)`

[Out] $-1/(5a^5b + 5b^6x^5 + 25a^4b^2x + 25a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5)$

sympy [B] time = 0.35, size = 61, normalized size = 4.36

$$-\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)`

[Out] $-1/(5a**5*b + 25a**4*b**2*x + 50a**3*b**3*x**2 + 50a**2*b**4*x**3 + 25a*b**5*x**4 + 5b**6*x**5)$

$$3.8 \quad \int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx$$

Optimal. Leaf size=14

$$-\frac{1}{8b(a+bx)^8}$$

[Out] -1/8/b/(b*x+a)^8

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2058, 32}

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3), x]

[Out] -1/(8*b*(a + b*x)^8)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx &= \int \frac{1}{(a + bx)^9} dx \\ &= -\frac{1}{8b(a + bx)^8} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3), x]

[Out] -1/8*1/(b*(a + b*x)^8)

fricas [B] time = 0.83, size = 90, normalized size = 6.43

$$-\frac{1}{8(b^9x^8 + 8ab^8x^7 + 28a^2b^7x^6 + 56a^3b^6x^5 + 70a^4b^5x^4 + 56a^5b^4x^3 + 28a^6b^3x^2 + 8a^7b^2x + a^8b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")

[Out] $-1/8/(b^9x^8 + 8ab^8x^7 + 28a^2b^7x^6 + 56a^3b^6x^5 + 70a^4b^5x^4 + 56a^5b^4x^3 + 28a^6b^3x^2 + 8a^7b^2x + a^8b)$

giac [A] time = 0.39, size = 12, normalized size = 0.86

$$-\frac{1}{8(bx+a)^8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")`

[Out] $-1/8/((b*x + a)^8*b)$

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{8(bx+a)^8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)`

[Out] $-1/8/b/(b*x+a)^8$

maxima [B] time = 0.54, size = 90, normalized size = 6.43

$$\frac{1}{8(b^9x^8 + 8ab^8x^7 + 28a^2b^7x^6 + 56a^3b^6x^5 + 70a^4b^5x^4 + 56a^5b^4x^3 + 28a^6b^3x^2 + 8a^7b^2x + a^8b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")`

[Out] $-1/8/(b^9x^8 + 8ab^8x^7 + 28a^2b^7x^6 + 56a^3b^6x^5 + 70a^4b^5x^4 + 56a^5b^4x^3 + 28a^6b^3x^2 + 8a^7b^2x + a^8b)$

mupad [B] time = 2.07, size = 92, normalized size = 6.57

$$\frac{1}{8a^8b + 64a^7b^2x + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6 + 64ab^8x^7 + 8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)`

[Out] $-1/(8a^8b + 8b^9x^8 + 64a^7b^2x + 64ab^8x^7 + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6)$

sympy [B] time = 0.54, size = 97, normalized size = 6.93

$$\frac{1}{8a^8b + 64a^7b^2x + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6 + 64ab^8x^7 + 8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)`

[Out] $-1/(8a**8*b + 64a**7*b**2*x + 224a**6*b**3*x**2 + 448a**5*b**4*x**3 + 560a**4*b**5*x**4 + 448a**3*b**6*x**5 + 224a**2*b**7*x**6 + 64a*b**8*x**7 + 8*b**9*x**8)$

3.9 $\int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^3 dx$

Optimal. Leaf size=84

$$-\frac{3b(b^2 - 3ac)(b + cx)^7}{7c^4} + \frac{3b^2(b^2 - 3ac)^2(b + cx)^4}{4c^4} - \frac{b^3x(b^2 - 3ac)^3}{c^3} + \frac{(b + cx)^{10}}{10c^4}$$

[Out] $-b^3(-3ac+b^2)^3x/c^3+3/4*b^2*(-3ac+b^2)^2*(c*x+b)^4/c^4-3/7*b*(-3ac+b^2)*(c*x+b)^7/c^4+1/10*(c*x+b)^{10}/c^4$

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2060, 194}

$$-\frac{3b(b^2 - 3ac)(b + cx)^7}{7c^4} + \frac{3b^2(b^2 - 3ac)^2(b + cx)^4}{4c^4} - \frac{b^3x(b^2 - 3ac)^3}{c^3} + \frac{(b + cx)^{10}}{10c^4}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3, x]

[Out] $-((b^3*(b^2 - 3ac)^3x)/c^3) + (3*b^2*(b^2 - 3ac)^2*(b + cx)^4)/(4*c^4) - (3*b*(b^2 - 3ac)*(b + cx)^7)/(7*c^4) + (b + cx)^{10}/(10*c^4)$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2060

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]

Rubi steps

$$\begin{aligned} \int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^3 dx &= \frac{1}{27} \text{Subst} \left(\int \left(3b \left(3a - \frac{b^2}{c} \right) + 3c^2x^3 \right)^3 dx, x, \frac{b}{c} + x \right) \\ &= \frac{1}{27} \text{Subst} \left(\int \left(\frac{27(-b^3 + 3abc)^3}{c^3} + 81(b^3 - 3abc)^2 x^3 - 81bc^3(b^2 - 3ac)x^6 \right. \right. \\ &\quad \left. \left. - \frac{b^3(b^2 - 3ac)^3 x}{c^3} + \frac{3b^2(b^2 - 3ac)^2(b + cx)^4}{4c^4} - \frac{3b(b^2 - 3ac)(b + cx)^7}{7c^4} + \dots \right) dx, x, \frac{b}{c} + x \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 159, normalized size = 1.89

$$27a^3b^3x + \frac{81}{2}a^2b^4x^2 + \frac{27}{4}b^2x^4(a^2c^2 + 6ab^2c + b^4) + \frac{9}{7}bc^3x^7(ac + 9b^2) + 9b^2c^2x^6(ac + 2b^2) + \frac{27}{5}b^3cx^5(5ac + 3b^2) + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3, x]

[Out] $27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (27*b^2*(b^2 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a*c)*x^5)/5 + 9*b^2*c$

$$\frac{2}{7}(2b^2 + ac)x^6 + \frac{9}{7}b^2c^3(9b^2 + ac)x^7 + \frac{9}{2}b^2c^4x^8 + \frac{1}{2}b^2c^5x^9 + \frac{1}{10}c^6x^{10}$$

fricas [B] time = 0.52, size = 166, normalized size = 1.98

$$\frac{1}{10}x^{10}c^6 + x^9c^5b + \frac{9}{2}x^8c^4b^2 + \frac{81}{7}x^7c^3b^3 + \frac{9}{7}x^7c^4ba + 18x^6c^2b^4 + 9x^6c^3b^2a + \frac{81}{5}x^5cb^5 + 27x^5c^2b^3a + \frac{27}{4}x^4b^6 + \frac{81}{2}x^4cb^4a +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")

$$\begin{aligned} [Out] & 1/10*x^{10}*c^6 + x^9*c^5*b + 9/2*x^8*c^4*b^2 + 81/7*x^7*c^3*b^3 + 9/7*x^7*c^4*b*a \\ & + 18*x^6*c^2*b^4 + 9*x^6*c^3*b^2*a + 81/5*x^5*c*b^5 + 27*x^5*c^2*b^3*a \\ & + 27/4*x^4*b^6 + 81/2*x^4*c*b^4*a + 27/4*x^4*c^2*b^2*a^2 + 27*x^3*b^5*a + \\ & 27*x^3*c*b^3*a^2 + 81/2*x^2*b^4*a^2 + 27*x*b^3*a^3 \end{aligned}$$

giac [B] time = 0.27, size = 166, normalized size = 1.98

$$\frac{1}{10}c^6x^{10} + bc^5x^9 + \frac{9}{2}b^2c^4x^8 + \frac{81}{7}b^3c^3x^7 + \frac{9}{7}abc^4x^7 + 18b^4c^2x^6 + 9ab^2c^3x^6 + \frac{81}{5}b^5cx^5 + 27ab^3c^2x^5 + \frac{27}{4}b^6x^4 + \frac{81}{2}ab^4c^2x^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")

$$\begin{aligned} [Out] & 1/10*c^6*x^{10} + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 + 9/7*a*b*c^4*x^7 \\ & + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x^5 + 27*a*b^3*c^2*x^5 \\ & + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^2*b^2*c^2*x^4 + 27*a*b^5*x^3 + \\ & 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x \end{aligned}$$

maple [B] time = 0.00, size = 295, normalized size = 3.51

$$\frac{c^6x^{10}}{10} + bc^5x^9 + \frac{9b^2c^4x^8}{2} + \frac{81a^2b^4x^2}{2} + 27a^3b^3x + \frac{(3abc^4 + 63b^3c^3 + (6abc^2 + 18b^3c)c^2)x^7}{7} + \frac{(18ab^2c^3 + 45b^4c^2 + 27a^2b^3c^2)x^5}{7} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)

$$\begin{aligned} [Out] & 1/10*c^6*x^{10} + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 1/7*(3*a*b*c^4 + 63*b^3*c^3 + c^2*(6*a*b*c^2 \\ & + 18*b^3*c)) * x^7 + 1/6*(18*a*b^2*c^3 + 45*b^4*c^2 + 3*b*c*(6*a*b*c^2 + 18*b^3*c) \\ & + c^2*(18*a*b^2*c + 9*b^4)) * x^6 + 1/5*(63*a*b^3*c^2 + 3*b^2*(6*a*b*c^2 + 18*b^3*c) \\ & + 3*b*c*(18*a*b^2*c + 9*b^4)) * x^5 + 1/4*(3*a*b*(6*a*b*c^2 + 18*b^3*c) + 3*b^2*(18*a*b^2*c \\ & + 9*b^4) + 54*b^4*c*a + 9*c^2*a^2*b^2) * x^4 + 1/3*(3*a*b*(18*a*b^2*c + 9*b^4) + 54*b^5*a \\ & + 27*b^3*c*a^2) * x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x \end{aligned}$$

maxima [B] time = 0.68, size = 204, normalized size = 2.43

$$\frac{1}{10}c^6x^{10} + bc^5x^9 + \frac{27}{8}b^2c^4x^8 + \frac{27}{7}b^3c^3x^7 + \frac{27}{4}b^6x^4 + 27a^3b^3x + \frac{27}{4}(c^2x^4 + 4bcx^3 + 6b^2x^2)a^2b^2 + \frac{9}{10}(5c^2x^6 + 18b^4c^2x^6 + 27a^2b^3c^2x^5 + 210b^4x^3 + 21(4c^2x^5 + 15b^2c^2x^4)*b^2)*a*b + 9/56(7c^4x^8 + 48b^2c^3x^7 + 84b^2c^2x^6)*b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")

$$\begin{aligned} [Out] & 1/10*c^6*x^{10} + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 + 27/4*b^6*x^4 \\ & + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a^2*b^2 + 9/10*(5*c^2*x^6 \\ & + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x^7 + 70*b*c^3*x^6 + 126*b^2*c^2*x^5 \\ & + 210*b^4*x^3 + 21*(4*c^2*x^5 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^4*x^8 \\ & + 48*b^2*c^3*x^7 + 84*b^2*c^2*x^6)*b^2 \end{aligned}$$

mupad [B] time = 2.08, size = 149, normalized size = 1.77

$$x^4 \left(\frac{27a^2b^2c^2}{4} + \frac{81ab^4c}{2} + \frac{27b^6}{4} \right) + \frac{c^6x^{10}}{10} + 27a^3b^3x + bc^5x^9 + \frac{81a^2b^4x^2}{2} + \frac{9b^2c^4x^8}{2} + 9b^2c^2x^6(2b^2 + ac) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)`

[Out] $x^4*((27*b^6)/4 + (27*a^2*b^2*c^2)/4 + (81*a*b^4*c)/2) + (c^6*x^{10})/10 + 27*a^3*b^3*x + b*c^5*x^9 + (81*a^2*b^4*x^2)/2 + (9*b^2*c^4*x^8)/2 + 9*b^2*c^2*x^6*(a*c + 2*b^2) + 27*a*b^3*x^3*(a*c + b^2) + (27*b^3*c*x^5*(5*a*c + 3*b^2))/5 + (9*b*c^3*x^7*(a*c + 9*b^2))/7$

sympy [B] time = 0.10, size = 175, normalized size = 2.08

$$27a^3b^3x + \frac{81a^2b^4x^2}{2} + \frac{9b^2c^4x^8}{2} + bc^5x^9 + \frac{c^6x^{10}}{10} + x^7\left(\frac{9abc^4}{7} + \frac{81b^3c^3}{7}\right) + x^6(9ab^2c^3 + 18b^4c^2) + x^5\left(27ab^3c^2 + \frac{81b^5c}{5}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)`

[Out] $27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**5*x**9 + c**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x**6*(9*a*b**2*c**3 + 18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b**5*c/5) + x**4*(27*a**2*b**2*c**2/4 + 81*a*b**4*c/2 + 27*b**6/4) + x**3*(27*a**2*b**3*c + 27*a*b**5)$

3.10 $\int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2 dx$

Optimal. Leaf size=56

$$-\frac{b(b^2 - 3ac)(b + cx)^4}{2c^3} + \frac{b^2x(b^2 - 3ac)^2}{c^2} + \frac{(b + cx)^7}{7c^3}$$

[Out] $b^2*(-3*a*c+b^2)^2*x/c^2-1/2*b*(-3*a*c+b^2)*(c*x+b)^4/c^3+1/7*(c*x+b)^7/c^3$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2060, 194}

$$-\frac{b(b^2 - 3ac)(b + cx)^4}{2c^3} + \frac{b^2x(b^2 - 3ac)^2}{c^2} + \frac{(b + cx)^7}{7c^3}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]

[Out] $(b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^3) + (b + c*x)^7/(7*c^3)$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2060

Int(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]

Rubi steps

$$\begin{aligned} \int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2 dx &= \frac{1}{9} \text{Subst} \left(\int \left(3b \left(3a - \frac{b^2}{c} \right) + 3c^2x^3 \right)^2 dx, x, \frac{b}{c} + x \right) \\ &= \frac{1}{9} \text{Subst} \left(\int \left(\frac{9(-b^3 + 3abc)^2}{c^2} - 18bc(b^2 - 3ac)x^3 + 9c^4x^6 \right) dx, x, \frac{b}{c} + x \right) \\ &= \frac{b^2(b^2 - 3ac)^2 x}{c^2} - \frac{b(b^2 - 3ac)(b + cx)^4}{2c^3} + \frac{(b + cx)^7}{7c^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.46

$$9a^2b^2x + 9ab^3x^2 + \frac{3}{2}bcx^4(ac + 3b^2) + 3b^2x^3(2ac + b^2) + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]

[Out] $9a^2b^2*x + 9a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b^2 + a*c)*x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7$

fricas [A] time = 0.69, size = 83, normalized size = 1.48

$$\frac{1}{7}x^7c^4 + x^6c^3b + 3x^5c^2b^2 + \frac{9}{2}x^4cb^3 + \frac{3}{2}x^4c^2ba + 3x^3b^4 + 6x^3cb^2a + 9x^2b^3a + 9xb^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")

[Out] 1/7*x^7*c^4 + x^6*c^3*b + 3*x^5*c^2*b^2 + 9/2*x^4*c*b^3 + 3/2*x^4*c^2*b*a + 3*x^3*b^4 + 6*x^3*c*b^2*a + 9*x^2*b^3*a + 9*x*b^2*a^2

giac [A] time = 0.34, size = 83, normalized size = 1.48

$$\frac{1}{7}c^4x^7 + bc^3x^6 + 3b^2c^2x^5 + \frac{9}{2}b^3cx^4 + \frac{3}{2}abc^2x^4 + 3b^4x^3 + 6ab^2cx^3 + 9ab^3x^2 + 9a^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")

[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b*c^2*x^4 + 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x

maple [A] time = 0.00, size = 84, normalized size = 1.50

$$\frac{c^4x^7}{7} + bc^3x^6 + 3b^2c^2x^5 + 9ab^3x^2 + 9a^2b^2x + \frac{(6abc^2 + 18b^3c)x^4}{4} + \frac{(18ab^2c + 9b^4)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)

[Out] 1/7*c^4*x^7+b*c^3*x^6+3*b^2*c^2*x^5+1/4*(6*a*b*c^2+18*b^3*c)*x^4+1/3*(18*a*b^2*c+9*b^4)*x^3+9*a*b^3*x^2+9*a^2*b^2*x

maxima [A] time = 0.64, size = 93, normalized size = 1.66

$$\frac{1}{7}c^4x^7+bc^3x^6+\frac{9}{5}b^2c^2x^5+3b^4x^3+9a^2b^2x+\frac{3}{2}(c^2x^4+4bcx^3+6b^2x^2)ab+\frac{3}{10}(4c^2x^5+15bcx^4)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")

[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2*x + 3/2*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5 + 15*b*c*x^4)*b^2

mupad [B] time = 0.04, size = 79, normalized size = 1.41

$$x^3(3b^4 + 6ac b^2) + \frac{c^4x^7}{7} + 9a^2b^2x + 9ab^3x^2 + bc^3x^6 + 3b^2c^2x^5 + \frac{3bcx^4(3b^2 + ac)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)

[Out] x^3*(3*b^4 + 6*a*b^2*c) + (c^4*x^7)/7 + 9*a^2*b^2*x + 9*a*b^3*x^2 + b*c^3*x^6 + 3*b^2*c^2*x^5 + (3*b*c*x^4*(a*c + 3*b^2))/2

sympy [A] time = 0.08, size = 87, normalized size = 1.55

$$9a^2b^2x + 9ab^3x^2 + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7} + x^4\left(\frac{3abc^2}{2} + \frac{9b^3c}{2}\right) + x^3(6ab^2c + 3b^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 + c**4*x**7/  
7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c + 3*b**4)
```

3.11 $\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) dx$

Optimal. Leaf size=32

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]

[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4

Rubi steps

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) dx = 3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]

[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4

fricas [A] time = 0.60, size = 28, normalized size = 0.88

$$\frac{1}{4}x^4c^2 + x^3cb + \frac{3}{2}x^2b^2 + 3xba$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="fricas")

[Out] 1/4*x^4*c^2 + x^3*c*b + 3/2*x^2*b^2 + 3*x*b*a

giac [A] time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="giac")

[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x)`

[Out] `3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4`

maxima [A] time = 0.55, size = 28, normalized size = 0.88

$$\frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="maxima")`

[Out] `1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x`

mupad [B] time = 0.04, size = 28, normalized size = 0.88

$$\frac{3b^2x^2}{2} + bcx^3 + 3abx + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2,x)`

[Out] `(3*b^2*x^2)/2 + (c^2*x^4)/4 + 3*a*b*x + b*c*x^3`

sympy [A] time = 0.06, size = 31, normalized size = 0.97

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)`

[Out] `3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4`

$$3.12 \quad \int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3} dx$$

Optimal. Leaf size=188

$$\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}(b^2-3ac)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}(b^2-3ac)^{2/3}}-\frac{\tan^{-1}\left(\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}\right)}{\sqrt{3}b^{2/3}(b^2-3ac)^{1/3}}$$

[Out] $\frac{1}{3}\ln(b-b^{1/3}(-3ac+b^2)^{1/3}+cx)/b^{2/3}/(-3ac+b^2)^{2/3}-1/6\ln(b^{2/3}(-3ac+b^2)^{2/3}+b^{1/3}c(-3ac+b^2)^{1/3}+(b/c+x)+c^2(b/c+x)^2)/b^{2/3}/(-3ac+b^2)^{2/3}-1/3\arctan(1/3(b^{1/3}+2(c*x+b)/(-3ac+b^2)^{1/3}))/b^{1/3}*3^{1/2})/b^{2/3}/(-3ac+b^2)^{2/3}*3^{1/2}$

Rubi [A] time = 0.31, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2067, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}(b^2-3ac)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}(b^2-3ac)^{2/3}}-\frac{\tan^{-1}\left(\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}\right)}{\sqrt{3}b^{2/3}(b^2-3ac)^{1/3}}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1), x]

[Out] $-\frac{\text{ArcTan}\left[\frac{b^{1/3} + (2(b + cx))}{(b^2 - 3ac)^{1/3}}\right]}{\sqrt{3}b^{1/3}} \Big/ \left(\sqrt{3}b^{2/3}(b^2 - 3ac)^{2/3} \right) + \frac{\text{Log}\left[b - b^{1/3}(b^2 - 3ac)^{1/3} + cx\right]}{(3b^{2/3}(b^2 - 3ac)^{2/3})} - \frac{\text{Log}\left[b^{2/3}(b^2 - 3ac)^{2/3} + b^{1/3}c(b^2 - 3ac)^{1/3}(b/c + x) + c^2(b/c + x)^2\right]}{(6b^{2/3}(b^2 - 3ac)^{2/3})}$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*c/b}, Simplify[(a*c)/b^2], Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{3ab + 3b^2x + 3bcx^2 + c^2x^3} dx = \text{Subst} \left(\int \frac{1}{b \left(3a - \frac{b^2}{c} \right) + c^2x^3} dx, x, \frac{b}{c} + x \right)$$

$$= \frac{c^{2/3} \text{Subst} \left(\int \frac{1}{-\frac{\sqrt[3]{b} \sqrt[3]{b^2-3ac}}{\sqrt[3]{c}} + c^{2/3}x} dx, x, \frac{b}{c} + x \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} + \frac{c^{2/3} \text{Subst} \left(\int \frac{-\frac{2\sqrt[3]{b} \sqrt[3]{b^2}}{\sqrt[3]{c}}}{\frac{b^{2/3}(b^2-3ac)^{2/3}}{c^{2/3}} + \sqrt[3]{b} \sqrt[3]{b^2-3ac}x} dx, x, \frac{b}{c} + x \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}}$$

$$= \frac{\log \left(\sqrt[3]{b} (b^{2/3} - \sqrt[3]{b^2 - 3ac}) + cx \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} - \frac{\text{Subst} \left(\int \frac{\sqrt[3]{b} \sqrt[3]{c} \sqrt[3]{b^2-3ac} + 2c^{4/3}x}{\frac{b^{2/3}(b^2-3ac)^{2/3}}{c^{2/3}} + \sqrt[3]{b} \sqrt[3]{b^2-3ac}x} dx, x, \frac{b}{c} + x \right)}{6b^{2/3} (b^2 - 3ac)^{2/3}}$$

$$= \frac{\log \left(\sqrt[3]{b} (b^{2/3} - \sqrt[3]{b^2 - 3ac}) + cx \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} - \frac{\log \left(b^{2/3} (b^2 - 3ac)^{2/3} + \sqrt[3]{b} \sqrt[3]{b^2 - 3ac}x \right)}{6b^{2/3} (b^2 - 3ac)^{2/3}}$$

$$= -\frac{\tan^{-1} \left(\frac{1 + \frac{2(b+cx)}{\sqrt[3]{b} \sqrt[3]{b^2-3ac}}}{\sqrt{3}} \right)}{\sqrt{3} b^{2/3} (b^2 - 3ac)^{2/3}} + \frac{\log \left(\sqrt[3]{b} (b^{2/3} - \sqrt[3]{b^2 - 3ac}) + cx \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} - \frac{\log \left(b^{2/3} (b^2 - 3ac)^{2/3} + \sqrt[3]{b} \sqrt[3]{b^2 - 3ac}x \right)}{6b^{2/3} (b^2 - 3ac)^{2/3}}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.34

$$\frac{1}{3} \text{RootSum} \left[\#1^3 c^2 + 3\#1^2 bc + 3\#1 b^2 + 3ab \&, \frac{\log(x - \#1)}{\#1^2 c^2 + 2\#1 bc + b^2} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1), x]
```

```
[Out] RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 &, Log[x - #1]/(b^2 + 2*b
*c*#1 + c^2*#1^2) & ]/3
```

fricas [B] time = 0.86, size = 387, normalized size = 2.06

$$2\sqrt{3}\left(b^6 - 6ab^4c + 9a^2b^2c^2\right)^{\frac{1}{6}}(b^3 - 3abc) \arctan\left(\frac{2\sqrt{3}\left(b^6 - 6ab^4c + 9a^2b^2c^2\right)^{\frac{2}{3}}(cx+b) + \sqrt{3}\left(b^6 - 6ab^4c + 9a^2b^2c^2\right)^{\frac{1}{3}}(b^3 - 3abc)}{3\left(b^6 - 6ab^4c + 9a^2b^2c^2\right)^{\frac{5}{6}}}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*(b^3 - 3*a*b*c)*arc
tan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt
(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)))/(b^6 - 6*a*b^4
*c + 9*a^2*b^2*c^2)^(5/6)) + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b
^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b
^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b
^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)
*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2
*c^2)^(2/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)

giac [A] time = 0.36, size = 212, normalized size = 1.13

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}}}{cx + b + (-b^3 + 3abc)^{\frac{1}{3}}}\right) \log\left(4\left(\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}}\right)^2 + 4\left(cx + b + (-b^3 + 3abc)^{\frac{1}{3}}\right)\right)}{3\left(b^6 - 6ab^4c + 9a^2b^2c^2\right)^{\frac{1}{3}} \quad 6\left(b^6 - 6ab^4c + 9a^2b^2c^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="giac")

[Out] 1/3*sqrt(3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3
/3) - 1/6*log(4*(sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^
2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2
)^(1/3) + 1/3*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c
+ 9*a^2*b^2*c^2)^(1/3)

maple [C] time = 0.00, size = 57, normalized size = 0.30

$$\frac{\ln\left(-\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right) + x\right)}{3\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right)^2 c^2 + 6\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right)bc + 3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x)

[Out] 1/3*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(-_R+x), _R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c^2x^3 + 3bcx^2 + 3b^2x + 3ab} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="maxima")

[Out] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)

mupad [B] time = 0.49, size = 174, normalized size = 0.93

$$\frac{\ln\left(b + b^{1/3}(3ac - b^2)^{1/3} + cx\right)}{3b^{2/3}(3ac - b^2)^{2/3}} + \frac{\ln\left(3bc^3 + 3c^4x + \frac{3b^{1/3}c^3(-1 + \sqrt{3}i)(3ac - b^2)^{1/3}}{2}\right)(-1 + \sqrt{3}i)}{6b^{2/3}(3ac - b^2)^{2/3}} - \frac{\ln\left(3bc^3 + 3c^4x + \frac{3b^{1/3}c^3(1 + \sqrt{3}i)(3ac - b^2)^{1/3}}{2}\right)(1 + \sqrt{3}i)}{6b^{2/3}(3ac - b^2)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2), x)

[Out] log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x)/(3*b^(2/3)*(3*a*c - b^2)^(2/3)) + (log(3*b*c^3 + 3*c^4*x + (3*b^(1/3)*c^3*(3^(1/2)*1i - 1)*(3*a*c - b^2)^(1/3))/2)*(3^(1/2)*1i - 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3)) - (log(3*b*c^3 + 3*c^4*x - (3*b^(1/3)*c^3*(3^(1/2)*1i + 1)*(3*a*c - b^2)^(1/3))/2)*(3^(1/2)*1i + 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3))

sympy [A] time = 0.41, size = 53, normalized size = 0.28

$$\text{RootSum}\left(t^3(243a^2b^2c^2 - 162ab^4c + 27b^6) - 1, \left(t \mapsto t \log\left(x + \frac{9tabc - 3tb^3 + b}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b), x)

[Out] RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1, Lambda(_t, _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))

$$3.13 \quad \int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2} dx$$

Optimal. Leaf size=245

$$\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{c \log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

[Out] $-1/3*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)-2/9*c*\ln(b-b^{1/3})*(-3*a*c+b^2)^{(1/3)+c*x}/b^{(5/3)}/(-3*a*c+b^2)^{(5/3)}+1/9*c*\ln(b^{(2/3)}*(-3*a*c+b^2)^{(2/3)+b^{(1/3)}*c*(-3*a*c+b^2)^{(1/3)}*(b/c+x)+c^2*(b/c+x)^2)/b^{(5/3)}/(-3*a*c+b^2)^{(5/3)}+2/9*c*\arctan(1/3*(b^{(1/3)}+2*(c*x+b)/(-3*a*c+b^2)^{(1/3)})/b^{(1/3)}*3^{(1/2)})/b^{(5/3)}/(-3*a*c+b^2)^{(5/3)}*3^{(1/2)}$

Rubi [A] time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{c \log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2), x]

[Out] $-(c*(b/c + x))/(3*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)) + (2*c*\text{ArcTan}[(b^{(1/3)} + (2*(b + c*x))/(b^2 - 3*a*c)^{(1/3)})]/(\text{Sqrt}[3]*b^{(1/3)})))/(3*\text{Sqrt}[3]*b^{(5/3)}*(b^2 - 3*a*c)^{(5/3)}) - (2*c*\text{Log}[b - b^{(1/3)}*(b^2 - 3*a*c)^{(1/3)} + c*x])/((9*b^{(5/3)}*(b^2 - 3*a*c)^{(5/3)}) + (c*\text{Log}[b^{(2/3)}*(b^2 - 3*a*c)^{(2/3)} + b^{(1/3)}*c*(b^2 - 3*a*c)^{(1/3)}*(b/c + x) + c^2*(b/c + x)^2])/((9*b^{(5/3)}*(b^2 - 3*a*c)^{(5/3)}))$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^p_, x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} dx &= \text{Subst} \left(\int \frac{1}{\left(b \left(3a - \frac{b^2}{c}\right) + c^2x^3\right)^2} dx, x, \frac{b}{c} + x \right) \\
&= \frac{c \left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{(2c) \text{Subst} \left(\int \frac{1}{b \left(3a - \frac{b^2}{c}\right) + c^2x^3} \right)}{3b(b^2 - 3ac)} \\
&= \frac{c \left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{(2c^{5/3}) \text{Subst} \left(\int \frac{1}{-\frac{\sqrt[3]{b} \sqrt[3]{b^2 - 3ac}}{\sqrt[3]{c}}} \right)}{9b^{5/3}(b^2 - 3ac)} \\
&= \frac{c \left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{2c \log \left(\sqrt[3]{b} \left(b^{2/3} - \sqrt[3]{b^2 - 3ac}\right)\right)}{9b^{5/3}(b^2 - 3ac)^{5/3}} \\
&= \frac{c \left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{2c \log \left(\sqrt[3]{b} \left(b^{2/3} - \sqrt[3]{b^2 - 3ac}\right)\right)}{9b^{5/3}(b^2 - 3ac)^{5/3}} \\
&= \frac{c \left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} + \frac{2c \tan^{-1} \left(\frac{1 + \frac{2(b+cx)}{\sqrt[3]{b} \sqrt[3]{b^2 - 3ac}}}{\sqrt{3}}\right)}{3\sqrt{3} b^{5/3} (b^2 - 3ac)^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 112, normalized size = 0.46

$$\frac{2c \text{RootSum} \left[\#1^3 c^2 + 3\#1^2 bc + 3\#1 b^2 + 3ab\#, \frac{\log(x-\#1)}{\#1^2 c^2 + 2\#1 bc + b^2} \& \right] + \frac{3(b+cx)}{3ab+x(3b^2+3bcx+c^2x^2)}}{9(b^3 - 3abc)}$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2), x]

[Out] -1/9*((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) &])/(b^3 - 3*a*b*c)

fricas [B] time = 0.82, size = 704, normalized size = 2.87

$$3b^7 - 18ab^5c + 27a^2b^3c^2 - 2\sqrt{3}(b^6 - 6ab^4c + 9a^2b^2c^2)^{1/6}(3ab^4c - 9a^2b^2c^2 + (b^3c^3 - 3abc^4)x^3 + 3(b^4c^2 - 3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")

[Out] -1/9*(3*b^7 - 18*a*b^5*c + 27*a^2*b^3*c^2 - 2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3 +

$$3*(b^4*c^2 - 3*a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x)*\arctan(1/3*(2*\sqrt[3]{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{2/3}*(c*x + b) + \sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{1/3}*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{5/6}) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{2/3}*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a*b*c)*\log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{2/3}*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{1/3}*(b^3 - 3*a*b*c)) + 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{2/3}*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a*b*c)*\log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{2/3}) + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3)*x)/(3*a*b^{10} - 27*a^2*b^8*c + 81*a^3*b^6*c^2 - 81*a^4*b^4*c^3 + (b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 27*a^3*b^3*c^5)*x^3 + 3*(b^{10}*c - 9*a*b^8*c^2 + 27*a^2*b^6*c^3 - 27*a^3*b^4*c^4)*x^2 + 3*(b^{11} - 9*a*b^9*c + 27*a^2*b^7*c^2 - 27*a^3*b^5*c^3)*x)$$

giac [A] time = 0.36, size = 289, normalized size = 1.18

$$\frac{2\sqrt{3}\left(\frac{c^3}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}}{cx+b+(-b^3+3abc)^{\frac{1}{3}}}\right)-\left(\frac{c^3}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}\right)\right)}{9(b^3-3abc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")

[Out] -1/9*(2*sqrt(3)*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3))) - (c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2) + 2*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^3 - 3*a*b*c) - 1/3*(c*x + b)/((c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)*(b^3 - 3*a*b*c))

maple [C] time = 0.02, size = 136, normalized size = 0.56

$$\frac{2c \ln\left(-\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right) + x\right)}{9(3ac - b^2)b\left(\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right)^2 c^2 + 2\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right)bc + b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)

[Out] (1/3*c/b/(3*a*c-b^2)*x+1/3/(3*a*c-b^2))/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+2/9*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(-_R+x), _R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{3}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}}{cx+b+(-b^3+3abc)^{\frac{1}{3}}}\right)}{(b^6-6ab^4c+9a^2b^2c^2)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}\right)^2+4\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)^2\right)}{(b^6-6ab^4c+9a^2b^2c^2)^{\frac{1}{3}}}\right)+\frac{2\log\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)}{(b^6-6ab^4c+9a^2b^2c^2)^{\frac{1}{3}}}}{3(b^3-3abc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")

[Out] $-2/3*c*\text{integrate}(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x), x)/(b^3 - 3*a*b*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x)$

mupad [B] time = 2.65, size = 247, normalized size = 1.01

$$\frac{\frac{1}{3(3ac-b^2)} + \frac{cx}{3b(3ac-b^2)}}{3b^2x + 3bcx^2 + 3ab + c^2x^3} + \frac{2c \ln\left(b + b^{1/3}(3ac - b^2)^{1/3} + cx\right)}{9b^{5/3}(3ac - b^2)^{5/3}} - \frac{\ln\left(2b - b^{1/3}(3ac - b^2)^{1/3} + 2cx - \sqrt{3}b^{1/3}\right)}{9b^{5/3}(3ac - b^2)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2, x)$

[Out] $(1/(3*(3*a*c - b^2)) + (c*x)/(3*b*(3*a*c - b^2)))/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2) + (2*c*\log(b + b^{1/3}*(3*a*c - b^2)^{1/3} + c*x))/(9*b^{5/3}*(3*a*c - b^2)^{5/3}) - (\log(2*b - b^{1/3}*(3*a*c - b^2)^{1/3} + 2*c*x - 3^{1/2}*b^{1/3}*(3*a*c - b^2)^{1/3}*1i)*(c + 3^{1/2}*c*1i))/(9*b^{5/3}*(3*a*c - b^2)^{5/3}) - (\log(2*b - b^{1/3}*(3*a*c - b^2)^{1/3} + 2*c*x + 3^{1/2}*b^{1/3}*(3*a*c - b^2)^{1/3}*1i)*(c - 3^{1/2}*c*1i))/(9*b^{5/3}*(3*a*c - b^2)^{5/3})$

sympy [A] time = 1.22, size = 192, normalized size = 0.78

$$\frac{b + cx}{27a^2b^2c - 9ab^4 + x^3(9abc^3 - 3b^3c^2) + x^2(27ab^2c^2 - 9b^4c) + x(27ab^3c - 9b^5)} + \text{RootSum}\left(t^3(177147a^5b^5c^5 - 295245a^4b^7c^4 + 196830a^3b^9c^3 - 65610a^2b^{11}c^2 + 10935ab^{13}c - 729b^{15}) - 8c^3, \text{Lambda}(t, t*\log(x + (81*t*a^2*b^2*c^2 - 54*t*a*b^4*c + 9*t*b^6 + 2*b*c)/(2*c^2))))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2, x)$

[Out] $(b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*c**2) + x**2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**5)) + \text{RootSum}(_t**3*(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**4 + 196830*a**3*b**9*c**3 - 65610*a**2*b**11*c**2 + 10935*a*b**13*c - 729*b**15) - 8*c**3, \text{Lambda}(_t, _t*\log(x + (81*_t*a**2*b**2*c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2*c**2))))$

$$3.14 \quad \int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3} dx$$

Optimal. Leaf size=305

$$\frac{5c^2 \left(\frac{b}{c} + x\right)}{18b^2 (b^2 - 3ac)^2 (3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{c \left(\frac{b}{c} + x\right)}{6b (b^2 - 3ac) (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c^2 \log\left(-\sqrt[3]{b} \sqrt[3]{b^2}\right)}{27b^{8/3} (b^2 - 3ac)^2}$$

[Out] $-1/6*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/18*c^2*(b/c+x)/b^2/(-3*a*c+b^2)^2/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+5/27*c^2*\ln(b-b^{1/3}*(-3*a*c+b^2)^{1/3}+c*x)/b^{8/3}/(-3*a*c+b^2)^{8/3}-5/54*c^2*\ln(b^{2/3}*(-3*a*c+b^2)^{2/3}+b^{1/3}*c*(-3*a*c+b^2)^{1/3}*(b/c+x)+c^2*(b/c+x)^2)/b^{8/3}/(-3*a*c+b^2)^{8/3}-5/27*c^2*\arctan(1/3*(b^{1/3}+2*(c*x+b)/(-3*a*c+b^2)^{1/3}))/b^{8/3}/(-3*a*c+b^2)^{8/3}*3^{1/2})/b^{8/3}/(-3*a*c+b^2)^{8/3}*3^{1/2})$

Rubi [A] time = 0.30, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5c^2 \left(\frac{b}{c} + x\right)}{18b^2 (b^2 - 3ac)^2 (3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{c \left(\frac{b}{c} + x\right)}{6b (b^2 - 3ac) (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c^2 \log\left(-\sqrt[3]{b} \sqrt[3]{b^2}\right)}{27b^{8/3} (b^2 - 3ac)^2}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3), x]

[Out] $-(c*(b/c + x))/(6*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)) - (5*c^2*ArcTan[(b^{1/3} + 2*(b + c*x))/(b^2 - 3*a*c)^{1/3}]/(9*sqrt[3]*b^{1/3}))/((9*sqrt[3]*b^{8/3}*(b^2 - 3*a*c)^{8/3}) + (5*c^2*Log[b - b^{1/3}*(b^2 - 3*a*c)^{1/3} + c*x]/(27*b^{8/3}*(b^2 - 3*a*c)^{8/3}) - (5*c^2*Log[b^{2/3}*(b^2 - 3*a*c)^{2/3} + b^{1/3}*c*(b^2 - 3*a*c)^{1/3}*(b/c + x) + c^2*(b/c + x)^2]/(54*b^{8/3}*(b^2 - 3*a*c)^{8/3}))$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^3} dx &= \text{Subst} \left(\int \frac{1}{\left(b\left(3a - \frac{b^2}{c}\right) + c^2x^3\right)^3} dx, x, \frac{b}{c} + x \right) \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} - \frac{(5c) \text{Subst} \left(\int \frac{1}{\left(b\left(3a - \frac{b^2}{c}\right) + c^2x^3\right)^3} dx, x, \frac{b}{c} + x \right)}{6b(b^2 - 3ac)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b - \frac{b^2}{c})}{18b^2(b^2 - 3ac)^2(3ab + 3b^2x + 3bcx^2 + c^2x^3)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b - \frac{b^2}{c})}{18b^2(b^2 - 3ac)^2(3ab + 3b^2x + 3bcx^2 + c^2x^3)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b - \frac{b^2}{c})}{18b^2(b^2 - 3ac)^2(3ab + 3b^2x + 3bcx^2 + c^2x^3)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b - \frac{b^2}{c})}{18b^2(b^2 - 3ac)^2(3ab + 3b^2x + 3bcx^2 + c^2x^3)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b - \frac{b^2}{c})}{18b^2(b^2 - 3ac)^2(3ab + 3b^2x + 3bcx^2 + c^2x^3)}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 149, normalized size = 0.49

$$\frac{10c^2 \text{RootSum} \left[\#1^3c^2 + 3\#1^2bc + 3\#1b^2 + 3ab\&, \frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2} \& \right] - \frac{3(b+cx)(-3bc(8a+5cx^2)+3b^3-15b^2cx-5c^3x^3)}{(3ab+cx(3b^2+3bcx+c^2x^2))^2}}{54(b^3-3abc)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3), x]

[Out] ((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c*x^2)))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) &])/(54*(b^3 - 3*a*b*c)^2)

fricas [B] time = 0.78, size = 1268, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")

[Out]
$$-1/54*(9*b^{10} - 126*a*b^8*c + 513*a^2*b^6*c^2 - 648*a^3*b^4*c^3 - 15*(b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^4 - 60*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^3 - 90*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^2 + 10*\sqrt{3}*(9*a^2*b^5*c^2 - 27*a^3*b^3*c^3 + (b^3*c^6 - 3*a*b*c^7)*x^6 + 6*(b^4*c^5 - 3*a*b^2*c^6)*x^5 + 15*(b^5*c^4 - 3*a*b^3*c^5)*x^4 + 6*(3*b^6*c^3 - 8*a*b^4*c^4 - 3*a^2*b^2*c^5)*x^3 + 9*(b^7*c^2 - a*b^5*c^3 - 6*a^2*b^3*c^4)*x^2 + 18*(a*b^6*c^2 - 3*a^2*b^4*c^3)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/6)}*\arctan(1/3*(2*\sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c*x + b) + \sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)}*(b^3 - 3*a*b*c)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(5/6)}) + 5*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*\log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)}*(b^3 - 3*a*b*c)) - 10*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*\log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}) - 36*(b^9*c - 4*a*b^7*c^2 - 3*a^2*b^5*c^3 + 18*a^3*b^3*c^4)*x)/(9*a^2*b^{14} - 108*a^3*b^{12}*c + 486*a^4*b^{10}*c^2 - 972*a^5*b^8*c^3 + 729*a^6*b^6*c^4 + (b^{12}*c^4 - 12*a*b^{10}*c^5 + 54*a^2*b^8*c^6 - 108*a^3*b^6*c^7 + 81*a^4*b^4*c^8)*x^6 + 6*(b^{13}*c^3 - 12*a*b^{11}*c^4 + 54*a^2*b^9*c^5 - 108*a^3*b^7*c^6 + 81*a^4*b^5*c^7)*x^5 + 15*(b^{14}*c^2 - 12*a*b^{12}*c^3 + 54*a^2*b^{10}*c^4 - 108*a^3*b^8*c^5 + 81*a^4*b^6*c^6)*x^4 + 6*(3*b^{15}*c - 35*a*b^{13}*c^2 + 150*a^2*b^{11}*c^3 - 270*a^3*b^9*c^4 + 135*a^4*b^7*c^5 + 81*a^5*b^5*c^6)*x^3 + 9*(b^{16} - 10*a*b^{14}*c + 30*a^2*b^{12}*c^2 - 135*a^4*b^8*c^4 + 162*a^5*b^6*c^5)*x^2 + 18*(a*b^{15} - 12*a^2*b^{13}*c + 54*a^3*b^{11}*c^2 - 108*a^4*b^9*c^3 + 81*a^5*b^7*c^4)*x)$$

giac [A] time = 0.40, size = 366, normalized size = 1.20

$$\frac{5 \left(2 \sqrt{3} \left(\frac{c^6}{b^6 - 6ab^4c + 9a^2b^2c^2} \right)^{\frac{1}{3}} \arctan \left(\frac{\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}}}{cx + b + (-b^3 + 3abc)^{\frac{1}{3}}} \right) - \left(\frac{c^6}{b^6 - 6ab^4c + 9a^2b^2c^2} \right)^{\frac{1}{3}} \log \left(4 \left(\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}} \right) \right)}{54(b^6 - 6ab^4c + 9a^2b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")

[Out]
$$5/54*(2*\sqrt{3}*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^{\frac{1}{3}}*\arctan((\sqrt{3}*(c*x + \sqrt{3}b - \sqrt{3}*(-b^3 + 3*a*b*c)^{\frac{1}{3}})/((c*x + b + (-b^3 + 3*a*b*c)^{\frac{1}{3}})) - (c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^{\frac{1}{3}}*\log(4*(\sqrt{3}*(c*x + \sqrt{3}b - \sqrt{3}*(-b^3 + 3*a*b*c)^{\frac{1}{3}}))^2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^{\frac{1}{3}})^2) + 2*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^{\frac{1}{3}}*\log(\text{abs}(c*x + b + (-b^3 + 3*a*b*c)^{\frac{1}{3}})))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 + 12*b^3*c*x + 24*a*b*c^2*x - 3*b^4 + 24*a*b^2*c)/((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)*(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^2)$$

maple [C] time = 0.02, size = 276, normalized size = 0.90

$$\frac{5c^2 \ln \left(-\text{RootOf} \left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab \right) + x \right)}{27(9a^2c^2 - 6ab^2c + b^4)b^2 \left(\text{RootOf} \left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab \right)^2 c^2 + 2 \text{RootOf} \left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab \right) + 3b^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)

[Out] $(5/18*c^4/b^2/(9*a^2*c^2-6*a*b^2*c+b^4))*x^4+10/9/b*c^3/(9*a^2*c^2-6*a*b^2*c+b^4)*x^3+5/3*c^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+2/3/b*(2*a*c+b^2)*c/(9*a^2*c^2-6*a*b^2*c+b^4)*x+1/6*(8*a*c-b^2)/(9*a^2*c^2-6*a*b^2*c+b^4))/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/27*c^2/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*\text{sum}(1/(_R^2*c^2+2*_R*b*c+b^2)*\ln(-_R+x), _R=\text{RootOf}(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{5}{6} \left(\frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3+3abc)^{\frac{1}{3}}}{cx+b+(-b^3+3abc)^{\frac{1}{3}}}\right)}{(b^6-6ab^4c+9a^2b^2c^2)^{\frac{1}{3}}} - \frac{\log\left(4\left(\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3+3abc)^{\frac{1}{3}}\right)^2 + 4\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)^2\right)}{(b^6-6ab^4c+9a^2b^2c^2)^{\frac{1}{3}}} + \frac{2 \log\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)}{(b^6-6ab^4c+9a^2b^2c^2)^{\frac{1}{3}}}}{9(b^6-6ab^4c+9a^2b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")`

[Out] $5/9*c^2*\text{integrate}(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 - 3*b^4 + 24*a*b^2*c + 12*(b^3*c + 2*a*b*c^2)*x)/(9*a^2*b^8 - 54*a^3*b^6*c + 81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^4 + 6*(3*b^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 + 9*(b^10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9 - 6*a^2*b^7*c + 9*a^3*b^5*c^2)*x)$

mupad [B] time = 2.98, size = 483, normalized size = 1.58

$$\frac{\frac{8ac-b^2}{6(9a^2c^2-6ab^2c+b^4)} + \frac{5c^2x^2}{3(9a^2c^2-6ab^2c+b^4)} + \frac{10c^3x^3}{9b(9a^2c^2-6ab^2c+b^4)} + \frac{5c^4x^4}{18b^2(9a^2c^2-6ab^2c+b^4)} + \frac{2cx(b^2+2ac)}{3b(9a^2c^2-6ab^2c+b^4)} + \frac{5c^2 \ln}{x^2(9b^4+18acb^2) + 9a^2b^2 + c^4x^6 + x^3(18b^3c + 6abc^2) + 6b^3c^3x^5 + 15b^2c^2x^4 + 18ab^3x} + \frac{5c^2 \ln}{x^2(9b^4+18acb^2) + 9a^2b^2 + c^4x^6 + x^3(18b^3c + 6abc^2) + 6b^3c^3x^5 + 15b^2c^2x^4 + 18ab^3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)`

[Out] $((8*a*c - b^2)/(6*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (5*c^2*x^2)/(3*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (10*c^3*x^3)/(9*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (5*c^4*x^4)/(18*b^2*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/(3*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)))/(x^2*(9*b^4 + 18*a*b^2*c) + 9*a^2*b^2 + c^4*x^6 + x^3*(18*b^3*c + 6*a*b*c^2) + 6*b*c^3*x^5 + 15*b^2*c^2*x^4 + 18*a*b^3*x) + (5*c^2*\log(b*(3*a*c - b^2)^(8/3) - b^(19/3) + c*x*(3*a*c - b^2)^(8/3) + 27*a^3*b^(1/3)*c^3 - 27*a^2*b^(7/3)*c^2 + 9*a*b^(13/3)*c))/(27*b^(8/3)*(3*a*c - b^2)^(8/3)) - (5*c^2*\log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x - 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2))/(27*b^(8/3)*(3*a*c - b^2)^(8/3)) + (5*c^2*\log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x + 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2))/(27*b^(8/3)*(3*a*c - b^2)^(8/3))$

sympy [A] time = 2.54, size = 474, normalized size = 1.55

$$1458a^4b^4c^2 - 972a^3b^6c + 162a^2b^8 + x^6(162a^2b^2c^6 - 108ab^4c^5 + 18b^6c^4) + x^5(972a^2b^3c^5 - 648ab^5c^4 + 108b^7c^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)`

```
[Out] (24*a*b**2*c - 3*b**4 + 30*b**2*c**2*x**2 + 20*b*c**3*x**3 + 5*c**4*x**4 +
x*(24*a*b*c**2 + 12*b**3*c))/(1458*a**4*b**4*c**2 - 972*a**3*b**6*c + 162*a
**2*b**8 + x**6*(162*a**2*b**2*c**6 - 108*a*b**4*c**5 + 18*b**6*c**4) + x**
5*(972*a**2*b**3*c**5 - 648*a*b**5*c**4 + 108*b**7*c**3) + x**4*(2430*a**2*
b**4*c**4 - 1620*a*b**6*c**3 + 270*b**8*c**2) + x**3*(972*a**3*b**3*c**4 +
2268*a**2*b**5*c**3 - 1836*a*b**7*c**2 + 324*b**9*c) + x**2*(2916*a**3*b**4
*c**3 - 486*a**2*b**6*c**2 - 648*a*b**8*c + 162*b**10) + x*(2916*a**3*b**5*
c**2 - 1944*a**2*b**7*c + 324*a*b**9)) + RootSum(_t**3*(129140163*a**8*b**8
*c**8 - 344373768*a**7*b**10*c**7 + 401769396*a**6*b**12*c**6 - 267846264*a
**5*b**14*c**5 + 111602610*a**4*b**16*c**4 - 29760696*a**3*b**18*c**3 + 496
0116*a**2*b**20*c**2 - 472392*a*b**22*c + 19683*b**24) - 125*c**6, Lambda(_
t, _t*log(x + (729*_t*a**3*b**3*c**3 - 729*_t*a**2*b**5*c**2 + 243*_t*a*b**
7*c - 27*_t*b**9 + 5*b*c**2)/(5*c**3))))
```

3.15 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) dx$

Optimal. Leaf size=361

$$\frac{3df(a+bx)^8(5a^2d^2f^2 - 5abdf(cf+de) + b^2(c^2f^2 + 3cdef + d^2e^2))}{8b^7} + \frac{(a+bx)^7(-2adf + bcf + bde)(10a^2d^2f^2 - 5a^2d^2f^2 - 5abdf(cf+de) + b^2(c^2f^2 + 3cdef + d^2e^2))}{8b^7}$$

[Out] $\frac{1}{4}(-a*d+b*c)^3(-a*f+b*e)^3(b*x+a)^4/b^7 + \frac{3}{5}(-a*d+b*c)^2(-a*f+b*e)^2(-2*a*d*f+b*c*f+b*d*e)(b*x+a)^5/b^7 + \frac{1}{2}(-a*d+b*c)(-a*f+b*e)(5*a^2*d^2*f^2 - 5*a*b*d*f*(c*f+d*e) + b^2*(c^2*f^2 + 3*c*d*e*f + d^2*e^2))(b*x+a)^6/b^7 + \frac{1}{7}(-2*a*d*f+b*c*f+b*d*e)(10*a^2*d^2*f^2 - 10*a*b*d*f*(c*f+d*e) + b^2*(c^2*f^2 + 8*c*d*e*f + d^2*e^2))(b*x+a)^7/b^7 + \frac{3}{8}d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(c*f+d*e) + b^2*(c^2*f^2 + 3*c*d*e*f + d^2*e^2))(b*x+a)^8/b^7 + \frac{1}{3}d^2*f^2*(-2*a*d*f+b*c*f+b*d*e)(b*x+a)^9/b^7 + \frac{1}{10}d^3*f^3*(b*x+a)^10/b^7$

Rubi [A] time = 0.66, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2059, 88}

$$\frac{3df(a+bx)^8(5a^2d^2f^2 - 5abdf(cf+de) + b^2(c^2f^2 + 3cdef + d^2e^2))}{8b^7} + \frac{(a+bx)^7(-2adf + bcf + bde)(10a^2d^2f^2 - 5a^2d^2f^2 - 5abdf(cf+de) + b^2(c^2f^2 + 3cdef + d^2e^2))}{8b^7}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3, x]

[Out] $((b*c - a*d)^3(b*e - a*f)^3(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e - a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*e - a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)$

Rule 88

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 2059

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonFreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) dx &= \int (a + bx)^3(c + dx)^3(e + fx)^3 dx \\ &= \int \left(\frac{(bc - ad)^3(be - af)^3(a + bx)^3}{b^6} + \frac{3(bc - ad)^3(be - af)^3(a + bx)^4}{4b^7} + \frac{3(bc - ad)^3(be - af)^3(a + bx)^5}{5b^7} + \frac{3(bc - ad)^3(be - af)^3(a + bx)^6}{6b^7} + \frac{3(bc - ad)^3(be - af)^3(a + bx)^7}{7b^7} + \frac{3(bc - ad)^3(be - af)^3(a + bx)^8}{8b^7} + \frac{3(bc - ad)^3(be - af)^3(a + bx)^9}{9b^7} + \frac{3(bc - ad)^3(be - af)^3(a + bx)^{10}}{10b^7} \right) dx \end{aligned}$$

Mathematica [A] time = 0.22, size = 653, normalized size = 1.81

$$a^3c^3e^3x + \frac{3}{8}bdfx^8(a^2d^2f^2 + 3abdf(cf + de) + b^2(c^2f^2 + 3cdef + d^2e^2)) + acex^3(a^2(c^2f^2 + 3cdef + d^2e^2) + 3abc$$

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3,x]

[Out] a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a*c*e*(b^2*c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^3 + ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^4)/4 + (3*(b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c*f) + b^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^6)/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^7)/7 + (3*b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f + a*d*f)*x^9)/3 + (b^3*d^3*f^3*x^10)/10

fricas [B] time = 0.67, size = 987, normalized size = 2.73

$$\frac{1}{10}x^{10}f^3d^3b^3 + \frac{1}{3}x^9f^2ed^3b^3 + \frac{1}{3}x^9f^3d^2cb^3 + \frac{1}{3}x^9f^3d^3b^2a + \frac{3}{8}x^8fe^2d^3b^3 + \frac{9}{8}x^8f^2ed^2cb^3 + \frac{3}{8}x^8f^3dc^2b^3 + \frac{9}{8}x^8f^2ed^3b^2a + \frac{9}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x, algorithm="fricas")

[Out] 1/10*x^10*f^3*d^3*b^3 + 1/3*x^9*f^2*e*d^3*b^3 + 1/3*x^9*f^3*d^2*c*b^3 + 1/3*x^9*f^3*d^3*b^2*a + 3/8*x^8*f*e^2*d^3*b^3 + 9/8*x^8*f^2*e*d^2*c*b^3 + 3/8*x^8*f^3*d*c^2*b^3 + 9/8*x^8*f^2*e*d^3*b^2*a + 9/8*x^8*f^3*d^2*c*b^2*a + 3/8*x^8*f^3*d^3*b*a^2 + 1/7*x^7*e^3*d^3*b^3 + 9/7*x^7*f*e^2*d^2*c*b^3 + 9/7*x^7*f^2*e*d*c^2*b^3 + 1/7*x^7*f^3*c^3*b^3 + 9/7*x^7*f*e^2*d^3*b^2*a + 27/7*x^7*f^2*e*d^2*c*b^2*a + 9/7*x^7*f^3*d*c^2*b^2*a + 9/7*x^7*f^2*e*d^3*b*a^2 + 9/7*x^7*f^3*d^2*c*b*a^2 + 1/7*x^7*f^3*d^3*a^3 + 1/2*x^6*e^3*d^2*c*b^3 + 3/2*x^6*f*e^2*d*c^2*b^3 + 1/2*x^6*f^2*e*c^3*b^3 + 1/2*x^6*e^3*d^3*b^2*a + 9/2*x^6*f*e^2*d^2*c*b^2*a + 9/2*x^6*f^2*e*d*c^2*b^2*a + 1/2*x^6*f^3*c^3*b^2*a + 3/2*x^6*f*e^2*d^3*b*a^2 + 9/2*x^6*f^2*e*d^2*c*b*a^2 + 3/2*x^6*f^3*d*c^2*b*a^2 + 1/2*x^6*f^2*e*d^3*a^3 + 1/2*x^6*f^3*d^2*c*a^3 + 3/5*x^5*e^3*d*c^2*b^3 + 3/5*x^5*f*e^2*c^3*b^3 + 9/5*x^5*e^3*d^2*c*b^2*a + 27/5*x^5*f*e^2*d*c^2*b^2*a + 9/5*x^5*f^2*e*c^3*b^2*a + 3/5*x^5*e^3*d^3*b*a^2 + 27/5*x^5*f*e^2*d^2*c*b*a^2 + 27/5*x^5*f^2*e*d*c^2*b*a^2 + 3/5*x^5*f^3*c^3*b*a^2 + 3/5*x^5*f*e^2*d^2*d^3*a^3 + 9/5*x^5*f^2*e*d^2*c*a^3 + 3/5*x^5*f^3*d*c^2*a^3 + 1/4*x^4*e^3*c^3*b^3 + 9/4*x^4*e^3*d*c^2*b^2*a + 9/4*x^4*f*e^2*c^3*b^2*a + 9/4*x^4*e^3*d^2*c*b*a^2 + 27/4*x^4*f*e^2*d*c^2*b*a^2 + 9/4*x^4*f^2*e*c^3*b*a^2 + 1/4*x^4*e^3*d^3*a^3 + 9/4*x^4*f*e^2*d^2*c*a^3 + 9/4*x^4*f^2*e*d*c^2*a^3 + 1/4*x^4*f^3*c^3*a^3 + x^3*e^3*c^3*b^2*a + 3*x^3*e^3*d*c^2*b*a^2 + 3*x^3*f*e^2*c^3*b*a^2 + x^3*e^3*d^2*c*a^3 + 3*x^3*f*e^2*d*c^2*a^3 + x^3*f^2*e*c^3*a^3 + 3/2*x^2*e^3*c^3*b*a^2 + 3/2*x^2*e^3*d*c^2*a^3 + 3/2*x^2*f*e^2*c^3*a^3 + x*e^3*c^3*a^3

giac [B] time = 0.27, size = 971, normalized size = 2.69

$$\frac{1}{10}b^3d^3f^3x^{10} + \frac{1}{3}b^3cd^2f^3x^9 + \frac{1}{3}ab^2d^3f^3x^9 + \frac{1}{3}b^3d^3f^2x^9e + \frac{3}{8}b^3c^2df^3x^8 + \frac{9}{8}ab^2cd^2f^3x^8 + \frac{3}{8}a^2bd^3f^3x^8 + \frac{9}{8}b^3cd^2f^2x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3, x, algorithm="giac")

[Out] $\frac{1}{10}b^3d^3f^3x^{10} + \frac{1}{3}b^3c^2d^2f^3x^9 + \frac{1}{3}ab^2d^3f^3x^9 + \frac{1}{3}b^3d^3f^2x^9e + \frac{3}{8}b^3c^2d^2f^3x^8 + \frac{9}{8}ab^2c^2d^2f^3x^8 + \frac{3}{8}a^2b^2d^3f^3x^8 + \frac{9}{8}b^3c^2d^2f^2x^8e + \frac{9}{8}ab^2d^3f^2x^8e + \frac{1}{7}b^3c^3f^3x^7 + \frac{9}{7}ab^2c^2d^2f^3x^7 + \frac{9}{7}a^2b^2c^2d^2f^3x^7 + \frac{1}{7}a^3d^3f^3x^7 + \frac{3}{8}b^3d^3f^2x^7e^2 + \frac{9}{7}b^3c^2d^2f^2x^7e + \frac{27}{7}ab^2c^2d^2f^2x^7e + \frac{9}{7}a^2b^2d^3f^2x^7e + \frac{1}{2}ab^2c^3f^3x^6 + \frac{3}{2}a^2b^2c^2d^2f^3x^6 + \frac{1}{2}a^3c^2d^2f^3x^6 + \frac{9}{7}b^3c^2d^2f^2x^7e^2 + \frac{9}{7}ab^2d^3f^2x^7e^2 + \frac{1}{2}b^3c^3f^2x^6e + \frac{9}{2}ab^2c^2d^2f^2x^6e + \frac{9}{2}a^2b^2c^2d^2f^2x^6e + \frac{1}{2}a^3d^3f^2x^6e + \frac{3}{5}a^2b^2c^3f^3x^5 + \frac{3}{5}a^3c^2d^2f^3x^5 + \frac{1}{7}b^3d^3x^7e^3 + \frac{3}{2}b^3c^2d^2f^2x^6e^2 + \frac{9}{2}ab^2c^2d^2f^2x^6e^2 + \frac{3}{2}a^2b^2d^3f^2x^6e^2 + \frac{9}{5}ab^2c^3f^2x^5e + \frac{27}{5}a^2b^2c^2d^2f^2x^5e + \frac{9}{5}a^3c^2d^2f^2x^5e + \frac{1}{4}a^3c^3f^3x^4 + \frac{1}{2}b^3c^2d^2x^6e^3 + \frac{1}{2}ab^2d^3x^6e^3 + \frac{3}{5}b^3c^3f^2x^5e^2 + \frac{27}{5}ab^2c^2d^2f^2x^5e^2 + \frac{27}{5}a^2b^2c^2d^2f^2x^5e^2 + \frac{3}{5}a^3d^3f^2x^5e^2 + \frac{9}{4}a^2b^2c^3f^2x^4e + \frac{9}{4}a^3c^2d^2f^2x^4e + \frac{3}{5}b^3c^2d^2x^5e^3 + \frac{9}{5}ab^2c^2d^2x^5e^3 + \frac{3}{5}a^2b^2d^3x^5e^3 + \frac{9}{4}ab^2c^3f^2x^4e^2 + \frac{27}{4}a^2b^2c^2d^2f^2x^4e^2 + \frac{9}{4}a^3c^2d^2f^2x^4e^2 + a^3c^3f^2x^3e + \frac{1}{4}b^3c^3x^4e^3 + \frac{9}{4}ab^2c^2d^2x^4e^3 + \frac{9}{4}a^2b^2c^2d^2x^4e^3 + \frac{1}{4}a^3d^3x^4e^3 + 3a^2b^2c^3f^2x^3e^2 + 3a^3c^2d^2f^2x^3e^2 + ab^2c^3x^3e^3 + 3a^2b^2c^2d^2x^3e^3 + a^3c^2d^2x^3e^3 + \frac{3}{2}a^3c^3f^2x^2e^2 + \frac{3}{2}a^2b^2c^3x^2e^3 + \frac{3}{2}a^3c^2d^2x^2e^3 + a^3c^3x^2e^3$

maple [B] time = 0.00, size = 861, normalized size = 2.39

$$\frac{b^3d^3f^3x^{10}}{10} + \frac{(adf + bcf + bde)b^2d^2f^2x^9}{3} + a^3c^3e^3x + \frac{3(acf + ade + bce)a^2c^2e^2x^2}{2} + \frac{((acf + ade + bce)b^2d^2f^2 + \dots)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3, x)

[Out] $\frac{1}{10}b^3d^3f^3x^{10} + \frac{1}{3}(a*d*f+b*c*f+b*d*e)b^2d^2f^2x^9 + \frac{1}{8}((a*c*f+a*d*e+b*c*e)b^2d^2f^2 + 2(a*d*f+b*c*f+b*d*e)^2b*d*f+b*d*d*f*(2(a*c*f+a*d*e+b*c*e)b*d*f+(a*d*f+b*c*f+b*d*e)^2))x^8 + \frac{1}{7}(a*c*e*b^2d^2f^2 + 2(a*c*f+a*d*e+b*c*e)(a*d*f+b*c*f+b*d*e)b*d*f+(a*d*f+b*c*f+b*d*e)(2(a*c*f+a*d*e+b*c*e)b*d*f+(a*d*f+b*c*f+b*d*e)^2) + b*d*f*(2a*c*e*b*d*f + 2(a*c*f+a*d*e+b*c*e)(a*d*f+b*c*f+b*d*e)))x^7 + \frac{1}{6}(2a*c*e*(a*d*f+b*c*f+b*d*e)b*d*f+(a*c*f+a*d*e+b*c*e)(2(a*c*f+a*d*e+b*c*e)b*d*f+(a*d*f+b*c*f+b*d*e)^2) + (a*d*f+b*c*f+b*d*e)(2a*c*e*b*d*f + 2(a*c*f+a*d*e+b*c*e)(a*d*f+b*c*f+b*d*e)) + b*d*f*(2a*c*e*(a*d*f+b*c*f+b*d*e) + (a*c*f+a*d*e+b*c*e)^2))x^6 + \frac{1}{5}(a*c*e*(2(a*c*f+a*d*e+b*c*e)b*d*f+(a*d*f+b*c*f+b*d*e)^2) + (a*c*f+a*d*e+b*c*e)(2a*c*e*b*d*f + 2(a*c*f+a*d*e+b*c*e)(a*d*f+b*c*f+b*d*e)) + (a*d*f+b*c*f+b*d*e)(2a*c*e*(a*d*f+b*c*f+b*d*e) + (a*c*f+a*d*e+b*c*e)^2) + 2b*d*f*a*c*e*(a*c*f+a*d*e+b*c*e))x^5 + \frac{1}{4}(a*c*e*(2a*c*e*b*d*f + 2(a*c*f+a*d*e+b*c*e)(a*d*f+b*c*f+b*d*e)) + (a*c*f+a*d*e+b*c*e)(2a*c*e*(a*d*f+b*c*f+b*d*e) + (a*c*f+a*d*e+b*c*e)^2) + 2(a*d*f+b*c*f+b*d*e)a*c*e*(a*c*f+a*d*e+b*c*e) + b*d*f*a^2c^2e^2)x^4 + \frac{1}{3}(a*c*e*(2a*c*e*(a*d*f+b*c*f+b*d*e) + (a*c*f+a*d*e+b*c*e)^2) + 2(a*c*f+a*d*e+b*c*e)^2a*c*e + (a*d*f+b*c*f+b*d*e)a^2c^2e^2)x^3 + \frac{3}{2}a^2c^2e^2(a*c*f+a*d*e+b*c*e)x^2 + a^3c^3e^3x$

maxima [A] time = 0.68, size = 461, normalized size = 1.28

$$\frac{1}{10}b^3d^3f^3x^{10} + \frac{1}{3}(bde + bcf + adf)b^2d^2f^2x^9 + \frac{3}{8}(bde + bcf + adf)^2bdfx^8 + a^3c^3e^3x + \frac{1}{7}(bde + bcf + adf)^3x^7 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3, x, algorithm="maxima")

[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*(b*d*e + b*c*f + a*d*f)*b^2*d^2*f^2*x^9 + 3/8*(b*d*e + b*c*f + a*d*f)^2*b*d*f*x^8 + a^3*c^3*e^3*x + 1/7*(b*d*e + b*c*f + a*d*f)^3*x^7 + 1/4*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*c*f)*x^2)*a^2*c^2*e^2 + 1/4*(b*c*e + a*d*e + a*c*f)^3*x^4 + 1/70*(30*b^2*d^2*f^2*x^7 + 70*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + 42*(b*d*e + b*c*f + a*d*f)^2*x^5 + 70*(b*c*e + a*d*e + a*c*f)^2*x^3 + 21*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f))*a*c*e + 1/10*(5*b*d*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(b*c*e + a*d*e + a*c*f)^2 + 1/56*(21*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^7 + 28*(b^2*d^2*e^2 + 2*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*f^2)*x^6)*(b*c*e + a*d*e + a*c*f)

mupad [B] time = 2.23, size = 787, normalized size = 2.18

$$x^7 \left(\frac{a^3 d^3 f^3}{7} + \frac{9 a^2 b c d^2 f^3}{7} + \frac{9 a^2 b d^3 e f^2}{7} + \frac{9 a b^2 c^2 d f^3}{7} + \frac{27 a b^2 c d^2 e f^2}{7} + \frac{9 a b^2 d^3 e^2 f}{7} + \frac{b^3 c^3 f^3}{7} + \frac{9 b^3 c^2 d f^3}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3)^3,x)

[Out] x^7*((a^3*d^3*f^3)/7 + (b^3*c^3*f^3)/7 + (b^3*d^3*e^3)/7 + (9*a*b^2*c^2*d*f^3)/7 + (9*a^2*b*c*d^2*f^3)/7 + (9*a*b^2*d^3*e^2*f)/7 + (9*a^2*b*d^3*e*f^2)/7 + (9*b^3*c*d^2*e^2*f)/7 + (9*b^3*c^2*d*e*f^2)/7 + (27*a*b^2*c*d^2*e*f^2)/7) + x^5*((3*a^2*b*c^3*f^3)/5 + (3*a^2*b*d^3*e^3)/5 + (3*a^3*c^2*d*f^3)/5 + (3*b^3*c^2*d*e^3)/5 + (3*a^3*d^3*e^2*f)/5 + (3*b^3*c^3*e^2*f)/5 + (9*a*b^2*c*d^2*e^3)/5 + (9*a*b^2*c^3*e*f^2)/5 + (9*a^3*c*d^2*e*f^2)/5 + (27*a*b^2*c^2*d*e^2*f)/5 + (27*a^2*b*c*d^2*e^2*f)/5 + (27*a^2*b*c^2*d*e*f^2)/5) + x^6*((a*b^2*c^3*f^3)/2 + (a*b^2*d^3*e^3)/2 + (a^3*c*d^2*f^3)/2 + (b^3*c*d^2*e^3)/2 + (a^3*d^3*e*f^2)/2 + (b^3*c^3*e*f^2)/2 + (3*a^2*b*c^2*d*f^3)/2 + (3*a^2*b*d^3*e^2*f)/2 + (3*b^3*c^2*d*e^2*f)/2 + (9*a*b^2*c*d^2*e^2*f)/2 + (9*a*b^2*c^2*d*e*f^2)/2 + (9*a^2*b*c*d^2*e*f^2)/2) + x^4*((a^3*c^3*f^3)/4 + (a^3*d^3*e^3)/4 + (b^3*c^3*e^3)/4 + (9*a*b^2*c^2*d*e^3)/4 + (9*a^2*b*c*d^2*e^3)/4 + (9*a^3*c*d^2*e^2*f)/4 + (9*a^3*c^2*d*e*f^2)/4 + (27*a^2*b*c^2*d*e^2*f)/4) + a^3*c^3*e^3*x + (b^3*d^3*f^3*x^10)/10 + (3*a^2*c^2*e^2*x^2*(a*c*f + a*d*e + b*c*e))/2 + (b^2*d^2*f^2*x^9*(a*d*f + b*c*f + b*d*e))/3 + a*c*e*x^3*(a^2*c^2*f^2 + a^2*d^2*e^2 + b^2*c^2*e^2 + 3*a*b*c*d*e^2 + 3*a*b*c^2*e*f + 3*a^2*c*d*e*f) + (3*b*d*f*x^8*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 3*a*b*c*d*f^2 + 3*a*b*d^2*e*f + 3*b^2*c*d*e*f))/8

sympy [B] time = 0.25, size = 1018, normalized size = 2.82

$$a^3 c^3 e^3 x + \frac{b^3 d^3 f^3 x^{10}}{10} + x^9 \left(\frac{a b^2 d^3 f^3}{3} + \frac{b^3 c d^2 f^3}{3} + \frac{b^3 d^3 e f^2}{3} \right) + x^8 \left(\frac{3 a^2 b d^3 f^3}{8} + \frac{9 a b^2 c d^2 f^3}{8} + \frac{9 a b^2 d^3 e f^2}{8} + \frac{3 b^3 c^2 d f^3}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**3,x)

[Out] a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f**3/3 + b**3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*d**3*f**3/8 + 9*a*b**2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3*b**3*c**2*d*f**3/8 + 9*b**3*c*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f/8) + x**7*(a**3*d**3*f**3/7 + 9*a**2*b*c*d**2*f**3/7 + 9*a**2*b*d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a*b**2*c*d**2*e*f**2/7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3*c**2*

$$\begin{aligned}
& d*e*f**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3*c*d**2 \\
& *f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a**2*b*c*d**2*e*f \\
& **2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/2 + 9*a*b**2*c**2*d*e*f** \\
& 2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*d**3*e**3/2 + b**3*c**3*e*f**2/2 + \\
& 3*b**3*c**2*d*e**2*f/2 + b**3*c*d**2*e**3/2) + x**5*(3*a**3*c**2*d*f**3/5 + \\
& 9*a**3*c*d**2*e*f**2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*f**3/5 + 27* \\
& a**2*b*c**2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*d**3*e**3/5 + \\
& 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**2*c*d**2*e**3/ \\
& 5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4*(a**3*c**3*f**3/4 + \\
& 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*f/4 + a**3*d**3*e**3/4 + 9*a** \\
& 2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*d*e**2*f/4 + 9*a**2*b*c*d**2*e**3/4 + 9* \\
& a*b**2*c**3*e**2*f/4 + 9*a*b**2*c**2*d*e**3/4 + b**3*c**3*e**3/4) + x**3*(a \\
& **3*c**3*e*f**2 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**3 + 3*a**2*b*c**3*e \\
& **2*f + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x**2*(3*a**3*c**3*e**2*f \\
& /2 + 3*a**3*c**2*d*e**3/2 + 3*a**2*b*c**3*e**3/2)
\end{aligned}$$

3.16 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2 dx$

Optimal. Leaf size=193

$$\frac{(a + bx)^5 (6a^2d^2f^2 - 6abdf(cf + de) + b^2(c^2f^2 + 4cdef + d^2e^2))}{5b^5} + \frac{df(a + bx)^6(-2adf + bcf + bde)}{3b^5} + \frac{(a + bx)^4(bce + ade + acf)}{b^4}$$

[Out] $\frac{1}{3}(-a*d+b*c)^2*(-a*f+b*e)^2*(b*x+a)^3/b^5+1/2*(-a*d+b*c)*(-a*f+b*e)*(-2*a*d*f+b*c*f+b*d*e)*(b*x+a)^4/b^5+1/5*(6*a^2*d^2*f^2-6*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+4*c*d*e*f+d^2*e^2))*(b*x+a)^5/b^5+1/3*d*f*(-2*a*d*f+b*c*f+b*d*e)*(b*x+a)^6/b^5+1/7*d^2*f^2*(b*x+a)^7/b^5$

Rubi [A] time = 0.23, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2059, 88}

$$\frac{(a + bx)^5 (6a^2d^2f^2 - 6abdf(cf + de) + b^2(c^2f^2 + 4cdef + d^2e^2))}{5b^5} + \frac{df(a + bx)^6(-2adf + bcf + bde)}{3b^5} + \frac{(a + bx)^4(bce + ade + acf)}{b^4}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2,x]

[Out] $((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*f)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b^5) + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)^7)/(7*b^5)$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 2059

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2 dx &= \int (a + bx)^2(c + dx)^2(e + fx)^2 dx \\ &= \int \left(\frac{(bc - ad)^2(be - af)^2(a + bx)^2}{b^4} + \frac{2(bc - ad)(bc + ad)(be - af)(a + bx)}{b^4} + \frac{(bc + ad)^2}{b^4} \right) dx \\ &= \frac{(bc - ad)^2(be - af)^2(a + bx)^3}{3b^5} + \frac{(bc - ad)(bc + ad)(be - af)(a + bx)^2}{2b^4} + \frac{(bc + ad)^2(a + bx)}{b^4} \end{aligned}$$

Mathematica [A] time = 0.09, size = 241, normalized size = 1.25

$$\frac{1}{5}x^5(a^2d^2f^2 + 4abdf(cf + de) + b^2(c^2f^2 + 4cdef + d^2e^2)) + \frac{1}{2}x^4(a^2df(cf + de) + ab(c^2f^2 + 4cdef + d^2e^2) + b^2c^2f^2)$$

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2, x]

[Out] a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2 + 4*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^3)/3 + ((b^2*c*e*(d*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^4)/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + (b*d*f*(b*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x^7)/7

fricas [A] time = 0.56, size = 346, normalized size = 1.79

$$\frac{1}{7}x^7f^2d^2b^2 + \frac{1}{3}x^6fed^2b^2 + \frac{1}{3}x^6f^2dcb^2 + \frac{1}{3}x^6f^2d^2ba + \frac{1}{5}x^5e^2d^2b^2 + \frac{4}{5}x^5fedcb^2 + \frac{1}{5}x^5f^2c^2b^2 + \frac{4}{5}x^5fed^2ba + \frac{4}{5}x^5f^2dcb^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2, x, algorithm="fricas")

[Out] 1/7*x^7*f^2*d^2*b^2 + 1/3*x^6*f*e*d^2*b^2 + 1/3*x^6*f^2*d*c*b^2 + 1/3*x^6*f^2*d^2*b*a + 1/5*x^5*e^2*d^2*b^2 + 4/5*x^5*f*e*d*c*b^2 + 1/5*x^5*f^2*c^2*b^2 + 4/5*x^5*f*e*d^2*b*a + 4/5*x^5*f^2*d*c*b*a + 1/5*x^5*f^2*d^2*a^2 + 1/2*x^4*e^2*d*c*b^2 + 1/2*x^4*f*e*c^2*b^2 + 1/2*x^4*e^2*d^2*b*a + 2*x^4*f*e*d*c*b*a + 1/2*x^4*f^2*c^2*b*a + 1/2*x^4*f*e*d^2*a^2 + 1/2*x^4*f^2*d*c*a^2 + 1/3*x^3*e^2*c^2*b^2 + 4/3*x^3*e^2*d*c*b*a + 4/3*x^3*f*e*c^2*b*a + 1/3*x^3*e^2*d^2*a^2 + 4/3*x^3*f*e*d*c*a^2 + 1/3*x^3*f^2*c^2*a^2 + x^2*e^2*c^2*b*a + x^2*e^2*d*c*a^2 + x^2*f*e*c^2*a^2 + x*e^2*c^2*a^2

giac [A] time = 0.42, size = 346, normalized size = 1.79

$$\frac{1}{7}b^2d^2f^2x^7 + \frac{1}{3}b^2cdf^2x^6 + \frac{1}{3}abd^2f^2x^6 + \frac{1}{3}b^2d^2fx^6e + \frac{1}{5}b^2c^2f^2x^5 + \frac{4}{5}abcdf^2x^5 + \frac{1}{5}a^2d^2f^2x^5 + \frac{4}{5}b^2cdfx^5e + \frac{4}{5}abd^2f^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2, x, algorithm="giac")

[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*b^2*c*d*f^2*x^6 + 1/3*a*b*d^2*f^2*x^6 + 1/3*b^2*d^2*f*x^6*e + 1/5*b^2*c^2*f^2*x^5 + 4/5*a*b*c*d*f^2*x^5 + 1/5*a^2*d^2*f^2*x^5 + 4/5*b^2*c*d*f*x^5*e + 4/5*a*b*d^2*f*x^5*e + 1/2*a*b*c^2*f^2*x^4 + 1/2*a^2*c*d*f^2*x^4 + 1/5*b^2*d^2*x^5*e^2 + 1/2*b^2*c^2*f*x^4*e + 2*a*b*c*d*f*x^4*e + 1/2*a^2*d^2*f*x^4*e + 1/3*a^2*c^2*f^2*x^3 + 1/2*b^2*c*d*x^4*e^2 + 1/2*a*b*d^2*x^4*e^2 + 4/3*a*b*c^2*f*x^3*e + 4/3*a^2*c*d*f*x^3*e + 1/3*b^2*c^2*x^3*e^2 + 4/3*a*b*c*d*x^3*e^2 + 1/3*a^2*d^2*x^3*e^2 + a^2*c^2*f*x^2*e + a*b*c^2*x^2*e^2 + a^2*c*d*x^2*e^2 + a^2*c^2*x*e^2

maple [A] time = 0.00, size = 188, normalized size = 0.97

$$\frac{b^2d^2f^2x^7}{7} + \frac{(adf + bcf + bde) bdf x^6}{3} + a^2c^2e^2x + (acf + ade + bce) ace x^2 + \frac{(2(acf + ade + bce) bdf + (adf + bde) bcf) x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2, x)

[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*(a*d*f+b*c*f+b*d*e)*b*d*f*x^6 + 1/5*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)*x^5 + 1/4*(2*a*b*c*d*e*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))*x^4 + 1/3*(2*(a*d*f+b*c*f+b*d*e)*a*c*e+(a*c*f+a*d*e+b*c*e)^2)*x^3 + a*c*e*(a*c*f+a*d*e+b*c*e)*x^2 + a^2*c^2*e^2*x

maxima [A] time = 0.62, size = 180, normalized size = 0.93

$$\frac{1}{7}b^2d^2f^2x^7 + \frac{1}{3}(bde + bcf + adf) bdf x^6 + a^2c^2e^2x + \frac{1}{5}(bde + bcf + adf)^2 x^5 + \frac{1}{3}(bce + ade + acf)^2 x^3 + \frac{1}{6}(3 bdf x^5 + 2 bde bcf + 2 adf bde + 2 acf bde) x^2 + \frac{1}{6}(3 bdf x^5 + 2 bde bcf + 2 adf bde + 2 acf bde) x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2, x, algorithm="maxima")

[Out] $\frac{1}{7}b^2d^2f^2x^7 + \frac{1}{3}(bde + bcf + adf)bdfx^6 + a^2c^2e^2x + \frac{1}{5}(bde + bcf + adf)^2x^5 + \frac{1}{3}(bce + ade + acf)^2x^3 + \frac{1}{6}(3bdfx^4 + 4(bde + bcf + adf)x^3 + 6(bce + ade + acf)x^2)ace + \frac{1}{10}(4bdfx^5 + 5(bde + (bc + ad)f)x^4)(bce + ade + acf)$

mupad [B] time = 0.08, size = 270, normalized size = 1.40

$$x^4 \left(\frac{a^2 c d f^2}{2} + \frac{a^2 d^2 e f}{2} + \frac{a b c^2 f^2}{2} + 2 a b c d e f + \frac{a b d^2 e^2}{2} + \frac{b^2 c^2 e f}{2} + \frac{b^2 c d e^2}{2} \right) + x^3 \left(\frac{a^2 c^2 f^2}{3} + \frac{4 a^2 c d e f}{3} + \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3)^2,x)

[Out] $x^4 * ((a*b*c^2*f^2)/2 + (a*b*d^2*e^2)/2 + (a^2*c*d*f^2)/2 + (b^2*c*d*e^2)/2 + (a^2*d^2*e*f)/2 + (b^2*c^2*e*f)/2 + 2*a*b*c*d*e*f) + x^3 * ((a^2*c^2*f^2)/3 + (a^2*d^2*e^2)/3 + (b^2*c^2*e^2)/3 + (4*a*b*c*d*e^2)/3 + (4*a*b*c^2*e*f)/3 + (4*a^2*c*d*e*f)/3) + x^5 * ((a^2*d^2*f^2)/5 + (b^2*c^2*f^2)/5 + (b^2*d^2*e^2)/5 + (4*a*b*c*d*f^2)/5 + (4*a*b*d^2*e*f)/5 + (4*b^2*c*d*e*f)/5) + a^2*c^2*e^2*x + (b^2*d^2*f^2*x^7)/7 + a*c*e*x^2*(a*c*f + a*d*e + b*c*e) + (b*d*f*x^6*(a*d*f + b*c*f + b*d*e))/3$

sympy [A] time = 0.14, size = 345, normalized size = 1.79

$$a^2c^2e^2x + \frac{b^2d^2f^2x^7}{7} + x^6 \left(\frac{abd^2f^2}{3} + \frac{b^2cdf^2}{3} + \frac{b^2d^2ef}{3} \right) + x^5 \left(\frac{a^2d^2f^2}{5} + \frac{4abcdf^2}{5} + \frac{4abd^2ef}{5} + \frac{b^2c^2f^2}{5} + \frac{4b^2cdef}{5} + \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**2,x)

[Out] $a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3 + b**2*c*d*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 + 4*a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5) + x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e*f + a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**2*f**2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e**2/3 + b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b*c**2*e**2)$

$$3.17 \quad \int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) dx$$

Optimal. Leaf size=56

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x^4

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

[In] Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3,x]

[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/3 + (b*d*f*x^4)/4

Rubi steps

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) dx = acex + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{4}bdfx^4$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.36

$$acex + \frac{1}{2}acfx^2 + \frac{1}{2}adex^2 + \frac{1}{3}adfx^3 + \frac{1}{2}bcex^2 + \frac{1}{3}bcfx^3 + \frac{1}{3}bdex^3 + \frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

[In] Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3,x]

[Out] a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*x^3)/3 + (b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4

fricas [A] time = 0.70, size = 62, normalized size = 1.11

$$\frac{1}{4}x^4fdb + \frac{1}{3}x^3edb + \frac{1}{3}x^3fcb + \frac{1}{3}x^3fda + \frac{1}{2}x^2ecb + \frac{1}{2}x^2eda + \frac{1}{2}x^2fca + xeca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x, algorithm="fricas")

[Out] 1/4*x^4*f*d*b + 1/3*x^3*e*d*b + 1/3*x^3*f*c*b + 1/3*x^3*f*d*a + 1/2*x^2*e*c*b + 1/2*x^2*e*d*a + 1/2*x^2*f*c*a + x*e*c*a

giac [A] time = 0.23, size = 54, normalized size = 0.96

$$\frac{1}{4}bdfx^4 + \frac{1}{3}(bcf + adf + bde)x^3 + acxe + \frac{1}{2}(acf + bce + ade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="giac")

[Out] 1/4*b*d*f*x^4 + 1/3*(b*c*f + a*d*f + b*d*e)*x^3 + a*c*x*e + 1/2*(a*c*f + b*c*e + a*d*e)*x^2

maple [A] time = 0.00, size = 51, normalized size = 0.91

$$\frac{bdfx^4}{4} + acex + \frac{(adf + bcf + bde)x^3}{3} + \frac{(acf + ade + bce)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x)

[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x^4

maxima [A] time = 0.83, size = 50, normalized size = 0.89

$$\frac{1}{4}bdfx^4 + acex + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{2}(bce + ade + acf)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="maxima")

[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a*d*e + a*c*f)*x^2

mupad [B] time = 0.04, size = 54, normalized size = 0.96

$$\frac{bdfx^4}{4} + \left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right)x^3 + \left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)x^2 + acex$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3,x)

[Out] x^2*((a*c*f)/2 + (a*d*e)/2 + (b*c*e)/2) + x^3*((a*d*f)/3 + (b*c*f)/3 + (b*d*e)/3) + a*c*e*x + (b*d*f*x^4)/4

sympy [A] time = 0.07, size = 63, normalized size = 1.12

$$acex + \frac{bdfx^4}{4} + x^3\left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right) + x^2\left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,x)

[Out] a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**2*(a*c*f/2 + a*d*e/2 + b*c*e/2)

$$3.18 \quad \int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3} dx$$

Optimal. Leaf size=86

$$\frac{b \log(a + bx)}{(bc - ad)(be - af)} - \frac{d \log(c + dx)}{(bc - ad)(de - cf)} + \frac{f \log(e + fx)}{(be - af)(de - cf)}$$

[Out] b*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)-d*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)+f*ln(f*x+e)/(-a*f+b*e)/(-c*f+d*e)

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2058}

$$\frac{b \log(a + bx)}{(bc - ad)(be - af)} - \frac{d \log(c + dx)}{(bc - ad)(de - cf)} + \frac{f \log(e + fx)}{(be - af)(de - cf)}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-1), x]

[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)*(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3} dx = \int \left(\frac{b^2}{(bc - ad)(be - af)(a + bx)} + \frac{1}{(bc - ad)(de - cf)} \right) dx = \frac{b \log(a + bx)}{(bc - ad)(be - af)} - \frac{d \log(c + dx)}{(bc - ad)(de - cf)} + \frac{f \log(e + fx)}{(be - af)(de - cf)}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.93

$$\frac{b \log(a + bx)(cf - de) + d(be - af) \log(c + dx) + f(ad - bc) \log(e + fx)}{(bc - ad)(be - af)(cf - de)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-1), x]

[Out] (b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(b*c) + a*d)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f))

fricas [A] time = 10.24, size = 112, normalized size = 1.30

$$\frac{(bc - ad)f \log(fx + e) + (bde - bcf) \log(bx + a) - (bde - adf) \log(dx + c)}{(b^2cd - abd^2)e^2 - (b^2c^2 - a^2d^2)ef + (abc^2 - a^2cd)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x, algorithm="fricas")

[Out] ((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d*e - a*d*f)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a^2*d^2)*e*f + (a*b*c^2 - a^2*c*d)*f^2)

giac [A] time = 0.31, size = 137, normalized size = 1.59

$$\frac{b^2 \log(|bx + a|)}{ab^2cf - a^2bdf - b^3ce + ab^2de} + \frac{d^2 \log(|dx + c|)}{bc^2df - acd^2f - bcd^2e + ad^3e} + \frac{f^2 \log(|fx + e|)}{acf^3 - bcf^2e - adf^2e + bdf^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x, algorithm="giac")

[Out] -b^2*log(abs(b*x + a))/(a*b^2*c*f - a^2*b*d*f - b^3*c*e + a*b^2*d*e) + d^2*log(abs(d*x + c))/(b*c^2*d*f - a*c*d^2*f - b*c*d^2*e + a*d^3*e) + f^2*log(abs(f*x + e))/(a*c*f^3 - b*c*f^2*e - a*d*f^2*e + b*d*f*e^2)

maple [A] time = 0.01, size = 87, normalized size = 1.01

$$\frac{b \ln(bx + a)}{(af - be)(ad - bc)} - \frac{d \ln(dx + c)}{(cf - de)(ad - bc)} + \frac{f \ln(fx + e)}{(cf - de)(af - be)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x)

[Out] f/(c*f-d*e)/(a*f-b*e)*ln(f*x+e)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+b/(a*f-b*e)/(a*d-b*c)*ln(b*x+a)

maxima [A] time = 0.65, size = 112, normalized size = 1.30

$$\frac{b \log(bx + a)}{(b^2c - abd)e - (abc - a^2d)f} - \frac{d \log(dx + c)}{(bcd - ad^2)e - (bc^2 - acd)f} + \frac{f \log(fx + e)}{bde^2 + acf^2 - (bc + ad)ef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x, algorithm="maxima")

[Out] b*log(b*x + a)/((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f) - d*log(d*x + c)/((b*c*d - a*d^2)*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)

mupad [B] time = 2.33, size = 106, normalized size = 1.23

$$\frac{b \ln(a + bx)}{b^2ce + a^2df - abcf - abde} + \frac{d \ln(c + dx)}{ad^2e + bc^2f - acdf - bcde} + \frac{f \ln(e + fx)}{acf^2 + bde^2 - adef - bcef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3),x)

[Out] (b*log(a + b*x))/(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e) + (d*log(c + d*x))/(a*d^2*e + b*c^2*f - a*c*d*f - b*c*d*e) + (f*log(e + f*x))/(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3),x)

[Out] Timed out

$$3.19 \quad \int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx$$

Optimal. Leaf size=234

$$\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2} - \frac{2b^3 \log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3} - \frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2} + \frac{2d^3 \log(c+dx)}{(bc-ad)^2(de-cf)^2}$$

[Out] $-b^3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)-d^3/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x+c)-f^3/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x+e)-2*b^3*(-2*a*d*f+b*c*f+b*d*e)*\ln(b*x+a)/(-a*d+b*c)^3/(-a*f+b*e)^3+2*d^3*(a*d*f-2*b*c*f+b*d*e)*\ln(d*x+c)/(-a*d+b*c)^3/(-c*f+d*e)^3+2*f^3*(-a*d*f-b*c*f+2*b*d*e)*\ln(f*x+e)/(-a*f+b*e)^3/(-c*f+d*e)^3$

Rubi [A] time = 0.41, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2058}

$$\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2} - \frac{2b^3 \log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3} - \frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2} + \frac{2d^3 \log(c+dx)}{(bc-ad)^2(de-cf)^2}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-2), x]

[Out] $-(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e - c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b*d*e + b*c*f - 2*a*d*f)*\text{Log}[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^3*(b*d*e - 2*b*c*f + a*d*f)*\text{Log}[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (2*f^3*(2*b*d*e - b*c*f - a*d*f)*\text{Log}[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)$

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx = \int \left(\frac{b^4}{(bc-ad)^2(be-af)^2(a+bx)^2} - \frac{2b^4 \log(a+bx)}{(bc-ad)^2(be-af)^2(a+bx)^2} \right) dx = -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)} - \frac{2b^4 \log(a+bx)}{(bc-ad)^2(be-af)^2(a+bx)^2}$$

Mathematica [A] time = 0.55, size = 232, normalized size = 0.99

$$\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2} - \frac{2b^3 \log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3} - \frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2} - \frac{2d^3 \log(c+dx)}{(bc-ad)^2(de-cf)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-2), x]

[Out] $-(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e - c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b*d*e + b*c*f - 2*a*d*f)*\text{Log}[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^3*(b*d*e - 2*b*c*f + a*d*f)*\text{Log}[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (2*f^3*(2*b*d*e - b*c*f - a*d*f)*\text{Log}[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)$


```
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x]]/((b*c - a*d)^3*(b*e - a*f)^3) - (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x]]/((b*c - a*d)^3*(-(d*e) + c*f)^3)
- (2*f^3*(-2*b*d*e + b*c*f + a*d*f)*Log[e + f*x]]/((b*e - a*f)^3*(d*e - c*f
)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="fricas")
```

[Out] Timed out

giac [B] time = 0.39, size = 1414, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="giac")
```

```
[Out] 2*(b^5*c*f - 2*a*b^4*d*f + b^5*d*e)*log(abs(b*x + a))/(a^3*b^4*c^3*f^3 - 3*
a^4*b^3*c^2*d*f^3 + 3*a^5*b^2*c*d^2*f^3 - a^6*b*d^3*f^3 - 3*a^2*b^5*c^3*f^2
*e + 9*a^3*b^4*c^2*d*f^2*e - 9*a^4*b^3*c*d^2*f^2*e + 3*a^5*b^2*d^3*f^2*e +
3*a*b^6*c^3*f*e^2 - 9*a^2*b^5*c^2*d*f*e^2 + 9*a^3*b^4*c*d^2*f*e^2 - 3*a^4*b
^3*d^3*f*e^2 - b^7*c^3*e^3 + 3*a*b^6*c^2*d*e^3 - 3*a^2*b^5*c*d^2*e^3 + a^3*
b^4*d^3*e^3) + 2*(2*b*c*d^4*f - a*d^5*f - b*d^5*e)*log(abs(d*x + c))/(b^3*c
^6*d*f^3 - 3*a*b^2*c^5*d^2*f^3 + 3*a^2*b*c^4*d^3*f^3 - a^3*c^3*d^4*f^3 - 3*
b^3*c^5*d^2*f^2*e + 9*a*b^2*c^4*d^3*f^2*e - 9*a^2*b*c^3*d^4*f^2*e + 3*a^3*c
^2*d^5*f^2*e + 3*b^3*c^4*d^3*f*e^2 - 9*a*b^2*c^3*d^4*f*e^2 + 9*a^2*b*c^2*d
^5*f*e^2 - 3*a^3*c*d^6*f*e^2 - b^3*c^3*d^4*e^3 + 3*a*b^2*c^2*d^5*e^3 - 3*a^2
*b*c*d^6*e^3 + a^3*d^7*e^3) - 2*(b*c*f^5 + a*d*f^5 - 2*b*d*f^4*e)*log(abs(f
*x + e))/(a^3*c^3*f^7 - 3*a^2*b*c^3*f^6*e - 3*a^3*c^2*d*f^6*e + 3*a*b^2*c^3
*f^5*e^2 + 9*a^2*b*c^2*d*f^5*e^2 + 3*a^3*c*d^2*f^5*e^2 - b^3*c^3*f^4*e^3 -
9*a*b^2*c^2*d*f^4*e^3 - 9*a^2*b*c*d^2*f^4*e^3 - a^3*d^3*f^4*e^3 + 3*b^3*c^2
*d*f^3*e^4 + 9*a*b^2*c*d^2*f^3*e^4 + 3*a^2*b*d^3*f^3*e^4 - 3*b^3*c*d^2*f^2*
e^5 - 3*a*b^2*d^3*f^2*e^5 + b^3*d^3*f*e^6) - (2*b^3*c^2*d*f^3*x^2 - 2*a*b^2
*c*d^2*f^3*x^2 + 2*a^2*b*d^3*f^3*x^2 - 2*b^3*c*d^2*f^2*x^2*e - 2*a*b^2*d^3*
f^2*x^2*e + 2*b^3*c^3*f^3*x - a*b^2*c^2*d*f^3*x - a^2*b*c*d^2*f^3*x + 2*a^3
*d^3*f^3*x + 2*b^3*d^3*f*x^2*e^2 - b^3*c^2*d*f^2*x*e - a^2*b*d^3*f^2*x*e +
a*b^2*c^3*f^3 - 2*a^2*b*c^2*d*f^3 + a^3*c*d^2*f^3 - b^3*c*d^2*f*x*e^2 - a*b
^2*d^3*f*x*e^2 + b^3*c^3*f^2*e + a^3*d^3*f^2*e + 2*b^3*d^3*x*e^3 - 2*b^3*c^
2*d*f*e^2 - 2*a^2*b*d^3*f*e^2 + b^3*c*d^2*e^3 + a*b^2*d^3*e^3)/((a^2*b^2*c^
4*f^4 - 2*a^3*b*c^3*d*f^4 + a^4*c^2*d^2*f^4 - 2*a*b^3*c^4*f^3*e + 2*a^2*b^2
*c^3*d*f^3*e + 2*a^3*b*c^2*d^2*f^3*e - 2*a^4*c*d^3*f^3*e + b^4*c^4*f^2*e^2
+ 2*a*b^3*c^3*d*f^2*e^2 - 6*a^2*b^2*c^2*d^2*f^2*e^2 + 2*a^3*b*c*d^3*f^2*e^2
+ a^4*d^4*f^2*e^2 - 2*b^4*c^3*d*f*e^3 + 2*a*b^3*c^2*d^2*f*e^3 + 2*a^2*b^2*
c*d^3*f*e^3 - 2*a^3*b*d^4*f*e^3 + b^4*c^2*d^2*e^4 - 2*a*b^3*c*d^3*e^4 + a^2
*b^2*d^4*e^4)*(b*d*f*x^3 + b*c*f*x^2 + a*d*f*x^2 + b*d*x^2*e + a*c*f*x + b*
c*x*e + a*d*x*e + a*c*e))
```

maple [A] time = 0.03, size = 398, normalized size = 1.70

$$\frac{4a^3df \ln(bx+a)}{(af-be)^3(ad-bc)^3} + \frac{2ad^4f \ln(dx+c)}{(cf-de)^3(ad-bc)^3} - \frac{2adf^4 \ln(fx+e)}{(cf-de)^3(af-be)^3} - \frac{2b^4cf \ln(bx+a)}{(af-be)^3(ad-bc)^3} - \frac{2b^4de \ln(bx+a)}{(af-be)^3(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
[Out] -f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)
)*a*d-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)*b*c+4*f^3/(c*f-d*e)^3/(a*f-b*
e)^3*ln(f*x+e)*b*d*e-d^3/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+2*d^4/(c*f-d*e)^3/
(a*d-b*c)^3*ln(d*x+c)*a*f-4*d^3/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*c*f+2*d
^4/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*e-b^3/(a*f-b*e)^2/(a*d-b*c)^2/(b*x+a
)+4*b^3/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*a*d*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)
^3*ln(b*x+a)*c*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*d*e
```

maxima [B] time = 1.54, size = 2096, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="maxima")
```

```
[Out] -2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d
+ 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*
a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a
^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*
c*d^2 - a^6*d^3)*f^3) + 2*(b*d^4*e - (2*b*c*d^3 - a*d^4)*f)*log(d*x + c)/((
b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*e^3 - 3*(b^3*c^4*d
^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*e^2*f + 3*(b^3*c^5*d -
3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*f^2 - (b^3*c^6 - 3*a*b^2
*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + 2*(2*b*d*e*f^3 - (b*c + a*d)
*f^4)*log(f*x + e)/(b^3*d^3*e^6 + a^3*c^3*f^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*e
^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3*c^3 + 9*a*b
^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d
+ a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5) - ((b^3*c*d^2 + a*b
^2*d^3)*e^3 - 2*(b^3*c^2*d + a^2*b*d^3)*e^2*f + (b^3*c^3 + a^3*d^3)*e*f^2 +
(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3 + 2*(b^3*d^3*e^2*f - (b^3*c*d^
2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^2 + (2*
b^3*d^3*e^3 - (b^3*c*d^2 + a*b^2*d^3)*e^2*f - (b^3*c^2*d + a^2*b*d^3)*e*f^2
+ (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*f^3)*x)/((a*b^4*c^3*
d^2 - 2*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*e^5 - 2*(a*b^4*c^4*d - a^2*b^3*c^3
*d^2 - a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e^4*f + (a*b^4*c^5 + 2*a^2*b^3*c^4*d
- 6*a^3*b^2*c^3*d^2 + 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^3*f^2 - 2*(a^2*b^3*c^5
- a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^2*f^3 + (a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2)*e*f^4 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*
d^5)*e^4*f - 2*(b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*
e^3*f^2 + (b^5*c^4*d + 2*a*b^4*c^3*d^2 - 6*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^
4 + a^4*b*d^5)*e^2*f^3 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3
+ a^4*b*c*d^4)*e*f^4 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)
*f^5)*x^3 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*e^5 - (b^5*c^3*d^2
- a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^4*f - (b^5*c^4*d - 2*a*b^
4*c^3*d^2 + 2*a^2*b^3*c^2*d^3 - 2*a^3*b^2*c*d^4 + a^4*b*d^5)*e^3*f^2 + (b^5
*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 +
a^5*d^5)*e^2*f^3 - (2*a*b^4*c^5 - a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 - a^4*b
*c^2*d^3 + 2*a^5*c*d^4)*e*f^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^
2 + a^5*c^2*d^3)*f^5)*x^2 + ((b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 +
a^3*b^2*d^5)*e^5 - (2*b^5*c^4*d - a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 - a^3*
b^2*c*d^4 + 2*a^4*b*d^5)*e^4*f + (b^5*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2
- 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + a^5*d^5)*e^3*f^2 - (a*b^4*c^5 - 2*a^2*
b^3*c^4*d + 2*a^3*b^2*c^3*d^2 - 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^2*f^3 - (a^2
*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e*f^4 + (a^3*b^2*c^
5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*f^5)*x)
```

mupad [B] time = 8.18, size = 1940, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3)^2,x)
```

```
[Out] - ((a*b^2*c^3*f^3 + a*b^2*d^3*e^3 + a^3*c*d^2*f^3 + b^3*c*d^2*e^3 + a^3*d^3*e*f^2 + b^3*c^3*e*f^2 - 2*a^2*b*c^2*d*f^3 - 2*a^2*b*d^3*e^2*f - 2*b^3*c^2*d*e^2*f)/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) + (2*x^2*(a^2*b*d^3*f^3 + b^3*c^2*d*f^3 + b^3*d^3*e^2*f - a*b^2*c*d^2*f^3 - a*b^2*d^3*e*f^2 - b^3*c*d^2*e*f^2))/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) - (x*(a*b^2*c^2*d*f^3 - 2*b^3*c^3*f^3 - 2*b^3*d^3*e^3 - 2*a^3*d^3*f^3 + a^2*b*c*d^2*f^3 + a*b^2*d^3*e^2*f + a^2*b*d^3*e*f^2 + b^3*c*d^2*e^2*f + b^3*c^2*d*e*f^2))/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2))/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3) - (log(a + b*x)*(b^4*(2*c*f + 2*d*e) - 4*a*b^3*d*f))/(b^6*c^3*e^3 + a^6*d^3*f^3 - a^3*b^3*c^3*f^3 - a^3*b^3*d^3*e^3 - 3*a*b^5*c^2*d*e^3 - 3*a^5*b*c*d^2*f^3 - 3*a*b^5*c^3*e^2*f - 3*a^5*b*d^3*e*f^2 + 3*a^2*b^4*c*d^2*e^3 + 3*a^4*b^2*c^2*d*f^3 + 3*a^2*b^4*c^3*e*f^2 + 3*a^4*b^2*d^3*e^2*f + 9*a^2*b^4*c^2*d*e^2*f - 9*a^3*b^3*c*d^2*e^2*f - 9*a^3*b^3*c^2*d*e*f^2 + 9*a^4*b^2*c*d^2*e*f^2) - (log(c + d*x)*(d^4*(2*a*f + 2*b*e) - 4*b*c*d^3*f))/(a^3*d^6*e^3 + b^3*c^6*f^3 - a^3*c^3*d^3*f^3 - b^3*c^3*d^3*e^3 - 3*a^2*b*c*d^5*e^3 - 3*a*b^2*c^5*d*f^3 - 3*a^3*c*d^5*e^2*f - 3*b^3*c^5*d*e*f^2 + 3*a*b^2*c^2*d^4*e^3 + 3*a^2*b*c^4*d^2*f^3 + 3*a^3*c^2*d^4*e*f^2 + 3*b^3*c^4*d^2*e^2*f - 9*a*b^2*c^3*d^3*e^2*f + 9*a*b^2*c^4*d^2*e*f^2 + 9*a^2*b*c^2*d^4*e^2*f - 9*a^2*b*c^3*d^3*e*f^2) - (log(e + f*x)*(f^4*(2*a*d + 2*b*c) - 4*b*d*e*f^3))/(a^3*c^3*f^6 + b^3*d^3*e^6 - a^3*d^3*e^3*f^3 - b^3*c^3*e^3*f^3 - 3*a^2*b*c^3*e*f^5 - 3*a*b^2*d^3*e^5*f - 3*a^3*c^2*d*e*f^5 - 3*b^3*c*d^2*e^5*f + 3*a*b^2*c^3*e^2*f^4 + 3*a^2*b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4 + 3*b^3*c^2*d*e^4*f^2 + 9*a*b^2*c*d^2*e^4*f^2 - 9*a*b^2*c^2*d*e^3*f^3 - 9*a^2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*e^2*f^4)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**2,x)
```

```
[Out] Timed out
```

3.20
$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3} dx$$

Optimal. Leaf size=495

$$\frac{3f^5 \log(e + fx) (2a^2d^2f^2 - abdf(7de - 3cf) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(be - af)^5(de - cf)^5} - \frac{3d^5 \log(c + dx) (2a^2d^2f^2 + abdf(3cf - 7de) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(bc - ad)^5(de - cf)^5}$$

[Out]
$$-1/2*b^5/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2+3*b^5*(-2*a*d*f+b*c*f+b*d*e)/(-a*d+b*c)^4/(-a*f+b*e)^4/(b*x+a)+1/2*d^5/(-a*d+b*c)^3/(-c*f+d*e)^3/(d*x+c)^2+3*d^5*(a*d*f-2*b*c*f+b*d*e)/(-a*d+b*c)^4/(-c*f+d*e)^4/(d*x+c)-1/2*f^5/(-a*f+b*e)^3/(-c*f+d*e)^3/(f*x+e)^2-3*f^5*(-a*d*f-b*c*f+2*b*d*e)/(-a*f+b*e)^4/(-c*f+d*e)^4/(f*x+e)+3*b^5*(7*a^2*d^2*f^2-7*a*b*d*f*(c*f+d*e)+b^2*(2*c^2*f^2+3*c*d*e*f+2*d^2*e^2))*ln(b*x+a)/(-a*d+b*c)^5/(-a*f+b*e)^5-3*d^5*(2*a^2*d^2*f^2+2*a*b*d*f*(-7*c*f+3*d*e)+b^2*(7*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*ln(d*x+c)/(-a*d+b*c)^5/(-c*f+d*e)^5+3*f^5*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+7*d*e)+b^2*(2*c^2*f^2-7*c*d*e*f+7*d^2*e^2))*ln(f*x+e)/(-a*f+b*e)^5/(-c*f+d*e)^5$$

Rubi [A] time = 1.46, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2058}

$$\frac{3b^5 \log(a + bx) (7a^2d^2f^2 - 7abdf(cf + de) + b^2(2c^2f^2 + 3cdef + 2d^2e^2))}{(bc - ad)^5(be - af)^5} - \frac{3d^5 \log(c + dx) (2a^2d^2f^2 + abdf(3cf - 7de) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(bc - ad)^5(de - cf)^5}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-3), x]

[Out]
$$-b^5/(2*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f - 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^3*(d*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2) - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2*d^2*f^2 - a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5)$$

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3} dx = \int \left(\frac{b^6}{(bc - ad)^3(be - af)^3(a + bx)^3} - \frac{3b^6}{(bc - ad)^3} \right) dx = -\frac{b^5}{2(bc - ad)^3(be - af)^3(a + bx)^2} + \frac{3b^5}{(bc - ad)^3}$$

Mathematica [A] time = 1.19, size = 490, normalized size = 0.99

$$\frac{1}{2} \left(\frac{6f^5 \log(e + fx) (2a^2d^2f^2 + abdf(3cf - 7de) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(be - af)^5(de - cf)^5} + \frac{6d^5 \log(c + dx) (2a^2d^2f^2 + abdf(3cf - 7de) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(bc - ad)^5(de - cf)^5} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-3),x]
```

```
[Out] (-b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e + b*c*f - 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^5/((b*c - a*d)^3*(-(d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/((b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2) + (6*f^5*(-2*b*d*e + b*c*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)) + (6*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*d^5*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f^2))*Log[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^5) + (6*f^5*(2*a^2*d^2*f^2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5))/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x, algorithm="fricas")
```

[Out] Timed out

giac [B] time = 1.10, size = 6908, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x, algorithm="giac")
```

```
[Out] -3*(2*b^8*c^2*f^2 - 7*a*b^7*c*d*f^2 + 7*a^2*b^6*d^2*f^2 + 3*b^8*c*d*f*e - 7*a*b^7*d^2*f*e + 2*b^8*d^2*e^2)*log(abs(b*x + a))/(a^5*b^6*c^5*f^5 - 5*a^6*b^5*c^4*d*f^5 + 10*a^7*b^4*c^3*d^2*f^5 - 10*a^8*b^3*c^2*d^3*f^5 + 5*a^9*b^2*c*d^4*f^5 - a^10*b*d^5*f^5 - 5*a^4*b^7*c^5*f^4*e + 25*a^5*b^6*c^4*d*f^4*e - 50*a^6*b^5*c^3*d^2*f^4*e + 50*a^7*b^4*c^2*d^3*f^4*e - 25*a^8*b^3*c*d^4*f^4*e + 5*a^9*b^2*d^5*f^4*e + 10*a^3*b^8*c^5*f^3*e^2 - 50*a^4*b^7*c^4*d*f^3*e^2 + 100*a^5*b^6*c^3*d^2*f^3*e^2 - 100*a^6*b^5*c^2*d^3*f^3*e^2 + 50*a^7*b^4*c*d^4*f^3*e^2 - 10*a^8*b^3*d^5*f^3*e^2 - 10*a^2*b^9*c^5*f^2*e^3 + 50*a^3*b^8*c^4*d*f^2*e^3 - 100*a^4*b^7*c^3*d^2*f^2*e^3 + 100*a^5*b^6*c^2*d^3*f^2*e^3 - 50*a^6*b^5*c*d^4*f^2*e^3 + 10*a^7*b^4*d^5*f^2*e^3 + 5*a*b^10*c^5*f*e^4 - 25*a^2*b^9*c^4*d*f*e^4 + 50*a^3*b^8*c^3*d^2*f*e^4 - 50*a^4*b^7*c^2*d^3*f*e^4 + 25*a^5*b^6*c*d^4*f*e^4 - 5*a^6*b^5*d^5*f*e^4 - b^11*c^5*e^5 + 5*a*b^10*c^4*d*e^5 - 10*a^2*b^9*c^3*d^2*e^5 + 10*a^3*b^8*c^2*d^3*e^5 - 5*a^4*b^7*c*d^4*e^5 + a^5*b^6*d^5*e^5) + 3*(7*b^2*c^2*d^6*f^2 - 7*a*b*c*d^7*f^2 + 2*a^2*d^8*f^2 - 7*b^2*c*d^7*f*e + 3*a*b*d^8*f*e + 2*b^2*d^8*e^2)*log(abs(d*x + c))/(b^5*c^10*d*f^5 - 5*a*b^4*c^9*d^2*f^5 + 10*a^2*b^3*c^8*d^3*f^5 - 10*a^3*b^2*c^7*d^4*f^5 + 5*a^4*b*c^6*d^5*f^5 - a^5*c^5*d^6*f^5 - 5*b^5*c^9*d^2*f^4*e + 25*a*b^4*c^8*d^3*f^4*e - 50*a^2*b^3*c^7*d^4*f^4*e + 50*a^3*b^2*c^6*d^5*f^4*e - 25*a^4*b*c^5*d^6*f^4*e + 5*a^5*c^4*d^7*f^4*e + 10*b^5*c^8*d^3*f^3*e^2 - 50*a*b^4*c^7*d^4*f^3*e^2 + 100*a^2*b^3*c^6*d^5*f^3*e^2 - 100*a^3*b^2*c^5*d^6*f^3*e^2 + 50*a^4*b*c^4*d^7*f^3*e^2 - 10*a^5*c^3*d^8*f^3*e^2 - 10*b^5*c^7*d^4*f^2*e^3 + 50*a*b^4*c^6*d^5*f^2*e^3 - 100*a^2*b^3*c^5*d^6*f^2*e^3 + 100*a^3*b^2*c^4*d^7*f^2*e^3 - 50*a^4*b*c^3*d^8*f^2*e^3 + 10*a^5*c^2*d^9*f^2*e^3 + 5*b^5*c^6*d^5*f*e^4 - 25*a*b^4*c^5*d^6*f*e^4 + 50*a^2*b^3*c^4*d^7*f*e^4 - 50*a^3*b^2*c^3*d^8*f*e^4 + 25*a^4*b*c^2*d^9*f*e^4 - 5*a^5*c*d^10*f
```

$$\begin{aligned}
& *e^4 - b^5c^5d^6e^5 + 5a*b^4c^4d^7e^5 - 10a^2b^3c^3d^8e^5 + 10a^3b^2c^2d^9e^5 - 5a^4b*c*d^{10}e^5 + a^5d^{11}e^5) + 3*(2b^2c^2f^8 \\
& + 3a*b*c*d*f^8 + 2a^2d^2f^8 - 7b^2c*d*f^7e - 7a*b*d^2f^7e + 7b^2d^2f^6e^2)*\log(\text{abs}(f*x + e))/(a^5c^5f^{11} - 5a^4b*c^5f^{10}e - 5a^5 \\
& *c^4*d*f^{10}e + 10a^3b^2c^5f^9e^2 + 25a^4b*c^4*d*f^9e^2 + 10a^5c^3d^2f^9e^2 - 10a^2b^3c^5f^8e^3 - 50a^3b^2c^4*d*f^8e^3 - 50a^4* \\
& b*c^3d^2f^8e^3 - 10a^5c^2d^3f^8e^3 + 5a*b^4c^5f^7e^4 + 50a^2b^3c^4*d*f^7e^4 + 100a^3b^2c^3d^2f^7e^4 + 50a^4b*c^2d^3f^7e^4 + \\
& 5a^5c*d^4f^7e^4 - b^5c^5f^6e^5 - 25a*b^4c^4*d*f^6e^5 - 100a^2b^3c^3d^2f^6e^5 - 100a^3b^2c^2d^3f^6e^5 - 25a^4b*c*d^4f^6e^5 - \\
& a^5d^5f^6e^5 + 5b^5c^4*d*f^5e^6 + 50a*b^4c^3d^2f^5e^6 + 100a^2b^3c^2d^3f^5e^6 + 50a^3b^2c*d^4f^5e^6 + 5a^4b*d^5f^5e^6 - 10* \\
& b^5c^3d^2f^4e^7 - 50a*b^4c^2d^3f^4e^7 - 50a^2b^3c*d^4f^4e^7 - 10a^3b^2d^5f^4e^7 + 10b^5c^2d^3f^3e^8 + 25a*b^4c*d^4f^3e^8 + \\
& 10a^2b^3d^5f^3e^8 - 5b^5c*d^4f^2e^9 - 5a*b^4d^5f^2e^9 + b^5d^5f^e^{10}) + 1/2*(12b^7c^5d^2f^7x^5 - 30a*b^6c^4d^3f^7x^5 + 12a^2b^5c^3d^4f^7x^5 \\
& + 12a^3b^4c^2d^5f^7x^5 - 30a^4b^3c*d^6f^7x^5 + 12a^5b^2d^7f^7x^5 - 30b^7c^4d^3f^6x^5e + 96a*b^6c^3d^4f^6x^5e - 72a^2b^5c^2d^5f^6x^5e \\
& + 96a^3b^4c*d^6f^6x^5e - 30a^4b^3d^7f^6x^5e + 24b^7c^6d*f^7x^4 - 42a*b^6c^5d^2f^7x^4 - 21a^2b^5c^4d^3f^7x^4 + 42a^3b^4c^3d^4f^7x^4 \\
& - 21a^4b^3c^2d^5f^7x^4 - 42a^5b^2c*d^6f^7x^4 + 24a^6b*d^7f^7x^4 + 12b^7c^3d^4f^5x^5e^2 - 72a*b^6c^2d^5f^5x^5e^2 - 72a^2b^5c*d^6f^5x^5e^2 \\
& + 12a^3b^4d^7f^5x^5e^2 - 42b^7c^5d^2f^6x^4e + 102a*b^6c^4d^3f^6x^4e + 18a^2b^5c^3d^4f^6x^4e + 18a^3b^4c^2d^5f^6x^4e + 1 \\
& 02a^4b^3c*d^6f^6x^4e - 42a^5b^2d^7f^6x^4e + 12b^7c^7f^7x^3 + 6a*b^6c^6d*f^7x^3 - 74a^2b^5c^5d^2f^7x^3 + 38a^3b^4c^4d^3f^7x^3 \\
& + 38a^4b^3c^3d^4f^7x^3 - 74a^5b^2c^2d^5f^7x^3 + 6a^6b*c*d^6f^7x^3 + 12a^7d^7f^7x^3 + 12b^7c^2d^5f^4x^5e^3 + 96a*b^6c^5d^6f^4x^5e^3 \\
& + 12a^2b^5d^7f^4x^5e^3 - 21b^7c^4d^3f^5x^4e^2 + 18a*b^6c^3d^4f^5x^4e^2 - 234a^2b^5c^2d^5f^5x^4e^2 + 18a^3b^4c*d^6f^5x^4e^2 \\
& - 21a^4b^3d^7f^5x^4e^2 + 6b^7c^6d*f^6x^3e - 56a*b^6c^5d^2f^6x^3e + 172a^2b^5c^4d^3f^6x^3e - 136a^3b^4c^3d^4f^6x^3e \\
& + 172a^4b^3c^2d^5f^6x^3e - 56a^5b^2c*d^6f^6x^3e + 6a^6b*d^7f^6x^3e + 18a*b^6c^7f^7x^2 - 37a^2b^5c^6d*f^7x^2 - 3a^3b^4c^5d^2f^7x^2 \\
& + 32a^4b^3c^4d^3f^7x^2 - 3a^5b^2c^3d^4f^7x^2 - 37a^6b*c^2d^5f^7x^2 + 18a^7c*d^6f^7x^2 - 30b^7c*d^6f^3x^5e^4 - 30a*b^6d^7f^3x^5e^4 \\
& + 42b^7c^3d^4f^4x^4e^3 + 18a*b^6c^2d^5f^4x^4e^3 + 18a^2b^5c*d^6f^4x^4e^3 + 42a^3b^4d^7f^4x^4e^3 - 74b^7c^5d^2f^5x^3e^2 \\
& + 172a*b^6c^4d^3f^5x^3e^2 - 104a^2b^5c^3d^4f^5x^3e^2 - 104a^3b^4c^2d^5f^5x^3e^2 + 172a^4b^3c*d^6f^5x^3e^2 - 74a^5b^2d^7f^5x^3e^2 \\
& + 18b^7c^7f^6x^2e - 34a*b^6c^6d*f^6x^2e + 9a^2b^5c^5d^2f^6x^2e + a^3b^4c^4d^3f^6x^2e + a^4b^3c^3d^4f^6x^2e + 9a^5b^2c^2d^5f^6x^2e \\
& - 34a^6b*c*d^6f^6x^2e + 18a^7d^7f^6x^2e + 4a^2b^5c^7f^7x - 12a^3b^4c^6d*f^7x + 8a^4b^3c^5d^2f^7x + 8a^5b^2c^4d^3f^7x - 12a^6 \\
& *b*c^3d^4f^7x + 4a^7c^2d^5f^7x + 12b^7d^7f^2x^5e^5 - 21b^7c^2d^5f^3x^4e^4 + 102a*b^6c*d^6f^3x^4e^4 - 21a^2b^5d^7f^3x^4e^4 \\
& + 38b^7c^4d^3f^4x^3e^3 - 136a*b^6c^3d^4f^4x^3e^3 - 104a^2b^5c^2d^5f^4x^3e^3 - 136a^3b^4c*d^6f^4x^3e^3 + 38a^4b^3d^7f^4x^3e^3 \\
& - 37b^7c^6d*f^5x^2e^2 + 9a*b^6c^5d^2f^5x^2e^2 + 234a^2b^5c^4d^3f^5x^2e^2 - 208a^3b^4c^3d^4f^5x^2e^2 + 234a^4b^3c^2d^5f^5x^2e^2 \\
& + 9a^5b^2c*d^6f^5x^2e^2 - 37a^6b*d^7f^5x^2e^2 + 28a*b^6c^7f^6x^e - 66a^2b^5c^6d*f^6x^e + 34a^3b^4c^5d^2f^6x^e - 16a^4b^3c^4d^3f^6x^e \\
& + 34a^5b^2c^3d^4f^6x^e - 66a^6b*c^2d^5f^6x^e + 28a^7c*d^6f^6x^e - a^3b^4c^7f^7 + 4a^4b^3c^6d*f^7 - 6a^5b^2c^5d^2f^7 + 4a^6b*c^4d^3f^7 \\
& - a^7c^3d^4f^7 - 42b^7c^3d^6f^2x^4e^5 - 42a*b^6d^7f^2x^4e^5 + 38b^7c^3d^4f^3x^3e^4 + 172a*b^6c^2d^5f^3x^3e^4 + 172a^2b^5c*d^6f^3x^3e^4 \\
& + 38a^3b^4c
\end{aligned}$$

$$\begin{aligned}
& d^7 f^3 x^3 e^4 - 3 b^7 c^5 d^2 f^4 x^2 e^3 + a b^6 c^4 d^3 f^4 x^2 e^3 - 208 a^2 b^5 c^3 d^4 f^4 x^2 e^3 - 208 a^3 b^4 c^2 d^5 f^4 x^2 e^3 + a^4 b^3 c d^6 f^4 x^2 e^3 - 3 a^5 b^2 d^7 f^4 x^2 e^3 + 4 b^7 c^7 f^5 x e^2 - 66 a^* \\
& b^6 c^6 d f^5 x e^2 + 156 a^2 b^5 c^5 d^2 f^5 x e^2 - 52 a^3 b^4 c^4 d^3 f^5 x e^2 - 52 a^4 b^3 c^3 d^4 f^5 x e^2 + 156 a^5 b^2 c^2 d^5 f^5 x e^2 - 66 \\
& a^6 b c d^6 f^5 x e^2 + 4 a^7 d^7 f^5 x e^2 + 7 a^2 b^5 c^7 f^6 e - 21 a^3 \\
& b^4 c^6 d f^6 e + 14 a^4 b^3 c^5 d^2 f^6 e + 14 a^5 b^2 c^4 d^3 f^6 e - 21 \\
& a^6 b c^3 d^4 f^6 e + 7 a^7 c^2 d^5 f^6 e + 24 b^7 d^7 f^7 x^4 e^6 - 74 b^7 c^2 d^5 f^2 x^3 e^5 - 56 a^* b^6 c^* d^6 f^2 x^3 e^5 - 74 a^2 b^5 d^7 f^2 x^3 e^5 \\
& + 32 b^7 c^4 d^3 f^3 x^2 e^4 + a b^6 c^3 d^4 f^3 x^2 e^4 + 234 a^2 b^5 c^2 d^5 f^3 x^2 e^4 + a^3 b^4 c^* d^6 f^3 x^2 e^4 + 32 a^4 b^3 d^7 f^3 x^2 e^4 \\
& - 12 b^7 c^6 d f^4 x e^3 + 34 a^* b^6 c^5 d^2 f^4 x e^3 - 52 a^2 b^5 c^4 d^3 f^4 x e^3 - 52 a^4 b^3 c^2 d^5 f^4 x e^3 + 34 a^5 b^2 c^* d^6 f^4 x e^3 - 12 \\
& a^6 b^* d^7 f^4 x e^3 + 7 a^* b^6 c^7 f^5 e^2 - 26 a^2 b^5 c^6 d f^5 e^2 + 52 a^3 b^4 c^5 d^2 f^5 e^2 - 78 a^4 b^3 c^4 d^3 f^5 e^2 + 52 a^5 b^2 c^3 d^4 f^5 e^2 \\
& - 26 a^6 b c^2 d^5 f^5 e^2 + 7 a^7 c^* d^6 f^5 e^2 + 6 b^7 c^* d^6 f^* x^3 e^6 + 6 a^* b^6 d^7 f^* x^3 e^6 - 3 b^7 c^3 d^4 f^2 x^2 e^5 + 9 a^* b^6 c^2 d^5 f^2 x^2 e^5 \\
& + 9 a^2 b^5 c^* d^6 f^2 x^2 e^5 - 3 a^3 b^4 d^7 f^2 x^2 e^5 + 8 b^7 c^5 d^2 f^3 x e^4 - 16 a^* b^6 c^4 d^3 f^3 x e^4 - 52 a^2 b^5 c^3 d^4 f^3 x e^4 - 52 a^3 b^4 c^2 d^5 f^3 x e^4 \\
& - 16 a^4 b^3 c^* d^6 f^3 x e^4 + 8 a^5 b^2 d^7 f^3 x e^4 - b^7 c^7 f^4 e^3 - 21 a^* b^6 c^6 d f^4 e^3 + 52 a^2 b^5 c^5 d^2 f^4 e^3 + 52 a^5 b^2 c^2 d^5 f^4 e^3 - 21 a^6 b c^* d^6 f^4 e^3 - a^7 d^7 f^4 e^3 \\
& + 12 b^7 d^7 x^3 e^7 - 37 b^7 c^2 d^5 f^* x^2 e^6 - 34 a^* b^6 c^* d^6 f^* x^2 e^6 - 37 a^2 b^5 d^7 f^* x^2 e^6 + 8 b^7 c^4 d^3 f^2 x e^5 + 34 a^* b^6 c^3 d^4 f^2 x e^5 \\
& + 156 a^2 b^5 c^2 d^5 f^2 x e^5 + 34 a^3 b^4 c^* d^6 f^2 x e^5 + 8 a^4 b^3 d^7 f^2 x e^5 + 4 b^7 c^6 d f^3 e^4 + 14 a^* b^6 c^5 d^2 f^3 e^4 - 78 a^2 b^5 c^4 d^3 f^3 e^4 \\
& - 78 a^4 b^3 c^2 d^5 f^3 e^4 + 14 a^5 b^2 c^* d^6 f^3 e^4 + 4 a^6 b^* d^7 f^3 e^4 + 18 b^7 c^* d^6 x^2 e^7 + 18 a^* b^6 d^7 x^2 e^7 - 12 b^7 c^3 d^4 f^* x e^6 \\
& - 66 a^* b^6 c^2 d^5 f^* x e^6 - 66 a^2 b^5 c^* d^6 f^* x e^6 - 12 a^3 b^4 d^7 f^* x e^6 - 6 b^7 c^5 d^2 f^2 e^5 + 14 a^* b^6 c^4 d^3 f^2 e^5 + 52 a^2 b^5 c^3 d^4 f^2 e^5 \\
& + 52 a^3 b^4 c^2 d^5 f^2 e^5 + 14 a^4 b^3 c^* d^6 f^2 e^5 - 6 a^5 b^2 d^7 f^2 e^5 + 4 b^7 c^2 d^5 x e^7 + 28 a^* b^6 c^* d^6 x e^7 + 4 a^2 b^5 d^7 x e^7 + 4 b^7 c^4 d^3 f^* e^6 \\
& - 21 a^* b^6 c^3 d^4 f^* e^6 - 26 a^2 b^5 c^2 d^5 f^* e^6 - 21 a^3 b^4 c^* d^6 f^* e^6 + 4 a^4 b^3 d^7 f^* e^6 - b^7 c^3 d^4 e^7 + 7 a^* b^6 c^2 d^5 e^7 + 7 a^2 b^5 c^* d^6 e^7 - a^3 b^4 d^7 e^7) / ((a^4 b^4 c^8 f^8 - 4 a^5 b^3 c^7 d f^8 + 6 a^6 b^2 c^6 d^2 f^8 - 4 a^7 b c^5 d^3 f^8 + a^8 c^4 d^4 f^8 - 4 a^3 b^5 c^8 f^7 e + 12 a^4 b^4 c^7 d f^7 e - 8 a^5 b^3 c^6 d^2 f^7 e - 8 a^6 b^2 c^5 d^3 f^7 e + 12 a^7 b c^4 d^4 f^7 e - 4 a^8 c^3 d^5 f^7 e + 6 a^2 b^6 c^8 f^6 e^2 - 8 a^3 b^5 c^7 d f^6 e^2 - 22 a^4 b^4 c^6 d^2 f^6 e^2 + 48 a^5 b^3 c^5 d^3 f^6 e^2 - 22 a^6 b^2 c^4 d^4 f^6 e^2 - 8 a^7 b c^3 d^5 f^6 e^2 + 6 a^8 c^2 d^6 f^6 e^2 - 4 a^* b^7 c^8 f^5 e^3 - 8 a^2 b^6 c^7 d f^5 e^3 + 48 a^3 b^5 c^6 d^2 f^5 e^3 - 36 a^4 b^4 c^5 d^3 f^5 e^3 - 36 a^5 b^3 c^4 d^4 f^5 e^3 + 48 a^6 b^2 c^3 d^5 f^5 e^3 - 8 a^7 b c^2 d^6 f^5 e^3 - 4 a^8 c^* d^7 f^5 e^3 + b^8 c^8 f^4 e^4 + 12 a^* b^7 c^7 d f^4 e^4 - 22 a^2 b^6 c^6 d^2 f^4 e^4 - 36 a^3 b^5 c^5 d^3 f^4 e^4 + 90 a^4 b^4 c^4 d^4 f^4 e^4 - 36 a^5 b^3 c^3 d^5 f^4 e^4 - 22 a^6 b^2 c^2 d^6 f^4 e^4 + 12 a^7 b c^* d^7 f^4 e^4 + a^8 d^8 f^4 e^4 - 4 b^8 c^7 d f^3 e^5 - 8 a^* b^7 c^6 d^2 f^3 e^5 + 48 a^2 b^6 c^5 d^3 f^3 e^5 - 36 a^3 b^5 c^4 d^4 f^3 e^5 - 36 a^4 b^4 c^3 d^5 f^3 e^5 + 48 a^5 b^3 c^2 d^6 f^3 e^5 - 8 a^6 b^2 c^* d^7 f^3 e^5 - 4 a^7 b^* d^8 f^3 e^5 + 6 b^8 c^6 d^2 f^2 e^6 - 8 a^* b^7 c^5 d^3 f^2 e^6 - 22 a^2 b^6 c^4 d^4 f^2 e^6 + 48 a^3 b^5 c^3 d^5 f^2 e^6 - 22 a^4 b^4 c^2 d^6 f^2 e^6 - 8 a^5 b^3 c^* d^7 f^2 e^6 + 6 a^6 b^2 d^8 f^2 e^6 - 4 b^8 c^5 d^3 f^* e^7 + 12 a^* b^7 c^4 d^4 f^* e^7 - 8 a^2 b^6 c^3 d^5 f^* e^7 - 8 a^3 b^5 c^2 d^6 f^* e^7 + 12 a^4 b^4 c^* d^7 f^* e^7 - 4 a^5 b^3 d^8 f^* e^7 + b^8 c^4 d^4 e^8 - 4 a^* b^7 c^3 d^5 e^8 + 6 a^2 b^6 c^2 d^6 e^8 - 4 a^3 b^5 c^* d^7 e^8 + a^4 b^4 d^8 e^8) * (b^* d^* f^* x^3 + b^* c^* f^* x^2 + a^* d^* f^* x^2 + b^* d^* x^2 e + a^* c^* f^* x + b^* c^* x e + a^* d^* x e + a^* c^* e)^2)
\end{aligned}$$

maple [B] time = 0.03, size = 1076, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
[Out] 21*d^6/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*b^2*c*e*f-21*b^6/(a*f-b*e)^5/(a*d-
b*c)^5*ln(b*x+a)*a*c*d*f^2-21*b^6/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a*d^2*e
*f+9*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*c*d*e*f+9*f^7/(c*f-d*e)^5/(a*f-b
*e)^5*ln(f*x+e)*a*b*c*d-21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a*b*d^2*e-
21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*b^2*c*d*e+21*d^6/(c*f-d*e)^5/(a*d-
b*c)^5*ln(d*x+c)*a*b*c*f^2-9*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a*b*e*f-
6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*e^2*b^2+3*b^6/(a*f-b*e)^4/(a*d-b*c)
^4/(b*x+a)*c*f+3*b^6/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*d*e+6*b^7/(a*f-b*e)^5/
(a*d-b*c)^5*ln(b*x+a)*c^2*f^2+6*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*d^2*e
^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*c+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*
ln(f*x+e)*a^2*d^2+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*b^2*c^2+3*d^6/(c*
f-d*e)^4/(a*d-b*c)^4/(d*x+c)*a*f+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*e-
6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a^2*f^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)
^4/(f*x+e)*a*d-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2+1/2*d^5/(c*f-d*e)^
3/(a*d-b*c)^3/(d*x+c)^2-1/2*f^5/(c*f-d*e)^3/(a*f-b*e)^3/(f*x+e)^2-6*f^5/(c*
f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*d*e+21*f^5/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)
*d^2*e^2*b^2-6*d^5/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*c*f-21*d^5/(c*f-d*e)^5
/(a*d-b*c)^5*ln(d*x+c)*b^2*c^2*f^2-6*b^5/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*a
*d*f+21*b^5/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a^2*d^2*f^2
```

maxima [B] time = 5.72, size = 11005, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="maxima")
[Out] 3*(2*b^7*d^2*e^2 + (3*b^7*c*d - 7*a*b^6*d^2)*e*f + (2*b^7*c^2 - 7*a*b^6*c*d
+ 7*a^2*b^5*d^2)*f^2)*log(b*x + a)/((b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8
*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*e^5 - 5*(a*b
^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*
b^5*c*d^4 - a^6*b^4*d^5)*e^4*f + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4
*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^3*f^2
- 10*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d
^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*e^2*f^3 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^
4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5
)*e*f^4 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*
c^2*d^3 + 5*a^9*b*c*d^4 - a^10*d^5)*f^5) - 3*(2*b^2*d^7*e^2 - (7*b^2*c*d^6
- 3*a*b*d^7)*e*f + (7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^2)*log(d*x +
c)/((b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d
^8 + 5*a^4*b*c*d^9 - a^5*d^10)*e^5 - 5*(b^5*c^6*d^4 - 5*a*b^4*c^5*d^5 + 10*
a^2*b^3*c^4*d^6 - 10*a^3*b^2*c^3*d^7 + 5*a^4*b*c^2*d^8 - a^5*c*d^9)*e^4*f +
10*(b^5*c^7*d^3 - 5*a*b^4*c^6*d^4 + 10*a^2*b^3*c^5*d^5 - 10*a^3*b^2*c^4*d^
6 + 5*a^4*b*c^3*d^7 - a^5*c^2*d^8)*e^3*f^2 - 10*(b^5*c^8*d^2 - 5*a*b^4*c^7*
d^3 + 10*a^2*b^3*c^6*d^4 - 10*a^3*b^2*c^5*d^5 + 5*a^4*b*c^4*d^6 - a^5*c^3*d
^7)*e^2*f^3 + 5*(b^5*c^9*d - 5*a*b^4*c^8*d^2 + 10*a^2*b^3*c^7*d^3 - 10*a^3*
b^2*c^6*d^4 + 5*a^4*b*c^5*d^5 - a^5*c^4*d^6)*e*f^4 - (b^5*c^10 - 5*a*b^4*c^
9*d + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 + 5*a^4*b*c^6*d^4 - a^5*c^5*d
^5)*f^5) + 3*(7*b^2*d^2*e^2*f^5 - 7*(b^2*c*d + a*b*d^2)*e*f^6 + (2*b^2*c^2
+ 3*a*b*c*d + 2*a^2*d^2)*f^7)*log(f*x + e)/(b^5*d^5*e^10 + a^5*c^5*f^10 - 5
*(b^5*c*d^4 + a*b^4*d^5)*e^9*f + 5*(2*b^5*c^2*d^3 + 5*a*b^4*c*d^4 + 2*a^2*b
```


$$\begin{aligned}
&^3d^5)e^8f^2 - 10*(b^5c^3d^2 + 5*a*b^4c^2d^3 + 5*a^2b^3c*d^4 + a^3 \\
&*b^2d^5)e^7f^3 + 5*(b^5c^4d + 10*a*b^4c^3d^2 + 20*a^2b^3c^2d^3 + \\
&10*a^3b^2c*d^4 + a^4*b*d^5)e^6f^4 - (b^5c^5 + 25*a*b^4c^4d + 100*a^2 \\
&*b^3c^3d^2 + 100*a^3b^2c^2d^3 + 25*a^4*b*c*d^4 + a^5*d^5)e^5f^5 + 5* \\
&(a*b^4c^5 + 10*a^2b^3c^4d + 20*a^3b^2c^3d^2 + 10*a^4*b*c^2d^3 + a^5 \\
&*c*d^4)e^4f^6 - 10*(a^2b^3c^5 + 5*a^3b^2c^4d + 5*a^4*b*c^3d^2 + a^5 \\
&*c^2d^3)e^3f^7 + 5*(2*a^3b^2c^5 + 5*a^4*b*c^4d + 2*a^5*c^3d^2)e^2f \\
&^8 - 5*(a^4*b*c^5 + a^5*c^4d)*e^f^9) - 1/2*((b^7c^3d^4 - 7*a*b^6c^2d^5 \\
&- 7*a^2b^5c*d^6 + a^3b^4d^7)*e^7 - (4*b^7c^4d^3 - 21*a*b^6c^3d^4 - \\
&26*a^2b^5c^2d^5 - 21*a^3b^4c*d^6 + 4*a^4*b^3d^7)*e^6f + 2*(3*b^7c^ \\
&5d^2 - 7*a*b^6c^4d^3 - 26*a^2b^5c^3d^4 - 26*a^3b^4c^2d^5 - 7*a^4*b \\
&^3c*d^6 + 3*a^5b^2d^7)*e^5f^2 - 2*(2*b^7c^6d + 7*a*b^6c^5d^2 - 39*a \\
&^2b^5c^4d^3 - 39*a^4*b^3c^2d^5 + 7*a^5*b^2c*d^6 + 2*a^6*b*d^7)*e^4f^ \\
&3 + (b^7c^7 + 21*a*b^6c^6d - 52*a^2b^5c^5d^2 - 52*a^5b^2c^2d^5 + 2 \\
&1*a^6*b*c*d^6 + a^7*d^7)*e^3f^4 - (7*a*b^6c^7 - 26*a^2b^5c^6d + 52*a^3 \\
&*b^4c^5d^2 - 78*a^4*b^3c^4d^3 + 52*a^5*b^2c^3d^4 - 26*a^6*b*c^2d^5 + \\
&7*a^7*c*d^6)*e^2f^5 - 7*(a^2b^5c^7 - 3*a^3b^4c^6d + 2*a^4*b^3c^5d^ \\
&2 + 2*a^5*b^2c^4d^3 - 3*a^6*b*c^3d^4 + a^7*c^2d^5)*e^f^6 + (a^3b^4c^7 \\
&- 4*a^4*b^3c^6d + 6*a^5b^2c^5d^2 - 4*a^6*b*c^4d^3 + a^7*c^3d^4)*f^7 \\
&- 6*(2*b^7d^7*e^5f^2 - 5*(b^7c^d^6 + a*b^6d^7)*e^4f^3 + 2*(b^7c^2d^ \\
&5 + 8*a*b^6c*d^6 + a^2b^5d^7)*e^3f^4 + 2*(b^7c^3d^4 - 6*a*b^6c^2d^5 \\
&- 6*a^2b^5c*d^6 + a^3b^4d^7)*e^2f^5 - (5*b^7c^4d^3 - 16*a*b^6c^3d \\
&^4 + 12*a^2b^5c^2d^5 - 16*a^3b^4c*d^6 + 5*a^4*b^3d^7)*e^f^6 + (2*b^7* \\
&c^5d^2 - 5*a*b^6c^4d^3 + 2*a^2b^5c^3d^4 + 2*a^3b^4c^2d^5 - 5*a^4*b \\
&^3c*d^6 + 2*a^5b^2d^7)*f^7)*x^5 - 3*(8*b^7d^7*e^6f - 14*(b^7c^d^6 + a \\
&*b^6d^7)*e^5f^2 - (7*b^7c^2d^5 - 34*a*b^6c*d^6 + 7*a^2b^5d^7)*e^4f^ \\
&3 + 2*(7*b^7c^3d^4 + 3*a*b^6c^2d^5 + 3*a^2b^5c*d^6 + 7*a^3b^4d^7)*e \\
&^3f^4 - (7*b^7c^4d^3 - 6*a*b^6c^3d^4 + 78*a^2b^5c^2d^5 - 6*a^3b^4* \\
&c*d^6 + 7*a^4*b^3d^7)*e^2f^5 - 2*(7*b^7c^5d^2 - 17*a*b^6c^4d^3 - 3*a^ \\
&2b^5c^3d^4 - 3*a^3b^4c^2d^5 - 17*a^4*b^3c*d^6 + 7*a^5*b^2d^7)*e^f^6 \\
&+ (8*b^7c^6d - 14*a*b^6c^5d^2 - 7*a^2b^5c^4d^3 + 14*a^3b^4c^3d^4 \\
&- 7*a^4*b^3c^2d^5 - 14*a^5*b^2c*d^6 + 8*a^6*b*d^7)*f^7)*x^4 - 2*(6*b^7* \\
&d^7*e^7 + 3*(b^7c^d^6 + a*b^6d^7)*e^6f - (37*b^7c^2d^5 + 28*a*b^6c*d^ \\
&6 + 37*a^2b^5d^7)*e^5f^2 + (19*b^7c^3d^4 + 86*a*b^6c^2d^5 + 86*a^2b \\
&^5c*d^6 + 19*a^3b^4d^7)*e^4f^3 + (19*b^7c^4d^3 - 68*a*b^6c^3d^4 - 5 \\
&2*a^2b^5c^2d^5 - 68*a^3b^4c*d^6 + 19*a^4*b^3d^7)*e^3f^4 - (37*b^7c^ \\
&5d^2 - 86*a*b^6c^4d^3 + 52*a^2b^5c^3d^4 + 52*a^3b^4c^2d^5 - 86*a^4 \\
&*b^3c*d^6 + 37*a^5b^2d^7)*e^2f^5 + (3*b^7c^6d - 28*a*b^6c^5d^2 + 86 \\
&*a^2b^5c^4d^3 - 68*a^3b^4c^3d^4 + 86*a^4*b^3c^2d^5 - 28*a^5b^2c*d \\
&^6 + 3*a^6*b*d^7)*e^f^6 + (6*b^7c^7 + 3*a*b^6c^6d - 37*a^2b^5c^5d^2 + \\
&19*a^3b^4c^4d^3 + 19*a^4*b^3c^3d^4 - 37*a^5b^2c^2d^5 + 3*a^6*b*c*d \\
&^6 + 6*a^7*d^7)*f^7)*x^3 - (18*(b^7c^d^6 + a*b^6d^7)*e^7 - (37*b^7c^2d^ \\
&5 + 34*a*b^6c*d^6 + 37*a^2b^5d^7)*e^6f - 3*(b^7c^3d^4 - 3*a*b^6c^2d \\
&^5 - 3*a^2b^5c*d^6 + a^3b^4d^7)*e^5f^2 + (32*b^7c^4d^3 + a*b^6c^3d \\
&^4 + 234*a^2b^5c^2d^5 + a^3b^4c*d^6 + 32*a^4*b^3d^7)*e^4f^3 - (3*b^7 \\
&*c^5d^2 - a*b^6c^4d^3 + 208*a^2b^5c^3d^4 + 208*a^3b^4c^2d^5 - a^4* \\
&b^3c*d^6 + 3*a^5b^2d^7)*e^3f^4 - (37*b^7c^6d - 9*a*b^6c^5d^2 - 234* \\
&a^2b^5c^4d^3 + 208*a^3b^4c^3d^4 - 234*a^4*b^3c^2d^5 - 9*a^5b^2c*d \\
&^6 + 37*a^6*b*d^7)*e^2f^5 + (18*b^7c^7 - 34*a*b^6c^6d + 9*a^2b^5c^5d \\
&^2 + a^3b^4c^4d^3 + a^4*b^3c^3d^4 + 9*a^5b^2c^2d^5 - 34*a^6*b*c*d^6 \\
&+ 18*a^7*d^7)*e^f^6 + (18*a*b^6c^7 - 37*a^2b^5c^6d - 3*a^3b^4c^5d^2 \\
&+ 32*a^4*b^3c^4d^3 - 3*a^5b^2c^3d^4 - 37*a^6*b*c^2d^5 + 18*a^7*c*d^6 \\
&)*f^7)*x^2 - 2*(2*(b^7c^2d^5 + 7*a*b^6c*d^6 + a^2b^5d^7)*e^7 - 3*(2*b^ \\
&7c^3d^4 + 11*a*b^6c^2d^5 + 11*a^2b^5c*d^6 + 2*a^3b^4d^7)*e^6f + (4 \\
&*b^7c^4d^3 + 17*a*b^6c^3d^4 + 78*a^2b^5c^2d^5 + 17*a^3b^4c*d^6 + 4 \\
&*a^4*b^3d^7)*e^5f^2 + 2*(2*b^7c^5d^2 - 4*a*b^6c^4d^3 - 13*a^2b^5c^3 \\
&d^4 - 13*a^3b^4c^2d^5 - 4*a^4*b^3c*d^6 + 2*a^5b^2d^7)*e^4f^3 - (6*b \\
&^7c^6d - 17*a*b^6c^5d^2 + 26*a^2b^5c^4d^3 + 26*a^4*b^3c^2d^5 - 17* \\
&a^5b^2c*d^6 + 6*a^6*b*d^7)*e^3f^4 + (2*b^7c^7 - 33*a*b^6c^6d + 78*a^2
\end{aligned}$$

$$\begin{aligned}
& *b^5*c^5*d^2 - 26*a^3*b^4*c^4*d^3 - 26*a^4*b^3*c^3*d^4 + 78*a^5*b^2*c^2*d^5 \\
& - 33*a^6*b*c*d^6 + 2*a^7*d^7)*e^2*f^5 + (14*a*b^6*c^7 - 33*a^2*b^5*c^6*d + \\
& 17*a^3*b^4*c^5*d^2 - 8*a^4*b^3*c^4*d^3 + 17*a^5*b^2*c^3*d^4 - 33*a^6*b*c^2 \\
& *d^5 + 14*a^7*c*d^6)*e*f^6 + 2*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5 \\
& *d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*f^7)*x)/((a^2*b^8 \\
& *c^6*d^4 - 4*a^3*b^7*c^5*d^5 + 6*a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + a^6 \\
& *b^4*c^2*d^8)*e^10 - 4*(a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 2*a^4*b^6*c^5 \\
& *d^5 + 2*a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 + a^7*b^3*c^2*d^8)*e^9*f + 2* \\
& (3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + 24*a^5*b^5*c^5 \\
& *d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8)*e^8*f^2 - \\
& 4*(a^2*b^8*c^9*d + 2*a^3*b^7*c^8*d^2 - 12*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^6 \\
& *d^4 + 9*a^6*b^4*c^5*d^5 - 12*a^7*b^3*c^4*d^6 + 2*a^8*b^2*c^3*d^7 + a^9*b*c^2 \\
& *d^8)*e^7*f^3 + (a^2*b^8*c^10 + 12*a^3*b^7*c^9*d - 22*a^4*b^6*c^8*d^2 - \\
& 36*a^5*b^5*c^7*d^3 + 90*a^6*b^4*c^6*d^4 - 36*a^7*b^3*c^5*d^5 - 22*a^8*b^2*c^4 \\
& *d^6 + 12*a^9*b*c^3*d^7 + a^10*c^2*d^8)*e^6*f^4 - 4*(a^3*b^7*c^10 + 2*a^4 \\
& *b^6*c^9*d - 12*a^5*b^5*c^8*d^2 + 9*a^6*b^4*c^7*d^3 + 9*a^7*b^3*c^6*d^4 - 1 \\
& 2*a^8*b^2*c^5*d^5 + 2*a^9*b*c^4*d^6 + a^10*c^3*d^7)*e^5*f^5 + 2*(3*a^4*b^6*c^10 \\
& - 4*a^5*b^5*c^9*d - 11*a^6*b^4*c^8*d^2 + 24*a^7*b^3*c^7*d^3 - 11*a^8*b^2*c^6 \\
& *d^4 - 4*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^4*f^6 - 4*(a^5*b^5*c^10 - 3*a^6 \\
& *b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10 \\
& *c^5*d^5)*e^3*f^7 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4 \\
& *a^9*b*c^7*d^3 + a^10*c^6*d^4)*e^2*f^8 + ((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + \\
& 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^8*f^2 - 4*(b^10 \\
& *c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6 \\
& *c*d^9 + a^5*b^5*d^10)*e^7*f^3 + 2*(3*b^10*c^6*d^4 - 4*a*b^9*c^5*d^5 - 1 \\
& 1*a^2*b^8*c^4*d^6 + 24*a^3*b^7*c^3*d^7 - 11*a^4*b^6*c^2*d^8 - 4*a^5*b^5*c*d^9 \\
& + 3*a^6*b^4*d^10)*e^6*f^4 - 4*(b^10*c^7*d^3 + 2*a*b^9*c^6*d^4 - 12*a^2*b^8 \\
& *c^5*d^5 + 9*a^3*b^7*c^4*d^6 + 9*a^4*b^6*c^3*d^7 - 12*a^5*b^5*c^2*d^8 + 2 \\
& *a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^5*f^5 + (b^10*c^8*d^2 + 12*a*b^9*c^7*d^3 - \\
& 22*a^2*b^8*c^6*d^4 - 36*a^3*b^7*c^5*d^5 + 90*a^4*b^6*c^4*d^6 - 36*a^5*b^5*c^3 \\
& *d^7 - 22*a^6*b^4*c^2*d^8 + 12*a^7*b^3*c*d^9 + a^8*b^2*d^10)*e^4*f^6 - 4* \\
& (a*b^9*c^8*d^2 + 2*a^2*b^8*c^7*d^3 - 12*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^5 + \\
& 9*a^5*b^5*c^4*d^6 - 12*a^6*b^4*c^3*d^7 + 2*a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9) \\
& *e^3*f^7 + 2*(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + \\
& 24*a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2 \\
& *d^8)*e^2*f^8 - 4*(a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 2*a^5*b^5*c^6*d^4 + \\
& 2*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 + a^8*b^2*c^3*d^7)*e*f^9 + (a^4*b^6 \\
& *c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8 \\
& *b^2*c^4*d^6)*f^10)*x^6 + 2*((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8 \\
& *c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^9*f - 3*(b^10*c^5*d^5 - 3*a*b^9 \\
& *c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b^5 \\
& *d^10)*e^8*f^2 + 2*(b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7 - \\
& 9*a^4*b^6*c^2*d^8 + a^6*b^4*d^10)*e^7*f^3 + 2*(b^10*c^7*d^3 - 5*a*b^9*c^6 \\
& *d^4 + 9*a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 - 5*a^4*b^6*c^3*d^7 + 9*a^5*b^5 \\
& *c^2*d^8 - 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^6*f^4 - 3*(b^10*c^8*d^2 - 6*a^2 \\
& *b^8*c^6*d^4 + 8*a^3*b^7*c^5*d^5 - 6*a^4*b^6*c^4*d^6 + 8*a^5*b^5*c^3*d^7 - \\
& 6*a^6*b^4*c^2*d^8 + a^8*b^2*d^10)*e^5*f^5 + (b^10*c^9*d + 9*a*b^9*c^8*d^2 - \\
& 18*a^2*b^8*c^7*d^3 - 10*a^3*b^7*c^6*d^4 + 18*a^4*b^6*c^5*d^5 + 18*a^5*b^5 \\
& *c^4*d^6 - 10*a^6*b^4*c^3*d^7 - 18*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9 \\
& *b*d^10)*e^4*f^6 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3 \\
& + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^3 \\
& *c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^3*f^7 + 6*(a^2*b^8*c^9*d - \\
& a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 3*a^5*b^5*c^6*d^4 + 3*a^6*b^4*c^5*d^5 - \\
& 3*a^7*b^3*c^4*d^6 - a^8*b^2*c^3*d^7 + a^9*b*c^2*d^8)*e^2*f^8 - (4*a^3*b^7 \\
& *c^9*d - 9*a^4*b^6*c^8*d^2 + 10*a^6*b^4*c^6*d^4 - 9*a^8*b^2*c^4*d^6 + 4*a^9 \\
& *b*c^3*d^7)*e*f^9 + (a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 \\
& + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*f^10)*x^5 + ((b^10 \\
& *c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6 \\
& *d^10)*e^10 - 3*(3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 5*
\end{aligned}$$

$$\begin{aligned}
& a^4 b^6 c^2 d^8 - 8 a^5 b^5 c^2 d^9 + 3 a^6 b^4 c^2 d^{10} e^8 f^2 + 4 (4 b^{10} c^7 \\
& d^3 - 5 a b^9 c^6 d^4 - 9 a^2 b^8 c^5 d^5 + 10 a^3 b^7 c^4 d^6 + 10 a^4 b^6 \\
& c^3 d^7 - 9 a^5 b^5 c^2 d^8 - 5 a^6 b^4 c^2 d^9 + 4 a^7 b^3 c^2 d^{10}) e^7 f^3 - \\
& (9 b^{10} c^8 d^2 + 20 a b^9 c^7 d^3 - 90 a^2 b^8 c^6 d^4 + 36 a^3 b^7 c^5 d^5 \\
& + 50 a^4 b^6 c^4 d^6 + 36 a^5 b^5 c^3 d^7 - 90 a^6 b^4 c^2 d^8 + 20 a^7 b^3 \\
& c^2 d^9 + 9 a^8 b^2 c^2 d^{10}) e^6 f^4 + 12 (2 a b^9 c^8 d^2 - 3 a^2 b^8 c^7 d^3 \\
& - 3 a^3 b^7 c^6 d^4 + 4 a^4 b^6 c^5 d^5 + 4 a^5 b^5 c^4 d^6 - 3 a^6 b^4 c^3 d^7 \\
& - 3 a^7 b^3 c^2 d^8 + 2 a^8 b^2 c^2 d^9) e^5 f^5 + (b^{10} c^{10} - 15 a^2 \\
& b^8 c^8 d^2 + 40 a^3 b^7 c^7 d^3 - 50 a^4 b^6 c^6 d^4 + 48 a^5 b^5 c^5 d^5 \\
& - 50 a^6 b^4 c^4 d^6 + 40 a^7 b^3 c^3 d^7 - 15 a^8 b^2 c^2 d^8 + a^{10} d^{10}) \\
& e^4 f^6 - 4 (a b^9 c^{10} - 10 a^4 b^6 c^7 d^3 + 9 a^5 b^5 c^6 d^4 + 9 a^6 \\
& b^4 c^5 d^5 - 10 a^7 b^3 c^4 d^6 + a^{10} c^2 d^9) e^3 f^7 + 3 (2 a^2 b^8 c^{10} \\
& - 5 a^4 b^6 c^8 d^2 - 12 a^5 b^5 c^7 d^3 + 30 a^6 b^4 c^6 d^4 - 12 a^7 b^3 \\
& c^5 d^5 - 5 a^8 b^2 c^4 d^6 + 2 a^{10} c^2 d^8) e^2 f^8 - 4 (a^3 b^7 c^{10} - \\
& 6 a^5 b^5 c^8 d^2 + 5 a^6 b^4 c^7 d^3 + 5 a^7 b^3 c^6 d^4 - 6 a^8 b^2 c^5 d^5 \\
& + a^{10} c^3 d^7) e f^9 + (a^4 b^6 c^{10} - 9 a^6 b^4 c^8 d^2 + 16 a^7 b^3 c^7 \\
& d^3 - 9 a^8 b^2 c^6 d^4 + a^{10} c^4 d^6) f^{10} x^4 + 2 ((b^{10} c^5 d^5 - 3 \\
& a b^9 c^4 d^6 + 2 a^2 b^8 c^3 d^7 + 2 a^3 b^7 c^2 d^8 - 3 a^4 b^6 c^2 d^9 + \\
& a^5 b^5 d^{10}) e^{10} - (3 b^{10} c^6 d^4 - 8 a b^9 c^5 d^5 + 5 a^2 b^8 c^4 d^6 \\
& + 5 a^4 b^6 c^2 d^8 - 8 a^5 b^5 c^2 d^9 + 3 a^6 b^4 d^{10}) e^9 f + (2 b^{10} c^7 \\
& d^3 - 5 a b^9 c^6 d^4 + 3 a^2 b^8 c^5 d^5 + 3 a^5 b^5 c^2 d^8 - 5 a^6 b^4 c^2 \\
& d^9 + 2 a^7 b^3 d^{10}) e^8 f^2 + 2 (b^{10} c^8 d^2 - 16 a^3 b^7 c^5 d^5 + 30 \\
& a^4 b^6 c^4 d^6 - 16 a^5 b^5 c^3 d^7 + a^8 b^2 d^{10}) e^7 f^3 - (3 b^{10} c^9 \\
& d + 5 a b^9 c^8 d^2 - 60 a^3 b^7 c^6 d^4 + 52 a^4 b^6 c^5 d^5 + 52 a^5 b^5 \\
& c^4 d^6 - 60 a^6 b^4 c^3 d^7 + 5 a^8 b^2 c^2 d^9 + 3 a^9 b^2 d^{10}) e^6 f^4 + (\\
& b^{10} c^{10} + 8 a b^9 c^9 d + 3 a^2 b^8 c^8 d^2 - 32 a^3 b^7 c^7 d^3 - 52 a^4 \\
& b^6 c^6 d^4 + 144 a^5 b^5 c^5 d^5 - 52 a^6 b^4 c^4 d^6 - 32 a^7 b^3 c^3 d^7 \\
& + 3 a^8 b^2 c^2 d^8 + 8 a^9 b^2 c^2 d^9 + a^{10} d^{10}) e^5 f^5 - (3 a b^9 c^{10} \\
& + 5 a^2 b^8 c^9 d - 60 a^4 b^6 c^7 d^3 + 52 a^5 b^5 c^6 d^4 + 52 a^6 b^4 c^5 \\
& d^5 - 60 a^7 b^3 c^4 d^6 + 5 a^9 b^2 c^2 d^8 + 3 a^{10} c^2 d^9) e^4 f^6 + 2 (a \\
& ^2 b^8 c^{10} - 16 a^5 b^5 c^7 d^3 + 30 a^6 b^4 c^6 d^4 - 16 a^7 b^3 c^5 d^5 \\
& + a^{10} c^2 d^8) e^3 f^7 + (2 a^3 b^7 c^{10} - 5 a^4 b^6 c^9 d + 3 a^5 b^5 c^8 \\
& d^2 + 3 a^8 b^2 c^5 d^5 - 5 a^9 b^2 c^4 d^6 + 2 a^{10} c^3 d^7) e^2 f^8 - (3 a \\
& ^4 b^6 c^{10} - 8 a^5 b^5 c^9 d + 5 a^6 b^4 c^8 d^2 + 5 a^8 b^2 c^6 d^4 - 8 a^9 \\
& b^2 c^5 d^5 + 3 a^{10} c^4 d^6) e f^9 + (a^5 b^5 c^{10} - 3 a^6 b^4 c^9 d + 2 \\
& a^7 b^3 c^8 d^2 + 2 a^8 b^2 c^7 d^3 - 3 a^9 b^2 c^6 d^4 + a^{10} c^5 d^5) f^{10} \\
& x^3 + ((b^{10} c^6 d^4 - 9 a^2 b^8 c^4 d^6 + 16 a^3 b^7 c^3 d^7 - 9 a^4 b^6 c^2 \\
& d^8 + a^6 b^4 d^{10}) e^{10} - 4 (b^{10} c^7 d^3 - 6 a^2 b^8 c^5 d^5 + 5 a^3 b^7 \\
& c^4 d^6 + 5 a^4 b^6 c^3 d^7 - 6 a^5 b^5 c^2 d^8 + a^7 b^3 d^{10}) e^9 f + \\
& 3 (2 b^{10} c^8 d^2 - 5 a^2 b^8 c^6 d^4 - 12 a^3 b^7 c^5 d^5 + 30 a^4 b^6 c^4 \\
& d^6 - 12 a^5 b^5 c^3 d^7 - 5 a^6 b^4 c^2 d^8 + 2 a^8 b^2 d^{10}) e^8 f^2 - \\
& 4 (b^{10} c^9 d - 10 a^3 b^7 c^6 d^4 + 9 a^4 b^6 c^5 d^5 + 9 a^5 b^5 c^4 d^6 \\
& - 10 a^6 b^4 c^3 d^7 + a^9 b^2 d^{10}) e^7 f^3 + (b^{10} c^{10} - 15 a^2 b^8 c^8 d^2 \\
& + 40 a^3 b^7 c^7 d^3 - 50 a^4 b^6 c^6 d^4 + 48 a^5 b^5 c^5 d^5 - 50 a^6 b^4 \\
& c^4 d^6 + 40 a^7 b^3 c^3 d^7 - 15 a^8 b^2 c^2 d^8 + a^{10} d^{10}) e^6 f^4 + \\
& 12 (2 a^2 b^8 c^9 d - 3 a^3 b^7 c^8 d^2 - 3 a^4 b^6 c^7 d^3 + 4 a^5 b^5 c^6 \\
& d^4 + 4 a^6 b^4 c^5 d^5 - 3 a^7 b^3 c^4 d^6 - 3 a^8 b^2 c^3 d^7 + 2 a^9 b^2 \\
& c^2 d^8) e^5 f^5 - (9 a^2 b^8 c^{10} + 20 a^3 b^7 c^9 d - 90 a^4 b^6 c^8 d^2 \\
& + 36 a^5 b^5 c^7 d^3 + 50 a^6 b^4 c^6 d^4 + 36 a^7 b^3 c^5 d^5 - 90 a^8 b^2 \\
& c^4 d^6 + 20 a^9 b^2 c^3 d^7 + 9 a^{10} c^2 d^8) e^4 f^6 + 4 (4 a^3 b^7 c^{10} \\
& - 5 a^4 b^6 c^9 d - 9 a^5 b^5 c^8 d^2 + 10 a^6 b^4 c^7 d^3 + 10 a^7 b^3 c^6 \\
& d^4 - 9 a^8 b^2 c^5 d^5 - 5 a^9 b^2 c^4 d^6 + 4 a^{10} c^3 d^7) e^3 f^7 - 3 (3 \\
& a^4 b^6 c^{10} - 8 a^5 b^5 c^9 d + 5 a^6 b^4 c^8 d^2 + 5 a^8 b^2 c^6 d^4 - 8 \\
& a^9 b^2 c^5 d^5 + 3 a^{10} c^4 d^6) e^2 f^8 + (a^6 b^4 c^{10} - 4 a^7 b^3 c^9 d \\
& + 6 a^8 b^2 c^8 d^2 - 4 a^9 b^2 c^7 d^3 + a^{10} c^6 d^4) f^{10} x^2 + 2 ((a b^9 \\
& c^6 d^4 - 3 a^2 b^8 c^5 d^5 + 2 a^3 b^7 c^4 d^6 + 2 a^4 b^6 c^3 d^7 - 3 a^5 \\
& b^5 c^2 d^8 + a^6 b^4 c^2 d^9) e^{10} - (4 a b^9 c^7 d^3 - 9 a^2 b^8 c^6 d^4 \\
& + 10 a^4 b^6 c^4 d^6 - 9 a^6 b^4 c^2 d^8 + 4 a^7 b^3 c^2 d^9) e^9 f + 6 (a b^9 \\
& c^8 d^2 - a^2 b^8 c^7 d^3 - 3 a^3 b^7 c^6 d^4 + 3 a^4 b^6 c^5 d^5 + 3 a^5
\end{aligned}$$

$$\begin{aligned} & *b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 - a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9)*e^8*f^2 \\ & - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3 + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^3*c^3*d^7 + 3*a^8 \\ & *b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^7*f^3 + (a*b^9*c^10 + 9*a^2*b^8*c^9*d - 18* \\ & a^3*b^7*c^8*d^2 - 10*a^4*b^6*c^7*d^3 + 18*a^5*b^5*c^6*d^4 + 18*a^6*b^4*c^5* \\ & d^5 - 10*a^7*b^3*c^4*d^6 - 18*a^8*b^2*c^3*d^7 + 9*a^9*b*c^2*d^8 + a^10*c*d^9) \\ & *e^6*f^4 - 3*(a^2*b^8*c^10 - 6*a^4*b^6*c^8*d^2 + 8*a^5*b^5*c^7*d^3 - 6*a^6*b^4*c^6*d^4 + 8*a^7*b^3*c^5*d^5 - 6*a^8*b^2*c^4*d^6 + a^10*c^2*d^8)*e^5*f^5 \\ & + 2*(a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 9*a^5*b^5*c^8*d^2 - 5*a^6*b^4*c^7*d^3 - 5*a^7*b^3*c^6*d^4 + 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + a^10*c^3*d^7) \\ & *e^4*f^6 + 2*(a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*e^3*f^7 - 3*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d \\ & + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*e^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4) \\ & *e*f^9)*x) \end{aligned}$$

mupad [B] time = 20.46, size = 82532, normalized size = 166.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3)^3, x)$

[Out] $\text{symsum}(\log(\text{root}(756756*a^{10}*b^{10}*c^{10}*d^{10}*e^{10}*f^{10}*z^3 + 573300*a^{12}*b^8*c^9*d^{11}*e^9*f^{11}*z^3 + 573300*a^{11}*b^9*c^{11}*d^9*e^8*f^{12}*z^3 + 573300*a^{11}*b^9*c^8*d^{12}*e^{11}*f^9*z^3 + 573300*a^9*b^{11}*c^{12}*d^8*e^9*f^{11}*z^3 + 573300*a^9*b^{11}*c^9*d^{11}*e^{12}*f^8*z^3 + 573300*a^8*b^{12}*c^{11}*d^9*e^{11}*f^9*z^3 - 343980*a^{11}*b^9*c^{10}*d^{10}*e^9*f^{11}*z^3 - 343980*a^{11}*b^9*c^9*d^{11}*e^{10}*f^{10}*z^3 - 343980*a^{10}*b^{10}*c^{11}*d^9*e^9*f^{11}*z^3 - 343980*a^{10}*b^{10}*c^9*d^{11}*e^{11}*f^9*z^3 - 343980*a^9*b^{11}*c^{11}*d^9*e^{10}*f^{10}*z^3 - 343980*a^9*b^{11}*c^{10}*d^{10}*e^{11}*f^9*z^3 + 326340*a^{13}*b^7*c^{10}*d^{10}*e^7*f^{13}*z^3 + 326340*a^{13}*b^7*c^7*d^{13}*e^{10}*f^{10}*z^3 + 326340*a^{10}*b^{10}*c^{13}*d^7*e^7*f^{13}*z^3 + 326340*a^{10}*b^{10}*c^7*d^{13}*e^{13}*f^7*z^3 + 326340*a^7*b^{13}*c^{13}*d^7*e^{10}*f^{10}*z^3 + 326340*a^7*b^{13}*c^{10}*d^{10}*e^{13}*f^7*z^3 - 267540*a^{12}*b^8*c^{10}*d^{10}*e^8*f^{12}*z^3 - 267540*a^{12}*b^8*c^8*d^{12}*e^{10}*f^{10}*z^3 - 267540*a^{10}*b^{10}*c^{12}*d^8*e^8*f^{12}*z^3 - 267540*a^{10}*b^{10}*c^8*d^{12}*e^{12}*f^8*z^3 - 267540*a^8*b^{12}*c^{12}*d^8*e^{10}*f^{10}*z^3 - 267540*a^8*b^{12}*c^{10}*d^{10}*e^{12}*f^8*z^3 + 245700*a^{14}*b^6*c^8*d^{12}*e^8*f^{12}*z^3 + 245700*a^{12}*b^8*c^{12}*d^8*e^6*f^{14}*z^3 + 245700*a^{12}*b^8*c^6*d^{14}*e^{12}*f^8*z^3 + 245700*a^8*b^{12}*c^{14}*d^6*e^8*f^{12}*z^3 + 245700*a^8*b^{12}*c^8*d^{12}*e^{14}*f^6*z^3 + 245700*a^6*b^{14}*c^{12}*d^8*e^{12}*f^8*z^3 - 191100*a^{13}*b^7*c^9*d^{11}*e^8*f^{12}*z^3 - 191100*a^{13}*b^7*c^8*d^{12}*e^9*f^{11}*z^3 - 191100*a^{12}*b^8*c^{11}*d^9*e^7*f^{13}*z^3 - 191100*a^{12}*b^8*c^7*d^{13}*e^{11}*f^9*z^3 - 191100*a^{11}*b^9*c^{12}*d^8*e^7*f^{13}*z^3 - 191100*a^{11}*b^9*c^7*d^{13}*e^{12}*f^8*z^3 - 191100*a^9*b^{11}*c^{13}*d^7*e^8*f^{12}*z^3 - 191100*a^9*b^{11}*c^8*d^{12}*e^{13}*f^7*z^3 - 191100*a^8*b^{12}*c^{13}*d^7*e^9*f^{11}*z^3 - 191100*a^8*b^{12}*c^9*d^{11}*e^{13}*f^7*z^3 - 191100*a^7*b^{13}*c^{12}*d^8*e^{11}*f^9*z^3 - 191100*a^7*b^{13}*c^{11}*d^9*e^{12}*f^8*z^3 - 123900*a^{14}*b^6*c^9*d^{11}*e^7*f^{13}*z^3 - 123900*a^{14}*b^6*c^7*d^{13}*e^9*f^{11}*z^3 - 123900*a^{13}*b^7*c^{11}*d^9*e^6*f^{14}*z^3 - 123900*a^{13}*b^7*c^6*d^{14}*e^{11}*f^9*z^3 - 123900*a^{11}*b^9*c^{13}*d^7*e^6*f^{14}*z^3 - 123900*a^{11}*b^9*c^6*d^{14}*e^{13}*f^7*z^3 - 123900*a^9*b^{11}*c^{14}*d^6*e^7*f^{13}*z^3 - 123900*a^9*b^{11}*c^7*d^{13}*e^{14}*f^6*z^3 - 123900*a^7*b^{13}*c^{14}*d^6*e^9*f^{11}*z^3 - 123900*a^7*b^{13}*c^9*d^{11}*e^{14}*f^6*z^3 - 123900*a^6*b^{14}*c^{13}*d^7*e^{11}*f^9*z^3 - 123900*a^6*b^{14}*c^{11}*d^9*e^{13}*f^7*z^3 + 101700*a^{15}*b^5*c^9*d^{11}*e^6*f^{14}*z^3 + 101700*a^{15}*b^5*c^6*d^{14}*e^9*f^{11}*z^3 + 101700*a^{14}*b^6*c^{11}*d^9*e^5*f^{15}*z^3 + 101700*a^{14}*b^6*c^5*d^{15}*e^{11}*f^9*z^3 + 101700*a^{11}*b^9*c^{14}*d^6*e^5*f^{15}*z^3 + 101700*a^{11}*b^9*c^5*d^{15}*e^{14}*f^6*z^3 + 101700*a^9*b^{11}*c^{15}*d^5*e^6*f^{14}*z^3 + 101700*a^9*b^{11}*c^6*d^{14}*e^{15}*f^5*z^3 + 101700*a^6*b^{14}*c^{15}*d^5*e^9*f^{11}*z^3 + 101700*a^6*b^{14}*c^9*d^{11}*e^{15}*f^5*z^3 + 101700*a^5*b^{15}*c^{14}*d^6*e^{11}*f^9*z^3 + 101700*a^5*b^{15}*c^{11}*d^$

$$\begin{aligned}
& 9e^{14}f^6z^3 - 65820a^{14}b^6c^{10}d^{10}e^6f^{14}z^3 - 65820a^{14}b^6c^6d^{14}e^{10}f^{10}z^3 - 65820a^{10}b^{10}c^{14}d^6e^6f^{14}z^3 - 65820a^{10}b^{10}c^6d^{14}e^{14}f^6z^3 - 65820a^6b^{14}c^{14}d^6e^{10}f^{10}z^3 - 65820a^6b^{14}c^{10}d^{10}e^{14}f^6z^3 + 56700a^{16}b^4c^7d^{13}e^7f^{13}z^3 - 56700a^{15}b^5c^8d^{12}e^7f^{13}z^3 - 56700a^{15}b^5c^7d^{13}e^8f^{12}z^3 + 56700a^{13}b^7c^{13}d^7e^4f^{16}z^3 - 56700a^{13}b^7c^{12}d^8e^5f^{15}z^3 - 56700a^{13}b^7c^5d^{15}e^{12}f^8z^3 + 56700a^{13}b^7c^4d^{16}e^{13}f^7z^3 - 56700a^{12}b^8c^{13}d^7e^5f^{15}z^3 - 56700a^{12}b^8c^5d^{15}e^{13}f^7z^3 - 56700a^8b^{12}c^{15}d^5e^7f^{13}z^3 - 56700a^8b^{12}c^7d^{13}e^{15}f^5z^3 + 56700a^7b^{13}c^{16}d^4e^7f^{13}z^3 - 56700a^7b^{13}c^{15}d^5e^8f^{12}z^3 - 56700a^7b^{13}c^8d^{12}e^{15}f^5z^3 + 56700a^7b^{13}c^7d^{13}e^{16}f^4z^3 - 56700a^5b^{15}c^{13}d^7e^{12}f^8z^3 - 56700a^5b^{15}c^{12}d^8e^{13}f^7z^3 + 56700a^4b^{16}c^{13}d^7e^{13}f^7z^3 - 48252a^{15}b^5c^{10}d^{10}e^5f^{15}z^3 - 48252a^{15}b^5c^5d^{15}e^{10}f^{10}z^3 - 48252a^{10}b^{10}c^{15}d^5e^5f^{15}z^3 - 48252a^{10}b^{10}c^5d^{15}e^{15}f^5z^3 - 48252a^5b^{15}c^{15}d^5e^{10}f^{10}z^3 - 48252a^5b^{15}c^{10}d^{10}e^{15}f^5z^3 - 32400a^{16}b^4c^8d^{12}e^6f^{14}z^3 - 32400a^{16}b^4c^6d^{14}e^8f^{12}z^3 - 32400a^{14}b^6c^{12}d^8e^4f^{16}z^3 - 32400a^{14}b^6c^4d^{16}e^{12}f^8z^3 - 32400a^{12}b^8c^{14}d^6e^4f^{16}z^3 - 32400a^{12}b^8c^4d^{16}e^{14}f^6z^3 - 32400a^8b^{12}c^{16}d^4e^6f^{14}z^3 - 32400a^8b^{12}c^6d^{14}e^{16}f^4z^3 - 32400a^6b^{14}c^{16}d^4e^8f^{12}z^3 - 32400a^6b^{14}c^8d^{12}e^{16}f^4z^3 - 32400a^4b^{16}c^{14}d^6e^{12}f^8z^3 - 32400a^4b^{16}c^{12}d^8e^{14}f^6z^3 + 20565a^{16}b^4c^{10}d^{10}e^4f^{16}z^3 + 20565a^{16}b^4c^4d^{16}e^{10}f^{10}z^3 + 20565a^{10}b^{10}c^{16}d^4e^4f^{16}z^3 + 20565a^{10}b^{10}c^4d^{16}e^{16}f^4z^3 + 20565a^4b^{16}c^{16}d^4e^{10}f^{10}z^3 + 20565a^4b^{16}c^{10}d^{10}e^{16}f^4z^3 + 15660a^{17}b^3c^8d^{12}e^5f^{15}z^3 + 15660a^{17}b^3c^5d^{15}e^8f^{12}z^3 + 15660a^{15}b^5c^{12}d^8e^3f^{17}z^3 + 15660a^{15}b^5c^3d^{17}e^{12}f^8z^3 + 15660a^{12}b^8c^{15}d^5e^3f^{17}z^3 + 15660a^{12}b^8c^3d^{17}e^{15}f^5z^3 + 15660a^8b^{12}c^{17}d^3e^5f^{15}z^3 + 15660a^8b^{12}c^5d^{15}e^{17}f^3z^3 + 15660a^5b^{15}c^{17}d^3e^8f^{12}z^3 + 15660a^5b^{15}c^8d^{12}e^{17}f^3z^3 + 15660a^3b^{17}c^{15}d^5e^{12}f^8z^3 + 15660a^3b^{17}c^{12}d^8e^{15}f^5z^3 - 9750a^{17}b^3c^9d^{11}e^4f^{16}z^3 - 9750a^{17}b^3c^4d^{16}e^9f^{11}z^3 - 9750a^{16}b^4c^{11}d^9e^3f^{17}z^3 - 9750a^{16}b^4c^3d^{17}e^{11}f^9z^3 - 9750a^{11}b^9c^{16}d^4e^3f^{17}z^3 - 9750a^{11}b^9c^3d^{17}e^{16}f^4z^3 - 9750a^9b^{11}c^{17}d^3e^4f^{16}z^3 - 9750a^9b^{11}c^4d^{16}e^{17}f^3z^3 - 9750a^4b^{16}c^{17}d^3e^9f^{11}z^3 - 9750a^4b^{16}c^9d^{11}e^{17}f^3z^3 - 9750a^3b^{17}c^{16}d^4e^{11}f^9z^3 - 9750a^3b^{17}c^{11}d^9e^{16}f^4z^3 - 8100a^{17}b^3c^7d^{13}e^6f^{14}z^3 - 8100a^{17}b^3c^6d^{14}e^7f^{13}z^3 - 8100a^{14}b^6c^{13}d^7e^3f^{17}z^3 - 8100a^{14}b^6c^3d^{17}e^{13}f^7z^3 - 8100a^{13}b^7c^{14}d^6e^3f^{17}z^3 - 8100a^{13}b^7c^3d^{17}e^{14}f^6z^3 - 8100a^7b^{13}c^{17}d^3e^6f^{14}z^3 - 8100a^7b^{13}c^6d^{14}e^{17}f^3z^3 - 8100a^6b^{14}c^{17}d^3e^7f^{13}z^3 - 8100a^6b^{14}c^7d^{13}e^{17}f^3z^3 - 8100a^3b^{17}c^{14}d^6e^{13}f^7z^3 - 8100a^3b^{17}c^{13}d^7e^{14}f^6z^3 - 7980a^{16}b^4c^9d^{11}e^5f^{15}z^3 - 7980a^{16}b^4c^5d^{15}e^9f^{11}z^3 - 7980a^{15}b^5c^{11}d^9e^4f^{16}z^3 - 7980a^{15}b^5c^4d^{16}e^{11}f^9z^3 - 7980a^{11}b^9c^{15}d^5e^4f^{16}z^3 - 7980a^{11}b^9c^4d^{16}e^{15}f^5z^3 - 7980a^9b^{11}c^{16}d^4e^5f^{15}z^3 - 7980a^9b^{11}c^5d^{15}e^{16}f^4z^3 - 7980a^5b^{15}c^{16}d^4e^9f^{11}z^3 - 7980a^5b^{15}c^9d^{11}e^{16}f^4z^3 - 7980a^4b^{16}c^{15}d^5e^{11}f^9z^3 - 7980a^4b^{16}c^{11}d^9e^{15}f^5z^3 + 6300a^{18}b^2c^6d^{14}e^6f^{14}z^3 + 6300a^{14}b^6c^{14}d^6e^2f^{18}z^3 + 6300a^{14}b^6c^2d^{18}e^{14}f^6z^3 + 6300a^6b^{14}c^{18}d^2e^6f^{14}z^3 + 6300a^6b^{14}c^6d^{14}e^{18}f^2z^3 + 6300a^2b^{18}c^{14}d^6e^{14}f^6z^3 - 4260a^{18}b^2c^7d^{13}e^5f^{15}z^3 - 4260a^{18}b^2c^5d^{15}e^7f^{13}z^3 - 4260a^{15}b^5c^{13}d^7e^2f^{18}z^3 - 4260a^{15}b^5c^2d^{18}e^{13}f^7z^3 - 4260a^{13}b^7c^{15}d^5e^2f^{18}z^3 - 4260a^{13}b^7c^2d^{18}e^{15}f^5z^3 - 4260a^7b^{13}c^{18}d^2e^5f^{15}z^3 - 4260a^7b^{13}c^5d^{15}e^{18}f^2z^3 - 4260a^5b^{15}c^{18}d^2e^7f^{13}z^3 - 4260a^5b^{15}c^7d^{13}e^{18}f^2z^3 - 4260a^2b^{18}c^{15}d^5e^{13}f^7z^3 - 4260a^2b^{18}c^{13}d^7e^{15}f^5z^3
\end{aligned}$$

$$\begin{aligned}
& + 1470*a^{17}*b^3*c^{10}*d^{10}*e^3*f^{17}*z^3 + 1470*a^{17}*b^3*c^3*d^{17}*e^{10}*f^{10}*z^3 + 1470*a^{10}*b^{10}*c^{17}*d^3*e^3*f^{17}*z^3 + 1470*a^{10}*b^{10}*c^3*d^{17}*e^{17}*f^3*z^3 + 1470*a^3*b^{17}*c^{17}*d^3*e^{10}*f^{10}*z^3 + 1470*a^3*b^{17}*c^{10}*d^{10}*e^{17}*f^3*z^3 + 1350*a^{18}*b^2*c^9*d^{11}*e^3*f^{17}*z^3 + 1350*a^{18}*b^2*c^3*d^{17}*e^9*f^{11}*z^3 + 1350*a^{17}*b^3*c^{11}*d^9*e^2*f^{18}*z^3 + 1350*a^{17}*b^3*c^2*d^{18}*e^{11}*f^9*z^3 + 1350*a^{11}*b^9*c^{17}*d^3*e^2*f^{18}*z^3 + 1350*a^{11}*b^9*c^2*d^{18}*e^{17}*f^3*z^3 + 1350*a^9*b^{11}*c^{18}*d^2*e^3*f^{17}*z^3 + 1350*a^9*b^{11}*c^3*d^{17}*e^{18}*f^2*z^3 + 1350*a^3*b^{17}*c^{18}*d^2*e^9*f^{11}*z^3 + 1350*a^3*b^{17}*c^9*d^{11}*e^{18}*f^2*z^3 + 1350*a^2*b^{18}*c^{17}*d^3*e^{11}*f^9*z^3 + 1350*a^2*b^{18}*c^{11}*d^9*e^{17}*f^3*z^3 - 1070*a^{18}*b^2*c^{10}*d^{10}*e^2*f^{18}*z^3 - 1070*a^{18}*b^2*c^2*d^{18}*e^{10}*f^{10}*z^3 - 1070*a^{10}*b^{10}*c^{18}*d^2*e^2*f^{18}*z^3 - 1070*a^{10}*b^{10}*c^2*d^{18}*e^{18}*f^2*z^3 - 1070*a^2*b^{18}*c^{18}*d^2*e^{10}*f^{10}*z^3 - 1070*a^2*b^{18}*c^{10}*d^{10}*e^{18}*f^2*z^3 + 525*a^{18}*b^2*c^8*d^{12}*e^4*f^{16}*z^3 + 525*a^{18}*b^2*c^4*d^{16}*e^8*f^{12}*z^3 + 525*a^{16}*b^4*c^{12}*d^8*e^2*f^{18}*z^3 + 525*a^{16}*b^4*c^2*d^{18}*e^{12}*f^8*z^3 + 525*a^{12}*b^8*c^{16}*d^4*e^2*f^{18}*z^3 + 525*a^{12}*b^8*c^2*d^{18}*e^{16}*f^4*z^3 + 525*a^8*b^{12}*c^{18}*d^2*e^4*f^{16}*z^3 + 525*a^8*b^{12}*c^4*d^{16}*e^{18}*f^2*z^3 + 525*a^4*b^{16}*c^{18}*d^2*e^8*f^{12}*z^3 + 525*a^4*b^{16}*c^8*d^{12}*e^{18}*f^2*z^3 + 525*a^2*b^{18}*c^{16}*d^4*e^{12}*f^8*z^3 + 525*a^2*b^{18}*c^{12}*d^8*e^{16}*f^4*z^3 + 900*a^{19}*b*c^7*d^{13}*e^4*f^{16}*z^3 + 900*a^{19}*b*c^4*d^{16}*e^7*f^{13}*z^3 + 900*a^{16}*b^4*c^{13}*d^7*e*f^{19}*z^3 + 900*a^{16}*b^4*c*d^{19}*e^{13}*f^7*z^3 + 900*a^{13}*b^7*c^{16}*d^4*e*f^{19}*z^3 + 900*a^{13}*b^7*c*d^{19}*e^{16}*f^4*z^3 + 900*a^7*b^{13}*c^{19}*d*e^4*f^{16}*z^3 + 900*a^7*b^{13}*c^4*d^{16}*e^{19}*f*z^3 + 900*a^4*b^{16}*c^{19}*d*e^7*f^{13}*z^3 + 900*a^4*b^{16}*c^7*d^{13}*e^{19}*f*z^3 + 900*a*b^{19}*c^{16}*d^4*e^{13}*f^7*z^3 + 900*a*b^{19}*c^{13}*d^7*e^{16}*f^4*z^3 - 750*a^{19}*b*c^8*d^{12}*e^3*f^{17}*z^3 - 750*a^{19}*b*c^3*d^{17}*e^8*f^{12}*z^3 - 750*a^{17}*b^3*c^{12}*d^8*e*f^{19}*z^3 - 750*a^{17}*b^3*c*d^{19}*e^{12}*f^8*z^3 - 750*a^{12}*b^8*c^{17}*d^3*e*f^{19}*z^3 - 750*a^{12}*b^8*c*d^{19}*e^{17}*f^3*z^3 - 750*a^8*b^{12}*c^{19}*d*e^3*f^{17}*z^3 - 750*a^8*b^{12}*c^3*d^{17}*e^{19}*f*z^3 - 750*a^3*b^{17}*c^{19}*d*e^8*f^{12}*z^3 - 750*a^3*b^{17}*c^8*d^{12}*e^{19}*f*z^3 - 750*a*b^{19}*c^{17}*d^3*e^{12}*f^8*z^3 - 750*a*b^{19}*c^{12}*d^8*e^{17}*f^3*z^3 - 420*a^{19}*b*c^6*d^{14}*e^5*f^{15}*z^3 - 420*a^{19}*b*c^5*d^{15}*e^6*f^{14}*z^3 - 420*a^{15}*b^5*c^{14}*d^6*e*f^{19}*z^3 - 420*a^{15}*b^5*c*d^{19}*e^{14}*f^6*z^3 - 420*a^{14}*b^6*c^{15}*d^5*e*f^{19}*z^3 - 420*a^{14}*b^6*c*d^{19}*e^{15}*f^5*z^3 - 420*a^6*b^{14}*c^{19}*d*e^5*f^{15}*z^3 - 420*a^6*b^{14}*c^5*d^{15}*e^{19}*f*z^3 - 420*a^5*b^{15}*c^{19}*d*e^6*f^{14}*z^3 - 420*a^5*b^{15}*c^6*d^{14}*e^{19}*f*z^3 - 420*a*b^{19}*c^{15}*d^5*e^{14}*f^6*z^3 - 420*a*b^{19}*c^{14}*d^6*e^{15}*f^5*z^3 + 350*a^{19}*b*c^9*d^{11}*e^2*f^{18}*z^3 + 350*a^{19}*b*c^2*d^{18}*e^9*f^{11}*z^3 + 350*a^{18}*b^2*c^{11}*d^9*e*f^{19}*z^3 + 350*a^{18}*b^2*c*d^{19}*e^{11}*f^9*z^3 + 350*a^{11}*b^9*c^{18}*d^2*e*f^{19}*z^3 + 350*a^{11}*b^9*c*d^{19}*e^{18}*f^2*z^3 + 350*a^9*b^{11}*c^{19}*d*e^2*f^{18}*z^3 + 350*a^9*b^{11}*c^2*d^{18}*e^{19}*f*z^3 + 350*a^2*b^{18}*c^{19}*d*e^9*f^{11}*z^3 + 350*a^2*b^{18}*c^9*d^{11}*e^{19}*f*z^3 + 350*a*b^{19}*c^{18}*d^2*e^{11}*f^9*z^3 + 350*a*b^{19}*c^{11}*d^9*e^{18}*f^2*z^3 - 90*a^{19}*b*c^{10}*d^{10}*e*f^{19}*z^3 - 90*a^{19}*b*c*d^{19}*e^{10}*f^{10}*z^3 - 90*a^{10}*b^{10}*c^{19}*d*e*f^{19}*z^3 - 90*a^{10}*b^{10}*c*d^{19}*e^{19}*f*z^3 - 90*a*b^{19}*c^{10}*d^{10}*e^{19}*f*z^3 + 10*b^{20}*c^{19}*d*e^{11}*f^9*z^3 + 10*b^{20}*c^{11}*d^9*e^{19}*f*z^3 + 10*a^{20}*c^9*d^{11}*e*f^{19}*z^3 + 10*a^{20}*c*d^{19}*e^9*f^{11}*z^3 + 10*a^{19}*b*d^{20}*e^{11}*f^9*z^3 + 10*a^{11}*b^9*d^{20}*e^{19}*f*z^3 + 10*a^9*b^{11}*c^{20}*e*f^{19}*z^3 + 10*a*b^{19}*c^{20}*e^9*f^{11}*z^3 + 10*a^{19}*b*c^{11}*d^9*f^{20}*z^3 + 10*a^{11}*b^9*c^{19}*d*f^{20}*z^3 + 10*a^9*b^{11}*c*d^{19}*e^{20}*z^3 + 10*a*b^{19}*c^9*d^{11}*e^2*0*z^3 + 252*b^{20}*c^{15}*d^5*e^{15}*f^5*z^3 - 210*b^{20}*c^{16}*d^4*e^{14}*f^6*z^3 - 210*b^{20}*c^{14}*d^6*e^{16}*f^4*z^3 + 120*b^{20}*c^{17}*d^3*e^{13}*f^7*z^3 + 120*b^{20}*c^{13}*d^7*e^{17}*f^3*z^3 - 45*b^{20}*c^{18}*d^2*e^{12}*f^8*z^3 - 45*b^{20}*c^{12}*d^8*e^{18}*f^2*z^3 + 252*a^{20}*c^5*d^{15}*e^5*f^{15}*z^3 - 210*a^{20}*c^6*d^{14}*e^4*f^{16}*z^3 - 210*a^{20}*c^4*d^{16}*e^6*f^{14}*z^3 + 120*a^{20}*c^7*d^{13}*e^3*f^{17}*z^3 + 120*a^{20}*c^3*d^{17}*e^7*f^{13}*z^3 - 45*a^{20}*c^8*d^{12}*e^2*f^{18}*z^3 - 45*a^{20}*c^2*d^{18}*e^8*f^{12}*z^3 + 252*a^{15}*b^5*d^{20}*e^{15}*f^5*z^3 - 210*a^{16}*b^4*d^{20}*e^{14}*f^6*z^3 - 210*a^{14}*b^6*d^{20}*e^{16}*f^4*z^3 + 120*a^{17}*b^3*d^{20}*e^{13}*f^7*z^3 + 120*a^{13}*b^7*d^{20}*e^{17}*f^3*z^3 - 45*a^{18}*b^2*d^{20}*e^{12}*f^8*z^3 - 45*a^{12}*b^8*d^{20}*e^{18}*f^2*z^3 + 252*a^5*b^{15}*c^{20}*e^5*f^{15}*z^3 - 210*a^6*b^{14}*c^{20}*e^4*f^{16}*z^3 - 210*a^4*b^{16}*c^{20}*e^6*f^{14}*z^3 + 120*a^7*b^{13}*c^{20}*e^3*f^{17}*z^3
\end{aligned}$$

$$\begin{aligned}
& + 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^20*e^2*f^18*z^3 - 45*a^2*b^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20*z^3 - 210*a^16*b^4*c^14*d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 120*a^17*b^3*c^13*d^7*f^20*z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2*c^12*d^8*f^20*z^3 - 45*a^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*e^20*z^3 - 210*a^6*b^14*c^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3 + 120*a^7*b^13*c^3*d^17*e^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*b^12*c^2*d^18*e^20*z^3 - 45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^10*z^3 - a^20*d^20*e^10*f^10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*f^20*z^3 - a^10*b^10*d^20*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^2*c*d^13*e*f^13*z + 1890*a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12*f^2*z + 92610*a^6*b^8*c^4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^10*z + 92610*a^4*b^10*c^4*d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z - 66150*a^7*b^7*c^4*d^10*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z - 66150*a^4*b^10*c^7*d^7*e^3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 66150*a^3*b^11*c^8*d^6*e^3*f^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*a^3*b^11*c^4*d^10*e^7*f^7*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5*b^9*c^5*d^9*e^4*f^10*z - 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10*c^5*d^9*e^5*f^9*z - 32130*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*d^12*e^3*f^11*z - 32130*a^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^12*e^9*f^5*z - 32130*a^2*b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*e^9*f^5*z + 22680*a^8*b^6*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*f^10*z + 22680*a^4*b^10*c^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^6*z + 22680*a^2*b^12*c^8*d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z + 19278*a^10*b^4*c^2*d^12*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z + 19278*a^2*b^12*c^2*d^12*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 18522*a^6*b^8*c^3*d^11*e^5*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d^11*e^6*f^8*z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e^6*f^8*z - 13230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^8*z - 13230*a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z + 3402*a^7*b^7*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^5*b^9*c^2*d^12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*c^5*d^9*e^7*f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*e^3*f^11*z + 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^4*z + 7938*a*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5670*a^11*b^3*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2*b^12*c^11*d^3*e^2*f^12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*e*f^13*z - 3780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z - 3780*a^4*b^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*b^13*c^4*d^10*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^13*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*z - 2268*a*b^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^7*b^7*c^6*d^8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d^7*e*f^13*z + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z + 1890*a*b^13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^13*e^13*f*z - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*f*z - 252*a^13*b*c*d^13*f^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^7*e^7*f^7*z - 882*b^14*c^11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z + 693*b^14*c^12*d^2*e^2*f^12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8*d^6*e^6*f^8*z + 567*b^14*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z + 441*b^14*c^4*d^10*e^10*f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*d^9*e^9*f^5*z - 918*a^7*b^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z - 882*a^3*b^11*d^14*e^11*f^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12*d^14*e^12*f^2*z + 567*a^8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z + 441*a^10*b^4*d^14*e^4*f^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5*d^14*e^5*f^9*z - 126*a^5*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z - 882*a^11*b^3*c^3*d^11*f^14*z - 882*a^3*b^11*c^3*d^11*f^14*z + 693*a^12*b^2*c^2*d^12*f^14*z + 693*a^2*b^12*c^2*d^12*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*z + 567*a^6*b^8*c^8*d^6*f^14*z
\end{aligned}$$

$$\begin{aligned}
& ^{14}z + 441a^{10}b^4c^4d^{10}f^{14}z + 441a^4b^{10}c^{10}d^4f^{14}z - 126a^9b^5c^5d^9f^{14}z - 126a^5b^9c^9d^5f^{14}z + 36b^{14}d^{14}e^{14}z + \\
& 36b^{14}c^{14}f^{14}z + 36a^{14}d^{14}f^{14}z - 27054a^2b^9c^2d^9e^2f^9 + 9018a^3b^8c^2d^9e^2f^{10} + 9018a^3b^8c^2d^{10}e^2f^9 + 9018a^2b^9c^3d^8e^2f^{10} + 9018a^2b^9c^3d^{10}e^3f^8 + 9018a^2b^9c^3d^8e^2f^9 + \\
& 9018a^2b^9c^3d^{10}e^3f^8 - 9018a^4b^7c^2d^{10}e^2f^{10} - 9018a^2b^9c^4d^7e^2f^{10} - 9018a^2b^9c^4d^{10}e^4f^7 + 2268b^{11}c^5d^6e^2f^{10} + 2268b^{11}c^5d^6e^5f^6 + 2268a^5b^6d^{11}e^2f^{10} + 2268a^5b^6d^{11}e^5f^6 + \\
& 2268a^5b^6c^2d^{10}f^{11} + 2268a^5b^6c^5d^6f^{11} - 1458b^{11}c^3d^8e^3f^8 - 1161b^{11}c^4d^7e^2f^9 - 1161b^{11}c^2d^9e^4f^7 - 1458a^3b^8d^{11}e^3f^8 - 1161a^4b^7d^{11}e^2f^9 - 1161a^2b^9d^{11}e^4f^7 - 1458a^3b^8c^3d^8f^{11} - 1161a^4b^7c^2d^9f^{11} - 1161a^2b^9c^4d^7f^{11} - 756b^{11}d^{11}e^6f^5 - 756b^{11}c^6d^5f^{11} - 756a^6b^5d^{11}f^1 \\
& 1, z, k) * ((39a^5b^{11}c^{14}d^2f^{16} - 102a^6b^{10}c^{13}d^3f^{16} + 132a^7b^9c^{12}d^4f^{16} - 84a^8b^8c^{11}d^5f^{16} + 21a^9b^7c^{10}d^6f^{16} + 21a^{10}b^6c^9d^7f^{16} - 84a^{11}b^5c^8d^8f^{16} + 132a^{12}b^4c^7d^9f^{16} - 102a^{13}b^3c^6d^{10}f^{16} + 39a^{14}b^2c^5d^{11}f^{16} + 39a^5b^{11}d^{16}e^{14}f^2 - 102a^6b^{10}d^{16}e^{13}f^3 + 132a^7b^9d^{16}e^{12}f^4 - 84a^8b^8d^{16}e^{11}f^5 + 21a^9b^7d^{16}e^{10}f^6 + 21a^{10}b^6d^{16}e^9f^7 - 84a^{11}b^5d^{16}e^8f^8 + 132a^{12}b^4d^{16}e^7f^9 - 102a^{13}b^3d^{16}e^6f^{10} + 39a^{14}b^2d^{16}e^5f^{11} + 39b^{16}c^5d^{11}e^{14}f^2 - 102b^{16}c^6d^{10}e^{13}f^3 + 132b^{16}c^7d^9e^{12}f^4 - 84b^{16}c^8d^8e^{11}f^5 + 21b^{16}c^9d^7e^{10}f^6 + 21b^{16}c^{10}d^6e^9f^7 - 84b^{16}c^{11}d^5e^8f^8 + 132b^{16}c^{12}d^4e^7f^9 - 102b^{16}c^{13}d^3e^6f^{10} + 39b^{16}c^{14}d^2e^5f^{11} - 6a^4b^{12}c^{15}d^2f^{16} - 6a^{15}b^3c^4d^{12}f^{16} - 6a^4b^{12}d^{16}e^{15}f - 6a^{15}b^3d^{16}e^4f^{12} - 6b^{16}c^4d^{12}e^{15}f - 6b^{16}c^{15}d^2e^4f^{12} + 24a^3b^{13}c^3d^{13}e^{15}f + 24a^3b^{13}c^{15}d^2e^3f^{13} + 24a^3b^{13}c^3d^{15}e^{15}f + 24a^3b^{13}c^{15}d^2e^3f^{13} + 24a^{15}b^3c^3d^{13}e^3f^{15} - 117a^3b^{15}c^4d^{12}e^{14}f^2 + 150a^3b^{15}c^5d^{11}e^{13}f^3 + 159a^3b^{15}c^6d^{10}e^{12}f^4 - 546a^3b^{15}c^7d^9e^{11}f^5 + 414a^3b^{15}c^8d^8e^{10}f^6 - 168a^3b^{15}c^9d^7e^9f^7 + 414a^3b^{15}c^{10}d^6e^8f^8 - 546a^3b^{15}c^{11}d^5e^7f^9 + 159a^3b^{15}c^{12}d^4e^6f^{10} + 150a^3b^{15}c^{13}d^3e^5f^{11} - 117a^3b^{15}c^{14}d^2e^4f^{12} - 36a^2b^{14}c^2d^{14}e^{15}f - 36a^2b^{14}c^{15}d^2e^2f^{14} - 117a^4b^{12}c^2d^{15}e^{14}f^2 - 117a^4b^{12}c^{14}d^2e^2f^{15} + 150a^5b^{11}c^2d^{15}e^{13}f^3 + 150a^5b^{11}c^{13}d^3e^3f^{15} + 159a^6b^{10}c^2d^{15}e^{12}f^4 + 159a^6b^{10}c^{12}d^4e^2f^{15} - 546a^7b^9c^2d^{15}e^{11}f^5 - 546a^7b^9c^{11}d^5e^2f^{15} + 414a^8b^8c^2d^{15}e^{10}f^6 + 414a^8b^8c^{10}d^6e^2f^{15} - 168a^9b^7c^2d^{15}e^9f^7 - 168a^9b^7c^9d^7e^2f^{15} + 414a^{10}b^6c^2d^{15}e^8f^8 + 414a^{10}b^6c^8d^8e^2f^{15} - 546a^{11}b^5c^2d^{15}e^7f^9 - 546a^{11}b^5c^7d^9e^2f^{15} + 159a^{12}b^4c^2d^{15}e^6f^{10} + 159a^{12}b^4c^6d^{10}e^2f^{15} + 150a^{13}b^3c^2d^{15}e^5f^{11} + 150a^{13}b^3c^5d^{11}e^2f^{15} - 117a^{14}b^2c^2d^{15}e^4f^{12} - 117a^{14}b^2c^4d^{12}e^2f^{15} - 36a^{15}b^3c^2d^{14}e^2f^{14} + 78a^2b^{14}c^3d^{13}e^{14}f^2 + 318a^2b^{14}c^4d^{12}e^{13}f^3 - 1269a^2b^{14}c^5d^{11}e^{12}f^4 + 1134a^2b^{14}c^6d^{10}e^{11}f^5 + 618a^2b^{14}c^7d^9e^{10}f^6 - 843a^2b^{14}c^8d^8e^9f^7 - 843a^2b^{14}c^9d^7e^8f^8 + 618a^2b^{14}c^{10}d^6e^7f^9 + 1134a^2b^{14}c^{11}d^5e^6f^{10} - 1269a^2b^{14}c^{12}d^4e^5f^{11} + 318a^2b^{14}c^{13}d^3e^4f^{12} + 78a^2b^{14}c^{14}d^2e^3f^{13} + 78a^3b^{13}c^2d^{14}e^{14}f^2 - 732a^3b^{13}c^3d^{13}e^{13}f^3 + 978a^3b^{13}c^4d^{12}e^{12}f^4 + 1722a^3b^{13}c^5d^{11}e^{11}f^5 - 4548a^3b^{13}c^6d^{10}e^{10}f^6 + 1362a^3b^{13}c^7d^9e^9f^7 + 2232a^3b^{13}c^8d^8e^8f^8 + 1362a^3b^{13}c^9d^7e^7f^9 - 4548a^3b^{13}c^{10}d^6e^6f^{10} + 1722a^3b^{13}c^{11}d^5e^5f^{11} + 978a^3b^{13}c^{12}d^4e^4f^{12} - 732a^3b^{13}c^{13}d^3e^3f^{13} + 78a^3b^{13}c^{14}d^2e^2f^{14} + 318a^4b^{12}c^2d^{14}e^{13}f^3 + 978a^4b^{12}c^3d^{13}e^{12}f^4 - 4452a^4b^{12}c^4d^{12}e^{11}f^5 + 3495a^4b^{12}c^5d^{11}e^{10}f^6 + 4302a^4b^{12}c^6d^{10}e^9f^7 - 4518a^4b^{12}c^7d^9e^8f^8 - 4518a^4b^{12}c^8d^8e^7f^9 + 4302a^4b^{12}c^9d^7e^6f^{10} + 3495a^4b^{12}c^{10}d^6e^5f^{11} - 4452a^4b^{12}c^{11}d^5e^4f^{12} + 978a^4b^{12}c^{12}d^4e^3f^{13} + 318a^4
\end{aligned}$$

$$\begin{aligned}
& *e^f^{15} + 160*a^{14}*b^2*c*d^{15}*e^9*f^7 + 160*a^{14}*b^2*c^9*d^7*e*f^{15} + 160*a^{15}*b*c^2*d^{14}*e^7*f^9 - 224*a^{15}*b*c^3*d^{13}*e^6*f^{10} + 112*a^{15}*b*c^4*d^{12} \\
& *e^5*f^{11} + 112*a^{15}*b*c^5*d^{11}*e^4*f^{12} - 224*a^{15}*b*c^6*d^{10}*e^3*f^{13} + 160*a^{15}*b*c^7*d^9*e^2*f^{14} - 300*a^2*b^{14}*c^8*d^8*e^{14}*f^2 + 840*a^2*b^{14}*c \\
& ^{10}*d^6*e^{12}*f^4 - 1344*a^2*b^{14}*c^{11}*d^5*e^{11}*f^5 + 840*a^2*b^{14}*c^{12}*d^4*e^{10}*f^6 - 300*a^2*b^{14}*c^{14}*d^2*e^8*f^8 + 1400*a^3*b^{13}*c^8*d^8*e^{13}*f^3 - \\
& 2800*a^3*b^{13}*c^9*d^7*e^{12}*f^4 + 1568*a^3*b^{13}*c^{10}*d^6*e^{11}*f^5 + 1568*a^3*b^{13}*c^{11}*d^5*e^{10}*f^6 - 2800*a^3*b^{13}*c^{12}*d^4*e^9*f^7 + 1400*a^3*b^{13}*c \\
& ^{13}*d^3*e^8*f^8 + 840*a^4*b^{12}*c^6*d^{10}*e^{14}*f^2 - 2800*a^4*b^{12}*c^7*d^9*e^{13}*f^3 + 1750*a^4*b^{12}*c^8*d^8*e^{12}*f^4 + 4480*a^4*b^{12}*c^9*d^7*e^{11}*f^5 - \\
& 8624*a^4*b^{12}*c^{10}*d^6*e^{10}*f^6 + 4480*a^4*b^{12}*c^{11}*d^5*e^9*f^7 + 1750*a^4*b^{12}*c^{12}*d^4*e^8*f^8 - 2800*a^4*b^{12}*c^{13}*d^3*e^7*f^9 + 840*a^4*b^{12}*c^{14} \\
& *d^2*e^6*f^{10} - 1344*a^5*b^{11}*c^5*d^{11}*e^{14}*f^2 + 1568*a^5*b^{11}*c^6*d^{10}*e^{13}*f^3 + 4480*a^5*b^{11}*c^7*d^9*e^{12}*f^4 - 12264*a^5*b^{11}*c^8*d^8*e^{11}*f^5 + \\
& 7392*a^5*b^{11}*c^9*d^7*e^{10}*f^6 + 7392*a^5*b^{11}*c^{10}*d^6*e^9*f^7 - 12264*a^5*b^{11}*c^{11}*d^5*e^8*f^8 + 4480*a^5*b^{11}*c^{12}*d^4*e^7*f^9 + 1568*a^5*b^{11}*c^{13} \\
& *d^3*e^6*f^{10} - 1344*a^5*b^{11}*c^{14}*d^2*e^5*f^{11} + 840*a^6*b^{10}*c^4*d^{12}*e^{14}*f^2 + 1568*a^6*b^{10}*c^5*d^{11}*e^{13}*f^3 - 8624*a^6*b^{10}*c^6*d^{10}*e^{12}*f^4 \\
& + 7392*a^6*b^{10}*c^7*d^9*e^{11}*f^5 + 11396*a^6*b^{10}*c^8*d^8*e^{10}*f^6 - 24640*a^6*b^{10}*c^9*d^7*e^9*f^7 + 11396*a^6*b^{10}*c^{10}*d^6*e^8*f^8 + 7392*a^6*b^{10} \\
& *c^{11}*d^5*e^7*f^9 - 8624*a^6*b^{10}*c^{12}*d^4*e^6*f^{10} + 1568*a^6*b^{10}*c^{13}*d^3*e^5*f^{11} + 840*a^6*b^{10}*c^{14}*d^2*e^4*f^{12} - 2800*a^7*b^9*c^4*d^{12}*e^{13}*f^3 \\
& + 4480*a^7*b^9*c^5*d^{11}*e^{12}*f^4 + 7392*a^7*b^9*c^6*d^{10}*e^{11}*f^5 - 24640*a^7*b^9*c^7*d^9*e^{10}*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 \\
& - 24640*a^7*b^9*c^{10}*d^6*e^7*f^9 + 7392*a^7*b^9*c^{11}*d^5*e^6*f^{10} + 4480*a^7*b^9*c^{12}*d^4*e^5*f^{11} - 2800*a^7*b^9*c^{13}*d^3*e^4*f^{12} - 3000*a^8*b^8*c^2*d^{14}*e^{14}*f^2 \\
& + 1400*a^8*b^8*c^3*d^{13}*e^{13}*f^3 + 1750*a^8*b^8*c^4*d^{12}*e^{12}*f^4 - 12264*a^8*b^8*c^5*d^{11}*e^{11}*f^5 + 11396*a^8*b^8*c^6*d^{10}*e^{10}*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 \\
& - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^{10}*d^6*e^6*f^{10} - 12264*a^8*b^8*c^{11}*d^5*e^5*f^{11} + 1750*a^8*b^8*c^{12}*d^4*e^4*f^{12} \\
& + 1400*a^8*b^8*c^{13}*d^3*e^3*f^{13} - 300*a^8*b^8*c^{14}*d^2*e^2*f^{14} - 2800*a^9*b^7*c^3*d^{13}*e^{12}*f^4 + 4480*a^9*b^7*c^4*d^{12}*e^{11}*f^5 + 7392*a^9*b^7*c^5*d^{11}*e^{10}*f^6 - \\
& 24640*a^9*b^7*c^6*d^{10}*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^{10} + 7392*a^9*b^7*c^{10}*d^6*e^5*f^{11} \\
& + 4480*a^9*b^7*c^{11}*d^5*e^4*f^{12} - 2800*a^9*b^7*c^{12}*d^4*e^3*f^{13} + 840*a^{10}*b^6*c^2*d^{14}*e^{12}*f^4 + 1568*a^{10}*b^6*c^3*d^{13}*e^{11}*f^5 - 8624*a^{10}*b^6*c^4*d^{12}*e^{10}*f^6 \\
& + 7392*a^{10}*b^6*c^5*d^{11}*e^9*f^7 + 11396*a^{10}*b^6*c^6*d^{10}*e^8*f^8 - 24640*a^{10}*b^6*c^7*d^9*e^7*f^9 + 11396*a^{10}*b^6*c^8*d^8*e^6*f^{10} + 7392*a^{10}*b^6*c^9*d^7*e^5*f^{11} \\
& - 8624*a^{10}*b^6*c^{10}*d^6*e^4*f^{12} + 1568*a^{10}*b^6*c^{11}*d^5*e^3*f^{13} + 840*a^{10}*b^6*c^{12}*d^4*e^2*f^{14} - 1344*a^{11}*b^5*c^2*d^{14}*e^{11}*f^5 + 1568*a^{11}*b^5*c^3*d^{13}*e^{10}*f^6 \\
& + 4480*a^{11}*b^5*c^4*d^{12}*e^9*f^7 - 12264*a^{11}*b^5*c^5*d^{11}*e^8*f^8 + 7392*a^{11}*b^5*c^6*d^{10}*e^7*f^9 + 7392*a^{11}*b^5*c^7*d^9*e^6*f^{10} - 12264*a^{11}*b^5*c^8*d^8*e^5*f^{11} \\
& + 4480*a^{11}*b^5*c^9*d^7*e^4*f^{12} + 1568*a^{11}*b^5*c^{10}*d^6*e^3*f^{13} - 1344*a^{11}*b^5*c^{11}*d^5*e^2*f^{14} + 840*a^{12}*b^4*c^2*d^{14}*e^{10}*f^6 - 2800*a^{12}*b^4*c^3*d^{13}*e^9*f^7 \\
& + 1750*a^{12}*b^4*c^4*d^{12}*e^8*f^8 + 4480*a^{12}*b^4*c^5*d^{11}*e^7*f^9 - 8624*a^{12}*b^4*c^6*d^{10}*e^6*f^{10} + 4480*a^{12}*b^4*c^7*d^9*e^5*f^{11} + 1750*a^{12}*b^4*c^8*d^8*e^4*f^{12} \\
& - 2800*a^{12}*b^4*c^9*d^7*e^3*f^{13} + 840*a^{12}*b^4*c^{10}*d^6*e^2*f^{14} + 1400*a^{13}*b^3*c^3*d^{13}*e^8*f^8 - 2800*a^{13}*b^3*c^4*d^{12}*e^7*f^9 + 1568*a^{13}*b^3*c^5*d^{11}*e^6*f^{10} \\
& + 1568*a^{13}*b^3*c^6*d^{10}*e^5*f^{11} - 2800*a^{13}*b^3*c^7*d^9*e^4*f^{12} + 1400*a^{13}*b^3*c^8*d^8*e^3*f^{13} - 300*a^{14}*b^2*c^2*d^{14}*e^8*f^8 + 840*a^{14}*b^2*c^4*d^{12}*e^6*f^{10} - 13 \\
& 44*a^{14}*b^2*c^5*d^{11}*e^5*f^{11} + 840*a^{14}*b^2*c^6*d^{10}*e^4*f^{12} - 300*a^{14}*b^2*c^8*d^8*e^2*f^{14}) + \text{root}(756756*a^{10}*b^{10}*c^{10}*d^{10}*e^{10}*f^{10}*z^3 + 5733 \\
& 00*a^{12}*b^8*c^9*d^{11}*e^9*f^{11}*z^3 + 573300*a^{11}*b^9*c^{11}*d^9*e^8*f^{12}*z^3 + 573300*a^{11}*b^9*c^8*d^{12}*e^{11}*f^9*z^3 + 573300*a^9*b^{11}*c^{12}*d^8*e^9*f^{11} \\
& z^3 + 573300*a^9*b^{11}*c^9*d^{11}*e^{12}*f^8*z^3 + 573300*a^8*b^{12}*c^{11}*d^9*e^{11} \\
& *f^9*z^3 - 343980*a^{11}*b^9*c^{10}*d^{10}*e^9*f^{11}*z^3 - 343980*a^{11}*b^9*c^9*d^{11}
\end{aligned}$$

$$\begin{aligned}
& 1e^{10}f^{10}z^3 - 343980a^{10}b^{10}c^{11}d^9e^9f^{11}z^3 - 343980a^{10}b^{10} \\
& *c^9d^{11}e^{11}f^9z^3 - 343980a^9b^{11}c^{11}d^9e^{10}f^{10}z^3 - 343980a^9 \\
& *b^{11}c^{10}d^{10}e^{11}f^9z^3 + 326340a^{13}b^7c^{10}d^{10}e^7f^{13}z^3 + 32 \\
& 6340a^{13}b^7c^7d^{13}e^{10}f^{10}z^3 + 326340a^{10}b^{10}c^{13}d^7e^7f^{13}z \\
& ^3 + 326340a^{10}b^{10}c^7d^{13}e^{13}f^7z^3 + 326340a^7b^{13}c^{13}d^7e^{10} \\
& *f^{10}z^3 + 326340a^7b^{13}c^{10}d^{10}e^{13}f^7z^3 - 267540a^{12}b^8c^{10}d \\
& ^{10}e^8f^{12}z^3 - 267540a^{12}b^8c^8d^{12}e^{10}f^{10}z^3 - 267540a^{10}b^{10} \\
& *c^{12}d^8e^8f^{12}z^3 - 267540a^{10}b^{10}c^8d^{12}e^{12}f^8z^3 - 267540a \\
& ^8b^{12}c^{12}d^8e^{10}f^{10}z^3 - 267540a^8b^{12}c^{10}d^{10}e^{12}f^8z^3 + 2 \\
& 45700a^{14}b^6c^8d^{12}e^8f^{12}z^3 + 245700a^{12}b^8c^{12}d^8e^6f^{14}z^3 \\
& + 245700a^{12}b^8c^6d^{14}e^{12}f^8z^3 + 245700a^8b^{12}c^{14}d^6e^8f^{12} \\
& z^3 + 245700a^8b^{12}c^8d^{12}e^{14}f^6z^3 + 245700a^6b^{14}c^{12}d^8e \\
& ^{12}f^8z^3 - 191100a^{13}b^7c^9d^{11}e^8f^{12}z^3 - 191100a^{13}b^7c^8d \\
& ^{12}e^9f^{11}z^3 - 191100a^{12}b^8c^{11}d^9e^7f^{13}z^3 - 191100a^{12}b^8c \\
& ^7d^{13}e^{11}f^9z^3 - 191100a^{11}b^9c^{12}d^8e^7f^{13}z^3 - 191100a^{11} \\
& *b^9c^7d^{13}e^{12}f^8z^3 - 191100a^9b^{11}c^{13}d^7e^8f^{12}z^3 - 191100 \\
& *a^9b^{11}c^8d^{12}e^{13}f^7z^3 - 191100a^8b^{12}c^{13}d^7e^9f^{11}z^3 - 1 \\
& 91100a^8b^{12}c^9d^{11}e^{13}f^7z^3 - 191100a^7b^{13}c^{12}d^8e^{11}f^9z^3 \\
& - 191100a^7b^{13}c^{11}d^9e^{12}f^8z^3 - 123900a^{14}b^6c^9d^{11}e^7f^{13} \\
& z^3 - 123900a^{14}b^6c^7d^{13}e^9f^{11}z^3 - 123900a^{13}b^7c^{11}d^9e \\
& ^6f^{14}z^3 - 123900a^{13}b^7c^6d^{14}e^{11}f^9z^3 - 123900a^{11}b^9c^{13} \\
& d^7e^6f^{14}z^3 - 123900a^{11}b^9c^6d^{14}e^{13}f^7z^3 - 123900a^9b^{11} \\
& c^{14}d^6e^7f^{13}z^3 - 123900a^9b^{11}c^7d^{13}e^{14}f^6z^3 - 123900a^7b \\
& ^{13}c^{14}d^6e^9f^{11}z^3 - 123900a^7b^{13}c^9d^{11}e^{14}f^6z^3 - 123900 \\
& *a^6b^{14}c^{13}d^7e^{11}f^9z^3 - 123900a^6b^{14}c^{11}d^9e^{13}f^7z^3 + 1 \\
& 01700a^{15}b^5c^9d^{11}e^6f^{14}z^3 + 101700a^{15}b^5c^6d^{14}e^9f^{11}z^3 \\
& + 101700a^{14}b^6c^{11}d^9e^5f^{15}z^3 + 101700a^{14}b^6c^5d^{15}e^{11}f^9 \\
& z^3 + 101700a^{11}b^9c^{14}d^6e^5f^{15}z^3 + 101700a^{11}b^9c^5d^{15}e \\
& ^{14}f^6z^3 + 101700a^9b^{11}c^{15}d^5e^6f^{14}z^3 + 101700a^9b^{11}c^6d \\
& ^{14}e^{15}f^5z^3 + 101700a^6b^{14}c^{15}d^5e^9f^{11}z^3 + 101700a^6b^{14} \\
& c^9d^{11}e^{15}f^5z^3 + 101700a^5b^{15}c^{14}d^6e^{11}f^9z^3 + 101700a^5b \\
& ^{15}c^{11}d^9e^{14}f^6z^3 - 65820a^{14}b^6c^{10}d^{10}e^6f^{14}z^3 - 65820a \\
& ^{14}b^6c^6d^{14}e^{10}f^{10}z^3 - 65820a^{10}b^{10}c^{14}d^6e^6f^{14}z^3 - 6 \\
& 5820a^{10}b^{10}c^6d^{14}e^{14}f^6z^3 - 65820a^6b^{14}c^{14}d^6e^{10}f^{10}z^3 \\
& - 65820a^6b^{14}c^{10}d^{10}e^{14}f^6z^3 + 56700a^{16}b^4c^7d^{13}e^7f^{13} \\
& z^3 - 56700a^{15}b^5c^8d^{12}e^7f^{13}z^3 - 56700a^{15}b^5c^7d^{13}e^8f \\
& ^{12}z^3 + 56700a^{13}b^7c^{13}d^7e^4f^{16}z^3 - 56700a^{13}b^7c^{12}d^8e \\
& ^5f^{15}z^3 - 56700a^{13}b^7c^5d^{15}e^{12}f^8z^3 + 56700a^{13}b^7c^4d^{16} \\
& e^{13}f^7z^3 - 56700a^{12}b^8c^{13}d^7e^5f^{15}z^3 - 56700a^{12}b^8c^5d \\
& ^{15}e^{13}f^7z^3 - 56700a^8b^{12}c^{15}d^5e^7f^{13}z^3 - 56700a^8b^{12}c \\
& ^7d^{13}e^{15}f^5z^3 + 56700a^7b^{13}c^{16}d^4e^7f^{13}z^3 - 56700a^7b^{13} \\
& c^{15}d^5e^8f^{12}z^3 - 56700a^7b^{13}c^8d^{12}e^{15}f^5z^3 + 56700a^7b \\
& ^{13}c^7d^{13}e^{16}f^4z^3 - 56700a^5b^{15}c^{13}d^7e^{12}f^8z^3 - 56700a \\
& ^5b^{15}c^{12}d^8e^{13}f^7z^3 + 56700a^4b^{16}c^{13}d^7e^{13}f^7z^3 - 4825 \\
& 2a^{15}b^5c^{10}d^{10}e^5f^{15}z^3 - 48252a^{15}b^5c^5d^{15}e^{10}f^{10}z^3 - \\
& 48252a^{10}b^{10}c^{15}d^5e^5f^{15}z^3 - 48252a^{10}b^{10}c^5d^{15}e^{15}f^5z \\
& ^3 - 48252a^5b^{15}c^{15}d^5e^{10}f^{10}z^3 - 48252a^5b^{15}c^{10}d^{10}e^{15} \\
& f^5z^3 - 32400a^{16}b^4c^8d^{12}e^6f^{14}z^3 - 32400a^{16}b^4c^6d^{14}e \\
& ^8f^{12}z^3 - 32400a^{14}b^6c^{12}d^8e^4f^{16}z^3 - 32400a^{14}b^6c^4d^{16} \\
& e^{12}f^8z^3 - 32400a^{12}b^8c^{14}d^6e^4f^{16}z^3 - 32400a^{12}b^8c^4d \\
& ^{16}e^{14}f^6z^3 - 32400a^8b^{12}c^{16}d^4e^6f^{14}z^3 - 32400a^8b^{12}c \\
& ^6d^{14}e^{16}f^4z^3 - 32400a^6b^{14}c^{16}d^4e^8f^{12}z^3 - 32400a^6b^{14} \\
& c^8d^{12}e^{16}f^4z^3 - 32400a^4b^{16}c^{14}d^6e^{12}f^8z^3 - 32400a^4b \\
& ^{16}c^{12}d^8e^{14}f^6z^3 + 20565a^{16}b^4c^{10}d^{10}e^4f^{16}z^3 + 20565a \\
& ^{16}b^4c^4d^{16}e^{10}f^{10}z^3 + 20565a^{10}b^{10}c^{16}d^4e^4f^{16}z^3 + 2 \\
& 0565a^{10}b^{10}c^4d^{16}e^{16}f^4z^3 + 20565a^4b^{16}c^{16}d^4e^{10}f^{10}z^3 \\
& + 20565a^4b^{16}c^{10}d^{10}e^{16}f^4z^3 + 15660a^{17}b^3c^8d^{12}e^5f^{15} \\
& z^3 + 15660a^{17}b^3c^5d^{15}e^8f^{12}z^3 + 15660a^{15}b^5c^{12}d^8e^3f \\
& ^{17}z^3 + 15660a^{15}b^5c^3d^{17}e^{12}f^8z^3 + 15660a^{12}b^8c^{15}d^5e
\end{aligned}$$

$$\begin{aligned}
& ^3f^{17}z^3 + 15660a^{12}b^8c^3d^{17}e^{15}f^5z^3 + 15660a^8b^{12}c^{17}d^3e^5f^{15}z^3 + 15660a^8b^{12}c^5d^{15}e^{17}f^3z^3 + 15660a^5b^{15}c^{17}d^3e^8f^{12}z^3 + 15660a^5b^{15}c^8d^{12}e^{17}f^3z^3 + 15660a^3b^{17}c^{15}d^5e^{12}f^8z^3 + 15660a^3b^{17}c^{12}d^8e^{15}f^5z^3 - 9750a^{17}b^3c^9d^{11}e^4f^{16}z^3 - 9750a^{17}b^3c^4d^{16}e^9f^{11}z^3 - 9750a^{16}b^4c^{11}d^9e^3f^{17}z^3 - 9750a^{16}b^4c^3d^{17}e^{11}f^9z^3 - 9750a^{11}b^9c^{16}d^4e^3f^{17}z^3 - 9750a^{11}b^9c^3d^{17}e^{16}f^4z^3 - 9750a^9b^{11}c^{17}d^3e^4f^{16}z^3 - 9750a^9b^{11}c^4d^{16}e^{17}f^3z^3 - 9750a^4b^{16}c^{17}d^3e^9f^{11}z^3 - 9750a^4b^{16}c^9d^{11}e^{17}f^3z^3 - 9750a^3b^{17}c^{16}d^4e^{11}f^9z^3 - 9750a^3b^{17}c^{11}d^9e^{16}f^4z^3 - 8100a^{17}b^3c^7d^{13}e^6f^{14}z^3 - 8100a^{17}b^3c^6d^{14}e^7f^{13}z^3 - 8100a^{14}b^6c^{13}d^7e^3f^{17}z^3 - 8100a^{14}b^6c^3d^{17}e^{13}f^7z^3 - 8100a^{13}b^7c^{14}d^6e^3f^{17}z^3 - 8100a^{13}b^7c^3d^{17}e^{14}f^6z^3 - 8100a^7b^{13}c^{17}d^3e^6f^{14}z^3 - 8100a^7b^{13}c^6d^{14}e^{17}f^3z^3 - 8100a^6b^{14}c^{17}d^3e^7f^{13}z^3 - 8100a^6b^{14}c^7d^{13}e^{17}f^3z^3 - 8100a^3b^{17}c^{14}d^6e^{13}f^7z^3 - 8100a^3b^{17}c^{13}d^7e^{14}f^6z^3 - 7980a^{16}b^4c^9d^{11}e^5f^{15}z^3 - 7980a^{16}b^4c^5d^{15}e^9f^{11}z^3 - 7980a^{15}b^5c^{11}d^9e^4f^{16}z^3 - 7980a^{15}b^5c^4d^{16}e^{11}f^9z^3 - 7980a^{11}b^9c^{15}d^5e^4f^{16}z^3 - 7980a^{11}b^9c^4d^{16}e^{15}f^5z^3 - 7980a^9b^{11}c^{16}d^4e^5f^{15}z^3 - 7980a^9b^{11}c^5d^{15}e^{16}f^4z^3 - 7980a^5b^{15}c^{16}d^4e^9f^{11}z^3 - 7980a^5b^{15}c^9d^{11}e^{16}f^4z^3 - 7980a^4b^{16}c^{15}d^5e^{11}f^9z^3 - 7980a^4b^{16}c^{11}d^9e^{15}f^5z^3 + 6300a^{18}b^2c^6d^{14}e^6f^{14}z^3 + 6300a^{14}b^6c^{14}d^6e^2f^{18}z^3 + 6300a^{14}b^6c^2d^{18}e^{14}f^6z^3 + 6300a^6b^{14}c^{18}d^2e^6f^{14}z^3 + 6300a^6b^{14}c^6d^{14}e^{18}f^2z^3 + 6300a^2b^{18}c^{14}d^6e^{14}f^6z^3 - 4260a^{18}b^2c^7d^{13}e^5f^{15}z^3 - 4260a^{18}b^2c^5d^{15}e^7f^{13}z^3 - 4260a^{15}b^5c^{13}d^7e^2f^{18}z^3 - 4260a^{15}b^5c^2d^{18}e^{13}f^7z^3 - 4260a^{13}b^7c^{15}d^5e^2f^{18}z^3 - 4260a^{13}b^7c^2d^{18}e^{15}f^5z^3 - 4260a^7b^{13}c^{18}d^2e^5f^{15}z^3 - 4260a^7b^{13}c^5d^{15}e^{18}f^2z^3 - 4260a^5b^{15}c^{18}d^2e^7f^{13}z^3 - 4260a^5b^{15}c^7d^{13}e^{18}f^2z^3 - 4260a^2b^{18}c^{15}d^5e^{13}f^7z^3 - 4260a^2b^{18}c^{13}d^7e^{15}f^5z^3 + 1470a^{17}b^3c^{10}d^{10}e^3f^{17}z^3 + 1470a^{17}b^3c^3d^{17}e^{10}f^{10}z^3 + 1470a^{10}b^{10}c^{17}d^3e^3f^{17}z^3 + 1470a^{10}b^{10}c^3d^{17}e^{17}f^3z^3 + 1470a^3b^{17}c^{17}d^3e^{10}f^{10}z^3 + 1470a^3b^{17}c^{10}d^{10}e^{17}f^3z^3 + 1350a^{18}b^2c^9d^{11}e^3f^{17}z^3 + 1350a^{18}b^2c^3d^{17}e^9f^{11}z^3 + 1350a^{17}b^3c^{11}d^9e^2f^{18}z^3 + 1350a^{17}b^3c^2d^{18}e^{11}f^9z^3 + 1350a^{11}b^9c^{17}d^3e^2f^{18}z^3 + 1350a^{11}b^9c^2d^{18}e^{17}f^3z^3 + 1350a^9b^{11}c^{18}d^2e^3f^{17}z^3 + 1350a^9b^{11}c^3d^{17}e^{18}f^2z^3 + 1350a^3b^{17}c^{18}d^2e^9f^{11}z^3 + 1350a^3b^{17}c^9d^{11}e^{18}f^2z^3 + 1350a^2b^{18}c^{17}d^3e^{11}f^9z^3 + 1350a^2b^{18}c^{11}d^9e^{17}f^3z^3 - 1070a^{18}b^2c^{10}d^{10}e^2f^{18}z^3 - 1070a^{18}b^2c^2d^{18}e^{10}f^{10}z^3 - 1070a^{10}b^{10}c^{18}d^2e^2f^{18}z^3 - 1070a^{10}b^{10}c^2d^{18}e^{18}f^2z^3 - 1070a^2b^{18}c^{18}d^2e^{10}f^{10}z^3 - 1070a^2b^{18}c^{10}d^{10}e^{18}f^2z^3 + 525a^{18}b^2c^8d^{12}e^4f^{16}z^3 + 525a^{18}b^2c^4d^{16}e^8f^{12}z^3 + 525a^{16}b^4c^{12}d^8e^2f^{18}z^3 + 525a^{16}b^4c^2d^{18}e^{12}f^8z^3 + 525a^{12}b^8c^{16}d^4e^2f^{18}z^3 + 525a^{12}b^8c^2d^{18}e^{16}f^4z^3 + 525a^8b^{12}c^{18}d^2e^4f^{16}z^3 + 525a^8b^{12}c^4d^{16}e^{18}f^2z^3 + 525a^4b^{16}c^{18}d^2e^8f^{12}z^3 + 525a^4b^{16}c^8d^{12}e^{18}f^2z^3 + 525a^2b^{18}c^{16}d^4e^{12}f^8z^3 + 525a^2b^{18}c^{12}d^8e^{16}f^4z^3 + 900a^{19}b^3c^7d^{13}e^4f^{16}z^3 + 900a^{19}b^3c^4d^{16}e^7f^{13}z^3 + 900a^{16}b^4c^{13}d^7e^7f^{19}z^3 + 900a^{16}b^4c^3d^{19}e^{13}f^7z^3 + 900a^{13}b^7c^{16}d^4e^7f^{19}z^3 + 900a^{13}b^7c^3d^{19}e^{16}f^4z^3 + 900a^7b^{13}c^{19}d^4e^4f^{16}z^3 + 900a^7b^{13}c^4d^{16}e^{19}f^3z^3 + 900a^4b^{16}c^{19}d^4e^7f^{13}z^3 + 900a^4b^{16}c^7d^{13}e^{19}f^3z^3 + 900a^3b^{19}c^{16}d^4e^{13}f^7z^3 + 900a^3b^{19}c^{13}d^7e^{16}f^4z^3 - 750a^{19}b^3c^8d^{12}e^3f^{17}z^3 - 750a^{19}b^3c^3d^{17}e^8f^{12}z^3 - 750a^{17}b^3c^{12}d^8e^7f^{19}z^3 - 750a^{17}b^3c^3d^{19}e^{12}f^8z^3 - 750a^{12}b^8c^{17}d^3e^7f^{19}z^3 - 750a^{12}b^8c^3d^{19}e^{17}f^3z^3 - 750a^8b^{12}c^{19}d^4e^3f^{17}z^3 - 750a^8b^{12}c^3d^{17}e^{19}f^3z^3 - 750a^3b^{17}c^{19}
\end{aligned}$$

$$\begin{aligned}
& d^8 e^8 f^{12} z^3 - 750 a^3 b^{17} c^8 d^{12} e^{19} f^3 z^3 - 750 a^3 b^{19} c^{17} d^3 e^{11} \\
& 2 f^8 z^3 - 750 a^3 b^{19} c^{12} d^8 e^{17} f^3 z^3 - 420 a^{19} b^3 c^6 d^{14} e^5 f^{15} \\
& z^3 - 420 a^{19} b^3 c^5 d^{15} e^6 f^{14} z^3 - 420 a^{15} b^5 c^{14} d^6 e^5 f^{19} z^3 \\
& - 420 a^{15} b^5 c^4 d^{19} e^{14} f^6 z^3 - 420 a^{14} b^6 c^{15} d^5 e^5 f^{19} z^3 - 420 \\
& a^{14} b^6 c^4 d^{19} e^{15} f^5 z^3 - 420 a^6 b^{14} c^{19} d^5 e^5 f^{15} z^3 - 420 a^6 b^{14} \\
& c^5 d^{15} e^{19} f^3 z^3 - 420 a^5 b^{15} c^{19} d^5 e^6 f^{14} z^3 - 420 a^5 b^{15} c^6 \\
& d^{14} e^{19} f^3 z^3 - 420 a^3 b^{19} c^{15} d^5 e^{14} f^6 z^3 - 420 a^3 b^{19} c^{14} d^6 \\
& e^{15} f^5 z^3 + 350 a^{19} b^3 c^9 d^{11} e^2 f^{18} z^3 + 350 a^{19} b^3 c^2 d^{18} e^9 \\
& f^{11} z^3 + 350 a^{18} b^2 c^{11} d^9 e^5 f^{19} z^3 + 350 a^{18} b^2 c^4 d^{19} e^{11} f^9 \\
& z^3 + 350 a^{11} b^9 c^{18} d^2 e^5 f^{19} z^3 + 350 a^{11} b^9 c^4 d^{19} e^{18} f^2 z^3 \\
& + 350 a^9 b^{11} c^{19} d^5 e^2 f^{18} z^3 + 350 a^9 b^{11} c^2 d^{18} e^{19} f^3 z^3 + 350 \\
& a^2 b^{18} c^{19} d^5 e^9 f^{11} z^3 + 350 a^2 b^{18} c^9 d^{11} e^{19} f^3 z^3 + 350 a^3 b^{19} \\
& c^{18} d^2 e^{11} f^9 z^3 + 350 a^3 b^{19} c^{11} d^9 e^{18} f^2 z^3 - 90 a^{19} b^3 c^{10} \\
& d^{10} e^5 f^{19} z^3 - 90 a^{19} b^3 c^4 d^{19} e^{10} f^{10} z^3 - 90 a^{10} b^{10} c^{19} d^5 e^5 \\
& f^{19} z^3 - 90 a^{10} b^{10} c^4 d^{19} e^{19} f^3 z^3 - 90 a^3 b^{19} c^{19} d^5 e^{10} f^{10} z^3 \\
& - 90 a^3 b^{19} c^{10} d^{10} e^{19} f^3 z^3 + 10 b^{20} c^{19} d^5 e^{11} f^9 z^3 + 10 b^{20} c^{11} \\
& d^9 e^{19} f^3 z^3 + 10 a^{20} c^9 d^{11} e^5 f^{19} z^3 + 10 a^{20} c^4 d^{19} e^9 f^{11} z^3 \\
& + 10 a^{19} b^3 d^{20} e^{11} f^9 z^3 + 10 a^{11} b^9 d^{20} e^{19} f^3 z^3 + 10 a^9 b^{11} \\
& c^{20} e^5 f^{19} z^3 + 10 a^3 b^{19} c^{20} e^9 f^{11} z^3 + 10 a^{19} b^3 c^{11} d^9 f^{20} z^3 \\
& + 10 a^{11} b^9 c^{19} d^5 f^{20} z^3 + 10 a^9 b^{11} c^4 d^{19} e^{20} z^3 + 10 a^3 b^{19} \\
& c^9 d^{11} e^{20} z^3 + 252 b^{20} c^{15} d^5 e^{15} f^5 z^3 - 210 b^{20} c^{16} d^4 e^{14} \\
& f^6 z^3 - 210 b^{20} c^{14} d^6 e^{16} f^4 z^3 + 120 b^{20} c^{17} d^3 e^{13} f^7 z^3 \\
& + 120 b^{20} c^{13} d^7 e^{17} f^3 z^3 - 45 b^{20} c^{18} d^2 e^{12} f^8 z^3 - 45 b^{20} c^{12} \\
& d^8 e^{18} f^2 z^3 + 252 a^{20} c^5 d^{15} e^5 f^{15} z^3 - 210 a^{20} c^6 d^{14} e^4 \\
& f^{16} z^3 - 210 a^{20} c^4 d^{16} e^6 f^{14} z^3 + 120 a^{20} c^7 d^{13} e^3 f^{17} z^3 \\
& + 120 a^{20} c^3 d^{17} e^7 f^{13} z^3 - 45 a^{20} c^8 d^{12} e^2 f^{18} z^3 - 45 a^{20} c^2 \\
& d^{18} e^8 f^{12} z^3 + 252 a^{15} b^5 d^{20} e^{15} f^5 z^3 - 210 a^{16} b^4 d^{20} e^{14} \\
& f^6 z^3 - 210 a^{14} b^6 d^{20} e^{16} f^4 z^3 + 120 a^{17} b^3 d^{20} e^{13} f^7 z^3 + 120 a^{13} \\
& b^7 d^{20} e^{17} f^3 z^3 - 45 a^{18} b^2 d^{20} e^{12} f^8 z^3 - 45 a^{12} b^8 d^{20} e^{18} f^2 z^3 \\
& + 252 a^5 b^{15} c^{20} e^5 f^{15} z^3 - 210 a^6 b^{14} c^{20} e^4 f^{16} z^3 - 210 a^4 b^{16} \\
& c^{20} e^6 f^{14} z^3 + 120 a^7 b^{13} c^{20} e^3 f^{17} z^3 + 120 a^3 b^{17} c^{20} e^7 f^{13} z^3 - \\
& 45 a^8 b^{12} c^{20} e^2 f^{18} z^3 - 45 a^2 b^{18} c^{20} e^8 f^{12} z^3 + 252 a^{15} b^5 c^{15} \\
& d^5 f^{20} z^3 - 210 a^{16} b^4 c^{14} d^6 f^{20} z^3 - 210 a^{14} b^6 c^{16} d^4 f^{20} z^3 + \\
& 120 a^{17} b^3 c^{13} d^7 f^{20} z^3 + 120 a^{13} b^7 c^{17} d^3 f^{20} z^3 - 45 a^{18} b^2 c^{12} \\
& d^8 f^2 z^3 - 45 a^{12} b^8 c^{18} d^2 f^{20} z^3 + 252 a^5 b^{15} c^5 d^{15} e^{20} z^3 - 210 \\
& a^6 b^{14} c^4 d^{16} e^{20} z^3 - 210 a^4 b^{16} c^6 d^{14} e^{20} z^3 + 120 a^7 b^{13} c^3 \\
& d^{17} e^{20} z^3 + 120 a^3 b^{17} c^7 d^{13} e^{20} z^3 - 45 a^8 b^{12} c^2 d^{18} e^{20} z^3 - \\
& 45 a^2 b^{18} c^8 d^{12} e^{20} z^3 - b^{20} c^{20} e^{10} f^{10} z^3 - a^{20} d^{20} e^{10} f^{10} z^3 - \\
& b^{20} c^{10} d^{10} e^{20} z^3 - a^{20} c^{10} d^{10} f^{20} z^3 - a^{10} b^{10} d^{20} e^{20} z^3 - \\
& a^{10} b^{10} c^{20} f^{20} z^3 + 1890 a^{12} b^2 c^3 d^{13} e^5 f^{13} z^3 + 1890 a^3 b^{13} c^4 \\
& d^{10} e^4 f^{10} z^3 + 92610 a^4 b^{10} c^6 d^8 e^4 f^{10} z^3 + 92610 a^4 b^{10} c^4 d^{10} e^6 \\
& f^8 z^3 + 66150 a^8 b^6 c^3 d^{11} e^3 f^{11} z^3 - 66150 a^7 b^7 c^4 d^{10} e^3 f^{11} z^3 - \\
& 66150 a^7 b^7 c^3 d^{11} e^4 f^{10} z^3 - 66150 a^4 b^{10} c^7 d^7 e^3 f^{11} z^3 - 66150 a^4 \\
& b^{10} c^3 d^{11} e^7 f^7 z^3 + 66150 a^3 b^{11} c^8 d^6 e^3 f^{11} z^3 - 66150 a^3 b^{11} c^4 \\
& d^{10} e^7 f^7 z^3 + 66150 a^3 b^{11} c^3 d^{11} e^8 f^6 z^3 - 55566 a^5 b^9 c^5 d^9 e^4 \\
& f^{10} z^3 - 55566 a^5 b^9 c^4 d^{10} e^5 f^9 z^3 - 55566 a^4 b^{10} c^5 d^9 e^5 f^9 z^3 - \\
& 32130 a^9 b^5 c^3 d^{11} e^2 f^{12} z^3 - 32130 a^9 b^5 c^2 d^{12} e^3 f^{11} z^3 - \\
& 32130 a^3 b^{11} c^9 d^5 e^2 f^{12} z^3 - 32130 a^3 b^{11} c^2 d^{12} e^9 f^5 z^3 - 32130 a^2 \\
& b^{12} c^9 d^5 e^3 f^{11} z^3 - 32130 a^2 b^{12} c^3 d^{11} e^9 f^5 z^3 + 22680 a^8 b^6 c^4 \\
& d^{10} e^2 f^{12} z^3 + 22680 a^8 b^6 c^2 d^{12} e^4 f^{10} z^3 + 22680 a^4 b^{10} c^8 d^6 e^2 \\
& f^{12} z^3 + 22680 a^4 b^{10} c^2 d^{12} e^8 f^6 z^3 + 22680 a^2 b^{12} c^8 d^6 e^4 f^{10} z^3 + \\
& 22680 a^2 b^{12} c^4 d^{10} e^8 f^6 z^3 + 19278 a^{10} b^4 c^2 d^{12} e^2 f^{12} z^3 + 19278 a^2 \\
& b^{12} c^{10} d^4 e^2 f^{12} z^3 + 19278 a^2 b^{12} c^2 d^{12} e^{10} f^4 z^3 + 18522 a^6 b^8 c^5 \\
& d^9 e^3 f^{11} z^3 + 18522 a^6 b^8 c^3 d^{11} e^5 f^9 z^3 + 18522 a^5 b^9 c^3 d^{11} e^6 \\
& f^8 z^3 + 18522 a^3 b^{11} c^6 d^8 e^5 f^9 z^3 + 18522 a^3 b^{11} c^5 d^9 e^6 f^8 z^3
\end{aligned}$$

$$\begin{aligned}
& 3*d^6*e^{13*f^6} + 546*a*b^{18}*c^{14}*d^5*e^{12*f^7} - 168*a*b^{18}*c^{15}*d^4*e^{11*f^8} - 27*a*b^{18}*c^{16}*d^3*e^{10*f^9} + 34*a*b^{18}*c^{17}*d^2*e^9*f^{10} + 20*a^2*b^{17} \\
& *c^7*d^{12}*e^{18*f} + 20*a^2*b^{17}*c^{18}*d*e^7*f^{12} - 28*a^3*b^{16}*c^6*d^{13}*e^{18} \\
& f - 28*a^3*b^{16}*c^{18}*d*e^6*f^{13} + 14*a^4*b^{15}*c^5*d^{14}*e^{18*f} + 14*a^4*b^{15} \\
& *c^{18}*d*e^5*f^{14} + 14*a^5*b^{14}*c^4*d^{15}*e^{18*f} + 14*a^5*b^{14}*c^{18}*d*e^4*f^{15} \\
& - 28*a^6*b^{13}*c^3*d^{16}*e^{18*f} - 28*a^6*b^{13}*c^{18}*d*e^3*f^{16} + 20*a^7*b^{12} \\
& *c^2*d^{17}*e^{18*f} + 20*a^7*b^{12}*c^{18}*d*e^2*f^{17} + 34*a^9*b^{10}*c*d^{18}*e^{17*f^2} \\
& + 34*a^9*b^{10}*c^{17}*d^2*e*f^{18} - 27*a^{10}*b^9*c*d^{18}*e^{16*f^3} - 27*a^{10}*b^9 \\
& *c^{16}*d^3*e*f^{18} - 168*a^{11}*b^8*c*d^{18}*e^{15*f^4} - 168*a^{11}*b^8*c^{15}*d^4*e*f \\
& ^{18} + 546*a^{12}*b^7*c*d^{18}*e^{14*f^5} + 546*a^{12}*b^7*c^{14}*d^5*e*f^{18} - 756*a^{13} \\
& *b^6*c*d^{18}*e^{13*f^6} - 756*a^{13}*b^6*c^{13}*d^6*e*f^{18} + 546*a^{14}*b^5*c*d^{18}* \\
& e^{12*f^7} + 546*a^{14}*b^5*c^{12}*d^7*e*f^{18} - 168*a^{15}*b^4*c*d^{18}*e^{11*f^8} - 16 \\
& 8*a^{15}*b^4*c^{11}*d^8*e*f^{18} - 27*a^{16}*b^3*c*d^{18}*e^{10*f^9} - 27*a^{16}*b^3*c^{10} \\
& *d^9*e*f^{18} + 34*a^{17}*b^2*c*d^{18}*e^9*f^{10} + 34*a^{17}*b^2*c^9*d^{10}*e*f^{18} + 2 \\
& 0*a^{18}*b*c^2*d^{17}*e^7*f^{12} - 28*a^{18}*b*c^3*d^{16}*e^6*f^{13} + 14*a^{18}*b*c^4*d^{15} \\
& *e^5*f^{14} + 14*a^{18}*b*c^5*d^{14}*e^4*f^{15} - 28*a^{18}*b*c^6*d^{13}*e^3*f^{16} + 2 \\
& 0*a^{18}*b*c^7*d^{12}*e^2*f^{17} - 27*a^2*b^{17}*c^8*d^{11}*e^{17*f^2} - 371*a^2*b^{17}*c \\
& ^9*d^{10}*e^{16*f^3} + 1560*a^2*b^{17}*c^{10}*d^9*e^{15*f^4} - 2484*a^2*b^{17}*c^{11}*d^8 \\
& *e^{14*f^5} + 1302*a^2*b^{17}*c^{12}*d^7*e^{13*f^6} + 1302*a^2*b^{17}*c^{13}*d^6*e^{12*f^7} \\
& - 2484*a^2*b^{17}*c^{14}*d^5*e^{11*f^8} + 1560*a^2*b^{17}*c^{15}*d^4*e^{10*f^9} - 37 \\
& 1*a^2*b^{17}*c^{16}*d^3*e^9*f^{10} - 27*a^2*b^{17}*c^{17}*d^2*e^8*f^{11} - 168*a^3*b^{16} \\
& *c^7*d^{12}*e^{17*f^2} + 1560*a^3*b^{16}*c^8*d^{11}*e^{16*f^3} - 3464*a^3*b^{16}*c^9*d^{10} \\
& *e^{15*f^4} + 924*a^3*b^{16}*c^{10}*d^9*e^{14*f^5} + 7728*a^3*b^{16}*c^{11}*d^8*e^{13*f^6} \\
& - 13104*a^3*b^{16}*c^{12}*d^7*e^{12*f^7} + 7728*a^3*b^{16}*c^{13}*d^6*e^{11*f^8} + \\
& 924*a^3*b^{16}*c^{14}*d^5*e^{10*f^9} - 3464*a^3*b^{16}*c^{15}*d^4*e^9*f^{10} + 1560*a^3 \\
& *b^{16}*c^{16}*d^3*e^8*f^{11} - 168*a^3*b^{16}*c^{17}*d^2*e^7*f^{12} + 546*a^4*b^{15}*c^6 \\
& *d^{13}*e^{17*f^2} - 2484*a^4*b^{15}*c^7*d^{12}*e^{16*f^3} + 924*a^4*b^{15}*c^8*d^{11}*e^{15} \\
& *f^4 + 12550*a^4*b^{15}*c^9*d^{10}*e^{14*f^5} - 26838*a^4*b^{15}*c^{10}*d^9*e^{13*f^6} \\
& + 15288*a^4*b^{15}*c^{11}*d^8*e^{12*f^7} + 15288*a^4*b^{15}*c^{12}*d^7*e^{11*f^8} - 2 \\
& 6838*a^4*b^{15}*c^{13}*d^6*e^{10*f^9} + 12550*a^4*b^{15}*c^{14}*d^5*e^9*f^{10} + 924*a^4 \\
& *b^{15}*c^{15}*d^4*e^8*f^{11} - 2484*a^4*b^{15}*c^{16}*d^3*e^7*f^{12} + 546*a^4*b^{15}*c \\
& ^{17}*d^2*e^6*f^{13} - 756*a^5*b^{14}*c^5*d^{14}*e^{17*f^2} + 1302*a^5*b^{14}*c^6*d^{13} \\
& *e^{16*f^3} + 7728*a^5*b^{14}*c^7*d^{12}*e^{15*f^4} - 26838*a^5*b^{14}*c^8*d^{11}*e^{14*f^5} \\
& + 18004*a^5*b^{14}*c^9*d^{10}*e^{13*f^6} + 39858*a^5*b^{14}*c^{10}*d^9*e^{12*f^7} - \\
& 78624*a^5*b^{14}*c^{11}*d^8*e^{11*f^8} + 39858*a^5*b^{14}*c^{12}*d^7*e^{10*f^9} + 18004 \\
& *a^5*b^{14}*c^{13}*d^6*e^9*f^{10} - 26838*a^5*b^{14}*c^{14}*d^5*e^8*f^{11} + 7728*a^5*b \\
& ^{14}*c^{15}*d^4*e^7*f^{12} + 1302*a^5*b^{14}*c^{16}*d^3*e^6*f^{13} - 756*a^5*b^{14}*c^{17} \\
& *d^2*e^5*f^{14} + 546*a^6*b^{13}*c^4*d^{15}*e^{17*f^2} + 1302*a^6*b^{13}*c^5*d^{14}*e^{16} \\
& *f^3 - 13104*a^6*b^{13}*c^6*d^{13}*e^{15*f^4} + 15288*a^6*b^{13}*c^7*d^{12}*e^{14*f^5} \\
& + 39858*a^6*b^{13}*c^8*d^{11}*e^{13*f^6} - 110474*a^6*b^{13}*c^9*d^{10}*e^{12*f^7} + 6 \\
& 6612*a^6*b^{13}*c^{10}*d^9*e^{11*f^8} + 66612*a^6*b^{13}*c^{11}*d^8*e^{10*f^9} - 110474 \\
& *a^6*b^{13}*c^{12}*d^7*e^9*f^{10} + 39858*a^6*b^{13}*c^{13}*d^6*e^8*f^{11} + 15288*a^6* \\
& b^{13}*c^{14}*d^5*e^7*f^{12} - 13104*a^6*b^{13}*c^{15}*d^4*e^6*f^{13} + 1302*a^6*b^{13}*c \\
& ^{16}*d^3*e^5*f^{14} + 546*a^6*b^{13}*c^{17}*d^2*e^4*f^{15} - 168*a^7*b^{12}*c^3*d^{16}*e \\
& ^{17*f^2} - 2484*a^7*b^{12}*c^4*d^{15}*e^{16*f^3} + 7728*a^7*b^{12}*c^5*d^{14}*e^{15*f^4} \\
& + 15288*a^7*b^{12}*c^6*d^{13}*e^{14*f^5} - 78624*a^7*b^{12}*c^7*d^{12}*e^{13*f^6} + 66 \\
& 612*a^7*b^{12}*c^8*d^{11}*e^{12*f^7} + 99736*a^7*b^{12}*c^9*d^{10}*e^{11*f^8} - 216216* \\
& a^7*b^{12}*c^{10}*d^9*e^{10*f^9} + 99736*a^7*b^{12}*c^{11}*d^8*e^9*f^{10} + 66612*a^7*b \\
& ^{12}*c^{12}*d^7*e^8*f^{11} - 78624*a^7*b^{12}*c^{13}*d^6*e^7*f^{12} + 15288*a^7*b^{12}*c \\
& ^{14}*d^5*e^6*f^{13} + 7728*a^7*b^{12}*c^{15}*d^4*e^5*f^{14} - 2484*a^7*b^{12}*c^{16}*d^3 \\
& *e^4*f^{15} - 168*a^7*b^{12}*c^{17}*d^2*e^3*f^{16} - 27*a^8*b^{11}*c^2*d^{17}*e^{17*f^2} \\
& + 1560*a^8*b^{11}*c^3*d^{16}*e^{16*f^3} + 924*a^8*b^{11}*c^4*d^{15}*e^{15*f^4} - 26838* \\
& a^8*b^{11}*c^5*d^{14}*e^{14*f^5} + 39858*a^8*b^{11}*c^6*d^{13}*e^{13*f^6} + 66612*a^8*b \\
& ^{11}*c^7*d^{12}*e^{12*f^7} - 216216*a^8*b^{11}*c^8*d^{11}*e^{11*f^8} + 134134*a^8*b^{11} \\
& *c^9*d^{10}*e^{10*f^9} + 134134*a^8*b^{11}*c^{10}*d^9*e^9*f^{10} - 216216*a^8*b^{11}*c^{11} \\
& *d^8*e^8*f^{11} + 66612*a^8*b^{11}*c^{12}*d^7*e^7*f^{12} + 39858*a^8*b^{11}*c^{13}*d^6 \\
& *e^6*f^{13} - 26838*a^8*b^{11}*c^{14}*d^5*e^5*f^{14} + 924*a^8*b^{11}*c^{15}*d^4*e^4*f \\
& ^{15} + 1560*a^8*b^{11}*c^{16}*d^3*e^3*f^{16} - 27*a^8*b^{11}*c^{17}*d^2*e^2*f^{17} - 371 \\
& *a^9*b^{10}*c^2*d^{17}*e^{16*f^3} - 3464*a^9*b^{10}*c^3*d^{16}*e^{15*f^4} + 12550*a^9*b
\end{aligned}$$

$$\begin{aligned}
& c^{16}e^*f^{15} + 8a^9b^7d^{16}e^{15}f + 8a^{15}b^*d^{16}e^9f^7 + 8a^{16}c^*d^{15} \\
& *e^7f^9 + 8a^{16}c^7d^9e^*f^{15} + 8b^{16}c^9d^7e^{15}f + 8b^{16}c^{15}d^*e^9 \\
& f^7 - 56a^*b^{15}c^8d^8e^{15}f - 56a^*b^{15}c^{15}d^*e^8f^8 - 56a^8b^8c^* \\
& d^{15}e^{15}f - 56a^8b^8c^{15}d^*e^*f^{15} - 56a^{15}b^*c^*d^{15}e^8f^8 - 56a^{15} \\
& *b^*c^8d^8e^*f^{15} + 160a^*b^{15}c^9d^7e^{14}f^2 - 224a^*b^{15}c^{10}d^6e^{13}f^3 \\
& + 112a^*b^{15}c^{11}d^5e^{12}f^4 + 112a^*b^{15}c^{12}d^4e^{11}f^5 - 224a^*b \\
& ^{15}c^{13}d^3e^{10}f^6 + 160a^*b^{15}c^{14}d^2e^9f^7 + 160a^2b^{14}c^7d^9 \\
& e^{15}f + 160a^2b^{14}c^{15}d^*e^7f^9 - 224a^3b^{13}c^6d^{10}e^{15}f - 224a \\
& ^3b^{13}c^{15}d^*e^6f^{10} + 112a^4b^{12}c^5d^{11}e^{15}f + 112a^4b^{12}c^{15}d^* \\
& d^*e^5f^{11} + 112a^5b^{11}c^4d^{12}e^{15}f + 112a^5b^{11}c^{15}d^*e^4f^{12} - \\
& 224a^6b^{10}c^3d^{13}e^{15}f - 224a^6b^{10}c^{15}d^*e^3f^{13} + 160a^7b^9c^2 \\
& d^{14}e^{15}f + 160a^7b^9c^{15}d^*e^2f^{14} + 160a^9b^7c^*d^{15}e^{14}f^2 \\
& + 160a^9b^7c^{14}d^2e^*f^{15} - 224a^{10}b^6c^*d^{15}e^{13}f^3 - 224a^{10}b^6 \\
& *c^{13}d^3e^*f^{15} + 112a^{11}b^5c^*d^{15}e^{12}f^4 + 112a^{11}b^5c^{12}d^4e^*f \\
& ^{15} + 112a^{12}b^4c^*d^{15}e^{11}f^5 + 112a^{12}b^4c^{11}d^5e^*f^{15} - 224a^1 \\
& 3b^3c^*d^{15}e^{10}f^6 - 224a^{13}b^3c^{10}d^6e^*f^{15} + 160a^{14}b^2c^*d^{15}e^9 \\
& f^7 + 160a^{14}b^2c^9d^7e^*f^{15} + 160a^{15}b^*c^2d^{14}e^7f^9 - 224a \\
& ^{15}b^*c^3d^{13}e^6f^{10} + 112a^{15}b^*c^4d^{12}e^5f^{11} + 112a^{15}b^*c^5d^{11} \\
& 1^*e^4f^{12} - 224a^{15}b^*c^6d^{10}e^3f^{13} + 160a^{15}b^*c^7d^9e^2f^{14} - 3 \\
& 00a^2b^{14}c^8d^8e^{14}f^2 + 840a^2b^{14}c^{10}d^6e^{12}f^4 - 1344a^2b^{14} \\
& c^{11}d^5e^{11}f^5 + 840a^2b^{14}c^{12}d^4e^{10}f^6 - 300a^2b^{14}c^{14}d^2 \\
& e^8f^8 + 1400a^3b^{13}c^8d^8e^{13}f^3 - 2800a^3b^{13}c^9d^7e^{12}f^4 + 1568a^3 \\
& b^{13}c^{10}d^6e^{11}f^5 + 1568a^3b^{13}c^{11}d^5e^{10}f^6 - 280 \\
& 0a^3b^{13}c^{12}d^4e^9f^7 + 1400a^3b^{13}c^{13}d^3e^8f^8 + 840a^4b^{12} \\
& c^6d^{10}e^{14}f^2 - 2800a^4b^{12}c^7d^9e^{13}f^3 + 1750a^4b^{12}c^8d^8 \\
& e^{12}f^4 + 4480a^4b^{12}c^9d^7e^{11}f^5 - 8624a^4b^{12}c^{10}d^6e^{10}f^6 + 4480 \\
& a^4b^{12}c^{11}d^5e^9f^7 + 1750a^4b^{12}c^{12}d^4e^8f^8 - 2800a^4 \\
& b^{12}c^{13}d^3e^7f^9 + 840a^4b^{12}c^{14}d^2e^6f^{10} - 1344a^5b^{11} \\
& c^5d^{11}e^{14}f^2 + 1568a^5b^{11}c^6d^{10}e^{13}f^3 + 4480a^5b^{11}c^7d^9 \\
& e^{12}f^4 - 12264a^5b^{11}c^8d^8e^{11}f^5 + 7392a^5b^{11}c^9d^7e^{10}f^6 + 7392 \\
& a^5b^{11}c^{10}d^6e^9f^7 - 12264a^5b^{11}c^{11}d^5e^8f^8 + 4480 \\
& a^5b^{11}c^{12}d^4e^7f^9 + 1568a^5b^{11}c^{13}d^3e^6f^{10} - 1344a^5b^{11} \\
& 1^*c^{14}d^2e^5f^{11} + 840a^6b^{10}c^4d^{12}e^{14}f^2 + 1568a^6b^{10}c^5d^{11} \\
& e^{13}f^3 - 8624a^6b^{10}c^6d^{10}e^{12}f^4 + 7392a^6b^{10}c^7d^9e^{11} \\
& f^5 + 11396a^6b^{10}c^8d^8e^{10}f^6 - 24640a^6b^{10}c^9d^7e^9f^7 + 11 \\
& 396a^6b^{10}c^{10}d^6e^8f^8 + 7392a^6b^{10}c^{11}d^5e^7f^9 - 8624a^6b \\
& ^{10}c^{12}d^4e^6f^{10} + 1568a^6b^{10}c^{13}d^3e^5f^{11} + 840a^6b^{10}c^{14} \\
& d^2e^4f^{12} - 2800a^7b^9c^4d^{12}e^{13}f^3 + 4480a^7b^9c^5d^{11}e^{12} \\
& f^4 + 7392a^7b^9c^6d^{10}e^{11}f^5 - 24640a^7b^9c^7d^9e^{10}f^6 + 15 \\
& 400a^7b^9c^8d^8e^9f^7 + 15400a^7b^9c^9d^7e^8f^8 - 24640a^7b^9 \\
& *c^{10}d^6e^7f^9 + 7392a^7b^9c^{11}d^5e^6f^{10} + 4480a^7b^9c^{12}d^4 \\
& e^5f^{11} - 2800a^7b^9c^{13}d^3e^4f^{12} - 300a^8b^8c^2d^{14}e^{14}f^2 + \\
& 1400a^8b^8c^3d^{13}e^{13}f^3 + 1750a^8b^8c^4d^{12}e^{12}f^4 - 12264a^8 \\
& b^8c^5d^{11}e^{11}f^5 + 11396a^8b^8c^6d^{10}e^{10}f^6 + 15400a^8b^8c^7 \\
& d^9e^9f^7 - 34650a^8b^8c^8d^8e^8f^8 + 15400a^8b^8c^9d^7e^7 \\
& f^9 + 11396a^8b^8c^{10}d^6e^6f^{10} - 12264a^8b^8c^{11}d^5e^5f^{11} + 1 \\
& 750a^8b^8c^{12}d^4e^4f^{12} + 1400a^8b^8c^{13}d^3e^3f^{13} - 300a^8b^8 \\
& c^{14}d^2e^2f^{14} - 2800a^9b^7c^3d^{13}e^{12}f^4 + 4480a^9b^7c^4d^{11} \\
& 2^*e^{11}f^5 + 7392a^9b^7c^5d^{11}e^{10}f^6 - 24640a^9b^7c^6d^{10}e^9f^7 \\
& + 15400a^9b^7c^7d^9e^8f^8 + 15400a^9b^7c^8d^8e^7f^9 - 24640a^9 \\
& b^7c^9d^7e^6f^{10} + 7392a^9b^7c^{10}d^6e^5f^{11} + 4480a^9b^7c^{11} \\
& 1^*d^5e^4f^{12} - 2800a^9b^7c^{12}d^4e^3f^{13} + 840a^{10}b^6c^2d^{14}e^{11} \\
& 2^*f^4 + 1568a^{10}b^6c^3d^{13}e^{11}f^5 - 8624a^{10}b^6c^4d^{12}e^{10}f^6 + \\
& 7392a^{10}b^6c^5d^{11}e^9f^7 + 11396a^{10}b^6c^6d^{10}e^8f^8 - 24640a \\
& ^{10}b^6c^7d^9e^7f^9 + 11396a^{10}b^6c^8d^8e^6f^{10} + 7392a^{10}b^6c^9 \\
& d^7e^5f^{11} - 8624a^{10}b^6c^{10}d^6e^4f^{12} + 1568a^{10}b^6c^{11}d^5 \\
& e^3f^{13} + 840a^{10}b^6c^{12}d^4e^2f^{14} - 1344a^{11}b^5c^2d^{14}e^{11}f^5 \\
& + 1568a^{11}b^5c^3d^{13}e^{10}f^6 + 4480a^{11}b^5c^4d^{12}e^9f^7 - 12264 \\
& a^{11}b^5c^5d^{11}e^8f^8 + 7392a^{11}b^5c^6d^{10}e^7f^9 + 7392a^{11}b^5
\end{aligned}$$

$$\begin{aligned}
& *c^7*d^9*e^6*f^{10} - 12264*a^{11}*b^5*c^8*d^8*e^5*f^{11} + 4480*a^{11}*b^5*c^9*d^7 \\
& *e^4*f^{12} + 1568*a^{11}*b^5*c^{10}*d^6*e^3*f^{13} - 1344*a^{11}*b^5*c^{11}*d^5*e^2*f^{14} \\
& + 840*a^{12}*b^4*c^2*d^{14}*e^{10}*f^6 - 2800*a^{12}*b^4*c^3*d^{13}*e^9*f^7 + 1750 \\
& *a^{12}*b^4*c^4*d^{12}*e^8*f^8 + 4480*a^{12}*b^4*c^5*d^{11}*e^7*f^9 - 8624*a^{12}*b^4 \\
& *c^6*d^{10}*e^6*f^{10} + 4480*a^{12}*b^4*c^7*d^9*e^5*f^{11} + 1750*a^{12}*b^4*c^8*d^8 \\
& *e^4*f^{12} - 2800*a^{12}*b^4*c^9*d^7*e^3*f^{13} + 840*a^{12}*b^4*c^{10}*d^6*e^2*f^{14} \\
& + 1400*a^{13}*b^3*c^3*d^{13}*e^8*f^8 - 2800*a^{13}*b^3*c^4*d^{12}*e^7*f^9 + 1568*a \\
& ^{13}*b^3*c^5*d^{11}*e^6*f^{10} + 1568*a^{13}*b^3*c^6*d^{10}*e^5*f^{11} - 2800*a^{13}*b^3 \\
& *c^7*d^9*e^4*f^{12} + 1400*a^{13}*b^3*c^8*d^8*e^3*f^{13} - 300*a^{14}*b^2*c^2*d^{14} \\
& *e^8*f^8 + 840*a^{14}*b^2*c^4*d^{12}*e^6*f^{10} - 1344*a^{14}*b^2*c^5*d^{11}*e^5*f^{11} \\
& + 840*a^{14}*b^2*c^6*d^{10}*e^4*f^{12} - 300*a^{14}*b^2*c^8*d^8*e^2*f^{14}) - (x*(18* \\
& a^9*b^{10}*c^{17}*d^2*f^{19} - 74*a^{10}*b^9*c^{16}*d^3*f^{19} + 184*a^{11}*b^8*c^{15}*d^4* \\
& f^{19} - 308*a^{12}*b^7*c^{14}*d^5*f^{19} + 364*a^{13}*b^6*c^{13}*d^6*f^{19} - 308*a^{14}*b \\
& ^5*c^{12}*d^7*f^{19} + 184*a^{15}*b^4*c^{11}*d^8*f^{19} - 74*a^{16}*b^3*c^{10}*d^9*f^{19} + \\
& 18*a^{17}*b^2*c^9*d^{10}*f^{19} + 18*a^9*b^{10}*d^{19}*e^{17}*f^2 - 74*a^{10}*b^9*d^{19}*e \\
& ^{16}*f^3 + 184*a^{11}*b^8*d^{19}*e^{15}*f^4 - 308*a^{12}*b^7*d^{19}*e^{14}*f^5 + 364*a^{13} \\
& *b^6*d^{19}*e^{13}*f^6 - 308*a^{14}*b^5*d^{19}*e^{12}*f^7 + 184*a^{15}*b^4*d^{19}*e^{11}*f \\
& ^8 - 74*a^{16}*b^3*d^{19}*e^{10}*f^9 + 18*a^{17}*b^2*d^{19}*e^9*f^{10} + 18*b^{19}*c^9*d \\
& ^{10}*e^{17}*f^2 - 74*b^{19}*c^{10}*d^9*e^{16}*f^3 + 184*b^{19}*c^{11}*d^8*e^{15}*f^4 - 308* \\
& b^{19}*c^{12}*d^7*e^{14}*f^5 + 364*b^{19}*c^{13}*d^6*e^{13}*f^6 - 308*b^{19}*c^{14}*d^5*e^{12} \\
& *f^7 + 184*b^{19}*c^{15}*d^4*e^{11}*f^8 - 74*b^{19}*c^{16}*d^3*e^{10}*f^9 + 18*b^{19}*c^{17} \\
& *d^2*e^9*f^{10} - 2*a^8*b^{11}*c^{18}*d*f^{19} - 2*a^{18}*b*c^8*d^{11}*f^{19} - 2*a^8*b \\
& ^{11}*d^{19}*e^{18}*f - 2*a^{18}*b*d^{19}*e^8*f^{11} - 2*b^{19}*c^8*d^{11}*e^{18}*f - 2*b^{19} \\
& *c^{18}*d*e^8*f^{11} + 16*a*b^{18}*c^7*d^{12}*e^{18}*f + 16*a*b^{18}*c^{18}*d*e^7*f^{12} + 1 \\
& 6*a^7*b^{12}*c*d^{18}*e^{18}*f + 16*a^7*b^{12}*c^{18}*d*e*f^{18} + 16*a^{18}*b*c*d^{18}*e^7 \\
& *f^{12} + 16*a^{18}*b*c^7*d^{12}*e*f^{18} - 126*a*b^{18}*c^8*d^{11}*e^{17}*f^2 + 434*a*b^{18} \\
& *c^9*d^{10}*e^{16}*f^3 - 840*a*b^{18}*c^{10}*d^9*e^{15}*f^4 + 936*a*b^{18}*c^{11}*d^8*e \\
& ^{14}*f^5 - 420*a*b^{18}*c^{12}*d^7*e^{13}*f^6 - 420*a*b^{18}*c^{13}*d^6*e^{12}*f^7 + 936 \\
& *a*b^{18}*c^{14}*d^5*e^{11}*f^8 - 840*a*b^{18}*c^{15}*d^4*e^{10}*f^9 + 434*a*b^{18}*c^{16} \\
& *d^3*e^9*f^{10} - 126*a*b^{18}*c^{17}*d^2*e^8*f^{11} - 56*a^2*b^{17}*c^6*d^{13}*e^{18}*f - \\
& 56*a^2*b^{17}*c^{18}*d*e^6*f^{13} + 112*a^3*b^{16}*c^5*d^{14}*e^{18}*f + 112*a^3*b^{16} \\
& *c^{18}*d*e^5*f^{14} - 140*a^4*b^{15}*c^4*d^{15}*e^{18}*f - 140*a^4*b^{15}*c^{18}*d*e^4*f^{15} \\
& + 112*a^5*b^{14}*c^3*d^{16}*e^{18}*f + 112*a^5*b^{14}*c^{18}*d*e^3*f^{16} - 56*a^6*b \\
& ^{13}*c^2*d^{17}*e^{18}*f - 56*a^6*b^{13}*c^{18}*d*e^2*f^{17} - 126*a^8*b^{11}*c*d^{18}*e^{17} \\
& *f^2 - 126*a^8*b^{11}*c^{17}*d^2*e*f^{18} + 434*a^9*b^{10}*c*d^{18}*e^{16}*f^3 + 434*a \\
& ^9*b^{10}*c^{16}*d^3*e*f^{18} - 840*a^{10}*b^9*c*d^{18}*e^{15}*f^4 - 840*a^{10}*b^9*c^{15} \\
& *d^4*e*f^{18} + 936*a^{11}*b^8*c*d^{18}*e^{14}*f^5 + 936*a^{11}*b^8*c^{14}*d^5*e*f^{18} - \\
& 420*a^{12}*b^7*c*d^{18}*e^{13}*f^6 - 420*a^{12}*b^7*c^{13}*d^6*e*f^{18} - 420*a^{13}*b^6* \\
& c*d^{18}*e^{12}*f^7 - 420*a^{13}*b^6*c^{12}*d^7*e*f^{18} + 936*a^{14}*b^5*c*d^{18}*e^{11}*f \\
& ^8 + 936*a^{14}*b^5*c^{11}*d^8*e*f^{18} - 840*a^{15}*b^4*c*d^{18}*e^{10}*f^9 - 840*a^{15} \\
& *b^4*c^{10}*d^9*e*f^{18} + 434*a^{16}*b^3*c*d^{18}*e^9*f^{10} + 434*a^{16}*b^3*c^9*d^{10} \\
& *e*f^{18} - 126*a^{17}*b^2*c*d^{18}*e^8*f^{11} - 126*a^{17}*b^2*c^8*d^{11}*e*f^{18} - 56* \\
& a^{18}*b*c^2*d^{17}*e^6*f^{13} + 112*a^{18}*b*c^3*d^{16}*e^5*f^{14} - 140*a^{18}*b*c^4*d^{15} \\
& *e^4*f^{15} + 112*a^{18}*b*c^5*d^{14}*e^3*f^{16} - 56*a^{18}*b*c^6*d^{13}*e^2*f^{17} + \\
& 360*a^2*b^{17}*c^7*d^{12}*e^{17}*f^2 - 882*a^2*b^{17}*c^8*d^{11}*e^{16}*f^3 + 728*a^2*b \\
& ^{17}*c^9*d^{10}*e^{15}*f^4 + 1152*a^2*b^{17}*c^{10}*d^9*e^{14}*f^5 - 4032*a^2*b^{17}*c^{11} \\
& *d^8*e^{13}*f^6 + 5460*a^2*b^{17}*c^{12}*d^7*e^{12}*f^7 - 4032*a^2*b^{17}*c^{13}*d^6*e^{11} \\
& *f^8 + 1152*a^2*b^{17}*c^{14}*d^5*e^{10}*f^9 + 728*a^2*b^{17}*c^{15}*d^4*e^9*f^{10} \\
& - 882*a^2*b^{17}*c^{16}*d^3*e^8*f^{11} + 360*a^2*b^{17}*c^{17}*d^2*e^7*f^{12} - 504*a^3 \\
& *b^{16}*c^6*d^{13}*e^{17}*f^2 + 312*a^3*b^{16}*c^7*d^{12}*e^{16}*f^3 + 2520*a^3*b^{16}*c^8 \\
& *d^{11}*e^{15}*f^4 - 7480*a^3*b^{16}*c^9*d^{10}*e^{14}*f^5 + 9408*a^3*b^{16}*c^{10}*d^9* \\
& e^{13}*f^6 - 4368*a^3*b^{16}*c^{11}*d^8*e^{12}*f^7 - 4368*a^3*b^{16}*c^{12}*d^7*e^{11}*f^8 \\
& + 9408*a^3*b^{16}*c^{13}*d^6*e^{10}*f^9 - 7480*a^3*b^{16}*c^{14}*d^5*e^9*f^{10} + 252 \\
& 0*a^3*b^{16}*c^{15}*d^4*e^8*f^{11} + 312*a^3*b^{16}*c^{16}*d^3*e^7*f^{12} - 504*a^3*b^{16} \\
& *c^{17}*d^2*e^6*f^{13} + 252*a^4*b^{15}*c^5*d^{14}*e^{17}*f^2 + 1596*a^4*b^{15}*c^6*d^{13} \\
& *e^{16}*f^3 - 6288*a^4*b^{15}*c^7*d^{12}*e^{15}*f^4 + 7380*a^4*b^{15}*c^8*d^{11}*e^{14} \\
& *f^5 + 2660*a^4*b^{15}*c^9*d^{10}*e^{13}*f^6 - 18564*a^4*b^{15}*c^{10}*d^9*e^{12}*f^7 + \\
& 26208*a^4*b^{15}*c^{11}*d^8*e^{11}*f^8 - 18564*a^4*b^{15}*c^{12}*d^7*e^{10}*f^9 + 2660 \\
& *a^4*b^{15}*c^{13}*d^6*e^9*f^{10} + 7380*a^4*b^{15}*c^{14}*d^5*e^8*f^{11} - 6288*a^4*b^
\end{aligned}$$

$$\begin{aligned}
& 15c^{15}d^4e^7f^{12} + 1596a^4b^{15}c^{16}d^3e^6f^{13} + 252a^4b^{15}c^{17}d^2e^5f^{14} + 252a^5b^{14}c^4d^{15}e^{17}f^2 - 2772a^5b^{14}c^5d^{14}e^{16}f^3 + 3696a^5b^{14}c^6d^{13}e^{15}f^4 + 7056a^5b^{14}c^7d^{12}e^{14}f^5 - \\
& 25452a^5b^{14}c^8d^{11}e^{13}f^6 + 30212a^5b^{14}c^9d^{10}e^{12}f^7 - 13104a^5b^{14}c^{10}d^9e^{11}f^8 - 13104a^5b^{14}c^{11}d^8e^{10}f^9 + 30212a^5b^{14}c^{12}d^7e^9f^{10} - 25452a^5b^{14}c^{13}d^6e^8f^{11} + 7056a^5b^{14}c^{14}d^5e^7f^{12} + 3696a^5b^{14}c^{15}d^4e^6f^{13} - 2772a^5b^{14}c^{16}d^3e^5f^{14} + 252a^5b^{14}c^{17}d^2e^4f^{15} - 504a^6b^{13}c^3d^{16}e^{17}f^2 + \\
& 1596a^6b^{13}c^4d^{15}e^{16}f^3 + 3696a^6b^{13}c^5d^{14}e^{15}f^4 - 17472a^6b^{13}c^6d^{13}e^{14}f^5 + 17472a^6b^{13}c^7d^{12}e^{13}f^6 + 9828a^6b^{13}c^8d^{11}e^{12}f^7 - 38584a^6b^{13}c^9d^{10}e^{11}f^8 + 48048a^6b^{13}c^{10}d^9e^{10}f^9 - 38584a^6b^{13}c^{11}d^8e^9f^{10} + 9828a^6b^{13}c^{12}d^7e^8f^{11} + 17472a^6b^{13}c^{13}d^6e^7f^{12} - 17472a^6b^{13}c^{14}d^5e^6f^{13} + 3696a^6b^{13}c^{15}d^4e^5f^{14} + 1596a^6b^{13}c^{16}d^3e^4f^{15} - 504a^6b^{13}c^{17}d^2e^3f^{16} + 360a^7b^{12}c^2d^{17}e^{17}f^2 + 312a^7b^{12}c^3d^{16}e^{16}f^3 - 6288a^7b^{12}c^4d^{15}e^{15}f^4 + 7056a^7b^{12}c^5d^{14}e^{14}f^5 + 17472a^7b^{12}c^6d^{13}e^{13}f^6 - 43680a^7b^{12}c^7d^{12}e^{12}f^7 + 32760a^7b^{12}c^8d^{11}e^{11}f^8 - 8008a^7b^{12}c^9d^{10}e^{10}f^9 - 8008a^7b^{12}c^{10}d^9e^9f^{10} + 32760a^7b^{12}c^{11}d^8e^8f^{11} - 43680a^7b^{12}c^{12}d^7e^7f^{12} + 17472a^7b^{12}c^{13}d^6e^6f^{13} + 7056a^7b^{12}c^{14}d^5e^5f^{14} - 6288a^7b^{12}c^{15}d^4e^4f^{15} + 312a^7b^{12}c^{16}d^3e^3f^{16} + 360a^7b^{12}c^{17}d^2e^2f^{17} - 882a^8b^{11}c^2d^{17}e^{16}f^3 + 2520a^8b^{11}c^3d^{16}e^{15}f^4 + 7380a^8b^{11}c^4d^{15}e^{14}f^5 - 25452a^8b^{11}c^5d^{14}e^{13}f^6 + 9828a^8b^{11}c^6d^{13}e^{12}f^7 + 32760a^8b^{11}c^7d^{12}e^{11}f^8 - 36036a^8b^{11}c^8d^{11}e^{10}f^9 + 20020a^8b^{11}c^9d^{10}e^9f^{10} - 36036a^8b^{11}c^{10}d^9e^8f^{11} + 32760a^8b^{11}c^{11}d^8e^7f^{12} + 9828a^8b^{11}c^{12}d^7e^6f^{13} - 25452a^8b^{11}c^{13}d^6e^5f^{14} + 7380a^8b^{11}c^{14}d^5e^4f^{15} + 2520a^8b^{11}c^{15}d^4e^3f^{16} - 882a^8b^{11}c^{16}d^3e^2f^{17} + 728a^9b^{10}c^2d^{17}e^{15}f^4 - 7480a^9b^{10}c^3d^{16}e^{14}f^5 + 2660a^9b^{10}c^4d^{15}e^{13}f^6 + 30212a^9b^{10}c^5d^{14}e^{12}f^7 - 38584a^9b^{10}c^6d^{13}e^{11}f^8 - 8008a^9b^{10}c^7d^{12}e^{10}f^9 + 20020a^9b^{10}c^8d^{11}e^9f^{10} + 20020a^9b^{10}c^9d^{10}e^8f^{11} - 8008a^9b^{10}c^{10}d^9e^7f^{12} - 38584a^9b^{10}c^{11}d^8e^6f^{13} + 30212a^9b^{10}c^{12}d^7e^5f^{14} + 2660a^9b^{10}c^{13}d^6e^4f^{15} - 7480a^9b^{10}c^{14}d^5e^3f^{16} + 728a^9b^{10}c^{15}d^4e^2f^{17} + 1152a^{10}b^9c^2d^{17}e^{14}f^5 + 9408a^{10}b^9c^3d^{16}e^{13}f^6 - 18564a^{10}b^9c^4d^{15}e^{12}f^7 - 13104a^{10}b^9c^5d^{14}e^{11}f^8 + 48048a^{10}b^9c^6d^{13}e^{10}f^9 - 8008a^{10}b^9c^7d^{12}e^9f^{10} - 36036a^{10}b^9c^8d^{11}e^8f^{11} - 8008a^{10}b^9c^9d^{10}e^7f^{12} + 48048a^{10}b^9c^{10}d^9e^6f^{13} - 13104a^{10}b^9c^{11}d^8e^5f^{14} - 18564a^{10}b^9c^{12}d^7e^4f^{15} + 9408a^{10}b^9c^{13}d^6e^3f^{16} + 1152a^{10}b^9c^{14}d^5e^2f^{17} - 4032a^{11}b^8c^2d^{17}e^{13}f^6 - 4368a^{11}b^8c^3d^{16}e^{12}f^7 + 26208a^{11}b^8c^4d^{15}e^{11}f^8 - 13104a^{11}b^8c^5d^{14}e^{10}f^9 - 38584a^{11}b^8c^6d^{13}e^9f^{10} + 32760a^{11}b^8c^7d^{12}e^8f^{11} + 32760a^{11}b^8c^8d^{11}e^7f^{12} - 38584a^{11}b^8c^9d^{10}e^6f^{13} - 13104a^{11}b^8c^{10}d^9e^5f^{14} + 26208a^{11}b^8c^{11}d^8e^4f^{15} - 4368a^{11}b^8c^{12}d^7e^3f^{16} - 4032a^{11}b^8c^{13}d^6e^2f^{17} + 5460a^{12}b^7c^2d^{17}e^{12}f^7 - 4368a^{12}b^7c^3d^{16}e^{11}f^8 - 18564a^{12}b^7c^4d^{15}e^{10}f^9 + 30212a^{12}b^7c^5d^{14}e^9f^{10} + 9828a^{12}b^7c^6d^{13}e^8f^{11} - 43680a^{12}b^7c^7d^{12}e^7f^{12} + 9828a^{12}b^7c^8d^{11}e^6f^{13} + 30212a^{12}b^7c^9d^{10}e^5f^{14} - 18564a^{12}b^7c^{10}d^9e^4f^{15} - 4368a^{12}b^7c^{11}d^8e^3f^{16} + 5460a^{12}b^7c^{12}d^7e^2f^{17} - 4032a^{13}b^6c^2d^{17}e^{11}f^8 + 9408a^{13}b^6c^3d^{16}e^{10}f^9 + 2660a^{13}b^6c^4d^{15}e^9f^{10} - 25452a^{13}b^6c^5d^{14}e^8f^{11} + 17472a^{13}b^6c^6d^{13}e^7f^{12} + 17472a^{13}b^6c^7d^{12}e^6f^{13} - 25452a^{13}b^6c^8d^{11}e^5f^{14} + 2660a^{13}b^6c^9d^{10}e^4f^{15} + 9408a^{13}b^6c^{10}d^9e^3f^{16} - 4032a^{13}b^6c^{11}d^8e^2f^{17} + 1152a^{14}b^5c^2d^{17}e^{10}f^9 - 7480a^{14}b^5c^3d^{16}e^9f^{10} + 7380a^{14}b^5c^4d^{15}e^8f^{11} + 7056a^{14}b^5c^5d^{14}e^7f^{12} - 17472a^{14}b^5c^6d^{13}e^6f^{13} + 7056a^{14}b^5c^7d^{12}e^5f^{14} + 7380a
\end{aligned}$$

$$\begin{aligned}
& ^{14}b^5c^8d^{11}e^4f^{15} - 7480a^{14}b^5c^9d^{10}e^3f^{16} + 1152a^{14}b^5 \\
& *c^{10}d^9e^2f^{17} + 728a^{15}b^4c^2d^{17}e^9f^{10} + 2520a^{15}b^4c^3d^1 \\
& 6e^8f^{11} - 6288a^{15}b^4c^4d^{15}e^7f^{12} + 3696a^{15}b^4c^5d^{14}e^6f \\
& ^{13} + 3696a^{15}b^4c^6d^{13}e^5f^{14} - 6288a^{15}b^4c^7d^{12}e^4f^{15} + 2 \\
& 520a^{15}b^4c^8d^{11}e^3f^{16} + 728a^{15}b^4c^9d^{10}e^2f^{17} - 882a^{16} \\
& b^3c^2d^{17}e^8f^{11} + 312a^{16}b^3c^3d^{16}e^7f^{12} + 1596a^{16}b^3c^4* \\
& d^{15}e^6f^{13} - 2772a^{16}b^3c^5d^{14}e^5f^{14} + 1596a^{16}b^3c^6d^{13}e^ \\
& 4f^{15} + 312a^{16}b^3c^7d^{12}e^3f^{16} - 882a^{16}b^3c^8d^{11}e^2f^{17} + \\
& 360a^{17}b^2c^2d^{17}e^7f^{12} - 504a^{17}b^2c^3d^{16}e^6f^{13} + 252a^{17} \\
& b^2c^4d^{15}e^5f^{14} + 252a^{17}b^2c^5d^{14}e^4f^{15} - 504a^{17}b^2c^6d \\
& ^{13}e^3f^{16} + 360a^{17}b^2c^7d^{12}e^2f^{17})) / (56a^3b^{13}c^5d^{11}e^{16} \\
& - a^8b^8d^{16}e^{16} - a^{16}c^8d^8e^{16} - b^{16}c^8d^8e^{16} - a^{16}d^{16}e^8 \\
& *f^8 - b^{16}c^{16}e^8f^8 - 28a^2b^{14}c^6d^{10}e^{16} - a^8b^8c^{16}f^{16} - \\
& 70a^4b^{12}c^4d^{12}e^{16} + 56a^5b^{11}c^3d^{13}e^{16} - 28a^6b^{10}c^2d^{14} \\
& 4e^{16} - 28a^{10}b^6c^{14}d^2f^{16} + 56a^{11}b^5c^{13}d^3f^{16} - 70a^{12}b^ \\
& 4c^{12}d^4f^{16} + 56a^{13}b^3c^{11}d^5f^{16} - 28a^{14}b^2c^{10}d^6f^{16} - 2 \\
& 8a^2b^{14}c^{16}e^6f^{10} + 56a^3b^{13}c^{16}e^5f^{11} - 70a^4b^{12}c^{16}e^4 \\
& *f^{12} + 56a^5b^{11}c^{16}e^3f^{13} - 28a^6b^{10}c^{16}e^2f^{14} - 28a^{10}b^6 \\
& *d^{16}e^{14}f^2 + 56a^{11}b^5d^{16}e^{13}f^3 - 70a^{12}b^4d^{16}e^{12}f^4 + 56 \\
& *a^{13}b^3d^{16}e^{11}f^5 - 28a^{14}b^2d^{16}e^{10}f^6 - 28a^{16}c^2d^{14}e^6* \\
& f^{10} + 56a^{16}c^3d^{13}e^5f^{11} - 70a^{16}c^4d^{12}e^4f^{12} + 56a^{16}c^5* \\
& d^{11}e^3f^{13} - 28a^{16}c^6d^{10}e^2f^{14} - 28b^{16}c^{10}d^6e^{14}f^2 + 56* \\
& b^{16}c^{11}d^5e^{13}f^3 - 70b^{16}c^{12}d^4e^{12}f^4 + 56b^{16}c^{13}d^3e^{11} \\
& f^5 - 28b^{16}c^{14}d^2e^{10}f^6 + 8a*b^{15}c^7d^9e^{16} + 8a^7b^9c*d^{15} \\
& e^{16} + 8a^9b^7c^{15}d*f^{16} + 8a^{15}b*c^9d^7f^{16} + 8a*b^{15}c^{16}e^7f^ \\
& 9 + 8a^7b^9c^{16}e*f^{15} + 8a^9b^7d^{16}e^{15}f + 8a^{15}b*d^{16}e^9f^7 + \\
& 8a^{16}c*d^{15}e^7f^9 + 8a^{16}c^7d^9e*f^{15} + 8b^{16}c^9d^7e^{15}f + 8* \\
& b^{16}c^{15}d*e^9f^7 - 56a*b^{15}c^8d^8e^{15}f - 56a*b^{15}c^{15}d*e^8f^8 - \\
& 56a^8b^8c*d^{15}e^{15}f - 56a^8b^8c^{15}d*e*f^{15} - 56a^{15}b*c*d^{15}e^8 \\
& *f^8 - 56a^{15}b*c^8d^8e*f^{15} + 160a*b^{15}c^9d^7e^{14}f^2 - 224a*b^{15} \\
& c^{10}d^6e^{13}f^3 + 112a*b^{15}c^{11}d^5e^{12}f^4 + 112a*b^{15}c^{12}d^4e^{11} \\
& *f^5 - 224a*b^{15}c^{13}d^3e^{10}f^6 + 160a*b^{15}c^{14}d^2e^9f^7 + 160a^2 \\
& *b^{14}c^7d^9e^{15}f + 160a^2b^{14}c^{15}d*e^7f^9 - 224a^3b^{13}c^6d^{10} \\
& e^{15}f - 224a^3b^{13}c^{15}d*e^6f^{10} + 112a^4b^{12}c^5d^{11}e^{15}f + 112* \\
& a^4b^{12}c^{15}d*e^5f^{11} + 112a^5b^{11}c^4d^{12}e^{15}f + 112a^5b^{11}c^{15} \\
& *d*e^4f^{12} - 224a^6b^{10}c^3d^{13}e^{15}f - 224a^6b^{10}c^{15}d*e^3f^{13} + \\
& 160a^7b^9c^2d^{14}e^{15}f + 160a^7b^9c^{15}d*e^2f^{14} + 160a^9b^7c* \\
& d^{15}e^{14}f^2 + 160a^9b^7c^{14}d^2e*f^{15} - 224a^{10}b^6c*d^{15}e^{13}f^3 \\
& - 224a^{10}b^6c^{13}d^3e*f^{15} + 112a^{11}b^5c*d^{15}e^{12}f^4 + 112a^{11}b^ \\
& 5c^{12}d^4e*f^{15} + 112a^{12}b^4c*d^{15}e^{11}f^5 + 112a^{12}b^4c^{11}d^5e* \\
& f^{15} - 224a^{13}b^3c*d^{15}e^{10}f^6 - 224a^{13}b^3c^{10}d^6e*f^{15} + 160a^ \\
& 14b^2c*d^{15}e^9f^7 + 160a^{14}b^2c^9d^7e*f^{15} + 160a^{15}b*c^2d^{14}e \\
& ^7f^9 - 224a^{15}b*c^3d^{13}e^6f^{10} + 112a^{15}b*c^4d^{12}e^5f^{11} + 112* \\
& a^{15}b*c^5d^{11}e^4f^{12} - 224a^{15}b*c^6d^{10}e^3f^{13} + 160a^{15}b*c^7d^ \\
& 9e^2f^{14} - 300a^2b^{14}c^8d^8e^{14}f^2 + 840a^2b^{14}c^{10}d^6e^{12}f^4 \\
& - 1344a^2b^{14}c^{11}d^5e^{11}f^5 + 840a^2b^{14}c^{12}d^4e^{10}f^6 - 300a \\
& ^2b^{14}c^{14}d^2e^8f^8 + 1400a^3b^{13}c^8d^8e^{13}f^3 - 2800a^3b^{13}c \\
& ^9d^7e^{12}f^4 + 1568a^3b^{13}c^{10}d^6e^{11}f^5 + 1568a^3b^{13}c^{11}d^5* \\
& e^{10}f^6 - 2800a^3b^{13}c^{12}d^4e^9f^7 + 1400a^3b^{13}c^{13}d^3e^8f^8 \\
& + 840a^4b^{12}c^6d^{10}e^{14}f^2 - 2800a^4b^{12}c^7d^9e^{13}f^3 + 1750a^ \\
& 4b^{12}c^8d^8e^{12}f^4 + 4480a^4b^{12}c^9d^7e^{11}f^5 - 8624a^4b^{12}c^ \\
& 10d^6e^{10}f^6 + 4480a^4b^{12}c^{11}d^5e^9f^7 + 1750a^4b^{12}c^{12}d^4e \\
& ^8f^8 - 2800a^4b^{12}c^{13}d^3e^7f^9 + 840a^4b^{12}c^{14}d^2e^6f^{10} - \\
& 1344a^5b^{11}c^5d^{11}e^{14}f^2 + 1568a^5b^{11}c^6d^{10}e^{13}f^3 + 4480a^ \\
& 5b^{11}c^7d^9e^{12}f^4 - 12264a^5b^{11}c^8d^8e^{11}f^5 + 7392a^5b^{11}c \\
& ^9d^7e^{10}f^6 + 7392a^5b^{11}c^{10}d^6e^9f^7 - 12264a^5b^{11}c^{11}d^5* \\
& e^8f^8 + 4480a^5b^{11}c^{12}d^4e^7f^9 + 1568a^5b^{11}c^{13}d^3e^6f^{10} \\
& - 1344a^5b^{11}c^{14}d^2e^5f^{11} + 840a^6b^{10}c^4d^{12}e^{14}f^2 + 1568a \\
& ^6b^{10}c^5d^{11}e^{13}f^3 - 8624a^6b^{10}c^6d^{10}e^{12}f^4 + 7392a^6b^{10}
\end{aligned}$$

$$\begin{aligned}
& *c^7*d^9*e^{11*f^5} + 11396*a^6*b^{10}*c^8*d^8*e^{10*f^6} - 24640*a^6*b^{10}*c^9*d^7*e^9*f^7 + 11396*a^6*b^{10}*c^{10}*d^6*e^8*f^8 + 7392*a^6*b^{10}*c^{11}*d^5*e^7*f^9 \\
& - 8624*a^6*b^{10}*c^{12}*d^4*e^6*f^{10} + 1568*a^6*b^{10}*c^{13}*d^3*e^5*f^{11} + 840*a^6*b^{10}*c^{14}*d^2*e^4*f^{12} - 2800*a^7*b^9*c^4*d^{12}*e^{13*f^3} + 4480*a^7*b^9*c^5*d^{11}*e^{12*f^4} \\
& + 7392*a^7*b^9*c^6*d^{10}*e^{11*f^5} - 24640*a^7*b^9*c^7*d^9*e^{10*f^6} + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^{10}*d^6*e^7*f^9 \\
& + 7392*a^7*b^9*c^{11}*d^5*e^6*f^{10} + 4480*a^7*b^9*c^{12}*d^4*e^5*f^{11} - 2800*a^7*b^9*c^{13}*d^3*e^4*f^{12} - 300*a^8*b^8*c^2*d^{14}*e^{14*f^2} + 1400*a^8*b^8*c^3*d^{13}*e^{13*f^3} \\
& + 1750*a^8*b^8*c^4*d^{12}*e^{12*f^4} - 12264*a^8*b^8*c^5*d^{11}*e^{11*f^5} + 11396*a^8*b^8*c^6*d^{10}*e^{10*f^6} + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 \\
& + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^{10}*d^6*e^6*f^{10} - 12264*a^8*b^8*c^{11}*d^5*e^5*f^{11} + 1750*a^8*b^8*c^{12}*d^4*e^4*f^{12} + 1400*a^8*b^8*c^{13}*d^3*e^3*f^{13} \\
& - 300*a^8*b^8*c^{14}*d^2*e^2*f^{14} - 2800*a^9*b^7*c^3*d^{13}*e^{12*f^4} + 4480*a^9*b^7*c^4*d^{12}*e^{11*f^5} + 7392*a^9*b^7*c^5*d^{11}*e^{10*f^6} - 24640*a^9*b^7*c^6*d^{10}*e^9*f^7 \\
& + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^{10} + 7392*a^9*b^7*c^{10}*d^6*e^5*f^{11} + 4480*a^9*b^7*c^{11}*d^5*e^4*f^{12} \\
& - 2800*a^9*b^7*c^{12}*d^4*e^3*f^{13} + 840*a^{10}*b^6*c^2*d^{14}*e^{12*f^4} + 1568*a^{10}*b^6*c^3*d^{13}*e^{11*f^5} - 8624*a^{10}*b^6*c^4*d^{12}*e^{10*f^6} + 7392*a^{10}*b^6*c^5*d^{11}*e^9*f^7 \\
& + 11396*a^{10}*b^6*c^6*d^{10}*e^8*f^8 - 24640*a^{10}*b^6*c^7*d^9*e^7*f^9 + 11396*a^{10}*b^6*c^8*d^8*e^6*f^{10} + 7392*a^{10}*b^6*c^9*d^7*e^5*f^{11} - 8624*a^{10}*b^6*c^{10}*d^6*e^4*f^{12} \\
& + 1568*a^{10}*b^6*c^{11}*d^5*e^3*f^{13} + 840*a^{10}*b^6*c^{12}*d^4*e^2*f^{14} - 1344*a^{11}*b^5*c^2*d^{14}*e^{11*f^5} + 1568*a^{11}*b^5*c^3*d^{13}*e^{10*f^6} + 4480*a^{11}*b^5*c^4*d^{12}*e^9*f^7 \\
& - 12264*a^{11}*b^5*c^5*d^{11}*e^8*f^8 + 7392*a^{11}*b^5*c^6*d^{10}*e^7*f^9 + 7392*a^{11}*b^5*c^7*d^9*e^6*f^{10} - 12264*a^{11}*b^5*c^8*d^8*e^5*f^{11} + 4480*a^{11}*b^5*c^9*d^7*e^4*f^{12} \\
& + 1568*a^{11}*b^5*c^{10}*d^6*e^3*f^{13} - 1344*a^{11}*b^5*c^{11}*d^5*e^2*f^{14} + 840*a^{12}*b^4*c^2*d^{14}*e^{10*f^6} - 2800*a^{12}*b^4*c^3*d^{13}*e^9*f^7 + 1750*a^{12}*b^4*c^4*d^{12}*e^8*f^8 \\
& + 4480*a^{12}*b^4*c^5*d^{11}*e^7*f^9 - 8624*a^{12}*b^4*c^6*d^{10}*e^6*f^{10} + 4480*a^{12}*b^4*c^7*d^9*e^5*f^{11} + 1750*a^{12}*b^4*c^8*d^8*e^4*f^{12} - 2800*a^{12}*b^4*c^9*d^7*e^3*f^{13} \\
& + 840*a^{12}*b^4*c^{10}*d^6*e^2*f^{14} + 1400*a^{13}*b^3*c^3*d^{13}*e^8*f^8 - 2800*a^{13}*b^3*c^4*d^{12}*e^7*f^9 + 1568*a^{13}*b^3*c^5*d^{11}*e^6*f^{10} + 1568*a^{13}*b^3*c^6*d^{10}*e^5*f^{11} \\
& - 2800*a^{13}*b^3*c^7*d^9*e^4*f^{12} + 1400*a^{13}*b^3*c^8*d^8*e^3*f^{13} - 300*a^{14}*b^2*c^2*d^{14}*e^8*f^8 + 840*a^{14}*b^2*c^4*d^{12}*e^6*f^{10} - 1344*a^{14}*b^2*c^5*d^{11}*e^5*f^{11} \\
& + 840*a^{14}*b^2*c^6*d^{10}*e^4*f^{12} - 300*a^{14}*b^2*c^8*d^8*e^2*f^{14}) + (x*(84*a^5*b^{11}*c^{13}*d^3*f^{16} - 12*a^4*b^{12}*c^{14}*d^2*f^{16} - 243*a^6*b^{10}*c^{12}*d^4*f^{16} \\
& + 366*a^7*b^9*c^{11}*d^5*f^{16} - 321*a^8*b^8*c^{10}*d^6*f^{16} + 252*a^9*b^7*c^9*d^7*f^{16} - 321*a^{10}*b^6*c^8*d^8*f^{16} + 366*a^{11}*b^5*c^7*d^9*f^{16} - 243*a^{12}*b^4*c^6*d^{10}*f^{16} \\
& + 84*a^{13}*b^3*c^5*d^{11}*f^{16} - 12*a^{14}*b^2*c^4*d^{12}*f^{16} - 12*a^4*b^{12}*d^{16}*e^{14*f^2} + 84*a^5*b^{11}*d^{16}*e^{13*f^3} - 243*a^6*b^{10}*d^{16}*e^{12*f^4} \\
& + 366*a^7*b^9*d^{16}*e^{11*f^5} - 321*a^8*b^8*d^{16}*e^{10*f^6} + 252*a^9*b^7*d^{16}*e^9*f^7 - 321*a^{10}*b^6*d^{16}*e^8*f^8 + 366*a^{11}*b^5*d^{16}*e^7*f^9 - 243*a^{12}*b^4*d^{16}*e^6*f^{10} \\
& + 84*a^{13}*b^3*d^{16}*e^5*f^{11} - 12*a^{14}*b^2*d^{16}*e^4*f^{12} - 12*b^{16}*c^4*d^{12}*e^{14*f^2} + 84*b^{16}*c^5*d^{11}*e^{13*f^3} - 243*b^{16}*c^6*d^{10}*e^{12*f^4} \\
& + 366*b^{16}*c^7*d^9*e^{11*f^5} - 321*b^{16}*c^8*d^8*e^{10*f^6} + 252*b^{16}*c^9*d^7*e^9*f^7 - 321*b^{16}*c^{10}*d^6*e^8*f^8 + 366*b^{16}*c^{11}*d^5*e^7*f^9 - 243*b^{16}*c^{12}*d^4*e^6*f^{10} \\
& + 84*b^{16}*c^{13}*d^3*e^5*f^{11} - 12*b^{16}*c^{14}*d^2*e^4*f^{12} + 48*a*b^{15}*c^3*d^{13}*e^{14*f^2} - 252*a*b^{15}*c^4*d^{12}*e^{13*f^3} + 366*a*b^{15}*c^5*d^{11}*e^{12*f^4} \\
& + 354*a*b^{15}*c^6*d^{10}*e^{11*f^5} - 1458*a*b^{15}*c^7*d^9*e^{10*f^6} + 942*a*b^{15}*c^8*d^8*e^9*f^7 + 942*a*b^{15}*c^9*d^7*e^8*f^8 - 1458*a*b^{15}*c^{10}*d^6*e^7*f^9 \\
& + 354*a*b^{15}*c^{11}*d^5*e^6*f^{10} + 366*a*b^{15}*c^{12}*d^4*e^5*f^{11} - 252*a*b^{15}*c^{13}*d^3*e^4*f^{12} + 48*a*b^{15}*c^{14}*d^2*e^3*f^{13} + 48*a^3*b^{13}*c*d^{15}*e^{14*f^2} \\
& + 48*a^3*b^{13}*c^14*d^2*e*f^{15} - 252*a^4*b^{12}*c*d^{15}*e^{13*f^3} - 252*a^4*b^{12}*c^{13}*d^3*e*f^{15} + 366*a^5*b^{11}*c*d^{15}*e^{12*f^4} + 366*a^5*b^{11}*c^{12}*d^4*e*f^{15} \\
& + 354*a^6*b^{10}*c*d^{15}*e^{11*f^5} + 354*a^6*b^{10}*c^{11}*d^5*e*f^{15} - 1458*a^7*b^9*c*d^{15}*e^{10*f^6} - 1458*a^7*b^9*c^{10}*d^6*e*f^{15} + 942*a^8*b^8*c*d^{15}*e^9*f^7 \\
& + 942*a^8*b^8*c^9*d^7*e*f^{15} + 942*a^9*b^7*c*d^{15}*e^8*f^8 + 942*a^9*b^7*c^8*d^8*e*f^{15}
\end{aligned}$$

$$\begin{aligned}
& 5 - 1458a^{10}b^6c^4d^{15}e^7f^9 - 1458a^{10}b^6c^7d^9e^6f^{15} + 354a^{11}b^5c^4d^{15}e^6f^{10} + 354a^{11}b^5c^6d^{10}e^6f^{15} + 366a^{12}b^4c^4d^{15}e^5f^{11} + 366a^{12}b^4c^5d^{11}e^6f^{15} - 252a^{13}b^3c^4d^{15}e^4f^{12} - 252a^{13}b^3c^4d^{12}e^6f^{15} + 48a^{14}b^2c^4d^{15}e^3f^{13} + 48a^{14}b^2c^3d^{13}e^6f^{15} - 72a^{14}b^2c^2d^{14}e^4f^{12} + 168a^{14}b^2c^3d^{13}e^3f^{13} + 723a^{14}b^2c^4d^{12}e^6f^{15} - 3258a^{14}b^2c^5d^{11}e^6f^{15} + 3156a^{14}b^2c^6d^{10}e^6f^{15} + 3522a^{14}b^2c^7d^9e^6f^{15} - 8478a^{14}b^2c^8d^8e^6f^{15} + 3522a^{14}b^2c^9d^7e^6f^{15} + 3156a^{14}b^2c^{10}d^6e^6f^{15} - 3258a^{14}b^2c^{11}d^5e^6f^{15} + 723a^{14}b^2c^{12}d^4e^6f^{15} + 168a^{14}b^2c^{13}d^3e^6f^{15} - 72a^{14}b^2c^{14}d^2e^6f^{15} + 168a^{14}b^2c^{13}d^3e^5f^{14} - 1692a^{14}b^2c^{13}d^3e^4f^{14} + 2538a^{14}b^2c^{13}d^3e^3f^{14} - 1692a^{14}b^2c^{13}d^3e^2f^{14} + 2538a^{14}b^2c^{13}d^3e^1f^{14} + 5634a^{14}b^2c^{13}d^3e^0f^{14} - 18738a^{14}b^2c^{13}d^3e^0f^{14} + 12042a^{14}b^2c^{13}d^3e^0f^{14} + 12042a^{14}b^2c^{13}d^3e^0f^{14} - 18738a^{14}b^2c^{13}d^3e^0f^{14} + 5634a^{14}b^2c^{13}d^3e^0f^{14} + 2538a^{14}b^2c^{13}d^3e^0f^{14} - 18738a^{14}b^2c^{13}d^3e^0f^{14} + 2538a^{14}b^2c^{13}d^3e^0f^{14} - 1692a^{14}b^2c^{13}d^3e^0f^{14} + 168a^{14}b^2c^{13}d^3e^0f^{14} + 723a^{14}b^2c^{13}d^3e^0f^{14} + 2538a^{14}b^2c^{13}d^3e^0f^{14} - 14022a^{14}b^2c^{13}d^3e^0f^{14} + 14022a^{14}b^2c^{13}d^3e^0f^{14} + 21087a^{14}b^2c^{13}d^3e^0f^{14} - 48168a^{14}b^2c^{13}d^3e^0f^{14} + 21087a^{14}b^2c^{13}d^3e^0f^{14} + 14022a^{14}b^2c^{13}d^3e^0f^{14} - 14022a^{14}b^2c^{13}d^3e^0f^{14} + 2538a^{14}b^2c^{13}d^3e^0f^{14} + 723a^{14}b^2c^{13}d^3e^0f^{14} + 2538a^{14}b^2c^{13}d^3e^0f^{14} - 14022a^{14}b^2c^{13}d^3e^0f^{14} + 14022a^{14}b^2c^{13}d^3e^0f^{14} - 50544a^{14}b^2c^{13}d^3e^0f^{14} + 33696a^{14}b^2c^{13}d^3e^0f^{14} + 33696a^{14}b^2c^{13}d^3e^0f^{14} - 50544a^{14}b^2c^{13}d^3e^0f^{14} + 14022a^{14}b^2c^{13}d^3e^0f^{14} + 5634a^{14}b^2c^{13}d^3e^0f^{14} - 3258a^{14}b^2c^{13}d^3e^0f^{14} + 3156a^{14}b^2c^{13}d^3e^0f^{14} - 8478a^{14}b^2c^{13}d^3e^0f^{14} + 12042a^{14}b^2c^{13}d^3e^0f^{14} - 48168a^{14}b^2c^{13}d^3e^0f^{14} + 33696a^{14}b^2c^{13}d^3e^0f^{14} - 78624a^{14}b^2c^{13}d^3e^0f^{14} + 33696a^{14}b^2c^{13}d^3e^0f^{14} + 21087a^{14}b^2c^{13}d^3e^0f^{14} - 18738a^{14}b^2c^{13}d^3e^0f^{14} + 3156a^{14}b^2c^{13}d^3e^0f^{14} + 3522a^{14}b^2c^{13}d^3e^0f^{14} + 9c^2d^{14}e^9f^7 + 12042a^{14}b^2c^3d^{13}e^8f^8 - 48168a^{14}b^2c^4d^{12}e^7f^9 + 33696a^{14}b^2c^5d^{11}e^6f^{10} + 33696a^{14}b^2c^6d^{10}e^5f^{11} - 48168a^{14}b^2c^7d^9e^4f^{12} + 12042a^{14}b^2c^8d^8e^3f^{13} + 3522a^{14}b^2c^9d^7e^2f^{14} - 8478a^{14}b^2c^8b^8c^2d^{14}e^8f^8 + 12042a^{14}b^2c^8b^8c^3d^{13}e^7f^9 + 21087a^{14}b^2c^8b^8c^4d^{12}e^6f^{10} - 50544a^{14}b^2c^8b^8c^5d^{11}e^5f^{11} + 21087a^{14}b^2c^8b^8c^6d^{10}e^4f^{12} + 12042a^{14}b^2c^8b^8c^7d^9e^3f^{13} - 8478a^{14}b^2c^8b^8c^8d^8e^2f^{14} + 3522a^{14}b^2c^9b^7c^2d^{14}e^7f^9 - 18738a^{14}b^2c^9b^7c^3d^{13}e^6f^{10} + 14022a^{14}b^2c^9b^7c^4d^{12}e^5f^{11} + 14022a^{14}b^2c^9b^7c^5d^{11}e^4f^{12} - 18738a^{14}b^2c^9b^7c^6d^{10}e^3f^{13} + 3522a^{14}b^2c^9b^7c^7d^9e^2f^{14} + 3156a^{14}b^2c^{10}b^6c^2d^{14}e^6f^{10} + 5634a^{14}b^2c^{10}b^6c^3d^{13}e^5f^{11} - 14022a^{14}b^2c^{10}b^6c^4d^{12}e^4f^{12} + 5634a^{14}b^2c^{10}b^6c^5d^{11}e^3f^{13} + 3156a^{14}b^2c^{10}b^6c^6d^{10}e^2f^{14} - 3258a^{14}b^2c^{11}b^5c^2d^{14}e^5f^{11} + 2538a^{14}b^2c^{11}b^5c^3d^{13}e^4f^{12} + 2538a^{14}b^2c^{11}b^5c^4d^{12}e^3f^{13} - 3258a^{14}b^2c^{11}b^5c^5d^{11}e^2f^{14} + 723a^{14}b^2c^{12}b^4c^2d^{14}e^4f^{12} - 1692a^{14}b^2c^{12}b^4c^3d^{13}e^3f^{13} + 723a^{14}b^2c^{12}b^4c^4d^{12}e^2f^{14} + 168a^{14}b^2c^{13}b^3c^2d^{14}e^3f^{13} + 168a^{14}b^2c^{13}b^3c^3d^{13}e^2f^{14} - 72a^{14}b^2c^{14}b^2c^2d^{14}e^2f^{14}))/((56a^3b^13c^5d^{11}e^{16} - a^8b^8d^{16}e^{16} - a^{16}c^8d^8e^{16} - b^{16}c^8d^8e^{16} - a^{16}d^{16}e^8f^8 - b^{16}c^{16}e^8f^8 - 28a^2b^{14}c^6d^{10}e^{16} - a^8b^8c^{16}f^{16} - 70a^4b^{12}c^4d^{12}e^{16} + 56a^5b^{11}c^3d^{13}e^{16} - 28a^6b^{10}c^2d^{14}e^{16} - 28a^{10}b^6c^{14}d^2f^{16} + 56a^{11}b^5c^{13}d^3f^{16} - 70a^{12}b^4c^{12}d^4f^{16} + 56a^{13}b^3c^{11}d^5f^{16} - 28a^{14}b^2c^{10}d^6f^{16} - 28a^2b^{14}c^{16}e^6f^{10} + 56a^3b^{13}c^{16}e^5f^{11} - 70a^4b^{12}c^{16}e^4f^{12} + 56a^5b^{11}c^{16}e^3f^{13} - 28a^6b^{10}c^{16}e^2f^{14} - 28a^{10}b^6d^{16}e^{14}f^2 + 56a^{11}b^5d^{16}e^{13}f^3 - 70a^{12}b^4d^{16}e^{12}f^4 + 56a^{13}b^3d^{16}e^{11}f^5 - 28a^{14}b^2d^{16}e^{10}f^6 - 28a^{16}c^2d^{14}e^6f^{10} + 56a^{16}c^3d^{13}e^5f^{11} - 70a^{16}c^4d^{12}e^4f^{12} + 56a^{16}c^5d^{11}e^3f^{13} - 28a^{16}c^6d^{10}e^2f^{14} - 28b^{16}c^{10}d^6e^{14}f^2 + 56b^{16}c^{11}d^5e^{13}f^3 - 70b^{16}c^{12}d^4e^{12}f^4 + 56b^{16}c^{13}d^3e^{11}f^5 - 28b^{16}c^{14}d^2e^{10}f^6 + 8a^8b^{15}c^7d^9e^{16} + 8a^7b^9c^d^{15}e^{16} + 8a^9b^7c^{15}d^f^{16} + 8a^{15}b^c^9d^7f^{16} + 8a
\end{aligned}$$

$$\begin{aligned}
& *b^{15}c^{16}e^{7f^9} + 8*a^7*b^9*c^{16}e^{f^{15}} + 8*a^9*b^7*d^{16}e^{15f} + 8*a^{15} \\
& *b*d^{16}e^9*f^7 + 8*a^{16}*c*d^{15}e^7*f^9 + 8*a^{16}*c^7*d^9*e^{f^{15}} + 8*b^{16}*c^9 \\
& *d^7*e^{15f} + 8*b^{16}*c^{15}*d^8*e^9*f^7 - 56*a*b^{15}*c^8*d^8*e^{15f} - 56*a*b^{15} \\
& *c^{15}*d^8*e^8*f^8 - 56*a^8*b^8*c*d^{15}e^{15f} - 56*a^8*b^8*c^{15}*d*e^{f^{15}} - 56* \\
& a^{15}*b*c*d^{15}e^8*f^8 - 56*a^{15}*b*c^8*d^8*e^{f^{15}} + 160*a*b^{15}*c^9*d^7*e^{14} \\
& f^2 - 224*a*b^{15}*c^{10}*d^6*e^{13f^3} + 112*a*b^{15}*c^{11}*d^5*e^{12f^4} + 112*a*b \\
& ^{15}*c^{12}*d^4*e^{11f^5} - 224*a*b^{15}*c^{13}*d^3*e^{10f^6} + 160*a*b^{15}*c^{14}*d^2* \\
& e^9*f^7 + 160*a^2*b^{14}*c^7*d^9*e^{15f} + 160*a^2*b^{14}*c^{15}*d^8*e^7*f^9 - 224*a \\
& ^3*b^{13}*c^6*d^{10}e^{15f} - 224*a^3*b^{13}*c^{15}*d^8*e^6*f^{10} + 112*a^4*b^{12}*c^5*d \\
& ^{11}e^{15f} + 112*a^4*b^{12}*c^{15}*d^8*e^5*f^{11} + 112*a^5*b^{11}*c^4*d^{12}e^{15f} + \\
& 112*a^5*b^{11}*c^{15}*d^8*e^4*f^{12} - 224*a^6*b^{10}*c^3*d^{13}e^{15f} - 224*a^6*b^{10}* \\
& c^{15}*d^8*e^3*f^{13} + 160*a^7*b^9*c^2*d^{14}e^{15f} + 160*a^7*b^9*c^{15}*d^8*e^2*f^{14} \\
& + 160*a^9*b^7*c*d^{15}e^{14f^2} + 160*a^9*b^7*c^{14}*d^2*e^{f^{15}} - 224*a^{10}*b^6 \\
& *c*d^{15}e^{13f^3} - 224*a^{10}*b^6*c^{13}*d^3*e^{f^{15}} + 112*a^{11}*b^5*c*d^{15}e^{12} \\
& f^4 + 112*a^{11}*b^5*c^{12}*d^4*e^{f^{15}} + 112*a^{12}*b^4*c*d^{15}e^{11f^5} + 112*a^{12} \\
& *b^4*c^{11}*d^5*e^{f^{15}} - 224*a^{13}*b^3*c*d^{15}e^{10f^6} - 224*a^{13}*b^3*c^{10}*d^6 \\
& *e^{f^{15}} + 160*a^{14}*b^2*c*d^{15}e^9*f^7 + 160*a^{14}*b^2*c^9*d^7*e^{f^{15}} + 160* \\
& a^{15}*b*c^2*d^{14}e^7*f^9 - 224*a^{15}*b*c^3*d^{13}e^6*f^{10} + 112*a^{15}*b*c^4*d^{12} \\
& e^5*f^{11} + 112*a^{15}*b*c^5*d^{11}e^4*f^{12} - 224*a^{15}*b*c^6*d^{10}e^3*f^{13} + \\
& 160*a^{15}*b*c^7*d^9*e^2*f^{14} - 300*a^2*b^{14}*c^8*d^8*e^{14f^2} + 840*a^2*b^{14} \\
& *c^{10}*d^6*e^{12f^4} - 1344*a^2*b^{14}*c^{11}*d^5*e^{11f^5} + 840*a^2*b^{14}*c^{12}*d^4 \\
& *e^{10f^6} - 300*a^2*b^{14}*c^{14}*d^2*e^8*f^8 + 1400*a^3*b^{13}*c^8*d^8*e^{13f^3} \\
& - 2800*a^3*b^{13}*c^9*d^7*e^{12f^4} + 1568*a^3*b^{13}*c^{10}*d^6*e^{11f^5} + 1568*a \\
& ^3*b^{13}*c^{11}*d^5*e^{10f^6} - 2800*a^3*b^{13}*c^{12}*d^4*e^9*f^7 + 1400*a^3*b^{13} \\
& *c^{13}*d^3*e^8*f^8 + 840*a^4*b^{12}*c^6*d^{10}e^{14f^2} - 2800*a^4*b^{12}*c^7*d^9*e \\
& ^{13f^3} + 1750*a^4*b^{12}*c^8*d^8*e^{12f^4} + 4480*a^4*b^{12}*c^9*d^7*e^{11f^5} - \\
& 8624*a^4*b^{12}*c^{10}*d^6*e^{10f^6} + 4480*a^4*b^{12}*c^{11}*d^5*e^9*f^7 + 1750*a^4 \\
& *b^{12}*c^{12}*d^4*e^8*f^8 - 2800*a^4*b^{12}*c^{13}*d^3*e^7*f^9 + 840*a^4*b^{12}*c^{14} \\
& *d^2*e^6*f^{10} - 1344*a^5*b^{11}*c^5*d^{11}e^{14f^2} + 1568*a^5*b^{11}*c^6*d^{10}e \\
& ^{13f^3} + 4480*a^5*b^{11}*c^7*d^9*e^{12f^4} - 12264*a^5*b^{11}*c^8*d^8*e^{11f^5} \\
& + 7392*a^5*b^{11}*c^9*d^7*e^{10f^6} + 7392*a^5*b^{11}*c^{10}*d^6*e^9*f^7 - 12264*a \\
& ^5*b^{11}*c^{11}*d^5*e^8*f^8 + 4480*a^5*b^{11}*c^{12}*d^4*e^7*f^9 + 1568*a^5*b^{11}*c \\
& ^{13}*d^3*e^6*f^{10} - 1344*a^5*b^{11}*c^{14}*d^2*e^5*f^{11} + 840*a^6*b^{10}*c^4*d^{12} \\
& e^{14f^2} + 1568*a^6*b^{10}*c^5*d^{11}e^{13f^3} - 8624*a^6*b^{10}*c^6*d^{10}e^{12f^4} \\
& + 7392*a^6*b^{10}*c^7*d^9*e^{11f^5} + 11396*a^6*b^{10}*c^8*d^8*e^{10f^6} - 2464 \\
& 0*a^6*b^{10}*c^9*d^7*e^9*f^7 + 11396*a^6*b^{10}*c^{10}*d^6*e^8*f^8 + 7392*a^6*b^{10} \\
& *c^{11}*d^5*e^7*f^9 - 8624*a^6*b^{10}*c^{12}*d^4*e^6*f^{10} + 1568*a^6*b^{10}*c^{13}*d \\
& ^3*e^5*f^{11} + 840*a^6*b^{10}*c^{14}*d^2*e^4*f^{12} - 2800*a^7*b^9*c^4*d^{12}e^{13f^3} \\
& + 4480*a^7*b^9*c^5*d^{11}e^{12f^4} + 7392*a^7*b^9*c^6*d^{10}e^{11f^5} - 2464 \\
& 0*a^7*b^9*c^7*d^9*e^{10f^6} + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9* \\
& c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^{10}*d^6*e^7*f^9 + 7392*a^7*b^9*c^{11}*d^5*e^6 \\
& f^{10} + 4480*a^7*b^9*c^{12}*d^4*e^5*f^{11} - 2800*a^7*b^9*c^{13}*d^3*e^4*f^{12} - \\
& 300*a^8*b^8*c^2*d^{14}e^{14f^2} + 1400*a^8*b^8*c^3*d^{13}e^{13f^3} + 1750*a^8*b \\
& ^8*c^4*d^{12}e^{12f^4} - 12264*a^8*b^8*c^5*d^{11}e^{11f^5} + 11396*a^8*b^8*c^6* \\
& d^{10}e^{10f^6} + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 \\
& + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^{10}*d^6*e^6*f^{10} - 1226 \\
& 4*a^8*b^8*c^{11}*d^5*e^5*f^{11} + 1750*a^8*b^8*c^{12}*d^4*e^4*f^{12} + 1400*a^8*b^8 \\
& *c^{13}*d^3*e^3*f^{13} - 300*a^8*b^8*c^{14}*d^2*e^2*f^{14} - 2800*a^9*b^7*c^3*d^{13} \\
& e^{12f^4} + 4480*a^9*b^7*c^4*d^{12}e^{11f^5} + 7392*a^9*b^7*c^5*d^{11}e^{10f^6} \\
& - 24640*a^9*b^7*c^6*d^{10}e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9 \\
& *b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^{10} + 7392*a^9*b^7*c^{10}* \\
& d^6*e^5*f^{11} + 4480*a^9*b^7*c^{11}*d^5*e^4*f^{12} - 2800*a^9*b^7*c^{12}*d^4*e^3*f \\
& ^{13} + 840*a^{10}*b^6*c^2*d^{14}e^{12f^4} + 1568*a^{10}*b^6*c^3*d^{13}e^{11f^5} - 86 \\
& 24*a^{10}*b^6*c^4*d^{12}e^{10f^6} + 7392*a^{10}*b^6*c^5*d^{11}e^9*f^7 + 11396*a^{10} \\
& *b^6*c^6*d^{10}e^8*f^8 - 24640*a^{10}*b^6*c^7*d^9*e^7*f^9 + 11396*a^{10}*b^6*c^8 \\
& *d^8*e^6*f^{10} + 7392*a^{10}*b^6*c^9*d^7*e^5*f^{11} - 8624*a^{10}*b^6*c^{10}*d^6*e^4 \\
& *f^{12} + 1568*a^{10}*b^6*c^{11}*d^5*e^3*f^{13} + 840*a^{10}*b^6*c^{12}*d^4*e^2*f^{14} - \\
& 1344*a^{11}*b^5*c^2*d^{14}e^{11f^5} + 1568*a^{11}*b^5*c^3*d^{13}e^{10f^6} + 4480*a^{11} \\
& *b^5*c^4*d^{12}e^9*f^7 - 12264*a^{11}*b^5*c^5*d^{11}e^8*f^8 + 7392*a^{11}*b^5*c
\end{aligned}$$

$$\begin{aligned}
& \cdot 6^d \cdot 10^e \cdot 7^f \cdot 9 + 7392 \cdot a^{11} \cdot b^5 \cdot c^7 \cdot d^9 \cdot e^6 \cdot f^{10} - 12264 \cdot a^{11} \cdot b^5 \cdot c^8 \cdot d^8 \cdot e^5 \cdot f^{11} \\
& + 4480 \cdot a^{11} \cdot b^5 \cdot c^9 \cdot d^7 \cdot e^4 \cdot f^{12} + 1568 \cdot a^{11} \cdot b^5 \cdot c^{10} \cdot d^6 \cdot e^3 \cdot f^{13} \\
& - 1344 \cdot a^{11} \cdot b^5 \cdot c^{11} \cdot d^5 \cdot e^2 \cdot f^{14} + 840 \cdot a^{12} \cdot b^4 \cdot c^2 \cdot d^{14} \cdot e^{10} \cdot f^6 - 2800 \cdot a^{12} \cdot b^4 \cdot c^3 \cdot d^{13} \cdot e^9 \cdot f^7 \\
& + 1750 \cdot a^{12} \cdot b^4 \cdot c^4 \cdot d^{12} \cdot e^8 \cdot f^8 + 4480 \cdot a^{12} \cdot b^4 \cdot c^5 \cdot d^{11} \cdot e^7 \cdot f^9 - 8624 \cdot a^{12} \cdot b^4 \cdot c^6 \cdot d^{10} \cdot e^6 \cdot f^{10} \\
& + 4480 \cdot a^{12} \cdot b^4 \cdot c^7 \cdot d^9 \cdot e^5 \cdot f^{11} + 1750 \cdot a^{12} \cdot b^4 \cdot c^8 \cdot d^8 \cdot e^4 \cdot f^{12} - 2800 \cdot a^{12} \cdot b^4 \cdot c^9 \cdot d^7 \cdot e^3 \cdot f^{13} \\
& + 840 \cdot a^{12} \cdot b^4 \cdot c^{10} \cdot d^6 \cdot e^2 \cdot f^{14} + 1400 \cdot a^{13} \cdot b^3 \cdot c^3 \cdot d^{13} \cdot e^8 \cdot f^8 - 2800 \cdot a^{13} \cdot b^3 \cdot c^4 \cdot d^{12} \cdot e^7 \cdot f^9 \\
& + 1568 \cdot a^{13} \cdot b^3 \cdot c^5 \cdot d^{11} \cdot e^6 \cdot f^{10} + 1568 \cdot a^{13} \cdot b^3 \cdot c^6 \cdot d^{10} \cdot e^5 \cdot f^{11} - 2800 \cdot a^{13} \cdot b^3 \cdot c^7 \cdot d^9 \cdot e^4 \cdot f^{12} \\
& + 1400 \cdot a^{13} \cdot b^3 \cdot c^8 \cdot d^8 \cdot e^3 \cdot f^{13} - 300 \cdot a^{14} \cdot b^2 \cdot c^2 \cdot d^{14} \cdot e^8 \cdot f^8 + 840 \cdot a^{14} \cdot b^2 \cdot c^4 \cdot d^{12} \cdot e^6 \cdot f^{10} - 1344 \cdot a^{14} \cdot b^2 \cdot c^5 \cdot d^{11} \cdot e^5 \cdot f^{11} \\
& + 840 \cdot a^{14} \cdot b^2 \cdot c^6 \cdot d^{10} \cdot e^4 \cdot f^{12} - 300 \cdot a^{14} \cdot b^2 \cdot c^8 \cdot d^8 \cdot e^2 \cdot f^{14}) - (36 \cdot a^{11} \cdot b^2 \cdot d^{13} \cdot f^{13} + 36 \cdot b^{13} \cdot c^{11} \cdot d^2 \cdot f^{13} + 36 \cdot b^{13} \cdot d^{13} \cdot e^{11} \cdot f^2 \\
& + 297 \cdot a^2 \cdot b^{11} \cdot c^9 \cdot d^4 \cdot f^{13} - 108 \cdot a^3 \cdot b^{10} \cdot c^8 \cdot d^5 \cdot f^{13} - 198 \cdot a^4 \cdot b^9 \cdot c^7 \cdot d^6 \cdot f^{13} + 153 \cdot a^5 \cdot b^8 \cdot c^6 \cdot d^7 \cdot f^{13} \\
& + 153 \cdot a^6 \cdot b^7 \cdot c^5 \cdot d^8 \cdot f^{13} - 198 \cdot a^7 \cdot b^6 \cdot c^4 \cdot d^9 \cdot f^{13} - 108 \cdot a^8 \cdot b^5 \cdot c^3 \cdot d^{10} \cdot f^{13} + 297 \cdot a^9 \cdot b^4 \cdot c^2 \cdot d^{11} \cdot f^{13} \\
& + 297 \cdot a^2 \cdot b^{11} \cdot d^{13} \cdot e^9 \cdot f^4 - 108 \cdot a^3 \cdot b^{10} \cdot d^{13} \cdot e^8 \cdot f^5 - 198 \cdot a^4 \cdot b^9 \cdot d^{13} \cdot e^7 \cdot f^6 + 153 \cdot a^5 \cdot b^8 \cdot d^{13} \cdot e^6 \cdot f^7 \\
& + 153 \cdot a^6 \cdot b^7 \cdot d^{13} \cdot e^5 \cdot f^8 - 198 \cdot a^7 \cdot b^6 \cdot d^{13} \cdot e^4 \cdot f^9 - 108 \cdot a^8 \cdot b^5 \cdot d^{13} \cdot e^3 \cdot f^{10} + 297 \cdot a^9 \cdot b^4 \cdot d^{13} \cdot e^2 \cdot f^{11} \\
& + 297 \cdot b^{13} \cdot c^2 \cdot d^{11} \cdot e^9 \cdot f^4 - 108 \cdot b^{13} \cdot c^3 \cdot d^{10} \cdot e^8 \cdot f^5 - 198 \cdot b^{13} \cdot c^4 \cdot d^9 \cdot e^7 \cdot f^6 + 153 \cdot b^{13} \cdot c^5 \cdot d^8 \cdot e^6 \cdot f^7 \\
& + 153 \cdot b^{13} \cdot c^6 \cdot d^7 \cdot e^5 \cdot f^8 - 198 \cdot b^{13} \cdot c^7 \cdot d^6 \cdot e^4 \cdot f^9 - 108 \cdot b^{13} \cdot c^8 \cdot d^5 \cdot e^3 \cdot f^{10} + 297 \cdot b^{13} \cdot c^9 \cdot d^4 \cdot e^2 \cdot f^{11} \\
& - 180 \cdot a \cdot b^{12} \cdot c^{10} \cdot d^3 \cdot f^{13} - 180 \cdot a^{10} \cdot b^3 \cdot c \cdot d^{12} \cdot f^{13} - 180 \cdot a \cdot b^{12} \cdot d^{13} \cdot e^{10} \cdot f^3 - 180 \cdot a^{10} \cdot b^3 \cdot d^{13} \cdot e \cdot f^{12} \\
& - 180 \cdot b^{13} \cdot c \cdot d^{12} \cdot e^{10} \cdot f^3 - 180 \cdot b^{13} \cdot c^{10} \cdot d^3 \cdot e \cdot f^{12} + 1026 \cdot a \cdot b^{12} \cdot c \cdot d^{12} \cdot e^9 \cdot f^4 + 1026 \cdot a \cdot b^{12} \cdot c^9 \cdot d^4 \cdot e \cdot f^{12} \\
& + 1026 \cdot a^9 \cdot b^4 \cdot c \cdot d^{12} \cdot e \cdot f^{12} - 2052 \cdot a \cdot b^{12} \cdot c^2 \cdot d^{11} \cdot e^8 \cdot f^5 + 1548 \cdot a \cdot b^{12} \cdot c^3 \cdot d^{10} \cdot e^7 \cdot f^6 + 297 \cdot a \cdot b^{12} \cdot c^4 \cdot d^9 \cdot e^6 \cdot f^7 \\
& - 1242 \cdot a \cdot b^{12} \cdot c^5 \cdot d^8 \cdot e^5 \cdot f^8 + 297 \cdot a \cdot b^{12} \cdot c^6 \cdot d^7 \cdot e^4 \cdot f^9 + 1548 \cdot a \cdot b^{12} \cdot c^7 \cdot d^6 \cdot e^3 \cdot f^{10} - 2052 \cdot a \cdot b^{12} \cdot c^8 \cdot d^5 \cdot e^2 \cdot f^{11} \\
& - 2052 \cdot a^2 \cdot b^{11} \cdot c \cdot d^{12} \cdot e^8 \cdot f^5 - 2052 \cdot a^2 \cdot b^{11} \cdot c^8 \cdot d^5 \cdot e \cdot f^{12} + 1548 \cdot a^3 \cdot b^{10} \cdot c \cdot d^{12} \cdot e^7 \cdot f^6 + 1548 \cdot a^3 \cdot b^{10} \cdot c^7 \cdot d^6 \cdot e \cdot f^{12} \\
& + 297 \cdot a^4 \cdot b^9 \cdot c \cdot d^{12} \cdot e^6 \cdot f^7 + 297 \cdot a^4 \cdot b^9 \cdot c^6 \cdot d^7 \cdot e \cdot f^{12} - 1242 \cdot a^5 \cdot b^8 \cdot c \cdot d^{12} \cdot e^5 \cdot f^8 - 1242 \cdot a^5 \cdot b^8 \cdot c^5 \cdot d^8 \cdot e \cdot f^{12} \\
& + 297 \cdot a^6 \cdot b^7 \cdot c \cdot d^{12} \cdot e^4 \cdot f^9 + 297 \cdot a^6 \cdot b^7 \cdot c^4 \cdot d^9 \cdot e \cdot f^{12} + 1548 \cdot a^7 \cdot b^6 \cdot c \cdot d^{12} \cdot e^3 \cdot f^{10} + 1548 \cdot a^7 \cdot b^6 \cdot c^3 \cdot d^{10} \cdot e \cdot f^{12} \\
& - 2052 \cdot a^8 \cdot b^5 \cdot c \cdot d^{12} \cdot e^2 \cdot f^{11} - 2052 \cdot a^8 \cdot b^5 \cdot c^2 \cdot d^{11} \cdot e \cdot f^{12} + 4860 \cdot a^2 \cdot b^{11} \cdot c^2 \cdot d^{11} \cdot e^7 \cdot f^6 - 4986 \cdot a^2 \cdot b^{11} \cdot c^3 \cdot d^{10} \cdot e^6 \cdot f^7 \\
& + 1701 \cdot a^2 \cdot b^{11} \cdot c^4 \cdot d^9 \cdot e^5 \cdot f^8 + 1701 \cdot a^2 \cdot b^{11} \cdot c^5 \cdot d^8 \cdot e^4 \cdot f^9 - 4986 \cdot a^2 \cdot b^{11} \cdot c^6 \cdot d^7 \cdot e^3 \cdot f^{10} + 4860 \cdot a^2 \cdot b^{11} \cdot c^7 \cdot d^6 \cdot e^2 \cdot f^{11} \\
& - 4986 \cdot a^3 \cdot b^{10} \cdot c^2 \cdot d^{11} \cdot e^6 \cdot f^7 + 6336 \cdot a^3 \cdot b^{10} \cdot c^3 \cdot d^{10} \cdot e^5 \cdot f^8 - 3960 \cdot a^3 \cdot b^{10} \cdot c^4 \cdot d^9 \cdot e^4 \cdot f^9 + 6336 \cdot a^3 \cdot b^{10} \cdot c^5 \cdot d^8 \cdot e^3 \cdot f^{10} \\
& - 4986 \cdot a^3 \cdot b^{10} \cdot c^6 \cdot d^7 \cdot e^2 \cdot f^{11} + 1701 \cdot a^4 \cdot b^9 \cdot c^2 \cdot d^{11} \cdot e^5 \cdot f^8 - 3960 \cdot a^4 \cdot b^9 \cdot c^3 \cdot d^{10} \cdot e^4 \cdot f^9 - 3960 \cdot a^4 \cdot b^9 \cdot c^4 \cdot d^9 \cdot e^3 \cdot f^{10} \\
& + 1701 \cdot a^4 \cdot b^9 \cdot c^5 \cdot d^8 \cdot e^2 \cdot f^{11} + 1701 \cdot a^5 \cdot b^8 \cdot c^2 \cdot d^{11} \cdot e^4 \cdot f^9 + 6336 \cdot a^5 \cdot b^8 \cdot c^3 \cdot d^{10} \cdot e^3 \cdot f^{10} + 1701 \cdot a^5 \cdot b^8 \cdot c^4 \cdot d^9 \cdot e^2 \cdot f^{11} \\
& - 4986 \cdot a^6 \cdot b^7 \cdot c^2 \cdot d^{11} \cdot e^3 \cdot f^{10} - 4986 \cdot a^6 \cdot b^7 \cdot c^3 \cdot d^{10} \cdot e^2 \cdot f^{11} + 4860 \cdot a^7 \cdot b^6 \cdot c^2 \cdot d^{11} \cdot e^2 \cdot f^{11}) \\
& / (56 \cdot a^3 \cdot b^{13} \cdot c^5 \cdot d^{11} \cdot e^{16} - a^8 \cdot b^8 \cdot d^{16} \cdot e^{16} - a^{16} \cdot c^8 \cdot d^8 \cdot e^8 \cdot f^{16} - b^{16} \cdot c^8 \cdot d^8 \cdot e^8 \cdot f^{16} - a^{16} \cdot d^{16} \cdot e^8 \cdot f^8 \\
& - b^{16} \cdot c^{16} \cdot e^8 \cdot f^8 - 28 \cdot a^2 \cdot b^{14} \cdot c^6 \cdot d^{10} \cdot e^{16} - a^8 \cdot b^8 \cdot c^{16} \cdot f^{16} - 70 \cdot a^4 \cdot b^{12} \cdot c^4 \cdot d^{12} \cdot e^{16} + 56 \cdot a^5 \cdot b^{11} \cdot c^3 \cdot d^{13} \cdot e^{16} \\
& - 28 \cdot a^6 \cdot b^{10} \cdot c^2 \cdot d^{14} \cdot e^{16} - 28 \cdot a^{10} \cdot b^6 \cdot c^{14} \cdot d^2 \cdot e^{16} + 56 \cdot a^{11} \cdot b^5 \cdot c^{13} \cdot d^3 \cdot e^{16} - 70 \cdot a^{12} \cdot b^4 \cdot c^{12} \cdot d^4 \cdot e^{16} \\
& + 56 \cdot a^{13} \cdot b^3 \cdot c^{11} \cdot d^5 \cdot e^{16} - 28 \cdot a^{14} \cdot b^2 \cdot c^{10} \cdot d^6 \cdot e^{16} - 28 \cdot a^2 \cdot b^{14} \cdot c^{16} \cdot e^6 \cdot f^{10} + 56 \cdot a^3 \cdot b^{13} \cdot c^{16} \cdot e^5 \cdot f^{11} \\
& - 70 \cdot a^4 \cdot b^{12} \cdot c^{16} \cdot e^4 \cdot f^{12} + 56 \cdot a^5 \cdot b^{11} \cdot c^{16} \cdot e^3 \cdot f^{13} - 28 \cdot a^6 \cdot b^{10} \cdot c^{16} \cdot e^2 \cdot f^{14} - 28 \cdot a^{10} \cdot b^6 \cdot d^{16} \cdot e^{14} \cdot f^2 \\
& + 56 \cdot a^{11} \cdot b^5 \cdot d^{16} \cdot e^{13} \cdot f^3 - 70 \cdot a^{12} \cdot b^4 \cdot d^{16} \cdot e^{12} \cdot f^4 + 56 \cdot a^{13} \cdot b^3 \cdot d^{16} \cdot e^{11} \cdot f^5 - 28 \cdot a^{14} \cdot b^2 \cdot d^{16} \cdot e^{10} \cdot f^6 \\
& - 28 \cdot a^{16} \cdot c^2 \cdot d^{14} \cdot e^6 \cdot f^{10} + 56 \cdot a^{16} \cdot c^3 \cdot d^{13} \cdot e^5 \cdot f^{11} - 70 \cdot a^{16} \cdot c^4 \cdot d^{12} \cdot e^4 \cdot f^{12} + 56 \cdot a^{16} \cdot c^5 \cdot d^{11} \cdot e^3 \cdot f^{13} \\
& - 28 \cdot a^{16} \cdot c^6 \cdot d^{10} \cdot e^2 \cdot f^{14} - 28 \cdot b^{16} \cdot c^{10} \cdot d^6 \cdot e^{14} \cdot f^2 + 56 \cdot b^{16} \cdot c^{11} \cdot d^5 \cdot e^{13} \cdot f^3 - 70 \cdot b^{16} \cdot c^{12} \cdot d^4 \cdot e^{12} \cdot f^4 \\
& + 56 \cdot b^{16} \cdot c^{13} \cdot d^3 \cdot e^{11} \cdot f^5 - 28 \cdot b^{16} \cdot c^{14} \cdot d^2 \cdot e^{10} \cdot f^6 + 8 \cdot a \cdot b^{15} \cdot c^7 \cdot d^9 \cdot e^{16} + 8 \cdot a^7 \cdot b^9 \cdot c \cdot d^{15} \cdot e^{16} \\
& + 8 \cdot a^9 \cdot b^7 \cdot c^{15} \cdot d \cdot e^{16} + 8 \cdot a^{15} \cdot b \cdot c^9 \cdot d^7 \cdot e^{16} + 8 \cdot a \cdot b^{15} \cdot c^{16} \cdot e^7 \cdot f^9 + 8 \cdot a^7 \cdot b^9 \cdot c^{16} \cdot e \cdot f^{15} \\
& + 8 \cdot a^9 \cdot b^7 \cdot d^{16} \cdot e^{15} \cdot f + 8 \cdot a^{15} \cdot b \cdot d^{16} \cdot e^9 \cdot f^7 + 8 \cdot a^{16} \cdot c \cdot d^{15} \cdot e^7 \cdot f^9 + 8 \cdot a^{16} \cdot c^7 \cdot d^9 \cdot e \cdot f^{15} \\
& + 8 \cdot b^{16} \cdot c^9 \cdot d^7 \cdot e^{15} \cdot f + 8 \cdot b^{16} \cdot c^{15} \cdot d \cdot e^9 \cdot f^7 - 56 \cdot a \cdot b^{15} \cdot c^8 \cdot d^8 \cdot e^{15} \cdot f
\end{aligned}$$

$$\begin{aligned}
& - 56*a*b^{15}*c^{15}*d^8*f^8 - 56*a^8*b^8*c*d^{15}*e^{15}*f - 56*a^8*b^8*c^{15}*d*e \\
& *f^{15} - 56*a^{15}*b*c*d^{15}*e^8*f^8 - 56*a^{15}*b*c^8*d^8*e*f^{15} + 160*a*b^{15}*c^9*d^7*e^{14}*f^2 \\
& - 224*a*b^{15}*c^{10}*d^6*e^{13}*f^3 + 112*a*b^{15}*c^{11}*d^5*e^{12}*f^4 + 112*a*b^{15}*c^{12}*d^4*e^{11}*f^5 \\
& - 224*a*b^{15}*c^{13}*d^3*e^{10}*f^6 + 160*a*b^{15}*c^{14}*d^2*e^9*f^7 + 160*a^2*b^{14}*c^7*d^9*e^{15}*f \\
& + 160*a^2*b^{14}*c^{15}*d*e^7*f^9 - 224*a^3*b^{13}*c^6*d^{10}*e^{15}*f - 224*a^3*b^{13}*c^{15}*d*e^6*f^{10} \\
& + 112*a^4*b^{12}*c^5*d^{11}*e^{15}*f + 112*a^4*b^{12}*c^{15}*d*e^5*f^{11} + 112*a^5*b^{11}*c^4*d^{12}*e^{15}*f \\
& + 112*a^5*b^{11}*c^{15}*d*e^4*f^{12} - 224*a^6*b^{10}*c^3*d^{13}*e^{15}*f - 224*a^6*b^{10}*c^{15}*d*e^3*f^{13} \\
& + 160*a^7*b^9*c^2*d^{14}*e^{15}*f + 160*a^7*b^9*c^{15}*d*e^2*f^{14} + 160*a^9*b^7*c*d^{15}*e^{14}*f^2 \\
& + 160*a^9*b^7*c^{14}*d^2*e*f^{15} - 224*a^{10}*b^6*c*d^{15}*e^{13}*f^3 - 224*a^{10}*b^6*c^{13}*d^3*e*f^{15} \\
& + 112*a^{11}*b^5*c*d^{15}*e^{12}*f^4 + 112*a^{11}*b^5*c^{12}*d^4*e*f^{15} + 112*a^{12}*b^4*c*d^{15}*e^{11}*f^5 \\
& + 112*a^{12}*b^4*c^{11}*d^5*e*f^{15} - 224*a^{13}*b^3*c*d^{15}*e^{10}*f^6 - 224*a^{13}*b^3*c^{10}*d^6*e*f^{15} \\
& + 160*a^{14}*b^2*c*d^{15}*e^9*f^7 + 160*a^{14}*b^2*c^9*d^7*e*f^{15} + 160*a^{15}*b*c^2*d^{14}*e^7*f^9 \\
& - 224*a^{15}*b*c^3*d^{13}*e^6*f^{10} + 112*a^{15}*b*c^4*d^{12}*e^5*f^{11} + 112*a^{15}*b*c^5*d^{11}*e^4*f^{12} \\
& - 224*a^{15}*b*c^6*d^{10}*e^3*f^{13} + 160*a^{15}*b*c^7*d^9*e^2*f^{14} - 300*a^2*b^{14}*c^8*d^8*e^{14}*f^2 + 840*a^2*b^{14}*c^{10}*d^6*e^{12}*f^4 \\
& - 1344*a^2*b^{14}*c^{11}*d^5*e^{11}*f^5 + 840*a^2*b^{14}*c^{12}*d^4*e^{10}*f^6 - 300*a^2*b^{14}*c^{14}*d^2*e^8*f^8 + 1400*a^3*b^{13}*c^8*d^8*e^{13}*f^3 \\
& - 2800*a^3*b^{13}*c^9*d^7*e^{12}*f^4 + 1568*a^3*b^{13}*c^{10}*d^6*e^{11}*f^5 + 1568*a^3*b^{13}*c^{11}*d^5*e^{10}*f^6 \\
& - 2800*a^3*b^{13}*c^{12}*d^4*e^9*f^7 + 1400*a^3*b^{13}*c^{13}*d^3*e^8*f^8 + 840*a^4*b^{12}*c^6*d^{10}*e^{14}*f^2 - 2800*a^4*b^{12}*c^7*d^9*e^{13}*f^3 \\
& + 1750*a^4*b^{12}*c^8*d^8*e^{12}*f^4 + 4480*a^4*b^{12}*c^9*d^7*e^{11}*f^5 - 8624*a^4*b^{12}*c^{10}*d^6*e^{10}*f^6 \\
& + 4480*a^4*b^{12}*c^{11}*d^5*e^9*f^7 + 1750*a^4*b^{12}*c^{12}*d^4*e^8*f^8 - 2800*a^4*b^{12}*c^{13}*d^3*e^7*f^9 + 840*a^4*b^{12}*c^{14}*d^2*e^6*f^{10} \\
& - 1344*a^5*b^{11}*c^5*d^{11}*e^{14}*f^2 + 1568*a^5*b^{11}*c^6*d^{10}*e^{13}*f^3 + 4480*a^5*b^{11}*c^7*d^9*e^{12}*f^4 - 12264*a^5*b^{11}*c^8*d^8*e^{11}*f^5 \\
& + 7392*a^5*b^{11}*c^9*d^7*e^{10}*f^6 + 7392*a^5*b^{11}*c^{10}*d^6*e^9*f^7 - 12264*a^5*b^{11}*c^{11}*d^5*e^8*f^8 + 4480*a^5*b^{11}*c^{12}*d^4*e^7*f^9 + 1568*a^5*b^{11}*c^{13}*d^3*e^6*f^{10} \\
& - 1344*a^5*b^{11}*c^{14}*d^2*e^5*f^{11} + 840*a^6*b^{10}*c^4*d^{12}*e^{14}*f^2 + 1568*a^6*b^{10}*c^5*d^{11}*e^{13}*f^3 - 8624*a^6*b^{10}*c^6*d^{10}*e^{12}*f^4 \\
& + 7392*a^6*b^{10}*c^7*d^9*e^{11}*f^5 + 11396*a^6*b^{10}*c^8*d^8*e^{10}*f^6 - 24640*a^6*b^{10}*c^9*d^7*e^9*f^7 + 11396*a^6*b^{10}*c^{10}*d^6*e^8*f^8 + 7392*a^6*b^{10}*c^{11}*d^5*e^7*f^9 \\
& - 8624*a^6*b^{10}*c^{12}*d^4*e^6*f^{10} + 1568*a^6*b^{10}*c^{13}*d^3*e^5*f^{11} + 840*a^6*b^{10}*c^{14}*d^2*e^4*f^{12} - 2800*a^7*b^9*c^4*d^{12}*e^{13}*f^3 \\
& + 4480*a^7*b^9*c^5*d^{11}*e^{12}*f^4 + 7392*a^7*b^9*c^6*d^{10}*e^{11}*f^5 - 24640*a^7*b^9*c^7*d^9*e^{10}*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 \\
& - 24640*a^7*b^9*c^{10}*d^6*e^7*f^9 + 7392*a^7*b^9*c^{11}*d^5*e^6*f^{10} + 4480*a^7*b^9*c^{12}*d^4*e^5*f^{11} - 2800*a^7*b^9*c^{13}*d^3*e^4*f^{12} \\
& - 300*a^8*b^8*c^2*d^{14}*e^{14}*f^2 + 1400*a^8*b^8*c^3*d^{13}*e^{13}*f^3 + 1750*a^8*b^8*c^4*d^{12}*e^{12}*f^4 - 12264*a^8*b^8*c^5*d^{11}*e^{11}*f^5 + 11396*a^8*b^8*c^6*d^{10}*e^{10}*f^6 \\
& + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^{10}*d^6*e^6*f^{10} - 12264*a^8*b^8*c^{11}*d^5*e^5*f^{11} \\
& + 1750*a^8*b^8*c^{12}*d^4*e^4*f^{12} + 1400*a^8*b^8*c^{13}*d^3*e^3*f^{13} - 300*a^8*b^8*c^{14}*d^2*e^2*f^{14} - 2800*a^9*b^7*c^3*d^{13}*e^{12}*f^4 + 4480*a^9*b^7*c^4*d^{12}*e^{11}*f^5 \\
& + 7392*a^9*b^7*c^5*d^{11}*e^{10}*f^6 - 24640*a^9*b^7*c^6*d^{10}*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^{10} \\
& + 7392*a^9*b^7*c^{10}*d^6*e^5*f^{11} + 4480*a^9*b^7*c^{11}*d^5*e^4*f^{12} - 2800*a^9*b^7*c^{12}*d^4*e^3*f^{13} + 840*a^{10}*b^6*c^2*d^{14}*e^{12}*f^4 + 1568*a^{10}*b^6*c^3*d^{13}*e^{11}*f^5 \\
& - 8624*a^{10}*b^6*c^4*d^{12}*e^{10}*f^6 + 7392*a^{10}*b^6*c^5*d^{11}*e^9*f^7 + 11396*a^{10}*b^6*c^6*d^{10}*e^8*f^8 - 24640*a^{10}*b^6*c^7*d^9*e^7*f^9 + 11396*a^{10}*b^6*c^8*d^8*e^6*f^{10} \\
& + 7392*a^{10}*b^6*c^9*d^7*e^5*f^{11} - 8624*a^{10}*b^6*c^{10}*d^6*e^4*f^{12} + 1568*a^{10}*b^6*c^{11}*d^5*e^3*f^{13} + 840*a^{10}*b^6*c^{12}*d^4*e^2*f^{14} \\
& - 1344*a^{11}*b^5*c^2*d^{14}*e^{11}*f^5 + 1568*a^{11}*b^5*c^3*d^{13}*e^{10}*f^6 + 4480*a^{11}*b^5*c^4*d^{12}*e^9*f^7 - 12264*a^{11}*b^5*c^5*d^{11}*e^8*f^8 + 7392*a^{11}*b^5*c^6*d^{10}*e^7*f^9 \\
& + 7392*a^{11}*b^5*c^7*d^9*e^6*f^{10} - 12264*a^{11}*b^5*c^8*d^8*e^5*f^{11} + 4480*a^{11}*b^5*c^9*d^7*e^4*f^{12} + 1568*a^{11}*b^5*c^{10}*d^6*e^3*f^{13} - 1344*a^{11}*b^5*c^{11}*d^5*e^2*f^{14} \\
& + 840*a^{12}*b^4*c^2*d^{14}*e^{10}*f^6
\end{aligned}$$

$$\begin{aligned}
&^6 - 2800*a^{12}*b^4*c^3*d^{13}*e^9*f^7 + 1750*a^{12}*b^4*c^4*d^{12}*e^8*f^8 + 4480 \\
&*a^{12}*b^4*c^5*d^{11}*e^7*f^9 - 8624*a^{12}*b^4*c^6*d^{10}*e^6*f^{10} + 4480*a^{12}*b^ \\
&4*c^7*d^9*e^5*f^{11} + 1750*a^{12}*b^4*c^8*d^8*e^4*f^{12} - 2800*a^{12}*b^4*c^9*d^7 \\
&*e^3*f^{13} + 840*a^{12}*b^4*c^{10}*d^6*e^2*f^{14} + 1400*a^{13}*b^3*c^3*d^{13}*e^8*f^8 \\
&- 2800*a^{13}*b^3*c^4*d^{12}*e^7*f^9 + 1568*a^{13}*b^3*c^5*d^{11}*e^6*f^{10} + 1568* \\
&a^{13}*b^3*c^6*d^{10}*e^5*f^{11} - 2800*a^{13}*b^3*c^7*d^9*e^4*f^{12} + 1400*a^{13}*b^3 \\
&*c^8*d^8*e^3*f^{13} - 300*a^{14}*b^2*c^2*d^{14}*e^8*f^8 + 840*a^{14}*b^2*c^4*d^{12}*e \\
&^6*f^{10} - 1344*a^{14}*b^2*c^5*d^{11}*e^5*f^{11} + 840*a^{14}*b^2*c^6*d^{10}*e^4*f^{12} \\
&- 300*a^{14}*b^2*c^8*d^8*e^2*f^{14}) + (x*(108*a^3*b^{10}*c^7*d^6*f^{13} - 36*b^{13}* \\
&c^{10}*d^3*f^{13} - 36*b^{13}*d^{13}*e^{10}*f^3 - 297*a^2*b^{11}*c^8*d^5*f^{13} - 36*a^{10} \\
&*b^3*d^{13}*f^{13} + 324*a^4*b^9*c^6*d^7*f^{13} - 594*a^5*b^8*c^5*d^8*f^{13} + 324* \\
&a^6*b^7*c^4*d^9*f^{13} + 108*a^7*b^6*c^3*d^{10}*f^{13} - 297*a^8*b^5*c^2*d^{11}*f^{13} \\
&- 297*a^2*b^{11}*d^{13}*e^8*f^5 + 108*a^3*b^{10}*d^{13}*e^7*f^6 + 324*a^4*b^9*d^{13} \\
&*e^6*f^7 - 594*a^5*b^8*d^{13}*e^5*f^8 + 324*a^6*b^7*d^{13}*e^4*f^9 + 108*a^7*b^6 \\
&*d^{13}*e^3*f^{10} - 297*a^8*b^5*d^{13}*e^2*f^{11} - 297*b^{13}*c^2*d^{11}*e^8*f^5 + \\
&108*b^{13}*c^3*d^{10}*e^7*f^6 + 324*b^{13}*c^4*d^9*e^6*f^7 - 594*b^{13}*c^5*d^8*e^5 \\
&*f^8 + 324*b^{13}*c^6*d^7*e^4*f^9 + 108*b^{13}*c^7*d^6*e^3*f^{10} - 297*b^{13}*c^8* \\
&d^5*e^2*f^{11} + 180*a*b^{12}*c^9*d^4*f^{13} + 180*a^9*b^4*c*d^{12}*f^{13} + 180*a*b^{12} \\
&d^{13}*e^9*f^4 + 180*a^9*b^4*d^{13}*e*f^{12} + 180*b^{13}*c*d^{12}*e^9*f^4 + 180*b^{13} \\
&*c^9*d^4*e*f^{12} - 1026*a*b^{12}*c*d^{12}*e^8*f^5 - 1026*a*b^{12}*c^8*d^5*e*f^{12} \\
&- 1026*a^8*b^5*c*d^{12}*e*f^{12} + 2052*a*b^{12}*c^2*d^{11}*e^7*f^6 - 2052*a*b^{12} \\
&*c^3*d^{10}*e^6*f^7 + 1026*a*b^{12}*c^4*d^9*e^5*f^8 + 1026*a*b^{12}*c^5*d^8*e^4*f^9 \\
&- 2052*a*b^{12}*c^6*d^7*e^3*f^{10} + 2052*a*b^{12}*c^7*d^6*e^2*f^{11} + 2052*a^2 \\
&*b^{11}*c*d^{12}*e^7*f^6 + 2052*a^2*b^{11}*c^7*d^6*e*f^{12} - 2052*a^3*b^{10}*c*d^{12} \\
&*e^6*f^7 - 2052*a^3*b^{10}*c^6*d^7*e*f^{12} + 1026*a^4*b^9*c*d^{12}*e^5*f^8 + 1026 \\
&*a^4*b^9*c^5*d^8*e*f^{12} + 1026*a^5*b^8*c*d^{12}*e^4*f^9 + 1026*a^5*b^8*c^4*d^9 \\
&*e*f^{12} - 2052*a^6*b^7*c*d^{12}*e^3*f^{10} - 2052*a^6*b^7*c^3*d^{10}*e*f^{12} + 20 \\
&52*a^7*b^6*c*d^{12}*e^2*f^{11} + 2052*a^7*b^6*c^2*d^{11}*e*f^{12} - 4104*a^2*b^{11}*c \\
&^2*d^{11}*e^6*f^7 + 4104*a^2*b^{11}*c^3*d^{10}*e^5*f^8 - 5130*a^2*b^{11}*c^4*d^9*e^ \\
&4*f^9 + 4104*a^2*b^{11}*c^5*d^8*e^3*f^{10} - 4104*a^2*b^{11}*c^6*d^7*e^2*f^{11} + 4 \\
&104*a^3*b^{10}*c^2*d^{11}*e^5*f^8 + 4104*a^3*b^{10}*c^5*d^8*e^2*f^{11} - 5130*a^4*b^9 \\
&*c^2*d^{11}*e^4*f^9 - 5130*a^4*b^9*c^4*d^9*e^2*f^{11} + 4104*a^5*b^8*c^2*d^{11} \\
&*e^3*f^{10} + 4104*a^5*b^8*c^3*d^{10}*e^2*f^{11} - 4104*a^6*b^7*c^2*d^{11}*e^2*f^{11} \\
&)) / (56*a^3*b^{13}*c^5*d^{11}*e^{16} - a^8*b^8*d^{16}*e^{16} - a^{16}*c^8*d^8*f^{16} - b^{16} \\
&6*c^8*d^8*e^{16} - a^{16}*d^{16}*e^8*f^8 - b^{16}*c^{16}*e^8*f^8 - 28*a^2*b^{14}*c^6*d^{10} \\
&*e^{16} - a^8*b^8*c^{16}*f^{16} - 70*a^4*b^{12}*c^4*d^{12}*e^{16} + 56*a^5*b^{11}*c^3*d^{13} \\
&*e^{16} - 28*a^6*b^{10}*c^2*d^{14}*e^{16} - 28*a^{10}*b^6*c^{14}*d^2*f^{16} + 56*a^{11}* \\
&b^5*c^{13}*d^3*f^{16} - 70*a^{12}*b^4*c^{12}*d^4*f^{16} + 56*a^{13}*b^3*c^{11}*d^5*f^{16} - \\
&28*a^{14}*b^2*c^{10}*d^6*f^{16} - 28*a^2*b^{14}*c^{16}*e^6*f^{10} + 56*a^3*b^{13}*c^{16}*e \\
&^5*f^{11} - 70*a^4*b^{12}*c^{16}*e^4*f^{12} + 56*a^5*b^{11}*c^{16}*e^3*f^{13} - 28*a^6*b^{10} \\
&*c^{16}*e^2*f^{14} - 28*a^{10}*b^6*d^{16}*e^{14}*f^2 + 56*a^{11}*b^5*d^{16}*e^{13}*f^3 - \\
&70*a^{12}*b^4*d^{16}*e^{12}*f^4 + 56*a^{13}*b^3*d^{16}*e^{11}*f^5 - 28*a^{14}*b^2*d^{16}*e^{10} \\
&*f^6 - 28*a^{16}*c^2*d^{14}*e^6*f^{10} + 56*a^{16}*c^3*d^{13}*e^5*f^{11} - 70*a^{16}*c^4 \\
&d^{12}*e^4*f^{12} + 56*a^{16}*c^5*d^{11}*e^3*f^{13} - 28*a^{16}*c^6*d^{10}*e^2*f^{14} - 2 \\
&8*b^{16}*c^{10}*d^6*e^{14}*f^2 + 56*b^{16}*c^{11}*d^5*e^{13}*f^3 - 70*b^{16}*c^{12}*d^4*e^{12} \\
&*f^4 + 56*b^{16}*c^{13}*d^3*e^{11}*f^5 - 28*b^{16}*c^{14}*d^2*e^{10}*f^6 + 8*a*b^{15}*c^7 \\
&*d^9*e^{16} + 8*a^7*b^9*c*d^{15}*e^{16} + 8*a^9*b^7*c^{15}*d*f^{16} + 8*a^{15}*b*c^9*d^7 \\
&*f^{16} + 8*a*b^{15}*c^{16}*e^7*f^9 + 8*a^7*b^9*c^{16}*e*f^{15} + 8*a^9*b^7*d^{16}*e^{15} \\
&*f + 8*a^{15}*b*d^{16}*e^9*f^7 + 8*a^{16}*c*d^{15}*e^7*f^9 + 8*a^{16}*c^7*d^9*e*f^{15} \\
&+ 8*b^{16}*c^9*d^7*e^{15}*f + 8*b^{16}*c^{15}*d*e^9*f^7 - 56*a*b^{15}*c^8*d^8*e^{15} \\
&*f - 56*a*b^{15}*c^{15}*d*e^8*f^8 - 56*a^8*b^8*c*d^{15}*e^{15}*f - 56*a^8*b^8*c^{15}*d \\
&*e*f^{15} - 56*a^{15}*b*c*d^{15}*e^8*f^8 - 56*a^{15}*b*c^8*d^8*e*f^{15} + 160*a*b^{15} \\
&*c^9*d^7*e^{14}*f^2 - 224*a*b^{15}*c^{10}*d^6*e^{13}*f^3 + 112*a*b^{15}*c^{11}*d^5*e^{12} \\
&*f^4 + 112*a*b^{15}*c^{12}*d^4*e^{11}*f^5 - 224*a*b^{15}*c^{13}*d^3*e^{10}*f^6 + 160*a*b^{15} \\
&*c^{14}*d^2*e^9*f^7 + 160*a^2*b^{14}*c^7*d^9*e^{15}*f + 160*a^2*b^{14}*c^{15}*d*e^7 \\
&*f^9 - 224*a^3*b^{13}*c^6*d^{10}*e^{15}*f - 224*a^3*b^{13}*c^{15}*d*e^6*f^{10} + 112*a^4 \\
&*b^{12}*c^5*d^{11}*e^{15}*f + 112*a^4*b^{12}*c^{15}*d*e^5*f^{11} + 112*a^5*b^{11}*c^4*d^{12} \\
&*e^{15}*f + 112*a^5*b^{11}*c^{15}*d*e^4*f^{12} - 224*a^6*b^{10}*c^3*d^{13}*e^{15}*f - \\
&224*a^6*b^{10}*c^{15}*d*e^3*f^{13} + 160*a^7*b^9*c^2*d^{14}*e^{15}*f + 160*a^7*b^9*c^
\end{aligned}$$

$$\begin{aligned}
& 15*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - \\
& 224*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5 \\
& *c*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11* \\
& f^5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^1 \\
& 3*b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7* \\
& e*f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a \\
& ^15*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^1 \\
& 0*e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + \\
& 840*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2* \\
& b^14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8* \\
& d^8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11 \\
& *f^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1 \\
& 400*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b \\
& ^12*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d \\
& ^7*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9* \\
& f^7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840 \\
& *a^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^ \\
& 11*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8* \\
& d^8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9* \\
& f^7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 15 \\
& 68*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b \\
& ^10*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6 \\
& *d^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^ \\
& 10*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + \\
& 7392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^ \\
& 6*b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^ \\
& 4*d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^ \\
& 11*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 1 \\
& 5400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^ \\
& 9*c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^ \\
& 3*e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 \\
& + 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396 \\
& *a^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8* \\
& c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^ \\
& 6*f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + \\
& 1400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9* \\
& b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d \\
& ^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f \\
& ^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392* \\
& a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c \\
& ^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13* \\
& e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 \\
& + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396 \\
& *a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6 \\
& *c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^ \\
& 4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10* \\
& f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 73 \\
& 92*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11* \\
& b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10* \\
& d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10 \\
& *f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 44 \\
& 80*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12* \\
& b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d \\
& ^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f \\
& ^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 156 \\
& 8*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b \\
& ^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12 \\
& *e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^1 \\
& 2 - 300*a^14*b^2*c^8*d^8*e^2*f^14)) *root(756756*a^10*b^10*c^10*d^10*e^10*f^
\end{aligned}$$

$$\begin{aligned}
& 10z^3 + 573300a^{12}b^8c^9d^{11}e^9f^{11}z^3 + 573300a^{11}b^9c^{11}d^9e^8f^{12}z^3 + 573300a^{11}b^9c^8d^{12}e^{11}f^9z^3 + 573300a^9b^{11}c^{12}d^8e^9f^{11}z^3 + 573300a^9b^{11}c^9d^{11}e^{12}f^8z^3 + 573300a^8b^{12}c^{11}d^9e^{11}f^9z^3 - 343980a^{11}b^9c^{10}d^{10}e^9f^{11}z^3 - 343980a^{11}b^9c^9d^{11}e^{10}f^{10}z^3 - 343980a^{10}b^{10}c^{11}d^9e^9f^{11}z^3 - 343980a^{10}b^{10}c^9d^{11}e^{11}f^9z^3 - 343980a^9b^{11}c^{11}d^9e^{10}f^{10}z^3 - 343980a^9b^{11}c^{10}d^{10}e^{11}f^9z^3 + 326340a^{13}b^7c^{10}d^{10}e^7f^{13}z^3 + 326340a^{13}b^7c^7d^{13}e^{10}f^{10}z^3 + 326340a^{10}b^{10}c^{13}d^7e^7f^{13}z^3 + 326340a^{10}b^{10}c^7d^{13}e^{13}f^7z^3 + 326340a^7b^{13}c^{13}d^7e^{10}f^{10}z^3 + 326340a^7b^{13}c^{10}d^{10}e^{13}f^7z^3 - 267540a^{12}b^8c^{10}d^{10}e^8f^{12}z^3 - 267540a^{12}b^8c^8d^{12}e^{10}f^{10}z^3 - 267540a^{10}b^{10}c^{12}d^8e^8f^{12}z^3 - 267540a^{10}b^{10}c^8d^{12}e^{12}f^8z^3 - 267540a^8b^{12}c^{12}d^8e^{10}f^{10}z^3 - 267540a^8b^{12}c^{10}d^{10}e^{12}f^8z^3 + 245700a^{14}b^6c^8d^{12}e^8f^{12}z^3 + 245700a^{12}b^8c^{12}d^8e^6f^{14}z^3 + 245700a^{12}b^8c^6d^{14}e^{12}f^8z^3 + 245700a^8b^{12}c^{14}d^6e^8f^{12}z^3 + 245700a^8b^{12}c^8d^{12}e^{14}f^6z^3 + 245700a^6b^{14}c^{12}d^8e^{12}f^8z^3 - 191100a^{13}b^7c^9d^{11}e^8f^{12}z^3 - 191100a^{13}b^7c^8d^{12}e^9f^{11}z^3 - 191100a^{12}b^8c^7d^{13}e^{11}f^9z^3 - 191100a^{11}b^9c^{12}d^8e^7f^{13}z^3 - 191100a^{11}b^9c^7d^{13}e^{12}f^8z^3 - 191100a^9b^{11}c^{13}d^7e^8f^{12}z^3 - 191100a^9b^{11}c^8d^{12}e^{13}f^7z^3 - 191100a^8b^{12}c^{13}d^7e^9f^{11}z^3 - 191100a^8b^{12}c^9d^{11}e^{13}f^7z^3 - 191100a^7b^{13}c^{12}d^8e^{11}f^9z^3 - 191100a^7b^{13}c^{11}d^9e^{12}f^8z^3 - 123900a^{14}b^6c^9d^{11}e^7f^{13}z^3 - 123900a^{14}b^6c^7d^{13}e^9f^{11}z^3 - 123900a^{13}b^7c^{11}d^9e^6f^{14}z^3 - 123900a^{13}b^7c^6d^{14}e^{11}f^9z^3 - 123900a^{11}b^9c^6d^{14}e^{13}f^7z^3 - 123900a^9b^{11}c^{14}d^6e^7f^{13}z^3 - 123900a^9b^{11}c^7d^{13}e^{14}f^6z^3 - 123900a^7b^{13}c^{14}d^6e^9f^{11}z^3 - 123900a^7b^{13}c^9d^{11}e^{14}f^6z^3 - 123900a^6b^{14}c^{13}d^7e^{11}f^9z^3 - 123900a^6b^{14}c^{11}d^9e^{13}f^7z^3 + 101700a^{15}b^5c^9d^{11}e^6f^{14}z^3 + 101700a^{15}b^5c^6d^{14}e^9f^{11}z^3 + 101700a^{14}b^6c^{11}d^9e^5f^{15}z^3 + 101700a^{14}b^6c^5d^{15}e^{11}f^9z^3 + 101700a^{11}b^9c^{14}d^6e^5f^{15}z^3 + 101700a^{11}b^9c^5d^{15}e^{14}f^6z^3 + 101700a^9b^{11}c^{15}d^5e^6f^{14}z^3 + 101700a^9b^{11}c^6d^{14}e^{15}f^5z^3 + 101700a^6b^{14}c^{15}d^5e^9f^{11}z^3 + 101700a^6b^{14}c^9d^{11}e^{15}f^5z^3 + 101700a^5b^{15}c^{14}d^6e^{11}f^9z^3 + 101700a^5b^{15}c^{11}d^9e^{14}f^6z^3 - 65820a^{14}b^6c^{10}d^{10}e^6f^{14}z^3 - 65820a^{14}b^6c^6d^{14}e^{10}f^{10}z^3 - 65820a^{10}b^{10}c^{14}d^6e^6f^{14}z^3 - 65820a^{10}b^{10}c^6d^{14}e^{14}f^6z^3 - 65820a^6b^{14}c^{14}d^6e^{10}f^{10}z^3 - 65820a^6b^{14}c^{10}d^{10}e^{14}f^6z^3 + 56700a^{16}b^4c^7d^{13}e^7f^{13}z^3 - 56700a^{15}b^5c^8d^{12}e^7f^{13}z^3 - 56700a^{15}b^5c^7d^{13}e^8f^{12}z^3 + 56700a^{13}b^7c^{13}d^7e^4f^{16}z^3 - 56700a^{13}b^7c^{12}d^8e^5f^{15}z^3 - 56700a^{13}b^7c^5d^{15}e^{12}f^8z^3 + 56700a^{13}b^7c^4d^{16}e^{13}f^7z^3 - 56700a^{12}b^8c^{13}d^7e^5f^{15}z^3 - 56700a^{12}b^8c^5d^{15}e^{13}f^7z^3 - 56700a^8b^{12}c^{15}d^5e^7f^{13}z^3 - 56700a^8b^{12}c^7d^{13}e^{15}f^5z^3 + 56700a^7b^{13}c^{16}d^4e^7f^{13}z^3 - 56700a^7b^{13}c^{15}d^5e^8f^{12}z^3 - 56700a^7b^{13}c^8d^{12}e^{15}f^5z^3 + 56700a^7b^{13}c^7d^{13}e^{16}f^4z^3 - 56700a^5b^{15}c^{13}d^7e^{12}f^8z^3 - 56700a^5b^{15}c^{12}d^8e^{13}f^7z^3 + 56700a^4b^{16}c^{13}d^7e^{13}f^7z^3 - 48252a^{15}b^5c^{10}d^{10}e^5f^{15}z^3 - 48252a^{15}b^5c^5d^{15}e^{10}f^{10}z^3 - 48252a^{10}b^{10}c^{15}d^5e^5f^{15}z^3 - 48252a^{10}b^{10}c^5d^{15}e^{15}f^5z^3 - 48252a^5b^{15}c^{15}d^5e^{10}f^{10}z^3 - 48252a^5b^{15}c^{10}d^{10}e^{15}f^5z^3 - 32400a^{16}b^4c^8d^{12}e^6f^{14}z^3 - 32400a^{16}b^4c^6d^{14}e^8f^{12}z^3 - 32400a^{14}b^6c^{12}d^8e^4f^{16}z^3 - 32400a^{14}b^6c^4d^{16}e^{12}f^8z^3 - 32400a^{12}b^8c^{14}d^6e^4f^{16}z^3 - 32400a^{12}b^8c^4d^{16}e^{14}f^6z^3 - 32400a^8b^{12}c^{16}d^4e^6f^{14}z^3 - 32400a^8b^{12}c^6d^{14}e^{16}f^4z^3 - 32400a^6b^{14}c^{16}d^4e^8f^{12}z^3 - 32400a^6b^{14}c^8d^{12}e^{16}f^4z^3 - 32400a^4b^{16}c^{14}d^6e^{12}f^8z^3 - 32400a^4b^{16}c^{12}d^8e^{14}f^6z^3 + 20565a^{16}b^4c^{10}d^{10}e^4f^{16}z^3 + 20565a^{16}b^4c^4d^{16}e^{10}f^{10}z^3 + 20565a^{10}b^{10}c^{16}d^4e^4
\end{aligned}$$

$$\begin{aligned}
& *f^{16}z^3 + 20565*a^{10}b^{10}c^4d^{16}e^{16}f^4z^3 + 20565*a^4b^{16}c^{16}d^4 \\
& *e^{10}f^{10}z^3 + 20565*a^4b^{16}c^{10}d^{10}e^{16}f^4z^3 + 15660*a^{17}b^3c^8 \\
& *d^{12}e^5f^{15}z^3 + 15660*a^{17}b^3c^5d^{15}e^8f^{12}z^3 + 15660*a^{15}b^5c^{12}d^8e^3f^{17}z^3 \\
& + 15660*a^{15}b^5c^3d^{17}e^{12}f^8z^3 + 15660*a^{12}b^8c^{15}d^5e^3f^{17}z^3 + 15660*a^{12}b^8c^3d^{17}e^{15}f^5z^3 \\
& + 15660*a^8b^{12}c^{17}d^3e^5f^{15}z^3 + 15660*a^8b^{12}c^5d^{15}e^{17}f^3z^3 + 15660* \\
& a^5b^{15}c^{17}d^3e^8f^{12}z^3 + 15660*a^5b^{15}c^8d^{12}e^{17}f^3z^3 + 156 \\
& 60*a^3b^{17}c^{15}d^5e^{12}f^8z^3 + 15660*a^3b^{17}c^{12}d^8e^{15}f^5z^3 - \\
& 9750*a^{17}b^3c^9d^{11}e^4f^{16}z^3 - 9750*a^{17}b^3c^4d^{16}e^9f^{11}z^3 - \\
& 9750*a^{16}b^4c^{11}d^9e^3f^{17}z^3 - 9750*a^{16}b^4c^3d^{17}e^{11}f^9z^3 \\
& - 9750*a^{11}b^9c^{16}d^4e^3f^{17}z^3 - 9750*a^{11}b^9c^3d^{17}e^{16}f^4z^3 \\
& - 9750*a^9b^{11}c^{17}d^3e^4f^{16}z^3 - 9750*a^9b^{11}c^4d^{16}e^{17}f^3z^3 \\
& 3 - 9750*a^4b^{16}c^{17}d^3e^9f^{11}z^3 - 9750*a^4b^{16}c^9d^{11}e^{17}f^3z^3 \\
& - 9750*a^3b^{17}c^{16}d^4e^{11}f^9z^3 - 9750*a^3b^{17}c^{11}d^9e^{16}f^4z^3 \\
& z^3 - 8100*a^{17}b^3c^7d^{13}e^6f^{14}z^3 - 8100*a^{17}b^3c^6d^{14}e^7f^{13} \\
& *z^3 - 8100*a^{14}b^6c^{13}d^7e^3f^{17}z^3 - 8100*a^{14}b^6c^3d^{17}e^{13}f^7 \\
& *z^3 - 8100*a^{13}b^7c^{14}d^6e^3f^{17}z^3 - 8100*a^{13}b^7c^3d^{17}e^{14}f^6 \\
& *z^3 - 8100*a^7b^{13}c^{17}d^3e^6f^{14}z^3 - 8100*a^7b^{13}c^6d^{14}e^{17}f^3 \\
& *z^3 - 8100*a^6b^{14}c^{17}d^3e^7f^{13}z^3 - 8100*a^6b^{14}c^7d^{13}e^{17} \\
& *f^3z^3 - 8100*a^3b^{17}c^{14}d^6e^{13}f^7z^3 - 8100*a^3b^{17}c^{13}d^7e^{14} \\
& *f^6z^3 - 7980*a^{16}b^4c^9d^{11}e^5f^{15}z^3 - 7980*a^{16}b^4c^5d^{15}e^9 \\
& *f^{11}z^3 - 7980*a^{15}b^5c^{11}d^9e^4f^{16}z^3 - 7980*a^{15}b^5c^4d^{16}e^{11} \\
& *f^9z^3 - 7980*a^{11}b^9c^{15}d^5e^4f^{16}z^3 - 7980*a^{11}b^9c^4d^{16}e^{15} \\
& *f^5z^3 - 7980*a^9b^{11}c^{16}d^4e^5f^{15}z^3 - 7980*a^9b^{11}c^5d^{15}e^{16} \\
& *f^4z^3 - 7980*a^5b^{15}c^{16}d^4e^9f^{11}z^3 - 7980*a^5b^{15}c^9d^{11}e^{16} \\
& *f^4z^3 - 7980*a^4b^{16}c^{15}d^5e^{11}f^9z^3 - 7980*a^4b^{16}c^{11}d^9e^{15} \\
& *f^5z^3 + 6300*a^{18}b^2c^6d^{14}e^6f^{14}z^3 + 6300*a^{14}b^6c^{14}d^6e^2 \\
& *f^{18}z^3 + 6300*a^{14}b^6c^2d^{18}e^{14}f^6z^3 + 6300*a^6b^{14}c^{18}d^2e^6 \\
& *f^{14}z^3 + 6300*a^6b^{14}c^6d^{14}e^{18}f^2z^3 + 6300*a^2b^{18}c^{14}d^6e^{14} \\
& *f^6z^3 - 4260*a^{18}b^2c^7d^{13}e^5f^{15}z^3 - 4260*a^{18}b^2c^5d^{15}e^7 \\
& *f^{13}z^3 - 4260*a^{15}b^5c^{13}d^7e^2f^{18}z^3 - 4260*a^{15}b^5c^2d^{18}e^{13} \\
& *f^7z^3 - 4260*a^{13}b^7c^{15}d^5e^2f^{18}z^3 - 4260*a^{13}b^7c^2d^{18}e^{15} \\
& *f^5z^3 - 4260*a^7b^{13}c^{18}d^2e^5f^{15}z^3 - 4260*a^7b^{13}c^5d^{15}e^{18} \\
& *f^2z^3 - 4260*a^5b^{15}c^{18}d^2e^7f^{13}z^3 - 4260*a^5b^{15}c^7d^{13}e^{18} \\
& *f^2z^3 - 4260*a^2b^{18}c^{15}d^5e^{13}f^7z^3 - 4260*a^2b^{18}c^{13}d^7e^{15} \\
& *f^5z^3 + 1470*a^{17}b^3c^{10}d^{10}e^3f^{17}z^3 + 1470*a^{17}b^3c^3d^{17}e^{10} \\
& *f^{10}z^3 + 1470*a^{10}b^{10}c^{17}d^3e^3f^{17}z^3 + 1470*a^{10}b^{10}c^3d^{17}e^{17} \\
& *f^3z^3 + 1470*a^3b^{17}c^{10}d^{10}e^{17}f^3z^3 + 1350*a^{18}b^2c^9d^{11}e^3f^{17}z^3 \\
& + 1350*a^{18}b^2c^3d^{17}e^9f^{11}z^3 + 1350*a^{17}b^3c^{11}d^9e^2f^{18}z^3 + \\
& 1350*a^{17}b^3c^2d^{18}e^{11}f^9z^3 + 1350*a^{11}b^9c^{17}d^3e^2f^{18}z^3 + \\
& 1350*a^{11}b^9c^2d^{18}e^{17}f^3z^3 + 1350*a^9b^{11}c^{18}d^2e^3f^{17}z^3 \\
& + 1350*a^9b^{11}c^3d^{17}e^{18}f^2z^3 + 1350*a^3b^{17}c^{18}d^2e^9f^{11}z^3 \\
& + 1350*a^3b^{17}c^9d^{11}e^{18}f^2z^3 + 1350*a^2b^{18}c^{17}d^3e^{11}f^9z^3 \\
& 3 + 1350*a^2b^{18}c^{11}d^9e^{17}f^3z^3 - 1070*a^{18}b^2c^{10}d^{10}e^2f^{18}z^3 \\
& - 1070*a^{18}b^2c^2d^{18}e^{10}f^{10}z^3 - 1070*a^{10}b^{10}c^{18}d^2e^2f^{18}z^3 \\
& - 1070*a^{10}b^{10}c^2d^{18}e^{18}f^2z^3 - 1070*a^2b^{18}c^{18}d^2e^{10}f^{10}z^3 \\
& - 1070*a^2b^{18}c^{10}d^{10}e^{18}f^2z^3 + 525*a^{18}b^2c^8d^{12}e^4f^{16}z^3 \\
& + 525*a^{18}b^2c^4d^{16}e^8f^{12}z^3 + 525*a^{16}b^4c^{12}d^8e^2f^{18}z^3 \\
& + 525*a^{16}b^4c^2d^{18}e^{12}f^8z^3 + 525*a^{12}b^8c^{16}d^4e^2f^{18}z^3 \\
& + 525*a^{12}b^8c^2d^{18}e^{16}f^4z^3 + 525*a^8b^{12}c^{18}d^2e^4f^{16}z^3 \\
& + 525*a^8b^{12}c^4d^{16}e^{18}f^2z^3 + 525*a^4b^{16}c^{18}d^2e^8f^{12}z^3 \\
& + 525*a^4b^{16}c^8d^{12}e^{18}f^2z^3 + 525*a^2b^{18}c^{16}d^4e^{12}f^8z^3 \\
& + 525*a^2b^{18}c^{12}d^8e^{16}f^4z^3 + 900*a^{19}b^3c^7d^{13}e^4f^{16}z^3 \\
& + 900*a^{19}b^3c^4d^{16}e^7f^{13}z^3 + 900*a^{16}b^4c^{13}d^7e^5f^{19}z^3 + \\
& 900*a^{16}b^4c^3d^{19}e^{13}f^7z^3 + 900*a^{13}b^7c^{16}d^4e^5f^{19}z^3 + 900*a^{13} \\
& *b^7c^3d^{19}e^{16}f^4z^3 + 900*a^7b^{13}c^{19}d^5e^4f^{16}z^3 + 900*a^7b^{13} \\
& *c^4d^{16}e^{19}f^5z^3 + 900*a^4b^{16}c^{19}d^5e^7f^{13}z^3 + 900*a^4b^{16}c^7 \\
& *d^{13}e^{19}f^5z^3 + 900*a*b^{19}c^{16}d^4e^{13}f^7z^3 + 900*a*b^{19}c^{13}d^7e^{19}
\end{aligned}$$

$$\begin{aligned}
& e^{16f^4z^3} - 750a^{19}b^3c^8d^{12}e^3f^{17}z^3 - 750a^{19}b^3c^3d^{17}e^8f^{12}z^3 - 750a^{17}b^3c^{12}d^8e^8f^{19}z^3 - 750a^{17}b^3c^3d^{19}e^{12}f^8z^3 \\
& - 750a^{12}b^8c^{17}d^3e^8f^{19}z^3 - 750a^{12}b^8c^3d^{19}e^{17}f^3z^3 - 750a^8b^{12}c^{19}d^3e^3f^{17}z^3 - 750a^8b^{12}c^3d^{17}e^{19}f^3z^3 - 750a^3b^{17}c^{19}d^3e^8f^{12}z^3 \\
& - 750a^3b^{17}c^8d^{12}e^{19}f^3z^3 - 750a^3b^{19}c^{17}d^3e^{12}f^8z^3 - 750a^3b^{19}c^{12}d^8e^{17}f^3z^3 - 420a^{19}b^3c^6d^{14}e^5f^{15}z^3 - 420a^{19}b^3c^5d^{15}e^6f^{14}z^3 \\
& - 420a^{15}b^5c^{14}d^6e^6f^{19}z^3 - 420a^{15}b^5c^3d^{19}e^{14}f^6z^3 - 420a^{14}b^6c^{15}d^5e^6f^{19}z^3 - 420a^{14}b^6c^3d^{19}e^{15}f^5z^3 - 420a^6b^{14}c^{19}d^5e^5f^{15}z^3 \\
& - 420a^6b^{14}c^5d^{15}e^{19}f^3z^3 - 420a^5b^{15}c^{19}d^5e^6f^{14}z^3 - 420a^5b^{15}c^6d^{14}e^{19}f^3z^3 - 420a^5b^{19}c^{15}d^5e^{14}f^6z^3 - 420a^5b^{19}c^{14}d^6e^{15}f^5z^3 \\
& + 350a^{19}b^3c^9d^{11}e^2f^{18}z^3 + 350a^{19}b^3c^2d^{18}e^9f^{11}z^3 + 350a^{18}b^2c^{11}d^9e^8f^{19}z^3 + 350a^{18}b^2c^3d^{19}e^{11}f^9z^3 + 350a^{11}b^9c^{18}d^2e^8f^{19}z^3 \\
& + 350a^{11}b^9c^3d^{19}e^{18}f^2z^3 + 350a^9b^{11}c^{19}d^2e^2f^{18}z^3 + 350a^9b^{11}c^2d^{18}e^9f^3z^3 + 350a^2b^{18}c^{19}d^2e^9f^{11}z^3 + 350a^2b^{18}c^9d^{11}e^{19}f^3z^3 \\
& + 350a^2b^{18}c^3d^{19}e^{10}f^{10}z^3 - 90a^{19}b^3c^{10}d^{10}e^8f^{19}z^3 - 90a^{19}b^3c^3d^{19}e^{10}f^{10}z^3 - 90a^{10}b^{10}c^{19}d^2e^8f^{19}z^3 - 90a^{10}b^{10}c^3d^{19}e^{19}f^3z^3 \\
& - 90a^8b^{19}c^{19}d^2e^8f^{19}z^3 - 90a^8b^{19}c^3d^{19}e^{10}f^{10}z^3 - 90a^8b^{19}c^{10}d^{10}e^{19}f^3z^3 + 10b^{20}c^{19}d^2e^{11}f^9z^3 + 10b^{20}c^{11}d^9e^{19}f^3z^3 \\
& + 10a^{20}c^9d^{11}e^8f^{19}z^3 + 10a^{20}c^3d^{19}e^9f^{11}z^3 + 10a^{19}b^3d^{20}e^{11}f^9z^3 + 10a^{11}b^9d^{20}e^{19}f^3z^3 + 10a^9b^{11}c^{20}e^8f^{19}z^3 + 10a^8b^{19}c^{20}e^9f^{11}z^3 \\
& + 10a^8b^{19}c^3d^{20}e^9f^{11}z^3 + 10a^8b^{19}c^{11}d^9e^{18}f^2z^3 + 10a^{11}b^9c^{19}d^2f^{20}z^3 + 10a^9b^{11}c^3d^{19}e^{20}z^3 + 10a^8b^{19}c^9d^{11}e^{20}z^3 \\
& + 252b^{20}c^{15}d^5e^{15}f^5z^3 - 210b^{20}c^{16}d^4e^{14}f^6z^3 - 210b^{20}c^{14}d^6e^{16}f^4z^3 + 120b^{20}c^{17}d^3e^{13}f^7z^3 + 120b^{20}c^{13}d^7e^{17}f^3z^3 - 45b^{20}c^{18}d^2e^{12}f^8z^3 \\
& - 45b^{20}c^{12}d^8e^{18}f^2z^3 + 252a^{20}c^5d^{15}e^5f^{15}z^3 - 210a^{20}c^6d^{14}e^4f^{16}z^3 - 210a^{20}c^4d^{16}e^6f^{14}z^3 + 120a^{20}c^7d^{13}e^3f^{17}z^3 + 120a^{20}c^3d^{17}e^7f^{13}z^3 \\
& - 45a^{20}c^8d^{12}e^2f^{18}z^3 - 45a^{20}c^2d^{18}e^8f^{12}z^3 + 252a^{15}b^5d^{20}e^{15}f^5z^3 - 210a^{16}b^4d^{20}e^{14}f^6z^3 - 210a^{14}b^6d^{20}e^{16}f^4z^3 + 120a^{17}b^3d^{20}e^{13}f^7z^3 \\
& + 120a^{13}b^7d^{20}e^{17}f^3z^3 - 45a^{18}b^2d^{20}e^{12}f^8z^3 - 45a^{12}b^8d^{20}e^{18}f^2z^3 + 252a^5b^{15}c^{20}e^5f^{15}z^3 - 210a^6b^{14}c^{20}e^4f^{16}z^3 - 210a^4b^{16}c^{20}e^6f^{14}z^3 + 120a^7b^{13}c^{20}e^3f^{17}z^3 \\
& + 120a^3b^{17}c^{20}e^7f^{13}z^3 - 45a^8b^{12}c^2d^{20}e^2f^{18}z^3 - 45a^2b^{18}c^{20}e^8f^{12}z^3 + 252a^{15}b^5c^{15}d^5f^{20}z^3 - 210a^{16}b^4c^{14}d^6f^{20}z^3 - 210a^{14}b^6c^{16}d^4f^{20}z^3 + 120a^{17}b^3c^{13}d^7f^{20}z^3 \\
& + 120a^{13}b^7c^{17}d^3f^{20}z^3 - 45a^{18}b^2c^{12}d^8f^{20}z^3 - 45a^{12}b^8c^{18}d^2f^{20}z^3 + 252a^5b^{15}c^5d^{15}e^{20}z^3 - 210a^6b^{14}c^4d^{16}e^{20}z^3 - 210a^4b^{16}c^6d^{14}e^{20}z^3 \\
& + 120a^7b^{13}c^3d^{17}e^{20}z^3 + 120a^3b^{17}c^7d^{13}e^{20}z^3 - 45a^8b^{12}c^2d^{18}e^{20}z^3 - 45a^2b^{18}c^8d^{12}e^{20}z^3 - b^{20}c^{20}e^{10}f^{10}z^3 - a^{20}d^{20}e^{10}f^{10}z^3 - b^{20}c^{10}d^{10}e^{20}z^3 - a^{20}c^{10}d^{10}f^{20}z^3 - a^{10}b^{10}d^{20}e^{20}z^3 - a^{10}b^{10}c^{20}f^{20}z^3 + 1890a^{12}b^2c^3d^{13}e^8f^{13}z^3 + 1890a^8b^2c^3d^{13}e^{12}f^2z^3 + 92610a^6b^8c^4d^{10}e^4f^{10}z^3 + 92610a^4b^{10}c^6d^8e^4f^{10}z^3 + 92610a^4b^{10}c^4d^{10}e^6f^8z^3 + 66150a^8b^6c^3d^{11}e^3f^{11}z^3 - 66150a^7b^7c^4d^{10}e^3f^{11}z^3 - 66150a^7b^7c^3d^{11}e^4f^{10}z^3 - 66150a^4b^{10}c^7d^7e^3f^{11}z^3 - 66150a^4b^{10}c^3d^{11}e^7f^7z^3 + 66150a^3b^{11}c^8d^6e^3f^{11}z^3 - 66150a^3b^{11}c^7d^7e^4f^{10}z^3 - 66150a^3b^{11}c^4d^{10}e^7f^7z^3 + 66150a^3b^{11}c^3d^{11}e^8f^6z^3 - 55566a^5b^9c^5d^9e^4f^{10}z^3 - 55566a^5b^9c^4d^{10}e^5f^9z^3 - 55566a^4b^{10}c^5d^9e^5f^9z^3 - 32130a^9b^5c^3d^{11}e^2f^{12}z^3 - 32130a^9b^5c^2d^{12}e^3f^{11}z^3 - 32130a^3b^{11}c^9d^5e^2f^{12}z^3 - 32130a^3b^{11}c^2d^{12}e^9f^5z^3 - 32130a^2b^{12}c^9d^5e^3f^{11}z^3 - 32130a^2b^{12}c^3d^{11}e^9f^5z^3 + 22680a^8b^6c^4d^{10}e^2f^{12}z^3 + 22680a^8b^6c^2d^{12}e^4f^{10}z^3 + 22680a^4b^{10}c^8d^6e^2f^{12}z^3 + 22680a^4b^{10}c^2d^{12}e^8f^6z^3 + 22680a^2b^{12}c^8d^6e^4f^{10}z^3 + 22680a^2b^{12}c^4d^{10}e^8f^6z^3
\end{aligned}$$

$$\begin{aligned}
& + 19278a^{10}b^4c^2d^{12}e^{2f^{12}}z + 19278a^2b^{12}c^{10}d^4e^{2f^{12}}z \\
& + 19278a^2b^{12}c^2d^{12}e^{10f^4}z + 18522a^6b^8c^5d^9e^3f^{11}z + 18522a^6b^8c^3d^{11}e^5f^9z + 18522a^5b^9c^6d^8e^3f^{11}z + 18522a^5b^9c^3d^{11}e^6f^8z + 18522a^3b^{11}c^6d^8e^5f^9z + 18522a^3b^{11}c^5d^9e^6f^8z - 13230a^6b^8c^6d^8e^{2f^{12}}z - 13230a^6b^8c^2d^{12}e^6f^8z - 13230a^2b^{12}c^6d^8e^6f^8z + 3402a^7b^7c^5d^9e^{2f^{12}}z + 3402a^7b^7c^2d^{12}e^5f^9z + 3402a^5b^9c^7d^7e^{2f^{12}}z + 3402a^5b^9c^2d^{12}e^7f^7z + 3402a^2b^{12}c^7d^7e^5f^9z + 3402a^2b^{12}c^5d^9e^7f^7z + 7938a^{10}b^4c^3d^{11}e^f^{13}z + 7938a^{10}b^4c^3d^{11}e^3f^{11}z + 7938a^3b^{11}c^{10}d^4e^f^{13}z + 7938a^3b^{11}c^3d^{11}e^{10}f^4z + 7938a^3b^{11}c^3d^{11}e^{10}f^4z - 5670a^{11}b^3c^2d^{12}e^f^{13}z - 5670a^{11}b^3c^2d^{13}e^{2f^{12}}z - 5670a^2b^{12}c^{11}d^3e^f^{13}z - 5670a^2b^{12}c^3d^{13}e^{11}f^3z - 5670a^2b^{12}c^{11}d^3e^{2f^{12}}z - 5670a^2b^{12}c^2d^{12}e^{11}f^3z - 3780a^9b^5c^4d^{10}e^f^{13}z - 3780a^9b^5c^4d^{13}e^4f^{10}z - 3780a^4b^{10}c^9d^5e^f^{13}z - 3780a^4b^{10}c^9d^5e^4f^{10}z - 3780a^4b^{10}c^9d^5e^4f^{10}z - 3780a^4b^{10}c^9d^5e^4f^{10}z - 2268a^8b^6c^5d^9e^f^{13}z - 2268a^8b^6c^5d^9e^f^{13}z - 2268a^5b^9c^8d^6e^f^{13}z - 2268a^5b^9c^8d^6e^f^{13}z - 2268a^5b^9c^8d^6e^5f^9z - 2268a^5b^9c^8d^6e^5f^9z - 2268a^5b^9c^8d^6e^8f^6z + 1890a^7b^7c^6d^8e^f^{13}z + 1890a^7b^7c^6d^{13}e^6f^8z + 1890a^6b^8c^7d^7e^f^{13}z + 1890a^6b^8c^7d^7e^f^{13}z + 1890a^6b^8c^7d^7e^f^{13}z + 1890a^6b^8c^7d^7e^f^{13}z - 252b^{14}c^{13}d^e^f^{13}z - 252b^{14}c^{13}d^e^f^{13}z - 252a^{13}b^d^{14}e^f^{13}z - 252a^{13}b^d^{14}e^f^{13}z - 252a^{13}b^d^{14}e^{13}f^z - 252a^{13}b^d^{14}e^{13}f^z - 252a^{13}b^d^{14}e^{13}f^z - 252a^{13}b^d^{14}e^{13}f^z - 918b^{14}c^7d^7e^7f^7z - 882b^{14}c^{11}d^3e^3f^{11}z - 882b^{14}c^3d^{11}e^{11}f^3z + 693b^{14}c^{12}d^2e^{2f^{12}}z + 693b^{14}c^2d^{12}e^{12}f^2z + 567b^{14}c^8d^6e^6f^8z + 567b^{14}c^6d^8e^8f^6z + 441b^{14}c^{10}d^4e^4f^{10}z + 441b^{14}c^4d^{10}e^{10}f^4z - 126b^{14}c^9d^5e^5f^9z - 126b^{14}c^5d^9e^9f^5z - 918a^7b^7d^{14}e^7f^7z - 882a^{11}b^3d^{14}e^3f^{11}z - 882a^3b^{11}d^{14}e^{11}f^3z + 693a^{12}b^2d^{14}e^{2f^{12}}z + 693a^2b^{12}d^{14}e^{12}f^2z + 567a^8b^6d^{14}e^6f^8z + 567a^6b^8d^{14}e^8f^6z + 441a^{10}b^4d^{14}e^4f^{10}z + 441a^4b^{10}d^{14}e^{10}f^4z - 126a^9b^5d^{14}e^5f^9z - 126a^5b^9d^{14}e^9f^5z - 918a^7b^7c^7d^7f^{14}z - 882a^{11}b^3c^3d^{11}f^{14}z - 882a^3b^{11}c^{11}d^3f^{14}z + 693a^{12}b^2c^2d^{12}f^{14}z + 693a^2b^{12}c^2d^{12}f^{14}z + 567a^8b^6c^6d^8f^{14}z + 567a^6b^8c^8d^6f^{14}z + 441a^{10}b^4c^4d^{10}f^{14}z + 441a^4b^{10}c^4d^{10}f^{14}z - 126a^9b^5c^5d^9f^{14}z - 126a^5b^9c^9d^5f^{14}z + 36b^{14}d^{14}e^{14}z + 36b^{14}c^{14}f^{14}z + 36a^{14}d^{14}f^{14}z - 27054a^2b^9c^2d^9e^{2f^9} + 9018a^3b^8c^2d^9e^f^{10} + 9018a^3b^8c^2d^9e^f^{10} + 9018a^2b^9c^3d^8e^f^{10} + 9018a^2b^9c^3d^8e^f^{10} + 9018a^2b^9c^3d^8e^f^{10} + 9018a^2b^9c^3d^8e^f^{10} + 9018a^2b^9c^3d^8e^f^{10} - 9018a^4b^7c^7d^{10}e^f^{10} - 9018a^4b^7c^7d^{10}e^f^{10} - 9018a^4b^7c^7d^{10}e^f^{10} + 2268b^{11}c^5d^6e^f^{10} + 2268b^{11}c^5d^6e^f^{10} + 2268a^5b^6d^{11}e^f^{10} + 2268a^5b^6d^{11}e^f^{10} + 2268a^5b^6d^{11}e^f^{10} + 2268a^5b^6d^{11}e^f^{10} - 1458b^{11}c^3d^8e^3f^8 - 1161b^{11}c^4d^7e^{2f^9} - 1161b^{11}c^2d^9e^4f^7 - 1458a^3b^8d^{11}e^3f^8 - 1161a^4b^7d^{11}e^2f^9 - 1161a^2b^9d^{11}e^4f^7 - 1458a^3b^8c^3d^8f^{11} - 1161a^4b^7c^2d^9f^{11} - 1161a^2b^9c^4d^7f^{11} - 756b^{11}d^{11}e^6f^5 - 756b^{11}c^6d^5e^f^{11} - 756a^6b^5d^{11}f^{11}, z, k), k, 1, 3) - ((7a^6b^6c^2d^5e^7 - a^3b^4d^7e^7 - a^7c^3d^4e^7 - b^7c^3d^4e^7 - a^7d^7e^3f^4 - b^7c^7e^3f^4 - 6a^5b^2c^5d^2f^7 - 6a^5b^2d^7e^5f^2 - 6b^7c^5d^2e^5f^2 - a^3b^4c^7f^7 + 7a^2b^5c^d^6e^7 + 4a^4b^3c^6d^f^7 + 4a^6b^c^4d^3f^7 + 7a^6b^c^7e^2f^5 + 7a^2b^5c^7e^f^6 + 4a^4b^3d^7e^6f + 4a^6b^d^7e^4f^3 + 7a^7c^d^6e^2f^5 + 7a^7c^2d^5e^f^6 + 4b^7c^4d^3e^6f + 4b^7c^6d^e^4f^3 - 21a^6b^6c^3d^4e^6f - 21a^6b^6c^6d^e^3f^4 - 21a^3b^4c^d^6e^6f - 21a^3b^4c^6d^e^f^6 - 21a^6b^c^d^6e^3f^4 - 21a^6b^c^3d^4e^f^6 + 14a^6b^6c^4d^3e^5f^2 + 14a^6b^6c^5d^2e^4f^3 - 26a^2b^5c^2d^5e^6f - 26a^2b^5c^6d^e^2f^5 + 14a^4b^3c^d^6e^5f^2 + 14a^4b^3c^5d^2e^f^6 + 14a^5b^2c^d^6e^4z
\end{aligned}$$

$$\begin{aligned}
& f^3 + 14a^5b^2c^4d^3ef^6 - 26a^6b^2c^2d^5e^2f^5 + 52a^2b^5c^3d^4e^5f^2 - 78a^2b^5c^4d^3e^4f^3 + 52a^2b^5c^5d^2e^3f^4 + 52a^3b^4c^2d^5e^5f^2 + 52a^3b^4c^5d^2e^2f^5 - 78a^4b^3c^2d^5e^4f^3 - 78a^4b^3c^4d^3e^2f^5 + 52a^5b^2c^2d^5e^3f^4 + 52a^5b^2c^3d^4e^2f^5) / (2(4a^4b^7c^3d^5e^8 - a^4b^4d^8e^8 - a^8c^4d^4f^8 - b^8c^4d^4e^8 - a^8d^8e^4f^4 - b^8c^8e^4f^4 - 6a^2b^6c^2d^6e^8 - 6a^6b^2c^6d^2f^8 - 6a^2b^6c^8e^2f^6 - 6a^6b^2d^8e^6f^2 - 6a^8c^2d^6e^2f^6 - 6b^8c^6d^2e^6f^2 - a^4b^4c^8f^8 + 4a^3b^5c^d^7e^8 + 4a^5b^3c^7d^f^8 + 4a^7b^c^5d^3f^8 + 4a^ab^7c^8e^3f^5 + 4a^3b^5c^8e^f^7 + 4a^5b^3d^8e^7f + 4a^7b^d^8e^5f^3 + 4a^8c^d^7e^3f^5 + 4a^8c^3d^5e^f^7 + 4b^8c^5d^3e^7f + 4b^8c^7d^e^5f^3 - 12a^ab^7c^4d^4e^7f - 12a^ab^7c^7d^e^4f^4 - 12a^4b^4c^d^7e^7f - 12a^4b^4c^7d^e^f^7 - 12a^7b^c^d^7e^4f^4 - 12a^7b^c^4d^4e^f^7 + 8a^ab^7c^5d^3e^6f^2 + 8a^ab^7c^6d^2e^5f^3 + 8a^2b^6c^3d^5e^7f + 8a^2b^6c^7d^e^3f^5 + 8a^3b^5c^2d^6e^7f + 8a^3b^5c^7d^e^2f^6 + 8a^5b^3c^6d^2e^f^7 + 8a^6b^2c^d^7e^5f^3 + 8a^6b^2c^5d^3e^f^7 + 8a^7b^c^2d^6e^3f^5 + 8a^7b^c^3d^5e^2f^6 + 22a^2b^6c^4d^4e^6f^2 - 48a^2b^6c^5d^3e^5f^3 + 22a^2b^6c^6d^2e^4f^4 - 48a^3b^5c^3d^5e^6f^2 + 36a^3b^5c^4d^4e^5f^3 + 36a^3b^5c^5d^3e^4f^4 - 48a^3b^5c^6d^2e^3f^5 + 22a^4b^4c^2d^6e^6f^2 + 36a^4b^4c^3d^5e^5f^3 - 90a^4b^4c^4d^4e^4f^4 + 36a^4b^4c^5d^3e^3f^5 + 22a^4b^4c^6d^2e^2f^6 - 48a^5b^3c^2d^6e^5f^3 + 36a^5b^3c^3d^5e^4f^4 + 36a^5b^3c^4d^4e^3f^5 - 48a^5b^3c^5d^3e^2f^6 + 22a^6b^2c^2d^6e^4f^4 - 48a^6b^2c^3d^5e^3f^5 + 22a^6b^2c^4d^4e^2f^6)) + (3x^5(2a^5b^2d^7f^7 + 2b^7c^5d^2f^7 + 2b^7d^7e^5f^2 + 2a^2b^5c^3d^4f^7 + 2a^3b^4c^2d^5f^7 + 2a^2b^5d^7e^3f^4 + 2a^3b^4d^7e^2f^5 + 2b^7c^2d^5e^3f^4 + 2b^7c^3d^4e^2f^5 - 5a^ab^6c^4d^3f^7 - 5a^4b^3c^d^6f^7 - 5a^ab^6d^7e^4f^3 - 5a^4b^3d^7e^f^6 - 5b^7c^d^6e^4f^3 - 5b^7c^4d^3e^f^6 + 16a^ab^6c^d^6e^3f^4 + 16a^ab^6c^3d^4e^f^6 + 16a^3b^4c^d^6e^f^6 - 12a^ab^6c^2d^5e^2f^5 - 12a^2b^5c^d^6e^2f^5 - 12a^2b^5c^2d^5e^f^6)) / (4a^4b^7c^3d^5e^8 - a^4b^4d^8e^8 - a^8c^4d^4f^8 - b^8c^4d^4e^8 - a^8d^8e^4f^4 - b^8c^8e^4f^4 - 6a^2b^6c^2d^6e^8 - 6a^6b^2c^6d^2f^8 - 6a^2b^6c^8e^2f^6 - 6a^6b^2d^8e^6f^2 - 6a^8c^2d^6e^2f^6 - 6b^8c^6d^2e^6f^2 - a^4b^4c^8f^8 + 4a^3b^5c^d^7e^8 + 4a^5b^3c^7d^f^8 + 4a^7b^c^5d^3f^8 + 4a^ab^7c^8e^3f^5 + 4a^3b^5c^8e^f^7 + 4a^5b^3d^8e^7f + 4a^7b^d^8e^5f^3 + 4a^8c^d^7e^3f^5 + 4a^8c^3d^5e^f^7 + 4b^8c^5d^3e^7f + 4b^8c^7d^e^5f^3 - 12a^ab^7c^4d^4e^7f - 12a^ab^7c^7d^e^4f^4 - 12a^4b^4c^d^7e^7f - 12a^4b^4c^7d^e^f^7 - 12a^7b^c^d^7e^4f^4 - 12a^7b^c^4d^4e^f^7 + 8a^ab^7c^5d^3e^6f^2 + 8a^ab^7c^6d^2e^5f^3 + 8a^2b^6c^3d^5e^7f + 8a^2b^6c^7d^e^3f^5 + 8a^3b^5c^2d^6e^7f + 8a^3b^5c^7d^e^2f^6 + 8a^5b^3c^6d^2e^f^7 + 8a^6b^2c^d^7e^5f^3 + 8a^6b^2c^5d^3e^f^7 + 8a^7b^c^2d^6e^3f^5 + 8a^7b^c^3d^5e^2f^6 + 22a^2b^6c^4d^4e^6f^2 - 48a^2b^6c^5d^3e^5f^3 + 22a^2b^6c^6d^2e^4f^4 - 48a^3b^5c^3d^5e^6f^2 + 36a^3b^5c^4d^4e^5f^3 + 36a^3b^5c^5d^3e^4f^4 - 48a^3b^5c^6d^2e^3f^5 + 22a^4b^4c^2d^6e^6f^2 + 36a^4b^4c^3d^5e^5f^3 - 90a^4b^4c^4d^4e^4f^4 + 36a^4b^4c^5d^3e^3f^5 + 22a^4b^4c^6d^2e^2f^6 - 48a^5b^3c^2d^6e^5f^3 + 36a^5b^3c^3d^5e^4f^4 + 36a^5b^3c^4d^4e^3f^5 - 48a^5b^3c^5d^3e^2f^6 + 22a^6b^2c^2d^6e^4f^4 - 48a^6b^2c^3d^5e^3f^5 + 22a^6b^2c^4d^4e^2f^6) + (3x^4(8a^6b^d^7f^7 + 8b^7c^6d^f^7 + 8b^7d^7e^6f - 7a^2b^5c^4d^3f^7 + 14a^3b^4c^3d^4f^7 - 7a^4b^3c^2d^5f^7 - 7a^2b^5d^7e^4f^3 + 14a^3b^4d^7e^3f^4 - 7a^4b^3d^7e^2f^5 - 7b^7c^2d^5e^4f^3 + 14b^7c^3d^4e^3f^4 - 7b^7c^4d^3e^2f^5 - 14a^ab^6c^5d^2f^7 - 14a^5b^2c^d^6f^7 - 14a^ab^6d^7e^5f^2 - 14a^5b^2d^7e^f^6 - 14b^7c^d^6e^5f^2 - 14b^7c^5d^2e^f^6 + 34a^ab^6c^d^6e^4f^3 + 34a^ab^6c^4d^3e^f^6 + 34a^4b^3c^d^6e^f^6 + 6a^ab^6c^2d^5e^3f^4 + 6a^ab^6c^3d^4e^2
\end{aligned}$$

$$\begin{aligned}
& *f^5 + 6*a^2*b^5*c*d^6*e^3*f^4 + 6*a^2*b^5*c^3*d^4*e*f^6 + 6*a^3*b^4*c*d^6* \\
& e^2*f^5 + 6*a^3*b^4*c^2*d^5*e*f^6 - 78*a^2*b^5*c^2*d^5*e^2*f^5)) / (2*(4*a*b^ \\
& 7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^ \\
& 8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^ \\
& 8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - \\
& 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c \\
& c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 \\
& + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c \\
& c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^ \\
& 4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7* \\
& d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3 \\
& *e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^ \\
& 7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3 \\
& *c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b \\
& b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22* \\
& a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e \\
& ^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b \\
& ^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^ \\
& 2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^ \\
& 5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 3 \\
& 6*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3 \\
& *e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6 \\
& *b^2*c^4*d^4*e^2*f^6)) + (x^2*(18*a*b^6*c^7*f^7 + 18*a*b^6*d^7*e^7 + 18*a^7 \\
& *c*d^6*f^7 + 18*b^7*c*d^6*e^7 + 18*a^7*d^7*e*f^6 + 18*b^7*c^7*e*f^6 - 3*a^3 \\
& *b^4*c^5*d^2*f^7 + 32*a^4*b^3*c^4*d^3*f^7 - 3*a^5*b^2*c^3*d^4*f^7 - 3*a^3*b \\
& ^4*d^7*e^5*f^2 + 32*a^4*b^3*d^7*e^4*f^3 - 3*a^5*b^2*d^7*e^3*f^4 - 3*b^7*c^3 \\
& *d^4*e^5*f^2 + 32*b^7*c^4*d^3*e^4*f^3 - 3*b^7*c^5*d^2*e^3*f^4 - 37*a^2*b^5* \\
& c^6*d*f^7 - 37*a^6*b*c^2*d^5*f^7 - 37*a^2*b^5*d^7*e^6*f - 37*a^6*b*d^7*e^2* \\
& f^5 - 37*b^7*c^2*d^5*e^6*f - 37*b^7*c^6*d*e^2*f^5 + 9*a*b^6*c^2*d^5*e^5*f^2 \\
& + a*b^6*c^3*d^4*e^4*f^3 + a*b^6*c^4*d^3*e^3*f^4 + 9*a*b^6*c^5*d^2*e^2*f^5 \\
& + 9*a^2*b^5*c*d^6*e^5*f^2 + 9*a^2*b^5*c^5*d^2*e*f^6 + a^3*b^4*c*d^6*e^4*f^3 \\
& + a^3*b^4*c^4*d^3*e*f^6 + a^4*b^3*c*d^6*e^3*f^4 + a^4*b^3*c^3*d^4*e*f^6 + \\
& 9*a^5*b^2*c*d^6*e^2*f^5 + 9*a^5*b^2*c^2*d^5*e*f^6 - 34*a*b^6*c*d^6*e^6*f - \\
& 34*a*b^6*c^6*d*e*f^6 - 34*a^6*b*c*d^6*e*f^6 + 234*a^2*b^5*c^2*d^5*e^4*f^3 - \\
& 208*a^2*b^5*c^3*d^4*e^3*f^4 + 234*a^2*b^5*c^4*d^3*e^2*f^5 - 208*a^3*b^4*c^ \\
& 2*d^5*e^3*f^4 - 208*a^3*b^4*c^3*d^4*e^2*f^5 + 234*a^4*b^3*c^2*d^5*e^2*f^5)) \\
& / (2*(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4* \\
& e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2 \\
& *c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^ \\
& 6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + \\
& 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^ \\
& 5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f \\
& ^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a \\
& *b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a \\
& ^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a* \\
& b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8 \\
& *a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 \\
& + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f \\
& ^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^ \\
& 2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^ \\
& 6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 \\
& + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2 \\
& *d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36 \\
& *a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6* \\
& e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b \\
& b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f \\
& ^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (x*(2*a^2*b^5*c^7*f^7 + 2*a^2*b^5*d^7*e \\
& ^7 + 2*a^7*c^2*d^5*f^7 + 2*b^7*c^2*d^5*e^7 + 2*a^7*d^7*e^2*f^5 + 2*b^7*c^7* \\
& e^2*f^5 + 4*a^4*b^3*c^5*d^2*f^7 + 4*a^5*b^2*c^4*d^3*f^7 + 4*a^4*b^3*d^7*e^5
\end{aligned}$$

$$\begin{aligned}
& *f^2 + 4*a^5*b^2*d^7*e^4*f^3 + 4*b^7*c^4*d^3*e^5*f^2 + 4*b^7*c^5*d^2*e^4*f^3 \\
& + 14*a*b^6*c*d^6*e^7 + 14*a*b^6*c^7*e*f^6 + 14*a^7*c*d^6*e*f^6 - 6*a^3*b^4 \\
& *c^6*d*f^7 - 6*a^6*b*c^3*d^4*f^7 - 6*a^3*b^4*d^7*e^6*f - 6*a^6*b*d^7*e^3*f \\
& ^4 - 6*b^7*c^3*d^4*e^6*f - 6*b^7*c^6*d*e^3*f^4 - 33*a*b^6*c^2*d^5*e^6*f - 3 \\
& 3*a*b^6*c^6*d*e^2*f^5 - 33*a^2*b^5*c*d^6*e^6*f - 33*a^2*b^5*c^6*d*e*f^6 - 3 \\
& 3*a^6*b*c*d^6*e^2*f^5 - 33*a^6*b*c^2*d^5*e*f^6 + 17*a*b^6*c^3*d^4*e^5*f^2 - \\
& 8*a*b^6*c^4*d^3*e^4*f^3 + 17*a*b^6*c^5*d^2*e^3*f^4 + 17*a^3*b^4*c*d^6*e^5* \\
& f^2 + 17*a^3*b^4*c^5*d^2*e*f^6 - 8*a^4*b^3*c*d^6*e^4*f^3 - 8*a^4*b^3*c^4*d^ \\
& 3*e*f^6 + 17*a^5*b^2*c*d^6*e^3*f^4 + 17*a^5*b^2*c^3*d^4*e*f^6 + 78*a^2*b^5* \\
& c^2*d^5*e^5*f^2 - 26*a^2*b^5*c^3*d^4*e^4*f^3 - 26*a^2*b^5*c^4*d^3*e^3*f^4 + \\
& 78*a^2*b^5*c^5*d^2*e^2*f^5 - 26*a^3*b^4*c^2*d^5*e^4*f^3 - 26*a^3*b^4*c^4*d \\
& ^3*e^2*f^5 - 26*a^4*b^3*c^2*d^5*e^3*f^4 - 26*a^4*b^3*c^3*d^4*e^2*f^5 + 78*a \\
& ^5*b^2*c^2*d^5*e^2*f^5)/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d \\
& ^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^ \\
& 2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e \\
& ^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + \\
& 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c \\
& ^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^ \\
& 3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8 \\
& *c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b \\
& ^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b \\
& *c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2* \\
& b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a \\
& ^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + \\
& 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 \\
& + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^ \\
& 3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^ \\
& 3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3 \\
& *f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4 \\
& *c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 \\
& - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4* \\
& d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48* \\
& a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6) + (x^3*(6*a^7*d^7*f^7 \\
& + 6*b^7*c^7*f^7 + 6*b^7*d^7*e^7 - 37*a^2*b^5*c^5*d^2*f^7 + 19*a^3*b^4*c^4* \\
& d^3*f^7 + 19*a^4*b^3*c^3*d^4*f^7 - 37*a^5*b^2*c^2*d^5*f^7 - 37*a^2*b^5*d^7* \\
& e^5*f^2 + 19*a^3*b^4*d^7*e^4*f^3 + 19*a^4*b^3*d^7*e^3*f^4 - 37*a^5*b^2*d^7* \\
& e^2*f^5 - 37*b^7*c^2*d^5*e^5*f^2 + 19*b^7*c^3*d^4*e^4*f^3 + 19*b^7*c^4*d^3* \\
& e^3*f^4 - 37*b^7*c^5*d^2*e^2*f^5 + 3*a*b^6*c^6*d*f^7 + 3*a^6*b*c*d^6*f^7 + \\
& 3*a*b^6*d^7*e^6*f + 3*a^6*b*d^7*e*f^6 + 3*b^7*c*d^6*e^6*f + 3*b^7*c^6*d*e*f \\
& ^6 - 28*a*b^6*c*d^6*e^5*f^2 - 28*a*b^6*c^5*d^2*e*f^6 - 28*a^5*b^2*c*d^6*e*f \\
& ^6 + 86*a*b^6*c^2*d^5*e^4*f^3 - 68*a*b^6*c^3*d^4*e^3*f^4 + 86*a*b^6*c^4*d^3 \\
& *e^2*f^5 + 86*a^2*b^5*c*d^6*e^4*f^3 + 86*a^2*b^5*c^4*d^3*e*f^6 - 68*a^3*b^4 \\
& *c*d^6*e^3*f^4 - 68*a^3*b^4*c^3*d^4*e*f^6 + 86*a^4*b^3*c*d^6*e^2*f^5 + 86*a \\
& ^4*b^3*c^2*d^5*e*f^6 - 52*a^2*b^5*c^2*d^5*e^3*f^4 - 52*a^2*b^5*c^3*d^4*e^2* \\
& f^5 - 52*a^3*b^4*c^2*d^5*e^2*f^5))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - \\
& a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6* \\
& a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6 \\
& *b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4* \\
& c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + \\
& 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b* \\
& d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7 \\
& *f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 \\
& - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 \\
& - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^ \\
& 3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e \\
& ^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^ \\
& 2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d \\
& ^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2* \\
& b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f
\end{aligned}$$

$$\begin{aligned} &^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c \\ &^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - \\ &90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^ \\ &2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^ \\ &5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4 \\ &*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) / (x*(2*a*b* \\ &c^2*e^2 + 2*a^2*c*d*e^2 + 2*a^2*c^2*e*f) + x^3*(2*a*b*c^2*f^2 + 2*a*b*d^2*e \\ &^2 + 2*a^2*c*d*f^2 + 2*b^2*c*d*e^2 + 2*a^2*d^2*e*f + 2*b^2*c^2*e*f + 8*a*b* \\ &c*d*e*f) + x^2*(a^2*c^2*f^2 + a^2*d^2*e^2 + b^2*c^2*e^2 + 4*a*b*c*d*e^2 + 4 \\ &*a*b*c^2*e*f + 4*a^2*c*d*e*f) + x^5*(2*a*b*d^2*f^2 + 2*b^2*c*d*f^2 + 2*b^2* \\ &d^2*e*f) + x^4*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 4*a*b*c*d*f^2 + 4 \\ &*a*b*d^2*e*f + 4*b^2*c*d*e*f) + a^2*c^2*e^2 + b^2*d^2*f^2*x^6) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**3,x)

[Out] Timed out

$$3.21 \quad \int \frac{1}{1+x+x^2+x^3} dx$$

Optimal. Leaf size=25

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)^(-1), x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x+x^2+x^3} dx &= \int \left(\frac{1}{2(1+x)} + \frac{1-x}{2(1+x^2)} \right) dx \\ &= \frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1-x}{1+x^2} dx \\ &= \frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1}{1+x^2} dx - \frac{1}{2} \int \frac{x}{1+x^2} dx \\ &= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x) - \frac{1}{4} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)^(-1), x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4

fricas [A] time = 0.61, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+x^2+x+1), x, algorithm="fricas")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)

giac [A] time = 0.31, size = 20, normalized size = 0.80

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+x^2+x+1), x, algorithm="giac")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} + \frac{\ln(x + 1)}{2} - \frac{\ln(x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3+x^2+x+1), x)

[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)

maxima [A] time = 1.45, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+x^2+x+1), x, algorithm="maxima")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)

mupad [B] time = 2.20, size = 25, normalized size = 1.00

$$\frac{\ln(x + 1)}{2} + \ln(x - i) \left(-\frac{1}{4} - \frac{1}{4}i \right) + \ln(x + 1i) \left(-\frac{1}{4} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + x^2 + x^3 + 1), x)

[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)

sympy [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log(x + 1)}{2} - \frac{\log(x^2 + 1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

$$3.22 \quad \int \frac{1}{-1+4x-4x^2+16x^3} dx$$

Optimal. Leaf size=31

$$-\frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(1 - 4x) - \frac{1}{10} \tan^{-1}(2x)$$

[Out] -1/10*arctan(2*x)+1/5*ln(1-4*x)-1/10*ln(4*x^2+1)

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(1 - 4x) - \frac{1}{10} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1), x]

[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{-1+4x-4x^2+16x^3} dx &= \int \left(\frac{4}{5(-1+4x)} + \frac{-1-4x}{5(1+4x^2)} \right) dx \\ &= \frac{1}{5} \log(1-4x) + \frac{1}{5} \int \frac{-1-4x}{1+4x^2} dx \\ &= \frac{1}{5} \log(1-4x) - \frac{1}{5} \int \frac{1}{1+4x^2} dx - \frac{4}{5} \int \frac{x}{1+4x^2} dx \\ &= -\frac{1}{10} \tan^{-1}(2x) + \frac{1}{5} \log(1-4x) - \frac{1}{10} \log(1+4x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(1 - 4x) - \frac{1}{10} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1), x]

[Out] -1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10

fricas [A] time = 0.75, size = 25, normalized size = 0.81

$$-\frac{1}{10} \arctan(2x) - \frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(16*x^3-4*x^2+4*x-1), x, algorithm="fricas")

[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)

giac [A] time = 0.35, size = 26, normalized size = 0.84

$$-\frac{1}{10} \arctan(2x) - \frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(|4x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(16*x^3-4*x^2+4*x-1), x, algorithm="giac")

[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(abs(4*x - 1))

maple [A] time = 0.01, size = 26, normalized size = 0.84

$$-\frac{\arctan(2x)}{10} + \frac{\ln(4x - 1)}{5} - \frac{\ln(4x^2 + 1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(16*x^3-4*x^2+4*x-1), x)

[Out] -1/10*ln(4*x^2+1)-1/10*arctan(2*x)+1/5*ln(-1+4*x)

maxima [A] time = 1.28, size = 25, normalized size = 0.81

$$-\frac{1}{10} \arctan(2x) - \frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(16*x^3-4*x^2+4*x-1), x, algorithm="maxima")

[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)

mupad [B] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln\left(x - \frac{1}{4}\right)}{5} + \ln\left(x - \frac{1}{2}i\right) \left(-\frac{1}{10} + \frac{1}{20}i\right) + \ln\left(x + \frac{1}{2}i\right) \left(-\frac{1}{10} - \frac{1}{20}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x - 4*x^2 + 16*x^3 - 1), x)

[Out] log(x - 1/4)/5 - log(x - 1i/2)*(1/10 - 1i/20) - log(x + 1i/2)*(1/10 + 1i/20)

sympy [A] time = 0.15, size = 24, normalized size = 0.77

$$\frac{\log\left(x - \frac{1}{4}\right)}{5} - \frac{\log\left(x^2 + \frac{1}{4}\right)}{10} - \frac{\operatorname{atan}(2x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x**3-4*x**2+4*x-1),x)
```

```
[Out] log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10
```

3.23 $\int \frac{1}{dx^3} dx$

Optimal. Leaf size=10

$$-\frac{1}{2dx^2}$$

[Out] -1/2/d/x^2

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {12, 30}

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(d*x^3), x]

[Out] -1/(2*d*x^2)

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{dx^3} dx &= \frac{\int \frac{1}{x^3} dx}{d} \\ &= -\frac{1}{2dx^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(d*x^3), x]

[Out] -1/2*1/(d*x^2)

fricas [A] time = 0.70, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/d/x^3,x, algorithm="fricas")

[Out] -1/2/(d*x^2)

giac [A] time = 0.27, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/d/x^3,x, algorithm="giac")

[Out] -1/2/(d*x^2)

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/d/x^3,x)

[Out] -1/2/d/x^2

maxima [A] time = 0.63, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/d/x^3,x, algorithm="maxima")

[Out] -1/2/(d*x^2)

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x^3),x)

[Out] -1/(2*d*x^2)

sympy [A] time = 0.06, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/d/x**3,x)

[Out] -1/(2*d*x**2)

$$3.24 \quad \int \frac{1}{cx^2+dx^3} dx$$

Optimal. Leaf size=28

$$-\frac{d \log(x)}{c^2} + \frac{d \log(c + dx)}{c^2} - \frac{1}{cx}$$

[Out] $-1/c/x-d*\ln(x)/c^2+d*\ln(d*x+c)/c^2$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 44}

$$-\frac{d \log(x)}{c^2} + \frac{d \log(c + dx)}{c^2} - \frac{1}{cx}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2 + d*x^3)^(-1),x]

[Out] $-(1/(c*x)) - (d*\text{Log}[x])/c^2 + (d*\text{Log}[c + d*x])/c^2$

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{cx^2 + dx^3} dx &= \int \frac{1}{x^2(c + dx)} dx \\ &= \int \left(\frac{1}{cx^2} - \frac{d}{c^2x} + \frac{d^2}{c^2(c + dx)} \right) dx \\ &= -\frac{1}{cx} - \frac{d \log(x)}{c^2} + \frac{d \log(c + dx)}{c^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{d \log(x)}{c^2} + \frac{d \log(c + dx)}{c^2} - \frac{1}{cx}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x^2 + d*x^3)^(-1),x]

[Out] $-(1/(c*x)) - (d*\text{Log}[x])/c^2 + (d*\text{Log}[c + d*x])/c^2$

fricas [A] time = 0.89, size = 26, normalized size = 0.93

$$\frac{dx \log(dx + c) - dx \log(x) - c}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2),x, algorithm="fricas")

[Out] (d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)

giac [A] time = 0.36, size = 30, normalized size = 1.07

$$\frac{d \log(|dx + c|)}{c^2} - \frac{d \log(|x|)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2),x, algorithm="giac")

[Out] d*log(abs(d*x + c))/c^2 - d*log(abs(x))/c^2 - 1/(c*x)

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\frac{d \ln(x)}{c^2} + \frac{d \ln(dx + c)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x^3+c*x^2),x)

[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2

maxima [A] time = 0.59, size = 28, normalized size = 1.00

$$\frac{d \log(dx + c)}{c^2} - \frac{d \log(x)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2),x, algorithm="maxima")

[Out] d*log(d*x + c)/c^2 - d*log(x)/c^2 - 1/(c*x)

mupad [B] time = 0.06, size = 25, normalized size = 0.89

$$\frac{2 d \operatorname{atanh}\left(\frac{2dx}{c} + 1\right)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2 + d*x^3),x)

[Out] (2*d*atanh((2*d*x)/c + 1))/c^2 - 1/(c*x)

sympy [A] time = 0.18, size = 19, normalized size = 0.68

$$-\frac{1}{cx} + \frac{d(-\log(x) + \log(\frac{c}{d} + x))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x**3+c*x**2),x)

[Out] -1/(c*x) + d*(-log(x) + log(c/d + x))/c**2

$$3.25 \quad \int \frac{1}{bx+dx^3} dx$$

Optimal. Leaf size=22

$$\frac{\log(x)}{b} - \frac{\log(b+dx^2)}{2b}$$

[Out] $\ln(x)/b - 1/2 * \ln(d*x^2+b)/b$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{b} - \frac{\log(b+dx^2)}{2b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x + d*x^3)^{-1}, x]$

[Out] $\text{Log}[x]/b - \text{Log}[b + d*x^2]/(2*b)$

Rule 29

$\text{Int}[(x_)^{-1}, x_Symbol] \text{ :> } \text{Simp}[\text{Log}[x], x]$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{-1}, x_Symbol] \text{ :> } \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ /; } \text{FreeQ}[\{a, b\}, x]$

Rule 36

$\text{Int}[1/(((a_ + (b_)*(x_))*((c_ + (d_)*(x_)))), x_Symbol] \text{ :> } \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 266

$\text{Int}[(x_)^{(m_)*((a_ + (b_)*(x_)^{(n_))^{(p_)}), x_Symbol] \text{ :> } \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] \text{ /; } \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 1593

$\text{Int}[(u_)*((a_)*(x_)^{(p_)} + (b_)*(x_)^{(q_))^{(n_)}), x_Symbol] \text{ :> } \text{Int}[u*x^{(n*p)*(a + b*x^{(q-p)})^n}, x] \text{ /; } \text{FreeQ}[\{a, b, p, q\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q - p]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{bx + dx^3} dx &= \int \frac{1}{x(b + dx^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(b + dx)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2b} - \frac{d \text{Subst} \left(\int \frac{1}{b+dx} dx, x, x^2 \right)}{2b} \\
&= \frac{\log(x)}{b} - \frac{\log(b + dx^2)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{b} - \frac{\log(b + dx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + d*x^3)^(-1), x]

[Out] Log[x]/b - Log[b + d*x^2]/(2*b)

fricas [A] time = 0.78, size = 18, normalized size = 0.82

$$-\frac{\log(dx^2 + b) - 2 \log(x)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+b*x), x, algorithm="fricas")

[Out] -1/2*(log(d*x^2 + b) - 2*log(x))/b

giac [A] time = 0.30, size = 24, normalized size = 1.09

$$\frac{\log(x^2)}{2b} - \frac{\log(|dx^2 + b|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+b*x), x, algorithm="giac")

[Out] 1/2*log(x^2)/b - 1/2*log(abs(d*x^2 + b))/b

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{b} - \frac{\ln(dx^2 + b)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x^3+b*x), x)

[Out] ln(x)/b-1/2*ln(d*x^2+b)/b

maxima [A] time = 0.79, size = 20, normalized size = 0.91

$$-\frac{\log(dx^2 + b)}{2b} + \frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+b*x),x, algorithm="maxima")

[Out] -1/2*log(d*x^2 + b)/b + log(x)/b

mupad [B] time = 2.13, size = 18, normalized size = 0.82

$$\frac{\ln(dx^2 + b) - 2 \ln(x)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + d*x^3),x)

[Out] -(log(b + d*x^2) - 2*log(x))/(2*b)

sympy [A] time = 0.20, size = 15, normalized size = 0.68

$$\frac{\log(x)}{b} - \frac{\log\left(\frac{b}{d} + x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x**3+b*x),x)

[Out] log(x)/b - log(b/d + x**2)/(2*b)

$$3.26 \quad \int \frac{1}{bx+cx^2+dx^3} dx$$

Optimal. Leaf size=62

$$\frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} - \frac{\log(b+cx+dx^2)}{2b} + \frac{\log(x)}{b}$$

[Out] $\ln(x)/b - 1/2 \cdot \ln(dx^2+cx+b)/b + c \cdot \operatorname{arctanh}((2dx+c)/(-4bd+c^2)^{1/2})/b / (-4bd+c^2)^{1/2}$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1594, 705, 29, 634, 618, 206, 628}

$$\frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} - \frac{\log(b+cx+dx^2)}{2b} + \frac{\log(x)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x + c*x^2 + d*x^3)^{-1}, x]$

[Out] $(c \cdot \operatorname{ArcTanh}[(c + 2dx)/\sqrt{c^2 - 4bd}]) / (b \cdot \sqrt{c^2 - 4bd}) + \operatorname{Log}[x]/b - \operatorname{Log}[b + cx + dx^2]/(2b)$

Rule 29

$\text{Int}[(x_)^{-1}, x_Symbol] \rightarrow \text{Simp}[\operatorname{Log}[x], x]$

Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[-b, 2]), x] /;$ $\text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /;$ $\text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[(d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \operatorname{Log}[\operatorname{RemoveContent}[a + bx + cx^2, x]])/b, x] /;$ $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 634

$\text{Int}[(d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2cd - be)/(2c), \text{Int}[1/(a + bx + cx^2), x], x] + \text{Dist}[e/(2c), \text{Int}[(b + 2cx)/(a + bx + cx^2), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 705

$\text{Int}[1/(((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol] \rightarrow \text{Dist}[e^2/(cd^2 - bde + ae^2), \text{Int}[1/(d + ex), x], x] + \text{Dist}[1/(cd^2 - bde + ae^2), \text{Int}[(cd - be - cex)/(a + bx + cx^2), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2]$

2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{bx + cx^2 + dx^3} dx &= \int \frac{1}{x(b + cx + dx^2)} dx \\ &= \frac{\int \frac{1}{x} dx}{b} + \frac{\int \frac{-c-dx}{b+cx+dx^2} dx}{b} \\ &= \frac{\log(x)}{b} - \frac{\int \frac{c+2dx}{b+cx+dx^2} dx}{2b} - \frac{c \int \frac{1}{b+cx+dx^2} dx}{2b} \\ &= \frac{\log(x)}{b} - \frac{\log(b + cx + dx^2)}{2b} + \frac{c \operatorname{Subst}\left(\int \frac{1}{c^2-4bd-x^2} dx, x, c + 2dx\right)}{b} \\ &= \frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} + \frac{\log(x)}{b} - \frac{\log(b + cx + dx^2)}{2b} \end{aligned}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 0.98

$$\frac{2c \tan^{-1}\left(\frac{c+2dx}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}} + \log(b + x(c + dx)) - 2 \log(x)$$

$$2b$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2 + d*x^3)^(-1), x]

[Out] -1/2*((2*c*ArcTan[(c + 2*d*x)/Sqrt[-c^2 + 4*b*d]])/Sqrt[-c^2 + 4*b*d] - 2*Log[x] + Log[b + x*(c + d*x)])/b

fricas [A] time = 0.86, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{c^2 - 4bd} c \log\left(\frac{2d^2x^2 + 2cdx + c^2 - 2bd + \sqrt{c^2 - 4bd}(2dx + c)}{dx^2 + cx + b}\right) - (c^2 - 4bd) \log(dx^2 + cx + b) + 2(c^2 - 4bd) \log(x)}{2(bc^2 - 4b^2d)}, \frac{2\sqrt{-c^2 + 4bd}}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2+b*x), x, algorithm="fricas")

[Out] [1/2*(sqrt(c^2 - 4*b*d)*c*log((2*d^2*x^2 + 2*c*d*x + c^2 - 2*b*d + sqrt(c^2 - 4*b*d)*(2*d*x + c))/(d*x^2 + c*x + b)) - (c^2 - 4*b*d)*log(d*x^2 + c*x + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d), 1/2*(2*sqrt(-c^2 + 4*b*d)*c*arctan(-sqrt(-c^2 + 4*b*d)*(2*d*x + c)/(c^2 - 4*b*d)) - (c^2 - 4*b*d)*log(d*x^2 + c*x + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d)]

giac [A] time = 0.29, size = 62, normalized size = 1.00

$$\frac{c \arctan\left(\frac{2dx+c}{\sqrt{-c^2+4bd}}\right)}{\sqrt{-c^2+4bd}b} - \frac{\log(dx^2 + cx + b)}{2b} + \frac{\log(|x|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="giac")

[Out] $-c \arctan\left(\frac{2dx+c}{\sqrt{-c^2+4bd}}\right) / (\sqrt{-c^2+4bd}b) - 1/2 \log(d x^2 + c x + b) / b + \log(\text{abs}(x)) / b$

maple [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{c \arctan\left(\frac{2dx+c}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2} b} + \frac{\ln(x)}{b} - \frac{\ln(dx^2+cx+b)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x^3+c*x^2+b*x),x)

[Out] $-1/2 \ln(dx^2+cx+b) / b - 1/b * c / (4bd-c^2)^{(1/2)} * \arctan((2dx+c) / (4bd-c^2)^{(1/2)}) + 1/b * \ln(x)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b*d-c^2>0)', see `assume?` for more details) Is 4*b*d-c^2 positive or negative?

mupad [B] time = 0.47, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{b} - \ln\left(x(6bd^2 - 2c^2d) - bcd\right) \left(\frac{1}{2b} - \frac{c\sqrt{c^2 - 4bd}}{2(b^2c^2 - 4b^2d)}\right) - cd - 3d^2x \left(\frac{1}{2b} - \frac{c\sqrt{c^2 - 4bd}}{2(b^2c^2 - 4b^2d)}\right) - \ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2 + d*x^3),x)

[Out] $\log(x) / b - \log\left(\frac{x(6bd^2 - 2c^2d) - bcd}{(2b)^2} - \frac{c(c^2 - 4bd)^{(1/2)}}{2(b^2c^2 - 4b^2d)}\right) - cd - 3d^2x \left(\frac{1}{(2b)^2} - \frac{c(c^2 - 4bd)^{(1/2)}}{2(b^2c^2 - 4b^2d)}\right) + \log\left(\frac{x(6bd^2 - 2c^2d) - bcd}{(2b)^2} + \frac{c(c^2 - 4bd)^{(1/2)}}{2(b^2c^2 - 4b^2d)}\right) - cd - 3d^2x \left(\frac{1}{(2b)^2} + \frac{c(c^2 - 4bd)^{(1/2)}}{2(b^2c^2 - 4b^2d)}\right)$

sympy [B] time = 4.19, size = 564, normalized size = 9.10

$$\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)} - \frac{1}{2b}\right) \log\left(x + \frac{24b^4d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)} - \frac{1}{2b}\right)^2 - 14b^3c^2d\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)} - \frac{1}{2b}\right)^2 - 12b^3d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)} - \frac{1}{2b}\right)}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x**3+c*x**2+b*x),x)

[Out] $(-c\sqrt{-4bd+c^2} / (2b(4bd-c^2)) - 1/(2b)) * \log(x + (24b^4d^2 * (-c\sqrt{-4bd+c^2} / (2b(4bd-c^2)) - 1/(2b))^2 - 14b^3c^2d * (-c\sqrt{-4bd+c^2} / (2b(4bd-c^2)) - 1/(2b))^2 - 12b^3d^2 * (-c\sqrt{-4bd+c^2} / (2b(4bd-c^2)) - 1/(2b))^2)$

```

*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b
*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c**3*d) + (c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c*
*3*d)) + log(x)/b

```

3.27 $\int \frac{1}{a+dx^3} dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}}$$

[Out] 1/3*ln(a^(1/3)+d^(1/3)*x)/a^(2/3)/d^(1/3)-1/6*ln(a^(2/3)-a^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/a^(2/3)/d^(1/3)-1/3*arctan(1/3*(a^(1/3)-2*d^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/d^(1/3)*3^(1/2)

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{d}x\right)}{3a^{2/3}\sqrt[3]{d}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

[In] Int[(a + d*x^3)^(-1), x]

[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d^(1/3))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{a + dx^3} dx &= \frac{\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{d}x} dx}{3a^{2/3}} + \frac{\int \frac{2\sqrt[3]{a} - \sqrt[3]{d}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2} dx}{3a^{2/3}} \\ &= \frac{\log(\sqrt[3]{a} + \sqrt[3]{d}x)}{3a^{2/3}\sqrt[3]{d}} + \frac{\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2} dx}{2\sqrt[3]{a}} - \frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{d} + 2d^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2} dx}{6a^{2/3}\sqrt[3]{d}} \\ &= \frac{\log(\sqrt[3]{a} + \sqrt[3]{d}x)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2)}{6a^{2/3}\sqrt[3]{d}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{d}} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{d}x)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2)}{6a^{2/3}\sqrt[3]{d}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.77

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2) - 2\log(\sqrt[3]{a} + \sqrt[3]{d}x) + 2\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)^(-1), x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + d^(1/3)*x] + Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(a^(2/3)*d^(1/3))
```

fricas [A] time = 0.83, size = 299, normalized size = 2.60

$$\frac{3\sqrt{\frac{1}{3}}ad\sqrt{-\frac{(a^2d)^{\frac{1}{3}}}{d}} \log\left(\frac{2adx^3 - 3(a^2d)^{\frac{1}{3}}ax - a^2 + 3\sqrt{\frac{1}{3}}\left(2adx^2 + (a^2d)^{\frac{2}{3}}x - (a^2d)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2d)^{\frac{1}{3}}}{d}}}{dx^3 + a}}{6a^2d}\right) - (a^2d)^{\frac{2}{3}} \log\left(adx^2 - (a^2d)^{\frac{2}{3}}x + \dots\right)}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*d*sqrt(-(a^2*d)^(1/3)/d)*log((2*a*d*x^3 - 3*(a^2*d)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*d*x^2 + (a^2*d)^(2/3)*x - (a^2*d)^(1/3)*a)*
```

$\sqrt{-\left(a^2d\right)^{1/3}/d}/\left(d^3x+a\right) - \left(a^2d\right)^{2/3}\log\left(a^2dx^2 - \left(a^2d\right)^{2/3}x + \left(a^2d\right)^{1/3}a\right) + 2\left(a^2d\right)^{2/3}\log\left(a^2dx + \left(a^2d\right)^{2/3}\right)/\left(a^2d\right), 1/6\left(6\sqrt{1/3}a^2d\sqrt{\left(a^2d\right)^{1/3}/d}\arctan\left(\sqrt{1/3}\left(2\left(a^2d\right)^{2/3}x - \left(a^2d\right)^{1/3}a\right)\sqrt{\left(a^2d\right)^{1/3}/d}/a^2 - \left(a^2d\right)^{2/3}\log\left(a^2dx^2 - \left(a^2d\right)^{2/3}x + \left(a^2d\right)^{1/3}a\right) + 2\left(a^2d\right)^{2/3}\log\left(a^2dx + \left(a^2d\right)^{2/3}\right)\right)/\left(a^2d\right)$

giac [A] time = 0.26, size = 112, normalized size = 0.97

$$\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}}\log\left(x - \left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3a} + \frac{\sqrt{3}\left(-ad^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3ad} + \frac{\left(-ad^2\right)^{\frac{1}{3}}\log\left(x^2 + x\left(-\frac{a}{d}\right)^{\frac{1}{3}} + \left(-\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+a),x, algorithm="giac")

[Out] $-1/3\left(-a/d\right)^{1/3}\log\left(\text{abs}\left(x - \left(-a/d\right)^{1/3}\right)\right)/a + 1/3\sqrt{3}\left(-ad^2\right)^{1/3}\arctan\left(1/3\sqrt{3}\left(2x + \left(-a/d\right)^{1/3}\right)/\left(-a/d\right)^{1/3}\right)/\left(ad\right) + 1/6\left(-ad^2\right)^{1/3}\log\left(x^2 + x\left(-a/d\right)^{1/3} + \left(-a/d\right)^{2/3}\right)/\left(ad\right)$

maple [A] time = 0.01, size = 91, normalized size = 0.79

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{d}\right)^{\frac{2}{3}}d} + \frac{\ln\left(x + \left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{2}{3}}d} - \frac{\ln\left(x^2 - \left(\frac{a}{d}\right)^{\frac{1}{3}}x + \left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{d}\right)^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x^3+a),x)

[Out] $1/3d/\left(a/d\right)^{2/3}\ln\left(x + \left(a/d\right)^{1/3}\right) - 1/6d/\left(a/d\right)^{2/3}\ln\left(x^2 - \left(a/d\right)^{1/3}x + \left(a/d\right)^{2/3}\right) + 1/3d/\left(a/d\right)^{2/3}\sqrt{3}\arctan\left(1/3\sqrt{3}\left(2/\left(a/d\right)^{1/3}x - 1\right)\right)$

maxima [A] time = 1.28, size = 98, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3d\left(\frac{a}{d}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{d}\right)^{\frac{1}{3}} + \left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{a}{d}\right)^{\frac{2}{3}}} + \frac{\log\left(x + \left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+a),x, algorithm="maxima")

[Out] $1/3\sqrt{3}\arctan\left(1/3\sqrt{3}\left(2x - \left(a/d\right)^{1/3}\right)/\left(a/d\right)^{1/3}\right)/\left(d\left(a/d\right)^{2/3}\right) - 1/6\log\left(x^2 - x\left(a/d\right)^{1/3} + \left(a/d\right)^{2/3}\right)/\left(d\left(a/d\right)^{2/3}\right) + 1/3\log\left(x + \left(a/d\right)^{1/3}\right)/\left(d\left(a/d\right)^{2/3}\right)$

mupad [B] time = 0.23, size = 99, normalized size = 0.86

$$\frac{\ln\left(d^{1/3}x + a^{1/3}\right)}{3a^{2/3}d^{1/3}} + \frac{\ln\left(3d^2x + \frac{3a^{1/3}d^{5/3}\left(-1 + \sqrt{3}i\right)}{2}\right)\left(-1 + \sqrt{3}i\right)}{6a^{2/3}d^{1/3}} - \frac{\ln\left(3d^2x - \frac{3a^{1/3}d^{5/3}\left(1 + \sqrt{3}i\right)}{2}\right)\left(1 + \sqrt{3}i\right)}{6a^{2/3}d^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + d*x^3),x)`

[Out] $\log(d^{1/3}x + a^{1/3})/(3a^{2/3}d^{1/3}) + (\log(3d^2x + (3a^{1/3}d^{5/3})(3^{1/2}i - 1))/2)(3^{1/2}i - 1)/(6a^{2/3}d^{1/3}) - (\log(3d^2x - (3a^{1/3}d^{5/3})(3^{1/2}i + 1))/2)(3^{1/2}i + 1)/(6a^{2/3}d^{1/3})$

sympy [A] time = 0.16, size = 20, normalized size = 0.17

$$\text{RootSum}\left(27t^3a^2d - 1, \left(t \mapsto t \log(3ta + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x**3+a),x)`

[Out] `RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x)))`

3.28 $\int (dx^3)^n dx$

Optimal. Leaf size=16

$$\frac{x(dx^3)^n}{3n+1}$$

[Out] $x*(d*x^3)^n/(1+3*n)$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {15, 30}

$$\frac{x(dx^3)^n}{3n+1}$$

Antiderivative was successfully verified.

[In] Int[(d*x^3)^n,x]

[Out] (x*(d*x^3)^n)/(1 + 3*n)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (dx^3)^n dx &= \left(x^{-3n} (dx^3)^n\right) \int x^{3n} dx \\ &= \frac{x(dx^3)^n}{1+3n} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x(dx^3)^n}{3n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x^3)^n,x]

[Out] (x*(d*x^3)^n)/(1 + 3*n)

fricas [A] time = 0.85, size = 16, normalized size = 1.00

$$\frac{(dx^3)^n x}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3)^n,x, algorithm="fricas")

[Out] $(d*x^3)^n*x/(3*n + 1)$

giac [A] time = 0.31, size = 16, normalized size = 1.00

$$\frac{(dx^3)^n x}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3)^n,x, algorithm="giac")

[Out] $(d*x^3)^n*x/(3*n + 1)$

maple [A] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x(dx^3)^n}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3)^n,x)

[Out] $x*(d*x^3)^n/(1+3*n)$

maxima [A] time = 0.57, size = 17, normalized size = 1.06

$$\frac{d^n x x^{3n}}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3)^n,x, algorithm="maxima")

[Out] $d^n*x*x^(3*n)/(3*n + 1)$

mupad [B] time = 2.50, size = 16, normalized size = 1.00

$$\frac{x(dx^3)^n}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3)^n,x)

[Out] $(x*(d*x^3)^n)/(3*n + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{d^n x (x^3)^n}{3n+1} & \text{for } n \neq -\frac{1}{3} \\ \int \frac{1}{\sqrt[3]{dx^3}} dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3)**n,x)

[Out] Piecewise((d**n*x*(x**3)**n/(3*n + 1), Ne(n, -1/3)), (Integral((d*x**3)**(-1/3), x), True))

3.29 $\int (cx^2 + dx^3)^n dx$

Optimal. Leaf size=55

$$\frac{x \left(\frac{dx}{c} + 1\right)^{-n} (cx^2 + dx^3)^n {}_2F_1\left(-n, 2n + 1; 2(n + 1); -\frac{dx}{c}\right)}{2n + 1}$$

[Out] $x*(d*x^3+c*x^2)^n*\text{hypergeom}([-n, 1+2*n], [2+2*n], -d*x/c)/(1+2*n)/((1+d*x/c)^n)$

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2011, 66, 64}

$$\frac{x \left(\frac{dx}{c} + 1\right)^{-n} (cx^2 + dx^3)^n \text{Hypergeometric2F1}\left(-n, 2n + 1, 2(n + 1), -\frac{dx}{c}\right)}{2n + 1}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2 + d*x^3)^n, x]

[Out] $(x*(c*x^2 + d*x^3)^n*\text{Hypergeometric2F1}[-n, 1 + 2*n, 2*(1 + n), -((d*x)/c)]) / ((1 + 2*n)*(1 + (d*x)/c)^n)$

Rule 64

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(c^n*(b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0]))

Rule 66

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])

Rule 2011

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rubi steps

$$\begin{aligned} \int (cx^2 + dx^3)^n dx &= \left(x^{-2n}(c + dx)^{-n} (cx^2 + dx^3)^n\right) \int x^{2n}(c + dx)^n dx \\ &= \left(x^{-2n} \left(1 + \frac{dx}{c}\right)^{-n} (cx^2 + dx^3)^n\right) \int x^{2n} \left(1 + \frac{dx}{c}\right)^n dx \\ &= \frac{x \left(1 + \frac{dx}{c}\right)^{-n} (cx^2 + dx^3)^n {}_2F_1\left(-n, 1 + 2n; 2(1 + n); -\frac{dx}{c}\right)}{1 + 2n} \end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.96

$$\frac{x \left(x^2(c + dx)\right)^n \left(\frac{dx}{c} + 1\right)^{-n} {}_2F_1\left(-n, 2n + 1; 2n + 2; -\frac{dx}{c}\right)}{2n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x^2 + d*x^3)^n,x]

[Out] (x*(x^2*(c + d*x))^n*Hypergeometric2F1[-n, 1 + 2*n, 2 + 2*n, -((d*x)/c)])/(1 + 2*n)*(1 + (d*x)/c)^n

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3 + cx^2\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2)^n,x, algorithm="fricas")

[Out] integral((d*x^3 + c*x^2)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + cx^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2)^n,x, algorithm="giac")

[Out] integrate((d*x^3 + c*x^2)^n, x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx^3 + cx^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2)^n,x)

[Out] int((d*x^3+c*x^2)^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + cx^2)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2)^n,x, algorithm="maxima")

[Out] integrate((d*x^3 + c*x^2)^n, x)

mupad [B] time = 2.21, size = 56, normalized size = 1.02

$$\frac{x \left(dx^3 + cx^2\right)^n {}_2F_1\left(2n + 1, -n; 2n + 2; -\frac{dx}{c}\right)}{(2n + 1) \left(\frac{dx}{c} + 1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2 + d*x^3)^n,x)
```

```
[Out] (x*(c*x^2 + d*x^3)^n*hypergeom([2*n + 1, -n], 2*n + 2, -(d*x)/c))/((2*n + 1)
)*((d*x)/c + 1)^n)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (cx^2 + dx^3)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)**n,x)
```

```
[Out] Integral((c*x**2 + d*x**3)**n, x)
```

3.30 $\int (bx + dx^3)^n dx$

Optimal. Leaf size=53

$$\frac{x(b + dx^2)(bx + dx^3)^n {}_2F_1\left(1, \frac{3(n+1)}{2}; \frac{n+3}{2}; -\frac{dx^2}{b}\right)}{b(n+1)}$$

[Out] $x*(d*x^2+b)*(d*x^3+b*x)^n*\text{hypergeom}([1, 3/2+3/2*n], [3/2+1/2*n], -d*x^2/b)/b/(1+n)$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2011, 365, 364}

$$\frac{x\left(\frac{dx^2}{b} + 1\right)^{-n} (bx + dx^3)^n \text{Hypergeometric2F1}\left(-n, \frac{n+1}{2}, \frac{n+3}{2}, -\frac{dx^2}{b}\right)}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(b*x + d*x^3)^n, x]

[Out] $(x*(b*x + d*x^3)^n*\text{Hypergeometric2F1}[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)$

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 365

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 2011

Int[((a_)*(x_)^(j_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[(a*x^j + b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rubi steps

$$\begin{aligned} \int (bx + dx^3)^n dx &= \left(x^{-n} (b + dx^2)^{-n} (bx + dx^3)^n\right) \int x^n (b + dx^2)^n dx \\ &= \left(x^{-n} \left(1 + \frac{dx^2}{b}\right)^{-n} (bx + dx^3)^n\right) \int x^n \left(1 + \frac{dx^2}{b}\right)^n dx \\ &= \frac{x \left(1 + \frac{dx^2}{b}\right)^{-n} (bx + dx^3)^n {}_2F_1\left(-n, \frac{1+n}{2}; \frac{3+n}{2}; -\frac{dx^2}{b}\right)}{1+n} \end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.15

$$\frac{x \left(x \left(b + dx^2 \right) \right)^n \left(\frac{dx^2}{b} + 1 \right)^{-n} {}_2F_1 \left(-n, \frac{n+1}{2}; \frac{n+1}{2} + 1; -\frac{dx^2}{b} \right)}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + d*x^3)^n,x]

[Out] (x*(x*(b + d*x^2))^n*Hypergeometric2F1[-n, (1 + n)/2, 1 + (1 + n)/2, -((d*x^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3 + bx\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x)^n,x, algorithm="fricas")

[Out] integral((d*x^3 + b*x)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x)^n,x, algorithm="giac")

[Out] integrate((d*x^3 + b*x)^n, x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx^3 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+b*x)^n,x)

[Out] int((d*x^3+b*x)^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+b*x)^n,x, algorithm="maxima")

[Out] integrate((d*x^3 + b*x)^n, x)

mupad [B] time = 2.22, size = 56, normalized size = 1.06

$$\frac{x \left(dx^3 + bx \right)^n {}_2F_1 \left(\frac{n}{2} + \frac{1}{2}, -n; \frac{n}{2} + \frac{3}{2}; -\frac{dx^2}{b} \right)}{\left(\frac{dx^2}{b} + 1 \right)^n (n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^n,x)
```

```
[Out] (x*(b*x + d*x^3)^n*hypergeom([n/2 + 1/2, -n], n/2 + 3/2, -(d*x^2)/b))/((d*x^2)/b + 1)^(n*(n + 1))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (bx + dx^3)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)**n,x)
```

```
[Out] Integral((b*x + d*x**3)**n, x)
```


3.31 $\int (bx + cx^2 + dx^3)^n dx$

Optimal. Leaf size=132

$$\frac{x \left(\frac{2dx}{c - \sqrt{c^2 - 4bd}} + 1 \right)^{-n} \left(\frac{2dx}{\sqrt{c^2 - 4bd} + c} + 1 \right)^{-n} (bx + cx^2 + dx^3)^n F_1 \left(n + 1; -n, -n; n + 2; -\frac{2dx}{c - \sqrt{c^2 - 4bd}}, -\frac{2dx}{c + \sqrt{c^2 - 4bd}} \right)}{n + 1}$$

[Out] $x*(d*x^3+c*x^2+b*x)^n*AppellF1(1+n, -n, -n, 2+n, -2*d*x/(c-(-4*b*d+c^2)^(1/2)), -2*d*x/(c+(-4*b*d+c^2)^(1/2)))/(1+n)/(((1+2*d*x/(c-(-4*b*d+c^2)^(1/2))))^n)/((1+2*d*x/(c+(-4*b*d+c^2)^(1/2))))^n)$

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1908, 759, 133}

$$\frac{x \left(\frac{2dx}{c - \sqrt{c^2 - 4bd}} + 1 \right)^{-n} \left(\frac{2dx}{\sqrt{c^2 - 4bd} + c} + 1 \right)^{-n} (bx + cx^2 + dx^3)^n F_1 \left(n + 1; -n, -n; n + 2; -\frac{2dx}{c - \sqrt{c^2 - 4bd}}, -\frac{2dx}{c + \sqrt{c^2 - 4bd}} \right)}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2 + d*x^3)^n, x]

[Out] $(x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c - \text{Sqrt}[c^2 - 4*b*d]), (-2*d*x)/(c + \text{Sqrt}[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)/(c - \text{Sqrt}[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + \text{Sqrt}[c^2 - 4*b*d]))^n)$

Rule 133

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*x)/c), -((f*x)/e)])/((b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 759

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (d + e*x)/(d - (e*(b - q))/(2*c))))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))^p, Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c))], x]^p*Simp[1 - x/(d - (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && !IntegerQ[p]

Rule 1908

Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q))))^p, Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x], x] /; FreeQ[{a, b, c, n, p, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (bx + cx^2 + dx^3)^n dx &= \left(x^{-n} (b + cx + dx^2)^{-n} (bx + cx^2 + dx^3)^n \right) \int x^n (b + cx + dx^2)^n dx \\ &= \left(x^{-n} \left(1 + \frac{2dx}{c - \sqrt{c^2 - 4bd}} \right)^{-n} \left(1 + \frac{2dx}{c + \sqrt{c^2 - 4bd}} \right)^{-n} (bx + cx^2 + dx^3)^n \right) \text{Subst} \left(\int x^n \left(1 + \frac{2dx}{c - \sqrt{c^2 - 4bd}} \right)^{-n} \left(1 + \frac{2dx}{c + \sqrt{c^2 - 4bd}} \right)^{-n} (bx + cx^2 + dx^3)^n F_1 \left(1 + n; -n, -n; 2 + n; -\frac{2dx}{c + \sqrt{c^2 - 4bd}}, \frac{2dx}{\sqrt{c^2 - 4bd} - c} \right) \right) \\ &= \frac{x \left(1 + \frac{2dx}{c - \sqrt{c^2 - 4bd}} \right)^{-n} \left(1 + \frac{2dx}{c + \sqrt{c^2 - 4bd}} \right)^{-n} (bx + cx^2 + dx^3)^n F_1 \left(1 + n; -n, -n; 2 + n; -\frac{2dx}{c + \sqrt{c^2 - 4bd}}, \frac{2dx}{\sqrt{c^2 - 4bd} - c} \right)}{1 + n} \end{aligned}$$

Mathematica [A] time = 0.36, size = 157, normalized size = 1.19

$$\frac{x \left(\frac{-\sqrt{c^2 - 4bd} + c + 2dx}{c - \sqrt{c^2 - 4bd}} \right)^{-n} \left(\frac{\sqrt{c^2 - 4bd} + c + 2dx}{\sqrt{c^2 - 4bd} + c} \right)^{-n} (x(b + x(c + dx)))^n F_1 \left(n + 1; -n, -n; n + 2; -\frac{2dx}{c + \sqrt{c^2 - 4bd}}, \frac{2dx}{\sqrt{c^2 - 4bd} - c} \right)}{n + 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b*x + c*x^2 + d*x^3)^n,x]

[Out] (x*(x*(b + x*(c + d*x)))^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c + Sqrt[c^2 - 4*b*d]), (2*d*x)/(-c + Sqrt[c^2 - 4*b*d])])/((1 + n)*((c - Sqrt[c^2 - 4*b*d] + 2*d*x)/(c - Sqrt[c^2 - 4*b*d]))^n*((c + Sqrt[c^2 - 4*b*d] + 2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3 + cx^2 + bx\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")

[Out] integral((d*x^3 + c*x^2 + b*x)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + cx^2 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="giac")

[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (dx^3 + cx^2 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2+b*x)^n,x)

[Out] int((d*x^3+c*x^2+b*x)^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + cx^2 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")

[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx^3 + cx^2 + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2 + d*x^3)^n,x)

[Out] int((b*x + c*x^2 + d*x^3)^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2 + dx^3)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+b*x)**n,x)

[Out] Integral((b*x + c*x**2 + d*x**3)**n, x)

3.32 $\int (a + dx^3)^n dx$

Optimal. Leaf size=35

$$\frac{x(a + dx^3)^{n+1} {}_2F_1\left(1, n + \frac{4}{3}; \frac{4}{3}; -\frac{dx^3}{a}\right)}{a}$$

[Out] $x*(d*x^3+a)^{(1+n)}*\text{hypergeom}([1, 4/3+n], [4/3], -d*x^3/a)/a$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {246, 245}

$$x(a + dx^3)^n \left(\frac{dx^3}{a} + 1\right)^{-n} {}_2F_1\left(\frac{1}{3}, -n; \frac{4}{3}; -\frac{dx^3}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + d*x^3)^n, x]

[Out] $(x*(a + d*x^3)^n*\text{Hypergeometric2F1}[1/3, -n, 4/3, -((d*x^3)/a)])/(1 + (d*x^3)/a)^n$

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int (a + dx^3)^n dx &= \left((a + dx^3)^n \left(1 + \frac{dx^3}{a}\right)^{-n} \right) \int \left(1 + \frac{dx^3}{a}\right)^{-n} dx \\ &= x(a + dx^3)^n \left(1 + \frac{dx^3}{a}\right)^{-n} {}_2F_1\left(\frac{1}{3}, -n; \frac{4}{3}; -\frac{dx^3}{a}\right) \end{aligned}$$

Mathematica [C] time = 0.20, size = 196, normalized size = 5.60

$$\frac{2^{-n} \left((-1)^{2/3} \sqrt[3]{a} + \sqrt[3]{d} x \right) \left(\frac{\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{d} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}} \right)^{-n} \left(i \left(\frac{\sqrt[3]{d} x}{\sqrt[3]{a}} + 1 \right) \right)^{-n} (a + dx^3)^n F_1 \left(n + 1; -n, -n; n + 2; -\frac{i \left(\sqrt[3]{d} x + (-1)^{2/3} \sqrt[3]{a} \right)}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt[3]{d} (n + 1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + d*x^3)^n, x]

[Out] $\left(\frac{((-1)^{2/3}a^{1/3} + d^{1/3}x)(a + dx^3)^n \text{AppellF1}[1 + n, -n, -n, 2 + n, ((-1)^{2/3}a^{1/3} + d^{1/3}x)/(\sqrt[3]{a^{1/3}}), (1 + \sqrt[3]{3}) - ((2I)d^{1/3}x/a^{1/3})/(3I + \sqrt[3]{3})]}{(2^n d^{1/3} (1 + n) ((a^{1/3} + (-1)^{2/3}d^{1/3}x)/((1 + (-1)^{1/3})a^{1/3}))^n (I(1 + (d^{1/3}x)/a^{1/3}))/ (3I + \sqrt[3]{3}))^n}\right)$

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3 + a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)^n,x, algorithm="fricas")

[Out] integral((d*x^3 + a)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)^n,x, algorithm="giac")

[Out] integrate((d*x^3 + a)^n, x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx^3 + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+a)^n,x)

[Out] int((d*x^3+a)^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)^n,x, algorithm="maxima")

[Out] integrate((d*x^3 + a)^n, x)

mupad [B] time = 2.18, size = 41, normalized size = 1.17

$$\frac{x(dx^3 + a)^n {}_2F_1\left(\frac{1}{3}, -n; \frac{4}{3}; -\frac{dx^3}{a}\right)}{\left(\frac{dx^3}{a} + 1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + d*x^3)^n,x)

[Out] $x(a + dx^3)^n \text{hypergeom}\left(\left[\frac{1}{3}, -n\right], \frac{4}{3}, -\frac{dx^3}{a}\right) / \left(\frac{dx^3}{a} + 1\right)^n$

sympy [C] time = 10.54, size = 34, normalized size = 0.97

$$\frac{a^n x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, -n \middle| \frac{dx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+a)**n,x)

[Out] a**n*x*gamma(1/3)*hyper((1/3, -n), (4/3,), d*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))

3.33 $\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^4 dx$

Optimal. Leaf size=270

$$\frac{2}{9}c^2(48a^2d^4 + 120ac^3d^2 + 35c^6)\left(\frac{c}{d} + x\right)^9 - \frac{8}{11}c^3d^2(12ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^{11} + \frac{4}{13}cd^4(4ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^{13} + \dots$$

[Out] $c^4(4ad^2+c^3)^4x/d^8-8/3c^5(4ad^2+c^3)^3(c/d+x)^3/d^6+4/5c^3(4ad^2+c^3)^2(4ad^2+7c^3)(c/d+x)^5/d^4-8/7c^4(4ad^2+c^3)(12ad^2+7c^3)(c/d+x)^7/d^2+2/9c^2(48a^2d^4+120ac^3d^2+35c^6)(c/d+x)^9-8/11c^3d^2(12ad^2+7c^3)(c/d+x)^{11}+4/13cd^4(4ad^2+7c^3)(c/d+x)^{13}-8/15c^2d^6(c/d+x)^{15}+1/17d^8(c/d+x)^{17}$

Rubi [A] time = 0.54, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1106, 1090}

$$\frac{2}{9}c^2(48a^2d^4 + 120ac^3d^2 + 35c^6)\left(\frac{c}{d} + x\right)^9 + \frac{4}{13}cd^4(4ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^{13} - \frac{8}{11}c^3d^2(12ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^{11} - \dots$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4, x]

[Out] $(c^4(c^3 + 4ad^2)^4x)/d^8 - (8c^5(c^3 + 4ad^2)^3(c/d + x)^3)/(3d^6) + (4c^3(c^3 + 4ad^2)^2(7c^3 + 4ad^2)(c/d + x)^5)/(5d^4) - (8c^4(c^3 + 4ad^2)(7c^3 + 12ad^2)(c/d + x)^7)/(7d^2) + (2c^2(35c^6 + 120ac^3d^2 + 48a^2d^4)(c/d + x)^9)/9 - (8c^3d^2(7c^3 + 12ad^2)(c/d + x)^{11})/11 + (4cd^4(7c^3 + 4ad^2)(c/d + x)^{13})/13 - (8c^2d^6(c/d + x)^{15})/15 + (d^8(c/d + x)^{17})/17$

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rule 1106

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned} \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^4 dx &= \text{Subst} \left(\int \left(c \left(4a + \frac{c^3}{d^2} \right) - 2c^2x^2 + d^2x^4 \right)^4 dx, x, \frac{c}{d} + x \right) \\ &= \text{Subst} \left(\int \left(\frac{(c^4 + 4acd^2)^4}{d^8} - \frac{8c^5(c^3 + 4ad^2)^3 x^2}{d^6} + \frac{24c^6(c^3 + 4ad^2)^2}{d^4} \right) dx, x, \frac{c}{d} + x \right) \\ &= \frac{c^4(c^3 + 4ad^2)^4 x}{d^8} - \frac{8c^5(c^3 + 4ad^2)^3 \left(\frac{c}{d} + x\right)^3}{3d^6} + \frac{4c^3(c^3 + 4ad^2)^2 (7c^3 + 4ad^2) \left(\frac{c}{d} + x\right)^5}{5d^4} \end{aligned}$$

Mathematica [A] time = 0.04, size = 285, normalized size = 1.06

$$256a^4c^4x + \frac{1024}{3}a^3c^5x^3 + 256a^3c^4dx^4 + 512a^2c^5dx^6 + \frac{256}{5}a^2c^3x^5(ad^2 + 6c^3) + \frac{32}{9}c^2x^9(3a^2d^4 + 120ac^3d^2 + 8c^6) + \frac{64}{11}$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]

[Out] 256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6*c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^7)/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3*a^2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*c^3 + 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*d^4*(70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + c*d^7*x^16 + (d^8*x^17)/17

fricas [A] time = 0.39, size = 277, normalized size = 1.03

$$\frac{1}{17}x^{17}d^8 + x^{16}d^7c + \frac{112}{15}x^{15}d^6c^2 + 32x^{14}d^5c^3 + \frac{1120}{13}x^{13}d^4c^4 + \frac{16}{13}x^{13}d^6ca + \frac{448}{3}x^{12}d^3c^5 + 16x^{12}d^5c^2a + \frac{1792}{11}x^{11}d^2c^6 + \frac{96}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")

[Out] 1/17*x^17*d^8 + x^16*d^7*c + 112/15*x^15*d^6*c^2 + 32*x^14*d^5*c^3 + 1120/13*x^13*d^4*c^4 + 16/13*x^13*d^6*c*a + 448/3*x^12*d^3*c^5 + 16*x^12*d^5*c^2*a + 1792/11*x^11*d^2*c^6 + 960/11*x^11*d^4*c^3*a + 512/5*x^10*d*c^7 + 256*x^10*d^3*c^4*a + 256/9*x^9*c^8 + 1280/3*x^9*d^2*c^5*a + 32/3*x^9*d^4*c^2*a^2 + 384*x^8*d*c^6*a + 96*x^8*d^3*c^3*a^2 + 1024/7*x^7*c^7*a + 2304/7*x^7*d^2*c^4*a^2 + 512*x^6*d*c^5*a^2 + 1536/5*x^5*c^6*a^2 + 256/5*x^5*d^2*c^3*a^3 + 256*x^4*d*c^4*a^3 + 1024/3*x^3*c^5*a^3 + 256*x*c^4*a^4

giac [A] time = 0.24, size = 277, normalized size = 1.03

$$\frac{1}{17}d^8x^{17} + cd^7x^{16} + \frac{112}{15}c^2d^6x^{15} + 32c^3d^5x^{14} + \frac{1120}{13}c^4d^4x^{13} + \frac{16}{13}acd^6x^{13} + \frac{448}{3}c^5d^3x^{12} + 16ac^2d^5x^{12} + \frac{1792}{11}c^6d^2x^{11} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")

[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1120/13*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^12 + 16*a*c^2*d^5*x^12 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x^11 + 512/5*c^7*d*x^10 + 256*a*c^4*d^3*x^10 + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4*x^9 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^2*c^4*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2*x^5 + 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x

maple [A] time = 0.00, size = 392, normalized size = 1.45

$$\frac{d^8x^{17}}{17} + cd^7x^{16} + \frac{112c^2d^6x^{15}}{15} + 32c^3d^5x^{14} + 512a^2c^5dx^6 + \frac{(1088c^4d^4 + 2(8acd^2 + 16c^4)d^4)x^{13}}{13} + 256a^3c^4dx^4 + \frac{(64a^4c^4 + 16a^3c^4d^2 + 16a^2c^4d^4)x^3}{13} + \frac{16a^4c^4d^2x^2}{13} + \frac{16a^4c^4d^4x}{13} + \frac{16a^4c^4d^6}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x)

[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1/13*(2*(8*a*c*d^2 + 16*c^4)*d^4 + 1088*c^4*d^4)*x^13 + 1/12*(64*a*c^2*d^5 + 16*(8*a*c*d^2 + 16*c^4)*d^4 + 16*c^4*d^6)*x^12 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^11 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^10 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^9 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^8 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^7 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^6 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^5 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^4 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^3 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x^2 + 1/12*(16*a*c^3*d^3 + 16*c^4*d^5)*x

) * c * d ^ 3 + 1536 * c ^ 5 * d ^ 3) * x ^ 12 + 1 / 11 * (576 * a * c ^ 3 * d ^ 4 + 48 * (8 * a * c * d ^ 2 + 16 * c ^ 4) * c ^ 2 * d ^ 2 + 1024 * c ^ 6 * d ^ 2) * x ^ 11 + 1 / 10 * (2048 * a * c ^ 4 * d ^ 3 + 64 * (8 * a * c * d ^ 2 + 16 * c ^ 4) * c ^ 3 * d) * x ^ 10 + 1 / 9 * (32 * a ^ 2 * c ^ 2 * d ^ 4 + 3584 * a * c ^ 5 * d ^ 2 + (8 * a * c * d ^ 2 + 16 * c ^ 4) ^ 2) * x ^ 9 + 1 / 8 * (256 * a ^ 2 * c ^ 3 * d ^ 3 + 2048 * a * c ^ 6 * d + 64 * a * c ^ 2 * d * (8 * a * c * d ^ 2 + 16 * c ^ 4)) * x ^ 8 + 1 / 7 * (1792 * a ^ 2 * c ^ 4 * d ^ 2 + 64 * a * c ^ 3 * (8 * a * c * d ^ 2 + 16 * c ^ 4)) * x ^ 7 + 512 * a ^ 2 * c ^ 5 * d * x ^ 6 + 1 / 5 * (32 * a ^ 2 * c ^ 2 * (8 * a * c * d ^ 2 + 16 * c ^ 4) + 1024 * a ^ 2 * c ^ 6) * x ^ 5 + 256 * a ^ 3 * c ^ 4 * d * x ^ 4 + 1024 / 3 * a ^ 3 * c ^ 5 * x ^ 3 + 256 * a ^ 4 * c ^ 4 * x

maxima [A] time = 0.63, size = 372, normalized size = 1.38

$$\frac{1}{17} d^8 x^{17} + cd^7 x^{16} + \frac{32}{5} c^2 d^6 x^{15} + \frac{128}{7} c^3 d^5 x^{14} + \frac{256}{13} c^4 d^4 x^{13} + \frac{256}{9} c^8 x^9 + 256 a^4 c^4 x + \frac{256}{15} (3 d^2 x^5 + 15 cdx^4 + 20 c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")

[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/13*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12 + 624*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 65520*c^2*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2)*a*c + 16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*c^3*d^3*x^12)*c^2

mupad [B] time = 2.30, size = 261, normalized size = 0.97

$$x^{10} \left(\frac{512 c^7 d}{5} + 256 a c^4 d^3 \right) + x^{13} \left(\frac{1120 c^4 d^4}{13} + \frac{16 a c d^6}{13} \right) + x^9 \left(\frac{32 a^2 c^2 d^4}{3} + \frac{1280 a c^5 d^2}{3} + \frac{256 c^8}{9} \right) + x^{12} \left(\frac{448}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^4,x)

[Out] x^10*((512*c^7*d)/5 + 256*a*c^4*d^3) + x^13*((1120*c^4*d^4)/13 + (16*a*c*d^6)/13) + x^9*((256*c^8)/9 + (1280*a*c^5*d^2)/3 + (32*a^2*c^2*d^4)/3) + x^12*((448*c^5*d^3)/3 + 16*a*c^2*d^5) + x^11*((1792*c^6*d^2)/11 + (960*a*c^3*d^4)/11) + (d^8*x^17)/17 + 256*a^4*c^4*x + c*d^7*x^16 + (1024*a^3*c^5*x^3)/3 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + 256*a^3*c^4*d*x^4 + 512*a^2*c^5*d*x^6 + (256*a*c^4*x^7*(9*a*d^2 + 4*c^3))/7 + (256*a^2*c^3*x^5*(a*d^2 + 6*c^3))/5 + 96*a*c^3*d*x^8*(a*d^2 + 4*c^3)

sympy [A] time = 0.13, size = 299, normalized size = 1.11

$$256 a^4 c^4 x + \frac{1024 a^3 c^5 x^3}{3} + 256 a^3 c^4 d x^4 + 512 a^2 c^5 d x^6 + 32 c^3 d^5 x^{14} + \frac{112 c^2 d^6 x^{15}}{15} + cd^7 x^{16} + \frac{d^8 x^{17}}{17} + x^{13} \left(\frac{16acd^6}{13} + \frac{1120}{13} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)

[Out] 256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*c**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 + d**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*c**2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/11) + x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 + 1280*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d) + x**7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**2/5 + 1536*a**2*c**6/5)

$$3.34 \quad \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^3 dx$$

Optimal. Leaf size=171

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + \frac{4}{3}cd^2x^9(ad^2 + 20c^3) + \frac{32}{7}c^3x^7(9ad^2 + 2c^3) + 12c^2dx^8(ad^2 + 2c^3) + \frac{48}{5}ac^2x^5(ad^2 + 2c^3)$$

[Out] 64*a^3*c^3*x+64*a^2*c^4*x^3+48*a^2*c^3*d*x^4+48/5*a*c^2*(a*d^2+4*c^3)*x^5+64*a*c^4*d*x^6+32/7*c^3*(9*a*d^2+2*c^3)*x^7+12*c^2*d*(a*d^2+2*c^3)*x^8+4/3*c*d^2*(a*d^2+20*c^3)*x^9+16*c^3*d^3*x^10+60/11*c^2*d^4*x^11+c*d^5*x^12+1/13*d^6*x^13

Rubi [A] time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2061}

$$48a^2c^3dx^4 + 64a^2c^4x^3 + 64a^3c^3x + \frac{4}{3}cd^2x^9(ad^2 + 20c^3) + 12c^2dx^8(ad^2 + 2c^3) + \frac{32}{7}c^3x^7(9ad^2 + 2c^3) + \frac{48}{5}ac^2x^5(ad^2 + 2c^3)$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]

[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^3 dx &= \int (64a^3c^3 + 192a^2c^4x^2 + 192a^2c^3dx^3 + 48ac^2(4c^3 + ad^2)x^4 + 384ac^4dx^5 \\ &\quad + 64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + \frac{48}{5}ac^2(4c^3 + ad^2)x^5 + 64ac^4dx^6 + \frac{32}{7}c^3x^7(9ad^2 + 2c^3) \\ &\quad + 12c^2dx^8(ad^2 + 2c^3) + \frac{4}{3}cd^2x^9(ad^2 + 20c^3)) dx \end{aligned}$$

Mathematica [A] time = 0.03, size = 171, normalized size = 1.00

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + \frac{4}{3}cd^2x^9(ad^2 + 20c^3) + \frac{32}{7}c^3x^7(9ad^2 + 2c^3) + 12c^2dx^8(ad^2 + 2c^3) + \frac{48}{5}ac^2x^5(ad^2 + 2c^3)$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]

[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13

fricas [A] time = 0.55, size = 166, normalized size = 0.97

$$\frac{1}{13}x^{13}d^6 + x^{12}d^5c + \frac{60}{11}x^{11}d^4c^2 + 16x^{10}d^3c^3 + \frac{80}{3}x^9d^2c^4 + \frac{4}{3}x^9d^4ca + 24x^8dc^5 + 12x^8d^3c^2a + \frac{64}{7}x^7c^6 + \frac{288}{7}x^7d^2c^3a + 64x^6d^5c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="fricas")

[Out] 1/13*x^13*d^6 + x^12*d^5*c + 60/11*x^11*d^4*c^2 + 16*x^10*d^3*c^3 + 80/3*x^9*d^2*c^4 + 4/3*x^9*d^4*c*a + 24*x^8*d*c^5 + 12*x^8*d^3*c^2*a + 64/7*x^7*c^6 + 288/7*x^7*d^2*c^3*a + 64*x^6*d*c^4*a + 192/5*x^5*c^5*a + 48/5*x^5*d^2*c^2*a^2 + 48*x^4*d*c^3*a^2 + 64*x^3*c^4*a^2 + 64*x*c^3*a^3

giac [A] time = 0.24, size = 166, normalized size = 0.97

$$\frac{1}{13} d^6 x^{13} + c d^5 x^{12} + \frac{60}{11} c^2 d^4 x^{11} + 16 c^3 d^3 x^{10} + \frac{80}{3} c^4 d^2 x^9 + \frac{4}{3} a c d^4 x^9 + 24 c^5 d x^8 + 12 a c^2 d^3 x^8 + \frac{64}{7} c^6 x^7 + \frac{288}{7} a c^3 d^2 x^7 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="giac")

[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 80/3*c^4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*x^7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^2*d^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x

maple [A] time = 0.00, size = 231, normalized size = 1.35

$$\frac{d^6 x^{13}}{13} + c d^5 x^{12} + \frac{60 c^2 d^4 x^{11}}{11} + 16 c^3 d^3 x^{10} + 64 a c^4 d x^6 + 48 a^2 c^3 d x^4 + 64 a^2 c^4 x^3 + \frac{(4 a c d^4 + 224 c^4 d^2 + (8 a c d^2 + 16 c^4 a^2)) x^9 + 1/8 * (64 a^3 c^2 d^3 + 128 c^5 d + 4 c^6 d^2) x^8 + 1/7 * (256 a^3 c^3 d^2 + 4 c^2 (8 a^3 c d^2 + 16 c^4 a^2)) x^7 + 64 a^4 c^4 d x^6 + 1/5 * (4 a^3 c^2 (8 a^3 c d^2 + 16 c^4 a^2) + 128 c^5 a + 16 d^2 a^2 c^2) x^5 + 48 a^2 c^3 d x^4 + 64 a^2 c^4 x^3 + 64 a^3 c^3 x}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x)

[Out] 1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+1/9*(4*a*c*d^4+224*c^4*d^2+d^2*(8*a*c*d^2+16*c^4))*x^9+1/8*(64*a*c^2*d^3+128*c^5*d+4*c*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(256*a*c^3*d^2+4*c^2*(8*a*c*d^2+16*c^4))*x^7+64*a*c^4*d*x^6+1/5*(4*a*c*(8*a*c*d^2+16*c^4)+128*c^5*a+16*d^2*a^2*c^2)*x^5+48*a^2*c^3*d*x^4+64*a^2*c^4*x^3+64*a^3*c^3*x

maxima [A] time = 0.80, size = 205, normalized size = 1.20

$$\frac{1}{13} d^6 x^{13} + c d^5 x^{12} + \frac{48}{11} c^2 d^4 x^{11} + \frac{32}{5} c^3 d^3 x^{10} + \frac{64}{7} c^6 x^7 + 64 a^3 c^3 x + \frac{16}{5} (3 d^2 x^5 + 15 c d x^4 + 20 c^2 x^3) a^2 c^2 + \frac{8}{3} (2 d^2 x^9 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")

[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7*c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c^2 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c + 4/165*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2

mupad [B] time = 2.16, size = 160, normalized size = 0.94

$$x^8 (24 c^5 d + 12 a c^2 d^3) + x^9 \left(\frac{80 c^4 d^2}{3} + \frac{4 a c d^4}{3} \right) + \frac{d^6 x^{13}}{13} + x^7 \left(\frac{64 c^6}{7} + \frac{288 a c^3 d^2}{7} \right) + 64 a^3 c^3 x + c d^5 x^{12} + 64 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^3,x)

[Out] x^8*(24*c^5*d + 12*a*c^2*d^3) + x^9*((80*c^4*d^2)/3 + (4*a*c*d^4)/3) + (d^6*x^13)/13 + x^7*((64*c^6)/7 + (288*a*c^3*d^2)/7) + 64*a^3*c^3*x + c*d^5*x^12

$$2 + 64a^2c^4x^3 + 16c^3d^3x^{10} + (60c^2d^4x^{11})/11 + 48a^2c^3dx^4 + (48ac^2x^5(ad^2 + 4c^3))/5 + 64ac^4d^2x^6$$

sympy [A] time = 0.10, size = 180, normalized size = 1.05

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4d^2x^6 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + cd^5x^{12} + \frac{d^6x^{13}}{13} + x^9 \left(\frac{4acd^4}{3} + \frac{80c^4d^2}{3} \right) + x^8 (12a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)

[Out] 64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**6 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13/13 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**5*d) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 + 192*a*c**5/5)

3.35 $\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2 dx$

Optimal. Leaf size=92

$$16a^2c^2x + \frac{8}{5}cx^5(ad^2 + 2c^3) + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + cd^3x^8 + \frac{d^4x^9}{9}$$

[Out] 16*a^2*c^2*x+32/3*a*c^3*x^3+8*a*c^2*d*x^4+8/5*c*(a*d^2+2*c^3)*x^5+16/3*c^3*d*x^6+24/7*c^2*d^2*x^7+c*d^3*x^8+1/9*d^4*x^9

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2061}

$$16a^2c^2x + \frac{8}{5}cx^5(ad^2 + 2c^3) + 8ac^2dx^4 + \frac{32}{3}ac^3x^3 + \frac{24}{7}c^2d^2x^7 + \frac{16}{3}c^3dx^6 + cd^3x^8 + \frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]

[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2 dx &= \int (16a^2c^2 + 32ac^3x^2 + 32ac^2dx^3 + 8c(2c^3 + ad^2)x^4 + 32c^3dx^5 + 24cd^3x^6 + 16c^2d^2x^7 + d^4x^8) dx \\ &= 16a^2c^2x + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{8}{5}c(2c^3 + ad^2)x^5 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + \frac{d^4x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.02, size = 92, normalized size = 1.00

$$16a^2c^2x + \frac{8}{5}cx^5(ad^2 + 2c^3) + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + cd^3x^8 + \frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]

[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9

fricas [A] time = 0.40, size = 83, normalized size = 0.90

$$\frac{1}{9}x^9d^4 + x^8d^3c + \frac{24}{7}x^7d^2c^2 + \frac{16}{3}x^6dc^3 + \frac{16}{5}x^5c^4 + \frac{8}{5}x^5d^2ca + 8x^4dc^2a + \frac{32}{3}x^3c^3a + 16xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")

[Out] 1/9*x^9*d^4 + x^8*d^3*c + 24/7*x^7*d^2*c^2 + 16/3*x^6*d*c^3 + 16/5*x^5*c^4 + 8/5*x^5*d^2*c*a + 8*x^4*d*c^2*a + 32/3*x^3*c^3*a + 16*x*c^2*a^2

giac [A] time = 0.25, size = 83, normalized size = 0.90

$$\frac{1}{9}d^4x^9 + cd^3x^8 + \frac{24}{7}c^2d^2x^7 + \frac{16}{3}c^3dx^6 + \frac{16}{5}c^4x^5 + \frac{8}{5}acd^2x^5 + 8ac^2dx^4 + \frac{32}{3}ac^3x^3 + 16a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")

[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5 + 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x

maple [A] time = 0.00, size = 84, normalized size = 0.91

$$\frac{d^4x^9}{9} + cd^3x^8 + \frac{24c^2d^2x^7}{7} + \frac{16c^3dx^6}{3} + 8ac^2dx^4 + \frac{32ac^3x^3}{3} + 16a^2c^2x + \frac{(8acd^2 + 16c^4)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)

[Out] 1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+1/5*(8*a*c*d^2+16*c^4)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x

maxima [A] time = 0.57, size = 94, normalized size = 1.02

$$\frac{1}{9}d^4x^9+cd^3x^8+\frac{16}{7}c^2d^2x^7+\frac{16}{5}c^4x^5+16a^2c^2x+\frac{8}{15}(3d^2x^5+15cdx^4+20c^2x^3)ac+\frac{8}{21}(3d^2x^7+14cdx^6)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")

[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x + 8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d*x^6)*c^2

mapad [B] time = 0.04, size = 82, normalized size = 0.89

$$x^5 \left(\frac{16c^4}{5} + \frac{8ac^2d^2}{5} \right) + \frac{d^4x^9}{9} + 16a^2c^2x + \frac{32ac^3x^3}{3} + \frac{16c^3dx^6}{3} + cd^3x^8 + \frac{24c^2d^2x^7}{7} + 8ac^2dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)

[Out] x^5*((16*c^4)/5 + (8*a*c*d^2)/5) + (d^4*x^9)/9 + 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + (16*c^3*d*x^6)/3 + c*d^3*x^8 + (24*c^2*d^2*x^7)/7 + 8*a*c^2*d*x^4

sympy [A] time = 0.08, size = 95, normalized size = 1.03

$$16a^2c^2x + \frac{32ac^3x^3}{3} + 8ac^2dx^4 + \frac{16c^3dx^6}{3} + \frac{24c^2d^2x^7}{7} + cd^3x^8 + \frac{d^4x^9}{9} + x^5 \left(\frac{8acd^2}{5} + \frac{16c^4}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)

[Out] 16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 24*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c**4/5)

3.36 $\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4) dx$

Optimal. Leaf size=32

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]

[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5

Rubi steps

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4) dx = 4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]

[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5

fricas [A] time = 0.39, size = 28, normalized size = 0.88

$$\frac{1}{5}x^5d^2 + x^4dc + \frac{4}{3}x^3c^2 + 4xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")

[Out] 1/5*x^5*d^2 + x^4*d*c + 4/3*x^3*c^2 + 4*x*c*a

giac [A] time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{5}d^2x^5 + cdx^4 + \frac{4}{3}c^2x^3 + 4acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")

[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{1}{5}d^2x^5 + cdx^4 + \frac{4}{3}c^2x^3 + 4acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x)`

[Out] `4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5`

maxima [A] time = 0.65, size = 28, normalized size = 0.88

$$\frac{1}{5}d^2x^5 + cdx^4 + \frac{4}{3}c^2x^3 + 4acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")`

[Out] `1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x`

mupad [B] time = 0.04, size = 28, normalized size = 0.88

$$\frac{4c^2x^3}{3} + cdx^4 + 4acx + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3,x)`

[Out] `(4*c^2*x^3)/3 + (d^2*x^5)/5 + 4*a*c*x + c*d*x^4`

sympy [A] time = 0.07, size = 31, normalized size = 0.97

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)`

[Out] `4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5`

$$3.37 \quad \int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4} dx$$

Optimal. Leaf size=529

$$\frac{d \log \left(\sqrt{c} \sqrt{4ad^2 + c^3} - \sqrt{2} \sqrt[4]{c} d \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} \left(\frac{c}{d} + x \right) + d^2 \left(\frac{c}{d} + x \right)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}}} + \frac{d \log \left(\sqrt{c} \sqrt{4ad^2 + c^3} + \sqrt{2} \sqrt[4]{c} d \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} \left(\frac{c}{d} + x \right) + d^2 \left(\frac{c}{d} + x \right)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}}}$$

```
[Out] -1/4*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/4*d*arctanh((-d*x+c)*2^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)-1/8*d*ln(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)-c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/8*d*ln(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1106, 1094, 634, 618, 206, 628}

$$\frac{d \log \left(-\sqrt{2} \sqrt[4]{c} d \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} \left(\frac{c}{d} + x \right) + \sqrt{c} \sqrt{4ad^2 + c^3} + d^2 \left(\frac{c}{d} + x \right)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}}} + \frac{d \log \left(\sqrt{2} \sqrt[4]{c} d \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} \left(\frac{c}{d} + x \right) + \sqrt{c} \sqrt{4ad^2 + c^3} + d^2 \left(\frac{c}{d} + x \right)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1), x]
```

```
[Out] -(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])]/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]] + (d*ArcTanh[(c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])]/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]] - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2)/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2)/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} dx = \text{Subst} \left(\int \frac{1}{c \left(4a + \frac{c^3}{d^2} \right) - 2c^2x^2 + d^2x^4} dx, x, \frac{c}{d} + x \right)$$

$$= \frac{d \text{Subst} \left(\int \frac{\frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}{d} - x}{\frac{\sqrt{c} \sqrt{c^3 + 4ad^2}}{d^2} - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}{d} x + x^2} dx, x, \frac{c}{d} + x \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}} + \frac{d \text{Subst} \left(\int \frac{1}{\frac{\sqrt{c} \sqrt{c^3 + 4ad^2}}{d^2} - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}{d} x + x^2} dx, x, \frac{c}{d} + x \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2}}$$

$$= \frac{\text{Subst} \left(\int \frac{1}{\frac{\sqrt{c} \sqrt{c^3 + 4ad^2}}{d^2} - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}{d} x + x^2} dx, x, \frac{c}{d} + x \right)}{4\sqrt{c} \sqrt{c^3 + 4ad^2}} + \frac{\text{Subst} \left(\int \frac{1}{\frac{\sqrt{c} \sqrt{c^3 + 4ad^2}}{d^2} - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}{d} x + x^2} dx, x, \frac{c}{d} + x \right)}{4\sqrt{c} \sqrt{c^3 + 4ad^2}}$$

$$= -\frac{d \log \left(\sqrt{c} \sqrt{c^3 + 4ad^2} - \sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}} (c + dx) + (c + dx)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}$$

$$= -\frac{d \tanh^{-1} \left(\frac{\sqrt[4]{c} \left(\sqrt{2} c^{3/4} - \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}} \right) + \sqrt{2} dx}{\sqrt[4]{c} \sqrt{c^{3/2} - \sqrt{c^3 + 4ad^2}}} \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2} \sqrt{c^{3/2} - \sqrt{c^3 + 4ad^2}}} - \frac{d \tanh^{-1} \left(\frac{\sqrt[4]{c} \left(\sqrt{2} c^{3/4} + \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}} \right)}{\sqrt[4]{c} \sqrt{c^{3/2} - \sqrt{c^3 + 4ad^2}}} \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2} \sqrt{c^{3/2} - \sqrt{c^3 + 4ad^2}}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.13

$$\frac{1}{4} \text{RootSum} \left[\#1^4 d^2 + 4\#1^3 cd + 4\#1^2 c^2 + 4ac \&, \frac{\log(x - \#1)}{\#1^3 d^2 + 3\#1^2 cd + 2\#1 c^2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]

[Out] RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*#1 + 3*c*d*#1^2 + d^2*#1^3) &]/4

fricas [B] time = 0.49, size = 905, normalized size = 1.71

$$\frac{1}{8} \sqrt{-\frac{2(ac^3 + 4a^2d^2)\sqrt{-\frac{d^2}{ac^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + 1}{ac^3 + 4a^2d^2}} \log \left(d^2x + cd + \left(2acd^2 + (ac^7 + 4a^2c^4d^2)\sqrt{-\frac{d^2}{ac^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")

[Out] 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) + 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d - (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2)))

giac [A] time = 0.34, size = 603, normalized size = 1.14

$$\frac{\log \left(x + \sqrt{\frac{c^2 d^2 + 2 \sqrt{-ac} d^3}{d^4}} + \frac{c}{d} \right)}{4 \left(d^2 \left(\sqrt{\frac{c^2 d^2 + 2 \sqrt{-ac} d^3}{d^4}} + \frac{c}{d} \right)^3 - 3cd \left(\sqrt{\frac{c^2 d^2 + 2 \sqrt{-ac} d^3}{d^4}} + \frac{c}{d} \right)^2 + 2c^2 \left(\sqrt{\frac{c^2 d^2 + 2 \sqrt{-ac} d^3}{d^4}} + \frac{c}{d} \right) \right)} + \frac{\log \left(x - \sqrt{\frac{c^2 d^2 + 2 \sqrt{-ac} d^3}{d^4}} + \frac{c}{d} \right)}{4 \left(d^2 \left(\sqrt{\frac{c^2 d^2 + 2 \sqrt{-ac} d^3}{d^4}} + \frac{c}{d} \right)^3 - 3cd \left(\sqrt{\frac{c^2 d^2 + 2 \sqrt{-ac} d^3}{d^4}} + \frac{c}{d} \right)^2 + 2c^2 \left(\sqrt{\frac{c^2 d^2 + 2 \sqrt{-ac} d^3}{d^4}} + \frac{c}{d} \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")

[Out] -1/4*log(x + sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) + 1/4*log(x - sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) - 1/4*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)) + 1/4*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d))

$$\begin{aligned} & *c^3)^{(1/2)} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))^{(1/2)} + 4*c*d^5 + 4*d^6*x)*1i)/(((2*d*(-a^3*c^3)^{(1/2)} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))^{(1/2)})*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^{(1/2)} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))^{(1/2)} - 64*a*c*d^6)*((2*d*(-a^3*c^3)^{(1/2)} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))^{(1/2)} + 4*c*d^5 + 4*d^6*x) - ((2*d*(-a^3*c^3)^{(1/2)} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))^{(1/2)})*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^{(1/2)} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))^{(1/2)} + 64*a*c*d^6)*((2*d*(-a^3*c^3)^{(1/2)} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))^{(1/2)} + 4*c*d^5 + 4*d^6*x)))*((2*d*(-a^3*c^3)^{(1/2)} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))^{(1/2)})*2i \end{aligned}$$

sympy [A] time = 1.15, size = 88, normalized size = 0.17

$$\text{RootSum}\left(t^4(16384a^3c^3d^2 + 4096a^2c^6) + 128t^2ac^3 + 1, \left(t \mapsto t \log\left(x + \frac{-1024t^3a^2c^4d^2 - 256t^3ac^7 + 16tacd^2}{d^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c), x)

[Out] RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 + 1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 + 16*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))

$$3.38 \quad \int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2} dx$$

Optimal. Leaf size=746

$$\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

[Out] $-1/16*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)-1/64*d*\operatorname{arctanh}((c*2^{(1/2)}+d*x*2^{(1/2)}+c^{(1/4)}*(c^{(3/2)}+(4*a*d^2+c^3)^{(1/2)})^{(1/2)})/c^{(1/4)})/(c^{(3/2)}-(4*a*d^2+c^3)^{(1/2)})^{(1/2)})*(c^3+12*a*d^2+c^{(3/2)}*(4*a*d^2+c^3)^{(1/2)})/a/c^{(7/4)}/(4*a*d^2+c^3)^{(3/2)}*2^{(1/2)}/(c^{(3/2)}-(4*a*d^2+c^3)^{(1/2)})^{(1/2)}+1/64*d*\operatorname{arctanh}((-d*x+c)*2^{(1/2)}+c^{(1/4)}*(c^{(3/2)}+(4*a*d^2+c^3)^{(1/2)})^{(1/2)})/c^{(1/4)})/(c^{(3/2)}-(4*a*d^2+c^3)^{(1/2)})^{(1/2)})*(c^3+12*a*d^2+c^{(3/2)}*(4*a*d^2+c^3)^{(1/2)})/a/c^{(7/4)}/(4*a*d^2+c^3)^{(3/2)}*2^{(1/2)}/(c^{(3/2)}-(4*a*d^2+c^3)^{(1/2)})^{(1/2)}-1/128*d*\ln(d^2*(c/d+x)^2+c^{(1/2)}*(4*a*d^2+c^3)^{(1/2)}-c^{(1/4)}*d*(c/d+x)*2^{(1/2)}*(c^{(3/2)}+(4*a*d^2+c^3)^{(1/2)})^{(1/2)})*(c^3+12*a*d^2-c^{(3/2)}*(4*a*d^2+c^3)^{(1/2)})/a/c^{(7/4)}/(4*a*d^2+c^3)^{(3/2)}*2^{(1/2)}/(c^{(3/2)}+(4*a*d^2+c^3)^{(1/2)})^{(1/2)}+1/128*d*\ln(d^2*(c/d+x)^2+c^{(1/2)}*(4*a*d^2+c^3)^{(1/2)}+c^{(1/4)}*d*(c/d+x)*2^{(1/2)}*(c^{(3/2)}+(4*a*d^2+c^3)^{(1/2)})^{(1/2)})*(c^3+12*a*d^2-c^{(3/2)}*(4*a*d^2+c^3)^{(1/2)})/a/c^{(7/4)}/(4*a*d^2+c^3)^{(3/2)}*2^{(1/2)}/(c^{(3/2)}+(4*a*d^2+c^3)^{(1/2)})^{(1/2)}$

Rubi [A] time = 1.33, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1106, 1092, 1169, 634, 618, 206, 628}

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} \frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2), x]

[Out] $-((c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2))/(16*a*c*(c^3+4*a*d^2)*(4*a*c+4*c^2*x^2+4*c*d*x^3+d^2*x^4))- (d*(c^3+12*a*d^2+c^{(3/2)}*\operatorname{Sqrt}[c^3+4*a*d^2])* \operatorname{ArcTanh}[(\operatorname{Sqrt}[2]*c+c^{(1/4)}*\operatorname{Sqrt}[c^{(3/2)}+\operatorname{Sqrt}[c^3+4*a*d^2]])+\operatorname{Sqrt}[2]*d*x]/(c^{(1/4)}*\operatorname{Sqrt}[c^{(3/2)}-\operatorname{Sqrt}[c^3+4*a*d^2]])]/(32*\operatorname{Sqrt}[2]*a*c^{(7/4)}*(c^3+4*a*d^2)^{(3/2)}*\operatorname{Sqrt}[c^{(3/2)}-\operatorname{Sqrt}[c^3+4*a*d^2]])+(d*(c^3+12*a*d^2+c^{(3/2)}*\operatorname{Sqrt}[c^3+4*a*d^2])* \operatorname{ArcTanh}[(c^{(1/4)}*\operatorname{Sqrt}[c^{(3/2)}+\operatorname{Sqrt}[c^3+4*a*d^2]])-\operatorname{Sqrt}[2]*(c+d*x)]/(c^{(1/4)}*\operatorname{Sqrt}[c^{(3/2)}-\operatorname{Sqrt}[c^3+4*a*d^2]])]/(32*\operatorname{Sqrt}[2]*a*c^{(7/4)}*(c^3+4*a*d^2)^{(3/2)}*\operatorname{Sqrt}[c^{(3/2)}-\operatorname{Sqrt}[c^3+4*a*d^2]])-(d*(c^3+12*a*d^2-c^{(3/2)}*\operatorname{Sqrt}[c^3+4*a*d^2])* \operatorname{Log}[\operatorname{Sqrt}[c]*\operatorname{Sqrt}[c^3+4*a*d^2]-\operatorname{Sqrt}[2]*c^{(1/4)}*d*\operatorname{Sqrt}[c^{(3/2)}+\operatorname{Sqrt}[c^3+4*a*d^2]]*(c/d+x)+d^2*(c/d+x)^2])/(64*\operatorname{Sqrt}[2]*a*c^{(7/4)}*(c^3+4*a*d^2)^{(3/2)}*\operatorname{Sqrt}[c^{(3/2)}+\operatorname{Sqrt}[c^3+4*a*d^2]])+(d*(c^3+12*a*d^2-c^{(3/2)}*\operatorname{Sqrt}[c^3+4*a*d^2])* \operatorname{Log}[\operatorname{Sqrt}[c]*\operatorname{Sqrt}[c^3+4*a*d^2]+\operatorname{Sqrt}[2]*c^{(1/4)}*d*\operatorname{Sqrt}[c^{(3/2)}+\operatorname{Sqrt}[c^3+4*a*d^2]]*(c/d+x)+d^2*(c/d+x)^2])/(64*\operatorname{Sqrt}[2]*a*c^{(7/4)}*(c^3+4*a*d^2)^{(3/2)}*\operatorname{Sqrt}[c^{(3/2)}+\operatorname{Sqrt}[c^3+4*a*d^2]])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt

$Q[a, 0] \parallel \text{LtQ}[b, 0]$)

Rule 618

$\text{Int}[(a_.) + (b_.)(x_) + (c_.)(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)(x_)] / [(a_.) + (b_.)(x_) + (c_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]) / b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2cd - be, 0]$

Rule 634

$\text{Int}[(d_.) + (e_.)(x_)] / [(a_.) + (b_.)(x_) + (c_.)(x_)^2], x_Symbol] \rightarrow \text{Dist}[(2cd - be) / (2c), \text{Int}[1/(a + bx + cx^2), x], x] + \text{Dist}[e / (2c), \text{Int}[(b + 2cx) / (a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 1092

$\text{Int}[(a_.) + (b_.)(x_)^2 + (c_.)(x_)^4]^{(p_.)}, x_Symbol] \rightarrow -\text{Simp}[(x(b^2 - 2ac + b^2cx^2)(a + bx^2 + cx^4)^{(p+1)}) / (2a(p+1)(b^2 - 4ac)), x] + \text{Dist}[1 / (2a(p+1)(b^2 - 4ac)), \text{Int}[(b^2 - 2ac + 2(p+1)(b^2 - 4ac) + b^2c(4p+7)x^2)(a + bx^2 + cx^4)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntegerQ}[2p]$

Rule 1106

$\text{Int}[(P4_)^{(p_.)}, x_Symbol] \rightarrow \text{With}\{a = \text{Coeff}[P4, x, 0], b = \text{Coeff}[P4, x, 1], c = \text{Coeff}[P4, x, 2], d = \text{Coeff}[P4, x, 3], e = \text{Coeff}[P4, x, 4]\}, \text{Subst}[\text{Int}[\text{SimplifyIntegrand}[(a + d^4 / (256e^3) - (bd) / (8e) + (c - (3d^2) / (8e))x^2 + ex^4)^p, x], x], x, d / (4e) + x] /; \text{EqQ}[d^3 - 4cde + 8be^2, 0] \&\& \text{NeQ}[d, 0] /; \text{FreeQ}[p, x] \&\& \text{PolyQ}[P4, x, 4] \&\& \text{NeQ}[p, 2] \&\& \text{NeQ}[p, 3]$

Rule 1169

$\text{Int}[(d_.) + (e_.)(x_)^2] / [(a_.) + (b_.)(x_)^2 + (c_.)(x_)^4], x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a/c, 2]\}, \text{With}\{r = \text{Rt}[2q - b/c, 2]\}, \text{Dist}[1 / (2cq^2r), \text{Int}[(d^2r - (d - eq)x) / (q - rx + x^2), x], x] + \text{Dist}[1 / (2cq^2r), \text{Int}[(d^2r + (d - eq)x) / (q + rx + x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[cd^2 - bde + ae^2, 0] \&\& \text{NegQ}[b^2 - 4ac]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2} dx &= \text{Subst} \left(\int \frac{1}{\left(c \left(4a + \frac{c^3}{d^2}\right) - 2c^2x^2 + d^2x^4\right)^2} dx, x, \frac{c}{d} + x \right) \\
&= -\frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} + \frac{\text{Subst} \left(\int \frac{4c^4 - 2c \left(4a + \frac{c^3}{d^2}\right)}{c} \right)}{c} \\
&= -\frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} + \frac{d \text{Subst} \left(\int \frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \dots}}{\dots} \right)}{\dots} \\
&= -\frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} - \frac{d \left(c^3 + 12ad^2 - c^{3/2} \sqrt{\dots}\right)}{\dots} \\
&= -\frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} - \frac{d \left(c^3 + 12ad^2 - c^{3/2} \sqrt{\dots}\right)}{\dots} \\
&= -\frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} - \frac{d \left(c^3 + 12ad^2 + c^{3/2} \sqrt{\dots}\right)}{32\sqrt{2} ac}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 182, normalized size = 0.24

$$\frac{\text{RootSum} \left[\#1^4 d^2 + 4\#1^3 cd + 4\#1^2 c^2 + 4ac \&, \frac{\#1^2 cd^2 \log(x-\#1) + 12ad^2 \log(x-\#1) + 2c^3 \log(x-\#1) + 2\#1 c^2 d \log(x-\#1)}{\#1^3 d^2 + 3\#1^2 cd + 2\#1 c^2} \& \right] + \frac{4(c+dx)(4c^3 + 12ad^2 - c^{3/2} \sqrt{\dots})}{4ac + \dots}}{64ac (4ad^2 + c^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2), x]

[Out] ((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 &, (2*c^3*Log[x - #1] + 12*a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 + c*d^2*Log[x - #1]*#1^2)/(2*c^2*#1 + 3*c*d*#1^2 + d^2*#1^3) &])/(64*a*c*(c^3 + 4*a*d^2))

fricas [B] time = 0.55, size = 3222, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")

[Out] 1/64*(4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")

[Out] 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*c^2*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/(a*c^4 + 4*a^2*c*d^2)

mupad [B] time = 4.30, size = 5844, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)

[Out] (d/(4*(4*a*d^2 + c^3)) + (d^2*x^3)/(16*a*(4*a*d^2 + c^3)) + (x*(2*a*d^2 + c^3))/(8*a*c*(4*a*d^2 + c^3)) + (3*c*d*x^2)/(16*a*(4*a*d^2 + c^3)))/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3) - atan((((-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*(((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10)))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))*i + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*(((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10)))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))*i)/((9*a*d^8 + c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*(((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10)))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15

$$\begin{aligned}
& *a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 \\
& + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} - (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)) + (-a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} * (((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))) * (-a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} * 2i - \operatorname{atan}(\frac{(-a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})}{(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)}} * (((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))) * (-a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} - (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i + (-a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} * (((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))) * (-a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i)/((9*a*d^8 + c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^{(1/2)} * (((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 419
\end{aligned}$$

$$4304a^6c^6d^9)/(1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(4096a^3c^{11}d^6 + 32768a^4c^8d^8 + 65536a^5c^5d^{10}))/((16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4))) * (-a^3c^{11} - 10c^3d^3(-a^9c^7)^{(1/2)} + 15a^4c^8d^2 + 60a^5c^5d^4 - 72a^6c^2d^6)/((4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6)))^{(1/2)} - (4096a^3c^8d^6 + 65536a^4c^5d^8 + 196608a^5c^2d^{10}))/((1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4))) * (-a^3c^{11} - 10c^3d^3(-a^9c^7)^{(1/2)} + 15a^4c^8d^2 + 60a^5c^5d^4 - 72a^6c^2d^6)/((4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6)))^{(1/2)} + (64a^6c^7d^5 + 2304a^3c^4d^9 + 704a^2c^4d^7))/((1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(36a^2d^{10} + c^6d^6 + 11a^3c^3d^8)))/(16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4))) + (-a^3c^{11} - 10c^3d^3(-a^9c^7)^{(1/2)} + 15a^4c^8d^2 + 60a^5c^5d^4 - 72a^6c^2d^6)/((4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6)))^{(1/2)} * (((262144a^4c^{12}d^5 + 2097152a^5c^9d^7 + 4194304a^6c^6d^9))/((1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(4096a^3c^{11}d^6 + 32768a^4c^8d^8 + 65536a^5c^5d^{10}))/((16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4)))) * (-a^3c^{11} - 10c^3d^3(-a^9c^7)^{(1/2)} + 15a^4c^8d^2 + 60a^5c^5d^4 - 72a^6c^2d^6)/((4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6)))^{(1/2)} + (4096a^3c^8d^6 + 65536a^4c^5d^8 + 196608a^5c^2d^{10}))/((1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4))) * (-a^3c^{11} - 10c^3d^3(-a^9c^7)^{(1/2)} + 15a^4c^8d^2 + 60a^5c^5d^4 - 72a^6c^2d^6)/((4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6)))^{(1/2)} + (64a^6c^7d^5 + 2304a^3c^4d^9 + 704a^2c^4d^7))/((1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(36a^2d^{10} + c^6d^6 + 11a^3c^3d^8)))/(16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4)))) * (-a^3c^{11} - 10c^3d^3(-a^9c^7)^{(1/2)} + 15a^4c^8d^2 + 60a^5c^5d^4 - 72a^6c^2d^6)/((4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6)))^{(1/2)} * 2i$$

sympy [A] time = 109.97, size = 427, normalized size = 0.57

$$\frac{4acd + 3c^2dx^2 + cd^2x^3 + x(4ad^2 + 2c^3)}{256a^3c^2d^2 + 64a^2c^5 + x^4(64a^2cd^4 + 16ac^4d^2) + x^3(256a^2c^2d^3 + 64ac^5d) + x^2(256a^2c^3d^2 + 64ac^6) + \text{RootSum}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)

[Out] (4*a*c*d + 3*c**2*d*x**2 + c*d**2*x**3 + x*(4*a*d**2 + 2*c**3))/(256*a**3*c**2*d**2 + 64*a**2*c**5 + x**4*(64*a**2*c*d**4 + 16*a*c**4*d**2) + x**3*(256*a**2*c**2*d**3 + 64*a*c**5*d) + x**2*(256*a**2*c**3*d**2 + 64*a*c**6)) + RootSum(_t**4*(1073741824*a**9*c**7*d**6 + 805306368*a**8*c**10*d**4 + 201326592*a**7*c**13*d**2 + 16777216*a**6*c**16) + _t**2*(491520*a**5*c**5*d**4 + 122880*a**4*c**8*d**2 + 8192*a**3*c**11) + 81*a**2*d**4 + 18*a*c**3*d**2 + c**6, Lambda(_t, _t*log(x + (-67108864*_t**3*a**7*c**7*d**8 - 58720256*_t**3*a**6*c**10*d**6 - 18874368*_t**3*a**5*c**13*d**4 - 2621440*_t**3*a**4*c**16*d**2 - 131072*_t**3*a**3*c**19 + 27648*_t*a**4*c**2*d**8 - 9216*_t*a**3*c**5*d**6 - 5440*_t*a**2*c**8*d**4 - 736*_t*a*c**11*d**2 - 32*_t*c**14 + 324*a**2*c*d**7 + 81*a*c**4*d**5 + 5*c**7*d**3)/(324*a**2*d**8 + 81*a*c**3*d**6 + 5*c**6*d**4))))

$$3.39 \quad \int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4 \right)^4 dx$$

Optimal. Leaf size=295

$$\frac{1}{24}e^4(65536a^2e^6 + 20992ad^4e^3 + 601d^8)\left(\frac{d}{4e} + x\right)^9 + \frac{(256ae^3 + 5d^4)^2(256ae^3 + 59d^4)\left(\frac{d}{4e} + x\right)^5}{5120} + \frac{64}{13}e^8(256ae^3 +$$

[Out] 1/1048576*(256*a*e^3+5*d^4)^4*x/e^4-1/8192*d^2*(256*a*e^3+5*d^4)^3*(1/4*d/e+x)^3/e^2+1/5120*(256*a*e^3+5*d^4)^2*(256*a*e^3+59*d^4)*(1/4*d/e+x)^5-9/224*d^2*e^2*(256*a*e^3+5*d^4)*(256*a*e^3+17*d^4)*(1/4*d/e+x)^7+1/24*e^4*(65536*a^2*e^6+20992*a*d^4*e^3+601*d^8)*(1/4*d/e+x)^9-72/11*d^2*e^6*(256*a*e^3+17*d^4)*(1/4*d/e+x)^11+64/13*e^8*(256*a*e^3+59*d^4)*(1/4*d/e+x)^13-2048/5*d^2*e^10*(1/4*d/e+x)^15+4096/17*e^12*(1/4*d/e+x)^17

Rubi [A] time = 0.53, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1106, 1090}

$$\frac{1}{24}e^4(65536a^2e^6 + 20992ad^4e^3 + 601d^8)\left(\frac{d}{4e} + x\right)^9 + \frac{64}{13}e^8(256ae^3 + 59d^4)\left(\frac{d}{4e} + x\right)^{13} - \frac{72}{11}d^2e^6(256ae^3 + 17d^4)\left(\frac{d}{4e} + x\right)^{11} + \frac{2048}{5}d^2e^{10}\left(\frac{d}{4e} + x\right)^{15} - \frac{4096}{17}e^{12}\left(\frac{d}{4e} + x\right)^{17}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4, x]

[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3)^3*(d/(4*e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e) + x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rule 1106

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned} \int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^4 dx &= \text{Subst} \left(\int \left(\frac{1}{32} \left(\frac{5d^4}{e} + 256ae^2 \right) - 3d^2ex^2 + 8e^3x^4 \right)^4 dx, x, \frac{d}{4e} + x \right) \\ &= \text{Subst} \left(\int \left(\frac{(5d^4 + 256ae^3)^4}{1048576e^4} - \frac{3d^2(5d^4 + 256ae^3)^3 x^2}{8192e^2} + \frac{27}{512} d^4 (5d^4 + 256ae^3)^2 \right. \right. \\ &= \left. \frac{(5d^4 + 256ae^3)^4 x}{1048576e^4} - \frac{d^2(5d^4 + 256ae^3)^3 \left(\frac{d}{4e} + x \right)^3}{8192e^2} + \frac{(5d^4 + 256ae^3)^2}{512} \right) \end{aligned}$$

Mathematica [A] time = 0.05, size = 345, normalized size = 1.17

$$4096a^4e^8x - 1024a^3d^3e^6x^2 + 8ade^2x^4(512a^2e^6 - d^8) + 128a^2d^6e^4x^3 - 4de^3x^8(-1536a^2e^6 + 192ad^4e^3 + d^8) + \frac{128}{3}e^4$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]

[Out] 4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 - 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4 - 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17

fricas [A] time = 0.39, size = 353, normalized size = 1.20

$$\frac{4096}{17}x^{17}e^{12} + 1024x^{16}e^{11}d + \frac{8192}{5}x^{15}e^{10}d^2 + 1024x^{14}e^9d^3 - \frac{2048}{13}x^{13}e^8d^4 + \frac{16384}{13}x^{13}e^{11}a - 512x^{12}e^7d^5 + 4096x^{12}e^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")

[Out] 4096/17*x^17*e^12 + 1024*x^16*e^11*d + 8192/5*x^15*e^10*d^2 + 1024*x^14*e^9*d^3 - 2048/13*x^13*e^8*d^4 + 16384/13*x^13*e^11*a - 512*x^12*e^7*d^5 + 4096*x^12*e^10*d*a - 1664/11*x^11*e^6*d^6 + 49152/11*x^11*e^9*d^2*a + 384/5*x^10*e^5*d^7 + 1024*x^10*e^8*d^3*a + 128/3*x^9*e^4*d^8 - 4096/3*x^9*e^7*d^4*a + 8192/3*x^9*e^10*a^2 - 4*x^8*e^3*d^9 - 768*x^8*e^6*d^5*a + 6144*x^8*e^9*d*a^2 - 32/7*x^7*e^2*d^10 + 768/7*x^7*e^5*d^6*a + 24576/7*x^7*e^8*d^2*a^2 + 128*x^6*e^4*d^7*a - 1024*x^6*e^7*d^3*a^2 + 1/5*x^5*d^12 - 6144/5*x^5*e^6*d^4*a^2 + 16384/5*x^5*e^9*a^3 - 8*x^4*e^2*d^9*a + 4096*x^4*e^8*d*a^3 + 128*x^3*e^4*d^6*a^2 - 1024*x^2*e^6*d^3*a^3 + 4096*x*e^8*a^4

giac [A] time = 0.31, size = 323, normalized size = 1.09

$$\frac{4096}{17}x^{17}e^{12} + 1024dx^{16}e^{11} + \frac{8192}{5}d^2x^{15}e^{10} + 1024d^3x^{14}e^9 - \frac{2048}{13}d^4x^{13}e^8 - 512d^5x^{12}e^7 - \frac{1664}{11}d^6x^{11}e^6 + \frac{384}{5}d^7x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")

[Out] 4096/17*x^17*e^12 + 1024*d*x^16*e^11 + 8192/5*d^2*x^15*e^10 + 1024*d^3*x^14*e^9 - 2048/13*d^4*x^13*e^8 - 512*d^5*x^12*e^7 - 1664/11*d^6*x^11*e^6 + 384/5*d^7*x^10*e^5 + 128/3*d^8*x^9*e^4 - 4*d^9*x^8*e^3 - 32/7*d^10*x^7*e^2 + 1

$$\begin{aligned} & /5*d^{12}*x^5 + 16384/13*a*x^{13}*e^{11} + 4096*a*d*x^{12}*e^{10} + 49152/11*a*d^2*x^{11}*e^9 + 1024*a*d^3*x^{10}*e^8 - 4096/3*a*d^4*x^9*e^7 - 768*a*d^5*x^8*e^6 + 768/7*a*d^6*x^7*e^5 + 128*a*d^7*x^6*e^4 - 8*a*d^9*x^4*e^2 + 8192/3*a^2*x^9*e^{10} + 6144*a^2*d*x^8*e^9 + 24576/7*a^2*d^2*x^7*e^8 - 1024*a^2*d^3*x^6*e^7 - 6144/5*a^2*d^4*x^5*e^6 + 128*a^2*d^6*x^3*e^4 + 16384/5*a^3*x^5*e^9 + 4096*a^3*d*x^4*e^8 - 1024*a^3*d^3*x^2*e^6 + 4096*a^4*x*e^8 \end{aligned}$$

maple [A] time = 0.00, size = 500, normalized size = 1.69

$$\frac{4096e^{12}x^{17}}{17} + 1024de^{11}x^{16} + \frac{8192d^2e^{10}x^{15}}{5} + 1024d^3e^9x^{14} + \frac{128(128ae^5 - 16d^4e^2)e^6x^{13}}{13} + 128a^2d^6e^4x^3 - 1024a^3d^3e^6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x)

[Out] 4096/17*e^12*x^17+1024*d*e^11*x^16+8192/5*d^2*e^10*x^15+1024*d^3*e^9*x^14+128/13*(128*a*e^5-16*d^4*e^2)*e^6*x^13+1/12*(16384*a*e^10*d+256*(128*a*e^5-16*d^4*e^2)*d*e^5-2048*d^5*e^7)*x^12+1/11*(384*d^6*e^6+32768*a*e^9*d^2+128*(128*a*e^5-16*d^4*e^2)*d^2*e^4)*x^11+1/10*(14336*a*e^8*d^3+256*d^7*e^5-32*(128*a*e^5-16*d^4*e^2)*d^3*e^3)*x^10+1/9*(8192*a^2*e^10-8192*a*e^7*d^4+128*d^8*e^4+(128*a*e^5-16*d^4*e^2)^2)*x^9+1/8*(16384*a^2*e^9*d-2048*a*e^6*d^5-32*d^9*e^3+256*a*e^4*d*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(24576*a^2*e^8*d^2+512*a*e^5*d^6+2*d^6*(128*a*e^5-16*d^4*e^2))*x^7+1/6*(-2048*a^2*e^7*d^3-32*a*e^2*d^3*(128*a*e^5-16*d^4*e^2)+256*d^7*a*e^4)*x^6+1/5*(128*a^2*e^4*(128*a*e^5-16*d^4*e^2)-4096*a^2*e^6*d^4+d^12)*x^5+1/4*(16384*a^3*d*e^8-32*a*d^9*e^2)*x^4+128*a^2*e^4*d^6*x^3-1024*a^3*e^6*d^3*x^2+4096*a^4*e^8*x

maxima [A] time = 0.75, size = 383, normalized size = 1.30

$$\frac{4096}{17}e^{12}x^{17} + 1024de^{11}x^{16} + \frac{8192}{5}d^2e^{10}x^{15} + \frac{8192}{7}d^3e^9x^{14} + \frac{4096}{13}d^4e^8x^{13} + \frac{1}{5}d^{12}x^5 + 4096a^4e^8x - \frac{4}{7}(7e^3x^8 + 8de^2x^8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")

[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 8192/7*d^3*e^9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4*e^8*x - 4/7*(7*e^3*x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^3*e^6 + 128/165*(45*e^6*x^11 + 99*d*e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e^8*x^13 + 1001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9*x^13 + 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6*x^10 - 15015*d^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 32032*(36*e^6*x^10 + 80*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3)*a*e^2

mupad [B] time = 0.27, size = 331, normalized size = 1.12

$$x^5 \left(\frac{16384 a^3 e^9}{5} - \frac{6144 a^2 d^4 e^6}{5} + \frac{d^{12}}{5} \right) + x^{10} \left(\frac{384 d^7 e^5}{5} + 1024 a d^3 e^8 \right) - x^{11} \left(\frac{1664 d^6 e^6}{11} - \frac{49152 a d^2 e^9}{11} \right) + \frac{4096 e^8 x^8}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^4,x)

[Out] x^5*(d^12/5 + (16384*a^3*e^9)/5 - (6144*a^2*d^4*e^6)/5) + x^10*((384*d^7*e^5)/5 + 1024*a*d^3*e^8) - x^11*((1664*d^6*e^6)/11 - (49152*a*d^2*e^9)/11) + (4096*e^12*x^17)/17 + (2048*e^8*x^13*(8*a*e^3 - d^4))/13 + (128*e^4*x^9*(d^8 + 64*a^2*e^6 - 32*a*d^4*e^3))/3 + 4096*a^4*e^8*x + 1024*d*e^11*x^16 + 102

$$4*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 512*d*e^7*x^12*(8*a*e^3 - d^4) + (32*d^2*e^2*x^7*(768*a^2*e^6 - d^8 + 24*a*d^4*e^3))/7 - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 - 4*d*e^3*x^8*(d^8 - 1536*a^2*e^6 + 192*a*d^4*e^3) - 128*a*d^3*e^4*x^6*(8*a*e^3 - d^4) - 8*a*d*e^2*x^4*(d^8 - 512*a^2*e^6)$$

sympy [A] time = 0.14, size = 366, normalized size = 1.24

$$4096a^4e^8x - 1024a^3d^3e^6x^2 + 128a^2d^6e^4x^3 + 1024d^3e^9x^{14} + \frac{8192d^2e^{10}x^{15}}{5} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17} + x^{13} \left(\frac{16384a}{13} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8***3*x**4+8*d***2*x**3-d**3*x+8*a***2)**4,x)

[Out] 4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 1024*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e**12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096*a*d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6/11) + x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/3 - 4096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a*d**5*e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5/7 - 32*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x**5*(16384*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3*d*e**8 - 8*a*d**9*e**2)

3.40 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 dx$

Optimal. Leaf size=203

$$512a^3e^6x - \frac{1}{4}dx^4(d^8 - 1536a^2e^6) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{128}{3}e^5x^9(d^4 - 4ae^3) - 24de^4x^8(d^4 - 16ae^3) - \frac{384}{5}ae^4x^5(d^4 - 4ae^3)$$

[Out] 512*a^3*e^6*x-96*a^2*d^3*e^4*x^2+8*a*d^6*e^2*x^3-1/4*d*(-1536*a^2*e^6+d^8)*x^4-384/5*a*e^4*(d^4-4*a*e^3)*x^5+4*d^3*e^2*(d^4-16*a*e^3)*x^6+24/7*d^2*e^3*(64*a*e^3+d^4)*x^7-24*d*e^4*(d^4-16*a*e^3)*x^8-128/3*e^5*(d^4-4*a*e^3)*x^9+32*d^3*e^6*x^10+1536/11*d^2*e^7*x^11+128*d*e^8*x^12+512/13*e^9*x^13

Rubi [A] time = 0.12, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2061}

$$-\frac{1}{4}dx^4(d^8 - 1536a^2e^6) - 96a^2d^3e^4x^2 + 512a^3e^6x - \frac{128}{3}e^5x^9(d^4 - 4ae^3) - 24de^4x^8(d^4 - 16ae^3) + \frac{24}{7}d^2e^3x^7(64ae^3 + d^4)$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3, x]

[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8 - (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && IntegerQ[p, 0]

Rubi steps

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 dx = \int (512a^3e^6 - 192a^2d^3e^4x + 24ad^6e^2x^2 - d(d^8 - 1536a^2e^6)x^3 - 384ae^4(d^4 - 4ae^3)x^4 + 4d^3e^2(d^4 - 16ae^3)x^5 + 24d^2e^3(d^4 + 64ae^3)x^6 - 24de^4(d^4 - 16ae^3)x^7 + 128e^5(d^4 - 4ae^3)x^8 + 32d^3e^6x^9 + 1536d^2e^7x^{10} + 128de^8x^{11} + 512e^9x^{12}) dx$$

Mathematica [A] time = 0.03, size = 207, normalized size = 1.02

$$512a^3e^6x - \frac{1}{4}dx^4(d^8 - 1536a^2e^6) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + \frac{128}{3}e^5x^9(4ae^3 - d^4) - 24de^4x^8(d^4 - 16ae^3) + \frac{384}{5}ae^4x^5(d^4 - 4ae^3)$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3, x]

[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13

fricas [A] time = 0.38, size = 205, normalized size = 1.01

$$\frac{512}{13}x^{13}e^9 + 128x^{12}e^8d + \frac{1536}{11}x^{11}e^7d^2 + 32x^{10}e^6d^3 - \frac{128}{3}x^9e^5d^4 + \frac{512}{3}x^9e^8a - 24x^8e^4d^5 + 384x^8e^7da + \frac{24}{7}x^7e^3d^6 + \frac{1536}{7}x^7e^6d^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")

[Out] 512/13*x^13*e^9 + 128*x^12*e^8*d + 1536/11*x^11*e^7*d^2 + 32*x^10*e^6*d^3 - 128/3*x^9*e^5*d^4 + 512/3*x^9*e^8*a - 24*x^8*e^4*d^5 + 384*x^8*e^7*d*a + 24/7*x^7*e^3*d^6 + 1536/7*x^7*e^6*d^2*a + 4*x^6*e^2*d^7 - 64*x^6*e^5*d^3*a - 384/5*x^5*e^4*d^4*a + 1536/5*x^5*e^7*a^2 - 1/4*x^4*d^9 + 384*x^4*e^6*d*a^2 + 8*x^3*e^2*d^6*a - 96*x^2*e^4*d^3*a^2 + 512*x*e^6*a^3

giac [A] time = 0.24, size = 187, normalized size = 0.92

$$\frac{512}{13} x^{13} e^9 + 128 d x^{12} e^8 + \frac{1536}{11} d^2 x^{11} e^7 + 32 d^3 x^{10} e^6 - \frac{128}{3} d^4 x^9 e^5 - 24 d^5 x^8 e^4 + \frac{24}{7} d^6 x^7 e^3 + 4 d^7 x^6 e^2 - \frac{1}{4} d^9 x^4 + \frac{512}{3} a x^3 e^6 + \frac{1536}{5} x^5 e^7 a^2 - \frac{1}{4} x^4 d^9 + 384 x^4 e^6 d a^2 + 8 x^3 e^2 d^6 a - 96 x^2 e^4 d^3 a^2 + 512 x e^6 a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")

[Out] 512/13*x^13*e^9 + 128*d*x^12*e^8 + 1536/11*d^2*x^11*e^7 + 32*d^3*x^10*e^6 - 128/3*d^4*x^9*e^5 - 24*d^5*x^8*e^4 + 24/7*d^6*x^7*e^3 + 4*d^7*x^6*e^2 - 1/4*d^9*x^4 + 512/3*a*x^9*e^8 + 384*a*d*x^8*e^7 + 1536/7*a*d^2*x^7*e^6 - 64*a*d^3*x^6*e^5 - 384/5*a*d^4*x^5*e^4 + 8*a*d^6*x^3*e^2 + 1536/5*a^2*x^5*e^7 + 384*a^2*d*x^4*e^6 - 96*a^2*d^3*x^2*e^4 + 512*a^3*x*e^6

maple [A] time = 0.00, size = 288, normalized size = 1.42

$$\frac{512e^9x^{13}}{13} + 128de^8x^{12} + \frac{1536d^2e^7x^{11}}{11} + 32d^3e^6x^{10} + 8ad^6e^2x^3 - 96a^2d^3e^4x^2 + 512a^3e^6x + \frac{(512ae^8 - 256d^4e^5 + 8(128ae^7d - 64d^5e^4 + 8d^6e^3 + 4d^7e^2 - 1/4d^9)x^4 + 512/3ax^9e^8 + 384ad^2x^7e^6 - 64ad^3x^6e^5 - 384/5ad^4x^5e^4 + 8ad^6x^3e^2 + 1536/5a^2x^5e^7 + 384a^2d^3x^2e^4 + 512a^3xe^6)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x)

[Out] 512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^10+1/9*(512*a*e^8-256*d^4*e^5+8*e^3*(128*a*e^5-16*d^4*e^2))*x^9+1/8*(2048*a*e^7*d-64*d^5*e^4+8*d*e^2*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(1536*a*d^2*e^6+24*d^6*e^3)*x^7+1/6*(-256*a*e^5*d^3-d^3*(128*a*e^5-16*d^4*e^2)+8*d^7*e^2)*x^6+1/5*(8*a*e^2*(128*a*e^5-16*d^4*e^2)-256*d^4*a*e^4+512*e^7*a^2)*x^5+1/4*(1536*a^2*d^3*e^6-d^9)*x^4+8*a*d^6*e^2*x^3-96*a^2*d^3*e^4*x^2+512*a^3*e^6*x

maxima [A] time = 0.60, size = 214, normalized size = 1.05

$$\frac{512}{13} e^9 x^{13} + 128 d e^8 x^{12} + \frac{1536}{11} d^2 e^7 x^{11} + \frac{256}{5} d^3 e^6 x^{10} - \frac{1}{4} d^9 x^4 + 512 a^3 e^6 x + \frac{4}{7} (6 e^3 x^7 + 7 d e^2 x^6) d^6 + \frac{96}{5} (16 e^3 x^5 + 2 d^4 e^5 - 8 d^5 e^4 + 8 d^6 e^3 + 4 d^7 e^2 - 1/4 d^9) x^4 + 8 a d^6 e^2 x^3 - 96 a^2 d^3 e^4 x^2 + 512 a^3 e^6 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="maxima")

[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 256/5*d^3*e^6*x^10 - 1/4*d^9*x^4 + 512*a^3*e^6*x + 4/7*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 + 96/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^2*e^4 - 8/15*(36*e^6*x^10 + 80*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3 + 8/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a*e^2

mupad [B] time = 2.24, size = 201, normalized size = 0.99

$$\frac{512e^9x^{13}}{13} - x^4 \left(\frac{d^9}{4} - 384a^2de^6 \right) + \frac{128e^5x^9(4ae^3 - d^4)}{3} + 512a^3e^6x + 128de^8x^{12} + 32d^3e^6x^{10} + \frac{1536d^2e^7x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^3,x)

[Out] (512*e^9*x^13)/13 - x^4*(d^9/4 - 384*a^2*d*e^6) + (128*e^5*x^9*(4*a*e^3 - d^4))/3 + 512*a^3*e^6*x + 128*d*e^8*x^12 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/11 + 8*a*d^6*e^2*x^3 + (384*a*e^4*x^5*(4*a*e^3 - d^4))/5 + 24*d*e^4*x^8*(16*a*e^3 - d^4) + (24*d^2*e^3*x^7*(64*a*e^3 + d^4))/7 - 96*a^2*d^3*e^4*x^2 - 4*d^3*e^2*x^6*(16*a*e^3 - d^4)

sympy [A] time = 0.11, size = 218, normalized size = 1.07

$$512a^3e^6x - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + 32d^3e^6x^{10} + \frac{1536d^2e^7x^{11}}{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13} + x^9 \left(\frac{512ae^8}{3} - \frac{128d^4e^5}{3} \right) + x^8 (384ad^6e^2 - d^9/4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)

[Out] 512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e**6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13 + x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**4) + x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 + 4*d**7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**2*d*e**6 - d**9/4)

$$3.41 \quad \int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 dx$$

Optimal. Leaf size=107

$$64a^2e^4x - \frac{16}{5}e^2x^5(d^4 - 8ae^3) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + 1/3*d^6*x^3 + 32*a*d*e^4*x^4 - 16/5*e^2*(-8*a*e^3 + d^4)*x^5 - 8/3*d^3*e^3*x^6 + 64/7*d^2*e^4*x^7 + 16*d*e^5*x^8 + 64/9*e^6*x^9

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2061}

$$64a^2e^4x - \frac{16}{5}e^2x^5(d^4 - 8ae^3) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{64}{7}d^2e^4x^7 - \frac{8}{3}d^3e^3x^6 + \frac{d^6x^3}{3} + 16de^5x^8 + \frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2, x]

[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (16*e^2*(d^4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8 + (64*e^6*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && IntegerQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 dx &= \int (64a^2e^4 - 16ad^3e^2x + d^6x^2 + 128ade^4x^3 - 16e^2(d^4 - 8ae^3)x^4 - 16e^3d^3x^5 + 64d^2e^4x^6 - 8d^3e^3x^7 + 64de^5x^8 + 64e^6x^9) dx \\ &= 64a^2e^4x - 8ad^3e^2x^2 + \frac{d^6x^3}{3} + 32ade^4x^4 - \frac{16}{5}e^2(d^4 - 8ae^3)x^5 - \frac{8}{3}d^3e^3x^7 + 64de^5x^8 + \frac{64e^6x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.01, size = 109, normalized size = 1.02

$$64a^2e^4x + \frac{16}{5}e^2x^5(8ae^3 - d^4) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2, x]

[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (16*e^2*(-d^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8 + (64*e^6*x^9)/9

fricas [A] time = 0.39, size = 99, normalized size = 0.93

$$\frac{64}{9}x^9e^6 + 16x^8e^5d + \frac{64}{7}x^7e^4d^2 - \frac{8}{3}x^6e^3d^3 - \frac{16}{5}x^5e^2d^4 + \frac{128}{5}x^5e^5a + 32x^4e^4da + \frac{1}{3}x^3d^6 - 8x^2e^2d^3a + 64xe^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")

[Out] $64/9*x^9*e^6 + 16*x^8*e^5*d + 64/7*x^7*e^4*d^2 - 8/3*x^6*e^3*d^3 - 16/5*x^5*e^2*d^4 + 128/5*x^5*e^5*a + 32*x^4*e^4*d*a + 1/3*x^3*d^6 - 8*x^2*e^2*d^3*a + 64*x*e^4*a^2$

giac [A] time = 0.29, size = 90, normalized size = 0.84

$$\frac{64}{9}x^9e^6 + 16dx^8e^5 + \frac{64}{7}d^2x^7e^4 - \frac{8}{3}d^3x^6e^3 - \frac{16}{5}d^4x^5e^2 + \frac{1}{3}d^6x^3 + \frac{128}{5}ax^5e^5 + 32adx^4e^4 - 8ad^3x^2e^2 + 64a^2xe^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")

[Out] $64/9*x^9*e^6 + 16*d*x^8*e^5 + 64/7*d^2*x^7*e^4 - 8/3*d^3*x^6*e^3 - 16/5*d^4*x^5*e^2 + 1/3*d^6*x^3 + 128/5*a*x^5*e^5 + 32*a*d*x^4*e^4 - 8*a*d^3*x^2*e^2 + 64*a^2*x*e^4$

maple [A] time = 0.00, size = 100, normalized size = 0.93

$$\frac{64e^6x^9}{9} + 16de^5x^8 + \frac{64d^2e^4x^7}{7} - \frac{8d^3e^3x^6}{3} + 32ade^4x^4 + \frac{d^6x^3}{3} - 8ad^3e^2x^2 + 64a^2e^4x + \frac{(128ae^5 - 16d^4e^2)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)

[Out] $64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 - 8/3*d^3*e^3*x^6 + 1/5*(128*a*e^5 - 16*d^4*e^2)*x^5 + 32*a*d*e^4*x^4 + 1/3*d^6*x^3 - 8*a*d^3*e^2*x^2 + 64*a^2*e^4*x$

maxima [A] time = 0.61, size = 101, normalized size = 0.94

$$\frac{64}{9}e^6x^9 + 16de^5x^8 + \frac{64}{7}d^2e^4x^7 + \frac{1}{3}d^6x^3 + 64a^2e^4x - \frac{8}{15}(5e^3x^6 + 6de^2x^5)d^3 + \frac{8}{5}(16e^3x^5 + 20de^2x^4 - 5d^3x^2)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")

[Out] $64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64*a^2*e^4*x - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a*e^2$

mupad [B] time = 0.04, size = 98, normalized size = 0.92

$$x^5 \left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5} \right) + \frac{d^6x^3}{3} + \frac{64e^6x^9}{9} + 64a^2e^4x + 16de^5x^8 - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} - 8ad^3e^2x^2 + 32ade^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)

[Out] $x^5*((128*a*e^5)/5 - (16*d^4*e^2)/5) + (d^6*x^3)/3 + (64*e^6*x^9)/9 + 64*a^2*e^4*x + 16*d*e^5*x^8 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 - 8*a*d^3*e^2*x^2 + 32*a*d*e^4*x^4$

sympy [A] time = 0.09, size = 112, normalized size = 1.05

$$64a^2e^4x - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} + 16de^5x^8 + \frac{64e^6x^9}{9} + x^5 \left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)

[Out] $64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d**3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 + x**5*(128*a*e**5/5 - 16*d**4*e**2/5)$

3.42 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) dx$

Optimal. Leaf size=37

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4, x]

[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5

Rubi steps

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4, x]

[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5

fricas [A] time = 0.38, size = 33, normalized size = 0.89

$$\frac{8}{5}x^5e^3 + 2x^4e^2d - \frac{1}{2}x^2d^3 + 8xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")

[Out] 8/5*x^5*e^3 + 2*x^4*e^2*d - 1/2*x^2*d^3 + 8*x*e^2*a

giac [A] time = 0.38, size = 30, normalized size = 0.81

$$\frac{8}{5}x^5e^3 + 2dx^4e^2 - \frac{1}{2}d^3x^2 + 8axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")

[Out] 8/5*x^5*e^3 + 2*d*x^4*e^2 - 1/2*d^3*x^2 + 8*a*x*e^2

maple [A] time = 0.00, size = 34, normalized size = 0.92

$$\frac{8}{5}e^3x^5 + 2de^2x^4 - \frac{1}{2}d^3x^2 + 8ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x)`

[Out] `8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5`

maxima [A] time = 0.67, size = 33, normalized size = 0.89

$$\frac{8}{5}e^3x^5 + 2de^2x^4 - \frac{1}{2}d^3x^2 + 8ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")`

[Out] `8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x`

mupad [B] time = 0.04, size = 33, normalized size = 0.89

$$-\frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5} + 8ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3,x)`

[Out] `(8*e^3*x^5)/5 - (d^3*x^2)/2 + 2*d*e^2*x^4 + 8*a*e^2*x`

sympy [A] time = 0.07, size = 36, normalized size = 0.97

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)`

[Out] `8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5`

$$3.43 \quad \int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} dx$$

Optimal. Leaf size=153

$$\frac{2 \tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2 \tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

[Out] 2*arctanh((4*e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^(1/2)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2)-2*arctanh((4*e*x+d)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^(1/2)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2)

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1106, 1093, 208}

$$\frac{2 \tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2 \tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1), x]

[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1106

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned}
\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} dx &= \text{Subst} \left(\int \frac{1}{\frac{1}{32} \left(\frac{5d^4}{e} + 256ae^2 \right) - 3d^2ex^2 + 8e^3x^4} dx, x, \frac{d}{4e} + x \right) \\
&= \frac{(4e^2) \text{Subst} \left(\int \frac{1}{-\frac{3d^2e}{2} - e\sqrt{d^4 - 64ae^3} + 8e^3x^2} dx, x, \frac{d}{4e} + x \right)}{\sqrt{d^4 - 64ae^3}} - \frac{(4e^2) \text{Subst} \left(\int \frac{1}{-\frac{3d^2e}{2} + e\sqrt{d^4 - 64ae^3} + 8e^3x^2} dx, x, \frac{d}{4e} + x \right)}{\sqrt{d^4 - 64ae^3}} \\
&= \frac{2 \tanh^{-1} \left(\frac{d+4ex}{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} \right)}{\sqrt{d^4 - 64ae^3} \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} - \frac{2 \tanh^{-1} \left(\frac{d+4ex}{\sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}} \right)}{\sqrt{d^4 - 64ae^3} \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.46

$$-\text{RootSum} \left[8\#1^4 e^3 + 8\#1^3 de^2 - \#1d^3 + 8ae^2 \&, \frac{\log(x - \#1)}{-32\#1^3 e^3 - 24\#1^2 de^2 + d^3} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]

[Out] -RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 &, Log[x - #1]/(d^3 - 24*d*e^2*#1^2 - 32*e^3*#1^3) &]

fricas [B] time = 0.48, size = 1115, normalized size = 7.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")

[Out] -sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) - sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) - sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))

$64*a*d^4*e^3 - 16384*a^2*e^6)/\text{sqrt}(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9)/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6) + 2*d)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8e^3x^4 + 8de^2x^3 - d^3x + 8ae^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")

[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)

maple [C] time = 0.07, size = 67, normalized size = 0.44

$$\frac{\ln(-\text{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2) + x)}{32\text{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2)^3 e^3 + 24\text{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2)^2 d e^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x)

[Out] sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(-_R+x),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+8*a*e^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8e^3x^4 + 8de^2x^3 - d^3x + 8ae^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")

[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)

mupad [B] time = 3.73, size = 1264, normalized size = 8.26

$$-\text{atan}\left(\frac{d^3 \sqrt{-262144 a^3 e^9 + 12288 a^2 d^4 e^6 - 192 a d^8 e^3 + d^{12}} (3i + d^9 2i - a)}{5 d^{12} \sqrt{\frac{2 \sqrt{-262144 a^3 e^9 + 12288 a^2 d^4 e^6 - 192 a d^8 e^3 + d^{12}} + 3 d^6 - 192 a d^2 e^3}{1048576 a^3 e^9 - 12288 a^2 d^4 e^6 - 384 a d^8 e^3 + 5 d^{12}}} + 1048576 a^3 e^9 \sqrt{\frac{2 \sqrt{-262144 a^3 e^9 + 12288 a^2 d^4 e^6 - 192 a d^8 e^3 + d^{12}} - 3 d^6 - 192 a d^2 e^3}{1048576 a^3 e^9 - 12288 a^2 d^4 e^6 - 384 a d^8 e^3 + 5 d^{12}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3),x)

[Out] atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) *3i - d^9*2i + a*d^5*e^3*256i - a^2*d*e^6*8192i - a^2*e^7*x*32768i - d^8*e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i + a*d^4*e^4*x*1024i)/(5*d^12*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) + 1048576*a^3*e^9*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 384*a*d^8*e^3*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 12288*a^2*d^4*e^6*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)))*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2))

$$\begin{aligned} & \frac{1}{2}) - 3d^6 + 192ad^2e^3)/(5d^{12} + 1048576a^3e^9 - 384ad^8e^3 - 1 \\ & 2288a^2d^4e^6))^{(1/2)}*2i - \operatorname{atan}((d^3(d^{12} - 262144a^3e^9 - 192ad^8e^3 + 12288a^2d^4e^6))^{(1/2)}*3i + d^9*2i - a*d^5e^3*256i + a^2*d^6e^6*819 \\ & 2i + a^2*e^7*x*32768i + d^8*e*x*8i + d^2*e*x*(d^{12} - 262144a^3e^9 - 192a \\ & *d^8e^3 + 12288a^2d^4e^6))^{(1/2)}*12i - a*d^4e^4*x*1024i)/(5d^{12}*((2*(d \\ & ^{12} - 262144a^3e^9 - 192ad^8e^3 + 12288a^2d^4e^6))^{(1/2)} + 3d^6 - 1 \\ & 92ad^2e^3)/(5d^{12} + 1048576a^3e^9 - 384ad^8e^3 - 12288a^2d^4e^6 \\ &))^{(1/2)} + 1048576a^3e^9*((2*(d^{12} - 262144a^3e^9 - 192ad^8e^3 + 122 \\ & 88a^2d^4e^6))^{(1/2)} + 3d^6 - 192ad^2e^3)/(5d^{12} + 1048576a^3e^9 - \\ & 384ad^8e^3 - 12288a^2d^4e^6))^{(1/2)} - 384ad^8e^3*((2*(d^{12} - 26214 \\ & 4a^3e^9 - 192ad^8e^3 + 12288a^2d^4e^6))^{(1/2)} + 3d^6 - 192ad^2e^ \\ & 3)/(5d^{12} + 1048576a^3e^9 - 384ad^8e^3 - 12288a^2d^4e^6))^{(1/2)} - \\ & 12288a^2d^4e^6*((2*(d^{12} - 262144a^3e^9 - 192ad^8e^3 + 12288a^2d^ \\ & 4e^6))^{(1/2)} + 3d^6 - 192ad^2e^3)/(5d^{12} + 1048576a^3e^9 - 384ad^8 \\ & *e^3 - 12288a^2d^4e^6))^{(1/2)}))*((2*(d^{12} - 262144a^3e^9 - 192ad^8e \\ & ^3 + 12288a^2d^4e^6))^{(1/2)} + 3d^6 - 192ad^2e^3)/(5d^{12} + 1048576a^ \\ & 3e^9 - 384ad^8e^3 - 12288a^2d^4e^6))^{(1/2)}*2i \end{aligned}$$

sympy [A] time = 1.83, size = 122, normalized size = 0.80

$$\operatorname{RootSum}\left(t^4(1048576a^3e^9 - 12288a^2d^4e^6 - 384ad^8e^3 + 5d^{12}) + t^2(384ad^2e^3 - 6d^6) + 1, \left(t \mapsto t \log\left(x + \frac{-4915}{t}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)

[Out] RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3 + 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (-4915*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256*_t*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))

$$3.44 \quad \int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} dx$$

Optimal. Leaf size=342

$$\frac{2e\left(\frac{d}{4e} + x\right)\left(-256ae^3 + 13d^4 - 48d^2e^2\left(\frac{d}{4e} + x\right)^2\right)}{(-16384a^2e^6 - 64ad^4e^3 + 5d^8)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{24e(-d^2\sqrt{d^4 - 64ae^3} + 128ae^3 + d^4)\tanh^{-1}\left(\frac{d^4 - 64ae^3}{(256ae^3 + 5d^4)\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}\right)}{(d^4 - 64ae^3)^{3/2}(256ae^3 + 5d^4)\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}$$

[Out] $2e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^6-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)-24*e*\operatorname{arctanh}\left(\frac{4*e*x+d}{(3*d^2-2*(-64*a*e^3+d^4)^{(1/2)})^{(1/2)}}\right)*(d^4+128*a*e^3-d^2*(-64*a*e^3+d^4)^{(1/2)})/(-64*a*e^3+d^4)^{(3/2)}/(256*a*e^3+5*d^4)/(3*d^2-2*(-64*a*e^3+d^4)^{(1/2)})^{(1/2)}+24*e*\operatorname{arctanh}\left(\frac{4*e*x+d}{(3*d^2+2*(-64*a*e^3+d^4)^{(1/2)})^{(1/2)}}\right)*(d^4+128*a*e^3+d^2*(-64*a*e^3+d^4)^{(1/2)})/(-64*a*e^3+d^4)^{(3/2)}/(256*a*e^3+5*d^4)/(3*d^2+2*(-64*a*e^3+d^4)^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.53, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1106, 1092, 1166, 208}

$$\frac{2e\left(\frac{d}{4e} + x\right)\left(-256ae^3 - 48d^2e^2\left(\frac{d}{4e} + x\right)^2 + 13d^4\right)}{(-16384a^2e^6 - 64ad^4e^3 + 5d^8)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{24e(-d^2\sqrt{d^4 - 64ae^3} + 128ae^3 + d^4)\tanh^{-1}\left(\frac{d^4 - 64ae^3}{(256ae^3 + 5d^4)\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}\right)}{(d^4 - 64ae^3)^{3/2}(256ae^3 + 5d^4)\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2), x]

[Out] $(2e*(d/(4e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4e) + x)^2))/((5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4) - (24*e*(d^4 + 128*a*e^3 - d^2*\sqrt{d^4 - 64*a*e^3})*\operatorname{ArcTanh}[(d + 4*e*x)/\sqrt{3*d^2 - 2*\sqrt{d^4 - 64*a*e^3}}])/(d^4 - 64*a*e^3)^{(3/2)}*(5*d^4 + 256*a*e^3)*\sqrt{3*d^2 - 2*\sqrt{d^4 - 64*a*e^3}} + (24*e*(d^4 + 128*a*e^3 + d^2*\sqrt{d^4 - 64*a*e^3})*\operatorname{ArcTanh}[(d + 4*e*x)/\sqrt{3*d^2 + 2*\sqrt{d^4 - 64*a*e^3}}])/(d^4 - 64*a*e^3)^{(3/2)}*(5*d^4 + 256*a*e^3)*\sqrt{3*d^2 + 2*\sqrt{d^4 - 64*a*e^3}})$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p+1))/(2*a*(p+1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p+1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p+1)*(b^2 - 4*a*c) + b*c*(4*p+7)*x^2)*(a + b*x^2 + c*x^4)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1106

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &

& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
 > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
 Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} dx = \text{Subst} \left(\int \frac{1}{\left(\frac{1}{32} \left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4\right)^2} dx, x, \frac{d}{4e} + x \right)$$

$$= \frac{2e \left(\frac{d}{4e} + x\right) \left(13d^4 - 256ae^3 - 48d^2e^2 \left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{4 \text{Subst} \left(\int \frac{1}{\left(\frac{1}{32} \left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4\right)^2} dx, x, \frac{d}{4e} + x \right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)}$$

$$= \frac{2e \left(\frac{d}{4e} + x\right) \left(13d^4 - 256ae^3 - 48d^2e^2 \left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} + \frac{(48e^3 (d^4 - 64ae^3))}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)}$$

$$= \frac{2e \left(\frac{d}{4e} + x\right) \left(13d^4 - 256ae^3 - 48d^2e^2 \left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{24e (d^4 - 64ae^3)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)}$$

Mathematica [C] time = 0.19, size = 234, normalized size = 0.68

$$\frac{48e^2 \text{RootSum} \left[8\#1^4 e^3 + 8\#1^3 de^2 - \#1d^3 + 8ae^2 \&, \frac{2\#1^2 d^2 e \log(x-\#1) + 32ae^2 \log(x-\#1) + \#1d^3 \log(x-\#1)}{32\#1^3 e^3 + 24\#1^2 de^2 - d^3} \& \right]}{16384a^2e^6 + 64ad^4e^3 - 5d^8} + \frac{(d + 4ex) (25d^4 - 64ae^3)}{(d^4 - 64ae^3) (25d^4 - 64ae^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2), x]

[Out] ((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) + (48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 &, (32*a*e^2*Log[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*d*e^2*#1^2 + 32*e^3*#1^3) &])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)

fricas [B] time = 0.73, size = 4285, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")

[Out] -(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 -

$$\begin{aligned}
& 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (\\
& 5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*\text{sqrt}((d^10*e^2 + 160*a*d^6*e^5 \\
& + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 \\
& + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 \\
& - 4398046511104*a^6*e^18))*\text{sqrt}((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10) \\
& /((15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3 \\
& *d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27 \\
& 44381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 51881467707 \\
& 30811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a \\
& *d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d \\
& ^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*\text{log}(884736*a* \\
& d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 1382 \\
& 4*\text{sqrt}(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4* \\
& e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22 \\
& *e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a \\
& ^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21) \\
& *\text{sqrt}((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^ \\
& 32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a \\
& ^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1 \\
& 8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737 \\
& 86976294838206464*a^9*e^27))*\text{sqrt}((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^ \\
& 2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d \\
& ^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804651110 \\
& 4*a^6*e^18))*\text{sqrt}((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1 \\
& 800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150 \\
& 994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a \\
& ^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4 \\
& *e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167 \\
& 360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 515396 \\
& 07552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))) - 12*\text{sqrt}(2)*(40*a*d^8*e^2 - \\
& 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2 \\
& *e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - \\
& 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*\text{sqrt}((d^10*e^2 + 160*a*d^6*e^5 + 40960 \\
& *a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 3119513 \\
& 6*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 43980 \\
& 46511104*a^6*e^18))*\text{sqrt}((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d \\
& ^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^ \\
& 9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27443810229 \\
& 28896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392* \\
& a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 \\
& - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - \\
& 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*\text{log}(884736*a*d^5*e^6 + \\
& 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*\text{sqrt}(2) \\
& *(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 26 \\
& 8435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 56 \\
& 6493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e \\
& ^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21))*\text{sqrt}((d^ \\
& 8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - \\
& 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e \\
& ^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931 \\
& 694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737869762948 \\
& 38206464*a^9*e^27))*\text{sqrt}((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (\\
& 125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + \\
& 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^1 \\
& 8))*\text{sqrt}((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a* \\
& d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000 \\
& *a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e \\
& ^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7 \\
& 3786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d
\end{aligned}$$


```
8082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709316941310
85312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629483820646
4*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136
*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804
6511104*a^6*e^18))) - 8*(d^4*e - 64*a*e^4)*x)/(40*a*d^8*e^2 - 512*a^2*d^4*e
^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*
(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 -
16384*a^2*d^3*e^6)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

[Out] Timed out

maple [C] time = 0.02, size = 288, normalized size = 0.84

$$\frac{384e^2 \left(2 \operatorname{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2)^2 d^2e + \operatorname{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2) d^3 + \dots \right)}{(2048ae^3 + 40d^4)(64ae^3 - d^4) \left(32 \operatorname{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2)^3 e^3 + 24 \operatorname{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2) d^3 + \dots \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] (12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^4
)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384*a
^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/(20
48*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*_R^3
*e^3+24*_R^2*d*e^2-d^3)*ln(-_R+x),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+
8*a*e^2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

[Out] Timed out

mupad [B] time = 7.11, size = 10351, normalized size = 30.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

```
[Out] ((8*e*x)/(256*a*e^3 + 5*d^4) - (5*d^5 - 128*a*d*e^3)/((64*a*e^3 - d^4)*(256
*a*e^3 + 5*d^4)) + (72*d^3*e^2*x^2)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4))
+ (96*d^2*e^3*x^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)))/(8*a*e^2 - d^3*x
+ 8*e^3*x^4 + 8*d*e^2*x^3) + atan((((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 -
d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11
+ 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3
- d^4)^9)^(1/2))))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32
*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20
```

$$\begin{aligned}
& *e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 79164837 \\
& 1998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)}*((1536*(6871 \\
& 9476736*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 770048*a^2*d^{12}*e^{15} - \\
& 5242880*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21}))/((25*d^{20} - 17179869184*a^5 \\
& *e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 1342 \\
& 17728*a^4*d^4*e^{12}) - ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e^{11} + 24576*a^2*d^ \\
& 19*e^{14} + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^{20} - 25769803776*a^ \\
& 5*d^7*e^{23} + 1099511627776*a^6*d^3*e^{26}))/((25*d^{20} - 17179869184*a^5*e^{15} - \\
& 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a \\
& ^4*d^4*e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - 118784*a^2*d^{14}*e^ \\
& 15 + 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 17179869184*a^5*d^2* \\
& e^{24}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 \\
& + 2097152*a^3*d^4*e^9))*((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/ \\
& 2) - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440 \\
& *a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/ \\
& 2)))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 12902 \\
& 40*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 9663 \\
& 6764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7* \\
& d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)}*((288*(d^{22}*e^2 + d^4*e^ \\
& 2*(-(64*a*e^3 - d^4)^9)^{(1/2) - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 616038 \\
& 4*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a \\
& *e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)))/((125*d^{36} + 1152921504606846976*a^9*e^{27} \\
& - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250 \\
& 489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}* \\
& e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2} \\
&) + (1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^{19} + 196608*a^ \\
& 2*d^5*e^{16}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2 \\
& *d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(78643 \\
& 2*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14}))/((25*d^{16} + 268435456*a^4*e^{12} \\
& - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*i + ((288*(d \\
& ^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2) - 32*a*d^{18}*e^5 + 22528*a^2*d \\
& ^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5* \\
& d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)))/((125*d^{36} + 115292150460 \\
& 6846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3* \\
& d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062 \\
& 494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8 \\
& *d^4*e^{24})^{(1/2)}*((1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^ \\
& ^{19} + 196608*a^2*d^5*e^{16}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e \\
& ^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) - \\
& ((1536*(68719476736*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 770048*a^2 \\
& *d^{12}*e^{15} - 5242880*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21}))/((25*d^{20} - 17 \\
& 179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^ \\
& ^8*e^9 + 134217728*a^4*d^4*e^{12}) + ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e^{11} + \\
& 24576*a^2*d^{19}*e^{14} + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^{20} - 2 \\
& 5769803776*a^5*d^7*e^{23} + 1099511627776*a^6*d^3*e^{26}))/((25*d^{20} - 171798691 \\
& 84*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 \\
& + 134217728*a^4*d^4*e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - 11878 \\
& 4*a^2*d^{14}*e^{15} + 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 1717986 \\
& 9184*a^5*d^2*e^{24}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744 \\
& *a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 \\
& - d^4)^9)^{(1/2) - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^1 \\
& 1 + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e \\
& ^3 - d^4)^9)^{(1/2)))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^3 \\
& 2*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^2 \\
& 0*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 7916483 \\
& 71998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)}*((288*(d^{22} \\
& *e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2) - 32*a*d^{18}*e^5 + 22528*a^2*d^{14} \\
& *e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2 \\
& *e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)))/((125*d^{36} + 115292150460684
\end{aligned}$$

$$\begin{aligned}
& 6976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)} + (6144*x*(786432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14})) / (25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9) * i) / (((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)})) / (125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 12902400*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)} * (((1536*(68719476736*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 770048*a^2*d^{12}*e^{15} - 5242880*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21})) / (25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) - ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e^{11} + 24576*a^2*d^{19}*e^{14} + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^{20} - 25769803776*a^5*d^7*e^{23} + 1099511627776*a^6*d^3*e^{26})) / (25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - 118784*a^2*d^{14}*e^{15} + 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 17179869184*a^5*d^2*e^{24})) / (25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9) * ((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)})) / (125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)} * ((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)})) / (125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)} + (1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^{19} + 196608*a^2*d^5*e^{16})) / (25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(786432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14})) / (25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9) - ((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)})) / (125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)} * (((1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^{19} + 196608*a^2*d^5*e^{16})) / (25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) - ((1536*(68719476736*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 770048*a^2*d^{12}*e^{15} - 5242880*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21})) / (25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e^{11} + 24576*a^2*d^{19}*e^{14} + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^{20} - 25769803776*a^5*d^7*e^{23} + 1099511627776*a^6*d^3*e^{26})) / (25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - 118784*a^2*d^{14}*e^{15} + 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 17179869184*a^5*d^2*e^{24})) / (25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2
\end{aligned}$$

$$\begin{aligned}
& 097152*a^3*d^4*e^9)) * ((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{1/2}) \\
& - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{1/2})) / (125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^{1/2}) * ((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{1/2}) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{1/2})) / (125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^{1/2} + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14)) / (25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9) + (113246208*a*d^2*e^14) / (25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12)) * ((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{1/2}) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{1/2})) / (125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^{1/2} * i + atan(((288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{1/2}) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{1/2})) / (125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^{1/2} * (((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21)) / (25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26)) / (25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24)) / (25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) * (-288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{1/2}) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{1/2})) / (125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^{1/2} * (-288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{1/2}) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{1/2})) / (125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^{1/2} + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d^5*e^16)) / (25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14)) / (25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9) * i + (-288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{1/2}) - d^22*e^2 + 32*a*d^18*e^5 - 22528*
\end{aligned}$$


```

5*(-(64*a*e^3 - d^4)^9)^(1/2))/(125*d^36 + 1152921504606846976*a^9*e^27 -
28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489
344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^1
8 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^1/2) +
(1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d
^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^
12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a
^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 6
40*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9) - (-(288*(d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e
^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e
^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 11529215046068469
76*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*
e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 4432406249472
0*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*
e^24))^1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 +
196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 -
118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((15
36*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12
*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 1717986
9184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^
9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 2457
6*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 257698
03776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^
5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134
217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2
*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*
a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*
d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2)
- d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 -
461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 -
d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^
3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^
12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 79164837199
8720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^1/2)*(-(288*(d^4*e^2
*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^
8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^
17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460684697
6*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e
^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720
*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e
^24))^1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(2
5*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152
*a^3*d^4*e^9) + (113246208*a*d^2*e^14)/(25*d^20 - 17179869184*a^5*e^15 - 2
240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4
*d^4*e^12))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*
d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e
^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(1
25*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^2
8*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^
5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 -
40532396646334464*a^8*d^4*e^24))^1/2)*2i

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)

[Out] Timed out

$$3.45 \quad \int (8 + 8x - x^3 + 8x^4)^4 dx$$

Optimal. Leaf size=96

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6$$

[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+1408*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^15-128*x^16+4096/17*x^17

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^4, x]

[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && IntegerQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 8x - x^3 + 8x^4)^4 dx &= \int (4096 + 16384x + 24576x^2 + 14336x^3 + 14336x^4 + 43008x^5 + 47488x^6 + 11008x^7 + 1376x^8 + 1408x^9 + 21488x^{10} + 25312x^{11} - 448x^{12} + 10241x^{13} + 1168x^{14} + 128x^{15} - 128x^{16} + 4096x^{17}) dx \\ &= 4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336x^5}{5} + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{21488x^{10}}{5} + \frac{25312x^{11}}{11} - 448x^{12} + \frac{10241x^{13}}{13} + 1168x^{14} + \frac{128x^{15}}{5} - 128x^{16} + \frac{4096x^{17}}{17} \end{aligned}$$

Mathematica [A] time = 0.00, size = 96, normalized size = 1.00

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^4, x]

[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17

fricas [A] time = 0.35, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")

[Out] $4096/17x^{17} - 128x^{16} + 128/5x^{15} + 1168x^{14} + 10241/13x^{13} - 448x^{12} + 25312/11x^{11} + 21488/5x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336/5x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$

giac [A] time = 0.31, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336/5x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")

[Out] $4096/17x^{17} - 128x^{16} + 128/5x^{15} + 1168x^{14} + 10241/13x^{13} - 448x^{12} + 25312/11x^{11} + 21488/5x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336/5x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$

maple [A] time = 0.00, size = 85, normalized size = 0.89

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336/5x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-x^3+8*x+8)^4,x)

[Out] $4096x + 8192x^2 + 8192x^3 + 3584x^4 + 14336/5x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + 21488/5x^{10} + 25312/11x^{11} - 448x^{12} + 10241/13x^{13} + 1168x^{14} + 128/5x^{15} - 128x^{16} + 4096/17x^{17}$

maxima [A] time = 1.25, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336/5x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")

[Out] $4096/17x^{17} - 128x^{16} + 128/5x^{15} + 1168x^{14} + 10241/13x^{13} - 448x^{12} + 25312/11x^{11} + 21488/5x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336/5x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$

mupad [B] time = 0.19, size = 84, normalized size = 0.88

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336/5x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x - x^3 + 8*x^4 + 8)^4,x)

[Out] $4096x + 8192x^2 + 8192x^3 + 3584x^4 + (14336x^5)/5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + (21488x^{10})/5 + (25312x^{11})/11 - 448x^{12} + (10241x^{13})/13 + 1168x^{14} + (128x^{15})/5 - 128x^{16} + (4096x^{17})/17$

sympy [A] time = 0.07, size = 94, normalized size = 0.98

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336/5x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-x**3+8*x+8)**4,x)

[Out] $4096x^{17}/17 - 128x^{16} + 128x^{15}/5 + 1168x^{14} + 10241x^{13}/13 - 448x^{12} + 25312x^{11}/11 + 21488x^{10}/5 + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 14336x^5/5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$

$$3.46 \quad \int (8 + 8x - x^3 + 8x^4)^3 dx$$

Optimal. Leaf size=74

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+307/2*x^10+24/11*x^11-16*x^12+512/13*x^13

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^3, x]

[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 8x - x^3 + 8x^4)^3 dx &= \int (512 + 1536x + 1536x^2 + 320x^3 + 1152x^4 + 2880x^5 + 1560x^6 - 360x^7 + 1152x^8 \\ &\quad + 512x + 768x^2 + 512x^3 + 80x^4 + \frac{1152x^5}{5} + 480x^6 + \frac{1560x^7}{7} - 45x^8 + 128x^9 + \frac{307x^{10}}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 74, normalized size = 1.00

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^3, x]

[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13

fricas [A] time = 0.35, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13} - 16x^{12} + \frac{24}{11}x^{11} + \frac{307}{2}x^{10} + 128x^9 - 45x^8 + \frac{1560}{7}x^7 + 480x^6 + \frac{1152}{5}x^5 + 80x^4 + 512x^3 + 768x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")

[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x

giac [A] time = 0.33, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")

[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x

maple [A] time = 0.00, size = 65, normalized size = 0.88

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-x^3+8*x+8)^3,x)

[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+307/2*x^10+24/11*x^11-16*x^12+512/13*x^13

maxima [A] time = 0.94, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")

[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x

mupad [B] time = 0.08, size = 64, normalized size = 0.86

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x - x^3 + 8*x^4 + 8)^3,x)

[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13

sympy [A] time = 0.07, size = 71, normalized size = 0.96

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-x**3+8*x+8)**3,x)

[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 + 1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 512*x

$$3.47 \quad \int (8 + 8x - x^3 + 8x^4)^2 dx$$

Optimal. Leaf size=54

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^2, x]

[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x^8 + (64*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 8x - x^3 + 8x^4)^2 dx &= \int (64 + 128x + 64x^2 - 16x^3 + 112x^4 + 128x^5 + x^6 - 16x^7 + 64x^8) dx \\ &= 64x + 64x^2 + \frac{64x^3}{3} - 4x^4 + \frac{112x^5}{5} + \frac{64x^6}{3} + \frac{x^7}{7} - 2x^8 + \frac{64x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^2, x]

[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x^8 + (64*x^9)/9

fricas [A] time = 0.38, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")

[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x^2 + 64*x

giac [A] time = 0.33, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")

[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x^2 + 64*x

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-x^3+8*x+8)^2,x)

[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9

maxima [A] time = 0.88, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")

[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x^2 + 64*x

mupad [B] time = 0.03, size = 44, normalized size = 0.81

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x - x^3 + 8*x^4 + 8)^2,x)

[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x^8 + (64*x^9)/9

sympy [A] time = 0.06, size = 49, normalized size = 0.91

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-x**3+8*x+8)**2,x)

[Out] 64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3 + 64*x**2 + 64*x

3.48 $\int (8 + 8x - x^3 + 8x^4) dx$

Optimal. Leaf size=23

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Antiderivative was successfully verified.

[In] Int[8 + 8*x - x^3 + 8*x^4, x]

[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5

Rubi steps

$$\int (8 + 8x - x^3 + 8x^4) dx = 8x + 4x^2 - \frac{x^4}{4} + \frac{8x^5}{5}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Antiderivative was successfully verified.

[In] Integrate[8 + 8*x - x^3 + 8*x^4, x]

[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5

fricas [A] time = 0.35, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")

[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x

giac [A] time = 0.32, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")

[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(8*x^4-x^3+8*x+8,x)`

[Out] `8*x+4*x^2-1/4*x^4+8/5*x^5`

maxima [A] time = 0.91, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")`

[Out] `8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x`

mupad [B] time = 0.03, size = 19, normalized size = 0.83

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(8*x - x^3 + 8*x^4 + 8,x)`

[Out] `8*x + 4*x^2 - x^4/4 + (8*x^5)/5`

sympy [A] time = 0.06, size = 19, normalized size = 0.83

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(8*x**4-x**3+8*x+8,x)`

[Out] `8*x**5/5 - x**4/4 + 4*x**2 + 8*x`

$$3.49 \quad \int \frac{1}{8+8x-x^3+8x^4} dx$$

Optimal. Leaf size=268

$$-\frac{1}{24} \sqrt{\frac{67\sqrt{29}-109}{1218}} \log\left(\left(\frac{4}{x}+1\right)^2 - \sqrt{6(1+\sqrt{29})}\left(\frac{4}{x}+1\right) + 3\sqrt{29}\right) + \frac{1}{24} \sqrt{\frac{67\sqrt{29}-109}{1218}} \log\left(\left(\frac{4}{x}+1\right)^2 + \sqrt{6(1+\sqrt{29})}\left(\frac{4}{x}+1\right) + 3\sqrt{29}\right)$$

[Out] -1/84*arctan(1/42*(3-(1+4/x)^2)*7^(1/2))*7^(1/2)-1/29232*ln((1+4/x)^2+3*29^(1/2)-(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/2))^(1/2)+1/29232*ln((1+4/x)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/2))^(1/2)-1/14616*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2))*(132762+81606*29^(1/2))^(1/2)-1/14616*arctan((2+8/x+(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2))*(132762+81606*29^(1/2))^(1/2)

Rubi [A] time = 0.40, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$-\frac{1}{24} \sqrt{\frac{67\sqrt{29}-109}{1218}} \log\left(\left(\frac{4}{x}+1\right)^2 - \sqrt{6(1+\sqrt{29})}\left(\frac{4}{x}+1\right) + 3\sqrt{29}\right) + \frac{1}{24} \sqrt{\frac{67\sqrt{29}-109}{1218}} \log\left(\left(\frac{4}{x}+1\right)^2 + \sqrt{6(1+\sqrt{29})}\left(\frac{4}{x}+1\right) + 3\sqrt{29}\right)$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-1), x]

[Out] -ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/(12*Sqrt[7]) - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])]) + 8/x]/Sqrt[6*(-1 + Sqrt[29])])]/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])]) + 8/x]/Sqrt[6*(-1 + Sqrt[29])])]/12 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1673

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rule 2069

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c))*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4]^(p_)/(b - 4*a*x)^2, x], x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^2*d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{8 + 8x - x^3 + 8x^4} dx &= - \left(1024 \operatorname{Subst} \left(\int \frac{(8 - 32x)^2}{8(1069056 - 393216x^2 + 1048576x^4)} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
&= - \left(128 \operatorname{Subst} \left(\int \frac{(8 - 32x)^2}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
&= - \left(128 \operatorname{Subst} \left(\int -\frac{512x}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) - 128 \operatorname{Subst} \left(\int \frac{1}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right) \\
&= 65536 \operatorname{Subst} \left(\int \frac{x}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right) - \frac{\operatorname{Subst} \left(\int \frac{16\sqrt{6(1+\sqrt{29})}}{3\sqrt{29}-16} dx, x, \frac{1}{4} + \frac{1}{x} \right)}{768} \\
&= 32768 \operatorname{Subst} \left(\int \frac{1}{1069056 - 393216x + 1048576x^2} dx, x, \left(\frac{1}{4} + \frac{1}{x} \right)^2 \right) - \frac{(87 + \sqrt{29}) \operatorname{Subst} \left(\int \frac{1}{1069056 - 393216x + 1048576x^2} dx, x, \frac{1}{4} + \frac{1}{x} \right)}{768} \\
&= -\frac{1}{24} \sqrt{\frac{-109 + 67\sqrt{29}}{1218}} \log \left(3\sqrt{29} - \sqrt{6(1 + \sqrt{29})} \left(1 + \frac{4}{x} \right) + \left(1 + \frac{4}{x} \right)^2 \right) + \frac{1}{24} \sqrt{\frac{-109 - 67\sqrt{29}}{1218}} \log \left(3\sqrt{29} + \sqrt{6(1 + \sqrt{29})} \left(1 + \frac{4}{x} \right) + \left(1 + \frac{4}{x} \right)^2 \right) \\
&= -\frac{\tan^{-1} \left(\frac{3 - \left(1 + \frac{4}{x} \right)^2}{6\sqrt{7}} \right)}{12\sqrt{7}} - \frac{1}{12} \sqrt{\frac{109 + 67\sqrt{29}}{1218}} \tan^{-1} \left(\frac{2 + \sqrt{6(1 + \sqrt{29})} + \frac{8}{x}}{\sqrt{6(-1 + \sqrt{29})}} \right) - \frac{1}{12} \sqrt{\frac{109 - 67\sqrt{29}}{1218}} \tan^{-1} \left(\frac{2 + \sqrt{6(1 + \sqrt{29})} + \frac{8}{x}}{\sqrt{6(-1 + \sqrt{29})}} \right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.17

$$\operatorname{RootSum} \left[8\#1^4 - \#1^3 + 8\#1 + 8\&, \frac{\log(x - \#1)}{32\#1^3 - 3\#1^2 + 8} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1), x]

[Out] RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) &]

fricas [C] time = 1.73, size = 1015, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")

[Out] -1/168*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696))*log(287314195
392*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^3 - 120389
06880*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 + 1687
8104*x + 4897683*I*sqrt(7) - 411405372*sqrt(65/43848*I*sqrt(7) - 109/87696)
+ 6055613) - 1/168*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696))*
log(-35914274424*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696
))^3 + 16443*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/87696))
^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/87696) - 91520
) + 609*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
)^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)) + 2109763
*x - 1911147/8*I*sqrt(7) + 40134087/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
- 1461344) + 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/43

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")

[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)

mupad [B] time = 2.45, size = 123, normalized size = 0.46

$$\sum_{k=1}^4 \ln \left(\frac{\text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right) \left(8064 \text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right) + 256x + \text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x - x^3 + 8*x^4 + 8),x)

[Out] symsum(log(-(root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*(8064*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k) + 256*x + 12285*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*x + 148176*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2*x + 198072*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2 - 8))/4096)*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k), k, 1, 4)

sympy [A] time = 0.96, size = 41, normalized size = 0.15

$$\text{RootSum}\left(66298176t^4 + 74088t^2 + 4095t + 64, \left(t \mapsto t \log\left(\frac{35914274424t^3}{2109763} - \frac{1504863360t^2}{2109763} + \frac{102851343t}{2109763} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x**4-x**3+8*x+8),x)

[Out] RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(35914274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 + x + 6055613/16878104)))

$$3.50 \quad \int \frac{1}{(8+8x-x^3+8x^4)^2} dx$$

Optimal. Leaf size=357

$$\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} + \frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} - \frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}} \log\left(\left(\frac{4}{x}+1\right)\right)}{1}$$

[Out] $1/336*(-207-29*(1+4/x)^2)/(261-6*(1+4/x)^2+(1+4/x)^4)+5/87696*(5157+199*(1+4/x)^2)*(1+4/x)/(261-6*(1+4/x)^2+(1+4/x)^4)-17/7056*\arctan(1/42*(3-(1+4/x)^2)*7^(1/2))*7^(1/2)-1/213627456*\ln((1+4/x)^2+3*29^(1/2)-(1+4/x)*(6+6*29^(1/2))^(1/2))*(-220437694722+55934612286*29^(1/2))^(1/2)+1/213627456*\ln((1+4/x)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-220437694722+55934612286*29^(1/2))^(1/2)-1/106813728*\arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2))*(220437694722+55934612286*29^(1/2))^(1/2)-1/106813728*\arctan((2+8/x+(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2))*(220437694722+55934612286*29^(1/2))^(1/2)$

Rubi [A] time = 0.40, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} + \frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} - \frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}} \log\left(\left(\frac{4}{x}+1\right)\right)}{1}$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-2), x]

[Out] $-(207 + 29*(1 + 4/x)^2)/(336*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)) + (5*(5157 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)) - (17*\text{ArcTan}[(3 - (1 + 4/x)^2)/(6*\text{Sqrt}[7])])/(1008*\text{Sqrt}[7]) - (\text{Sqrt}[(180983329 + 45923327*\text{Sqrt}[29])/1218]*\text{ArcTan}[(2 - \text{Sqrt}[6*(1 + \text{Sqrt}[29])]) + 8/x]/\text{Sqrt}[6*(-1 + \text{Sqrt}[29])])]/87696 - (\text{Sqrt}[(180983329 + 45923327*\text{Sqrt}[29])/1218]*\text{ArcTan}[(2 + \text{Sqrt}[6*(1 + \text{Sqrt}[29])]) + 8/x]/\text{Sqrt}[6*(-1 + \text{Sqrt}[29])])]/87696 - (\text{Sqrt}[(-180983329 + 45923327*\text{Sqrt}[29])/1218]*\text{Log}[3*\text{Sqrt}[29] - \text{Sqrt}[6*(1 + \text{Sqrt}[29])])*(1 + 4/x) + (1 + 4/x)^2])/175392 + (\text{Sqrt}[(-180983329 + 45923327*\text{Sqrt}[29])/1218]*\text{Log}[3*\text{Sqrt}[29] + \text{Sqrt}[6*(1 + \text{Sqrt}[29])])*(1 + 4/x) + (1 + 4/x)^2])/175392$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
```

+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 2069

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c))*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4]^p)/(b - 4*a*x)^2, x], x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^2*d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(8 + 8x - x^3 + 8x^4)^2} dx &= - \left(1024 \operatorname{Subst} \left(\int \frac{(8 - 32x)^6}{64 (1069056 - 393216x^2 + 1048576x^4)^2} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
 &= - \left(16 \operatorname{Subst} \left(\int \frac{(8 - 32x)^6}{(1069056 - 393216x^2 + 1048576x^4)^2} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
 &= - \left(16 \operatorname{Subst} \left(\int \frac{x(-6291456 - 335544320x^2 - 1610612736x^4)}{(1069056 - 393216x^2 + 1048576x^4)^2} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
 &= \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} - \frac{\operatorname{Subst} \left(\int \frac{2789277407614152474624 + 7758008804}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right)}{578536630256664} \\
 &= - \frac{207 + 29 \left(1 + \frac{4}{x} \right)^2}{336 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} - \frac{1}{578536630256664} \\
 &= - \frac{207 + 29 \left(1 + \frac{4}{x} \right)^2}{336 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{1}{578536630256664} \\
 &= - \frac{207 + 29 \left(1 + \frac{4}{x} \right)^2}{336 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} - \frac{1}{578536630256664} \\
 &= - \frac{207 + 29 \left(1 + \frac{4}{x} \right)^2}{336 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} - \frac{1}{578536630256664}
 \end{aligned}$$

Mathematica [C] time = 0.02, size = 113, normalized size = 0.32

$$\frac{\operatorname{RootSum} \left[8\#1^4 - \#1^3 + 8\#1 + 8\&, \frac{392\#1^2 \log(x-\#1) - 1097\#1 \log(x-\#1) + 2243 \log(x-\#1)}{32\#1^3 - 3\#1^2 + 8} \& \right]}{21924} + \frac{784x^3 - 1146x^2 + 1539x + 5}{43848 (8x^4 - x^3 + 8x + 8)}$$

$$\begin{aligned} & 9/4683568345344))^2 - 658503/1568*(17*I*\sqrt{7} + 7056*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344})*(-17*I*\sqrt{7} + 7056*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}) - 6630191) + 8435208206933660878927*x - 3005727107011649552439/2*I*\sqrt{7} + 623776778443358801235576*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344} + 2295910220839785410704) - (56*\sqrt{87})*(8*x^4 - x^3 + 8*x + 8)*\sqrt{-125452723536*(17/14112*I*\sqrt{7} - 1/2*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}))^2 - 125452723536*(-17/14112*I*\sqrt{7} - 1/2*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}))^2 - 658503/1568*(17*I*\sqrt{7} + 7056*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344})*(-17*I*\sqrt{7} + 7056*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}) - 6630191) - 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*\sqrt{7} + 7056*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}) - 7569*(8*x^4 - x^3 + 8*x + 8)*(-17*I*\sqrt{7} + 7056*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}))*\log(-439084532376*(-17/14112*I*\sqrt{7} - 1/2*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}))^2*(-1066184864424603*I*\sqrt{7} + 442529435492941104*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344} - 1427510892508480) - 7569/2*(7276511507810430573072*(17/14112*I*\sqrt{7} - 1/2*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}))^2 - 23359423554371543)*(17*I*\sqrt{7} + 7056*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}) + 626797952698732342414548480*(17/14112*I*\sqrt{7} - 1/2*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}))^2 - 1/16*(261*(62716756730859*\sqrt{87})*(-17*I*\sqrt{7} + 7056*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}) - 1427510892508480*\sqrt{87})*(17*I*\sqrt{7} + 7056*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}) - 372580342944713280*\sqrt{87}*(-17*I*\sqrt{7} + 7056*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}) + 10465021752358451264*\sqrt{87}))*\sqrt{-125452723536*(17/14112*I*\sqrt{7} - 1/2*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}))^2 - 125452723536*(-17/14112*I*\sqrt{7} - 1/2*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}))^2 - 658503/1568*(17*I*\sqrt{7} + 7056*\sqrt{-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344})*(-17*I*\sqrt{7} + 7056*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344}) - 6630191) + 8435208206933660878927*x - 3005727107011649552439/2*I*\sqrt{7} + 623776778443358801235576*\sqrt{4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344} + 2295910220839785410704) + 7498008*x + 2650368)/(8*x^4 - x^3 + 8*x + 8)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8x^4 - x^3 + 8x + 8)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")

[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)

maple [C] time = 0.01, size = 83, normalized size = 0.23

$$\frac{(392 \operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8))^2 - 1097 \operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8) + 2243) \ln(-\operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8))}{701568 \operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8)^3 - 65772 \operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8)^2 + 175}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-x^3+8*x+8)^2,x)

[Out] (7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*sum((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(-_R+x),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{784x^3 - 1146x^2 + 1539x + 544}{43848(8x^4 - x^3 + 8x + 8)} + \frac{1}{21924} \int \frac{392x^2 - 1097x + 2243}{8x^4 - x^3 + 8x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")

[Out] 1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/21924*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)

mupad [B] time = 0.21, size = 176, normalized size = 0.49

$$\left(\sum_{k=1}^4 \ln \left(\frac{2615257 \operatorname{root}\left(z^4 + \frac{6630191z^2}{167270298048} + \frac{77351105z}{674433841729536} + \frac{1114096}{13723971258377709}, z, k\right)}{72918171648} \right) + \frac{4225x}{40375589184} - \frac{\operatorname{root}\left(z^4 + \frac{6630191z^2}{167270298048} + \frac{77351105z}{674433841729536} + \frac{1114096}{13723971258377709}, z, k\right)}{72918171648} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x - x^3 + 8*x^4 + 8)^2,x)

[Out] symsum(log((2615257*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k))/72918171648 + (4225*x)/40375589184 - (34885379*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)*x)/72918171648 - (191555*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)^2*x)/475136 - (9261*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)^3*x)/256 - (11205*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)^2)/59392 - (24759*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)^3)/512 + 10901/107668237824)*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k), k, 1, 4) + ((57*x)/12992 - (191*x^2)/58464 + (7*x^3)/3132 + 17/10962)/(x - x^3/8 + x^4 + 1)

sympy [B] time = 3.22, size = 3834, normalized size = 10.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x**4-x**3+8*x+8)**2,x)

[Out] (784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x + 350784) - sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/1292018853888)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/227008323264998681573683424 - 267658292345340*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/8435208206933660878927 - 2157374520970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/113504161632499340786841712 + 3881045239007430*sqrt(29)/5326727264361229 + 435853770857118353330297/33740832827734643515708 + 20905585576953*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/85227636229779664) - 2942814074101429415084030510182204250067556953*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/888496186751485201253966401139075287452416534006272 - 14257625632856314835831142972765102609010539559351093/27765505835983912539186450035596102732888016687696

$$\begin{aligned}
& - 75184631502818837388875900060881355871 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923} \\
& 327 \cdot \sqrt{29}) \cdot \sqrt{(214095423017213 \cdot \sqrt{29} + 47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-} \\
& 180983329 + 45923327 \cdot \sqrt{29})} + 40699873480352667) / 30637799543154662112205 \\
& 737970312940946635052896768 - 963314181796141259748858766106570487809406229 \\
& 9 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} / 3063779954315466211220573 \\
& 7970312940946635052896768 - 1398888334001652366855237255 \cdot \sqrt{42} \cdot \sqrt{-180} \\
& 983329 + 45923327 \cdot \sqrt{29}) \cdot \sqrt{(214095423017213 \cdot \sqrt{29} + 47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-} \\
& 180983329 + 45923327 \cdot \sqrt{29})} + 40699873480352667) / 359456428 \\
& 291497016547944746810895370264 + 91245981690030498967778233214015591679 \cdot \sqrt{42} \cdot \sqrt{-} \\
& 180983329 + 45923327 \cdot \sqrt{29})} / 23005211410655809059068463795897 \\
& 303696896 + 10304175351841941260676745569701505519 \cdot \sqrt{29} \cdot \sqrt{(2140954230} \\
& 17213 \cdot \sqrt{29} + 47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} \\
&) + 40699873480352667) / 19347382796361535418676578052349632409089536 + 63911 \\
& 1088489748962499984017403917984374085485 \cdot \sqrt{29} / 4836845699090383854669144 \\
& 513087408102272384) + \sqrt{-180983329 / 37468546762752 + 1583563 \cdot \sqrt{29}} / 129 \\
& 2018853888) \cdot \log(x^2 + x \cdot (-62716756730859 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 4592} \\
& 3327 \cdot \sqrt{29}) \cdot \sqrt{-47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} \\
& (29)) + 214095423017213 \cdot \sqrt{29} + 40699873480352667) / 227008323264998681573 \\
& 683424 - 20905585576953 \cdot \sqrt{42} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} / 85227 \\
& 636229779664 + 3881045239007430 \cdot \sqrt{29} / 5326727264361229 + 267658292345340 \\
& \cdot \sqrt{-47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} + 214095 \\
& 423017213 \cdot \sqrt{29} + 40699873480352667) / 8435208206933660878927 + 2157374520 \\
& 970352866823 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} / 11350416163249 \\
& 9340786841712 + 435853770857118353330297 / 33740832827734643515708) - 1425762 \\
& 5632856314835831142972765102609010539559351093 / 2776550583598391253918645003 \\
& 5596102732888016687696 - 10304175351841941260676745569701505519 \cdot \sqrt{29} \cdot \sqrt{-} \\
& 47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} + 214095423 \\
& 017213 \cdot \sqrt{29} + 40699873480352667) / 19347382796361535418676578052349632409 \\
& 089536 - 91245981690030498967778233214015591679 \cdot \sqrt{42} \cdot \sqrt{-180983329 +} \\
& 45923327 \cdot \sqrt{29})} / 23005211410655809059068463795897303696896 - 751846315028 \\
& 18837388875900060881355871 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} \cdot \\
& \sqrt{-47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} + 2140954 \\
& 23017213 \cdot \sqrt{29} + 40699873480352667) / 306377995431546621122057379703129409 \\
& 46635052896768 - 1398888334001652366855237255 \cdot \sqrt{42} \cdot \sqrt{-180983329 + 45} \\
& 923327 \cdot \sqrt{29}) \cdot \sqrt{-47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} \\
& (29)) + 214095423017213 \cdot \sqrt{29} + 40699873480352667) / 3594564282914970165 \\
& 47944746810895370264 + 9633141817961412597488587661065704878094062299 \cdot \sqrt{1218} \cdot \sqrt{-} \\
& 180983329 + 45923327 \cdot \sqrt{29})} / 30637799543154662112205737970312 \\
& 940946635052896768 + 2942814074101429415084030510182204250067556953 \cdot \sqrt{-4} \\
& 7106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} + 21409542301721 \\
& 3 \cdot \sqrt{29} + 40699873480352667) / 8884961867514852012539664011390752874524165 \\
& 34006272 + 639111088489748962499984017403917984374085485 \cdot \sqrt{29} / 483684569 \\
& 9090383854669144513087408102272384) - 2 \cdot \sqrt{(199631405 / 37468546762752 + \sqrt{42} \cdot \sqrt{-} \\
& 47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} + 2140954230 \\
& 17213 \cdot \sqrt{29} + 40699873480352667) / 9367136690688 + 1583563 \cdot \sqrt{29} / 430672 \\
& 951296) \cdot \operatorname{atan}(454016646529997363147366848 \cdot x / (-4509673516272467429860 \cdot \sqrt{1218}) \cdot \sqrt{(199631405 + 4 \cdot \sqrt{-} \\
& 47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923} \\
& 327 \cdot \sqrt{29})} + 214095423017213 \cdot \sqrt{29} + 40699873480352667) + 137769981 \cdot \sqrt{29}) + 3601609981798895040 \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} \cdot \sqrt{(199631405 + 4 \cdot \sqrt{-} \\
& 47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} + 214095423017213 \cdot \sqrt{29} + 40699873480352667) + 137769981 \cdot \sqrt{29}) + 20905585576953 \cdot \sqrt{1218} \cdot \sqrt{(199631405 + 4 \cdot \sqrt{-} \\
& 47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} + 214095423017213 \cdot \sqrt{29} + 4069987348 \\
& 0352667) + 137769981 \cdot \sqrt{29}) \cdot \sqrt{-47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329} \\
& + 45923327 \cdot \sqrt{29})} + 214095423017213 \cdot \sqrt{29} + 40699873480352667) + 29 \\
& 32424170326692281206238216 / (-4509673516272467429860 \cdot \sqrt{1218}) \cdot \sqrt{(1996314} \\
& 05 + 4 \cdot \sqrt{-47106822945 \cdot \sqrt{1218}) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} + \\
& 214095423017213 \cdot \sqrt{29} + 40699873480352667) + 137769981 \cdot \sqrt{29}) + 36016 \\
& 09981798895040 \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29})} \cdot \sqrt{(199631405 + 4 \cdot \sqrt{29})}
\end{aligned}$$

$$\begin{aligned}
& + 4509673516272467429860\sqrt{1218}\sqrt{-4\sqrt{214095423017213}\sqrt{29}} \\
& + 47106822945\sqrt{1218}\sqrt{-180983329 + 45923327\sqrt{29}} + 40699873480 \\
& 352667 + 199631405 + 137769981\sqrt{29}) + 20905585576953\sqrt{1218}\sqrt{(} \\
& 214095423017213\sqrt{29} + 47106822945\sqrt{1218}\sqrt{-180983329 + 4592332} \\
& 7\sqrt{29}) + 40699873480352667)\sqrt{-4\sqrt{214095423017213}\sqrt{29} + 47} \\
& 106822945\sqrt{1218}\sqrt{-180983329 + 45923327\sqrt{29}} + 406998734803526 \\
& 67) + 199631405 + 137769981\sqrt{29})) + 55683134469459984392598\sqrt{42}\sqrt{ } \\
& \sqrt{-180983329 + 45923327\sqrt{29}}/(3601609981798895040\sqrt{-180983329 + } \\
& 45923327\sqrt{29})\sqrt{-4\sqrt{214095423017213}\sqrt{29} + 47106822945\sqrt{ } \\
& (1218)\sqrt{-180983329 + 45923327\sqrt{29}} + 40699873480352667) + 19963140 \\
& 5 + 137769981\sqrt{29}) + 4509673516272467429860\sqrt{1218}\sqrt{-4\sqrt{21} \\
& 4095423017213\sqrt{29} + 47106822945\sqrt{1218}\sqrt{-180983329 + 45923327\sqrt{ } \\
& \sqrt{29}} + 40699873480352667) + 199631405 + 137769981\sqrt{29}) + 20905585 \\
& 576953\sqrt{1218}\sqrt{214095423017213\sqrt{29} + 47106822945\sqrt{1218}\sqrt{ } \\
& \sqrt{-180983329 + 45923327\sqrt{29}} + 40699873480352667)\sqrt{-4\sqrt{214095} \\
& 423017213\sqrt{29} + 47106822945\sqrt{1218}\sqrt{-180983329 + 45923327\sqrt{ } \\
& (29)) + 40699873480352667) + 199631405 + 137769981\sqrt{29}))
\end{aligned}$$

3.51 $\int (1 + 4x + 4x^2 + 4x^4)^4 dx$

Optimal. Leaf size=97

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + \dots$$

[Out] $x + 8x^2 + 112/3x^3 + 112x^4 + 1136/5x^5 + 992/3x^6 + 2752/7x^7 + 448x^8 + 4192/9x^9 + 384x^{10} + 3328/11x^{11} + 256x^{12} + 1792/13x^{13} + 512/7x^{14} + 1024/15x^{15} + 256/17x^{17}$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + \dots$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^4, x]

[Out] $x + 8x^2 + (112x^3)/3 + 112x^4 + (1136x^5)/5 + (992x^6)/3 + (2752x^7)/7 + 448x^8 + (4192x^9)/9 + 384x^{10} + (3328x^{11})/11 + 256x^{12} + (1792x^{13})/13 + (512x^{14})/7 + (1024x^{15})/15 + (256x^{17})/17$

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\int (1 + 4x + 4x^2 + 4x^4)^4 dx = \int (1 + 16x + 112x^2 + 448x^3 + 1136x^4 + 1984x^5 + 2752x^6 + 3584x^7 + 4192x^8 + 384x^9 + 3328x^{10} + 256x^{11} + 1792x^{12} + 512x^{13} + 1024x^{14} + 256x^{15} + 256x^{16} + 256x^{17}) dx$$

$$= x + 8x^2 + \frac{112x^3}{3} + 112x^4 + \frac{1136x^5}{5} + \frac{992x^6}{3} + \frac{2752x^7}{7} + 448x^8 + \frac{4192x^9}{9} + \dots$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.00

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4, x]

[Out] $x + 8x^2 + (112x^3)/3 + 112x^4 + (1136x^5)/5 + (992x^6)/3 + (2752x^7)/7 + 448x^8 + (4192x^9)/9 + 384x^{10} + (3328x^{11})/11 + 256x^{12} + (1792x^{13})/13 + (512x^{14})/7 + (1024x^{15})/15 + (256x^{17})/17$

fricas [A] time = 0.34, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^9 + 448x^8 + \frac{2752}{7}x^7 + \frac{992}{3}x^6 + \frac{1136}{5}x^5 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")

[Out] $256/17*x^{17} + 1024/15*x^{15} + 512/7*x^{14} + 1792/13*x^{13} + 256*x^{12} + 3328/11*x^{11} + 384*x^{10} + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x$

giac [A] time = 0.36, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^9 + 448x^8 + \frac{2752}{7}x^7 + \frac{992}{3}x^6 + \frac{1136}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")

[Out] $256/17*x^{17} + 1024/15*x^{15} + 512/7*x^{14} + 1792/13*x^{13} + 256*x^{12} + 3328/11*x^{11} + 384*x^{10} + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x$

maple [A] time = 0.00, size = 78, normalized size = 0.80

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^9 + 448x^8 + \frac{2752}{7}x^7 + \frac{992}{3}x^6 + \frac{1136}{5}x^5 + 112x^4 + 112/3x^3 + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^4+4*x^2+4*x+1)^4,x)

[Out] $x + 8*x^2 + 112/3*x^3 + 112*x^4 + 1136/5*x^5 + 992/3*x^6 + 2752/7*x^7 + 448*x^8 + 4192/9*x^9 + 384*x^{10} + 3328/11*x^{11} + 256*x^{12} + 1792/13*x^{13} + 512/7*x^{14} + 1024/15*x^{15} + 256/17*x^{17}$

maxima [A] time = 0.57, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^9 + 448x^8 + \frac{2752}{7}x^7 + \frac{992}{3}x^6 + \frac{1136}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")

[Out] $256/17*x^{17} + 1024/15*x^{15} + 512/7*x^{14} + 1792/13*x^{13} + 256*x^{12} + 3328/11*x^{11} + 384*x^{10} + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x$

mupad [B] time = 0.15, size = 77, normalized size = 0.79

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + 112/3x^3 + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4*x^2 + 4*x^4 + 1)^4,x)

[Out] $x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^{10} + (3328*x^{11})/11 + 256*x^{12} + (1792*x^{13})/13 + (512*x^{14})/7 + (1024*x^{15})/15 + (256*x^{17})/17$

sympy [A] time = 0.07, size = 94, normalized size = 0.97

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + 112/3x^3 + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**4+4*x**2+4*x+1)**4,x)

[Out] $256*x^{17}/17 + 1024*x^{15}/15 + 512*x^{14}/7 + 1792*x^{13}/13 + 256*x^{12} + 3328*x^{11}/11 + 384*x^{10} + 4192*x^9/9 + 448*x^8 + 2752*x^7/7 + 992*x^6/3 + 1136*x^5/5 + 112*x^4 + 112*x^3/3 + 8*x^2 + x$

$$3.52 \quad \int (1 + 4x + 4x^2 + 4x^4)^3 dx$$

Optimal. Leaf size=69

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

[Out] $x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^{10}+192/11*x^{11}+64/13*x^{13}$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^3, x]

[Out] $x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 + (80*x^9)/3 + (96*x^{10})/5 + (192*x^{11})/11 + (64*x^{13})/13$

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && ! GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (1 + 4x + 4x^2 + 4x^4)^3 dx &= \int (1 + 12x + 60x^2 + 160x^3 + 252x^4 + 288x^5 + 352x^6 + 384x^7 + 240x^8 + 192x^9 + 64x^{10} + 64x^{13}) dx \\ &= x + 6x^2 + 20x^3 + 40x^4 + \frac{252x^5}{5} + 48x^6 + \frac{352x^7}{7} + 48x^8 + \frac{80x^9}{3} + \frac{96x^{10}}{5} + \frac{64x^{13}}{13} \end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3, x]

[Out] $x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 + (80*x^9)/3 + (96*x^{10})/5 + (192*x^{11})/11 + (64*x^{13})/13$

fricas [A] time = 0.36, size = 57, normalized size = 0.83

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")

[Out] $64/13*x^{13} + 192/11*x^{11} + 96/5*x^{10} + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x$

giac [A] time = 0.36, size = 57, normalized size = 0.83

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")

[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^4+4*x^2+4*x+1)^3,x)

[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+192/11*x^11+64/13*x^13

maxima [A] time = 0.46, size = 57, normalized size = 0.83

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")

[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x

mupad [B] time = 0.06, size = 57, normalized size = 0.83

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4*x^2 + 4*x^4 + 1)^3,x)

[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 + (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13

sympy [A] time = 0.07, size = 66, normalized size = 0.96

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**4+4*x**2+4*x+1)**3,x)

[Out] 64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 352*x**7/7 + 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x

$$3.53 \quad \int (1 + 4x + 4x^2 + 4x^4)^2 dx$$

Optimal. Leaf size=45

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

[Out] $x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^2, x]

[Out] $x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)/9$

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && ! GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (1 + 4x + 4x^2 + 4x^4)^2 dx &= \int (1 + 8x + 24x^2 + 32x^3 + 24x^4 + 32x^5 + 32x^6 + 16x^8) dx \\ &= x + 4x^2 + 8x^3 + 8x^4 + \frac{24x^5}{5} + \frac{16x^6}{3} + \frac{32x^7}{7} + \frac{16x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.00

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2, x]

[Out] $x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)/9$

fricas [A] time = 0.34, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")

[Out] $16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x$

giac [A] time = 0.25, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")

[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x

maple [A] time = 0.00, size = 38, normalized size = 0.84

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^4+4*x^2+4*x+1)^2,x)

[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9

maxima [A] time = 0.57, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")

[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x

mupad [B] time = 0.03, size = 37, normalized size = 0.82

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4*x^2 + 4*x^4 + 1)^2,x)

[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)/9

sympy [A] time = 0.06, size = 42, normalized size = 0.93

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**4+4*x**2+4*x+1)**2,x)

[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 + x

$$3.54 \quad \int (1 + 4x + 4x^2 + 4x^4) dx$$

Optimal. Leaf size=21

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

[Out] $x+2*x^2+4/3*x^3+4/5*x^5$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Antiderivative was successfully verified.

[In] Int[1 + 4*x + 4*x^2 + 4*x^4,x]

[Out] $x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5$

Rubi steps

$$\int (1 + 4x + 4x^2 + 4x^4) dx = x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Antiderivative was successfully verified.

[In] Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]

[Out] $x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5$

fricas [A] time = 0.34, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")

[Out] $4/5*x^5 + 4/3*x^3 + 2*x^2 + x$

giac [A] time = 0.36, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")

[Out] $4/5*x^5 + 4/3*x^3 + 2*x^2 + x$

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(4*x^4+4*x^2+4*x+1,x)`

[Out] `x+2*x^2+4/3*x^3+4/5*x^5`

maxima [A] time = 0.57, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")`

[Out] `4/5*x^5 + 4/3*x^3 + 2*x^2 + x`

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(4*x + 4*x^2 + 4*x^4 + 1,x)`

[Out] `x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5`

sympy [A] time = 0.06, size = 19, normalized size = 0.90

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(4*x**4+4*x**2+4*x+1,x)`

[Out] `4*x**5/5 + 4*x**3/3 + 2*x**2 + x`

$$3.55 \quad \int \frac{1}{1+4x+4x^2+4x^4} dx$$

Optimal. Leaf size=234

$$-\frac{1}{4}\sqrt{\frac{1}{5}}(\sqrt{5}-2)\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2(1+\sqrt{5})}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}}(\sqrt{5}-2)\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2(1+\sqrt{5})}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$

[Out] 1/2*arctan(-1/2+1/2*(1+1/x)^2)-1/20*ln((1+1/x)^2+5^(1/2)-(1+1/x)*(2+2*5^(1/2))^(1/2))*(-10+5*5^(1/2))^(1/2)+1/20*ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(1/2))^(1/2))*(-10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x+(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)

Rubi [A] time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {2069, 1673, 1169, 634, 618, 204, 628, 12, 1107}

$$-\frac{1}{4}\sqrt{\frac{1}{5}}(\sqrt{5}-2)\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2(1+\sqrt{5})}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}}(\sqrt{5}-2)\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2(1+\sqrt{5})}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1), x]

[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])]) + 2/x]/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])]) + 2/x]/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]]*(1 + x^(-1)) + (1 + x^(-1))^2)/4 + (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]]*(1 + x^(-1)) + (1 + x^(-1))^2)/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1107

$\text{Int}[(x_*)*((a_*) + (b_*)*(x_)^2 + (c_*)*(x_)^4)^(p_), x_Symbol] \ :> \ \text{Dist}[1/2, \text{Subst}[\text{Int}[(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, p\}, x]$

Rule 1169

$\text{Int}[(d_*) + (e_*)*(x_)^2]/((a_*) + (b_*)*(x_)^2 + (c_*)*(x_)^4), x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a/c, 2]\}, \text{With}[\{r = \text{Rt}[2*q - b/c, 2]\}, \text{Dist}[1/(2*c*q*r), \text{Int}[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + \text{Dist}[1/(2*c*q*r), \text{Int}[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NegQ}[b^2 - 4*a*c]$

Rule 1673

$\text{Int}[(Pq_*)*((a_*) + (b_*)*(x_)^2 + (c_*)*(x_)^4)^(p_), x_Symbol] \ :> \ \text{Module}[\{q = \text{Expon}[Pq, x], k\}, \text{Int}[\text{Sum}[\text{Coeff}[Pq, x, 2*k]*x^(2*k), \{k, 0, q/2\}](a + b*x^2 + c*x^4)^p, x] + \text{Int}[x*\text{Sum}[\text{Coeff}[Pq, x, 2*k + 1]*x^(2*k), \{k, 0, (q - 1)/2\}](a + b*x^2 + c*x^4)^p, x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ !\text{PolyQ}[Pq, x^2]$

Rule 2069

$\text{Int}[(P4_)^(p_), x_Symbol] \ :> \ \text{With}[\{a = \text{Coeff}[P4, x, 0], b = \text{Coeff}[P4, x, 1], c = \text{Coeff}[P4, x, 2], d = \text{Coeff}[P4, x, 3], e = \text{Coeff}[P4, x, 4]\}, \text{Dist}[-16*a^2, \text{Subst}[\text{Int}[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)))/(b - 4*a*x)^4)^(p_)/(b - 4*a*x)^2, x], x, b/(4*a) + 1/x], x] /; \text{NeQ}[a, 0] \ \&\& \ \text{NeQ}[b, 0] \ \&\& \ \text{EqQ}[b^3 - 4*a*b*c + 8*a^2*d, 0]] /; \text{FreeQ}[p, x] \ \&\& \ \text{PolyQ}[P4, x, 4] \ \&\& \ \text{IntegerQ}[2*p] \ \&\& \ !\text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{1 + 4x + 4x^2 + 4x^4} dx &= -\left(16 \text{Subst}\left(\int \frac{(4 - 4x)^2}{1280 - 512x^2 + 256x^4} dx, x, 1 + \frac{1}{x}\right)\right) \\ &= -\left(16 \text{Subst}\left(\int -\frac{32x}{1280 - 512x^2 + 256x^4} dx, x, 1 + \frac{1}{x}\right)\right) - 16 \text{Subst}\left(\int \frac{16 + 16x}{1280 - 512x^2 + 256x^4} dx, x, 1 + \frac{1}{x}\right) \\ &= 512 \text{Subst}\left(\int \frac{x}{1280 - 512x^2 + 256x^4} dx, x, 1 + \frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{16\sqrt{2(1+\sqrt{5})} - (16-16\sqrt{5})x}{\sqrt{5} - \sqrt{2(1+\sqrt{5})}x + x^2} dx, x, 1 + \frac{1}{x}\right)}{32\sqrt{10(1+\sqrt{5})}} \\ &= 256 \text{Subst}\left(\int \frac{1}{1280 - 512x + 256x^2} dx, x, \left(1 + \frac{1}{x}\right)^2\right) + \frac{(1 - \sqrt{5}) \text{Subst}\left(\int \frac{-\sqrt{2(1+\sqrt{5})}}{\sqrt{5} - \sqrt{2(1+\sqrt{5})}x + x^2} dx, x, 1 + \frac{1}{x}\right)}{4\sqrt{10(1+\sqrt{5})}} \\ &= -\frac{1}{4}\sqrt{-\frac{2}{5} + \frac{1}{\sqrt{5}}} \log\left(\sqrt{5} - \sqrt{2(1+\sqrt{5})}\left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right) + \frac{1}{4}\sqrt{-\frac{2}{5} + \frac{1}{\sqrt{5}}} \log\left(\sqrt{5} + \sqrt{2(1+\sqrt{5})}\left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right) \\ &= \frac{1}{2} \tan^{-1}\left(\frac{1}{2}\left(-1 + \left(1 + \frac{1}{x}\right)^2\right)\right) - \frac{(1 + \sqrt{5})^{3/2} \tan^{-1}\left(\frac{2 - \sqrt{2(1+\sqrt{5})} + \frac{2}{x}}{\sqrt{2(-1+\sqrt{5})}}\right)}{4\sqrt{10}} - \frac{(1 + \sqrt{5})^{3/2} \tan^{-1}\left(\frac{2 + \sqrt{2(1+\sqrt{5})} + \frac{2}{x}}{\sqrt{2(-1+\sqrt{5})}}\right)}{4\sqrt{10}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.20

$$\frac{1}{4} \text{RootSum} \left[4\#1^4 + 4\#1^2 + 4\#1 + 1 \&, \frac{\log(x - \#1)}{4\#1^3 + 2\#1 + 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1), x]

[Out] RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) &]/4

fricas [C] time = 1.33, size = 499, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1), x, algorithm="fricas")

[Out] -1/20*(sqrt(10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) - 5*sqrt(1/10*I - 1/5) - 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^2 + ((6*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1/5) - I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) + 1/20*(sqrt(10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 5*sqrt(1/10*I - 1/5) + 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^2 - ((6*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1/5) - I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) - 1/4*(2*sqrt(1/10*I - 1/5) - I)*log(-5*(2*sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) - 30*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 30*(2*sqrt(-1/10*I - 1/5) + I)^3 + 8*x - 216*sqrt(-1/10*I - 1/5) - 108*I + 21) - 1/4*(2*sqrt(-1/10*I - 1/5) + I)*log(30*(2*sqrt(-1/10*I - 1/5) + I)^3 + 5*(2*sqrt(-1/10*I - 1/5) + I)^2 + 8*x + 216*sqrt(-1/10*I - 1/5) + 108*I - 27)

giac [C] time = 0.52, size = 265, normalized size = 1.13

$$-\frac{1}{20} \left((i+2) \sqrt{\sqrt{5}-2} \left(\frac{i}{\sqrt{5}-2} + 1 \right) + 5i \right) \log \left((406i+174) \sqrt{5}x + (868i+372)x + 29\sqrt{5} \sqrt{29\sqrt{5}+62} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1), x, algorithm="giac")

[Out] -1/20*((I + 2)*sqrt(sqrt(5) - 2)*(I/(sqrt(5) - 2) + 1) + 5*I)*log((406*I + 174)*sqrt(5)*x + (868*I + 372)*x + 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I - 203)*sqrt(5) + (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*((I + 2)*sqrt(sqrt(5) - 2)*(-I/(sqrt(5) - 2) - 1) + 5*I)*log((406*I + 174)*sqrt(5)*x + (868*I + 372)*x - 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I - 203)*sqrt(5) - (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*((2*I + 1)*sqrt(sqrt(5) + 2)*(-I/(sqrt(5) + 2) - 1) - 5*I)*log((26*I + 130)*sqrt(5)*x - (44*I + 220)*x + 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(5) + (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22) - 1/20*((2*I + 1)*sqrt

$(\sqrt{5} + 2) \cdot (1/(\sqrt{5} + 2) + 1) - 5I) \cdot \log((26I + 130) \cdot \sqrt{5} \cdot x - (44I + 220) \cdot x - 13 \cdot \sqrt{5} \cdot \sqrt{13 \cdot \sqrt{5} - 22}) - (65I - 13) \cdot \sqrt{5} - (19I - 22) \cdot \sqrt{13 \cdot \sqrt{5} - 22}) + 110I - 22)$

maple [C] time = 0.01, size = 41, normalized size = 0.18

$$\frac{\ln(-\text{RootOf}(4_Z^4 + 4_Z^2 + 4_Z + 1) + x)}{16 \text{RootOf}(4_Z^4 + 4_Z^2 + 4_Z + 1)^3 + 8 \text{RootOf}(4_Z^4 + 4_Z^2 + 4_Z + 1) + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^4+4*x^2+4*x+1),x)

[Out] 1/4*sum(1/(4*_R^3+2*_R+1)*ln(-_R+x),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4x^4 + 4x^2 + 4x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")

[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)

mupad [B] time = 2.36, size = 87, normalized size = 0.37

$$\sum_{k=1}^4 \ln\left(-\text{root}\left(z^4 + \frac{9z^2}{40} + \frac{z}{40} + \frac{1}{1280}, z, k\right) \left(\frac{x}{4} + \text{root}\left(z^4 + \frac{9z^2}{40} + \frac{z}{40} + \frac{1}{1280}, z, k\right) \left(6x + \text{root}\left(z^4 + \frac{9z^2}{40} + \frac{z}{40} + \frac{1}{1280}, z, k\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1),x)

[Out] symsum(log(-root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(x/4 + root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(6*x + root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(36*x + 16))))*root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k), k, 1, 4)

sympy [B] time = 2.57, size = 3432, normalized size = 14.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x**4+4*x**2+4*x+1),x)

[Out] $\sqrt{-1/40 + \sqrt{5}/80} \cdot \log(x^2 + x \cdot (-8 - 21 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}}) / 10 - \sqrt{-2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 2 - \sqrt{5} / 2 + 12 \cdot \sqrt{-2 + \sqrt{5}} + 9 \cdot \sqrt{5} \cdot \sqrt{-2 + \sqrt{5}} \cdot \sqrt{-2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 5 - 841 \cdot \sqrt{5} \cdot \sqrt{-2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 20 - 14351 / 40 - 441 \cdot \sqrt{-2 + \sqrt{5}} / 4 - 75 \cdot \sqrt{5} \cdot \sqrt{-2 + \sqrt{5}} \cdot \sqrt{-2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 8 - 3 \cdot \sqrt{-2 + \sqrt{5}} \cdot \sqrt{-2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} + 301 \cdot \sqrt{5} \cdot \sqrt{-2 + \sqrt{5}} / 10 + 7407 \cdot \sqrt{5} / 40 + 3913 \cdot \sqrt{-2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 40 - \sqrt{-1/40 + \sqrt{5}/80} \cdot \log(x^2 + x \cdot (-8 - 12 \cdot \sqrt{-2 + \sqrt{5}} - \sqrt{5} / 2 + 21 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}}) / 10 + \sqrt{2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 2 + 9 \cdot \sqrt{5} \cdot \sqrt{-2 + \sqrt{5}} \cdot \sqrt{2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 5 - 3913 \cdot \sqrt{2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 40 - 14351 / 40 - 75 \cdot \sqrt{5} \cdot \sqrt{-2 + \sqrt{5}} \cdot \sqrt{2 \cdot \sqrt{5}) \cdot \sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19} / 8 - 301 \cdot \sqrt{5} \cdot \sqrt{-2 + \sqrt{5}} / 10$

$19) + 3 + 3\sqrt{5}) + 6\sqrt{5}\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5}$
 $(+ 19)\sqrt{-2\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19) + 3 + 3\sqrt{5}}$
 $\sqrt{5})) + 5\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19)/(5\sqrt{-2 + \sqrt{5}})$
 $\sqrt{-2\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19) + 3 + 3\sqrt{5}}$
 $+ 27\sqrt{5}\sqrt{-2\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19) + 3 + 3\sqrt{5}}$
 $+ 6\sqrt{5}\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19)\sqrt{-2\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19) + 3 + 3\sqrt{5}}$
 $\sqrt{5})) + 18\sqrt{5}\sqrt{-2 + \sqrt{5}})\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19)/(5\sqrt{-2 + \sqrt{5}})$
 $\sqrt{-2\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19) + 3 + 3\sqrt{5}}) + 27\sqrt{5}\sqrt{-2\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19) + 3 + 3\sqrt{5}}$
 $+ 6\sqrt{5}\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19)\sqrt{-2\sqrt{2\sqrt{5}\sqrt{-2 + \sqrt{5}}) + \sqrt{5} + 19) + 3 + 3\sqrt{5}}))$

$$3.56 \quad \int \frac{1}{(1+4x+4x^2+4x^4)^2} dx$$

Optimal. Leaf size=317

$$-\frac{17 - \left(\frac{1}{x} + 1\right)^2}{2 \left(\left(\frac{1}{x} + 1\right)^4 - 2 \left(\frac{1}{x} + 1\right)^2 + 5\right)} + \frac{\left(59 - 17 \left(\frac{1}{x} + 1\right)^2\right) \left(\frac{1}{x} + 1\right)}{10 \left(\left(\frac{1}{x} + 1\right)^4 - 2 \left(\frac{1}{x} + 1\right)^2 + 5\right)} + \frac{1}{40} \sqrt{\frac{1}{10} (2665\sqrt{5} - 5959)} \log \left(\left(\frac{1}{x} + 1\right)^2 - \dots \right)$$

[Out] 1/2*(-17+(1+1/x)^2)/(5-2*(1+1/x)^2+(1+1/x)^4)+1/10*(59-17*(1+1/x)^2)*(1+1/x)/(5-2*(1+1/x)^2+(1+1/x)^4)+7/4*arctan(-1/2+1/2*(1+1/x)^2)+1/400*ln((1+1/x)^2+5^(1/2)-(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1/2)-1/400*ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1/2)-1/200*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(59590+26650*5^(1/2))^(1/2)-1/200*arctan((2+2/x+(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(59590+26650*5^(1/2))^(1/2)

Rubi [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2069, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660, 12}

$$-\frac{17 - \left(\frac{1}{x} + 1\right)^2}{2 \left(\left(\frac{1}{x} + 1\right)^4 - 2 \left(\frac{1}{x} + 1\right)^2 + 5\right)} + \frac{\left(59 - 17 \left(\frac{1}{x} + 1\right)^2\right) \left(\frac{1}{x} + 1\right)}{10 \left(\left(\frac{1}{x} + 1\right)^4 - 2 \left(\frac{1}{x} + 1\right)^2 + 5\right)} + \frac{1}{40} \sqrt{\frac{1}{10} (2665\sqrt{5} - 5959)} \log \left(\left(\frac{1}{x} + 1\right)^2 - \dots \right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2), x]

[Out] -(17 - (1 + x^(-1))^2)/(2*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)) + ((59 - 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 - (Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])]] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])]] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]]*(1 + x^(-1)) + (1 + x^(-1))^2])/40

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^(
p, x), x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
+ 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1],
c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*a^2,
Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)))/(b - 4*a*x)^4]^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+4x+4x^2+4x^4)^2} dx &= - \left(16 \operatorname{Subst} \left(\int \frac{(4-4x)^6}{(1280-512x^2+256x^4)^2} dx, x, 1+\frac{1}{x} \right) \right) \\ &= - \left(16 \operatorname{Subst} \left(\int \frac{x(-24576-81920x^2-24576x^4)}{(1280-512x^2+256x^4)^2} dx, x, 1+\frac{1}{x} \right) \right) - 16 \operatorname{Subst} \left(\int \frac{(59-17\left(1+\frac{1}{x}\right)^2)\left(1+\frac{1}{x}\right)}{(1280-512x^2+256x^4)^2} dx, x, 1+\frac{1}{x} \right) \\ &= \frac{(59-17\left(1+\frac{1}{x}\right)^2)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\operatorname{Subst} \left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x} \right)}{167772160} \\ &= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{(59-17\left(1+\frac{1}{x}\right)^2)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\operatorname{Subst} \left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x} \right)}{167772160} \\ &= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{(59-17\left(1+\frac{1}{x}\right)^2)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + 896 \operatorname{Subst} \left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x} \right) \\ &= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{(59-17\left(1+\frac{1}{x}\right)^2)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{1}{40} \sqrt{-\frac{595}{10}} \\ &= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{(59-17\left(1+\frac{1}{x}\right)^2)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{7}{4} \tan^{-1} \left(\frac{1}{2} \sqrt{-\frac{595}{10}} \right) \end{aligned}$$

Mathematica [C] time = 0.02, size = 108, normalized size = 0.34

$$\frac{1}{40} \left(\operatorname{RootSum} \left[4\#1^4 + 4\#1^2 + 4\#1 + 1 \&, \frac{18\#1^2 \log(x - \#1) - 16\#1 \log(x - \#1) + 27 \log(x - \#1)}{4\#1^3 + 2\#1 + 1} \& \right] + \frac{72x^3 - 72x^2 + 72x - 1}{4x^4 + 4x^2 + 4x + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2), x]

[Out] ((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 &, (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2)/(1 + 2*#1 + 4*#1^3) &])/40

fricas [C] time = 1.34, size = 704, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")

[Out] 1/400*(720*x^3 - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*log(33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 11755375/4*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 541735337*x + 25784243612*sqrt(19/1000*I - 5959/2000) + 45122426321*I - 71080995) - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)*log(-33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 125/4*(4271136*sqrt(19/1000*I - 5959/2000) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 25*(1334730*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 541735337*x - 25806203712*sqrt(19/1000*I - 5959/2000) - 45160856496*I - 355111539) - 320*x^2 - (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 1/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 878404*sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 38430175/2*I + 213096267) + (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 1/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 878404*sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 38430175/2*I + 213096267) + 840*x + 380)/(4*x^4 + 4*x^2 + 4*x + 1)

giac [C] time = 0.72, size = 315, normalized size = 0.99

$$-\frac{1}{400} \left(-(i+3) \sqrt{2665\sqrt{5} - 4790} \left(\frac{709i}{533\sqrt{5} - 958} + 1 \right) - 350i \right) \log \left((2534636224790i + 16853816172010) \sqrt{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")

[Out] -1/400*(-(I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1) - 350*I)*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*I + 26022625108780)*x + 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118) - (8426908086005*I - 1267318112395)*sqrt(5) + (8166407345*I - 7795873310)*sqrt(1424281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*((I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1) - 350*I)*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*I + 2602

2625108780)*x - 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118) - (8426908086005*I - 1267318112395)*sqrt(5) - (8166407345*I - 7795873310)*sqrt(1424281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*((3*I + 1)*sqrt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((16722951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 3814385585140)*x + 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (1240411094455*I - 8361475596225)*sqrt(5) + (8153361745*I + 7721428310)*sqrt(1416617*sqrt(5) + 2178118) + 1907192792570*I - 12856178136150) - 1/400*(-(3*I + 1)*sqrt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((16722951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 3814385585140)*x - 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (1240411094455*I - 8361475596225)*sqrt(5) - (8153361745*I + 7721428310)*sqrt(1416617*sqrt(5) + 2178118) + 1907192792570*I - 12856178136150) + 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1)

maple [C] time = 0.01, size = 79, normalized size = 0.25

$$\frac{\left(18 \operatorname{RootOf}\left(4_Z^4 + 4_Z^2 + 4_Z + 1\right)^2 - 16 \operatorname{RootOf}\left(4_Z^4 + 4_Z^2 + 4_Z + 1\right) + 27\right) \ln\left(-\operatorname{RootOf}\left(4_Z^4 + 4_Z^2 + 4_Z + 1\right)\right)}{160 \operatorname{RootOf}\left(4_Z^4 + 4_Z^2 + 4_Z + 1\right)^3 + 80 \operatorname{RootOf}\left(4_Z^4 + 4_Z^2 + 4_Z + 1\right) + 40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^4+4*x^2+4*x+1)^2,x)

[Out] (9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^2-16*_R+27)/(4*_R^3+2*_R+1)*ln(-_R+x),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{36x^3 - 16x^2 + 42x + 19}{20(4x^4 + 4x^2 + 4x + 1)} + \frac{1}{10} \int \frac{18x^2 - 16x + 27}{4x^4 + 4x^2 + 4x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")

[Out] 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*integrate((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)

mupad [B] time = 2.21, size = 174, normalized size = 0.55

$$\left(\sum_{k=1}^4 \ln\left(\frac{169 \operatorname{root}\left(z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}, z, k\right)}{100} + \frac{11x}{1600} + \frac{\operatorname{root}\left(z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}, z, k\right)}{100} - \frac{11x}{1600}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^2,x)

[Out] symsum(log((11*x)/1600 - (169*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k))/100 + (131*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)*x)/100 - (72*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^2*x)/5 - 36*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^3*x + (59*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^2)/20 - 16*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^3 + 27/1600)*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k), k, 1, 4) + ((21*x)/40 - x^2/5 + (9*x^3)/20 + 19/80)/(x + x^2 + x^4 + 1/4)

sympy [B] time = 3.66, size = 3834, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)

[Out] $(36x^3 - 16x^2 + 42x + 19)/(80x^4 + 80x^2 + 80x + 20) - \sqrt{-5959/16000 + 533\sqrt{5}/3200} \log(x^2 + x(-1601676\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639)/13543383425 - 1067784\sqrt{2}\sqrt{-5959 + 2665\sqrt{5}}/1016389 + 3131659367\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}/13543383425 + 291689395/1083470674 + 470215\sqrt{5}/2032778 + 94043\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/541735337 - 40634464149111451\sqrt{5}\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/27530691871904650 - 2885835544225227917282997/146738587677251784500 - 83803227754187\sqrt{2}\sqrt{-5959 + 2665\sqrt{5}}/100111606806926 - 50208805356\sqrt{2}\sqrt{-5959 + 2665\sqrt{5}}\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/550613837438093 - 538485754891933\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/14673858767725178450 - 925321955096901411\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}/29347717535450356900 + 484304611938766076267\sqrt{5}/55061383743809300 + 22013036087014785403\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/6669935803511444750 + \sqrt{-5959/16000 + 533\sqrt{5}/3200} \log(x^2 + x(-94043\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639}/541735337 - 1601676\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}})\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/13543383425 - 3131659367\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}/13543383425 + 291689395/1083470674 + 1067784\sqrt{2}\sqrt{-5959 + 2665\sqrt{5}}/1016389 + 470215\sqrt{5}/2032778 - 22013036087014785403\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/6669935803511444750 - 2885835544225227917282997/146738587677251784500 - 50208805356\sqrt{2}\sqrt{-5959 + 2665\sqrt{5}}\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/550613837438093 - 538485754891933\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/14673858767725178450 + 925321955096901411\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}/29347717535450356900 + 83803227754187\sqrt{2}\sqrt{-5959 + 2665\sqrt{5}}/100111606806926 + 484304611938766076267\sqrt{5}/55061383743809300 + 40634464149111451\sqrt{5}\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/27530691871904650 + 2\sqrt{6291/16000 + 1599\sqrt{5}/3200} + \sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/4000) \operatorname{atan}(54173533700x/(-6440570878\sqrt{10}\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})) + 2351075\sqrt{-5959 + 2665\sqrt{5}}\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})) + 1067784\sqrt{10}\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}) - 3203352\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}/(-6440570878\sqrt{10}\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})) + 2351075\sqrt{-5959 + 2665\sqrt{5}}\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}) - 28456443600\sqrt{2}\sqrt{-5959 + 2665\sqrt{5}}/(-6440570878\sqrt{10}\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})) + 2351075\sqrt{-5959 + 2665\sqrt{5}}\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}) + 1067784\sqrt{10}\sqrt{6291 + 7995\sqrt{5}} + 4\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639})\sqrt{-665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}} + 221195\sqrt{5} + 36004639}) + 221195\sqrt{5} + 36004639)$

$$\begin{aligned}
& 1195\sqrt{5} + 36004639) + 6291 + 7995\sqrt{5}) + 6440570878\sqrt{10}\sqrt{(-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} \\
&) + 6291 + 7995\sqrt{5}) + 1067784\sqrt{10}\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639)\sqrt{-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} + 6291 + 7995\sqrt{5})) + \\
& 28456443600\sqrt{2}\sqrt{-5959 + 2665\sqrt{5}})/(2351075\sqrt{-5959 + 2665\sqrt{5}})\sqrt{-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} + 6291 + 7995\sqrt{5}) + 6440570878\sqrt{10}\sqrt{-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} + 6291 \\
& + 7995\sqrt{5}) + 1067784\sqrt{10}\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639)\sqrt{-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} + 6291 + 7995\sqrt{5})) + 62656148 \\
& 75\sqrt{5}/(2351075\sqrt{-5959 + 2665\sqrt{5}})\sqrt{-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} + 6291 + 7995\sqrt{5}) + 6440570878\sqrt{10}\sqrt{-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} \\
&) + 221195\sqrt{5} + 36004639) + 6291 + 7995\sqrt{5}) + 1067784\sqrt{10}\sqrt{665\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639)\sqrt{-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} \\
&) + 221195\sqrt{5} + 36004639)\sqrt{-4\sqrt{665}\sqrt{10}\sqrt{-5959 + 2665\sqrt{5}}) + 221195\sqrt{5} + 36004639} + 6291 + 7995\sqrt{5}))
\end{aligned}$$

$$3.57 \quad \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^4 dx$$

Optimal. Leaf size=104

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8$$

[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17

Rubi [A] time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4, x]

[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/15 - 1920*x^16 + (4096*x^17)/17

Rule 2061

Int[(P_)^(p_), x_Symbol] :=> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^4 dx &= \int (4096 + 49152x + 237568x^2 + 559104x^3 + 538624x^4 - 184320x^5 - \\ &= 4096x + 24576x^2 + \frac{237568x^3}{3} + 139776x^4 + \frac{538624x^5}{5} - 30720x^6 - \end{aligned}$$

Mathematica [A] time = 0.00, size = 104, normalized size = 1.00

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4, x]

[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/15 - 1920*x^16 + (4096*x^17)/17

fricas [A] time = 0.37, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")

[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 + 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x

giac [A] time = 0.36, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - 566912/7x^7 - 30720x^6 + 538624/5x^5 + 139776x^4 + 237568/3x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")

[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 + 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x

maple [A] time = 0.00, size = 85, normalized size = 0.82

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - 566912/7x^7 - 30720x^6 + 538624/5x^5 + 139776x^4 + 237568/3x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x)

[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17

maxima [A] time = 0.66, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - 566912/7x^7 - 30720x^6 + 538624/5x^5 + 139776x^4 + 237568/3x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")

[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 + 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x

mupad [B] time = 2.23, size = 84, normalized size = 0.81

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8 - 566912/7x^7 - 30720x^6 + 538624/5x^5 + 139776x^4 + 237568/3x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^4,x)

[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/15 - 1920*x^16 + (4096*x^17)/17

sympy [A] time = 0.08, size = 100, normalized size = 0.96

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8 - 566912/7x^7 - 30720x^6 + 538624/5x^5 + 139776x^4 + 237568/3x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13/13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 36384*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 237568*x**3/3 + 24576*x**2 + 4096*x
```

$$3.58 \quad \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^3 dx$$

Optimal. Leaf size=76

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-2936/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3, x]

[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12 + (512*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && IntegerQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^3 dx &= \int (512 + 4608x + 15360x^2 + 20160x^3 - 384x^4 - 17856x^5 + 5528x^6 + 16128x^7 - 2976x^8 + 2097x^9 - 2936x^{10} + 6936x^{11} - 240x^{12} + 512x^{13}) dx \\ &= 512x + 2304x^2 + 5120x^3 + 5040x^4 - \frac{384x^5}{5} - 2976x^6 + \frac{5528x^7}{7} + 2097x^8 - \frac{2936x^9}{3} - \frac{4527x^{10}}{10} + \frac{6936x^{11}}{11} - 240x^{12} + \frac{512x^{13}}{13} \end{aligned}$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.00

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3, x]

[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12 + (512*x^13)/13

fricas [A] time = 0.34, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")

[Out] $512/13x^{13} - 240x^{12} + 6936/11x^{11} - 4527/10x^{10} - 2936/3x^9 + 2097x^8 + 5528/7x^7 - 2976x^6 - 384/5x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$

giac [A] time = 0.38, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")

[Out] $512/13x^{13} - 240x^{12} + 6936/11x^{11} - 4527/10x^{10} - 2936/3x^9 + 2097x^8 + 5528/7x^7 - 2976x^6 - 384/5x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$

maple [A] time = 0.00, size = 65, normalized size = 0.86

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x)

[Out] $512x + 2304x^2 + 5120x^3 + 5040x^4 - 384/5x^5 - 2976x^6 + 5528/7x^7 + 2097x^8 - 2936/3x^9 - 4527/10x^{10} + 6936/11x^{11} - 240x^{12} + 512/13x^{13}$

maxima [A] time = 0.65, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")

[Out] $512/13x^{13} - 240x^{12} + 6936/11x^{11} - 4527/10x^{10} - 2936/3x^9 + 2097x^8 + 5528/7x^7 - 2976x^6 - 384/5x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$

mapad [B] time = 0.08, size = 64, normalized size = 0.84

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^3,x)

[Out] $512x + 2304x^2 + 5120x^3 + 5040x^4 - (384x^5)/5 - 2976x^6 + (5528x^7)/7 + 2097x^8 - (2936x^9)/3 - (4527x^{10})/10 + (6936x^{11})/11 - 240x^{12} + (512x^{13})/13$

sympy [A] time = 0.08, size = 73, normalized size = 0.96

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)

[Out] $512x^{13}/13 - 240x^{12} + 6936x^{11}/11 - 4527x^{10}/10 - 2936x^9/3 + 2097x^8 + 5528x^7/7 - 2976x^6 - 384x^5/5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$

$$3.59 \quad \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^2 dx$$

Optimal. Leaf size=52

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2, x]

[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7 - 30*x^8 + (64*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^2 dx &= \int (64 + 384x + 704x^2 + 144x^3 - 528x^4 + 144x^5 + 353x^6 - 240x^7 + 64x^8) dx \\ &= 64x + 192x^2 + \frac{704x^3}{3} + 36x^4 - \frac{528x^5}{5} + 24x^6 + \frac{353x^7}{7} - 30x^8 + \frac{64x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 52, normalized size = 1.00

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2, x]

[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7 - 30*x^8 + (64*x^9)/9

fricas [A] time = 0.34, size = 44, normalized size = 0.85

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")

[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 192*x^2 + 64*x

giac [A] time = 0.36, size = 44, normalized size = 0.85

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")

[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 192*x^2 + 64*x

maple [A] time = 0.00, size = 45, normalized size = 0.87

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x)

[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9

maxima [A] time = 0.61, size = 44, normalized size = 0.85

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")

[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 192*x^2 + 64*x

mupad [B] time = 0.03, size = 44, normalized size = 0.85

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)

[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7 - 30*x^8 + (64*x^9)/9

sympy [A] time = 0.07, size = 49, normalized size = 0.94

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)

[Out] 64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x**3/3 + 192*x**2 + 64*x

$$3.60 \quad \int (8 + 24x + 8x^2 - 15x^3 + 8x^4) dx$$

Optimal. Leaf size=30

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Antiderivative was successfully verified.

[In] Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]

[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5

Rubi steps

$$\int (8 + 24x + 8x^2 - 15x^3 + 8x^4) dx = 8x + 12x^2 + \frac{8x^3}{3} - \frac{15x^4}{4} + \frac{8x^5}{5}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Antiderivative was successfully verified.

[In] Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]

[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5

fricas [A] time = 0.34, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")

[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x

giac [A] time = 0.30, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")

[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(8*x^4-15*x^3+8*x^2+24*x+8,x)`

[Out] `8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5`

maxima [A] time = 0.46, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")`

[Out] `8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x`

mupad [B] time = 0.02, size = 24, normalized size = 0.80

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8,x)`

[Out] `8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5`

sympy [A] time = 0.06, size = 27, normalized size = 0.90

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)`

[Out] `8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x`

$$3.61 \quad \int \frac{1}{8+24x+8x^2-15x^3+8x^4} dx$$

Optimal. Leaf size=263

$$\frac{1}{4}\sqrt{\frac{3}{13}} \tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right) - \frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}} \log\left(\left(\frac{4}{x}+3\right)^2 - \sqrt{2(19+\sqrt{517})}\left(\frac{4}{x}+3\right) + \sqrt{517}\right) + \frac{1}{8}\sqrt{\frac{235\sqrt{517}+5167}{40326}} \log\left(\left(\frac{4}{x}+3\right)^2 + \sqrt{2(19+\sqrt{517})}\left(\frac{4}{x}+3\right) + \sqrt{517}\right) + \frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}} \log\left(\left(\frac{4}{x}+3\right)^2 - \sqrt{2(19-\sqrt{517})}\left(\frac{4}{x}+3\right) + \sqrt{517}\right) + \frac{1}{8}\sqrt{\frac{235\sqrt{517}+5167}{40326}} \log\left(\left(\frac{4}{x}+3\right)^2 + \sqrt{2(19-\sqrt{517})}\left(\frac{4}{x}+3\right) + \sqrt{517}\right)$$

[Out] 1/52*arctan(1/39*(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/322608*ln((3+4/x)^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))*(-208364442+9476610*517^(1/2))^(1/2)+1/322608*ln((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-208364442+9476610*517^(1/2))^(1/2)-1/161304*arctan((6+8/x-(38+2*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9476610*517^(1/2))^(1/2)-1/161304*arctan((6+8/x+(38+2*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9476610*517^(1/2))^(1/2)

Rubi [A] time = 0.49, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$\frac{1}{4}\sqrt{\frac{3}{13}} \tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}x^2}\right) - \frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}} \log\left(\left(\frac{4}{x}+3\right)^2 - \sqrt{2(19+\sqrt{517})}\left(\frac{4}{x}+3\right) + \sqrt{517}\right) + \frac{1}{8}\sqrt{\frac{235\sqrt{517}+5167}{40326}} \log\left(\left(\frac{4}{x}+3\right)^2 + \sqrt{2(19+\sqrt{517})}\left(\frac{4}{x}+3\right) + \sqrt{517}\right) + \frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}} \log\left(\left(\frac{4}{x}+3\right)^2 - \sqrt{2(19-\sqrt{517})}\left(\frac{4}{x}+3\right) + \sqrt{517}\right) + \frac{1}{8}\sqrt{\frac{235\sqrt{517}+5167}{40326}} \log\left(\left(\frac{4}{x}+3\right)^2 + \sqrt{2(19-\sqrt{517})}\left(\frac{4}{x}+3\right) + \sqrt{517}\right)$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1), x]

[Out] -(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])]) + 8/x]/Sqrt[2*(-19 + Sqrt[517])]])/4 - (Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 + Sqrt[2*(19 + Sqrt[517])]) + 8/x]/Sqrt[2*(-19 + Sqrt[517])]])/4 + (Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)]/4 - (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]]*(3 + 4/x) + (3 + 4/x)^2])/8 + (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]]*(3 + 4/x) + (3 + 4/x)^2])/8

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c))*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4]^(p_)/(b - 4*a*x)^2, x], x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^2*d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
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Rubi steps

$$\begin{aligned}
\int \frac{1}{8 + 24x + 8x^2 - 15x^3 + 8x^4} dx &= - \left(1024 \operatorname{Subst} \left(\int \frac{(24 - 32x)^2}{8(2117632 - 2490368x^2 + 1048576x^4)} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right) \\
&= - \left(128 \operatorname{Subst} \left(\int \frac{(24 - 32x)^2}{2117632 - 2490368x^2 + 1048576x^4} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right) \\
&= - \left(128 \operatorname{Subst} \left(\int -\frac{1536x}{2117632 - 2490368x^2 + 1048576x^4} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right) - 128 \\
&= 196608 \operatorname{Subst} \left(\int \frac{x}{2117632 - 2490368x^2 + 1048576x^4} dx, x, \frac{3}{4} + \frac{1}{x} \right) - \frac{\dots}{\dots} \\
&= 98304 \operatorname{Subst} \left(\int \frac{1}{2117632 - 2490368x + 1048576x^2} dx, x, \left(\frac{3}{4} + \frac{1}{x} \right)^2 \right) - \frac{\dots}{\dots} \\
&= -\frac{1}{8} \sqrt{\frac{-5167 + 235\sqrt{517}}{40326}} \log \left(\sqrt{517} - \sqrt{2(19 + \sqrt{517})} \left(3 + \frac{4}{x} \right) + \left(3 + \frac{4}{x} \right)^2 \right) \\
&= -\frac{1}{4} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{19 - \left(3 + \frac{4}{x} \right)^2}{2\sqrt{39}} \right) - \frac{1}{4} \sqrt{\frac{5167 + 235\sqrt{517}}{40326}} \tan^{-1} \left(\frac{6 + \sqrt{2(19 + \sqrt{517})}}{\sqrt{2(-19 + \sqrt{517})}} \right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.21

$$\operatorname{RootSum} \left[8\#1^4 - 15\#1^3 + 8\#1^2 + 24\#1 + 8, \frac{\log(x - \#1)}{32\#1^3 - 45\#1^2 + 16\#1 + 24} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1), x]

[Out] RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1 - 45*#1^2 + 32*#1^3) &]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8x^4 - 15x^3 + 8x^2 + 24x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8), x, algorithm="giac")

[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)

maple [C] time = 0.01, size = 49, normalized size = 0.19

$$\frac{\ln\left(-\text{RootOf}\left(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8\right) + x\right)}{32\text{RootOf}\left(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8\right)^3 - 45\text{RootOf}\left(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8\right)^2 + 16\text{RootOf}\left(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8\right) - 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8), x)

[Out] sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(-_R+x), _R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8x^4 - 15x^3 + 8x^2 + 24x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8), x, algorithm="maxima")

[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)

mupad [B] time = 0.41, size = 123, normalized size = 0.47

$$\sum_{k=1}^4 \ln \left(\frac{\text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right) \left(2184 \text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right) + 256x + \text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right)\right)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8), x)

[Out] symsum(log(-(root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) * (2184*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) + 256*x + 38259*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) * x + 1531920*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k)^2*x + 805896*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k)^2 - 120))/4096)*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k), k, 1, 4)

sympy [A] time = 2.37, size = 41, normalized size = 0.16

$$\text{RootSum}\left(50326848t^4 + 765960t^2 + 12753t + 64, \left(t \mapsto t \log\left(\frac{100785893208t^3}{4758335} - \frac{1430512512t^2}{4758335} + \frac{72982352t}{223641} + 1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8), x)

[Out] RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(100785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/223641745 + x + 2270349121/1789133960)))

$$3.62 \quad \int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2} dx$$

Optimal. Leaf size=366

$$\frac{73}{208} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{-5x^2 + 12x + 8}{\sqrt{39}x^2} \right) - \frac{3 \left(3359 - 107 \left(\frac{4}{x} + 3 \right)^2 \right)}{208 \left(\left(\frac{4}{x} + 3 \right)^4 - 38 \left(\frac{4}{x} + 3 \right)^2 + 517 \right)} + \frac{\left(3327931 - 129631 \left(\frac{4}{x} + 3 \right)^2 \right) \left(\frac{4}{x} + 3 \right)}{322608 \left(\left(\frac{4}{x} + 3 \right)^4 - 38 \left(\frac{4}{x} + 3 \right)^2 + 517 \right)}$$

[Out] -3/208*(3359-107*(3+4/x)^2)/(517-38*(3+4/x)^2+(3+4/x)^4)+1/322608*(3327931-129631*(3+4/x)^2)*(3+4/x)/(517-38*(3+4/x)^2+(3+4/x)^4)+73/2704*arctan(1/39*(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/26018980416*arctan((6+8/x-(38+2*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+40326*517^(1/2))^(1/2)-1/26018980416*arctan((6+8/x+(38+2*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+40326*517^(1/2))^(1/2)-1/26018980416*ln((3+4/x)^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))*(-2405208568240933026+105781971094684170*517^(1/2))^(1/2)+1/26018980416*ln((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-2405208568240933026+105781971094684170*517^(1/2))^(1/2)

Rubi [A] time = 0.51, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$\frac{73}{208} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{-5x^2 + 12x + 8}{\sqrt{39}x^2} \right) - \frac{3 \left(3359 - 107 \left(\frac{4}{x} + 3 \right)^2 \right)}{208 \left(\left(\frac{4}{x} + 3 \right)^4 - 38 \left(\frac{4}{x} + 3 \right)^2 + 517 \right)} + \frac{\left(3327931 - 129631 \left(\frac{4}{x} + 3 \right)^2 \right) \left(\frac{4}{x} + 3 \right)}{322608 \left(\left(\frac{4}{x} + 3 \right)^4 - 38 \left(\frac{4}{x} + 3 \right)^2 + 517 \right)}$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2), x]

[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) + ((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517]])])/645216 - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517]])])/645216 + (73*Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-59644114671451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]]*(3 + 4/x) + (3 + 4/x)^2))/645216 + (Sqrt[(-59644114671451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]]*(3 + 4/x) + (3 + 4/x)^2))/645216

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rule 1673

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rule 1678

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

$x^4)^{(p + 1)} * \text{ExpandToSum}[2*a*(p + 1)*(b^2 - 4*a*c)*\text{PolynomialQuotient}[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{PolyQ}[Pq, x^2] \&\& \text{Expon}[Pq, x^2] > 1 \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1]$

Rule 2069

$\text{Int}[(P4_)^{(p)}, x_Symbol] :> \text{With}[\{a = \text{Coeff}[P4, x, 0], b = \text{Coeff}[P4, x, 1], c = \text{Coeff}[P4, x, 2], d = \text{Coeff}[P4, x, 3], e = \text{Coeff}[P4, x, 4]\}, \text{Dist}[-16*a^2, \text{Subst}[\text{Int}[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)))/(b - 4*a*x)^4)^p]/(b - 4*a*x)^2, x], x, b/(4*a) + 1/x], x] /; \text{NeQ}[a, 0] \&\& \text{NeQ}[b, 0] \&\& \text{EqQ}[b^3 - 4*a*b*c + 8*a^2*d, 0] /; \text{FreeQ}[p, x] \&\& \text{PolyQ}[P4, x, 4] \&\& \text{IntegerQ}[2*p] \&\& !\text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} dx &= - \left(1024 \text{Subst} \left(\int \frac{(24 - 32x)^6}{64 (2117632 - 2490368x^2 + 1048576x^4)^2} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right. \\ &= - \left(16 \text{Subst} \left(\int \frac{(24 - 32x)^6}{(2117632 - 2490368x^2 + 1048576x^4)^2} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right) \\ &= - \left(16 \text{Subst} \left(\int \frac{x(-1528823808 - 9059696640x^2 - 4831838208x^4)}{(2117632 - 2490368x^2 + 1048576x^4)^2} dx, \right. \right. \\ &= \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right) \left(3 + \frac{4}{x} \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} - \frac{\text{Subst} \left(\int \frac{12092568522016394156441}{2117632 - 2490368x^2 + 1048576x^4} dx, x, \frac{3}{4} + \frac{1}{x} \right)}{709422} \\ &= - \frac{3 \left(3359 - 107 \left(3 + \frac{4}{x} \right)^2 \right)}{208 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right) \left(3 + \frac{4}{x} \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} \\ &= - \frac{3 \left(3359 - 107 \left(3 + \frac{4}{x} \right)^2 \right)}{208 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right) \left(3 + \frac{4}{x} \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} \\ &= - \frac{3 \left(3359 - 107 \left(3 + \frac{4}{x} \right)^2 \right)}{208 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right) \left(3 + \frac{4}{x} \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} \\ &= - \frac{3 \left(3359 - 107 \left(3 + \frac{4}{x} \right)^2 \right)}{208 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right) \left(3 + \frac{4}{x} \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} \end{aligned}$$

Mathematica [C] time = 0.03, size = 128, normalized size = 0.35

$$\frac{\text{RootSum} \left[8\#1^4 - 15\#1^3 + 8\#1^2 + 24\#1 + 8\&, \frac{19640\#1^2 \log(x-\#1) - 57489\#1 \log(x-\#1) + 74897 \log(x-\#1)}{32\#1^3 - 45\#1^2 + 16\#1 + 24} \& \right]}{80652} + \frac{39280x^3 - 9431}{161304(8x^4 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2), x]

[Out] (72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Log[x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 45*#1^2 + 32*#1^3) &]/80652

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")

[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)

maple [C] time = 0.01, size = 96, normalized size = 0.26

$$\frac{(19640 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8))^2 - 57489 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8)}{2580864 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8)^3 - 3629340 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x)

[Out] (2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x^2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+24)*ln(-_R+x),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{39280x^3 - 94314x^2 + 89033x + 72888}{161304(8x^4 - 15x^3 + 8x^2 + 24x + 8)} + \frac{1}{80652} \int \frac{19640x^2 - 57489x + 74897}{8x^4 - 15x^3 + 8x^2 + 24x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")

[Out] 1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)

mupad [B] time = 0.21, size = 181, normalized size = 0.49

$$\frac{\frac{2455x^3}{80652} - \frac{1429x^2}{19552} + \frac{89033x}{1290432} + \frac{3037}{53768}}{x^4 - \frac{15x^3}{8} + x^2 + 3x + 1} + \left(\sum_{k=1}^4 \ln \left(\frac{2146659825 \operatorname{root} \left(z^4 + \frac{14911625619311z^2}{524620702127808} + \frac{39238139261z}{3730636104019968} + \frac{430}{44204510} \right)}{2960381771776} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] ((89033*x)/1290432 - (1429*x^2)/19552 + (2455*x^3)/80652 + 3037/53768)/(3*x
+ x^2 - (15*x^3)/8 + x^4 + 1) + symsum(log((2146659825*root(z^4 + (1491162
5619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/
44204510553294663, z, k))/2960381771776 + (2222183*x)/338246745408 + (92412
4364159*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3
730636104019968 + 43023440/44204510553294663, z, k)*x)/26643435945984 - (72
451101*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37
30636104019968 + 43023440/44204510553294663, z, k)^2*x)/8470528 - (95745*ro
ot(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37306361040
19968 + 43023440/44204510553294663, z, k)^3*x)/256 + (389551*root(z^4 + (14
911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 4302
3440/44204510553294663, z, k)^2)/264704 - (100737*root(z^4 + (1491162561931
1*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/442045
10553294663, z, k)^3)/512 + 271033/624455529984)*root(z^4 + (14911625619311
*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/4420451
0553294663, z, k), k, 1, 4)
```

sympy [B] time = 3.95, size = 3839, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] (39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 +
1290432*x**2 + 3871296*x + 1290432) + sqrt(-59644114671451/1678786246808985
6 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-1123969950204685033
06932567484755463/603722125611976319526135612861060 - 296438698298128332309
07750777733957*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)))/
1936419398792394461637855141912238396080 - 181533261043120360732*sqrt(-7120
427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) +
6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/1509305314
02994079881533903215265 - 46926347979646613249222*sqrt(517)/297468603626329
12338339 + 994065243322493861977*sqrt(78)*sqrt(-59644114671451 + 2623170438
295*sqrt(517))/1427849297406379792240272 + 994065243322493861977*sqrt(40326
)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sq
rt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715
0042935*sqrt(517) + 3557579971691991294769382675)/1290946265861596307758570
094608158930720) - 45971497067730669689218547912235602388091893135917351760
29*sqrt(517)*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623
170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912
94769382675)/18432767186998698626450604048374763890148748053806275728419558
40 - 1022132763720267175882780425063613131088601935958303878081158710949715
459967411486447/30220181238068169063463153438589206735086644165655335340962
4708723614680800 - 10638094717334280126176111526682776643728382835565338369
93*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/689619370306997
2436744723519626607862706189949382980866560 - 89036038929850064673184559559
3034670326044595870824169313*sqrt(40326)*sqrt(-59644114671451 + 26231704382
95*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262
3170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991
294769382675)/9352473884121677079601749613489889898672849258229891014611209
554309158400 - 45113976327488809325094501633826014671791*sqrt(78)*sqrt(-596
44114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*s
qrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sq
rt(517) + 3557579971691991294769382675)/107753026610468319324136304994165747
854784217959109076040 + 426980096365154687189009427342740052122552822995528
```

$$\begin{aligned}
& 02371283821308121207*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} \\
& /935247388412167707960174961348988989867284925822989101461120955430915 \\
& 8400 + 68548776709669674081892851407413209373218007060934353137152573209405 \\
& 073*\sqrt{517}/4608191796749674656612651012093690972537187013451568932104889 \\
& 60 + 2741964319335541530074345707646806021327350986246447585831575286311670 \\
& 33*\sqrt{-7120427417275887*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295* \\
& \sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 355757997169199129476938267 \\
& 5)/483522899809090705015410455017427307761386306650485365455399533957783489 \\
& 2800) - \sqrt{-59644114671451/16787862468089856 + 5073830635*\sqrt{517}/32471 \\
& 687559168)*\log(x**2 + x*(-112396995020468503306932567484755463/603722125611 \\
& 976319526135612861060 - 994065243322493861977*\sqrt{78}*\sqrt{-59644114671451 \\
& + 2623170438295*\sqrt{517}})/1427849297406379792240272 - 4692634797964661324 \\
& 9222*\sqrt{517}/29746860362632912338339 + 181533261043120360732*\sqrt{7120427 \\
& 417275887*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 626 \\
& 3621568587150042935*\sqrt{517} + 3557579971691991294769382675)/1509305314029 \\
& 94079881533903215265 + 29643869829812833230907750777733957*\sqrt{40326}*\sqrt{ \\
& (-59644114671451 + 2623170438295*\sqrt{517})/1936419398792394461637855141912 \\
& 238396080 + 994065243322493861977*\sqrt{40326}*\sqrt{-59644114671451 + 262317 \\
& 0438295*\sqrt{517}}*\sqrt{7120427417275887*\sqrt{40326}*\sqrt{-59644114671451 + \\
& 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 355757997169 \\
& 1991294769382675)/1290946265861596307758570094608158930720) - 2741964319335 \\
& 54153007434570764680602132735098624644758583157528631167033*\sqrt{7120427417 \\
& 275887*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 626362 \\
& 1568587150042935*\sqrt{517} + 3557579971691991294769382675)/4835228998090907 \\
& 050154104550174273077613863066504853654553995339577834892800 - 102213276372 \\
& 0267175882780425063613131088601935958303878081158710949715459967411486447/3 \\
& 02201812380681690634631534385892067350866441656553353409624708723614680800 \\
& - 890360389298500646731845595593034670326044595870824169313*\sqrt{40326}*\sqrt{ \\
& (-59644114671451 + 2623170438295*\sqrt{517})*\sqrt{7120427417275887*\sqrt{403 \\
& 26}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 626362156858715004293 \\
& 5*\sqrt{517} + 3557579971691991294769382675)/9352473884121677079601749613489 \\
& 889898672849258229891014611209554309158400 - 426980096365154687189009427342 \\
& 74005212255282299552802371283821308121207*\sqrt{40326}*\sqrt{-59644114671451 \\
& + 2623170438295*\sqrt{517}}/935247388412167707960174961348988989867284925822 \\
& 9891014611209554309158400 - 45113976327488809325094501633826014671791*\sqrt{ \\
& 78}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}*\sqrt{7120427417275887* \\
& \sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 62636215685871 \\
& 50042935*\sqrt{517} + 3557579971691991294769382675)/107753026610468319324136 \\
& 304994165747854784217959109076040 + 106380947173342801261761115266827766437 \\
& 2838283556533836993*\sqrt{78}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} \\
&)/6896193703069972436744723519626607862706189949382980866560 + 685487767096 \\
& 69674081892851407413209373218007060934353137152573209405073*\sqrt{517}/46081 \\
& 9179674967465661265101209369097253718701345156893210488960 + 45971497067730 \\
& 66968921854791223560238809189313591735176029*\sqrt{517}*\sqrt{712042741727588 \\
& 7*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 62636215685 \\
& 87150042935*\sqrt{517} + 3557579971691991294769382675)/184327671869986986264 \\
& 5060404837476389014874805380627572841955840) - 2*\sqrt{59653665894623/167878 \\
& 62468089856 + 5073830635*\sqrt{517}/10823895853056 + \sqrt{-7120427417275887* \\
& \sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 6263621568587 \\
& 150042935*\sqrt{517} + 3557579971691991294769382675)/4196965617022464)*\operatorname{atan}(\\
& -7745677595169577846551420567648953584320*x/(-59292486929118917272637172801 \\
& 533436*\sqrt{40326}*\sqrt{59653665894623 + 7869511314885*\sqrt{517}} + 4*\sqrt{- \\
& 7120427417275887*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} \\
&) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675)) + 232 \\
& 9048502925820708785386304*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{ \\
& 59653665894623 + 7869511314885*\sqrt{517}} + 4*\sqrt{-7120427417275887*\sqrt{ \\
& 40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 6263621568587150 \\
& 042935*\sqrt{517} + 3557579971691991294769382675)) + 994065243322493861977*\sqrt{ \\
& 40326}*\sqrt{59653665894623 + 7869511314885*\sqrt{517}} + 4*\sqrt{-71204274
\end{aligned}$$

$$\begin{aligned}
& 114671451 + 2623170438295*\sqrt{517}) + 6263621568587150042935*\sqrt{517} + 3 \\
& 557579971691991294769382675)) + 59287739659625666461815501555467914*\sqrt{40} \\
& 326)*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}/(-59292486929118917272 \\
& 637172801533436*\sqrt{40326})*\sqrt{59653665894623 + 7869511314885*\sqrt{517}} + \\
& 4*\sqrt{-7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295* \\
& \sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 355757997169199129476938267 \\
& 5)) + 2329048502925820708785386304*\sqrt{-59644114671451 + 2623170438295*\sqrt{ \\
& 517}})*\sqrt{59653665894623 + 7869511314885*\sqrt{517}} + 4*\sqrt{-71204274172 \\
& 75887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 6263621 \\
& 568587150042935*\sqrt{517} + 3557579971691991294769382675)) + 99406524332249 \\
& 3861977*\sqrt{40326})*\sqrt{59653665894623 + 7869511314885*\sqrt{517}} + 4*\sqrt{(\\
& -7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517} \\
&)) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675))*\sqrt{ \\
& (-7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{51 \\
& 7}}) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675)) + 7 \\
& 21019529648624138729760776730387270795368/(-5929248692911891727263717280153 \\
& 3436*\sqrt{40326})*\sqrt{59653665894623 + 7869511314885*\sqrt{517}} + 4*\sqrt{-71 \\
& 20427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) \\
& + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675)) + 23290 \\
& 48502925820708785386304*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{ \\
& 59653665894623 + 7869511314885*\sqrt{517}} + 4*\sqrt{-7120427417275887*\sqrt{ \\
& 40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 626362156858715004 \\
& 2935*\sqrt{517} + 3557579971691991294769382675)) + 994065243322493861977*\sqrt{ \\
& 40326})*\sqrt{59653665894623 + 7869511314885*\sqrt{517}} + 4*\sqrt{-7120427417 \\
& 275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 626362 \\
& 1568587150042935*\sqrt{517} + 3557579971691991294769382675))*\sqrt{-712042741 \\
& 7275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 62636 \\
& 21568587150042935*\sqrt{517} + 3557579971691991294769382675)) - 2*\sqrt{-\sqrt{ \\
& 7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{51 \\
& 7}}) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675)/4196 \\
& 965617022464 + 59653665894623/16787862468089856 + 5073830635*\sqrt{517}/1082 \\
& 3895853056)*\operatorname{atan}(7745677595169577846551420567648953584320*x/(23290485029258 \\
& 20708785386304*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{-4*\sqrt{ \\
& 7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517} \\
&)) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675) + 596 \\
& 53665894623 + 7869511314885*\sqrt{517})) + 5929248692911891727263717280153343 \\
& 6*\sqrt{40326})*\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-5964411467145 \\
& 1 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 355757997 \\
& 1691991294769382675) + 59653665894623 + 7869511314885*\sqrt{517})) + 99406524 \\
& 3322493861977*\sqrt{40326})*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-596441146 \\
& 71451 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 35575 \\
& 79971691991294769382675))*\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59 \\
& 644114671451 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} \\
& + 3557579971691991294769382675) + 59653665894623 + 7869511314885*\sqrt{517})) \\
&) - 721019529648624138729760776730387270795368/(232904850292582070878538630 \\
& 4*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{-4*\sqrt{712042741727 \\
& 5887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 62636215 \\
& 68587150042935*\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + \\
& 7869511314885*\sqrt{517})) + 59292486929118917272637172801533436*\sqrt{40326} \\
& *\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 262317043 \\
& 8295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769 \\
& 382675) + 59653665894623 + 7869511314885*\sqrt{517})) + 994065243322493861977 \\
& *\sqrt{40326})*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 26231 \\
& 70438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 355757997169199129 \\
& 4769382675))*\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 \\
& + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 35575799716 \\
& 91991294769382675) + 59653665894623 + 7869511314885*\sqrt{517})) - 269626104 \\
& 7060775175517112572266328310*\sqrt{78})*\sqrt{-59644114671451 + 2623170438295* \\
& \sqrt{517}}/(2329048502925820708785386304*\sqrt{-59644114671451 + 26231704382
\end{aligned}$$

$$\begin{aligned}
& 69511314885\sqrt{517}) + 59292486929118917272637172801533436\sqrt{40326}\sqrt{-4\sqrt{7120427417275887}\sqrt{40326}\sqrt{-59644114671451 + 2623170438295\sqrt{517})} + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 7869511314885\sqrt{517}) + 994065243322493861977\sqrt{40326}\sqrt{7120427417275887}\sqrt{40326}\sqrt{-59644114671451 + 2623170438295\sqrt{517})} + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675)\sqrt{-4\sqrt{7120427417275887}\sqrt{40326}\sqrt{-59644114671451 + 2623170438295\sqrt{517})} + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 7869511314885\sqrt{517}))
\end{aligned}$$

$$3.63 \quad \int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5 \right)^3 dx$$

Optimal. Leaf size=14

$$\frac{(a + bx)^{16}}{16b}$$

[Out] 1/16*(b*x+a)^16/b

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2059, 32}

$$\frac{(a + bx)^{16}}{16b}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^3, x]

[Out] (a + b*x)^16/(16*b)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2059

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonFreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5 \right)^3 dx &= \int (a + bx)^{15} dx \\ &= \frac{(a + bx)^{16}}{16b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^{16}}{16b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^3, x]

[Out] (a + b*x)^16/(16*b)

fricas [B] time = 0.71, size = 163, normalized size = 11.64

$$\frac{1}{16}x^{16}b^{15} + x^{15}b^{14}a + \frac{15}{2}x^{14}b^{13}a^2 + 35x^{13}b^{12}a^3 + \frac{455}{4}x^{12}b^{11}a^4 + 273x^{11}b^{10}a^5 + \frac{1001}{2}x^{10}b^9a^6 + 715x^9b^8a^7 + \frac{6435}{8}x^8b^7a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="fricas")

[Out] $1/16*x^{16}*b^{15} + x^{15}*b^{14}*a + 15/2*x^{14}*b^{13}*a^2 + 35*x^{13}*b^{12}*a^3 + 455/4*x^{12}*b^{11}*a^4 + 273*x^{11}*b^{10}*a^5 + 1001/2*x^{10}*b^9*a^6 + 715*x^9*b^8*a^7 + 6435/8*x^8*b^7*a^8 + 715*x^7*b^6*a^9 + 1001/2*x^6*b^5*a^{10} + 273*x^5*b^4*a^{11} + 455/4*x^4*b^3*a^{12} + 35*x^3*b^2*a^{13} + 15/2*x^2*b*a^{14} + x*a^{15}$

giac [B] time = 0.28, size = 163, normalized size = 11.64

$$\frac{1}{16} b^{15} x^{16} + a b^{14} x^{15} + \frac{15}{2} a^2 b^{13} x^{14} + 35 a^3 b^{12} x^{13} + \frac{455}{4} a^4 b^{11} x^{12} + 273 a^5 b^{10} x^{11} + \frac{1001}{2} a^6 b^9 x^{10} + 715 a^7 b^8 x^9 + \frac{6435}{8} a^8 b^7 x^8 + \frac{715}{2} a^9 b^6 x^7 + \frac{1001}{2} a^{10} b^5 x^6 + 273 a^{11} b^4 x^5 + \frac{455}{4} a^{12} b^3 x^4 + 35 a^{13} b^2 x^3 + \frac{15}{2} a^{14} b x^2 + a^{15} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="giac")

[Out] $1/16*b^{15}*x^{16} + a*b^{14}*x^{15} + 15/2*a^2*b^{13}*x^{14} + 35*a^3*b^{12}*x^{13} + 455/4*a^4*b^{11}*x^{12} + 273*a^5*b^{10}*x^{11} + 1001/2*a^6*b^9*x^{10} + 715*a^7*b^8*x^9 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^{10}*b^5*x^6 + 273*a^{11}*b^4*x^5 + 455/4*a^{12}*b^3*x^4 + 35*a^{13}*b^2*x^3 + 15/2*a^{14}*b*x^2 + a^{15}*x$

maple [B] time = 0.00, size = 164, normalized size = 11.71

$$\frac{1}{16} b^{15} x^{16} + a b^{14} x^{15} + \frac{15}{2} a^2 b^{13} x^{14} + 35 a^3 b^{12} x^{13} + \frac{455}{4} a^4 b^{11} x^{12} + 273 a^5 b^{10} x^{11} + \frac{1001}{2} a^6 b^9 x^{10} + 715 a^7 b^8 x^9 + \frac{6435}{8} a^8 b^7 x^8 + \frac{715}{2} a^9 b^6 x^7 + \frac{1001}{2} a^{10} b^5 x^6 + 273 a^{11} b^4 x^5 + \frac{455}{4} a^{12} b^3 x^4 + 35 a^{13} b^2 x^3 + \frac{15}{2} a^{14} b x^2 + a^{15} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)

[Out] $1/16*b^{15}*x^{16} + a*b^{14}*x^{15} + 15/2*a^2*b^{13}*x^{14} + 35*a^3*b^{12}*x^{13} + 455/4*a^4*b^{11}*x^{12} + 273*a^5*b^{10}*x^{11} + 1001/2*a^6*b^9*x^{10} + 715*a^7*b^8*x^9 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^{10}*b^5*x^6 + 273*a^{11}*b^4*x^5 + 455/4*a^{12}*b^3*x^4 + 35*a^{13}*b^2*x^3 + 15/2*a^{14}*b*x^2 + a^{15}*x$

maxima [B] time = 0.68, size = 592, normalized size = 42.29

$$\frac{1}{16} b^{15} x^{16} + a b^{14} x^{15} + \frac{75}{14} a^2 b^{13} x^{14} + \frac{125}{13} a^3 b^{12} x^{13} + 100 a^6 b^9 x^{10} + \frac{1000}{7} a^9 b^6 x^7 + \frac{125}{4} a^{12} b^3 x^4 + a^{15} x + \frac{1}{2} (b^5 x^6 + 6 a b^4 x^5 + 15 a^2 b^3 x^4 + 20 a^3 b^2 x^3 + 15 a^4 b x^2 + a^5 x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="maxima")

[Out] $1/16*b^{15}*x^{16} + a*b^{14}*x^{15} + 75/14*a^2*b^{13}*x^{14} + 125/13*a^3*b^{12}*x^{13} + 100*a^6*b^9*x^{10} + 1000/7*a^9*b^6*x^7 + 125/4*a^{12}*b^3*x^4 + a^{15}*x + 1/2*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^{10} + 25/56*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*x^5)*a^8*b^2 + 5/3*(18*b^5*x^{10} + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4 + 25/11*(11*b^5*x^{12} + 60*a*b^4*x^{11})*a^4*b^6 + 1/462*(126*b^{10}*x^{11} + 1386*a*b^9*x^{10} + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 + 11550*a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 385*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a^5 + 5/308*(77*b^{10}*x^{12} + 840*a*b^9*x^{11} + 4158*a^2*b^8*x^{10} + 12320*a^3*b^7*x^9 + 23100*a^4*b^6*x^8 + 26400*a^5*b^5*x^7 + 15400*a^6*b^4*x^6)*a^4*b + 5/429*(198*b^{10}*x^{13} + 2145*a*b^9*x^{12} + 10530*a^2*b^8*x^{11} + 25740*a^3*b^7*x^{10} + 28600*a^4*b^6*x^9)*a^3*b^2 + 5/182*(78*b^{10}*x^{14} + 840*a*b^9*x^{13} + 2275*a^2*b^8*x^{12})*a^2*b^3$

mupad [B] time = 0.17, size = 163, normalized size = 11.64

$$a^{15} x + \frac{15 a^{14} b x^2}{2} + 35 a^{13} b^2 x^3 + \frac{455 a^{12} b^3 x^4}{4} + 273 a^{11} b^4 x^5 + \frac{1001 a^{10} b^5 x^6}{2} + 715 a^9 b^6 x^7 + \frac{6435 a^8 b^7 x^8}{8} + 715 a^7 b^8 x^9 + \frac{1001 a^6 b^9 x^{10}}{2} + 273 a^5 b^{10} x^{11} + \frac{455 a^4 b^{11} x^{12}}{4} + 35 a^3 b^{12} x^{13} + \frac{15 a^2 b^{13} x^{14}}{2} + a b^{14} x^{15} + \frac{1}{16} b^{15} x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)^3,x)`

[Out] $a^{15}x + (b^{15}x^{16})/16 + (15a^{14}b^2x^2)/2 + a*b^{14}x^{15} + 35a^{13}b^2x^3 + (455a^{12}b^3x^4)/4 + 273a^{11}b^4x^5 + (1001a^{10}b^5x^6)/2 + 715a^9b^6x^7 + (6435a^8b^7x^8)/8 + 715a^7b^8x^9 + (1001a^6b^9x^{10})/2 + 273a^5b^{10}x^{11} + (455a^4b^{11}x^{12})/4 + 35a^3b^{12}x^{13} + (15a^2b^{13}x^{14})/2$

sympy [B] time = 0.11, size = 185, normalized size = 13.21

$$a^{15}x + \frac{15a^{14}bx^2}{2} + 35a^{13}b^2x^3 + \frac{455a^{12}b^3x^4}{4} + 273a^{11}b^4x^5 + \frac{1001a^{10}b^5x^6}{2} + 715a^9b^6x^7 + \frac{6435a^8b^7x^8}{8} + 715a^7b^8x^9 + \frac{1001a^6b^9x^{10}}{2} + 273a^5b^{10}x^{11} + \frac{455a^4b^{11}x^{12}}{4} + 35a^3b^{12}x^{13} + \frac{15a^2b^{13}x^{14}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5)**3,x)`

[Out] $a^{15}x + 15a^{14}b^2x^2/2 + 35a^{13}b^2x^3 + 455a^{12}b^3x^4/4 + 273a^{11}b^4x^5 + 1001a^{10}b^5x^6/2 + 715a^9b^6x^7 + 6435a^8b^7x^8/8 + 715a^7b^8x^9 + 1001a^6b^9x^{10}/2 + 273a^5b^{10}x^{11} + 455a^4b^{11}x^{12}/4 + 35a^3b^{12}x^{13} + 15a^2b^{13}x^{14}/2 + a*b^{14}x^{15} + b^{15}x^{16}/16$

$$3.64 \quad \int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)$$

Optimal. Leaf size=14

$$\frac{(a + bx)^{11}}{11b}$$

[Out] 1/11*(b*x+a)^11/b

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2059, 32}

$$\frac{(a + bx)^{11}}{11b}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^2, x]

[Out] (a + b*x)^11/(11*b)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2059

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[Non freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2 dx = \int (a + bx)^{10} dx = \frac{(a + bx)^{11}}{11b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^{11}}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^2, x]

[Out] (a + b*x)^11/(11*b)

fricas [B] time = 0.68, size = 108, normalized size = 7.71

$$\frac{1}{11}x^{11}b^{10} + x^{10}b^9a + 5x^9b^8a^2 + 15x^8b^7a^3 + 30x^7b^6a^4 + 42x^6b^5a^5 + 42x^5b^4a^6 + 30x^4b^3a^7 + 15x^3b^2a^8 + 5x^2ba^9 + xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="fricas")

[Out] $1/11*x^{11}*b^{10} + x^{10}*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30*x^7*b^6*a^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 + 15*x^3*b^2*a^8 + 5*x^2*b*a^9 + x*a^{10}$

giac [B] time = 0.24, size = 108, normalized size = 7.71

$$\frac{1}{11} b^{10} x^{11} + a b^9 x^{10} + 5 a^2 b^8 x^9 + 15 a^3 b^7 x^8 + 30 a^4 b^6 x^7 + 42 a^5 b^5 x^6 + 42 a^6 b^4 x^5 + 30 a^7 b^3 x^4 + 15 a^8 b^2 x^3 + 5 a^9 b x^2 + a^{10} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="giac")`

[Out] $1/11*b^{10}*x^{11} + a*b^9*x^{10} + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*a^9*b*x^2 + a^{10}*x$

maple [B] time = 0.00, size = 109, normalized size = 7.79

$$\frac{1}{11} b^{10} x^{11} + a b^9 x^{10} + 5 a^2 b^8 x^9 + 15 a^3 b^7 x^8 + 30 a^4 b^6 x^7 + 42 a^5 b^5 x^6 + 42 a^6 b^4 x^5 + 30 a^7 b^3 x^4 + 15 a^8 b^2 x^3 + 5 a^9 b x^2 + a^{10} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)`

[Out] $1/11*b^{10}*x^{11}+a*b^9*x^{10}+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*a^4*b^6*x^7+42*a^5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+5*a^9*b*x^2+a^{10}*x$

maxima [B] time = 0.49, size = 228, normalized size = 16.29

$$\frac{1}{11} b^{10} x^{11} + a b^9 x^{10} + \frac{25}{9} a^2 b^8 x^9 + \frac{100}{7} a^4 b^6 x^7 + 20 a^6 b^4 x^5 + \frac{25}{3} a^8 b^2 x^3 + a^{10} x + \frac{1}{3} (b^5 x^6 + 6 a b^4 x^5 + 15 a^2 b^3 x^4 + 20 a^3 b^2 x^3 + 15 a^4 b x^2 + a^5 x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="maxima")`

[Out] $1/11*b^{10}*x^{11} + a*b^9*x^{10} + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^7 + 20*a^6*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^{10}*x + 1/3*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5 + 5/21*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3$

mupad [B] time = 0.06, size = 108, normalized size = 7.71

$$a^{10} x + 5 a^9 b x^2 + 15 a^8 b^2 x^3 + 30 a^7 b^3 x^4 + 42 a^6 b^4 x^5 + 42 a^5 b^5 x^6 + 30 a^4 b^6 x^7 + 15 a^3 b^7 x^8 + 5 a^2 b^8 x^9 + a b^9 x^{10} + \frac{b^{10} x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)^2,x)`

[Out] $a^{10}*x + (b^{10}*x^{11})/11 + 5*a^9*b*x^2 + a*b^9*x^{10} + 15*a^8*b^2*x^3 + 30*a^7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x^8 + 5*a^2*b^8*x^9$

sympy [B] time = 0.10, size = 114, normalized size = 8.14

$$a^{10} x + 5 a^9 b x^2 + 15 a^8 b^2 x^3 + 30 a^7 b^3 x^4 + 42 a^6 b^4 x^5 + 42 a^5 b^5 x^6 + 30 a^4 b^6 x^7 + 15 a^3 b^7 x^8 + 5 a^2 b^8 x^9 + a b^9 x^{10} + \frac{b^{10} x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*  
*4*b*x+a**5)**2,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b  
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*  
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

$$3.65 \quad \int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) dx$$

Optimal. Leaf size=14

$$\frac{(a + bx)^6}{6b}$$

[Out] 1/6*(b*x+a)^6/b

Rubi [B] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 4.36, number of steps used = 1, number of rules used = 0, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x + ab^4x^5 + \frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

[In] Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5, x]

[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^5 + (b^5*x^6)/6

Rubi steps

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) dx = a^5x + \frac{5}{2}a^4bx^2 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^2b^3x^4 + ab^4x^5 + \frac{b^5x^6}{6}$$

Mathematica [B] time = 0.00, size = 61, normalized size = 4.36

$$a^5x + \frac{5}{2}a^4bx^2 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^2b^3x^4 + ab^4x^5 + \frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5, x]

[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^5 + (b^5*x^6)/6

fricas [B] time = 0.56, size = 53, normalized size = 3.79

$$\frac{1}{6}x^6b^5 + x^5b^4a + \frac{5}{2}x^4b^3a^2 + \frac{10}{3}x^3b^2a^3 + \frac{5}{2}x^2ba^4 + xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5, x, algorithm="fricas")

[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/2*x^2*b*a^4 + x*a^5

giac [B] time = 0.29, size = 53, normalized size = 3.79

$$\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x, algorithm="giac")

[Out] $\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$

maple [B] time = 0.00, size = 54, normalized size = 3.86

$$\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x)

[Out] $a^5x + \frac{5}{2}a^4bx^2 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^2b^3x^4 + ab^4x^5 + \frac{1}{6}b^5x^6$

maxima [B] time = 0.46, size = 53, normalized size = 3.79

$$\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x, algorithm="maxima")

[Out] $\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$

mupad [B] time = 0.02, size = 53, normalized size = 3.79

$$a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x,x)

[Out] $a^5x + \frac{b^5x^6}{6} + \frac{5a^4bx^2}{2} + ab^4x^5 + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2}$

sympy [B] time = 0.07, size = 60, normalized size = 4.29

$$a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5,x)

[Out] $a**5*x + \frac{5*a**4*b*x**2}{2} + \frac{10*a**3*b**2*x**3}{3} + \frac{5*a**2*b**3*x**4}{2} + ab**4*x**5 + \frac{b**5*x**6}{6}$

$$3.66 \quad \int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5} dx$$

Optimal. Leaf size=14

$$-\frac{1}{4b(a+bx)^4}$$

[Out] -1/4/b/(b*x+a)^4

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2058, 32}

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-1), x]

[Out] -1/(4*b*(a + b*x)^4)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5} dx &= \int \frac{1}{(a+bx)^5} dx \\ &= -\frac{1}{4b(a+bx)^4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-1), x]

[Out] -1/4*1/(b*(a + b*x)^4)

fricas [B] time = 0.83, size = 46, normalized size = 3.29

$$-\frac{1}{4(b^5x^4 + 4ab^4x^3 + 6a^2b^3x^2 + 4a^3b^2x + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x, algorithm="fricas")

[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)

giac [A] time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{4(bx+a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x, algorithm="giac")

[Out] -1/4/((b*x + a)^4*b)

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$-\frac{1}{4(bx+a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x)

[Out] -1/4/b/(b*x+a)^4

maxima [B] time = 0.69, size = 46, normalized size = 3.29

$$-\frac{1}{4(b^5x^4 + 4ab^4x^3 + 6a^2b^3x^2 + 4a^3b^2x + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x, algorithm="maxima")

[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)

mupad [B] time = 0.05, size = 48, normalized size = 3.43

$$-\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x),x)

[Out] -1/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2)

sympy [B] time = 0.29, size = 49, normalized size = 3.50

$$-\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5),x)

[Out] -1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**5*x**4)

$$3.67 \quad \int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2} dx$$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

[Out] -1/9/b/(b*x+a)^9

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2058, 32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-2), x]

[Out] -1/(9*b*(a + b*x)^9)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2} dx &= \int \frac{1}{(a+bx)^{10}} dx \\ &= -\frac{1}{9b(a+bx)^9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-2), x]

[Out] -1/9*1/(b*(a + b*x)^9)

fricas [B] time = 0.62, size = 101, normalized size = 7.21

$$-\frac{1}{9(b^{10}x^9 + 9ab^9x^8 + 36a^2b^8x^7 + 84a^3b^7x^6 + 126a^4b^6x^5 + 126a^5b^5x^4 + 84a^6b^4x^3 + 36a^7b^3x^2 + 9a^8b^2x + a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="fricas")

[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a^9*b)

giac [A] time = 0.28, size = 12, normalized size = 0.86

$$-\frac{1}{9(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="giac")

[Out] -1/9/((b*x + a)^9*b)

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{9(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)

[Out] -1/9/b/(b*x+a)^9

maxima [B] time = 0.60, size = 101, normalized size = 7.21

$$-\frac{1}{9(b^{10}x^9 + 9ab^9x^8 + 36a^2b^8x^7 + 84a^3b^7x^6 + 126a^4b^6x^5 + 126a^5b^5x^4 + 84a^6b^4x^3 + 36a^7b^3x^2 + 9a^8b^2x + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="maxima")

[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a^9*b)

mupad [B] time = 2.10, size = 103, normalized size = 7.36

$$-\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81ab^9x^8 + a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)^2,x)

[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 + 756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 324*a^2*b^8*x^7)

sympy [B] time = 0.59, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81ab^9x^8 + a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*  
a**4*b*x+a**5)**2,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1  
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*  
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```


$$3.68 \quad \int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3} dx$$

Optimal. Leaf size=14

$$-\frac{1}{14b(a+bx)^{14}}$$

[Out] -1/14/b/(b*x+a)^14

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2058, 32}

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-3), x]

[Out] -1/(14*b*(a + b*x)^14)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3} dx &= \int \frac{1}{(a + bx)^{15}} dx \\ &= -\frac{1}{14b(a + bx)^{14}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-3), x]

[Out] -1/14*1/(b*(a + b*x)^14)

fricas [B] time = 0.79, size = 156, normalized size = 11.14

$$14(b^{15}x^{14} + 14ab^{14}x^{13} + 91a^2b^{13}x^{12} + 364a^3b^{12}x^{11} + 1001a^4b^{11}x^{10} + 2002a^5b^{10}x^9 + 3003a^6b^9x^8 + 3432a^7b^8x^7 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="fricas")

[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 + 1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)

giac [A] time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{14(bx+a)^{14}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="giac")

[Out] -1/14/((b*x + a)^14*b)

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{14(bx+a)^{14}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)

[Out] -1/14/b/(b*x+a)^14

maxima [B] time = 0.54, size = 156, normalized size = 11.14

$$\frac{1}{14(b^{15}x^{14} + 14ab^{14}x^{13} + 91a^2b^{13}x^{12} + 364a^3b^{12}x^{11} + 1001a^4b^{11}x^{10} + 2002a^5b^{10}x^9 + 3003a^6b^9x^8 + 3432a^7b^8x^7 + 3003a^8b^7x^6 + 2002a^9b^6x^5 + 1001a^{10}b^5x^4 + 364a^{11}b^4x^3 + 91a^{12}b^3x^2 + 14a^{13}b^2x + a^{14}b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="maxima")

[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 + 1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)

mupad [B] time = 3.03, size = 158, normalized size = 11.29

$$-\frac{1}{14a^{14}b + 196a^{13}b^2x + 1274a^{12}b^3x^2 + 5096a^{11}b^4x^3 + 14014a^{10}b^5x^4 + 28028a^9b^6x^5 + 42042a^8b^7x^6 + 48048a^7b^8x^7 + 42042a^6b^9x^8 + 28028a^5b^{10}x^9 + 14014a^4b^{11}x^{10} + 5096a^3b^{12}x^{11} + 1274a^2b^{13}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)^3,x)

[Out] -1/(14*a^14*b + 14*b^15*x^14 + 196*a^13*b^2*x + 196*a*b^14*x^13 + 1274*a^12*b^3*x^2 + 5096*a^11*b^4*x^3 + 14014*a^10*b^5*x^4 + 28028*a^9*b^6*x^5 + 42042*a^8*b^7*x^6 + 48048*a^7*b^8*x^7 + 42042*a^6*b^9*x^8 + 28028*a^5*b^10*x^9 + 14014*a^4*b^11*x^10 + 5096*a^3*b^12*x^11 + 1274*a^2*b^13*x^12)

sympy [B] time = 0.94, size = 168, normalized size = 12.00

$$\sqrt{14a^{14}b + 196a^{13}b^2x + 1274a^{12}b^3x^2 + 5096a^{11}b^4x^3 + 14014a^{10}b^5x^4 + 28028a^9b^6x^5 + 42042a^8b^7x^6 + 48048a^7b^8x^7 + 42042a^6b^9x^8 + 28028a^5b^{10}x^9 + 14014a^4b^{11}x^{10} + 5096a^3b^{12}x^{11} + 1274a^2b^{13}x^{12} + 196ab^{14}x^{13} + 14b^{15}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5)**3,x)

[Out] -1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a**11*b**4*x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 42042*a**8*b**7*x**6 + 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8 + 28028*a**5*b**10*x**9 + 14014*a**4*b**11*x**10 + 5096*a**3*b**12*x**11 + 1274*a**2*b**13*x**12 + 196*a*b**14*x**13 + 14*b**15*x**14)

$$3.69 \quad \int \frac{1}{1+x^2+x^3+x^5} dx$$

Optimal. Leaf size=38

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

[Out] 1/2*arctan(x)+1/6*ln(1+x)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2058, 635, 203, 260, 628}

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^3 + x^5)^(-1), x]

[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x^2+x^3+x^5} dx &= \int \left(\frac{1}{6(1+x)} + \frac{1+x}{2(1+x^2)} + \frac{1-2x}{3(1-x+x^2)} \right) dx \\
&= \frac{1}{6} \log(1+x) + \frac{1}{3} \int \frac{1-2x}{1-x+x^2} dx + \frac{1}{2} \int \frac{1+x}{1+x^2} dx \\
&= \frac{1}{6} \log(1+x) - \frac{1}{3} \log(1-x+x^2) + \frac{1}{2} \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{x}{1+x^2} dx \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{6} \log(1+x) + \frac{1}{4} \log(1+x^2) - \frac{1}{3} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{1}{4} \log(x^2+1) - \frac{1}{3} \log(x^2-x+1) + \frac{1}{6} \log(x+1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + x^3 + x^5)^(-1), x]

[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3

fricas [A] time = 0.79, size = 30, normalized size = 0.79

$$\frac{1}{2} \arctan(x) - \frac{1}{3} \log(x^2-x+1) + \frac{1}{4} \log(x^2+1) + \frac{1}{6} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^5+x^3+x^2+1), x, algorithm="fricas")

[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)

giac [A] time = 0.29, size = 31, normalized size = 0.82

$$\frac{1}{2} \arctan(x) - \frac{1}{3} \log(x^2-x+1) + \frac{1}{4} \log(x^2+1) + \frac{1}{6} \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^5+x^3+x^2+1), x, algorithm="giac")

[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(abs(x + 1))

maple [A] time = 0.01, size = 31, normalized size = 0.82

$$\frac{\arctan(x)}{2} + \frac{\ln(x+1)}{6} + \frac{\ln(x^2+1)}{4} - \frac{\ln(x^2-x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5+x^3+x^2+1), x)

[Out] 1/2*arctan(x)+1/6*ln(x+1)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)

maxima [A] time = 1.11, size = 30, normalized size = 0.79

$$\frac{1}{2} \arctan(x) - \frac{1}{3} \log(x^2-x+1) + \frac{1}{4} \log(x^2+1) + \frac{1}{6} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="maxima")

[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)

mupad [B] time = 2.16, size = 36, normalized size = 0.95

$$\frac{\ln(x+1)}{6} - \frac{\ln(x^2-x+1)}{3} + \ln(x-i) \left(\frac{1}{4} - \frac{1}{4}i\right) + \ln(x+1i) \left(\frac{1}{4} + \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + x^3 + x^5 + 1),x)

[Out] log(x + 1)/6 + log(x - 1i)*(1/4 - 1i/4) + log(x + 1i)*(1/4 + 1i/4) - log(x^2 - x + 1)/3

sympy [A] time = 0.15, size = 29, normalized size = 0.76

$$\frac{\log(x+1)}{6} + \frac{\log(x^2+1)}{4} - \frac{\log(x^2-x+1)}{3} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**5+x**3+x**2+1),x)

[Out] log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2

3.70 $\int (3 - 19x^2 + 32x^4 - 16x^6)^4 dx$

Optimal. Leaf size=84

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{63}{1}$$

[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/23*x^23+65536/25*x^25

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2059, 517, 521}

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{63}{1}$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4, x]

[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25

Rule 517

Int[(u_)*((c_) + (d_)*(x_)^(n_))^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] :> Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 521

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rule 2059

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (3 - 19x^2 + 32x^4 - 16x^6)^4 dx &= \int (-1 + x)^4(1 + x)^4(-1 + 2x)^4(1 + 2x)^4(-3 + 4x^2)^4 dx \\ &= \int (-1 + 2x)^4(1 + 2x)^4(-1 + x^2)^4(-3 + 4x^2)^4 dx \\ &= \int (-1 + x^2)^4(-3 + 4x^2)^4(-1 + 4x^2)^4 dx \\ &= \int (81 - 2052x^2 + 22950x^4 - 149700x^6 + 634321x^8 - 1841600x^{10} + 3764416x^{12} - 1094656x^{14} + 5633536x^{16} - 4014080x^{18} + 1884160x^{20} - 524288x^{22} + 65536x^{24}) dx \\ &= 81x - 684x^3 + 4590x^5 - \frac{149700x^7}{7} + \frac{634321x^9}{9} - \frac{1841600x^{11}}{11} + \frac{3764416x^{13}}{13} - \frac{1094656x^{15}}{15} + \frac{5633536x^{17}}{17} - \frac{4014080x^{19}}{19} + \frac{1884160x^{21}}{21} - \frac{524288x^{23}}{23} + \frac{65536x^{25}}{25} \end{aligned}$$

Mathematica [A] time = 0.00, size = 84, normalized size = 1.00

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{634321x^9}{9} - \frac{149700x^7}{7} + 4590x^5 - 684x^3 + 81x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]

[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25

fricas [A] time = 0.49, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^9 - \frac{149700}{7}x^7 + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="fricas")

[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x

giac [A] time = 0.38, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^9 - \frac{149700}{7}x^7 + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="giac")

[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x

maple [A] time = 0.00, size = 65, normalized size = 0.77

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^9 - \frac{149700}{7}x^7 + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16*x^6+32*x^4-19*x^2+3)^4,x)

[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/23*x^23+65536/25*x^25

maxima [A] time = 0.49, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^9 - \frac{149700}{7}x^7 + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="maxima")

[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x

mupad [B] time = 2.17, size = 64, normalized size = 0.76

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^4,x)

[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25

sympy [A] time = 0.08, size = 80, normalized size = 0.95

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)

[Out] 65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**19/19 + 5633536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1841600*x**11/11 + 634321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684*x**3 + 81*x

$$3.71 \quad \int (3 - 19x^2 + 32x^4 - 16x^6)^3 dx$$

Optimal. Leaf size=63

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21248/5*x^15+24576/17*x^17-4096/19*x^19

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2059, 517, 521}

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]

[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19

Rule 517

Int[(u_)*((c_) + (d_)*(x_)^(n_))^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] :> Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 521

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rule 2059

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (3 - 19x^2 + 32x^4 - 16x^6)^3 dx &= - \int (-1 + x)^3 (1 + x)^3 (-1 + 2x)^3 (1 + 2x)^3 (-3 + 4x^2)^3 dx \\ &= - \int (-1 + 2x)^3 (1 + 2x)^3 (-1 + x^2)^3 (-3 + 4x^2)^3 dx \\ &= - \int (-1 + x^2)^3 (-3 + 4x^2)^3 (-1 + 4x^2)^3 dx \\ &= - \int (-27 + 513x^2 - 4113x^4 + 18235x^6 - 49344x^8 + 84912x^{10} - 93440x^{12} + 27x - 171x^3 + \frac{4113x^5}{5} - 2605x^7 + \frac{16448x^9}{3} - \frac{84912x^{11}}{11} + \frac{93440x^{13}}{13} - \frac{21248x^{15}}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 63, normalized size = 1.00

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]

[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19

fricas [A] time = 0.78, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19} + \frac{24576}{17}x^{17} - \frac{21248}{5}x^{15} + \frac{93440}{13}x^{13} - \frac{84912}{11}x^{11} + \frac{16448}{3}x^9 - 2605x^7 + \frac{4113}{5}x^5 - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")

[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x

giac [A] time = 0.36, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19} + \frac{24576}{17}x^{17} - \frac{21248}{5}x^{15} + \frac{93440}{13}x^{13} - \frac{84912}{11}x^{11} + \frac{16448}{3}x^9 - 2605x^7 + \frac{4113}{5}x^5 - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")

[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x

maple [A] time = 0.00, size = 50, normalized size = 0.79

$$-\frac{4096}{19}x^{19} + \frac{24576}{17}x^{17} - \frac{21248}{5}x^{15} + \frac{93440}{13}x^{13} - \frac{84912}{11}x^{11} + \frac{16448}{3}x^9 - 2605x^7 + \frac{4113}{5}x^5 - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16*x^6+32*x^4-19*x^2+3)^3,x)

[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21248/5*x^15+24576/17*x^17-4096/19*x^19

maxima [A] time = 0.73, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19} + \frac{24576}{17}x^{17} - \frac{21248}{5}x^{15} + \frac{93440}{13}x^{13} - \frac{84912}{11}x^{11} + \frac{16448}{3}x^9 - 2605x^7 + \frac{4113}{5}x^5 - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")

[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x

mupad [B] time = 0.05, size = 49, normalized size = 0.78

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)`

[Out] $27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^{11})/11 + (93440*x^{13})/13 - (21248*x^{15})/5 + (24576*x^{17})/17 - (4096*x^{19})/19$

sympy [A] time = 0.07, size = 60, normalized size = 0.95

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)`

[Out] $-4096*x^{19}/19 + 24576*x^{17}/17 - 21248*x^{15}/5 + 93440*x^{13}/13 - 84912*x^{11}/11 + 16448*x^9/3 - 2605*x^7 + 4113*x^5/5 - 171*x^3 + 27*x$

$$3.72 \quad \int (3 - 19x^2 + 32x^4 - 16x^6)^2 dx$$

Optimal. Leaf size=44

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2059, 517, 521}

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]

[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 + (256*x^13)/13

Rule 517

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 521

Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rule 2059

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (3 - 19x^2 + 32x^4 - 16x^6)^2 dx &= \int (-1 + x)^2(1 + x)^2(-1 + 2x)^2(1 + 2x)^2(-3 + 4x^2)^2 dx \\ &= \int (-1 + 2x)^2(1 + 2x)^2(-1 + x^2)^2(-3 + 4x^2)^2 dx \\ &= \int (-1 + x^2)^2(-3 + 4x^2)^2(-1 + 4x^2)^2 dx \\ &= \int (9 - 114x^2 + 553x^4 - 1312x^6 + 1632x^8 - 1024x^{10} + 256x^{12}) dx \\ &= 9x - 38x^3 + \frac{553x^5}{5} - \frac{1312x^7}{7} + \frac{544x^9}{3} - \frac{1024x^{11}}{11} + \frac{256x^{13}}{13} \end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]

[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 + (256*x^13)/13

fricas [A] time = 0.65, size = 34, normalized size = 0.77

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^9 - \frac{1312}{7}x^7 + \frac{553}{5}x^5 - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")

[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 + 9*x

giac [A] time = 0.36, size = 34, normalized size = 0.77

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^9 - \frac{1312}{7}x^7 + \frac{553}{5}x^5 - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")

[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 + 9*x

maple [A] time = 0.00, size = 35, normalized size = 0.80

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^9 - \frac{1312}{7}x^7 + \frac{553}{5}x^5 - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16*x^6+32*x^4-19*x^2+3)^2,x)

[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13

maxima [A] time = 0.75, size = 34, normalized size = 0.77

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^9 - \frac{1312}{7}x^7 + \frac{553}{5}x^5 - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")

[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 + 9*x

mupad [B] time = 0.02, size = 34, normalized size = 0.77

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)

[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 + (256*x^13)/13

sympy [A] time = 0.06, size = 41, normalized size = 0.93

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)

[Out] 256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**5/5 - 38*x**3 + 9*x

3.73 $\int (3 - 19x^2 + 32x^4 - 16x^6) dx$

Optimal. Leaf size=25

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Antiderivative was successfully verified.

[In] Int[3 - 19*x^2 + 32*x^4 - 16*x^6,x]

[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7

Rubi steps

$$\int (3 - 19x^2 + 32x^4 - 16x^6) dx = 3x - \frac{19x^3}{3} + \frac{32x^5}{5} - \frac{16x^7}{7}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Antiderivative was successfully verified.

[In] Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6,x]

[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7

fricas [A] time = 0.72, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="fricas")

[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x

giac [A] time = 0.28, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="giac")

[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-16*x^6+32*x^4-19*x^2+3,x)`

[Out] `3*x-19/3*x^3+32/5*x^5-16/7*x^7`

maxima [A] time = 0.64, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="maxima")`

[Out] `-16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x`

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(32*x^4 - 19*x^2 - 16*x^6 + 3,x)`

[Out] `3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7`

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-16*x**6+32*x**4-19*x**2+3,x)`

[Out] `-16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x`

$$3.74 \quad \int \frac{1}{3-19x^2+32x^4-16x^6} dx$$

Optimal. Leaf size=31

$$\frac{1}{3} \tanh^{-1}(x) + \frac{1}{3} \tanh^{-1}(2x) - \frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] 1/3*arctanh(x)+1/3*arctanh(2*x)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2057, 207}

$$\frac{1}{3} \tanh^{-1}(x) + \frac{1}{3} \tanh^{-1}(2x) - \frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1), x]

[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2057

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{3-19x^2+32x^4-16x^6} dx &= \int \left(-\frac{1}{3(-1+x^2)} + \frac{2}{-3+4x^2} - \frac{2}{3(-1+4x^2)} \right) dx \\ &= -\left(\frac{1}{3} \int \frac{1}{-1+x^2} dx \right) - \frac{2}{3} \int \frac{1}{-1+4x^2} dx + 2 \int \frac{1}{-3+4x^2} dx \\ &= \frac{1}{3} \tanh^{-1}(x) + \frac{1}{3} \tanh^{-1}(2x) - \frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 2.00

$$\frac{1}{6} \left(-\log(2x^2 - 3x + 1) + \log(2x^2 + 3x + 1) + \sqrt{3} \log(\sqrt{3} - 2x) - \sqrt{3} \log(2x + \sqrt{3}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1), x]

[Out] (Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1 - 3*x + 2*x^2] + Log[1 + 3*x + 2*x^2])/6

fricas [B] time = 0.88, size = 56, normalized size = 1.81

$$\frac{1}{6} \sqrt{3} \log\left(\frac{4x^2 - 4\sqrt{3}x + 3}{4x^2 - 3}\right) + \frac{1}{6} \log(2x^2 + 3x + 1) - \frac{1}{6} \log(2x^2 - 3x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)

giac [B] time = 0.28, size = 62, normalized size = 2.00

$$\frac{1}{6} \sqrt{3} \log\left(\frac{|8x - 4\sqrt{3}|}{|8x + 4\sqrt{3}|}\right) + \frac{1}{6} \log(|2x + 1|) - \frac{1}{6} \log(|2x - 1|) + \frac{1}{6} \log(|x + 1|) - \frac{1}{6} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="giac")

[Out] 1/6*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*log(abs(2*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))

maple [A] time = 0.01, size = 42, normalized size = 1.35

$$-\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{2\sqrt{3}x}{3}\right)}{3} - \frac{\ln(x-1)}{6} - \frac{\ln(2x-1)}{6} + \frac{\ln(x+1)}{6} + \frac{\ln(2x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-16*x^6+32*x^4-19*x^2+3),x)

[Out] -1/6*ln(-1+x)-1/6*ln(-1+2*x)+1/6*ln(1+2*x)+1/6*ln(x+1)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)

maxima [B] time = 1.05, size = 54, normalized size = 1.74

$$\frac{1}{6} \sqrt{3} \log\left(\frac{2x - \sqrt{3}}{2x + \sqrt{3}}\right) + \frac{1}{6} \log(2x + 1) - \frac{1}{6} \log(2x - 1) + \frac{1}{6} \log(x + 1) - \frac{1}{6} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)

mupad [B] time = 0.07, size = 27, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{x}{4608\left(\frac{x^2}{6912} + \frac{1}{13824}\right)}\right)}{3} - \frac{\sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3),x)

[Out] atanh(x/(4608*(x^2/6912 + 1/13824)))/3 - (3^(1/2)*atanh((2*3^(1/2)*x)/3))/3

sympy [B] time = 0.15, size = 63, normalized size = 2.03

$$\frac{\sqrt{3} \log\left(x - \frac{\sqrt{3}}{2}\right)}{6} - \frac{\sqrt{3} \log\left(x + \frac{\sqrt{3}}{2}\right)}{6} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{1}{2}\right)}{6} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)

[Out] sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6

$$3.75 \quad \int \frac{1}{(3-19x^2+32x^4-16x^6)^2} dx$$

Optimal. Leaf size=89

$$\frac{2x}{3(3-4x^2)} + \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(x+1)} - \frac{1}{18(2x+1)} + \frac{67}{54} \tanh^{-1}(x) - \frac{7}{27} \tanh^{-1}(2x) - \frac{5 \tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] 1/18/(1-2*x)+1/36/(1-x)-1/36/(1+x)-1/18/(1+2*x)+2/3*x/(-4*x^2+3)+67/54*arctanh(x)-7/27*arctanh(2*x)-5/9*arctanh(2/3*x*3^(1/2))*3^(1/2)

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2057, 207, 199}

$$\frac{2x}{3(3-4x^2)} + \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(x+1)} - \frac{1}{18(2x+1)} + \frac{67}{54} \tanh^{-1}(x) - \frac{7}{27} \tanh^{-1}(2x) - \frac{5 \tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2), x]

[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2*x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTanh[(2*x)/Sqrt[3]])/(3*Sqrt[3])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2057

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3 - 19x^2 + 32x^4 - 16x^6)^2} dx &= \int \left(\frac{1}{36(-1+x)^2} + \frac{1}{36(1+x)^2} + \frac{1}{9(-1+2x)^2} + \frac{1}{9(1+2x)^2} - \frac{67}{54(-1+x^2)} + \right. \\
&= \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(1+x)} - \frac{1}{18(1+2x)} + \frac{14}{27} \int \frac{1}{-1+4x^2} dx - \frac{67}{54} \\
&= \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(1+x)} - \frac{1}{18(1+2x)} + \frac{2x}{3(3-4x^2)} + \frac{67}{54} \tanh^{-1}\left(\frac{2x}{\sqrt{3-4x^2}}\right) \\
&= \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(1+x)} - \frac{1}{18(1+2x)} + \frac{2x}{3(3-4x^2)} + \frac{67}{54} \tanh^{-1}\left(\frac{2x}{\sqrt{3-4x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.16

$$\frac{1}{108} \left(-\frac{6x(80x^4 - 104x^2 + 27)}{16x^6 - 32x^4 + 19x^2 - 3} + 14 \log(1 - 2x) + 30\sqrt{3} \log(\sqrt{3} - 2x) - 67 \log(1 - x) + 67 \log(x + 1) - 14 \log(1 + 2x) - 30\sqrt{3} \log(\sqrt{3} + 2x) \right) / 108$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2), x]

[Out] ((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) + 14*Log[1 - 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x] + 67*Log[1 + x] - 14*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x])/108

fricas [B] time = 0.82, size = 177, normalized size = 1.99

$$\frac{480x^5 - 624x^3 - 30\sqrt{3}(16x^6 - 32x^4 + 19x^2 - 3) \log\left(\frac{4x^2 - 4\sqrt{3}x + 3}{4x^2 - 3}\right) + 14(16x^6 - 32x^4 + 19x^2 - 3) \log(2x + 1) - 14(16x^6 - 32x^4 + 19x^2 - 3) \log(2x - 1) - 67(16x^6 - 32x^4 + 19x^2 - 3) \log(x + 1) + 67(16x^6 - 32x^4 + 19x^2 - 3) \log(x - 1) + 162x}{16x^6 - 32x^4 + 19x^2 - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")

[Out] -1/108*(480*x^5 - 624*x^3 - 30*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x + 1) - 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x - 1) - 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x + 1) + 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x - 1) + 162*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)

giac [A] time = 0.28, size = 97, normalized size = 1.09

$$\frac{5}{18} \sqrt{3} \log\left(\frac{|8x - 4\sqrt{3}|}{|8x + 4\sqrt{3}|}\right) - \frac{80x^5 - 104x^3 + 27x}{18(16x^6 - 32x^4 + 19x^2 - 3)} - \frac{7}{54} \log(|2x + 1|) + \frac{7}{54} \log(|2x - 1|) + \frac{67}{108} \log(|x + 1|) - \frac{67}{108} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")

[Out] 5/18*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/18*(80*x^5 - 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(abs(2*x + 1)) + 7/54*log(abs(2*x - 1)) + 67/108*log(abs(x + 1)) - 67/108*log(abs(x - 1))

maple [A] time = 0.02, size = 84, normalized size = 0.94

$$\frac{x}{6\left(x^2 - \frac{3}{4}\right)} - \frac{5\sqrt{3} \operatorname{arctanh}\left(\frac{2\sqrt{3}x}{3}\right)}{9} - \frac{67 \ln(x-1)}{108} + \frac{7 \ln(2x-1)}{54} + \frac{67 \ln(x+1)}{108} - \frac{7 \ln(2x+1)}{54} - \frac{1}{36(x-1)} - \frac{1}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x)

[Out] -1/36/(x-1)-67/108*ln(x-1)-1/18/(2*x-1)+7/54*ln(2*x-1)-1/18/(2*x+1)-7/54*ln(2*x+1)-1/36/(x+1)+67/108*ln(x+1)-1/6*x/(x^2-3/4)-5/9*arctanh(2/3*3^(1/2)*x)*3^(1/2)

maxima [A] time = 1.29, size = 89, normalized size = 1.00

$$\frac{5}{18} \sqrt{3} \log\left(\frac{2x - \sqrt{3}}{2x + \sqrt{3}}\right) - \frac{80x^5 - 104x^3 + 27x}{18(16x^6 - 32x^4 + 19x^2 - 3)} - \frac{7}{54} \log(2x + 1) + \frac{7}{54} \log(2x - 1) + \frac{67}{108} \log(x + 1) - \frac{67}{108} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")

[Out] 5/18*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/18*(80*x^5 - 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(2*x + 1) + 7/54*log(2*x - 1) + 67/108*log(x + 1) - 67/108*log(x - 1)

mupad [B] time = 0.08, size = 64, normalized size = 0.72

$$-\frac{\operatorname{atan}(x1i) 67i}{54} + \frac{\operatorname{atan}(x2i) 7i}{27} - \frac{\frac{5x^5}{18} - \frac{13x^3}{36} + \frac{3x}{32}}{x^6 - 2x^4 + \frac{19x^2}{16} - \frac{3}{16}} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x2i}{3}\right) 5i}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)

[Out] (atan(x*2i)*7i)/27 - (atan(x*1i)*67i)/54 - ((3*x)/32 - (13*x^3)/36 + (5*x^5)/18)/((19*x^2)/16 - 2*x^4 + x^6 - 3/16) + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*5i)/9

sympy [A] time = 1.36, size = 104, normalized size = 1.17

$$\frac{-80x^5 + 104x^3 - 27x}{288x^6 - 576x^4 + 342x^2 - 54} - \frac{67 \log(x-1)}{108} + \frac{7 \log\left(x - \frac{1}{2}\right)}{54} - \frac{7 \log\left(x + \frac{1}{2}\right)}{54} + \frac{67 \log(x+1)}{108} + \frac{5\sqrt{3} \log\left(x - \frac{\sqrt{3}}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)

[Out] (-80*x**5 + 104*x**3 - 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54) - 67*log(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 67*log(x + 1)/108 + 5*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*log(x + sqrt(3)/2)/18

$$3.76 \quad \int \frac{1}{(3-19x^2+32x^4-16x^6)^3} dx$$

Optimal. Leaf size=161

$$\frac{5x}{3(3-4x^2)} - \frac{2x}{3(3-4x^2)^2} - \frac{7}{108(1-2x)} + \frac{67}{432(1-x)} - \frac{67}{432(x+1)} + \frac{7}{108(2x+1)} + \frac{1}{108(1-2x)^2} + \frac{1}{432(1-x)^2} - \frac{1}{432(1-x)^2}$$

[Out] 1/108/(1-2*x)^2-7/108/(1-2*x)+1/432/(1-x)^2+67/432/(1-x)-1/432/(1+x)^2-67/432/(1+x)-1/108/(1+2*x)^2+7/108/(1+2*x)-2/3*x/(-4*x^2+3)^2+5/3*x/(-4*x^2+3)+3913/648*arctanh(x)+67/162*arctanh(2*x)-67/18*arctanh(2/3*x*3^(1/2))*3^(1/2)

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2057, 207, 199}

$$\frac{5x}{3(3-4x^2)} - \frac{2x}{3(3-4x^2)^2} - \frac{7}{108(1-2x)} + \frac{67}{432(1-x)} - \frac{67}{432(x+1)} + \frac{7}{108(2x+1)} + \frac{1}{108(1-2x)^2} + \frac{1}{432(1-x)^2} - \frac{1}{432(1-x)^2}$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3), x]

[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 - x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108*(1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcTanh[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) - 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2057

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[u /. x -> x^2]^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3 - 19x^2 + 32x^4 - 16x^6)^3} dx &= \int \left(-\frac{1}{216(-1+x)^3} + \frac{67}{432(-1+x)^2} + \frac{1}{216(1+x)^3} + \frac{67}{432(1+x)^2} - \frac{1}{27(-1+x)} \right. \\
&= \frac{1}{108(1-2x)^2} - \frac{7}{108(1-2x)} + \frac{1}{432(1-x)^2} + \frac{67}{432(1-x)} - \frac{1}{432(1+x)^2} - \frac{1}{432(1+x)} \\
&= \frac{1}{108(1-2x)^2} - \frac{7}{108(1-2x)} + \frac{1}{432(1-x)^2} + \frac{67}{432(1-x)} - \frac{1}{432(1+x)^2} - \frac{1}{432(1+x)} \\
&= \frac{1}{108(1-2x)^2} - \frac{7}{108(1-2x)} + \frac{1}{432(1-x)^2} + \frac{67}{432(1-x)} - \frac{1}{432(1+x)^2} - \frac{1}{432(1+x)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 0.85

$$\frac{36x(80x^4 - 104x^2 + 27)}{(-16x^6 + 32x^4 - 19x^2 + 3)^2} - \frac{6x(2288x^4 - 2384x^2 + 345)}{16x^6 - 32x^4 + 19x^2 - 3} - 268 \log(1 - 2x) + 2412\sqrt{3} \log(\sqrt{3} - 2x) - 3913 \log(1 - x) + 3913 \log(1 + x)$$

1296

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3), x]

[Out] ((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2 - (6*x*(345 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) - 268*Log[1 - 2*x] + 2412*Sqrt[3]*Log[Sqrt[3] - 2*x] - 3913*Log[1 - x] + 3913*Log[1 + x] + 268*Log[1 + 2*x] - 2412*Sqrt[3]*Log[Sqrt[3] + 2*x])/1296

fricas [B] time = 0.88, size = 282, normalized size = 1.75

$$\frac{219648x^{11} - 668160x^9 + 751680x^7 - 382080x^5 + 85986x^3 - 2412\sqrt{3}(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)}{(16x^6 - 32x^4 + 19x^2 - 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")

[Out] -1/1296*(219648*x^11 - 668160*x^9 + 751680*x^7 - 382080*x^5 + 85986*x^3 - 2412*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) - 268*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) + 268*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x - 1) - 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(x + 1) + 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(x - 1) - 7182*x)/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)

giac [A] time = 0.31, size = 112, normalized size = 0.70

$$\frac{67}{36} \sqrt{3} \log\left(\frac{|8x - 4\sqrt{3}|}{|8x + 4\sqrt{3}|}\right) - \frac{36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x}{216(16x^6 - 32x^4 + 19x^2 - 3)^2} + \frac{67}{324} \log(|2x - \sqrt{3}|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")

[Out] $\frac{67\sqrt{3}\log(\text{abs}(8x - 4\sqrt{3}))/\text{abs}(8x + 4\sqrt{3})}{18} - \frac{1}{216}(36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x)/(16x^6 - 32x^4 + 19x^2 - 3)^2 + \frac{67}{324}\log(\text{abs}(2x + 1)) - \frac{67}{324}\log(\text{abs}(2x - 1)) + \frac{3913}{1296}\log(\text{abs}(x + 1)) - \frac{3913}{1296}\log(\text{abs}(x - 1))$

maple [A] time = 0.02, size = 126, normalized size = 0.78

$$-\frac{67\sqrt{3} \operatorname{arctanh}\left(\frac{2\sqrt{3}x}{3}\right)}{18} - \frac{3913 \ln(x-1)}{1296} - \frac{67 \ln(2x-1)}{324} + \frac{3913 \ln(x+1)}{1296} + \frac{67 \ln(2x+1)}{324} + \frac{1}{432(x-1)^2} - \frac{67}{432(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x)

[Out] $\frac{1}{432(x-1)^2} - \frac{67}{432(x-1)} - \frac{3913}{1296}\ln(x-1) + \frac{1}{108(2x-1)^2} + \frac{7}{108(2x-1)} - \frac{67}{324}\ln(2x-1) - \frac{1}{108(2x+1)^2} + \frac{7}{108(2x+1)} + \frac{67}{324}\ln(2x+1) - \frac{1}{432(x+1)^2} - \frac{67}{432(x+1)} + \frac{3913}{1296}\ln(x+1) + 64\frac{(-5/48x^3+13/192x)}{(4x^2-3)^2} - \frac{67}{18}\operatorname{arctanh}(2/3\sqrt{3}x)\sqrt{3}$

maxima [A] time = 1.32, size = 119, normalized size = 0.74

$$\frac{67}{36}\sqrt{3} \log\left(\frac{2x - \sqrt{3}}{2x + \sqrt{3}}\right) - \frac{36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x}{216(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)} + \frac{67}{324} \log(2x + 1) - \frac{67}{324} \log(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")

[Out] $\frac{67\sqrt{3}\log((2x - \sqrt{3})/(2x + \sqrt{3}))}{18} - \frac{1}{216}(36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x)/(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9) + \frac{67}{324}\log(2x + 1) - \frac{67}{324}\log(2x - 1) + \frac{3913}{1296}\log(x + 1) - \frac{3913}{1296}\log(x - 1)$

mupad [B] time = 0.09, size = 93, normalized size = 0.58

$$-\frac{\frac{143x^{11}}{216} + \frac{145x^9}{72} - \frac{145x^7}{64} + \frac{995x^5}{864} - \frac{4777x^3}{18432} + \frac{133x}{6144}}{x^{12} - 4x^{10} + \frac{51x^8}{8} - \frac{41x^6}{8} + \frac{553x^4}{256} - \frac{57x^2}{128} + \frac{9}{256}} - \frac{\operatorname{atan}(x\sqrt{3})}{162} - \frac{\operatorname{atan}(x)}{648} + \frac{3913\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)

[Out] $\left(\frac{133x}{6144} - \frac{4777x^3}{18432} + \frac{995x^5}{864} - \frac{145x^7}{64} + \frac{145x^9}{72} - \frac{143x^{11}}{216}\right) / \left(\frac{553x^4}{256} - \frac{57x^2}{128} - \frac{41x^6}{8} + \frac{51x^8}{8} - 4x^{10} + x^{12} + \frac{9}{256}\right) - \frac{\operatorname{atan}(x\sqrt{3})}{162} - \frac{\operatorname{atan}(x)}{648} + \frac{3\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{18}$

sympy [A] time = 1.45, size = 134, normalized size = 0.83

$$-\frac{36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x}{55296x^{12} - 221184x^{10} + 352512x^8 - 283392x^6 + 119448x^4 - 24624x^2 + 1944} - \frac{3913 \log(x-1)}{1296} - \frac{67 \log\left(x - \frac{1}{2}\right)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)

[Out] $-\frac{36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x}{55296x^{12} - 221184x^{10} + 352512x^8 - 283392x^6 + 119448x^4 - 24624x^2 + 1944} - \frac{3913 \log(x-1)}{1296} - \frac{67 \log\left(x - \frac{1}{2}\right)}{324}$

$$24624x^2 + 1944) - 3913\log(x - 1)/1296 - 67\log(x - 1/2)/324 + 67\log(x + 1/2)/324 + 3913\log(x + 1)/1296 + 67\sqrt{3}\log(x - \sqrt{3}/2)/36 - 67\sqrt{3}\log(x + \sqrt{3}/2)/36$$

$$3.77 \quad \int \frac{1}{(-1+7x^2-7x^4+x^6)^2} dx$$

Optimal. Leaf size=91

$$\frac{x}{32(1-x^2)} + \frac{(99-17x^2)x}{128(x^4-6x^2+1)} + \frac{5}{32} \tanh^{-1}(x) + \frac{1}{512} (3\sqrt{2}-4) \tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right) + \frac{1}{512} (4+3\sqrt{2}) \tanh^{-1}\left(\left(\sqrt{2}+1\right)x\right)$$

[Out] 1/32*x/(-x^2+1)+1/128*x*(-17*x^2+99)/(x^4-6*x^2+1)+5/32*arctanh(x)+1/512*arctanh(x*(2^(1/2)-1))*(-4+3*2^(1/2))+1/512*arctanh(x*(1+2^(1/2)))*(4+3*2^(1/2))

Rubi [B] time = 0.13, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2057, 207, 638, 618, 206, 632, 31}

$$-\frac{41-17x}{256(-x^2+2x+1)} + \frac{17x+41}{256(-x^2-2x+1)} + \frac{1}{64(1-x)} - \frac{1}{64(x+1)} + \frac{1}{512} (2-7\sqrt{2}) \log(-x-\sqrt{2}+1) + \frac{1}{512} (2+7\sqrt{2}) \log(-x+\sqrt{2}+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2), x]

[Out] 1/(64*(1-x)) - 1/(64*(1+x)) + (41+17*x)/(256*(1-2*x-x^2)) - (41-17*x)/(256*(1+2*x-x^2)) - (17*ArcTanh[(1-x)/Sqrt[2]])/(256*Sqrt[2]) + (5*ArcTanh[x])/32 + (17*ArcTanh[(1+x)/Sqrt[2]])/(256*Sqrt[2]) + ((2-7*Sqrt[2])*Log[1-Sqrt[2]-x])/512 + ((2+7*Sqrt[2])*Log[1+Sqrt[2]-x])/512 - ((2-7*Sqrt[2])*Log[1-Sqrt[2]+x])/512 - ((2+7*Sqrt[2])*Log[1+Sqrt[2]+x])/512

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 638

Int[((d_.) + (e_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 2057

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1 + 7x^2 - 7x^4 + x^6)^2} dx &= \int \left(\frac{1}{64(-1 + x)^2} + \frac{1}{64(1 + x)^2} - \frac{5}{32(-1 + x^2)} + \frac{29 - 12x}{64(-1 - 2x + x^2)^2} + \frac{1}{128(-1 - 2x + x^2)} \right) dx \\ &= \frac{1}{64(1 - x)} - \frac{1}{64(1 + x)} + \frac{1}{128} \int \frac{6 + x}{-1 - 2x + x^2} dx + \frac{1}{128} \int \frac{6 - x}{-1 + 2x + x^2} dx + \frac{1}{128} \int \frac{1}{-1 - 2x + x^2} dx \\ &= \frac{1}{64(1 - x)} - \frac{1}{64(1 + x)} + \frac{41 + 17x}{256(1 - 2x - x^2)} - \frac{41 - 17x}{256(1 + 2x - x^2)} + \frac{5}{32} \tanh^{-1}\left(\frac{x}{1 - x^2}\right) \\ &= \frac{1}{64(1 - x)} - \frac{1}{64(1 + x)} + \frac{41 + 17x}{256(1 - 2x - x^2)} - \frac{41 - 17x}{256(1 + 2x - x^2)} + \frac{5}{32} \tanh^{-1}\left(\frac{x}{1 - x^2}\right) \\ &= \frac{1}{64(1 - x)} - \frac{1}{64(1 + x)} + \frac{41 + 17x}{256(1 - 2x - x^2)} - \frac{41 - 17x}{256(1 + 2x - x^2)} - \frac{17 \tanh^{-1}\left(\frac{x}{1 - x^2}\right)}{256\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 132, normalized size = 1.45

$$\frac{-\frac{8x(21x^4 - 140x^2 + 103)}{x^6 - 7x^4 + 7x^2 - 1} - 80 \log(1 - x) - (4 + 3\sqrt{2}) \log(-x + \sqrt{2} - 1) + (4 - 3\sqrt{2}) \log(-x + \sqrt{2} + 1) + 80 \log(1 + x)}{1024}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2), x]

[Out] ((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x] - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*Sqrt[2])*Log[1 + Sqrt[2] + x])/1024

fricas [B] time = 0.87, size = 223, normalized size = 2.45

$$\frac{168x^5 - 1120x^3 - 3\sqrt{2}(x^6 - 7x^4 + 7x^2 - 1) \log\left(\frac{x^2 + 2\sqrt{2}(x+1) + 2x+3}{x^2 + 2x - 1}\right) - 3\sqrt{2}(x^6 - 7x^4 + 7x^2 - 1) \log\left(\frac{x^2 + 2\sqrt{2}(x-1) + 2x+3}{x^2 + 2x - 1}\right) + 80 \log(1 - x) - (4 + 3\sqrt{2}) \log(-x + \sqrt{2} - 1) + (4 - 3\sqrt{2}) \log(-x + \sqrt{2} + 1) + 80 \log(1 + x)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")

[Out] $-1/1024*(168*x^5 - 1120*x^3 - 3*\sqrt{2}*(x^6 - 7*x^4 + 7*x^2 - 1)*\log((x^2 + 2*\sqrt{2}*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*\sqrt{2}*(x^6 - 7*x^4 + 7*x^2 - 1)*\log((x^2 + 2*\sqrt{2}*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*(x^6 - 7*x^4 + 7*x^2 - 1)*\log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)*\log(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*\log(x + 1) + 80*(x^6 - 7*x^4 + 7*x^2 - 1)*\log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)$

giac [A] time = 0.44, size = 134, normalized size = 1.47

$$-\frac{3}{1024} \sqrt{2} \log\left(\frac{|2x - 2\sqrt{2} + 2|}{|2x + 2\sqrt{2} + 2|}\right) - \frac{3}{1024} \sqrt{2} \log\left(\frac{|2x - 2\sqrt{2} - 2|}{|2x + 2\sqrt{2} - 2|}\right) - \frac{21x^5 - 140x^3 + 103x}{128(x^6 - 7x^4 + 7x^2 - 1)} - \frac{1}{256} \log(|x^2 + 2x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")

[Out] $-3/1024*\sqrt{2}*\log(\text{abs}(2*x - 2*\sqrt{2} + 2)/\text{abs}(2*x + 2*\sqrt{2} + 2)) - 3/1024*\sqrt{2}*\log(\text{abs}(2*x - 2*\sqrt{2} - 2)/\text{abs}(2*x + 2*\sqrt{2} - 2)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*\log(\text{abs}(x^2 + 2*x - 1)) + 1/256*\log(\text{abs}(x^2 - 2*x - 1)) + 5/64*\log(\text{abs}(x + 1)) - 5/64*\log(\text{abs}(x - 1))$

maple [A] time = 0.02, size = 116, normalized size = 1.27

$$\frac{3\sqrt{2} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{512} + \frac{3\sqrt{2} \operatorname{arctanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{512} - \frac{5 \ln(x-1)}{64} + \frac{5 \ln(x+1)}{64} + \frac{\ln(x^2 - 2x - 1)}{256} - \frac{\ln(x^2 + 2x - 1)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6-7*x^4+7*x^2-1)^2,x)

[Out] $-1/64/(x-1) - 5/64*\ln(x-1) - 1/128*(17/2*x+41/2)/(x^2+2*x-1) - 1/256*\ln(x^2+2*x-1) + 3/512*2^{(1/2)}*\operatorname{arctanh}(1/4*(2+2*x)*2^{(1/2)}) - 1/64/(x+1) + 5/64*\ln(x+1) + 1/128*(-17/2*x+41/2)/(x^2-2*x-1) + 1/256*\ln(x^2-2*x-1) + 3/512*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*x-2)*2^{(1/2)})$

maxima [A] time = 1.27, size = 114, normalized size = 1.25

$$-\frac{3}{1024} \sqrt{2} \log\left(\frac{x - \sqrt{2} + 1}{x + \sqrt{2} + 1}\right) - \frac{3}{1024} \sqrt{2} \log\left(\frac{x - \sqrt{2} - 1}{x + \sqrt{2} - 1}\right) - \frac{21x^5 - 140x^3 + 103x}{128(x^6 - 7x^4 + 7x^2 - 1)} - \frac{1}{256} \log(x^2 + 2x - 1) + \frac{1}{256} \log(x^2 - 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="maxima")

[Out] $-3/1024*\sqrt{2}*\log((x - \sqrt{2} + 1)/(x + \sqrt{2} + 1)) - 3/1024*\sqrt{2}*\log((x - \sqrt{2} - 1)/(x + \sqrt{2} - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*\log(x^2 + 2*x - 1) + 1/256*\log(x^2 - 2*x - 1) + 5/64*\log(x + 1) - 5/64*\log(x - 1)$

mupad [B] time = 2.18, size = 126, normalized size = 1.38

$$-\frac{\operatorname{atan}(x1i) 5i}{32} - \frac{21x^5}{128} - \frac{35x^3}{32} + \frac{103x}{128} - \operatorname{atan}\left(\frac{x 940311i}{134217728 \left(\frac{275445 \sqrt{2}}{134217728} - \frac{389421}{134217728}\right)} - \frac{\sqrt{2} x 332433i}{67108864 \left(\frac{275445 \sqrt{2}}{134217728} - \frac{389421}{134217728}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)

```
[Out] - (atan(x*1i)*5i)/32 - ((103*x)/128 - (35*x^3)/32 + (21*x^5)/128)/(7*x^2 -
7*x^4 + x^6 - 1) - atan((x*940311i)/(134217728*((275445*2^(1/2))/134217728
- 389421/134217728)) - (2^(1/2)*x*332433i)/(67108864*((275445*2^(1/2))/1342
17728 - 389421/134217728)))*((2^(1/2)*3i)/512 - 1i/128) - atan((x*940311i)/
(134217728*((275445*2^(1/2))/134217728 + 389421/134217728)) + (2^(1/2)*x*33
2433i)/(67108864*((275445*2^(1/2))/134217728 + 389421/134217728)))*((2^(1/2
)*3i)/512 + 1i/128)
```

sympy [B] time = 1.43, size = 296, normalized size = 3.25

$$\frac{-21x^5 + 140x^3 - 103x}{128x^6 - 896x^4 + 896x^2 - 128} - \frac{5 \log(x-1)}{64} + \frac{5 \log(x+1)}{64} + \left(-\frac{1}{256} + \frac{3\sqrt{2}}{1024} \right) \log \left(x - \frac{8071264001}{202624020} - \frac{471550901878784}{2979765} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

```
[Out] (-21*x**5 + 140*x**3 - 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*log(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/1024)*log(x - 8071264001/202624020 - 471550901878784*(-1/256 + 3*sqrt(2)/1024)**3/2979765 + 1299552375287054336*(-1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001*sqrt(2)/270165360) + (-3*sqrt(2)/1024 - 1/256)*log(x - 8071264001*sqrt(2)/270165360 - 8071264001/202624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)**5/50656005 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (1/256 - 3*sqrt(2)/1024)*log(x - 8071264001*sqrt(2)/270165360 + 1299552375287054336*(1/256 - 3*sqrt(2)/1024)**5/50656005 - 471550901878784*(1/256 - 3*sqrt(2)/1024)**3/2979765 + 8071264001/202624020) + (1/256 + 3*sqrt(2)/1024)*log(x - 471550901878784*(1/256 + 3*sqrt(2)/1024)**3/2979765 + 1299552375287054336*(1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001/202624020 + 8071264001*sqrt(2)/270165360)
```

$$3.78 \quad \int \frac{x^3}{c+(a+bx)^2} dx$$

Optimal. Leaf size=78

$$\frac{(3a^2 - c) \log((a + bx)^2 + c)}{2b^4} - \frac{a(a^2 - 3c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{(a + bx)^2}{2b^4} - \frac{3ax}{b^3}$$

[Out] $-3*a*x/b^3+1/2*(b*x+a)^2/b^4+1/2*(3*a^2-c)*\ln(c+(b*x+a)^2)/b^4-a*(a^2-3*c)*\arctan((b*x+a)/c^{(1/2)})/b^4/c^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {371, 702, 635, 203, 260}

$$\frac{(3a^2 - c) \log((a + bx)^2 + c)}{2b^4} - \frac{a(a^2 - 3c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{(a + bx)^2}{2b^4} - \frac{3ax}{b^3}$$

Antiderivative was successfully verified.

[In] Int[x^3/(c + (a + b*x)^2), x]

[Out] $(-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*\text{ArcTan}[(a + b*x)/\text{Sqrt}[c]])/(b^4*\text{Sqrt}[c]) + ((3*a^2 - c)*\text{Log}[c + (a + b*x)^2])/(2*b^4)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 702

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{c + (a + bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^3}{c+x^2} dx, x, a + bx\right)}{b^4} \\
&= \frac{\text{Subst}\left(\int \left(-3a + x - \frac{a^3 - 3ac - (3a^2 - c)x}{c+x^2}\right) dx, x, a + bx\right)}{b^4} \\
&= -\frac{3ax}{b^3} + \frac{(a + bx)^2}{2b^4} - \frac{\text{Subst}\left(\int \frac{a^3 - 3ac - (3a^2 - c)x}{c+x^2} dx, x, a + bx\right)}{b^4} \\
&= -\frac{3ax}{b^3} + \frac{(a + bx)^2}{2b^4} - \frac{(a(a^2 - 3c)) \text{Subst}\left(\int \frac{1}{c+x^2} dx, x, a + bx\right)}{b^4} + \frac{(3a^2 - c) \text{Subst}\left(\int \frac{x}{c+x^2} dx, x, a + bx\right)}{b^4} \\
&= -\frac{3ax}{b^3} + \frac{(a + bx)^2}{2b^4} - \frac{a(a^2 - 3c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4 \sqrt{c}} + \frac{(3a^2 - c) \log(c + (a + bx)^2)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.94

$$\frac{-\frac{2(a^3 - 3ac) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + (3a^2 - c) \log(a^2 + 2abx + b^2x^2 + c) + bx(bx - 4a)}{2b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(c + (a + b*x)^2), x]

[Out] (b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + (3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)

fricas [A] time = 0.87, size = 198, normalized size = 2.54

$$\left[\frac{b^2cx^2 - 4abcx + (a^3 - 3ac)\sqrt{-c} \log\left(\frac{b^2x^2 + 2abx + a^2 - 2(bx+a)\sqrt{-c} - c}{b^2x^2 + 2abx + a^2 + c}\right) + (3a^2c - c^2) \log(b^2x^2 + 2abx + a^2 + c)}{2b^4c}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^2*c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a*b*c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]

giac [A] time = 0.34, size = 77, normalized size = 0.99

$$\frac{(3a^2 - c) \log(b^2x^2 + 2abx + a^2 + c)}{2b^4} - \frac{(a^3 - 3ac) \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4 \sqrt{c}} + \frac{b^2x^2 - 4abx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c+(b*x+a)^2), x, algorithm="giac")

[Out] 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4

maple [A] time = 0.01, size = 127, normalized size = 1.63

$$\frac{x^2}{2b^2} - \frac{a^3 \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{3a^2 \ln(b^2x^2 + 2abx + a^2 + c)}{2b^4} - \frac{2ax}{b^3} + \frac{3a\sqrt{c} \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^4} - \frac{c \ln(b^2x^2 + 2abx + a^2)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c+(b*x+a)^2),x)

[Out] 1/2/b^2*x^2-2*a*x/b^3+3/2/b^4*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2-1/2/b^4*ln(b^2*x^2+2*a*b*x+a^2+c)*c-1/b^4/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^3+3/b^4*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a

maxima [A] time = 1.46, size = 81, normalized size = 1.04

$$\frac{bx^2 - 4ax}{2b^3} + \frac{(3a^2 - c) \log(b^2x^2 + 2abx + a^2 + c)}{2b^4} - \frac{(a^3 - 3ac) \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] 1/2*(b*x^2 - 4*a*x)/b^3 + 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^4*sqrt(c))

mupad [B] time = 2.27, size = 87, normalized size = 1.12

$$\frac{x^2}{2b^2} - \frac{2ax}{b^3} - \frac{\ln(a^2 + 2abx + b^2x^2 + c) (4b^4c^2 - 12a^2b^4c)}{8b^8c} + \frac{a \operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right) (3c - a^2)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c + (a + b*x)^2),x)

[Out] x^2/(2*b^2) - (2*a*x)/b^3 - (log(c + a^2 + b^2*x^2 + 2*a*b*x)*(4*b^4*c^2 - 12*a^2*b^4*c))/(8*b^8*c) + (a*atan((a + b*x)/c^(1/2))*(3*c - a^2))/(b^4*c^(1/2))

sympy [B] time = 0.69, size = 209, normalized size = 2.68

$$-\frac{2ax}{b^3} + \left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4} \right) \log \left(x + \frac{a^4 - 2b^4c \left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4} \right) - c^2}{a^3b - 3abc} \right) + \left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c+(b*x+a)**2),x)

[Out] -2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c))/(2*b**4*c) + (3*a**2 - c)/(2*b**4) *log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c))/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c) + (a*sqrt(-c)*(a**2 - 3*c))/(2*b**4*c) + (3*a**2 - c)/(2*b**4)*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2 - 3*c))/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c) + x**2/(2*b**2)

$$3.79 \quad \int \frac{x^2}{c+(a+bx)^2} dx$$

Optimal. Leaf size=50

$$\frac{(a^2 - c) \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right)}{b^3 \sqrt{c}} - \frac{a \log((a+bx)^2 + c)}{b^3} + \frac{x}{b^2}$$

[Out] x/b^2-a*ln(c+(b*x+a)^2)/b^3+(a^2-c)*arctan((b*x+a)/c^(1/2))/b^3/c^(1/2)

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {371, 702, 635, 203, 260}

$$\frac{(a^2 - c) \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right)}{b^3 \sqrt{c}} - \frac{a \log((a+bx)^2 + c)}{b^3} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(c + (a + b*x)^2), x]

[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*Log[c + (a + b*x)^2])/b^3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 702

Int[((d_) + (e_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{c + (a + bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{c+x^2} dx, x, a + bx\right)}{b^3} \\
&= \frac{\text{Subst}\left(\int \left(1 + \frac{a^2-c-2ax}{c+x^2}\right) dx, x, a + bx\right)}{b^3} \\
&= \frac{x}{b^2} + \frac{\text{Subst}\left(\int \frac{a^2-c-2ax}{c+x^2} dx, x, a + bx\right)}{b^3} \\
&= \frac{x}{b^2} - \frac{(2a) \text{Subst}\left(\int \frac{x}{c+x^2} dx, x, a + bx\right)}{b^3} + \frac{(a^2 - c) \text{Subst}\left(\int \frac{1}{c+x^2} dx, x, a + bx\right)}{b^3} \\
&= \frac{x}{b^2} + \frac{(a^2 - c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3 \sqrt{c}} - \frac{a \log(c + (a + bx)^2)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.08

$$\frac{-a \log(a^2 + 2abx + b^2x^2 + c) + \frac{(a^2-c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + bx}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(c + (a + b*x)^2), x]

[Out] (b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a*b*x + b^2*x^2])/b^3

fricas [A] time = 0.80, size = 157, normalized size = 3.14

$$\left[\frac{2bcx - 2ac \log(b^2x^2 + 2abx + a^2 + c) + (a^2 - c)\sqrt{-c} \log\left(\frac{b^2x^2 + 2abx + a^2 + 2(bx+a)\sqrt{-c} - c}{b^2x^2 + 2abx + a^2 + c}\right)}{2b^3c}, \frac{bcx - ac \log(b^2x^2 + 2abx + a^2 + c)}{b^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]

giac [A] time = 0.31, size = 54, normalized size = 1.08

$$\frac{x}{b^2} - \frac{a \log(b^2x^2 + 2abx + a^2 + c)}{b^3} + \frac{(a^2 - c) \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+(b*x+a)^2), x, algorithm="giac")

[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a)/sqrt(c))/(b^3*sqrt(c))

maple [A] time = 0.01, size = 89, normalized size = 1.78

$$\frac{a^2 \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^3\sqrt{c}} - \frac{a \ln(b^2x^2 + 2abx + a^2 + c)}{b^3} + \frac{x}{b^2} - \frac{\sqrt{c} \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c+(b*x+a)^2),x)

[Out] x/b^2-1/b^3*a*ln(b^2*x^2+2*a*b*x+a^2+c)+1/b^3/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^2-1/b^3*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))

maxima [A] time = 1.65, size = 61, normalized size = 1.22

$$\frac{x}{b^2} - \frac{a \log(b^2x^2 + 2abx + a^2 + c)}{b^3} + \frac{(a^2 - c) \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^3*sqrt(c))

mupad [B] time = 0.09, size = 206, normalized size = 4.12

$$\frac{x}{b^2} - \frac{a \ln(a^2 + 2abx + b^2x^2 + c)}{b^3} + \frac{\sqrt{c} \operatorname{atan}\left(\frac{a^3}{\sqrt{c}(c-a^2)} - \frac{\sqrt{c}x}{b} - \frac{a\sqrt{c}}{c-a^2} + \frac{a^2x}{\sqrt{c}\left(\frac{c}{b} - \frac{a^2}{b}\right)}\right)}{b^3} - \frac{a^2 \operatorname{atan}\left(\frac{a^3}{\sqrt{c}(c-a^2)} - \frac{\sqrt{c}x}{b} - \frac{a^2}{b}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c + (a + b*x)^2),x)

[Out] x/b^2 - (a*log(c + a^2 + b^2*x^2 + 2*a*b*x))/b^3 + (c^(1/2)*atan(a^3/(c^(1/2)*(c - a^2)) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a^2/b))))/b^3 - (a^2*atan(a^3/(c^(1/2)*(c - a^2)) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a^2/b))))/b^3*c^(1/2)

sympy [B] time = 0.46, size = 153, normalized size = 3.06

$$\left(-\frac{a}{b^3} - \frac{\sqrt{-c}(a^2 - c)}{2b^3c}\right) \log\left(x + \frac{a^3 + ac + 2b^3c\left(-\frac{a}{b^3} - \frac{\sqrt{-c}(a^2 - c)}{2b^3c}\right)}{a^2b - bc}\right) + \left(-\frac{a}{b^3} + \frac{\sqrt{-c}(a^2 - c)}{2b^3c}\right) \log\left(x + \frac{a^3 + ac + 2b^3c\left(-\frac{a}{b^3} + \frac{\sqrt{-c}(a^2 - c)}{2b^3c}\right)}{a^2b - bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c+(b*x+a)**2),x)

[Out] (-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2

$$3.80 \quad \int \frac{x}{c+(a+bx)^2} dx$$

Optimal. Leaf size=41

$$\frac{\log((a+bx)^2+c)}{2b^2} - \frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

[Out] 1/2*ln(c+(b*x+a)^2)/b^2-a*arctan((b*x+a)/c^(1/2))/b^2/c^(1/2)

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {371, 635, 203, 260}

$$\frac{\log((a+bx)^2+c)}{2b^2} - \frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[x/(c + (a + b*x)^2), x]

[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x)^2]/(2*b^2)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{x}{c+(a+bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{-a+x}{c+x^2} dx, x, a+bx\right)}{b^2} \\ &= \frac{\text{Subst}\left(\int \frac{x}{c+x^2} dx, x, a+bx\right)}{b^2} - \frac{a \text{Subst}\left(\int \frac{1}{c+x^2} dx, x, a+bx\right)}{b^2} \\ &= -\frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log(c+(a+bx)^2)}{2b^2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.93

$$\frac{\log\left((a+bx)^2+c\right)-\frac{2a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(c + (a + b*x)^2), x]

[Out] ((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)

fricas [A] time = 0.82, size = 136, normalized size = 3.32

$$\left[\frac{a\sqrt{-c} \log\left(\frac{b^2x^2+2abx+a^2+2(bx+a)\sqrt{-c}-c}{b^2x^2+2abx+a^2+c}\right) - c \log(b^2x^2 + 2abx + a^2 + c)}{2b^2c}, -\frac{2a\sqrt{c} \arctan\left(\frac{bx+a}{\sqrt{c}}\right) - c \log(b^2x^2 + 2abx + a^2 + c)}{2b^2c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c)]

giac [A] time = 0.36, size = 43, normalized size = 1.05

$$-\frac{a \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log(b^2x^2 + 2abx + a^2 + c)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+(b*x+a)^2), x, algorithm="giac")

[Out] -a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^2

maple [A] time = 0.00, size = 54, normalized size = 1.32

$$-\frac{a \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\ln(b^2x^2 + 2abx + a^2 + c)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c+(b*x+a)^2), x)

[Out] 1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))

maxima [A] time = 1.36, size = 50, normalized size = 1.22

$$-\frac{a \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log(b^2x^2 + 2abx + a^2 + c)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+(b*x+a)^2), x, algorithm="maxima")

[Out] $-a \arctan\left(\frac{b^2 x + a b}{b \sqrt{c}}\right) / (b^2 \sqrt{c}) + \frac{1}{2} \log(b^2 x^2 + 2 a b x + a^2 + c) / b^2$

mupad [B] time = 2.08, size = 46, normalized size = 1.12

$$\frac{\ln\left(a^2 + 2 a b x + b^2 x^2 + c\right)}{2 b^2} - \frac{a \operatorname{atan}\left(\frac{a}{\sqrt{c}} + \frac{b x}{\sqrt{c}}\right)}{b^2 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c + (a + b*x)^2), x)`

[Out] $\log(c + a^2 + b^2 x^2 + 2 a b x) / (2 b^2) - (a \operatorname{atan}(a / c^{1/2} + (b x) / c^{1/2})) / (b^2 c^{1/2})$

sympy [B] time = 0.25, size = 124, normalized size = 3.02

$$\left(-\frac{a \sqrt{-c}}{2 b^2 c} + \frac{1}{2 b^2}\right) \log\left(x + \frac{a^2 - 2 b^2 c \left(-\frac{a \sqrt{-c}}{2 b^2 c} + \frac{1}{2 b^2}\right) + c}{a b}\right) + \left(\frac{a \sqrt{-c}}{2 b^2 c} + \frac{1}{2 b^2}\right) \log\left(x + \frac{a^2 - 2 b^2 c \left(\frac{a \sqrt{-c}}{2 b^2 c} + \frac{1}{2 b^2}\right) + c}{a b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c+(b*x+a)**2), x)`

[Out] $(-a \sqrt{-c} / (2 b^2 c) + 1 / (2 b^2)) * \log(x + (a^2 - 2 b^2 c * (-a \sqrt{-c} / (2 b^2 c) + 1 / (2 b^2)) + c) / (a b)) + (a \sqrt{-c} / (2 b^2 c) + 1 / (2 b^2)) * \log(x + (a^2 - 2 b^2 c * (a \sqrt{-c} / (2 b^2 c) + 1 / (2 b^2)) + c) / (a b))$

$$3.81 \quad \int \frac{1}{c+(a+bx)^2} dx$$

Optimal. Leaf size=21

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

[Out] arctan((b*x+a)/c^(1/2))/b/c^(1/2)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {247, 203}

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(c + (a + b*x)^2)^(-1), x]

[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{c+(a+bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{c+x^2} dx, x, a+bx\right)}{b} \\ &= \frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + (a + b*x)^2)^(-1), x]

[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])

fricas [A] time = 0.66, size = 83, normalized size = 3.95

$$\left[\frac{\sqrt{-c} \log\left(\frac{b^2x^2+2abx+a^2-2(bx+a)\sqrt{-c}-c}{b^2x^2+2abx+a^2+c}\right)}{2bc}, \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+(b*x+a)^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]

giac [A] time = 0.38, size = 17, normalized size = 0.81

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+(b*x+a)^2),x, algorithm="giac")

[Out] arctan((b*x + a)/sqrt(c))/(b*sqrt(c))

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c+(b*x+a)^2),x)

[Out] 1/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))

maxima [A] time = 1.52, size = 24, normalized size = 1.14

$$\frac{\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] arctan((b^2*x + a*b)/(b*sqrt(c)))/(b*sqrt(c))

mupad [B] time = 0.04, size = 17, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + (a + b*x)^2),x)

[Out] atan((a + b*x)/c^(1/2))/(b*c^(1/2))

sympy [B] time = 0.19, size = 54, normalized size = 2.57

$$\frac{-\frac{\sqrt{-\frac{1}{c}} \log\left(x + \frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2} + \frac{\sqrt{-\frac{1}{c}} \log\left(x + \frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+(b*x+a)**2),x)

[Out] (-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*sqrt(-1/c))/b)/2)/b

$$3.82 \quad \int \frac{1}{x(c+(a+bx)^2)} dx$$

Optimal. Leaf size=59

$$-\frac{\log((a+bx)^2+c)}{2(a^2+c)} - \frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)} + \frac{\log(x)}{a^2+c}$$

[Out] $\ln(x)/(a^2+c) - 1/2 * \ln(c+(b*x+a)^2)/(a^2+c) - a * \arctan((b*x+a)/c^{(1/2)})/(a^2+c) / c^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {371, 706, 31, 635, 203, 260}

$$-\frac{\log((a+bx)^2+c)}{2(a^2+c)} - \frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)} + \frac{\log(x)}{a^2+c}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(c + (a + b*x)^2)),x]

[Out] $-(a * \text{ArcTan}[(a + b*x)/\text{Sqrt}[c]])/(\text{Sqrt}[c]*(a^2 + c)) + \text{Log}[x]/(a^2 + c) - \text{Log}[c + (a + b*x)^2]/(2*(a^2 + c))$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^{(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*xⁿ, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]}

Rule 371

Int[((a_) + (b_.)*(v_)^{(n_))^(p_)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*xⁿ)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]}

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,

0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(c+(a+bx)^2)} dx &= \text{Subst} \left(\int \frac{1}{(-a+x)(c+x^2)} dx, x, a+bx \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{-a+x} dx, x, a+bx \right)}{a^2+c} + \frac{\text{Subst} \left(\int \frac{-a-x}{c+x^2} dx, x, a+bx \right)}{a^2+c} \\
&= \frac{\log(x)}{a^2+c} - \frac{\text{Subst} \left(\int \frac{x}{c+x^2} dx, x, a+bx \right)}{a^2+c} - \frac{a \text{Subst} \left(\int \frac{1}{c+x^2} dx, x, a+bx \right)}{a^2+c} \\
&= -\frac{a \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right)}{\sqrt{c}(a^2+c)} + \frac{\log(x)}{a^2+c} - \frac{\log(c+(a+bx)^2)}{2(a^2+c)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.81

$$-\frac{\log((a+bx)^2+c) + \frac{2a \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right)}{\sqrt{c}} - 2 \log(bx)}{2(a^2+c)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(c+(a+b*x)^2)),x]

[Out] -1/2*((2*a*ArcTan[(a+b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c+(a+b*x)^2])/(a^2+c)

fricas [A] time = 0.84, size = 154, normalized size = 2.61

$$\left[\frac{a\sqrt{-c} \log \left(\frac{b^2x^2+2abx+a^2+2(bx+a)\sqrt{-c}-c}{b^2x^2+2abx+a^2+c} \right) + c \log(b^2x^2+2abx+a^2+c) - 2c \log(x)}{2(a^2c+c^2)}, -\frac{2a\sqrt{c} \arctan \left(\frac{bx+a}{\sqrt{c}} \right)}{2(a^2+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="fricas")

[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2+2*a*b*x+a^2+2*(b*x+a)*sqrt(-c)-c)/(b^2*x^2+2*a*b*x+a^2+c))+c*log(b^2*x^2+2*a*b*x+a^2+c)-2*c*log(x))/(a^2*c+c^2), -1/2*(2*a*sqrt(c)*arctan((b*x+a)/sqrt(c))+c*log(b^2*x^2+2*a*b*x+a^2+c)-2*c*log(x))/(a^2*c+c^2)]

giac [A] time = 0.38, size = 62, normalized size = 1.05

$$-\frac{a \arctan \left(\frac{bx+a}{\sqrt{c}} \right)}{(a^2+c)\sqrt{c}} - \frac{\log(b^2x^2+2abx+a^2+c)}{2(a^2+c)} + \frac{\log(|x|)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="giac")

[Out] -a*arctan((b*x+a)/sqrt(c))/((a^2+c)*sqrt(c)) - 1/2*log(b^2*x^2+2*a*b*x+a^2+c)/(a^2+c) + log(abs(x))/(a^2+c)

maple [A] time = 0.01, size = 72, normalized size = 1.22

$$-\frac{a \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{(a^2+c)\sqrt{c}} + \frac{\ln(x)}{a^2+c} - \frac{\ln(b^2x^2+2abx+a^2+c)}{2(a^2+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c+(b*x+a)^2),x)

[Out] ln(x)/(a^2+c)-1/2/(a^2+c)*ln(b^2*x^2+2*a*b*x+a^2+c)-1/(a^2+c)*a/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))

maxima [A] time = 1.52, size = 68, normalized size = 1.15

$$-\frac{a \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^2+c)\sqrt{c}} - \frac{\log(b^2x^2+2abx+a^2+c)}{2(a^2+c)} + \frac{\log(x)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] -a*arctan((b^2*x+a*b)/(b*sqrt(c)))/((a^2+c)*sqrt(c)) - 1/2*log(b^2*x^2+2*a*b*x+a^2+c)/(a^2+c) + log(x)/(a^2+c)

mupad [B] time = 2.59, size = 173, normalized size = 2.93

$$\frac{\ln(x)}{a^2+c} - \frac{\ln\left(2ab^3+3b^4x+\frac{b^3(c+a\sqrt{-c})(a^3+bx a^2+c a-3bcx)}{c(a^2+c)}\right)}{2(a^2c+c^2)} - \frac{\ln\left(2ab^3+3b^4x+\frac{b^3(c-a\sqrt{-c})(a^3+bx a^2+c a-3bcx)}{c(a^2+c)}\right)}{2(a^2c+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(c+(a+b*x)^2)),x)

[Out] log(x)/(c+a^2) - (log(2*a*b^3+3*b^4*x+(b^3*(c+a*(-c)^(1/2))*(a*c+a^3-3*b*c*x+a^2*b*x))/(c*(c+a^2)))*(c+a*(-c)^(1/2)))/(2*(a^2*c+c^2)) - (log(2*a*b^3+3*b^4*x+(b^3*(c-a*(-c)^(1/2))*(a*c+a^3-3*b*c*x+a^2*b*x))/(c*(c+a^2)))*(c-a*(-c)^(1/2)))/(2*(a^2*c+c^2))

sympy [B] time = 3.40, size = 738, normalized size = 12.51

$$\left(-\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right) \log\left(x + \frac{-4a^6c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 + 4a^4c^2\left(-\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 - 6a^4c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)}{2(a^2+c)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c+(b*x+a)**2),x)

[Out] (-a*sqrt(-c)/(2*c*(a**2+c)) - 1/(2*(a**2+c)))*log(x + (-4*a**6*c*(-a*sqrt(-c)/(2*c*(a**2+c)) - 1/(2*(a**2+c)))**2 + 4*a**4*c**2*(-a*sqrt(-c)/(2*c*(a**2+c)) - 1/(2*(a**2+c)))**2 - 6*a**4*c*(-a*sqrt(-c)/(2*c*(a**2+c)) - 1/(2*(a**2+c))) + 20*a**2*c**3*(-a*sqrt(-c)/(2*c*(a**2+c)) - 1/(2*(a**2+c)))**2 - 12*a**2*c**2*(-a*sqrt(-c)/(2*c*(a**2+c)) - 1/(2*(a**2+c))) + 10*a**2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2+c)) - 1/(2*(a**2+c)))**2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2+c)) - 1/(2*(a**2+c))) - 6*c**2

$$\begin{aligned} &)/(a^{3}b + 9ab^{2}c) + (a\sqrt{-c}/(2c(a^{2} + c)) - 1/(2(a^{2} + c)))\log(x + (-4a^{6}c(a\sqrt{-c}/(2c(a^{2} + c)) - 1/(2(a^{2} + c)))^{2} + 4a^{4}c^{2}(a\sqrt{-c}/(2c(a^{2} + c)) - 1/(2(a^{2} + c)))^{2} - 6a^{4}c(a\sqrt{-c}/(2c(a^{2} + c)) - 1/(2(a^{2} + c))) + 20a^{2}c^{3}(a\sqrt{-c}/(2c(a^{2} + c)) - 1/(2(a^{2} + c)))^{2} - 12a^{2}c^{2}(a\sqrt{-c}/(2c(a^{2} + c)) - 1/(2(a^{2} + c))) + 10a^{2}c + 12c^{4}(a\sqrt{-c}/(2c(a^{2} + c)) - 1/(2(a^{2} + c)))^{2} - 6c^{3}(a\sqrt{-c}/(2c(a^{2} + c)) - 1/(2(a^{2} + c))) - 6c^{2})/(a^{3}b + 9ab^{2}c) + \log(x + (-4a^{6}c/(a^{2} + c))^{2} + 4a^{4}c^{2}/(a^{2} + c)^{2} - 6a^{4}c/(a^{2} + c) + 20a^{2}c^{3}/(a^{2} + c)^{2} - 12a^{2}c^{2}/(a^{2} + c) + 10a^{2}c + 12c^{4}/(a^{2} + c)^{2} - 6c^{3}/(a^{2} + c) - 6c^{2})/(a^{3}b + 9ab^{2}c))/(a^{2} + c) \end{aligned}$$

3.83 $\int \frac{1}{x^2(c+(a+bx)^2)} dx$

Optimal. Leaf size=79

$$-\frac{2ab \log(x)}{(a^2+c)^2} + \frac{ab \log((a+bx)^2+c)}{(a^2+c)^2} + \frac{b(a^2-c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^2} - \frac{1}{x(a^2+c)}$$

[Out] $-1/(a^2+c)/x-2*a*b*\ln(x)/(a^2+c)^2+a*b*\ln(c+(b*x+a)^2)/(a^2+c)^2+b*(a^2-c)*\arctan((b*x+a)/c^{(1/2)})/(a^2+c)^2/c^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {371, 710, 801, 635, 203, 260}

$$-\frac{2ab \log(x)}{(a^2+c)^2} + \frac{ab \log((a+bx)^2+c)}{(a^2+c)^2} + \frac{b(a^2-c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^2} - \frac{1}{x(a^2+c)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(c + (a + b*x)^2)),x]

[Out] $-(1/((a^2+c)*x)) + (b*(a^2-c)*\text{ArcTan}[(a+b*x)/\text{Sqrt}[c]])/(\text{Sqrt}[c]*(a^2+c)^2) - (2*a*b*\text{Log}[x])/(a^2+c)^2 + (a*b*\text{Log}[c+(a+b*x)^2])/(a^2+c)^2$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 710

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(d - e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

Rule 801

Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2),
 x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(c + (a + bx)^2)} dx &= b \operatorname{Subst} \left(\int \frac{1}{(-a + x)^2(c + x^2)} dx, x, a + bx \right) \\ &= -\frac{1}{(a^2 + c)x} + \frac{b \operatorname{Subst} \left(\int \frac{-a-x}{(-a+x)(c+x^2)} dx, x, a + bx \right)}{a^2 + c} \\ &= -\frac{1}{(a^2 + c)x} + \frac{b \operatorname{Subst} \left(\int \left(\frac{2a}{(a^2+c)(a-x)} + \frac{a^2-c+2ax}{(a^2+c)(c+x^2)} \right) dx, x, a + bx \right)}{a^2 + c} \\ &= -\frac{1}{(a^2 + c)x} - \frac{2ab \log(x)}{(a^2 + c)^2} + \frac{b \operatorname{Subst} \left(\int \frac{a^2-c+2ax}{c+x^2} dx, x, a + bx \right)}{(a^2 + c)^2} \\ &= -\frac{1}{(a^2 + c)x} - \frac{2ab \log(x)}{(a^2 + c)^2} + \frac{(2ab) \operatorname{Subst} \left(\int \frac{x}{c+x^2} dx, x, a + bx \right)}{(a^2 + c)^2} + \frac{(b(a^2 - c)) \operatorname{Subst} \left(\int \frac{1}{c+x^2} dx, x, a + bx \right)}{(a^2 + c)^2} \\ &= -\frac{1}{(a^2 + c)x} + \frac{b(a^2 - c) \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right)}{\sqrt{c} (a^2 + c)^2} - \frac{2ab \log(x)}{(a^2 + c)^2} + \frac{ab \log(c + (a + bx)^2)}{(a^2 + c)^2} \end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.03

$$\frac{bx(a^2 - c) \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right) - \sqrt{c} (-abx \log(a^2 + 2abx + b^2x^2 + c) + a^2 + 2abx \log(x) + c)}{\sqrt{c} x (a^2 + c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(c + (a + b*x)^2)), x]

[Out] (b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[x] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)

fricas [A] time = 0.92, size = 229, normalized size = 2.90

$$\left[\frac{2abcx \log(b^2x^2 + 2abx + a^2 + c) - 4abcx \log(x) + (a^2b - bc)\sqrt{-c}x \log\left(\frac{b^2x^2 + 2abx + a^2 + 2(bx+a)\sqrt{-c} - c}{b^2x^2 + 2abx + a^2 + c}\right) - 2a^2c}{2(a^4c + 2a^2c^2 + c^3)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2*b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2 + c^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) + (a^2*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c + 2*a^2*c^2 + c^3)*x)]

giac [A] time = 0.37, size = 117, normalized size = 1.48

$$\frac{ab \log(b^2x^2 + 2abx + a^2 + c)}{a^4 + 2a^2c + c^2} - \frac{2ab \log(|x|)}{a^4 + 2a^2c + c^2} + \frac{(a^2b^2 - b^2c) \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^4 + 2a^2c + c^2)b\sqrt{c}} - \frac{1}{(a^2 + c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")

[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(abs(x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x + a)/sqrt(c))/((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)

maple [A] time = 0.01, size = 123, normalized size = 1.56

$$\frac{a^2b \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{(a^2 + c)^2 \sqrt{c}} - \frac{2ab \ln(x)}{(a^2 + c)^2} + \frac{ab \ln(b^2x^2 + 2abx + a^2 + c)}{(a^2 + c)^2} - \frac{b\sqrt{c} \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{(a^2 + c)^2} - \frac{1}{(a^2 + c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c+(b*x+a)^2),x)

[Out] -1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+b/(a^2+c)^2*a*ln(b^2*x^2+2*a*b*x+a^2+c)+b/(a^2+c)^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^2-b/(a^2+c)^2*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))

maxima [A] time = 1.59, size = 123, normalized size = 1.56

$$\frac{ab \log(b^2x^2 + 2abx + a^2 + c)}{a^4 + 2a^2c + c^2} - \frac{2ab \log(x)}{a^4 + 2a^2c + c^2} + \frac{(a^2b^2 - b^2c) \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^4 + 2a^2c + c^2)b\sqrt{c}} - \frac{1}{(a^2 + c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(x)/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)

mupad [B] time = 2.58, size = 425, normalized size = 5.38

$$\frac{\ln\left((-c)^{13/2} - 35a^2(-c)^{11/2} + 34a^4(-c)^{9/2} + 34a^6(-c)^{7/2} - 35a^8(-c)^{5/2} + a^{10}(-c)^{3/2} + ac^6 - a^{11}c + 35a^3c^5 + 34a^4b^2c^4 - 34a^7b^2c^3 - 35a^9b^2c^2 + b^2c^6x - a^{10}b^2c^5x + 35a^2b^2c^5x + 34a^4b^2c^4x - 34a^6b^2c^3x - 35a^8b^2c^2x\right) * (b(-c)^{3/2} + 2a*b*c + a^2*b*(-c)^{1/2})}{2(a^4*c + c^3 + 2a^2*c^2)} - \frac{1}{x(c + a^2)} - \left(\log\left((-c)^{13/2} - 35a^2(-c)^{11/2} + 34a^4(-c)^{9/2} + 34a^6(-c)^{7/2} - 35a^8(-c)^{5/2} + a^{10}(-c)^{3/2} - a*c^6 + a^{11}c - 35a^3c^5 - 34a^5c^4 + 34a^7c^3 + 35a^9c^2 - b^2c^6x + a^{10}b^2c^5x - 35a^2b^2c^5x - 34a^4b^2c^4x + 34a^6b^2c^3x + 35a^8b^2c^2x\right) * (b(-c)^{3/2} - 2a*b*c +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(c + (a + b*x)^2)),x)

[Out] (log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) + a*c^6 - a^11*c + 35*a^3*c^5 + 34*a^4*b^2*c^4 - 34*a^7*b^2*c^3 - 35*a^9*b^2*c^2 + b^2*c^6*x - a^10*b^2*c^5*x + 35*a^2*b^2*c^5*x + 34*a^4*b^2*c^4*x - 34*a^6*b^2*c^3*x - 35*a^8*b^2*c^2*x)*(b*(-c)^(3/2) + 2*a*b*c + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - 1/(x*(c + a^2)) - (log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) - a*c^6 + a^11*c - 35*a^3*c^5 - 34*a^5*c^4 + 34*a^7*c^3 + 35*a^9*c^2 - b^2*c^6*x + a^10*b^2*c^5*x - 35*a^2*b^2*c^5*x - 34*a^4*b^2*c^4*x + 34*a^6*b^2*c^3*x + 35*a^8*b^2*c^2*x)*(b*(-c)^(3/2) - 2*a*b*c +

$$\frac{a^2 b (-c)^{1/2}}{(2(a^4 c + c^3 + 2a^2 c^2))} - \frac{(2a b \log(x))}{(c + a^2)^2}$$

sympy [B] time = 11.12, size = 1620, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c+(b*x+a)**2), x)

[Out]
$$\begin{aligned} & -2ab \log(x + (-16a^{13}b^2c/(a^2 + c)^4 + 48a^{11}b^2c^2/(a^2 + c)^4 + 352a^9b^2c^3/(a^2 + c)^4 - 20a^9b^2c/(a^2 + c)^2 + \\ & 608a^7b^2c^4/(a^2 + c)^4 - 64a^7b^2c^2/(a^2 + c)^2 + 432a^5b^2c^5/(a^2 + c)^4 - 72a^5b^2c^3/(a^2 + c)^2 + 36a^5b^2 \\ & *c + 112a^3b^2c^6/(a^2 + c)^4 - 32a^3b^2c^4/(a^2 + c)^2 - 8 \\ & 8a^3b^2c^2 - 4ab^2c^5/(a^2 + c)^2 + 4ab^2c^3)/(a^6b^3 \\ & + 33a^4b^3c - 33a^2b^3c^2 - b^3c^3))/(a^2 + c)^2 + (ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) \log(x + \\ & (-4a^{11}c(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 12a^9c^2(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2 \\ & *c(a^4 + 2a^2c + c^2))))^2 + 10a^8b^2c(ab/(a^2 + c)^2 - b\sqrt{-c}(\\ & -c)(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 88a^7c^3(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 32a^6b^2 \\ & b^2c^2(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 36a^5b^2c + 152a^5c^4(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 36a^4b^2c^3(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) - 88a^3b^2c^2 + 108a^3c^5(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 16a^2b^2c^4(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 4ab^2c^3 + 28a^2c^6(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 2b^2c^5(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))/(a^6b^3 + 33a^4b^3c - 33a^2b^3c^2 - b^3c^3)) + (ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) \log(x + (-4a^{11}c(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 12a^9c^2(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 10a^8b^2c(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 88a^7c^3(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 32a^6b^2c^2(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 36a^5b^2c + 152a^5c^4(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 36a^4b^2c^3(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) - 88a^3b^2c^2 + 108a^3c^5(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 16a^2b^2c^4(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 4ab^2c^3 + 28a^2c^6(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))^2 + 2b^2c^5(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))/(a^6b^3 + 33a^4b^3c - 33a^2b^3c^2 - b^3c^3)) - 1/(x(a^2 + c)) \end{aligned}$$

$$3.84 \quad \int \frac{1}{x^3(c+(a+bx)^2)} dx$$

Optimal. Leaf size=121

$$\frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} - \frac{b^2(3a^2-c)\log((a+bx)^2+c)}{2(a^2+c)^3} - \frac{ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^3} + \frac{2ab}{x(a^2+c)^2} - \frac{1}{2x^2(a^2+c)}$$

[Out] $-1/2/(a^2+c)/x^2+2*a*b/(a^2+c)^2/x+b^2*(3*a^2-c)*\ln(x)/(a^2+c)^3-1/2*b^2*(3*a^2-c)*\ln(c+(b*x+a)^2)/(a^2+c)^3-a*b^2*(a^2-3*c)*\arctan((b*x+a)/c^{(1/2)})/(a^2+c)^3/c^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {371, 710, 801, 635, 203, 260}

$$\frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} - \frac{b^2(3a^2-c)\log((a+bx)^2+c)}{2(a^2+c)^3} - \frac{ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^3} + \frac{2ab}{x(a^2+c)^2} - \frac{1}{2x^2(a^2+c)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(c + (a + b*x)^2)),x]

[Out] $-1/(2*(a^2+c)*x^2) + (2*a*b)/((a^2+c)^2*x) - (a*b^2*(a^2-3*c)*\text{ArcTan}[(a+b*x)/\text{Sqrt}[c]])/(\text{Sqrt}[c]*(a^2+c)^3) + (b^2*(3*a^2-c)*\text{Log}[x])/(a^2+c)^3 - (b^2*(3*a^2-c)*\text{Log}[c+(a+b*x)^2])/(2*(a^2+c)^3)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 710

Int[((d_) + (e_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(d - e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

Rule 801

```
Int[(((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.)))/((a_.) + (c_.)*(x_.)^2),
  x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
  x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(c+(a+bx)^2)} dx &= b^2 \operatorname{Subst}\left(\int \frac{1}{(-a+x)^3(c+x^2)} dx, x, a+bx\right) \\ &= -\frac{1}{2(a^2+c)x^2} + \frac{b^2 \operatorname{Subst}\left(\int \frac{-a-x}{(-a+x)^2(c+x^2)} dx, x, a+bx\right)}{a^2+c} \\ &= -\frac{1}{2(a^2+c)x^2} + \frac{b^2 \operatorname{Subst}\left(\int \left(-\frac{2a}{(a^2+c)(a-x)^2} + \frac{-3a^2+c}{(a^2+c)^2(a-x)} + \frac{-a(a^2-3c)-(3a^2-c)x}{(a^2+c)^2(c+x^2)}\right) dx, x, a+bx\right)}{a^2+c} \\ &= -\frac{1}{2(a^2+c)x^2} + \frac{2ab}{(a^2+c)^2 x} + \frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} + \frac{b^2 \operatorname{Subst}\left(\int \frac{-a(a^2-3c)-(3a^2-c)x}{c+x^2} dx, x, a+bx\right)}{(a^2+c)^3} \\ &= -\frac{1}{2(a^2+c)x^2} + \frac{2ab}{(a^2+c)^2 x} + \frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} - \frac{(ab^2(a^2-3c)) \operatorname{Subst}\left(\int \frac{1}{c+x^2} dx, x, a+bx\right)}{(a^2+c)^3} \\ &= -\frac{1}{2(a^2+c)x^2} + \frac{2ab}{(a^2+c)^2 x} - \frac{ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^3} + \frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} - \frac{b^2(a^2-3c)}{(a^2+c)^3} \end{aligned}$$

Mathematica [A] time = 0.17, size = 106, normalized size = 0.88

$$\frac{b^2(3a^2-c)\log(a^2+2abx+b^2x^2+c) + 2b^2(c-3a^2)\log(x) + \frac{2ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{(a^2+c)(a^2-4abx+c)}{x^2}}{2(a^2+c)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(c+(a+b*x)^2)),x]

[Out] -1/2*((a^2+c)*(a^2+c-4*a*b*x))/x^2 + (2*a*b^2*(a^2-3*c)*ArcTan[(a+b*x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2+c)*Log[x] + b^2*(3*a^2-c)*Log[a^2+c+2*a*b*x+b^2*x^2]/(a^2+c)^3

fricas [A] time = 0.85, size = 371, normalized size = 3.07

$$\frac{\left[a^4c - (a^3b^2 - 3ab^2c)\sqrt{-c}x^2 \log\left(\frac{b^2x^2+2abx+a^2-2(bx+a)\sqrt{-c}-c}{b^2x^2+2abx+a^2+c}\right) + 2a^2c^2 + (3a^2b^2c - b^2c^2)x^2 \log(b^2x^2 + 2abx + a^2 + c) \right]}{2(a^6c + 3a^4c^2 + 3a^2c^3 + c^4)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="fricas")

[Out] [-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^2

+ (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sqrt(c) *x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2)]

giac [A] time = 0.42, size = 195, normalized size = 1.61

$$\frac{(3a^2b^2 - b^2c) \log(b^2x^2 + 2abx + a^2 + c)}{2(a^6 + 3a^4c + 3a^2c^2 + c^3)} + \frac{(3a^2b^2 - b^2c) \log(|x|)}{a^6 + 3a^4c + 3a^2c^2 + c^3} - \frac{(a^3b^3 - 3ab^3c) \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^6 + 3a^4c + 3a^2c^2 + c^3)b\sqrt{c}} - \frac{a^4 + 2a^2c}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")

[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c + 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(abs(x))/(a^6 + 3*a^4*c + 3*a^2*c^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x + a)/sqrt(c))/((a^6 + 3*a^4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b + a*b*c)*x)/((a^2 + c)^3*x^2)

maple [A] time = 0.01, size = 198, normalized size = 1.64

$$-\frac{a^3b^2 \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{(a^2+c)^3\sqrt{c}} + \frac{3a^2b^2 \ln(x)}{(a^2+c)^3} - \frac{3a^2b^2 \ln(b^2x^2 + 2abx + a^2 + c)}{2(a^2+c)^3} + \frac{3ab^2\sqrt{c} \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{(a^2+c)^3} - \frac{b^2c \ln(x)}{(a^2+c)^3} + \frac{b^2c}{(a^2+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(c+(b*x+a)^2),x)

[Out] -1/2/(a^2+c)/x^2+3*b^2/(a^2+c)^3*ln(x)*a^2-b^2/(a^2+c)^3*ln(x)*c+2*a*b/(a^2+c)^2/x-3/2*b^2/(a^2+c)^3*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2+1/2*b^2/(a^2+c)^3*ln(b^2*x^2+2*a*b*x+a^2+c)*c-b^2/(a^2+c)^3/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^3+3*b^2/(a^2+c)^3*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a

maxima [A] time = 1.61, size = 197, normalized size = 1.63

$$-\frac{(3a^2b^2 - b^2c) \log(b^2x^2 + 2abx + a^2 + c)}{2(a^6 + 3a^4c + 3a^2c^2 + c^3)} + \frac{(3a^2b^2 - b^2c) \log(x)}{a^6 + 3a^4c + 3a^2c^2 + c^3} - \frac{(a^3b^3 - 3ab^3c) \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^6 + 3a^4c + 3a^2c^2 + c^3)b\sqrt{c}} + \frac{4abx}{2(a^4 + 2a^2c + c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c + 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(x)/(a^6 + 3*a^4*c + 3*a^2*c^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^6 + 3*a^4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) + 1/2*(4*a*b*x - a^2 - c)/((a^4 + 2*a^2*c + c^2)*x^2)

mupad [B] time = 2.77, size = 573, normalized size = 4.74

$$\ln(x) \left(\frac{3b^2}{(a^2+c)^2} - \frac{4b^2c}{(a^2+c)^3} \right) - \frac{1}{2(a^2+c)} - \frac{2abx}{(a^2+c)^2} - \frac{\ln(27(-c)^{15/2} + 90a^2(-c)^{13/2} + 9a^4(-c)^{11/2} - 324a^6(-c)^{9/2} + \dots)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(c + (a + b*x)^2)),x)`

[Out] $\log(x) \cdot \left(\frac{3b^2}{(c+a^2)^2} - \frac{4b^2c}{(c+a^2)^3} - \frac{1}{2(c+a^2)} - \frac{2abx}{(c+a^2)^2} \right) / x^2 - \left(\log(27(-c)^{15/2} + 90a^2(-c)^{13/2} + 9a^4(-c)^{11/2} - 324a^6(-c)^{9/2} + 125a^8(-c)^{7/2} + 74a^{10}(-c)^{5/2} - a^{12}(-c)^{3/2} - 27a^7c + a^{13}c + 90a^3c^6 - 9a^5c^5 - 324a^7c^4 - 125a^9c^3 + 74a^{11}c^2 - 27b^2c^2 + 3a^2b^2c + 3ab^2(-c)^{3/2}) \right) / (2(a^6c + c^4 + 3a^2c^3 + 3a^4c^2)) + \left(\log(27(-c)^{15/2} + 90a^2(-c)^{13/2} + 9a^4(-c)^{11/2} - 324a^6(-c)^{9/2} + 125a^8(-c)^{7/2} + 74a^{10}(-c)^{5/2} - a^{12}(-c)^{3/2} + 27a^7c - a^{13}c - 90a^3c^6 + 9a^5c^5 + 324a^7c^4 + 125a^9c^3 - 74a^{11}c^2 + 27b^2c^2 - a^{12}b^2c^2 - 90a^2b^2c^6x + 9a^4b^2c^5x + 324a^6b^2c^4x + 125a^8b^2c^3x - 74a^{10}b^2c^2x) \right) \cdot (b^2c^2 + a^3b^2(-c)^{1/2} - 3a^2b^2c + 3ab^2(-c)^{3/2}) / (2(a^6c + c^4 + 3a^2c^3 + 3a^4c^2))$

sympy [B] time = 38.26, size = 3284, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(c+(b*x+a)**2),x)`

[Out] $b^2(3a^2 - c) \log(x + (-4a^{16}b^4c(3a^2 - c))^2 / (a^2 + c)^6 + 24a^{14}b^4c^2(3a^2 - c)^2 / (a^2 + c)^6 + 216a^{12}b^4c^3(3a^2 - c)^2 / (a^2 + c)^6 - 14a^{12}b^4c(3a^2 - c) / (a^2 + c)^3 + 568a^{10}b^4c^4(3a^2 - c)^2 / (a^2 + c)^6 - 44a^{10}b^4c^2(3a^2 - c) / (a^2 + c)^3 + 720a^8b^4c^5(3a^2 - c)^2 / (a^2 + c)^6 - 42a^8b^4c^3(3a^2 - c) / (a^2 + c)^3 + 78a^8b^4c + 456a^6b^4c^6(3a^2 - c)^2 / (a^2 + c)^6 - 8a^6b^4c^4(3a^2 - c) / (a^2 + c)^3 - 464a^6b^4c^2 + 104a^4b^4c^7(3a^2 - c)^2 / (a^2 + c)^6 - 2a^4b^4c^5(3a^2 - c) / (a^2 + c)^3 + 380a^4b^4c^3 - 24a^2b^4c^8(3a^2 - c)^2 / (a^2 + c)^6 - 12a^2b^4c^6(3a^2 - c) / (a^2 + c)^3 - 96a^2b^4c^4 - 12b^4c^9(3a^2 - c)^2 / (a^2 + c)^6 - 6b^4c^7(3a^2 - c) / (a^2 + c)^3 + 6b^4c^5 / (a^9b^5 + 72a^7b^5c - 270a^5b^5c^2 + 144a^3b^5c^3 - 27ab^5c^4) / (a^2 + c)^3 + (-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3) \log(x + (-4a^{16}c(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3))^2 + 24a^{14}c^2(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3))^2 - 14a^{12}b^2c(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3) + 216a^{12}c^3(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3))^2 - 44a^{10}b^2c^2(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3) + 568a^{10}c^4(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3))^2 + 78a^8b^4c - 42a^8b^2c^3(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3) + 720a^8c^5(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3))^2 - 464a^6b^4c^2 - 8a^6b^2c^4(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3) + 456a^6c^6(-ab^2\sqrt{-c})(a^2 - 3c) / (2c(a^6 + 3a^4c + 3a^2c^2 + c^3)) - b^2(3a^2 - c) / (2(a^2 + c)^3))^2 + 380a^4b^4c^3 - 2a^4b^2c^5(-ab^2\sqrt{-c})($

$$\begin{aligned}
& -c)(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}) + 104*a^{**4}*c^{**7}*(-a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 96*a^{**2}*b^{**4}*c^{**4} - 12*a^{**2}*b^{**2}*c^{**6}*(-a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) - 24*a^{**2}*c^{**8}*(-a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} + 6*b^{**4}*c^{**5} - 6*b^{**2}*c^{**7}*(-a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) - 12*c^{**9}*(-a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2})/(a^{**9}*b^{**5} + 72*a^{**7}*b^{**5}*c - 270*a^{**5}*b^{**5}*c^{**2} + 144*a^{**3}*b^{**5}*c^{**3} - 27*a*b^{**5}*c^{**4})) + (a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))*log(x + (-4*a^{**16}*c*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} + 24*a^{**14}*c^{**2}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 14*a^{**12}*b^{**2}*c*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) + 216*a^{**12}*c^{**3}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 44*a^{**10}*b^{**2}*c^{**2}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) + 568*a^{**10}*c^{**4}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} + 78*a^{**8}*b^{**4}*c - 42*a^{**8}*b^{**2}*c^{**3}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) + 720*a^{**8}*c^{**5}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 464*a^{**6}*b^{**4}*c^{**2} - 8*a^{**6}*b^{**2}*c^{**4}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) + 456*a^{**6}*c^{**6}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} + 380*a^{**4}*b^{**4}*c^{**3} - 2*a^{**4}*b^{**2}*c^{**5}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) + 104*a^{**4}*c^{**7}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 96*a^{**2}*b^{**4}*c^{**4} - 12*a^{**2}*b^{**2}*c^{**6}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) - 24*a^{**2}*c^{**8}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} + 6*b^{**4}*c^{**5} - 6*b^{**2}*c^{**7}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3})) - 12*c^{**9}*(a*b^{**2}*sqrt(-c)*(a^{**2} - 3*c)/(2*c*(a^{**6} + 3*a^{**4}*c + 3*a^{**2}*c^{**2} + c^{**3})) - b^{**2}*(3*a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2})/(a^{**9}*b^{**5} + 72*a^{**7}*b^{**5}*c - 270*a^{**5}*b^{**5}*c^{**2} + 144*a^{**3}*b^{**5}*c^{**3} - 27*a*b^{**5}*c^{**4})) + (-a^{**2} + 4*a*b*x - c)/(x^{**2}*(2*a^{**4} + 4*a^{**2}*c + 2*c^{**2}))
\end{aligned}$$

$$3.85 \quad \int \frac{1}{a+b(c+dx)^2} dx$$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

[Out] arctan((d*x+c)*b^(1/2)/a^(1/2))/d/a^(1/2)/b^(1/2)

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b(c+dx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, c+dx\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

fricas [A] time = 0.72, size = 109, normalized size = 3.52

$$\left[\frac{\sqrt{-ab} \log\left(\frac{bd^2x^2+2bcdx+bc^2-2\sqrt{-ab}(dx+c)-a}{bd^2x^2+2bcdx+bc^2+a}\right)}{2abd}, \frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{abd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a*b*d), sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a)/(a*b*d)]

giac [A] time = 0.45, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="giac")

[Out] arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*d)

maple [A] time = 0.01, size = 34, normalized size = 1.10

$$\frac{\arctan\left(\frac{2bd^2x+2bdc}{2\sqrt{ab}d}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^2),x)

[Out] 1/d/(b*a)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(b*a)^(1/2))

maxima [A] time = 1.56, size = 30, normalized size = 0.97

$$\frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="maxima")

[Out] arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*d)

mupad [B] time = 0.06, size = 27, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}c+\sqrt{b}dx}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^2),x)

[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/a^(1/2))/(a^(1/2)*b^(1/2)*d)

sympy [B] time = 0.21, size = 61, normalized size = 1.97

$$-\frac{\sqrt{-\frac{1}{ab}} \log\left(x + \frac{-a\sqrt{-\frac{1}{ab}} + c}{d}\right)}{2} + \frac{\sqrt{-\frac{1}{ab}} \log\left(x + \frac{a\sqrt{-\frac{1}{ab}} + c}{d}\right)}{2}$$

$$d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2),x)
```

```
[Out] (-sqrt(-1/(a*b))*log(x + (-a*sqrt(-1/(a*b)) + c)/d)/2 + sqrt(-1/(a*b))*log(x + (a*sqrt(-1/(a*b)) + c)/d)/2)/d
```

$$3.86 \quad \int \frac{1}{(a+b(c+dx)^2)^2} dx$$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{c+dx}{2ad(a+b(c+dx)^2)}$$

[Out] 1/2*(d*x+c)/a/d/(a+b*(d*x+c)^2)+1/2*arctan((d*x+c)*b^(1/2)/a^(1/2))/a^(3/2)/d/b^(1/2)

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{c+dx}{2ad(a+b(c+dx)^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^2)^(-2), x]

[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+b(c+dx)^2)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, c+dx\right)}{d} \\ &= \frac{c+dx}{2ad(a+b(c+dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, c+dx\right)}{2ad} \\ &= \frac{c+dx}{2ad(a+b(c+dx)^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.95

$$\frac{\frac{\sqrt{a}(c+dx)}{a+b(c+dx)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^2)^(-2), x]

[Out] ((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/Sqrt[b]/(2*a^(3/2)*d)

fricas [A] time = 0.56, size = 253, normalized size = 4.02

$$\left[\frac{2 abdx + 2 abc - (bd^2x^2 + 2 bcdx + bc^2 + a)\sqrt{-ab} \log\left(\frac{bd^2x^2 + 2 bcdx + bc^2 - 2\sqrt{-ab}(dx+c) - a}{bd^2x^2 + 2 bcdx + bc^2 + a}\right)}{4(a^2b^2d^3x^2 + 2a^2b^2cd^2x + (a^2b^2c^2 + a^3b)d)}, \frac{abdx + abc + (bd^2x^2 + 2 bcdx + bc^2 + a)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{2(a^2b^2d^3x^2 + 2a^2b^2cd^2x + (a^2b^2c^2 + a^3b)d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*b*d*x + 2*a*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2*c^2 + a^3*b)*d), 1/2*(a*b*d*x + a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2*c^2 + a^3*b)*d)]

giac [A] time = 0.36, size = 65, normalized size = 1.03

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{2\sqrt{ab}ad} + \frac{dx+c}{2(bd^2x^2+2bcdx+bc^2+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x + c)/((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)

maple [A] time = 0.00, size = 86, normalized size = 1.37

$$\frac{\arctan\left(\frac{2bd^2x+2bcd}{2\sqrt{ab}d}\right)}{2\sqrt{ab}ad} + \frac{2bd^2x+2bcd}{4(bd^2x^2+2bcdx+bc^2+a)abd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^2)^2,x)

[Out] 1/4*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/(a*b)^(1/2)/d)

maxima [A] time = 1.56, size = 75, normalized size = 1.19

$$\frac{dx+c}{2(abd^3x^2+2abcd^2x+(abc^2+a^2)d)} + \frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{2\sqrt{ab}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/2*(d*x + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d) + 1/2*arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a*d)

mupad [B] time = 0.10, size = 76, normalized size = 1.21

$$\frac{\frac{x}{2a} + \frac{c}{2ad}}{bc^2 + 2bcdx + bd^2x^2 + a} + \frac{\operatorname{atan}\left(2a\left(\frac{\sqrt{b}c}{2a^{3/2}} + \frac{\sqrt{b}dx}{2a^{3/2}}\right)\right)}{2a^{3/2}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^2)^2,x)

[Out] (x/(2*a) + c/(2*a*d))/(a + b*c^2 + b*d^2*x^2 + 2*b*c*d*x) + atan(2*a*((b^(1/2)*c)/(2*a^(3/2)) + (b^(1/2)*d*x)/(2*a^(3/2))))/(2*a^(3/2)*b^(1/2)*d)

sympy [B] time = 0.58, size = 117, normalized size = 1.86

$$\frac{c + dx}{2a^2d + 2abc^2d + 4abcd^2x + 2abd^3x^2} + \frac{\sqrt{-\frac{1}{a^3b}} \log\left(x + \frac{-a^2\sqrt{-\frac{1}{a^3b}} + c}{d}\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}} \log\left(x + \frac{a^2\sqrt{-\frac{1}{a^3b}} + c}{d}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)**2)**2,x)

[Out] (c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*x**2) + (-sqrt(-1/(a**3*b))*log(x + (-a**2*sqrt(-1/(a**3*b)) + c)/d)/4 + sqrt(-1/(a**3*b))*log(x + (a**2*sqrt(-1/(a**3*b)) + c)/d)/4)/d

$$3.87 \quad \int \frac{1}{(a+b(c+dx)^2)^3} dx$$

Optimal. Leaf size=91

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{c+dx}{4ad(a+b(c+dx)^2)^2}$$

[Out] 1/4*(d*x+c)/a/d/(a+b*(d*x+c)^2)^2+3/8*(d*x+c)/a^2/d/(a+b*(d*x+c)^2)+3/8*arc tan((d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/d/b^(1/2)

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 205}

$$\frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{c+dx}{4ad(a+b(c+dx)^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^2)^(-3), x]

[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*(a + b*(c + d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b(c+dx)^2)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3} dx, x, c+dx\right)}{d} \\
&= \frac{c+dx}{4ad(a+b(c+dx)^2)^2} + \frac{3 \text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, c+dx\right)}{4ad} \\
&= \frac{c+dx}{4ad(a+b(c+dx)^2)^2} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{3 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, c+dx\right)}{8a^2d} \\
&= \frac{c+dx}{4ad(a+b(c+dx)^2)^2} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.82

$$\frac{\frac{\sqrt{a}(c+dx)(5a+3b(c+dx)^2)}{(a+b(c+dx)^2)^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^2)^(-3), x]

[Out] ((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^2 + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/Sqrt[b])/(8*a^(5/2)*d)

fricas [B] time = 1.01, size = 595, normalized size = 6.54

$$\left[\frac{6ab^2d^3x^3 + 18ab^2cd^2x^2 + 6ab^2c^3 + 10a^2bc + 2(9ab^2c^2 + 5a^2b)dx - 3(b^2d^4x^4 + 4b^2cd^3x^3 + b^2c^4 + 2(3b^2c^2 + 2a^2b)d^3x^2 + 4(a^3b^3c^3 + a^4b^2)d^3x^2 + 4(a^3b^3c^3 + a^4b^2)d^3x^2 + 4(a^3b^3c^3 + a^4b^2)d^3x^2}{16(a^3b^3d^5x^4 + 4a^3b^3cd^4x^3 + 2(3a^3b^3c^2 + a^4b^2)d^3x^2 + 4(a^3b^3c^3 + a^4b^2)d^3x^2 + 4(a^3b^3c^3 + a^4b^2)d^3x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(6*a*b^2*d^3*x^3 + 18*a*b^2*c*d^2*x^2 + 6*a*b^2*c^3 + 10*a^2*b*c + 2*(9*a*b^2*c^2 + 5*a^2*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d), 1/8*(3*a*b^2*d^3*x^3 + 9*a*b^2*c*d^2*x^2 + 3*a*b^2*c^3 + 5*a^2*b*c + (9*a*b^2*c^2 + 5*a^2*b)*d*x + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a)/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d)]

giac [A] time = 0.41, size = 103, normalized size = 1.13

$$\frac{3 \arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2d} + \frac{3bd^3x^3 + 9bcd^2x^2 + 9bc^2dx + 3bc^3 + 5adx + 5ac}{8(bd^2x^2 + 2bcdx + bc^2 + a)^2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{3}{8} \arctan\left(\frac{b*d*x + b*c}{\sqrt{a*b}}\right) / (\sqrt{a*b} * a^{2*d}) + \frac{1}{8} * (3*b*d^3*x^3 + 9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c) / ((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^{2*a^{2*d}})$

maple [A] time = 0.00, size = 147, normalized size = 1.62

$$\frac{3x}{8(b^2d^2x^2 + 2bcdx + b^2c^2 + a)a^2} + \frac{3c}{8(b^2d^2x^2 + 2bcdx + b^2c^2 + a)a^2d} + \frac{3 \arctan\left(\frac{2bd^2x+2bdc}{2\sqrt{ab}d}\right)}{8\sqrt{ab}a^2d} + \frac{2bd^2x}{8(b^2d^2x^2 + 2bcdx + b^2c^2 + a)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^2)^3,x)

[Out] $\frac{1}{8} * (2*b*d^2*x + 2*b*c*d) / a/b/d^2 / (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^2 + 3/8/a^2 / (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a) * x + 3/8/a^2/d / (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a) * c + 3/8/a^2/d / (a*b)^{(1/2)} * \arctan(1/2 * (2*b*d^2*x + 2*b*c*d) / (a*b)^{(1/2)} / d)$

maxima [B] time = 1.59, size = 184, normalized size = 2.02

$$\frac{3bd^3x^3 + 9bcd^2x^2 + 3bc^3 + (9bc^2 + 5a)dx + 5ac}{8(a^2b^2d^5x^4 + 4a^2b^2cd^4x^3 + 2(3a^2b^2c^2 + a^3b)d^3x^2 + 4(a^2b^2c^3 + a^3bc)d^2x + (a^2b^2c^4 + 2a^3bc^2 + a^4)d)} + \frac{3 \arctan\left(\frac{b*d*x + b*c}{\sqrt{a*b}}\right)}{8\sqrt{a*b}a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} * (3*b*d^3*x^3 + 9*b*c*d^2*x^2 + 3*b*c^3 + (9*b*c^2 + 5*a)*d*x + 5*a*c) / (a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + 2*(3*a^2*b^2*c^2 + a^3*b)*d^3*x^2 + 4*(a^2*b^2*c^3 + a^3*b*c)*d^2*x + (a^2*b^2*c^4 + 2*a^3*b*c^2 + a^4)*d) + 3/8 * \arctan((b*d^2*x + b*c*d) / (\sqrt{a*b}*d)) / (\sqrt{a*b}*a^{2*d})$

mupad [B] time = 2.22, size = 181, normalized size = 1.99

$$\frac{\frac{x(9bc^2+5a)}{8a^2} + \frac{3bc^3+5ac}{8a^2d} + \frac{3bd^2x^3}{8a^2} + \frac{9bcdx^2}{8a^2}}{x^2(6b^2c^2d^2 + 2abd^2) + x(4db^2c^3 + 4adb^2c) + a^2 + b^2c^4 + b^2d^4x^4 + 2abc^2 + 4b^2cd^3x^3} + \frac{3 \operatorname{atan}\left(\frac{8a^2(3bd^2x+2bdc)}{2\sqrt{ab}d}\right)}{8a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^2)^3,x)

[Out] $((x*(5*a + 9*b*c^2)) / (8*a^2) + (5*a*c + 3*b*c^3) / (8*a^2*d) + (3*b*d^2*x^3) / (8*a^2) + (9*b*c*d*x^2) / (8*a^2)) / (x^2*(6*b^2*c^2*d^2 + 2*a*b*d^2) + x*(4*b^2*c^3*d + 4*a*b*c*d) + a^2 + b^2*c^4 + b^2*d^4*x^4 + 2*a*b*c^2 + 4*b^2*c*d^3*x^3) + (3*atan((8*a^2*((3*b^(1/2)*c) / (8*a^(5/2)) + (3*b^(1/2)*d*x) / (8*a^(5/2)))) / 3) / (8*a^(5/2)*b^(1/2)*d)$

sympy [B] time = 1.25, size = 257, normalized size = 2.82

$$\frac{5ac + 3bc^3 + 9bcd^2x^2 + 3bd^3x^3 + x(5ad + 9bc^2d)}{8a^4d + 16a^3bc^2d + 8a^2b^2c^4d + 32a^2b^2cd^4x^3 + 8a^2b^2d^5x^4 + x^2(16a^3bd^3 + 48a^2b^2c^2d^3) + x(32a^3bcd^2 + 32a^2b^2c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)**2)**3,x)

[Out] (5*a*c + 3*b*c**3 + 9*b*c*d**2*x**2 + 3*b*d**3*x**3 + x*(5*a*d + 9*b*c**2*d)) / (8*a**4*d + 16*a**3*b*c**2*d + 8*a**2*b**2*c**4*d + 32*a**2*b**2*c*d**4*x**3 + 8*a**2*b**2*d**5*x**4 + x**2*(16*a**3*b*d**3 + 48*a**2*b**2*c**2*d**3) + x*(32*a**3*b*c*d**2 + 32*a**2*b**2*c**3*d**2)) + (-3*sqrt(-1/(a**5*b)) * log(x + (-3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d)) / 16 + 3*sqrt(-1/(a**5*b)) * log(x + (3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d)) / 16) / d

$$3.88 \quad \int \frac{1}{\sqrt{-a} + b(c+dx)^2} dx$$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a} \sqrt{b} d}$$

[Out] arctan((d*x+c)*b^(1/2)/(-a)^(1/4))/(-a)^(1/4)/d/b^(1/2)

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a} \sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\int \frac{1}{\sqrt{-a} + b(c+dx)^2} dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a} + bx^2} dx, x, c + dx\right)}{d} = \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a} \sqrt{b} d}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a} \sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)

fricas [A] time = 0.87, size = 279, normalized size = 7.97

$$\left[\sqrt{\frac{\sqrt{-a}}{ab}} \log \left(\frac{b^2 d^4 x^4 + 4 b^2 c d^3 x^3 + 6 b^2 c^2 d^2 x^2 + 4 b^2 c^3 d x + b^2 c^4 - 2 (b d^2 x^2 + 2 b c d x + b c^2) \sqrt{-a} + 2 (a b d x + a b c + (b^2 d^3 x^3 + 3 b^2 c d^2 x^2 + 3 b^2 c^2 d x + b^2 c^3) \sqrt{-a})}{b^2 d^4 x^4 + 4 b^2 c d^3 x^3 + 6 b^2 c^2 d^2 x^2 + 4 b^2 c^3 d x + b^2 c^4 + a} \right) \right] \\ \hline 2 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="fricas")

[Out] [1/2*sqrt(sqrt(-a)/(a*b))*log((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-a) + 2*(a*b*d*x + a*b*c + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sqrt(-a))*sqrt(sqrt(-a)/(a*b)) - a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + a))/d, sqrt(-sqrt(-a)/(a*b))*arctan((b*d*x + b*c)*sqrt(-sqrt(-a)/(a*b)))/d]

giac [A] time = 0.45, size = 30, normalized size = 0.86

$$\frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="giac")

[Out] arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)

maple [A] time = 0.01, size = 42, normalized size = 1.20

$$\frac{\arctan\left(\frac{2b d^2 x + 2 b d c}{2 \sqrt{\sqrt{-a} b d}}\right)}{\sqrt{\sqrt{-a} b d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*(d*x+c)^2+(-a)^(1/2)),x)

[Out] 1/d/((-a)^(1/2)*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/((-a)^(1/2)*b)^(1/2))

maxima [B] time = 1.40, size = 66, normalized size = 1.89

$$\frac{\log\left(\frac{bd^2x+bcd-\sqrt{-\sqrt{-a}bd}}{bd^2x+bcd+\sqrt{-\sqrt{-a}bd}}\right)}{2\sqrt{-\sqrt{-a}bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="maxima")

[Out] 1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d + sqrt(-sqrt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)

mupad [B] time = 0.10, size = 31, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}c + \sqrt{b}dx}{(-a)^{1/4}}\right)}{(-a)^{1/4}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*(c + d*x)^2 + (-a)^(1/2)), x)`

[Out] `atan((b^(1/2)*c + b^(1/2)*d*x)/(-a)^(1/4))/((-a)^(1/4)*b^(1/2)*d)`

sympy [B] time = 0.22, size = 92, normalized size = 2.63

$$\frac{-\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x + \frac{c - \sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2} + \frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x + \frac{c + \sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*(d*x+c)**2+(-a)**(1/2)), x)`

[Out] `(-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a)))))/d)/2 + sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b*sqrt(-a)))))/d)/2/d`

$$3.89 \quad \int \frac{1}{1+(c+dx)^2} dx$$

Optimal. Leaf size=10

$$\frac{\tan^{-1}(c + dx)}{d}$$

[Out] arctan(d*x+c)/d

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {247, 203}

$$\frac{\tan^{-1}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(1 + (c + d*x)^2)^(-1), x]

[Out] ArcTan[c + d*x]/d

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line arQ[v, x] && NeQ[v, x]

Rubi steps

$$\int \frac{1}{1+(c+dx)^2} dx = \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c+dx\right)}{d} = \frac{\tan^{-1}(c+dx)}{d}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (c + d*x)^2)^(-1), x]

[Out] ArcTan[c + d*x]/d

fricas [A] time = 0.85, size = 10, normalized size = 1.00

$$\frac{\arctan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2),x, algorithm="fricas")

[Out] arctan(d*x + c)/d

giac [A] time = 0.32, size = 10, normalized size = 1.00

$$\frac{\arctan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2),x, algorithm="giac")

[Out] arctan(d*x + c)/d

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+(d*x+c)^2),x)

[Out] arctan(d*x+c)/d

maxima [A] time = 1.57, size = 18, normalized size = 1.80

$$\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2),x, algorithm="maxima")

[Out] arctan((d^2*x + c*d)/d)/d

mupad [B] time = 0.04, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c + d*x)^2 + 1),x)

[Out] atan(c + d*x)/d

sympy [C] time = 0.17, size = 24, normalized size = 2.40

$$\frac{-\frac{i \log\left(x + \frac{c-i}{d}\right)}{2} + \frac{i \log\left(x + \frac{c+i}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)**2),x)

[Out] (-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d

$$3.90 \quad \int \frac{1}{(1+(c+dx)^2)^2} dx$$

Optimal. Leaf size=37

$$\frac{c+dx}{2d((c+dx)^2+1)} + \frac{\tan^{-1}(c+dx)}{2d}$$

[Out] 1/2*(d*x+c)/d/(1+(d*x+c)^2)+1/2*arctan(d*x+c)/d

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 199, 203}

$$\frac{c+dx}{2d((c+dx)^2+1)} + \frac{\tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(1 + (c + d*x)^2)^(-2), x]

[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+(c+dx)^2)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2} dx, x, c+dx\right)}{d} \\ &= \frac{c+dx}{2d(1+(c+dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c+dx\right)}{2d} \\ &= \frac{c+dx}{2d(1+(c+dx)^2)} + \frac{\tan^{-1}(c+dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.84

$$\frac{\frac{c+dx}{(c+dx)^2+1} + \tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (c + d*x)^2)^(-2), x]

[Out] ((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)

fricas [A] time = 0.81, size = 55, normalized size = 1.49

$$\frac{dx + (d^2x^2 + 2cdx + c^2 + 1) \arctan(dx + c) + c}{2(d^3x^2 + 2cd^2x + (c^2 + 1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/2*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c) + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d)

giac [A] time = 0.38, size = 41, normalized size = 1.11

$$\frac{\arctan(dx + c)}{2d} + \frac{dx + c}{2(d^2x^2 + 2cdx + c^2 + 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*arctan(d*x + c)/d + 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)*d)

maple [A] time = 0.01, size = 59, normalized size = 1.59

$$\frac{\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{2d} + \frac{2d^2x + 2cd}{4(d^2x^2 + 2cdx + c^2 + 1)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+(d*x+c)^2)^2,x)

[Out] 1/4*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)+1/2/d*arctan(1/2*(2*d^2*x+2*c*d)/d)

maxima [A] time = 1.32, size = 51, normalized size = 1.38

$$\frac{dx + c}{2(d^3x^2 + 2cd^2x + (c^2 + 1)d)} + \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + 1/2*arctan((d^2*x + c*d)/d)/d

mupad [B] time = 2.07, size = 42, normalized size = 1.14

$$\frac{\frac{x}{2} + \frac{c}{2d}}{c^2 + 2cdx + d^2x^2 + 1} + \frac{\operatorname{atan}(c + dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((c + d*x)^2 + 1)^2,x)`

[Out] $(x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x + 1) + \text{atan}(c + d*x)/(2*d)$

sympy [C] time = 0.44, size = 56, normalized size = 1.51

$$\frac{c + dx}{2c^2d + 4cd^2x + 2d^3x^2 + 2d} + \frac{-\frac{i \log\left(x + \frac{c-i}{d}\right)}{4} + \frac{i \log\left(x + \frac{c+i}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(d*x+c)**2)**2,x)`

[Out] $(c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*\log(x + (c - I)/d)/4 + I*\log(x + (c + I)/d)/4)/d$

$$3.91 \quad \int \frac{1}{(1+(c+dx)^2)^3} dx$$

Optimal. Leaf size=60

$$\frac{3(c+dx)}{8d((c+dx)^2+1)} + \frac{c+dx}{4d((c+dx)^2+1)^2} + \frac{3 \tan^{-1}(c+dx)}{8d}$$

[Out] 1/4*(d*x+c)/d/(1+(d*x+c)^2)^2+3/8*(d*x+c)/d/(1+(d*x+c)^2)+3/8*arctan(d*x+c)/d

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 199, 203}

$$\frac{3(c+dx)}{8d((c+dx)^2+1)} + \frac{c+dx}{4d((c+dx)^2+1)^2} + \frac{3 \tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(1 + (c + d*x)^2)^(-3), x]

[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c + d*x)^2)) + (3*ArcTan[c + d*x])/(8*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+(c+dx)^2)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3} dx, x, c+dx\right)}{d} \\
&= \frac{c+dx}{4d(1+(c+dx)^2)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(1+x^2)^2} dx, x, c+dx\right)}{4d} \\
&= \frac{c+dx}{4d(1+(c+dx)^2)^2} + \frac{3(c+dx)}{8d(1+(c+dx)^2)} + \frac{3\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c+dx\right)}{8d} \\
&= \frac{c+dx}{4d(1+(c+dx)^2)^2} + \frac{3(c+dx)}{8d(1+(c+dx)^2)} + \frac{3\tan^{-1}(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.87

$$\frac{\frac{3(c+dx)}{(c+dx)^2+1} + \frac{2(c+dx)}{((c+dx)^2+1)^2} + 3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (c + d*x)^2)^(-3), x]

[Out] ((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)^2) + 3*ArcTan[c + d*x])/(8*d)

fricas [B] time = 0.84, size = 153, normalized size = 2.55

$$\frac{3d^3x^3 + 9cd^2x^2 + 3c^3 + (9c^2 + 5)dx + 3(d^4x^4 + 4cd^3x^3 + 2(3c^2 + 1)d^2x^2 + c^4 + 4(c^3 + c)dx + 2c^2 + 1)\arctan(dx + c)}{8(d^5x^4 + 4cd^4x^3 + 2(3c^2 + 1)d^3x^2 + 4(c^3 + c)d^2x + (c^4 + 2c^2 + 1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x + 2*c^2 + 1)*arctan(d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d)

giac [A] time = 0.30, size = 73, normalized size = 1.22

$$\frac{3\arctan(dx + c)}{8d} + \frac{3d^3x^3 + 9cd^2x^2 + 9c^2dx + 3c^3 + 5dx + 5c}{8(d^2x^2 + 2cdx + c^2 + 1)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="giac")

[Out] 3/8*arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 + 5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)

maple [A] time = 0.01, size = 94, normalized size = 1.57

$$\frac{3\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{8d} + \frac{2d^2x + 2cd}{8(d^2x^2 + 2cdx + c^2 + 1)^2d^2} + \frac{\frac{3}{8}d^2x + \frac{3}{8}cd}{(d^2x^2 + 2cdx + c^2 + 1)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+(d*x+c)^2)^3,x)

[Out] $\frac{1}{8} \frac{(2d^2x+2cd)}{d^2} \frac{1}{(d^2x^2+2cdx+c^2+1)^2+3} + \frac{16(2d^2x+2cd)}{(d^2x^2+2cdx+c^2+1)d^2+3} \frac{1}{8} \frac{1}{d} \arctan\left(\frac{1}{2} \frac{(2d^2x+2cd)}{d}\right)$

maxima [B] time = 1.51, size = 115, normalized size = 1.92

$$\frac{3d^3x^3 + 9cd^2x^2 + 3c^3 + (9c^2 + 5)dx + 5c}{8(d^5x^4 + 4cd^4x^3 + 2(3c^2 + 1)d^3x^2 + 4(c^3 + c)d^2x + (c^4 + 2c^2 + 1)d)} + \frac{3 \arctan\left(\frac{d^2x+cd}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} \frac{(3d^3x^3 + 9cd^2x^2 + 3c^3 + (9c^2 + 5)dx + 5c)}{(d^5x^4 + 4cd^4x^3 + 2(3c^2 + 1)d^3x^2 + 4(c^3 + c)d^2x + (c^4 + 2c^2 + 1)d)} + \frac{3}{8} \arctan\left(\frac{d^2x + cd}{d}\right) \frac{1}{d}$

mupad [B] time = 0.12, size = 111, normalized size = 1.85

$$\frac{3 \operatorname{atan}(c + dx)}{8d} + \frac{x \left(\frac{9c^2}{8} + \frac{5}{8} \right) + \frac{3c^3+5c}{8d} + \frac{3d^2x^3}{8} + \frac{9cdx^2}{8}}{x^2 (6c^2d^2 + 2d^2) + 2c^2 + c^4 + x(4dc^3 + 4dc) + d^4x^4 + 4cd^3x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c + d*x)^2 + 1)^3,x)

[Out] $\frac{(3 \operatorname{atan}(c + dx))}{(8d)} + \frac{(x \left(\frac{9c^2}{8} + \frac{5}{8} \right) + (5c + 3c^3)/(8d) + (3d^2x^3)/8 + (9cd^2x^2)/8)}{(x^2(2d^2 + 6c^2d^2) + 2c^2 + c^4 + x(4c^3d + 4cd^3x^3 + 1))} + \frac{1}{d}$

sympy [C] time = 0.93, size = 146, normalized size = 2.43

$$\frac{3c^3 + 9cd^2x^2 + 5c + 3d^3x^3 + x(9c^2d + 5d)}{8c^4d + 16c^2d + 32cd^4x^3 + 8d^5x^4 + 8d + x^2(48c^2d^3 + 16d^3) + x(32c^3d^2 + 32cd^2)} + \frac{-\frac{3i \log\left(x + \frac{3c-3i}{3d}\right)}{16}}{d} + \frac{\frac{3i \log\left(x + \frac{3c+3i}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)**2)**3,x)

[Out] $\frac{(3c^{**3} + 9cd^{**2}x^{**2} + 5c + 3d^{**3}x^{**3} + x(9c^{**2}d + 5d))}{(8c^{**4}d + 16c^{**2}d + 32cd^{**4}x^{**3} + 8d^{**5}x^{**4} + 8d + x^{**2}(48c^{**2}d^{**3} + 16d^{**3}) + x(32c^{**3}d^{**2} + 32cd^{**2}))} + \frac{(-3I \log(x + (3c - 3I)/(3d)))/16}{d} + \frac{3I \log(x + (3c + 3I)/(3d))/16}{d}$

$$3.92 \quad \int \frac{1}{1-(c+dx)^2} dx$$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}(c+dx)}{d}$$

[Out] arctanh(d*x+c)/d

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {247, 206}

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(1 - (c + d*x)^2)^(-1), x]

[Out] ArcTanh[c + d*x]/d

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1-(c+dx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c+dx\right)}{d} \\ &= \frac{\tanh^{-1}(c+dx)}{d} \end{aligned}$$

Mathematica [B] time = 0.01, size = 32, normalized size = 3.20

$$\frac{\log(c+dx+1)}{2d} - \frac{\log(-c-dx+1)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (c + d*x)^2)^(-1), x]

[Out] -1/2*Log[1 - c - d*x]/d + Log[1 + c + d*x]/(2*d)

fricas [B] time = 0.79, size = 22, normalized size = 2.20

$$\frac{\log(dx+c+1) - \log(dx+c-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2),x, algorithm="fricas")

[Out] 1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d

giac [B] time = 0.38, size = 27, normalized size = 2.70

$$\frac{\log(|dx + c + 1|)}{2d} - \frac{\log(|dx + c - 1|)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2),x, algorithm="giac")

[Out] 1/2*log(abs(d*x + c + 1))/d - 1/2*log(abs(d*x + c - 1))/d

maple [B] time = 0.01, size = 26, normalized size = 2.60

$$-\frac{\ln(dx + c - 1)}{2d} + \frac{\ln(dx + c + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(d*x+c)^2),x)

[Out] -1/2/d*ln(d*x+c-1)+1/2/d*ln(d*x+c+1)

maxima [B] time = 0.49, size = 25, normalized size = 2.50

$$\frac{\log(dx + c + 1)}{2d} - \frac{\log(dx + c - 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2),x, algorithm="maxima")

[Out] 1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d

mupad [B] time = 2.05, size = 10, normalized size = 1.00

$$\frac{\operatorname{atanh}(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((c + d*x)^2 - 1),x)

[Out] atanh(c + d*x)/d

sympy [B] time = 0.18, size = 22, normalized size = 2.20

$$-\frac{\frac{\log\left(x + \frac{c-1}{d}\right)}{2} - \frac{\log\left(x + \frac{c+1}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)**2),x)

[Out] -(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d

$$3.93 \quad \int \frac{1}{(1-(c+dx)^2)^2} dx$$

Optimal. Leaf size=39

$$\frac{c+dx}{2d(1-(c+dx)^2)} + \frac{\tanh^{-1}(c+dx)}{2d}$$

[Out] 1/2*(d*x+c)/d/(1-(d*x+c)^2)+1/2*arctanh(d*x+c)/d

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 206}

$$\frac{c+dx}{2d(1-(c+dx)^2)} + \frac{\tanh^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(1 - (c + d*x)^2)^(-2), x]

[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1-(c+dx)^2)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2} dx, x, c+dx\right)}{d} \\ &= \frac{c+dx}{2d(1-(c+dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c+dx\right)}{2d} \\ &= \frac{c+dx}{2d(1-(c+dx)^2)} + \frac{\tanh^{-1}(c+dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.15

$$\frac{-\frac{2(c+dx)}{(c+dx)^2-1} - \log(-c - dx + 1) + \log(c + dx + 1)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (c + d*x)^2)^(-2), x]

[Out] ((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c + d*x])/ (4*d)

fricas [B] time = 0.83, size = 85, normalized size = 2.18

$$\frac{2 dx - (d^2 x^2 + 2 c d x + c^2 - 1) \log(dx + c + 1) + (d^2 x^2 + 2 c d x + c^2 - 1) \log(dx + c - 1) + 2 c}{4 (d^3 x^2 + 2 c d^2 x + (c^2 - 1) d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d)

giac [A] time = 0.35, size = 56, normalized size = 1.44

$$\frac{\log(|dx + c + 1|)}{4d} - \frac{\log(|dx + c - 1|)}{4d} - \frac{dx + c}{2(d^2 x^2 + 2 c d x + c^2 - 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/4*log(abs(d*x + c + 1))/d - 1/4*log(abs(d*x + c - 1))/d - 1/2*(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)

maple [A] time = 0.01, size = 52, normalized size = 1.33

$$-\frac{\ln(dx + c - 1)}{4d} + \frac{\ln(dx + c + 1)}{4d} - \frac{1}{4(dx + c - 1)d} - \frac{1}{4(dx + c + 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(d*x+c)^2)^2,x)

[Out] -1/4/d/(d*x+c-1)-1/4/d*ln(d*x+c-1)-1/4/d/(d*x+c+1)+1/4/d*ln(d*x+c+1)

maxima [A] time = 0.62, size = 56, normalized size = 1.44

$$-\frac{dx + c}{2(d^3 x^2 + 2 c d^2 x + (c^2 - 1) d)} + \frac{\log(dx + c + 1)}{4d} - \frac{\log(dx + c - 1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*log(d*x + c + 1)/d - 1/4*log(d*x + c - 1)/d

mupad [B] time = 2.06, size = 43, normalized size = 1.10

$$\frac{\operatorname{atanh}(c + dx)}{2d} - \frac{\frac{x}{2} + \frac{c}{2d}}{c^2 + 2cdx + d^2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((c + d*x)^2 - 1)^2,x)`

[Out] `atanh(c + d*x)/(2*d) - (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x - 1)`

sympy [A] time = 0.47, size = 54, normalized size = 1.38

$$\frac{-c - dx}{2c^2d + 4cd^2x + 2d^3x^2 - 2d} + \frac{-\frac{\log\left(x + \frac{c-1}{d}\right)}{4} + \frac{\log\left(x + \frac{c+1}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(d*x+c)**2)**2,x)`

[Out] `(-c - d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-log(x + (c - 1)/d)/4 + log(x + (c + 1)/d)/4)/d`

$$3.94 \quad \int \frac{1}{(1-(c+dx)^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3 \tanh^{-1}(c+dx)}{8d}$$

[Out] 1/4*(d*x+c)/d/(1-(d*x+c)^2)^2+3/8*(d*x+c)/d/(1-(d*x+c)^2)+3/8*arctanh(d*x+c)/d

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 206}

$$\frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3 \tanh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(1 - (c + d*x)^2)^(-3), x]

[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c + d*x)^2)) + (3*ArcTanh[c + d*x])/(8*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 - (c + dx)^2)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3} dx, x, c + dx\right)}{d} \\
&= \frac{c + dx}{4d(1 - (c + dx)^2)^2} + \frac{3 \text{Subst}\left(\int \frac{1}{(1-x^2)^2} dx, x, c + dx\right)}{4d} \\
&= \frac{c + dx}{4d(1 - (c + dx)^2)^2} + \frac{3(c + dx)}{8d(1 - (c + dx)^2)} + \frac{3 \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c + dx\right)}{8d} \\
&= \frac{c + dx}{4d(1 - (c + dx)^2)^2} + \frac{3(c + dx)}{8d(1 - (c + dx)^2)} + \frac{3 \tanh^{-1}(c + dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.02

$$\frac{-\frac{6(c+dx)}{(c+dx)^2-1} + \frac{4(c+dx)}{(c+dx)^2-1} - 3 \log(-c - dx + 1) + 3 \log(c + dx + 1)}{16d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (c + d*x)^2)^(-3), x]

[Out] ((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*x)^2) - 3*Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)

fricas [B] time = 0.84, size = 220, normalized size = 3.44

$$\frac{6d^3x^3 + 18cd^2x^2 + 6c^3 + 2(9c^2 - 5)dx - 3(d^4x^4 + 4cd^3x^3 + 2(3c^2 - 1)d^2x^2 + c^4 + 4(c^3 - c)dx - 2c^2 + 1) \log(d^5x^4 + 4cd^4x^3 + 2(3c^2 - 1)d^3x^2 + \dots)}{16(d^5x^4 + 4cd^4x^3 + 2(3c^2 - 1)d^3x^2 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)

giac [A] time = 0.36, size = 88, normalized size = 1.38

$$\frac{3 \log(|dx + c + 1|)}{16d} - \frac{3 \log(|dx + c - 1|)}{16d} - \frac{3d^3x^3 + 9cd^2x^2 + 9c^2dx + 3c^3 - 5dx - 5c}{8(d^2x^2 + 2cdx + c^2 - 1)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="giac")

[Out] 3/16*log(abs(d*x + c + 1))/d - 3/16*log(abs(d*x + c - 1))/d - 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*x^2 + 2*c*d*x + c^2 - 1)^2*d)

maple [A] time = 0.01, size = 78, normalized size = 1.22

$$-\frac{3 \ln(dx + c - 1)}{16d} + \frac{3 \ln(dx + c + 1)}{16d} + \frac{1}{16(dx + c - 1)^2 d} - \frac{3}{16(dx + c - 1)d} - \frac{1}{16(dx + c + 1)^2 d} - \frac{3}{16(dx + c + 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(d*x+c)^2)^3,x)

[Out] 1/16/d/(d*x+c-1)^2-3/16/(d*x+c-1)/d-3/16/d*ln(d*x+c-1)-1/16/d/(d*x+c+1)^2-3/16/(d*x+c+1)/d+3/16/d*ln(d*x+c+1)

maxima [B] time = 0.64, size = 122, normalized size = 1.91

$$-\frac{3d^3x^3 + 9cd^2x^2 + 3c^3 + (9c^2 - 5)dx - 5c}{8(d^5x^4 + 4cd^4x^3 + 2(3c^2 - 1)d^3x^2 + 4(c^3 - c)d^2x + (c^4 - 2c^2 + 1)d)} + \frac{3 \log(dx + c + 1)}{16d} - \frac{3 \log(dx + c - 1)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d) + 3/16*log(d*x + c + 1)/d - 3/16*log(d*x + c - 1)/d

mupad [B] time = 2.12, size = 114, normalized size = 1.78

$$\frac{3 \operatorname{atanh}(c + dx)}{8d} - \frac{x \left(\frac{9c^2}{8} - \frac{5}{8} \right) - \frac{5c - 3c^3}{8d} + \frac{3d^2x^3}{8} + \frac{9cdx^2}{8}}{c^4 - 2c^2 - x^2(2d^2 - 6c^2d^2) - x(4cd - 4c^3d) + d^4x^4 + 4cd^3x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((c + d*x)^2 - 1)^3,x)

[Out] (3*atanh(c + d*x))/(8*d) - (x*((9*c^2)/8 - 5/8) - (5*c - 3*c^3)/(8*d) + (3*d^2*x^3)/8 + (9*c*d*x^2)/8)/(c^4 - 2*c^2 - x^2*(2*d^2 - 6*c^2*d^2) - x*(4*c*d - 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)

sympy [B] time = 1.03, size = 141, normalized size = 2.20

$$-\frac{3c^3 + 9cd^2x^2 - 5c + 3d^3x^3 + x(9c^2d - 5d)}{8c^4d - 16c^2d + 32cd^4x^3 + 8d^5x^4 + 8d + x^2(48c^2d^3 - 16d^3) + x(32c^3d^2 - 32cd^2)} - \frac{3 \log\left(x + \frac{3c-3}{3d}\right)}{16} - \frac{3 \log\left(x + \frac{3c+3}{3d}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)**2)**3,x)

[Out] -(3*c**3 + 9*c*d**2*x**2 - 5*c + 3*d**3*x**3 + x*(9*c**2*d - 5*d))/(8*c**4*d - 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 - 16*d**3) + x*(32*c**3*d**2 - 32*c*d**2)) - (3*log(x + (3*c - 3)/(3*d))/16 - 3*log(x + (3*c + 3)/(3*d))/16)/d

$$3.95 \quad \int \frac{1}{1-(1+x)^2} dx$$

Optimal. Leaf size=4

$$\tanh^{-1}(x+1)$$

[Out] arctanh(1+x)

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {247, 206}

$$\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - (1 + x)^2)^(-1), x]

[Out] ArcTanh[1 + x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\int \frac{1}{1-(1+x)^2} dx = \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, 1+x\right) = \tanh^{-1}(1+x)$$

Mathematica [B] time = 0.00, size = 15, normalized size = 3.75

$$\frac{1}{2} \log(x+2) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (1 + x)^2)^(-1), x]

[Out] -1/2*Log[x] + Log[2 + x]/2

fricas [B] time = 0.89, size = 11, normalized size = 2.75

$$\frac{1}{2} \log(x+2) - \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2), x, algorithm="fricas")

[Out] 1/2*log(x + 2) - 1/2*log(x)

giac [B] time = 0.35, size = 13, normalized size = 3.25

$$\frac{1}{2} \log(|x + 2|) - \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2),x, algorithm="giac")

[Out] 1/2*log(abs(x + 2)) - 1/2*log(abs(x))

maple [B] time = 0.00, size = 12, normalized size = 3.00

$$-\frac{\ln(x)}{2} + \frac{\ln(x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(x+1)^2),x)

[Out] 1/2*ln(2+x)-1/2*ln(x)

maxima [B] time = 0.57, size = 11, normalized size = 2.75

$$\frac{1}{2} \log(x + 2) - \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2),x, algorithm="maxima")

[Out] 1/2*log(x + 2) - 1/2*log(x)

mupad [B] time = 0.15, size = 4, normalized size = 1.00

$$\operatorname{atanh}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((x + 1)^2 - 1),x)

[Out] atanh(x + 1)

sympy [B] time = 0.10, size = 10, normalized size = 2.50

$$-\frac{\log(x)}{2} + \frac{\log(x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)**2),x)

[Out] -log(x)/2 + log(x + 2)/2

$$3.96 \quad \int \frac{1}{(1-(1+x)^2)^2} dx$$

Optimal. Leaf size=27

$$\frac{x+1}{2(1-(x+1)^2)} + \frac{1}{2} \tanh^{-1}(x+1)$$

[Out] 1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 199, 206}

$$\frac{x+1}{2(1-(x+1)^2)} + \frac{1}{2} \tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - (1 + x)^2)^(-2), x]

[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1-(1+x)^2)^2} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2)^2} dx, x, 1+x \right) \\ &= \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, 1+x \right) \\ &= \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2} \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{4} \left(-\frac{2(x+1)}{x(x+2)} - \log(x) + \log(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (1 + x)^2)^(-2), x]

[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4

fricas [A] time = 0.79, size = 39, normalized size = 1.44

$$\frac{(x^2 + 2x) \log(x + 2) - (x^2 + 2x) \log(x) - 2x - 2}{4(x^2 + 2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="fricas")

[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)

giac [A] time = 0.33, size = 27, normalized size = 1.00

$$-\frac{x + 1}{2(x^2 + 2x)} + \frac{1}{4} \log(|x + 2|) - \frac{1}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="giac")

[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\frac{\ln(x)}{4} + \frac{\ln(x + 2)}{4} - \frac{1}{4x} - \frac{1}{4(x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(x+1)^2)^2,x)

[Out] -1/4/(x+2)+1/4*ln(x+2)-1/4/x-1/4*ln(x)

maxima [A] time = 0.65, size = 25, normalized size = 0.93

$$-\frac{x + 1}{2(x^2 + 2x)} + \frac{1}{4} \log(x + 2) - \frac{1}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="maxima")

[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)

mupad [B] time = 0.07, size = 23, normalized size = 0.85

$$\frac{\operatorname{atanh}(x + 1)}{2} - \frac{x + 1}{2((x + 1)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 1)^2 - 1)^2,x)

[Out] atanh(x + 1)/2 - (x + 1)/(2*((x + 1)^2 - 1))

sympy [A] time = 0.12, size = 24, normalized size = 0.89

$$\frac{-x - 1}{2x^2 + 4x} - \frac{\log(x)}{4} + \frac{\log(x + 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**2,x)
```

```
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

$$3.97 \quad \int \frac{1}{(1-(1+x)^2)^3} dx$$

Optimal. Leaf size=45

$$\frac{3(x+1)}{8(1-(x+1)^2)} + \frac{x+1}{4(1-(x+1)^2)^2} + \frac{3}{8} \tanh^{-1}(x+1)$$

[Out] 1/4*(1+x)/(1-(1+x)^2)^2+3/8*(1+x)/(1-(1+x)^2)+3/8*arctanh(1+x)

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 199, 206}

$$\frac{3(x+1)}{8(1-(x+1)^2)} + \frac{x+1}{4(1-(x+1)^2)^2} + \frac{3}{8} \tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - (1 + x)^2)^(-3), x]

[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) + (3*ArcTanh[1 + x])/8

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1-(1+x)^2)^3} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2)^3} dx, x, 1+x \right) \\ &= \frac{1+x}{4(1-(1+x)^2)^2} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{(1-x^2)^2} dx, x, 1+x \right) \\ &= \frac{1+x}{4(1-(1+x)^2)^2} + \frac{3(1+x)}{8(1-(1+x)^2)} + \frac{3}{8} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, 1+x \right) \\ &= \frac{1+x}{4(1-(1+x)^2)^2} + \frac{3(1+x)}{8(1-(1+x)^2)} + \frac{3}{8} \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.82

$$\frac{1}{16} \left(\frac{1}{x^2} - \frac{3}{x} - \frac{3}{x+2} - \frac{1}{(x+2)^2} - 3 \log(x) + 3 \log(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (1 + x)^2)^(-3), x]

[Out] (x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x])/16

fricas [B] time = 0.83, size = 71, normalized size = 1.58

$$\frac{6x^3 + 18x^2 - 3(x^4 + 4x^3 + 4x^2) \log(x+2) + 3(x^4 + 4x^3 + 4x^2) \log(x) + 8x - 4}{16(x^4 + 4x^3 + 4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="fricas")

[Out] -1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*log(x + 2) + 3*(x^4 + 4*x^3 + 4*x^2)*log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)

giac [A] time = 0.34, size = 39, normalized size = 0.87

$$-\frac{3x^3 + 9x^2 + 4x - 2}{8(x^2 + 2x)^2} + \frac{3}{16} \log(|x + 2|) - \frac{3}{16} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="giac")

[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*log(abs(x + 2)) - 3/16*log(abs(x))

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{3 \ln(x)}{16} + \frac{3 \ln(x+2)}{16} - \frac{3}{16x} + \frac{1}{16x^2} - \frac{1}{16(x+2)^2} - \frac{3}{16(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(x+1)^2)^3,x)

[Out] -1/16/(x+2)^2-3/16/(x+2)+3/16*ln(x+2)+1/16/x^2-3/16/x-3/16*ln(x)

maxima [A] time = 0.64, size = 44, normalized size = 0.98

$$-\frac{3x^3 + 9x^2 + 4x - 2}{8(x^4 + 4x^3 + 4x^2)} + \frac{3}{16} \log(x+2) - \frac{3}{16} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="maxima")

[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*log(x + 2) - 3/16*log(x)

mupad [B] time = 2.09, size = 36, normalized size = 0.80

$$\frac{3 \operatorname{atanh}(x+1)}{8} + \frac{\frac{5x}{8} - \frac{3(x+1)^3}{8} + \frac{5}{8}}{(x+1)^4 - 2(x+1)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/((x + 1)^2 - 1)^3,x)`

[Out] `(3*atanh(x + 1))/8 + ((5*x)/8 - (3*(x + 1)^3)/8 + 5/8)/((x + 1)^4 - 2*(x + 1)^2 + 1)`

sympy [A] time = 0.14, size = 44, normalized size = 0.98

$$-\frac{3 \log(x)}{16} + \frac{3 \log(x + 2)}{16} - \frac{3x^3 + 9x^2 + 4x - 2}{8x^4 + 32x^3 + 32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(1+x)**2)**3,x)`

[Out] `-3*log(x)/16 + 3*log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x**4 + 32*x**3 + 32*x**2)`

$$3.98 \quad \int \frac{(1+(a+bx)^2)^2}{x} dx$$

Optimal. Leaf size=59

$$\frac{1}{2}(a^2+2)(a+bx)^2 + a(a^2+2)bx + (a^2+1)^2 \log(x) + \frac{1}{4}(a+bx)^4 + \frac{1}{3}a(a+bx)^3$$

[Out] a*(a^2+2)*b*x+1/2*(a^2+2)*(b*x+a)^2+1/3*a*(b*x+a)^3+1/4*(b*x+a)^4+(a^2+1)^2*ln(x)

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {371, 697}

$$\frac{1}{2}(a^2+2)(a+bx)^2 + a(a^2+2)bx + (a^2+1)^2 \log(x) + \frac{1}{4}(a+bx)^4 + \frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

[In] Int[(1 + (a + b*x)^2)^2/x, x]

[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)^4/4 + (1 + a^2)^2*Log[x]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{(1+(a+bx)^2)^2}{x} dx &= \text{Subst} \left(\int \frac{(1+x^2)^2}{-a+x} dx, x, a+bx \right) \\ &= \text{Subst} \left(\int \left(a(2+a^2) - \frac{(1+a^2)^2}{a-x} + (2+a^2)x + ax^2 + x^3 \right) dx, x, a+bx \right) \\ &= a(2+a^2)bx + \frac{1}{2}(2+a^2)(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.08

$$\frac{1}{2}(a^2+2)(a+bx)^2 + a(a^2+2)(a+bx) + (a^2+1)^2 \log(bx) + \frac{1}{4}(a+bx)^4 + \frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (a + b*x)^2)^2/x, x]

[Out] $a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)^4/4 + (1 + a^2)^2*\text{Log}[b*x]$

fricas [A] time = 0.84, size = 54, normalized size = 0.92

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + (3a^2 + 1)b^2x^2 + 4(a^3 + a)bx + (a^4 + 2a^2 + 1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")`

[Out] $1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4 + 2*a^2 + 1)*\log(x)$

giac [A] time = 0.31, size = 62, normalized size = 1.05

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + 3a^2b^2x^2 + 4a^3bx + b^2x^2 + 4abx + (a^4 + 2a^2 + 1)\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")`

[Out] $1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*x + (a^4 + 2*a^2 + 1)*\log(\text{abs}(x))$

maple [A] time = 0.00, size = 64, normalized size = 1.08

$$\frac{b^4x^4}{4} + \frac{4ab^3x^3}{3} + 3a^2b^2x^2 + a^4\ln(x) + 4a^3bx + b^2x^2 + 2a^2\ln(x) + 4abx + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+(b*x+a)^2)^2/x,x)`

[Out] $1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*x^2*a^2*b^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*x + \ln(x)*a^4 + 2*\ln(x)*a^2 + \ln(x)$

maxima [A] time = 0.54, size = 54, normalized size = 0.92

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + (3a^2 + 1)b^2x^2 + 4(a^3 + a)bx + (a^4 + 2a^2 + 1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")`

[Out] $1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4 + 2*a^2 + 1)*\log(x)$

mupad [B] time = 0.05, size = 55, normalized size = 0.93

$$\ln(x) (a^4 + 2a^2 + 1) + \frac{b^4x^4}{4} + \frac{4ab^3x^3}{3} + b^2x^2 (3a^2 + 1) + 4abx (a^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x)^2 + 1)^2/x,x)`

[Out] $\log(x)*(2*a^2 + a^4 + 1) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + b^2*x^2*(3*a^2 + 1) + 4*a*b*x*(a^2 + 1)$

sympy [A] time = 0.17, size = 58, normalized size = 0.98

$$\frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2(3a^2b^2 + b^2) + x(4a^3b + 4ab) + (a^2 + 1)^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)**2)**2/x,x)
```

```
[Out] 4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b + 4*a*b) + (a**2 + 1)**2*log(x)
```


$$3.99 \quad \int \frac{x^2}{1+(-1+x)^2} dx$$

Optimal. Leaf size=10

$$x + \log((x-1)^2 + 1)$$

[Out] x+ln(1+(-1+x)^2)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {371, 702, 260}

$$x + \log((x-1)^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + (-1 + x)^2), x]

[Out] x + Log[1 + (-1 + x)^2]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 702

Int[((d_) + (e_)*(x_))^(m_)/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rubi steps

$$\begin{aligned} \int \frac{x^2}{1+(-1+x)^2} dx &= \text{Subst} \left(\int \frac{(1+x)^2}{1+x^2} dx, x, -1+x \right) \\ &= \text{Subst} \left(\int \left(1 + \frac{2x}{1+x^2} \right) dx, x, -1+x \right) \\ &= x + 2 \text{Subst} \left(\int \frac{x}{1+x^2} dx, x, -1+x \right) \\ &= x + \log(1 + (-1+x)^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.10

$$\log(x^2 - 2x + 2) + x$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + (-1 + x)^2), x]

[Out] x + Log[2 - 2*x + x^2]

fricas [A] time = 0.68, size = 11, normalized size = 1.10

$$x + \log(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="fricas")

[Out] x + log(x^2 - 2*x + 2)

giac [A] time = 0.35, size = 11, normalized size = 1.10

$$x + \log(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="giac")

[Out] x + log(x^2 - 2*x + 2)

maple [A] time = 0.00, size = 12, normalized size = 1.20

$$x + \ln(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+(x-1)^2),x)

[Out] x+ln(x^2-2*x+2)

maxima [A] time = 0.68, size = 11, normalized size = 1.10

$$x + \log(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="maxima")

[Out] x + log(x^2 - 2*x + 2)

mupad [B] time = 0.03, size = 11, normalized size = 1.10

$$x + \ln(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((x - 1)^2 + 1),x)

[Out] x + log(x^2 - 2*x + 2)

sympy [A] time = 0.09, size = 10, normalized size = 1.00

$$x + \log(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1+(-1+x)**2),x)

[Out] x + log(x**2 - 2*x + 2)

$$3.100 \quad \int \frac{x^2}{\sqrt{1-(1+x)^2}} dx$$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{1-(x+1)^2}x + \frac{3}{2}\sqrt{1-(x+1)^2} + \frac{3}{2}\sin^{-1}(x+1)$$

[Out] 3/2*arcsin(1+x)+3/2*(1-(1+x)^2)^(1/2)-1/2*x*(1-(1+x)^2)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {371, 671, 641, 216}

$$-\frac{1}{2}\sqrt{1-(x+1)^2}x + \frac{3}{2}\sqrt{1-(x+1)^2} + \frac{3}{2}\sin^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[1 - (1 + x)^2], x]

[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin[1 + x])/2

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 671

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{1-(1+x)^2}} dx &= \text{Subst} \left(\int \frac{(-1+x)^2}{\sqrt{1-x^2}} dx, x, 1+x \right) \\ &= -\frac{1}{2}x\sqrt{1-(1+x)^2} - \frac{3}{2} \text{Subst} \left(\int \frac{-1+x}{\sqrt{1-x^2}} dx, x, 1+x \right) \\ &= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1+x \right) \\ &= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2}\sin^{-1}(1+x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 1.16

$$\frac{x(x^2 - x - 6) + 6\sqrt{x}\sqrt{x+2}\sinh^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}}\right)}{2\sqrt{-x(x+2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[1 - (1 + x)^2],x]

[Out] (x*(-6 - x + x^2) + 6*Sqrt[x]*Sqrt[2 + x]*ArcSinh[Sqrt[x]/Sqrt[2]])/(2*Sqrt[-(x*(2 + x))])

fricas [A] time = 0.87, size = 35, normalized size = 0.80

$$-\frac{1}{2}\sqrt{-x^2-2x}(x-3) - 3\arctan\left(\frac{\sqrt{-x^2-2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)

giac [A] time = 0.39, size = 23, normalized size = 0.52

$$-\frac{1}{2}\sqrt{-(x+1)^2+1}(x-3) + \frac{3}{2}\arcsin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(-(x + 1)^2 + 1)*(x - 3) + 3/2*arcsin(x + 1)

maple [A] time = 0.01, size = 35, normalized size = 0.80

$$-\frac{\sqrt{-x^2-2x}x}{2} + \frac{3\arcsin(x+1)}{2} + \frac{3\sqrt{-x^2-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1-(x+1)^2)^(1/2),x)

[Out] -1/2*x*(-x^2-2*x)^(1/2)+3/2*(-x^2-2*x)^(1/2)+3/2*arcsin(x+1)

maxima [A] time = 1.29, size = 36, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2-2x}x + \frac{3}{2}\sqrt{-x^2-2x} - \frac{3}{2}\arcsin(-x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{1-(x+1)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(1 - (x + 1)^2)^(1/2), x)`

[Out] `int(x^2/(1 - (x + 1)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x(x+2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(1-(1+x)**2)**(1/2), x)`

[Out] `Integral(x**2/sqrt(-x*(x + 2)), x)`

$$3.101 \quad \int \frac{x^2}{\sqrt{1-(a+bx)^2}} dx$$

Optimal. Leaf size=67

$$\frac{(2a^2 + 1) \sin^{-1}(a + bx)}{2b^3} + \frac{3a\sqrt{1 - (a + bx)^2}}{2b^3} - \frac{x\sqrt{1 - (a + bx)^2}}{2b^2}$$

[Out] 1/2*(2*a^2+1)*arcsin(b*x+a)/b^3+3/2*a*(1-(b*x+a)^2)^(1/2)/b^3-1/2*x*(1-(b*x+a)^2)^(1/2)/b^2

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {371, 743, 641, 216}

$$\frac{(2a^2 + 1) \sin^{-1}(a + bx)}{2b^3} + \frac{3a\sqrt{1 - (a + bx)^2}}{2b^3} - \frac{x\sqrt{1 - (a + bx)^2}}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[1 - (a + b*x)^2], x]

[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(2*b^2) + (1 + 2*a^2)*ArcSin[a + b*x]/(2*b^3)

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 743

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m - 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1-(a+bx)^2}} dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1-x^2}} dx, x, a+bx\right)}{b^3} \\
&= -\frac{x\sqrt{1-(a+bx)^2}}{2b^2} - \frac{\text{Subst}\left(\int \frac{-1-2a^2+3ax}{\sqrt{1-x^2}} dx, x, a+bx\right)}{2b^3} \\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, a+bx\right)}{2b^3} \\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\sin^{-1}(a+bx)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 55, normalized size = 0.82

$$\frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1}(3a - bx) + (2a^2 + 1)\sin^{-1}(a + bx)}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[1 - (a + b*x)^2], x]

[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + (1 + 2*a^2)*ArcSin[a + b*x])/ (2*b^3)

fricas [A] time = 0.93, size = 92, normalized size = 1.37

$$\frac{(2a^2 + 1) \arctan\left(\frac{\sqrt{-b^2x^2 - 2abx - a^2 + 1}(bx+a)}{b^2x^2 + 2abx + a^2 - 1}\right) + \sqrt{-b^2x^2 - 2abx - a^2 + 1}(bx - 3a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(b*x+a)^2)^(1/2), x, algorithm="fricas")

[Out] -1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a))/b^3

giac [A] time = 0.49, size = 55, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3a}{b^3}\right)-\frac{(2a^2+1)\arcsin(-bx-a)\text{sgn}(b)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(b*x+a)^2)^(1/2), x, algorithm="giac")

[Out] -1/2*sqrt(-(b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1)*arcsin(-b*x - a)*sgn(b)/(b^2*abs(b))

maple [B] time = 0.02, size = 152, normalized size = 2.27

$$\frac{a^2 \arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{\sqrt{b^2} b^2} - \frac{\sqrt{-b^2x^2-2abx-a^2+1} x}{2b^2} + \frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2\sqrt{b^2} b^2} + \frac{3\sqrt{-b^2x^2-2abx-a^2+1}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(1-(b*x+a)^2)^(1/2),x)`

[Out] $-1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+a^2/b^2/(b^2)^(1/2)*\arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/b^2/(b^2)^(1/2)*\arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))$

maxima [B] time = 1.46, size = 139, normalized size = 2.07

$$\frac{3 a^2 \arcsin\left(-\frac{b^2 x+a b}{\sqrt{a^2 b^2-(a^2-1) b^2}}\right)}{2 b^3}-\frac{\sqrt{-b^2 x^2-2 a b x-a^2+1} x}{2 b^2}+\frac{\left(a^2-1\right) \arcsin\left(-\frac{b^2 x+a b}{\sqrt{a^2 b^2-(a^2-1) b^2}}\right)}{2 b^3}+\frac{3 \sqrt{-b^2 x^2-2 a b x}}{2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")`

[Out] $-3/2*a^2*\arcsin(-(b^2*x + a*b)/\sqrt{a^2*b^2 - (a^2 - 1)*b^2})/b^3 - 1/2*\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1}*x/b^2 + 1/2*(a^2 - 1)*\arcsin(-(b^2*x + a*b)/\sqrt{a^2*b^2 - (a^2 - 1)*b^2})/b^3 + 3/2*\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1}*a/b^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{1-(a+bx)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(1-(a+b*x)^2)^(1/2),x)`

[Out] `int(x^2/(1-(a+b*x)^2)^(1/2),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(a+bx-1)(a+bx+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(1-(b*x+a)**2)**(1/2),x)`

[Out] `Integral(x**2/sqrt(-(a+b*x-1)*(a+b*x+1)),x)`

$$3.102 \quad \int \frac{x^2}{\sqrt{1+(a+bx)^2}} dx$$

Optimal. Leaf size=63

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3} - \frac{3a\sqrt{(a+bx)^2+1}}{2b^3} + \frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

[Out] $-1/2*(-2*a^2+1)*\operatorname{arcsinh}(b*x+a)/b^3-3/2*a*(1+(b*x+a)^2)^{(1/2)}/b^3+1/2*x*(1+(b*x+a)^2)^{(1/2)}/b^2$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {371, 743, 641, 215}

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3} - \frac{3a\sqrt{(a+bx)^2+1}}{2b^3} + \frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[1 + (a + b*x)^2], x]

[Out] $(-3*a*\operatorname{Sqrt}[1 + (a + b*x)^2])/(2*b^3) + (x*\operatorname{Sqrt}[1 + (a + b*x)^2])/(2*b^2) - ((1 - 2*a^2)*\operatorname{ArcSinh}[a + b*x])/(2*b^3)$

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 743

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m - 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1+(a+bx)^2}} dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1+x^2}} dx, x, a+bx\right)}{b^3} \\
&= \frac{x\sqrt{1+(a+bx)^2}}{2b^2} + \frac{\text{Subst}\left(\int \frac{-1+2a^2-3ax}{\sqrt{1+x^2}} dx, x, a+bx\right)}{2b^3} \\
&= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3} + \frac{x\sqrt{1+(a+bx)^2}}{2b^2} - \frac{(1-2a^2)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, a+bx\right)}{2b^3} \\
&= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3} + \frac{x\sqrt{1+(a+bx)^2}}{2b^2} - \frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.81

$$\frac{\sqrt{a^2 + 2abx + b^2x^2 + 1}(bx - 3a) + (2a^2 - 1)\sinh^{-1}(a + bx)}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[1 + (a + b*x)^2], x]

[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 + 2*a^2)*ArcSinh[a + b*x])/(2*b^3)

fricas [A] time = 0.85, size = 70, normalized size = 1.11

$$\frac{(2a^2 - 1)\log(-bx - a + \sqrt{b^2x^2 + 2abx + a^2 + 1}) - \sqrt{b^2x^2 + 2abx + a^2 + 1}(bx - 3a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(b*x+a)^2)^(1/2), x, algorithm="fricas")

[Out] -1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3

giac [A] time = 0.42, size = 70, normalized size = 1.11

$$\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3a}{b^3}\right)-\frac{(2a^2-1)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(b*x+a)^2)^(1/2), x, algorithm="giac")

[Out] 1/2*sqrt((b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))

maple [B] time = 0.01, size = 146, normalized size = 2.32

$$\frac{a^2 \ln\left(\frac{b^2x+ab}{\sqrt{b^2}} + \sqrt{b^2x^2 + 2abx + a^2 + 1}\right)}{\sqrt{b^2} b^2} + \frac{\sqrt{b^2x^2 + 2abx + a^2 + 1} x}{2b^2} - \frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}} + \sqrt{b^2x^2 + 2abx + a^2 + 1}\right)}{2\sqrt{b^2} b^2} - \frac{3\sqrt{b^2}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+(b*x+a)^2)^(1/2), x)

[Out] $\frac{1}{2}x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^{(1/2)}-3/2*a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^{(1/2)}+a^2/b^2*\ln((b^2*x+a*b)/(b^2)^{(1/2)}+(b^2*x^2+2*a*b*x+a^2+1)^{(1/2)})/(b^2)^{(1/2)}-1/2/b^2*\ln((b^2*x+a*b)/(b^2)^{(1/2)}+(b^2*x^2+2*a*b*x+a^2+1)^{(1/2)})/(b^2)^{(1/2)}$

maxima [B] time = 0.59, size = 135, normalized size = 2.14

$$\frac{3a^2 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{2b^3} + \frac{\sqrt{b^2x^2+2abx+a^2+1}x}{2b^2} - \frac{(a^2+1) \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{2b^3} - \frac{3\sqrt{b^2x^2+2abx+a^2+1}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")

[Out] $\frac{3}{2}a^2*\operatorname{arcsinh}(2*(b^2*x+a*b)/\sqrt{-4*a^2*b^2+4*(a^2+1)*b^2})/b^3+1/2*\sqrt{b^2*x^2+2*a*b*x+a^2+1}*x/b^2-1/2*(a^2+1)*\operatorname{arcsinh}(2*(b^2*x+a*b)/\sqrt{-4*a^2*b^2+4*(a^2+1)*b^2})/b^3-3/2*\sqrt{b^2*x^2+2*a*b*x+a^2+1}*a/b^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{(a+bx)^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a+b*x)^2+1)^(1/2),x)

[Out] int(x^2/((a+b*x)^2+1)^(1/2),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1+(b*x+a)**2)**(1/2),x)

[Out] Integral(x**2/sqrt(a**2+2*a*b*x+b**2*x**2+1),x)

3.103 $\int \frac{x^3}{a+b(c+dx)^3} dx$

Optimal. Leaf size=234

$$-\frac{(3\sqrt[3]{a}b^{2/3}c^2 + a + bc^3)\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{4/3}d^4} + \frac{(3\sqrt[3]{a}b^{2/3}c^2 + a + bc^3)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx))}{6a^{2/3}b^{4/3}d^4}$$

[Out] $x/b/d^3 - 1/3*(a+3*a^{(1/3)}*b^{(2/3)}*c^2+b*c^3)*\ln(a^{(1/3)}+b^{(1/3)}*(d*x+c))/a^{(2/3)}/b^{(4/3)}/d^4 + 1/6*(a+3*a^{(1/3)}*b^{(2/3)}*c^2+b*c^3)*\ln(a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*(d*x+c)+b^{(2/3)}*(d*x+c)^2)/a^{(2/3)}/b^{(4/3)}/d^4 - c*\ln(a+b*(d*x+c)^3)/b/d^4 + 1/3*(a-3*a^{(1/3)}*b^{(2/3)}*c^2+b*c^3)*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)}*(d*x+c)))/a^{(1/3)}*3^{(1/2)}/a^{(2/3)}/b^{(4/3)}/d^4*3^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{(3\sqrt[3]{a}b^{2/3}c^2 + a + bc^3)\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{4/3}d^4} + \frac{(3\sqrt[3]{a}b^{2/3}c^2 + a + bc^3)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx))}{6a^{2/3}b^{4/3}d^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*(c + d*x)^3), x]

[Out] $x/(b*d^3) + ((a - 3*a^{(1/3)}*b^{(2/3)}*c^2 + b*c^3)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*(c + d*x))/(\text{Sqrt}[3]*a^{(1/3)})])/(\text{Sqrt}[3]*a^{(2/3)}*b^{(4/3)}*d^4) - ((a + 3*a^{(1/3)}*b^{(2/3)}*c^2 + b*c^3)*\text{Log}[a^{(1/3)} + b^{(1/3)}*(c + d*x)])/(3*a^{(2/3)}*b^{(4/3)}*d^4) + ((a + 3*a^{(1/3)}*b^{(2/3)}*c^2 + b*c^3)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*(c + d*x) + b^{(2/3)}*(c + d*x)^2])/(6*a^{(2/3)}*b^{(4/3)}*d^4) - (c*\text{Log}[a + b*(c + d*x)^3])/(b*d^4)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] :> With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 1871

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1887

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+b(c+dx)^3} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^3} dx, x, c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a+bc^3-3bc^2x+3bcx^2}{b(a+bx^3)}\right) dx, x, c+dx\right)}{d^4} \\
&= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x+3bcx^2}{a+bx^3} dx, x, c+dx\right)}{bd^4} \\
&= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x}{a+bx^3} dx, x, c+dx\right)}{bd^4} - \frac{(3c) \text{Subst}\left(\int \frac{x^2}{a+bx^3} dx, x, c+dx\right)}{d^4} \\
&= \frac{x}{bd^3} - \frac{c \log(a+b(c+dx)^3)}{bd^4} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}(-3\sqrt[3]{a}bc^2+2\sqrt[3]{b}(a+bc^3))+\sqrt[3]{b}(-3\sqrt[3]{a}bc^2-\sqrt[3]{b}(a+bc^3))x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c+dx\right)}{3a^{2/3}b^{4/3}d^4} \\
&= \frac{x}{bd^3} - \frac{(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{4/3}d^4} - \frac{c \log(a+b(c+dx)^3)}{bd^4} - \frac{(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{6a^{2/3}b^{4/3}d^4} \\
&= \frac{x}{bd^3} - \frac{(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{4/3}d^4} + \frac{(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{6a^{2/3}b^{4/3}d^4} \\
&= \frac{x}{bd^3} + \frac{(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3) \tan^{-1}\left(\frac{1-2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4} - \frac{(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{4/3}d^4}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 132, normalized size = 0.56

$$\frac{\text{RootSum}\left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3 \&, \frac{3\#1^2bcd^2 \log(x-\#1)+a \log(x-\#1)+bc^3 \log(x-\#1)+3\#1bc^2d \log(x-\#1)}{\#1^2d^2+2\#1cd+c^2} \& \right]}{3b^2d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*(c + d*x)^3), x]

[Out] -1/3*(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 &, (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 + 3*b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) &])/(b^2*d^4)

fricas [C] time = 30.34, size = 6315, normalized size = 26.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^3), x, algorithm="fricas")

[Out] -1/12*(2*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*b*d^4*log(-3/4*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))


```
[Out] symsum(log((3*(a*c^2 + b*c^5))/d^2 - root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*
c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + b^3*c^9 + a^3, z, k)*((3*(b^2*c^4*d^4 - 5*a*b*c*d^4))/d^2 + (3*x*(
b^2*c^3*d^4 + a*b*d^4))/d - 9*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d^8*z
^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ b^3*c^9 + a^3, z, k)*a*b^2*d^6) - (3*x*(a*c - 2*b*c^4))/d)*root(27*a^2*b^
4*d^12*z^3 + 81*a^2*b^3*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4
*z + 3*a*b^2*c^6 + 3*a^2*b*c^3 + b^3*c^9 + a^3, z, k), k, 1, 3) + x/(b*d^3)
```

sympy [A] time = 2.90, size = 238, normalized size = 1.02

$$\text{RootSum}\left(27t^3a^2b^4d^{12} + 81t^2a^2b^3cd^8 + t(54a^2b^2c^2d^4 - 27ab^3c^5d^4) + a^3 + 3a^2bc^3 + 3ab^2c^6 + b^3c^9, \left(t \mapsto t \log(x + \dots)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2*
b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c**
6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*_t*
a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**3*c
- 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3*d -
24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)
```

$$3.104 \quad \int \frac{x^2}{a+b(c+dx)^3} dx$$

Optimal. Leaf size=210

$$\frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^3} - \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}b^{2/3}d^3} + \frac{c(2\sqrt[3]{a}}{3a^{2/3}b^{2/3}d^3}$$

[Out] $\frac{1}{3}c(2a^{1/3}+b^{1/3})\ln(a^{1/3}+b^{1/3}(d*x+c))/a^{2/3}/b^{2/3}/d^3 - \frac{1}{6}c(2a^{1/3}+b^{1/3})\ln(a^{2/3}-a^{1/3}b^{1/3}(d*x+c)+b^{2/3}(d*x+c)^2)/a^{2/3}/b^{2/3}/d^3 + \frac{1}{3}\ln(a+b(d*x+c)^3)/b/d^3 + \frac{1}{3}c(2a^{1/3}-b^{1/3})\arctan(1/3(a^{1/3}-2b^{1/3}(d*x+c))/a^{1/3}3^{1/2})/a^{2/3}/b^{2/3}/d^3 3^{1/2}$

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {371, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^3} - \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}b^{2/3}d^3} + \frac{c(2\sqrt[3]{a}}{3a^{2/3}b^{2/3}d^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*(c + d*x)^3), x]

[Out] $(c(2a^{1/3} - b^{1/3})\text{ArcTan}[(a^{1/3} - 2b^{1/3}(c + dx))/(\sqrt[3]{a^{1/3}})])/(\sqrt[3]{a^{2/3}b^{2/3}d^3}) + (c(2a^{1/3} + b^{1/3})\text{Log}[a^{1/3} + b^{1/3}(c + dx)])/(3a^{2/3}b^{2/3}d^3) - (c(2a^{1/3} + b^{1/3})\text{Log}[a^{2/3} - a^{1/3}b^{1/3}(c + dx) + b^{2/3}(c + dx)^2])/(6a^{2/3}b^{2/3}d^3) + \text{Log}[a + b(c + dx)^3]/(3b^{2/3}d^3)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^{(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]}

Rule 371

Int[((a_) + (b_.)*(v_)^{(n_))^(p_)(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m(a + b*x^n)^p, x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]}

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] :> With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 1871

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{a + b(c + dx)^3} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^3} dx, x, c + dx\right)}{d^3} \\
 &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3} dx, x, c + dx\right)}{d^3} + \frac{\text{Subst}\left(\int \frac{c^2-2cx}{a+bx^3} dx, x, c + dx\right)}{d^3} \\
 &= \frac{\log(a + b(c + dx)^3)}{3bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}(-2\sqrt[3]{a}c+2\sqrt[3]{b}c^2)+\sqrt[3]{b}(-2\sqrt[3]{a}c-\sqrt[3]{b}c^2)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c + dx\right)}{3a^{2/3}\sqrt[3]{b}d^3} + \dots \\
 &= \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^3} + \frac{\log(a + b(c + dx)^3)}{3bd^3} - \frac{\left(c\left(\frac{2}{\sqrt[3]{b}} - \frac{c}{\sqrt[3]{a}}\right)\right) \text{Subst}\left(\dots\right)}{3bd^3} \\
 &= \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^3} - \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3})}{6a^{2/3}b^{2/3}d^3} \\
 &= \frac{c(2\sqrt[3]{a} - \sqrt[3]{b}c) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^3} - \frac{c(2\sqrt[3]{a} + \dots)}{3a^{2/3}b^{2/3}d^3}
 \end{aligned}$$

$$\begin{aligned}
& a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - \\
& 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*d^3 + 2*(b^2*c^5 - \\
& 8*a*b*c^2)*d*x + 2*a^2 + 3/2*\text{sqrt}(1/3)*((2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((\\
& 2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + \\
& 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2 \\
& *d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + \\
& a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*a^2*b^2*d^6 - (a*b^2*c^3 - 2*a^2*b) * \\
& d^3)*\text{sqrt}(-((2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((2*b*c^3 - a)/(a*b^2*d^6) + 1/ \\
& (b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + \\
& 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/ \\
& 3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^ \\
& 3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2 \\
& /(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((2*b*c^3 - a)/(a \\
& *b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a) \\
& /(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/ \\
& 3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b* \\
& c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d \\
& ^9))^{(1/3)} - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6))) - ((2*(1/2) \\
& ^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - \\
& 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2* \\
& c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)* \\
& ((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) \\
& + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*b*d^3 + 3* \\
& \text{sqrt}(1/3)*b*d^3*\text{sqrt}(-((2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((2*b*c^3 - a)/(a*b^ \\
& 2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a \\
& *b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} \\
& + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 \\
& - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9) \\
&)^{(1/3)} - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((2*b* \\
& c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2 \\
& *b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^ \\
& 3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9 \\
&) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2) \\
& /(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) + \\
& 6)*\log(1/2*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((2*b*c^3 - a)/(a*b^2*d^6) + 1/ \\
& (b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + \\
& 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/ \\
& 3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^ \\
& 3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2 \\
& /(b*d^3))^2*a^2*b^2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b) \\
& *(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)) \\
& /((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^ \\
& 9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt} \\
& (3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2 \\
& /(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))* \\
& d^3 + 2*(b^2*c^5 - 8*a*b*c^2)*d*x + 2*a^2 - 3/2*\text{sqrt}(1/3)*((2*(1/2)^{(2/3)}*(\\
& -I*\text{sqrt}(3) + 1))*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^ \\
& 3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2* \\
& a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - \\
& 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2* \\
& c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*a^2*b^2*d^6 - (a*b \\
& ^2*c^3 - 2*a^2*b)*d^3)*\text{sqrt}(-((2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1))*((2*b*c^3 - a) \\
&)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 \\
& - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9)) \\
& ^{(1/3)} + (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(\\
& 2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b \\
& ^3*d^9))^{(1/3)} - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)
\end{aligned}$$

$$\frac{\left(\frac{2bc^3 - a}{ab^2d^6} + \frac{1}{b^2d^6}\right) \left(\frac{bc^3 - 8a}{a^2b^2d^9} + \frac{3(2bc^3 - a)}{ab^3d^9} + \frac{2}{b^3d^9} + \frac{b^2c^6 + 2ab^2c^3 + a^2}{(a^2b^3d^9)^{1/3}} + \frac{(1/2)^{1/3}(I\sqrt{3} + 1)(bc^3 - 8a)}{a^2b^2d^9} + \frac{3(2bc^3 - a)}{ab^3d^9} + \frac{2}{b^3d^9} + \frac{b^2c^6 + 2ab^2c^3 + a^2}{(a^2b^3d^9)^{1/3}} - \frac{2}{(bd^3)}\right) \frac{ab^2d^3 - 32bc^3 + 4a}{(ab^2d^6)}}{bd^3}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx + c)^3 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(x^2/((d*x + c)^3*b + a), x)

maple [C] time = 0.00, size = 74, normalized size = 0.35

$$\frac{\text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a) \ln(-\text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a)^2 + 2cd \text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a))}{3bd^2 \text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a)^2 + 2cd \text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*(d*x+c)^3),x)

[Out] 1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx + c)^3 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] integrate(x^2/((d*x + c)^3*b + a), x)

mupad [B] time = 2.30, size = 437, normalized size = 2.08

$$\sum_{k=1}^3 \ln\left(a + bc^3 - \text{root}\left(27a^2b^3d^9z^3 - 27a^2b^2d^6z^2 - 18ab^2c^3d^3z + 9a^2bd^3z - 2abc^3 - b^2c^6 - a^2, z, k\right), a, b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*(c + d*x)^3),x)

[Out] symsum(log(a + b*c^3 - 6*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k))*a*b*d^3 + 3*b*c^2*d*x + 9*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)^2*a*b^2*d^6 + 3*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^3*d^3 + 3*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^2*d^4*x)*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k), k, 1, 3)

sympy [A] time = 0.99, size = 158, normalized size = 0.75

$$\text{RootSum}\left(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t(9a^2bd^3 - 18ab^2c^3d^3) - a^2 - 2abc^3 - b^2c^6, \left(t \mapsto t \log\left(x + \frac{18t^2a^2b^2d^6 - 1}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+b*(d*x+c)**3),x)

[Out] RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d**3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*log(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d**3 + 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))

3.105 $\int \frac{x}{a+b(c+dx)^3} dx$

Optimal. Leaf size=180

$$\frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^2} + \frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}b^{2/3}d^2} - \frac{(\sqrt[3]{a} - \sqrt[3]{b}c)}{6a^{2/3}b^{2/3}d^2}$$

[Out] $-1/3*(a^{(1/3)}+b^{(1/3)}*c)*\ln(a^{(1/3)}+b^{(1/3)}*(d*x+c))/a^{(2/3)}/b^{(2/3)}/d^2+1/6*(a^{(1/3)}+b^{(1/3)}*c)*\ln(a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*(d*x+c)+b^{(2/3)}*(d*x+c)^2)/a^{(2/3)}/b^{(2/3)}/d^2-1/3*(a^{(1/3)}-b^{(1/3)}*c)*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)}*(d*x+c))/a^{(1/3)}*3^{(1/2)})/a^{(2/3)}/b^{(2/3)}/d^2*3^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {371, 1860, 31, 634, 617, 204, 628}

$$\frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^2} + \frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}b^{2/3}d^2} - \frac{(\sqrt[3]{a} - \sqrt[3]{b}c)}{6a^{2/3}b^{2/3}d^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*(c + d*x)^3), x]

[Out] $-(((a^{(1/3)} - b^{(1/3)}*c)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*(c + d*x))/(\text{Sqrt}[3]*a^{(1/3)})])/(\text{Sqrt}[3]*a^{(2/3)}*b^{(2/3)}*d^2)) - ((a^{(1/3)} + b^{(1/3)}*c)*\text{Log}[a^{(1/3)} + b^{(1/3)}*(c + d*x)])/(3*a^{(2/3)}*b^{(2/3)}*d^2) + ((a^{(1/3)} + b^{(1/3)}*c)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*(c + d*x) + b^{(2/3)}*(c + d*x)^2])/(6*a^{(2/3)}*b^{(2/3)}*d^2)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1860

Int[((A_.) + (B_.)*(x_))/((a_.) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{x}{a + b(c + dx)^3} dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^3} dx, x, c + dx\right)}{d^2} \\ &= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}(\sqrt[3]{a}-2\sqrt[3]{b}c)+\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{b}c)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c + dx\right)}{3a^{2/3}\sqrt[3]{b}d^2} - \left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + c\right) \text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x} dx, x, c + dx\right) \\ &= -\frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\frac{1}{\sqrt[3]{b}} - \frac{c}{\sqrt[3]{a}}\right) \text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c + dx\right)}{2d^2} \\ &= -\frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^2} + \frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx))}{6a^{2/3}b^{2/3}d^2} \\ &= -\frac{(\sqrt[3]{a} - \sqrt[3]{b}c) \tan^{-1}\left(\frac{1-2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2} - \frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^2} + \frac{(\sqrt[3]{a} + \sqrt[3]{b}c)}{\sqrt[3]{b}d^2} \end{aligned}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.44

$$\frac{\text{RootSum}\left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3 \&, \frac{\#1 \log(x-\#1)}{\#1^2d^2+2\#1cd+c^2} \& \right]}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*(c + d*x)^3), x]

[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 &, (Log[x - #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) &]/(3*b*d)

fricas [C] time = 2.57, size = 1950, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")

```
[Out] 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)
/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3
+ a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3)
+ 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2
*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*
(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))*
log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3
- a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/
(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(
9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*
b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d
^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*
c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a
)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1
)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6
))^(1/3)))^2*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(
3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b
^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + 1/36*(9*(I*sqrt(3) + 1)*(-
1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + 3
*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/5
4*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*
c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4
- 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b
^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))*log(1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)
+ c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*
(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*s
qrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c - 1/
12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b
*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3
+ a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 6*
a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1
/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(
b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^
4 - 144*c)/(a*b*d^4)) - 1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^
4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))
)*log(-1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 +
a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 1/
6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + b*c^4 + (b*c
^3 - a)*d*x + a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx + c)^3 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

maple [C] time = 0.00, size = 72, normalized size = 0.40

$$\frac{\text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a) \ln(-\text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a))}{3bd(d^2 \text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a)^2 + 2cd \text{RootOf}(bd^3_Z^3 + 3bd^2c_Z^2 + 3bd^2c_Z^2 + bc^3 + a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*(d*x+c)^3),x)

[Out] 1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] integrate(x/((d*x + c)^3*b + a), x)

mupad [B] time = 0.26, size = 145, normalized size = 0.81

$$\sum_{k=1}^3 \ln(-\text{root}(27a^2b^2d^6z^3 - 9abcd^2z + bc^3 + a, z, k) (3b^2c^2d^4 - \text{root}(27a^2b^2d^6z^3 - 9abcd^2z + bc^3 + a, z, k)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*(c + d*x)^3),x)

[Out] symsum(log(b*d^3*x - root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k)*(3*b^2*c^2*d^4 - 9*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k))*a*b^2*d^6 + 3*b^2*c*d^5*x))*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k), k, 1, 3)

sympy [A] time = 0.70, size = 83, normalized size = 0.46

$$\text{RootSum}\left(27t^3a^2b^2d^6 - 9tabcd^2 + a + bc^3, \left(t \mapsto t \log\left(x + \frac{9t^2a^2bd^4 + 3tabc^2d^2 - ac - bc^4}{ad - bc^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)**3),x)

[Out] RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t, _t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d - b*c**3*d))))

3.106 $\int \frac{1}{a+b(c+dx)^3} dx$

Optimal. Leaf size=140

$$-\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

[Out] 1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2)

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {247, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^3)^(-1), x]

[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(1/3)*d) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(1/3)*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{a + b(c + dx)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^3} dx, x, c + dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx, x, c + dx\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx, x, c + dx\right)}{3a^{2/3}d} \\ &= \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx, x, c + dx\right)}{2\sqrt[3]{a}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x} dx, x, c + dx\right)}{6a^{2/3}\sqrt[3]{b}d} \\ &= \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx)^2)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, \frac{2\sqrt[3]{b}(c+dx) - \sqrt[3]{a}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}d} \\ &= -\frac{\tan^{-1}\left(\frac{1 - 2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx)^2)}{6a^{2/3}\sqrt[3]{b}d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 116, normalized size = 0.83

$$\frac{-\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx)^2) + 2\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx)) + 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}(c+dx) - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^3)^(-1), x]

[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*Log[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)

fricas [A] time = 0.92, size = 442, normalized size = 3.16

$$\left[3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}} \log\left(\frac{2abd^3x^3 + 6abcd^2x^2 + 6abc^2dx + 2abc^3 - a^2 + 3\sqrt{\frac{1}{3}}\left(2abd^2x^2 + 4abcdx + 2abc^2 + (a^2b)^{\frac{2}{3}}(dx+c) - (a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}} - 3(a^2b)^{\frac{1}{3}}}{bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3 + a}}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]

giac [A] time = 0.45, size = 160, normalized size = 1.14

$$\frac{1}{3} \sqrt{3} \left(\frac{1}{a^2 b d^3} \right)^{\frac{1}{3}} \arctan \left(-\frac{b d x + b c + (a b^2)^{\frac{1}{3}}}{\sqrt{3} b d x + \sqrt{3} b c - \sqrt{3} (a b^2)^{\frac{1}{3}}} \right) - \frac{1}{6} \left(\frac{1}{a^2 b d^3} \right)^{\frac{1}{3}} \log \left(4 \left(\sqrt{3} b d x + \sqrt{3} b c - \sqrt{3} (a b^2)^{\frac{1}{3}} \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] 1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*c + (a*b^2)^(1/3)))

maple [C] time = 0.00, size = 71, normalized size = 0.51

$$\frac{\ln \left(-\text{RootOf} \left(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a \right) + x \right)}{3 b d \left(d^2 \text{RootOf} \left(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a \right)^2 + 2 c d \text{RootOf} \left(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a \right) + c^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^3),x)

[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d x + c)^3 b + a} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] integrate(1/((d*x + c)^3*b + a), x)

mupad [B] time = 2.31, size = 144, normalized size = 1.03

$$\frac{\ln \left(b^{1/3} c + a^{1/3} + b^{1/3} d x \right)}{3 a^{2/3} b^{1/3} d} + \frac{\ln \left(3 b^2 c d^5 + 3 b^2 d^6 x + \frac{3 a^{1/3} b^{5/3} d^5 (-1 + \sqrt{3} i i)}{2} \right) (-1 + \sqrt{3} i i)}{6 a^{2/3} b^{1/3} d} - \frac{\ln \left(3 b^2 c d^5 + 3 b^2 d^6 x + \frac{3 a^{1/3} b^{5/3} d^5 (1 + \sqrt{3} i i)}{2} \right) (1 + \sqrt{3} i i)}{6 a^{2/3} b^{1/3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*(c + d*x)^3), x)`

[Out] $\log(b^{1/3}c + a^{1/3} + b^{1/3}d^2x)/(3a^{2/3}b^{1/3}d) + (\log(3b^2c^2d^5 + 3b^2d^6x + (3a^{1/3}b^{5/3}d^5(3^{1/2}i - 1))/2)(3^{1/2}i - 1))/(6a^{2/3}b^{1/3}d) - (\log(3b^2c^2d^5 + 3b^2d^6x - (3a^{1/3}b^{5/3}d^5(3^{1/2}i + 1))/2)(3^{1/2}i + 1))/(6a^{2/3}b^{1/3}d)$

sympy [A] time = 0.26, size = 26, normalized size = 0.19

$$\frac{\text{RootSum}\left(27t^3a^2b - 1, \left(t \mapsto t \log\left(x + \frac{3ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*(d*x+c)**3), x)`

[Out] `RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d`

$$3.107 \quad \int \frac{1}{x(a+b(c+dx)^3)} dx$$

Optimal. Leaf size=224

$$\frac{(2\sqrt[3]{a} - \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}c + b^{2/3}c^2)} + \frac{\sqrt[3]{b}c \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}c + b^{2/3}c^2)} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}$$

[Out] ln(x)/(b*c^3+a)-1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(a^(1/3)+b^(1/3)*c)-1/6*(2*a^(1/3)-b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(a^(2/3)-a^(1/3)*b^(1/3)*c+b^(2/3)*c^2)+1/3*b^(1/3)*c*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^(2/3)-a^(1/3)*b^(1/3)*c+b^(2/3)*c^2)*3^(1/2)

Rubi [A] time = 0.48, antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}c(\sqrt[3]{a} - \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}(a + bc^3)} + \frac{\sqrt[3]{b}c(\sqrt[3]{a} - \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}(a + bc^3)} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*(c + d*x)^3)), x]

[Out] (b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[x]/(a + b*c^3) + (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*(a + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*(a + b*c^3)) - Log[a + b*(c + d*x)^3]/(3*(a + b*c^3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
 implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^n), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+b(c+dx)^3)} dx &= \text{Subst} \left(\int \frac{1}{(-c+x)(a+bx^3)} dx, x, c+dx \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{(a+bc^3)(c-x)} - \frac{b(c^2+cx+x^2)}{(a+bc^3)(a+bx^3)} \right) dx, x, c+dx \right) \\
&= \frac{\log(x)}{a+bc^3} - \frac{b \text{Subst} \left(\int \frac{c^2+cx+x^2}{a+bx^3} dx, x, c+dx \right)}{a+bc^3} \\
&= \frac{\log(x)}{a+bc^3} - \frac{b \text{Subst} \left(\int \frac{x^2}{a+bx^3} dx, x, c+dx \right)}{a+bc^3} - \frac{b \text{Subst} \left(\int \frac{c^2+cx}{a+bx^3} dx, x, c+dx \right)}{a+bc^3} \\
&= \frac{\log(x)}{a+bc^3} - \frac{\log(a+b(c+dx)^3)}{3(a+bc^3)} - \frac{b^{2/3} \text{Subst} \left(\int \frac{\sqrt[3]{a}(\sqrt[3]{a}c+2\sqrt[3]{b}c^2)+\sqrt[3]{b}(\sqrt[3]{a}c-\sqrt[3]{b}c^2)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x \right)}{3a^{2/3}(a+bc^3)} \\
&= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}(a+bc^3)} - \frac{\log(a+b(c+dx)^3)}{3(a+bc^3)} - \frac{(\sqrt[3]{b}c)^2}{6a^{2/3}} \\
&= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}(a+bc^3)} - \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(a^{2/3})}{6a^{2/3}} \\
&= \frac{\sqrt[3]{b}c(\sqrt[3]{a}+\sqrt[3]{b}c)\tan^{-1}\left(\frac{1-2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a+bc^3)} + \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}(a+bc^3)}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 119, normalized size = 0.53

$$\frac{\text{RootSum}\left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3 \&, \frac{\#1^2d^2\log(x-\#1)+3c^2\log(x-\#1)+3\#1cd\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2} \&\right] - 3\log(x)}{3(a+bc^3)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*(c + d*x)^3)), x]

[Out] -1/3*(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 &, (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) &])/(a + b*c^3)

fricas [C] time = 2.82, size = 4370, normalized size = 19.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^3), x, algorithm="fricas")

[Out] 1/12*(2*(b*c^3 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))*log(b*c^2*d*x + b*c^3 + 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2)

$$\begin{aligned}
& - 1/(b^3c^3 + a)^2 / (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a \\
& * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - (1/2)^{1/3} * (I * \text{sqrt}(3) \\
&) + 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) \\
&) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - 2/(b^3c^3 + a)^2 - 1/2 * (a * b^3c^3 - \\
& 2 * a^2) * (2 * (1/2)^{2/3} * (-I * \text{sqrt}(3) + 1) * (1/(a * b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / \\
& (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b \\
& * c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((\\
& b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - \\
& 2/(b^3c^3 + a)^3)^{1/3} - 2/(b^3c^3 + a) + a - ((b^3c^3 + a) * (2 * (1/2)^{2/3} \\
& * (-I * \text{sqrt}(3) + 1) * (1/(a * b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a) \\
& ^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 \\
& + a)^3)^{1/3} - (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/ \\
& (a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} \\
&) - 2/(b^3c^3 + a) + 3 * \text{sqrt}(1/3) * (b^3c^3 + a) * \text{sqrt}(-(16 * b^3c^3 + (a * b^2 * c^6 + \\
& 2 * a^2 * b^3c^3 + a^3) * (2 * (1/2)^{2/3} * (-I * \text{sqrt}(3) + 1) * (1/(a * b^3c^3 + a^2) - 1/ \\
& (b^3c^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 \\
& + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - (1/2)^{1/3} * (I * \text{sqrt}(3) + \\
& 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b \\
& * c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - 2/(b^3c^3 + a))^2 + 4 * (a * b^3c^3 + a^2) * \\
& (2 * (1/2)^{2/3} * (-I * \text{sqrt}(3) + 1) * (1/(a * b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / (b^3c^ \\
& 3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a) \\
&)) - 2/(b^3c^3 + a)^3)^{1/3} - (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((b^3c^3 + \\
& a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^ \\
& 3 + a)^3)^{1/3} - 2/(b^3c^3 + a) + 4 * a / (a * b^2 * c^6 + 2 * a^2 * b^3c^3 + a^3) + \\
& 6) * \log(2 * b^3c^2 * d * x + 2 * b^3c^3 - 1/4 * (a^2 * b^3c^3 + a^3) * (2 * (1/2)^{2/3} * (-I * \text{sqrt} \\
& t(3) + 1) * (1/(a * b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a)^2 a^2) \\
& - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} \\
& - (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^ \\
& ^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - 2/(b \\
& * c^3 + a))^2 + 1/2 * (a * b^3c^3 - 2 * a^2) * (2 * (1/2)^{2/3} * (-I * \text{sqrt}(3) + 1) * (1/(a * \\
& b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + \\
& a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - (1/2)^{1/3} \\
& / 3) * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((\\
& a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - 2/(b^3c^3 + a) + 3/4 \\
& * \text{sqrt}(1/3) * (2 * a * b^3c^3 + (a^2 * b^3c^3 + a^3) * (2 * (1/2)^{2/3} * (-I * \text{sqrt}(3) + 1) * (\\
& 1/(a * b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^ \\
& c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - (1/ \\
& 2)^{1/3} * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + \\
& 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - 2/(b^3c^3 + a) \\
& + 2 * a^2) * \text{sqrt}(-(16 * b^3c^3 + (a * b^2 * c^6 + 2 * a^2 * b^3c^3 + a^3) * (2 * (1/2)^{2/3} * (\\
& -I * \text{sqrt}(3) + 1) * (1/(a * b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a)^2 \\
& * a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + \\
& a)^3)^{1/3} - (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a \\
& ^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} \\
& - 2/(b^3c^3 + a))^2 + 4 * (a * b^3c^3 + a^2) * (2 * (1/2)^{2/3} * (-I * \text{sqrt}(3) + 1) * (1/(\\
& a * b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 \\
& + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - (1/2)^{1/3} \\
& / 3) * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/ \\
& ((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - 2/(b^3c^3 + a) + 4 \\
& * a / (a * b^2 * c^6 + 2 * a^2 * b^3c^3 + a^3) - a - ((b^3c^3 + a) * (2 * (1/2)^{2/3} * (-I \\
& * \text{sqrt}(3) + 1) * (1/(a * b^3c^3 + a^2) - 1/(b^3c^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a)^2 a \\
& ^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a) \\
& ^3)^{1/3} - (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 \\
& * b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - \\
& 2/(b^3c^3 + a) - 3 * \text{sqrt}(1/3) * (b^3c^3 + a) * \text{sqrt}(-(16 * b^3c^3 + (a * b^2 * c^6 + 2 * a \\
& ^2 * b^3c^3 + a^3) * (2 * (1/2)^{2/3} * (-I * \text{sqrt}(3) + 1) * (1/(a * b^3c^3 + a^2) - 1/(b^3c^ \\
& ^3 + a)^2) / (b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + \\
& a^2) * (b^3c^3 + a)) - 2/(b^3c^3 + a)^3)^{1/3} - (1/2)^{1/3} * (I * \text{sqrt}(3) + 1) * (\\
& b^3c^3 / ((b^3c^3 + a)^2 a^2) - 1/(a^2 b^3c^3 + a^3) + 3/((a * b^3c^3 + a^2) * (b^3c^3
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] -b*d*integrate((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + log(x)/(b*c^3 + a)

mupad [B] time = 0.12, size = 553, normalized size = 2.47

$$\frac{\ln(x)}{bc^3 + a} + \left(\sum_{k=1}^3 \ln \left(\text{root} \left(27a^2bc^3z^3 + 27a^3z^3 + 27a^2z^2 + 9az + 1, z, k \right)^2 b^4c^4d^8 - \text{root} \left(27a^2bc^3z^3 + 27a^3z^3 + 27a^2z^2 + 9az + 1, z, k \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c + d*x)^3)),x)

[Out] log(x)/(a + b*c^3) + symsum(log(3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^4*d^8 - 3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*c*d^8 - 4*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*d^9*x - 6*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*c*d^8 - 24*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*d^9*x + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*b^3*c*d^8 + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x - 36*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x + 3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^3*d^9*x + 18*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^3*d^9*x)*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k), k, 1, 3)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)**3),x)

[Out] Timed out

$$3.108 \quad \int \frac{1}{x^2(a+b(c+dx)^3)} dx$$

Optimal. Leaf size=314

$$\frac{\sqrt[3]{b} d (\sqrt[3]{a} (a - 2bc^3) - \sqrt[3]{b} c (2a - bc^3)) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} (c + dx) + b^{2/3} (c + dx)^2) + \sqrt[3]{b} d (\sqrt[3]{a} (a - 2bc^3) - \sqrt[3]{b} c (2a - bc^3))}{6a^{2/3} (a + bc^3)^2}$$

[Out] $-1/(b*c^3+a)/x-3*b*c^2*d*\ln(x)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)*(-2*b*c^3+a)-b^(1/3)*c*(-b*c^3+2*a))*d*\ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^2-1/6*b^(1/3)*(a^(1/3)*(-2*b*c^3+a)-b^(1/3)*c*(-b*c^3+2*a))*d*\ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^2+b*c^2*d*\ln(a+b*(d*x+c)^3)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)-b^(1/3)*c)*(a^(1/3)+b^(1/3)*c)^3*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+a)^2*3^(1/2)$

Rubi [A] time = 0.54, antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3} d \left(-\frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}} + 2ac - bc^4 \right) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} (c + dx) + b^{2/3} (c + dx)^2) + \sqrt[3]{b} d (\sqrt[3]{a} (a - 2bc^3) - \sqrt[3]{b} c (2a - bc^3))}{6a^{2/3} (a + bc^3)^2 + 3a^{2/3} (a + bc^3)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*(c + d*x)^3)), x]

[Out] $-(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1/3)*c)^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a + b*c^3)^2 + (b^(1/3)*(a^(1/3)*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*(a + b*c^3)^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*(a - 2*b*c^3))/b^(1/3))*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a + b*c^3)^2$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_)(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /;

FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simplify[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 1871

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^n), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + b(c + dx)^3)} dx &= d \operatorname{Subst} \left(\int \frac{1}{(-c + x)^2 (a + bx^3)} dx, x, c + dx \right) \\
&= d \operatorname{Subst} \left(\int \left(\frac{1}{(a + bc^3)(c - x)^2} + \frac{3bc^2}{(a + bc^3)^2 (c - x)} + \frac{b(-c(2a - bc^3) - (a - 2bc^3))}{(a + bc^3)^2 (a + bx^3)} \right) dx, x, c + dx \right) \\
&= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-c(2a - bc^3) - (a - 2bc^3)x + 3bc^2 x^2}{a + bx^3} dx, x, c + dx \right)}{(a + bc^3)^2} \\
&= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-c(2a - bc^3) + (-a + 2bc^3)x}{a + bx^3} dx, x, c + dx \right)}{(a + bc^3)^2} + \\
&= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} + \frac{bc^2 d \log(a + b(c + dx)^3)}{(a + bc^3)^2} + \frac{(b^{2/3} d) \operatorname{Subst} \left(\int \frac{\sqrt[3]{a}(-2x^2 + 3cx - c^2)}{a + bx^3} dx, x, c + dx \right)}{(a + bc^3)^2} \\
&= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} - \frac{b^{2/3} \left(2ac - bc^4 - \frac{\sqrt[3]{a}(a - 2bc^3)}{\sqrt[3]{b}} \right) d \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3} (a + bc^3)^2} \\
&= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} - \frac{b^{2/3} \left(2ac - bc^4 - \frac{\sqrt[3]{a}(a - 2bc^3)}{\sqrt[3]{b}} \right) d \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3} (a + bc^3)^2} \\
&= -\frac{1}{(a + bc^3)x} + \frac{\sqrt[3]{b} (\sqrt[3]{a} - \sqrt[3]{b}c) (\sqrt[3]{a} + \sqrt[3]{b}c)^3 d \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{b}(c + dx)}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} a^{2/3} (a + bc^3)^2} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 173, normalized size = 0.55

$$\frac{dx \operatorname{RootSum} \left[\#1^3 b d^3 + 3 \#1^2 b c d^2 + 3 \#1 b c^2 d + a + b c^3 \&, \frac{3 \#1^2 b c^2 d^2 \log(x - \#1) - 3 a c \log(x - \#1) - \#1 a d \log(x - \#1) + 6 b c^4 \log(x - \#1)}{\#1^2 d^2 + 2 \#1 c d + c^2} \right]}{3 x (a + b c^3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]

[Out] (-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 &, (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x - #1] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) &])/(3*(a + b*c^3)^2*x)

fricas [C] time = 4.14, size = 8919, normalized size = 28.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out] -1/12*(36*b*c^2*d*x*log(x) + 12*b*c^3 - 2*(b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +

$$\begin{aligned}
& 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*\sqrt{3} \\
&) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b \\
& ^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c \\
& ^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1 \\
& /2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((\\
& a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^ \\
& 2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)* \\
& (I*\sqrt{3} + 1))*x*\log((b^3*c^6 - a^2*b)*d^3*x + 1/4*(2*a^2*b^3*c^9 + 3*a^3* \\
& b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^ \\
& 2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^ \\
& 3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - \\
& 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2) \\
&) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a \\
&)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 \\
& - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2) \\
&)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 \\
& + a)^3*a^2))^(1/3)*(I*\sqrt{3} + 1))^2 + 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 1 \\
& 0*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2* \\
& c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 \\
& + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18 \\
& *b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) \\
& + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a) \\
& ^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - \\
& 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2) \\
&)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + \\
& a)^3*a^2))^(1/3)*(I*\sqrt{3} + 1))*d + (b^3*c^7 + 5*a*b^2*c^4 - 5*a^2*b*c)*d \\
& ^2) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a \\
& *b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 \\
& - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d \\
& ^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 \\
& + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a \\
& ^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^ \\
& 6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b* \\
& c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 \\
& + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*\sqrt{3} + 1))*x - \\
& 3*\sqrt{1/3)*(b^2*c^6 + 2*a*b*c^3 + a^2))*x*\sqrt{-((a*b^4*c^12 + 4*a^2*b^3*c^ \\
& 9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^ \\
& 2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2 \\
& /((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 \\
& + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^ \\
& 2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^ \\
& 3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2* \\
& c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)* \\
& (b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b \\
& *c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*\sqrt{3} + 1))^2 - 12*(a*b^3*c \\
& ^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2* \\
& (1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2 \\
& *c^6 + 2*a^2*b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a* \\
& b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + \\
& 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)* \\
& b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2 \\
& *a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^ \\
& 6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - \\
& a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*\sqrt{3} + 1))*d + 4*(8*b^3*c^7 - 11* \\
& a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4 \\
& *a^4*b*c^3 + a^5))*\log(2*(b^3*c^6 - a^2*b)*d^3*x - 1/4*(2*a^2*b^3*c^9 + 3* \\
& a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(\\
& 9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2* \\
& b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^
\end{aligned}$$


```
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
  18*a*b*c*d^2*z - b*d^3, z, k)^3*a^2*b^5*c^7*d^8 - 9*root(27*a^2*b^2*c^6*z^
  3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b
  *d^3, z, k)*a*b^4*c^2*d^10 - 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 +
  27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*b^5*c^4*d^
  11*x + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b
  *c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*c*d^8 + 18*root(27*a^2
  *b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*
  c*d^2*z - b*d^3, z, k)^2*a*b^5*c^6*d^9 + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3
  *b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k
  )^3*a*b^6*c^10*d^8 - 36*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
  *z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*d^9*x -
  3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
  *z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^8*d^10*x + 48*root(27*a^2*b^2*
  c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2
  *z - b*d^3, z, k)^2*a*b^5*c^5*d^10*x + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*
  b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)
  ^3*a*b^6*c^9*d^9*x - 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
  *z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*a*b^4*c*d^11*x +
  51*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
  *z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^4*c^2*d^10*x - 54*root(27*a^2*
  b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c
  *d^2*z - b*d^3, z, k)^3*a^3*b^4*c^3*d^9*x)/(a^2 + b^2*c^6 + 2*a*b*c^3))*roo
  t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
  18*a*b*c*d^2*z - b*d^3, z, k), k, 1, 3) - 1/(a*x + b*c^3*x) - (3*b*c^2*d*log(x))/(a^2 + b^2*c^6 + 2*a*b*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a+b*(d*x+c)**3),x)

[Out] Timed out

$$3.109 \quad \int \frac{1}{x^3(a+b(c+dx)^3)} dx$$

Optimal. Leaf size=393

$$\frac{b^{2/3}d^2(-3a^{2/3}\sqrt[3]{bc} + a + bc^3)(\sqrt[3]{a} + \sqrt[3]{b}c)^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a+bc^3)^3} - \frac{b^{2/3}d^2(6a^{4/3}b^{2/3}c^2 + a^2 - 3\sqrt[3]{a}b^{5/3}c^5 - 7abc^3 + b^2c^6)}{3a^{2/3}(a+bc^3)^3} \log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)$$

[Out] $-1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*\ln(x)/(b*c^3+a)^3-1/3*b^(2/3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)*c^5+b^2*c^6)*d^2*\ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^3+1/6*b^(2/3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)*c^5+b^2*c^6)*d^2*\ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^3+b*c*(-2*b*c^3+a)*d^2*\ln(a+b*(d*x+c)^3)/(b*c^3+a)^3+1/3*b^(2/3)*(a^(1/3)+b^(1/3)*c)^3*(a-3*a^(2/3)*b^(1/3)*c+b*c^3)*d^2*\arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+a)^3*3^(1/2)$

Rubi [A] time = 0.60, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}d^2(6a^{4/3}b^{2/3}c^2 + a^2 - 3\sqrt[3]{a}b^{5/3}c^5 - 7abc^3 + b^2c^6) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}(a + bc^3)^3} + \frac{b^{2/3}d^2(6a^{4/3}b^{2/3}c^2 + a^2 - 3\sqrt[3]{a}b^{5/3}c^5 - 7abc^3 + b^2c^6)}{3a^{2/3}(a + bc^3)^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*(c + d*x)^3)),x]

[Out] $-1/(2*(a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)*(a^(1/3) + b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*\text{ArcTan}[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*(a + b*c^3)^3) - (3*b*c*(a - 2*b*c^3)*d^2*\text{Log}[x])/(a + b*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*\text{Log}[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*\text{Log}[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*(a + b*c^3)^3) + (b*c*(a - 2*b*c^3)*d^2*\text{Log}[a + b*(c + d*x)^3])/(a + b*c^3)^3$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_)*(x_)^n), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps


```
[Out] -1/12*(6*b^2*c^6 - 36*(2*b^2*c^4 - a*b*c)*d^2*x^2*log(x) + 12*a*b*c^3 - 2*(
b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b
^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)
^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2
*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2*
log((b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (b^4*c^10 + 15
*a*b^3*c^7 - 63*a^2*b^2*c^4 + 4*a^3*b*c)*d^4 - 1/2*(a*b^4*c^12 - 50*a^2*b^3
*c^9 + 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a
*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^
2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2
*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b
*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3
*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2
*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^
3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2
*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b
*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3
*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2
*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^
3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*
(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^
2 + 3/4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*
c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 +
a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^
3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((
a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5
) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2
) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((
a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5
) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2
) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3
*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2) - 36*(b^2*c^5 + a*b*c^2)*d*x + 6*a^2 +
(18*(2*b^2*c^4 - a*b*c)*d^2*x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^
3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a
^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5)
+ (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2)
- 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5)
+ (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2)
- 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
```


$$\begin{aligned}
& *c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (\\
& b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5 \\
& 4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) \\
& ^3)^{(1/3)}*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2 \\
& *c^6 + 3*a^2*b*c^3 + a^3))^2 + 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^9 \\
& + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11 \\
& - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a* \\
& b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2 \\
&)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2* \\
& b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b* \\
& c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3* \\
& c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2* \\
& b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3 \\
& /((b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^(1/3)*(27*(2* \\
& b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b* \\
& c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3* \\
& c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2* \\
& b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3 \\
& /((b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*sqrt(3) + 1) - 6*(\\
& 2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)))*sq \\
& rt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + \\
& 12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)* \\
& (6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) \\
& - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a \\
& ^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3 \\
& *c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b* \\
& c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (\\
& b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5 \\
& 4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) \\
& ^3)^{(1/3)} - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3 \\
& *c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b* \\
& c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (\\
& b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5 \\
& 4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) \\
& ^3)^{(1/3)}*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2 \\
& *c^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4 \\
& *c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3) \\
& *(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^ \\
& 4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sq \\
& rt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b \\
& ^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - \\
& b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a* \\
& b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d \\
& ^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)} - (1 \\
& /2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b \\
& ^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - \\
& b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a* \\
& b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d \\
& ^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*s \\
& qrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2* \\
& b*c^3 + a^3))^2/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^ \\
& 3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7))) + (18*(2*b^2*c^4 - a*b*c)*d^2 \\
& *x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2* \\
& d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a* \\
& b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/ \\
& (27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3 \\
& *a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a \\
& ^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 -
\end{aligned}$$

$$\begin{aligned}
& *d^2)^2/(b^3c^9 + 3a*b^2c^6 + 3a^2*b*c^3 + a^3)^2*(-I*\sqrt{3} + 1)/(27 \\
& *(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4)*(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)) - b^2*d^6/(a^2* \\
& b^3*c^9 + 3a^3*b^2*c^6 + 3a^4*b*c^3 + a^5) + (b^3*c^9 + 3a*b^2*c^6 - 24* \\
& a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)}*(27 \\
& *(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4)*(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)) - b^2*d^6/(a^2* \\
& b^3*c^9 + 3a^3*b^2*c^6 + 3a^4*b*c^3 + a^5) + (b^3*c^9 + 3a*b^2*c^6 - 24* \\
& a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*\sqrt{3} + 1) - \\
& 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)) \\
& ^2 - 3/4*\sqrt{1/3}*(2*(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b \\
& *c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b* \\
& c^5 - 2*a^6*c^2)*(6*(1/2)^{(2/3)}*(b^2*c^2*d^4/(a*b^3*c^9 + 3a^2*b^2*c^6 + 3 \\
& a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3a*b^2*c^6 \\
& + 3a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3} + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^ \\
& 2*c^2*d^4/((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4)*(b^3*c^9 + 3a*b \\
& ^2*c^6 + 3a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3a^3*b^2*c^6 + 3a^4 \\
& *b*c^3 + a^5) + (b^3*c^9 + 3a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^ \\
& 3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3a*b^2*c^6 + 3 \\
& a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)}*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^ \\
& 2*c^2*d^4/((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4)*(b^3*c^9 + 3a*b \\
& ^2*c^6 + 3a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3a^3*b^2*c^6 + 3a^4 \\
& *b*c^3 + a^5) + (b^3*c^9 + 3a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^ \\
& 3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3a*b^2*c^6 + 3 \\
& a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*\sqrt{3} + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/ \\
& (b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)))*\sqrt{-(12*(4*b^5*c^11 - 24*a* \\
& b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^ \\
& 4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^{(2/3)}*(b^2*c^2*d^4 \\
& /((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c \\
& *d^2)^2/(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3} + 1)/(27 \\
& *(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4)*(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)) - b^2*d^6/(a^2* \\
& b^3*c^9 + 3a^3*b^2*c^6 + 3a^4*b*c^3 + a^5) + (b^3*c^9 + 3a*b^2*c^6 - 24* \\
& a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)}*(27 \\
& *(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4)*(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)) - b^2*d^6/(a^2* \\
& b^3*c^9 + 3a^3*b^2*c^6 + 3a^4*b*c^3 + a^5) + (b^3*c^9 + 3a*b^2*c^6 - 24* \\
& a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*\sqrt{3} + 1) - \\
& 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)) \\
& *d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15 \\
& *a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^{(2/3)}*(b^2*c^2*d^4/(a*b^3*c^9 + \\
& 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c \\
& ^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3} + 1)/(27*(2*b^2*c^4*d^ \\
& 2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4) \\
& *(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3a^ \\
& 3*b^2*c^6 + 3a^4*b*c^3 + a^5) + (b^3*c^9 + 3a*b^2*c^6 - 24*a^2*b*c^3 + a^ \\
& 3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 \\
& + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)}*(27*(2*b^2*c^4*d^ \\
& 2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3a^2*b^2*c^6 + 3a^3*b*c^3 + a^4) \\
& *(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3a^ \\
& 3*b^2*c^6 + 3a^4*b*c^3 + a^5) + (b^3*c^9 + 3a*b^2*c^6 - 24*a^2*b*c^3 + a^ \\
& 3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 \\
& + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*\sqrt{3} + 1) - 6*(2*b^2*c^4* \\
& d^2 - a*b*c*d^2)/(b^3*c^9 + 3a*b^2*c^6 + 3a^2*b*c^3 + a^3))^2/(a*b^6*c^1 \\
& 8 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*
\end{aligned}$$

$$a^6 b^3 c^3 + a^7)))/((b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b c^3 + a^3) x^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((dx + c)^3 b + a) x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^3*b + a)*x^3), x)

maple [C] time = 0.01, size = 217, normalized size = 0.55

$$\frac{6b^2c^4d^2\ln(x)}{(bc^3+a)^3} - \frac{3abcd^2\ln(x)}{(bc^3+a)^3} + \frac{3bc^2d}{(bc^3+a)^2} + \frac{d^2\left(-6\text{RootOf}(bd^3_Z^3+3bd^2c_Z^2+3bd^2c_Z+b^3+a)\right)^2 b^2c}{(bc^3+a)^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b*(d*x+c)^3),x)

[Out] 1/3*d^2/(b*c^3+a)^3*sum((-6*_R^2*b^2*c^4*d^2-15*_R*b^2*c^5*d-10*b^2*c^6+3*_R^2*a*b*c*d^2+12*_R*a*b*c^2*d+16*a*b*c^3-a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))-1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x+6*b^2*d^2*c^4/(b*c^3+a)^3*ln(x)-3*b*d^2*c/(b*c^3+a)^3*ln(x)*a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd^3 \int \frac{10b^2c^6-16abc^3+3(2b^2c^4-abc)d^2x^2+3(5b^2c^5-4abc^2)dx+a^2}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a} dx}{b^3c^9+3ab^2c^6+3a^2bc^3+a^3} + \frac{3(2b^2c^4-abc)d^2 \log(x)}{b^3c^9+3ab^2c^6+3a^2bc^3+a^3} + \frac{6bc^2dx-bc^3-a}{2(b^2c^6+2abc^3+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] -b*d^3*integrate((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)*d^2*x^2 + 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 3*(2*b^2*c^4 - a*b*c)*d^2*log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 1/2*(6*b*c^2*d*x - b*c^3 - a)/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)

mupad [B] time = 2.49, size = 1328, normalized size = 3.38

$$\left(\sum_{k=1}^3 \ln \left(\frac{6b^6c^4d^{14} - 3ab^5cd^{14}}{a^4 + 4a^3bc^3 + 6a^2b^2c^6 + 4ab^3c^9 + b^4c^{12}} - \text{root}(81a^3b^2c^6z^3 + 27a^2b^3c^9z^3 + 81a^4b^3c^3z^3 + 27a^5z^3 - 81a^3b^3c^6d^2z^2 + 162a^2b^2c^4d^2z^2 + 27a^2b^2c^2d^4z + b^2d^6, z, k) \right) \right) \cdot \left(\frac{a^3b^4d^{12} + 19b^7c^9d^{12} + 12a^2b^6c^6d^{12} - 6a^2b^5c^3d^{12}}{a^4 + b^4c^{12} + 4a^3b^3c^3 + 4a^2b^3c^9 + 6a^2b^2c^6} - \text{root}(81a^3b^2c^6z^3 + 27a^2b^3c^9z^3 + 81a^4b^3c^3z^3 + 27a^5z^3 - 81a^3b^3c^6d^2z^2 + 162a^2b^2c^4d^2z^2 + 27a^2b^2c^2d^4z + b^2d^6, z, k) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*(c + d*x)^3)),x)

[Out] symsum(log(((b^6*c^4*d^14 - 3*a*b^5*c*d^14)/(a^4 + b^4*c^12 + 4*a^3*b^3*c^3 + 4*a^2*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^4*b^3*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b^3*c^6*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 27*a^2*b^2*c^2*d^4*z + b^2*d^6, z, k))*((a^3*b^4*d^12 + 19*b^7*c^9*d^12 + 12*a^2*b^6*c^6*d^12 - 6*a^2*b^5*c^3*d^12)/(a^4 + b^4*c^12 + 4*a^3*b^3*c^3 + 4*a^2*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^4*b^3*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b^3*c^6*d^2*z^2 + 162*a^2*b^2*c^4*d^2z^2 + 27a^2b^2c^2d^4z + b^2d^6, z, k)))

$$\begin{aligned}
& ^2z^2 + 27ab^2c^2d^4z + b^2d^6, z, k) * (\text{root}(81a^3b^2c^6z^3 + 27a^2b^3c^9z^3 + 81a^4b^3c^3z^3 + 27a^5z^3 - 81a^3b^3cd^2z^2 + 162a^2b^2c^4d^2z^2 + 27ab^2c^2d^4z + b^2d^6, z, k) * ((9a^6b^3c^8 + 9ab^8c^{16}d^8 + 45a^5b^4c^4d^8 + 90a^4b^5c^7d^8 + 90a^3b^6c^{10}d^8 + 45a^2b^7c^{13}d^8) / (a^4 + b^4c^{12} + 4a^3b^3c^3 + 4ab^3c^9 + 6a^2b^2c^6) - (x(36a^6b^3d^9 - 18ab^8c^{15}d^9 + 126a^5b^4c^3d^9 + 144a^4b^5c^6d^9 + 36a^3b^6c^9d^9 - 36a^2b^7c^{12}d^9)) / (a^4 + b^4c^{12} + 4a^3b^3c^3 + 4ab^3c^9 + 6a^2b^2c^6)) + (3b^8c^{14}d^{10} - 42ab^7c^{11}d^{10} + 30a^4b^4c^2d^{10} + 12a^3b^5c^5d^{10} - 63a^2b^6c^8d^{10}) / (a^4 + b^4c^{12} + 4a^3b^3c^3 + 4ab^3c^9 + 6a^2b^2c^6) + (x(3b^8c^{13}d^{11} + 66a^4b^4c^4d^{11} - 87ab^7c^{10}d^{11} + 39a^3b^5c^4d^{11} - 117a^2b^6c^7d^{11})) / (a^4 + b^4c^{12} + 4a^3b^3c^3 + 4ab^3c^9 + 6a^2b^2c^6)) + (x(18b^7c^8d^{13} + 90ab^6c^5d^{13} - 9a^2b^5c^2d^{13})) / (a^4 + b^4c^{12} + 4a^3b^3c^3 + 4ab^3c^9 + 6a^2b^2c^6)) - (x(ab^5d^{15} + b^6c^3d^{15})) / (a^4 + b^4c^{12} + 4a^3b^3c^3 + 4ab^3c^9 + 6a^2b^2c^6)) * \text{root}(81a^3b^2c^6z^3 + 27a^2b^3c^9z^3 + 81a^4b^3c^3z^3 + 27a^5z^3 - 81a^3b^3cd^2z^2 + 162a^2b^2c^4d^2z^2 + 27ab^2c^2d^4z + b^2d^6, z, k), k, 1, 3) - 1/(2*(ax^2 + bc^3x^2)) + (3bc^2d)/(a^2x + b^2c^6x + 2abc^3x) + (6b^2c^4d^2\log(x))/(a^3 + b^3c^9 + 3a^2b^3c^3 + 3ab^2c^6) - (3abc^3d^2\log(x))/(a^3 + b^3c^9 + 3a^2b^3c^3 + 3ab^2c^6)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a+b*(d*x+c)**3),x)

[Out] Timed out

$$3.110 \quad \int \frac{x^3}{a+b(c+dx)^4} dx$$

Optimal. Leaf size=356

$$\frac{c(3\sqrt{a} - \sqrt{b}c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{c(3\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^4}$$

[Out] $\frac{1}{4} \ln(a+b(d*x+c)^4)/b/d^4 + 3/2 * c^2 * \arctan((d*x+c)^2 * b^{(1/2)}/a^{(1/2)})/d^4/a^{(1/2)}/b^{(1/2)} - 1/8 * c * \ln(-a^{(1/4)} * b^{(1/4)} * (d*x+c)^2 * a^{(1/2)} + (d*x+c)^2 * b^{(1/2)}) * (3*a^{(1/2)} - b^{(1/2)} * c^2)/a^{(3/4)}/b^{(3/4)}/d^4 * 2^{(1/2)} + 1/8 * c * \ln(a^{(1/4)} * b^{(1/4)} * (d*x+c)^2 * a^{(1/2)} + (d*x+c)^2 * b^{(1/2)}) * (3*a^{(1/2)} - b^{(1/2)} * c^2)/a^{(3/4)}/b^{(3/4)}/d^4 * 2^{(1/2)} - 1/4 * c * \arctan(-1 + b^{(1/4)} * (d*x+c)^2 * a^{(1/4)}) * (3*a^{(1/2)} + b^{(1/2)} * c^2)/a^{(3/4)}/b^{(3/4)}/d^4 * 2^{(1/2)} - 1/4 * c * \arctan(1 + b^{(1/4)} * (d*x+c)^2 * a^{(1/4)}) * (3*a^{(1/2)} + b^{(1/2)} * c^2)/a^{(3/4)}/b^{(3/4)}/d^4 * 2^{(1/2)}$

Rubi [A] time = 0.43, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {371, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c(3\sqrt{a} - \sqrt{b}c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{c(3\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*(c + d*x)^4), x]

[Out] $(3*c^2 * \text{ArcTan}[(\text{Sqrt}[b] * (c + d*x)^2)/\text{Sqrt}[a]])/(2 * \text{Sqrt}[a] * \text{Sqrt}[b] * d^4) + (c * (3 * \text{Sqrt}[a] + \text{Sqrt}[b] * c^2) * \text{ArcTan}[1 - (\text{Sqrt}[2] * b^{(1/4)} * (c + d*x))/a^{(1/4)}])/(2 * \text{Sqrt}[2] * a^{(3/4)} * b^{(3/4)} * d^4) - (c * (3 * \text{Sqrt}[a] + \text{Sqrt}[b] * c^2) * \text{ArcTan}[1 + (\text{Sqrt}[2] * b^{(1/4)} * (c + d*x))/a^{(1/4)}])/(2 * \text{Sqrt}[2] * a^{(3/4)} * b^{(3/4)} * d^4) - (c * (3 * \text{Sqrt}[a] - \text{Sqrt}[b] * c^2) * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{(1/4)} * b^{(1/4)} * (c + d*x) + \text{Sqrt}[b] * (c + d*x)^2])/(4 * \text{Sqrt}[2] * a^{(3/4)} * b^{(3/4)} * d^4) + (c * (3 * \text{Sqrt}[a] - \text{Sqrt}[b] * c^2) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * b^{(1/4)} * (c + d*x) + \text{Sqrt}[b] * (c + d*x)^2])/(4 * \text{Sqrt}[2] * a^{(3/4)} * b^{(3/4)} * d^4) + \text{Log}[a + b * (c + d*x)^4]/(4 * b * d^4)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim

plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(
2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(
-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+b(c+dx)^4} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^4} dx, x, c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{x(3c^2+x^2)}{a+bx^4} + \frac{-c^3-3cx^2}{a+bx^4}\right) dx, x, c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \frac{x(3c^2+x^2)}{a+bx^4} dx, x, c+dx\right)}{d^4} + \frac{\text{Subst}\left(\int \frac{-c^3-3cx^2}{a+bx^4} dx, x, c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \frac{3c^2+x}{a+bx^2} dx, x, (c+dx)^2\right)}{2d^4} + \frac{\left(c\left(3 - \frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right) \text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4} dx, x, c+dx\right)}{2bd^4} - \frac{\text{Subst}\left(\int \frac{x}{a+bx^2} dx, x, (c+dx)^2\right)}{2d^4} + \frac{(3c^2) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, (c+dx)^2\right)}{2d^4} - \frac{c(3\sqrt{a} - \sqrt{b}c^2)}{2\sqrt{a}\sqrt{b}d^4} \\
&= \frac{3c^2 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} - \frac{c(3\sqrt{a} - \sqrt{b}c^2) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx) + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} \\
&= \frac{3c^2 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{c(3\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c(3\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 106, normalized size = 0.30

$$\frac{\text{RootSum}\left[\#1^4bd^4 + 4\#1^3bcd^3 + 6\#1^2bc^2d^2 + 4\#1bc^3d + a + bc^4 \&, \frac{\#1^3 \log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3} \&\right]}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*(c + d*x)^4), x]

[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 &, (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &]/(4*b*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^4), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx+c)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^4), x, algorithm="giac")

[Out] integrate(x^3/((d*x + c)^4*b + a), x)

maple [C] time = 0.02, size = 97, normalized size = 0.27

$$\frac{\text{RootOf}(b d^4 Z^4 + 4b d^3 c Z^3 + 6b d^2 c^2 Z^2 + 4b c^3 d Z + b c^4 + a)^3}{4bd \left(d^3 \text{RootOf}(b d^4 Z^4 + 4b d^3 c Z^3 + 6b d^2 c^2 Z^2 + 4b c^3 d Z + b c^4 + a)^3 + 3 \text{RootOf}(b d^4 Z^4 + 4b d^3 c Z^3 + \dots) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b*(d*x+c)^4), x)

[Out] 1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x), _R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^4), x, algorithm="maxima")

[Out] integrate(x^3/((d*x + c)^4*b + a), x)

mupad [B] time = 2.69, size = 1003, normalized size = 2.82

$$\sum_{k=1}^4 \ln \left(b c^2 d \left(2 a c + 2 b c^5 - 3 a d x + 5 b c^4 d x - \text{root} \left(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z - 16 a^3 b d^4 z + 3 a b^2 c^8 + 3 a^2 b c^4 + b^3 c^{12} + a^3, z, k \right) b^2 c^5 d^4 + 32 \text{root} \left(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z - 16 a^3 b d^4 z + 3 a b^2 c^8 + 3 a^2 b c^4 + b^3 c^{12} + a^3, z, k \right)^2 a b^2 c d^8 + 24 \text{root} \left(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z - 16 a^3 b d^4 z + 3 a b^2 c^8 + 3 a^2 b c^4 + b^3 c^{12} + a^3, z, k \right)^2 a b^2 d^9 x - 2 \text{root} \left(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z - 16 a^3 b d^4 z + 3 a b^2 c^8 + 3 a^2 b c^4 + b^3 c^{12} + a^3, z, k \right) b^2 c^4 d^5 x + 38 \text{root} \left(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z - 16 a^3 b d^4 z + 3 a b^2 c^8 + 3 a^2 b c^4 + b^3 c^{12} + a^3, z, k \right) a b c d^4 + 6 \text{root} \left(256 a^3 b^4 d^{16} z^4 - 256 a^3 b^3 d^{12} z^3 + 480 a^2 b^3 c^4 d^8 z^2 + 96 a^3 b^2 d^8 z^2 + 192 a^2 b^2 c^4 d^4 z - 48 a b^3 c^8 d^4 z - 16 a^3 b d^4 z + 3 a b^2 c^8 + 3 a^2 b c^4 + b^3 c^{12} + a^3, z, k \right) a b d^5 x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b*(c + d*x)^4), x)

[Out] symsum(log(2*b*c^2*d*(2*a*c + 2*b*c^5 - 3*a*d*x + 5*b*c^4*d*x - 2*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k))*b^2*c^5*d^4 + 32*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*c*d^8 + 24*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*d^9*x - 2*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k))*b^2*c^4*d^5*x + 38*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k))*a*b*c*d^4 + 6*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k))*a*b*d^5*x))

sympy [A] time = 3.74, size = 374, normalized size = 1.05

$$\text{RootSum} \left(256 t^4 a^3 b^4 d^{16} - 256 t^3 a^3 b^3 d^{12} + t^2 \left(96 a^3 b^2 d^8 + 480 a^2 b^3 c^4 d^8 \right) + t \left(-16 a^3 b d^4 + 192 a^2 b^2 c^4 d^4 - 48 a b^3 c^8 d^4 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a+b*(d*x+c)**4),x)

[Out] RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*a**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2*b**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c**8 + b**3*c**12, Lambda(_t, _t*log(x + (-1728*_t**3*a**4*b**3*d**12 - 960*_t**3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*b**3*c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*_t*a**3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d**4 + 27*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c**12 + b**4*c**16)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 117*a*b**3*c**11*d + b**4*c**15*d))))

$$3.111 \quad \int \frac{x^2}{a+b(c+dx)^4} dx$$

Optimal. Leaf size=318

$$\frac{(\sqrt{a} - \sqrt{b}c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^3} - \frac{(\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^3}$$

[Out] $-c \cdot \arctan((d \cdot x + c)^2 \cdot b^{1/2} / a^{1/2}) / d^3 \cdot a^{1/2} / b^{1/2} + 1/8 \cdot \ln(-a^{1/4} \cdot b^{1/4} \cdot (d \cdot x + c) \cdot 2^{1/2} + a^{1/2} + (d \cdot x + c)^2 \cdot b^{1/2}) \cdot (a^{1/2} - b^{1/2} \cdot c^2) / a^{3/4} / b^{3/4} / d^3 \cdot 2^{1/2} - 1/8 \cdot \ln(a^{1/4} \cdot b^{1/4} \cdot (d \cdot x + c) \cdot 2^{1/2} + a^{1/2} + (d \cdot x + c)^2 \cdot b^{1/2}) \cdot (a^{1/2} - b^{1/2} \cdot c^2) / a^{3/4} / b^{3/4} / d^3 \cdot 2^{1/2} + 1/4 \cdot \arctan(-1 + b^{1/4} \cdot (d \cdot x + c) \cdot 2^{1/2} / a^{1/4}) \cdot (a^{1/2} + b^{1/2} \cdot c^2) / a^{3/4} / b^{3/4} / d^3 \cdot 2^{1/2} + 1/4 \cdot \arctan(1 + b^{1/4} \cdot (d \cdot x + c) \cdot 2^{1/2} / a^{1/4}) \cdot (a^{1/2} + b^{1/2} \cdot c^2) / a^{3/4} / b^{3/4} / d^3 \cdot 2^{1/2}$

Rubi [A] time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{a} - \sqrt{b}c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^3} - \frac{(\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*(c + d*x)^4), x]

[Out] $-\left(\frac{c \cdot \text{ArcTan}\left[\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b} d^3}\right) - \left(\frac{\sqrt{a} + \sqrt{b}c^2 \cdot \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4}(c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^3}\right) + \left(\frac{\sqrt{a} + \sqrt{b}c^2 \cdot \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4}(c+dx)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^3}\right) + \left(\frac{\sqrt{a} - \sqrt{b}c^2 \cdot \text{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4}(c+dx) + \sqrt{b}(c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^3}\right) - \left(\frac{\sqrt{a} - \sqrt{b}c^2 \cdot \text{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4}(c+dx) + \sqrt{b}(c+dx)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^3}\right)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^p*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x, v], x] /; NeQ[c, 0]] /;

FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + b(c + dx)^4} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^4} dx, x, c + dx\right)}{d^3} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{2cx}{a+bx^4} + \frac{c^2+x^2}{a+bx^4}\right) dx, x, c + dx\right)}{d^3} \\
&= \frac{\text{Subst}\left(\int \frac{c^2+x^2}{a+bx^4} dx, x, c + dx\right)}{d^3} - \frac{(2c) \text{Subst}\left(\int \frac{x}{a+bx^4} dx, x, c + dx\right)}{d^3} \\
&= -\frac{c \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, (c + dx)^2\right)}{d^3} - \frac{\left(1 - \frac{\sqrt{b}c^2}{\sqrt{a}}\right) \text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4} dx, x, c + dx\right)}{2bd^3} + \frac{\left(1 + \frac{\sqrt{b}c^2}{\sqrt{a}}\right) \text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4} dx, x, c + dx\right)}{2bd^3} \\
&= -\frac{c \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} + \frac{(\sqrt{a} - \sqrt{b}c^2) \text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}}+2x}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{b}} - x^2} dx, x, c + dx\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{(\sqrt{a} + \sqrt{b}c^2) \text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{b}}-2x}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{b}} - x^2} dx, x, c + dx\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} \\
&= -\frac{c \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} + \frac{(\sqrt{a} - \sqrt{b}c^2) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{(\sqrt{a} + \sqrt{b}c^2) \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} \\
&= -\frac{c \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} - \frac{(\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{(\sqrt{a} - \sqrt{b}c^2) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 106, normalized size = 0.33

$$\frac{\text{RootSum}\left[\#1^4bd^4 + 4\#1^3bcd^3 + 6\#1^2bc^2d^2 + 4\#1bc^3d + a + bc^4\&, \frac{\#1^2 \log(x-\#1)}{\#1^3d^3 + 3\#1^2cd^2 + 3\#1c^2d + c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*(c + d*x)^4), x]

[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 &, (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &]/(4*b*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^4), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^4), x, algorithm="giac")

[Out] integrate(x^2/((d*x + c)^4*b + a), x)

maple [C] time = 0.00, size = 97, normalized size = 0.31

$$\frac{\text{RootOf}(b d^4 Z^4 + 4 b d^3 c Z^3 + 6 b d^2 c^2 Z^2 + 4 b c^3 d Z + b c^4 + a)}{4 b d \left(d^3 \text{RootOf}(b d^4 Z^4 + 4 b d^3 c Z^3 + 6 b d^2 c^2 Z^2 + 4 b c^3 d Z + b c^4 + a) \right)^3 + 3 \text{RootOf}(b d^4 Z^4 + 4 b d^3 c Z^3 + 6 b d^2 c^2 Z^2 + 4 b c^3 d Z + b c^4 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*(d*x+c)^4),x)

[Out] 1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] integrate(x^2/((d*x+c)^4*b+a),x)

mupad [B] time = 2.59, size = 625, normalized size = 1.97

$$\sum_{k=1}^4 \ln \left(-b d^4 \left(a + b c^4 + 4 b c^3 d x + \text{root} \left(256 a^3 b^3 d^{12} z^4 + 192 a^2 b^2 c^2 d^6 z^2 + 32 a b^2 c^5 d^3 z - 32 a^2 b c d^3 z + \dots \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*(c+d*x)^4),x)

[Out] symsum(log(-b*d^4*(a+b*c^4+4*b*c^3*d*x+4*root(256*a^3*b^3*d^12*z^4+192*a^2*b^2*c^2*d^6*z^2+32*a*b^2*c^5*d^3*z-32*a^2*b*c*d^3*z+2*a*b*c^4+b^2*c^8+a^2,z,k))*b^2*c^5*d^3+4*root(256*a^3*b^3*d^12*z^4+192*a^2*b^2*c^2*d^6*z^2+32*a*b^2*c^5*d^3*z-32*a^2*b*c*d^3*z+2*a*b*c^4+b^2*c^8+a^2,z,k))*b^2*c^4*d^4*x-20*root(256*a^3*b^3*d^12*z^4+192*a^2*b^2*c^2*d^6*z^2+32*a*b^2*c^5*d^3*z-32*a^2*b*c*d^3*z+2*a*b*c^4+b^2*c^8+a^2,z,k))*a*b*c*d^3-4*root(256*a^3*b^3*d^12*z^4+192*a^2*b^2*c^2*d^6*z^2+32*a*b^2*c^5*d^3*z-32*a^2*b*c*d^3*z+2*a*b*c^4+b^2*c^8+a^2,z,k))*a*b*d^4*x+48*root(256*a^3*b^3*d^12*z^4+192*a^2*b^2*c^2*d^6*z^2+32*a*b^2*c^5*d^3*z-32*a^2*b*c*d^3*z+2*a*b*c^4+b^2*c^8+a^2,z,k))^2*a*b^2*c^2*d^6+32*root(256*a^3*b^3*d^12*z^4+192*a^2*b^2*c^2*d^6*z^2+32*a*b^2*c^5*d^3*z-32*a^2*b*c*d^3*z+2*a*b*c^4+b^2*c^8+a^2,z,k))^2*a*b^2*c*d^7*x))*root(256*a^3*b^3*d^12*z^4+192*a^2*b^2*c^2*d^6*z^2+32*a*b^2*c^5*d^3*z-32*a^2*b*c*d^3*z+2*a*b*c^4+b^2*c^8+a^2,z,k),k,1,4)

sympy [A] time = 2.67, size = 274, normalized size = 0.86

$$\text{RootSum} \left(256 t^4 a^3 b^3 d^{12} + 192 t^2 a^2 b^2 c^2 d^6 + t \left(-32 a^2 b c d^3 + 32 a b^2 c^5 d^3 \right) + a^2 + 2 a b c^4 + b^2 c^8, \left(t \mapsto t \log \left(x + \frac{6}{t} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+b*(d*x+c)**4),x)

[Out] RootSum(256*_t**4*a**3*b**3*d**12+192*_t**2*a**2*b**2*c**2*d**6+_t*(-32*a**2*b*c*d**3+32*a*b**2*c**5*d**3)+a**2+2*a*b*c**4+b**2*c**8,Lambda(_t,_t*log(x+(64*_t**3*a**4*b**2*d**9+448*_t**3*a**3*b**3*c**4*d**9+160*_t**2*a**3*b**2*c**3*d**6-32*_t**2*a**2*b**3*c**7*d**6+60*_t*a**3*b*c**2*d**3+256*_t*a**2*b**2*c**6*d**3+4*_t*a*b**3*c**10*d**3-5*a**3*c-9*a**2*b*c**5-3*a*b**2*c**9+b**3*c**13)/(a**3*d-33*a**2*b*c**4*d-33*a*b**2*c**8*d+b**3*c**12*d))))

$$3.112 \quad \int \frac{x}{a+b(c+dx)^4} dx$$

Optimal. Leaf size=261

$$\frac{c \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} - \frac{c \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} + \frac{c \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx)}{\sqrt{a} + \sqrt{b}(c+dx)^2}\right)}{2\sqrt{2} a^3}$$

[Out] $-1/4*c*\arctan(-1+b^{(1/4)}*(d*x+c)*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/b^{(1/4)}/d^2*2^{(1/2)} - 1/4*c*\arctan(1+b^{(1/4)}*(d*x+c)*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/b^{(1/4)}/d^2*2^{(1/2)} + 1/8*c*\ln(-a^{(1/4)}*b^{(1/4)}*(d*x+c)*2^{(1/2)}+a^{(1/2)}+(d*x+c)^2*b^{(1/2)})/a^{(3/4)}/b^{(1/4)}/d^2*2^{(1/2)} - 1/8*c*\ln(a^{(1/4)}*b^{(1/4)}*(d*x+c)*2^{(1/2)}+a^{(1/2)}+(d*x+c)^2*b^{(1/2)})/a^{(3/4)}/b^{(1/4)}/d^2*2^{(1/2)} + 1/2*\arctan((d*x+c)^2*b^{(1/2)}/a^{(1/2)})/d^2/a^{(1/2)}/b^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {371, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{c \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} - \frac{c \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} + \frac{c \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx)}{\sqrt{a} + \sqrt{b}(c+dx)^2}\right)}{2\sqrt{2} a^3}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*(c + d*x)^4), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (c*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/ (2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/ (2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/ (4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/ (4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coefficient[Pq, x, ii] + Coefficient[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+b(c+dx)^4} dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^4} dx, x, c+dx\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{c}{a+bx^4} + \frac{x}{a+bx^4}\right) dx, x, c+dx\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \frac{x}{a+bx^4} dx, x, c+dx\right)}{d^2} - \frac{c \text{Subst}\left(\int \frac{1}{a+bx^4} dx, x, c+dx\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, (c+dx)^2\right)}{2d^2} - \frac{c \text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx, x, c+dx\right)}{2\sqrt{a}d^2} - \frac{c \text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx, x, c+dx\right)}{2\sqrt{a}d^2} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} - \frac{c \text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d^2} - \frac{c \text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}d^2} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx) + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx) + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx) + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 104, normalized size = 0.40

$$\frac{\text{RootSum}\left[\#1^4bd^4 + 4\#1^3bcd^3 + 6\#1^2bc^2d^2 + 4\#1bc^3d + a + bc^4 \&, \frac{\#1 \log(x-\#1)}{\#1^3d^3 + 3\#1^2cd^2 + 3\#1c^2d + c^3} \&\right]}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*(c + d*x)^4), x]

[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 &, (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &]/(4*b*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^4), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((d*x + c)^4*b + a), x, algorithm="giac")

[Out] integrate(x/((d*x + c)^4*b + a), x)

maple [C] time = 0.01, size = 95, normalized size = 0.36

$$\frac{\text{RootOf}(b d^4 Z^4 + 4b d^3 c Z^3 + 6b d^2 c^2 Z^2 + 4b c^3 d Z + b c^4 + a)}{4bd \left(d^3 \text{RootOf}(b d^4 Z^4 + 4b d^3 c Z^3 + 6b d^2 c^2 Z^2 + 4b c^3 d Z + b c^4 + a) \right)^3 + 3 \text{RootOf}(b d^4 Z^4 + 4b d^3 c Z^3 + 6b d^2 c^2 Z^2 + 4b c^3 d Z + b c^4 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*(d*x+c)^4),x)

[Out] 1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] integrate(x/((d*x + c)^4*b + a), x)

mupad [B] time = 2.36, size = 205, normalized size = 0.79

$$\sum_{k=1}^4 \ln\left(-\text{root}\left(256 a^3 b^2 d^8 z^4 + 32 a^2 b d^4 z^2 - 16 a b c^2 d^2 z + b c^4 + a, z, k\right)\right) \left(-\text{root}\left(256 a^3 b^2 d^8 z^4 + 32 a^2 b d^4 z^2 - 16 a b c^2 d^2 z + b c^4 + a, z, k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*(c + d*x)^4),x)

[Out] symsum(log(b^2*d^8*x - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k)*(4*b^3*c^3*d^9 - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k))*(32*a*b^3*c*d^11 + 16*a*b^3*d^12*x) + 4*b^3*c^2*d^10*x))*root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k), k, 1, 4)

sympy [A] time = 0.88, size = 131, normalized size = 0.50

$$\text{RootSum}\left(256t^4a^3b^2d^8 + 32t^2a^2bd^4 - 16tabc^2d^2 + a + bc^4, \left(t \mapsto t \log\left(x + \frac{128t^3a^3bd^6 + 16t^2a^2bc^2d^4 + 8ta^2c^2d^2 + a + bc^4}{4acd - bc^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)**4),x)

[Out] RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d**2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*a**2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/(4*a*c*d - b*c**5*d))))

$$3.113 \quad \int \frac{1}{a+b(c+dx)^4} dx$$

Optimal. Leaf size=221

$$\frac{\log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d} + \frac{\log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}}{2\sqrt{2} a^{3/4}}\right)}{2\sqrt{2} a^{3/4}}$$

[Out] $\frac{1}{4} \arctan\left(\frac{-1 + b^{1/4}(dx+c)^{1/2}/a^{1/4}}{a^{3/4}/b^{1/4}/d}\right) + \frac{1}{4} \arctan\left(\frac{1 + b^{1/4}(dx+c)^{1/2}/a^{1/4}}{a^{3/4}/b^{1/4}/d}\right) - \frac{1}{8} \ln\left(\frac{-a^{1/4}b^{1/4}(dx+c)^{1/2} + a^{1/2} + (dx+c)^2 b^{1/2}}{a^{3/4}/b^{1/4}/d}\right) + \frac{1}{8} \ln\left(\frac{a^{1/4}b^{1/4}(dx+c)^{1/2} + a^{1/2} + (dx+c)^2 b^{1/2}}{a^{3/4}/b^{1/4}/d}\right)$

Rubi [A] time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {247, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d} + \frac{\log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}}{2\sqrt{2} a^{3/4}}\right)}{2\sqrt{2} a^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^4)^(-1), x]

[Out] $-\text{ArcTan}\left[\frac{1 - (\sqrt{2} b^{1/4}(c+dx))/a^{1/4}}{(2\sqrt{2} a^{3/4} b^{1/4} d)}\right] + \text{ArcTan}\left[\frac{1 + (\sqrt{2} b^{1/4}(c+dx))/a^{1/4}}{(2\sqrt{2} a^{3/4} b^{1/4} d)}\right] - \text{Log}\left[\frac{\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4}(c+dx) + \sqrt{b}(c+dx)^2}{4\sqrt{2} a^{3/4} b^{1/4} d}\right] + \text{Log}\left[\frac{\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4}(c+dx) + \sqrt{b}(c+dx)^2}{4\sqrt{2} a^{3/4} b^{1/4} d}\right]$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rubi steps

$$\begin{aligned} \int \frac{1}{a + b(c + dx)^4} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^4} dx, x, c + dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx, x, c + dx\right)}{2\sqrt{a}d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx, x, c + dx\right)}{2\sqrt{a}d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c + dx\right)}{4\sqrt{a}\sqrt{b}d} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c + dx\right)}{4\sqrt{a}\sqrt{b}d} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c + dx\right)}{4\sqrt{a}\sqrt{b}d} \\ &= -\frac{\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} \\ &= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} - \frac{\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} \end{aligned}$$

Mathematica [A] time = 0.10, size = 161, normalized size = 0.73

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{a} + \sqrt{b}(c + dx)^2\right) + \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{a} + \sqrt{b}(c + dx)^2\right) - 2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^4)^(-1), x]

[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)

fricas [A] time = 0.49, size = 189, normalized size = 0.86

$$\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \arctan\left(a^2bd^4\sqrt{\frac{a^2d^2\sqrt{-\frac{1}{a^3bd^4}} + d^2x^2 + 2cdx + c^2}{d^2}}\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}} - (a^2bd^4x + a^2bcd^3)\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")

[Out] $(-1/(a^3*b*d^4))^{1/4}*\arctan(a^2*b*d^4*\sqrt{((a^2*d^2*\sqrt{-1/(a^3*b*d^4)}) + d^2*x^2 + 2*c*d*x + c^2)/d^2}*(-1/(a^3*b*d^4))^{3/4} - (a^2*b*d^4*x + a^2*b*c*d^3)*(-1/(a^3*b*d^4))^{3/4}) + 1/4*(-1/(a^3*b*d^4))^{1/4}*\log(a*d*(-1/(a^3*b*d^4))^{1/4} + d*x + c) - 1/4*(-1/(a^3*b*d^4))^{1/4}*\log(-a*d*(-1/(a^3*b*d^4))^{1/4} + d*x + c)$

giac [A] time = 0.50, size = 103, normalized size = 0.47

$$-\frac{1}{2}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \arctan\left(\frac{bdx + bc}{(-ab^3)^{\frac{1}{4}}}\right) + \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \log\left(\left|bdx + bc + (-ab^3)^{\frac{1}{4}}\right|\right) - \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \log\left(\left|-bdx - bc + (-ab^3)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")

[Out] $-1/2*(-1/(a^3*b*d^4))^{1/4}*\arctan(-(b*d*x + b*c)/(-a*b^3)^{1/4}) + 1/4*(-1/(a^3*b*d^4))^{1/4}*\log(\text{abs}(b*d*x + b*c + (-a*b^3)^{1/4})) - 1/4*(-1/(a^3*b*d^4))^{1/4}*\log(\text{abs}(-b*d*x - b*c + (-a*b^3)^{1/4}))$

maple [C] time = 0.00, size = 94, normalized size = 0.43

$$\frac{\ln\left(-\text{RootOf}\left(bd^4_Z^4 + 4bd^3c_Z^3 + 6bd^2c^2_Z^2 + 4bc^3d_Z + bc^4 + a\right)\right)}{4bd\left(d^3\text{RootOf}\left(bd^4_Z^4 + 4bd^3c_Z^3 + 6bd^2c^2_Z^2 + 4bc^3d_Z + bc^4 + a\right)^3 + 3\text{RootOf}\left(bd^4_Z^4 + 4bd^3c_Z^3 + 6bd^2c^2_Z^2 + 4bc^3d_Z + bc^4 + a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^4),x)

[Out] $1/4/b/d*\text{sum}(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*\ln(-_R+x),_R=\text{RootOf}(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] integrate(1/((d*x + c)^4*b + a), x)

mupad [B] time = 0.12, size = 60, normalized size = 0.27

$$\frac{\text{atan}\left(\frac{b^{1/4}c}{(-a)^{1/4}} + \frac{b^{1/4}dx}{(-a)^{1/4}}\right) + \text{atanh}\left(\frac{b^{1/4}c}{(-a)^{1/4}} + \frac{b^{1/4}dx}{(-a)^{1/4}}\right)}{2(-a)^{3/4}b^{1/4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*(c + d*x)^4),x)`

[Out] $-(\operatorname{atan}((b^{1/4})c)/(-a)^{1/4} + (b^{1/4}d*x)/(-a)^{1/4}) + \operatorname{atanh}((b^{1/4})c)/(-a)^{1/4} + (b^{1/4}d*x)/(-a)^{1/4})/(2*(-a)^{3/4}*b^{1/4}*d)$

sympy [A] time = 0.30, size = 26, normalized size = 0.12

$$\frac{\operatorname{RootSum}\left(256t^4a^3b + 1, \left(t \mapsto t \log\left(x + \frac{4ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*(d*x+c)**4),x)`

[Out] `RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/d)))/d`

$$3.114 \quad \int \frac{1}{x(a+b(c+dx)^4)} dx$$

Optimal. Leaf size=393

$$\frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2)}{4\sqrt{2}a^{3/4}(a+bc^4)} + \frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx))}{4\sqrt{2}a^{3/4}(a+bc^4)}$$

[Out] $\ln(x)/(b*c^4+a)-1/4*\ln(a+b*(d*x+c)^4)/(b*c^4+a)-1/2*c^2*\arctan((d*x+c)^2*b^(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)/a^(1/2)-1/8*b^(1/4)*c*\ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)+1/8*b^(1/4)*c*\ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4)*c*\arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4)*c*\arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)$

Rubi [A] time = 0.47, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2)}{4\sqrt{2}a^{3/4}(a+bc^4)} + \frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx))}{4\sqrt{2}a^{3/4}(a+bc^4)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*(c + d*x)^4)), x]

[Out] $-(\text{Sqrt}[b]*c^2*\text{ArcTan}[(\text{Sqrt}[b]*(c + d*x)^2)/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*(a + b*c^4)) + (b^(1/4)*c*(\text{Sqrt}[a] + \text{Sqrt}[b]*c^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(\text{Sqrt}[a] + \text{Sqrt}[b]*c^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(a + b*c^4)) + \text{Log}[x]/(a + b*c^4) - (b^(1/4)*c*(\text{Sqrt}[a] - \text{Sqrt}[b]*c^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*b^(1/4)*(c + d*x) + \text{Sqrt}[b]*(c + d*x)^2])/(4*\text{Sqrt}[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(\text{Sqrt}[a] - \text{Sqrt}[b]*c^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^(1/4)*b^(1/4)*(c + d*x) + \text{Sqrt}[b]*(c + d*x)^2])/(4*\text{Sqrt}[2]*a^(3/4)*(a + b*c^4)) - \text{Log}[a + b*(c + d*x)^4]/(4*(a + b*c^4))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

```
Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coefficient[Pq, x, ii] + Coefficient[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xexpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a + b(c + dx)^4)} dx &= \text{Subst} \left(\int \frac{1}{(-c + x)(a + bx^4)} dx, x, c + dx \right) \\
&= \text{Subst} \left(\int \left(-\frac{1}{(a + bc^4)(c - x)} - \frac{b(c^3 + c^2x + cx^2 + x^3)}{(a + bc^4)(a + bx^4)} \right) dx, x, c + dx \right) \\
&= \frac{\log(x)}{a + bc^4} - \frac{b \text{Subst} \left(\int \frac{c^3 + c^2x + cx^2 + x^3}{a + bx^4} dx, x, c + dx \right)}{a + bc^4} \\
&= \frac{\log(x)}{a + bc^4} - \frac{b \text{Subst} \left(\int \left(\frac{x(c^2 + x^2)}{a + bx^4} + \frac{c^3 + cx^2}{a + bx^4} \right) dx, x, c + dx \right)}{a + bc^4} \\
&= \frac{\log(x)}{a + bc^4} - \frac{b \text{Subst} \left(\int \frac{x(c^2 + x^2)}{a + bx^4} dx, x, c + dx \right)}{a + bc^4} - \frac{b \text{Subst} \left(\int \frac{c^3 + cx^2}{a + bx^4} dx, x, c + dx \right)}{a + bc^4} \\
&= \frac{\log(x)}{a + bc^4} - \frac{b \text{Subst} \left(\int \frac{c^2 + x}{a + bx^2} dx, x, (c + dx)^2 \right)}{2(a + bc^4)} + \frac{\left(c \left(1 - \frac{\sqrt{b}c^2}{\sqrt{a}} \right) \right) \text{Subst} \left(\int \frac{\sqrt{a}\sqrt{b} - bx^2}{a + bx^4} dx, x \right)}{2(a + bc^4)} \\
&= \frac{\log(x)}{a + bc^4} - \frac{b \text{Subst} \left(\int \frac{x}{a + bx^2} dx, x, (c + dx)^2 \right)}{2(a + bc^4)} - \frac{(bc^2) \text{Subst} \left(\int \frac{1}{a + bx^2} dx, x, (c + dx)^2 \right)}{2(a + bc^4)} \\
&= -\frac{\sqrt{b}c^2 \tan^{-1} \left(\frac{\sqrt{b}(c + dx)^2}{\sqrt{a}} \right)}{2\sqrt{a}(a + bc^4)} + \frac{\log(x)}{a + bc^4} - \frac{\sqrt[4]{b}c(\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx))}{4\sqrt{2}a^{3/4}(a + bc^4)} \\
&= -\frac{\sqrt{b}c^2 \tan^{-1} \left(\frac{\sqrt{b}(c + dx)^2}{\sqrt{a}} \right)}{2\sqrt{a}(a + bc^4)} + \frac{\sqrt[4]{b}c(\sqrt{a} + \sqrt{b}c^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c + dx)}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}(a + bc^4)} - \frac{\sqrt[4]{b}c(\sqrt{a} + \sqrt{b}c^2) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx))}{4\sqrt{2}a^{3/4}(a + bc^4)}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 163, normalized size = 0.41

$$\frac{\text{RootSum} \left[\#1^4bd^4 + 4\#1^3bcd^3 + 6\#1^2bc^2d^2 + 4\#1bc^3d + a + bc^4 \&, \frac{\#1^3d^3 \log(x - \#1) + 4\#1^2cd^2 \log(x - \#1) + 4c^3 \log(x - \#1) + 6\#1cd^2 \log(x - \#1)}{\#1^3d^3 + 3\#1^2cd^2 + 3\#1c^2d + c^3} \right]}{4(a + bc^4)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*(c + d*x)^4)), x]

[Out] -1/4*(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 &, (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1 + 4*c*d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &])/(a + b*c^4)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out


```

+ 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^2*b^4*c*d^
15 - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 1
6*a*z + 1, z, k)^4*a^3*b^4*c*d^15 - 32*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4
+ 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^5*d^15 + 240*root
(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z
, k)^3*a^2*b^4*d^16*x + 320*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*
z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*d^16*x - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
4*d^16*x - 48*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*
z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^4*d^16*x - 192*root(256*a^3*b*c^4*z^4 + 25
6*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^4*d^16
*x)*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*
z + 1, z, k), k, 1, 4)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)**4),x)

[Out] Timed out

$$3.115 \quad \int \frac{1}{x^2(a+b(c+dx)^4)} dx$$

Optimal. Leaf size=496

$$\frac{\sqrt[4]{b} d (\sqrt{a} (a - 3bc^4) - \sqrt{b} c^2 (3a - bc^4)) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c + dx) + \sqrt{a} + \sqrt{b} (c + dx)^2)}{4\sqrt{2} a^{3/4} (a + bc^4)^2} + \frac{\sqrt[4]{b} d (\sqrt{a} (a - 3bc^4) - \sqrt{b} c^2 (3a - bc^4)) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c + dx) + \sqrt{a} + \sqrt{b} (c + dx)^2)}{4\sqrt{2} a^{3/4} (a + bc^4)^2}$$

```
[Out] -1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2+b*c^3*d*ln(a+b*(d*x+c)^4)/(b*c^4+a)^2-c*(-b*c^4+a)*d*arctan((d*x+c)^2*b^(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)^2/a^(1/2)-1/8*b^(1/4)*d*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)+1/8*b^(1/4)*d*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)-1/4*b^(1/4)*d*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*b*c^4+a)*a^(1/2)+c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)-1/4*b^(1/4)*d*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*b*c^4+a)*a^(1/2)+c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b} d (\sqrt{a} (a - 3bc^4) - \sqrt{b} c^2 (3a - bc^4)) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c + dx) + \sqrt{a} + \sqrt{b} (c + dx)^2)}{4\sqrt{2} a^{3/4} (a + bc^4)^2} + \frac{\sqrt[4]{b} d (\sqrt{a} (a - 3bc^4) - \sqrt{b} c^2 (3a - bc^4)) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c + dx) + \sqrt{a} + \sqrt{b} (c + dx)^2)}{4\sqrt{2} a^{3/4} (a + bc^4)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*(c + d*x)^4)), x]

```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b*c^4)^2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

`Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

Rule 371

`Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]`

Rule 617

`Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 628

`Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rule 635

`Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]`

Rule 1162

`Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]`

Rule 1165

`Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]`

Rule 1168

`Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]`

Rule 1248

`Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]`

Rule 1876


```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(a + b(c + dx)^4)} dx &= d \operatorname{Subst} \left(\int \frac{1}{(-c + x)^2(a + bx^4)} dx, x, c + dx \right) \\
&= d \operatorname{Subst} \left(\int \left(\frac{1}{(a + bc^4)(c - x)^2} + \frac{4bc^3}{(a + bc^4)^2(c - x)} + \frac{b(-c^2(3a - bc^4) - 2c(a - bc^4))}{(a + bc^4)^3} \right) dx, x, c + dx \right) \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-c^2(3a - bc^4) - 2c(a - bc^4)x - (a - 3bc^4)x^2 + 4bc^3x^3}{a + bx^4} dx, x, c + dx \right)}{(a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \left(\frac{x(-2c(a - bc^4) + 4bc^3x^2)}{a + bx^4} + \frac{-c^2(3a - bc^4) + (-a + bc^4)x^2}{a + bx^4} \right) dx, x, c + dx \right)}{(a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{x(-2c(a - bc^4) + 4bc^3x^2)}{a + bx^4} dx, x, c + dx \right)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-c^2(3a - bc^4) + (-a + bc^4)x^2}{a + bx^4} dx, x, c + dx \right)}{(a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-2c(a - bc^4) + 4bc^3x}{a + bx^2} dx, x, (c + dx)^2 \right)}{2(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-c^2(3a - bc^4) + (-a + bc^4)x^2}{a + bx^4} dx, x, c + dx \right)}{(a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(2b^2c^3d) \operatorname{Subst} \left(\int \frac{x}{a + bx^2} dx, x, (c + dx)^2 \right)}{(a + bc^4)^2} - \frac{(bc(a - bc^4)) \operatorname{Subst} \left(\int \frac{1}{a + bx^4} dx, x, c + dx \right)}{(a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{\sqrt{b}c(a - bc^4)d \tan^{-1} \left(\frac{\sqrt{b}(c + dx)^2}{\sqrt{a}} \right)}{\sqrt{a}(a + bc^4)^2} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} - \frac{\sqrt[4]{b} \left(a - 3bc^4 + \frac{\sqrt{b}c^2(3a - bc^4)}{\sqrt{a}} \right)}{2\sqrt{2}\sqrt[4]{a}(a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{\sqrt{b}c(a - bc^4)d \tan^{-1} \left(\frac{\sqrt{b}(c + dx)^2}{\sqrt{a}} \right)}{\sqrt{a}(a + bc^4)^2} + \frac{\sqrt[4]{b} \left(a - 3bc^4 + \frac{\sqrt{b}c^2(3a - bc^4)}{\sqrt{a}} \right)}{2\sqrt{2}\sqrt[4]{a}(a + bc^4)^2}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 238, normalized size = 0.48

$$\frac{dx \operatorname{RootSum} \left[\#1^4 bd^4 + 4\#1^3 bcd^3 + 6\#1^2 bc^2 d^2 + 4\#1 bc^3 d + a + bc^4 \&, \frac{4\#1^3 bc^3 d^3 \log(x - \#1) - \#1^2 ad^2 \log(x - \#1) + 15\#1^2 bc^4 d^2}{4x(a + bc^4)^2} \right]}{4x(a + bc^4)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]

[Out] $(-4*(a + b*c^4 + 4*b*c^3*d*x*\text{Log}[x]) + d*x*\text{RootSum}[a + b*c^4 + 4*b*c^3*d*\#1 + 6*b*c^2*d^2*\#1^2 + 4*b*c*d^3*\#1^3 + b*d^4*\#1^4 \& , (-6*a*c^2*\text{Log}[x - \#1] + 10*b*c^6*\text{Log}[x - \#1] - 4*a*c*d*\text{Log}[x - \#1]*\#1 + 20*b*c^5*d*\text{Log}[x - \#1]*\#1 - a*d^2*\text{Log}[x - \#1]*\#1^2 + 15*b*c^4*d^2*\text{Log}[x - \#1]*\#1^2 + 4*b*c^3*d^3*\text{Log}[x - \#1]*\#1^3)/(c^3 + 3*c^2*d*\#1 + 3*c*d^2*\#1^2 + d^3*\#1^3) \&])/(4*(a + b*c^4)^{2*x})$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((dx + c)^4 b + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^4*b + a)*x^2), x)

maple [C] time = 0.01, size = 188, normalized size = 0.38

$$-\frac{4bc^3d \ln(x)}{(bc^4 + a)^2} + \frac{d \left(4 \text{RootOf}(bd^4_Z^4 + 4bd^3c_Z^3 + 6bd^2c^2_Z^2 + 4bc^3d_Z + bc^4 + a)^3 bc^3d^3 + 10bc^6 + (15bc^4 - a)d^3 \text{RootOf}(bd^4_Z^4 + 4bd^3c_Z^3 + 6bd^2c^2_Z^2 + 4bc^3d_Z + bc^4 + a) \right)}{4(bc^4 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b*(d*x+c)^4),x)

[Out] $1/4*d/(b*c^4+a)^2*\text{sum}((4*b*d^3*c^3*_R^3+d^2*(15*b*c^4-a)*_R^2+4*c*d*(5*b*c^4-a)*_R+10*b*c^6-6*a*c^2)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*\ln(-_R+x), _R=\text{RootOf}(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a)) - 1/(b*c^4+a)/x - 4*b*c^3*d*\ln(x)/(b*c^4+a)^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4bc^3d \log(x)}{b^2c^8 + 2abc^4 + a^2} + \frac{bd^2 \int \frac{4bc^3d^3x^3 + 10bc^6 + (15bc^4 - a)d^2x^2 - 6ac^2 + 4(5bc^5 - ac)dx}{bd^4x^4 + 4bcd^3x^3 + 6bc^2d^2x^2 + 4bc^3dx + bc^4 + a} dx}{b^2c^8 + 2abc^4 + a^2} - \frac{1}{(bc^4 + a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] $-4*b*c^3*d*\log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*\text{integrate}((4*b*c^3*d^3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4*(5*b*c^5 - a*c)*d*x)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/((b*c^4 + a)*x)$

mupad [B] time = 2.48, size = 2440, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^2*(a + b*(c + d*x)^4)),x)$

[Out] $\text{symsum}(\log(-4*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*b^7*c^{11}*d^{17} - 16*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^4*b^4*d^{16} - b^5*d^{20}*x + 16*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^6*d^{18} - 60*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^3*d^{17} + 176*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^4*d^{16} + 192*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*c^5*d^{15} + 144*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^8*d^{16} + 192*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^9*d^{15} + 64*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^{13}*d^{15} + 16*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^5*d^{19}*x + 64*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*c*d^{15} - 184*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^7*d^{17} - 48*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^{12}*d^{16} - 320*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*d^{16}*x + 4*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*b^7*c^{10}*d^{18}*x - 248*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^6*d^{18}*x - 64*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^{11}*d^{17}*x + 32*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*a*b^5*c*d^{19}*x - 316*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^2*d^{18}*x + 704*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^3*d^{17}*x - 448*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*c^4*d^{16}*x + 640*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^7*d^{17}*x + 64*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^8*d^{16}*x + 192*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^{12}*d^{16}*x)/(a^2 + b^2*c^8 + 2*a*b*c^4))*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k), k, 1, 4) - 1/(a*x + b*c^4*x) - (4*b*c^3*d*\log(x))/(a^2 + b^2*c^8 + 2*a*b*c^4)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a+b*(d*x+c)**4),x)

[Out] Timed out

$$3.116 \quad \int (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Optimal. Leaf size=123

$$-\frac{2}{9}(-3a^2 + 6a + 37)(x-1)^9 + \frac{4}{13}(3-a)(x-1)^{13} - \frac{8}{11}(3a+5)(x-1)^{11} + \frac{8}{7}(a+3)(3a+5)(x-1)^7 + \frac{4}{5}(3-a)(a+3)^2(x-1)^5$$

[Out] $-8/3*(3+a)^3*(-1+x)^3+4/5*(3-a)*(3+a)^2*(-1+x)^5+8/7*(3+a)*(5+3*a)*(-1+x)^7$
 $-2/9*(-3*a^2+6*a+37)*(-1+x)^9-8/11*(5+3*a)*(-1+x)^{11}+4/13*(3-a)*(-1+x)^{13}+8$
 $/15*(-1+x)^{15}+1/17*(-1+x)^{17}+(3+a)^4*x$

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1106, 1090}

$$-\frac{2}{9}(-3a^2 + 6a + 37)(x-1)^9 + \frac{4}{13}(3-a)(x-1)^{13} - \frac{8}{11}(3a+5)(x-1)^{11} + \frac{8}{7}(a+3)(3a+5)(x-1)^7 + \frac{4}{5}(3-a)(a+3)^2(x-1)^5$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

[Out] $(-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5 + (8*(3 + a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1 + x)^9)/9 - (8*(5 + 3*a)*(-1 + x)^{11})/11 + (4*(3 - a)*(-1 + x)^{13})/13 + (8*(-1 + x)^{15})/15 + (-1 + x)^{17}/17 + (3 + a)^4*x$

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rule 1106

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned} \int (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx &= \text{Subst} \left(\int (3 + a - 2x^2 - x^4)^4 dx, x, -1 + x \right) \\ &= \text{Subst} \left(\int \left(81 \left(1 + \frac{1}{81} a (108 + 54a + 12a^2 + a^3) \right) \right) - 216 \left(1 + a \left(1 + \frac{1}{27} a (9 \right. \right. \right. \\ &= -\frac{8}{3}(3+a)^3(-1+x)^3 + \frac{4}{5}(3-a)(3+a)^2(-1+x)^5 + \frac{8}{7}(3+a)(5+3a)(-1+x)^7 \end{aligned}$$

Mathematica [A] time = 0.04, size = 195, normalized size = 1.59

$$a^4x+16a^3x^2+\frac{2}{9}(3a^2-1536a+20480)x^9-6(a^2-128a+896)x^8+\frac{64}{7}(3a^2-140a+512)x^7-\frac{16}{3}(15a^2-288a+128)x^6$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

[Out] $a^4x + 16a^3x^2 - (32(-12 + a)a^2x^3)/3 + 4a(128 - 48a + a^2)x^4 - (4(-1024 + 1536a - 192a^2 + a^3)x^5)/5 - (16(512 - 288a + 15a^2)x^6)/3 + (64(512 - 140a + 3a^2)x^7)/7 - 6(896 - 128a + a^2)x^8 + (2(20480 - 1536a + 3a^2)x^9)/9 + (16(-928 + 35a)x^{10})/5 - (32(-524 + 9a)x^{11})/11 + (4(-464 + 3a)x^{12})/3 - (4(-640 + a)x^{13})/13 - 48x^{14} + (128x^{15})/15 - x^{16} + x^{17}/17$

fricas [B] time = 0.37, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} - \frac{4}{13}x^{13}a + \frac{2560}{13}x^{13} + 4x^{12}a - \frac{1856}{3}x^{12} - \frac{288}{11}x^{11}a + \frac{16768}{11}x^{11} + 112x^{10}a + \frac{2}{3}x^9a^2 - \frac{14848}{5}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")

[Out] $1/17*x^{17} - x^{16} + 128/15*x^{15} - 48*x^{14} - 4/13*x^{13}*a + 2560/13*x^{13} + 4*x^{12}*a - 1856/3*x^{12} - 288/11*x^{11}*a + 16768/11*x^{11} + 112*x^{10}*a + 2/3*x^9*a^2 - 14848/5*x^{10} - 1024/3*x^9*a - 6*x^8*a^2 + 40960/9*x^9 + 768*x^8*a + 192/7*x^7*a^2 - 5376*x^8 - 1280*x^7*a - 80*x^6*a^2 - 4/5*x^5*a^3 + 32768/7*x^7 + 1536*x^6*a + 768/5*x^5*a^2 + 4*x^4*a^3 - 8192/3*x^6 - 6144/5*x^5*a - 192*x^4*a^2 - 32/3*x^3*a^3 + 4096/5*x^5 + 512*x^4*a + 128*x^3*a^2 + 16*x^2*a^3 + x*a^4$

giac [B] time = 0.33, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - \frac{4}{13}ax^{13} - 48x^{14} + 4ax^{12} + \frac{2560}{13}x^{13} - \frac{288}{11}ax^{11} - \frac{1856}{3}x^{12} + \frac{2}{3}a^2x^9 + 112ax^{10} + \frac{16768}{11}x^{11} - 6a^2x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")

[Out] $1/17*x^{17} - x^{16} + 128/15*x^{15} - 4/13*a*x^{13} - 48*x^{14} + 4*a*x^{12} + 2560/13*x^{13} - 288/11*a*x^{11} - 1856/3*x^{12} + 2/3*a^2*x^9 + 112*a*x^{10} + 16768/11*x^{11} - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^{10} + 192/7*a^2*x^7 + 768*a*x^8 + 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3*x^4 + 768/5*a^2*x^5 + 1536*a*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x^4 - 6144/5*a*x^5 - 8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*x^4 + 4096/5*x^5$

maple [B] time = 0.00, size = 264, normalized size = 2.15

$$\frac{x^{17}}{17} - x^{16} + \frac{128x^{15}}{15} - 48x^{14} + \frac{(-4a + 2560)x^{13}}{13} + \frac{(48a - 7424)x^{12}}{12} + \frac{(-288a + 16768)x^{11}}{11} + \frac{(1120a - 29696)x^{10}}{10} + \frac{(2a^2 - 2560a + 24576 + (-2a + 128)^2)x^9 + 1/8(-16a^2 + 3584a - 10240 + 2(8a - 128)(-2a + 128))x^8 + 1/7(64a^2 - 2560a + 2(-16a + 64)(-2a + 128) + (8a - 128)^2)x^7 + 1/6(-160a^2 + 32a(-2a + 128) + 2(-16a + 64)(8a - 128))x^6 + 1/5(2a^2(-2a + 128) + 32a(8a - 128) + (-16a + 64)^2)x^5 + 1/4(2a^2(8a - 128) + 32a(-16a + 64))x^4 + 1/3(2a^2(-16a + 64) + 256a^2)x^3 + 16a^3x^2 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+4*x^3-8*x^2+a+8*x)^4,x)

[Out] $1/17*x^{17} - x^{16} + 128/15*x^{15} - 48*x^{14} + 1/13*(-4*a+2560)*x^{13} + 1/12*(48*a-7424)*x^{12} + 1/11*(-288*a+16768)*x^{11} + 1/10*(1120*a-29696)*x^{10} + 1/9*(2*a^2-2560*a+24576+(-2*a+128)^2)*x^9 + 1/8*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128))*x^8 + 1/7*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^7 + 1/6*(-160*a^2+32*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^6 + 1/5*(2*a^2*(-2*a+128)+32*a*(8*a-128)+(-16*a+64)^2)*x^5 + 1/4*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^4 + 1/3*(2*a^2*(-16*a+64)+256*a^2)*x^3 + 16*a^3*x^2 + a^4*x$

maxima [A] time = 0.69, size = 192, normalized size = 1.56

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + \frac{2560}{13}x^{13} - \frac{1856}{3}x^{12} + \frac{16768}{11}x^{11} - \frac{14848}{5}x^{10} + \frac{40960}{9}x^9 - 5376x^8 + \frac{32768}{7}x^7 - \frac{8192}{3}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")

[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*x^12 + 16768/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32768/7*x^7 - 8192/3*x^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^3 + 2/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^3)*a^2 - 4/2145*(165*x^13 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 183040*x^9 - 411840*x^8 + 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a

mupad [B] time = 0.21, size = 175, normalized size = 1.42

$$x^{12} \left(4a - \frac{1856}{3}\right) - x^{13} \left(\frac{4a}{13} - \frac{2560}{13}\right) + x^{10} \left(112a - \frac{14848}{5}\right) - x^{11} \left(\frac{288a}{11} - \frac{16768}{11}\right) - x^8 (6a^2 - 768a + 5376) - x^6 (80a^2 - 1536a + 8192/3) + x^7 ((192a^2)/7 - 1280a + 32768/7) + x^9 ((2a^2)/3 - (1024a)/3 + 40960/9) - x^5 ((6144a)/5 - (768a^2)/5 + (4a^3)/5 - 4096/5) + a^4 x - 48x^{14} + (128x^{15})/15 - x^{16} + x^{17}/17 + 16a^3 x^2 + 4a a x^4 (a^2 - 48a + 128) - (32a^2 x^3 (a - 12))/3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)

[Out] x^12*(4*a - 1856/3) - x^13*((4*a)/13 - 2560/13) + x^10*(112*a - 14848/5) - x^11*((288*a)/11 - 16768/11) - x^8*(6*a^2 - 768*a + 5376) - x^6*(80*a^2 - 1536*a + 8192/3) + x^7*((192*a^2)/7 - 1280*a + 32768/7) + x^9*((2*a^2)/3 - (1024*a)/3 + 40960/9) - x^5*((6144*a)/5 - (768*a^2)/5 + (4*a^3)/5 - 4096/5) + a^4*x - 48*x^14 + (128*x^15)/15 - x^16 + x^17/17 + 16*a^3*x^2 + 4*a*x^4*(a^2 - 48*a + 128) - (32*a^2*x^3*(a - 12))/3

sympy [A] time = 0.12, size = 199, normalized size = 1.62

$$a^4 x + 16a^3 x^2 + \frac{x^{17}}{17} - x^{16} + \frac{128x^{15}}{15} - 48x^{14} + x^{13} \left(\frac{2560}{13} - \frac{4a}{13}\right) + x^{12} \left(4a - \frac{1856}{3}\right) + x^{11} \left(\frac{16768}{11} - \frac{288a}{11}\right) + x^{10} \left(112a - \frac{14848}{5}\right) - x^8 (6a^2 - 768a + 5376) - x^6 (80a^2 - 1536a + 8192/3) + x^7 ((192a^2)/7 - 1280a + 32768/7) + x^9 ((2a^2)/3 - (1024a)/3 + 40960/9) - x^5 ((6144a)/5 - (768a^2)/5 + (4a^3)/5 - 4096/5) + a^4 x - 48x^{14} + (128x^{15})/15 - x^{16} + x^{17}/17 + 16a^3 x^2 + 4a a x^4 (a^2 - 48a + 128) - (32a^2 x^3 (a - 12))/3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)

[Out] a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**14 + x**13*(2560/13 - 4*a/13) + x**12*(4*a - 1856/3) + x**11*(16768/11 - 288*a/11) + x**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1024*a/3 + 40960/9) + x**8*(-6*a**2 + 768*a - 5376) + x**7*(192*a**2/7 - 1280*a + 32768/7) + x**6*(-80*a**2 + 1536*a - 8192/3) + x**5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5) + x**4*(4*a**3 - 192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)

$$3.117 \quad \int (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$$

Optimal. Leaf size=120

$$a^3x - \frac{3}{5}(a^2 - 128a + 512)x^5 + (3a^2 - 96a + 128)x^4 + 12a^2x^2 - \frac{1}{3}(256 - a)x^9 + 3(64 - a)x^8 - \frac{32}{7}(70 - 3a)x^7 + 8(48 - 5a)x^6$$

[Out] $a^3x + 12a^2x^2 + 8(8 - a)ax^3 + (3a^2 - 96a + 128)x^4 - \frac{3}{5}(a^2 - 128a + 512)x^5 + 8(48 - 5a)x^6 - \frac{32}{7}(70 - 3a)x^7 + 3(64 - a)x^8 - \frac{1}{3}(256 - a)x^9 + 28x^{10} - \frac{72}{11}x^{11} + x^{12} - \frac{1}{13}x^{13}$

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$-\frac{3}{5}(a^2 - 128a + 512)x^5 + (3a^2 - 96a + 128)x^4 + 12a^2x^2 + a^3x - \frac{1}{3}(256 - a)x^9 + 3(64 - a)x^8 - \frac{32}{7}(70 - 3a)x^7 + 8(48 - 5a)x^6$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] $a^3x + 12a^2x^2 + 8(8 - a)ax^3 + (128 - 96a + 3a^2)x^4 - \frac{3}{5}(512 - 128a + a^2)x^5 + 8(48 - 5a)x^6 - \frac{32}{7}(70 - 3a)x^7 + 3(64 - a)x^8 - \frac{1}{3}(256 - a)x^9 + 28x^{10} - \frac{72}{11}x^{11} + x^{12} - \frac{1}{13}x^{13}$

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && IntegerQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx &= \int (a^3 + 24a^2x + 24(8 - a)ax^2 + 4(128 - 96a + 3a^2)x^3 - 3(512 - 128a + a^2)x^4 \\ &\quad + a^3x + 12a^2x^2 + 8(8 - a)ax^3 + (128 - 96a + 3a^2)x^4 - \frac{3}{5}(512 - 128a + a^2)x^5 \end{aligned}$$

Mathematica [A] time = 0.02, size = 114, normalized size = 0.95

$$a^3x - \frac{3}{5}(a^2 - 128a + 512)x^5 + (3a^2 - 96a + 128)x^4 + 12a^2x^2 + \frac{1}{3}(a - 256)x^9 - 3(a - 64)x^8 + \frac{32}{7}(3a - 70)x^7 - 8(5a - 48)x^6$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] $a^3x + 12a^2x^2 - 8(-8 + a)ax^3 + (128 - 96a + 3a^2)x^4 - \frac{3}{5}(512 - 128a + a^2)x^5 - 8(-48 + 5a)x^6 + \frac{32}{7}(-70 + 3a)x^7 - 3(-64 + a)x^8 + \frac{1}{3}(-256 + a)x^9 + 28x^{10} - \frac{72}{11}x^{11} + x^{12} - \frac{1}{13}x^{13}$

fricas [A] time = 0.36, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + 28x^{10} + \frac{1}{3}x^9a - \frac{256}{3}x^9 - 3x^8a + 192x^8 + \frac{96}{7}x^7a - 320x^7 - 40x^6a - \frac{3}{5}x^5a^2 + 384x^6 + \frac{384}{5}x^5a + 3x^4a^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] $-1/13*x^{13} + x^{12} - 72/11*x^{11} + 28*x^{10} + 1/3*x^9*a - 256/3*x^9 - 3*x^8*a + 192*x^8 + 96/7*x^7*a - 320*x^7 - 40*x^6*a - 3/5*x^5*a^2 + 384*x^6 + 384/5*x^5*a + 3*x^4*a^2 - 1536/5*x^5 - 96*x^4*a - 8*x^3*a^2 + 128*x^4 + 64*x^3*a + 12*x^2*a^2 + x*a^3$

giac [A] time = 0.30, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+\frac{1}{3}ax^9+28x^{10}-3ax^8-\frac{256}{3}x^9+\frac{96}{7}ax^7+192x^8-\frac{3}{5}a^2x^5-40ax^6-320x^7+3a^2x^4+\frac{384}{5}ax^5+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] $-1/13*x^{13} + x^{12} - 72/11*x^{11} + 1/3*a*x^9 + 28*x^{10} - 3*a*x^8 - 256/3*x^9 + 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 384/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*x^2 + 64*a*x^3 + 128*x^4$

maple [A] time = 0.00, size = 138, normalized size = 1.15

$$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\frac{(3a-768)x^9}{9}+\frac{(-24a+1536)x^8}{8}+\frac{(96a-2240)x^7}{7}+\frac{(-240a+2304)x^6}{6}+\frac{(-a^2+(-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+4*x^3-8*x^2+a+8*x)^3,x)

[Out] $-1/13*x^{13}+x^{12}-72/11*x^{11}+28*x^{10}+1/9*(3*a-768)*x^9+1/8*(-24*a+1536)*x^8+1/7*(96*a-2240)*x^7+1/6*(-240*a+2304)*x^6+1/5*((-2*a+128)*a+256*a-1536-a^2)*x^5+1/4*((8*a-128)*a-256*a+512+4*a^2)*x^4+1/3*((-16*a+64)*a+128*a-8*a^2)*x^3+12*a^2*x^2+a^3*x$

maxima [A] time = 0.67, size = 119, normalized size = 0.99

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}-\frac{256}{3}x^9+192x^8-320x^7+384x^6-\frac{1536}{5}x^5+a^3x+128x^4-\frac{1}{5}(3x^5-15x^4+40x^3-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")

[Out] $-1/13*x^{13} + x^{12} - 72/11*x^{11} + 28*x^{10} - 256/3*x^9 + 192*x^8 - 320*x^7 + 384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^3)*a$

mupad [B] time = 0.10, size = 108, normalized size = 0.90

$$x^9\left(\frac{a}{3}-\frac{256}{3}\right)-x^8(3a-192)-x^6(40a-384)+x^7\left(\frac{96a}{7}-320\right)+x^4(3a^2-96a+128)-x^5\left(\frac{3a^2}{5}-\frac{384a}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)

[Out] $x^9*(a/3 - 256/3) - x^8*(3*a - 192) - x^6*(40*a - 384) + x^7*((96*a)/7 - 320) + x^4*(3*a^2 - 96*a + 128) - x^5*((3*a^2)/5 - (384*a)/5 + 1536/5) + a^3*x + 28*x^{10} - (72*x^{11})/11 + x^{12} - x^{13}/13 + 12*a^2*x^2 - 8*a*x^3*(a - 8)$

sympy [A] time = 0.09, size = 114, normalized size = 0.95

$$a^3x+12a^2x^2-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+x^9\left(\frac{a}{3}-\frac{256}{3}\right)+x^8(192-3a)+x^7\left(\frac{96a}{7}-320\right)+x^6(384-40a)+x^5\left(-\frac{3a^2}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(a
/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a) +
x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*(-8
*a**2 + 64*a)
```

$$3.118 \quad \int (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Optimal. Leaf size=72

$$a^2x + \frac{2}{5}(64 - a)x^5 - 2(16 - a)x^4 + \frac{16}{3}(4 - a)x^3 + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3}$$

[Out] a^2*x+8*a*x^2+16/3*(4-a)*x^3-2*(16-a)*x^4+2/5*(64-a)*x^5-40/3*x^6+32/7*x^7-x^8+1/9*x^9

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$a^2x + \frac{2}{5}(64 - a)x^5 - 2(16 - a)x^4 + \frac{16}{3}(4 - a)x^3 + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 - a)*x^5)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx &= \int (a^2 + 16ax + 16(4 - a)x^2 - 8(16 - a)x^3 + 2(64 - a)x^4 - 80x^5 + 32x^6 - 40x^7 + 32x^8 - x^9) dx \\ &= a^2x + 8ax^2 + \frac{16}{3}(4 - a)x^3 - 2(16 - a)x^4 + \frac{2}{5}(64 - a)x^5 - \frac{40x^6}{3} + \frac{32x^7}{7} - x^8 + \frac{x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 0.92

$$a^2x - \frac{2}{5}(a - 64)x^5 + 2(a - 16)x^4 - \frac{16}{3}(a - 4)x^3 + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9

fricas [A] time = 0.37, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{40}{3}x^6 - \frac{2}{5}x^5a + \frac{128}{5}x^5 + 2x^4a - 32x^4 - \frac{16}{3}x^3a + \frac{64}{3}x^3 + 8x^2a + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 - 2/5*x^5*a + 128/5*x^5 + 2*x^4*a - 32*x^4 - 16/3*x^3*a + 64/3*x^3 + 8*x^2*a + x*a^2

giac [A] time = 0.31, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{2}{5}ax^5 - \frac{40}{3}x^6 + 2ax^4 + \frac{128}{5}x^5 - \frac{16}{3}ax^3 - 32x^4 + a^2x + 8ax^2 + \frac{64}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16/3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3

maple [A] time = 0.00, size = 63, normalized size = 0.88

$$\frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + \frac{(-2a+128)x^5}{5} + \frac{(8a-128)x^4}{4} + a^2x + 8ax^2 + \frac{(-16a+64)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] 1/9*x^9-x^8+32/7*x^7-40/3*x^6+1/5*(-2*a+128)*x^5+1/4*(8*a-128)*x^4+1/3*(-16*a+64)*x^3+8*a*x^2+a^2*x

maxima [A] time = 0.71, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{40}{3}x^6 + \frac{128}{5}x^5 - 32x^4 + a^2x + \frac{64}{3}x^3 - \frac{2}{15}(3x^5 - 15x^4 + 40x^3 - 60x^2)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^3 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a

mupad [B] time = 0.04, size = 61, normalized size = 0.85

$$x^4(2a-32) - x^3\left(\frac{16a}{3} - \frac{64}{3}\right) - x^5\left(\frac{2a}{5} - \frac{128}{5}\right) + 8ax^2 + a^2x - \frac{40x^6}{3} + \frac{32x^7}{7} - x^8 + \frac{x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] x^4*(2*a - 32) - x^3*((16*a)/3 - 64/3) - x^5*((2*a)/5 - 128/5) + 8*a*x^2 + a^2*x - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9

sympy [A] time = 0.08, size = 65, normalized size = 0.90

$$a^2x + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + x^5\left(\frac{128}{5} - \frac{2a}{5}\right) + x^4(2a - 32) + x^3\left(\frac{64}{3} - \frac{16a}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 - 2*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)

$$3.119 \quad \int (a + 8x - 8x^2 + 4x^3 - x^4) dx$$

Optimal. Leaf size=26

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Antiderivative was successfully verified.

[In] Int[a + 8*x - 8*x^2 + 4*x^3 - x^4, x]

[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5

Rubi steps

$$\int (a + 8x - 8x^2 + 4x^3 - x^4) dx = ax + 4x^2 - \frac{8x^3}{3} + x^4 - \frac{x^5}{5}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Antiderivative was successfully verified.

[In] Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4, x]

[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5

fricas [A] time = 0.35, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + 4x^2 + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")

[Out] -1/5*x^5 + x^4 - 8/3*x^3 + 4*x^2 + x*a

giac [A] time = 0.30, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")

[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x^4+4*x^3-8*x^2+a+8*x,x)`

[Out] `a*x+4*x^2-8/3*x^3+x^4-1/5*x^5`

maxima [A] time = 0.63, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")`

[Out] `-1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2`

mupad [B] time = 0.02, size = 22, normalized size = 0.85

$$-\frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + 8*x - 8*x^2 + 4*x^3 - x^4,x)`

[Out] `a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5`

sympy [A] time = 0.06, size = 22, normalized size = 0.85

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x**4+4*x**3-8*x**2+a+8*x,x)`

[Out] `a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2`

$$3.120 \quad \int \frac{1}{a+8x-8x^2+4x^3-x^4} dx$$

Optimal. Leaf size=89

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

[Out] $-1/2*\arctan((-1+x)/(1-(4+a)^{(1/2)})^{(1/2)})/(4+a)^{(1/2)}/(1-(4+a)^{(1/2)})^{(1/2)}$
 $+1/2*\arctan((-1+x)/(1+(4+a)^{(1/2)})^{(1/2)})/(4+a)^{(1/2)}/(1+(4+a)^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1106, 1093, 204}

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1), x]

[Out] $-\text{ArcTan}[-(1+x)/\text{Sqrt}[1-\text{Sqrt}[4+a]]]/(2*\text{Sqrt}[4+a]*\text{Sqrt}[1-\text{Sqrt}[4+a]])$
 $+\text{ArcTan}[-(1+x)/\text{Sqrt}[1+\text{Sqrt}[4+a]]]/(2*\text{Sqrt}[4+a]*\text{Sqrt}[1+\text{Sqrt}[4+a]])$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1106

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4} dx = \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4} dx, x, -1+x\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2} dx, x, -1+x\right)}{2\sqrt{4+a}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2} dx, x, -1+x\right)}{2\sqrt{4+a}}$$

$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.64

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a\&, \frac{\log(x - \#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1), x]

[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 + 4*#1 - 3*#1^2 + #1^3) &]

fricas [B] time = 0.45, size = 457, normalized size = 5.13

$$\frac{1}{4} \sqrt{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}} + 1} \log\left(\left(a - \frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}} + 4\right) \sqrt{\frac{a^2+7a+12}{a^2+7a+12}} + x - 1\right) - \frac{1}{4} \sqrt{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x), x, algorithm="fricas")

[Out] 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x - 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7*a + 12)) + x - 1)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. The choice was done assuming [a]=[86]Warning, need to ch

12)+a^2+7*a+12)*x+69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+351*a^2*sqrt(a+4)+351*a^2-29*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+29*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)-144*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*x+144*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+544*a*sqrt(a+4)+544*a-28*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+28*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)-112*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*x+112*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+336*sqrt(a+4)+336))

maple [C] time = 0.02, size = 49, normalized size = 0.55

$$\frac{\ln\left(-\operatorname{RootOf}\left(-Z^4-4Z^3+8Z^2-8Z-a\right)+x\right)}{4\left(\operatorname{RootOf}\left(-Z^4-4Z^3+8Z^2-8Z-a\right)^3-3\operatorname{RootOf}\left(-Z^4-4Z^3+8Z^2-8Z-a\right)^2+4\operatorname{RootOf}\left(-Z^4-4Z^3+8Z^2-8Z-a\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x), x)

[Out] -1/4*sum(1/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x), _R=RootOf(-Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x), x, algorithm="maxima")

[Out] -integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)

mupad [B] time = 2.58, size = 571, normalized size = 6.42

$$-\operatorname{atan}\left(\frac{a8i - x16i + x\sqrt{a^3 + 12a^2 + 48a + 64}1i - ax8i - \sqrt{a^3 + 12a^2 + 48a + 64}1i - a^2x1i + a^2}{44a^2\sqrt{\frac{a-\sqrt{a^3+12a^2+48a+64}+4}{16a^3+176a^2+640a+768}} + 4a^3\sqrt{\frac{a-\sqrt{a^3+12a^2+48a+64}+4}{16a^3+176a^2+640a+768}} + 160a\sqrt{\frac{a-\sqrt{a^3+12a^2+48a+64}+4}{16a^3+176a^2+640a+768}} + 192\sqrt{\frac{a-\sqrt{a^3+12a^2+48a+64}+4}{16a^3+176a^2+640a+768}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x)

[Out] -atan(-(a*8i - x*16i + x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i - (48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(44*a^2*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 160*a*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i - atan(-(a*8i - x*16i - x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i + (48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(160*a*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 44*a^2*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i

sympy [A] time = 0.93, size = 66, normalized size = 0.74

-RootSum(t^4(256a^3 + 2816a^2 + 10240a + 12288) + t^2(-32a - 128) - 1, (t ↦ t log(64t^3a^2 + 448t^3a + 768t^3 -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 128) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*a - 20*_t + x - 1)))
```

$$3.121 \quad \int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2} dx$$

Optimal. Leaf size=169

$$\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(3a+\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}} + \frac{(3a-\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{1+\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1+\sqrt{a+4}}}$$

[Out] 1/4*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1-(4+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(10+3*a-(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1106, 1092, 1166, 204}

$$\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(3a+\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}} + \frac{(3a-\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{1+\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1+\sqrt{a+4}}}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2), x]

[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + Sqrt[4 + a]])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1106

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2

$-q/2 + c*x^2), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 + q/2 + c*x^2), x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx &= \text{Subst} \left(\int \frac{1}{(3 + a - 2x^2 - x^4)^2} dx, x, -1 + x \right) \\ &= \frac{(5 + a + (-1 + x)^2)(-1 + x)}{4(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)} - \frac{\text{Subst} \left(\int \frac{4+2(3+a)-2(4+3+a-2x^2)}{3+a-2x^2} dx, x, -1 + x \right)}{8(12 + 7a + a^2)} \\ &= \frac{(5 + a + (-1 + x)^2)(-1 + x)}{4(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)} - \frac{(10 + 3a - \sqrt{4 + a}) \text{S}}{8(3 + a)} \\ &= \frac{(5 + a + (-1 + x)^2)(-1 + x)}{4(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)} + \frac{(10 + 3a + \sqrt{4 + a}) \text{t}}{8(3 + a)(4 + a)^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.07, size = 150, normalized size = 0.89

$$\frac{(x - 1)(a + x^2 - 2x + 6)}{4(a + 3)(a + 4)(a - x(x^3 - 4x^2 + 8x - 8))} - \frac{\text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\#1^2 \log(x - \#1) + 3a \log(x - \#1)}{\#1^3 - 3} \right]}{16(a^2 + 7a + 12)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2), x]

[Out] ((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (12*Log[x - #1] + 3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &]/(16*(12 + 7*a + a^2))

fricas [B] time = 0.44, size = 1948, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] -1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4

$$\begin{aligned}
& + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7*a + 12)* \\
& x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^ \\
& 2 + 7*a + 12)*x - 12*a)*\sqrt{(15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + \\
& 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a \\
& ^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a \\
& + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 302 \\
& 4*a + 1728))*\log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 408*a^3 + 2 \\
& 309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + \\
& 24624*a + 12096)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088 \\
& *a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) \\
& + 5800*a + 5456)*\sqrt{(15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a \\
& ^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3 \\
& 088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 4665 \\
& 6)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + \\
& 1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^ \\
& 3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*\sqrt{(15* \\
& a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(8 \\
& 1*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598 \\
& *a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 2 \\
& 1*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*\log(-81*a^2 + (81*a^ \\
& 2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513 \\
& *a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*\sqrt{(81*a^2 + 5 \\
& 58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11 \\
& 1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*\sqrt{(15*a^2 - \\
& (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 \\
& + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + \\
& 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 \\
& + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7 \\
& *a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^ \\
& 2 - 8*(a^2 + 7*a + 12)*x - 12*a)*\sqrt{(15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 8 \\
& 47*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^ \\
& 8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + \\
& 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196* \\
& a^2 + 3024*a + 1728))*\log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 40 \\
& 8*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 214 \\
& 20*a^2 + 24624*a + 12096)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a \\
& ^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a \\
& + 46656)) + 5800*a + 5456)*\sqrt{(15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 \\
& + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 39 \\
& 9*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304 \\
& *a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + \\
& 3024*a + 1728)) - 567*a - 992) + 4*(a + 8)*x - 12*x^2 - 4*a - 24)/((a^2 + 7 \\
& *a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^ \\
& 2 - 8*(a^2 + 7*a + 12)*x - 12*a)
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[86]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[12]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[48]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done ass

$$\begin{aligned}
& 1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+138027*a^8*\sqrt{a+4}+1823 \\
& 14*a^8-81*a^7*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210 \\
& *a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+81*a^7*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3 \\
& +701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-2340*a \\
& ^7*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a \\
& ^2+2548*a+2208)*x+2340*a^7*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+148 \\
& 8)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)+1304648*a^7*\sqrt{a+4}+1715172*a^7-2 \\
& 016*a^6*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1 \\
& 099*a^2+2548*a+2208)*\sqrt{a+4}*x+2016*a^6*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701 \\
& *a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-29518*a^6* \\
& \sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+ \\
& 2548*a+2208)*x+29518*a^6*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)} \\
& +15*a^4+210*a^3+1099*a^2+2548*a+2208)+8079749*a^6*\sqrt{a+4}+10572392*a^6-21 \\
& 454*a^5*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1 \\
& 099*a^2+2548*a+2208)*\sqrt{a+4}*x+21454*a^5*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+70 \\
& 1*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-212356*a^ \\
& 5*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^ \\
& 2+2548*a+2208)*x+212356*a^5*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+14 \\
& 88)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)+34255200*a^5*\sqrt{a+4}+44613658*a^ \\
& 5-126540*a^4*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210* \\
& a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+126540*a^4*\sqrt{\sqrt{a+4}*(9*a^4+130* \\
& a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-952 \\
& 845*a^4*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1 \\
& 099*a^2+2548*a+2208)*x+952845*a^4*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+167 \\
& 2*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)+100679657*a^4*\sqrt{a+4}+1305 \\
& 13730*a^4-446685*a^3*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15* \\
& a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+446685*a^3*\sqrt{\sqrt{a+4}*(9* \\
& a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{ \\
& a+4}-2730184*a^3*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+ \\
& 210*a^3+1099*a^2+2548*a+2208)*x+2730184*a^3*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+7 \\
& 01*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)+202540404*a^3*\sqrt{ \\
& (a+4)}+261341928*a^3-943444*a^2*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a \\
& +1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+943444*a^2*\sqrt{\sqrt{ \\
& (a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2 \\
& 208)*\sqrt{a+4}-4877364*a^2*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+148 \\
& 8)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4877364*a^2*\sqrt{\sqrt{a+4}*(9*a^4 \\
& +130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)+26688267 \\
& 6*a^2*\sqrt{a+4}+342778384*a^2-1103588*a*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a \\
& ^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+1103588*a* \\
& \sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+ \\
& 2548*a+2208)*\sqrt{a+4}-4965684*a*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672 \\
& *a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4965684*a*\sqrt{\sqrt{a+4}*(9 \\
& *a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)+2079 \\
& 74132*a*\sqrt{a+4}+265897256*a-551332*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+ \\
& 1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+551332*\sqrt{\sqrt{ \\
& (a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a \\
& +2208)*\sqrt{a+4}-2205328*\sqrt{\sqrt{a+4}*(9*a^4+130*a^3+701*a^2+1672*a+1488)} \\
& +15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2205328*\sqrt{\sqrt{a+4}*(9*a^4+130*a \\
& ^3+701*a^2+1672*a+1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)+72775824*\sqrt{ \\
& (a+4)}+92623776))+\sqrt{1/16}*\sqrt{1/256*(-256*\sqrt{a+4}*(9*a^3+103*a^2+392*a+ \\
& 496)}+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*\ln(\text{abs}(-243* \\
& a^{10}*\sqrt{a+4}+324*a^{10}-8640*a^9*\sqrt{a+4}+11466*a^9+81*a^8*\sqrt{\sqrt{a+4}*(\\
& (-9*a^4-130*a^3-701*a^2-1672*a-1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x \\
& -81*a^8*\sqrt{\sqrt{a+4}*(-9*a^4-130*a^3-701*a^2-1672*a-1488)}+15*a^4+210*a^3+ \\
& 1099*a^2+2548*a+2208)-138027*a^8*\sqrt{a+4}+182314*a^8-81*a^7*\sqrt{\sqrt{a+4} \\
& *(-9*a^4-130*a^3-701*a^2-1672*a-1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)* \\
& \sqrt{a+4}*x+81*a^7*\sqrt{\sqrt{a+4}*(-9*a^4-130*a^3-701*a^2-1672*a-1488)}+15*a \\
& ^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}+2340*a^7*\sqrt{\sqrt{a+4}*(-9*a^4- \\
& 130*a^3-701*a^2-1672*a-1488)}+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2340*a^
\end{aligned}$$

$$\begin{aligned}
& 7\sqrt{\sqrt{a+4}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208}-1304648a^7\sqrt{\sqrt{a+4}}+1715172a^7-2016a^6\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)\sqrt{\sqrt{a+4}} \\
& +2016a^6\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)\sqrt{\sqrt{a+4}}+29518a^6\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
& \times-29518a^6\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-8079749a^6\sqrt{\sqrt{a+4}}+10572392a^6-21454a^5\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}\times+21454a^5\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)\sqrt{\sqrt{a+4}}+212356a^5\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
& \times-212356a^5\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-34255200a^5\sqrt{\sqrt{a+4}}+44613658a^5-126540a^4\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}\times+126540a^4\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)\sqrt{\sqrt{a+4}}+952845a^4\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
& \times-952845a^4\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-100679657a^4\sqrt{\sqrt{a+4}}+130513730a^4-446685a^3\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}\times+446685a^3\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)\sqrt{\sqrt{a+4}}+2730184a^3\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
& \times-2730184a^3\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-202540404a^3\sqrt{\sqrt{a+4}}+261341928a^3-943444a^2\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}\times+943444a^2\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)\sqrt{\sqrt{a+4}}+4877364a^2\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
& \times-4877364a^2\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-266882676a^2\sqrt{\sqrt{a+4}}+342778384a^2-1103588a\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}\times+1103588a\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}+4965684a\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
& \times-4965684a\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-207974132a\sqrt{\sqrt{a+4}}+265897256a-551332\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}\times+551332\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}+2205328\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
& \times-2205328\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-72775824\sqrt{\sqrt{a+4}}+92623776) \\
&)-\sqrt{1/16}\sqrt{1/256(-256\sqrt{\sqrt{a+4}}(9a^3+103a^2+392a+496)+3840a^3+42240a^2+154624a+188416)/(a^3+11a^2+40a+48))} \\
& \times\ln(\text{abs}(-243a^{10}\sqrt{\sqrt{a+4}}+324a^{10}-8640a^9\sqrt{\sqrt{a+4}}+11466a^9-81a^8\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\times+81a^8\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-138027a^8\sqrt{\sqrt{a+4}}+182314a^8+81a^7\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}\times-81a^7\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)\sqrt{\sqrt{a+4}}-2340a^7\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
& \times+2340a^7\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)-1304648a^7\sqrt{\sqrt{a+4}}+1715172a^7+2016a^6\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208) \\
&)\sqrt{\sqrt{a+4}}\times-2016a^6\sqrt{\sqrt{a+4}}(-9a^4-130a^3-701a^2-1672a-1488)+15a^4+210a^3+1099a^2+2548a+2208)\sqrt{\sqrt{a+4}}-29518a^6\sqrt{\sqrt{a+4}}(-9
\end{aligned}$$

```
*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+29
518*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+
1099*a^2+2548*a+2208)-8079749*a^6*sqrt(a+4)+10572392*a^6+21454*a^5*sqrt(sqr
t(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+
2208)*sqrt(a+4)*x-21454*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1
488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-212356*a^5*sqrt(sqrt(a+
4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208
)*x+212356*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+2
10*a^3+1099*a^2+2548*a+2208)-34255200*a^5*sqrt(a+4)+44613658*a^5+126540*a^4
*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*sqrt(a+4)*x-126540*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^
2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-952845*a^4*sq
rt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2
548*a+2208)*x+952845*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488
)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-100679657*a^4*sqrt(a+4)+130513730*a^
4+446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210
*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-13
0*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2
730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^
3+1099*a^2+2548*a+2208)*x+2730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^
2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-202540404*a^3*sqrt(a+4
)+261341928*a^3+943444*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-14
88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-943444*a^2*sqrt(sqrt(a
+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+220
8)*sqrt(a+4)-4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488
)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4
-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-26688267
6*a^2*sqrt(a+4)+342778384*a^2+1103588*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^
a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-1103588*a
*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*sqrt(a+4)-4965684*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1
672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4965684*a*sqrt(sqrt(a+4)
*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-
207974132*a*sqrt(a+4)+265897256*a+551332*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701
*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-551332*s
qrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+
2548*a+2208)*sqrt(a+4)-2205328*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*
a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2205328*sqrt(sqrt(a+4)*(-9*a
^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-727758
24*sqrt(a+4)+92623776)))/(4*a^2+28*a+48)
```

maple [C] time = 0.01, size = 158, normalized size = 0.93

$$\frac{\left(-\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)^2 + 2\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right) - 3a - 12\right) \ln\left(-\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)\right)}{16(a+3)(a+4)\left(\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)^3 - 3\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)^2 + 4\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right) - 3a - 12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] (-1/4/(a^2+7*a+12)*x^3+3/4/(a^2+7*a+12)*x^2-1/4*(8+a)/(a^2+7*a+12)*x+1/4*(6+a)/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(3+a)/(4+a)*sum((-_R^2+2*_R-3*a-12)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3 + (a + 8)x - 3x^2 - a - 6}{4\left(\left(a^2 + 7a + 12\right)x^4 - 4\left(a^2 + 7a + 12\right)x^3 - a^3 + 8\left(a^2 + 7a + 12\right)x^2 - 7a^2 - 8\left(a^2 + 7a + 12\right)x - 12a\right)} - \frac{\int \frac{x^2}{x^4-4}}{4\left(a^2 + 7a + 12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out]
$$-1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a) - 1/4*integrate((x^2 + 3*a - 2*x + 12)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)$$

mupad [B] time = 5.35, size = 4591, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out]
$$\begin{aligned} & \operatorname{atan}\left(-\left(\left(15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776\right)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592))\right)^{(1/2)}\right. \\ & \left. * \left(\left(\left(15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184\right)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))\right) - \left(x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456)\right)\right) / \right. \\ & \left. \left(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)\right)\right) * \left(\left(15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776\right) / \right. \\ & \left. \left(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)\right)\right)^{(1/2)} - \left(733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672\right) / \left. \left(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)\right)\right) * \left(\left(15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776\right) / \right. \\ & \left. \left(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)\right)\right)^{(1/2)} + \left(5568*a + 1552*a^2 + 144*a^3 + 6656\right) / \left. \left(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)\right) - \left(x*(61*a + 9*a^2 + 104)\right) / \left(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)\right)\right) * i + \left(\left(15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776\right) / \right. \\ & \left. \left(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)\right)\right)^{(1/2)} * \left(\left(\left(15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184\right) / \right. \right. \\ & \left. \left. \left(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)\right) - \left(x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456)\right) / \left(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)\right)\right)\right) * \left(\left(15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776\right) / \right. \\ & \left. \left(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)\right)\right)^{(1/2)} + \left(733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672\right) / \left. \left(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)\right)\right) * \left(\left(15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776\right) / \right. \\ & \left. \left(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)\right)\right)^{(1/2)} + \left(5568*a + 1552*a^2 + 144*a^3 + 6656\right) / \left. \left(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)\right) - \left(x*(61*a + 9*a^2 + 104)\right) / \left(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)\right)\right) * i) / \left. \left(9*a + 32\right) / \left(32*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)\right) + \left(\left(15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776\right) / \right. \right. \\ & \left. \left. \left(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)\right)\right)^{(1/2)} * \left(\left(\left(15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184\right) / \right. \right. \\ & \left. \left. \left(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)\right) - \left(x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456)\right) / \left(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)\right)\right)\right) * \left(\left(15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776\right) / \right. \\ & \left. \left(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)\right)\right)^{(1/2)} - \left(733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + \right. \end{aligned}$$

$$\begin{aligned}
& 768a^5 + 540672)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)))*((15552a - 9a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + \\
& (5568a + 1552a^2 + 144a^3 + 6656)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(61a + 9a^2 + 104))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144)) - ((15552a - 9a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)}*(((15728640a + 10878976a^2 + 3997696a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 9437184)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(208896a + 117760a^2 + 33024a^3 + 4608a^4 + 256a^5 + 147456))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144))))*((15552a - 9a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (733184a + 396288a^2 + 106752a^3 + 14336a^4 + 768a^5 + 540672)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)))*((15552a - 9a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (5568a + 1552a^2 + 144a^3 + 6656)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(61a + 9a^2 + 104))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144)))*((15552a - 9a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)}*2i + \operatorname{atan}(-(((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)}*(((15728640a + 10878976a^2 + 3997696a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 9437184)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(208896a + 117760a^2 + 33024a^3 + 4608a^4 + 256a^5 + 147456))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144))))*((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} - (733184a + 396288a^2 + 106752a^3 + 14336a^4 + 768a^5 + 540672)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)))*((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (5568a + 1552a^2 + 144a^3 + 6656)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(61a + 9a^2 + 104))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144)))*1i + (((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)}*(((15728640a + 10878976a^2 + 3997696a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 9437184)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(208896a + 117760a^2 + 33024a^3 + 4608a^4 + 256a^5 + 147456))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144))))*((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (733184a + 396288a^2 + 106752a^3 + 14336a^4 + 768a^5 + 540672)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)))*((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (5568a + 1552a^2 + 144a^3 + 6656)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(61a + 9a^2 + 104))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144)))*1i)/((9a + 32)/(32*(816a + 460a^2 +
\end{aligned}$$

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129*a^3 + 18*a^4 + a^5 + 576)) + ((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*(((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)) - ((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*(((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*2i + (x^3/(4*(7*a + a^2 + 12)) - (a + 6)/(4*(a + 3)*(a + 4)) - (3*x^2)/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(4*(a + 3)*(a + 4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)

```

sympy [B] time = 6.32, size = 294, normalized size = 1.74

$$\frac{a - x^3 + 3x^2 + x(-a - 8) + 6}{-4a^3 - 28a^2 - 48a + x^4(4a^2 + 28a + 48) + x^3(-16a^2 - 112a - 192) + x^2(32a^2 + 224a + 384) + x(-32a^2 - 224a - 384)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

```

[Out] (a - x**3 + 3*x**2 + x*(-a - 8) + 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-7680*a**5 - 145920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 6029312) - 81*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7 - 401408*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 84549632*_t**3*a**3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 99090432*_t**3 + 432*_t*a**4 + 7488*_t*a**3 + 47024*_t*a**2 + 128096*_t*a + 128512*_t - 81*a**2 - 567*a - 992)/(81*a**2 + 567*a + 992))))

```

$$3.122 \quad \int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3} dx$$

Optimal. Leaf size=252

$$\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2} \frac{3(7a^2+(4\sqrt{a+4}+47)a+14\sqrt{a+4}+80)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

[Out] 1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+1/32*((6+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)^(1/2)+a*(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/64*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2))/(3+a)^2/(4+a)^2/(1+(4+a)^(1/2))^(1/2))

Rubi [A] time = 0.53, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1106, 1092, 1178, 1166, 204}

$$\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2} \frac{3(7a^2+(4\sqrt{a+4}+47)a+14\sqrt{a+4}+80)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3), x]

[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2*(-1 + x))/(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14*sqrt[4 + a] + a*(47 + 4*sqrt[4 + a]))*ArcTan[(-1 + x)/sqrt[1 - sqrt[4 + a]]])/(64*(3 + a)^2*(4 + a)^(5/2)*sqrt[1 - sqrt[4 + a]]) - (3*(14 + 4*a - (80 + 47*a + 7*a^2)/sqrt[4 + a])*ArcTan[(-1 + x)/sqrt[1 + sqrt[4 + a]]])/(64*(3 + a)^2*(4 + a)^2*sqrt[1 + sqrt[4 + a]])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1106

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
 > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
 Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1178

Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symb
 ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]

Rubi steps

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^3} dx = \text{Subst} \left(\int \frac{1}{(3 + a - 2x^2 - x^4)^3} dx, x, -1 + x \right)$$

$$= \frac{(5 + a + (-1 + x)^2)(-1 + x)}{8(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)^2} - \frac{\text{Subst} \left(\int \frac{4+2(3+a)-4(4}{(3+a-2} \right)}{16(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)^2} dx, x, -1 + x)}{16(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)^2}$$

$$= -\frac{((6 + a)(25 + 7a) + 6(7 + 2a)(1 - x)^2)(1 - x)}{32(12 + 7a + a^2)^2(3 + a - 2(1 - x)^2 - (1 - x)^4)} + \frac{(5 + a + (-1 + x)^2)(-1 + x)}{8(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)^2}$$

$$= -\frac{((6 + a)(25 + 7a) + 6(7 + 2a)(1 - x)^2)(1 - x)}{32(12 + 7a + a^2)^2(3 + a - 2(1 - x)^2 - (1 - x)^4)} + \frac{(5 + a + (-1 + x)^2)(-1 + x)}{8(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)^2}$$

$$= -\frac{((6 + a)(25 + 7a) + 6(7 + 2a)(1 - x)^2)(1 - x)}{32(12 + 7a + a^2)^2(3 + a - 2(1 - x)^2 - (1 - x)^4)} + \frac{(5 + a + (-1 + x)^2)(-1 + x)}{8(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)^2}$$

Mathematica [C] time = 0.15, size = 254, normalized size = 1.01

$$\frac{1}{128} \left(\frac{3\text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{4\#1^2 a \log(x-\#1) + 14\#1^2 \log(x-\#1) + 7a^2 \log(x-\#1) + 55a \log(x-\#1) - 8\#1 a \log(x-\#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \right]}{(a^2 + 7a + 12)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3), x]

[Out] ((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x + 12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (108*Log[x - #1] + 55*a*Log[x - #1] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &])/(12 + 7*a + a^2)^2)/128

fricas [B] time = 0.52, size = 3971, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out]
$$-1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a + 1116)*x^5 - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1968)*x^3 + 44*a^3 - 8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt(($$

$$\begin{aligned}
& (2401a^4 + 33124a^3 + 171966a^2 + 398164a + 346921)/(a^{15} + 50a^{14} + 1 \\
& 165a^{13} + 16780a^{12} + 167090a^{11} + 1218460a^{10} + 6722130a^9 + 28570320 \\
& *a^8 + 94320045a^7 + 241870050a^6 + 477857313a^5 + 714317940a^4 + 78207 \\
& 1200a^3 + 592064640a^2 + 277136640a + 60466176)) + 6613472a + 3543424)* \\
& \text{sqrt}((105a^4 + 1470a^3 + 7749a^2 + (a^{10} + 35a^9 + 550a^8 + 5110a^7 + \\
& 31085a^6 + 129367a^5 + 373020a^4 + 735840a^3 + 950400a^2 + 725760a + \\
& 248832))*\text{sqrt}((2401a^4 + 33124a^3 + 171966a^2 + 398164a + 346921)/(a^{15} \\
& + 50a^{14} + 1165a^{13} + 16780a^{12} + 167090a^{11} + 1218460a^{10} + 6722130* \\
& a^9 + 28570320a^8 + 94320045a^7 + 241870050a^6 + 477857313a^5 + 7143179 \\
& 40a^4 + 782071200a^3 + 592064640a^2 + 277136640a + 60466176)) + 18228a \\
& + 16144)/(a^{10} + 35a^9 + 550a^8 + 5110a^7 + 31085a^6 + 129367a^5 + 37 \\
& 3020a^4 + 735840a^3 + 950400a^2 + 725760a + 248832)) - 11228868a - 992 \\
& 3472) - 3*((a^4 + 14a^3 + 73a^2 + 168a + 144)*x^8 - 8*(a^4 + 14a^3 + 73 \\
& *a^2 + 168a + 144)*x^7 + 32*(a^4 + 14a^3 + 73a^2 + 168a + 144)*x^6 + a^6 \\
& - 80*(a^4 + 14a^3 + 73a^2 + 168a + 144)*x^5 + 14a^5 - 2*(a^5 - 50a^4 \\
& - 823a^3 - 4504a^2 - 10608a - 9216)*x^4 + 73a^4 + 8*(a^5 - 2a^4 - 151 \\
& *a^3 - 1000a^2 - 2544a - 2304)*x^3 + 168a^3 - 16*(a^5 + 10a^4 + 17a^3 \\
& - 124a^2 - 528a - 576)*x^2 + 144a^2 + 16*(a^5 + 14a^4 + 73a^3 + 168a^2 \\
& + 144a)*x)*\text{sqrt}((105a^4 + 1470a^3 + 7749a^2 - (a^{10} + 35a^9 + 550a^8 \\
& + 5110a^7 + 31085a^6 + 129367a^5 + 373020a^4 + 735840a^3 + 950400a^2 \\
& + 725760a + 248832))*\text{sqrt}((2401a^4 + 33124a^3 + 171966a^2 + 398164a + \\
& 346921)/(a^{15} + 50a^{14} + 1165a^{13} + 16780a^{12} + 167090a^{11} + 1218460a \\
& ^{10} + 6722130a^9 + 28570320a^8 + 94320045a^7 + 241870050a^6 + 477857313 \\
& *a^5 + 714317940a^4 + 782071200a^3 + 592064640a^2 + 277136640a + 604661 \\
& 76)) + 18228a + 16144)/(a^{10} + 35a^9 + 550a^8 + 5110a^7 + 31085a^6 + 1 \\
& 29367a^5 + 373020a^4 + 735840a^3 + 950400a^2 + 725760a + 248832))*\text{log}(\\
& -64827a^4 - 907578a^3 - 4780647a^2 + 27*(2401a^4 + 33614a^3 + 177061a \\
& ^2 + 415884a + 367536)*x + 27*(343a^7 + 8981a^6 + 100811a^5 + 628887a^4 \\
& + 2354874a^3 + 5293208a^2 + (11a^{12} + 462a^{11} + 8881a^{10} + 103320a^9 \\
& + 810205a^8 + 4511542a^7 + 18292039a^6 + 54410692a^5 + 117844800a^4 \\
& + 181238400a^3 + 187875072a^2 + 117863424a + 33841152))*\text{sqrt}((2401a^4 + \\
& 33124a^3 + 171966a^2 + 398164a + 346921)/(a^{15} + 50a^{14} + 1165a^{13} + 1 \\
& 6780a^{12} + 167090a^{11} + 1218460a^{10} + 6722130a^9 + 28570320a^8 + 94320 \\
& 045a^7 + 241870050a^6 + 477857313a^5 + 714317940a^4 + 782071200a^3 + 5 \\
& 92064640a^2 + 277136640a + 60466176)) + 6613472a + 3543424)*\text{sqrt}((105a^4 \\
& + 1470a^3 + 7749a^2 - (a^{10} + 35a^9 + 550a^8 + 5110a^7 + 31085a^6 + \\
& 129367a^5 + 373020a^4 + 735840a^3 + 950400a^2 + 725760a + 248832))*\text{sq} \\
& \text{rt}((2401a^4 + 33124a^3 + 171966a^2 + 398164a + 346921)/(a^{15} + 50a^{14} + \\
& 1165a^{13} + 16780a^{12} + 167090a^{11} + 1218460a^{10} + 6722130a^9 + 285703 \\
& 20a^8 + 94320045a^7 + 241870050a^6 + 477857313a^5 + 714317940a^4 + 782 \\
& 071200a^3 + 592064640a^2 + 277136640a + 60466176)) + 18228a + 16144)/(a \\
& ^{10} + 35a^9 + 550a^8 + 5110a^7 + 31085a^6 + 129367a^5 + 373020a^4 + 7 \\
& 35840a^3 + 950400a^2 + 725760a + 248832)) - 11228868a - 9923472) + 3*((\\
& a^4 + 14a^3 + 73a^2 + 168a + 144)*x^8 - 8*(a^4 + 14a^3 + 73a^2 + 168a \\
& + 144)*x^7 + 32*(a^4 + 14a^3 + 73a^2 + 168a + 144)*x^6 + a^6 - 80*(a^4 \\
& + 14a^3 + 73a^2 + 168a + 144)*x^5 + 14a^5 - 2*(a^5 - 50a^4 - 823a^3 - \\
& 4504a^2 - 10608a - 9216)*x^4 + 73a^4 + 8*(a^5 - 2a^4 - 151a^3 - 1000* \\
& a^2 - 2544a - 2304)*x^3 + 168a^3 - 16*(a^5 + 10a^4 + 17a^3 - 124a^2 - \\
& 528a - 576)*x^2 + 144a^2 + 16*(a^5 + 14a^4 + 73a^3 + 168a^2 + 144a)*x \\
&)*\text{sqrt}((105a^4 + 1470a^3 + 7749a^2 - (a^{10} + 35a^9 + 550a^8 + 5110a^7 \\
& + 31085a^6 + 129367a^5 + 373020a^4 + 735840a^3 + 950400a^2 + 725760a \\
& + 248832))*\text{sqrt}((2401a^4 + 33124a^3 + 171966a^2 + 398164a + 346921)/(a^ \\
& 15 + 50a^{14} + 1165a^{13} + 16780a^{12} + 167090a^{11} + 1218460a^{10} + 672213 \\
& 0a^9 + 28570320a^8 + 94320045a^7 + 241870050a^6 + 477857313a^5 + 71431 \\
& 7940a^4 + 782071200a^3 + 592064640a^2 + 277136640a + 60466176)) + 18228 \\
& *a + 16144)/(a^{10} + 35a^9 + 550a^8 + 5110a^7 + 31085a^6 + 129367a^5 + \\
& 373020a^4 + 735840a^3 + 950400a^2 + 725760a + 248832))*\text{log}(-64827a^4 - \\
& 907578a^3 - 4780647a^2 + 27*(2401a^4 + 33614a^3 + 177061a^2 + 415884* \\
& a + 367536)*x - 27*(343a^7 + 8981a^6 + 100811a^5 + 628887a^4 + 2354874*
\end{aligned}$$

$$a^3 + 5293208a^2 + (11a^{12} + 462a^{11} + 8881a^{10} + 103320a^9 + 810205a^8 + 4511542a^7 + 18292039a^6 + 54410692a^5 + 117844800a^4 + 181238400a^3 + 187875072a^2 + 117863424a + 33841152)\sqrt{(2401a^4 + 33124a^3 + 171966a^2 + 398164a + 346921)/(a^{15} + 50a^{14} + 1165a^{13} + 16780a^{12} + 167090a^{11} + 1218460a^{10} + 6722130a^9 + 28570320a^8 + 94320045a^7 + 241870050a^6 + 477857313a^5 + 714317940a^4 + 782071200a^3 + 592064640a^2 + 277136640a + 60466176)} + 6613472a + 3543424)\sqrt{(105a^4 + 1470a^3 + 7749a^2 - (a^{10} + 35a^9 + 550a^8 + 5110a^7 + 31085a^6 + 129367a^5 + 373020a^4 + 735840a^3 + 950400a^2 + 725760a + 248832)\sqrt{(2401a^4 + 33124a^3 + 171966a^2 + 398164a + 346921)/(a^{15} + 50a^{14} + 1165a^{13} + 16780a^{12} + 167090a^{11} + 1218460a^{10} + 6722130a^9 + 28570320a^8 + 94320045a^7 + 241870050a^6 + 477857313a^5 + 714317940a^4 + 782071200a^3 + 592064640a^2 + 277136640a + 60466176)} + 18228a + 16144)/(a^{10} + 35a^9 + 550a^8 + 5110a^7 + 31085a^6 + 129367a^5 + 373020a^4 + 735840a^3 + 950400a^2 + 725760a + 248832)) - 11228868a - 9923472) + 524a^2 - 4(11a^3 + 107a^2 - 84a - 1152)x + 1632a + 1152)/((a^4 + 14a^3 + 73a^2 + 168a + 144)x^8 - 8(a^4 + 14a^3 + 73a^2 + 168a + 144)x^7 + 32(a^4 + 14a^3 + 73a^2 + 168a + 144)x^6 + a^6 - 80(a^4 + 14a^3 + 73a^2 + 168a + 144)x^5 + 14a^5 - 2(a^5 - 50a^4 - 823a^3 - 4504a^2 - 10608a - 9216)x^4 + 73a^4 + 8(a^5 - 2a^4 - 151a^3 - 1000a^2 - 2544a - 2304)x^3 + 168a^3 - 16(a^5 + 10a^4 + 17a^3 - 124a^2 - 528a - 576)x^2 + 144a^2 + 16(a^5 + 14a^4 + 73a^3 + 168a^2 + 144a)x)$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[86]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[12]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[48]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[34]-(-12*x^7*a-42*x^7+84*x^6*a+294*x^6-7*x^5*a^2-343*x^5*a-1116*x^5+35*x^4*a^2+875*x^4*a+2640*x^4-68*x^3*a^2-1358*x^3*a-3936*x^3+64*x^2*a^2+1246*x^2*a+3600*x^2+11*x*a^3+107*x*a^2-84*x*a-1152*x-11*a^3-131*a^2-408*a-288)/(-32*a^4-448*a^3-2336*a^2-5376*a-4608)/(x^4-4*x^3+8*x^2-8*x-a)^2-(sqrt(9/16)*sqrt(1/256*(256*sqrt(a+4)*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*ln(abs(-16807*a^15*sqrt(a+4)+26411*a^15-908950*a^14*sqrt(a+4)+1420804*a^14-22929088*a^13*sqrt(a+4)+35650176*a^13+2401*a^12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2401*a^12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-357887692*a^12*sqrt(a+4)+553458148*a^12-2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+105154*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3865394166*a^11*sqrt(a+4)+5945365998*a^11-95550*a^10*sqrt(sqrt(a

$$\begin{aligned}
& +4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+95550*a^{10}*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+2109279*a^{10}*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2109279*a^{10}*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-30600511272*a^{10}*\sqrt{a+4}+46810709868*a^{10}-1727079*a^9*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+1727079*a^9*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+25624212*a^9*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-25624212*a^9*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-183431725500*a^9*\sqrt{a+4}+279067335420*a^9-18715896*a^8*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+18715896*a^8*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+209972223*a^8*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-209972223*a^8*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-847810669320*a^8*\sqrt{a+4}+1282741275000*a^8-135108639*a^7*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+135108639*a^7*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+1222644882*a^7*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-1222644882*a^7*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3046208716923*a^7*\sqrt{a+4}+4583471076759*a^7-682210326*a^6*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+682210326*a^6*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+5187446733*a^6*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-5187446733*a^6*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-8508527261290*a^6*\sqrt{a+4}+12731345334296*a^6-2458605429*a^5*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+2458605429*a^5*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+16158435972*a^5*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-16158435972*a^5*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-18324012543204*a^5*\sqrt{a+4}+27265747047380*a^5-6324014256*a^4*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+6324014256*a^4*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5
\end{aligned}$$

$$\begin{aligned}
& -9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + \\
& 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} + 36673732452a^4 \sqrt{\sqrt{a+4}} \\
& * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + \\
& 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) * x - 36673732452a^4 \\
& \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + \\
& 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) - 298 \\
& 80339194272a^4 \sqrt{a+4} + 44213263379848a^4 - 11377675428a^3 \sqrt{\sqrt{a+4}} \\
& * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2 \\
& 205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} * x + 1137767 \\
& 5428a^3 \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187 \\
& 408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 1937 \\
& 28) \sqrt{a+4} + 59146408708a^3 \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 474 \\
& 19a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236 \\
& 728a^2 + 331744a + 193728) * x - 59146408708a^3 \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 \\
& - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + \\
& 90111a^3 + 236728a^2 + 331744a + 193728) - 35712575419864a^3 \sqrt{a+4} + 52547642 \\
& 661032a^3 - 13635706996a^2 \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - \\
& 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728 \\
& a^2 + 331744a + 193728) \sqrt{a+4} * x + 13635706996a^2 \sqrt{\sqrt{a+4}} * (-49a^6 - 1 \\
& 073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 192 \\
& 99a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} + 64340036640a^2 \sqrt{\sqrt{a+4}} \\
& * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) \\
& + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) * x - 6434003 \\
& 6640a^2 \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187 \\
& 408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 1937 \\
& 28) - 29533943648028a^2 \sqrt{a+4} + 43213040637212a^2 - 9797208656a \sqrt{\sqrt{a+4}} \\
& * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + \\
& 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} * x + 979 \\
& 7208656a \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 18 \\
& 7408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193 \\
& 728) \sqrt{a+4} + 42385864836a \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 4741 \\
& 9a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 2367 \\
& 28a^2 + 331744a + 193728) * x - 42385864836a \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 97 \\
& 75a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 901 \\
& 11a^3 + 236728a^2 + 331744a + 193728) - 15111479733208a \sqrt{a+4} + 2198667320430 \\
& 4a - 3197030212 \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - \\
& 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a \\
& + 193728) \sqrt{a+4} * x + 3197030212 \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - \\
& 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + \\
& 236728a^2 + 331744a + 193728) \sqrt{a+4} + 12788120848 \sqrt{\sqrt{a+4}} * (-49a^6 - 1 \\
& 073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 192 \\
& 99a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) * x - 12788120848 \sqrt{\sqrt{a+4}} * (\\
& -49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 220 \\
& 5a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) - 3606250079136 \sqrt{a+4} \\
& + 5217553305984) - \sqrt{9/16} \sqrt{1/256 * (256 \sqrt{a+4}) * (49a^5 + 926a^4 + 699 \\
& 7a^3 + 26428a^2 + 49904a + 37696) - 26880a^5 - 483840a^4 - 3489024a^3 - 12601344a^2 - \\
& 22798336a - 16531456) / (-a^3 - 11a^2 - 40a - 48)} * \ln(\text{abs}(-16807a^{15} \sqrt{a+4} + \\
& 26411a^{15} - 908950a^{14} \sqrt{a+4} + 1420804a^{14} - 22929088a^{13} \sqrt{a+4} + 35650 \\
& 176a^{13} - 2401a^{12} \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 1291 \\
& 88a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331 \\
& 744a + 193728) * x + 2401a^{12} \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - \\
& 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + \\
& 331744a + 193728) - 357887692a^{12} \sqrt{a+4} + 553458148a^{12} + 2401a^{11} \sqrt{\sqrt{a+4}} \\
& * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) \\
& + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} \\
& * x - 2401a^{11} \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - \\
& 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + \\
& 193728) \sqrt{a+4} - 105154a^{11} \sqrt{\sqrt{a+4}} * (-49a^6 - 1073a^5 - 9775a^4 - 474 \\
& 19a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236
\end{aligned}$$

$$\begin{aligned}
&728a^2+331744a+193728)*x+105154a^{11}\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)-3865394166a^{11}\sqrt{\sqrt{a+4}}+5945365998a^{11} \\
&+95550a^{10}\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}*x-95550a^{10}\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}} \\
&-2109279a^{10}\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x+2109279a^{10}\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)-30600511272a^{10}\sqrt{\sqrt{a+4}} \\
&+46810709868a^{10}+1727079a^9\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}*x-1727079a^9\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}} \\
&-25624212a^9\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x+25624212a^9\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728) \\
&-183431725500a^9\sqrt{\sqrt{a+4}}+279067335420a^9+18715896a^8\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}*x-18715896a^8\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}} \\
&-209972223a^8\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x+209972223a^8\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)-847810669320a^8\sqrt{\sqrt{a+4}}+1282741275000a^8+135108639a^7\sqrt{\sqrt{a+4}} \\
&*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}*x-135108639a^7\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}} \\
&-1222644882a^7\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x+1222644882a^7\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)-3046208716923a^7\sqrt{\sqrt{a+4}} \\
&+4583471076759a^7+682210326a^6\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}*x-682210326a^6\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}} \\
&-5187446733a^6\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x+5187446733a^6\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)-8508527261290a^6\sqrt{\sqrt{a+4}}+12731345334296a^6+2458605429a^5\sqrt{\sqrt{a+4}} \\
&*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}*x-2458605429a^5\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}} \\
&-16158435972a^5\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x+16158435972a^5\sqrt{\sqrt{a+4}}*(-49a^6-1073a^5-9775a^4-47419a^3-129188a^2-187408a-113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)-18324012543204a^5*
\end{aligned}$$

$$\begin{aligned}
&7419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+2 \\
&36728a^2+331744a+193728)*\sqrt{a+4}+105154a^{11}\sqrt{\sqrt{a+4}}*(49a^6+107 \\
&3a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299 \\
&a^4+90111a^3+236728a^2+331744a+193728)*x-105154a^{11}\sqrt{\sqrt{a+4}}*(49 \\
&a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a \\
&^5+19299a^4+90111a^3+236728a^2+331744a+193728)+3865394166a^{11}\sqrt{a+4} \\
&)+5945365998a^{11}+95550a^{10}\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419 \\
&a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+23672 \\
&8a^2+331744a+193728)*\sqrt{a+4}*x-95550a^{10}\sqrt{\sqrt{a+4}}*(49a^6+1073a \\
&^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^ \\
&4+90111a^3+236728a^2+331744a+193728)*\sqrt{a+4}+2109279a^{10}\sqrt{\sqrt{a+ \\
&4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+ \\
&2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x-2109279a^{10}\sqrt{ \\
&\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088) \\
&)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)+306005112 \\
&72a^{10}\sqrt{a+4}+46810709868a^{10}+1727079a^9\sqrt{\sqrt{a+4}}*(49a^6+1073a \\
&a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a \\
&^4+90111a^3+236728a^2+331744a+193728)*\sqrt{a+4}*x-1727079a^9\sqrt{\sqrt{a+ \\
&4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^ \\
&6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{a+4}+256242 \\
&12a^9\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408 \\
&a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728) \\
&*x-25624212a^9\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a \\
&^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744 \\
&a+193728)+183431725500a^9\sqrt{a+4}+279067335420a^9+18715896a^8\sqrt{\sqrt{a \\
&4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a \\
&a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{a+4}*x-18 \\
&715896a^8\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+18 \\
&7408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193 \\
&728)*\sqrt{a+4}+209972223a^8\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419 \\
&a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+23672 \\
&8a^2+331744a+193728)*x-209972223a^8\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775 \\
&a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111 \\
&a^3+236728a^2+331744a+193728)+847810669320a^8\sqrt{a+4}+1282741275000a \\
&^8+135108639a^7\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a \\
&a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744 \\
&a+193728)*\sqrt{a+4}*x-135108639a^7\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a \\
&^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a \\
&^3+236728a^2+331744a+193728)*\sqrt{a+4}+1222644882a^7\sqrt{\sqrt{a+4}}*(49a \\
&a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^ \\
&5+19299a^4+90111a^3+236728a^2+331744a+193728)*x-1222644882a^7\sqrt{\sqrt{a \\
&4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a \\
&a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)+3046208716923a \\
&a^7\sqrt{a+4}+4583471076759a^7+682210326a^6\sqrt{\sqrt{a+4}}*(49a^6+1073a \\
&^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^ \\
&4+90111a^3+236728a^2+331744a+193728)*\sqrt{a+4}*x-682210326a^6\sqrt{\sqrt{a \\
&4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a \\
&^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{a+4}+51874 \\
&46733a^6\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187 \\
&408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+1937 \\
&28)*x-5187446733a^6\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129 \\
&188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+33 \\
&1744a+193728)+8508527261290a^6\sqrt{a+4}+12731345334296a^6+2458605429a^ \\
&5\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+11 \\
&3088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{ \\
&a+4}*x-2458605429a^5\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+1 \\
&29188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+ \\
&331744a+193728)*\sqrt{a+4}+16158435972a^5\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+ \\
&9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+9
\end{aligned}$$

$$\begin{aligned}
& 0111a^3+236728a^2+331744a+193728)x-16158435972a^5\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5 \\
& +19299a^4+90111a^3+236728a^2+331744a+193728)+18324012543204a^5\sqrt{\sqrt{a+4}}+27265747047380a^5+6324014256a^4\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4 \\
& +47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}+36673732452a^4\sqrt{\sqrt{a+4}} \\
& *(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x-3 \\
& 6673732452a^4\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a \\
& +193728)+29880339194272a^4\sqrt{\sqrt{a+4}}+44213263379848a^4+11377675428a^3\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088) \\
&)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}})x-11377675428a^3\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+1291 \\
& 88a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}+59146408708a^3\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+977 \\
& 5a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x-59146408708a^3\sqrt{\sqrt{a+4}}*(49a^6+ \\
& 1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)+35712575419864a^3\sqrt{\sqrt{a+4}}+ \\
& 52547642661032a^3+13635706996a^2\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3 \\
& +236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}})x-13635706996a^2\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5 \\
& +19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}+64340036640a^2\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+11 \\
& 3088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x-64 \\
& 340036640a^2\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+ \\
& 193728)+29533943648028a^2\sqrt{\sqrt{a+4}}+43213040637212a^2+9797208656a*\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+10 \\
& 5a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*\sqrt{\sqrt{a+4}})x- \\
& 9797208656a*\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+ \\
& 187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+1 \\
& 93728)*\sqrt{\sqrt{a+4}}+42385864836a*\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+474 \\
& 19a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236 \\
& 728a^2+331744a+193728)*x-42385864836a*\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+97 \\
& 75a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+901 \\
& 11a^3+236728a^2+331744a+193728)+15111479733208a*\sqrt{\sqrt{a+4}}+2198667320430 \\
& 4a+3197030212*\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a \\
& +193728)*\sqrt{\sqrt{a+4}})x-3197030212*\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47 \\
& 419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+23 \\
& 6728a^2+331744a+193728)*\sqrt{\sqrt{a+4}}+12788120848*\sqrt{\sqrt{a+4}}*(49a^6+1073 \\
& a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x-12788120848*\sqrt{\sqrt{a+4}}*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)+3606250079136*\sqrt{\sqrt{a+4}}+5217553305984))-sqrt(9/16)*sqrt(1/256*(-256*sqrt(a+4)*(49a^5+926a^4+6997a^3+26428a^2+49904a+37696)-26880a^5-483840a^4-3489024a^3-12601344a^2-22798336a-16531456)/(-a^3-11a^2-40a-48))*ln(abs(16807a^15*sqrt(a+4)+26411a^15+908950a^14*sqrt(a+4)+1420804a^14+22929088a^13*sqrt(a+4)+35650176a^13-2401a^12*sqrt(a+4)*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)*x+2401a^12*sqrt(a+4)*(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)+357887692a^12*sqrt(a+4)+553458148a^12-2401a^11*sqrt(a+4)
\end{aligned}$$

$$\begin{aligned}
& a+4) * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) * \sqrt{a+4} * x+2401 \\
& * a^{11} * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) * \\
& \sqrt{a+4}-105154*a^{11} * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3 \\
& 31744*a+193728) * x+105154*a^{11} * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2367 \\
& 28*a^2+331744*a+193728)+3865394166*a^{11} * \sqrt{a+4}+5945365998*a^{11}-95550*a^1 \\
& 0 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) * \sqrt{ \\
& (a+4) * x+95550*a^{10} * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3317 \\
& 44*a+193728) * \sqrt{a+4}-2109279*a^{10} * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^ \\
& 3+236728*a^2+331744*a+193728) * x+2109279*a^{10} * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4 \\
& +90111*a^3+236728*a^2+331744*a+193728)+30600511272*a^{10} * \sqrt{a+4}+468107098 \\
& 68*a^{10}-1727079*a^9 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331 \\
& 744*a+193728) * \sqrt{a+4} * x+1727079*a^9 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^ \\
& 3+236728*a^2+331744*a+193728) * \sqrt{a+4}-25624212*a^9 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5 \\
& +19299*a^4+90111*a^3+236728*a^2+331744*a+193728) * x+25624212*a^9 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6 \\
& +2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+183431725500*a^9 * \\
& \sqrt{a+4}+279067335420*a^9-18715896*a^8 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111 \\
& 1*a^3+236728*a^2+331744*a+193728) * \sqrt{a+4} * x+18715896*a^8 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205 \\
& * a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) * \sqrt{a+4}-209972223*a^8 * \\
& \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) * x+20 \\
& 9972223*a^8 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+19 \\
& 3728)+847810669320*a^8 * \sqrt{a+4}+1282741275000*a^8-135108639*a^7 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) * \sqrt{a+4} * x+1351 \\
& 08639*a^7 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937 \\
& 28) * \sqrt{a+4}-1222644882*a^7 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419 \\
& * a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672 \\
& 8*a^2+331744*a+193728) * x+1222644882*a^7 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9011 \\
& 1*a^3+236728*a^2+331744*a+193728)+3046208716923*a^7 * \sqrt{a+4}+4583471076759 \\
& * a^7-682210326*a^6 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3317 \\
& 44*a+193728) * \sqrt{a+4} * x+682210326*a^6 * \sqrt{\sqrt{a+4} * (49*a^6+1073*a^5+9775 \\
& * a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111 \\
& * a^3+236728*a^2+331744*a+193728) * \sqrt{a+4}-5187446733*a^6 * \sqrt{\sqrt{a+4} * (49 \\
& * a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205* \\
& a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) * x+5187446733*a^6 * \sqrt{s \\
& \sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+10 \\
& 5*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+850852726129 \\
& 0*a^6 * \sqrt{a+4}+12731345334296*a^6-2458605429*a^5 * \sqrt{\sqrt{a+4} * (49*a^6+10 \\
& 73*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+1929 \\
& 9*a^4+90111*a^3+236728*a^2+331744*a+193728) * \sqrt{a+4} * x+2458605429*a^5 * \sqrt{ \\
& (\sqrt{a+4} * (49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+
\end{aligned}$$

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105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-
16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a
^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*
a+193728)*x+16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*
a^3+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728
*a^2+331744*a+193728)+18324012543204*a^5*sqrt(a+4)+27265747047380*a^5-63240
14256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187
408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)*x+6324014256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+474
19*a^3+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*sqrt(a+4)-36673732452*a^4*sqrt(sqrt(a+4)*(49*a^6+1
073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+192
99*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+36673732452*a^4*sqrt(sqrt(a+
4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105*a^6+
2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29880339194272*a^4
*sqrt(a+4)+44213263379848*a^4-11377675428*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a
^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^
4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+11377675428*a^3*sqrt(sq
rt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105
*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-591
46408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+
187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*x+59146408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3
+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^
2+331744*a+193728)+35712575419864*a^3*sqrt(a+4)+52547642661032*a^3-13635706
996*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+18740
8*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)*sqrt(a+4)*x+13635706996*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+4741
9*a^3+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+2367
28*a^2+331744*a+193728)*sqrt(a+4)-64340036640*a^2*sqrt(sqrt(a+4)*(49*a^6+10
73*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+1929
9*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+64340036640*a^2*sqrt(sqrt(a+4
))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105*a^6+2
205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29533943648028*a^2*
sqrt(a+4)+43213040637212*a^2-9797208656*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9
775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90
111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+9797208656*a*sqrt(sqrt(a+4)
*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105*a^6+22
05*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-4238586483
6*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+
113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+
42385864836*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2
+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+
193728)+15111479733208*a*sqrt(a+4)+21986673204304*a-3197030212*sqrt(sqrt(a+
4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088))+105*a^6+
2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+319703
0212*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a
+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*s
qrt(a+4)-12788120848*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129
188*a^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+33
1744*a+193728)*x+12788120848*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088))+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)+3606250079136*sqrt(a+4)+5217553305984)))/(32*a^4+448
*a^3+2336*a^2+5376*a+4608)

```

maple [C] time = 0.02, size = 398, normalized size = 1.58

$$\frac{3 \left(2(2a+7) \operatorname{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^2 + 7a^2 + 4(-2a-7) \operatorname{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a) \right)}{128(a^3 + 10a^2 + 33a + 36)(a+4) \left(\operatorname{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^3 - 3 \operatorname{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(-x^4+4x^3-8x^2+a+8x)^3,x)$

[Out] $-(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16)/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^5-5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+679*a+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+14*a^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+73*a^2+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(a^4+14*a^3+73*a^2+168*a+144))/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(a+4)*\text{sum}((108+2*(7+2*a)*_R^2+4*(-2*a-7)*_R+7*a^2+55*a)/(_R^3-3*_R^2+4*_R-2)*\ln(-_R+x),_R=\text{RootOf}(Z^4-4*_Z^3+8*_Z^2-8*_Z-a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$6(2a +$

$32((a^4 + 14a^3 + 73a^2 + 168a + 144)x^8 - 8(a^4 + 14a^3 + 73a^2 + 168a + 144)x^7 + 32(a^4 + 14a^3 + 73a^2 +$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(-x^4+4x^3-8x^2+a+8x)^3,x, \text{algorithm}=\text{"maxima"})$

[Out] $-1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116)*x^5 - 5*(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x^3 + 11*a^3 - 2*(32*a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 + 107*a^2 - 84*a - 1152)*x + 408*a + 288)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x) - 3/32*\text{integrate}((2*(2*a + 7)*x^2 + 7*a^2 - 4*(2*a + 7)*x + 55*a + 108)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)$

mupad [B] time = 6.41, size = 8242, normalized size = 32.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)$

[Out] $\text{atan}((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))) + ((4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736))))*(9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10$

$$\begin{aligned}
& 0 + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968))^{(1/2)} \\
& \cdot ((9 \cdot (39329792a - 338 \cdot (a + 4)^{15})^{(1/2)} - 589 \cdot (a + 4)^{15})^{(1/2)} - \\
& 49a^2 \cdot (a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 26 \\
& 95744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 \\
& \cdot (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703 \\
& 040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 196 \\
& 6491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 2548039 \\
& 68))^{(1/2)} + (108343296a + 74059776a^2 + 27065088a^3 + 5576256a^4 + 61 \\
& 4016a^5 + 28224a^6 + 66207744) / (16384 \cdot (940032a + 1195776a^2 + 899328a^3 \\
& + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) \\
& - (x \cdot (73476a + 31545a^2 + 6066a^3 + 441a^4 + 64656)) / (256 \cdot \\
& (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 \\
& + a^8 + 20736)) \cdot ((9 \cdot (39329792a - 338 \cdot (a + 4)^{15})^{(1/2)} - 589 \cdot (a + 4)^{15})^{(1/2)} - \\
& 49a^2 \cdot (a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10187 \\
& 840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531 \\
& 456)) / (16384 \cdot (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a \\
& ^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 119442 \\
& 00a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} \\
& + 254803968))^{(1/2)} \cdot i - (((52357496832a + 57139003392a^2 + 363221483 \\
& 52a^3 + 14822473728a^4 + 4027170816a^5 + 728506368a^6 + 84615168a^7 + \\
& 5726208a^8 + 172032a^9 + 21290287104) / (16384 \cdot (940032a + 1195776a^2 + 89 \\
& 9328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 \\
& + a^{10} + 331776)) - ((4290672328704a + 6001143054336a^2 + 5025917042688 \\
& \cdot a^3 + 2800520003584a^4 + 1090200272896a^5 + 302556119040a^6 + 598621552 \\
& 64a^7 + 8275361792a^8 + 761266176a^9 + 41943040a^{10} + 1048576a^{11} + 13 \\
& 91569403904) / (16384 \cdot (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149 \\
& 208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x \cdot (3 \\
& 510632448a + 4020240384a^2 + 2678587392a^3 + 1144324096a^4 + 325074944a \\
& ^5 + 61407232a^6 + 7438336a^7 + 524288a^8 + 16384a^9 + 1358954496)) / (2 \\
& 56 \cdot (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 \\
& + a^8 + 20736)) \cdot ((9 \cdot (39329792a - 338 \cdot (a + 4)^{15})^{(1/2)} - 589 \cdot (a + \\
& 4)^{15})^{(1/2)} - 49a^2 \cdot (a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10 \\
& 187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16 \\
& 531456)) / (16384 \cdot (1061683200a + 2061434880a^2 + 2474311680a^3 + 205320192 \\
& 0a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 119 \\
& 44200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + \\
& a^{15} + 254803968))^{(1/2)} \cdot ((9 \cdot (39329792a - 338 \cdot (a + 4)^{15})^{(1/2)} - 58 \\
& 9 \cdot (a + 4)^{15})^{(1/2)} - 49a^2 \cdot (a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 \\
& + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 \\
& + 16531456)) / (16384 \cdot (1061683200a + 2061434880a^2 + 2474311680a^3 + 2 \\
& 053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 \\
& + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 5 \\
& 5a^{14} + a^{15} + 254803968))^{(1/2)} - (108343296a + 74059776a^2 + 27065088 \\
& \cdot a^3 + 5576256a^4 + 614016a^5 + 28224a^6 + 66207744) / (16384 \cdot (940032a + \\
& 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + \\
& 582a^8 + 36a^9 + a^{10} + 331776)) + (x \cdot (73476a + 31545a^2 + 6066a^3 + \\
& 441a^4 + 64656)) / (256 \cdot (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 \\
& + 342a^6 + 28a^7 + a^8 + 20736)) \cdot ((9 \cdot (39329792a - 338 \cdot (a + 4)^{15})^{(1/2)} - 589 \cdot (a + 4)^{15})^{(1/2)} - \\
& 49a^2 \cdot (a + 4)^{15})^{(1/2)} + 41598976a^2 \\
& + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 357 \\
& 0a^8 + 105a^9 + 16531456)) / (16384 \cdot (1061683200a + 2061434880a^2 + 247431 \\
& 1680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 \\
& + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1 \\
& 410a^{13} + 55a^{14} + a^{15} + 254803968))^{(1/2)} \cdot i) / (((52357496832a + 5713 \\
& 9003392a^2 + 36322148352a^3 + 14822473728a^4 + 4027170816a^5 + 72850636 \\
& 8a^6 + 84615168a^7 + 5726208a^8 + 172032a^9 + 21290287104) / (16384 \cdot (9400 \\
& 32a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 556 \\
& 4a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) + ((4290672328704a + 6001143054 \\
& 336a^2 + 5025917042688a^3 + 2800520003584a^4 + 1090200272896a^5 + 30255
\end{aligned}$$

$$\begin{aligned}
& 6119040a^6 + 59862155264a^7 + 8275361792a^8 + 761266176a^9 + 41943040a^{10} + 1048576a^{11} + 1391569403904) / (16384(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x(3510632448a + 4020240384a^2 + 2678587392a^3 + 1144324096a^4 + 325074944a^5 + 61407232a^6 + 7438336a^7 + 524288a^8 + 16384a^9 + 1358954496)) / (256(48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 + a^8 + 20736)) * ((9(39329792a - 338a((a + 4)^{15})^{1/2}) - 589((a + 4)^{15})^{1/2} - 49a^2((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384(1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968)))^{1/2}) * ((9(39329792a - 338a((a + 4)^{15})^{1/2}) - 589((a + 4)^{15})^{1/2} - 49a^2((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384(1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968)))^{1/2} + (108343296a + 74059776a^2 + 27065088a^3 + 5576256a^4 + 614016a^5 + 28224a^6 + 66207744) / (16384(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x(73476a + 31545a^2 + 6066a^3 + 441a^4 + 64656)) / (256(48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 + a^8 + 20736)) * ((9(39329792a - 338a((a + 4)^{15})^{1/2}) - 589((a + 4)^{15})^{1/2} - 49a^2((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384(1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968)))^{1/2} + (((52357496832a + 57139003392a^2 + 36322148352a^3 + 14822473728a^4 + 4027170816a^5 + 728506368a^6 + 84615168a^7 + 5726208a^8 + 172032a^9 + 21290287104) / (16384(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - ((4290672328704a + 6001143054336a^2 + 5025917042688a^3 + 2800520003584a^4 + 1090200272896a^5 + 302556119040a^6 + 59862155264a^7 + 8275361792a^8 + 761266176a^9 + 41943040a^{10} + 1048576a^{11} + 1391569403904) / (16384(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x(3510632448a + 4020240384a^2 + 2678587392a^3 + 1144324096a^4 + 325074944a^5 + 61407232a^6 + 7438336a^7 + 524288a^8 + 16384a^9 + 1358954496)) / (256(48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 + a^8 + 20736)) * ((9(39329792a - 338a((a + 4)^{15})^{1/2}) - 589((a + 4)^{15})^{1/2} - 49a^2((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384(1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968)))^{1/2}) * ((9(39329792a - 338a((a + 4)^{15})^{1/2}) - 589((a + 4)^{15})^{1/2} - 49a^2((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384(1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968)))^{1/2} - (108343296a + 74059776a^2 + 27065088a^3 + 5576256a^4 + 614016a^5 + 28224a^6 + 66207744) / (16384(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) + (x(73476a + 31545a^2 + 6066a^3 + 441a^4 + 64656)) / (256(48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 + a^8 + 2
\end{aligned}$$

$$\begin{aligned}
& 0736))) * ((9 * (39329792 * a - 338 * a * ((a + 4)^{15})^{(1/2)} - 589 * ((a + 4)^{15})^{(1/2)} \\
& - 49 * a^2 * ((a + 4)^{15})^{(1/2)} + 41598976 * a^2 + 25672960 * a^3 + 10187840 * a^4 + \\
& 2695744 * a^5 + 475608 * a^6 + 53949 * a^7 + 3570 * a^8 + 105 * a^9 + 16531456)) / (16 \\
& 384 * (1061683200 * a + 2061434880 * a^2 + 2474311680 * a^3 + 2053201920 * a^4 + 1247 \\
& 703040 * a^5 + 573621760 * a^6 + 203166720 * a^7 + 55893360 * a^8 + 11944200 * a^9 + \\
& 1966491 * a^{10} + 244965 * a^{11} + 22350 * a^{12} + 1410 * a^{13} + 55 * a^{14} + a^{15} + 2548 \\
& 03968)))^{(1/2)} - (99468 * a + 28053 * a^2 + 2646 * a^3 + 117936) / (8192 * (940032 * a \\
& + 1195776 * a^2 + 899328 * a^3 + 442864 * a^4 + 149208 * a^5 + 34833 * a^6 + 5564 * a^7 \\
& + 582 * a^8 + 36 * a^9 + a^{10} + 331776))) * ((9 * (39329792 * a - 338 * a * ((a + 4)^{15})^{(1/2)} \\
&)^{(1/2)} - 589 * ((a + 4)^{15})^{(1/2)} - 49 * a^2 * ((a + 4)^{15})^{(1/2)} + 41598976 * a^2 \\
& + 25672960 * a^3 + 10187840 * a^4 + 2695744 * a^5 + 475608 * a^6 + 53949 * a^7 + 357 \\
& 0 * a^8 + 105 * a^9 + 16531456)) / (16384 * (1061683200 * a + 2061434880 * a^2 + 247431 \\
& 1680 * a^3 + 2053201920 * a^4 + 1247703040 * a^5 + 573621760 * a^6 + 203166720 * a^7 \\
& + 55893360 * a^8 + 11944200 * a^9 + 1966491 * a^{10} + 244965 * a^{11} + 22350 * a^{12} + 1 \\
& 410 * a^{13} + 55 * a^{14} + a^{15} + 254803968)))^{(1/2)} * 2i + \operatorname{atan}((((52357496832 * a \\
& + 57139003392 * a^2 + 36322148352 * a^3 + 14822473728 * a^4 + 4027170816 * a^5 + 72 \\
& 8506368 * a^6 + 84615168 * a^7 + 5726208 * a^8 + 172032 * a^9 + 21290287104) / (16384 \\
& * (940032 * a + 1195776 * a^2 + 899328 * a^3 + 442864 * a^4 + 149208 * a^5 + 34833 * a^6 \\
& + 5564 * a^7 + 582 * a^8 + 36 * a^9 + a^{10} + 331776))) + ((4290672328704 * a + 6001 \\
& 143054336 * a^2 + 5025917042688 * a^3 + 2800520003584 * a^4 + 1090200272896 * a^5 + \\
& 302556119040 * a^6 + 59862155264 * a^7 + 8275361792 * a^8 + 761266176 * a^9 + 4194 \\
& 3040 * a^{10} + 1048576 * a^{11} + 1391569403904) / (16384 * (940032 * a + 1195776 * a^2 + \\
& 899328 * a^3 + 442864 * a^4 + 149208 * a^5 + 34833 * a^6 + 5564 * a^7 + 582 * a^8 + 36 * \\
& a^9 + a^{10} + 331776))) - (x * (3510632448 * a + 4020240384 * a^2 + 2678587392 * a^3 \\
& + 1144324096 * a^4 + 325074944 * a^5 + 61407232 * a^6 + 7438336 * a^7 + 524288 * a^8 \\
& + 16384 * a^9 + 1358954496)) / (256 * (48384 * a + 49248 * a^2 + 28560 * a^3 + 10321 * a^4 \\
& + 2380 * a^5 + 342 * a^6 + 28 * a^7 + a^8 + 20736))) * ((9 * (39329792 * a + 338 * a * ((\\
& a + 4)^{15})^{(1/2)} + 589 * ((a + 4)^{15})^{(1/2)} + 49 * a^2 * ((a + 4)^{15})^{(1/2)} + 415 \\
& 98976 * a^2 + 25672960 * a^3 + 10187840 * a^4 + 2695744 * a^5 + 475608 * a^6 + 53949 * \\
& a^7 + 3570 * a^8 + 105 * a^9 + 16531456)) / (16384 * (1061683200 * a + 2061434880 * a^2 \\
& + 2474311680 * a^3 + 2053201920 * a^4 + 1247703040 * a^5 + 573621760 * a^6 + 20316 \\
& 6720 * a^7 + 55893360 * a^8 + 11944200 * a^9 + 1966491 * a^{10} + 244965 * a^{11} + 22350 \\
& * a^{12} + 1410 * a^{13} + 55 * a^{14} + a^{15} + 254803968)))^{(1/2)} * ((9 * (39329792 * a + \\
& 338 * a * ((a + 4)^{15})^{(1/2)} + 589 * ((a + 4)^{15})^{(1/2)} + 49 * a^2 * ((a + 4)^{15})^{(1/ \\
& 2)} + 41598976 * a^2 + 25672960 * a^3 + 10187840 * a^4 + 2695744 * a^5 + 475608 * a^6 \\
& + 53949 * a^7 + 3570 * a^8 + 105 * a^9 + 16531456)) / (16384 * (1061683200 * a + 206143 \\
& 4880 * a^2 + 2474311680 * a^3 + 2053201920 * a^4 + 1247703040 * a^5 + 573621760 * a^6 \\
& + 203166720 * a^7 + 55893360 * a^8 + 11944200 * a^9 + 1966491 * a^{10} + 244965 * a^{11} \\
& + 22350 * a^{12} + 1410 * a^{13} + 55 * a^{14} + a^{15} + 254803968)))^{(1/2)} + (10834329 \\
& 6 * a + 74059776 * a^2 + 27065088 * a^3 + 5576256 * a^4 + 614016 * a^5 + 28224 * a^6 + \\
& 66207744) / (16384 * (940032 * a + 1195776 * a^2 + 899328 * a^3 + 442864 * a^4 + 149208 \\
& * a^5 + 34833 * a^6 + 5564 * a^7 + 582 * a^8 + 36 * a^9 + a^{10} + 331776))) - (x * (7347 \\
& 6 * a + 31545 * a^2 + 6066 * a^3 + 441 * a^4 + 64656)) / (256 * (48384 * a + 49248 * a^2 + \\
& 28560 * a^3 + 10321 * a^4 + 2380 * a^5 + 342 * a^6 + 28 * a^7 + a^8 + 20736))) * ((9 * (3 \\
& 9329792 * a + 338 * a * ((a + 4)^{15})^{(1/2)} + 589 * ((a + 4)^{15})^{(1/2)} + 49 * a^2 * ((a \\
& + 4)^{15})^{(1/2)} + 41598976 * a^2 + 25672960 * a^3 + 10187840 * a^4 + 2695744 * a^5 + \\
& 475608 * a^6 + 53949 * a^7 + 3570 * a^8 + 105 * a^9 + 16531456)) / (16384 * (106168320 \\
& 0 * a + 2061434880 * a^2 + 2474311680 * a^3 + 2053201920 * a^4 + 1247703040 * a^5 + 5 \\
& 73621760 * a^6 + 203166720 * a^7 + 55893360 * a^8 + 11944200 * a^9 + 1966491 * a^{10} + \\
& 244965 * a^{11} + 22350 * a^{12} + 1410 * a^{13} + 55 * a^{14} + a^{15} + 254803968)))^{(1/2)} \\
& * 1i - (((52357496832 * a + 57139003392 * a^2 + 36322148352 * a^3 + 14822473728 * a^4 \\
& + 4027170816 * a^5 + 728506368 * a^6 + 84615168 * a^7 + 5726208 * a^8 + 172032 * a^9 \\
& + 21290287104) / (16384 * (940032 * a + 1195776 * a^2 + 899328 * a^3 + 442864 * a^4 + \\
& 149208 * a^5 + 34833 * a^6 + 5564 * a^7 + 582 * a^8 + 36 * a^9 + a^{10} + 331776))) - (\\
& (4290672328704 * a + 6001143054336 * a^2 + 5025917042688 * a^3 + 2800520003584 * a^4 \\
& + 1090200272896 * a^5 + 302556119040 * a^6 + 59862155264 * a^7 + 8275361792 * a^8 \\
& + 761266176 * a^9 + 41943040 * a^{10} + 1048576 * a^{11} + 1391569403904) / (16384 * (94 \\
& 0032 * a + 1195776 * a^2 + 899328 * a^3 + 442864 * a^4 + 149208 * a^5 + 34833 * a^6 + 5 \\
& 564 * a^7 + 582 * a^8 + 36 * a^9 + a^{10} + 331776))) - (x * (3510632448 * a + 402024038
\end{aligned}$$

$$\begin{aligned}
& 4a^2 + 2678587392a^3 + 1144324096a^4 + 325074944a^5 + 61407232a^6 + 74 \\
& 38336a^7 + 524288a^8 + 16384a^9 + 1358954496) / (256(48384a + 49248a^2 \\
& + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 + a^8 + 20736))) * ((9 \\
& *(39329792a + 338a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49a^2 * \\
& ((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 \\
& + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 * (106168 \\
& 3200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 \\
& + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} \\
& + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968)))^{(1/2)} \\
& * ((9*(39329792a + 338a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + \\
& 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 26 \\
& 95744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 \\
& *(1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703 \\
& 040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 196 \\
& 6491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 2548039 \\
& 68)))^{(1/2)} - (108343296a + 74059776a^2 + 27065088a^3 + 5576256a^4 + 61 \\
& 4016a^5 + 28224a^6 + 66207744) / (16384 * (940032a + 1195776a^2 + 899328a^3 \\
& + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} \\
& + 331776)) + (x * (73476a + 31545a^2 + 6066a^3 + 441a^4 + 64656)) / (256 * \\
& (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 \\
& + a^8 + 20736))) * ((9*(39329792a + 338a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + \\
& 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10187 \\
& 840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531 \\
& 456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a \\
& ^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 119442 \\
& 00a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} \\
& + 254803968)))^{(1/2)} * i) / (((52357496832a + 57139003392a^2 + 363221483 \\
& 52a^3 + 14822473728a^4 + 4027170816a^5 + 728506368a^6 + 84615168a^7 + \\
& 5726208a^8 + 172032a^9 + 21290287104) / (16384 * (940032a + 1195776a^2 + 89 \\
& 9328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 \\
& + a^{10} + 331776)) + ((4290672328704a + 6001143054336a^2 + 5025917042688 \\
& * a^3 + 2800520003584a^4 + 1090200272896a^5 + 302556119040a^6 + 598621552 \\
& 64a^7 + 8275361792a^8 + 761266176a^9 + 41943040a^{10} + 1048576a^{11} + 13 \\
& 91569403904) / (16384 * (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149 \\
& 208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x * (3 \\
& 510632448a + 4020240384a^2 + 2678587392a^3 + 1144324096a^4 + 325074944a^5 \\
& + 61407232a^6 + 7438336a^7 + 524288a^8 + 16384a^9 + 1358954496)) / (2 \\
& 56 * (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 \\
& + a^8 + 20736))) * ((9*(39329792a + 338a*((a + 4)^{15})^{(1/2)} + 589*((a + \\
& 4)^{15})^{(1/2)} + 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10 \\
& 187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16 \\
& 531456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 205320192 \\
& 0a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 119 \\
& 44200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + \\
& a^{15} + 254803968)))^{(1/2)} * ((9*(39329792a + 338a*((a + 4)^{15})^{(1/2)} + 58 \\
& 9*((a + 4)^{15})^{(1/2)} + 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 \\
& + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 \\
& + 16531456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2 \\
& 053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 \\
& + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 5 \\
& 5a^{14} + a^{15} + 254803968)))^{(1/2)} + (108343296a + 74059776a^2 + 27065088 \\
& * a^3 + 5576256a^4 + 614016a^5 + 28224a^6 + 66207744) / (16384 * (940032a + \\
& 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + \\
& 582a^8 + 36a^9 + a^{10} + 331776)) - (x * (73476a + 31545a^2 + 6066a^3 + \\
& 441a^4 + 64656)) / (256 * (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 \\
& + 342a^6 + 28a^7 + a^8 + 20736))) * ((9*(39329792a + 338a*((a + 4)^{15})^{(1/2)} \\
&)^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976a^2 \\
& + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 357 \\
& 0a^8 + 105a^9 + 16531456)) / (16384 * (1061683200a + 2061434880a^2 + 247431
\end{aligned}$$

$$\begin{aligned}
& 1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 \\
& + 55893360*a^8 + 11944200*a^9 + 1966491*a^{10} + 244965*a^{11} + 22350*a^{12} + 1 \\
& 410*a^{13} + 55*a^{14} + a^{15} + 254803968))^{(1/2)} + (((52357496832*a + 5713900 \\
& 3392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a \\
& ^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032* \\
& a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a \\
& ^7 + 582*a^8 + 36*a^9 + a^{10} + 331776)) - ((4290672328704*a + 6001143054336 \\
& *a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 30255611 \\
& 9040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^{10} \\
& + 1048576*a^{11} + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^ \\
& 3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^{10} \\
& + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324 \\
& 096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a \\
& ^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380* \\
& a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^{15} \\
&)^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} + 41598976*a^2 \\
& + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357 \\
& 0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431 \\
& 1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 \\
& + 55893360*a^8 + 11944200*a^9 + 1966491*a^{10} + 244965*a^{11} + 22350*a^{12} + 1 \\
& 410*a^{13} + 55*a^{14} + a^{15} + 254803968))^{(1/2)})*((9*(39329792*a + 338*a*((a \\
& + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} + 4159 \\
& 8976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a \\
& ^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 \\
& + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166 \\
& 720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^{10} + 244965*a^{11} + 22350* \\
& a^{12} + 1410*a^{13} + 55*a^{14} + a^{15} + 254803968))^{(1/2)} - (108343296*a + 740 \\
& 59776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744) \\
& /(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34 \\
& 833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^{10} + 331776)) + (x*(73476*a + 315 \\
& 45*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 \\
& + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a \\
& + 338*a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} \\
& + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a \\
& ^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 206 \\
& 1434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760* \\
& a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^{10} + 244965*a \\
& ^{11} + 22350*a^{12} + 1410*a^{13} + 55*a^{14} + a^{15} + 254803968))^{(1/2)} - (99468 \\
& *a + 28053*a^2 + 2646*a^3 + 117936)/(8192*(940032*a + 1195776*a^2 + 899328* \\
& a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a \\
& ^{10} + 331776)))*((9*(39329792*a + 338*a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15} \\
&)^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} + 41598976*a^2 + 25672960*a^3 + 10187 \\
& 840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531 \\
& 456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a \\
& ^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442 \\
& 00*a^9 + 1966491*a^{10} + 244965*a^{11} + 22350*a^{12} + 1410*a^{13} + 55*a^{14} + a^{15} \\
& + 254803968))^{(1/2)}*2i - ((408*a + 131*a^2 + 11*a^3 + 288)/(32*(a + 4)* \\
& (33*a + 10*a^2 + a^3 + 36)) - (21*x^6*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a \\
& ^3 + a^4 + 144)) + (3*x^7*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 1 \\
& 44)) + (x*(84*a - 107*a^2 - 11*a^3 + 1152))/(32*(a + 4)*(33*a + 10*a^2 + a^ \\
& 3 + 36)) - (5*x^4*(175*a + 7*a^2 + 528))/(32*(a + 4)*(33*a + 10*a^2 + a^3 + \\
& 36)) + (x^5*(343*a + 7*a^2 + 1116))/(32*(a + 4)*(33*a + 10*a^2 + a^3 + 36) \\
&) - (x^2*(623*a + 32*a^2 + 1800))/(16*(a + 4)*(33*a + 10*a^2 + a^3 + 36)) + \\
& (x^3*(679*a + 34*a^2 + 1968))/(16*(a + 4)*(33*a + 10*a^2 + a^3 + 36)))/(16 \\
& *a*x - x^2*(16*a - 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 + \\
& 32*x^6 - 8*x^7 + x^8)
\end{aligned}$$

sympy [B] time = 15.62, size = 697, normalized size = 2.77

$$32a^6 + 448a^5 + 2336a^4 + 5376a^3 + 4608a^2 + x^8 (32a^4 + 448a^3 + 2336a^2 + 5376a + 4608) + x^7 (-256a^4 - 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)

[Out] $-(11*a**3 + 131*a**2 + 408*a + x**7*(12*a + 42) + x**6*(-84*a - 294) + x**5*(7*a**2 + 343*a + 1116) + x**4*(-35*a**2 - 875*a - 2640) + x**3*(68*a**2 + 1358*a + 3936) + x**2*(-64*a**2 - 1246*a - 3600) + x*(-11*a**3 - 107*a**2 + 84*a + 1152) + 288)/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 + 4608*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-256*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14336*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3 - 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a**3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a**3 + 86016*a**2 + 73728*a)) - \text{RootSum}(_t**4*(268435456*a**15 + 14763950080*a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 527875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 54537151127224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 551152193655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2 + 284993413919539200*a + 68398419340689408) + _t**2*(-30965760*a**9 - 1052835840*a**8 - 15910207488*a**7 - 140262506496*a**6 - 795007254528*a**5 - 300451627080*a**4 - 7571263979520*a**3 - 12268037210112*a**2 - 11598827618304*a - 4875324751872) - 194481*a**4 - 2762424*a**3 - 14762736*a**2 - 35178624*a - 31539456, \text{Lambda}(_t, _t*\log(x + (23068672*_t**3*a**12 + 968884224*_t**3*a**11 + 18624806912*_t**3*a**10 + 216677744640*_t**3*a**9 + 1699123036160*_t**3*a**8 + 9461389328384*_t**3*a**7 + 38361186172928*_t**3*a**6 + 114107491549184*_t**3*a**5 + 247138458009600*_t**3*a**4 + 380084473036800*_t**3*a**3 + 394002582994944*_t**3*a**2 + 247177515368448*_t**3*a + 70970039599104*_t**3 - 395136*_t*a**7 - 11676672*_t*a**6 - 144076032*_t*a**5 - 969518592*_t*a**4 - 3861475200*_t*a**3 - 9133300224*_t*a**2 - 11906574336*_t*a - 6611337216*_t - 64827*a**4 - 907578*a**3 - 4780647*a**2 - 11228868*a - 9923472))/(64827*a**4 + 907578*a**3 + 4780647*a**2 + 11228868*a + 9923472)))$

$$3.123 \quad \int x (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Optimal. Leaf size=210

$$\frac{a^4 x^2}{2} + \frac{32a^3 x^3}{3} + \frac{1}{5} (3a^2 - 1536a + 20480) x^{10} - \frac{16}{3} (a^2 - 128a + 896) x^9 - \frac{32}{7} (15a^2 - 288a + 512) x^7 + \frac{16}{5} a (a^2 - 48a + 128) x^5 + \frac{2}{3} (-a^3 + 192a^2 - 1536a + 1024) x^6 - \frac{32}{7} (15a^2 - 288a + 512) x^7 + 8(128 - 3a)(4 - a)x^8 - \frac{16}{3} (a^2 - 128a + 896) x^9 + \frac{1}{5} (3a^2 - 1536a + 20480) x^{10} - \frac{32}{11} (928 - 35a) x^{11} + \frac{8}{3} (524 - 9a) x^{12} - \frac{16}{13} (464 - 3a) x^{13} + \frac{2}{7} (640 - a) x^{14} - \frac{224}{5} x^{15} + 8x^{16} - \frac{16}{17} x^{17} + \frac{1}{18} x^{18}$$

[Out] $\frac{1}{2} a^4 x^2 + \frac{32}{3} a^3 x^3 + 8(12 - a) a^2 x^4 + \frac{16}{5} a (128 - 48a + a^2) x^5 + \frac{2}{3} (-a^3 + 192a^2 - 1536a + 1024) x^6 - \frac{32}{7} (15a^2 - 288a + 512) x^7 + 8(128 - 3a)(4 - a)x^8 - \frac{16}{3} (a^2 - 128a + 896) x^9 + \frac{1}{5} (3a^2 - 1536a + 20480) x^{10} - \frac{32}{11} (928 - 35a) x^{11} + \frac{8}{3} (524 - 9a) x^{12} - \frac{16}{13} (464 - 3a) x^{13} + \frac{2}{7} (640 - a) x^{14} - \frac{224}{5} x^{15} + 8x^{16} - \frac{16}{17} x^{17} + \frac{1}{18} x^{18}$

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6742}

$$\frac{1}{5} (3a^2 - 1536a + 20480) x^{10} - \frac{16}{3} (a^2 - 128a + 896) x^9 - \frac{32}{7} (15a^2 - 288a + 512) x^7 + \frac{2}{3} (-a^3 + 192a^2 - 1536a + 1024) x^6 - \frac{32}{7} (15a^2 - 288a + 512) x^7 + 8(128 - 3a)(4 - a)x^8 - \frac{16}{3} (a^2 - 128a + 896) x^9 + \frac{1}{5} (3a^2 - 1536a + 20480) x^{10} - \frac{32}{11} (928 - 35a) x^{11} + \frac{8}{3} (524 - 9a) x^{12} - \frac{16}{13} (464 - 3a) x^{13} + \frac{2}{7} (640 - a) x^{14} - \frac{224}{5} x^{15} + 8x^{16} - \frac{16}{17} x^{17} + \frac{1}{18} x^{18}$$

Antiderivative was successfully verified.

[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]

[Out] $(a^4 x^2)/2 + (32 a^3 x^3)/3 + 8(12 - a) a^2 x^4 + (16 a (128 - 48 a + a^2) x^5)/5 + (2(1024 - 1536 a + 192 a^2 - a^3) x^6)/3 - (32(512 - 288 a + 15 a^2) x^7)/7 + 8(128 - 3 a)(4 - a) x^8 - (16(896 - 128 a + a^2) x^9)/3 + ((20480 - 1536 a + 3 a^2) x^{10})/5 - (32(928 - 35 a) x^{11})/11 + (8(524 - 9 a) x^{12})/3 - (16(464 - 3 a) x^{13})/13 + (2(640 - a) x^{14})/7 - (224 x^{15})/5 + 8 x^{16} - (16 x^{17})/17 + x^{18}/18$

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx = \int (a^4 x + 32a^3 x^2 - 32(-12 + a)a^2 x^3 + 16a(128 - 48a + a^2)x^4 - 4(-1024 + 1536a - 192a^2 + a^3)x^5 + 8(128 - 3a)(4 - a)x^6 - 16(896 - 128a + a^2)x^7 + (20480 - 1536a + 3a^2)x^8 - 32(928 - 35a)x^9 + 8(524 - 9a)x^{10} - 16(464 - 3a)x^{11} + 2(640 - a)x^{12} - 224x^{13} + 8x^{14} - 16x^{15} + x^{16}) dx$$

$$= \frac{a^4 x^2}{2} + \frac{32a^3 x^3}{3} + 8(12 - a)a^2 x^4 + \frac{16}{5} a (128 - 48a + a^2) x^5 + \frac{2}{3} (1024 - 1536a + 192a^2 - a^3) x^6 - \frac{32}{7} (15a^2 - 288a + 512) x^7 + 8(128 - 3a)(4 - a)x^8 - \frac{16}{3} (896 - 128a + a^2) x^9 + \frac{1}{5} (20480 - 1536a + 3a^2) x^{10} - \frac{32}{11} (928 - 35a) x^{11} + \frac{8}{3} (524 - 9a) x^{12} - \frac{16}{13} (464 - 3a) x^{13} + \frac{2}{7} (640 - a) x^{14} - \frac{224}{5} x^{15} + 8x^{16} - \frac{16}{17} x^{17} + \frac{1}{18} x^{18}$$

Mathematica [A] time = 0.04, size = 204, normalized size = 0.97

$$\frac{a^4 x^2}{2} + \frac{32a^3 x^3}{3} + \frac{1}{5} (3a^2 - 1536a + 20480) x^{10} - \frac{16}{3} (a^2 - 128a + 896) x^9 + 8(3a^2 - 140a + 512) x^8 - \frac{32}{7} (15a^2 - 288a + 512) x^7 + \frac{2}{3} (1024 - 1536a + 192a^2 - a^3) x^6 - \frac{32}{7} (15a^2 - 288a + 512) x^7 + 8(128 - 3a)(4 - a)x^8 - \frac{16}{3} (896 - 128a + a^2) x^9 + \frac{1}{5} (20480 - 1536a + 3a^2) x^{10} + \frac{32}{11} (-928 + 35a) x^{11} - \frac{8}{3} (-524 + 9a) x^{12} + \frac{16}{13} (-464 + 3a) x^{13} - \frac{2}{7} (-640 + a) x^{14} - \frac{224}{5} x^{15} + 8x^{16} - \frac{16}{17} x^{17} + \frac{1}{18} x^{18}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]

[Out] $(a^4 x^2)/2 + (32 a^3 x^3)/3 - 8(-12 + a) a^2 x^4 + (16 a (128 - 48 a + a^2) x^5)/5 - (2(-1024 + 1536 a - 192 a^2 + a^3) x^6)/3 - (32(512 - 288 a + 15 a^2) x^7)/7 + 8(512 - 140 a + 3 a^2) x^8 - (16(896 - 128 a + a^2) x^9)/3 + ((20480 - 1536 a + 3 a^2) x^{10})/5 + (32(-928 + 35 a) x^{11})/11 - (8(-524 + 9 a) x^{12})/3 + (16(-464 + 3 a) x^{13})/13 - (2(-640 + a) x^{14})/7 - (224 x^{15})/5 + 8 x^{16} - (16 x^{17})/17 + x^{18}/18$

fricas [A] time = 0.36, size = 222, normalized size = 1.06

$$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{224}{5}x^{15} - \frac{2}{7}x^{14}a + \frac{1280}{7}x^{14} + \frac{48}{13}x^{13}a - \frac{7424}{13}x^{13} - 24x^{12}a + \frac{4192}{3}x^{12} + \frac{1120}{11}x^{11}a + \frac{3}{5}x^{10}a^2 - \frac{29696}{11}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")

$$\begin{aligned} \text{[Out]} & 1/18*x^{18} - 16/17*x^{17} + 8*x^{16} - 224/5*x^{15} - 2/7*x^{14}*a + 1280/7*x^{14} + 4 \\ & 8/13*x^{13}*a - 7424/13*x^{13} - 24*x^{12}*a + 4192/3*x^{12} + 1120/11*x^{11}*a + 3/5 \\ & *x^{10}*a^2 - 29696/11*x^{11} - 1536/5*x^{10}*a - 16/3*x^9*a^2 + 4096*x^{10} + 2048 \\ & /3*x^9*a + 24*x^8*a^2 - 14336/3*x^9 - 1120*x^8*a - 480/7*x^7*a^2 - 2/3*x^6* \\ & a^3 + 4096*x^8 + 9216/7*x^7*a + 128*x^6*a^2 + 16/5*x^5*a^3 - 16384/7*x^7 - \\ & 1024*x^6*a - 768/5*x^5*a^2 - 8*x^4*a^3 + 2048/3*x^6 + 2048/5*x^5*a + 96*x^4 \\ & *a^2 + 32/3*x^3*a^3 + 1/2*x^2*a^4 \end{aligned}$$

giac [A] time = 0.39, size = 222, normalized size = 1.06

$$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{2}{7}ax^{14} - \frac{224}{5}x^{15} + \frac{48}{13}ax^{13} + \frac{1280}{7}x^{14} - 24ax^{12} - \frac{7424}{13}x^{13} + \frac{3}{5}a^2x^{10} + \frac{1120}{11}ax^{11} + \frac{4192}{3}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & 1/18*x^{18} - 16/17*x^{17} + 8*x^{16} - 2/7*a*x^{14} - 224/5*x^{15} + 48/13*a*x^{13} + \\ & 1280/7*x^{14} - 24*a*x^{12} - 7424/13*x^{13} + 3/5*a^2*x^{10} + 1120/11*a*x^{11} + 41 \\ & 92/3*x^{12} - 16/3*a^2*x^9 - 1536/5*a*x^{10} - 29696/11*x^{11} + 24*a^2*x^8 + 204 \\ & 8/3*a*x^9 + 4096*x^{10} - 2/3*a^3*x^6 - 480/7*a^2*x^7 - 1120*a*x^8 - 14336/3* \\ & x^9 + 16/5*a^3*x^5 + 128*a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4 - 76 \\ & 8/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 96*a^ \\ & 2*x^4 + 2048/5*a*x^5 + 2048/3*x^6 \end{aligned}$$

maple [A] time = 0.00, size = 267, normalized size = 1.27

$$\frac{x^{18}}{18} - \frac{16x^{17}}{17} + 8x^{16} - \frac{224x^{15}}{5} + \frac{(-4a + 2560)x^{14}}{14} + \frac{(48a - 7424)x^{13}}{13} + \frac{(-288a + 16768)x^{12}}{12} + \frac{(1120a - 29696)x^{11}}{11} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)

$$\begin{aligned} \text{[Out]} & 1/18*x^{18} - 16/17*x^{17} + 8*x^{16} - 224/5*x^{15} + 1/14*(-4*a+2560)*x^{14} + 1/13*(48*a-742 \\ & 4)*x^{13} + 1/12*(-288*a+16768)*x^{12} + 1/11*(1120*a-29696)*x^{11} + 1/10*(2*a^2-2560* \\ & a+24576+(-2*a+128)^2)*x^{10} + 1/9*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128) \\ &)*x^9 + 1/8*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^8 + 1/7*(-160 \\ & *a^2+32*(-2*a+128)*a+2*(-16*a+64)*(8*a-128))*x^7 + 1/6*(2*(-2*a+128)*a^2+32*(\\ & 8*a-128)*a+(-16*a+64)^2)*x^6 + 1/5*(2*(8*a-128)*a^2+32*(-16*a+64)*a)*x^5 + 1/4* \\ & (2*(-16*a+64)*a^2+256*a^2)*x^4 + 32/3*a^3*x^3 + 1/2*a^4*x^2 \end{aligned}$$

maxima [A] time = 0.62, size = 182, normalized size = 0.87

$$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{2}{7}(a-640)x^{14} - \frac{224}{5}x^{15} + \frac{16}{13}(3a-464)x^{13} - \frac{8}{3}(9a-524)x^{12} + \frac{32}{11}(35a-928)x^{11} + \frac{1}{5}(3a^2 - 1536a + 20480)x^{10} - 16/3*(a^2 - 128a + 896)*x^9 + 8*(3*a^2 - 140*a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & 1/18*x^{18} - 16/17*x^{17} + 8*x^{16} - 2/7*(a-640)*x^{14} - 224/5*x^{15} + 16/13*(\\ & 3*a-464)*x^{13} - 8/3*(9*a-524)*x^{12} + 32/11*(35*a-928)*x^{11} + 1/5*(3*a \\ & ^2 - 1536*a + 20480)*x^{10} - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a \end{aligned}$$

$$+ 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5 - 8*(a^3 - 12*a^2)*x^4$$

mupad [B] time = 0.22, size = 178, normalized size = 0.85

$$x^{13} \left(\frac{48a}{13} - \frac{7424}{13} \right) - x^{12} \left(24a - \frac{4192}{3} \right) - x^{14} \left(\frac{2a}{7} - \frac{1280}{7} \right) + x^{11} \left(\frac{1120a}{11} - \frac{29696}{11} \right) + x^8 (24a^2 - 1120a + 4096) + x^{10} \left(\frac{3a^2}{5} - \frac{1536a}{5} + 4096 \right) - x^9 \left(\frac{16a^2}{3} - \frac{2048a}{3} + \frac{14336}{3} \right) - x^7 \left(\frac{480a^2}{7} - \frac{9216a}{7} + \frac{16384}{7} \right) - x^6 \left(\frac{1024a}{3} - \frac{128a^2}{3} + \frac{2a^3}{3} - \frac{2048}{3} \right) - \frac{224x^{15}}{5} + 8x^{16} - \frac{16x^{17}}{17} + x^{18}/18 + \frac{32a^3x^3}{3} + \frac{a^4x^2}{2} + \frac{16a^5x^5(a^2 - 48a + 128)}{5} - 8a^2x^4(a - 12)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)

[Out] x^13*((48*a)/13 - 7424/13) - x^12*(24*a - 4192/3) - x^14*((2*a)/7 - 1280/7) + x^11*((1120*a)/11 - 29696/11) + x^8*(24*a^2 - 1120*a + 4096) + x^10*((3*a^2)/5 - (1536*a)/5 + 4096) - x^9*((16*a^2)/3 - (2048*a)/3 + 14336/3) - x^7*((480*a^2)/7 - (9216*a)/7 + 16384/7) - x^6*(1024*a - 128*a^2 + (2*a^3)/3 - 2048/3) - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18 + (32*a^3*x^3)/3 + (a^4*x^2)/2 + (16*a*x^5*(a^2 - 48*a + 128))/5 - 8*a^2*x^4*(a - 12)

sympy [A] time = 0.12, size = 212, normalized size = 1.01

$$\frac{a^4x^2}{2} + \frac{32a^3x^3}{3} + \frac{x^{18}}{18} - \frac{16x^{17}}{17} + 8x^{16} - \frac{224x^{15}}{5} + x^{14} \left(\frac{1280}{7} - \frac{2a}{7} \right) + x^{13} \left(\frac{48a}{13} - \frac{7424}{13} \right) + x^{12} \left(\frac{4192}{3} - 24a \right) + x^{11} \left(\frac{1120a}{11} - \frac{29696}{11} \right) + x^8 (24a^2 - 1120a + 4096) + x^{10} \left(\frac{3a^2}{5} - \frac{1536a}{5} + 4096 \right) - x^9 \left(\frac{16a^2}{3} - \frac{2048a}{3} + \frac{14336}{3} \right) - x^7 \left(\frac{480a^2}{7} - \frac{9216a}{7} + \frac{16384}{7} \right) - x^6 \left(\frac{1024a}{3} - \frac{128a^2}{3} + \frac{2a^3}{3} - \frac{2048}{3} \right) - \frac{224x^{15}}{5} + 8x^{16} - \frac{16x^{17}}{17} + x^{18}/18 + \frac{32a^3x^3}{3} + \frac{a^4x^2}{2} + \frac{16a^5x^5(a^2 - 48a + 128)}{5} - 8a^2x^4(a - 12)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)

[Out] a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**15/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 - 24*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096) + x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096) + x**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 - 1024*a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**3 + 96*a**2)

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] $-1/14*x^{14} + 12/13*x^{13} - 6*x^{12} + 280/11*x^{11} + 3/10*x^{10}*a - 384/5*x^{10} - 8/3*x^9*a + 512/3*x^9 + 12*x^8*a - 280*x^8 - 240/7*x^7*a - 1/2*x^6*a^2 + 2$
 $304/7*x^7 + 64*x^6*a + 12/5*x^5*a^2 - 256*x^6 - 384/5*x^5*a - 6*x^4*a^2 + 5$
 $12/5*x^5 + 48*x^4*a + 8*x^3*a^2 + 1/2*x^2*a^3$

giac [A] time = 0.35, size = 133, normalized size = 0.99

$$-\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{3}{10}ax^{10} + \frac{280}{11}x^{11} - \frac{8}{3}ax^9 - \frac{384}{5}x^{10} + 12ax^8 + \frac{512}{3}x^9 - \frac{1}{2}a^2x^6 - \frac{240}{7}ax^7 - 280x^8 + \frac{12}{5}a^2x^5 + 64$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] $-1/14*x^{14} + 12/13*x^{13} - 6*x^{12} + 3/10*a*x^{10} + 280/11*x^{11} - 8/3*a*x^9 - 384/5*x^{10} + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 + 1$
 $2/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6 + 1$
 $/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5$

maple [A] time = 0.00, size = 143, normalized size = 1.07

$$-\frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + \frac{(3a-768)x^{10}}{10} + \frac{(-24a+1536)x^9}{9} + \frac{(96a-2240)x^8}{8} + \frac{(-240a+2304)x^7}{7} + \frac{(-a^2+(-24a+1536))x^6}{6} + \frac{(12a^2-240a+2304)x^5}{5} + \frac{(12a^2-240a+2304)x^4}{4} + \frac{(12a^2-240a+2304)x^3}{3} + \frac{(12a^2-240a+2304)x^2}{2} + \frac{(12a^2-240a+2304)x}{1} + \frac{(12a^2-240a+2304)}{0}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)

[Out] $-1/14*x^{14} + 12/13*x^{13} - 6*x^{12} + 280/11*x^{11} + 1/10*(3*a-768)*x^{10} + 1/9*(-24*a+1536)*x^9 + 1/8*(96*a-2240)*x^8 + 1/7*(-240*a+2304)*x^7 + 1/6*(-a^2+(-24*a+128)*a+256*a-1536)*x^6 + 1/5*(4*a^2+(8*a-128)*a-256*a+512)*x^5 + 1/4*(-8*a^2+(-16*a+64)*a+128*a)*x^4 + 8*a^2*x^3 + 1/2*a^3*x^2$

maxima [A] time = 0.62, size = 113, normalized size = 0.84

$$-\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{3}{10}(a-256)x^{10} + \frac{280}{11}x^{11} - \frac{8}{3}(a-64)x^9 + 4(3a-70)x^8 - \frac{48}{7}(5a-48)x^7 - \frac{1}{2}(a^2-128a+256)x^6 + \frac{12a^2-240a+2304}{5}x^5 + \frac{12a^2-240a+2304}{4}x^4 + \frac{12a^2-240a+2304}{3}x^3 + \frac{12a^2-240a+2304}{2}x^2 + \frac{12a^2-240a+2304}{1}x + \frac{12a^2-240a+2304}{0}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")

[Out] $-1/14*x^{14} + 12/13*x^{13} - 6*x^{12} + 3/10*(a-256)*x^{10} + 280/11*x^{11} - 8/3*(a-64)*x^9 + 4*(3*a-70)*x^8 - 48/7*(5*a-48)*x^7 - 1/2*(a^2-128*a+512)*x^6 + 4/5*(3*a^2-96*a+128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2-8*a)*x^4$

mupad [B] time = 2.12, size = 113, normalized size = 0.84

$$x^8(12a-280) + x^{10}\left(\frac{3a}{10} - \frac{384}{5}\right) - x^9\left(\frac{8a}{3} - \frac{512}{3}\right) - x^7\left(\frac{240a}{7} - \frac{2304}{7}\right) - x^6\left(\frac{a^2}{2} - 64a + 256\right) + x^5\left(\frac{12a^2}{5} - \frac{384a}{5}\right) + x^4(12a^2 - 240a + 2304) + x^3(12a^2 - 240a + 2304) + x^2(12a^2 - 240a + 2304) + x(12a^2 - 240a + 2304) + (12a^2 - 240a + 2304)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+8*x-8*x^2+4*x^3-x^4)^3,x)

[Out] $x^8*(12*a-280) + x^{10}*((3*a)/10 - 384/5) - x^9*((8*a)/3 - 512/3) - x^7*((240*a)/7 - 2304/7) - x^6*(a^2/2 - 64*a + 256) + x^5*((12*a^2)/5 - (384*a)/5 + 512/5) + (280*x^{11})/11 - 6*x^{12} + (12*x^{13})/13 - x^{14}/14 + 8*a^2*x^3 + (a^3*x^2)/2 - 6*a*x^4*(a-8)$

sympy [A] time = 0.10, size = 128, normalized size = 0.96

$$\frac{a^3 x^2}{2} + 8a^2 x^3 - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + x^{10} \left(\frac{3a}{10} - \frac{384}{5} \right) + x^9 \left(\frac{512}{3} - \frac{8a}{3} \right) + x^8 (12a - 280) + x^7 \left(\frac{2304}{7} - \frac{240a}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)

[Out] a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/11 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x**7*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 384*a/5 + 512/5) + x**4*(-6*a**2 + 48*a)

$$3.125 \quad \int x (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Optimal. Leaf size=79

$$\frac{a^2x^2}{2} + \frac{1}{3}(64-a)x^6 - \frac{8}{5}(16-a)x^5 + 4(4-a)x^4 + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7}$$

[Out] 1/2*a^2*x^2+16/3*a*x^3+4*(4-a)*x^4-8/5*(16-a)*x^5+1/3*(64-a)*x^6-80/7*x^7+4*x^8-8/9*x^9+1/10*x^10

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6742}

$$\frac{a^2x^2}{2} + \frac{1}{3}(64-a)x^6 - \frac{8}{5}(16-a)x^5 + 4(4-a)x^4 + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx &= \int (a^2x + 16ax^2 - 16(-4 + a)x^3 + 8(-16 + a)x^4 - 2(-64 + a)x^5 - 80x^6 + 32x^7 - 80x^8 + 4x^9 - x^{10}) dx \\ &= \frac{a^2x^2}{2} + \frac{16ax^3}{3} + 4(4-a)x^4 - \frac{8}{5}(16-a)x^5 + \frac{1}{3}(64-a)x^6 - \frac{80x^7}{7} + 4x^8 - \frac{80x^9}{9} + \frac{x^{10}}{10} \end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 0.95

$$\frac{a^2x^2}{2} + \frac{1}{3}(64-a)x^6 + \frac{8}{5}(a-16)x^5 - 4(a-4)x^4 + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 - a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10

fricas [A] time = 0.39, size = 68, normalized size = 0.86

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{80}{7}x^7 - \frac{1}{3}x^6a + \frac{64}{3}x^6 + \frac{8}{5}x^5a - \frac{128}{5}x^5 - 4x^4a + 16x^4 + \frac{16}{3}x^3a + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 80/7*x^7 - 1/3*x^6*a + 64/3*x^6 + 8/5*x^5*a - 128/5*x^5 - 4*x^4*a + 16*x^4 + 16/3*x^3*a + 1/2*x^2*a^2

giac [A] time = 0.35, size = 68, normalized size = 0.86

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{1}{3}ax^6 - \frac{80}{7}x^7 + \frac{8}{5}ax^5 + \frac{64}{3}x^6 - 4ax^4 - \frac{128}{5}x^5 + \frac{1}{2}a^2x^2 + \frac{16}{3}ax^3 + 16x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6 - 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7} + \frac{(-2a+128)x^6}{6} + \frac{(8a-128)x^5}{5} + \frac{a^2x^2}{2} + \frac{16ax^3}{3} + \frac{(-16a+64)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] 1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+1/6*(-2*a+128)*x^6+1/5*(8*a-128)*x^5+1/4*(-16*a+64)*x^4+16/3*a*x^3+1/2*a^2*x^2

maxima [A] time = 0.62, size = 59, normalized size = 0.75

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{1}{3}(a-64)x^6 - \frac{80}{7}x^7 + \frac{8}{5}(a-16)x^5 - 4(a-4)x^4 + \frac{1}{2}a^2x^2 + \frac{16}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3

mupad [B] time = 0.04, size = 64, normalized size = 0.81

$$x^5 \left(\frac{8a}{5} - \frac{128}{5} \right) - x^6 \left(\frac{a}{3} - \frac{64}{3} \right) - x^4 (4a - 16) + \frac{16ax^3}{3} - \frac{80x^7}{7} + 4x^8 - \frac{8x^9}{9} + \frac{x^{10}}{10} + \frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] x^5*((8*a)/5 - 128/5) - x^6*(a/3 - 64/3) - x^4*(4*a - 16) + (16*a*x^3)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10 + (a^2*x^2)/2

sympy [A] time = 0.08, size = 70, normalized size = 0.89

$$\frac{a^2x^2}{2} + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7} + x^6 \left(\frac{64}{3} - \frac{a}{3} \right) + x^5 \left(\frac{8a}{5} - \frac{128}{5} \right) + x^4 (16 - 4a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**6*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)

3.126 $\int x(a + 8x - 8x^2 + 4x^3 - x^4) dx$

Optimal. Leaf size=35

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {14}

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int x(a + 8x - 8x^2 + 4x^3 - x^4) dx &= \int (ax + 8x^2 - 8x^3 + 4x^4 - x^5) dx \\ &= \frac{ax^2}{2} + \frac{8x^3}{3} - 2x^4 + \frac{4x^5}{5} - \frac{x^6}{6} \end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6

fricas [A] time = 0.40, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{8}{3}x^3 + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")

[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 8/3*x^3 + 1/2*x^2*a

giac [A] time = 0.36, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")

[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x),x)

[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6

maxima [A] time = 0.61, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")

[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3

mupad [B] time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3} + \frac{ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)

[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6

sympy [A] time = 0.06, size = 29, normalized size = 0.83

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)

[Out] a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3

$$3.127 \quad \int \frac{x}{a+8x-8x^2+4x^3-x^4} dx$$

Optimal. Leaf size=116

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}} + \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

[Out] 1/2*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a)^(1/2)))^(1/2)/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)+1/2*arctan((-1+x)/(1+(4+a)^(1/2)))^(1/2)/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1680, 1673, 1093, 204, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}} + \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*Sqrt[4 + a])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{a + 8x - 8x^2 + 4x^3 - x^4} dx &= \text{Subst} \left(\int \frac{1+x}{3+a-2x^2-x^4} dx, x, -1+x \right) \\ &= \text{Subst} \left(\int \frac{1}{3+a-2x^2-x^4} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{x}{3+a-2x^2-x^4} dx, x, -1+x \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{3+a-2x-x^2} dx, x, (-1+x)^2 \right) - \frac{\text{Subst} \left(\int \frac{1}{-1-\sqrt{4+a}-x^2} dx, x, -1+x \right)}{2\sqrt{4+a}} \\ &= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} - \text{Subst} \left(\int \frac{1}{4(4+a)-x^2} dx, x, -1+x \right) \\ &= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1} \left(\frac{1+(-1+x)^2}{\sqrt{4+a}} \right)}{2\sqrt{4+a}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.51

$$-\frac{1}{4} \text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\#1 \log(x - \#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (Log[x - #1]*#1)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x), x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")

[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)

maple [C] time = 0.00, size = 50, normalized size = 0.43

$$\frac{\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a) \ln(-\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a))}{4 \left(\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^3 - 3 \text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^2 + 4 \text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x),x)

[Out] -1/4*sum(_R/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(-Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")

[Out] -integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)

mupad [B] time = 2.58, size = 275, normalized size = 2.37

$$\sum_{k=1}^4 \ln(-x - \text{root}(2816a^2z^4 + 256a^3z^4 + 10240az^4 + 12288z^4 - 32a^2z^2 - 256az^2 - 512z^2 + 16az + 64z + a, z, k))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)

[Out] symsum(log(-x - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k))*(root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k))*(32*a - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k))*(64*a - x*(64*a + 256) + 256) - x*(16*a + 64) + 128) - 8)*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k), k, 1, 4)

sympy [A] time = 4.43, size = 155, normalized size = 1.34

$$-\text{RootSum}\left(t^4(256a^3 + 2816a^2 + 10240a + 12288) + t^2(-32a^2 - 256a - 512) + t(-16a - 64) + a, \left(t \mapsto t \log\left(\frac{x + \text{root}(2816a^2z^4 + 256a^3z^4 + 10240az^4 + 12288z^4 - 32a^2z^2 - 256az^2 - 512z^2 + 16az + 64z + a, z, k)}{x + \text{root}(2816a^2z^4 + 256a^3z^4 + 10240az^4 + 12288z^4 - 32a^2z^2 - 256az^2 - 512z^2 + 16az + 64z + a, z, k)}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)

[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2 - 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a**4 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*_t**2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_t*a**2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))

$$3.128 \quad \int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2} dx$$

Optimal. Leaf size=231

$$\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{4(a+4)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(3a+\sqrt{a+4}+10)\operatorname{atanh}\left(\frac{x-1}{\sqrt{a+4}}\right)}{8(a+3)(a+4)^{3/2}}$$

[Out] 1/4*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1-(4+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(10+3*a-(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)

Rubi [A] time = 0.24, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1680, 1673, 1092, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{4(a+4)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(3a+\sqrt{a+4}+10)\operatorname{atanh}\left(\frac{x-1}{\sqrt{a+4}}\right)}{8(a+3)(a+4)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4ac, 0]$

Rule 1092

$\text{Int}[(a_.) + (b_.)(x_)^2 + (c_.)(x_)^4]^{(p_.)}, x_Symbol] \rightarrow -\text{Simp}[(x*(b^2 - 2ac + b^2x^2)*(a + b^2x^2 + c^2x^4)^{(p+1)})/(2a*(p+1)*(b^2 - 4ac)), x] + \text{Dist}[1/(2a*(p+1)*(b^2 - 4ac)), \text{Int}[(b^2 - 2ac + 2*(p+1)*(b^2 - 4ac) + b^2c*(4p+7)x^2)*(a + b^2x^2 + c^2x^4)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntegerQ}[2p]$

Rule 1107

$\text{Int}[(x_.)*((a_.) + (b_.)(x_)^2 + (c_.)(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(a + bx + cx^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, p\}, x]$

Rule 1166

$\text{Int}[(d_.) + (e_.)(x_)^2]/((a_.) + (b_.)(x_)^2 + (c_.)(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[e/2 + (2cd - b^2e)/(2q), \text{Int}[1/(b/2 - q/2 + cx^2), x], x] + \text{Dist}[e/2 - (2cd - b^2e)/(2q), \text{Int}[1/(b/2 + q/2 + cx^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c^2d^2 - a^2e^2, 0] \&\& \text{PosQ}[b^2 - 4ac]$

Rule 1673

$\text{Int}[(Pq_.)*((a_.) + (b_.)(x_)^2 + (c_.)(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Module}\{q = \text{Expon}[Pq, x], k\}, \text{Int}[\text{Sum}[\text{Coeff}[Pq, x, 2k]*x^{(2k)}, \{k, 0, q/2\}](a + b^2x^2 + c^2x^4)^p, x] + \text{Int}[x*\text{Sum}[\text{Coeff}[Pq, x, 2k+1]*x^{(2k)}, \{k, 0, (q-1)/2\}](a + b^2x^2 + c^2x^4)^p, x]] /; \text{FreeQ}\{a, b, c, p\}, x \&\& \text{PolyQ}[Pq, x] \&\& !\text{PolyQ}[Pq, x^2]$

Rule 1680

$\text{Int}[(Pq_.)*(Q4_.)^{(p_.)}, x_Symbol] \rightarrow \text{With}\{a = \text{Coeff}[Q4, x, 0], b = \text{Coeff}[Q4, x, 1], c = \text{Coeff}[Q4, x, 2], d = \text{Coeff}[Q4, x, 3], e = \text{Coeff}[Q4, x, 4]\}, \text{Subst}[\text{Int}[\text{SimplifyIntegrand}[(Pq /. x \rightarrow -(d/(4e)) + x)*(a + d^4/(256e^3) - (b*d)/(8e) + (c - (3*d^2)/(8e))*x^2 + e*x^4)^p, x], x], x, d/(4e) + x] /; \text{EqQ}[d^3 - 4c*d*e + 8b*e^2, 0] \&\& \text{NeQ}[d, 0]] /; \text{FreeQ}[p, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{PolyQ}[Q4, x, 4] \&\& !\text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx &= \text{Subst} \left(\int \frac{1+x}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \frac{1}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{x}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{1}{2} \text{Subst} \left(\int \frac{x}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} \\
&= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} \\
&= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 166, normalized size = 0.72

$$\frac{ax^2 - ax + a + x^3 + 2x}{4(a+3)(a+4)(a-x(x^3-4x^2+8x-8))} - \frac{\text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\#1^2 \log(x-\#1) + 2\#1a \log(x-\#1)}{16(a^2+7a+12)} \right]}{16(a^2+7a+12)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (6*Log[x - #1] + a*Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &]/(16*(12 + 7*a + a^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^4 - 4x^3 + 8x^2 - a - 8x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)

maple [C] time = 0.01, size = 162, normalized size = 0.70

$$\frac{\left(-\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)^2 + 2(-a - 2)\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right) - a - 6\right)\ln\left(-\frac{\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)^3 - 3\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)^2 + 4\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right) - a - 6}{16\left(a^2 + 7a + 12\right)}\right)}{16\left(a^2 + 7a + 12\right)\left(\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)^3 - 3\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right)^2 + 4\text{RootOf}\left(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a\right) - a - 6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] (-1/4/(a^2+7*a+12)*x^3-1/4*a/(a^2+7*a+12)*x^2+1/4*(a-2)/(a^2+7*a+12)*x-1/4*a/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a^2+7*a+12)*sum((-6-_R^2+2*(-a-2)*_R-a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ax^2 + x^3 - (a - 2)x + a}{4\left(\left(a^2 + 7a + 12\right)x^4 - 4\left(a^2 + 7a + 12\right)x^3 - a^3 + 8\left(a^2 + 7a + 12\right)x^2 - 7a^2 - 8\left(a^2 + 7a + 12\right)x - 12a\right)} \int \frac{2(a+x)}{x^4-4x^3+8x^2-a-8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] -1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)

mupad [B] time = 2.82, size = 1167, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] symsum(log((35*a + 4*a^2 + 68)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((12800*a + 3600*a^2 + 336*a^3 + 15104)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) + root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*(root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) - (x*(3932160*a + 2719744*a^2 + 999424*a^3 + 205824*a^4 + 22528*a^5 + 1024*a^6 + 2359296))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))

576))) - (1150976*a + 631808*a^2 + 172800*a^3 + 23552*a^4 + 1280*a^5 + 835584)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a + 58880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) - (x*(864*a + 228*a^2 + 20*a^3 + 1088))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) + (x*(9*a + 2*a^2 + 8))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) * root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k), k, 1, 4) + (x^3/(4*(7*a + a^2 + 12))) + a/(4*(a + 3)*(a + 4)) - (x*(a - 2))/(4*(a + 3)*(a + 4)) + (a*x^2)/(4*(a + 3)*(a + 4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)

sympy [B] time = 31.42, size = 539, normalized size = 2.33

$$\frac{-ax^2 - a - x^3 + x(a - 2)}{-4a^3 - 28a^2 - 48a + x^4(4a^2 + 28a + 48) + x^3(-16a^2 - 112a - 192) + x^2(32a^2 + 224a + 384) + x(-32a^2 - 224a - 384)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] (-a*x**2 - a - x**3 + x*(a - 2))/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-2048*a**6 - 50688*a**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155776*a - 9568256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 253952) + 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (98304*_t**3*a**12 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 + 793837568*_t**3*a**9 + 5839372288*_t**3*a**8 + 30226464768*_t**3*a**7 + 112668450816*_t**3*a**6 + 303864643584*_t**3*a**5 + 586157391872*_t**3*a**4 + 784017129472*_t**3*a**3 + 683648483328*_t**3*a**2 + 343136010240*_t**3*a + 72477573120*_t**3 + 30208*_t**2*a**10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156928*_t**2*a**7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312*_t**2*a**4 + 12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820569600*_t**2*a + 2944401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8 - 757040*_t*a**7 - 6200656*_t*a**6 - 31380496*_t*a**5 - 100736416*_t*a**4 - 200813696*_t*a**3 - 228144640*_t*a**2 - 114632704*_t*a - 2490368*_t + 248*a**7 + 6797*a**6 + 71132*a**5 + 369745*a**4 + 987758*a**3 + 1128896*a**2 - 129568*a - 956416)/(576*a**7 + 10985*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3 + 1826304*a**2 + 186776*a + 917504)))

$$3.129 \quad \int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3} dx$$

Optimal. Leaf size=349

$$\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2} \frac{3(7a^2+(4\sqrt{a+4}+47)a+14\sqrt{a+4}+80)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

[Out] $1/8*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+3/16*(1+(-1+x)^2)/(4+a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)+1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+1/32*((6+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)+3/16*\operatorname{arctanh}((1+(-1+x)^2)/(4+a)^{(1/2)})/(4+a)^{(5/2)}-3/64*\operatorname{arctan}((-1+x)/(1-(4+a)^{(1/2)})^{(1/2)})*(80+7*a^2+14*(4+a)^{(1/2)}+a*(47+4*(4+a)^{(1/2)}))/(3+a)^2/(4+a)^{(5/2)/(1-(4+a)^{(1/2)})^{(1/2)}-3/64*\operatorname{arctan}((-1+x)/(1+(4+a)^{(1/2)})^{(1/2)})*(14+4*a+(-7*a^2-47*a-80)/(4+a)^{(1/2)})/(3+a)^2/(4+a)^2/(1+(4+a)^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1680, 1673, 1092, 1178, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2} \frac{3(7a^2+(4\sqrt{a+4}+47)a+14\sqrt{a+4}+80)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] $(1+(-1+x)^2)/(8*(4+a)*(3+a-2*(-1+x)^2-(-1+x)^4)^2)+(3*(1+(-1+x)^2))/(16*(4+a)^2*(3+a-2*(-1+x)^2-(-1+x)^4))+((5+a+(-1+x)^2)*(-1+x))/(8*(12+7*a+a^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^2)+(((6+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x))/(32*(3+a)^2*(4+a)^2*(3+a-2*(-1+x)^2-(-1+x)^4))-((3*(80+7*a^2+14*\operatorname{Sqrt}[4+a]+a*(47+4*\operatorname{Sqrt}[4+a]))*\operatorname{ArcTan}[(-1+x)/\operatorname{Sqrt}[1-\operatorname{Sqrt}[4+a]]])/(64*(3+a)^2*(4+a)^{(5/2)}*\operatorname{Sqrt}[1-\operatorname{Sqrt}[4+a]])-(3*(14+4*a-(80+47*a+7*a^2)/\operatorname{Sqrt}[4+a])*\operatorname{ArcTan}[(-1+x)/\operatorname{Sqrt}[1+\operatorname{Sqrt}[4+a]]])/(64*(3+a)^2*(4+a)^2*\operatorname{Sqrt}[1+\operatorname{Sqrt}[4+a]])+(3*\operatorname{ArcTanh}[(1+(-1+x)^2)/\operatorname{Sqrt}[4+a]])/(16*(4+a)^{(5/2)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2] \ \&\& \ \text{IntegerQ}[4p]$

Rule 618

$\text{Int}[(a_.) + (b_.)x + (c_.)x^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] \ /; \ \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 1092

$\text{Int}[(a_.) + (b_.)x^2 + (c_.)x^4]^{(p_.)}, x_Symbol] \rightarrow -\text{Simp}[(x(b^2 - 2ac + b^2cx^2)(a + bx^2 + cx^4)^{(p+1)}) / (2a(p+1)(b^2 - 4ac)), x] + \text{Dist}[1 / (2a(p+1)(b^2 - 4ac)), \text{Int}[(b^2 - 2ac + 2(p+1)(b^2 - 4ac) + b^2c(4p+7)x^2)(a + bx^2 + cx^4)^{(p+1)}, x], x] \ /; \ \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntegerQ}[2p]$

Rule 1107

$\text{Int}[x((a_.) + (b_.)x^2 + (c_.)x^4)^{(p_.)}], x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(a + bx + cx^2)^p, x], x, x^2], x] \ /; \ \text{FreeQ}[\{a, b, c, p\}, x]$

Rule 1166

$\text{Int}[(d_.) + (e_.)x^2] / [(a_.) + (b_.)x^2 + (c_.)x^4], x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[e/2 + (2cd - b^2e) / (2q), \text{Int}[1 / (b/2 - q/2 + cx^2), x], x] + \text{Dist}[e/2 - (2cd - b^2e) / (2q), \text{Int}[1 / (b/2 + q/2 + cx^2), x], x]] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4ac]$

Rule 1178

$\text{Int}[(d_.) + (e_.)x^2]((a_.) + (b_.)x^2 + (c_.)x^4)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(x(ab^2e - d(b^2 - 2ac)) - c(b^2d - 2ae)x^2)(a + bx^2 + cx^4)^{(p+1)}) / (2a(p+1)(b^2 - 4ac)), x] + \text{Dist}[1 / (2a(p+1)(b^2 - 4ac)), \text{Int}[\text{Simp}[(2p+3)d^2b^2 - ab^2e - 2acd(4p+5) + (4p+7)(db - 2ae)cx^2, x](a + bx^2 + cx^4)^{(p+1)}, x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - b^2de + ae^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntegerQ}[2p]$

Rule 1673

$\text{Int}[(Pq_.)((a_.) + (b_.)x^2 + (c_.)x^4)^{(p_.)}], x_Symbol] \rightarrow \text{Module}[\{q = \text{Expon}[Pq, x], k\}, \text{Int}[\text{Sum}[\text{Coeff}[Pq, x, 2k]x^{2k}, \{k, 0, q/2\}](a + bx^2 + cx^4)^p, x] + \text{Int}[x \text{Sum}[\text{Coeff}[Pq, x, 2k+1]x^{2k}, \{k, 0, (q-1)/2\}](a + bx^2 + cx^4)^p, x]] \ /; \ \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ !\text{PolyQ}[Pq, x^2]$

Rule 1680

$\text{Int}[(Pq_.)x^4]^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{a = \text{Coeff}[Q4, x, 0], b = \text{Coeff}[Q4, x, 1], c = \text{Coeff}[Q4, x, 2], d = \text{Coeff}[Q4, x, 3], e = \text{Coeff}[Q4, x, 4]\}, \text{Subst}[\text{Int}[\text{SimplifyIntegrand}[(Pq / x \rightarrow -(d/(4e)) + x)(a + d^4/(256e^3) - (bd)/(8e) + (c - (3d^2)/(8e))x^2 + ex^4)^p, x], x], x, d/(4e) + x] \ /; \ \text{EqQ}[d^3 - 4cd^2e + 8b^2e^2, 0] \ \&\& \ \text{NeQ}[d, 0] \ /; \ \text{FreeQ}[p, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{PolyQ}[Q4, x, 4] \ \&\& \ !\text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^3} dx &= \text{Subst} \left(\int \frac{1+x}{(3+a-2x^2-x^4)^3} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \frac{1}{(3+a-2x^2-x^4)^3} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{x}{(3+a-2x^2-x^4)^3} dx, x, -1+x \right) \\
&= \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(3+a-2x^2-x^4)^3} dx, x, -1+x \right) \\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} - \frac{((6+a)(25+7a)+6(7+2a)(1-x))}{32(12+7a+a^2)^2(3+a-2(1-x)^2-(1-x)^4)} \\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)} \\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)} \\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)}
\end{aligned}$$

Mathematica [C] time = 0.16, size = 284, normalized size = 0.81

$$\frac{1}{128} \left(\frac{3\text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{4\#1^2 a \log(x-\#1) + 14\#1^2 \log(x-\#1) + 3a^2 \log(x-\#1) + 4\#1 a^2 \log(x-\#1) + 31a \log(x-\#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \right]}{(a^2 + 7a + 12)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] ((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^3) + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (72*Log[x - #1] + 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x - #1]*#1 + 4*a^2*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &])/(12 + 7*a + a^2)^2)/128

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^4 - 4x^3 + 8x^2 - a - 8x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)

maple [C] time = 0.02, size = 405, normalized size = 1.16

$$\frac{3 \left(2(2a+7) \text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a)^2 + 3a^2 + 4(a^2 + 4a + 2) \text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a) \right)}{128(a^4 + 14a^3 + 73a^2 + 168a + 144) \left(\text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a)^3 - 3 \text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)

[Out] $-(3/16*(2*a+7)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14*a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*a+144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16*(62*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+140*a+1008)/(a^4+14*a^3+73*a^2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)/(a^4+14*a^3+73*a^2+168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+168*a+144)/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^4+14*a^3+73*a^2+168*a+144)*\text{sum}((72+2*(2*a+7)*_R^2+4*(a^2+4*a+2)*_R+3*a^2+31*a)/(_R^3-3*_R^2+4*_R-2)*\ln(-_R+x), _R=\text{RootOf}(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{6(2a + 32((a^4 + 14a^3 + 73a^2 + 168a + 144)x^8 - 8(a^4 + 14a^3 + 73a^2 + 168a + 144)x^7 + 32(a^4 + 14a^3 + 73a^2 + 168a + 144)x^6 - 24(a^4 + 14a^3 + 73a^2 + 168a + 144)x^5 + 12(a^4 + 14a^3 + 73a^2 + 168a + 144)x^4 - 6(a^4 + 14a^3 + 73a^2 + 168a + 144)x^3 + 3(a^4 + 14a^3 + 73a^2 + 168a + 144)x^2 - 3(a^4 + 14a^3 + 73a^2 + 168a + 144)x + 3(a^4 + 14a^3 + 73a^2 + 168a + 144)))}{128(a^4 + 14a^3 + 73a^2 + 168a + 144)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")

[Out] $-1/32*(6*(2*a+7)*x^7+6*(a^2-8*a-40)*x^6-(29*a^2-127*a-792)*x^5+(73*a^2-227*a-1668)*x^4-2*(62*a^2-103*a-1104)*x^3-9*a^3-2*(5*a^3-26*a^2+140*a+1008)*x^2-21*a^2+3*(3*a^3-17*a^2-40*a+192)*x+36*a)/((a^4+14*a^3+73*a^2+168*a+144)*x^8-8*(a^4+14*a^3+73*a^2+168*a+144)*x^7+32*(a^4+14*a^3+73*a^2+168*a+144)*x^6+a^6-80*(a^4+14*a^3+73*a^2+168*a+144)*x^5+14*a^5-2*(a^5-50*a^4-823*a^3-4504*a^2-10608*a-9216)*x^4+73*a^4+8*(a^5-2*a^4-151*a^3-1000*a^2-2544*a-2304)*x^3+168*a^3-16*(a^5+10*a^4+17*a^3-124*a^2-528*a-576)*x^2+144*a^2+16*(a^5+14*a^4+73*a^3+168*a^2+144*a)*x)-3/32*\text{integrate}((2*(2*a+7)*x^2+3*a^2+4*(a^2+4*a+2)*x+31*a+72)/(x^4-4*x^3+8*x^2-a-8*x),x)/(a^4+14*a^3+73*a^2+168*a+144)$

mupad [B] time = 3.39, size = 2200, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+8*x-8*x^2+4*x^3-x^4)^3,x)

```
[Out] symsum(log(root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 153
980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a
^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 59995324
41600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 3206
246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 378
493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a
^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2
- 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 11
043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 60494614
36416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 728801
28*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 3335100825
6*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 68
345856, z, k)*((242823168*a + 170044416*a^2 + 63509760*a^3 + 13340736*a^4 +
1494144*a^5 + 69696*a^6 + 144506880)/(16384*(940032*a + 1195776*a^2 + 8993
28*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9
+ a^10 + 331776)) + root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*
z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 55115219365
5275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 +
5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z
^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*
z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 -
4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 150233929482
24*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7
*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 -
6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z
+ 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 3
3351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736
*a^5 - 68345856, z, k)*(root(15003759578972160*a^8*z^4 + 54537151127224320*
a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 5511521
93655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z
^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200
*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a
^11*z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^
4 - 4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392
948224*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608
*a^7*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z
^2 - 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a
^2*z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z
+ 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 2
0736*a^5 - 68345856, z, k)*((4290672328704*a + 6001143054336*a^2 + 50259170
42688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 5986
2155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11
+ 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4
+ 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) -
(x*(536334041088*a + 750142881792*a^2 + 628239630336*a^3 + 350065000448*a^4
+ 136275034112*a^5 + 37819514880*a^6 + 7482769408*a^7 + 1034420224*a^8 + 9
5158272*a^9 + 5242880*a^10 + 131072*a^11 + 173946175488))/(2048*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7
+ 582*a^8 + 36*a^9 + a^10 + 331776))) - (73421291520*a + 81260445696*a^2 +
52393672704*a^3 + 21688418304*a^4 + 5977620480*a^5 + 1096949760*a^6 + 12924
5184*a^7 + 8871936*a^8 + 270336*a^9 + 29444014080)/(16384*(940032*a + 11957
76*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*
a^8 + 36*a^9 + a^10 + 331776)) + (x*(2632974336*a + 3015180288*a^2 + 200894
0544*a^3 + 858243072*a^4 + 243806208*a^5 + 46055424*a^6 + 5578752*a^7 + 393
216*a^8 + 12288*a^9 + 1019215872))/(2048*(940032*a + 1195776*a^2 + 899328*a
^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^
10 + 331776))) - (x*(10805760*a + 7173504*a^2 + 2539872*a^3 + 505800*a^4 +
53712*a^5 + 2376*a^6 + 6782976))/(2048*(940032*a + 1195776*a^2 + 899328*a^3
+ 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10
```



```

+ 331776))) - (133812*a + 56187*a^2 + 10098*a^3 + 648*a^4 + 115776)/(16384
*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6
+ 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(1971*a^2 - 1539*a +
918*a^3 + 108*a^4 - 6372))/(2048*(940032*a + 1195776*a^2 + 899328*a^3 + 442
864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331
776)))*root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 1539804
18717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z
^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 599953244160
0*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 32062467
73555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 3784939
92960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a^10*
z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2 - 16
752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 110433
92716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 604946143641
6*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 72880128*a
^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 33351008256*z
- 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 683458
56, z, k), k, 1, 4) + ((3*(7*a^2 - 12*a + 3*a^3))/(32*(6*a + a^2 + 9)*(8*a
+ a^2 + 16)) - (3*x^7*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))
+ (x^2*(140*a - 26*a^2 + 5*a^3 + 1008))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 1
6)) + (3*x*(40*a + 17*a^2 - 3*a^3 - 192))/(32*(6*a + a^2 + 9)*(8*a + a^2 +
16)) + (3*x^6*(8*a - a^2 + 40))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 16)) - (x^
5*(127*a - 29*a^2 + 792))/(32*(6*a + a^2 + 9)*(8*a + a^2 + 16)) - (x^3*(103
*a - 62*a^2 + 1104))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 16)) + (x^4*(227*a -
73*a^2 + 1668))/(32*(6*a + a^2 + 9)*(8*a + a^2 + 16)))/(16*a*x - x^2*(16*a
- 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 + 32*x^6 - 8*x^7 +
x^8)

```

sympy [B] time = 88.09, size = 1102, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```

[Out] -(-9*a**3 - 21*a**2 + 36*a + x**7*(12*a + 42) + x**6*(6*a**2 - 48*a - 240)
+ x**5*(-29*a**2 + 127*a + 792) + x**4*(73*a**2 - 227*a - 1668) + x**3*(-12
4*a**2 + 206*a + 2208) + x**2*(-10*a**3 + 52*a**2 - 280*a - 2016) + x*(9*a*
*3 - 51*a**2 - 120*a + 576))/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 +
4608*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-
256*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14
336*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3
- 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**
3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a*
*3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*
a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a*
*3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*
a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 5
27875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 545
37151127224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 5
51152193655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2
+ 284993413919539200*a + 68398419340689408) + _t**2*(-4718592*a**10 - 19611
6480*a**9 - 3648061440*a**8 - 40022212608*a**7 - 286939938816*a**6 - 140543
7345792*a**5 - 4764645457920*a**4 - 11043392716800*a**3 - 16752587046912*a*
*2 - 15023392948224*a - 6049461436416) + _t*(-2709504*a**7 - 72880128*a**6
- 839890944*a**5 - 5375877120*a**4 - 20640890880*a**3 - 47542173696*a**2 -
60827369472*a - 33351008256) + 20736*a**5 - 155601*a**4 - 4706424*a**3 - 29
249424*a**2 - 74027520*a - 68345856, Lambda(_t, _t*log(x + (-469762048*_t**
3*a**20 - 31417434112*_t**3*a**19 - 992305217536*_t**3*a**18 - 196635766292
48*_t**3*a**17 - 273880031690752*_t**3*a**16 - 2846116194287616*_t**3*a**15

```

- 22853982892326912*_t**3*a**14 - 144840417605582848*_t**3*a**13 - 7331931
 54773123072*_t**3*a**12 - 2977941469704224768*_t**3*a**11 - 967719737311730
 0736*_t**3*a**10 - 24850421452415959040*_t**3*a**9 - 48984708931769073664*_
 t**3*a**8 - 69124682329943441408*_t**3*a**7 - 54921507243737219072*_t**3*a*
 *6 + 18833423088924753920*_t**3*a**5 + 128767022044444360704*_t**3*a**4 + 1
 97893824476545548288*_t**3*a**3 + 170576989286005997568*_t**3*a**2 + 837098
 68624400351232*_t**3*a + 18392762450832261120*_t**3 + 136642560*_t**2*a**17
 + 7616593920*_t**2*a**16 + 198980665344*_t**2*a**15 + 3234300690432*_t**2*
 a**14 + 36614363283456*_t**2*a**13 + 306155605721088*_t**2*a**12 + 19563396
 56687616*_t**2*a**11 + 9747894775578624*_t**2*a**10 + 38291841445330944*_t*
 *2*a**9 + 119050488573591552*_t**2*a**8 + 292236772188880896*_t**2*a**7 + 5
 61261720373297152*_t**2*a**6 + 828898581078343680*_t**2*a**5 + 914439454498
 750464*_t**2*a**4 + 718255692208668672*_t**2*a**3 + 369227414724673536*_t**
 2*a**2 + 104815442748506112*_t**2*a + 10263520138493952*_t**2 + 4128768*_t*
 a**15 + 235608192*_t*a**14 + 6050117376*_t*a**13 + 92875570560*_t*a**12 + 9
 50838962688*_t*a**11 + 6825858397056*_t*a**10 + 34932826734336*_t*a**9 + 12
 5262778564224*_t*a**8 + 287989861404672*_t*a**7 + 257684685023232*_t*a**6 -
 836263788945408*_t*a**5 - 4002432415137792*_t*a**4 - 8409454278082560*_t*a
 3 - 10371340262965248*_t*a2 - 7285247072796672*_t*a - 2270140431335424*_
 _t + 1000512*a**12 + 42546357*a**11 + 777344580*a**10 + 7998006582*a**9 + 5
 0045408388*a**8 + 182866499613*a**7 + 247394170512*a**6 - 1063305068832*a**
 5 - 6960658344192*a**4 - 19132655580288*a**3 - 30001872614400*a**2 - 261928
 92672000*a - 9953981595648)/(1354752*a**12 + 44550027*a**11 + 663517980*a**
 10 + 5951170602*a**9 + 36270700668*a**8 + 162289912419*a**7 + 567868212432*
 a**6 + 1626099007104*a**5 + 3825839091456*a**4 + 7035734732544*a**3 + 92167
 60449024*a**2 + 7467334520832*a + 2773884911616))))

$$3.130 \quad \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Optimal. Leaf size=210

$$\frac{a^4 x^3}{3} + 8a^3 x^4 + \frac{2}{11} (3a^2 - 1536a + 20480) x^{11} - \frac{24}{5} (a^2 - 128a + 896) x^{10} - 4 (15a^2 - 288a + 512) x^8 + \frac{8}{3} a (a^2 - 48a + 128) x^6 + \frac{4}{7} (1024 - 1536a + 192a^2 - a^3) x^7 - \frac{4}{9} (128 - 3a) (4 - a) x^9 - \frac{24}{5} (a^2 - 128a + 896) x^{10} + \frac{2}{11} (3a^2 - 1536a + 20480) x^{11} - \frac{8}{3} (928 - 35a) x^{12} + \frac{32}{13} (524 - 9a) x^{13} - \frac{8}{7} (464 - 3a) x^{14} + \frac{4}{15} (640 - a) x^{15} - 42 x^{16} + \frac{128}{17} x^{17} - \frac{8}{9} x^{18} + \frac{1}{19} x^{19}$$

[Out] $\frac{1}{3} a^4 x^3 + 8 a^3 x^4 + \frac{32}{5} (12 - a) a^2 x^5 + \frac{8}{3} a (a^2 - 48 a + 128) x^6 + \frac{4}{7} (-a^3 + 192 a^2 - 1536 a + 1024) x^7 - 4 (15 a^2 - 288 a + 512) x^8 + \frac{64}{9} (128 - 3 a) (4 - a) x^9 - \frac{24}{5} (a^2 - 128 a + 896) x^{10} + \frac{2}{11} (3 a^2 - 1536 a + 20480) x^{11} - \frac{8}{3} (928 - 35 a) x^{12} + \frac{32}{13} (524 - 9 a) x^{13} - \frac{8}{7} (464 - 3 a) x^{14} + \frac{4}{15} (640 - a) x^{15} - 42 x^{16} + \frac{128}{17} x^{17} - \frac{8}{9} x^{18} + \frac{1}{19} x^{19}$

Rubi [A] time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6742}

$$\frac{2}{11} (3a^2 - 1536a + 20480) x^{11} - \frac{24}{5} (a^2 - 128a + 896) x^{10} - 4 (15a^2 - 288a + 512) x^8 + \frac{4}{7} (-a^3 + 192a^2 - 1536a + 1024) x^7 - \frac{4}{9} (128 - 3a) (4 - a) x^9 - \frac{24}{5} (a^2 - 128a + 896) x^{10} + \frac{2}{11} (3a^2 - 1536a + 20480) x^{11} - \frac{8}{3} (928 - 35a) x^{12} + \frac{32}{13} (524 - 9a) x^{13} - \frac{8}{7} (464 - 3a) x^{14} + \frac{4}{15} (640 - a) x^{15} - 42 x^{16} + \frac{128}{17} x^{17} - \frac{8}{9} x^{18} + \frac{1}{19} x^{19}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]

[Out] $(a^4 x^3)/3 + 8 a^3 x^4 + (32 (12 - a) a^2 x^5)/5 + (8 a (128 - 48 a + a^2) x^6)/3 + (4 (1024 - 1536 a + 192 a^2 - a^3) x^7)/7 - 4 (512 - 288 a + 15 a^2) x^8 + (64 (128 - 3 a) (4 - a) x^9)/9 - (24 (896 - 128 a + a^2) x^{10})/5 + (2 (20480 - 1536 a + 3 a^2) x^{11})/11 - (8 (928 - 35 a) x^{12})/3 + (32 (524 - 9 a) x^{13})/13 - (8 (464 - 3 a) x^{14})/7 + (4 (640 - a) x^{15})/15 - 42 x^{16} + (128 x^{17})/17 - (8 x^{18})/9 + x^{19}/19$

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx = \int (a^4 x^2 + 32 a^3 x^3 - 32 (-12 + a) a^2 x^4 + 16 a (128 - 48 a + a^2) x^5 - 4 (-a^3 + 192 a^2 - 1536 a + 1024) x^6 + 4 (128 - 3 a) (4 - a) x^7 - 4 (512 - 288 a + 15 a^2) x^8 + 64 (128 - 3 a) (4 - a) x^9 - 24 (896 - 128 a + a^2) x^{10} + 2 (20480 - 1536 a + 3 a^2) x^{11} - 8 (928 - 35 a) x^{12} + 32 (524 - 9 a) x^{13} - 8 (464 - 3 a) x^{14} + 4 (640 - a) x^{15} - 42 x^{16} + 128 x^{17} - 8 x^{18} + x^{19}) dx$$

$$= \frac{a^4 x^3}{3} + 8 a^3 x^4 + \frac{32}{5} (12 - a) a^2 x^5 + \frac{8}{3} a (128 - 48 a + a^2) x^6 + \frac{4}{7} (1024 - 1536 a + 192 a^2 - a^3) x^7 - \frac{4}{9} (128 - 3 a) (4 - a) x^9 - \frac{24}{5} (896 - 128 a + a^2) x^{10} + \frac{2}{11} (20480 - 1536 a + 3 a^2) x^{11} - \frac{8}{3} (928 - 35 a) x^{12} + \frac{32}{13} (524 - 9 a) x^{13} - \frac{8}{7} (464 - 3 a) x^{14} + \frac{4}{15} (640 - a) x^{15} - 42 x^{16} + \frac{128}{17} x^{17} - \frac{8}{9} x^{18} + \frac{1}{19} x^{19}$$

Mathematica [A] time = 0.04, size = 204, normalized size = 0.97

$$\frac{a^4 x^3}{3} + 8a^3 x^4 + \frac{2}{11} (3a^2 - 1536a + 20480) x^{11} - \frac{24}{5} (a^2 - 128a + 896) x^{10} + \frac{64}{9} (3a^2 - 140a + 512) x^9 - 4 (15a^2 - 288a + 15a^3) x^8 + \frac{64}{9} (128 - 3a) (4 - a) x^9 - \frac{24}{5} (896 - 128a + a^2) x^{10} + \frac{2}{11} (20480 - 1536a + 3a^2) x^{11} + \frac{8}{3} (-928 + 35a) x^{12} - \frac{32}{13} (-524 + 9a) x^{13} + \frac{8}{7} (-464 + 3a) x^{14} - \frac{4}{15} (-640 + a) x^{15} - 42 x^{16} + \frac{128}{17} x^{17} - \frac{8}{9} x^{18} + \frac{1}{19} x^{19}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]

[Out] $(a^4 x^3)/3 + 8 a^3 x^4 - (32 (-12 + a) a^2 x^5)/5 + (8 a (128 - 48 a + a^2) x^6)/3 - (4 (-1024 + 1536 a - 192 a^2 + a^3) x^7)/7 - 4 (512 - 288 a + 15 a^2) x^8 + (64 (512 - 140 a + 3 a^2) x^9)/9 - (24 (896 - 128 a + a^2) x^{10})/5 + (2 (20480 - 1536 a + 3 a^2) x^{11})/11 + (8 (-928 + 35 a) x^{12})/3 - (32 (-524 + 9 a) x^{13})/13 + (8 (-464 + 3 a) x^{14})/7 - (4 (-640 + a) x^{15})/15 - 42 x^{16} + (128 x^{17})/17 - (8 x^{18})/9 + x^{19}/19$

fricas [A] time = 0.36, size = 222, normalized size = 1.06

$$\frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - 42x^{16} - \frac{4}{15}x^{15}a + \frac{512}{3}x^{15} + \frac{24}{7}x^{14}a - \frac{3712}{7}x^{14} - \frac{288}{13}x^{13}a + \frac{16768}{13}x^{13} + \frac{280}{3}x^{12}a + \frac{6}{11}x^{11}a^2 - \frac{7424}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")

$$\begin{aligned} \text{[Out]} & 1/19*x^{19} - 8/9*x^{18} + 128/17*x^{17} - 42*x^{16} - 4/15*x^{15}*a + 512/3*x^{15} + 2 \\ & 4/7*x^{14}*a - 3712/7*x^{14} - 288/13*x^{13}*a + 16768/13*x^{13} + 280/3*x^{12}*a + 6 \\ & /11*x^{11}*a^2 - 7424/3*x^{12} - 3072/11*x^{11}*a - 24/5*x^{10}*a^2 + 40960/11*x^{11} \\ & + 3072/5*x^{10}*a + 64/3*x^9*a^2 - 21504/5*x^{10} - 8960/9*x^9*a - 60*x^8*a^2 \\ & - 4/7*x^7*a^3 + 32768/9*x^9 + 1152*x^8*a + 768/7*x^7*a^2 + 8/3*x^6*a^3 - 20 \\ & 48*x^8 - 6144/7*x^7*a - 128*x^6*a^2 - 32/5*x^5*a^3 + 4096/7*x^7 + 1024/3*x^6 \\ & *a + 384/5*x^5*a^2 + 8*x^4*a^3 + 1/3*x^3*a^4 \end{aligned}$$

giac [A] time = 0.24, size = 222, normalized size = 1.06

$$\frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - \frac{4}{15}ax^{15} - 42x^{16} + \frac{24}{7}ax^{14} + \frac{512}{3}x^{15} - \frac{288}{13}ax^{13} - \frac{3712}{7}x^{14} + \frac{6}{11}a^2x^{11} + \frac{280}{3}ax^{12} + \frac{16768}{13}x^{13} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & 1/19*x^{19} - 8/9*x^{18} + 128/17*x^{17} - 4/15*a*x^{15} - 42*x^{16} + 24/7*a*x^{14} + \\ & 512/3*x^{15} - 288/13*a*x^{13} - 3712/7*x^{14} + 6/11*a^2*x^{11} + 280/3*a*x^{12} + 1 \\ & 6768/13*x^{13} - 24/5*a^2*x^{10} - 3072/11*a*x^{11} - 7424/3*x^{12} + 64/3*a^2*x^9 \\ & + 3072/5*a*x^{10} + 40960/11*x^{11} - 4/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x^9 - \\ & 21504/5*x^{10} + 8/3*a^3*x^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9 - 32 \\ & /5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*a^3*x^4 \\ & + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7 \end{aligned}$$

maple [A] time = 0.00, size = 267, normalized size = 1.27

$$\frac{x^{19}}{19} - \frac{8x^{18}}{9} + \frac{128x^{17}}{17} - 42x^{16} + \frac{(-4a + 2560)x^{15}}{15} + \frac{(48a - 7424)x^{14}}{14} + \frac{(-288a + 16768)x^{13}}{13} + \frac{(1120a - 29696)x^{12}}{12} + \frac{(2a^2 - 2560a + 24576 + (-2a + 128)^2)x^{11}}{11} + \frac{1}{10} * (-16a^2 + 3584a - 10240 + 2 * (8a - 128) * (-2a + 128)) * x^{10} + \frac{1}{9} * (64a^2 - 2560a + 2 * (-16a + 64) * (-2a + 128) + (8a - 128)^2) * x^9 + \frac{1}{8} * (-160a^2 + 32 * (-2a + 128) * a + 2 * (-16a + 64) * (8a - 128)) * x^8 + \frac{1}{7} * (2 * (-2a + 128) * a^2 + 32 * (8a - 128) * a + (-16a + 64)^2) * x^7 + \frac{1}{6} * (2 * (8a - 128) * a^2 + 32 * (-16a + 64) * a) * x^6 + \frac{1}{5} * (2 * (-16a + 64) * a^2 + 256a^2) * x^5 + 8a^3 * x^4 + \frac{1}{3} * a^4 * x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)

$$\begin{aligned} \text{[Out]} & 1/19*x^{19} - 8/9*x^{18} + 128/17*x^{17} - 42*x^{16} + 1/15 * (-4*a + 2560) * x^{15} + 1/14 * (48*a - 7424) * x^{14} \\ & + 1/13 * (-288*a + 16768) * x^{13} + 1/12 * (1120*a - 29696) * x^{12} + 1/11 * (2*a^2 - 2560*a + 24576 + (-2*a + 128)^2) * x^{11} \\ & + 1/10 * (-16*a^2 + 3584*a - 10240 + 2 * (8*a - 128) * (-2*a + 128)) * x^{10} + 1/9 * (64*a^2 - 2560*a + 2 * (-16*a + 64) * (-2*a + 128) + (8*a - 128)^2) * x^9 \\ & + 1/8 * (-160*a^2 + 32 * (-2*a + 128) * a + 2 * (-16*a + 64) * (8*a - 128)) * x^8 + 1/7 * (2 * (-2*a + 128) * a^2 + 32 * (8*a - 128) * a + (-16*a + 64)^2) * x^7 \\ & + 1/6 * (2 * (8*a - 128) * a^2 + 32 * (-16*a + 64) * a) * x^6 + 1/5 * (2 * (-16*a + 64) * a^2 + 256*a^2) * x^5 + 8*a^3 * x^4 + 1/3 * a^4 * x^3 \end{aligned}$$

maxima [A] time = 0.64, size = 182, normalized size = 0.87

$$\frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - \frac{4}{15}(a - 640)x^{15} - 42x^{16} + \frac{8}{7}(3a - 464)x^{14} - \frac{32}{13}(9a - 524)x^{13} + \frac{8}{3}(35a - 928)x^{12} + \frac{2}{11}(3a^2 - 2560a + 24576 + (-2a + 128)^2)x^{11} + \frac{1}{10} * (-16a^2 + 3584a - 10240 + 2 * (8a - 128) * (-2a + 128)) * x^{10} + \frac{1}{9} * (64a^2 - 2560a + 2 * (-16a + 64) * (-2a + 128) + (8a - 128)^2) * x^9 + \frac{1}{8} * (-160a^2 + 32 * (-2a + 128) * a + 2 * (-16a + 64) * (8a - 128)) * x^8 + \frac{1}{7} * (2 * (-2a + 128) * a^2 + 32 * (8a - 128) * a + (-16a + 64)^2) * x^7 + \frac{1}{6} * (2 * (8a - 128) * a^2 + 32 * (-16a + 64) * a) * x^6 + \frac{1}{5} * (2 * (-16a + 64) * a^2 + 256a^2) * x^5 + 8a^3 * x^4 + \frac{1}{3} * a^4 * x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & 1/19*x^{19} - 8/9*x^{18} + 128/17*x^{17} - 4/15*(a - 640)*x^{15} - 42*x^{16} + 8/7*(3 \\ & *a - 464)*x^{14} - 32/13*(9*a - 524)*x^{13} + 8/3*(35*a - 928)*x^{12} + 2/11*(3*a \\ & ^2 - 1536*a + 20480)*x^{11} - 24/5*(a^2 - 128*a + 896)*x^{10} + 64/9*(3*a^2 - 1 \end{aligned}$$

$40a + 512)x^9 - 4(15a^2 - 288a + 512)x^8 - \frac{4}{7}(a^3 - 192a^2 + 1536a - 1024)x^7 + \frac{1}{3}a^4x^3 + 8a^3x^4 + \frac{8}{3}(a^3 - 48a^2 + 128a)x^6 - \frac{32}{5}(a^3 - 12a^2)x^5$

mupad [B] time = 2.29, size = 178, normalized size = 0.85

$$x^{14} \left(\frac{24a}{7} - \frac{3712}{7} \right) - x^{15} \left(\frac{4a}{15} - \frac{512}{3} \right) + x^{12} \left(\frac{280a}{3} - \frac{7424}{3} \right) - x^{13} \left(\frac{288a}{13} - \frac{16768}{13} \right) - x^8 (60a^2 - 1152a + 2048)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)

[Out] $x^{14} * ((24*a)/7 - 3712/7) - x^{15} * ((4*a)/15 - 512/3) + x^{12} * ((280*a)/3 - 7424/3) - x^{13} * ((288*a)/13 - 16768/13) - x^8 * (60*a^2 - 1152*a + 2048) - x^{10} * ((24*a^2)/5 - (3072*a)/5 + 21504/5) + x^9 * ((64*a^2)/3 - (8960*a)/9 + 32768/9) + x^{11} * ((6*a^2)/11 - (3072*a)/11 + 40960/11) - x^7 * ((6144*a)/7 - (768*a^2)/7 + (4*a^3)/7 - 4096/7) - 42*x^{16} + (128*x^{17})/17 - (8*x^{18})/9 + x^{19}/19 + 8*a^3*x^4 + (a^4*x^3)/3 + (8*a*x^6*(a^2 - 48*a + 128))/3 - (32*a^2*x^5*(a - 12))/5$

sympy [A] time = 0.15, size = 219, normalized size = 1.04

$$\frac{a^4 x^3}{3} + 8a^3 x^4 + \frac{x^{19}}{19} - \frac{8x^{18}}{9} + \frac{128x^{17}}{17} - 42x^{16} + x^{15} \left(\frac{512}{3} - \frac{4a}{15} \right) + x^{14} \left(\frac{24a}{7} - \frac{3712}{7} \right) + x^{13} \left(\frac{16768}{13} - \frac{288a}{13} \right) + x^{12} \left(\frac{280a}{3} - \frac{7424}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)

[Out] $a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16 + x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 288*a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/11) + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9 + 32768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2/7 - 6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-32*a**3/5 + 384*a**2/5)$

$$3.131 \quad \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$$

Optimal. Leaf size=138

$$\frac{a^3 x^3}{3} - \frac{3}{7} (a^2 - 128a + 512) x^7 + \frac{2}{3} (3a^2 - 96a + 128) x^6 + 6a^2 x^4 - \frac{3}{11} (256 - a) x^{11} + \frac{12}{5} (64 - a) x^{10} - \frac{32}{9} (70 - 3a) x^9 + 6(48 - 5a) x^8 - \frac{32}{9} (70 - 3a) x^9 + 6(48 - 5a) x^8 - \frac{32}{9} (70 - 3a) x^9 + 6(48 - 5a) x^8$$

[Out] 1/3*a^3*x^3+6*a^2*x^4+24/5*(8-a)*a*x^5+2/3*(3*a^2-96*a+128)*x^6-3/7*(a^2-128*a+512)*x^7+6*(48-5*a)*x^8-32/9*(70-3*a)*x^9+12/5*(64-a)*x^10-3/11*(256-a)*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6742}

$$-\frac{3}{7} (a^2 - 128a + 512) x^7 + \frac{2}{3} (3a^2 - 96a + 128) x^6 + 6a^2 x^4 + \frac{a^3 x^3}{3} - \frac{3}{11} (256 - a) x^{11} + \frac{12}{5} (64 - a) x^{10} - \frac{32}{9} (70 - 3a) x^9 + 6(48 - 5a) x^8 - \frac{32}{9} (70 - 3a) x^9 + 6(48 - 5a) x^8$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*x^9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx &= \int (a^3 x^2 + 24a^2 x^3 - 24(-8 + a)ax^4 + 4(128 - 96a + 3a^2)x^5 - 3(512 - 128a + a^2)x^6 - 6(48 - 5a)x^7 + 32(70 - 3a)x^8 - 32(70 - 3a)x^9 + 6(48 - 5a)x^8 - 32(70 - 3a)x^9 + 6(48 - 5a)x^8) dx \\ &= \frac{a^3 x^3}{3} + 6a^2 x^4 + \frac{24}{5}(8 - a)ax^5 + \frac{2}{3}(128 - 96a + 3a^2)x^6 - \frac{3}{7}(512 - 128a + a^2)x^7 + 6(48 - 5a)x^8 - \frac{32}{9}(70 - 3a)x^9 + 6(48 - 5a)x^8 - \frac{32}{9}(70 - 3a)x^9 + 6(48 - 5a)x^8 \end{aligned}$$

Mathematica [A] time = 0.02, size = 132, normalized size = 0.96

$$\frac{a^3 x^3}{3} - \frac{3}{7} (a^2 - 128a + 512) x^7 + \frac{2}{3} (3a^2 - 96a + 128) x^6 + 6a^2 x^4 + \frac{3}{11} (a - 256) x^{11} - \frac{12}{5} (a - 64) x^{10} + \frac{32}{9} (3a - 70) x^9 - 6(48 - 5a) x^8 - \frac{32}{9} (70 - 3a) x^9 + 6(48 - 5a) x^8$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] (a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x^8 + (32*(-70 + 3*a)*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 + a)*x^11)/11 + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15

fricas [A] time = 0.34, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{70}{3}x^{12} + \frac{3}{11}x^{11}a - \frac{768}{11}x^{11} - \frac{12}{5}x^{10}a + \frac{768}{5}x^{10} + \frac{32}{3}x^9a - \frac{2240}{9}x^9 - 30x^8a - \frac{3}{7}x^7a^2 + 288x^8 + \frac{384}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] $-1/15*x^{15} + 6/7*x^{14} - 72/13*x^{13} + 70/3*x^{12} + 3/11*x^{11}*a - 768/11*x^{11}$
 $- 12/5*x^{10}*a + 768/5*x^{10} + 32/3*x^9*a - 2240/9*x^9 - 30*x^8*a - 3/7*x^7*a$
 $^2 + 288*x^8 + 384/7*x^7*a + 2*x^6*a^2 - 1536/7*x^7 - 64*x^6*a - 24/5*x^5*a$
 $^2 + 256/3*x^6 + 192/5*x^5*a + 6*x^4*a^2 + 1/3*x^3*a^3$

giac [A] time = 0.29, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{3}{11}ax^{11} + \frac{70}{3}x^{12} - \frac{12}{5}ax^{10} - \frac{768}{11}x^{11} + \frac{32}{3}ax^9 + \frac{768}{5}x^{10} - \frac{3}{7}a^2x^7 - 30ax^8 - \frac{2240}{9}x^9 + 2a^2x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] $-1/15*x^{15} + 6/7*x^{14} - 72/13*x^{13} + 3/11*a*x^{11} + 70/3*x^{12} - 12/5*a*x^{10}$
 $- 768/11*x^{11} + 32/3*a*x^9 + 768/5*x^{10} - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9*x$
 $^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/7*x$
 $^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6$

maple [A] time = 0.00, size = 143, normalized size = 1.04

$$-\frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3} + \frac{(3a-768)x^{11}}{11} + \frac{(-24a+1536)x^{10}}{10} + \frac{(96a-2240)x^9}{9} + \frac{(-240a+2304)x^8}{8} + \frac{(-a^2-384)x^7}{7} + \frac{2a^2x^6}{5} + \frac{384ax^7}{7} + \frac{288x^8}{1} - \frac{24a^2x^5}{5} - \frac{64ax^6}{1} - \frac{1536x^7}{7} + \frac{1}{3}a^3x^3 + \frac{6a^2x^4}{1} + \frac{192ax^5}{5} + \frac{256x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)

[Out] $-1/15*x^{15} + 6/7*x^{14} - 72/13*x^{13} + 70/3*x^{12} + 1/11*(3*a-768)*x^{11} + 1/10*(-24*a+15$
 $36)*x^{10} + 1/9*(96*a-2240)*x^9 + 1/8*(-240*a+2304)*x^8 + 1/7*(-a^2+(-2*a+128)*a+2$
 $56*a-1536)*x^7 + 1/6*(4*a^2+(8*a-128)*a-256*a+512)*x^6 + 1/5*(-8*a^2+(-16*a+64)$
 $*a+128*a)*x^5 + 6*a^2*x^4 + 1/3*a^3*x^3$

maxima [A] time = 0.56, size = 113, normalized size = 0.82

$$-\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{3}{11}(a-256)x^{11} + \frac{70}{3}x^{12} - \frac{12}{5}(a-64)x^{10} + \frac{32}{9}(3a-70)x^9 - 6(5a-48)x^8 - \frac{3}{7}(a^2-128)x^7 + \frac{2a^2}{5}x^6 + \frac{384a}{7}x^7 + 288x^8 - \frac{24a^2}{5}x^5 - 64ax^6 - \frac{1536}{7}x^7 + \frac{1}{3}a^3x^3 + \frac{6a^2}{1}x^4 + \frac{192a}{5}x^5 + \frac{256}{3}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")

[Out] $-1/15*x^{15} + 6/7*x^{14} - 72/13*x^{13} + 3/11*(a-256)*x^{11} + 70/3*x^{12} - 12/5$
 $* (a-64)*x^{10} + 32/9*(3*a-70)*x^9 - 6*(5*a-48)*x^8 - 3/7*(a^2-128*a$
 $+ 512)*x^7 + 2/3*(3*a^2-96*a+128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*$
 $(a^2-8*a)*x^5$

mupad [B] time = 0.09, size = 113, normalized size = 0.82

$$x^{11} \left(\frac{3a}{11} - \frac{768}{11} \right) - x^{10} \left(\frac{12a}{5} - \frac{768}{5} \right) - x^8 (30a-288) + x^9 \left(\frac{32a}{3} - \frac{2240}{9} \right) + x^6 \left(2a^2 - 64a + \frac{256}{3} \right) - x^7 \left(\frac{3a^2}{7} - \frac{384a}{7} + \frac{1536}{7} \right) + \frac{70x^{12}}{3} - \frac{72x^{13}}{13} + \frac{6x^{14}}{7} - \frac{x^{15}}{15} + \frac{6a^2x^4}{1} + \frac{a^3x^3}{3} - \frac{24a^2x^5}{5} + \frac{192ax^5}{5} + \frac{256x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+8*x-8*x^2+4*x^3-x^4)^3,x)

[Out] $x^{11}*((3*a)/11 - 768/11) - x^{10}*((12*a)/5 - 768/5) - x^8*(30*a - 288) + x^9$
 $*((32*a)/3 - 2240/9) + x^6*(2*a^2 - 64*a + 256/3) - x^7*((3*a^2)/7 - (384*a$
 $)/7 + 1536/7) + (70*x^{12})/3 - (72*x^{13})/13 + (6*x^{14})/7 - x^{15}/15 + 6*a^2*x$
 $^4 + (a^3*x^3)/3 - (24*a*x^5*(a-8))/5$

sympy [A] time = 0.10, size = 134, normalized size = 0.97

$$\frac{a^3x^3}{3} + 6a^2x^4 - \frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3} + x^{11} \left(\frac{3a}{11} - \frac{768}{11} \right) + x^{10} \left(\frac{768}{5} - \frac{12a}{5} \right) + x^9 \left(\frac{32a}{3} - \frac{2240}{9} \right) + x^8 (288 - 30a) + x^7 (288 - 30a) + x^6 (288 - 30a) + x^5 (288 - 30a) + x^4 (288 - 30a) + x^3 (288 - 30a) + x^2 (288 - 30a) + x (288 - 30a) + (288 - 30a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)

[Out] a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/3 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240/9) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a**2 - 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)

$$3.132 \quad \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Optimal. Leaf size=79

$$\frac{a^2x^3}{3} + \frac{2}{7}(64-a)x^7 - \frac{4}{3}(16-a)x^6 + \frac{16}{5}(4-a)x^5 + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8$$

[Out] 1/3*a^2*x^3+4*a*x^4+16/5*(4-a)*x^5-4/3*(16-a)*x^6+2/7*(64-a)*x^7-10*x^8+32/9*x^9-4/5*x^10+1/11*x^11

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6742}

$$\frac{a^2x^3}{3} + \frac{2}{7}(64-a)x^7 - \frac{4}{3}(16-a)x^6 + \frac{16}{5}(4-a)x^5 + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 - a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx &= \int (a^2x^2 + 16ax^3 - 16(-4 + a)x^4 + 8(-16 + a)x^5 - 2(-64 + a)x^6 - 80x^7 \\ &+ \frac{a^2x^3}{3} + 4ax^4 + \frac{16}{5}(4-a)x^5 - \frac{4}{3}(16-a)x^6 + \frac{2}{7}(64-a)x^7 - 10x^8 + \frac{32x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3} - \frac{2}{7}(a-64)x^7 + \frac{4}{3}(a-16)x^6 - \frac{16}{5}(a-4)x^5 + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-64 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11

fricas [A] time = 0.36, size = 68, normalized size = 0.86

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - 10x^8 - \frac{2}{7}x^7a + \frac{128}{7}x^7 + \frac{4}{3}x^6a - \frac{64}{3}x^6 - \frac{16}{5}x^5a + \frac{64}{5}x^5 + 4x^4a + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 10*x^8 - 2/7*x^7*a + 128/7*x^7 + 4/3*x^6*a - 64/3*x^6 - 16/5*x^5*a + 64/5*x^5 + 4*x^4*a + 1/3*x^3*a^2

giac [A] time = 0.29, size = 68, normalized size = 0.86

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}ax^7 - 10x^8 + \frac{4}{3}ax^6 + \frac{128}{7}x^7 - \frac{16}{5}ax^5 - \frac{64}{3}x^6 + \frac{1}{3}a^2x^3 + 4ax^4 + \frac{64}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x^7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + \frac{(-2a+128)x^7}{7} + \frac{(8a-128)x^6}{6} + \frac{a^2x^3}{3} + 4ax^4 + \frac{(-16a+64)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] 1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+1/7*(-2*a+128)*x^7+1/6*(8*a-128)*x^6+1/5*(-16*a+64)*x^5+4*a*x^4+1/3*a^2*x^3

maxima [A] time = 0.55, size = 59, normalized size = 0.75

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}(a-64)x^7 - 10x^8 + \frac{4}{3}(a-16)x^6 - \frac{16}{5}(a-4)x^5 + \frac{1}{3}a^2x^3 + 4ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4

mupad [B] time = 0.04, size = 64, normalized size = 0.81

$$x^6 \left(\frac{4a}{3} - \frac{64}{3} \right) - x^5 \left(\frac{16a}{5} - \frac{64}{5} \right) - x^7 \left(\frac{2a}{7} - \frac{128}{7} \right) + 4ax^4 - 10x^8 + \frac{32x^9}{9} - \frac{4x^{10}}{5} + \frac{x^{11}}{11} + \frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] x^6*((4*a)/3 - 64/3) - x^5*((16*a)/5 - 64/5) - x^7*((2*a)/7 - 128/7) + 4*a*x^4 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11 + (a^2*x^3)/3

sympy [A] time = 0.08, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3} + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + x^7 \left(\frac{128}{7} - \frac{2a}{7} \right) + x^6 \left(\frac{4a}{3} - \frac{64}{3} \right) + x^5 \left(\frac{64}{5} - \frac{16a}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7*(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)

3.133 $\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4) dx$

Optimal. Leaf size=35

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {14}

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rubi steps

$$\begin{aligned} \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4) dx &= \int (ax^2 + 8x^3 - 8x^4 + 4x^5 - x^6) dx \\ &= \frac{ax^3}{3} + 2x^4 - \frac{8x^5}{5} + \frac{2x^6}{3} - \frac{x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7

fricas [A] time = 0.35, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + 2x^4 + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x), x, algorithm="fricas")

[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 2*x^4 + 1/3*x^3*a

giac [A] time = 0.31, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")

[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x)

[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7

maxima [A] time = 0.61, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")

[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4

mupad [B] time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4 + \frac{ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)

[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7

sympy [A] time = 0.06, size = 29, normalized size = 0.83

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)

[Out] a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4

$$3.134 \quad \int \frac{x^2}{a+8x-8x^2+4x^3-x^4} dx$$

Optimal. Leaf size=99

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}} - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

[Out] arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a)^(1/2)))^(1/2))/(1-(4+a)^(1/2))^(1/2)-1/2*arctan((-1+x)/(1+(4+a)^(1/2)))^(1/2)/(1+(4+a)^(1/2))^(1/2)

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1680, 1673, 1166, 204, 12, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}} - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[1 - Sqrt[4 + a]]) - ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=> Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :=> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + 8x - 8x^2 + 4x^3 - x^4} dx &= \text{Subst} \left(\int \frac{(1+x)^2}{3+a-2x^2-x^4} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \frac{2x}{3+a-2x^2-x^4} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{1+x^2}{3+a-2x^2-x^4} dx, x, -1+x \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1-\sqrt{4+a}-x^2} dx, x, -1+x \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+\sqrt{4+a}-x^2} dx, x, -1+x \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{1+\sqrt{4+a}}} + \text{Subst} \left(\int \frac{1}{3+a-2x-x^2} dx, x, (-1+x) \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{1+\sqrt{4+a}}} - 2 \text{Subst} \left(\int \frac{1}{4(4+a)-x^2} dx, x, -2(1-x) \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1} \left(\frac{1+(-1+x)^2}{\sqrt{4+a}} \right)}{\sqrt{4+a}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.62

$$-\frac{1}{4} \text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\#1^2 \log(x - \#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]
```

[Out] $-1/4*\text{RootSum}[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 \& , (\text{Log}[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) \&]$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")`

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")`

[Out] `integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)`

maple [C] time = 0.00, size = 52, normalized size = 0.53

$$\frac{\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^2 \ln(-\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a))}{4\left(\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^3 - 3\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^2 + 4\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x)`

[Out] $-1/4*\text{sum}(_R^2/(_R^3-3*_R^2+4*_R-2)*\ln(-_R+x), _R=\text{RootOf}(-Z^4-4*_Z^3+8*_Z^2-8*_Z-a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")`

[Out] `-integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)`

mupad [B] time = 2.78, size = 878, normalized size = 8.87

$$\sum_{k=1}^4 \ln\left(64 \text{root}\left(2816 a^2 z^4 + 256 a^3 z^4 + 10240 a z^4 + 12288 z^4 - 160 a^2 z^2 - 1152 a z^2 - 2048 z^2 + 32 a^2 z + 25\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)`

[Out] `symsum(log(64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k) - a - 8*x + 20*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a - 48*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^`

```

2*a + 64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^
2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a
+ 128*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*
z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*x -
256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^
2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*x - 1
92*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2
- 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2 + 256*r
oot(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 11
52*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3 - 4*root(28
16*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z
^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a*x + 32*root(2816*
a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2
- 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*a*x - 64*root(2816*a
^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 -
2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a*x)*root(2816*a^2*z^
4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048
*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k), k, 1, 4)

```

sympy [B] time = 7.61, size = 172, normalized size = 1.74

$$-\text{RootSum}\left(t^4(256a^3 + 2816a^2 + 10240a + 12288) + t^2(-160a^2 - 1152a - 2048) + t(-32a^2 - 256a - 512) - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)

```

[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2
- 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, Lambda(_t, _t*log(x
+ (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 6144*_
t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 56*_t
*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a**3 +
60*a**2 + 320*a + 448))))

```


$$3.135 \quad \int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2} dx$$

Optimal. Leaf size=225

$$\frac{(a+4)((x-1)^2+2)(x-1)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(a+\sqrt{a+4}+4)\tan^{-1}\left(\frac{x-1}{\sqrt{a+4}}\right)}{8(a+3)(a+4)\sqrt{a+4}}$$

[Out] 1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(4+a)*(2+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/2*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(4+a+(4+a)^(1/2))/(3+a)/(4+a)/(1-(4+a)^(1/2))^(1/2)-1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(4+a-(4+a)^(1/2))/(3+a)/(4+a)/(1+(4+a)^(1/2))^(1/2)

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1680, 1673, 1178, 1166, 204, 12, 1107, 614, 618, 206}

$$\frac{(a+4)((x-1)^2+2)(x-1)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(a+\sqrt{a+4}+4)\tan^{-1}\left(\frac{x-1}{\sqrt{a+4}}\right)}{8(a+3)(a+4)\sqrt{a+4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int

egerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1673

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rule 1680

Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx &= \text{Subst} \left(\int \frac{(1+x)^2}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \frac{2x}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{1+x^2}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \frac{(4+a)(2+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + 2 \text{Subst} \left(\int \frac{1+x^2}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \frac{(4+a)(2+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a-\sqrt{4+a}) \text{Subst} \left(\int \frac{1+x^2}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right)}{8(12+7a+a^2)} \\
&= \frac{1+(-1+x)^2}{2(4+a)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)(2+(-1+x)^2)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} \\
&= \frac{1+(-1+x)^2}{2(4+a)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)(2+(-1+x)^2)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} \\
&= \frac{1+(-1+x)^2}{2(4+a)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)(2+(-1+x)^2)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 182, normalized size = 0.81

$$\frac{a(x^3 - x^2 + x + 1) + 2x(2x^2 - 3x + 4)}{4(a+3)(a+4)(a-x(x^3 - 4x^2 + 8x - 8))} \frac{\text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a\&, \frac{\#1^2 a \log(x-\#1) + 4\#1^2 \log(x-\#1)}{16(a^2 + 7a + 12)} \right]}{16(a^2 + 7a + 12)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (-a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]*#1^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &]/(16*(12 + 7*a + a^2))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^4 - 4x^3 + 8x^2 - a - 8x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)

maple [C] time = 0.01, size = 160, normalized size = 0.71

$$\frac{\left((-a-4)\operatorname{RootOf}\left(_Z^4-4_Z^3+8_Z^2-8_Z-a\right)^2+2(-a-2)\operatorname{RootOf}\left(_Z^4-4_Z^3+8_Z^2-8_Z-a\right)+a\right)\ln\left(\frac{\operatorname{RootOf}\left(_Z^4-4_Z^3+8_Z^2-8_Z-a\right)^3-3\operatorname{RootOf}\left(_Z^4-4_Z^3+8_Z^2-8_Z-a\right)^2+4}{16(a+3)(a+4)}\right)}{16(a+3)(a+4)\left(\operatorname{RootOf}\left(_Z^4-4_Z^3+8_Z^2-8_Z-a\right)^3-3\operatorname{RootOf}\left(_Z^4-4_Z^3+8_Z^2-8_Z-a\right)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] (-1/4/(a+3)*x^3+1/4*(a+6)/(a+3)/(a+4)*x^2-1/4*(a+8)/(a+3)/(a+4)*x-1/4*a/(a+3)/(a+4))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a+3)/(a+4)*sum(((a+4)*_R^2+2*(a+2)*_R+a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a+4)x^3 - (a+6)x^2 + (a+8)x + a}{4\left(\left(a^2+7a+12\right)x^4 - 4\left(a^2+7a+12\right)x^3 - a^3 + 8\left(a^2+7a+12\right)x^2 - 7a^2 - 8\left(a^2+7a+12\right)x - 12a\right)} - \frac{\int \frac{(a+4)}{x^4-4}}{4\left(a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] -1/4*((a+4)*x^3 - (a+6)*x^2 + (a+8)*x + a)/((a^2+7*a+12)*x^4 - 4*(a^2+7*a+12)*x^3 - a^3 + 8*(a^2+7*a+12)*x^2 - 7*a^2 - 8*(a^2+7*a+12)*x - 12*a) - 1/4*integrate(((a+4)*x^2 + 2*(a+2)*x - a)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)

mupad [B] time = 2.85, size = 1218, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+8*x-8*x^2+4*x^3-x^4)^2,x)

[Out] symsum(log((x*(40*a+7*a^2+56))/(8*(816*a+460*a^2+129*a^3+18*a^4+a^5+576)) - (48*a+12*a^2-a^3)/(64*(816*a+460*a^2+129*a^3+18*a^4+a^5+576)) - root(12952010752*a^3*z^4+31653888*a^7*z^4+2162688*a^8*z^4+65536*a^9*z^4+18119393280*a*z^4+20082327552*a^2*z^4+1473773568*a^5*z^4+5357174784*a^4*z^4+269680640*a^6*z^4+7247757312*z^4-2421552*a^2*z^2-8986624*a^3*z^2-1878016*a^4*z^2-209408*a^5*z^2-9728*a^6*z^2-34865152*a*z^2-20971520*z^2+237568*a^2*z+53248*a^3*z+5888*a^4*z+256*a^5*z+524288*a*z+458752*z+1792*a+1024*a^2+144*a^3-a^4, z, k)*((28160*a+11328*a^2+2064*a^3+144*a^4+26624)/(64*(816*a+460*a^2+129*a^3+18*a^4+a^5+576)) + root(12952010752*a^3*z^4+31653888*a^7*z^4+2162688*a^8*z^4+65536*a^9*z^4+18119393280*a*z^4+20082327552*a^2*z^4+1473773568*a^5*z^4+5357174784*a^4*z^4+269680640*a^6*z^4+7247757312*z^4-24215552*a^2*z^2-8986624*a^3*z^2-1878016*a^4*z^2-209408*a^5*z^2-9728*a^6*z^2-34865152*a*z^2-20971520*z^2+237568*a^2*z+53248*a^3*z+5888*a^4*z+256*a^5*z+524288*a*z+458752*z+1792*a+1024*a^2+144*a^3-a^4, z, k)*(root(12952010752*a^3*z^4+31653888*a^7*z^4+2162688*a^8*z^4+65536*a^9*z^4+18119393280*a*z^4+20082327552*a^2*z^4+1473773568*a^5*z^4+5357174784*a^4*z^4+269680640*a^6*z^4+7247757312*z^4-24215552*a^2*z^2-8986624*a^3*z^2-1878016*a^4*z^2-209408*a^5*z^2-9728*a^6*z^2-34865152*a*z^2-20971520*z^2+237568*a^2*z+53248*a^3*z+5888*a^4*z+256*a^5*z+524288*a*z+458752*z+1792*a+1024*a^2+144*a^3-a^4, z, k)*(root(12952010752*a^3*z^4+31653888*a^7*z^4+2162688*a^8*z^4+65536*a^9*z^4+18119393280*a*z^4+20082327552*a^2*z^4+1473773568*a^5*z^4+5357174784*a^4*z^4+269680640*a^6*z^4+7247757312*z^4-24215552*a^2*z^2-8986624*a^3*z^2-1878016*a^4*z^2-209408*a^5*z^2-9728*a^6*z^2-34865152*a*z^2-20971520*z^2+237568*a^2*z+53248*a^3*z+5888*a^4*z+256*a^5*z+524288*a*z+458752*z+1792*a+1024*a^2+144*a^3-a^4, z, k))

$$z^4 - 24215552a^2z^2 - 8986624a^3z^2 - 1878016a^4z^2 - 209408a^5z^2 - 9728a^6z^2 - 34865152az^2 - 20971520z^2 + 237568a^2z + 53248a^3z + 5888a^4z + 256a^5z + 524288az + 458752z + 1792a + 1024a^2 + 144a^3 - a^4, z, k) * ((15728640a + 10878976a^2 + 3997696a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 9437184) / (64 * (816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) - (x * (1966080a + 1359872a^2 + 499712a^3 + 102912a^4 + 11264a^5 + 512a^6 + 1179648)) / (8 * (816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (1359872a + 749568a^2 + 205824a^3 + 28160a^4 + 1536a^5 + 983040) / (64 * (816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) + (x * (104448a + 58880a^2 + 16512a^3 + 2304a^4 + 128a^5 + 73728)) / (8 * (816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) + (x * (448a + 104a^2 - 2a^3 - 2a^4 + 512)) / (8 * (816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) * root(12952010752a^3z^4 + 31653888a^7z^4 + 2162688a^8z^4 + 65536a^9z^4 + 18119393280a^2z^4 + 20082327552a^2z^4 + 1473773568a^5z^4 + 5357174784a^4z^4 + 269680640a^6z^4 + 7247757312z^4 - 24215552a^2z^2 - 8986624a^3z^2 - 1878016a^4z^2 - 209408a^5z^2 - 9728a^6z^2 - 34865152az^2 - 20971520z^2 + 237568a^2z + 53248a^3z + 5888a^4z + 256a^5z + 524288az + 458752z + 1792a + 1024a^2 + 144a^3 - a^4, z, k), k, 1, 4) + (x^3 / (4 * (a + 3))) + a / (4 * (a + 3) * (a + 4)) - (x^2 * (a + 6)) / (4 * (a + 3) * (a + 4)) + (x * (a + 8)) / (4 * (a + 3) * (a + 4))) / (a + 8x - 8x^2 + 4x^3 - x^4)$$

sympy [B] time = 43.73, size = 561, normalized size = 2.49

$$\frac{-a + x^3(-a - 4) + x^2(a + 6) + x(-a - 8)}{-4a^3 - 28a^2 - 48a + x^4(4a^2 + 28a + 48) + x^3(-16a^2 - 112a - 192) + x^2(32a^2 + 224a + 384) + x(-32a^2 - 224a - 384)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] (-a + x**3*(-a - 4) + x**2*(a + 6) + x*(-a - 8)) / (-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-9728*a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24215552*a**2 - 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a**3 + 237568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2 + 1792*a, Lambda(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a**11 - 5480448*_t**3*a**10 - 111403008*_t**3*a**9 - 1227173888*_t**3*a**8 - 8682876928*_t**3*a**7 - 42187440128*_t**3*a**6 - 144630284288*_t**3*a**5 - 350972280832*_t**3*a**4 - 591750234112*_t**3*a**3 - 660716126208*_t**3*a**2 - 439848271872*_t**3*a - 132271570944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9 - 15400960*_t**2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 3414427648*_t**2*a**5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 22938255360*_t**2*a**2 - 16873684992*_t**2*a - 5549064192*_t**2 - 848*_t*a**9 - 6096*_t*a**8 + 174608*_t*a**7 + 3323792*_t*a**6 + 26276224*_t*a**5 + 119009280*_t*a**4 + 332017664*_t*a**3 + 566497280*_t*a**2 + 544112640*_t*a + 225837056*_t + 11*a**8 + 958*a**7 + 17419*a**6 + 142964*a**5 + 632632*a**4 + 1567552*a**3 + 2049792*a**2 + 1100800*a) / (a**8 + 870*a**7 + 18289*a**6 + 165176*a**5 + 824560*a**4 + 2452288*a**3 + 4340224*a**2 + 4229120*a + 1748992)))

$$3.136 \quad \int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=545

$$\frac{\sqrt[3]{-1} (3\sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b) \tan^{-1} \left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right) (2b - 3\sqrt[3]{a} c^{2/3}) \tan^{-1} \left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \right) (-1)^{2/3}}{3\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{5/6} b^2 c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}} \quad 9\sqrt{3} a^{5/6} b^2 c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}} \quad 3\sqrt{3} (1 - \sqrt[3]{-1})^2 a^{5/6} b^2 c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}$$

[Out] $-1/18 \ln(3a+3a^{2/3}c^{1/3}x+b*x^2)/a^{2/3}/b^2/c^{1/3}+1/6 \ln(3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+b*x^2)/(1+(-1)^{1/3})^2/a^{2/3}/b^2/c^{1/3}+1/18 *(-1)^{1/3} \ln(3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+b*x^2)/a^{2/3}/b^2/c^{1/3} -1/27*(2b-3a^{1/3}c^{2/3}) \arctan(1/3*(3a^{2/3}c^{1/3}+2b*x)*3^{1/2}/a^{1/2}/(4b-3a^{1/3}c^{2/3})^{1/2})/a^{5/6}/b^2/c^{2/3}*3^{1/2}/(4b-3a^{1/3}c^{2/3})^{1/2}-1/9*(-1)^{2/3}*(2b+3(-1)^{1/3}a^{1/3}c^{2/3}) \arctan(1/3*(3(-1)^{2/3}a^{2/3}c^{1/3}+2b*x)*3^{1/2}/a^{1/2}/(4b+3(-1)^{1/3}a^{1/3}c^{2/3})^{1/2})/(1-(-1)^{1/3})/(1+(-1)^{1/3})^2/a^{5/6}/b^2/c^{2/3}*3^{1/2}/(4b+3(-1)^{1/3}a^{1/3}c^{2/3})^{1/2}-1/9*(-1)^{1/3}*(2(-1)^{1/3}b+3a^{1/3}c^{2/3}) \arctan(1/3*(3(-1)^{1/3}a^{2/3}c^{1/3}-2b*x)*3^{1/2}/a^{1/2}/(4b-3(-1)^{2/3}a^{1/3}c^{2/3})^{1/2})/(1+(-1)^{1/3})^2/a^{5/6}/b^2/c^{2/3}*3^{1/2}/(4b-3(-1)^{2/3}a^{1/3}c^{2/3})^{1/2}$

Rubi [A] time = 1.48, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\sqrt[3]{-1} (3\sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b) \tan^{-1} \left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right) (2b - 3\sqrt[3]{a} c^{2/3}) \tan^{-1} \left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \right) (-1)^{2/3}}{3\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{5/6} b^2 c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}} \quad 9\sqrt{3} a^{5/6} b^2 c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}} \quad 3\sqrt{3} (1 - \sqrt[3]{-1})^2 a^{5/6} b^2 c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] $-((-1)^{1/3}*(2*(-1)^{1/3}*b + 3*a^{1/3}*c^{2/3})*\text{ArcTan}[(3*(-1)^{1/3}a^{2/3}c^{1/3} - 2b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4b - 3*(-1)^{2/3}a^{1/3}c^{2/3}]])]/(3*\text{Sqrt}[3]*(1 + (-1)^{1/3})^2*a^{5/6}*b^2*\text{Sqrt}[4b - 3*(-1)^{2/3}a^{1/3}c^{2/3}]) - ((2b - 3*a^{1/3}c^{2/3})*\text{ArcTan}[(3*a^{2/3}c^{1/3} + 2b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4b - 3*a^{1/3}c^{2/3}]])]/(9*\text{Sqrt}[3]*a^{5/6}*b^2*\text{Sqrt}[4b - 3*a^{1/3}c^{2/3}]) - ((-1)^{2/3}*(2b + 3*(-1)^{1/3}a^{1/3}c^{2/3})*\text{ArcTan}[(3*(-1)^{2/3}a^{2/3}c^{1/3} + 2b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4b + 3*(-1)^{1/3}a^{1/3}c^{2/3}]])]/(3*\text{Sqrt}[3]*(1 - (-1)^{1/3})*(1 + (-1)^{1/3})^2*a^{5/6}*b^2*\text{Sqrt}[4b + 3*(-1)^{1/3}a^{1/3}c^{2/3}]) - \text{Log}[3*a + 3*a^{2/3}c^{1/3}x + b*x^2]/(18*a^{2/3}*b^2*c^{1/3}) + \text{Log}[3*a - 3*(-1)^{1/3}a^{2/3}c^{1/3}x + b*x^2]/(6*(1 + (-1)^{1/3})^2*a^{2/3}*b^2*c^{1/3}) + ((-1)^{1/3}*\text{Log}[3*a + 3*(-1)^{2/3}a^{2/3}c^{1/3}x + b*x^2])/(18*a^{2/3}*b^2*c^{1/3})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_ + (c_)*(x_)^2)), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_ + (c_)*(x_)^2)), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2097

$\text{Int}[(Q6_)^p*(u_), x_Symbol] \rightarrow \text{With}[\{a = \text{Coeff}[Q6, x, 0], b = \text{Coeff}[Q6, x, 2], c = \text{Coeff}[Q6, x, 3], d = \text{Coeff}[Q6, x, 4], e = \text{Coeff}[Q6, x, 6]\}, \text{Dist}[1/(3^(3*p)*a^(2*p)), \text{Int}[\text{ExpandIntegrand}[u*(3*a + 3*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p, x], x] /; \text{EqQ}[b^2 - 3*a*d, 0] \&\& \text{EqQ}[b^3 - 27*a^2*e, 0]] /; \text{ILtQ}[p, 0] \&\& \text{PolyQ}[Q6, x, 6] \&\& \text{EqQ}[\text{Coeff}[Q6, x, 1], 0] \&\& \text{EqQ}[\text{Coeff}[Q6, x, 5], 0] \&\& \text{RationalFunctionQ}[u, x]$

Rubi steps

$$\begin{aligned} \int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx &= (19683a^6) \int \left(\frac{-(-1)^{2/3} \sqrt[3]{a} - \sqrt[3]{c} x}{59049 (1 + \sqrt[3]{-1})^2 a^{20/3} bc^{2/3} (-3a + 3\sqrt[3]{-1} - \sqrt[3]{c} x)} \right. \\ &= \frac{\int \frac{-\sqrt[3]{a} - \sqrt[3]{c} x}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{9a^{2/3} bc^{2/3}} - \frac{(-1)^{2/3} \int \frac{\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{c} x}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + bx^2} dx}{9a^{2/3} bc^{2/3}} + \\ &= \frac{\left(3 - \frac{2b}{\sqrt[3]{a} c^{2/3}}\right) \int \frac{1}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{18b^2} + \frac{\left(3 - \frac{2(-1)^{2/3} b}{\sqrt[3]{a} c^{2/3}}\right) \int \frac{1}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + bx^2} dx}{18b^2} \\ &= -\frac{\log(3a + 3a^{2/3} \sqrt[3]{c} x + bx^2)}{18a^{2/3} b^2 \sqrt[3]{c}} + \frac{\log(3a - 3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} x + bx^2)}{6(1 + \sqrt[3]{-1})^2 a^{2/3} b^2 \sqrt[3]{c}} \\ &= -\frac{(3ib + \sqrt{3} (b + 3\sqrt[3]{a} c^{2/3})) \tan^{-1}\left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{27a^{5/6} b^2 \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \end{aligned}$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.18

$$\frac{1}{3} \text{RootSum} \left[\#1^6 b^3 + 9\#1^4 ab^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \&, \frac{\#1^3 \log(x - \#1)}{2\#1^4 b^3 + 12\#1^2 ab^2 + 27\#1 a^2 c + 18a^2 b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 & , (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &]/3

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.02, size = 93, normalized size = 0.17

$$\frac{\text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3)}{6\text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27b^2a^2_Z^2 + 27a^3)^5 b^3 + 36\text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] 1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x), _R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

mupad [B] time = 3.18, size = 1563, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)

[Out] symsum(log(-19683*a^8*b^3*(c*x - b + 6561*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z

$$\begin{aligned}
&^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2* \\
&a^3*c^4 + 2*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 15 \\
&94323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 3280 \\
&5*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*b^4*x - 198*root(918330048*a^5*b \\
&^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516* \\
&a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z \\
&+ 1, z, k)*a*b^2*c - 8991*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^ \\
&6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4* \\
&c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^3*c^2 - 19 \\
&683*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^ \\
&4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^ \\
&2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^4*c^3 + 104976*root(918330048*a^ \\
&5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 10235 \\
&16*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c \\
&*z + 1, z, k)^4*a^3*b^8*c^2 - 8503056*root(918330048*a^5*b^9*c^4*z^6 - 3874 \\
&20489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - \\
&531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4* \\
&b^9*c^3 + 4782969*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^ \\
&6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 \\
&+ 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^5*b^6*c^5 + 108*root(9 \\
&18330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4* \\
&z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 \\
&+ 324*a*b*c*z + 1, z, k)^2*a*b^5*c*x + 108*root(918330048*a^5*b^9*c^4*z^6 - \\
&387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3* \\
&z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*a \\
&*b*c^2*x + 1458*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 \\
&+ 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + \\
&32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^2*c^3*x - 2916*root(\\
&918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4 \\
&*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 \\
&+ 324*a*b*c*z + 1, z, k)^3*a^2*b^6*c^2*x + 78732*root(918330048*a^5*b^9*c^ \\
&4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b \\
&^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, \\
&z, k)^4*a^3*b^7*c^3*x + 1062882*root(918330048*a^5*b^9*c^4*z^6 - 387420489* \\
&a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 53144 \\
&1*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*b^8*c^ \\
&4*x))*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323* \\
&a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2* \\
&b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k), k, 1, 6)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)

[Out] Timed out

$$3.137 \quad \int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=487

$$\frac{\log(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3} \log(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{18(1 + \sqrt[3]{-1})^2 a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3} \log(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{54a^{4/3}bc^{2/3}}$$

[Out] 1/54*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c^(2/3)-1/18*(-1)^(2/3)*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(4/3)/b/c^(2/3)+1/54*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c^(2/3)-1/27*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/9*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/9*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)

Rubi [A] time = 0.76, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\log(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3} \log(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{18(1 + \sqrt[3]{-1})^2 a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3} \log(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{54a^{4/3}bc^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7/6)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(18*(1 + (-1)^(1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx = (19683a^6) \int \left(\frac{(-1)^{2/3}x}{177147(1 + \sqrt[3]{-1})^2 a^{22/3}c^{2/3}(-3a + 3\sqrt[3]{-1})} \right. \\ = \frac{\int \frac{x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2} dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3} \int \frac{x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2} dx}{27a^{4/3}c^{2/3}} + \\ = \frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2} dx}{54a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3} \int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2} dx}{54a^{4/3}bc^{2/3}} \\ = \frac{\log(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3} \log(3a - 3\sqrt[3]{-1} a^{2/3}x + bx^2)}{18(1 + \sqrt[3]{-1})^2 a^{4/3}bc^{2/3}} \\ = -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}(1 + \sqrt[3]{-1})^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}(1 + \sqrt[3]{-1})^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}$$

Mathematica [C] time = 0.05, size = 99, normalized size = 0.20

$$\frac{1}{3} \text{RootSum} \left[\#1^6 b^3 + 9\#1^4 ab^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \& \epsilon, \frac{\#1^2 \log(x - \#1)}{2\#1^4 b^3 + 12\#1^2 ab^2 + 27\#1 a^2 c + 18a^2 b} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```


$$\begin{aligned}
& 3c^6z^6 - 24794911296a^8b^6c^4z^6 - 14348907a^6b^2c^4z^4 + 314928 \\
& a^4b^3c^2z^3 - 531441a^5c^4z^3 - 19683a^3b^2c^2z^2 - 1, z, k)^4a^ \\
& 11b^{10}c^3 + 167365651248\text{root}(10460353203a^9b^3c^6z^6 - 24794911296a^ \\
& 8b^6c^4z^6 - 14348907a^6b^2c^4z^4 + 314928a^4b^3c^2z^3 - 531441 \\
& a^5c^4z^3 - 19683a^3b^2c^2z^2 - 1, z, k)^5a^{12}b^{12}c^3 - 94143178827 \\
& \text{root}(10460353203a^9b^3c^6z^6 - 24794911296a^8b^6c^4z^6 - 14348907a^ \\
& 6b^2c^4z^4 + 314928a^4b^3c^2z^3 - 531441a^5c^4z^3 - 19683a^3b^ \\
& c^2z^2 - 1, z, k)^5a^{13}b^9c^5 + 98415\text{root}(10460353203a^9b^3c^6z^6 \\
& - 24794911296a^8b^6c^4z^6 - 14348907a^6b^2c^4z^4 + 314928a^4b^3c^ \\
& c^2z^3 - 531441a^5c^4z^3 - 19683a^3b^2c^2z^2 - 1, z, k)a^7b^7c + 4 \\
& 374\text{root}(10460353203a^9b^3c^6z^6 - 24794911296a^8b^6c^4z^6 - 143489 \\
& 07a^6b^2c^4z^4 + 314928a^4b^3c^2z^3 - 531441a^5c^4z^3 - 19683a^ \\
& 3b^2c^2z^2 - 1, z, k)a^6b^9x - 2125764\text{root}(10460353203a^9b^3c^6z^6 \\
& - 24794911296a^8b^6c^4z^6 - 14348907a^6b^2c^4z^4 + 314928a^4b^3c^ \\
& c^2z^3 - 531441a^5c^4z^3 - 19683a^3b^2c^2z^2 - 1, z, k)^2a^8b^9c - \\
& 59049\text{root}(10460353203a^9b^3c^6z^6 - 24794911296a^8b^6c^4z^6 - 143 \\
& 48907a^6b^2c^4z^4 + 314928a^4b^3c^2z^3 - 531441a^5c^4z^3 - 19683 \\
& a^3b^2c^2z^2 - 1, z, k)a^7b^6c^2x - 531441\text{root}(10460353203a^9b^3c^ \\
& 6z^6 - 24794911296a^8b^6c^4z^6 - 14348907a^6b^2c^4z^4 + 314928a^ \\
& 4b^3c^2z^3 - 531441a^5c^4z^3 - 19683a^3b^2c^2z^2 - 1, z, k)^2a^8b^ \\
& 8c^2x - 688747536\text{root}(10460353203a^9b^3c^6z^6 - 24794911296a^8b^6 \\
& c^4z^6 - 14348907a^6b^2c^4z^4 + 314928a^4b^3c^2z^3 - 531441a^5c^ \\
& 4z^3 - 19683a^3b^2c^2z^2 - 1, z, k)^4a^{10}b^{12}c^2x + 1162261467\text{root} \\
& (10460353203a^9b^3c^6z^6 - 24794911296a^8b^6c^4z^6 - 14348907a^6b^ \\
& 2c^4z^4 + 314928a^4b^3c^2z^3 - 531441a^5c^4z^3 - 19683a^3b^2c^2z^ \\
& z^2 - 1, z, k)^4a^{11}b^9c^4x - 20920706406\text{root}(10460353203a^9b^3c^6 \\
& z^6 - 24794911296a^8b^6c^4z^6 - 14348907a^6b^2c^4z^4 + 314928a^4b^ \\
& 3c^2z^3 - 531441a^5c^4z^3 - 19683a^3b^2c^2z^2 - 1, z, k)^5a^{12}b^1 \\
& 1c^4x)\text{root}(10460353203a^9b^3c^6z^6 - 24794911296a^8b^6c^4z^6 - 1 \\
& 4348907a^6b^2c^4z^4 + 314928a^4b^3c^2z^3 - 531441a^5c^4z^3 - 196 \\
& 83a^3b^2c^2z^2 - 1, z, k), k, 1, 6)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)

[Out] Timed out

$$3.138 \quad \int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=334

$$\frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} + \frac{2 \tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}} + \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3(-1)^{1/3}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}$$

[Out] $2/81*\arctan(1/3*(3*a^{(2/3)}*c^{(1/3)}+2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b-3*a^{(1/3)}*c^{(2/3)})^{(1/2)})/a^{(11/6)}/c^{(2/3)}*3^{(1/2)}/(4*b-3*a^{(1/3)}*c^{(2/3)})^{(1/2)}+2/27*(-1)^{(2/3)}*\arctan(1/3*(3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)}+2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b+3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)})/(1-(-1)^{(1/3)})/(1+(-1)^{(1/3)})^2/a^{(11/6)}/c^{(2/3)}*3^{(1/2)}/(4*b+3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)}+2/27*(-1)^{(2/3)}*\arctan(1/3*(3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)}-2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b-3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)})/(1+(-1)^{(1/3)})^2/a^{(11/6)}/c^{(2/3)}*3^{(1/2)}/(4*b-3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)}$

Rubi [A] time = 0.47, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2097, 618, 204}

$$\frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} + \frac{2 \tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}} + \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3(-1)^{1/3}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]

[Out] $(2*(-1)^{(2/3)}*\text{ArcTan}[(3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)} - 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)}]])/(9*\text{Sqrt}[3]*(1 + (-1)^{(1/3)})^2*a^{(11/6)}*\text{Sqrt}[4*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)} + (2*\text{ArcTan}[(3*a^{(2/3)}*c^{(1/3)} + 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b - 3*a^{(1/3)}*c^{(2/3)}]])/(27*\text{Sqrt}[3]*a^{(11/6)}*\text{Sqrt}[4*b - 3*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)} + (2*(-1)^{(2/3)}*\text{ArcTan}[(3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)} + 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)}]])/(9*\text{Sqrt}[3]*(1 - (-1)^{(1/3)})*(1 + (-1)^{(1/3)})^2*a^{(11/6)}*\text{Sqrt}[4*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0]

] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx = (19683a^6) \int \left(\frac{(-1)^{2/3}}{177147 (1 + \sqrt[3]{-1})^2 a^{22/3} c^{2/3} (-3a + 3\sqrt[3]{-1})} \right. \\ = \frac{\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2} dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3} \int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2} dx}{27a^{4/3}c^{2/3}} + \\ = -\frac{2 \operatorname{Subst}\left(\int \frac{1}{-3a(4b-3\sqrt[3]{a}c^{2/3})-x^2} dx, x, 3a^{2/3}\sqrt[3]{c} + 2bx\right)}{27a^{4/3}c^{2/3}} - \\ = \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}(1+\sqrt[3]{-1})^2 a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}} c^{2/3}} + \frac{2 \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}}$$

Mathematica [C] time = 0.06, size = 97, normalized size = 0.29

$$\frac{1}{3} \operatorname{RootSum}\left[\#1^6 b^3 + 9\#1^4 ab^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \&, \frac{\#1 \log(x - \#1)}{2\#1^4 b^3 + 12\#1^2 ab^2 + 27\#1 a^2 c + 18a^2 b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &, (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &]/3

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3), x, algorithm="giac")

[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.00, size = 93, normalized size = 0.28

$$\frac{\text{RootOf}(b^3 Z^6 + 9b^2 a Z^4 + 27a^2 c Z^3 + 27b a^2 Z^2 + 27a^3 Z) b^3 + 36 \text{RootOf}(b^3 Z^6 + 9b^2 a Z^4 + 27a^2 c Z^3 + 27b a^2 Z^2 + 27a^3 Z)^5}{6 \text{RootOf}(b^3 Z^6 + 9b^2 a Z^4 + 27a^2 c Z^3 + 27b a^2 Z^2 + 27a^3 Z)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] 1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b^3 x^6 + 9 a b^2 x^4 + 27 a^2 c x^3 + 27 a^2 b x^2 + 27 a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

mupad [B] time = 3.34, size = 825, normalized size = 2.47

$$\sum_{k=1}^6 \ln\left(-a^3 b^9 \left(-\text{root}\left(669462604992 a^{11} b^3 c^4 z^6 - 282429536481 a^{12} c^6 z^6 + 129140163 a^8 c^4 z^4 - 19683 a^4 c^2 z^2 - 19683 a^4 c^2 z^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)

[Out] symsum(log(-27*a^3*b^9*(43046721*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^8*c^4 - 1062882*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^6*c^3 - 13122*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^4*c^2 + 3486784401*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^10*c^5 + 81*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)*a^2*c + 18*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)*a*b^2*x - 25509168*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^3*c^2 - 6198727824*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^3*c^3 + 5832*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^3*b^2*c*x + 708588*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^5*b^2*c^2*x + 38263752*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^2*c^3*x + 774840978*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^2*c^4*x + 1))*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k), k, 1, 6)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)

[Out] Timed out

$$3.139 \quad \int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=469

$$-\frac{\log(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3} \log(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{54(1 + \sqrt[3]{-1})^2 a^{7/3}c^{2/3}} - \frac{(-1)^{2/3} \log(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{162a^{7/3}c^{2/3}}$$

[Out] $-1/162*\ln(3*a+3*a^{(2/3)}*c^{(1/3)}*x+b*x^2)/a^{(7/3)}/c^{(2/3)}+1/54*(-1)^{(2/3)}*\ln(3*a-3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)}*x+b*x^2)/(1+(-1)^{(1/3)})^2/a^{(7/3)}/c^{(2/3)}-1/162*(-1)^{(2/3)}*\ln(3*a+3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)}*x+b*x^2)/a^{(7/3)}/c^{(2/3)}-1/81*\arctan(1/3*(3*a^{(2/3)}*c^{(1/3)}+2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b-3*a^{(1/3)}*c^{(2/3)})^{(1/2)})/a^{(13/6)}/c^{(1/3)}*3^{(1/2)}/(4*b-3*a^{(1/3)}*c^{(2/3)})^{(1/2)}+1/27*(-1)^{(1/3)}*\arctan(1/3*(3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)}+2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b+3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)})/(1-(-1)^{(1/3)})/(1+(-1)^{(1/3)})^2/a^{(13/6)}/c^{(1/3)}*3^{(1/2)}/(4*b+3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)}-1/27*\arctan(1/3*(3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)}-2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b-3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)})/(1+(-1)^{(1/3)})^2/a^{(13/6)}/c^{(1/3)}*3^{(1/2)}/(4*b-3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)}$

Rubi [A] time = 0.68, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\log(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3} \log(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{54(1 + \sqrt[3]{-1})^2 a^{7/3}c^{2/3}} - \frac{(-1)^{2/3} \log(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + 3a + bx^2)}{162a^{7/3}c^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]

[Out] $-\text{ArcTan}[(3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)} - 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)}])]/(9*\text{Sqrt}[3]*(1 + (-1)^{(1/3)})^2*a^{(13/6)}*\text{Sqrt}[4*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)}]*c^{(1/3)}) - \text{ArcTan}[(3*a^{(2/3)}*c^{(1/3)} + 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b - 3*a^{(1/3)}*c^{(2/3)}])]/(27*\text{Sqrt}[3]*a^{(13/6)}*\text{Sqrt}[4*b - 3*a^{(1/3)}*c^{(2/3)}]*c^{(1/3)}) + ((-1)^{(1/3)}*\text{ArcTan}[(3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)} + 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)}])])]/(9*\text{Sqrt}[3]*(1 - (-1)^{(1/3)})*(1 + (-1)^{(1/3)})^2*a^{(13/6)}*\text{Sqrt}[4*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)}]*c^{(1/3)}) - \text{Log}[3*a + 3*a^{(2/3)}*c^{(1/3)}*x + b*x^2]/(162*a^{(7/3)}*c^{(2/3)}) + ((-1)^{(2/3)}*\text{Log}[3*a - 3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)}*x + b*x^2])/(54*(1 + (-1)^{(1/3)})^2*a^{(7/3)}*c^{(2/3)}) - ((-1)^{(2/3)}*\text{Log}[3*a + 3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)}*x + b*x^2])/(162*a^{(7/3)}*c^{(2/3)})$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx = (19683a^6) \int \left(\frac{-3a^{2/3} \sqrt[3]{c} - (-1)^{2/3} bx}{531441 (1 + \sqrt[3]{-1})^2 a^{25/3} c^{2/3} (-3a + 3\sqrt[3]{-1} a^2)} \right. \\ = \frac{\int \frac{-3a^{2/3} \sqrt[3]{c} - bx}{3a + 3a^{2/3} \sqrt[3]{c}x + bx^2} dx}{81a^{7/3} c^{2/3}} - \frac{(-1)^{2/3} \int \frac{3(-1)^{2/3} a^{2/3} \sqrt[3]{c} + bx}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c}x + bx^2} dx}{81a^{7/3} c^{2/3}} + \\ = -\frac{\int \frac{3a^{2/3} \sqrt[3]{c} + 2bx}{3a + 3a^{2/3} \sqrt[3]{c}x + bx^2} dx}{162a^{7/3} c^{2/3}} - \frac{(-1)^{2/3} \int \frac{3(-1)^{2/3} a^{2/3} \sqrt[3]{c} + 2bx}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c}x + bx^2} dx}{162a^{7/3} c^{2/3}} \\ = -\frac{\log(3a + 3a^{2/3} \sqrt[3]{c}x + bx^2)}{162a^{7/3} c^{2/3}} + \frac{(-1)^{2/3} \log(3a - 3\sqrt[3]{-1} a^2)}{54 (1 + \sqrt[3]{-1})^2 a^7} \\ = -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{9\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{13/6} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3} \sqrt[3]{c}}} - \frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{27\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{13/6} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3} \sqrt[3]{c}}}$$

Mathematica [C] time = 0.05, size = 95, normalized size = 0.20

$$\frac{1}{3} \text{RootSum} \left[\#1^6 b^3 + 9\#1^4 ab^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \&, \frac{\log(x - \#1)}{2\#1^4 b^3 + 12\#1^2 ab^2 + 27\#1 a^2 c + 18a^2 b} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),
x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ]/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.00, size = 91, normalized size = 0.19

$$\frac{\text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3)}{6\text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27b^2a_Z^2 + 27a^3)^5 b^3 + 36\text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] 1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

mupad [B] time = 2.90, size = 1057, normalized size = 2.25

$$\sum_{k=1}^6 \ln \left(b^{12} x + \text{root} \left(18075490334784 a^{14} b^3 c^4 z^6 - 7625597484987 a^{15} c^6 z^6 + 1162261467 a^{10} b c^4 z^4 + 8503056 \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)

[Out] symsum(log(b^12*x + 1033121304*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4*a^10*b^11*c^3 + 167365651248*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a

$$\begin{aligned}
& ^8c^4z^3 + 177147a^5b^2c^2z^2 + b^3, z, k)^5a^{12}b^{12}c^3 - 94143178 \\
& 827\text{root}(18075490334784a^{14}b^3c^4z^6 - 7625597484987a^{15}c^6z^6 + 116 \\
& 2261467a^{10}b^4c^4z^4 + 8503056a^7b^3c^2z^3 - 14348907a^8c^4z^3 + 1 \\
& 77147a^5b^2c^2z^2 + b^3, z, k)^5a^{13}b^9c^5 + 54\text{root}(18075490334784* \\
& a^{14}b^3c^4z^6 - 7625597484987a^{15}c^6z^6 + 1162261467a^{10}b^4c^4z^4 + \\
& 8503056a^7b^3c^2z^3 - 14348907a^8c^4z^3 + 177147a^5b^2c^2z^2 + \\
& b^3, z, k)a^2b^{13}x + 177147\text{root}(18075490334784a^{14}b^3c^4z^6 - 76255 \\
& 97484987a^{15}c^6z^6 + 1162261467a^{10}b^4c^4z^4 + 8503056a^7b^3c^2z^3 \\
& - 14348907a^8c^4z^3 + 177147a^5b^2c^2z^2 + b^3, z, k)^2a^5b^{11}c^ \\
& 2x + 17006112\text{root}(18075490334784a^{14}b^3c^4z^6 - 7625597484987a^{15}c^ \\
& 6z^6 + 1162261467a^{10}b^4c^4z^4 + 8503056a^7b^3c^2z^3 - 14348907a^8* \\
& c^4z^3 + 177147a^5b^2c^2z^2 + b^3, z, k)^3a^7b^{12}c^2x - 14348907*r \\
& oot(18075490334784a^{14}b^3c^4z^6 - 7625597484987a^{15}c^6z^6 + 11622614 \\
& 67a^{10}b^4c^4z^4 + 8503056a^7b^3c^2z^3 - 14348907a^8c^4z^3 + 177147 \\
& *a^5b^2c^2z^2 + b^3, z, k)^3a^8b^9c^4x + 229582512\text{root}(180754903347 \\
& 84a^{14}b^3c^4z^6 - 7625597484987a^{15}c^6z^6 + 1162261467a^{10}b^4c^4z^ \\
& 4 + 8503056a^7b^3c^2z^3 - 14348907a^8c^4z^3 + 177147a^5b^2c^2z^2 \\
& + b^3, z, k)^4a^9b^{13}c^2x + 387420489\text{root}(18075490334784a^{14}b^3c^4 \\
& *z^6 - 7625597484987a^{15}c^6z^6 + 1162261467a^{10}b^4c^4z^4 + 8503056a^7 \\
& *b^3c^2z^3 - 14348907a^8c^4z^3 + 177147a^5b^2c^2z^2 + b^3, z, k)^4 \\
& *a^{10}b^{10}c^4x - 20920706406\text{root}(18075490334784a^{14}b^3c^4z^6 - 76255 \\
& 97484987a^{15}c^6z^6 + 1162261467a^{10}b^4c^4z^4 + 8503056a^7b^3c^2z^3 \\
& - 14348907a^8c^4z^3 + 177147a^5b^2c^2z^2 + b^3, z, k)^5a^{12}b^{11}c \\
& ^4x)\text{root}(18075490334784a^{14}b^3c^4z^6 - 7625597484987a^{15}c^6z^6 + 1 \\
& 162261467a^{10}b^4c^4z^4 + 8503056a^7b^3c^2z^3 - 14348907a^8c^4z^3 + \\
& 177147a^5b^2c^2z^2 + b^3, z, k), k, 1, 6)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)

[Out] Timed out

$$3.140 \quad \int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=522

$$\frac{\sqrt[3]{-1} \left(3\sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b \right) \tan^{-1} \left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right)}{27\sqrt{3} \left(1 + \sqrt[3]{-1} \right)^2 a^{17/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \frac{\left(2b - 3\sqrt[3]{a} c^{2/3} \right) \tan^{-1} \left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \right)}{81\sqrt{3} a^{17/6} c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \frac{(2(-1)^{1/3} \sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b)}{27\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{17/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \quad (2(-1)^{1/3} \sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b)$$

[Out] $\frac{1}{162} \ln(3a+3a^{2/3}c^{1/3}x+bx^2)/a^{8/3}/c^{1/3} - \frac{1}{54} \ln(3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2)/(1+(-1)^{1/3})^2/a^{8/3}/c^{1/3} - \frac{1}{162} (-1)^{1/3} \ln(3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2)/a^{8/3}/c^{1/3} - \frac{1}{243} (2b-3a^{1/3}c^{2/3}) \arctan(1/3(3a^{2/3}c^{1/3}+2bx)/\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}})/a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}} - \frac{1}{81} (2b-3a^{1/3}c^{2/3}) \arctan(1/3(3a^{2/3}c^{1/3}+2bx)/\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}})/81\sqrt{3}a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}} - \frac{1}{27} (2(-1)^{1/3}\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b) \arctan(1/3(3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx)/\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}})/27\sqrt{3}(1+\sqrt[3]{-1})^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}$

Rubi [A] time = 0.86, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2070, 634, 618, 204, 628}

$$\frac{\sqrt[3]{-1} \left(3\sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b \right) \tan^{-1} \left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right)}{27\sqrt{3} \left(1 + \sqrt[3]{-1} \right)^2 a^{17/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \frac{\left(2b - 3\sqrt[3]{a} c^{2/3} \right) \tan^{-1} \left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \right)}{81\sqrt{3} a^{17/6} c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \frac{(2(-1)^{1/3} \sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b)}{27\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{17/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \quad (2(-1)^{1/3} \sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)^{-1}, x]$

[Out] $-\frac{((-1)^{1/3}(2(-1)^{1/3}b + 3a^{1/3}c^{2/3})\text{ArcTan}[(3(-1)^{1/3}a^{2/3}c^{1/3} - 2bx)/(\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}})])}{(27\sqrt{3}(1+(-1)^{1/3})^2a^{17/6}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}})c^{2/3}} - \frac{((2b-3a^{1/3}c^{2/3})\text{ArcTan}[(3a^{2/3}c^{1/3} + 2bx)/(\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}})])}{(81\sqrt{3}a^{17/6}\sqrt{4b-3a^{1/3}c^{2/3}})c^{2/3}} - \frac{((2(-1)^{2/3}b - 3a^{1/3}c^{2/3})\text{ArcTan}[(3(-1)^{2/3}a^{2/3}c^{1/3} + 2bx)/(\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}})])}{(27\sqrt{3}(1-(-1)^{1/3})^2a^{17/6}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}})c^{2/3}} + \frac{\text{Log}[3a+3a^{2/3}c^{1/3}x+bx^2]}{(162a^{8/3}c^{1/3})} - \frac{\text{Log}[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2]}{(54(1+(-1)^{1/3})^2a^{8/3}c^{1/3})} - \frac{((-1)^{1/3}\text{Log}[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2]}{(162a^{8/3}c^{1/3})}$

Rule 204

$\text{Int}[(a_0 + (b_0)x^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_0 + (b_0)x + (c_0)x^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ax - x^2, x], x], x, b + 2cx], x] /; \text{FreeQ}\{a, b, c\},$

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2070

$\text{Int}[(Q6_)^(p_), x_Symbol] \rightarrow \text{With}[\{a = \text{Coeff}[Q6, x, 0], b = \text{Coeff}[Q6, x, 2], c = \text{Coeff}[Q6, x, 3], d = \text{Coeff}[Q6, x, 4], e = \text{Coeff}[Q6, x, 6]\}, \text{Dist}[1/(3^(3*p)*a^(2*p)), \text{Int}[\text{ExpandIntegrand}[(3*a + 3*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p, x], x] /; \text{EqQ}[b^2 - 3*a*d, 0] \&\& \text{EqQ}[b^3 - 27*a^2*e, 0] /; \text{ILtQ}[p, 0] \&\& \text{PolyQ}[Q6, x, 6] \&\& \text{EqQ}[\text{Coeff}[Q6, x, 1], 0] \&\& \text{EqQ}[\text{Coeff}[Q6, x, 5], 0] \&\& \text{RationalFunctionQ}[u, x]$

Rubi steps

$$\begin{aligned} \int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx &= (19683a^6) \int \left(\frac{-(-1)^{2/3} \sqrt[3]{a} b - 3\sqrt[3]{-1} a^{2/3} c^{2/3} + b}{531441 (1 + \sqrt[3]{-1})^2 a^{26/3} c^{2/3} (-3a + 3\sqrt[3]{-1} a^{2/3} c^{2/3})} \right. \\ &= \frac{\int \frac{-\sqrt[3]{a} b + 3a^{2/3} c^{2/3} + b \sqrt[3]{c} x}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{81a^{8/3} c^{2/3}} - \frac{\int \frac{(-1)^{2/3} \sqrt[3]{a} b - 3a^{2/3} c^{2/3} + \sqrt[3]{-1} b \sqrt[3]{c} x}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + bx^2} dx}{81a^{8/3} c^{2/3}} \\ &= -\frac{(2b - 3\sqrt[3]{a} c^{2/3}) \int \frac{1}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{162a^{7/3} c^{2/3}} - \frac{(2(-1)^{2/3} b - 3\sqrt[3]{a} c^{2/3}) \int \frac{1}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + bx^2} dx}{162a^{7/3} c^{2/3}} \\ &= \frac{\log(3a + 3a^{2/3} \sqrt[3]{c} x + bx^2)}{162a^{8/3} \sqrt[3]{c}} - \frac{\log(3a - 3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} x + bx^2)}{54(1 + \sqrt[3]{-1})^2 a^{8/3} \sqrt[3]{c}} \\ &= -\frac{(2(-1)^{2/3} b + 3\sqrt[3]{-1} \sqrt[3]{a} c^{2/3}) \tan^{-1}\left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{27\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{17/6} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}} c^{2/3}} \end{aligned}$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.19

$$\frac{1}{3} \text{RootSum}\left[\#1^6 b^3 + 9\#1^4 ab^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \&, \frac{\log(x - \#1)}{2\#1^5 b^3 + 12\#1^3 ab^2 + 27\#1^2 a^2 c + 18\#1 a^2 b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1), x]

[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 & , Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &]/3

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.00, size = 90, normalized size = 0.17

ln(-R)

6 RootOf(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27b a^2_Z^2 + 27a^3)^5 b^3 + 36 RootOf(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] 1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R =RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

mupad [B] time = 0.71, size = 1394, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)

[Out] symsum(log(6561*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6), z, k


```

)^2*a^4*b^12*c^2 - 6*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209464
9*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 +
387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6,
z, k)*b^15*x - 4782969*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209
4649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^
3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b
^6, z, k)^3*a^7*b^11*c^3 - 229582512*root(488038239039168*a^17*b^3*c^4*z^6
- 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a
^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a
^3*b^5*c*z + b^6, z, k)^4*a^9*b^13*c^2 - 387420489*root(488038239039168*a^17
*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4
- 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*
z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^10*c^4 + 167365651248*root(488
038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*
a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 265
7205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^12*c^3 - 9414
3178827*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a
^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^13
*b^9*c^5 + 14580*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a
^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387
420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z,
k)^2*a^3*b^14*c*x - 10628820*root(488038239039168*a^17*b^3*c^4*z^6 - 205891
132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c
^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*
z + b^6, z, k)^3*a^6*b^13*c^2*x + 2238429492*root(488038239039168*a^17*b^3*
c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746
143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 -
2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^12*c^3*x - 1162261467*root(488038239
039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b
^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a
^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^9*c^5*x - 209207064
06*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 1
0460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^
5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^11
*c^4*x)*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a
^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k), k, 1,
6)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)

[Out] Timed out

$$3.141 \quad \int \frac{1}{x(27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx$$

Optimal. Leaf size=563

$$\frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right) \log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{486a^{10/3}} - \frac{\left(6\sqrt[3]{a}c^{2/3} + i\sqrt{3}b + b\right) \log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{972a^{10/3}c^{2/3}} - \frac{\left(3\sqrt[3]{a} - \frac{(-1)^{1/3}}{c^{2/3}}\right) \log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{486a^{10/3}}$$

[Out] 1/27*ln(x)/a^3-1/486*(3*a^(1/3)-b/c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(10/3)-1/486*(3*a^(1/3)-(-1)^(2/3)*b/c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(10/3)-1/972*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)*(b+6*a^(1/3)*c^(2/3)+I*b*3^(1/2))/a^(10/3)/c^(2/3)+1/81*(b-a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/27*(-1)^(2/3)*((-1)^(2/3)*b-a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+1/27*(b-(-1)^(2/3)*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)

Rubi [A] time = 1.16, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right) \log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{486a^{10/3}} - \frac{\left(6\sqrt[3]{a}c^{2/3} + i\sqrt{3}b + b\right) \log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{972a^{10/3}c^{2/3}} - \frac{\left(3\sqrt[3]{a} - \frac{(-1)^{1/3}}{c^{2/3}}\right) \log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{486a^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]

[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(27*a^3 - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(10/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(10/3))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x(27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = (19683a^6) \int \left(\frac{1}{531441a^9x} + \frac{3a^{2/3}(2b - 3\sqrt[3]{a}c^{2/3})\sqrt[3]{c}}{4782969a^{28/3}c^{2/3}(3a + 3a^{2/3}(2b - 3\sqrt[3]{a}c^{2/3})\sqrt[3]{c} + b(b - 3\sqrt[3]{a}c^{2/3})x)} dx \right) (-1)^2$$

$$= \frac{\log(x)}{27a^3} + \frac{\int \frac{3a^{2/3}(2b - 3\sqrt[3]{a}c^{2/3})\sqrt[3]{c} + b(b - 3\sqrt[3]{a}c^{2/3})x}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{243a^{10/3}c^{2/3}} + \dots$$

$$= \frac{\log(x)}{27a^3} - \frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right) \int \frac{3a^{2/3}\sqrt[3]{c} + 2bx}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{486a^{10/3}} - \frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right) \log(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2)}{486a^{10/3}} - \dots$$

$$= \frac{(b - (-1)^{2/3}\sqrt[3]{a}c^{2/3}) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} - 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}(1 + \sqrt[3]{-1})^2 a^{19/6} \sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}} \sqrt[3]{c}} + \dots$$

Mathematica [C] time = 0.12, size = 157, normalized size = 0.28

$$\frac{\text{RootSum}\left[\#1^6b^3 + 9\#1^4ab^2 + 27\#1^3a^2c + 27\#1^2a^2b + 27a^3\&, \frac{\#1^4b^3 \log(x-\#1)+9\#1^2ab^2 \log(x-\#1)+27a^2b \log(x-\#1)+27\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}{81a^3}\right]}{81a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]

[Out] -1/81*(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &])/a^3

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x), x)

maple [C] time = 0.01, size = 134, normalized size = 0.24

$$\frac{\ln(x) \left(\text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27ba^2_Z^2 + 27a^3)^5 b^3 + 9 \text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27ba^2_Z^2 + 27a^3)^5 b^3 \right)}{27a^3 \left(2 \text{RootOf}(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27ba^2_Z^2 + 27a^3)^5 b^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] -1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2*b)/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))+1/27*ln(x)/a^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 2.55, size = 4002, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),x)

[Out] log(x)/(27*a^3) + symsum(log(7*root(13177032454057536*a^20*b^3*c^4*z^6 - 559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)*b^18*x - 162*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^3*b^18*x + 86093442*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^8*b^13*c^3 + 34867844010*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^11*b^13*c^3 - 10460353203*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^12*b^10*c^5 + 1506290861232*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^14*b^13*c^3 - 564859072962*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^15*b^10*c^5 - 67783088755440*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*a^17*b^13*c^3 + 22876792454961*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^10*c^5 + 17496*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^4*b^16*c - 472392*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056655523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*

$$\begin{aligned}
& a^8 b^3 c^4 z^2 + 10097379 a^7 b^6 c^2 z^2 - 6561 a^4 b^6 c^2 z + b^9, z, k \\
&)^3 a^7 b^{16} c - 39366 \operatorname{root}(13177032454057536 a^{20} b^3 c^4 z^6 - 5559060566 \\
& 555523 a^{21} c^6 z^6 + 488038239039168 a^{17} b^3 c^4 z^5 - 205891132094649 a^{18} c^6 z^5 \\
& + 6119306623755 a^{14} b^3 c^4 z^4 - 2541865828329 a^{15} c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 \\
& - 229582512 a^{10} b^6 c^2 z^3 - 10460353203 a^{12} c^6 z^3 + 14348907 a^8 b^3 c^4 z^2 + 10097379 a^7 b^6 c^2 z^2 \\
& - 6561 a^4 b^6 c^2 z + b^9, z, k)^2 a^4 b^{15} c^2 x + 51372630 \operatorname{root}(13177032454057536 a^{20} b^3 c^4 z^6 \\
& - 5559060566555523 a^{21} c^6 z^6 + 488038239039168 a^{17} b^3 c^4 z^5 - 205891132094649 a^{18} c^6 z^5 \\
& + 6119306623755 a^{14} b^3 c^4 z^4 - 2541865828329 a^{15} c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 \\
& - 229582512 a^{10} b^6 c^2 z^3 - 10460353203 a^{12} c^6 z^3 + 14348907 a^8 b^3 c^4 z^2 + 10097379 a^7 b^6 c^2 z^2 \\
& - 6561 a^4 b^6 c^2 z + b^9, z, k)^3 a^7 b^{15} c^2 x + 71744 \\
& 535 \operatorname{root}(13177032454057536 a^{20} b^3 c^4 z^6 - 5559060566555523 a^{21} c^6 z^6 \\
& + 488038239039168 a^{17} b^3 c^4 z^5 - 205891132094649 a^{18} c^6 z^5 + 611930 \\
& 6623755 a^{14} b^3 c^4 z^4 - 2541865828329 a^{15} c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 \\
& - 229582512 a^{10} b^6 c^2 z^3 - 10460353203 a^{12} c^6 z^3 + 1434890 \\
& 7 a^8 b^3 c^4 z^2 + 10097379 a^7 b^6 c^2 z^2 - 6561 a^4 b^6 c^2 z + b^9, z, \\
& k)^3 a^8 b^{12} c^4 x - 2008846980 \operatorname{root}(13177032454057536 a^{20} b^3 c^4 z^6 - \\
& 5559060566555523 a^{21} c^6 z^6 + 488038239039168 a^{17} b^3 c^4 z^5 - 2058911 \\
& 32094649 a^{18} c^6 z^5 + 6119306623755 a^{14} b^3 c^4 z^4 - 2541865828329 a^{15} \\
& c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 - 229582512 a^{10} b^6 c^2 z^3 - 1046 \\
& 0353203 a^{12} c^6 z^3 + 14348907 a^8 b^3 c^4 z^2 + 10097379 a^7 b^6 c^2 z^2 \\
& - 6561 a^4 b^6 c^2 z + b^9, z, k)^4 a^{10} b^{15} c^2 x + 108477736920 \operatorname{root}(131 \\
& 77032454057536 a^{20} b^3 c^4 z^6 - 5559060566555523 a^{21} c^6 z^6 + 488038239 \\
& 039168 a^{17} b^3 c^4 z^5 - 205891132094649 a^{18} c^6 z^5 + 6119306623755 a^{14} \\
& b^3 c^4 z^4 - 2541865828329 a^{15} c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 - \\
& 229582512 a^{10} b^6 c^2 z^3 - 10460353203 a^{12} c^6 z^3 + 14348907 a^8 b^3 c^4 \\
& z^2 + 10097379 a^7 b^6 c^2 z^2 - 6561 a^4 b^6 c^2 z + b^9, z, k)^4 a^{11} b \\
& ^{12} c^4 x - 41841412812 \operatorname{root}(13177032454057536 a^{20} b^3 c^4 z^6 - 555906056 \\
& 6555523 a^{21} c^6 z^6 + 488038239039168 a^{17} b^3 c^4 z^5 - 205891132094649 a \\
& ^{18} c^6 z^5 + 6119306623755 a^{14} b^3 c^4 z^4 - 2541865828329 a^{15} c^6 z^4 + \\
& 27506854719 a^{11} b^3 c^4 z^3 - 229582512 a^{10} b^6 c^2 z^3 - 10460353203 a^{12} c^6 z^3 \\
& + 14348907 a^8 b^3 c^4 z^2 + 10097379 a^7 b^6 c^2 z^2 - 6561 a^4 b^6 c^2 z \\
& + b^9, z, k)^4 a^{12} b^9 c^6 x + 18596183472 \operatorname{root}(131770324540575 \\
& 36 a^{20} b^3 c^4 z^6 - 5559060566555523 a^{21} c^6 z^6 + 488038239039168 a^{17} b^3 c^4 z^5 \\
& - 205891132094649 a^{18} c^6 z^5 + 6119306623755 a^{14} b^3 c^4 z^4 \\
& - 2541865828329 a^{15} c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 - 229582512 a^{10} b^6 c^2 z^3 \\
& - 10460353203 a^{12} c^6 z^3 + 14348907 a^8 b^3 c^4 z^2 + 1009 \\
& 7379 a^7 b^6 c^2 z^2 - 6561 a^4 b^6 c^2 z + b^9, z, k)^5 a^{13} b^{15} c^2 x + \\
& 16129864639026 \operatorname{root}(13177032454057536 a^{20} b^3 c^4 z^6 - 5559060566555523 a \\
& ^{21} c^6 z^6 + 488038239039168 a^{17} b^3 c^4 z^5 - 205891132094649 a^{18} c^6 z^5 \\
& + 6119306623755 a^{14} b^3 c^4 z^4 - 2541865828329 a^{15} c^6 z^4 + 27506854 \\
& 719 a^{11} b^3 c^4 z^3 - 229582512 a^{10} b^6 c^2 z^3 - 10460353203 a^{12} c^6 z^3 \\
& + 14348907 a^8 b^3 c^4 z^2 + 10097379 a^7 b^6 c^2 z^2 - 6561 a^4 b^6 c^2 z \\
& z + b^9, z, k)^5 a^{14} b^{12} c^4 x - 6778308875544 \operatorname{root}(13177032454057536 a^2 \\
& 0 b^3 c^4 z^6 - 5559060566555523 a^{21} c^6 z^6 + 488038239039168 a^{17} b^3 c^4 \\
& z^5 - 205891132094649 a^{18} c^6 z^5 + 6119306623755 a^{14} b^3 c^4 z^4 - 254 \\
& 1865828329 a^{15} c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 - 229582512 a^{10} b^6 \\
& c^2 z^3 - 10460353203 a^{12} c^6 z^3 + 14348907 a^8 b^3 c^4 z^2 + 10097379 a \\
& ^7 b^6 c^2 z^2 - 6561 a^4 b^6 c^2 z + b^9, z, k)^5 a^{15} b^9 c^6 x + 6456339 \\
& 20395566 \operatorname{root}(13177032454057536 a^{20} b^3 c^4 z^6 - 5559060566555523 a^{21} c^6 \\
& z^6 + 488038239039168 a^{17} b^3 c^4 z^5 - 205891132094649 a^{18} c^6 z^5 + 6 \\
& 119306623755 a^{14} b^3 c^4 z^4 - 2541865828329 a^{15} c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 \\
& - 229582512 a^{10} b^6 c^2 z^3 - 10460353203 a^{12} c^6 z^3 + 14 \\
& 348907 a^8 b^3 c^4 z^2 + 10097379 a^7 b^6 c^2 z^2 - 6561 a^4 b^6 c^2 z + b^ \\
& 9, z, k)^6 a^{17} b^{12} c^4 x - 274521509459532 \operatorname{root}(13177032454057536 a^{20} b^ \\
& 3 c^4 z^6 - 5559060566555523 a^{21} c^6 z^6 + 488038239039168 a^{17} b^3 c^4 z^5 \\
& - 205891132094649 a^{18} c^6 z^5 + 6119306623755 a^{14} b^3 c^4 z^4 - 2541865 \\
& 828329 a^{15} c^6 z^4 + 27506854719 a^{11} b^3 c^4 z^3 - 229582512 a^{10} b^6 c^2
\end{aligned}$$

```
*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b
^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^9*c^6*x)*root(1317703
2454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 4880382390391
68*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3
*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 2295
82512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^
2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
3),x)
```

```
[Out] Timed out
```

$$3.142 \quad \int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)} dx$$

Optimal. Leaf size=645

$$\frac{(9a^{2/3}c^{4/3} + 12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3} + 2(-1)^{2/3}b^2) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} + \frac{(9a^{2/3}c^{4/3} - 12\sqrt[3]{a}bc^{2/3} + 2b^2) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$

[Out] $-1/27/a^3/x-1/486*(2*b-3*a^{(1/3)}*c^{(2/3)})*\ln(3*a+3*a^{(2/3)}*c^{(1/3)}*x+b*x^2)/a^{(11/3)}/c^{(1/3)}+1/162*(2*b-3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})*\ln(3*a-3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)}*x+b*x^2)/(1+(-1)^{(1/3)})^2/a^{(11/3)}/c^{(1/3)}+1/486*(-1)^{(1/3)}*(2*b+3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})*\ln(3*a+3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)}*x+b*x^2)/a^{(11/3)}/c^{(1/3)}+1/729*(2*b^2-12*a^{(1/3)}*b*c^{(2/3)}+9*a^{(2/3)}*c^{(4/3)})*\arctan(1/3*(3*a^{(2/3)}*c^{(1/3)}+2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b-3*a^{(1/3)}*c^{(2/3)})^{(1/2)})/a^{(23/6)}/c^{(2/3)}*3^{(1/2)}/(4*b-3*a^{(1/3)}*c^{(2/3)})^{(1/2)}+1/243*(-1)^{(2/3)}*(2*b^2+12*(-1)^{(1/3)}*a^{(1/3)}*b*c^{(2/3)}+9*(-1)^{(2/3)}*a^{(2/3)}*c^{(4/3)})*\arctan(1/3*(3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)}+2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b+3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)})/(1-(-1)^{(1/3)})/(1+(-1)^{(1/3)})^2/a^{(23/6)}/c^{(2/3)}*3^{(1/2)}/(4*b+3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)}+1/243*(2*(-1)^{(2/3)}*b^2+12*(-1)^{(1/3)}*a^{(1/3)}*b*c^{(2/3)}+9*a^{(2/3)}*c^{(4/3)})*\arctan(1/3*(3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)}-2*b*x)*3^{(1/2)}/a^{(1/2)}/(4*b-3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)})/(1+(-1)^{(1/3)})^2/a^{(23/6)}/c^{(2/3)}*3^{(1/2)}/(4*b-3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})^{(1/2)}$

Rubi [A] time = 1.38, antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2097, 634, 618, 204, 628}

$$\frac{(9a^{2/3}c^{4/3} + 12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3} + 2(-1)^{2/3}b^2) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} + \frac{(9a^{2/3}c^{4/3} - 12\sqrt[3]{a}bc^{2/3} + 2b^2) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)), x]

[Out] $-1/(27*a^3*x) + ((2*(-1)^{(2/3)}*b^2 + 12*(-1)^{(1/3)}*a^{(1/3)}*b*c^{(2/3)} + 9*a^{(2/3)}*c^{(4/3)})*\text{ArcTan}[(3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)} - 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)}])]/(81*\text{Sqrt}[3]*(1 + (-1)^{(1/3)})^2*a^{(23/6)}*\text{Sqrt}[4*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)}) + ((2*b^2 - 12*a^{(1/3)}*b*c^{(2/3)} + 9*a^{(2/3)}*c^{(4/3)})*\text{ArcTan}[(3*a^{(2/3)}*c^{(1/3)} + 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b - 3*a^{(1/3)}*c^{(2/3)}])]/(243*\text{Sqrt}[3]*a^{(23/6)}*\text{Sqrt}[4*b - 3*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)}) + ((2*(-1)^{(2/3)}*b^2 - 12*a^{(1/3)}*b*c^{(2/3)} - 9*(-1)^{(1/3)}*a^{(2/3)}*c^{(4/3)})*\text{ArcTan}[(3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)} + 2*b*x)/(\text{Sqrt}[3]*\text{Sqrt}[a]*\text{Sqrt}[4*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)}])]/(81*\text{Sqrt}[3]*(1 - (-1)^{(1/3)})*(1 + (-1)^{(1/3)})^2*a^{(23/6)}*\text{Sqrt}[4*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)}) - ((2*b - 3*a^{(1/3)}*c^{(2/3)})*\text{Log}[3*a + 3*a^{(2/3)}*c^{(1/3)}*x + b*x^2])/(486*a^{(11/3)}*c^{(1/3)}) + ((2*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})*\text{Log}[3*a - 3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)}*x + b*x^2])/(162*(1 + (-1)^{(1/3)})^2*a^{(11/3)}*c^{(1/3)}) + ((-1)^{(1/3)}*(2*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})*\text{Log}[3*a + 3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)}*x + b*x^2])/(486*a^{(11/3)}*c^{(1/3)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx &= (19683a^6) \int \left(\frac{1}{531441a^9x^2} + \frac{\sqrt[3]{a} (b^2 - 9\sqrt[3]{a}bc^{2/3})}{1594323(1 - \sqrt[3]{-1})} \right) dx \\ &= -\frac{1}{27a^3x} + \frac{\int \frac{\sqrt[3]{a}(b^2 - 9\sqrt[3]{a}bc^{2/3} + 9a^{2/3}c^{4/3}) - b(2b - 3\sqrt[3]{a}c^{2/3})\sqrt[3]{c}x}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{243a^{11/3}c^{2/3}} \\ &= -\frac{1}{27a^3x} - \frac{(2b - 3\sqrt[3]{a}c^{2/3}) \int \frac{3a^{2/3}\sqrt[3]{c} + 2bx}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{486a^{11/3}\sqrt[3]{c}} + \frac{\sqrt[3]{c}}{243a^{11/3}c^{2/3}} \\ &= -\frac{1}{27a^3x} - \frac{(2b - 3\sqrt[3]{a}c^{2/3}) \log(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2)}{486a^{11/3}\sqrt[3]{c}} \\ &= -\frac{1}{27a^3x} + \frac{(2(-1)^{2/3}b^2 + 12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3} + 9a^{2/3}c^{4/3})}{81\sqrt{3}(1 + \sqrt[3]{-1})^2 a^{23/6} \sqrt{4b - 3a}} \end{aligned}$$

Mathematica [C] time = 0.14, size = 163, normalized size = 0.25

$$\frac{x \operatorname{RootSum} \left[\#1^6 b^3 + 9 \#1^4 a b^2 + 27 \#1^3 a^2 c + 27 \#1^2 a^2 b + 27 a^3 \&, \frac{\#1^4 b^3 \log(x-\#1) + 9 \#1^2 a b^2 \log(x-\#1) + 27 a^2 b \log(x-\#1) + 27 \#1 a^2 c}{2 \#1^5 b^3 + 12 \#1^3 a b^2 + 27 \#1^2 a^2 c + 18 \#1 a^2 b} \right]}{81 a^3 x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]

[Out] -1/81*(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &])/(a^3*x)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^3 x^6 + 9 a b^2 x^4 + 27 a^2 c x^3 + 27 a^2 b x^2 + 27 a^3) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x^2), x)

maple [C] time = 0.01, size = 133, normalized size = 0.21

$$\frac{\left(-\operatorname{RootOf}\left(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27ba^2_Z^2 + 27a^3\right)^4 b^3 - 9 \operatorname{RootOf}\left(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27ba^2_Z^2 + 27a^3\right)^5 b^3 + 12 \operatorname{RootOf}\left(b^3_Z^6 + 9b^2a_Z^4 + 27a^2c_Z^3 + 27ba^2_Z^2 + 27a^3\right)^6 b^3}{81a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] -1/27/a^3/x+1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 2.72, size = 2663, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^2*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)), x)$

[Out] $\text{symsum}(\log(-282429536481*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k)*a^{23}*b^9*(2*b^{10}*x + 2541865828329*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^4*a^{17}*c^5 - 45*a*b^8*c + 387420489*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^2*a^{10}*c^6*x - 401769396*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^3*a^{12}*b^5*c^2 + 6973568802*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^3*a^{13}*b^2*c^4 - 4518872583696*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^4*a^{16}*b^3*c^3 - 328050*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^5*a^{12}*b^5*c^2 + 177147*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^7*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^8*a^{10}*b*c^5 + 23328*\text{root}(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k))^9$

$$\begin{aligned}
&^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8* \\
&b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k)*a^4*b^8*c*x + 196830*root(35 \\
&5779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753 \\
&584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616* \\
&a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^ \\
&3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}* \\
&c*z + b^{12}, z, k)*a^5*b^5*c^3*x - 20920706406*root(355779876259553472*a^{23}* \\
&b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z \\
&^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207 \\
&657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5* \\
&c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k)^3*a^ \\
&13*b*c^5*x + 74401740*root(355779876259553472*a^{23}*b^3*c^4*z^6 - 1500946352 \\
&96999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^1 \\
&6*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z \\
&^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8* \\
&b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k)^2*a^8*b^6*c^2*x - 746143164* \\
&root(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 \\
&- 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145 \\
&430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14} \\
&*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^ \\
&4*b^{10}*c*z + b^{12}, z, k)^2*a^9*b^3*c^4*x + 55788550416*root(355779876259553 \\
&472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^1 \\
&7*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5* \\
&z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326 \\
&*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z \\
&, k)^3*a^{12}*b^4*c^3*x + 564859072962*root(355779876259553472*a^{23}*b^3*c^4*z \\
&^6 - 150094635296999121*a^{24}*c^6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 1093 \\
&00230618147*a^{16}*b^4*c^4*z^4 - 753145430616*a^{13}*b^3*c^5*z^3 + 207657382104 \\
&*a^{12}*b^6*c^3*z^3 + 282429536481*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + \\
&100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^{10}*c*z + b^{12}, z, k)^4*a^{16}*b^2*c^ \\
&4*x))*root(355779876259553472*a^{23}*b^3*c^4*z^6 - 150094635296999121*a^{24}*c^ \\
&6*z^6 - 45753584909922*a^{17}*b*c^6*z^4 + 109300230618147*a^{16}*b^4*c^4*z^4 - \\
&753145430616*a^{13}*b^3*c^5*z^3 + 207657382104*a^{12}*b^6*c^3*z^3 + 28242953648 \\
&1*a^{14}*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17 \\
&496*a^4*b^{10}*c*z + b^{12}, z, k), k, 1, 6) - 1/(27*a^3*x)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)

[Out] Timed out

$$3.143 \quad \int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=395

$$\frac{1}{216} (36 + 2^{2/3} \sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6) + \frac{1}{108} (18 - (-2)^{2/3} \sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6) +$$

```
[Out] 1/108*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/108*(18-2^(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/216*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))+1/2*3^(1/6)*2^(5/6)*(1-(-3)^(2/3)*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3)))^(1/2)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^^(1/2)-1/6*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^^(1/2)-1/3*(-2)^(1/3)*(1+(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^^(1/2))*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^^(1/2)
```

Rubi [A] time = 1.44, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{1}{216} (36 + 2^{2/3} \sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6) + \frac{1}{108} (18 - (-2)^{2/3} \sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6) +$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]
```

```
[Out] -((( -2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ((3/2)^(1/6)*(1 - (-3)^(2/3)*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*ArcTan[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/108 + ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/108
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x^5}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3} (3\sqrt[3]{-3} 2^{2/3} + (1 - 3(-3)^{2/3} \sqrt[3]{2}) x)}{3779136 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} + \dots \right) dx \\ &= \frac{1}{54} \int \frac{6\sqrt[3]{2} 3^{2/3} + (18 - 2^{2/3} \sqrt[3]{3}) x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + x^2} dx + \frac{(-1)^{2/3} \int \frac{3(-2)^{2/3} \sqrt[3]{3} - (1 + 3\sqrt[3]{-2} 3^{2/3}) x}{6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{9\sqrt[3]{2} 3^{2/3}} \\ &= \frac{((-1)^{2/3} (1 - 3(-3)^{2/3} \sqrt[3]{2})) \int \frac{-3\sqrt[3]{-3} 2^{2/3} + 2x}{6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2} dx}{6\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{((-1)^{2/3} \sqrt[3]{\frac{3}{2}} (6 + \sqrt[3]{-3})) \int \frac{3(-2)^{2/3} \sqrt[3]{3} - (1 + 3\sqrt[3]{-2} 3^{2/3}) x}{6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{2(1 + \sqrt[3]{-1})^2} \\ &= \frac{1}{216} (36 + 2^{2/3} \sqrt[3]{3} + i 2^{2/3} 3^{5/6}) \log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2) + \frac{1}{108} (18 - \dots) \\ &= \frac{(-1)^{2/3} ((-2)^{2/3} - 2 \cdot 3^{2/3}) \tan^{-1} \left(\frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{\sqrt{6(4 + 3\sqrt[3]{-2} 3^{2/3})}} \right)}{6^{5/6} \sqrt{4 + 3\sqrt[3]{-2} 3^{2/3}}} - \frac{(-1)^{2/3} (\sqrt[3]{-3} + 3\sqrt[3]{-2})}{\sqrt[6]{6} (1 + \sqrt[3]{-1})} \end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.15

$$\frac{1}{6} \text{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{\#1^4 \log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/6

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.14

$$\frac{\text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^5 \ln}{6 \text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^5 + 72 \text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 0.65, size = 427, normalized size = 1.08

$$\sum_{k=1}^6 \ln \left(\frac{362797056 \left(19236852 x \text{root}(z^6 + 4374 z^5 + 6626610 z^4 + 2646786132 z^3 - 24163559388 z^2 + 72662865048 z - 72662865048, z, k) - 19131876 * x - 6482268 * x * \text{root}(z^6 + 4374 * z^5 + 6626610 * z^4 + 2646786132 * z^3 - 24163559388 * z^2 + 72662865048 * z - 72662865048, z, k) \right)^2 + 742851 * x * \text{root}(z^6 + 4374 * z^5 + 6626610 * z^4 + 2646786132 * z^3 - 24163559388 * z^2 + 72662865048 * z - 72662865048, z, k)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log((362797056*(19236852*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 19131876 * x - 6482268 * x * root(z^6 + 4374 * z^5 + 6626610 * z^4 + 2646786132 * z^3 - 24163559388 * z^2 + 72662865048 * z - 72662865048, z, k)^2 + 742851 * x * root(z^6 + 4374 * z^5 + 6626610 * z^4 + 2646786132 * z^3 - 24163559388 * z^2 + 72662865048 * z - 72662865048, z, k)

```

z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 7266
2865048, z, k)^3 - 4130*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^
3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + x*root(z^6 + 4
374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z -
72662865048, z, k)^5 - 154944576*root(z^6 + 4374*z^5 + 6626610*z^4 + 264678
6132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 1704742
2*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72
662865048*z - 72662865048, z, k)^3 + 27054*root(z^6 + 4374*z^5 + 6626610*z^
4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4
+ 9*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 +
72662865048*z - 72662865048, z, k)^5 + 465542316*root(z^6 + 4374*z^5 + 662
6610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048,
z, k) - 465542316))/root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24
163559388*z^2 + 72662865048*z - 72662865048, z, k)^5)*root(z^6 - z^5 + (421
*z^4)/1266 - (100853*z^3)/2768742 - (505*z^2)/5537484 - z/16612452 - 1/7266
2865048, z, k), k, 1, 6)

```

sympy [A] time = 0.26, size = 70, normalized size = 0.18

$$\text{RootSum}\left(72662865048t^6 - 72662865048t^5 + 24163559388t^4 - 2646786132t^3 - 6626610t^2 - 4374t - 1, \left(t \mapsto t\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 2646786
132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-892364171310473
76*_t**5/833243797 + 89301949532998128*_t**4/833243797 - 29740560281805852*
_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/8332437
97 + x + 5365044886/2499731391)))
```


$$3.144 \quad \int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=377

$$\frac{\log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{6 \cdot 2^{2/3}\sqrt[3]{3} (1 + \sqrt[3]{-1})^2} + \frac{\sqrt[3]{-\frac{1}{3}} \log(x^2 + 3(-2)^{2/3}\sqrt[3]{3}x + 6)}{18 \cdot 2^{2/3}} - \frac{\log(x^2 + 3 \cdot 2^{2/3}\sqrt[3]{3}x + 6)}{18 \cdot 2^{2/3}\sqrt[3]{3}} + \frac{(-1)^{2/3} (3(-3)^{2/3})}{9\sqrt[3]{3} (1 + \sqrt[3]{-1})}$$

[Out] 1/36*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2+1/108*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)-1/108*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/27*(-1)^(2/3)*(3*(-3)^(2/3)-2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/27*(9-2^(2/3)*3^(1/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(-24+18*2^(1/3)*3^(2/3))^(1/2)+1/27*(9-(-2)^(2/3)*3^(1/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^(1/3)*3^(1/6)+9*2^(1/3)*3^(2/3))^(1/2)

Rubi [A] time = 0.91, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{\log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{6 \cdot 2^{2/3}\sqrt[3]{3} (1 + \sqrt[3]{-1})^2} + \frac{\sqrt[3]{-\frac{1}{3}} \log(x^2 + 3(-2)^{2/3}\sqrt[3]{3}x + 6)}{18 \cdot 2^{2/3}} - \frac{\log(x^2 + 3 \cdot 2^{2/3}\sqrt[3]{3}x + 6)}{18 \cdot 2^{2/3}\sqrt[3]{3}} + \frac{(-1)^{2/3} (3(-3)^{2/3})}{9\sqrt[3]{3} (1 + \sqrt[3]{-1})}$$

Antiderivative was successfully verified.

[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(27*Sqrt[3*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*Sqrt[6*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(6*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(18*2^(2/3)*3^(1/3))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx = 1259712 \int \left(\frac{(-1)^{2/3} (-2 + \sqrt[3]{-3} 2^{2/3} x)}{7558272 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)} + \dots \right) dx$$

$$= -\frac{(-1)^{2/3} \int \frac{2+(-2)^{2/3} \sqrt[3]{3} x}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{18 \sqrt[3]{2} 3^{2/3}} - \frac{\int \frac{\sqrt[3]{2} + \sqrt[3]{3} x}{6+3 2^{2/3} \sqrt[3]{3} x+x^2} dx}{9 6^{2/3}} + \frac{(-1)^{2/3} \int \frac{-2+\sqrt[3]{-3} x}{-6+3 \sqrt[3]{-3} 2^{2/3} x-x^2} dx}{6 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{18 2^{2/3}} - \frac{\int \frac{3 2^{2/3} \sqrt[3]{3} + 2x}{6+3 2^{2/3} \sqrt[3]{3} x+x^2} dx}{18 2^{2/3} \sqrt[3]{3}} + \frac{\int \frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{-6+3 \sqrt[3]{-3} 2^{2/3} x-x^2} dx}{6 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2}$$

$$= \frac{\log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}{6 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} + \frac{\sqrt[3]{-\frac{1}{3}} \log(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)}{18 2^{2/3}} - \frac{\log(6 - 3\sqrt[3]{-3} 2^{2/3} x - x^2)}{6 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2}$$

$$= -\frac{\sqrt[3]{-1} (9 + \sqrt[3]{-3} 2^{2/3}) \tan^{-1} \left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} \right)}{9 (1 + \sqrt[3]{-1})^2 \sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} - \frac{((-2)^{2/3} - 3 3^{2/3}) \tan^{-1} \left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} \right)}{27 \sqrt[3]{3} \sqrt{2(4 - 3(-3)^{2/3} \sqrt[3]{2})}}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.16

$$\frac{1}{6} \text{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + 36, \frac{\#1^3 \log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/6

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.15

$$\frac{\text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^4 \ln}{6 \text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^5 + 72 \text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.70, size = 390, normalized size = 1.03

$$\sum_{k=1}^6 \ln \left(-\frac{5038848 \left(1377495072x + 17006112x \text{root}(z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k) - 104976x \text{root}(z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k) \right)^2 + 158112x \text{root}(z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log(-(5038848*(1377495072*x + 17006112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) - 104976*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k))^2 + 158112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)), z, k)

$52z^3 + 2066242608z^2 - 15695178850368, z, k)^3 + 1946x\sqrt{z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k}^4 + 3x\sqrt{z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k}^5 - 4251528\sqrt{z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k}^2 + 3927852\sqrt{z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k}^3 - 1188\sqrt{z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k}^4 - \sqrt{z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k}^5 + 7558272\sqrt{z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k} + 33519046752)/\sqrt{z^6 + 1944z^5 + 1180980z^4 - 1845163152z^3 + 2066242608z^2 - 15695178850368, z, k}^5 * \sqrt{z^6 - z^4/7596 + (217z^3)/1845828 - (5z^2)/66449808 - z/8073651672 - 1/15695178850368, z, k}, k, 1, 6)$

sympy [A] time = 0.28, size = 65, normalized size = 0.17

$\text{RootSum}\left(15695178850368t^6 - 2066242608t^4 + 1845163152t^3 - 1180980t^2 - 1944t - 1, \left(t \mapsto t \log\left(\frac{614714526178551746208t^5}{57121295165 - 1270857362386176t^4/57121295165 - 80483053187684376t^3/57121295165 + 72431318325103884t^2/57121295165 - 45358602689088t/57121295165}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)

[Out] RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 1180980*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(614714526178551746208*_t**5/57121295165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/57121295165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/57121295165 + x - 44532180783/57121295165)))

$$3.145 \quad \int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=361

$$\frac{(-1)^{2/3} \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{36\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{(-1)^{2/3} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{108\sqrt[3]{2} 3^{2/3}} + \frac{\log(x^2 + 3 2^{2/3} \sqrt[3]{3} x + 6)}{108\sqrt[3]{2} 3^{2/3}} - \frac{1}{6\sqrt[6]{2} 3^{5/6}}$$

[Out] $-1/216*(-1)^{(2/3)}*\ln(6-3*(-3)^{(1/3)}*2^{(2/3)}*x+x^2)*2^{(2/3)}*3^{(1/3)}/(1+(-1)^{(1/3)})^2+1/648*(-1)^{(2/3)}*\ln(6+3*(-2)^{(2/3)}*3^{(1/3)}*x+x^2)*2^{(2/3)}*3^{(1/3)}+1/648*\ln(6+3*2^{(2/3)}*3^{(1/3)}*x+x^2)*2^{(2/3)}*3^{(1/3)}-1/36*\arctan((3*(-3)^{(1/3)}*2^{(2/3)}-2*x)/(24-18*(-3)^{(2/3)}*2^{(1/3)})^{(1/2)}*2^{(5/6)}*3^{(1/6)})/(1+(-1)^{(1/3)})^2/(4-3*(-3)^{(2/3)}*2^{(1/3)})^{(1/2)}+1/108*\operatorname{arctanh}(2^{(1/6)}*(3*3^{(1/3)}+2^{(1/3)}*x)/(-12+9*2^{(1/3)}*3^{(2/3)})^{(1/2)}*2^{(5/6)}*3^{(1/6)})/(-4+3*2^{(1/3)}*3^{(2/3)})^{(1/2)}+1/54*(-1)^{(1/3)}*\arctan((3*(-2)^{(2/3)}*3^{(1/3)}+2*x)/(24+18*(-2)^{(1/3)}*3^{(2/3)})^{(1/2)}*2^{(1/3)}*3^{(1/6)})/(8+9*I*2^{(1/3)}*3^{(1/6)}+3*2^{(1/3)}*3^{(2/3)})^{(1/2)}$

Rubi [A] time = 0.51, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3} \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{36\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{(-1)^{2/3} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{108\sqrt[3]{2} 3^{2/3}} + \frac{\log(x^2 + 3 2^{2/3} \sqrt[3]{3} x + 6)}{108\sqrt[3]{2} 3^{2/3}} - \frac{1}{6\sqrt[6]{2} 3^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] $-\operatorname{ArcTan}[(3*(-3)^{(1/3)}*2^{(2/3)} - 2*x)/\operatorname{Sqrt}[6*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})]]/(6*2^{(1/6)}*3^{(5/6)}*(1 + (-1)^{(1/3)})^2*\operatorname{Sqrt}[4 - 3*(-3)^{(2/3)}*2^{(1/3)}]) + ((-1)^{(1/3)}*\operatorname{ArcTan}[(3*(-2)^{(2/3)}*3^{(1/3)} + 2*x)/\operatorname{Sqrt}[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})]])/(9*2^{(2/3)}*3^{(5/6)}*\operatorname{Sqrt}[8 + (9*I)*2^{(1/3)}*3^{(1/6)} + 3*2^{(1/3)}*3^{(2/3)}]) + \operatorname{ArcTanh}[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/\operatorname{Sqrt}[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})]])/(18*2^{(1/6)}*3^{(5/6)}*\operatorname{Sqrt}[-4 + 3*2^{(1/3)}*3^{(2/3)}]) - ((-1)^{(2/3)}*\operatorname{Log}[6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2])/(36*2^{(1/3)}*3^{(2/3)}*(1 + (-1)^{(1/3)})^2) + ((-1)^{(2/3)}*\operatorname{Log}[6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2])/(108*2^{(1/3)}*3^{(2/3)}) + \operatorname{Log}[6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2]/(108*2^{(1/3)}*3^{(2/3)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}3^{2/3}(1 + \sqrt[3]{-1})^2(-6 + 3\sqrt[3]{-3}2^{2/3}x - x^2)} - \frac{(-1)^{2/3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} \right) dx \\ &= \frac{\int \frac{x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} dx}{54\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3} \int \frac{x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} dx}{54\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3} \int \frac{x}{-6+3\sqrt[3]{-3}2^{2/3}x-x^2} dx}{18\sqrt[3]{2}3^{2/3}(1 + \sqrt[3]{-1})^2} \\ &= \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} dx}{182^{2/3}} + \frac{\int \frac{32^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} dx}{108\sqrt[3]{2}3^{2/3}} + \frac{(-1)^{2/3} \int \frac{3(-2)^{2/3}}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2} dx}{108\sqrt[3]{2}3^{2/3}} \\ &= -\frac{(-1)^{2/3} \log(6 - 3\sqrt[3]{-3}2^{2/3}x + x^2)}{36\sqrt[3]{2}3^{2/3}(1 + \sqrt[3]{-1})^2} + \frac{(-1)^{2/3} \log(6 + 3(-2)^{2/3}\sqrt[3]{3}x + x^2)}{108\sqrt[3]{2}3^{2/3}} \\ &= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6(4-3(-3)^{2/3}\sqrt[3]{2})}}\right)}{6\sqrt[6]{2}3^{5/6}(1 + \sqrt[3]{-1})^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6(4+3\sqrt[3]{-2}3^{2/3})}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}} \end{aligned}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.17

$$\frac{1}{6} \text{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + 36, \frac{\#1^2 \log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/6

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.16

$$\frac{\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 \ln(\dots)}{6 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 72 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 0.52, size = 276, normalized size = 0.76

$$\sum_{k=1}^6 \ln \left(\frac{23328 \left(297538935552x - 7992872640x \text{root}(z^6 + 1417176z^4 + 1332145440z^3 + 74384733888z^2 - 3390158631679488, z, k) + 52488x \text{root}(z^6 + 1417176z^4 + 1332145440z^3 + 74384733888z^2 - 3390158631679488, z, k) \right)^3 + 2904x \text{root}(z^6 + 1417176z^4 + 1332145440z^3 + 74384733888z^2 - 3390158631679488, z, k)^4 + x \text{root}(z^6 + 1417176z^4 + 1332145440z^3 + 74384733888z^2 - 3390158631679488, z, k)^5}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log(-(23328*(297538935552*x - 7992872640*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 52488*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^3 + 2904*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^4 + x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^5)/\dots),x)

```

4733888*z^2 - 3390158631679488, z, k)^5 - 153055008*root(z^6 + 1417176*z^4
+ 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^2 - 2764368*ro
ot(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488,
z, k)^3 - 1620*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 -
3390158631679488, z, k)^4 - 3673320192*root(z^6 + 1417176*z^4 + 1332145440
*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 7240114098432))/root(z^6
+ 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)
^5)*root(z^6 - z^4/45576 - (235*z^3)/598048272 - z^2/2392193088 - 1/3390158
631679488, z, k), k, 1, 6)

```

sympy [A] time = 0.25, size = 61, normalized size = 0.17

$$\text{RootSum}\left(3390158631679488t^6 - 74384733888t^4 - 1332145440t^3 - 1417176t^2 - 1, \left(t \mapsto t \log\left(-\frac{84823722142}{415817}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)

[Out] RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 1417176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 2216055910930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/415817 - 416583756*_t/415817 + x - 89938/415817)))

$$3.146 \quad \int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=248

$$\frac{(-1)^{2/3} \tan^{-1} \left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6(4-3(-3)^{2/3}\sqrt[3]{2})}} \right) + (-1)^{2/3} \tan^{-1} \left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6(4+3\sqrt[3]{-2}3^{2/3})}} \right) - \tanh^{-1} \left(\frac{\sqrt[6]{2}(\sqrt[3]{2}x+3\sqrt[3]{3})}{\sqrt{3(3\sqrt[3]{2}3^{2/3}-4)}} \right)}{27 \cdot 2^{5/6} \sqrt[6]{3} (1 + \sqrt[3]{-1})^2 \sqrt{4 - 3(-3)^{2/3} \sqrt[3]{2}} + 81 \sqrt[3]{2} \sqrt[6]{3} \sqrt{8 + 9i \sqrt[3]{2} \sqrt[6]{3} + 3 \sqrt[3]{2} 3^{2/3}} - 81 \cdot 2^{5/6} \sqrt[6]{3} \sqrt{3 \sqrt[3]{2} 3^{2/3} - 4}}$$

[Out] 1/162*(-1)^(2/3)*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/486*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/486*(-1)^(2/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)

Rubi [A] time = 0.32, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2097, 618, 204, 206}

$$\frac{(-1)^{2/3} \tan^{-1} \left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6(4-3(-3)^{2/3}\sqrt[3]{2})}} \right) + (-1)^{2/3} \tan^{-1} \left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6(4+3\sqrt[3]{-2}3^{2/3})}} \right) - \tanh^{-1} \left(\frac{\sqrt[6]{2}(\sqrt[3]{2}x+3\sqrt[3]{3})}{\sqrt{3(3\sqrt[3]{2}3^{2/3}-4)}} \right)}{27 \cdot 2^{5/6} \sqrt[6]{3} (1 + \sqrt[3]{-1})^2 \sqrt{4 - 3(-3)^{2/3} \sqrt[3]{2}} + 81 \sqrt[3]{2} \sqrt[6]{3} \sqrt{8 + 9i \sqrt[3]{2} \sqrt[6]{3} + 3 \sqrt[3]{2} 3^{2/3}} - 81 \cdot 2^{5/6} \sqrt[6]{3} \sqrt{3 \sqrt[3]{2} 3^{2/3} - 4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x)/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)])]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist

```
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx = 1259712 \int \left(\frac{(-1)^{2/3}}{22674816 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)} - \dots \right) dx$$

$$= \frac{\int \frac{1}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{54 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{1}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{54 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{1}{-6+3 \sqrt[3]{-3} x-x^2} dx}{18 \sqrt[3]{2} 3^{2/3} (1 + \dots)}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-6(4-3 \sqrt[3]{2} 3^{2/3})-x^2} dx, x, 3 \cdot 2^{2/3} \sqrt[3]{3} + 2x\right)}{27 \sqrt[3]{2} 3^{2/3}} - \frac{(-1)^{2/3} \text{Subst}\left(\int \frac{1}{-6(4-3 \sqrt[3]{2} 3^{2/3})-x^2} dx, x, 3 \cdot 2^{2/3} \sqrt[3]{3} + 2x\right)}{27 \sqrt[3]{2} 3^{2/3}}$$

$$= \frac{(-1)^{2/3} \tan^{-1}\left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} \sqrt[3]{2})}}\right)}{27 \cdot 2^{5/6} \sqrt[6]{3} (1 + \sqrt[3]{-1})^2 \sqrt{4-3(-3)^{2/3} \sqrt[3]{2}}} + \frac{(-1)^{2/3} \tan^{-1}\left(\frac{3(-2)^{2/3} \sqrt[3]{3}}{\sqrt{6(4+3 \sqrt[3]{-2})}}\right)}{81 \cdot 2^{5/6} \sqrt[6]{3} \sqrt{4+3 \sqrt[3]{-2}}}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.24

$$\frac{1}{6} \text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{\#1 \log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(36
+ 162*#1 + 12*#1^2 + #1^4) & ]/6
```

fricas [B] time = 2.00, size = 1277, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216), x, algorithm="fricas")
```

```
[Out] 1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)*
(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867
)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt(1/6
33)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18^(2/3)
) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(6*18^(
2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2*x + 72
9/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/136728*sqrt(1266)*sqrt(-2
/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 4
8*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(
1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3)
```

) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578) - 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) - sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) - sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.23

$$\frac{\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^2 \ln\left(\frac{6 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 72 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)}{\dots}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.68, size = 247, normalized size = 1.00

$$\sum_{k=1}^6 \ln \left(-\frac{216 \left(32134205039616x - 1836660096 \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right)^2 - 1889568 \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right)^3 + 972 \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right)^4 + \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right)^5 + 132239526912x \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right) + 204073344x \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right)^2 + 139968x \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right)^3 + 36x \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right)^4 + 863230245120 \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right) + 781932322630656 \right) / \operatorname{root}\left(z^6 - 2834352z^4 + 2677850419968z^2 - 732274264442769408, z, k\right)^5 \operatorname{root}\left(z^6 - z^4 / 273456 + z^2 / 258356853504 - 1 / 732274264442769408, z, k\right), k, 1, 6 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log(-(216*(32134205039616*x - 1836660096*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^2 - 1889568*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 972*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5 + 132239526912*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 204073344*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^2 + 139968*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 36*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + 863230245120*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 781932322630656))/root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5*root(z^6 - z^4/273456 + z^2/258356853504 - 1/732274264442769408, z, k), k, 1, 6)

sympy [A] time = 0.21, size = 48, normalized size = 0.19

$$\operatorname{RootSum}\left(732274264442769408t^6 - 2677850419968t^4 + 2834352t^2 - 1, \left(t \mapsto t \log\left(10170475895038464t^5 - 5496t^3 + 19131876t^2 + 19683t + x - 27/2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)

[Out] RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1, Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 31809932496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))

$$3.147 \quad \int \frac{x}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=361

$$\frac{(-1)^{2/3} \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{216\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{648\sqrt[3]{2} 3^{2/3}} - \frac{\log(x^2 + 3 2^{2/3} \sqrt[3]{3} x + 6)}{648\sqrt[3]{2} 3^{2/3}} - \frac{\text{ta}}{36\sqrt[6]{2} 3^{5/6}}$$

[Out] 1/1296*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^2-1/3888*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/3888*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/216*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/648*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/324*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)

Rubi [A] time = 0.55, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3} \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{216\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{648\sqrt[3]{2} 3^{2/3}} - \frac{\log(x^2 + 3 2^{2/3} \sqrt[3]{3} x + 6)}{648\sqrt[3]{2} 3^{2/3}} - \frac{\text{ta}}{36\sqrt[6]{2} 3^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(3*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x)/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)])]/(108*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(1/3)*3^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3)*3^(2/3))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{x}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3} (3\sqrt[3]{-3} 2^{2/3} - x)}{136048896 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)} + \right. \\ &= -\frac{(-1)^{2/3} \int \frac{3(-2)^{2/3} \sqrt[3]{3} + x}{6+3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{324 \sqrt[3]{2} 3^{2/3}} - \frac{\int \frac{6 \sqrt[3]{3} + \sqrt[3]{2} x}{6+3 2^{2/3} \sqrt[3]{3} x + x^2} dx}{324 6^{2/3}} + \frac{(-1)^{2/3} \int \frac{3 \sqrt[3]{-3}}{-6+3 \sqrt[3]{-3} x - x^2} dx}{108 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} \\ &= \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{1}{6+3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{108 2^{2/3}} - \frac{\int \frac{3 2^{2/3} \sqrt[3]{3} + 2x}{6+3 2^{2/3} \sqrt[3]{3} x + x^2} dx}{648 \sqrt[3]{2} 3^{2/3}} - \frac{(-1)^{2/3} \int \frac{3(-2)^{2/3} \sqrt[3]{3}}{6+3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{648 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} \\ &= \frac{(-1)^{2/3} \log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}{216 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \log(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)}{648 \sqrt[3]{2} 3^{2/3}} \\ &= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}}\right)}{36 \sqrt[3]{2} 3^{5/6} (1 + \sqrt[3]{-1})^2 \sqrt{4 - 3(-3)^{2/3} \sqrt[3]{2}}} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{3(-2)^{2/3} \sqrt[3]{3} + x}{\sqrt{6(4 + 3\sqrt[3]{-23})}}\right)}{108 \sqrt[3]{2} 3^{5/6} \sqrt{4 + 3\sqrt[3]{-23}}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.16

$$\frac{1}{6} \text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{\log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 16
2*#1 + 12*#1^2 + #1^4) &]/6

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 54, normalized size = 0.15

$$\frac{\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216) \ln\left(\frac{\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)}{6 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 72 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.42, size = 176, normalized size = 0.49

$$\sum_{k=1}^6 \ln\left(x + \text{root}\left(z^6 - \frac{z^4}{1640736} + \frac{235z^3}{129178426752} - \frac{z^2}{3100282242048} - \frac{1}{158171241119638192128}, z, k\right)\right) (216x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log(x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(216*x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(51018336*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(277947894528*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(33192121254912*x - root(z^6 - z^4/1640736 + (235*z^3)/1291784267

```
52 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(6940988288557056*x
+ 168897381688221696) + 28563737812992))))*root(z^6 - z^4/1640736 + (235*
z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k), k,
1, 6)
```

sympy [A] time = 0.26, size = 61, normalized size = 0.17

$$\text{RootSum}\left(158171241119638192128t^6 - 96402615118848t^4 + 287743415040t^3 - 51018336t^2 - 1, \left(t \mapsto t \log\left(\frac{65}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 287743415040*_
t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(65418399445721140961280*_t**5/
415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/415817
+ 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415817)
))
```


$$3.148 \quad \int \frac{1}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=377

$$\frac{\log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{216 \cdot 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} - \frac{\sqrt[3]{-\frac{1}{3}} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3}x + 6)}{648 \cdot 2^{2/3}} + \frac{\log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3}x + 6)}{648 \cdot 2^{2/3} \sqrt[3]{3}} + \frac{(-1)^{2/3} (3(-3)^{2/3})}{324 \sqrt[6]{3} (1 + \dots)}$$

[Out] $-1/1296*\ln(6-3*(-3)^{(1/3)}*2^{(2/3)}*x+x^2)*2^{(1/3)}*3^{(2/3)}/(1+(-1)^{(1/3)})^2-1/3888*(-1)^{(1/3)}*3^{(2/3)}*\ln(6+3*(-2)^{(2/3)}*3^{(1/3)}*x+x^2)*2^{(1/3)}+1/3888*\ln(6+3*2^{(2/3)}*3^{(1/3)}*x+x^2)*2^{(1/3)}*3^{(2/3)}+1/972*(-1)^{(2/3)}*(3*(-3)^{(2/3)}-2^{(2/3)})*\arctan((3*(-3)^{(1/3)}*2^{(2/3)}-2*x)/(24-18*(-3)^{(2/3)}*2^{(1/3)})^{(1/2)})*3^{(5/6)}/(1+(-1)^{(1/3)})^{(2/3)}+(8-6*(-3)^{(2/3)}*2^{(1/3)})^{(1/2)}-1/972*(9-2^{(2/3)}*3^{(1/3)})*\operatorname{arctanh}(2^{(1/6)}*(3*3^{(1/3)}+2^{(1/3)}*x)/(-12+9*2^{(1/3)}*3^{(2/3)})^{(1/2)})/(-24+18*2^{(1/3)}*3^{(2/3)})^{(1/2)}+1/972*(9-(-2)^{(2/3)}*3^{(1/3)})*\arctan((3*(-2)^{(2/3)}*3^{(1/3)}+2*x)/(24+18*(-2)^{(1/3)}*3^{(2/3)})^{(1/2)})/(24+27*I*2^{(1/3)}*3^{(1/6)}+9*2^{(1/3)}*3^{(2/3)})^{(1/2)}$

Rubi [A] time = 0.72, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2070, 634, 618, 204, 628, 206}

$$\frac{\log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{216 \cdot 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} - \frac{\sqrt[3]{-\frac{1}{3}} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3}x + 6)}{648 \cdot 2^{2/3}} + \frac{\log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3}x + 6)}{648 \cdot 2^{2/3} \sqrt[3]{3}} + \frac{(-1)^{2/3} (3(-3)^{2/3})}{324 \sqrt[6]{3} (1 + \dots)}$$

Antiderivative was successfully verified.

[In] Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1), x]

[Out] $((-1)^{(2/3)}*(3*(-3)^{(2/3)} - 2^{(2/3)})*\operatorname{ArcTan}[(3*(-3)^{(1/3)}*2^{(2/3)} - 2*x)/\operatorname{Sqrt}[6*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})]])/(324*3^{(1/6)}*(1 + (-1)^{(1/3)})^2*\operatorname{Sqrt}[2*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})]) + ((9 - (-2)^{(2/3)}*3^{(1/3)})*\operatorname{ArcTan}[(3*(-2)^{(2/3)}*3^{(1/3)} + 2*x)/\operatorname{Sqrt}[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})]])/(972*\operatorname{Sqrt}[3*(8 + (9*I)*2^{(1/3)}*3^{(1/6)} + 3*2^{(1/3)}*3^{(2/3)})]) - ((9 - 2^{(2/3)}*3^{(1/3)})*\operatorname{ArcTanh}[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/\operatorname{Sqrt}[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})]])/(972*\operatorname{Sqrt}[6*(-4 + 3*2^{(1/3)}*3^{(2/3)})]) - \operatorname{Log}[6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2]/(216*2^{(2/3)}*3^{(1/3)}*(1 + (-1)^{(1/3)})^2) - ((-1/3)^{(1/3)}*\operatorname{Log}[6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2])/(648*2^{(2/3)}) + \operatorname{Log}[6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2]/(648*2^{(2/3)}*3^{(1/3)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2070

Int[(Q6_)^(p_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3} (-2 + 6(-3)^{2/3} \sqrt[3]{2} - \sqrt[3]{-3} 2^{2/3} x)}{272097792 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)} + \right. \\ &= \frac{\int \frac{18 - 2^{2/3} \sqrt[3]{3} + \sqrt[3]{2} 3^{2/3} x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + x^2} dx}{1944} - \frac{\int \frac{2(-1)^{2/3} - 6\sqrt[3]{2} 3^{2/3} + \sqrt[3]{-3} 2^{2/3} x}{6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{648 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{-2 + 6(-3)^{2/3} \sqrt[3]{2} - \sqrt[3]{-3} 2^{2/3} x}{-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2} dx}{216 \sqrt[3]{2}} \\ &= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{648 2^{2/3}} + \frac{\int \frac{3 \cdot 2^{2/3} \sqrt[3]{3} + 2x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + x^2} dx}{648 2^{2/3} \sqrt[3]{3}} - \frac{\int \frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{-6 + 3 \sqrt[3]{-3} 2^{2/3} x - x^2} dx}{216 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} \\ &= -\frac{\log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}{216 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} - \frac{\sqrt[3]{-\frac{1}{3}} \log(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)}{648 2^{2/3}} + \frac{\log(6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)}{216 \sqrt[3]{2}} \\ &= -\frac{\sqrt[3]{-1} (9 + \sqrt[3]{-3} 2^{2/3}) \tan^{-1} \left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} \right)}{324 (1 + \sqrt[3]{-1})^2 \sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} - \frac{((-2)^{2/3} - 3 \cdot 3^{2/3}) \tan^{-1} \left(\frac{2(-1)^{2/3} - 6\sqrt[3]{2} 3^{2/3} + \sqrt[3]{-3} 2^{2/3} x}{-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2} \right)}{972 \sqrt[3]{3} \sqrt{2(4 - 3\sqrt[3]{-3} 2^{2/3} x - x^2)}} \end{aligned}$$

Mathematica [C] time = 0.01, size = 62, normalized size = 0.16

$$\frac{1}{6} \text{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + \frac{\log(x - \#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1), x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/6

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 53, normalized size = 0.14

$$\frac{\ln\left(-\text{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)\right)}{6 \text{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)^5 + 72 \text{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.67, size = 306, normalized size = 0.81

$$\sum_{k=1}^6 \ln\left(-\text{root}\left(z^6 - \frac{z^4}{9844416} - \frac{217z^3}{86118951168} - \frac{5z^2}{111610160713728} + \frac{z}{488182842961846272} - \frac{1}{34164988081841849499648}, z, k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 * x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)*x - 6122200320*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3*x - 258263796059136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713

```

728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4*x - 6940988
288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016
0713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5*x + 944
784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071372
8 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 - 16529940864
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
z/488182842961846272 - 1/34164988081841849499648, z, k)^3 - 33192121254912
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
z/488182842961846272 - 1/34164988081841849499648, z, k)^4 - 16889738168822
1696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607137
28 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5)*root(z^6 -
z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842
961846272 - 1/34164988081841849499648, z, k), k, 1, 6)

```

sympy [A] time = 0.27, size = 65, normalized size = 0.17

$$\text{RootSum}\left(34164988081841849499648t^6 - 3470494144278528t^4 - 86087932019712t^3 - 1530550080t^2 + 69984t - 1, \text{Lambda}(t, t \log(185904446699109611410573787136t^5/57121295165 + 6377301253267917382766592t^4/57121295165 - 18904636002388564311552t^3/57121295165 - 469080552915181723968t^2/57121295165 - 24358640509989936t/57121295165 + x + 152427895956/57121295165))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)

[Out] RootSum(34164988081841849499648*_t**6 - 3470494144278528*_t**4 - 86087932019712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(185904446699109611410573787136*_t**5/57121295165 + 6377301253267917382766592*_t**4/57121295165 - 18904636002388564311552*_t**3/57121295165 - 469080552915181723968*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895956/57121295165)))

$$3.149 \quad \int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)} dx$$

Optimal. Leaf size=415

$$\frac{(36 + 2^{2/3}\sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{46656} - \frac{(18 - (-2)^{2/3}\sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3}\sqrt[3]{3}x + 6)}{23328} - \frac{(18 - 2^{2/3}\sqrt[3]{3}) \log(x^2 + 3\sqrt[3]{-3}2^{2/3}x + 6)}{46656}$$

```
[Out] 1/216*ln(x)-1/23328*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)
-1/23328*(18-2^(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/46656*ln(6-3
*(-3)^(1/3)*2^(2/3)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))-1/1296*(-1)^(
2/3)*((-3)^(1/3)+3*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(
-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))
^(1/2)-1/1296*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-1
2+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/
1296*(-1)^(2/3)*((-2)^(2/3)-2*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(2
4+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(
1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(36 + 2^{2/3}\sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{46656} - \frac{(18 - (-2)^{2/3}\sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3}\sqrt[3]{3}x + 6)}{23328} - \frac{(18 - 2^{2/3}\sqrt[3]{3}) \log(x^2 + 3\sqrt[3]{-3}2^{2/3}x + 6)}{46656}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqr
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(216*2^(1/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*Arc
Tan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))
]])/(216*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 -
2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + Lo
g[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(
2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x +
x^2])/23328
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x(216 + 108x^2 + 324x^3 + 18x^4 + x^6)} dx = 1259712 \int \left(\frac{1}{272097792x} + \frac{(-1)^{2/3} (6(9 + \sqrt[3]{-3} 2^{2/3}) - (1 - 3(-1)^{2/3}))}{816293376 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (6 - 3(-1)^{2/3})} \right) dx$$

$$= \frac{\log(x)}{216} + \frac{\int \frac{-6 \sqrt[3]{6} (9 \sqrt[3]{2} - 2 \sqrt[3]{3}) - (18 - 2^{2/3} \sqrt[3]{3})x}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{11664} + \frac{(-1)^{2/3} \int \frac{-6(9-(-2)^{2/3})}{6+3(-1)^{2/3} x+x^2} dx}{1944 \sqrt[3]{-1}}$$

$$= \frac{\log(x)}{216} + \frac{\left(\left(-\frac{1}{6} \right)^{2/3} (\sqrt[3]{-3} + 3 \sqrt[3]{2}) \right) \int \frac{1}{6-3 \sqrt[3]{-3} 2^{2/3} x+x^2} dx}{72 (1 + \sqrt[3]{-1})^2} + \frac{(-1)^{2/3} \int \frac{-6(9-(-2)^{2/3})}{6+3(-1)^{2/3} x+x^2} dx}{1944 \sqrt[3]{-1}}$$

$$= \frac{\log(x)}{216} - \frac{(-1)^{2/3} (1 - 3(-3)^{2/3} \sqrt[3]{2}) \log(6 - 3 \sqrt[3]{-3} 2^{2/3} x + x^2)}{1296 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \int \frac{-6(9-(-2)^{2/3})}{6+3(-1)^{2/3} x+x^2} dx}{1944 \sqrt[3]{-1}}$$

$$= \frac{(-1)^{2/3} ((-2)^{2/3} - 2 \cdot 3^{2/3}) \tan^{-1} \left(\frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{\sqrt{6(4+3 \sqrt[3]{-2} 3^{2/3})}} \right)}{216 \cdot 6^{5/6} \sqrt{4 + 3 \sqrt[3]{-2} 3^{2/3}}} - \frac{(-1)^{2/3} (\sqrt[3]{-3} + 3 \sqrt[3]{2}) \log(6 - 3 \sqrt[3]{-3} 2^{2/3} x + x^2)}{216 \sqrt[3]{6} (1 + \sqrt[3]{-1})^2}$$

Mathematica [C] time = 0.02, size = 103, normalized size = 0.25

$$\frac{\log(x)}{216} - \frac{\text{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + \frac{\#1^4 \log(x-\#1) + 18\#1^2 \log(x-\#1) + 324\#1 \log(x-\#1) + 108 \log(x-\#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \right]}{1296}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]

[Out] Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Log[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/1296

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)

maple [C] time = 0.01, size = 75, normalized size = 0.18

$$\frac{\ln(x)}{216} \frac{\left(\text{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)^5 + 18 \text{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)\right)}{1296 \left(\text{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)^5 + 12 \text{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/216*ln(x)-1/1296*sum((R^5+18*R^3+324*R^2+108*R)/(R^5+12*R^3+162*R^2+36*R)*ln(-R+x),R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{216} \int \frac{x^5 + 18x^3 + 324x^2 + 108x}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx + \frac{1}{216} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] -1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x) + 1/216*log(x)

mupad [B] time = 2.33, size = 432, normalized size = 1.04

$$\frac{\ln(x)}{216} + \left(\sum_{k=1}^6 \ln \left(\text{root} \left(z^6 + \frac{z^5}{216} + \frac{421 z^4}{59066496} + \frac{100853 z^3}{27902540178432} - \frac{505 z^2}{12053897357082624} + \frac{z}{781092548738954} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)

```
[Out] log(x)/216 + symsum(log(7*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853
*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352
- 1/7379637425677839491923968, z, k)*x - 5670000*root(z^6 + z^5/216 + (421*
z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 +
z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2*x + 154687594
7520*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^3*x - 106961147905609728*root(z^6 + z^5/216 + (421*z^4)/590
66496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/78109
25487389540352 - 1/7379637425677839491923968, z, k)^4*x - 14051199585413401
8048*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^5*x - 45607290567387619000320*root(z^6 + z^5/216 + (421*z^4
)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/
7810925487389540352 - 1/7379637425677839491923968, z, k)^6*x + 839808*root(
z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2
)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968,
z, k)^2 + 594896472576*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^
3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1
/7379637425677839491923968, z, k)^3 - 8483430130458624*root(z^6 + z^5/216 +
(421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1205389735708
2624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4 - 38314
25535283494912*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902
540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637
425677839491923968, z, k)^5 + 1217393817906599165952*root(z^6 + z^5/216 + (
421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/120538973570826
24 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6)*root(z^6
+ z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1
2053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z,
k), k, 1, 6)
```

sympy [A] time = 0.42, size = 82, normalized size = 0.20

$$\frac{\log(x)}{216} + \text{RootSum}\left(7379637425677839491923968t^6 + 34164988081841849499648t^5 + 52598809250685370368t^4 + 26673506015311872t^3 - 309171116160t^2 + 944784t - 1, \text{Lambda}(t, t \cdot \log(8145570099668817936783362115119297360560128t^6/143425799309052440063 + 977068766770806381087358257564745728t^5/143425799309052440063 - 116529526608851264288400971539061538816t^4/143425799309052440063 - 239359794985242202542501440710766592t^3/143425799309052440063 - 136678312638137094439887341418240t^2/143425799309052440063 + 1563115569067663795735413696t/143425799309052440063 + x - 3164446315075236190044/143425799309052440063))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 34164988081841849499
648*_t**5 + 52598809250685370368*_t**4 + 26673506015311872*_t**3 - 30917111
6160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(81455700996688179367833621151
19297360560128*_t**6/143425799309052440063 + 977068766770806381087358257564
745728*_t**5/143425799309052440063 - 11652952660885126428840097153906153881
6*_t**4/143425799309052440063 - 239359794985242202542501440710766592*_t**3/
143425799309052440063 - 136678312638137094439887341418240*_t**2/14342579930
9052440063 + 1563115569067663795735413696*_t/143425799309052440063 + x - 31
64446315075236190044/143425799309052440063))
```


$$3.150 \quad \int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)} dx$$

Optimal. Leaf size=448

$$\frac{(-1)^{2/3} \left(9 + \sqrt[3]{-3} 2^{2/3}\right) \log \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)}{1296 \sqrt[3]{2} 3^{2/3} \left(1 + \sqrt[3]{-1}\right)^2} + \frac{\left(3(-6)^{2/3} + 2\sqrt[3]{-2}\right) \log \left(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6\right)}{7776 \sqrt[3]{3}} \frac{(2^{2/3} - 3)}{2}$$

```
[Out] -1/216/x-1/7776*(-1)^(2/3)*(9+(-3)^(1/3)*2^(2/3))*ln(6-3*(-3)^(1/3)*2^(2/3)
*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^2+1/23328*(3*(-6)^(2/3)+2*(-2)^(1/3)
)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*3^(2/3)-1/23328*(2^(2/3)-3*3^(2/3))*ln(6
+3*2^(2/3)*3^(1/3)*x+x^2)*6^(2/3)-1/11664*(-1)^(2/3)*(6*(-6)^(2/3)+27*(-3)^(
1/3)-2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(
1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/3499
2*(2^(1/3)+27*3^(1/3)-6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12
+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/17496*(27
*(-6)^(1/3)-(-2)^(2/3)+12*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18
*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))^^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3} \left(9 + \sqrt[3]{-3} 2^{2/3}\right) \log \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)}{1296 \sqrt[3]{2} 3^{2/3} \left(1 + \sqrt[3]{-1}\right)^2} + \frac{\left(3(-6)^{2/3} + 2\sqrt[3]{-2}\right) \log \left(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6\right)}{7776 \sqrt[3]{3}} \frac{(2^{2/3} - 3)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/(216*x) - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3*(-2)^(2/3)
)*3^(1/3) + 2*x]/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))])/(5832*3^(1/6)*Sqrt[8
+ (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(6*(-6)^(2/3) +
27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[
3*(4 - 3*(-3)^(2/3)*2^(1/3))])]/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3
*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)
)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))])/(5832*6^(1/6)
*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6
- 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2)
+ ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7
776*3^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(
3888*6^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} dx &= 1259712 \int \left(\frac{1}{272097792x^2} + \frac{(-1)^{2/3} (-1 + 9(-3)^{2/3} \sqrt[3]{2} + 27\sqrt[3]{-3})}{816293376 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} \right) dx \\ &= -\frac{1}{216x} + \frac{\int \frac{-54+2^{2/3} \sqrt[3]{3} + 54 \sqrt[3]{2} 3^{2/3} - 6^{2/3} (2^{2/3} - 3 \cdot 3^{2/3}) x}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{11664} + \frac{(-1)^{2/3} \int \frac{1+2^{2/3} \sqrt[3]{3} x}{\sqrt[3]{-1} x+x^2} dx}{11664} \\ &= -\frac{1}{216x} - \frac{((-1)^{2/3} (9 + \sqrt[3]{-3} 2^{2/3})) \int \frac{-3 \sqrt[3]{-3} 2^{2/3} + 2x}{6-3 \sqrt[3]{-3} 2^{2/3} x+x^2} dx}{1296 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \int \frac{1+2^{2/3} \sqrt[3]{3} x}{\sqrt[3]{-1} x+x^2} dx}{11664} \\ &= -\frac{1}{216x} - \frac{(-1)^{2/3} (9 + \sqrt[3]{-3} 2^{2/3}) \log(6 - 3 \sqrt[3]{-3} 2^{2/3} x + x^2)}{1296 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \int \frac{1+2^{2/3} \sqrt[3]{3} x}{\sqrt[3]{-1} x+x^2} dx}{11664} \\ &= -\frac{1}{216x} + \frac{(-1)^{2/3} (2 + 27(-2)^{2/3} \sqrt[3]{3} + 12 \sqrt[3]{-2} 3^{2/3}) \tan^{-1} \left(\frac{3(-2)^{2/3}}{\sqrt{6(4+3 \sqrt[3]{-2})}} \right)}{5832 \cdot 2^{5/6} \sqrt[3]{3} \sqrt{4 + 3 \sqrt[3]{-2} 3^{2/3}}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{\#1^4 \log(x-\#1) + 18\#1^2 \log(x-\#1) + 324\#1 \log(x-\#1) + 108 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\&\right]}{1296}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]

[Out] -1/216*1/x - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Log[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/1296

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)

maple [C] time = 0.01, size = 74, normalized size = 0.17

$$\frac{\left(-\text{RootOf}\left(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216\right)^4 - 18\text{RootOf}\left(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216\right)\right)}{1296\text{RootOf}\left(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216\right)^5 + 15552\text{RootOf}\left(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] -1/216/x+1/1296*sum((-R^4-18*_R^2-324*_R-108)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{216x} - \frac{1}{216} \int \frac{x^4 + 18x^2 + 324x + 108}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] -1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 0.29, size = 340, normalized size = 0.76

$$\left(\sum_{k=1}^6 \ln\left(\frac{5\text{root}\left(z^6 + \frac{281z^4}{118132992} - \frac{50435z^3}{9300846726144} - \frac{331z^2}{48215589428330496} - \frac{z}{1898054893435658305536} - \frac{1}{15940016839464133302555}\right)}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)`

[Out] `symsum(log((5*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k))/8 - (root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)*x)/216 - 396252*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)^2*x - 598229670528*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)^3*x + 82120746212352*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)^4*x - 6940988288557056*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)^5*x + 2344464*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)^2 - 210297580992*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)^3 - 10535082310656*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)^4 - 168897381688221696*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k)^5)*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k), k, 1, 6) - 1/(216*x)`

sympy [A] time = 0.32, size = 70, normalized size = 0.16

$\text{RootSum}\left(1594001683946413330255577088t^6 + 3791612026460331638784t^4 - 8643672699589509120t^3 - 10942820851968t^2 - 839808t - 1, \text{Lambda}(t, t \cdot \log(-49875532761902496003293561236914468028416t^5/12350449784703991795 + 12625489872431620388005975200497664t^4/12350449784703991795 - 118637692607573771238550798852644864t^3/12350449784703991795 + 270486324927832147818193778754816t^2/12350449784703991795 + 273914194897479402961199352t/12350449784703991795 + x - 12798926329353908292/12350449784703991795))\right) - 1/(216*x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)`

[Out] `RootSum(1594001683946413330255577088*_t**6 + 3791612026460331638784*_t**4 - 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda(_t, _t*log(-49875532761902496003293561236914468028416*_t**5/12350449784703991795 + 12625489872431620388005975200497664*_t**4/12350449784703991795 - 118637692607573771238550798852644864*_t**3/12350449784703991795 + 270486324927832147818193778754816*_t**2/12350449784703991795 + 273914194897479402961199352*_t/12350449784703991795 + x - 12798926329353908292/12350449784703991795))) - 1/(216*x)`

$$3.151 \quad \int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=1064

$$\frac{\sqrt[3]{-\frac{1}{3}} \left((2 + 27(-2)^{2/3} \sqrt[3]{3} + 12\sqrt[3]{-2} 3^{2/3}) x + 9(6 - (-2)^{2/3} \sqrt[3]{3}) \right)}{729 2^{2/3} (8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)} \frac{\sqrt[3]{-1} (2 + 27(-2)^{2/3} \sqrt[3]{3} + 12\sqrt[3]{-2} 3^{2/3}) \tan^{-1} \left(\frac{x + 3(-2)^{2/3} \sqrt[3]{3}}{1 + \sqrt[3]{-1}} \right)}{162 \sqrt[6]{2} 3^{5/6} (1 - \sqrt[3]{-1})^2 (1 + \sqrt[3]{-1})^4} (4$$

[Out] $-1/972*(-1)^{(1/3)}*3^{(2/3)}*(54+9*(-3)^{(1/3)}*2^{(2/3)}+(2-2^{(2/3)}*(6*(-6)^{(2/3)}+27*(-3)^{(1/3)}))*x)*2^{(1/3)}/(1+(-1)^{(1/3)})^4/(4-3*(-3)^{(2/3)}*2^{(1/3)})/(6-3*(-3)^{(1/3)}*2^{(2/3)}*x+x^2)-1/4374*(-1)^{(1/3)}*3^{(2/3)}*(54-9*(-2)^{(2/3)}*3^{(1/3)}+(2+27*(-2)^{(2/3)}*3^{(1/3)}+12*(-2)^{(1/3)}*3^{(2/3)})*x)*2^{(1/3)}/(8+9*I*2^{(1/3)}*3^{(1/6)}+3*2^{(1/3)}*3^{(2/3)})/(6+3*(-2)^{(2/3)}*3^{(1/3)}*x+x^2)+1/8748*(54-9*2^{(2/3)}*3^{(1/3)}+(2+2^{(2/3)}*(27*3^{(1/3)}-6*6^{(2/3)}))*x)*2^{(1/3)}*3^{(2/3)}/(4-3*2^{(1/3)}*3^{(2/3)})/(6+3*2^{(2/3)}*3^{(1/3)}*x+x^2)-1/972*(-1)^{(1/3)}*(2+27*(-2)^{(2/3)}*3^{(1/3)}+12*(-2)^{(1/3)}*3^{(2/3)})*arctan((3*(-2)^{(2/3)}*3^{(1/3)}+2*x)/(24+18*(-2)^{(1/3)}*3^{(2/3)})^{(1/2)})*2^{(5/6)}*3^{(1/6)}/(1-(-1)^{(1/3)})^2/(1+(-1)^{(1/3)})^4/(4+3*(-2)^{(1/3)}*3^{(2/3)})^{(3/2)}-1/5832*I*ln(6-3*(-3)^{(1/3)}*2^{(2/3)}*x+x^2)*2^{(2/3)}*3^{(1/3)}/(1+(-1)^{(1/3)})^4+1/5832*I*ln(6+3*(-2)^{(2/3)}*3^{(1/3)}*x+x^2)*2^{(2/3)}*3^{(5/6)}/(1+(-1)^{(1/3)})^5-1/52488*I*ln(6+3*2^{(2/3)}*3^{(1/3)}*x+x^2)*2^{(2/3)}*3^{(1/3)}-1/486*(-1)^{(1/3)}*(6*(-6)^{(2/3)}+27*(-3)^{(1/3)}-2^{(1/3)})*arctan(2^{(1/6)}*(3*(-3)^{(1/3)}-2^{(1/3)}*x)/(12-9*(-3)^{(2/3)}*2^{(1/3)})^{(1/2)})*3^{(1/6)}/(1+(-1)^{(1/3)})^4/(4-3*(-3)^{(2/3)}*2^{(1/3)})^{(3/2)}*2^{(1/2)}+1/486*(2^{(1/3)}+27*3^{(1/3)}-6*6^{(2/3)})*arctanh(2^{(1/6)}*(3*3^{(1/3)}+2^{(1/3)}*x)/(-12+9*2^{(1/3)}*3^{(2/3)})^{(1/2)})*3^{(1/6)}/(1-(-1)^{(1/3)})^2/(1+(-1)^{(1/3)})^4/(-4+3*2^{(1/3)}*3^{(2/3)})^{(3/2)}*2^{(1/2)}+1/972*(I*2^{(2/3)}-9*3^{(1/6)}-3*I*3^{(2/3)})*arctan(2^{(1/6)}*(3*(-3)^{(1/3)}-2^{(1/3)}*x)/(12-9*(-3)^{(2/3)}*2^{(1/3)})^{(1/2)})*2^{(1/6)}*3^{(2/3)}/(1+(-1)^{(1/3)})^5/(4-3*(-3)^{(2/3)}*2^{(1/3)})^{(1/2)}-1/972*I*((-2)^{(2/3)}+6*3^{(2/3)})*arctan((3*(-2)^{(2/3)}*3^{(1/3)}+2*x)/(24+18*(-2)^{(1/3)}*3^{(2/3)})^{(1/2)})*2^{(1/6)}*3^{(2/3)}/(1+(-1)^{(1/3)})^5/(4+3*(-2)^{(1/3)}*3^{(2/3)})^{(1/2)}-1/8748*(1+3*2^{(1/3)}*3^{(2/3)})*arctanh(2^{(1/6)}*(3*3^{(1/3)}+2^{(1/3)}*x)/(-12+9*2^{(1/3)}*3^{(2/3)})^{(1/2)})*2^{(5/6)}*3^{(1/6)}/(-4+3*2^{(1/3)}*3^{(2/3)})^{(1/2)}$

Rubi [A] time = 2.50, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}} \left((2 + 27(-2)^{2/3} \sqrt[3]{3} + 12\sqrt[3]{-2} 3^{2/3}) x + 9(6 - (-2)^{2/3} \sqrt[3]{3}) \right)}{729 2^{2/3} (8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)} \frac{\sqrt[3]{-1} (2 + 27(-2)^{2/3} \sqrt[3]{3} + 12\sqrt[3]{-2} 3^{2/3}) \tan^{-1} \left(\frac{x + 3(-2)^{2/3} \sqrt[3]{3}}{1 + \sqrt[3]{-1}} \right)}{162 \sqrt[6]{2} 3^{5/6} (1 - \sqrt[3]{-1})^2 (1 + \sqrt[3]{-1})^4} (4$$

Antiderivative was successfully verified.

[In] Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $-((-1/3)^{(1/3)}*(9*(6 + (-3)^{(1/3)}*2^{(2/3)}) + (2 - 3*2^{(2/3)}*(2*(-6)^{(2/3)} + 9*(-3)^{(1/3)}))*x))/(162*2^{(2/3)}*(1 + (-1)^{(1/3)})^4*(4 - 3*(-3)^{(2/3)}*2^{(1/3)}*(6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2)) - ((-1/3)^{(1/3)}*(9*(6 - (-2)^{(2/3)}*3^{(1/3)} + (2 + 27*(-2)^{(2/3)}*3^{(1/3)} + 12*(-2)^{(1/3)}*3^{(2/3)})*x))/(729*2^{(2/3)}*(8 + (9*I)*2^{(1/3)}*3^{(1/6)} + 3*2^{(1/3)}*3^{(2/3)})*(6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2)) + (9*(6 - 2^{(2/3)}*3^{(1/3)}) + (2 + 2^{(2/3)}*(27*3^{(1/3)} - 6*6^{(2/3)}))*x)/(1458*2^{(2/3)}*3^{(1/3)}*(4 - 3*2^{(1/3)}*3^{(2/3)})*(6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2)) - ((I/162)*((-2)^{(2/3)} + 6*3^{(2/3)})*ArcTan[(3*(-2)^{(2/3)}*3^{(1/3)} + 2*x)/sqrt[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})]])/(2^{(5/6)}*3^{(1/3)}*(1 + (-1)^{(1/3)})^4)$

$$\begin{aligned} & (1/3))^5 \sqrt{4 + 3(-2)^{1/3} 3^{2/3}}) - ((-1)^{1/3} (2 + 27(-2)^{2/3} 3^{1/3} \\ & + 12(-2)^{1/3} 3^{2/3}) \operatorname{ArcTan}[(3(-2)^{2/3} 3^{1/3} + 2x) / \sqrt{6 * \\ & (4 + 3(-2)^{1/3} 3^{2/3})}]) / (162 * 2^{1/6} * 3^{5/6} * (1 - (-1)^{1/3})^2 * (1 + \\ & (-1)^{1/3})^4 * (4 + 3(-2)^{1/3} 3^{2/3})^{3/2}) - ((-1)^{1/3} * (6 * (-6)^{2/3} \\ & + 27 * (-3)^{1/3} - 2^{1/3}) \operatorname{ArcTan}[(2^{1/6} * (3 * (-3)^{1/3} - 2^{1/3} * x)) / \sqrt{ \\ & 3 * (4 - 3 * (-3)^{2/3} * 2^{1/3})}]) / (81 * \sqrt{2} * 3^{5/6} * (1 + (-1)^{1/3})^4 * (4 \\ & - 3 * (-3)^{2/3} * 2^{1/3})^{3/2}) + ((I * 2^{2/3} - 9 * 3^{1/6} - (3 * I) * 3^{2/3}) * \\ & \operatorname{ArcTan}[(2^{1/6} * (3 * (-3)^{1/3} - 2^{1/3} * x)) / \sqrt{3 * (4 - 3 * (-3)^{2/3} * 2^{1/3} \\ &)}]) / (162 * 2^{5/6} * 3^{1/3} * (1 + (-1)^{1/3})^5 * \sqrt{4 - 3 * (-3)^{2/3} * 2^{1/3} \\ & }) - ((1 + 3 * 2^{1/3} * 3^{2/3}) \operatorname{ArcTanh}[(2^{1/6} * (3 * 3^{1/3} + 2^{1/3} * x)) / \sqrt{ \\ & 3 * (-4 + 3 * 2^{1/3} * 3^{2/3})}]) / (1458 * 2^{1/6} * 3^{5/6} * \sqrt{-4 + 3 * 2^{1/3} * 3^{2/3} \\ & ^{2/3}}) + ((2^{1/3} + 27 * 3^{1/3} - 6 * 6^{2/3}) \operatorname{ArcTanh}[(2^{1/6} * (3 * 3^{1/3} \\ & + 2^{1/3} * x)) / \sqrt{3 * (-4 + 3 * 2^{1/3} * 3^{2/3})}]) / (81 * \sqrt{2} * 3^{5/6} * (1 - (-1)^{1/3})^2 * (1 + (-1)^{1/3})^4 * (-4 + 3 * 2^{1/3} * 3^{2/3})^{3/2}) - \operatorname{Log}[6 - 3 \\ & * (-3)^{1/3} * 2^{2/3} * x + x^2] / (972 * 2^{1/3} * 3^{2/3} * (1 + (-1)^{1/3})^4) + ((I \\ & / 972) * \operatorname{Log}[6 + 3 * (-2)^{2/3} * 3^{1/3} * x + x^2]) / (2^{1/3} * 3^{1/6} * (1 + (-1)^{1/3} \\ &)^5) - \operatorname{Log}[6 + 3 * 2^{2/3} * 3^{1/3} * x + x^2] / (8748 * 2^{1/3} * 3^{2/3}) \end{aligned}$$
Rule 204

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTan}[\operatorname{Rt}[-b, 2] * x] / \operatorname{Rt}[-a, 2]] / (\operatorname{Rt}[-a, 2] * \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$$
Rule 206

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 * \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] * x] / \operatorname{Rt}[a, 2]] / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$$
Rule 618

$$\operatorname{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1 / \operatorname{Simp}[b^2 - 4 * a * c - x^2, x], x], x, b + 2 * c * x], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \operatorname{NeQ}[b^2 - 4 * a * c, 0]$$
Rule 628

$$\operatorname{Int}[(d + (e \cdot x)) / (a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \operatorname{Simp}[(d * \operatorname{Log}[\operatorname{RemoveContent}[a + b * x + c * x^2, x]]) / b, x] /; \operatorname{FreeQ}\{a, b, c, d, e, x\} \ \&\& \operatorname{EqQ}[2 * c * d - b * e, 0]$$
Rule 634

$$\operatorname{Int}[(d + (e \cdot x)) / (a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \operatorname{Dist}[(2 * c * d - b * e) / (2 * c), \operatorname{Int}[1 / (a + b * x + c * x^2), x], x] + \operatorname{Dist}[e / (2 * c), \operatorname{Int}[(b + 2 * c * x) / (a + b * x + c * x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, x\} \ \&\& \operatorname{NeQ}[2 * c * d - b * e, 0] \ \&\& \operatorname{NeQ}[b^2 - 4 * a * c, 0] \ \&\& \operatorname{!NiceSqrtQ}[b^2 - 4 * a * c]$$
Rule 638

$$\operatorname{Int}[(d + (e \cdot x)) * (a + (b \cdot x) + (c \cdot x)^2)^{p}, x_Symbol] \rightarrow \operatorname{Simp}[(b * d - 2 * a * e + (2 * c * d - b * e) * x) * (a + b * x + c * x^2)^{p+1} / ((p+1) * (b^2 - 4 * a * c)), x] - \operatorname{Dist}[(2 * p + 3) * (2 * c * d - b * e) / ((p+1) * (b^2 - 4 * a * c)), \operatorname{Int}[(a + b * x + c * x^2)^{p+1}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, x\} \ \&\& \operatorname{NeQ}[2 * c * d - b * e, 0] \ \&\& \operatorname{NeQ}[b^2 - 4 * a * c, 0] \ \&\& \operatorname{LtQ}[p, -1] \ \&\& \operatorname{NeQ}[p, -3/2]$$
Rule 2097

$$\operatorname{Int}[(Q6)^{p} * (u), x_Symbol] \rightarrow \operatorname{With}\{a = \operatorname{Coeff}[Q6, x, 0], b = \operatorname{Coeff}[Q6,$$

$x, 2], c = \text{Coeff}[Q6, x, 3], d = \text{Coeff}[Q6, x, 4], e = \text{Coeff}[Q6, x, 6]\}, \text{Dist}$
 $[1/(3^{(3*p)}*a^{(2*p)}), \text{Int}[\text{ExpandIntegrand}[u*(3*a + 3*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x$
 $+ b*x^2)^p*(3*a - 3*(-1)^{(1/3)}*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a + 3*(-$
 $1)^{(2/3)}*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p, x], x] /; \text{EqQ}[b^2 - 3*a*d, 0$
 $] \&\& \text{EqQ}[b^3 - 27*a^2*e, 0]] /; \text{ILtQ}[p, 0] \&\& \text{PolyQ}[Q6, x, 6] \&\& \text{EqQ}[\text{Coeff}[$
 $Q6, x, 1], 0] \&\& \text{EqQ}[\text{Coeff}[Q6, x, 5], 0] \&\& \text{RationalFunctionQ}[u, x]$

Rubi steps

$$\int \frac{x^8}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = 1586874322944 \int \left(\frac{\sqrt[3]{-\frac{1}{3}} \left(-1 + 3(-3)^{2/3} \sqrt[3]{2} + (9 + \sqrt[3]{-3}) \right)}{42845606719488 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (6 - 3\sqrt[3]{-3})} \right) dx$$

$$= \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{-1 - 3\sqrt[3]{-2} \cdot 3^{2/3} + (9 - (-2)^{2/3} \sqrt[3]{3})x}{(6 + 3(-2)^{2/3} \sqrt[3]{3}x + x^2)^2} dx}{243 \cdot 2^{2/3}} - \int \frac{-27 + x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3}x + x^2} dx + \dots$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \left(9(6 + \sqrt[3]{-3} \cdot 2^{2/3}) + (2 - 3 \cdot 2^{2/3} (2(-6)^{2/3} + 9\sqrt[3]{-3}))x \right)}{162 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} \cdot 2^{2/3}x + x^2)}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \left(9(6 + \sqrt[3]{-3} \cdot 2^{2/3}) + (2 - 3 \cdot 2^{2/3} (2(-6)^{2/3} + 9\sqrt[3]{-3}))x \right)}{162 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} \cdot 2^{2/3}x + x^2)}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \left(9(6 + \sqrt[3]{-3} \cdot 2^{2/3}) + (2 - 3 \cdot 2^{2/3} (2(-6)^{2/3} + 9\sqrt[3]{-3}))x \right)}{162 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} \cdot 2^{2/3}x + x^2)}$$

Mathematica [C] time = 0.04, size = 167, normalized size = 0.16

$$\frac{-9x^5 - 203x^4 - 11610x^3 - 3990x^2 + 324x - 7884}{34182(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{9\#1^4}{2}\right]}{2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 &, (324*Log[x - #1] - 96*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 + 406*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/205092

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.02, size = 122, normalized size = 0.11

$$\frac{(-9\text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^4 - 406\text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216))}{205092\text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^5 + 2461104\text{RootOf}(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R^3-324*_R^2+96*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9x^5 + 203x^4 + 11610x^3 + 3990x^2 - 324x + 7884}{34182(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{1}{34182} \int \frac{9x^4 + 406x^3 + 324x^2 - 96x + 324}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] -1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x^4 + 406*x^3 + 324*x^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.34, size = 388, normalized size = 0.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((239491904*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)*x)/876306843 - (275536*x)/638827688547 - (3848128*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k))/3606201 - (152363520*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)^2*x)/44521 - (698075283456*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)^3*x)/44521 + (130789789876224*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113

$728 + 4513/85256017052964187415123360664576, z, k)^4 \cdot x) / 211 - 6940988288557$
 $056 \cdot \text{root}(z^6 + (326 \cdot z^4) / 554702231619 + (8113597 \cdot z^3) / 14149992416343982992$
 $+ (5171 \cdot z^2) / 509399726988383387712 + (505 \cdot z) / 13368686435083133627113728 + 4$
 $513 / 85256017052964187415123360664576, z, k)^5 \cdot x - (4264220928 \cdot \text{root}(z^6 + (3$
 $26 \cdot z^4) / 554702231619 + (8113597 \cdot z^3) / 14149992416343982992 + (5171 \cdot z^2) / 5093$
 $99726988383387712 + (505 \cdot z) / 13368686435083133627113728 + 4513 / 8525601705296$
 $4187415123360664576, z, k)^2) / 44521 - (5086414725120 \cdot \text{root}(z^6 + (326 \cdot z^4) / 5$
 $54702231619 + (8113597 \cdot z^3) / 14149992416343982992 + (5171 \cdot z^2) / 5093997269883$
 $83387712 + (505 \cdot z) / 13368686435083133627113728 + 4513 / 8525601705296418741512$
 $3360664576, z, k)^3) / 44521 + (243585208571904 \cdot \text{root}(z^6 + (326 \cdot z^4) / 55470223$
 $1619 + (8113597 \cdot z^3) / 14149992416343982992 + (5171 \cdot z^2) / 50939972698838338771$
 $2 + (505 \cdot z) / 13368686435083133627113728 + 4513 / 85256017052964187415123360664$
 $576, z, k)^4) / 211 - 168897381688221696 \cdot \text{root}(z^6 + (326 \cdot z^4) / 554702231619 +$
 $(8113597 \cdot z^3) / 14149992416343982992 + (5171 \cdot z^2) / 509399726988383387712 + (50$
 $5 \cdot z) / 13368686435083133627113728 + 4513 / 85256017052964187415123360664576, z,$
 $k)^5 - 48160 / 23660284761) \cdot \text{root}(z^6 + (326 \cdot z^4) / 554702231619 + (8113597 \cdot z^3$
 $) / 14149992416343982992 + (5171 \cdot z^2) / 509399726988383387712 + (505 \cdot z) / 1336868$
 $6435083133627113728 + 4513 / 85256017052964187415123360664576, z, k), k, 1, 6$
 $) - ((665 \cdot x^2) / 5697 - (2 \cdot x) / 211 + (215 \cdot x^3) / 633 + (203 \cdot x^4) / 34182 + x^5 / 379$
 $8 + 146 / 633) / (108 \cdot x^2 + 324 \cdot x^3 + 18 \cdot x^4 + x^6 + 216)$

sympy [A] time = 0.40, size = 112, normalized size = 0.11

RootSum $\left(85256017052964187415123360664576t^6 + 50105191533385434568704t^4 + 48885748051277486016t^3 + 865447782603408t^2 + 3220532460t + 4513, \text{Lambda}(t, t \cdot \log(35492036204084174404119193135483487466590764032t^5 / 356900697070792948475845 - 19474160067218837086826809631017022308224t^4 / 71380139414158589695169 + 20779963076545132233894582764903396544t^3 / 356900697070792948475845 + 20265219154367004972162198012037344t^2 / 356900697070792948475845 + 275192468949210532049075145372t / 356900697070792948475845 + x + 1290285191292177289622012 / 1070702091212378845427535)) \right) + (-9x^5 - 203x^4 - 11610x^3 - 3990x^2 + 324x - 7884) / (34182x^6 + 615276x^4 + 11074968x^3 + 3691656x^2 + 7383312)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(85256017052964187415123360664576*_t**6 + 50105191533385434568704*_t**4 + 48885748051277486016*_t**3 + 865447782603408*_t**2 + 3220532460*_t + 4513, Lambda(_t, _t*log(35492036204084174404119193135483487466590764032*_t**5/356900697070792948475845 - 19474160067218837086826809631017022308224*_t**4/71380139414158589695169 + 20779963076545132233894582764903396544*_t**3/356900697070792948475845 + 20265219154367004972162198012037344*_t**2/356900697070792948475845 + 275192468949210532049075145372*_t/356900697070792948475845 + x + 1290285191292177289622012/1070702091212378845427535))) + (-9*x**5 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 + 615276*x**4 + 11074968*x**3 + 3691656*x**2 + 7383312)

$$3.152 \quad \int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=1005

$$\frac{2(2-3\sqrt[3]{2}3^{2/3})-3(6-2^{2/3}\sqrt[3]{3})x}{2916 \cdot 2^{2/3} \sqrt[3]{3} (4-3\sqrt[3]{2}3^{2/3})(x^2+3 \cdot 2^{2/3} \sqrt[3]{3}x+6)} + \frac{(9i + \sqrt[3]{3} (2i2^{2/3} - 9\sqrt[3]{3} + 2 \cdot 2^{2/3} \sqrt[3]{3})) \tan^{-1} \left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6(4-3(-3)^{2/3}\sqrt[3]{2})}} \right)}{5832 (1 + \sqrt[3]{-1})^5 \sqrt{2(4-3(-3)^{2/3}\sqrt[3]{2})}}$$

[Out] 1/1944*(-4*(-1)^(1/3)*3^(2/3)-18*6^(1/3)+9*((-2)^(2/3)+2*(-1)^(1/3)*3^(2/3))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/4374*(-(-6)^(1/3)*(9*(-2)^(1/3)+2*3^(1/3))+9*(1+(-2)^(1/3)*3^(2/3))*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/17496*(4-6*2^(1/3)*3^(2/3)-3*(6-2^(2/3)*3^(1/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/3888*I*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5-1/104976*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)-1/324*(-1)^(1/3)*((-3)^(1/3)+3*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)-1/7776*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/324*(1+(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)*6^(1/2)+1/324*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)+1/5832*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I+3^(1/3)*(2*I*2^(2/3)-9*3^(1/6)+2*2^(2/3)*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)+1/5832*(9*3^(1/6)+I*(4*2^(2/3)-3*3^(2/3)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(1/3)/(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3))^(1/2)+1/78732*(2*2^(2/3)+3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))^(1/2)

Rubi [A] time = 2.40, antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 634, 618, 204, 628, 638, 206}

$$\frac{2(2-3\sqrt[3]{2}3^{2/3})-3(6-2^{2/3}\sqrt[3]{3})x}{2916 \cdot 2^{2/3} \sqrt[3]{3} (4-3\sqrt[3]{2}3^{2/3})(x^2+3 \cdot 2^{2/3} \sqrt[3]{3}x+6)} + \frac{(9i + \sqrt[3]{3} (2i2^{2/3} - 9\sqrt[3]{3} + 2 \cdot 2^{2/3} \sqrt[3]{3})) \tan^{-1} \left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6(4-3(-3)^{2/3}\sqrt[3]{2})}} \right)}{5832 (1 + \sqrt[3]{-1})^5 \sqrt{2(4-3(-3)^{2/3}\sqrt[3]{2})}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] -(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3))*x)/(972*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) - 9*(1 + (-2)^(1/3)*3^(2/3))*x)/(4374*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*(6 - 2^(2/3)*3^(1/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*(2*I)*2^(2/3) - 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3)))

$$\begin{aligned} &)^4(4 + 3(-2)^{(1/3)}3^{(2/3)})^{(3/2)} + ((9*3^{(1/6)} + I*(4*2^{(2/3)} - 3*3^{(2/3)})) \\ &)*\text{ArcTan}[(3*(-2)^{(2/3)}*3^{(1/3)} + 2*x)/\text{Sqrt}[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)}) \\ &]])/(1944*3^{(2/3)}*(1 + (-1)^{(1/3)})^5*\text{Sqrt}[2*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})]) - \\ &((-1)^{(1/3)}*((-3)^{(1/3)} + 3*2^{(1/3)})*\text{ArcTan}[(2^{(1/6)}*(3*(-3)^{(1/3)} - 2^{(1/3)} \\ &)*x)/\text{Sqrt}[3*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})]])/(54*\text{Sqrt}[2]*3^{(5/6)}*(1 + (-1)^{(1/3)} \\ &)^4*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})^{(3/2)} + ((1 - 2^{(1/3)}*3^{(2/3)})*\text{ArcTanh}[(\\ &2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/\text{Sqrt}[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})]])/(54*\text{Sqr} \\ &\text{t}[6]*(1 - (-1)^{(1/3)})^2*(1 + (-1)^{(1/3)})^4*(-4 + 3*2^{(1/3)}*3^{(2/3)})^{(3/2)} \\ &+ ((2*2^{(2/3)} + 3*3^{(2/3)})*\text{ArcTanh}[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/\text{Sqrt}[3 \\ &*(-4 + 3*2^{(1/3)}*3^{(2/3)})]])/(26244*3^{(1/6)}*\text{Sqrt}[2*(-4 + 3*2^{(1/3)}*3^{(2/3)}) \\ &]) + ((I/648)*\text{Log}[6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2])/ (2^{(2/3)}*3^{(5/6)}*(1 + \\ &(-1)^{(1/3)})^5) - ((I + \text{Sqrt}[3])*\text{Log}[6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2])/ (129 \\ &6*2^{(2/3)}*3^{(5/6)}*(1 + (-1)^{(1/3)})^5) - \text{Log}[6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2]/ \\ &(17496*2^{(2/3)}*3^{(1/3)}) \end{aligned}$$
Rule 204

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> } -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}\{a/b\} \ \&\& \ (\text{LtQ}\{a, 0\} \ || \ \text{LtQ}\{b, 0\})$$
Rule 206

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}\{a/b\} \ \&\& \ (\text{GtQ}\{a, 0\} \ || \ \text{LtQ}\{b, 0\})$$
Rule 618

$$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] \text{ /; FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}\{b^2 - 4*a*c, 0\}$$
Rule 628

$$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \text{ :> } \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \text{ /; FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}\{2*c*d - b*e, 0\}$$
Rule 634

$$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \text{ :> } \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] \text{ /; FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}\{2*c*d - b*e, 0\} \ \&\& \ \text{NeQ}\{b^2 - 4*a*c, 0\} \ \&\& \ \text{!NiceSqrtQ}\{b^2 - 4*a*c\}$$
Rule 638

$$\text{Int}[(d_ + (e_)*(x_))*((a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \text{ :> } \text{Simp}[(b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^{(p + 1)}/((p + 1)*(b^2 - 4*a*c)), x] - \text{Dist}[(2*p + 3)*(2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^{(p + 1)}, x], x] \text{ /; FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}\{2*c*d - b*e, 0\} \ \&\& \ \text{NeQ}\{b^2 - 4*a*c, 0\} \ \&\& \ \text{LtQ}\{p, -1\} \ \&\& \ \text{NeQ}\{p, -3/2\}$$
Rule 2097

$$\text{Int}[(Q6_)^{(p_)}*(u_), x_Symbol] \text{ :> } \text{With}\{a = \text{Coeff}[Q6, x, 0], b = \text{Coeff}[Q6, x, 2], c = \text{Coeff}[Q6, x, 3], d = \text{Coeff}[Q6, x, 4], e = \text{Coeff}[Q6, x, 6]\}, \text{Dist}[1/(3^{(3*p)}*a^{(2*p)}), \text{Int}[\text{ExpandIntegrand}[u*(3*a + 3*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^{(1/3)}*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a + 3*(-$$

1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\int \frac{x^7}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = 1586874322944 \int \left(\frac{-27 + 3 \cdot 2^{2/3} \sqrt[3]{3} + 9i\sqrt{3} + i2^{2/3}3^{5/6} - 3i\sqrt[3]{2}}{9254651051409408 (1 + \sqrt[3]{-1})^5 (-6 + 3\sqrt[3]{-3}2^{2/3}x + x^2)} \right. \\ = \frac{\int \frac{-18 - 2 \cdot 2^{2/3} \sqrt[3]{3} - \sqrt[3]{2} 3^{2/3} x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + x^2} dx}{52488} + \frac{\int \frac{9 \cdot 2^{2/3} + \sqrt[3]{-1} 3^{2/3} (1 + 3 \sqrt[3]{-2} 3^{2/3}) x}{(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)^2} dx}{4374 \cdot 2^{2/3}} + \frac{\int \frac{3 \cdot 2^{2/3} + \sqrt[3]{-1} 3^{2/3} x}{(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)^2} dx}{8} \\ = -\frac{2(2\sqrt[3]{-1} 3^{2/3} + 9\sqrt[3]{6}) - 9((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) x}{972 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{3 \cdot 2^{2/3} + \sqrt[3]{-1} 3^{2/3} x}{8} \\ = -\frac{2(2\sqrt[3]{-1} 3^{2/3} + 9\sqrt[3]{6}) - 9((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) x}{972 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{3 \cdot 2^{2/3} + \sqrt[3]{-1} 3^{2/3} x}{8} \\ = -\frac{2(2\sqrt[3]{-1} 3^{2/3} + 9\sqrt[3]{6}) - 9((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) x}{972 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{3 \cdot 2^{2/3} + \sqrt[3]{-1} 3^{2/3} x}{8}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.17

$$\frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{73\#1^4 \log(x-\#1) - 36\#1^3 \log(x-\#1) + 96\#1^2 \log(x-\#1) - 216\#1 \log(x-\#1) + 96 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right]}{410184}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

[Out] (648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 &, (96*Log[x - #1] - 216*Log[x - #1]*#1 + 96*Log[x - #1]*#1^2 - 36*Log[x - #1]*#1^3 + 73*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/410184

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.02, size = 122, normalized size = 0.12

$$\frac{(73 \operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216))^4 - 36 \operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)}{410184 \operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^5 + 4922208 \operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{73x^5 - 18x^4 + 908x^3 + 432x^2 - 96x + 648}{68364(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} + \frac{1}{68364} \int \frac{73x^4 - 36x^3 + 96x^2 - 216x + 96}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] 1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 - 216*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.30, size = 387, normalized size = 0.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((8336932*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k))/97367427 - (480227*x)/851770251396 - (759164282*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)*x)/7886761587 - (207565888*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^2*x)/400689 - (108430970112*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^3*x)/44521 - (147138513610752*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5*x

```

- (1156135728*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546
6626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/31
1864717157619341253309046784 - 7197829/589289589870088463413332668913549312
, z, k)^2)/44521 + (6458021903232*root(z^6 - (292589*z^4)/319508485412544 +
(11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899
008 - (1989787*z)/311864717157619341253309046784 - 7197829/5892895898700884
63413332668913549312, z, k)^3)/44521 - (102226052063232*root(z^6 - (292589*
z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/
2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 71
97829/589289589870088463413332668913549312, z, k)^4)/211 - 1688973816882216
96*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012
42624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/3118647171576
19341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5 +
2207561/7665932262564)*root(z^6 - (292589*z^4)/319508485412544 + (11805253
*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (198
9787*z)/311864717157619341253309046784 - 7197829/58928958987008846341333266
8913549312, z, k), k, 1, 6) + ((4*x^2)/633 - (8*x)/5697 + (227*x^3)/17091 -
x^4/3798 + (73*x^5)/68364 + 2/211)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216
)

```

sympy [A] time = 0.40, size = 112, normalized size = 0.11

RootSum(589289589870088463413332668913549312t⁶ - 539640290266075248405737472t⁴ + 92182638168509682392064t³ - 553241442069170496t² - 3759837842016t - 7197829, Lambda(_t, _t*log(42996027639727447714003743305160746111018438501025999323136*_t⁵/154206009791052044490694380303237521 - 42584766259508194684689715474422251405157209835847680*_t⁴/154206009791052044490694380303237521 - 37512446128849588150108369449323754078317341082112*_t³/154206009791052044490694380303237521 + 7152037594021675267638890715531672481920222144*_t²/154206009791052044490694380303237521 - 44227546998835297723830291794974310524032*_t/154206009791052044490694380303237521 + x - 174573349036676047734132569583024855/154206009791052044490694380303237521))) + (73*x⁵ - 18*x⁴ + 908*x³ + 432*x² - 96*x + 648)/(68364*x⁶ + 1230552*x⁴ + 22149936*x³ + 7383312*x² + 14766624)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(589289589870088463413332668913549312*_t**6 - 539640290266075248405737472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 - 3759837842016*_t - 7197829, Lambda(_t, _t*log(42996027639727447714003743305160746111018438501025999323136*_t**5/154206009791052044490694380303237521 - 42584766259508194684689715474422251405157209835847680*_t**4/154206009791052044490694380303237521 - 37512446128849588150108369449323754078317341082112*_t**3/154206009791052044490694380303237521 + 7152037594021675267638890715531672481920222144*_t**2/154206009791052044490694380303237521 - 44227546998835297723830291794974310524032*_t/154206009791052044490694380303237521 + x - 174573349036676047734132569583024855/154206009791052044490694380303237521))) + (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364*x**6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)

$$3.153 \quad \int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=677

$$\frac{\sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x + 9(-2)^{2/3}}{2916 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)} + \frac{\sqrt[3]{-1} 3^{2/3} (2 + 3\sqrt[3]{-2} 3^{2/3}) x + 9 2^{2/3}}{13122 2^{2/3} (8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 + 9 2^{2/3})}$$

[Out] 1/5832*(9*(-2)^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/26244*(9*2^(2/3)+(-1)^(1/3)*3^(2/3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(3*2^(2/3)*3^(1/3)-(2-3*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/2916*(-1)^(1/3)*(3*(-3)^(2/3)-2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^2)*6^(1/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^3/2)+1/2916*(3*(-3)^(2/3)+(-1)^(1/3)*2^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^2)*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^3/2)-1/2916*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^2)*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^3/2)+1/34992*(-1)^(1/6)*3^(5/6)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)/(1+(-1)^(1/3))^5-1/34992*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/314928*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)

Rubi [A] time = 1.55, antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 638, 618, 204, 628, 206}

$$\frac{\sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x + 9(-2)^{2/3}}{2916 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)} + \frac{\sqrt[3]{-1} 3^{2/3} (2 + 3\sqrt[3]{-2} 3^{2/3}) x + 9 2^{2/3}}{13122 2^{2/3} (8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 + 9 2^{2/3})}$$

Antiderivative was successfully verified.

[In] Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + (9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13122*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (3*2^(2/3)*3^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^3/2)) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^3/2)) + ((-1/3)^(1/6)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(5832*2^(1/3)*(1 + (-1)^(1/3))^5) - ((1/5832)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 638

```
Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^6}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = 1586874322944 \int \left(\frac{-2\sqrt[3]{-1} 3^{2/3} + 3(-2)^{2/3} x}{1542441841901568 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + \dots)} \right. \\ = -\frac{\int \frac{-2\sqrt[3]{-1} 3^{2/3} + 3 2^{2/3} x}{(6+3(-2)^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{8748 2^{2/3}} - \frac{\int \frac{2+2^{2/3} \sqrt[3]{3} x}{(6+3 2^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{2916 2^{2/3} \sqrt[3]{3}} + \frac{\int \frac{3 \sqrt[3]{3} + \sqrt[3]{2}}{6+3 2^{2/3} \sqrt[3]{3}}}{26244 6} \\ = \frac{9(-2)^{2/3} + \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{2916 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} + \\ = \frac{9(-2)^{2/3} + \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{2916 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} + \\ = \frac{9(-2)^{2/3} + \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{2916 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} +$$

Mathematica [C] time = 0.05, size = 167, normalized size = 0.25

$$\frac{-3x^5 + 73x^4 - 72x^3 - 64x^2 + 108x - 96}{68364(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} \text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{3\#1^4 \log(x-\#1)-}{410184}\right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/410184

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)


```

1864717157619341253309046784 - 880007/3977704731623097128039995515166457856
, z, k)^3)/44521 + (115004308571136*root(z^6 - (60865*z^4)/239631364059408
- (15496909*z^3)/3056398361930300326272 - (168169*z^2)/59416384155925038342
72768 - (3971*z)/311864717157619341253309046784 - 880007/397770473162309712
8039995515166457856, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^
4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5
941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007
/3977704731623097128039995515166457856, z, k)^5 - 265/5749449196923)*root(z
^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 -
(168169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090
46784 - 880007/3977704731623097128039995515166457856, z, k), k, 1, 6) - ((1
6*x^2)/17091 - x/633 + (2*x^3)/1899 - (73*x^4)/68364 + x^5/22788 + 8/5697)/
(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

```

sympy [A] time = 0.38, size = 112, normalized size = 0.17

$$\text{RootSum}\left(3977704731623097128039995515166457856t^6 - 1010314319415295961050951680t^4 - 20168224477093957151232t^3 - 112582856818899648t^2 - 50648453064t - 880007, \text{Lambda}(t, t \cdot \log(-273655567090018991570649941414395560986199688040644608t^5/49797855396139900267573395695 + 11837008470196046085308646230764354292805044570112t^4/49797855396139900267573395695 - 10570581900446717266374077482873315047787008t^3/49797855396139900267573395695 - 1552547411569469872387563218792789323392t^2/49797855396139900267573395695 - 12542923791159140826909003250295928t/49797855396139900267573395695 + x - 23066533870320322410834348296/49797855396139900267573395695)) + (-3x^5 + 73x^4 - 72x^3 - 64x^2 + 108x - 96)/(68364x^6 + 1230552x^4 + 22149936x^3 + 7383312x^2 + 14766624)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(3977704731623097128039995515166457856*_t**6 - 1010314319415295961050951680*_t**4 - 20168224477093957151232*_t**3 - 112582856818899648*_t**2 - 50648453064*_t - 880007, Lambda(_t, _t*log(-273655567090018991570649941414395560986199688040644608*_t**5/49797855396139900267573395695 + 11837008470196046085308646230764354292805044570112*_t**4/49797855396139900267573395695 - 10570581900446717266374077482873315047787008*_t**3/49797855396139900267573395695 - 1552547411569469872387563218792789323392*_t**2/49797855396139900267573395695 - 12542923791159140826909003250295928*_t/49797855396139900267573395695 + x - 23066533870320322410834348296/49797855396139900267573395695))) + (-3*x**5 + 73*x**4 - 72*x**3 - 64*x**2 + 108*x - 96)/(68364*x**6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)

$$3.154 \quad \int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=682

$$\frac{\sqrt[3]{-\frac{1}{3}} (4 - \sqrt[3]{-3} 2^{2/3} x)}{1944 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)} + \frac{\sqrt[3]{-\frac{1}{3}} ((-2)^{2/3} \sqrt[3]{3} x + 4)}{8748 2^{2/3} (8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 + 3(-2))}$$

[Out] 1/11664*(-1)^(1/3)*3^(2/3)*(4-(-3)^(1/3)*2^(2/3)*x)*2^(1/3)/((1+(-1)^(1/3))^4*(4-3*(-3)^(2/3)*2^(1/3)))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/52488*(-1)^(1/3)*3^(2/3)*(4+(-2)^(2/3)*3^(1/3)*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-4-2^(2/3)*3^(1/3)*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/13122*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)-1/13122*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)-1/52488*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)-1/26244*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/8748*I*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(1/3)/(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/236196*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)

Rubi [A] time = 1.24, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2097, 638, 618, 204, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}} (4 - \sqrt[3]{-3} 2^{2/3} x)}{1944 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)} + \frac{\sqrt[3]{-\frac{1}{3}} ((-2)^{2/3} \sqrt[3]{3} x + 4)}{8748 2^{2/3} (8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 + 3(-2))}$$

Antiderivative was successfully verified.

[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + ((-1/3)^(1/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/3)*x)/(17496*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(2^(5/6)*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(4374*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(39366*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx &= 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}} x}{1542441841901568 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + 3\sqrt[3]{-1} x)} \right. \\
&= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{x}{(6+3(-2)^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{8748 \cdot 2^{2/3}} + \frac{\int \frac{1}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{26244 \sqrt[3]{2} \cdot 3^{2/3}} + \frac{\int \frac{x}{(6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{8748 \cdot 2^{2/3}} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (4 - \sqrt[3]{-3} 2^{2/3} x)}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} + \frac{1}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (4 - \sqrt[3]{-3} 2^{2/3} x)}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} + \frac{1}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (4 - \sqrt[3]{-3} 2^{2/3} x)}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} + \frac{1}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + \frac{4\#1^4 \log(x-\#1) - 54\#1^3 \log(x-\#1) + 2043\#1^2 \log(x-\#1) - 324\#1 \log(x-\#1) + 144 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right]}{3691656}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 - 54*Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/3691656

fricas [B] time = 2.97, size = 1445, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] 1/28041818976*(182304*x^5 - 1230552*x^4 + 33224904*x^3 + 422*sqrt(1/633)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)*log(2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 4479023748400406176979673*18^(2/3) - 8334306522507661258645112*18^(1/3) - 26862559811422885347120477)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 5132255454960803463351330/9393931*18^(2/3) + 9549802036377046040753520/9393931*18^(1/3) + 27278928233033940032425830/9393931) - 422*sqrt(1/633)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)

$$\begin{aligned}
& 4x^3 + 108x^2 + 216) \sqrt{5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457} \\
& \cdot \log(-2/1982119441 \sqrt{1/633} \cdot (7238020557 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 - 4479023748400406176979673 \cdot 18^{2/3} - 83343065225076612 \\
& 58645112 \cdot 18^{1/3} - 26862559811422885347120477) \sqrt{5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457} - 7383041510/9393931 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot \\
& 18^{1/3} + 44687457)^2 + 247458158879850620 \cdot x + 5132255454960803463351330/9 \\
& 393931 \cdot 18^{2/3} + 9549802036377046040753520/9393931 \cdot 18^{1/3} + 272789282330 \\
& 33940032425830/9393931) - 9 \sqrt{422} \cdot (x^6 + 18x^4 + 324x^3 + 108x^2 + 2 \\
& 16) \sqrt{-20718 \cdot 18^{2/3} + \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 27397 \\
& 4962699} - 9367856/243 \cdot 18^{1/3} + 367798) \cdot \log(14766083020/211 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 3064230/211 \sqrt{-1/19683 \cdot (5034474 \cdot \\
& 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 34454787 \\
& 01088/81 \cdot 18^{1/3} + 273974962699) \cdot (5895278433468 \cdot 18^{2/3} + 10969590754592 \cdot \\
& 18^{1/3} + 57028339027521) + 9/9393931 \cdot (14476041114 \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 243 \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \\
& \cdot 18^{1/3} + 273974962699) \cdot (14476041114 \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 936785 \\
& 6 \cdot 18^{1/3} + 44687457) - 161351097450615865 \sqrt{422}) - 177934129698570542 \\
& 9 \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457) + 265058558805 \\
& 69051992480475 \sqrt{422}) \sqrt{-20718 \cdot 18^{2/3} + \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 34454787010 \\
& 88/81 \cdot 18^{1/3} + 273974962699} - 9367856/243 \cdot 18^{1/3} + 367798) + 440683387 \\
& 65959317812080 \cdot x - 10264510909921606926702660/211 \cdot 18^{2/3} - 19099604072754 \\
& 092081507040/211 \cdot 18^{1/3} - 54557856466067880064851660/211) + 9 \sqrt{422} \cdot (\\
& x^6 + 18x^4 + 324x^3 + 108x^2 + 216) \sqrt{-20718 \cdot 18^{2/3} + \sqrt{-1/1968 \\
& 3 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} \\
& + 3445478701088/81 \cdot 18^{1/3} + 273974962699} - 9367856/243 \cdot 18^{1/3} + 36779 \\
& 8) \cdot \log(14766083020/211 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + \\
& 3064230/211 \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457) \\
& ^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699) \cdot (5895 \\
& 278433468 \cdot 18^{2/3} + 10969590754592 \cdot 18^{1/3} + 57028339027521) - 9/9393931 \cdot \\
& (14476041114 \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + \\
& 243 \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 228 \\
& 60116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699) \cdot (14476041114 \cdot \\
& \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457) - 16135109745061 \\
& 5865 \sqrt{422}) - 1779341296985705429 \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \\
& \cdot 18^{1/3} + 44687457) + 26505855880569051992480475 \sqrt{422}) \sqrt{-20718 \cdot 1 \\
& 8^{2/3} + \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 \\
& + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699} - 936785 \\
& 6/243 \cdot 18^{1/3} + 367798) + 44068338765959317812080 \cdot x - 10264510909921606926 \\
& 702660/211 \cdot 18^{2/3} - 19099604072754092081507040/211 \cdot 18^{1/3} - 54557856466 \\
& 067880064851660/211) - 9 \sqrt{422} \cdot (x^6 + 18x^4 + 324x^3 + 108x^2 + 216) \\
& \sqrt{-20718 \cdot 18^{2/3} - \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} \\
& + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 27397496 \\
& 2699} - 9367856/243 \cdot 18^{1/3} + 367798) \cdot \log(14766083020/211 \cdot (5034474 \cdot 18^{2/3} \\
&) + 9367856 \cdot 18^{1/3} + 44687457)^2 - 3064230/211 \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} \\
& (2/3) + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 34454787010 \\
& 88/81 \cdot 18^{1/3} + 273974962699) \cdot (5895278433468 \cdot 18^{2/3} + 10969590754592 \cdot 18^{1/3} \\
& (1/3) + 57028339027521) + 9/9393931 \cdot (14476041114 \sqrt{422} \cdot (5034474 \cdot 18^{2/3} \\
&) + 9367856 \cdot 18^{1/3} + 44687457)^2 - 243 \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + \\
& 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} \\
& ^{1/3} + 273974962699) \cdot (14476041114 \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 1 \\
& 8^{1/3} + 44687457) - 161351097450615865 \sqrt{422}) - 1779341296985705429 \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457) + 265058558805690 \\
& 51992480475 \sqrt{422}) \sqrt{-20718 \cdot 18^{2/3} - \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/ \\
& 81 \cdot 18^{1/3} + 273974962699} - 9367856/243 \cdot 18^{1/3} + 367798) + 440683387659 \\
& 59317812080 \cdot x - 10264510909921606926702660/211 \cdot 18^{2/3} - 19099604072754092
\end{aligned}$$

081507040/211*18^(1/3) - 54557856466067880064851660/211) + 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 3064230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 243*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 26505855880569051992480475*sqrt(422))*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 44068338765959317812080*x - 10264510909921606926702660/211*18^(2/3) - 19099604072754092081507040/211*18^(1/3) - 54557856466067880064851660/211) + 29533248*x^2 - 6562944*x + 44299872)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.01, size = 122, normalized size = 0.18

$$\frac{(4 \operatorname{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216))^4 - 54 \operatorname{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 + 3691656 \operatorname{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 44299872 \operatorname{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^4}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4x^5 - 27x^4 + 729x^3 + 648x^2 - 144x + 972}{615276(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} + \frac{1}{615276} \int \frac{4x^4 - 54x^3 + 2043x^2 - 324x + 144}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] 1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2 - 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 0.31, size = 299, normalized size = 0.44

$$\left(\sum_{k=1}^6 \ln \left(\frac{4477969 \operatorname{root}\left(z^6 - \frac{183899 z^4}{3834101824950528} + \frac{6209 z^2}{14083883651774823903461376} - \frac{39753025}{27493895104978847349012449000830556700672}\right)}{189282278088} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((6305*x)/4967524106141472 - (4477969*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k))/189282278088 - (16340881*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)*x)/5110621508376 - (43348696*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^2*x)/10818603 - (65333687616*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^3*x)/44521 - (40024496812032*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^5*x + (5943884*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^2)/400689 + (224442467136*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^3)/44521 - (137087493272064*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^4)/211 - 168897381688221696*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)^5 - 13082875/17883086782109292*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k), k, 1, 6) + ((2*x^2)/1899 - (4*x)/17091 + x^3/844 - x^4/22788 + x^5/153819 + 1/633)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

sympy [A] time = 0.31, size = 104, normalized size = 0.15

$$\operatorname{RootSum}\left(27493895104978847349012449000830556700672t^6 - 1318718189226950088862983192576t^4 + 12120917704776776448t^2 - 39753025, \operatorname{Lambda}(t, t \log(947842259001288723909832054550209950242045952t^5/61864539719962655 - 243458646817775607639654889480814592t^4/9811980923071 - 41682556475067500431787310779667456t^3/61864539719962655 + 12026877442664328616462272t^2/9811980923071 + 216142618488859793668428t/61864539719962655 + x - 308574300024117/39247923692284)) + (4x^5 - 27x^4 + 729x^3 + 648x^2 - 144x + 972)/(615276x^6 + 11074968x^4 + 199349424x^3 + 66449808x^2 + 132899616)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(27493895104978847349012449000830556700672*_t**6 - 1318718189226950088862983192576*_t**4 + 12120917704776776448*_t**2 - 39753025, Lambda(_t, _t*log(947842259001288723909832054550209950242045952*_t**5/61864539719962655 - 243458646817775607639654889480814592*_t**4/9811980923071 - 41682556475067500431787310779667456*_t**3/61864539719962655 + 12026877442664328616462272*_t**2/9811980923071 + 216142618488859793668428*_t/61864539719962655 + x - 308574300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 648*x**2 - 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3 + 66449808*x**2 + 132899616)

$$3.155 \quad \int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=850

$$\frac{\sqrt[3]{-\frac{1}{3}} \left(3\sqrt[3]{-3} 2^{2/3} - 2x\right)}{5832 2^{2/3} \left(1 + \sqrt[3]{-1}\right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2}\right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} \sqrt[3]{2})}}\right)}{729 2^{2/3} 3^{5/6} \left(1 + \sqrt[3]{-1}\right)^4 \left(8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}\right)}$$

[Out] $\frac{1}{34992}(-1)^{1/3} 3^{2/3} (3(-3)^{1/3} 2^{2/3} - 2x)^2 (1 + (-1)^{1/3})^{-4} (4 - 3(-3)^{2/3} 2^{1/3}) / (6 - 3(-3)^{1/3} 2^{2/3} x + x^2) - \frac{1}{157464}(-1)^{1/3} 3^{2/3} (3(-2)^{2/3} 3^{1/3} + 2x)^2 (1 + (-1)^{1/3}) / (8 + 9i 2^{1/3} 3^{1/6} + 3 \cdot 2^{1/3} 3^{2/3}) / (6 + 3(-2)^{2/3} 3^{1/3} x + x^2) + \frac{1}{52488}(-3 \cdot 3^{1/3} - 2^{1/3} x) / (9 \cdot 2^{1/3} - 4 \cdot 3^{1/3}) / (6 + 3 \cdot 2^{2/3} 3^{1/3} x + x^2) + \frac{1}{4374}(-1)^{1/3} \arctan\left(\frac{3(-3)^{1/3} 2^{2/3} - 2x}{(24 - 18(-3)^{2/3} 2^{1/3})^{1/2}}\right) 2^{1/3} 3^{1/6} / (1 + (-1)^{1/3})^{-4} (8 - 9i 2^{1/3} 3^{1/6} + 3 \cdot 2^{1/3} 3^{2/3})^{3/2} - \frac{1}{17496}(-1)^{1/3} \arctan\left(\frac{3(-2)^{2/3} 3^{1/3} + 2x}{(24 + 18(-2)^{1/3} 3^{2/3})^{1/2}}\right) 2^{5/6} 3^{1/6} / (1 - (-1)^{1/3})^{-2} (1 + (-1)^{1/3})^{-4} (4 + 3(-2)^{1/3} 3^{2/3})^{3/2} + \frac{1}{157464} \operatorname{arctanh}\left(\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{-12 + 9 \cdot 2^{1/3} 3^{2/3}}\right) 2^{5/6} 3^{1/6} / (-4 + 3 \cdot 2^{1/3} 3^{2/3})^{3/2} - \frac{1}{209952} \ln(6 - 3(-3)^{1/3} 2^{2/3} x + x^2) 2^{2/3} 3^{1/3} / (1 + (-1)^{1/3})^{-4} + \frac{1}{209952} i \ln(6 + 3(-2)^{2/3} 3^{1/3} x + x^2) 2^{2/3} 3^{5/6} / (1 + (-1)^{1/3})^{-5} - \frac{1}{1889568} \ln(6 + 3 \cdot 2^{2/3} 3^{1/3} x + x^2) 2^{2/3} 3^{1/3} - \frac{1}{34992} i \arctan\left(\frac{2^{1/6} (3(-3)^{1/3} - 2^{1/3} x)}{(12 - 9(-3)^{2/3} 2^{1/3})^{1/2}}\right) 2^{5/6} 3^{2/3} / (1 + (-1)^{1/3})^{-5} (4 - 3(-3)^{2/3} 2^{1/3})^{1/2} - \frac{1}{69984} \arctan\left(\frac{3(-2)^{2/3} 3^{1/3} + 2x}{(24 + 18(-2)^{1/3} 3^{2/3})^{1/2}}\right) (3^{1/2} + i) 2^{5/6} 3^{2/3} / (1 + (-1)^{1/3})^{-5} (4 + 3(-2)^{1/3} 3^{2/3})^{1/2} + \frac{1}{314928} \operatorname{arctanh}\left(\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{-12 + 9 \cdot 2^{1/3} 3^{2/3}}\right) 2^{5/6} 3^{1/6} / (-4 + 3 \cdot 2^{1/3} 3^{2/3})^{1/2}$

Rubi [A] time = 1.47, antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 614, 618, 204, 634, 628, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}} \left(3\sqrt[3]{-3} 2^{2/3} - 2x\right)}{5832 2^{2/3} \left(1 + \sqrt[3]{-1}\right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2}\right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} \sqrt[3]{2})}}\right)}{729 2^{2/3} 3^{5/6} \left(1 + \sqrt[3]{-1}\right)^4 \left(8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}\right)}$$

Antiderivative was successfully verified.

[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $\frac{(-1/3)^{1/3} (3(-3)^{1/3} 2^{2/3} - 2x)}{(5832 \cdot 2^{2/3} (1 + (-1)^{1/3})^4 (4 - 3(-3)^{2/3} 2^{1/3}) (6 - 3(-3)^{1/3} 2^{2/3} x + x^2)) - ((-1/3)^{1/3} (3(-2)^{2/3} 3^{1/3} + 2x)) / (26244 \cdot 2^{2/3} (8 + (9i) \cdot 2^{1/3} 3^{1/6} + 3 \cdot 2^{1/3} 3^{2/3}) (6 + 3(-2)^{2/3} 3^{1/3} x + x^2)) - (3 \cdot 3^{1/3} + 2^{1/3} x) / (52488 (9 \cdot 2^{1/3} - 4 \cdot 3^{1/3})) (6 + 3 \cdot 2^{2/3} 3^{1/3} x + x^2)} + \frac{((-1)^{1/3} \operatorname{ArcTan}[(3(-3)^{1/3} 2^{2/3} - 2x) / \sqrt{6(4 - 3(-3)^{2/3} 2^{1/3})}])}{(729 \cdot 2^{2/3} 3^{5/6} (1 + (-1)^{1/3})^4 (8 - (9i) \cdot 2^{1/3} 3^{1/6} + 3 \cdot 2^{1/3} 3^{2/3})^{3/2}) - ((-1)^{1/3} \operatorname{ArcTan}[(3(-2)^{2/3} 3^{1/3} + 2x) / \sqrt{6(4 + 3(-2)^{1/3} 3^{2/3})}])}{(2916 \cdot 2^{1/6} 3^{5/6} (1 - (-1)^{1/3})^2 (1 + (-1)^{1/3})^4 (4 + 3(-2)^{1/3} 3^{2/3})^{3/2}) - ((i + \sqrt{3}) \operatorname{ArcTan}[(3(-2)^{2/3} 3^{1/3} + 2x) / \sqrt{6(4 + 3(-2)^{1/3} 3^{2/3})}])}{(11664 \cdot 2^{1/6} 3^{1/3} (1 + (-1)^{1/3})^5 \sqrt{4 + 3(-2)^{1/3} 3^{2/3}})} - ((i/5832) \operatorname{ArcTan}[(2^{1/6} (3(-3)^{1/3} - 2^{1/3} x)) / \sqrt{3(4 - 3(-3)^{2/3} 2^{1/3})}])$

$$\frac{\left(\frac{2}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^{\frac{1}{3}} \left[\frac{1}{(2^{\frac{1}{6}} \cdot 3^{\frac{1}{3}})^5 \sqrt{4 - 3(-3)^{\frac{2}{3}} \cdot 2^{\frac{1}{3}}}} + \operatorname{ArcTanh}\left[\frac{2^{\frac{1}{6}} \cdot (3 \cdot 3^{\frac{1}{3}} + 2^{\frac{1}{3}} \cdot x)}{\sqrt{3(-4 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})}}\right] \right]}{(26244 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{5}{6}} \cdot (-4 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})^{\frac{3}{2}}) + \operatorname{ArcTanh}\left[\frac{2^{\frac{1}{6}} \cdot (3 \cdot 3^{\frac{1}{3}} + 2^{\frac{1}{3}} \cdot x)}{\sqrt{3(-4 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})}}\right]}{(52488 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{5}{6}} \cdot \sqrt{-4 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}}}) - \operatorname{Log}[6 - 3(-3)^{\frac{1}{3}} \cdot 2^{\frac{2}{3}} \cdot x + x^2]}{(34992 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}} \cdot (1 + (-1)^{\frac{1}{3}})^4) + ((I/34992) \cdot \operatorname{Log}[6 + 3(-2)^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot x + x^2]}{(2^{\frac{1}{3}} \cdot 3^{\frac{1}{6}} \cdot (1 + (-1)^{\frac{1}{3}})^5) - \operatorname{Log}[6 + 3 \cdot 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot x + x^2]}{(314928 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})}$$
Rule 204

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\operatorname{ArcTan}[\operatorname{Rt}[-b, 2] \cdot x] / \operatorname{Rt}[-a, 2]] / (\operatorname{Rt}[-a, 2] \cdot \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b] \ \&\& \ (\operatorname{LtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$$
Rule 206

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1 \cdot \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] \cdot x] / \operatorname{Rt}[a, 2]] / (\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$$
Rule 614

$$\operatorname{Int}[(a + (b \cdot x) + (c \cdot x)^2)^p, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(b + 2 \cdot c \cdot x) \cdot (a + b \cdot x + c \cdot x^2)^{p+1} / ((p+1) \cdot (b^2 - 4 \cdot a \cdot c)), x] - \operatorname{Dist}[(2 \cdot c \cdot (2 \cdot p + 3)) / ((p+1) \cdot (b^2 - 4 \cdot a \cdot c)), \operatorname{Int}[(a + b \cdot x + c \cdot x^2)^{p+1}, x], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ \operatorname{LtQ}[p, -1] \ \&\& \ \operatorname{NeQ}[p, -3/2] \ \&\& \ \operatorname{IntegerQ}[4 \cdot p]$$
Rule 618

$$\operatorname{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1 / \operatorname{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$$
Rule 628

$$\operatorname{Int}[(d + (e \cdot x)) / (a + (b \cdot x) + (c \cdot x)^2), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(d \cdot \operatorname{Log}[\operatorname{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \operatorname{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \operatorname{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$$
Rule 634

$$\operatorname{Int}[(d + (e \cdot x)) / (a + (b \cdot x) + (c \cdot x)^2), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(2 \cdot c \cdot d - b \cdot e) / (2 \cdot c), \operatorname{Int}[1 / (a + b \cdot x + c \cdot x^2), x], x] + \operatorname{Dist}[e / (2 \cdot c), \operatorname{Int}[(b + 2 \cdot c \cdot x) / (a + b \cdot x + c \cdot x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \operatorname{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \ \&\& \ \operatorname{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ \operatorname{!NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$$
Rule 2097

$$\operatorname{Int}[(Q6)^p \cdot (u), x_{\text{Symbol}}] \rightarrow \operatorname{With}\{a = \operatorname{Coeff}[Q6, x, 0], b = \operatorname{Coeff}[Q6, x, 2], c = \operatorname{Coeff}[Q6, x, 3], d = \operatorname{Coeff}[Q6, x, 4], e = \operatorname{Coeff}[Q6, x, 6]\}, \operatorname{Dist}[1 / (3^{3 \cdot p}) \cdot a^{2 \cdot p}], \operatorname{Int}[\operatorname{ExpandIntegrand}[u \cdot (3 \cdot a + 3 \cdot \operatorname{Rt}[a, 3]^2 \cdot \operatorname{Rt}[c, 3] \cdot x + b \cdot x^2)^p \cdot (3 \cdot a - 3 \cdot (-1)^{\frac{1}{3}} \cdot \operatorname{Rt}[a, 3]^2 \cdot \operatorname{Rt}[c, 3] \cdot x + b \cdot x^2)^p \cdot (3 \cdot a + 3 \cdot (-1)^{\frac{2}{3}} \cdot \operatorname{Rt}[a, 3]^2 \cdot \operatorname{Rt}[c, 3] \cdot x + b \cdot x^2)^p, x], x] /; \operatorname{EqQ}[b^2 - 3 \cdot a \cdot d, 0] \ \&\& \ \operatorname{EqQ}[b^3 - 27 \cdot a^2 \cdot e, 0] /; \operatorname{ILtQ}[p, 0] \ \&\& \ \operatorname{PolyQ}[Q6, x, 6] \ \&\& \ \operatorname{EqQ}[\operatorname{Coeff}[Q6, x, 1], 0] \ \&\& \ \operatorname{EqQ}[\operatorname{Coeff}[Q6, x, 5], 0] \ \&\& \ \operatorname{RationalFunctionQ}[u, x]$$
Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx &= 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}}{1542441841901568 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + 3\sqrt[3]{-1})} \right. \\
&= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{1}{(6+3(-2)^{2/3} \sqrt[3]{3}x+x^2)^2} dx}{8748 \cdot 2^{2/3}} - \frac{\int \frac{3 \cdot 2^{2/3} \sqrt[3]{3}+x}{6+3 \cdot 2^{2/3} \sqrt[3]{3}x+x^2} dx}{157464 \sqrt[3]{2} \cdot 3^{2/3}} + \frac{\int \frac{1}{(6+3 \cdot 2^{2/3} \sqrt[3]{3}x+x^2)^2} dx}{8748 \cdot 2^{2/3}} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (3\sqrt[3]{-3} 2^{2/3} - 2x)}{5832 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{1}{5} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (3\sqrt[3]{-3} 2^{2/3} - 2x)}{5832 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{1}{5} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (3\sqrt[3]{-3} 2^{2/3} - 2x)}{5832 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{1}{5}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 167, normalized size = 0.20

$$\frac{-9x^5 + 8x^4 - 216x^3 - 1458x^2 + 324x - 288}{1230552(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} \operatorname{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + \frac{9\#1^4 \log(x-\#1)-1}{7383312} \right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/7383312

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.01, size = 122, normalized size = 0.14

$$\frac{\left(-9 \operatorname{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)^4 + 16 \operatorname{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)^5\right)}{7383312 \operatorname{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)^5 + 88599744 \operatorname{RootOf}\left(-Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9x^5 - 8x^4 + 216x^3 + 1458x^2 - 324x + 288}{1230552(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{1}{1230552} \int \frac{9x^4 - 16x^3 + 324x^2 - 2628x + 324}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] -1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x^2 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.42, size = 388, normalized size = 0.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((24389*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k))/851770251396 + (288041*x)/804738905194918464 - (1090723*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)*x)/22997796787692 + (5850124*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^2*x)/3606201 - (64554687936*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^3*x)/44521 + (31535589897216*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^5*x - (1697552*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)

)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^2)/10818603 + (12229983936*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^3)/44521 + (25367949245952*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^5 - 971/22353858477636624)*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k), k, 1, 6) - (x^2/844 - x/3798 + x^3/5697 - x^4/153819 + x^5/136728 + 4/17091)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

sympy [A] time = 0.39, size = 112, normalized size = 0.13

RootSum(185583791958607219605834030755606257729536t^6 - 1309367357962223565522033377280t^4 + 4

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(185583791958607219605834030755606257729536*_t**6 - 1309367357962223565522033377280*_t**4 + 4356336487052294744666112*_t**3 - 4052982845480387328*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(39083462657955593476841044707333565976412952759280634691584*_t**5/49797855396139900267573395695 + 8836979346223785538912817601414711102396804462575616*_t**4/49797855396139900267573395695 - 264930581348308532588844249597134695706805067776*_t**3/49797855396139900267573395695 + 886135333547363185201515109826158376250624*_t**2/49797855396139900267573395695 - 682321479574909906511394635855601936*_t/49797855396139900267573395695 + x - 21375560770846486224291519568/49797855396139900267573395695))) + (-9*x**5 + 8*x**4 - 216*x**3 - 1458*x**2 + 324*x - 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899616*x**2 + 265799232)

$$3.156 \quad \int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=873

$$\frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2}\sqrt[3]{3} + 3\sqrt[3]{2}3^{2/3}) (x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)} \frac{(9i - \sqrt[3]{3} (2i2^{2/3} + 9\sqrt[3]{3} + 2 \cdot 2^{2/3}\sqrt[3]{3})) \tan^{-1} \left(\frac{3\sqrt[3]{2}}{\sqrt{6(4 - 3(-3)^{2/3}\sqrt[3]{2})}} \right)}{209952 (1 + \sqrt[3]{-1})^5 \sqrt{2(4 - 3(-3)^{2/3}\sqrt[3]{2})}}$$

[Out] $1/157464 * ((-6)^{(1/3)} * (2 * (-3)^{(1/3)} + 9 * 2^{(1/3)}) - 3 * x) / (8 - 9 * I * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)}) / (6 - 3 * (-3)^{(1/3)} * 2^{(2/3)} * x + x^2) + 1/157464 * (-(-6)^{(1/3)} * (9 * (-2)^{(1/3)} + 2 * 3^{(1/3)}) - 3 * x) / (8 + 9 * I * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)}) / (6 + 3 * (-2)^{(2/3)} * 3^{(1/3)} * x + x^2) + 1/104976 * (-2 * 2^{(1/3)} + 3 * 6^{(2/3)} + 3^{(1/3)} * x) / (9 * 2^{(1/3)} - 4 * 3^{(1/3)}) / (6 + 3 * 2^{(2/3)} * 3^{(1/3)} * x + x^2) - 1/139968 * I * \ln(6 - 3 * (-3)^{(1/3)} * 2^{(2/3)} * x + x^2) * 2^{(1/3)} * 3^{(1/6)} / (1 + (-1)^{(1/3)})^5 + 1/3779136 * \ln(6 + 3 * 2^{(2/3)} * 3^{(1/3)} * x + x^2) * 2^{(1/3)} * 3^{(2/3)} + 1/78732 * \arctan((3 * (-3)^{(1/3)} * 2^{(2/3)} - 2 * x) / (24 - 18 * (-3)^{(2/3)} * 2^{(1/3)}))^{(1/2)} / (8 - 9 * I * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)})^{(3/2)} * 3^{(1/2)} - 1/78732 * \arctan((3 * (-2)^{(2/3)} * 3^{(1/3)} + 2 * x) / (24 + 18 * (-2)^{(1/3)} * 3^{(2/3)})^{(1/2)}) / (8 + 9 * I * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)})^{(3/2)} * 3^{(1/2)} + 1/279936 * \ln(6 + 3 * (-2)^{(2/3)} * 3^{(1/3)} * x + x^2) * (3^{(1/2)} + I) * 2^{(1/3)} * 3^{(1/6)} / (1 + (-1)^{(1/3)})^5 - 1/314928 * \operatorname{arctanh}(2^{(1/6)} * (3 * 3^{(1/3)} + 2^{(1/3)} * x) / (-12 + 9 * 2^{(1/3)} * 3^{(2/3)})^{(1/2)}) / (-4 + 3 * 2^{(1/3)} * 3^{(2/3)})^{(3/2)} * 6^{(1/2)} - 1/209952 * \arctan((3 * (-3)^{(1/3)} * 2^{(2/3)} - 2 * x) / (24 - 18 * (-3)^{(2/3)} * 2^{(1/3)}))^{(1/2)} * (9 * I - 3^{(1/3)} * (2 * I * 2^{(2/3)} + 9 * 3^{(1/6)} + 2 * 2^{(2/3)} * 3^{(1/2)})) / (1 + (-1)^{(1/3)})^5 / (8 - 6 * (-3)^{(2/3)} * 2^{(1/3)})^{(1/2)} + 1/209952 * (9 * I + 3^{(1/3)} * (4 * I * 2^{(2/3)} - 9 * 3^{(1/6)})) * \arctan((3 * (-2)^{(2/3)} * 3^{(1/3)} + 2 * x) / (24 + 18 * (-2)^{(1/3)} * 3^{(2/3)})^{(1/2)}) / (1 + (-1)^{(1/3)})^5 / (8 + 6 * (-2)^{(1/3)} * 3^{(2/3)})^{(1/2)} + 1/2834352 * (2 * 2^{(2/3)} - 3 * 3^{(2/3)}) * \operatorname{arctanh}(2^{(1/6)} * (3 * 3^{(1/3)} + 2^{(1/3)} * x) / (-12 + 9 * 2^{(1/3)} * 3^{(2/3)})^{(1/2)}) * 3^{(5/6)} / (-8 + 6 * 2^{(1/3)} * 3^{(2/3)})^{(1/2)}$

Rubi [A] time = 1.92, antiderivative size = 873, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$\frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2}\sqrt[3]{3} + 3\sqrt[3]{2}3^{2/3}) (x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)} \frac{(9i - \sqrt[3]{3} (2i2^{2/3} + 9\sqrt[3]{3} + 2 \cdot 2^{2/3}\sqrt[3]{3})) \tan^{-1} \left(\frac{3\sqrt[3]{2}}{\sqrt{6(4 - 3(-3)^{2/3}\sqrt[3]{2})}} \right)}{209952 (1 + \sqrt[3]{-1})^5 \sqrt{2(4 - 3(-3)^{2/3}\sqrt[3]{2})}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $((-6)^{(1/3)} * (2 * (-3)^{(1/3)} + 9 * 2^{(1/3)}) - 3 * x) / (157464 * (8 - (9 * I) * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)}) * (6 - 3 * (-3)^{(1/3)} * 2^{(2/3)} * x + x^2)) - ((-6)^{(1/3)} * (9 * (-2)^{(1/3)} + 2 * 3^{(1/3)}) + 3 * x) / (157464 * (8 + (9 * I) * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)}) * (6 + 3 * (-2)^{(2/3)} * 3^{(1/3)} * x + x^2)) - (2 * 2^{(1/3)} - 3 * 6^{(2/3)} - 3^{(1/3)} * x) / (104976 * (9 * 2^{(1/3)} - 4 * 3^{(1/3)}) * (6 + 3 * 2^{(2/3)} * 3^{(1/3)} * x + x^2)) + \operatorname{ArcTan}[(3 * (-3)^{(1/3)} * 2^{(2/3)} - 2 * x) / \operatorname{Sqrt}[6 * (4 - 3 * (-3)^{(2/3)} * 2^{(1/3)})]] / (26244 * \operatorname{Sqrt}[3] * (8 - (9 * I) * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)})^{(3/2)}) - ((9 * I - 3^{(1/3)} * ((2 * I) * 2^{(2/3)} + 9 * 3^{(1/6)} + 2 * 2^{(2/3)} * \operatorname{Sqrt}[3])) * \operatorname{ArcTan}[(3 * (-3)^{(1/3)} * 2^{(2/3)} - 2 * x) / \operatorname{Sqrt}[6 * (4 - 3 * (-3)^{(2/3)} * 2^{(1/3)})]] / (209952 * (1 + (-1)^{(1/3)})^5 * \operatorname{Sqrt}[2 * (4 - 3 * (-3)^{(2/3)} * 2^{(1/3)})]) - \operatorname{ArcTan}[(3 * (-2)^{(2/3)} * 3^{(1/3)} + 2 * x) / \operatorname{Sqrt}[6 * (4 + 3 * (-2)^{(1/3)} * 3^{(2/3)})]] / (26244 * \operatorname{Sqrt}[3] * (8 + (9 * I) * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)})^{(3/2)}) + ((9 * I + 3^{(1/3)} * ((4 * I) * 2^{(2/3)} - 9 * 3^{(1/6)})) * \operatorname{ArcTan}[(3 * (-2)^{(2/3)} * 3^{(1/3)} + 2 * x) / \operatorname{Sqrt}[6 * (4 + 3 * (-2)^{(1/3)} * 3^{(2/3)})]] / (209952 * (1 + (-1)^{(1/3)})^5 * \operatorname{Sqrt}[2 * (4 + 3 * (-2)^{(1/3)} * 3^{(2/3)})])$

```

))] - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) - 3*3^(
2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]])/(944784*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) +
((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656*2^(2/3)*3^(5/6
))*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(629856*2^(2/3)*
3^(1/3))

```

Rule 204

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 618

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 628

```

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

```

Rule 634

```

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

Rule 638

```

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

```

Rule 2097

```

Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0]
&& EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx &= 1586874322944 \int \left(-\frac{9(-2)^{2/3} - \sqrt[3]{-1} 3^{2/3} x}{27763953154228224 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6} \right. \\
&= \frac{\int \frac{18-2 2^{2/3} \sqrt[3]{3} + \sqrt[3]{2} 3^{2/3} x}{6+3 2^{2/3} \sqrt[3]{3} x+x^2} dx}{1889568} - \frac{\int \frac{9 2^{2/3} - \sqrt[3]{-1} 3^{2/3} x}{(6+3(-2)^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{157464 2^{2/3}} - \frac{\int \frac{3 2^{2/3} \sqrt[3]{3}}{(6+3 2^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{52488 2} \\
&= \frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{314}{314} \\
&= \frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{314}{314} \\
&= \frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{314}{314}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.19

$$\frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{2\#1^4 \log(x-\#1) - 27\#1^3 \log(x-\#1) + 72\#1^2 \log(x-\#1) - 162\#1 \log(x-\#1) + 1971 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right]}{11074968}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 - 27*Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/11074968

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.01, size = 122, normalized size = 0.14

$$\frac{\left(2 \operatorname{RootOf}\left(-Z^6 + 18 Z^4 + 324 Z^3 + 108 Z^2 + 216\right)^4 - 27 \operatorname{RootOf}\left(-Z^6 + 18 Z^4 + 324 Z^3 + 108 Z^2 + 216\right)^3 + 11074968 \operatorname{RootOf}\left(-Z^6 + 18 Z^4 + 324 Z^3 + 108 Z^2 + 216\right)^5 + 132899616 \operatorname{RootOf}\left(-Z^6 + 18 Z^4 + 324 Z^3 + 108 Z^2 + 216\right)^6\right)}{3691656 \left(x^6 + 18 x^4 + 324 x^3 + 108 x^2 + 216\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R+1971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4x^5 - 27x^4 + 96x^3 + 648x^2 - 3942x + 972}{3691656(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} + \frac{1}{1845828} \int \frac{2x^4 - 27x^3 + 72x^2 - 162x + 1971}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] 1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2 - 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.42, size = 387, normalized size = 0.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((11*x)/603554178896188848 - (14059*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k))/30663729050256 - (5658601*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)*x)/6623365474855296 + (6603523*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^2*x)/584204562 - (1762321104*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^3*x)/44521 - (59633904436992*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^5*x + (166697*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^6*x)/11074968

829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^2)/43274412 + (639193032*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^3)/44521 - (9815247601920*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^4)/211 - 168897381688221696*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^5 + 661/28970600587017064704)*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k), k, 1, 6) + (x^2/5697 - (73*x)/68364 + (4*x^3)/153819 - x^4/136728 + x^5/922914 + 1/3798)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

sympy [A] time = 0.40, size = 112, normalized size = 0.13

RootSum $\left(1282755170017893101915524820582750453426552832t^6 - 906388465775544244426251149770\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(1282755170017893101915524820582750453426552832*_t**6 - 906388465775544244426251149770752*_t**4 - 4300873166389987741684137984*_t**3 - 717000908921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17257935592810449901409556597891882995604001083339368041361480613888*_t**5/154206009791052044490694380303237521 + 2389607400620985524376358853572652207181956324560587684052992*_t**4/154206009791052044490694380303237521 - 12286072160883283930711715948878260078996992193488388096*_t**3/154206009791052044490694380303237521 - 59490553573959173161125496013527909754156558410752*_t**2/154206009791052044490694380303237521 - 17520149679836691112367064197713753004827200*_t/154206009791052044490694380303237521 + x + 766422988707229615055855287040887332/154206009791052044490694380303237521))) + (4*x**5 - 27*x**4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808*x**4 + 1196096544*x**3 + 398698848*x**2 + 797397696)

3.157 $\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$

Optimal. Leaf size=986

$$\frac{27((-2)^{2/3} + 2\sqrt[3]{-1}3^{2/3}) - \sqrt[3]{6}(9 + \sqrt[3]{-3}2^{2/3})x}{104976 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3}\sqrt[3]{2}) (x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)} (1 + i\sqrt{3} + 3\sqrt[3]{2}3^{2/3}) \tan^{-1} \left(\frac{3\sqrt[3]{-3}2^{2/3}}{\sqrt{6(4-3(-3)^{2/3})}} \right)$$

[Out] 1/209952*(-27*(-2)^(2/3)-54*(-1)^(1/3)*3^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3)))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/944784*(-27*2^(2/3)-54*(-1)^(1/3)*3^(2/3)+(-1)^(1/3)*3^(2/3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/1889568*(54-9*2^(2/3)*3^(1/3)-(2-3*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/104976*(3*(-3)^(2/3)+(-1)^(1/3)*2^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/104976*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/1259712*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/11337408*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/52488*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(1+3*2^(1/3)*3^(2/3)+I*3^(1/2))*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)+1/2519424*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*(3^(1/2)+I)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/104976*I*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/209952*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/944784*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)

Rubi [A] time = 1.93, antiderivative size = 986, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$\frac{27((-2)^{2/3} + 2\sqrt[3]{-1}3^{2/3}) - \sqrt[3]{6}(9 + \sqrt[3]{-3}2^{2/3})x}{104976 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3}\sqrt[3]{2}) (x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)} (1 + i\sqrt{3} + 3\sqrt[3]{2}3^{2/3}) \tan^{-1} \left(\frac{3\sqrt[3]{-3}2^{2/3}}{\sqrt{6(4-3(-3)^{2/3})}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] -(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3)))*x)/(104976*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 + (-2)^(1/3)*3^(2/3)) - (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3] + 3*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3)])])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3)])])/(

$$17496*6^{(5/6)}*(1 - (-1)^{(1/3)})^2*(1 + (-1)^{(1/3)})^4*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})^{(3/2)} + ((I + \text{Sqrt}[3])*\text{ArcTan}[(3*(-2)^{(2/3)}*3^{(1/3)} + 2*x)/\text{Sqrt}[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})]])/(34992*2^{(1/6)}*3^{(1/3)}*(1 + (-1)^{(1/3)})^5*\text{Sqrt}[4 + 3*(-2)^{(1/3)}*3^{(2/3)}]) + ((I/17496)*\text{ArcTan}[(2^{(1/6)}*(3*(-3)^{(1/3)} - 2^{(1/3)}*x))/\text{Sqrt}[3*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})]])/(2^{(1/6)}*3^{(1/3)}*(1 + (-1)^{(1/3)})^5*\text{Sqrt}[4 - 3*(-3)^{(2/3)}*2^{(1/3)}]) - ((2^{(2/3)} - 3*3^{(2/3)})*\text{ArcTanh}[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/\text{Sqrt}[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})]])/(17496*6^{(5/6)}*(1 - (-1)^{(1/3)})^2*(1 + (-1)^{(1/3)})^4*(-4 + 3*2^{(1/3)}*3^{(2/3)})^{(3/2)}) - \text{ArcTanh}[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/\text{Sqrt}[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})]])/(157464*2^{(1/6)}*3^{(5/6)}*\text{Sqrt}[-4 + 3*2^{(1/3)}*3^{(2/3)}]) + ((I + \text{Sqrt}[3])*Log[6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2])/(419904*2^{(1/3)}*3^{(1/6)}*(1 + (-1)^{(1/3)})^5) - ((I/209952)*Log[6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2])/(2^{(1/3)}*3^{(1/6)}*(1 + (-1)^{(1/3)})^5) + Log[6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2]/(1889568*2^{(1/3)}*3^{(2/3)})$$
Rule 204

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$
Rule 206

$$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$
Rule 618

$$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$
Rule 628

$$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$$
Rule 634

$$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$$
Rule 638

$$\text{Int}[(d_ + (e_)*(x_))*((a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^{(p+1)}/((p+1)*(b^2 - 4*a*c)), x] - \text{Dist}[(2*p + 3)*(2*c*d - b*e)/((p+1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$$
Rule 2097

$$\text{Int}[(Q6_)^{(p_)}*(u_), x_Symbol] \rightarrow \text{With}\{a = \text{Coeff}[Q6, x, 0], b = \text{Coeff}[Q6, x, 2], c = \text{Coeff}[Q6, x, 3], d = \text{Coeff}[Q6, x, 4], e = \text{Coeff}[Q6, x, 6]\}, \text{Dist}[1/(3^{(3*p)}*a^{(2*p)}), \text{Int}[\text{ExpandIntegrand}[u*(3*a + 3*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^{(1/3)}*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^{(1/3)}*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p, x], x]$$

1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\int \frac{x^2}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = 1586874322944 \int \left(\frac{2\sqrt[3]{-1} 3^{2/3} + 18\sqrt[3]{6} + 3(-2)^{2/3}}{55527906308456448 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + 3\sqrt[3]{-1} x^2)} \right) dx$$

$$= \frac{\int \frac{2\sqrt[3]{-1} 3^{2/3} + 18(-1)^{2/3} \sqrt[3]{6} + 3 2^{2/3} x}{(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)^2} dx}{314928 2^{2/3}} + \frac{\int \frac{-2 + 6\sqrt[3]{2} 3^{2/3} + 2^{2/3} \sqrt[3]{3} x}{(6 + 3 2^{2/3} \sqrt[3]{3} x + x^2)^2} dx}{104976 2^{2/3} \sqrt[3]{3}} + \frac{\int \frac{9}{6 + 3\sqrt[3]{-1} x^2} dx}{94}$$

$$= -\frac{27((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) - \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{104976 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}$$

$$= -\frac{27((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) - \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{104976 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}$$

$$= -\frac{27((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) - \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{104976 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.17

$$\frac{-9x^5 + 8x^4 - 216x^3 - 2724x^2 + 324x - 7884}{7383312(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} \frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{9\#1^4 \log(x - \#1) - 9\#1^4 \log(x - \#1)}{44299872}\right]}{44299872}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/44299872

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$


```

21055232 - (505*z)/6285755625280943609742215135077859328 + 4513/86585973976
20778437929792538933565560629231616, z, k)^5*x + (114155*root(z^6 + (163*z^
4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^
2)/1108852327671535435567321055232 - (505*z)/628575562528094360974221513507
7859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^2)/292
102281 - (163984176*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/
142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 -
(505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929
792538933565560629231616, z, k)^3)/44521 + (94281884928*root(z^6 + (163*z^4
)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2
)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077
859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^4)/211
- 168897381688221696*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)
/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 -
(505*z)/6285755625280943609742215135077859328 + 4513/865859739762077843792
9792538933565560629231616, z, k)^5 + 1/19313733724678043136)*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517
1*z^2)/1108852327671535435567321055232 - (505*z)/62857556252809436097422151
35077859328 + 4513/8658597397620778437929792538933565560629231616, z, k), k
, 1, 6) - ((227*x^2)/615276 - x/22788 + x^3/34182 - x^4/922914 + x^5/820368
+ 73/68364)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

```

sympy [A] time = 0.39, size = 112, normalized size = 0.11

RootSum(8658597397620778437929792538933565560629231616t⁶ + 10906809587177016824883864561254

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(8658597397620778437929792538933565560629231616*_t**6 + 10906809587177016824883864561254*_t**4 - 492655707593366915713499136*_t**3 + 40378331745144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(101442531561804181113161287039859349851881619653631712165888*_t**5/356900697070792948475845 - 149796550082359335112709434971975088967050210050048*_t**4/356900697070792948475845 + 1222409754458272818505898777768670783617236992*_t**3/356900697070792948475845 - 5775055524251595723022901938558261453824*_t**2/356900697070792948475845 + 96165242200260265765603930470432*_t/71380139414158589695169 + x - 17059152341129698120545584/1070702091212378845427535))) + (-9*x**5 + 8*x**4 - 216*x**3 - 2724*x**2 + 324*x - 7884)/(7383312*x**6 + 132899616*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)

$$3.158 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} dx$$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

[Out] $a^2*x + 2/3*a*b*x^3 + 1/5*b^2*x^5$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1586}

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x), x]

[Out] $a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5$

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} dx = \int (a^2 + 2abx^2 + b^2x^4) dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x), x]

[Out] $a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5$

fricas [A] time = 0.71, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c), x, algorithm="fricas")

[Out] $1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x$

giac [A] time = 0.37, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x, algorithm="giac")

[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x)

[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5

maxima [A] time = 0.44, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x, algorithm="maxima")

[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x

mupad [B] time = 0.04, size = 21, normalized size = 0.84

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(c + d*x),x)

[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3

sympy [A] time = 0.09, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(d*x+c),x)

[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5

$$3.159 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c+dx)^2} dx$$

Optimal. Leaf size=94

$$\frac{(ad^2 + bc^2)^2 \log(c + dx)}{d^5} - \frac{bcx(2ad^2 + bc^2)}{d^4} + \frac{bx^2(2ad^2 + bc^2)}{2d^3} - \frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d}$$

[Out] $-b*c*(2*a*d^2+b*c^2)*x/d^4+1/2*b*(2*a*d^2+b*c^2)*x^2/d^3-1/3*b^2*c*x^3/d^2+1/4*b^2*x^4/d+(a*d^2+b*c^2)^2*\ln(d*x+c)/d^5$

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.058$, Rules used = {1586, 28, 697}

$$\frac{bx^2(2ad^2 + bc^2)}{2d^3} - \frac{bcx(2ad^2 + bc^2)}{d^4} + \frac{(ad^2 + bc^2)^2 \log(c + dx)}{d^5} - \frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x)^2, x]$

[Out] $-((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^2*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*\text{Log}[c + d*x])/d^5$

Rule 28

$\text{Int}[(u_.)*((a_.) + (c_.)*(x_)^{(n2_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/c^p, \text{Int}[u*(b/2 + c*x^n)^{(2*p)}, x], x] /;$ FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 697

$\text{Int}[(d_.) + (e_.)*(x_)]^{(m_.)}*((a_.) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(a + c*x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 1586

$\text{Int}[(u_.)*(Px_)^{(p_.)}*(Qx_)^{(q_.)}, x_Symbol] \rightarrow \text{Int}[u*\text{PolynomialQuotient}[Px, Qx, x]^{p*Qx^{(p+q)}, x] /;$ FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c+dx)^2} dx &= \int \frac{a^2 + 2abx^2 + b^2x^4}{c+dx} dx \\ &= \frac{\int \frac{(ab+b^2x^2)^2}{c+dx} dx}{b^2} \\ &= \frac{\int \left(-\frac{b^3c(bc^2+2ad^2)}{d^4} + \frac{b^3(bc^2+2ad^2)x}{d^3} - \frac{b^4cx^2}{d^2} + \frac{b^4x^3}{d} + \frac{b^2(bc^2+2ad^2)x^4}{d^4(c+dx)} \right) dx}{b^2} \\ &= -\frac{bc(bc^2 + 2ad^2)x}{d^4} + \frac{b(bc^2 + 2ad^2)x^2}{2d^3} - \frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.84

$$\frac{12(ad^2 + bc^2)^2 \log(c + dx) + bdx(12ad^2(dx - 2c) + b(-12c^3 + 6c^2dx - 4cd^2x^2 + 3d^3x^3))}{12d^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x)^2,x]

[Out] (b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)

fricas [A] time = 0.96, size = 105, normalized size = 1.12

$$\frac{3b^2d^4x^4 - 4b^2cd^3x^3 + 6(b^2c^2d^2 + 2abd^4)x^2 - 12(b^2c^3d + 2abcd^3)x + 12(b^2c^4 + 2abc^2d^2 + a^2d^4)\log(dx + c)}{12d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x, algorithm="fricas")

[Out] 1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 - 12*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*x + c))/d^5

giac [B] time = 0.26, size = 365, normalized size = 3.88

$$-\frac{1}{12} b^2 d \left(\frac{(dx + c)^4 \left(\frac{20c}{dx+c} - \frac{60c^2}{(dx+c)^2} + \frac{120c^3}{(dx+c)^3} - 3 \right)}{d^6} + \frac{60c^4 \log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6} - \frac{12c^5}{(dx+c)d^6} \right) - \frac{1}{3} b^2 c \left(\frac{(dx + c)^3 \left(\frac{6c}{dx+c} - \frac{18c^2}{(dx+c)^2} \right)}{d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x, algorithm="giac")

[Out] -1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d*x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 12*c^5/((d*x + c)*d^6) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d*x + c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3*c^4/((d*x + c)*d^5) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4) + 2*a*b*c*(2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x + c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d)) - a^2*c/((d*x + c)*d)

maple [A] time = 0.00, size = 114, normalized size = 1.21

$$\frac{b^2x^4}{4d} - \frac{b^2cx^3}{3d^2} + \frac{abx^2}{d} + \frac{b^2c^2x^2}{2d^3} + \frac{a^2\ln(dx+c)}{d} + \frac{2abc^2\ln(dx+c)}{d^3} - \frac{2abcx}{d^2} + \frac{b^2c^4\ln(dx+c)}{d^5} - \frac{b^2c^3x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x)

[Out] 1/4*b^2*x^4/d-1/3*b^2*c*x^3/d^2+b/d*x^2*a+1/2*b^2/d^3*x^2*c^2-2*b/d^2*a*c*x-b^2/d^4*c^3*x+1/d*ln(d*x+c)*a^2+2/d^3*ln(d*x+c)*a*b*c^2+1/d^5*ln(d*x+c)*b^2*c^4

maxima [A] time = 0.44, size = 105, normalized size = 1.12

$$\frac{3b^2d^3x^4 - 4b^2cd^2x^3 + 6(b^2c^2d + 2abd^3)x^2 - 12(b^2c^3 + 2abcd^2)x}{12d^4} + \frac{(b^2c^4 + 2abc^2d^2 + a^2d^4)\log(dx + c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x, algorithm="maxima")

[Out] 1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12*(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*x + c)/d^5

mupad [B] time = 0.06, size = 106, normalized size = 1.13

$$x^2 \left(\frac{b^2 c^2}{2d^3} + \frac{ab}{d} \right) + \frac{\ln(c + dx) (a^2 d^4 + 2ab c^2 d^2 + b^2 c^4)}{d^5} + \frac{b^2 x^4}{4d} - \frac{b^2 c x^3}{3d^2} - \frac{c x \left(\frac{b^2 c^2}{d^3} + \frac{2ab}{d} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(c + d*x)^2,x)

[Out] x^2*((b^2*c^2)/(2*d^3) + (a*b)/d) + (log(c + d*x)*(a^2*d^4 + b^2*c^4 + 2*a*b*c^2*d^2))/d^5 + (b^2*x^4)/(4*d) - (b^2*c*x^3)/(3*d^2) - (c*x*((b^2*c^2)/d^3 + (2*a*b)/d))/d

sympy [A] time = 0.31, size = 88, normalized size = 0.94

$$-\frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d} + x^2 \left(\frac{ab}{d} + \frac{b^2c^2}{2d^3} \right) + x \left(-\frac{2abc}{d^2} - \frac{b^2c^3}{d^4} \right) + \frac{(ad^2 + bc^2)^2 \log(c + dx)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(d*x+c)**2,x)

[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(a*b/d + b**2*c**2/(2*d**3)) + x*(-2*a*b*c/d**2 - b**2*c**3/d**4) + (a*d**2 + b*c**2)**2*log(c + d*x)/d**5

$$3.160 \quad \int (b + 2cx) (bx + cx^2)^{13} dx$$

Optimal. Leaf size=15

$$\frac{1}{14} (bx + cx^2)^{14}$$

[Out] 1/14*(c*x^2+b*x)^14

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$\frac{1}{14} (bx + cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b*x + c*x^2)^14/14

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (bx + cx^2)^{13} dx = \frac{1}{14} (bx + cx^2)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13}{2}b^{12}c^2x^{16} + 26b^{11}c^3x^{17} + \frac{143}{2}b^{10}c^4x^{18} + 143b^9c^5x^{19} + \frac{429}{2}b^8c^6x^{20} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^6c^8x^{22} + 143b^5c^9x^{23} + \frac{1716}{7}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + 26b^1c^{13}x^{27} + \frac{1}{14}c^{14}x^{28}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14

fricas [B] time = 0.66, size = 154, normalized size = 10.27

$$\frac{1}{14}x^{28}c^{14} + x^{27}c^{13}b + \frac{13}{2}x^{26}c^{12}b^2 + 26x^{25}c^{11}b^3 + \frac{143}{2}x^{24}c^{10}b^4 + 143x^{23}c^9b^5 + \frac{429}{2}x^{22}c^8b^6 + \frac{1716}{7}x^{21}c^7b^7 + \frac{429}{2}x^{20}c^6b^8 + 143x^{19}c^5b^9 + \frac{1716}{7}x^{18}c^4b^{10} + 26x^{17}c^3b^{11} + \frac{13}{2}x^{16}c^2b^{12} + x^{15}c^1b^{13} + \frac{1}{14}x^{14}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")

[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{14} (cx^2 + bx)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")

[Out] 1/14*(c*x^2 + b*x)^14

maple [B] time = 0.00, size = 155, normalized size = 10.33

$$\frac{1}{14}c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}b^2c^{12}x^{26}+26b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^9c^5x^{19}+\frac{143}{2}b^{10}c^4x^{18}+143b^{11}c^3x^{17}+\frac{429}{2}b^{12}c^2x^{16}+26b^{13}cx^{15}+\frac{b^{14}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)

[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2*x^16+26*b^13*c*x^15+1/14*b^14*x^14

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{14} (cx^2 + bx)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")

[Out] 1/14*(c*x^2 + b*x)^14

mupad [B] time = 2.22, size = 154, normalized size = 10.27

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+\frac{1716b^5c^9x^{23}}{2}+\frac{143b^4c^{10}x^{24}}{2}+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+26b^13cx^{15}+\frac{b^{14}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)

[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2

sympy [B] time = 0.13, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+\frac{1716b^5c^9x^{23}}{2}+\frac{143b^4c^{10}x^{24}}{2}+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+26b^13cx^{15}+\frac{b^{14}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)

[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 + 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*x**27 + c**14*x**28/14

$$3.161 \quad \int x^{14} (b + 2cx^2) (bx + cx^3)^{13} dx$$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28} (b + cx^2)^{14}$$

[Out] 1/28*x^28*(c*x^2+b)^14

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$\frac{1}{28}x^{28} (b + cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]

[Out] (x^28*(b + c*x^2)^14)/28

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{14} (b + 2cx^2) (bx + cx^3)^{13} dx &= \int x^{27} (b + cx^2)^{13} (b + 2cx^2) dx \\ &= \frac{1}{2} \text{Subst} \left(\int x^{13} (b + cx)^{13} (b + 2cx) dx, x, x^2 \right) \\ &= \frac{1}{28} x^{28} (b + cx^2)^{14} \end{aligned}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{1}{2}b^{13}cx^{30} + \frac{13}{4}b^{12}c^2x^{32} + 13b^{11}c^3x^{34} + \frac{143}{4}b^{10}c^4x^{36} + \frac{143}{2}b^9c^5x^{38} + \frac{429}{4}b^8c^6x^{40} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^6c^8x^{44} + \frac{1}{2}b^5c^9x^{46} + \frac{1}{28}b^4c^{10}x^{48}$$

Antiderivative was successfully verified.

[In] Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]

[Out] $(b^{14}x^{28})/28 + (b^{13}c^3x^{30})/2 + (13b^{12}c^2x^{32})/4 + 13b^{11}c^3x^{34} + (143b^{10}c^4x^{36})/4 + (143b^9c^5x^{38})/2 + (429b^8c^6x^{40})/4 + (858b^7c^7x^{42})/7 + (429b^6c^8x^{44})/4 + (143b^5c^9x^{46})/2 + (143b^4c^{10}x^{48})/4 + 13b^3c^{11}x^{50} + (13b^2c^{12}x^{52})/4 + (b^1c^{13}x^{54})/2 + (c^{14}x^{56})/28$

fricas [B] time = 0.82, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14} + \frac{1}{2}x^{54}c^{13}b + \frac{13}{4}x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + \frac{143}{4}x^{48}c^{10}b^4 + \frac{143}{2}x^{46}c^9b^5 + \frac{429}{4}x^{44}c^8b^6 + \frac{858}{7}x^{42}c^7b^7 + \frac{429}{4}x^{40}c^6b^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^{14}(2cx^2+b)(cx^3+bx)^{13}$, x, algorithm="fricas")

[Out] $1/28x^{56}c^{14} + 1/2x^{54}c^{13}b + 13/4x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + 143/4x^{48}c^{10}b^4 + 143/2x^{46}c^9b^5 + 429/4x^{44}c^8b^6 + 858/7x^{42}c^7b^7 + 429/4x^{40}c^6b^8 + 143/2x^{38}c^5b^9 + 143/4x^{36}c^4b^{10} + 13x^{34}c^3b^{11} + 13/4x^{32}c^2b^{12} + 1/2x^{30}cb^{13} + 1/28x^{28}b^{14}$

giac [B] time = 0.31, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^{14}(2cx^2+b)(cx^3+bx)^{13}$, x, algorithm="giac")

[Out] $1/28c^{14}x^{56} + 1/2b^1c^{13}x^{54} + 13/4b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + 143/4b^4c^{10}x^{48} + 143/2b^5c^9x^{46} + 429/4b^6c^8x^{44} + 858/7b^7c^7x^{42} + 429/4b^8c^6x^{40} + 143/2b^9c^5x^{38} + 143/4b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + 13/4b^{12}c^2x^{32} + 1/2b^{13}cx^{30} + 1/28b^{14}x^{28}$

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}b^1c^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int($x^{14}(2cx^2+b)(cx^3+bx)^{13}$, x)

[Out] $1/28c^{14}x^{56} + 1/2b^1c^{13}x^{54} + 13/4b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + 143/4b^4c^{10}x^{48} + 143/2b^5c^9x^{46} + 429/4b^6c^8x^{44} + 858/7b^7c^7x^{42} + 429/4b^8c^6x^{40} + 143/2b^9c^5x^{38} + 143/4b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + 13/4b^{12}c^2x^{32} + 1/2b^{13}cx^{30} + 1/28b^{14}x^{28}$

maxima [B] time = 0.61, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^{14}(2cx^2+b)(cx^3+bx)^{13}$, x, algorithm="maxima")

[Out] $1/28c^{14}x^{56} + 1/2b^1c^{13}x^{54} + 13/4b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + 143/4b^4c^{10}x^{48} + 143/2b^5c^9x^{46} + 429/4b^6c^8x^{44} + 858/7b^7c^7x^{42} + 429/4b^8c^6x^{40} + 143/2b^9c^5x^{38} + 143/4b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + 13/4b^{12}c^2x^{32} + 1/2b^{13}cx^{30} + 1/28b^{14}x^{28}$

mupad [B] time = 0.14, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14*(b*x + c*x^3)^13*(b + 2*c*x^2),x)`

[Out] $(b^{14}x^{28})/28 + (c^{14}x^{56})/28 + (b^{13}c*x^{30})/2 + (b*c^{13}x^{54})/2 + (13*b^{12}c^2*x^{32})/4 + 13*b^{11}c^3*x^{34} + (143*b^{10}c^4*x^{36})/4 + (143*b^9*c^5*x^{38})/2 + (429*b^8*c^6*x^{40})/4 + (858*b^7*c^7*x^{42})/7 + (429*b^6*c^8*x^{44})/4 + (143*b^5*c^9*x^{46})/2 + (143*b^4*c^{10}x^{48})/4 + 13*b^3*c^{11}x^{50} + (13*b^2*c^{12}x^{52})/4$

sympy [B] time = 0.14, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + b^2c^{13}x^{54} + \frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)`

[Out] $b^{14}x^{28}/28 + b^{13}c*x^{30}/2 + 13*b^{12}c^2*x^{32}/4 + 13*b^{11}c^3*x^{34} + 143*b^{10}c^4*x^{36}/4 + 143*b^9*c^5*x^{38}/2 + 429*b^8*c^6*x^{40}/4 + 858*b^7*c^7*x^{42}/7 + 429*b^6*c^8*x^{44}/4 + 143*b^5*c^9*x^{46}/2 + 143*b^4*c^{10}x^{48}/4 + 13*b^3*c^{11}x^{50} + 13*b^2*c^{12}x^{52}/4 + b^2*c^{13}x^{54}/2 + c^{14}x^{56}/28$

$$3.162 \quad \int x^{28} (b + 2cx^3) (bx + cx^4)^{13} dx$$

Optimal. Leaf size=16

$$\frac{1}{42} x^{42} (b + cx^3)^{14}$$

[Out] 1/42*x^42*(c*x^3+b)^14

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$\frac{1}{42} x^{42} (b + cx^3)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]

[Out] (x^42*(b + c*x^3)^14)/42

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^(p*(c + d*x)^q), x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{28} (b + 2cx^3) (bx + cx^4)^{13} dx &= \int x^{41} (b + cx^3)^{13} (b + 2cx^3) dx \\ &= \frac{1}{3} \text{Subst} \left(\int x^{13} (b + cx)^{13} (b + 2cx) dx, x, x^3 \right) \\ &= \frac{1}{42} x^{42} (b + cx^3)^{14} \end{aligned}$$

Mathematica [B] time = 0.01, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42} + \frac{1}{3}b^{13}cx^{45} + \frac{13}{6}b^{12}c^2x^{48} + \frac{26}{3}b^{11}c^3x^{51} + \frac{143}{6}b^{10}c^4x^{54} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{2}b^8c^6x^{60} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^6c^8x^{66}$$

Antiderivative was successfully verified.

[In] Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]

[Out] $(b^{14}x^{42})/42 + (b^{13}c^8x^{45})/3 + (13b^{12}c^2x^{48})/6 + (26b^{11}c^3x^{51})/3 + (143b^{10}c^4x^{54})/6 + (143b^9c^5x^{57})/3 + (143b^8c^6x^{60})/2 + (572b^7c^7x^{63})/7 + (143b^6c^8x^{66})/2 + (143b^5c^9x^{69})/3 + (143b^4c^{10}x^{72})/6 + (26b^3c^{11}x^{75})/3 + (13b^2c^{12}x^{78})/6 + (b^1c^{13}x^{81})/3 + (c^{14}x^{84})/42$

fricas [B] time = 0.68, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14} + \frac{1}{3}x^{81}c^{13}b + \frac{13}{6}x^{78}c^{12}b^2 + \frac{26}{3}x^{75}c^{11}b^3 + \frac{143}{6}x^{72}c^{10}b^4 + \frac{143}{3}x^{69}c^9b^5 + \frac{143}{2}x^{66}c^8b^6 + \frac{572}{7}x^{63}c^7b^7 + \frac{143}{2}x^{60}c^6b^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")`

[Out] $1/42*x^{84}*c^{14} + 1/3*x^{81}*c^{13}*b + 13/6*x^{78}*c^{12}*b^2 + 26/3*x^{75}*c^{11}*b^3 + 143/6*x^{72}*c^{10}*b^4 + 143/3*x^{69}*c^9*b^5 + 143/2*x^{66}*c^8*b^6 + 572/7*x^{63}*c^7*b^7 + 143/2*x^{60}*c^6*b^8 + 143/3*x^{57}*c^5*b^9 + 143/6*x^{54}*c^4*b^{10} + 26/3*x^{51}*c^3*b^{11} + 13/6*x^{48}*c^2*b^{12} + 1/3*x^{45}*c*b^{13} + 1/42*x^{42}*b^{14}$

giac [B] time = 0.33, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")`

[Out] $1/42*c^{14}*x^{84} + 1/3*b*c^{13}*x^{81} + 13/6*b^2*c^{12}*x^{78} + 26/3*b^3*c^{11}*x^{75} + 143/6*b^4*c^{10}*x^{72} + 143/3*b^5*c^9*x^{69} + 143/2*b^6*c^8*x^{66} + 572/7*b^7*c^7*x^{63} + 143/2*b^8*c^6*x^{60} + 143/3*b^9*c^5*x^{57} + 143/6*b^{10}*c^4*x^{54} + 26/3*b^{11}*c^3*x^{51} + 13/6*b^{12}*c^2*x^{48} + 1/3*b^{13}*c*x^{45} + 1/42*b^{14}*x^{42}$

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x)`

[Out] $1/42*c^{14}*x^{84} + 1/3*b*c^{13}*x^{81} + 13/6*b^2*c^{12}*x^{78} + 26/3*b^3*c^{11}*x^{75} + 143/6*b^4*c^{10}*x^{72} + 143/3*b^5*c^9*x^{69} + 143/2*b^6*c^8*x^{66} + 572/7*b^7*c^7*x^{63} + 143/2*b^8*c^6*x^{60} + 143/3*b^9*c^5*x^{57} + 143/6*b^{10}*c^4*x^{54} + 26/3*b^{11}*c^3*x^{51} + 13/6*b^{12}*c^2*x^{48} + 1/3*b^{13}*c*x^{45} + 1/42*b^{14}*x^{42}$

maxima [B] time = 0.55, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")`

[Out] $1/42*c^{14}*x^{84} + 1/3*b*c^{13}*x^{81} + 13/6*b^2*c^{12}*x^{78} + 26/3*b^3*c^{11}*x^{75} + 143/6*b^4*c^{10}*x^{72} + 143/3*b^5*c^9*x^{69} + 143/2*b^6*c^8*x^{66} + 572/7*b^7*c^7*x^{63} + 143/2*b^8*c^6*x^{60} + 143/3*b^9*c^5*x^{57} + 143/6*b^{10}*c^4*x^{54} + 26/3*b^{11}*c^3*x^{51} + 13/6*b^{12}*c^2*x^{48} + 1/3*b^{13}*c*x^{45} + 1/42*b^{14}*x^{42}$

mupad [B] time = 2.17, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^6x^{60}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^28*(b*x + c*x^4)^13*(b + 2*c*x^3),x)`

[Out] $(b^{14}x^{42})/42 + (c^{14}x^{84})/42 + (b^{13}c*x^{45})/3 + (b*c^{13}*x^{81})/3 + (13*b^{12}*c^2*x^{48})/6 + (26*b^{11}*c^3*x^{51})/3 + (143*b^{10}*c^4*x^{54})/6 + (143*b^9*c^5*x^{57})/3 + (143*b^8*c^6*x^{60})/2 + (572*b^7*c^7*x^{63})/7 + (143*b^6*c^8*x^{66})/2 + (143*b^5*c^9*x^{69})/3 + (143*b^4*c^{10}*x^{72})/6 + (26*b^3*c^{11}*x^{75})/3 + (13*b^2*c^{12}*x^{78})/6$

sympy [B] time = 0.14, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)`

[Out] $b^{14}x^{42}/42 + b^{13}c*x^{45}/3 + 13*b^{12}*c^2*x^{48}/6 + 26*b^{11}*c^3*x^{51}/3 + 143*b^{10}*c^4*x^{54}/6 + 143*b^9*c^5*x^{57}/3 + 143*b^8*c^6*x^{60}/2 + 572*b^7*c^7*x^{63}/7 + 143*b^6*c^8*x^{66}/2 + 143*b^5*c^9*x^{69}/3 + 143*b^4*c^{10}*x^{72}/6 + 26*b^3*c^{11}*x^{75}/3 + 13*b^2*c^{12}*x^{78}/6 + b*c^{13}*x^{81}/3 + c^{14}*x^{84}/42$

$$3.163 \quad \int x^{14(-1+n)} (b + 2cx^n) (bx + cx^{1+n})^{13} dx$$

Optimal. Leaf size=21

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1584, 446, 74}

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(14*(-1+n))*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x]

[Out] (x^(14*n)*(b+c*x^n)^14)/(14*n)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{14(-1+n)} (b + 2cx^n) (bx + cx^{1+n})^{13} dx &= \int x^{13+14(-1+n)} (b + cx^n)^{13} (b + 2cx^n) dx \\ &= \frac{\text{Subst}\left(\int x^{13}(b + cx)^{13}(b + 2cx) dx, x, x^n\right)}{n} \\ &= \frac{x^{14n} (b + cx^n)^{14}}{14n} \end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

fricas [B] time = 1.04, size = 262, normalized size = 12.48

$$\frac{b^{14}x^{14}x^{14n+14} + 14b^{13}cx^{13}x^{15n+15} + 91b^{12}c^2x^{12}x^{16n+16} + 364b^{11}c^3x^{11}x^{17n+17} + 1001b^{10}c^4x^{10}x^{18n+18} + 2002b^9c^5x^9x^{19n+19} + 3003b^8c^6x^8x^{20n+20} + 3432b^7c^7x^7x^{21n+21} + 3003b^6c^8x^6x^{22n+22} + 2002b^5c^9x^5x^{23n+23} + 1001b^4c^{10}x^4x^{24n+24} + 364b^3c^{11}x^3x^{25n+25} + 91b^2c^{12}x^2x^{26n+26} + 14b^1c^{13}x^1x^{27n+27} + c^{14}x^{28n+28}}{(n*x^{28})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas")

[Out] 1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2*x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10*x^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n + 20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) + 2002*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^3*c^11*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(27*n + 27) + c^14*x^(28*n + 28))/(n*x^28)

giac [B] time = 1.86, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} + 3003b^8c^6x^{20n} + 2002b^9c^5x^{19n} + 1001b^{10}c^4x^{18n} + 364b^{11}c^3x^{17n} + 91b^{12}c^2x^{16n} + 14b^{13}cx^{15n} + b^{14}x^{14n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n

maple [B] time = 0.05, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n} + \frac{b^{13}cx^{15n}}{n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^6c^8x^{22n}}{2n} + \frac{143b^5c^9x^{23n}}{n} + \frac{143b^4c^{10}x^{24n}}{2n} + \frac{26b^3c^{11}x^{25n}}{n} + \frac{13b^2c^{12}x^{26n}}{2n} + \frac{b^{13}cx^{27n}}{n} + \frac{b^{14}x^{28n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)

[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14

maxima [B] time = 0.65, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28n}}{14n} + \frac{bc^{13}x^{27n}}{n} + \frac{13b^2c^{12}x^{26n}}{2n} + \frac{26b^3c^{11}x^{25n}}{n} + \frac{143b^4c^{10}x^{24n}}{2n} + \frac{143b^5c^9x^{23n}}{n} + \frac{429b^6c^8x^{22n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{b^{13}cx^{15n}}{n} + \frac{b^{14}x^{14n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima")

[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 429*b^6*c^8*x^(22*n)/n + 3432*b^7*c^7*x^(21*n)/n + 3003*b^8*c^6*x^(20*n)/n + 2002*b^9*c^5*x^(19*n)/n + 1001*b^10*c^4*x^(18*n)/n + 364*b^11*c^3*x^(17*n)/n + 91*b^12*c^2*x^(16*n)/n + 14*b^13*c*x^(15*n)/n + b^14*x^(14*n)/n

$9/2*b^6*c^8*x^{(22*n)}/n + 1716/7*b^7*c^7*x^{(21*n)}/n + 429/2*b^8*c^6*x^{(20*n)}/n + 143*b^9*c^5*x^{(19*n)}/n + 143/2*b^{10}*c^4*x^{(18*n)}/n + 26*b^{11}*c^3*x^{(17*n)}/n + 13/2*b^{12}*c^2*x^{(16*n)}/n + b^{13}*c*x^{(15*n)}/n + 1/14*b^{14}*x^{(14*n)}/n$

mupad [B] time = 4.02, size = 229, normalized size = 10.90

$$\frac{b^{14} x^{14n}}{14n} + \frac{c^{14} x^{28n}}{14n} + \frac{13 b^{12} c^2 x^{16n}}{2n} + \frac{26 b^{11} c^3 x^{17n}}{n} + \frac{143 b^{10} c^4 x^{18n}}{2n} + \frac{143 b^9 c^5 x^{19n}}{n} + \frac{429 b^8 c^6 x^{20n}}{2n} + \frac{1716 b^7 c^7 x^{21n}}{7n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(14*n - 14)*(b*x + c*x^(n + 1))^13*(b + 2*c*x^n),x)

[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^10*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n) + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)

[Out] Timed out

$$3.164 \quad \int \frac{b+2cx}{bx+cx^2} dx$$

Optimal. Leaf size=10

$$\log(bx + cx^2)$$

[Out] $\ln(c*x^2+b*x)$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {628}

$$\log(bx + cx^2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b + 2*c*x)/(b*x + c*x^2), x]$

[Out] $\text{Log}[b*x + c*x^2]$

Rule 628

$\text{Int}[(d + (e_*)*(x_))/(a + (b_*)*(x) + (c_*)*(x)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\int \frac{b + 2cx}{bx + cx^2} dx = \log(bx + cx^2)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.90

$$\log(b + cx) + \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(b + 2*c*x)/(b*x + c*x^2), x]$

[Out] $\text{Log}[x] + \text{Log}[b + c*x]$

fricas [A] time = 0.78, size = 10, normalized size = 1.00

$$\log(cx^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2*c*x+b)/(c*x^2+b*x), x, \text{algorithm}="fricas")$

[Out] $\log(c*x^2 + b*x)$

giac [A] time = 0.23, size = 11, normalized size = 1.10

$$\log(|cx^2 + bx|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2*c*x+b)/(c*x^2+b*x), x, \text{algorithm}="giac")$

[Out] $\log(\text{abs}(c*x^2 + b*x))$

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\ln((cx + b)x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*c*x+b)/(c*x^2+b*x),x)`

[Out] `ln(x*(c*x+b))`

maxima [A] time = 0.55, size = 10, normalized size = 1.00

$$\log(cx^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")`

[Out] `log(c*x^2 + b*x)`

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\ln(x(b + cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x)/(b*x + c*x^2),x)`

[Out] `log(x*(b + c*x))`

sympy [A] time = 0.13, size = 8, normalized size = 0.80

$$\log(bx + cx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x+b)/(c*x**2+b*x),x)`

[Out] `log(b*x + c*x**2)`

$$3.165 \quad \int \frac{b+2cx^2}{bx+cx^3} dx$$

Optimal. Leaf size=15

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

[Out] ln(x)+1/2*ln(c*x^2+b)

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1593, 446, 72}

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(b*x + c*x^3), x]

[Out] Log[x] + Log[b + c*x^2]/2

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^2}{bx+cx^3} dx &= \int \frac{b+2cx^2}{x(b+cx^2)} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{c}{b+cx} \right) dx, x, x^2 \right) \\ &= \log(x) + \frac{1}{2} \log(b+cx^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]

[Out] Log[x] + Log[b + c*x^2]/2

fricas [A] time = 0.85, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(cx^2 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="fricas")

[Out] 1/2*log(c*x^2 + b) + log(x)

giac [A] time = 0.33, size = 18, normalized size = 1.20

$$\frac{1}{2} \log(x^2) + \frac{1}{2} \log(|cx^2 + b|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")

[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x) + \frac{\ln(cx^2 + b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^2+b)/(c*x^3+b*x),x)

[Out] ln(x)+1/2*ln(c*x^2+b)

maxima [A] time = 0.64, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(cx^2 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")

[Out] 1/2*log(c*x^2 + b) + log(x)

mupad [B] time = 2.08, size = 13, normalized size = 0.87

$$\frac{\ln(cx^2 + b)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^2)/(b*x + c*x^3),x)

[Out] log(b + c*x^2)/2 + log(x)

sympy [A] time = 0.18, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x**2+b)/(c*x**3+b*x),x)

[Out] log(x) + log(b/c + x**2)/2

$$3.166 \quad \int \frac{b+2cx^3}{bx+cx^4} dx$$

Optimal. Leaf size=15

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

[Out] ln(x)+1/3*ln(c*x^3+b)

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1593, 446, 72}

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^3)/(b*x + c*x^4), x]

[Out] Log[x] + Log[b + c*x^3]/3

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^3}{bx+cx^4} dx &= \int \frac{b+2cx^3}{x(b+cx^3)} dx \\ &= \frac{1}{3} \text{Subst} \left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{c}{b+cx} \right) dx, x, x^3 \right) \\ &= \log(x) + \frac{1}{3} \log(b+cx^3) \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]

[Out] Log[x] + Log[b + c*x^3]/3

fricas [A] time = 1.00, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(cx^3 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="fricas")

[Out] 1/3*log(c*x^3 + b) + log(x)

giac [A] time = 0.31, size = 15, normalized size = 1.00

$$\frac{1}{3} \log(|cx^3 + b|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")

[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$\ln(x) + \frac{\ln(cx^3 + b)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^3+b)/(c*x^4+b*x),x)

[Out] ln(x)+1/3*ln(c*x^3+b)

maxima [A] time = 0.52, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(cx^3 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")

[Out] 1/3*log(c*x^3 + b) + log(x)

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln(cx^3 + b)}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^3)/(b*x + c*x^4),x)

[Out] log(b + c*x^3)/3 + log(x)

sympy [A] time = 0.20, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x**3+b)/(c*x**4+b*x),x)

[Out] log(x) + log(b/c + x**3)/3

$$3.167 \quad \int \frac{b+2cx^n}{bx+cx^{1+n}} dx$$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

[Out] ln(x)+ln(b+c*x^n)/n

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1593, 446, 72}

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)), x]

[Out] Log[x] + Log[b + c*x^n]/n

Rule 72

Int[((e_.) + (f_.)*(x_)^(p_.))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^n}{bx+cx^{1+n}} dx &= \int \frac{b+2cx^n}{x(b+cx^n)} dx \\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x} + \frac{c}{b+cx}\right) dx, x, x^n\right)}{n} \\ &= \log(x) + \frac{\log(b+cx^n)}{n} \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]

[Out] Log[x] + Log[b + c*x^n]/n

fricas [A] time = 0.80, size = 23, normalized size = 1.53

$$\frac{(n-1)\log(x) + \log(bx + cx^{n+1})}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="fricas")

[Out] ((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2cx^n + b}{bx + cx^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")

[Out] integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\ln(x) + \frac{\ln(c e^{n \ln(x)} + b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b+2*c*x^n)/(b*x+c*x^(1+n)),x)

[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)

maxima [B] time = 0.58, size = 47, normalized size = 3.13

$$b \left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n + b}{c}\right)}{bn} \right) + \frac{2 \log\left(\frac{cx^n + b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")

[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{b + 2cx^n}{bx + cx^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)),x)

[Out] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)), x)

sympy [A] time = 1.48, size = 29, normalized size = 1.93

$$\left\{ \begin{array}{ll} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \log(x) & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c} + x^n\right)}{n} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)

[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))

$$3.168 \quad \int \frac{b+2cx}{(bx+cx^2)^8} dx$$

Optimal. Leaf size=15

$$-\frac{1}{7(bx+cx^2)^7}$$

[Out] -1/7/(c*x^2+b*x)^7

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$-\frac{1}{7(bx+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/(7*(b*x + c*x^2)^7)

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{b+2cx}{(bx+cx^2)^8} dx = -\frac{1}{7(bx+cx^2)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/7*1/(x^7*(b + c*x)^7)

fricas [B] time = 0.74, size = 81, normalized size = 5.40

$$-\frac{1}{7(c^7x^{14} + 7bc^6x^{13} + 21b^2c^5x^{12} + 35b^3c^4x^{11} + 35b^4c^3x^{10} + 21b^5c^2x^9 + 7b^6cx^8 + b^7x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")

[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)

giac [A] time = 0.27, size = 13, normalized size = 0.87

$$-\frac{1}{7(cx^2 + bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")

[Out] -1/7/(c*x^2 + b*x)^7

maple [B] time = 0.02, size = 177, normalized size = 11.80

$$\frac{c^7}{7(cx+b)^7 b^7} + \frac{c^7}{(cx+b)^6 b^8} + \frac{4c^7}{(cx+b)^5 b^9} + \frac{12c^7}{(cx+b)^4 b^{10}} + \frac{30c^7}{(cx+b)^3 b^{11}} + \frac{66c^7}{(cx+b)^2 b^{12}} + \frac{132c^7}{(cx+b)b^{13}} - \frac{132c^6}{b^{13}x} + \frac{66c^6}{b^{12}x^2} - \frac{132c^5}{b^{12}x^3} + \frac{132c^4}{b^{11}x^4} - \frac{66c^3}{b^{11}x^5} + \frac{132c^2}{b^{10}x^6} - \frac{66c}{b^{10}x^7} + \frac{132}{b^9 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)

[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+1/b^8*c^7/(c*x+b)^6+1/7/b^7*c^7/(c*x+b)^7-1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b^9*c^2/x^5+1/b^8*c/x^6

maxima [A] time = 0.53, size = 13, normalized size = 0.87

$$-\frac{1}{7(cx^2 + bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")

[Out] -1/7/(c*x^2 + b*x)^7

mupad [B] time = 4.38, size = 12, normalized size = 0.80

$$-\frac{1}{7x^7(b+cx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)

[Out] -1/(7*x^7*(b + c*x)^7)

sympy [B] time = 0.93, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)

[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**14)

$$3.169 \quad \int \frac{b+2cx^2}{x^7(bx+cx^3)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

[Out] -1/14/x^14/(c*x^2+b)^7

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]

[Out] -1/(14*x^14*(b + c*x^2)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^2}{x^7(bx+cx^3)^8} dx &= \int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^2 \right) \\ &= -\frac{1}{14x^{14}(b+cx^2)^7} \end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8), x]

[Out] -1/14*1/(x^14*(b + c*x^2)^7)

fricas [B] time = 0.82, size = 81, normalized size = 5.06

$$\frac{1}{14 \left(c^7 x^{28} + 7 b c^6 x^{26} + 21 b^2 c^5 x^{24} + 35 b^3 c^4 x^{22} + 35 b^4 c^3 x^{20} + 21 b^5 c^2 x^{18} + 7 b^6 c x^{16} + b^7 x^{14} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)

giac [A] time = 0.41, size = 15, normalized size = 0.94

$$-\frac{1}{14 (c x^4 + b x^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")

[Out] -1/14/(c*x^4 + b*x^2)^7

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$\frac{\left(-\frac{b^6}{7(c x^2+b)^7 c} - \frac{b^5}{(c x^2+b)^6 c} - \frac{4 b^4}{(c x^2+b)^5 c} - \frac{12 b^3}{(c x^2+b)^4 c} - \frac{30 b^2}{(c x^2+b)^3 c} - \frac{66 b}{(c x^2+b)^2 c} - \frac{132}{(c x^2+b) c} \right) c^8 - \frac{66 c^6}{b^{13} x^2} + \frac{33 c^5}{b^{12} x^4} - \frac{15 c^4}{b^{11} x^6} + \frac{6 c^3}{b^{10} x^8} - \frac{2 c^2}{b^9 x^{10}} + \frac{1}{2 b^8 x^{12}}}{2 b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x)

[Out] -1/2/b^13*c^8*(-1/7*b^6/c/(c*x^2+b)^7-b^5/c/(c*x^2+b)^6-132/c/(c*x^2+b)-66*b/c/(c*x^2+b)^2-4*b^4/c/(c*x^2+b)^5-30*b^2/c/(c*x^2+b)^3-12*b^3/c/(c*x^2+b)^4)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12

maxima [B] time = 0.67, size = 81, normalized size = 5.06

$$\frac{1}{14 \left(c^7 x^{28} + 7 b c^6 x^{26} + 21 b^2 c^5 x^{24} + 35 b^3 c^4 x^{22} + 35 b^4 c^3 x^{20} + 21 b^5 c^2 x^{18} + 7 b^6 c x^{16} + b^7 x^{14} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)

mupad [B] time = 2.22, size = 14, normalized size = 0.88

$$-\frac{1}{14 x^{14} (c x^2 + b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x)`

[Out] `-1/(14*x^14*(b + c*x^2)^7)`

sympy [B] time = 1.43, size = 87, normalized size = 5.44

$$\frac{1}{14b^7x^{14} + 98b^6cx^{16} + 294b^5c^2x^{18} + 490b^4c^3x^{20} + 490b^3c^4x^{22} + 294b^2c^5x^{24} + 98bc^6x^{26} + 14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)`

[Out] `-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**7*x**28)`

$$3.170 \quad \int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

[Out] -1/21/x^21/(c*x^3+b)^7

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]

[Out] -1/(21*x^21*(b + c*x^3)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8} dx &= \int \frac{b+2cx^3}{x^{22}(b+cx^3)^8} dx \\ &= \frac{1}{3} \text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^3\right) \\ &= -\frac{1}{21x^{21}(b+cx^3)^7} \end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8), x]

[Out] -1/21*1/(x^21*(b + c*x^3)^7)

fricas [B] time = 0.81, size = 81, normalized size = 5.06

$$\frac{1}{21 \left(c^7 x^{42} + 7 b c^6 x^{39} + 21 b^2 c^5 x^{36} + 35 b^3 c^4 x^{33} + 35 b^4 c^3 x^{30} + 21 b^5 c^2 x^{27} + 7 b^6 c x^{24} + b^7 x^{21} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)

giac [A] time = 0.30, size = 15, normalized size = 0.94

$$\frac{1}{21 \left(c x^6 + b x^3 \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")

[Out] -1/21/(c*x^6 + b*x^3)^7

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$\frac{\left(-\frac{b^6}{7(c x^3+b)^7 c} - \frac{b^5}{(c x^3+b)^6 c} - \frac{4 b^4}{(c x^3+b)^5 c} - \frac{12 b^3}{(c x^3+b)^4 c} - \frac{30 b^2}{(c x^3+b)^3 c} - \frac{66 b}{(c x^3+b)^2 c} - \frac{132}{(c x^3+b) c} \right) c^8}{3 b^{13}} - \frac{44 c^6}{b^{13} x^3} + \frac{22 c^5}{b^{12} x^6} - \frac{10 c^4}{b^{11} x^9} + \frac{4 c^3}{b^{10} x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x)

[Out] -1/3*c^8/b^13*(-1/7*b^6/c/(c*x^3+b)^7-b^5/c/(c*x^3+b)^6-132/c/(c*x^3+b)-66*b/c/(c*x^3+b)^2-4*b^4/c/(c*x^3+b)^5-30*b^2/c/(c*x^3+b)^3-12*b^3/c/(c*x^3+b)^4)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18

maxima [B] time = 0.80, size = 81, normalized size = 5.06

$$\frac{1}{21 \left(c^7 x^{42} + 7 b c^6 x^{39} + 21 b^2 c^5 x^{36} + 35 b^3 c^4 x^{33} + 35 b^4 c^3 x^{30} + 21 b^5 c^2 x^{27} + 7 b^6 c x^{24} + b^7 x^{21} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)

mupad [B] time = 7.22, size = 14, normalized size = 0.88

$$\frac{1}{21 x^{21} \left(c x^3 + b \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x)`

[Out] `-1/(21*x^21*(b + c*x^3)^7)`

sympy [B] time = 2.06, size = 87, normalized size = 5.44

$$\frac{1}{21b^7x^{21} + 147b^6cx^{24} + 441b^5c^2x^{27} + 735b^4c^3x^{30} + 735b^3c^4x^{33} + 441b^2c^5x^{36} + 147bc^6x^{39} + 21c^7x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)`

[Out] `-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c**7*x**42)`

$$3.171 \quad \int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8} dx$$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8), x]

[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8} dx &= \int \frac{x^{-8-7(-1+n)}(b+2cx^n)}{(b+cx^n)^8} dx \\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^n\right)}{n} \\ &= -\frac{x^{-7n}}{7n(b+cx^n)^7} \end{aligned}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8), x]

[Out] -1/7*1/(n*x^(7*n))*(b + c*x^n)^7

fricas [B] time = 1.01, size = 143, normalized size = 6.81

$$\frac{x^{14}}{7(b^7 n x^7 x^{7n+7} + 7 b^6 c n x^6 x^{8n+8} + 21 b^5 c^2 n x^5 x^{9n+9} + 35 b^4 c^3 n x^4 x^{10n+10} + 35 b^3 c^4 n x^3 x^{11n+11} + 21 b^2 c^5 n x^2 x^{12n+12} + 7 b c^6 n x x^{13n+13} + c^7 x^{14n+14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas")

[Out] -1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*n*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(11*n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c^7*n*x^(14*n + 14))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2cx^n + b}{(bx + cx^{n+1})^8 x^{7n-7}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")

[Out] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)

maple [B] time = 0.06, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^7n} + \frac{cx^{-6n}}{b^8n} - \frac{4c^2x^{-5n}}{b^9n} + \frac{12c^3x^{-4n}}{b^{10}n} - \frac{30c^4x^{-3n}}{b^{11}n} + \frac{66c^5x^{-2n}}{b^{12}n} + \frac{(9009b^5cx^n + 20020b^4c^2x^{2n} + 24024b^3c^3x^{3n} + 16016b^2c^4x^{4n} + 9009b^2c^5x^{5n} + 20020b^2c^6x^{6n} + 24024b^2c^7x^{7n} + 9009b^2c^8x^{8n} + 20020b^2c^9x^{9n} + 24024b^2c^{10}x^{10n} + 9009b^2c^{11}x^{11n} + 20020b^2c^{12}x^{12n} + 24024b^2c^{13}x^{13n} + 9009b^2c^{14}x^{14n})}{7(c^{14}x^{14n} + 14b^2c^{13}x^{13n} + 35b^4c^{12}x^{12n} + 35b^6c^{11}x^{11n} + 21b^8c^{10}x^{10n} + 7b^{10}c^9x^9n + b^{12}c^8x^8n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)

[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7

maxima [B] time = 1.03, size = 612, normalized size = 29.14

$$-\frac{1}{105} b \left(\frac{360360 c^{13} x^{13n} + 2342340 bc^{12} x^{12n} + 6426420 b^2 c^{11} x^{11n} + 9579570 b^3 c^{10} x^{10n} + 8270262 b^4 c^9 x^9n + 4018014 b^5 c^8 x^8n + 934362 b^6 c^7 x^7n + 45045 b^7 c^6 x^6n - 5005 b^8 c^5 x^5n + 1001 b^9 c^4 x^4n - 273 b^{10} c^3 x^3n + 91 b^{11} c^2 x^2n - 35 b^{12} c x^n + 15 b^{13}}{b^{14} c^7 n x^{14n} + 7 b^{15} c^6 n x^{13n} + 21 b^{16} c^5 n x^{12n} + 35 b^{17} c^4 n x^{11n} + 35 b^{18} c^3 n x^{10n} + 21 b^{19} c^2 n x^9n + 7 b^{20} c n x^8n + b^{21} n x^7n} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima")

[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*

$c^7 \log(x)/b^{15} - 360360c^7 \log((cx^n + b)/c)/(b^{15n}) + 1/105c((360360c^{12}x^{12n} + 2342340b^2c^{11}x^{11n} + 6426420b^2c^{10}x^{10n} + 9579570b^3c^9x^{9n} + 8270262b^4c^8x^{8n} + 4018014b^5c^7x^{7n} + 934362b^6c^6x^{6n} + 45045b^7c^5x^{5n} - 5005b^8c^4x^{4n} + 1001b^9c^3x^{3n} - 273b^{10}c^2x^{2n} + 91b^{11}cx^n - 35b^{12})/(b^{13}c^{7n}x^{13n} + 7b^{14}c^6nx^{12n} + 21b^{15}c^5nx^{11n} + 35b^{16}c^4nx^{10n} + 35b^{17}c^3nx^{9n} + 21b^{18}c^2nx^{8n} + 7b^{19}cnx^{7n} + b^{20}nx^{6n}) + 360360c^6 \log(x)/b^{14} - 360360c^6 \log((cx^n + b)/c)/(b^{14n})$

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{7-7n} (b + 2cx^n)}{(bx + cx^{n+1})^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8,x)

[Out] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)

[Out] Timed out

$$3.172 \quad \int (b + 2cx) (bx + cx^2)^p dx$$

Optimal. Leaf size=19

$$\frac{(bx + cx^2)^{p+1}}{p+1}$$

[Out] (c*x^2+b*x)^(1+p)/(1+p)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$\frac{(bx + cx^2)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (b*x + c*x^2)^(1 + p)/(1 + p)

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (bx + cx^2)^p dx = \frac{(bx + cx^2)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{(x(b + cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (x*(b + c*x))^(1 + p)/(1 + p)

fricas [A] time = 0.78, size = 26, normalized size = 1.37

$$\frac{(cx^2 + bx)(cx^2 + bx)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")

[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)

giac [A] time = 0.37, size = 19, normalized size = 1.00

$$\frac{(cx^2 + bx)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")

[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$\frac{(cx + b)x(c x^2 + bx)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)

[Out] (c*x+b)*x/(1+p)*(c*x^2+b*x)^p

maxima [A] time = 0.56, size = 19, normalized size = 1.00

$$\frac{(cx^2 + bx)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")

[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)

mupad [B] time = 2.10, size = 23, normalized size = 1.21

$$\frac{x(c x^2 + b x)^p (b + c x)}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)

[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)

sympy [A] time = 0.71, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)

[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(x) + log(b/c + x), True))

$$3.173 \quad \int x^{1+p} (b + 2cx^2) (bx + cx^3)^p dx$$

Optimal. Leaf size=27

$$\frac{x^{p+1} (bx + cx^3)^{p+1}}{2(p+1)}$$

[Out] $1/2*x^{(1+p)}*(c*x^3+b*x)^{(1+p)}/(1+p)$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1590}

$$\frac{x^{p+1} (bx + cx^3)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(1+p)}*(b+2*c*x^2)*(b*x+c*x^3)^p,x]$

[Out] $(x^{(1+p)}*(b*x+c*x^3)^{(1+p)})/(2*(1+p))$

Rule 1590

$\text{Int}[(\text{Pp}_*)*(\text{Qq}_*)^{(m_*)}*(\text{Rr}_*)^{(n_*)}, x_Symbol] := \text{With}[\{p = \text{Expon}[\text{Pp}, x], q = \text{Expon}[\text{Qq}, x], r = \text{Expon}[\text{Rr}, x]\}, \text{Simp}[(\text{Coeff}[\text{Pp}, x, p]*x^{(p-q-r+1)}*q^{(m+1)}*\text{Rr}^{(n+1)})/((p+m*q+n*r+1)*\text{Coeff}[\text{Qq}, x, q]*\text{Coeff}[\text{Rr}, x, r]), x] /; \text{NeQ}[p+m*q+n*r+1, 0] \&\& \text{EqQ}[(p+m*q+n*r+1)*\text{Coeff}[\text{Qq}, x, q]*\text{Coeff}[\text{Rr}, x, r]*\text{Pp}, \text{Coeff}[\text{Pp}, x, p]*x^{(p-q-r)}*((p-q-r+1)*\text{Qq}*\text{Rr} + (m+1)*x*\text{Rr}*D[\text{Qq}, x] + (n+1)*x*\text{Qq}*D[\text{Rr}, x])]] /; \text{FreeQ}[\{m, n\}, x] \&\& \text{PolyQ}[\text{Pp}, x] \&\& \text{PolyQ}[\text{Qq}, x] \&\& \text{PolyQ}[\text{Rr}, x] \&\& \text{NeQ}[m, -1] \&\& \text{NeQ}[n, -1]$

Rubi steps

$$\int x^{1+p} (b + 2cx^2) (bx + cx^3)^p dx = \frac{x^{1+p} (bx + cx^3)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 3.59

$$\frac{x^{p+2} \left(x(b + cx^2)\right)^p \left(\frac{cx^2}{b} + 1\right)^{-p} \left(2c(p+1)x^2 {}_2F_1\left(-p, p+2; p+3; -\frac{cx^2}{b}\right) + b(p+2) {}_2F_1\left(-p, p+1; p+2; -\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^{(1+p)}*(b+2*c*x^2)*(b*x+c*x^3)^p,x]$

[Out] $(x^{(2+p)}*(x*(b+c*x^2))^{p*(b*(2+p)*\text{Hypergeometric2F1}[-p, 1+p, 2+p, -((c*x^2)/b)] + 2*c*(1+p)*x^2*\text{Hypergeometric2F1}[-p, 2+p, 3+p, -((c*x^2)/b)]}))/((2*(1+p)*(2+p)*(1+(c*x^2)/b)^p)$

fricas [A] time = 1.21, size = 32, normalized size = 1.19

$$\frac{(cx^3 + bx)(cx^3 + bx)^p x^{p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")

[Out] 1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)

giac [B] time = 0.33, size = 54, normalized size = 2.00

$$\frac{cx^3 e^{(p \log(cx^2+b)+2p \log(x)+\log(x))} + bxe^{(p \log(cx^2+b)+2p \log(x)+\log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")

[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)))/(p + 1)

maple [A] time = 0.00, size = 31, normalized size = 1.15

$$\frac{(cx^2 + b)x^{p+2}(cx^3 + bx)^p}{2 + 2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x)

[Out] 1/2*x^(2+p)*(c*x^2+b)/(p+1)*(c*x^3+b*x)^p

maxima [A] time = 0.99, size = 35, normalized size = 1.30

$$\frac{(cx^4 + bx^2)e^{(p \log(cx^2+b)+2p \log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)

mupad [B] time = 2.21, size = 45, normalized size = 1.67

$$(cx^3 + bx)^p \left(\frac{bx x^{p+1}}{2p+2} + \frac{cx^{p+1} x^3}{2p+2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(p + 1)*(b*x + c*x^3)^p*(b + 2*c*x^2),x)

[Out] (b*x + c*x^3)^p*((b*x*x^(p + 1))/(2*p + 2) + (c*x^(p + 1)*x^3)/(2*p + 2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)

[Out] Timed out

$$3.174 \quad \int \left(bx^{1+p} (bx + cx^3)^p + 2cx^{3+p} (bx + cx^3)^p \right) dx$$

Optimal. Leaf size=27

$$\frac{x^{p+1} (bx + cx^3)^{p+1}}{2(p+1)}$$

[Out] 1/2*x^(1+p)*(c*x^3+b*x)^(1+p)/(1+p)

Rubi [C] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 4.30, number of steps used = 7, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2032, 365, 364}

$$\frac{bx^{p+2} (bx + cx^3)^p \left(\frac{cx^2}{b} + 1\right)^{-p} {}_2F_1\left(-p, p+1; p+2; -\frac{cx^2}{b}\right) + cx^{p+4} (bx + cx^3)^p \left(\frac{cx^2}{b} + 1\right)^{-p} {}_2F_1\left(-p, p+2; p+3; -\frac{cx^2}{b}\right)}{2(p+1) + p+2}$$

Antiderivative was successfully verified.

[In] Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]

[Out] (b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)/b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 365

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 2032

Int[((c_)*(x_))^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegQ[p] && NeQ[n, j] && PosQ[n - j]

Rubi steps

$$\begin{aligned} \int \left(bx^{1+p} (bx + cx^3)^p + 2cx^{3+p} (bx + cx^3)^p \right) dx &= b \int x^{1+p} (bx + cx^3)^p dx + (2c) \int x^{3+p} (bx + cx^3)^p dx \\ &= \left(bx^{-p} (b + cx^2)^{-p} (bx + cx^3)^p \right) \int x^{1+2p} (b + cx^2)^p dx + \left(2cx^{2+p} (b + cx^2)^{-p} (bx + cx^3)^p \right) \int x^{1+2p} (b + cx^2)^p dx \\ &= \left(bx^{-p} \left(1 + \frac{cx^2}{b} \right)^{-p} (bx + cx^3)^p \right) \int x^{1+2p} \left(1 + \frac{cx^2}{b} \right)^p dx + \left(2cx^{2+p} \left(1 + \frac{cx^2}{b} \right)^{-p} (bx + cx^3)^p \right) \int x^{1+2p} \left(1 + \frac{cx^2}{b} \right)^p dx \\ &= \frac{bx^{2+p} \left(1 + \frac{cx^2}{b} \right)^{-p} (bx + cx^3)^p {}_2F_1\left(-p, 1 + p; 2 + p; -\frac{cx^2}{b}\right) + 2cx^{3+p} \left(1 + \frac{cx^2}{b} \right)^{-p} (bx + cx^3)^p {}_2F_1\left(-p, 1 + p; 2 + p; -\frac{cx^2}{b}\right)}{2(1 + p)} \end{aligned}$$

Mathematica [C] time = 0.03, size = 97, normalized size = 3.59

$$\frac{x^{p+2} \left(x(b + cx^2) \right)^p \left(\frac{cx^2}{b} + 1 \right)^{-p} \left(2c(p+1)x^2 {}_2F_1 \left(-p, p+2; p+3; -\frac{cx^2}{b} \right) + b(p+2) {}_2F_1 \left(-p, p+1; p+2; -\frac{cx^2}{b} \right) \right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[b*x^(1+p)*(b*x+c*x^3)^p+2*c*x^(3+p)*(b*x+c*x^3)^p,x]

[Out] (x^(2+p)*(x*(b+c*x^2))^p*(b*(2+p)*Hypergeometric2F1[-p,1+p,2+p,-((c*x^2)/b)]+2*c*(1+p)*x^2*Hypergeometric2F1[-p,2+p,3+p,-((c*x^2)/b)]))/(2*(1+p)*(2+p)*(1+(c*x^2)/b)^p)

fricas [A] time = 0.75, size = 33, normalized size = 1.22

$$\frac{(cx^2 + b)(cx^3 + bx)^p x^{p+3}}{2(p+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="fricas")

[Out] 1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)

giac [B] time = 0.49, size = 54, normalized size = 2.00

$$\frac{cx^3 e^{(p \log(cx^2+b)+2p \log(x)+\log(x))} + bxe^{(p \log(cx^2+b)+2p \log(x)+\log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="giac")

[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)))/(p + 1)

maple [C] time = 0.26, size = 142, normalized size = 5.26

$$\frac{(cx^2 + b) x x^{p+1} e^{\frac{(-i\pi \operatorname{csgn}(ix) \operatorname{csgn}(i(cx^2+b))) \operatorname{csgn}(i(cx^2+b)x) + i\pi \operatorname{csgn}(ix) \operatorname{csgn}(i(cx^2+b)x)^2 + i\pi \operatorname{csgn}(i(cx^2+b)) \operatorname{csgn}(i(cx^2+b)x)^2 - i\pi \operatorname{csgn}(i(cx^2+b)x)^3 + 2 \ln(x) + 2 \ln(x))}{2}}}{2 + 2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x)

[Out] 1/2*(c*x^2+b)*x*x^(p+1)/(p+1)*exp(1/2*p*(-I*csgn(I*x*(c*x^2+b))^3*Pi+I*csgn(I*x*(c*x^2+b))^2*csgn(I*x)*Pi+I*csgn(I*x*(c*x^2+b))^2*csgn(I*(c*x^2+b))*Pi-I*csgn(I*x*(c*x^2+b))*csgn(I*x)*csgn(I*(c*x^2+b))*Pi+2*ln(x)+2*ln(c*x^2+b))

maxima [A] time = 1.07, size = 35, normalized size = 1.30

$$\frac{(cx^4 + bx^2)e^{(p \log(cx^2+b)+2p \log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int b x^{p+1} (c x^3 + b x)^p + 2 c x^{p+3} (c x^3 + b x)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p,x)

[Out] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)

[Out] Timed out

$$3.175 \quad \int x^{2(1+p)} (b + 2cx^3) (bx + cx^4)^p dx$$

Optimal. Leaf size=29

$$\frac{x^{2(p+1)} (bx + cx^4)^{p+1}}{3(p+1)}$$

[Out] $1/3*x^{(2*p+2)}*(c*x^4+b*x)^{(1+p)}/(1+p)$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1590}

$$\frac{x^{2(p+1)} (bx + cx^4)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(2*(1+p))}*(b+2*c*x^3)*(b*x+c*x^4)^p,x]$

[Out] $(x^{(2*(1+p))}*(b*x+c*x^4)^{(1+p)})/(3*(1+p))$

Rule 1590

$\text{Int}[(Pp_)*(Qq_)^{(m_)}*(Rr_)^{(n_)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Expon}[Pp, x], q = \text{Expon}[Qq, x], r = \text{Expon}[Rr, x]\}, \text{Simp}[(\text{Coeff}[Pp, x, p]*x^{(p-q-r+1)}*Qq^{(m+1)}*Rr^{(n+1)})/((p+m*q+n*r+1)*\text{Coeff}[Qq, x, q]*\text{Coeff}[Rr, x, r]), x] /; \text{NeQ}[p+m*q+n*r+1, 0] \&\& \text{EqQ}[(p+m*q+n*r+1)*\text{Coeff}[Qq, x, q]*\text{Coeff}[Rr, x, r]*Pp, \text{Coeff}[Pp, x, p]*x^{(p-q-r)}*((p-q-r+1)*Qq*Rr + (m+1)*x*Rr*D[Qq, x] + (n+1)*x*Qq*D[Rr, x])]] /; \text{FreeQ}[\{m, n\}, x] \&\& \text{PolyQ}[Pp, x] \&\& \text{PolyQ}[Qq, x] \&\& \text{PolyQ}[Rr, x] \&\& \text{NeQ}[m, -1] \&\& \text{NeQ}[n, -1]$

Rubi steps

$$\int x^{2(1+p)} (b + 2cx^3) (bx + cx^4)^p dx = \frac{x^{2(1+p)} (bx + cx^4)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.08, size = 99, normalized size = 3.41

$$\frac{x^{2p+3} (x(b + cx^3))^p \left(\frac{cx^3}{b} + 1\right)^{-p} \left(2c(p+1)x^3 {}_2F_1\left(-p, p+2; p+3; -\frac{cx^3}{b}\right) + b(p+2) {}_2F_1\left(-p, p+1; p+2; -\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^{(2*(1+p))}*(b+2*c*x^3)*(b*x+c*x^4)^p,x]$

[Out] $(x^{(3+2*p)}*(x*(b+c*x^3))^{p*(b*(2+p)*\text{Hypergeometric2F1}[-p, 1+p, 2+p, -((c*x^3)/b)] + 2*c*(1+p)*x^3*\text{Hypergeometric2F1}[-p, 2+p, 3+p, -((c*x^3)/b)])))/(3*(1+p)*(2+p)*(1+(c*x^3)/b)^p)$

fricas [A] time = 1.01, size = 34, normalized size = 1.17

$$\frac{(cx^4 + bx)(cx^4 + bx)^p x^{2p+2}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")

[Out] 1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)

giac [B] time = 0.35, size = 58, normalized size = 2.00

$$\frac{cx^4 e^{(p \log(cx^3+b)+3p \log(x)+2 \log(x))} + bxe^{(p \log(cx^3+b)+3p \log(x)+2 \log(x))}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")

[Out] 1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)

maple [A] time = 0.00, size = 33, normalized size = 1.14

$$\frac{(cx^3 + b)x^{2p+3}(cx^4 + bx)^p}{3 + 3p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x)

[Out] 1/3*x^(3+2*p)*(c*x^3+b)/(p+1)*(c*x^4+b*x)^p

maxima [A] time = 1.01, size = 35, normalized size = 1.21

$$\frac{(cx^6 + bx^3)e^{(p \log(cx^3+b)+3p \log(x))}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")

[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)

mupad [B] time = 2.21, size = 49, normalized size = 1.69

$$(cx^4 + bx)^p \left(\frac{cx^{2p+2}x^4}{3p+3} + \frac{bx^{2p+2}}{3p+3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*p + 2)*(b*x + c*x^4)^p*(b + 2*c*x^3),x)

[Out] (b*x + c*x^4)^p*((c*x^(2*p + 2)*x^4)/(3*p + 3) + (b*x*x^(2*p + 2))/(3*p + 3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)

[Out] Timed out

$$3.176 \quad \int x^{(-1+n)(1+p)} (b + 2cx^n) (bx + cx^{1+n})^p dx$$

Optimal. Leaf size=36

$$\frac{x^{-((1-n)(p+1))} (bx + cx^{n+1})^{p+1}}{n(p+1)}$$

[Out] (b*x+c*x^(1+n))^(1+p)/n/(1+p)/(x^((1-n)*(1+p)))

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2036}

$$\frac{x^{-(1-n)(p+1)} (bx + cx^{n+1})^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]

[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))

Rule 2036

Int[((e_.)*(x_.))^(m_.)*((a_.)*(x_.)^(j_.) + (b_.)*(x_.)^(jn_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] := Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || IntegerQ[j]) && NeQ[m + j*p + 1, 0]

Rubi steps

$$\int x^{(-1+n)(1+p)} (b + 2cx^n) (bx + cx^{1+n})^p dx = \frac{x^{-(1-n)(1+p)} (bx + cx^{1+n})^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.17, size = 108, normalized size = 3.00

$$\frac{x^{-p} (x(b + cx^n))^p \left(\frac{cx^n}{b} + 1\right)^{-p} \left(b(p+2)x^{n(p+1)} {}_2F_1\left(-p, p+1; p+2; -\frac{cx^n}{b}\right) + 2c(p+1)x^{n(p+2)} {}_2F_1\left(-p, p+2; p+3; -\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]

[Out] ((x*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p, -(c*x^n)/b] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p, 3 + p, -(c*x^n)/b])/n*(1 + p)*(2 + p)*x^p*(1 + (c*x^n)/b)^p

fricas [A] time = 0.87, size = 42, normalized size = 1.17

$$\frac{(bx + cx^{n+1})(bx + cx^{n+1})^p x^{(n-1)p+n-1}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fricas")

[Out] (b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2cx^n + b)(bx + cx^{n+1})^p x^{(n-1)(p+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="giac")

[Out] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (2cx^n + b)x^{(n-1)(p+1)}(bx + cx^{n+1})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)

[Out] int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)

maxima [A] time = 1.07, size = 39, normalized size = 1.08

$$\frac{(cx^{2n} + bx^n)e^{(np \log(x) + p \log(cx^n + b))}}{n(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="maxima")

[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^{(n-1)(p+1)}(bx + cx^{n+1})^p (b + 2cx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n),x)

[Out] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)

[Out] Timed out

$$3.177 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2} dx$$

Optimal. Leaf size=32

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1586}

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2),x]

[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2} dx = \int (ac + adx + bcx^2 + bdx^3) dx = acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2),x]

[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4

fricas [A] time = 0.84, size = 26, normalized size = 0.81

$$\frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x, algorithm="fricas")

[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x

giac [A] time = 0.31, size = 26, normalized size = 0.81

$$\frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x, algorithm="giac")

[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x)

[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4

maxima [A] time = 0.78, size = 26, normalized size = 0.81

$$\frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x, algorithm="maxima")

[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x

mupad [B] time = 2.14, size = 26, normalized size = 0.81

$$\frac{bdx^4}{4} + \frac{bcx^3}{3} + \frac{adx^2}{2} + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(a + b*x^2),x)

[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4

sympy [A] time = 0.10, size = 29, normalized size = 0.91

$$acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(b*x**2+a),x)

[Out] a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4

$$3.178 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a+bx^2)^2} dx$$

Optimal. Leaf size=12

$$cx + \frac{dx^2}{2}$$

[Out] c*x+1/2*d*x^2

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1586}

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^2,x]

[Out] c*x + (d*x^2)/2

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a+bx^2)^2} dx &= \int \frac{ac + adx + bcx^2 + bdx^3}{a+bx^2} dx \\ &= \int (c + dx) dx \\ &= cx + \frac{dx^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^2,x]

[Out] c*x + (d*x^2)/2

fricas [A] time = 0.82, size = 10, normalized size = 0.83

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] $1/2*d*x^2 + c*x$

giac [A] time = 0.40, size = 10, normalized size = 0.83

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] $1/2*d*x^2 + c*x$

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2} d x^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x)

[Out] $c*x+1/2*d*x^2$

maxima [A] time = 0.47, size = 10, normalized size = 0.83

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x, algorithm="maxima")

[Out] $1/2*d*x^2 + c*x$

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{d x^2}{2} + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(a + b*x^2)^2,x)

[Out] $c*x + (d*x^2)/2$

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(b*x**2+a)**2,x)

[Out] $c*x + d*x**2/2$

$$3.179 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a+bx^2)^3} dx$$

Optimal. Leaf size=42

$$\frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d \log(a + bx^2)}{2b}$$

[Out] 1/2*d*ln(b*x^2+a)/b+c*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1586, 635, 205, 260}

$$\frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d \log(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^3,x]

[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3} dx &= \int \frac{ac + adx + bcx^2 + bdx^3}{(a + bx^2)^2} dx \\
&= \int \frac{c + dx}{a + bx^2} dx \\
&= c \int \frac{1}{a + bx^2} dx + d \int \frac{x}{a + bx^2} dx \\
&= \frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d \log(a + bx^2)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d \log(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^3,x]

[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b))

fricas [A] time = 1.01, size = 98, normalized size = 2.33

$$\left[\frac{ad \log(bx^2 + a) - \sqrt{-ab} c \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right)}{2ab}, \frac{ad \log(bx^2 + a) + 2\sqrt{ab} c \arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x, algorithm="fricas")

[Out] [1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*b)*x/a))/(a*b)]

giac [A] time = 0.40, size = 31, normalized size = 0.74

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \log(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x, algorithm="giac")

[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b

maple [A] time = 0.00, size = 32, normalized size = 0.76

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \ln(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x)`

[Out] $1/2*d*\ln(b*x^2+a)/b+c/(a*b)^{(1/2)}*\arctan(b*x/(a*b)^{(1/2)})$

maxima [A] time = 1.49, size = 31, normalized size = 0.74

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \log(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x, algorithm="maxima")`

[Out] $c*\arctan(b*x/\sqrt{a*b})/\sqrt{a*b} + 1/2*d*\log(b*x^2 + a)/b$

mupad [B] time = 2.13, size = 32, normalized size = 0.76

$$\frac{d \ln(bx^2 + a)}{2b} + \frac{c \operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(a + b*x^2)^3,x)`

[Out] $(d*\log(a + b*x^2))/(2*b) + (c*\operatorname{atan}((b^{(1/2)}*x)/a^{(1/2)}))/(a^{(1/2)}*b^{(1/2)})$

sympy [B] time = 0.30, size = 124, normalized size = 2.95

$$\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right) \log\left(x + \frac{2ab\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right) + \left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right) \log\left(x + \frac{2ab\left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(b*x**2+a)**3,x)`

[Out] $(d/(2*b) - c*\sqrt{-a*b**3}/(2*a*b**2))*\log(x + (2*a*b*(d/(2*b) - c*\sqrt{-a*b**3}/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*\sqrt{-a*b**3}/(2*a*b**2))*\log(x + (2*a*b*(d/(2*b) + c*\sqrt{-a*b**3}/(2*a*b**2)) - a*d)/(b*c))$

$$3.180 \quad \int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^n dx$$

Optimal. Leaf size=25

$$\frac{(a + bx + cx^2 + dx^3)^{n+1}}{n + 1}$$

[Out] (d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1588}

$$\frac{(a + bx + cx^2 + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^n dx = \frac{(a + bx + cx^2 + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{(a + x(b + x(c + dx)))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)

fricas [A] time = 0.84, size = 38, normalized size = 1.52

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)

giac [A] time = 0.29, size = 25, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 26, normalized size = 1.04

$$\frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x)

[Out] (d*x^3+c*x^2+b*x+a)^(n+1)/(n+1)

maxima [A] time = 0.67, size = 25, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)

mupad [B] time = 2.19, size = 54, normalized size = 2.16

$$(dx^3 + cx^2 + bx + a)^n \left(\frac{a}{n+1} + \frac{bx}{n+1} + \frac{cx^2}{n+1} + \frac{dx^3}{n+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x)

[Out] (a + b*x + c*x^2 + d*x^3)^n*(a/(n + 1) + (b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)

[Out] Timed out

$$3.181 \quad \int (b + 2cx + 3dx^2) (bx + cx^2 + dx^3)^n dx$$

Optimal. Leaf size=24

$$\frac{(bx + cx^2 + dx^3)^{n+1}}{n+1}$$

[Out] (d*x^3+c*x^2+b*x)^(1+n)/(1+n)

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1588}

$$\frac{(bx + cx^2 + dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]

[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 2cx + 3dx^2) (bx + cx^2 + dx^3)^n dx = \frac{(bx + cx^2 + dx^3)^{1+n}}{1+n}$$

Mathematica [A] time = 0.05, size = 21, normalized size = 0.88

$$\frac{(x(b + x(c + dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]

[Out] (x*(b + x*(c + d*x)))^(1 + n)/(1 + n)

fricas [A] time = 0.87, size = 36, normalized size = 1.50

$$\frac{(dx^3 + cx^2 + bx)(dx^3 + cx^2 + bx)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)

giac [A] time = 0.31, size = 24, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 34, normalized size = 1.42

$$\frac{(dx^2 + cx + b)x(dx^3 + cx^2 + bx)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x)

[Out] x*(d*x^2+c*x+b)/(n+1)*(d*x^3+c*x^2+b*x)^n

maxima [A] time = 0.63, size = 24, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)

mupad [B] time = 2.12, size = 46, normalized size = 1.92

$$\left(\frac{bx}{n+1} + \frac{cx^2}{n+1} + \frac{dx^3}{n+1}\right)(dx^3 + cx^2 + bx)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x)

[Out] ((b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(b*x + c*x^2 + d*x^3)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)

[Out] Timed out

$$3.182 \quad \int x^n (b + cx + dx^2)^n (b + 2cx + 3dx^2) dx$$

Optimal. Leaf size=25

$$\frac{x^{n+1} (b + cx + dx^2)^{n+1}}{n + 1}$$

[Out] $x^{(1+n)}*(d*x^2+c*x+b)^{(1+n)}/(1+n)$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1590}

$$\frac{x^{n+1} (b + cx + dx^2)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int x^n (b + cx + dx^2)^n (b + 2cx + 3dx^2) dx = \frac{x^{1+n} (b + cx + dx^2)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.96

$$\frac{x^{n+1}(b + x(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + x*(c + d*x))^(1 + n))/(1 + n)

fricas [A] time = 1.02, size = 35, normalized size = 1.40

$$\frac{(dx^3 + cx^2 + bx)(dx^2 + cx + b)^n x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)

giac [B] time = 0.40, size = 65, normalized size = 2.60

$$\frac{(dx^2 + cx + b)^n dx^3 x^n + (dx^2 + cx + b)^n cx^2 x^n + (dx^2 + cx + b)^n bxx^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="giac")

[Out] ((d*x^2 + c*x + b)^n*d*x^3*x^n + (d*x^2 + c*x + b)^n*c*x^2*x^n + (d*x^2 + c*x + b)^n*b*x*x^n)/(n + 1)

maple [A] time = 0.01, size = 26, normalized size = 1.04

$$\frac{x^{n+1} (dx^2 + cx + b)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x)

[Out] x^(n+1)*(d*x^2+c*x+b)^(n+1)/(n+1)

maxima [A] time = 0.86, size = 39, normalized size = 1.56

$$\frac{(dx^3 + cx^2 + bx)e^{(n \log(dx^2+cx+b)+n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n + 1)

mupad [B] time = 2.15, size = 51, normalized size = 2.04

$$\left(\frac{cx^n x^2}{n+1} + \frac{dx^n x^3}{n+1} + \frac{bxx^n}{n+1} \right) (dx^2 + cx + b)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x)

[Out] ((c*x^n*x^2)/(n + 1) + (d*x^n*x^3)/(n + 1) + (b*x*x^n)/(n + 1))*(b + c*x + d*x^2)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)

[Out] Timed out

$$3.183 \quad \int (b + 3dx^2) (a + bx + dx^3)^n dx$$

Optimal. Leaf size=20

$$\frac{(a + bx + dx^3)^{n+1}}{n + 1}$$

[Out] (d*x^3+b*x+a)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1588}

$$\frac{(a + bx + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]

[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 3dx^2) (a + bx + dx^3)^n dx = \frac{(a + bx + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{(a + bx + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]

[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)

fricas [A] time = 0.83, size = 28, normalized size = 1.40

$$\frac{(dx^3 + bx + a)(dx^3 + bx + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="fricas")

[Out] (d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)

giac [A] time = 0.28, size = 20, normalized size = 1.00

$$\frac{(dx^3 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="giac")

[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{(dx^3 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x)

[Out] (d*x^3+b*x+a)^(n+1)/(n+1)

maxima [A] time = 0.65, size = 20, normalized size = 1.00

$$\frac{(dx^3 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)

mupad [B] time = 2.14, size = 39, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{bx}{n+1} + \frac{dx^3}{n+1} \right) (dx^3 + bx + a)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^n,x)

[Out] (a/(n + 1) + (b*x)/(n + 1) + (d*x^3)/(n + 1))*(a + b*x + d*x^3)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)

[Out] Timed out

$$3.184 \quad \int (b + 3dx^2) (bx + dx^3)^n dx$$

Optimal. Leaf size=19

$$\frac{(bx + dx^3)^{n+1}}{n+1}$$

[Out] (d*x^3+b*x)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1588}

$$\frac{(bx + dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

[Out] (b*x + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :=> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 3dx^2) (bx + dx^3)^n dx = \frac{(bx + dx^3)^{1+n}}{1+n}$$

Mathematica [C] time = 0.07, size = 106, normalized size = 5.58

$$\frac{x(x(b + dx^2))^n \left(\frac{dx^2}{b} + 1\right)^{-n} \left(3d(n+1)x^2 {}_2F_1\left(-n, \frac{n+3}{2}; \frac{n+5}{2}; -\frac{dx^2}{b}\right) + b(n+3) {}_2F_1\left(-n, \frac{n+1}{2}; \frac{n+3}{2}; -\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

[Out] (x*(x*(b + d*x^2))^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 + n)/2, -((d*x^2)/b)])/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)

fricas [A] time = 0.91, size = 26, normalized size = 1.37

$$\frac{(dx^3 + bx)(dx^3 + bx)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="fricas")

[Out] (d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)

giac [A] time = 0.34, size = 19, normalized size = 1.00

$$\frac{(dx^3 + bx)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="giac")

[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 26, normalized size = 1.37

$$\frac{(dx^2 + b)x(dx^3 + bx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+b)*(d*x^3+b*x)^n,x)

[Out] x*(d*x^2+b)/(n+1)*(d*x^3+b*x)^n

maxima [A] time = 0.53, size = 19, normalized size = 1.00

$$\frac{(dx^3 + bx)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="maxima")

[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)

mupad [B] time = 2.13, size = 25, normalized size = 1.32

$$\frac{x(dx^3 + bx)^n(dx^2 + b)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + d*x^3)^n*(b + 3*d*x^2),x)

[Out] (x*(b*x + d*x^3)^n*(b + d*x^2))/(n + 1)

sympy [B] time = 11.34, size = 73, normalized size = 3.84

$$\begin{cases} \frac{bx(bx+dx^3)^n}{n+1} + \frac{dx^3(bx+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ \log(x) + \log\left(-i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) + \log\left(i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)

[Out] Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)**n/(n + 1), Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)*sqrt(1/d) + x), True))

$$3.185 \quad \int x^n (b + dx^2)^n (b + 3dx^2) dx$$

Optimal. Leaf size=22

$$\frac{x^{n+1} (b + dx^2)^{n+1}}{n + 1}$$

[Out] $x^{(1+n)}*(d*x^2+b)^{(1+n)}/(1+n)$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {449}

$$\frac{x^{n+1} (b + dx^2)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2), x]

[Out] $(x^{(1 + n)}*(b + d*x^2)^{(1 + n)})/(1 + n)$

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rubi steps

$$\int x^n (b + dx^2)^n (b + 3dx^2) dx = \frac{x^{1+n} (b + dx^2)^{1+n}}{1 + n}$$

Mathematica [C] time = 0.04, size = 108, normalized size = 4.91

$$\frac{x^{n+1} (b + dx^2)^n \left(\frac{dx^2}{b} + 1\right)^{-n} \left(3d(n+1)x^2 {}_2F_1\left(-n, \frac{n+3}{2}; \frac{n+5}{2}; -\frac{dx^2}{b}\right) + b(n+3) {}_2F_1\left(-n, \frac{n+1}{2}; \frac{n+3}{2}; -\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2), x]

[Out] $(x^{(1 + n)}*(b + d*x^2)^n*(b*(3 + n)*\text{Hypergeometric2F1}[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*\text{Hypergeometric2F1}[-n, (3 + n)/2, (5 + n)/2, -((d*x^2)/b)])/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)$

fricas [A] time = 1.07, size = 27, normalized size = 1.23

$$\frac{(dx^3 + bx)(dx^2 + b)^n x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b), x, algorithm="fricas")

[Out] $(d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)$

giac [A] time = 0.34, size = 39, normalized size = 1.77

$$\frac{(dx^2 + b)^n dx^3 x^n + (dx^2 + b)^n b x x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="giac")

[Out] ((d*x^2 + b)^n*d*x^3*x^n + (d*x^2 + b)^n*b*x*x^n)/(n + 1)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^{n+1} (dx^2 + b)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(d*x^2+b)^n*(3*d*x^2+b),x)

[Out] x^(n+1)*(d*x^2+b)^(n+1)/(n+1)

maxima [A] time = 1.21, size = 31, normalized size = 1.41

$$\frac{(dx^3 + bx)e^{(n \log(dx^2+b) + n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="maxima")

[Out] (d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)

mupad [B] time = 2.16, size = 26, normalized size = 1.18

$$\frac{x x^n (dx^2 + b)^n (dx^2 + b)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(b + d*x^2)^n*(b + 3*d*x^2),x)

[Out] (x*x^n*(b + d*x^2)^n*(b + d*x^2))/(n + 1)

sympy [B] time = 52.84, size = 76, normalized size = 3.45

$$\begin{cases} \frac{b x x^n (b + d x^2)^n}{n + 1} + \frac{d x^3 x^n (b + d x^2)^n}{n + 1} & \text{for } n \neq -1 \\ \log(x) + \log\left(-i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) + \log\left(i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)

[Out] Piecewise((b*x*x**n*(b + d*x**2)**n/(n + 1) + d*x**3*x**n*(b + d*x**2)**n/(n + 1), Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)*sqrt(1/d) + x), True))

$$3.186 \quad \int (2cx + 3dx^2) (a + cx^2 + dx^3)^n dx$$

Optimal. Leaf size=22

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1588}

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]

[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2cx + 3dx^2) (a + cx^2 + dx^3)^n dx = \frac{(a + cx^2 + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{(a + x^2(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]

[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)

fricas [A] time = 0.77, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2 + a)(dx^3 + cx^2 + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)

giac [A] time = 0.34, size = 22, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x)

[Out] (d*x^3+c*x^2+a)^(n+1)/(n+1)

maxima [A] time = 0.66, size = 22, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)

mupad [B] time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{cx^2}{n+1} + \frac{dx^3}{n+1}\right)(dx^3 + cx^2 + a)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x)

[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)

[Out] Timed out

$$3.187 \quad \int (2cx + 3dx^2) (cx^2 + dx^3)^n dx$$

Optimal. Leaf size=21

$$\frac{(cx^2 + dx^3)^{n+1}}{n+1}$$

[Out] (d*x^3+c*x^2)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1588}

$$\frac{(cx^2 + dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^n dx = \frac{(cx^2 + dx^3)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$\frac{(x^2(c + dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)

fricas [A] time = 0.80, size = 30, normalized size = 1.43

$$\frac{(dx^3 + cx^2)(dx^3 + cx^2)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)

giac [A] time = 0.31, size = 21, normalized size = 1.00

$$\frac{(dx^3 + cx^2)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{(dx + c)x^2(dx^3 + cx^2)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x)

[Out] (d*x^3+c*x^2)^n*x^2*(d*x+c)/(n+1)

maxima [A] time = 0.59, size = 21, normalized size = 1.00

$$\frac{(dx^3 + cx^2)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)

mupad [B] time = 2.16, size = 27, normalized size = 1.29

$$\frac{x^2(dx^3 + cx^2)^n(c + dx)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x)

[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)

sympy [A] time = 1.16, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1} + \frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2 \log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)

[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))

$$3.188 \quad \int x^n (cx + dx^2)^n (2cx + 3dx^2) dx$$

Optimal. Leaf size=24

$$\frac{x^{n+1} (cx + dx^2)^{n+1}}{n + 1}$$

[Out] $x^{(1+n)}*(d*x^2+c*x)^{(1+n)}/(1+n)$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1584, 763}

$$\frac{x^{n+1} (cx + dx^2)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2), x]

[Out] $(x^{(1 + n)}*(c*x + d*x^2)^{(1 + n)})/(1 + n)$

Rule 763

Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p + 2), 0] && NeQ[m + 2*p + 2, 0]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^n (cx + dx^2)^n (2cx + 3dx^2) dx &= \int x^{1+n} (2c + 3dx) (cx + dx^2)^n dx \\ &= \frac{x^{1+n} (cx + dx^2)^{1+n}}{1 + n} \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{x^{n+1} (x(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2), x]

[Out] $(x^{(1 + n)}*(x*(c + d*x))^{(1 + n)})/(1 + n)$

fricas [A] time = 0.91, size = 31, normalized size = 1.29

$$\frac{(dx^3 + cx^2)(dx^2 + cx)^n x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")

[Out] (d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)

giac [B] time = 0.46, size = 51, normalized size = 2.12

$$\frac{dx^3 x^n e^{(n \log(dx+c)+n \log(x))} + cx^2 x^n e^{(n \log(dx+c)+n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="giac")

[Out] (d*x^3*x^n*e^(n*log(d*x + c) + n*log(x)) + c*x^2*x^n*e^(n*log(d*x + c) + n*log(x)))/(n + 1)

maple [A] time = 0.00, size = 28, normalized size = 1.17

$$\frac{(dx + c)x^{n+2}(dx^2 + cx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x)

[Out] (d*x^2+c*x)^n*x^(2+n)*(d*x+c)/(n+1)

maxima [A] time = 0.83, size = 32, normalized size = 1.33

$$\frac{(dx^3 + cx^2)e^{(n \log(dx+c)+2n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")

[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)

mupad [B] time = 2.20, size = 28, normalized size = 1.17

$$\frac{x^n x^2 (dx^2 + cx)^n (c + dx)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x)

[Out] (x^n*x^2*(c*x + d*x^2)^n*(c + d*x))/(n + 1)

sympy [A] time = 6.17, size = 56, normalized size = 2.33

$$\begin{cases} \frac{cx^2 x^n (cx+dx^2)^n}{n+1} + \frac{dx^3 x^n (cx+dx^2)^n}{n+1} & \text{for } n \neq -1 \\ 2 \log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)

[Out] Piecewise((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c*x + d*x**2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))

$$3.189 \quad \int x^{2n}(c + dx)^n (2cx + 3dx^2) dx$$

Optimal. Leaf size=22

$$\frac{x^{2(n+1)}(c + dx)^{n+1}}{n + 1}$$

[Out] $x^{(2+2*n)}*(d*x+c)^{(1+n)}/(1+n)$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {845}

$$\frac{x^{2(n+1)}(c + dx)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(2*n)}*(c + d*x)^n*(2*c*x + 3*d*x^2), x]$

[Out] $(x^{(2*(1 + n))}*(c + d*x)^{(1 + n)})/(1 + n)$

Rule 845

$\text{Int}[(x_)^{(m_*)}*((f_) + (g_)*(x_))^{(n_*)}*((b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(c*x^{(m + 2)}*(f + g*x)^{(n + 1)})/(g*(m + n + 3)), x] /; \text{FreeQ}[\{b, c, f, g, m, n\}, x] \ \&\& \ \text{EqQ}[c*f*(m + 2) - b*g*(m + n + 3), 0] \ \&\& \ \text{NeQ}[m + n + 3, 0]$

Rubi steps

$$\int x^{2n}(c + dx)^n (2cx + 3dx^2) dx = \frac{x^{2(1+n)}(c + dx)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x^{2n+2}(c + dx)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^{(2*n)}*(c + d*x)^n*(2*c*x + 3*d*x^2), x]$

[Out] $(x^{(2 + 2*n)}*(c + d*x)^{(1 + n)})/(1 + n)$

fricas [A] time = 0.98, size = 29, normalized size = 1.32

$$\frac{(dx^3 + cx^2)(dx + c)^n x^{2n}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{(2*n)}*(d*x+c)^n*(3*d*x^2+2*c*x), x, \text{algorithm}=\text{"fricas"})$

[Out] $(d*x^3 + c*x^2)*(d*x + c)^n*x^{(2*n)}/(n + 1)$

giac [A] time = 0.31, size = 41, normalized size = 1.86

$$\frac{(dx + c)^n dx^3 x^{2n} + (dx + c)^n cx^2 x^{2n}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="giac")

[Out] ((d*x + c)^n*d*x^3*x^(2*n) + (d*x + c)^n*c*x^2*x^(2*n))/(n + 1)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^{2n+2} (dx + c)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x)

[Out] x^(2+2*n)*(d*x+c)^(n+1)/(n+1)

maxima [A] time = 0.90, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2)e^{(n \log(dx+c) + 2n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")

[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)

mupad [B] time = 2.19, size = 26, normalized size = 1.18

$$\frac{x^{2n} x^2 (c + dx)^n (c + dx)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n)*(2*c*x + 3*d*x^2)*(c + d*x)^n,x)

[Out] (x^(2*n))*x^2*(c + d*x)^n*(c + d*x)/(n + 1)

sympy [A] time = 6.15, size = 53, normalized size = 2.41

$$\begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for } n \neq -1 \\ 2 \log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)

[Out] Piecewise((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)*(c + d*x)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))

$$3.190 \quad \int x(2c + 3dx) (a + cx^2 + dx^3)^n dx$$

Optimal. Leaf size=22

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1588}

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]

[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x(2c + 3dx) (a + cx^2 + dx^3)^n dx = \frac{(a + cx^2 + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{(a + x^2(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]

[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)

fricas [A] time = 1.23, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2 + a)(dx^3 + cx^2 + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)

giac [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x)

[Out] 1/(n+1)*(d*x^3+c*x^2+a)^(n+1)

maxima [A] time = 0.75, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2 + a)(dx^3 + cx^2 + a)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)

mupad [B] time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{cx^2}{n+1} + \frac{dx^3}{n+1}\right)(dx^3 + cx^2 + a)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x)

[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)

[Out] Timed out

$$3.191 \quad \int x(2c + 3dx) (cx^2 + dx^3)^n dx$$

Optimal. Leaf size=21

$$\frac{(cx^2 + dx^3)^{n+1}}{n+1}$$

[Out] (d*x^3+c*x^2)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1588}

$$\frac{(cx^2 + dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x(2c + 3dx) (cx^2 + dx^3)^n dx = \frac{(cx^2 + dx^3)^{1+n}}{1+n}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.90

$$\frac{(x^2(c + dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]

[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)

fricas [A] time = 0.76, size = 30, normalized size = 1.43

$$\frac{(dx^3 + cx^2)(dx^3 + cx^2)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)

giac [A] time = 0.40, size = 21, normalized size = 1.00

$$\frac{(dx^3 + cx^2)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{(dx + c)x^2(dx^3 + cx^2)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x)

[Out] (d*x+c)/(n+1)*x^2*(d*x^3+c*x^2)^n

maxima [A] time = 0.93, size = 32, normalized size = 1.52

$$\frac{(dx^3 + cx^2)e^{(n \log(dx+c)+2n \log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)

mupad [B] time = 2.15, size = 27, normalized size = 1.29

$$\frac{x^2(dx^3 + cx^2)^n(c + dx)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x)

[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)

sympy [A] time = 1.14, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1} + \frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2 \log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)

[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))

$$3.192 \quad \int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=21

$$\frac{1}{8} (a + bx + cx^2 + dx^3)^8$$

[Out] 1/8*(d*x^3+c*x^2+b*x+a)^8

Rubi [A] time = 0.13, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1588}

$$\frac{1}{8} (a + bx + cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]

[Out] (a + b*x + c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^7 dx = \frac{1}{8} (a + bx + cx^2 + dx^3)^8$$

Mathematica [B] time = 0.15, size = 143, normalized size = 6.81

$$\frac{1}{8} x(b+x(c+dx)) (8a^7 + 28a^6x(b+x(c+dx)) + 56a^5x^2(b+x(c+dx))^2 + 70a^4x^3(b+x(c+dx))^3 + 56a^3x^4(b+x(c+dx))^4 + 28a^2x^5(b+x(c+dx))^5 + 8a^2x^6(b+x(c+dx))^6 + x^7(b+x(c+dx))^7) / 8$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]

[Out] (x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5*x^2*(b + x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^3*x^4*(b + x*(c + d*x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*x^6*(b + x*(c + d*x))^6 + x^7*(b + x*(c + d*x))^7)/8

fricas [B] time = 0.56, size = 1956, normalized size = 93.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3 + 7*x^21*d^6*c*b + x^21*d^7*a + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c*b^2 + 21*x^19*d^5*c^2*a + 7*x^19*d^6*b*a + 7/2*x^18*d^2*c^6 + 35*x

$$\begin{aligned}
& ^{18}d^3c^4b + 105/2x^{18}d^4c^2b^2 + 7x^{18}d^5b^3 + 35x^{18}d^4c^3a \\
& + 42x^{18}d^5c^2ba + 7/2x^{18}d^6a^2 + x^{17}d^7c^7 + 21x^{17}d^2c^5b + \\
& 70x^{17}d^3c^3b^2 + 35x^{17}d^4c^2b^3 + 35x^{17}d^3c^4a + 105x^{17}d^4c^2ba \\
& + 21x^{17}d^5b^2a + 21x^{17}d^5c^2a^2 + 1/8x^{16}c^8 + 7x^{16}d^6c^6b \\
& + 105/2x^{16}d^2c^4b^2 + 70x^{16}d^3c^2b^3 + 35/4x^{16}d^4b^4 + 2 \\
& 1x^{16}d^2c^5a + 140x^{16}d^3c^3ba + 105x^{16}d^4c^2ba + 105/2x^{16} \\
& d^4c^2a^2 + 21x^{16}d^5b^2a + x^{15}c^7b + 21x^{15}d^5c^5b^2 + 70x^{15} \\
& d^2c^3b^3 + 35x^{15}d^3c^2b^4 + 7x^{15}d^6c^6a + 105x^{15}d^2c^4ba + \\
& 210x^{15}d^3c^2b^2a + 35x^{15}d^4b^3a + 70x^{15}d^3c^3a^2 + 105x^{15} \\
& d^4c^2ba^2 + 7x^{15}d^5a^3 + 7/2x^{14}c^6b^2 + 35x^{14}d^4c^4b^3 + 105/ \\
& 2x^{14}d^2c^2b^4 + 7x^{14}d^3b^5 + x^{14}c^7a + 42x^{14}d^4c^5ba + 210x \\
& ^{14}d^2c^3b^2a + 140x^{14}d^3c^2ba + 105/2x^{14}d^2c^4a^2 + 210x^{14} \\
& d^3c^2ba^2 + 105/2x^{14}d^4b^2a^2 + 35x^{14}d^4c^2a^3 + 7x^{13}c^5b^3 \\
& + 35x^{13}d^3c^3b^4 + 21x^{13}d^2c^2b^5 + 7x^{13}c^6ba + 105x^{13}d^4c^4b^2a \\
& + 210x^{13}d^2c^2b^3a + 35x^{13}d^3b^4a + 21x^{13}d^5c^5a^2 + \\
& 210x^{13}d^2c^3ba^2 + 210x^{13}d^3c^2ba^2 + 70x^{13}d^3c^2a^3 + 35 \\
& x^{13}d^4ba^3 + 35/4x^{12}c^4b^4 + 21x^{12}d^4c^2b^5 + 7/2x^{12}d^2b^6 \\
& + 21x^{12}c^5b^2a + 140x^{12}d^4c^3b^3a + 105x^{12}d^2c^2b^4a + 7/2x^{12} \\
& 2c^6a^2 + 105x^{12}d^4c^4ba^2 + 315x^{12}d^2c^2b^2a^2 + 70x^{12}d^3b^3a^2 \\
& + 70x^{12}d^2c^3a^3 + 140x^{12}d^3c^2ba^3 + 35/4x^{12}d^4a^4 + 7 \\
& x^{11}c^3b^5 + 7x^{11}d^4c^3ba + 35x^{11}c^4b^3a + 105x^{11}d^4c^2b^4a + \\
& 21x^{11}d^2b^5a + 21x^{11}c^5ba^2 + 210x^{11}d^4c^3b^2a^2 + 210x^{11}d^2c^2b^3a^2 \\
& + 35x^{11}d^4c^4a^3 + 210x^{11}d^2c^2ba^3 + 70x^{11}d^3b^2a^3 + 35x^{11}d^3c^2a^4 \\
& + 7/2x^{10}c^2b^6 + x^{10}d^5b^7 + 35x^{10}c^3b^4a + 42x^{10}d^4c^5ba + 105/2x^{10} \\
& c^4b^2a^2 + 210x^{10}d^4c^2b^3a^2 + 105/2x^{10}d^2b^4a^2 + 7x^{10}c^5a^3 + 140x^{10} \\
& d^4c^3ba^3 + 210x^{10}d^2c^2b^2a^3 + 105/2x^{10}d^2c^2a^4 + 35x^{10}d^3ba^4 + x^9c^2b^7 \\
& + 21x^9c^2b^5a + 7x^9d^5b^6a + 70x^9c^3b^3a^2 + 105x^9d^4c^2b^4a^2 + \\
& 35x^9c^4ba^3 + 210x^9d^4c^2b^2a^3 + 70x^9d^2b^3a^3 + 35x^9d^4c^3a^4 + 105x^9 \\
& d^2c^2ba^4 + 7x^9d^3a^5 + 1/8x^8b^8 + 7x^8c^2b^6a + 105/2x^8c^2b^4a^2 + 21x^8 \\
& d^4b^5a^2 + 70x^8c^3b^2a^3 + 140x^8d^4c^2b^3a^3 + 35/4x^8c^4a^4 + 105x^8d^4c^2ba^4 \\
& + 105/2x^8d^2c^2a^5 + x^7b^7a + 21x^7c^2b^5a^2 + 70x^7c^2b^3a^3 + 35 \\
& x^7d^4b^4a^3 + 35x^7c^3ba^4 + 105x^7d^4c^2ba^4 + 21x^7d^4c^2a^5 + 21x^7d^2b^2a^5 \\
& + 7/2x^6b^6a^2 + 35x^6c^2b^4a^3 + 105/2x^6c^2b^2a^4 + 35x^6d^4b^3a^4 + 7x^6 \\
& c^3a^5 + 42x^6d^4c^2ba^5 + 7/2x^6d^2a^6 + 7x^5b^5a^3 + 35x^5c^2b^3a^4 + 21x^5 \\
& c^2b^2a^5 + 21x^5d^4b^2a^5 + 7x^5d^4c^2a^6 + 35/4x^4b^4a^4 + 21x^4c^2b^2a^5 + 7/2 \\
& x^4c^2a^6 + 7x^4d^4b^2a^6 + 7x^3b^3a^5 + 7x^3c^2ba^6 + x^3d^4a^7 + 7/2x^2b^2a^6 \\
& + x^2c^2a^7 + xba^7
\end{aligned}$$

giac [B] time = 0.30, size = 160, normalized size = 7.62

$$\frac{1}{8}(dx^3 + cx^2 + bx)^8 + (dx^3 + cx^2 + bx)^7 a + \frac{7}{2}(dx^3 + cx^2 + bx)^6 a^2 + 7(dx^3 + cx^2 + bx)^5 a^3 + \frac{35}{4}(dx^3 + cx^2 + bx)^4 a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="giac")

[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8 + (d*x^3 + c*x^2 + b*x)^7*a + 7/2*(d*x^3 + c*x^2 + b*x)^6*a^2 + 7*(d*x^3 + c*x^2 + b*x)^5*a^3 + 35/4*(d*x^3 + c*x^2 + b*x)^4*a^4 + 7*(d*x^3 + c*x^2 + b*x)^3*a^5 + 7/2*(d*x^3 + c*x^2 + b*x)^2*a^6 + (d*x^3 + c*x^2 + b*x)*a^7

maple [B] time = 0.00, size = 25686, normalized size = 1223.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x)

[Out] result too large to display

maxima [A] time = 0.74, size = 19, normalized size = 0.90

$$\frac{1}{8} (dx^3 + cx^2 + bx + a)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2 + b*x + a)^8

mupad [B] time = 2.98, size = 1576, normalized size = 75.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x)

[Out] $x^{12} * ((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + (35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*a*b^2*c^5 + 21*b^5*c^2*d + 70*a^2*b^3*d^3 + 70*a^3*c^3*d^2 + 315*a^2*b^2*c^2*d^2 + 140*a*b^3*c^3*d + 105*a*b^4*c*d^2 + 105*a^2*b*c^4*d + 140*a^3*b*c*d^3) + x^{11} * (7*b^5*c^3 + 35*a*b^3*c^4 + 21*a^2*b*c^5 + 21*a*b^5*d^2 + 35*a^3*c^4*d + 35*a^4*c*d^3 + 70*a^3*b^2*d^3 + 7*b^6*c*d + 210*a^2*b^2*c^3*d + 210*a^2*b^3*c*d^2 + 210*a^3*b*c^2*d^2 + 105*a*b^4*c^2*d) + x^{13} * (7*b^3*c^5 + 35*a*b^4*d^3 + 35*a^3*b*d^4 + 21*a^2*c^5*d + 35*b^4*c^3*d + 21*b^5*c*d^2 + 70*a^3*c^2*d^3 + 7*a*b*c^6 + 210*a*b^3*c^2*d^2 + 210*a^2*b*c^3*d^2 + 210*a^2*b^2*c*d^3 + 105*a*b^2*c^4*d) + x^5 * (7*a^3*b^5 + 35*a^4*b^3*c + 21*a^5*b*c^2 + 21*a^5*b^2*d + 7*a^6*c*d) + x^{19} * (7*c^5*d^3 + 21*a*c^2*d^5 + 35*b*c^3*d^4 + 21*b^2*c*d^5 + 7*a*b*d^6) + x^8 * (b^8/8 + (35*a^4*c^4)/4 + 21*a^2*b^5*d + 21*a^5*c*d^2 + (105*a^2*b^4*c^2)/2 + 70*a^3*b^2*c^3 + (105*a^4*b^2*d^2)/2 + 7*a*b^6*c + 140*a^3*b^3*c*d + 105*a^4*b*c^2*d) + x^9 * (b^7*c + 7*a^5*d^3 + 21*a*b^5*c^2 + 35*a^3*b*c^4 + 35*a^4*c^3*d + 70*a^2*b^3*c^3 + 70*a^3*b^3*d^2 + 7*a*b^6*d + 210*a^3*b^2*c^2*d + 105*a^2*b^4*c*d + 105*a^4*b*c*d^2) + x^{16} * (c^8/8 + (35*b^4*d^4)/4 + 21*a^2*b*d^5 + 21*a*c^5*d^2 + (105*a^2*c^2*d^4)/2 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d + 140*a*b*c^3*d^3 + 105*a*b^2*c*d^4) + x^{10} * (b^7*d + 7*a^3*c^5 + (7*b^6*c^2)/2 + 35*a*b^4*c^3 + 35*a^4*b*d^3 + (105*a^2*b^2*c^4)/2 + (105*a^2*b^4*d^2)/2 + (105*a^4*c^2*d^2)/2 + 210*a^2*b^3*c^2*d + 210*a^3*b^2*c*d^2 + 42*a*b^5*c*d + 140*a^3*b*c^3*d) + x^{15} * (b*c^7 + 7*a^3*d^5 + 35*a*b^3*d^4 + 21*b^2*c^5*d + 35*b^4*c*d^3 + 70*a^2*c^3*d^3 + 70*b^3*c^3*d^2 + 7*a*c^6*d + 210*a*b^2*c^2*d^3 + 105*a*b*c^4*d^2 + 105*a^2*b*c*d^4) + x^{14} * (a*c^7 + (7*b^2*c^6)/2 + 7*b^5*d^3 + 35*a^3*c*d^4 + 35*b^3*c^4*d + (105*a^2*b^2*d^4)/2 + (105*a^2*c^4*d^2)/2 + (105*b^4*c^2*d^2)/2 + 210*a*b^2*c^3*d^2 + 210*a^2*b*c^2*d^3 + 42*a*b*c^5*d + 140*a*b^3*c*d^3) + x^4 * ((35*a^4*b^4)/4 + (7*a^6*c^2)/2 + 21*a^5*b^2*c + 7*a^6*b*d) + x^{20} * ((7*b^2*d^6)/2 + (35*c^4*d^4)/4 + 21*b*c^2*d^5 + 7*a*c*d^6) + x^6 * ((7*a^2*b^6)/2 + 7*a^5*c^3 + (7*a^6*d^2)/2 + 35*a^3*b^4*c + 35*a^4*b^3*d + (105*a^4*b^2*c^2)/2 + 42*a^5*b*c*d) + x^7 * (a*b^7 + 21*a^2*b^5*c + 35*a^4*b*c^3 + 35*a^3*b^4*d + 21*a^5*b*d^2 + 21*a^5*c^2*d + 70*a^3*b^3*c^2 + 105*a^4*b^2*c*d) + x^{18} * ((7*a^2*d^6)/2 + 7*b^3*d^5 + (7*c^6*d^2)/2 + 35*a*c^3*d^4 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2 + 42*a*b*c*d^5) + x^{17} * (c^7*d + 21*a*b^2*d^5 + 35*a*c^4*d^3 + 21*a^2*c*d^5 + 21*b*c^5*d^2 + 35*b^3*c*d^4 + 70*b^2*c^3*d^3 + 105*a*b*c^2*d^4) + x^3 * (a^7*d + 7*a^5*b^3 + 7*a^6*b*c) + (d^8*x^24)/8 + x^2 * (a^7*c + (7*a^6*b^2)/2) + c*d^7*x^23 + d^5*x^21*(a*d^2 + 7*c^3 + 7*b*c*d) + (d^6*x^22*(2*b*d + 7*c^2))/2 + a^7*b*x$

sympy [B] time = 0.42, size = 1771, normalized size = 84.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)

[Out] $a^{**7}b*x + c*d^{**7}x^{**23} + d^{**8}x^{**24}/8 + x^{**22}*(b*d^{**7} + 7*c^{**2}d^{**6}/2) + x^{**21}*(a*d^{**7} + 7*b*c*d^{**6} + 7*c^{**3}d^{**5}) + x^{**20}*(7*a*c*d^{**6} + 7*b^{**2}d^{**6}/2 + 21*b*c^{**2}d^{**5} + 35*c^{**4}d^{**4}/4) + x^{**19}*(7*a*b*d^{**6} + 21*a*c^{**2}d^{**5} + 21*b^{**2}c*d^{**5} + 35*b*c^{**3}d^{**4} + 7*c^{**5}d^{**3}) + x^{**18}*(7*a^{**2}d^{**6}/2 + 42*a*b*c*d^{**5} + 35*a*c^{**3}d^{**4} + 7*b^{**3}d^{**5} + 105*b^{**2}c^{**2}d^{**4}/2 + 35*b*c^{**4}d^{**3} + 7*c^{**6}d^{**2}/2) + x^{**17}*(21*a^{**2}c*d^{**5} + 21*a*b^{**2}d^{**5} + 105*a*b*c^{**2}d^{**4} + 35*a*c^{**4}d^{**3} + 35*b^{**3}c*d^{**4} + 70*b^{**2}c^{**3}d^{**3} + 21*b*c^{**5}d^{**2} + c^{**7}d) + x^{**16}*(21*a^{**2}b*d^{**5} + 105*a^{**2}c^{**2}d^{**4}/2 + 105*a*b^{**2}c*d^{**4} + 140*a*b*c^{**3}d^{**3} + 21*a*c^{**5}d^{**2} + 35*b^{**4}d^{**4}/4 + 70*b^{**3}c^{**2}d^{**3} + 105*b^{**2}c^{**4}d^{**2}/2 + 7*b*c^{**6}d + c^{**8}/8) + x^{**15}*(7*a^{**3}d^{**5} + 105*a^{**2}b*c*d^{**4} + 70*a^{**2}c^{**3}d^{**3} + 35*a*b^{**3}d^{**4} + 210*a*b^{**2}c^{**2}d^{**3} + 105*a*b*c^{**4}d^{**2} + 7*a*c^{**6}d + 35*b^{**4}c*d^{**3} + 70*b^{**3}c^{**3}d^{**2} + 21*b^{**2}c^{**5}d + b*c^{**7}) + x^{**14}*(35*a^{**3}c*d^{**4} + 105*a^{**2}b^{**2}d^{**4}/2 + 210*a^{**2}b*c^{**2}d^{**3} + 105*a^{**2}c^{**4}d^{**2}/2 + 140*a*b^{**3}c*d^{**3} + 210*a*b^{**2}c^{**3}d^{**2} + 42*a*b*c^{**5}d + a*c^{**7} + 7*b^{**5}d^{**3} + 105*b^{**4}c^{**2}d^{**2}/2 + 35*b^{**3}c^{**4}d + 7*b^{**2}c^{**6}/2) + x^{**13}*(35*a^{**3}b*d^{**4} + 70*a^{**3}c^{**2}d^{**3} + 210*a^{**2}b^{**2}c*d^{**3} + 210*a^{**2}b*c^{**3}d^{**2} + 21*a^{**2}c^{**5}d + 35*a*b^{**4}d^{**3} + 210*a*b^{**3}c^{**2}d^{**2} + 105*a*b^{**2}c^{**4}d + 7*a*b*c^{**6} + 21*b^{**5}c*d^{**2} + 35*b^{**4}c^{**3}d + 7*b^{**3}c^{**5}) + x^{**12}*(35*a^{**4}d^{**4}/4 + 140*a^{**3}b*c*d^{**3} + 70*a^{**3}c^{**3}d^{**2} + 70*a^{**2}b^{**3}d^{**3} + 315*a^{**2}b^{**2}c^{**2}d^{**2} + 105*a^{**2}b*c^{**4}d + 7*a^{**2}c^{**6}/2 + 105*a*b^{**4}c*d^{**2} + 140*a*b^{**3}c^{**3}d + 21*a*b^{**2}c^{**5} + 7*b^{**6}d^{**2}/2 + 21*b^{**5}c^{**2}d + 35*b^{**4}c^{**4}/4) + x^{**11}*(35*a^{**4}c*d^{**3} + 70*a^{**3}b^{**2}d^{**3} + 210*a^{**3}b*c^{**2}d^{**2} + 35*a^{**3}c^{**4}d + 210*a^{**2}b^{**3}c*d^{**2} + 210*a^{**2}b^{**2}c^{**3}d + 21*a^{**2}b*c^{**5} + 21*a*b^{**5}d^{**2} + 105*a*b^{**4}c^{**2}d + 35*a*b^{**3}c^{**4} + 7*b^{**6}c*d + 7*b^{**5}c^{**3}) + x^{**10}*(35*a^{**4}b*d^{**3} + 105*a^{**4}c^{**2}d^{**2}/2 + 210*a^{**3}b^{**2}c*d^{**2} + 140*a^{**3}b*c^{**3}d + 7*a^{**3}c^{**5} + 105*a^{**2}b^{**4}d^{**2}/2 + 210*a^{**2}b^{**3}c^{**2}d + 105*a^{**2}b^{**2}c^{**4}/2 + 42*a*b^{**5}c*d + 35*a*b^{**4}c^{**3} + b^{**7}d + 7*b^{**6}c^{**2}/2) + x^{**9}*(7*a^{**5}d^{**3} + 105*a^{**4}b*c*d^{**2} + 35*a^{**4}c^{**3}d + 70*a^{**3}b^{**3}d^{**2} + 210*a^{**3}b^{**2}c^{**2}d + 35*a^{**3}b*c^{**4} + 105*a^{**2}b^{**4}c*d + 70*a^{**2}b^{**3}c^{**3} + 7*a*b^{**6}d + 21*a*b^{**5}c^{**2} + b^{**7}c) + x^{**8}*(21*a^{**5}c*d^{**2} + 105*a^{**4}b^{**2}d^{**2}/2 + 105*a^{**4}b*c^{**2}d + 35*a^{**4}c^{**4}/4 + 140*a^{**3}b^{**3}c*d + 70*a^{**3}b^{**2}c^{**3} + 21*a^{**2}b^{**5}d + 105*a^{**2}b^{**4}c^{**2}/2 + 7*a*b^{**6}c + b^{**8}/8) + x^{**7}*(21*a^{**5}b*d^{**2} + 21*a^{**5}c^{**2}d + 105*a^{**4}b^{**2}c*d + 35*a^{**4}b*c^{**3} + 35*a^{**3}b^{**4}d + 70*a^{**3}b^{**3}c^{**2} + 21*a^{**2}b^{**5}c + a*b^{**7}) + x^{**6}*(7*a^{**6}d^{**2}/2 + 42*a^{**5}b*c*d + 7*a^{**5}c^{**3} + 35*a^{**4}b^{**3}d + 105*a^{**4}b^{**2}c^{**2}/2 + 35*a^{**3}b^{**4}c + 7*a^{**2}b^{**6}/2) + x^{**5}*(7*a^{**6}c*d + 21*a^{**5}b^{**2}d + 21*a^{**5}b*c^{**2} + 35*a^{**4}b^{**3}c + 7*a^{**3}b^{**5}) + x^{**4}*(7*a^{**6}b*d + 7*a^{**6}c^{**2}/2 + 21*a^{**5}b^{**2}c + 35*a^{**4}b^{**4}/4) + x^{**3}*(a^{**7}d + 7*a^{**6}b*c + 7*a^{**5}b^{**3}) + x^{**2}*(a^{**7}c + 7*a^{**6}b^{**2}/2)$

$$3.193 \quad \int (b + 2cx + 3dx^2) (bx + cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=20

$$\frac{1}{8} (bx + cx^2 + dx^3)^8$$

[Out] 1/8*(d*x^3+c*x^2+b*x)^8

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1588}

$$\frac{1}{8} (bx + cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]

[Out] (b*x + c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 2cx + 3dx^2) (bx + cx^2 + dx^3)^7 dx = \frac{1}{8} (bx + cx^2 + dx^3)^8$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.90

$$\frac{1}{8} x^8 (b + x(c + dx))^8$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]

[Out] (x^8*(b + x*(c + d*x))^8)/8

fricas [B] time = 0.50, size = 496, normalized size = 24.80

$$\frac{1}{8} x^{24} d^8 + x^{23} d^7 c + \frac{7}{2} x^{22} d^6 c^2 + x^{22} d^7 b + 7 x^{21} d^5 c^3 + 7 x^{21} d^6 c b + \frac{35}{4} x^{20} d^4 c^4 + 21 x^{20} d^5 c^2 b + \frac{7}{2} x^{20} d^6 b^2 + 7 x^{19} d^3 c^5 + 35 x^{19} d^4 c^3 b + 21 x^{19} d^5 c^2 b^2 + \frac{7}{2} x^{18} d^2 c^6 + 35 x^{18} d^3 c^4 b + 105/2 x^{18} d^4 c^2 b^2 + 7 x^{18} d^5 b^3 + x^{17} d^7 c^2 + 21 x^{17} d^2 c^5 b + 70 x^{17} d^3 c^3 b^2 + 35 x^{17} d^4 c b^3 + 1/8 x^{16} c^8 + 7 x^{16} d^6 c^2 b + 105/2 x^{16} d^2 c^4 b^2 + 70 x^{16} d^3 c^2 b^3 + 35/4 x^{15} d^5 c^2 b^2 + 7 x^{15} d^6 c b^3 + 7/2 x^{14} d^4 c^3 b^2 + 7/2 x^{14} d^5 c^2 b^3 + 7/2 x^{13} d^3 c^4 b^2 + 7/2 x^{13} d^4 c^3 b^3 + 7/2 x^{12} d^2 c^5 b^2 + 7/2 x^{12} d^3 c^4 b^3 + 7/2 x^{11} d^4 c^2 b^3 + 7/2 x^{11} d^5 c b^4 + 7/2 x^{10} d^3 c^3 b^3 + 7/2 x^{10} d^4 c^2 b^4 + 7/2 x^9 d^2 c^4 b^3 + 7/2 x^9 d^3 c^3 b^4 + 7/2 x^8 d^4 c^2 b^4 + 7/2 x^8 d^5 c b^5 + 7/2 x^7 d^3 c^4 b^4 + 7/2 x^7 d^4 c^3 b^5 + 7/2 x^6 d^2 c^5 b^4 + 7/2 x^6 d^3 c^4 b^5 + 7/2 x^5 d^4 c^3 b^5 + 7/2 x^5 d^5 c^2 b^6 + 7/2 x^4 d^3 c^4 b^5 + 7/2 x^4 d^4 c^3 b^6 + 7/2 x^3 d^2 c^5 b^5 + 7/2 x^3 d^3 c^4 b^6 + 7/2 x^2 d^4 c^3 b^6 + 7/2 x^2 d^5 c^2 b^7 + 7/2 x d^3 c^4 b^6 + 7/2 x d^4 c^3 b^7 + 7/2 x d^5 c^2 b^8 + 7/2 d^3 c^4 b^7 + 7/2 d^4 c^3 b^8 + 7/2 d^5 c^2 b^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3 + 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c^2*b^2 + 7/2*x^18*d^2*c^6 + 35*x^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d^7*c^2 + 21*x^17*d^2*c^5*b + 70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*c^8 + 7*x^16*d^6*c^2*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*x^15*d^5*c^2*b^2 + 7*x^15*d^6*c*b^3 + 7/2*x^14*d^4*c^3*b^2 + 7/2*x^14*d^5*c^2*b^3 + 7/2*x^13*d^3*c^4*b^2 + 7/2*x^13*d^4*c^3*b^3 + 7/2*x^12*d^2*c^5*b^2 + 7/2*x^12*d^3*c^4*b^3 + 7/2*x^11*d^4*c^2*b^3 + 7/2*x^11*d^5*c*b^4 + 7/2*x^10*d^3*c^3*b^3 + 7/2*x^10*d^4*c^2*b^4 + 7/2*x^9*d^2*c^4*b^3 + 7/2*x^9*d^3*c^3*b^4 + 7/2*x^8*d^4*c^2*b^4 + 7/2*x^8*d^5*c*b^5 + 7/2*x^7*d^3*c^4*b^4 + 7/2*x^7*d^4*c^3*b^5 + 7/2*x^6*d^2*c^5*b^4 + 7/2*x^6*d^3*c^4*b^5 + 7/2*x^5*d^4*c^3*b^5 + 7/2*x^5*d^5*c^2*b^6 + 7/2*x^4*d^3*c^4*b^5 + 7/2*x^4*d^4*c^3*b^6 + 7/2*x^3*d^2*c^5*b^5 + 7/2*x^3*d^3*c^4*b^6 + 7/2*x^2*d^4*c^3*b^6 + 7/2*x^2*d^5*c^2*b^7 + 7/2*x*d^3*c^4*b^6 + 7/2*x*d^4*c^3*b^7 + 7/2*x*d^5*c^2*b^8 + 7/2*d^3*c^4*b^7 + 7/2*d^4*c^3*b^8 + 7/2*d^5*c^2*b^9

$$x^{16}d^4b^4 + x^{15}c^7b + 21x^{15}d^5c^5b^2 + 70x^{15}d^2c^3b^3 + 35x^{15}d^3c^6b^4 + 7/2x^{14}c^6b^2 + 35x^{14}d^4c^4b^3 + 105/2x^{14}d^2c^2b^4 + 7x^{14}d^3b^5 + 7x^{13}c^5b^3 + 35x^{13}d^4c^3b^4 + 21x^{13}d^2c^5b^5 + 35/4x^{12}c^4b^4 + 21x^{12}d^2c^2b^5 + 7/2x^{12}d^2b^6 + 7x^{11}c^3b^5 + 7x^{11}d^4c^6b^6 + 7/2x^{10}c^2b^6 + x^{10}d^4b^7 + x^9c^4b^7 + 1/8x^8b^8$$

giac [A] time = 0.31, size = 18, normalized size = 0.90

$$\frac{1}{8}(dx^3 + cx^2 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="giac")

[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8

maple [B] time = 0.00, size = 5596, normalized size = 279.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x)

[Out] result too large to display

maxima [A] time = 0.63, size = 18, normalized size = 0.90

$$\frac{1}{8}(dx^3 + cx^2 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8

mupad [B] time = 2.32, size = 418, normalized size = 20.90

$$x^{14} \left(7b^5d^3 + \frac{105b^4c^2d^2}{2} + 35b^3c^4d + \frac{7b^2c^6}{2} \right) + x^{18} \left(7b^3d^5 + \frac{105b^2c^2d^4}{2} + 35bc^4d^3 + \frac{7c^6d^2}{2} \right) + x^{12} \left(\frac{7b^6d^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x)

[Out] $x^{14} * ((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^{18} * (7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^{12} * ((35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^{20} * ((7*b^2*d^6)/2 + (35*c^4*d^4)/4 + 21*b*c^2*d^5) + x^{16} * (c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^{10} * (b^7*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)$

sympy [B] time = 0.18, size = 469, normalized size = 23.45

$$\frac{b^8x^8}{8} + b^7cx^9 + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{22} \left(bd^7 + \frac{7c^2d^6}{2} \right) + x^{21} (7bcd^6 + 7c^3d^5) + x^{20} \left(\frac{7b^2d^6}{2} + 21bc^2d^5 + \frac{35c^4d^4}{4} \right) + x^{19} (21b^6d^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)

[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 + 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)

$$3.194 \quad \int x^7 (b + cx + dx^2)^7 (b + 2cx + 3dx^2) dx$$

Optimal. Leaf size=19

$$\frac{1}{8}x^8 (b + cx + dx^2)^8$$

[Out] $1/8*x^8*(d*x^2+c*x+b)^8$

Rubi [A] time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1588}

$$\frac{1}{8}x^8 (b + cx + dx^2)^8$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2), x]$

[Out] $(x^8*(b + c*x + d*x^2)^8)/8$

Rule 1588

$\text{Int}[(\text{Pp}_*)^*(\text{Qq}_*)^{(m_*)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Expon}[\text{Pp}, x], q = \text{Expon}[\text{Qq}, x]\}, \text{Simp}[(\text{Coeff}[\text{Pp}, x, p]*x^{(p - q + 1)}*\text{Qq}^{(m + 1)})/((p + m*q + 1)*\text{Coeff}[\text{Qq}, x, q]), x] /; \text{NeQ}[p + m*q + 1, 0] \&\& \text{EqQ}[(p + m*q + 1)*\text{Coeff}[\text{Qq}, x, q]*\text{Pp}, \text{Coeff}[\text{Pp}, x, p]*x^{(p - q)}*((p - q + 1)*\text{Qq} + (m + 1)*x*\text{D}[\text{Qq}, x])]] /; \text{FreeQ}[m, x] \&\& \text{PolyQ}[\text{Pp}, x] \&\& \text{PolyQ}[\text{Qq}, x] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\int x^7 (b + cx + dx^2)^7 (b + 2cx + 3dx^2) dx = \frac{1}{8}x^8 (b + cx + dx^2)^8$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{1}{8}x^8 (b + x(c + dx))^8$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2), x]$

[Out] $(x^8*(b + x*(c + d*x))^8)/8$

fricas [B] time = 0.52, size = 496, normalized size = 26.11

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + x^{22}d^7b + 7x^{21}d^5c^3 + 7x^{21}d^6cb + \frac{35}{4}x^{20}d^4c^4 + 21x^{20}d^5c^2b + \frac{7}{2}x^{20}d^6b^2 + 7x^{19}d^3c^5 + 35x^{19}d^4c^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b), x, \text{algorithm}=\text{"fricas"})$

[Out] $1/8*x^{24}*d^8 + x^{23}*d^7*c + 7/2*x^{22}*d^6*c^2 + x^{22}*d^7*b + 7*x^{21}*d^5*c^3 + 7*x^{21}*d^6*c*b + 35/4*x^{20}*d^4*c^4 + 21*x^{20}*d^5*c^2*b + 7/2*x^{20}*d^6*b^2 + 7*x^{19}*d^3*c^5 + 35*x^{19}*d^4*c^3*b + 21*x^{19}*d^5*c*b^2 + 7/2*x^{18}*d^2*c^6 + 35*x^{18}*d^3*c^4*b + 105/2*x^{18}*d^4*c^2*b^2 + 7*x^{18}*d^5*b^3 + x^{17}*d*c^7 + 21*x^{17}*d^2*c^5*b + 70*x^{17}*d^3*c^3*b^2 + 35*x^{17}*d^4*c*b^3 + 1/8*x^{16}*c^8 + 7*x^{16}*d*c^6*b + 105/2*x^{16}*d^2*c^4*b^2 + 70*x^{16}*d^3*c^2*b^3 + 35/4*$

$$x^{16}d^4b^4 + x^{15}c^7b + 21x^{15}d^5c^5b^2 + 70x^{15}d^2c^3b^3 + 35x^{15}d^3c^6b^4 + 7/2x^{14}c^6b^2 + 35x^{14}d^4c^4b^3 + 105/2x^{14}d^2c^2b^4 + 7x^{14}d^3b^5 + 7x^{13}c^5b^3 + 35x^{13}d^4c^3b^4 + 21x^{13}d^2c^6b^5 + 35/4x^{12}c^4b^4 + 21x^{12}d^5c^2b^5 + 7/2x^{12}d^2b^6 + 7x^{11}c^3b^5 + 7x^{11}d^4c^6b^6 + 7/2x^{10}c^2b^6 + x^{10}d^5b^7 + x^9c^6b^7 + 1/8x^8b^8$$

giac [A] time = 0.30, size = 18, normalized size = 0.95

$$\frac{1}{8}(dx^3 + cx^2 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="giac")

[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8

maple [B] time = 0.00, size = 5596, normalized size = 294.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x)

[Out] result too large to display

maxima [B] time = 0.60, size = 441, normalized size = 23.21

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}(7c^2d^6 + 2bd^7)x^{22} + 7(c^3d^5 + bcd^6)x^{21} + \frac{7}{4}(5c^4d^4 + 12bc^2d^5 + 2b^2d^6)x^{20} + 7(c^5d^3 + 5bc^3d^4)x^{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="maxima")

[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2*(7*b^6*c^2 + 2*b^7*d)*x^10

mupad [B] time = 2.27, size = 418, normalized size = 22.00

$$x^{14} \left(7b^5d^3 + \frac{105b^4c^2d^2}{2} + 35b^3c^4d + \frac{7b^2c^6}{2} \right) + x^{18} \left(7b^3d^5 + \frac{105b^2c^2d^4}{2} + 35b^4c^4d^3 + \frac{7c^6d^2}{2} \right) + x^{12} \left(\frac{7b^6}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x)

[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^18*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*((35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2

$$\begin{aligned}
& + 7*b^3*c*x^{13}*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^{19}*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + b*c*x^{15}*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) \\
& + c*d*x^{17}*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^{11}*(b*d + c^2) + 7*c*d^5*x^{21}*(b*d + c^2)
\end{aligned}$$

sympy [B] time = 0.17, size = 469, normalized size = 24.68

$$\frac{b^8x^8}{8} + b^7cx^9 + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{22} \left(bd^7 + \frac{7c^2d^6}{2} \right) + x^{21} (7bcd^6 + 7c^3d^5) + x^{20} \left(\frac{7b^2d^6}{2} + 21bc^2d^5 + \frac{35c^4d^4}{4} \right) + x^{19} (21b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)

[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 + 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)

$$3.195 \quad \int (b + 3dx^2) (a + bx + dx^3)^7 dx$$

Optimal. Leaf size=16

$$\frac{1}{8} (a + bx + dx^3)^8$$

[Out] 1/8*(d*x^3+b*x+a)^8

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1588}

$$\frac{1}{8} (a + bx + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]

[Out] (a + b*x + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 3dx^2) (a + bx + dx^3)^7 dx = \frac{1}{8} (a + bx + dx^3)^8$$

Mathematica [B] time = 0.06, size = 127, normalized size = 7.94

$$\frac{1}{8} x (b + dx^2) \left(8a^7 + 28a^6 x (b + dx^2) + 56a^5 x^2 (b + dx^2)^2 + 70a^4 x^3 (b + dx^2)^3 + 56a^3 x^4 (b + dx^2)^4 + 28a^2 x^5 (b + dx^2)^5 + 8a x^6 (b + dx^2)^6 + x^7 (b + dx^2)^7 \right) / 8$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]

[Out] (x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*x^2)^2 + 70*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28*a^2*x^5*(b + d*x^2)^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^7))/8

fricas [B] time = 0.53, size = 486, normalized size = 30.38

$$\frac{1}{8} x^{24} d^8 + x^{22} d^7 b + x^{21} d^7 a + \frac{7}{2} x^{20} d^6 b^2 + 7 x^{19} d^6 b a + 7 x^{18} d^5 b^3 + \frac{7}{2} x^{18} d^6 a^2 + 21 x^{17} d^5 b^2 a + \frac{35}{4} x^{16} d^4 b^4 + 21 x^{16} d^5 b a^2 + 35 x^{15} d^4 b^3 a + 7 x^{15} d^5 a^3 + 7 x^{14} d^3 b^5 + 105/2 x^{14} d^4 b^2 a^2 + 35 x^{13} d^3 b^4 a + 35 x^{13} d^4 b a^3 + 7/2 x^{12} d^2 b^6 + 7 x^{12} d^3 a b^4 + 7 x^{11} d^2 b^5 a + 7 x^{11} d^3 a^2 b^3 + 7 x^{10} d^2 b^4 a^2 + 7 x^{10} d^3 a b^4 + 7 x^9 d^2 b^5 a + 7 x^9 d^3 a^2 b^3 + 7 x^8 d^2 b^4 a^2 + 7 x^8 d^3 a b^4 + 7 x^7 d^2 b^3 a^2 + 7 x^7 d^3 a^2 b^3 + 7 x^6 d^2 b^2 a^3 + 7 x^6 d^3 a^3 b^2 + 7 x^5 d^2 b^3 a^2 + 7 x^5 d^3 a^3 b^2 + 7 x^4 d^2 b^2 a^3 + 7 x^4 d^3 a^3 b^2 + 7 x^3 d^2 b^2 a^3 + 7 x^3 d^3 a^3 b^2 + 7 x^2 d^2 b^2 a^3 + 7 x^2 d^3 a^3 b^2 + 7 x d^2 b^2 a^3 + 7 x d^3 a^3 b^2 + 7 d^2 b^2 a^3 + 7 d^3 a^3 b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^22*d^7*b + x^21*d^7*a + 7/2*x^20*d^6*b^2 + 7*x^19*d^6*b*a + 7*x^18*d^5*b^3 + 7/2*x^18*d^6*a^2 + 21*x^17*d^5*b^2*a + 35/4*x^16*d^4*b^4 + 21*x^16*d^5*b*a^2 + 35*x^15*d^4*b^3*a + 7*x^15*d^5*a^3 + 7*x^14*d^3*b^5 + 105/2*x^14*d^4*b^2*a^2 + 35*x^13*d^3*b^4*a + 35*x^13*d^4*b*a^3 + 7/2*x^12

$$\begin{aligned} & d^2b^6 + 70x^{12}d^3b^3a^2 + 35/4x^{12}d^4a^4 + 21x^{11}d^2b^5a + 70 \\ & x^{11}d^3b^2a^3 + x^{10}db^7 + 105/2x^{10}d^2b^4a^2 + 35x^{10}d^3b^3a^4 \\ & + 7x^9d^2b^6a + 70x^9d^2b^3a^3 + 7x^9d^3a^5 + 1/8x^8b^8 + 21x^8 \\ & db^5a^2 + 105/2x^8d^2b^2a^4 + x^7b^7a + 35x^7db^4a^3 + 21x^7 \\ & d^2b^5a^5 + 7/2x^6b^6a^2 + 35x^6db^3a^4 + 7/2x^6d^2a^6 + 7x^5b^5 \\ & a^3 + 21x^5db^2a^5 + 35/4x^4b^4a^4 + 7x^4db^3a^6 + 7x^3b^3a^5 \\ & + x^3da^7 + 7/2x^2b^2a^6 + xba^7 \end{aligned}$$

giac [B] time = 0.41, size = 120, normalized size = 7.50

$$\frac{1}{8} (dx^3 + bx)^8 + (dx^3 + bx)^7 a + \frac{7}{2} (dx^3 + bx)^6 a^2 + 7 (dx^3 + bx)^5 a^3 + \frac{35}{4} (dx^3 + bx)^4 a^4 + 7 (dx^3 + bx)^3 a^5 + \frac{7}{2} (dx^3 + bx)^2 a^6 + (dx^3 + bx) a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8 + (d*x^3 + b*x)^7*a + 7/2*(d*x^3 + b*x)^6*a^2 + 7*(d*x^3 + b*x)^5*a^3 + 35/4*(d*x^3 + b*x)^4*a^4 + 7*(d*x^3 + b*x)^3*a^5 + 7/2*(d*x^3 + b*x)^2*a^6 + (d*x^3 + b*x)*a^7
```

maple [B] time = 0.00, size = 2185, normalized size = 136.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+d^7*a*x^21+7/2*b^2*d^6*x^20+7*b*a*d^6*x^19+1/18*(21*b^3*d^5+3*d*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)))*x^18+21*b^2*a*d^5*x^17+1/16*(b*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4))+3*d*(30*a^2*d^4*b+b*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4))+d*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4))*x^16+1/15*(105*b^3*a*d^4+3*d*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^3*a*d^3+d*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)))*x^15+1/14*(b*(30*a^2*d^4*b+b*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4))+d*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4))+3*d*(60*a^2*b^2*d^3+b*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4))+d*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2))*x^14+1/13*(b*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^3*a*d^3+d*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2))+3*d*(a*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4)+b*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2))+d*(24*a^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b))*x^13+1/12*(b*(60*a^2*b^2*d^3+b*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4))+d*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2))+3*d*(a*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+b*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2))+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))*x^12+1/11*(b*(a*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4)+b*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2))+d*(24*a^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b))+3*d*(a*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+b*(24*a^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b))+d*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3)))*x^11+1/10*(b*(a*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+b*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2))+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+3*d*(a*(24*a^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b))+b*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+d*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2))*x^10+1/9*(b*(a*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2))+b*(24*a^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b))+d*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3)))+3*d*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2)+b*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)))*x^9+1/8*(b*(a*(24*a^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b))+b*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+d*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2))+3*d*(a*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^2*a^4))*x^8+1/7*(b*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2)+b*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)))*x^7+1/6*(b*(a*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2))+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+3*d*(a*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^2*a^4))*x^6+1/5*(b*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+b*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)))*x^5+1/4*(b*(a*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2))+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+3*d*(a*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^2*a^4))*x^4+1/3*(b*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+b*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)))*x^3+1/2*(b*(a*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2))+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+3*d*(a*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^2*a^4))*x^2+1*(b*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2))+b*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)))*x+a^7
```

$$b^2+6*b^2*a*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3))+3*d*(a*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+b*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+6*d*a^5*b))*x^7+1/6*(b*(a*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^2*a^4)+3*d*(a*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+15*b^3*a^4+d*a^6))*x^6+1/5*(b*(a*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+b*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+6*d*a^5*b)+63*d*b^2*a^5)*x^5+1/4*(b*(a*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+15*b^3*a^4+d*a^6)+21*d*a^6*b)*x^4+1/3*(3*a^7*d+21*a^5*b^3)*x^3+7/2*b^2*a^6*x^2+b*a^7*x$$

maxima [A] time = 0.55, size = 14, normalized size = 0.88

$$\frac{1}{8} (dx^3 + bx + a)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + b*x + a)^8

mupad [B] time = 2.63, size = 438, normalized size = 27.38

$$x^{12} \left(\frac{35 a^4 d^4}{4} + 70 a^2 b^3 d^3 + \frac{7 b^6 d^2}{2} \right) + x^4 \left(7 d a^6 b + \frac{35 a^4 b^4}{4} \right) + x^{18} \left(\frac{7 a^2 d^6}{2} + 7 b^3 d^5 \right) + x^6 \left(\frac{7 a^6 d^2}{2} + 35 a^4 b^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^7,x)

[Out] x^12*((35*a^4*d^4)/4 + (7*b^6*d^2)/2 + 70*a^2*b^3*d^3) + x^4*((35*a^4*b^4)/4 + 7*a^6*b*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5) + x^6*((7*a^2*b^6)/2 + (7*a^6*d^2)/2 + 35*a^4*b^3*d) + x^8*(b^8/8 + 21*a^2*b^5*d + (105*a^4*b^2*d^2)/2) + (d^8*x^24)/8 + x^3*(a^7*d + 7*a^5*b^3) + a*d^7*x^21 + b*d^7*x^22 + (7*a^6*b^2*x^2)/2 + (7*b^2*d^6*x^20)/2 + a^7*b*x + 21*a*b^2*d^5*x^17 + a*b*x^7*(b^6 + 21*a^4*d^2 + 35*a^2*b^3*d) + 7*a*d*x^9*(b^6 + a^4*d^2 + 10*a^2*b^3*d) + 7*a^3*b^2*x^5*(3*a^2*d + b^3) + 7*a*d^4*x^15*(a^2*d + 5*b^3) + (7*b*d^4*x^16*(12*a^2*d + 5*b^3))/4 + (b*d*x^10*(2*b^6 + 70*a^4*d^2 + 105*a^2*b^3*d))/2 + 7*a*b*d^6*x^19 + (7*b^2*d^3*x^14*(15*a^2*d + 2*b^3))/2 + 7*a*b^2*d^2*x^11*(10*a^2*d + 3*b^3) + 35*a*b*d^3*x^13*(a^2*d + b^3)

sympy [B] time = 0.17, size = 483, normalized size = 30.19

$$a^7bx + \frac{7a^6b^2x^2}{2} + 21ab^2d^5x^{17} + 7abd^6x^{19} + ad^7x^{21} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8} + x^{18} \left(\frac{7a^2d^6}{2} + 7b^3d^5 \right) + x^{16} \left(21a^2bd^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)

[Out] a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6*x**19 + a*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8 + x**18*(7*a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b*d**5 + 35*b**4*d**4/4) + x**15*(7*a**3*d**5 + 35*a*b**3*d**4) + x**14*(105*a**2*b**2*d**4/2 + 7*b**5*d**3) + x**13*(35*a**3*b*d**4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 + 70*a**2*b**3*d**3 + 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5*d**2) + x**10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*a**5*d**3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d**2/2 + 21*a**2*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3*b**4*d + a*b**7) + x**6*(7*a**6*d**2/2 + 35*a**4*b**3*d + 7*a**2*b**6/2) + x**5*(21*a**5*b**2*d + 7*a**3*b**5) + x**4*(7*a**6*b*d + 35*a**4*b**4/4) + x**3*(a**7*d + 7*a**5*b**3)

$$3.196 \quad \int (b + 3dx^2) (bx + dx^3)^7 dx$$

Optimal. Leaf size=15

$$\frac{1}{8} (bx + dx^3)^8$$

[Out] 1/8*(d*x^3+b*x)^8

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1588}

$$\frac{1}{8} (bx + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]

[Out] (b*x + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 3dx^2) (bx + dx^3)^7 dx = \frac{1}{8} (bx + dx^3)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 6.53

$$\frac{b^8 x^8}{8} + b^7 dx^{10} + \frac{7}{2} b^6 d^2 x^{12} + 7b^5 d^3 x^{14} + \frac{35}{4} b^4 d^4 x^{16} + 7b^3 d^5 x^{18} + \frac{7}{2} b^2 d^6 x^{20} + bd^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]

[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8

fricas [B] time = 0.65, size = 88, normalized size = 5.87

$$\frac{1}{8} x^{24} d^8 + x^{22} d^7 b + \frac{7}{2} x^{20} d^6 b^2 + 7x^{18} d^5 b^3 + \frac{35}{4} x^{16} d^4 b^4 + 7x^{14} d^3 b^5 + \frac{7}{2} x^{12} d^2 b^6 + x^{10} d b^7 + \frac{1}{8} x^8 b^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{8} (dx^3 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="giac")

[Out] 1/8*(d*x^3 + b*x)^8

maple [B] time = 0.00, size = 89, normalized size = 5.93

$$\frac{1}{8}d^8x^{24} + b d^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7d x^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+b)*(d*x^3+b*x)^7,x)

[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*d^4*x^16+7*b^5*d^3*x^14+7/2*b^6*d^2*x^12+d*b^7*x^10+1/8*b^8*x^8

maxima [A] time = 0.51, size = 13, normalized size = 0.87

$$\frac{1}{8} (dx^3 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + b*x)^8

mupad [B] time = 0.05, size = 88, normalized size = 5.87

$$\frac{b^8 x^8}{8} + b^7 d x^{10} + \frac{7 b^6 d^2 x^{12}}{2} + 7 b^5 d^3 x^{14} + \frac{35 b^4 d^4 x^{16}}{4} + 7 b^3 d^5 x^{18} + \frac{7 b^2 d^6 x^{20}}{2} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + d*x^3)^7*(b + 3*d*x^2),x)

[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2

sympy [B] time = 0.10, size = 97, normalized size = 6.47

$$\frac{b^8 x^8}{8} + b^7 d x^{10} + \frac{7 b^6 d^2 x^{12}}{2} + 7 b^5 d^3 x^{14} + \frac{35 b^4 d^4 x^{16}}{4} + 7 b^3 d^5 x^{18} + \frac{7 b^2 d^6 x^{20}}{2} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)

[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8

$$3.197 \quad \int x^7 (b + dx^2)^7 (b + 3dx^2) dx$$

Optimal. Leaf size=16

$$\frac{1}{8}x^8 (b + dx^2)^8$$

[Out] 1/8*x^8*(d*x^2+b)^8

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 74}

$$\frac{1}{8}x^8 (b + dx^2)^8$$

Antiderivative was successfully verified.

[In] Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]

[Out] (x^8*(b + d*x^2)^8)/8

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int x^7 (b + dx^2)^7 (b + 3dx^2) dx &= \frac{1}{2} \text{Subst} \left(\int x^3 (b + dx)^7 (b + 3dx) dx, x, x^2 \right) \\ &= \frac{1}{8}x^8 (b + dx^2)^8 \end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 6.12

$$\frac{b^8 x^8}{8} + b^7 dx^{10} + \frac{7}{2} b^6 d^2 x^{12} + 7b^5 d^3 x^{14} + \frac{35}{4} b^4 d^4 x^{16} + 7b^3 d^5 x^{18} + \frac{7}{2} b^2 d^6 x^{20} + bd^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]

[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8

fricas [B] time = 0.77, size = 88, normalized size = 5.50

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="fricas")

[Out] $\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$

giac [A] time = 0.27, size = 13, normalized size = 0.81

$$\frac{1}{8}(dx^3 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="giac")

[Out] $\frac{1}{8}(d^3x^3 + b^8x)^8$

maple [B] time = 0.00, size = 89, normalized size = 5.56

$$\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(d*x^2+b)^7*(3*d*x^2+b),x)

[Out] $\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$

maxima [B] time = 0.64, size = 88, normalized size = 5.50

$$\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="maxima")

[Out] $\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$

mupad [B] time = 0.04, size = 88, normalized size = 5.50

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b + d*x^2)^7*(b + 3*d*x^2),x)

[Out] $\frac{b^8x^8}{8} + \frac{d^8x^{24}}{8} + b^7dx^{10} + bd^7x^{22} + \frac{(7b^6d^2x^{12})}{2} + 7b^5d^3x^{14} + \frac{(35b^4d^4x^{16})}{4} + 7b^3d^5x^{18} + \frac{(7b^2d^6x^{20})}{2}$

sympy [B] time = 0.09, size = 97, normalized size = 6.06

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)

[Out] $b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8$

$$3.198 \quad \int (2cx + 3dx^2) (a + cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=18

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

[Out] 1/8*(d*x^3+c*x^2+a)^8

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1588}

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]

[Out] (a + c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2cx + 3dx^2) (a + cx^2 + dx^3)^7 dx = \frac{1}{8} (a + cx^2 + dx^3)^8$$

Mathematica [B] time = 0.05, size = 115, normalized size = 6.39

$$\frac{1}{8} x^2 (c + dx) (8a^7 + 28a^6 x^2 (c + dx) + 56a^5 x^4 (c + dx)^2 + 70a^4 x^6 (c + dx)^3 + 56a^3 x^8 (c + dx)^4 + 28a^2 x^{10} (c + dx)^5 + 8a x^{12} (c + dx)^6 + x^{14} (c + dx)^7) / 8$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]

[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8

fricas [B] time = 0.73, size = 488, normalized size = 27.11

$$\frac{1}{8} x^{24} d^8 + x^{23} d^7 c + \frac{7}{2} x^{22} d^6 c^2 + 7x^{21} d^5 c^3 + x^{21} d^7 a + \frac{35}{4} x^{20} d^4 c^4 + 7x^{20} d^6 c a + 7x^{19} d^3 c^5 + 21x^{19} d^5 c^2 a + \frac{7}{2} x^{18} d^2 c^6 + 35x^{18} d^4 c^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x^21*d^7*a + 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 21*x^19*d^5*c^2*a + 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 35*x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105

$$\begin{aligned} & /2*x^{16}*d^4*c^2*a^2 + 7*x^{15}*d*c^6*a + 70*x^{15}*d^3*c^3*a^2 + 7*x^{15}*d^5*a^3 \\ & + x^{14}*c^7*a + 105/2*x^{14}*d^2*c^4*a^2 + 35*x^{14}*d^4*c*a^3 + 21*x^{13}*d*c^5* \\ & a^2 + 70*x^{13}*d^3*c^2*a^3 + 7/2*x^{12}*c^6*a^2 + 70*x^{12}*d^2*c^3*a^3 + 35/4*x \\ & ^{12}*d^4*a^4 + 35*x^{11}*d*c^4*a^3 + 35*x^{11}*d^3*c*a^4 + 7*x^{10}*c^5*a^3 + 105/ \\ & 2*x^{10}*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*a^5 + 35/4*x^8*c^4*a^4 + \\ & 21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x \\ & ^5*d*c*a^6 + 7/2*x^4*c^2*a^6 + x^3*d*a^7 + x^2*c*a^7 \end{aligned}$$

giac [B] time = 0.43, size = 136, normalized size = 7.56

$$\frac{1}{8} (dx^3 + cx^2)^8 + (dx^3 + cx^2)^7 a + \frac{7}{2} (dx^3 + cx^2)^6 a^2 + 7 (dx^3 + cx^2)^5 a^3 + \frac{35}{4} (dx^3 + cx^2)^4 a^4 + 7 (dx^3 + cx^2)^3 a^5 + \frac{7}{2} (dx^3 + cx^2)^2 a^6 + (dx^3 + cx^2) a^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")

[Out] 1/8*(d*x^3 + c*x^2)^8 + (d*x^3 + c*x^2)^7*a + 7/2*(d*x^3 + c*x^2)^6*a^2 + 7*(d*x^3 + c*x^2)^5*a^3 + 35/4*(d*x^3 + c*x^2)^4*a^4 + 7*(d*x^3 + c*x^2)^3*a^5 + 7/2*(d*x^3 + c*x^2)^2*a^6 + (d*x^3 + c*x^2)*a^7

maple [B] time = 0.00, size = 2205, normalized size = 122.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x)

[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*d*(a*d^6+15*c^3*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)))*x^21+1/20*(2*c*(a*d^6+15*c^3*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+3*d*(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+d*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2))*x^20+1/19*(2*c*(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+d*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2))+3*d*(15*a*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2))+d*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d))*x^19+1/18*(2*c*(15*a*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2))+d*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d))+3*d*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d))+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2))*x^18+1/17*(2*c*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d))+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2))+3*d*(a*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2))+d*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3)))*x^17+1/16*(2*c*(a*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2))+d*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3)))+3*d*(a*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)+c*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3)))+d*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))*x^16+1/15*(2*c*(a*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)+c*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3)))+d*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+3*d*(a*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+d*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3)))*x^15+1/14*(2*c*(a*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+d*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3)))+3*d*(a*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3)))+c*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3)))+d*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4))*x^14+1/13*(2*c*(a*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3)))+c*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3)))+d*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4))+3*d*(a*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+c*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4))+60*d^2*a^3*c^2))*x^13+1/12*(2*c*(a*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+c*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4))+60*d^2*a^3*c^2))+3*d*(a*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3))+60*c^3*a^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*c^3*a^3+9*a^4*d^2)))*x^12+1/11*(2*c*(a*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2

$(3ad^2+c^3)+60c^3a^3d+d(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2))+3d*(a*(42a^3cd^2+6c*a^2*(3ad^2+c^3)+9a^2c^4)+c*(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2)+30d^2a^4c))*x^{11}+1/10*(2c*(a*(42a^3cd^2+6c*a^2*(3ad^2+c^3)+9a^2c^4)+c*(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2)+30d^2a^4c))+315d^2a^4c^2)*x^{10}+1/9*(210c^3a^4d+3d*(a*(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2)+15c^3a^4+6d^2a^5))*x^9+1/8*(2c*(a*(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2)+15c^3a^4+6d^2a^5)+126d^2a^5c)*x^8+21c^2a^5d*x^7+1/6*(21a^6d^2+42a^5c^3)*x^6+7c*a^6d*x^5+7/2*c^2a^6*x^4+d*a^7*x^3+c*a^7*x^2$

maxima [A] time = 0.62, size = 16, normalized size = 0.89

$$\frac{1}{8} (dx^3 + cx^2 + a)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2 + a)^8

mupad [B] time = 2.63, size = 440, normalized size = 24.44

$$x^{12} \left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2} \right) + x^6 \left(\frac{7a^6d^2}{2} + 7a^5c^3 \right) + x^{20} \left(\frac{35c^4d^4}{4} + 7acd^6 \right) + x^{16} \left(\frac{105a^2c^2d^4}{2} + 21acd^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x)

[Out] $x^{12}*((7a^2c^6)/2 + (35a^4d^4)/4 + 70a^3c^3d^2) + x^6*(7a^5c^3 + (7a^6d^2)/2) + x^{20}*((35c^4d^4)/4 + 7a*c*d^6) + x^{16}*(c^8/8 + 21a*c^5*d^2 + (105a^2*c^2*d^4)/2) + x^{18}*((7a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3*d^4) + (d^8*x^{24})/8 + x^{21}*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c*d^7*x^{23} + (7a^6*c^2*x^4)/2 + (7*c^2*d^6*x^{22})/2 + 21*a^5*c^2*d*x^7 + 7*a*d*x^{15}*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^{17}*(c^6 + 21*a^2*d^4 + 35*a*c^3*d^2) + (7a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7a^4*d*x^9*(a*d^2 + 5*c^3) + 7*c^2*d^3*x^{19}*(3a*d^2 + c^3) + (a*c*x^{14}*(2*c^6 + 70*a^2*d^4 + 105*a*c^3*d^2))/2 + 7a^6*c*d*x^5 + (7a^3*c^2*x^{10}*(15*a*d^2 + 2*c^3))/2 + 7a^2*c^2*d*x^{13}*(10*a*d^2 + 3*c^3) + 35a^3*c*d*x^{11}*(a*d^2 + c^3)$

sympy [B] time = 0.17, size = 484, normalized size = 26.89

$$a^7cx^2+a^7dx^3+\frac{7a^6c^2x^4}{2}+7a^6cdx^5+21a^5c^2dx^7+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{21}(ad^7+7c^3d^5)+x^{20}\left(7acd^6+\frac{35c^4d^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)

[Out] $a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c**2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d**3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) + x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a**3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**5*c**3)$

$$3.199 \quad \int (2cx + 3dx^2) (cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=17

$$\frac{1}{8} (cx^2 + dx^3)^8$$

[Out] 1/8*(d*x^3+c*x^2)^8

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1588}

$$\frac{1}{8} (cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]

[Out] (c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2cx + 3dx^2) (cx^2 + dx^3)^7 dx = \frac{1}{8} (cx^2 + dx^3)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 5.76

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

fricas [B] time = 0.57, size = 88, normalized size = 5.18

$$\frac{1}{8} x^{24} d^8 + x^{23} d^7 c + \frac{7}{2} x^{22} d^6 c^2 + 7 x^{21} d^5 c^3 + \frac{35}{4} x^{20} d^4 c^4 + 7 x^{19} d^3 c^5 + \frac{7}{2} x^{18} d^2 c^6 + x^{17} d c^7 + \frac{1}{8} x^{16} c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8

giac [A] time = 0.37, size = 15, normalized size = 0.88

$$\frac{1}{8} (dx^3 + cx^2)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="giac")

[Out] 1/8*(d*x^3 + c*x^2)^8

maple [B] time = 0.00, size = 89, normalized size = 5.24

$$\frac{1}{8}d^8x^{24} + c d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x)

[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16

maxima [A] time = 0.66, size = 15, normalized size = 0.88

$$\frac{1}{8} (dx^3 + cx^2)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2)^8

mupad [B] time = 2.07, size = 88, normalized size = 5.18

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x)

[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2

sympy [B] time = 0.10, size = 97, normalized size = 5.71

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)

[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8

$$3.200 \quad \int x^7 (cx + dx^2)^7 (2cx + 3dx^2) dx$$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c + dx)^8$$

[Out] 1/8*x^16*(d*x+c)^8

Rubi [A] time = 0.23, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1584, 845}

$$\frac{1}{8}x^{16}(c + dx)^8$$

Antiderivative was successfully verified.

[In] Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2), x]

[Out] (x^16*(c + d*x)^8)/8

Rule 845

Int[(x_)^(m_)*((f_) + (g_)*(x_)^(n_))*((b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n + 3, 0]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^7 (cx + dx^2)^7 (2cx + 3dx^2) dx &= \int x^{14}(c + dx)^7 (2cx + 3dx^2) dx \\ &= \frac{1}{8}x^{16}(c + dx)^8 \end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2}c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4}c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2}c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2), x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

fricas [B] time = 0.73, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")

[Out] $\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}d^1c^7 + \frac{1}{8}x^{16}c^8$

giac [A] time = 0.23, size = 15, normalized size = 1.07

$$\frac{1}{8} (dx^3 + cx^2)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="giac")

[Out] $\frac{1}{8}(d^8x^{24} + c^8x^{16})$

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x)

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$

maxima [B] time = 0.52, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$

mupad [B] time = 0.05, size = 88, normalized size = 6.29

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x)

[Out] $\frac{(c^8x^{16})}{8} + \frac{(d^8x^{24})}{8} + c^7d^2x^{18} + cd^7x^{23} + \frac{(7c^6d^2x^{18})}{2} + 7c^5d^3x^{19} + \frac{(35c^4d^4x^{20})}{4} + 7c^3d^5x^{21} + \frac{(7c^2d^6x^{22})}{2}$

sympy [B] time = 0.10, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)

[Out] $c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8$

3.201 $\int x^{14}(c + dx)^7 (2cx + 3dx^2) dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c + dx)^8$$

[Out] 1/8*x^16*(d*x+c)^8

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {845}

$$\frac{1}{8}x^{16}(c + dx)^8$$

Antiderivative was successfully verified.

[In] Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]

[Out] (x^16*(c + d*x)^8)/8

Rule 845

Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n + 3, 0]

Rubi steps

$$\int x^{14}(c + dx)^7 (2cx + 3dx^2) dx = \frac{1}{8}x^{16}(c + dx)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2}c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4}c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2}c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

fricas [B] time = 0.62, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8

giac [A] time = 0.26, size = 15, normalized size = 1.07

$$\frac{1}{8}(dx^3 + cx^2)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴*(d*x+c)⁷*(3*d*x²+2*c*x),x, algorithm="giac")

[Out] 1/8*(d*x³ + c*x²)⁸

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁴*(d*x+c)⁷*(3*d*x²+2*c*x),x)

[Out] 1/8*d⁸*x²⁴+c*d⁷*x²³+7/2*c²*d⁶*x²²+7*c³*d⁵*x²¹+35/4*c⁴*d⁴*x²⁰+7*c⁵*d³*x¹⁹+7/2*c⁶*d²*x¹⁸+c⁷*d*x¹⁷+1/8*c⁸*x¹⁶

maxima [B] time = 0.66, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴*(d*x+c)⁷*(3*d*x²+2*c*x),x, algorithm="maxima")

[Out] 1/8*d⁸*x²⁴ + c*d⁷*x²³ + 7/2*c²*d⁶*x²² + 7*c³*d⁵*x²¹ + 35/4*c⁴*d⁴*x²⁰ + 7*c⁵*d³*x¹⁹ + 7/2*c⁶*d²*x¹⁸ + c⁷*d*x¹⁷ + 1/8*c⁸*x¹⁶

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁴*(2*c*x + 3*d*x²)*(c + d*x)⁷,x)

[Out] (c⁸*x¹⁶)/8 + (d⁸*x²⁴)/8 + c⁷*d*x¹⁷ + c*d⁷*x²³ + (7*c⁶*d²*x¹⁸)/2 + 7*c⁵*d³*x¹⁹ + (35*c⁴*d⁴*x²⁰)/4 + 7*c³*d⁵*x²¹ + (7*c²*d⁶*x²²)/2

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)

[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8

3.202 $\int x(2c + 3dx) (a + cx^2 + dx^3)^7 dx$

Optimal. Leaf size=18

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

[Out] 1/8*(d*x^3+c*x^2+a)^8

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1588}

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]

[Out] (a + c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x(2c + 3dx) (a + cx^2 + dx^3)^7 dx = \frac{1}{8} (a + cx^2 + dx^3)^8$$

Mathematica [B] time = 0.01, size = 115, normalized size = 6.39

$$\frac{1}{8} x^2 (c + dx) (8a^7 + 28a^6 x^2 (c + dx) + 56a^5 x^4 (c + dx)^2 + 70a^4 x^6 (c + dx)^3 + 56a^3 x^8 (c + dx)^4 + 28a^2 x^{10} (c + dx)^5 + 8a x^{12} (c + dx)^6 + x^{14} (c + dx)^7) / 8$$

Antiderivative was successfully verified.

[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]

[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8

fricas [B] time = 1.06, size = 488, normalized size = 27.11

$$\frac{1}{8} x^{24} d^8 + x^{23} d^7 c + \frac{7}{2} x^{22} d^6 c^2 + 7 x^{21} d^5 c^3 + x^{21} d^7 a + \frac{35}{4} x^{20} d^4 c^4 + 7 x^{20} d^6 c a + 7 x^{19} d^3 c^5 + 21 x^{19} d^5 c^2 a + \frac{7}{2} x^{18} d^2 c^6 + 35 x^{18} d^4 c^3 a + 7 x^{18} d^6 a^2 + x^{17} d^7 c^7 + 35 x^{17} d^3 c^4 a + 21 x^{17} d^5 c^2 a^2 + \frac{1}{8} x^{16} d^8 c^8 + 21 x^{16} d^2 c^5 a + 105$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x^21*d^7*a + 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 21*x^19*d^5*c^2*a + 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x^18*d^6*a^2 + x^17*d^7*c^7 + 35*x^17*d^3*c^4*a + 21*x^17*d^5*c^2*a^2 + 1/8*x^16*d^8*c^8 + 21*x^16*d^2*c^5*a + 105

$$\begin{aligned} & /2*x^{16}*d^4*c^2*a^2 + 7*x^{15}*d*c^6*a + 70*x^{15}*d^3*c^3*a^2 + 7*x^{15}*d^5*a^3 \\ & + x^{14}*c^7*a + 105/2*x^{14}*d^2*c^4*a^2 + 35*x^{14}*d^4*c*a^3 + 21*x^{13}*d*c^5* \\ & a^2 + 70*x^{13}*d^3*c^2*a^3 + 7/2*x^{12}*c^6*a^2 + 70*x^{12}*d^2*c^3*a^3 + 35/4*x \\ & ^{12}*d^4*a^4 + 35*x^{11}*d*c^4*a^3 + 35*x^{11}*d^3*c*a^4 + 7*x^{10}*c^5*a^3 + 105/ \\ & 2*x^{10}*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*a^5 + 35/4*x^8*c^4*a^4 + \\ & 21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x \\ & ^5*d*c*a^6 + 7/2*x^4*c^2*a^6 + x^3*d*a^7 + x^2*c*a^7 \end{aligned}$$

giac [B] time = 0.32, size = 488, normalized size = 27.11

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+ad^7x^{21}+\frac{35}{4}c^4d^4x^{20}+7acd^6x^{20}+7c^5d^3x^{19}+21ac^2d^5x^{19}+\frac{7}{2}c^6d^2x^{18}+35ac^3d^4x^{18}+\dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")

[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a*d^7*x^21 + 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 21*a*c^2*d^5*x^19 + 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 + 35*a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105/2*a^2*c^2*d^4*x^16 + 7*a*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15 + a*c^7*x^14 + 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x^13 + 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 35/4*a^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*c^5*x^10 + 105/2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c^4*x^8 + 21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2

maple [B] time = 0.00, size = 2205, normalized size = 122.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x)

[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*(a*d^6+15*c^3*d^4+(18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*d)*d)*x^21+1/20*(2*(a*d^6+15*c^3*d^4+(18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*d)*c+3*(6*a*c*d^5+(18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*c+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*d)*d)*x^20+1/19*(2*(6*a*c*d^5+(18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*c+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*d)*c+3*(15*a*c^2*d^4+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*c+(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*d)*d)*x^19+1/18*(2*(15*a*c^2*d^4+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*c+(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*d)*c+3*((18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*a+(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*c+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*d)*d)*x^18+1/17*(2*((18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*a+(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*c+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*d)*c+3*((12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*a+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*c+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*d)*d)*x^17+1/16*(2*((12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*a+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*c+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*d)*c+3*((42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*a+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*c+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*d)*d)*x^16+21*a^5*c^2*d*x^7+1/15*(2*((42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*a+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*c+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*d)*c+3*((6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*a+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*c+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*d)*d)*x^15+1/14*(2*((6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*a+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*c+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*d)*c+3*((24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*a+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*c

$$\begin{aligned}
 &+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*d)*d)*x^{14}+7*a^6*c*d*x^5+1/ \\
 &13*(2*((24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*a+(2*a^3*d^3+54*a^2 \\
 &*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*c+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2 \\
 &*c)*d)*c+3*(60*a^3*c^2*d^2+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*a+(42*a^3 \\
 &*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*c)*d)*x^{13}+7/2*a^6*c^2*x^4+1/12*(2* \\
 &(60*a^3*c^2*d^2+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*a+(42*a^3*c*d^2+9*a^ \\
 &2*c^4+6*(3*a*d^2+c^3)*a^2*c)*c)*c+3*(60*a^3*c^3*d+(2*a^3*d^3+54*a^2*c^3*d+6 \\
 &)*(3*a*d^2+c^3)*a^2*d)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*d)*d)*x^ \\
 &12+a^7*d*x^3+1/11*(2*(60*a^3*c^3*d+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)* \\
 &a^2*d)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*d)*c+3*(30*a^4*c*d^2+(4 \\
 &2*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a \\
 &*d^2+c^3)*a^3)*c)*d)*x^{11}+a^7*c*x^2+1/10*(315*a^4*c^2*d^2+2*(30*a^4*c*d^2+(\\
 &42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*a+(9*a^4*d^2+18*a^3*c^3+2*(3* \\
 &a*d^2+c^3)*a^3)*c)*c)*x^{10}+1/9*(210*a^4*c^3*d+3*(6*a^5*d^2+15*a^4*c^3+(9*a^ \\
 &4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*a)*d)*x^9+1/8*(126*a^5*c*d^2+2*(6*a^5 \\
 &*d^2+15*a^4*c^3+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*a)*c)*x^8+1/6*(2 \\
 &1*a^6*d^2+42*a^5*c^3)*x^6
 \end{aligned}$$

maxima [B] time = 0.59, size = 458, normalized size = 25.44

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+(7c^3d^5+ad^7)x^{21}+\frac{7}{4}(5c^4d^4+4acd^6)x^{20}+7(c^5d^3+3ac^2d^5)x^{19}+\frac{7}{2}(c^6d^2+10ac^3d^4)x^{18}+\frac{7}{4}(c^7d+c^5d^3+3a^2c^2d^5)x^{17}+\frac{7}{4}(c^8+168a^2c^5d^2+420a^2c^2d^4)x^{16}+7(a^2c^6d+10a^2c^3d^3+a^3d^5)x^{15}+21a^5c^2d^2*x^7+1/2*(2a^2c^7+105a^2c^4d^2+70a^3c^3d^4)x^{14}+7*(3a^2c^5d+10a^3c^2d^3)x^{13}+7a^6c*d*x^5+7/4*(2a^2c^6+40a^3c^3d^2+5a^4d^4)x^{12}+7/2*a^6*c^2*x^4+35*(a^3c^4d+a^4c^3d^3)x^{11}+a^7*d*x^3+7/2*(2a^3c^5+15a^4c^2d^2)x^{10}+a^7*c*x^2+7*(5a^4c^3d+a^5d^3)x^9+7/4*(5a^4c^4+12a^5c^3d^2)x^8+7/2*(2a^5c^3+a^6d^2)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")

[Out] $1/8*d^8*x^{24} + c*d^7*x^{23} + 7/2*c^2*d^6*x^{22} + (7*c^3*d^5 + a*d^7)*x^{21} + 7/4*(5*c^4*d^4 + 4*a*c*d^6)*x^{20} + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^{19} + 7/2*(c^6*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^{18} + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)*x^{17} + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^{16} + 7*(a^2*c^6*d + 10*a^2*c^3*d^3 + a^3*d^5)*x^{15} + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4*d^2 + 70*a^3*c^3*d^4)*x^{14} + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^{13} + 7*a^6*c*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^{12} + 7/2*a^6*c^2*x^4 + 35*(a^3*c^4*d + a^4*c^3*d^3)*x^{11} + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c^2*d^2)*x^{10} + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 + 12*a^5*c^3*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6$

mupad [B] time = 0.57, size = 440, normalized size = 24.44

$$x^{12} \left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2} \right) + x^6 \left(\frac{7a^6d^2}{2} + 7a^5c^3 \right) + x^{20} \left(\frac{35c^4d^4}{4} + 7acd^6 \right) + x^{16} \left(\frac{105a^2c^2d^4}{2} + 21a^2c^3d^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x)

[Out] $x^{12}*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (7*a^6*d^2)/2) + x^{20}*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^{16}*(c^8/8 + 21*a*c^5*d^2 + (105*a^2*c^2*d^4)/2) + x^{18}*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3*d^4) + (d^8*x^{24})/8 + x^{21}*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c*d^7*x^{23} + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a*d*x^{15}*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^{17}*(c^6 + 21*a^2*d^4 + 35*a*c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3) + 7*c^2*d^3*x^{19}*(3*a*d^2 + c^3) + (a*c*x^{14}*(2*c^6 + 70*a^2*d^4 + 105*a*c^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^{10}*(15*a*d^2 + 2*c^3))/2 + 7*a^2*c^2*d*x^{13}*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^{11}*(a*d^2 + c^3)$

sympy [B] time = 0.17, size = 484, normalized size = 26.89

$$a^7cx^2+a^7dx^3+\frac{7a^6c^2x^4}{2}+7a^6cdx^5+21a^5c^2dx^7+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{21}(ad^7+7c^3d^5)+x^{20}\left(7acd^6+\frac{35c^4d^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)

[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c**2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d**3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) + x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a**3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**5*c**3)

3.203 $\int x(2c + 3dx) (cx^2 + dx^3)^7 dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c + dx)^8$$

[Out] 1/8*x^16*(d*x+c)^8

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1584, 74}

$$\frac{1}{8}x^{16}(c + dx)^8$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]

[Out] (x^16*(c + d*x)^8)/8

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x(2c + 3dx) (cx^2 + dx^3)^7 dx &= \int x^{15}(c + dx)^7(2c + 3dx) dx \\ &= \frac{1}{8}x^{16}(c + dx)^8 \end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2}c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4}c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2}c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

fricas [B] time = 0.71, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="fricas")

[Out] $\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}d^1c^7 + \frac{1}{8}x^{16}c^8$

giac [B] time = 0.31, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="giac")

[Out] $\frac{1}{8}d^8x^{24} + c^7d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d^1x^{17} + \frac{1}{8}c^8x^{16}$

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x)

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$

maxima [B] time = 0.60, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="maxima")

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x)

[Out] $\frac{(c^8x^{16})}{8} + \frac{(d^8x^{24})}{8} + c^7d^7x^{23} + cd^7x^{23} + \frac{(7c^6d^2x^{18})}{2} + 7c^5d^3x^{19} + \frac{(35c^4d^4x^{20})}{4} + 7c^3d^5x^{21} + \frac{(7c^2d^6x^{22})}{2}$

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)

[Out] $c^{**8}x^{**16}/8 + c^{**7}d^7x^{**17} + 7c^{**6}d^2x^{**18}/2 + 7c^{**5}d^3x^{**19} + 35c^{**4}d^4x^{**20}/4 + 7c^{**3}d^5x^{**21} + 7c^{**2}d^6x^{**22}/2 + c^7d^7x^{**23} + d^{**8}x^{**24}/8$

$$3.204 \quad \int x^8(2c + 3dx)(cx + dx^2)^7 dx$$

Optimal. Leaf size=18

$$\frac{1}{8}x^8(cx + dx^2)^8$$

[Out] 1/8*x^8*(d*x^2+c*x)^8

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {763}

$$\frac{1}{8}x^8(cx + dx^2)^8$$

Antiderivative was successfully verified.

[In] Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]

[Out] (x^8*(c*x + d*x^2)^8)/8

Rule 763

Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p + 2), 0] && NeQ[m + 2*p + 2, 0]

Rubi steps

$$\int x^8(2c + 3dx)(cx + dx^2)^7 dx = \frac{1}{8}x^8(cx + dx^2)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 5.44

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7}{2}c^6d^2x^{18} + 7c^5d^3x^{19} + \frac{35}{4}c^4d^4x^{20} + 7c^3d^5x^{21} + \frac{7}{2}c^2d^6x^{22} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

fricas [B] time = 0.56, size = 88, normalized size = 4.89

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8

giac [B] time = 0.29, size = 88, normalized size = 4.89

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="giac")

[Out] $1/8*d^8*x^{24} + c*d^7*x^{23} + 7/2*c^2*d^6*x^{22} + 7*c^3*d^5*x^{21} + 35/4*c^4*d^4*x^{20} + 7*c^5*d^3*x^{19} + 7/2*c^6*d^2*x^{18} + c^7*d*x^{17} + 1/8*c^8*x^{16}$

maple [B] time = 0.00, size = 89, normalized size = 4.94

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x)

[Out] $1/8*d^8*x^{24} + c*d^7*x^{23} + 7/2*c^2*d^6*x^{22} + 7*c^3*d^5*x^{21} + 35/4*c^4*d^4*x^{20} + 7*c^5*d^3*x^{19} + 7/2*c^6*d^2*x^{18} + c^7*d*x^{17} + 1/8*c^8*x^{16}$

maxima [B] time = 0.57, size = 88, normalized size = 4.89

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="maxima")

[Out] $1/8*d^8*x^{24} + c*d^7*x^{23} + 7/2*c^2*d^6*x^{22} + 7*c^3*d^5*x^{21} + 35/4*c^4*d^4*x^{20} + 7*c^5*d^3*x^{19} + 7/2*c^6*d^2*x^{18} + c^7*d*x^{17} + 1/8*c^8*x^{16}$

mupad [B] time = 0.04, size = 88, normalized size = 4.89

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(c*x + d*x^2)^7*(2*c + 3*d*x),x)

[Out] $(c^8*x^{16})/8 + (d^8*x^{24})/8 + c^7*d*x^{17} + c*d^7*x^{23} + (7*c^6*d^2*x^{18})/2 + 7*c^5*d^3*x^{19} + (35*c^4*d^4*x^{20})/4 + 7*c^3*d^5*x^{21} + (7*c^2*d^6*x^{22})/2$

sympy [B] time = 0.10, size = 97, normalized size = 5.39

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)

[Out] $c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8$

3.205 $\int x^{15}(c + dx)^7(2c + 3dx) dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c + dx)^8$$

[Out] 1/8*x^16*(d*x+c)^8

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {74}

$$\frac{1}{8}x^{16}(c + dx)^8$$

Antiderivative was successfully verified.

[In] Int[x^15*(c + d*x)^7*(2*c + 3*d*x), x]

[Out] (x^16*(c + d*x)^8)/8

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rubi steps

$$\int x^{15}(c + dx)^7(2c + 3dx) dx = \frac{1}{8}x^{16}(c + dx)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2}c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4}c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2}c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^15*(c + d*x)^7*(2*c + 3*d*x), x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

fricas [B] time = 0.79, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c), x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8

giac [B] time = 0.24, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁵*(d*x+c)⁷*(3*d*x+2*c),x, algorithm="giac")

[Out] 1/8*d⁸*x²⁴ + c*d⁷*x²³ + 7/2*c²*d⁶*x²² + 7*c³*d⁵*x²¹ + 35/4*c⁴*d⁴*x²⁰ + 7*c⁵*d³*x¹⁹ + 7/2*c⁶*d²*x¹⁸ + c⁷*d*x¹⁷ + 1/8*c⁸*x¹⁶

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁵*(d*x+c)⁷*(3*d*x+2*c),x)

[Out] 1/8*d⁸*x²⁴+c*d⁷*x²³+7/2*c²*d⁶*x²²+7*c³*d⁵*x²¹+35/4*c⁴*d⁴*x²⁰+7*c⁵*d³*x¹⁹+7/2*c⁶*d²*x¹⁸+c⁷*d*x¹⁷+1/8*c⁸*x¹⁶

maxima [B] time = 0.67, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁵*(d*x+c)⁷*(3*d*x+2*c),x, algorithm="maxima")

[Out] 1/8*d⁸*x²⁴ + c*d⁷*x²³ + 7/2*c²*d⁶*x²² + 7*c³*d⁵*x²¹ + 35/4*c⁴*d⁴*x²⁰ + 7*c⁵*d³*x¹⁹ + 7/2*c⁶*d²*x¹⁸ + c⁷*d*x¹⁷ + 1/8*c⁸*x¹⁶

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁵*(2*c + 3*d*x)*(c + d*x)⁷,x)

[Out] (c⁸*x¹⁶)/8 + (d⁸*x²⁴)/8 + c⁷*d*x¹⁷ + c*d⁷*x²³ + (7*c⁶*d²*x¹⁸)/2 + 7*c⁵*d³*x¹⁹ + (35*c⁴*d⁴*x²⁰)/4 + 7*c³*d⁵*x²¹ + (7*c²*d⁶*x²²)/2

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)

[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8

$$3.206 \quad \int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^4 \right) dx$$

Optimal. Leaf size=28

$$\frac{1}{160}x^5(2a + bx)^5 + ax + \frac{bx^2}{2}$$

[Out] a*x+1/2*b*x^2+1/160*x^5*(b*x+2*a)^5

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1591}

$$\frac{1}{160}x^5(2a + bx)^5 + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4), x]

[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^4 \right) dx &= \text{Subst} \left(\int (1 + x^4) dx, x, ax + \frac{bx^2}{2} \right) \\ &= ax + \frac{bx^2}{2} + \frac{1}{160}x^5(2a + bx)^5 \end{aligned}$$

Mathematica [B] time = 0.01, size = 80, normalized size = 2.86

$$\frac{a^5x^5}{5} + \frac{1}{2}a^4bx^6 + \frac{1}{2}a^3b^2x^7 + \frac{1}{4}a^2b^3x^8 + \frac{1}{16}ab^4x^9 + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4), x]

[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160

fricas [B] time = 0.71, size = 66, normalized size = 2.36

$$\frac{1}{160}x^{10}b^5 + \frac{1}{16}x^9b^4a + \frac{1}{4}x^8b^3a^2 + \frac{1}{2}x^7b^2a^3 + \frac{1}{2}x^6ba^4 + \frac{1}{5}x^5a^5 + \frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4), x, algorithm="fricas")

[Out] $1/160*x^{10}*b^5 + 1/16*x^9*b^4*a + 1/4*x^8*b^3*a^2 + 1/2*x^7*b^2*a^3 + 1/2*x^6*b*a^4 + 1/5*x^5*a^5 + 1/2*x^2*b + x*a$

giac [A] time = 0.37, size = 24, normalized size = 0.86

$$\frac{1}{160} (bx^2 + 2ax)^5 + \frac{1}{2} bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="giac")`

[Out] $1/160*(b*x^2 + 2*a*x)^5 + 1/2*b*x^2 + a*x$

maple [B] time = 0.00, size = 67, normalized size = 2.39

$$\frac{1}{160}b^5x^{10} + \frac{1}{16}ab^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4bx^6 + \frac{1}{5}a^5x^5 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x)`

[Out] $1/160*b^5*x^{10}+1/16*a*b^4*x^9+1/4*a^2*b^3*x^8+1/2*a^3*b^2*x^7+1/2*a^4*b*x^6+1/5*a^5*x^5+1/2*b*x^2+a*x$

maxima [B] time = 0.61, size = 66, normalized size = 2.36

$$\frac{1}{160}b^5x^{10} + \frac{1}{16}ab^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4bx^6 + \frac{1}{5}a^5x^5 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")`

[Out] $1/160*b^5*x^{10} + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x$

mupad [B] time = 0.05, size = 66, normalized size = 2.36

$$\frac{a^5x^5}{5} + \frac{a^4bx^6}{2} + \frac{a^3b^2x^7}{2} + \frac{a^2b^3x^8}{4} + \frac{ab^4x^9}{16} + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)`

[Out] $a*x + (b*x^2)/2 + (a^5*x^5)/5 + (b^5*x^{10})/160 + (a^4*b*x^6)/2 + (a*b^4*x^9)/16 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4$

sympy [B] time = 0.09, size = 70, normalized size = 2.50

$$\frac{a^5x^5}{5} + \frac{a^4bx^6}{2} + \frac{a^3b^2x^7}{2} + \frac{a^2b^3x^8}{4} + \frac{ab^4x^9}{16} + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)`

[Out] $a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4*x**9/16 + a*x + b**5*x**10/160 + b*x**2/2$

$$3.207 \quad \int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^4 \right) dx$$

Optimal. Leaf size=31

$$\frac{1}{5} \left(ax + \frac{bx^2}{2} + c \right)^5 + ax + \frac{bx^2}{2}$$

[Out] a*x+1/2*b*x^2+1/5*(c+a*x+1/2*b*x^2)^5

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1591}

$$\frac{1}{5} \left(ax + \frac{bx^2}{2} + c \right)^5 + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4), x]

[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^4 \right) dx &= \text{Subst} \left(\int (1 + x^4) dx, x, c + ax + \frac{bx^2}{2} \right) \\ &= ax + \frac{bx^2}{2} + \frac{1}{5} \left(c + ax + \frac{bx^2}{2} \right)^5 \end{aligned}$$

Mathematica [B] time = 0.04, size = 108, normalized size = 3.48

$$\frac{1}{160} x(2a+bx) \left(16a^4x^4 + 32a^3bx^5 + 24a^2b^2x^6 + 8ab^3x^7 + 80c^3x(2a+bx) + 40c^2x^2(2a+bx)^2 + 10cx^3(2a+bx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4), x]

[Out] (x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 + 8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 + 10*c*x^3*(2*a + b*x)^3))/160

fricas [B] time = 0.55, size = 208, normalized size = 6.71

$$\frac{1}{160} x^{10} b^5 + \frac{1}{16} x^9 b^4 a + \frac{1}{16} x^8 c b^4 + \frac{1}{4} x^8 b^3 a^2 + \frac{1}{2} x^7 c b^3 a + \frac{1}{2} x^7 b^2 a^3 + \frac{1}{4} x^6 c^2 b^3 + \frac{3}{2} x^6 c b^2 a^2 + \frac{1}{2} x^6 b a^4 + \frac{3}{2} x^5 c^2 b^2 a + 2 x^5 c b a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="fricas")

[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/16*x^8*c*b^4 + 1/4*x^8*b^3*a^2 + 1/2*x^7*c*b^3*a + 1/2*x^7*b^2*a^3 + 1/4*x^6*c^2*b^3 + 3/2*x^6*c*b^2*a^2 + 1/2*x^6*b*a^4 + 3/2*x^5*c^2*b^2*a + 2*x^5*c*b*a^3 + 1/5*x^5*a^5 + 1/2*x^4*c^3*b^2 + 3*x^4*c^2*b*a^2 + x^4*c*a^4 + 2*x^3*c^3*b*a + 2*x^3*c^2*a^3 + 1/2*x^2*c^4*b + 2*x^2*c^3*a^2 + x*c^4*a + 1/2*x^2*b + x*a

giac [B] time = 0.37, size = 88, normalized size = 2.84

$$\frac{1}{160} (bx^2 + 2ax)^5 + \frac{1}{16} (bx^2 + 2ax)^4 c + \frac{1}{4} (bx^2 + 2ax)^3 c^2 + \frac{1}{2} (bx^2 + 2ax)^2 c^3 + \frac{1}{2} (bx^2 + 2ax) c^4 + \frac{1}{2} bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="giac")

[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/16*(b*x^2 + 2*a*x)^4*c + 1/4*(b*x^2 + 2*a*x)^3*c^2 + 1/2*(b*x^2 + 2*a*x)^2*c^3 + 1/2*(b*x^2 + 2*a*x)*c^4 + 1/2*b*x^2 + a*x

maple [B] time = 0.00, size = 325, normalized size = 10.48

$$\frac{b^5 x^{10}}{160} + \frac{a b^4 x^9}{16} + \frac{\left(\frac{a^2 b^3}{2} + \left(a^2 b^2 + \frac{(a^2 + bc)b^2}{2}\right)b\right) x^8}{8} + \frac{\left(\left(a^2 b^2 + \frac{(a^2 + bc)b^2}{2}\right)a + (a b^2 c + 2(a^2 + bc)ab)\right) x^7}{7} + \frac{\left(a b^2 c + 2(a^2 + bc)ab\right) x^6}{6} + \frac{a^2 b^2 c + 2(a^2 + bc)ab}{5} x^5 + \frac{a^2 b^2 c + 2(a^2 + bc)ab}{4} x^4 + \frac{a^2 b^2 c + 2(a^2 + bc)ab}{3} x^3 + \frac{a^2 b^2 c + 2(a^2 + bc)ab}{2} x^2 + a^2 b^2 c + 2(a^2 + bc)ab$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x)

[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/8*(1/2*a^2*b^3+b*(1/2*(a^2+b*c)*b^2+a^2*b^2))*x^8+1/7*(a*(1/2*(a^2+b*c)*b^2+a^2*b^2)+b*(a*c*b^2+2*(a^2+b*c)*a*b))*x^7+1/6*(a*(a*c*b^2+2*(a^2+b*c)*a*b)+b*(1/2*c^2*b^2+4*a^2*c*b+(a^2+b*c)^2))*x^6+1/5*(a*(1/2*c^2*b^2+4*a^2*c*b+(a^2+b*c)^2)+b*(2*c^2*a*b+4*a*c*(a^2+b*c)))*x^5+1/4*(a*(2*c^2*a*b+4*a*c*(a^2+b*c))+b*(2*c^2*(a^2+b*c)+4*a^2*c^2))*x^4+1/3*(a*(2*c^2*(a^2+b*c)+4*a^2*c^2)+4*a*b*c^3)*x^3+1/2*(4*a^2*c^3+b*(c^4+1))*x^2+a*(c^4+1)*x

maxima [B] time = 0.44, size = 187, normalized size = 6.03

$$\frac{1}{160} b^5 x^{10} + \frac{1}{16} a b^4 x^9 + \frac{1}{16} (4 a^2 b^3 + b^4 c) x^8 + \frac{1}{2} (a^3 b^2 + a b^3 c) x^7 + \frac{1}{4} (2 a^4 b + 6 a^2 b^2 c + b^3 c^2) x^6 + \frac{1}{10} (2 a^5 + 20 a^3 b c + 15 a^2 b^2 c^2) x^5 + \frac{1}{2} (2 a^4 c + 6 a^2 b^2 c^2 + b^2 c^3) x^4 + 2 (a^3 c^2 + a b^2 c^3) x^3 + \frac{1}{2} (4 a^2 c^3 + b^2 c^4 + b) x^2 + (a^2 c^4 + a) x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")

[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b^2*c^2 + b^2*c^3)*x^4 + 2*(a^3*c^2 + a*b^2*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 + a)*x

mupad [B] time = 0.10, size = 180, normalized size = 5.81

$$x^6 \left(\frac{a^4 b}{2} + \frac{3 a^2 b^2 c}{2} + \frac{b^3 c^2}{4} \right) + x^4 \left(a^4 c + 3 a^2 b c^2 + \frac{b^2 c^3}{2} \right) + x^2 \left(2 a^2 c^3 + \frac{b c^4}{2} + \frac{b}{2} \right) + x^5 \left(\frac{a^5}{5} + 2 a^3 b c + \frac{3 a b^2 c^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((c + a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)

[Out] x^6*((a^4*b)/2 + (b^3*c^2)/4 + (3*a^2*b^2*c)/2) + x^4*(a^4*c + (b^2*c^3)/2 + 3*a^2*b*c^2) + x^2*(b/2 + (b*c^4)/2 + 2*a^2*c^3) + x^5*(a^5/5 + (3*a*b^2*c

$$3.208 \quad \int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^n \right) dx$$

Optimal. Leaf size=34

$$\frac{\left(ax + \frac{bx^2}{2} \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1591}

$$\frac{\left(ax + \frac{bx^2}{2} \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n), x]

[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^n \right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, ax + \frac{bx^2}{2} \right) \\ &= ax + \frac{bx^2}{2} + \frac{\left(ax + \frac{bx^2}{2} \right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.00

$$\frac{x(2a + bx) \left(\left(ax + \frac{bx^2}{2} \right)^n + n + 1 \right)}{2(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n), x]

[Out] (x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))

fricas [A] time = 0.79, size = 48, normalized size = 1.41

$$\frac{(bn + b)x^2 + (bx^2 + 2ax) \left(\frac{1}{2} bx^2 + ax \right)^n + 2(an + a)x}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="fricas")

[Out] $\frac{1}{2} * ((b * n + b) * x^2 + (b * x^2 + 2 * a * x) * (1/2 * b * x^2 + a * x)^n + 2 * (a * n + a) * x) / (n + 1)$

giac [A] time = 0.40, size = 30, normalized size = 0.88

$$\frac{1}{2} b x^2 + a x + \frac{\left(\frac{1}{2} b x^2 + a x\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="giac")

[Out] $\frac{1}{2} * b * x^2 + a * x + (1/2 * b * x^2 + a * x)^{(n + 1)} / (n + 1)$

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{b x^2}{2} + a x + \frac{\left(\frac{1}{2} b x^2 + a x\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x)

[Out] $a * x + 1/2 * b * x^2 + (a * x + 1/2 * b * x^2)^{(n+1)} / (n+1)$

maxima [A] time = 1.13, size = 52, normalized size = 1.53

$$\frac{1}{2} b x^2 + a x + \frac{(b x^2 + 2 a x) e^{(n \log(b x + 2 a) + n \log(x))}}{2^{n+1} n + 2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="maxima")

[Out] $\frac{1}{2} * b * x^2 + a * x + (b * x^2 + 2 * a * x) * e^{(n * \log(b * x + 2 * a) + n * \log(x))} / (2^{(n + 1)} * n + 2^{(n + 1)})$

mupad [B] time = 2.12, size = 31, normalized size = 0.91

$$\frac{x (2 a + b x) \left(n + \left(\frac{b x^2}{2} + a x\right)^n + 1\right)}{2 (n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)

[Out] $(x * (2 * a + b * x) * (n + (a * x + (b * x^2) / 2)^n + 1)) / (2 * (n + 1))$

sympy [A] time = 50.75, size = 230, normalized size = 6.76

$$\left\{ \begin{array}{ll} a \left(x + \frac{\log(x)}{a}\right) & \text{for } b = 0 \wedge n = -1 \\ a \left(\frac{a^n x x^n}{n+1} + \frac{n x}{n+1} + \frac{x}{n+1}\right) & \text{for } b = 0 \\ a x + \frac{b x^2}{2} + \log(x) + \log\left(\frac{2 a}{b} + x\right) & \text{for } n = -1 \\ \frac{2 \cdot 2^n a b n x}{2 \cdot 2^n b n + 2 \cdot 2^n b} + \frac{2 \cdot 2^n a b x}{2 \cdot 2^n b n + 2 \cdot 2^n b} + \frac{2^n b^2 n x^2}{2 \cdot 2^n b n + 2 \cdot 2^n b} + \frac{2^n b^2 x^2}{2 \cdot 2^n b n + 2 \cdot 2^n b} + \frac{2 a b x (2 a x + b x^2)^n}{2 \cdot 2^n b n + 2 \cdot 2^n b} + \frac{b^2 x^2 (2 a x + b x^2)^n}{2 \cdot 2^n b n + 2 \cdot 2^n b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)`

[Out] `Piecewise((a*(x + log(x))/a, Eq(b, 0) & Eq(n, -1)), (a*(a**n*x*x**n/(n + 1) + n*x/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a/b + x), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*2**n*b) + 2*2**n*a*b*x/(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b), True))`

$$3.209 \quad \int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^n \right) dx$$

Optimal. Leaf size=35

$$\frac{\left(ax + \frac{bx^2}{2} + c \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

[Out] a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1591}

$$\frac{\left(ax + \frac{bx^2}{2} + c \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n), x]

[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^n \right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, c + ax + \frac{bx^2}{2} \right) \\ &= ax + \frac{bx^2}{2} + \frac{\left(c + ax + \frac{bx^2}{2} \right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 1.00

$$\frac{\left(ax + \frac{bx^2}{2} + c \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n), x]

[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)

fricas [A] time = 0.95, size = 52, normalized size = 1.49

$$\frac{(bn + b)x^2 + (bx^2 + 2ax + 2c) \left(\frac{1}{2}bx^2 + ax + c \right)^n + 2(an + a)x}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="fricas")

[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*n + a)*x)/(n + 1)

giac [A] time = 0.25, size = 32, normalized size = 0.91

$$\frac{1}{2}bx^2 + ax + c + \frac{\left(\frac{1}{2}bx^2 + ax + c\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="giac")

[Out] 1/2*b*x^2 + a*x + c + (1/2*b*x^2 + a*x + c)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 33, normalized size = 0.94

$$\frac{bx^2}{2} + ax + c + \frac{\left(\frac{1}{2}bx^2 + ax + c\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x)

[Out] c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(n+1)/(n+1)

maxima [A] time = 1.16, size = 54, normalized size = 1.54

$$\frac{1}{2}bx^2 + ax + \frac{(bx^2 + 2ax + 2c)(bx^2 + 2ax + 2c)^n}{2^{n+1}n + 2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")

[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^(n + 1)*n + 2^(n + 1))

mupad [B] time = 2.11, size = 58, normalized size = 1.66

$$ax + \left(\frac{bx^2}{2} + ax + c\right)^n \left(\frac{2c}{2n+2} + \frac{bx^2}{2n+2} + \frac{2ax}{2n+2}\right) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c + a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)

[Out] a*x + (c + a*x + (b*x^2)/2)^n*((2*c)/(2*n + 2) + (b*x^2)/(2*n + 2) + (2*a*x)/(2*n + 2)) + (b*x^2)/2

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)

[Out] Timed out

$$3.210 \quad \int (a + cx^2) \left(1 + \left(ax + \frac{cx^3}{3}\right)^5\right) dx$$

Optimal. Leaf size=30

$$\frac{1}{6} \left(ax + \frac{cx^3}{3}\right)^6 + ax + \frac{cx^3}{3}$$

[Out] a*x+1/3*c*x^3+1/6*(a*x+1/3*c*x^3)^6

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1591}

$$\frac{1}{6} \left(ax + \frac{cx^3}{3}\right)^6 + ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]

[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + cx^2) \left(1 + \left(ax + \frac{cx^3}{3}\right)^5\right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, ax + \frac{cx^3}{3} \right) \\ &= ax + \frac{cx^3}{3} + \frac{1}{6} \left(ax + \frac{cx^3}{3}\right)^6 \end{aligned}$$

Mathematica [B] time = 0.01, size = 93, normalized size = 3.10

$$\frac{a^6 x^6}{6} + \frac{1}{3} a^5 c x^8 + \frac{5}{18} a^4 c^2 x^{10} + \frac{10}{81} a^3 c^3 x^{12} + \frac{5}{162} a^2 c^4 x^{14} + \frac{1}{243} a c^5 x^{16} + ax + \frac{c^6 x^{18}}{4374} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]

[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/4374

fricas [B] time = 0.76, size = 77, normalized size = 2.57

$$\frac{1}{4374} x^{18} c^6 + \frac{1}{243} x^{16} c^5 a + \frac{5}{162} x^{14} c^4 a^2 + \frac{10}{81} x^{12} c^3 a^3 + \frac{5}{18} x^{10} c^2 a^4 + \frac{1}{3} x^8 c a^5 + \frac{1}{6} x^6 a^6 + \frac{1}{3} x^3 c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="fricas")

[Out] $1/4374*x^{18}*c^6 + 1/243*x^{16}*c^5*a + 5/162*x^{14}*c^4*a^2 + 10/81*x^{12}*c^3*a^3 + 5/18*x^{10}*c^2*a^4 + 1/3*x^8*c*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + x*a$

giac [A] time = 0.22, size = 24, normalized size = 0.80

$$\frac{1}{4374} (cx^3 + 3ax)^6 + \frac{1}{3} cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="giac")`

[Out] $1/4374*(c*x^3 + 3*a*x)^6 + 1/3*c*x^3 + a*x$

maple [B] time = 0.00, size = 78, normalized size = 2.60

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5cx^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x)`

[Out] $1/4374*c^6*x^{18}+1/243*a*c^5*x^{16}+5/162*a^2*c^4*x^{14}+10/81*a^3*c^3*x^{12}+5/18*a^4*c^2*x^{10}+1/3*a^5*c*x^8+1/6*a^6*x^6+1/3*c*x^3+a*x$

maxima [B] time = 0.44, size = 77, normalized size = 2.57

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5cx^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")`

[Out] $1/4374*c^6*x^{18} + 1/243*a*c^5*x^{16} + 5/162*a^2*c^4*x^{14} + 10/81*a^3*c^3*x^{12} + 5/18*a^4*c^2*x^{10} + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x$

mupad [B] time = 0.05, size = 77, normalized size = 2.57

$$\frac{a^6x^6}{6} + \frac{a^5cx^8}{3} + \frac{5a^4c^2x^{10}}{18} + \frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162} + \frac{ac^5x^{16}}{243} + ax + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)*((a*x + (c*x^3)/3)^5 + 1),x)`

[Out] $a*x + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^{18})/4374 + (a^5*c*x^8)/3 + (a*c^5*x^{16})/243 + (5*a^4*c^2*x^{10})/18 + (10*a^3*c^3*x^{12})/81 + (5*a^2*c^4*x^{14})/162$

sympy [B] time = 0.09, size = 87, normalized size = 2.90

$$\frac{a^6x^6}{6} + \frac{a^5cx^8}{3} + \frac{5a^4c^2x^{10}}{18} + \frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162} + \frac{ac^5x^{16}}{243} + ax + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)`

[Out] $a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81 + 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x**3/3$

$$3.211 \quad \int (a + cx^2) \left(1 + \left(d + ax + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=31

$$\frac{1}{6} \left(ax + \frac{cx^3}{3} + d \right)^6 + ax + \frac{cx^3}{3}$$

[Out] a*x+1/3*c*x^3+1/6*(d+a*x+1/3*c*x^3)^6

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1591}

$$\frac{1}{6} \left(ax + \frac{cx^3}{3} + d \right)^6 + ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5), x]

[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + cx^2) \left(1 + \left(d + ax + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, d + ax + \frac{cx^3}{3} \right) \\ &= ax + \frac{cx^3}{3} + \frac{1}{6} \left(d + ax + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.05, size = 140, normalized size = 4.52

$$\frac{x(3a + cx^2) \left(243a^5x^5 + 405a^4cx^7 + 270a^3c^2x^9 + 90a^2c^3x^{11} + 15ac^4x^{13} + 1215d^4(3ax + cx^3) + 540d^3(3ax + cx^3) \right)}{4374}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5), x]

[Out] (x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*c^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c*x^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*x + c*x^3)^4))/4374

fricas [B] time = 0.99, size = 291, normalized size = 9.39

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{243}x^{16}c^5a + \frac{1}{243}x^{15}dc^5 + \frac{5}{162}x^{14}c^4a^2 + \frac{5}{81}x^{13}dc^4a + \frac{5}{162}x^{12}d^2c^4 + \frac{10}{81}x^{12}c^3a^3 + \frac{10}{27}x^{11}dc^3a^2 + \frac{10}{27}x^{10}d^2c^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")

[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 1/243*x^15*d*c^5 + 5/162*x^14*c^4*a^2 + 5/81*x^13*d*c^4*a + 5/162*x^12*d^2*c^4 + 10/81*x^12*c^3*a^3 + 10/27*x^11*d*c^3*a^2 + 10/27*x^10*d^2*c^3*a + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 + 10/9*x^9*d*c^2*a^3 + 5/3*x^8*d^2*c^2*a^2 + 1/3*x^8*c*a^5 + 10/9*x^7*d^3*c^2*a + 5/3*x^7*d*c*a^4 + 5/18*x^6*d^4*c^2 + 10/3*x^6*d^2*c*a^3 + 1/6*x^6*a^6 + 10/3*x^5*d^3*c*a^2 + x^5*d*a^5 + 5/3*x^4*d^4*c*a + 5/2*x^4*d^2*a^4 + 1/3*x^3*d^5*c + 10/3*x^3*d^3*a^3 + 5/2*x^2*d^4*a^2 + x*d^5*a + 1/3*x^3*c + x*a

giac [B] time = 0.34, size = 105, normalized size = 3.39

$$\frac{1}{4374} (cx^3 + 3ax)^6 + \frac{1}{243} (cx^3 + 3ax)^5 d + \frac{5}{162} (cx^3 + 3ax)^4 d^2 + \frac{10}{81} (cx^3 + 3ax)^3 d^3 + \frac{5}{18} (cx^3 + 3ax)^2 d^4 + \frac{1}{3} (cx^3 + 3ax) d^5 + \frac{1}{3} cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")

[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/243*(c*x^3 + 3*a*x)^5*d + 5/162*(c*x^3 + 3*a*x)^4*d^2 + 10/81*(c*x^3 + 3*a*x)^3*d^3 + 5/18*(c*x^3 + 3*a*x)^2*d^4 + 1/3*(c*x^3 + 3*a*x)*d^5 + 1/3*c*x^3 + a*x

maple [B] time = 0.00, size = 618, normalized size = 19.94

$$\frac{c^6 x^{18}}{4374} + \frac{a c^5 x^{16}}{243} + \frac{c^5 d x^{15}}{243} + \frac{5 a^2 c^4 x^{14}}{162} + \frac{5 a c^4 d x^{13}}{81} + \frac{10 a^2 c^3 d x^{11}}{27} + \frac{\left(\frac{10 a^3 c^3}{27} + \left(\frac{2 a^3 c^2}{3} + \frac{4 c^3 d^2}{27} + \frac{\left(\frac{4}{3} a^3 c + \frac{2}{3} c^2 d^2 \right) c}{3} \right) c \right) x^{12}}{12} + \left(\frac{2 a^3 c^2}{3} + \frac{4 c^3 d^2}{27} + \frac{\left(\frac{4}{3} a^3 c + \frac{2}{3} c^2 d^2 \right) c}{3} \right) x^{12} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x)

[Out] 1/4374*c^6*x^18+1/243*a*c^5*x^16+1/243*c^5*d*x^15+5/162*a^2*c^4*x^14+5/81*a*c^4*d*x^13+1/12*(10/27*a^3*c^3+c*(4/27*d^2*c^3+2/3*a^3*c^2+1/3*c*(2/3*d^2*c^2+4/3*a^3*c))) *x^12+10/27*a^2*c^3*d*x^11+1/10*(a*(4/27*d^2*c^3+2/3*a^3*c^2+1/3*c*(2/3*d^2*c^2+4/3*a^3*c))+c*(4/3*d^2*a*c^2+a*(2/3*d^2*c^2+4/3*a^3*c)+1/3*c*(a^4+4*a*c*d^2))) *x^10+1/9*(10/3*a^3*d*c^2+c*(d*(2/3*d^2*c^2+4/3*a^3*c)+4*a^3*d*c+1/3*c*(4/3*d^3*c+4*a^3*d))) *x^9+1/8*(a*(4/3*d^2*a*c^2+a*(2/3*d^2*c^2+4/3*a^3*c)+1/3*c*(a^4+4*a*c*d^2))+c*(6*d^2*a^2*c+a*(a^4+4*a*c*d^2))) *x^8+1/7*(a*(d*(2/3*d^2*c^2+4/3*a^3*c)+4*a^3*d*c+1/3*c*(4/3*d^3*c+4*a^3*d))+c*(d*(a^4+4*a*c*d^2)+a*(4/3*d^3*c+4*a^3*d)+4/3*c*d^3*a)) *x^7+1/6*(a*(6*d^2*a^2*c+a*(a^4+4*a*c*d^2))+c*(d*(4/3*d^3*c+4*a^3*d)+6*a^3*d^2+1/3*c*d^4)) *x^6+1/5*(a*(d*(a^4+4*a*c*d^2)+a*(4/3*d^3*c+4*a^3*d)+4/3*c*d^3*a)+10*a^2*c*d^3)*x^5+1/4*(a*(d*(4/3*d^3*c+4*a^3*d)+6*a^3*d^2+1/3*c*d^4)+5*a*c*d^4)*x^4+1/3*(10*a^3*d^3+c*(d^5+1))*x^3+5/2*a^2*d^4*x^2+a*(d^5+1)*x

maxima [B] time = 0.46, size = 280, normalized size = 9.03

$$\frac{1}{4374} c^6 x^{18} + \frac{1}{243} a c^5 x^{16} + \frac{1}{243} c^5 d x^{15} + \frac{5}{162} a^2 c^4 x^{14} + \frac{5}{81} a c^4 d x^{13} + \frac{10}{27} a^2 c^3 d x^{11} + \frac{5}{162} (4 a^3 c^3 + c^4 d^2) x^{12} + \frac{5}{54} (3 a^4 c^2 + 4 a^3 c d^2) x^{10} + \frac{10}{81} (9 a^3 c^2 d + c^3 d^3) x^9 + \frac{1}{3} (a^5 c + 5 a^2 c^2 d^2) x^8 + \frac{5}{2} a^2 d^4 x^2 + \frac{5}{9} (3 a^4 c d + 2 a^3 d^2) x^7 + \frac{1}{3} (a^4 c^2 + 4 a^3 c d^2) x^6 + \frac{1}{3} (a^4 c^2 + 4 a^3 c d^2) x^5 + \frac{1}{3} (a^4 c^2 + 4 a^3 c d^2) x^4 + \frac{1}{3} (a^4 c^2 + 4 a^3 c d^2) x^3 + \frac{1}{3} (a^4 c^2 + 4 a^3 c d^2) x^2 + \frac{1}{3} (a^4 c^2 + 4 a^3 c d^2) x + \frac{1}{3} (a^4 c^2 + 4 a^3 c d^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="maxima")

[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14 + 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*x^12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)*x^9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*a^3*d^2)*x^7 + 1/3*(a^4*c^2 + 4*a^3*c*d^2)*x^6 + 1/3*(a^4*c^2 + 4*a^3*c*d^2)*x^5 + 1/3*(a^4*c^2 + 4*a^3*c*d^2)*x^4 + 1/3*(a^4*c^2 + 4*a^3*c*d^2)*x^3 + 1/3*(a^4*c^2 + 4*a^3*c*d^2)*x^2 + 1/3*(a^4*c^2 + 4*a^3*c*d^2)*x + 1/3*(a^4*c^2 + 4*a^3*c*d^2)

$$a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 + c*d^5 + c)*x^3 + (a*d^5 + a)*x$$

mupad [B] time = 2.27, size = 266, normalized size = 8.58

$$x^5 \left(a^5 d + \frac{10 c a^2 d^3}{3} \right) + x^4 \left(\frac{5 a^4 d^2}{2} + \frac{5 c a d^4}{3} \right) + x^3 \left(\frac{10 a^3 d^3}{3} + \frac{c d^5}{3} + \frac{c}{3} \right) + x^6 \left(\frac{a^6}{6} + \frac{10 a^3 c d^2}{3} + \frac{5 c^2 d^4}{18} \right) + \frac{c^6 x}{4374}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + a*x + (c*x^3)/3)^5 + 1)*(a + c*x^2), x)

[Out] $x^5*(a^5*d + (10*a^2*c*d^3)/3) + x^4*((5*a^4*d^2)/2 + (5*a*c*d^4)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3) + x^6*(a^6/6 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3) + (c^6*x^18)/4374 + (a*c^5*x^16)/243 + a*x*(d^5 + 1) + (c^5*d*x^15)/243 + (5*a^2*c^4*x^14)/162 + (5*a^2*d^4*x^2)/2 + (5*c^3*x^12*(c*d^2 + 4*a^3))/162 + (a^2*c*x^8*(5*c*d^2 + a^3))/3 + (10*a^2*c^3*d*x^11)/27 + (5*a*c^2*x^10*(4*c*d^2 + 3*a^3))/54 + (10*c^2*d*x^9*(c*d^2 + 9*a^3))/81 + (5*a*c^4*d*x^13)/81 + (5*a*c*d*x^7*(2*c*d^2 + 3*a^3))/9$

sympy [B] time = 0.14, size = 314, normalized size = 10.13

$$\frac{5a^2c^4x^{14}}{162} + \frac{10a^2c^3dx^{11}}{27} + \frac{5a^2d^4x^2}{2} + \frac{ac^5x^{16}}{243} + \frac{5ac^4dx^{13}}{81} + \frac{c^6x^{18}}{4374} + \frac{c^5dx^{15}}{243} + x^{12} \left(\frac{10a^3c^3}{81} + \frac{5c^4d^2}{162} \right) + x^{10} \left(\frac{5a^4c^2}{18} + \frac{10a^3c^2}{27} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5), x)

[Out] $5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c**5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 + x**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*c**3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a**6/6 + 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/3) + x**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3 + c/3) + x*(a*d**5 + a)$

$$3.212 \quad \int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=34

$$\frac{x^{12}(3b + 2cx)^6}{279936} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

[Out] 1/2*b*x^2+1/3*c*x^3+1/279936*x^12*(2*c*x+3*b)^6

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {1591}

$$\frac{x^{12}(3b + 2cx)^6}{279936} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{x^{12}(3b + 2cx)^6}{279936} \end{aligned}$$

Mathematica [B] time = 0.01, size = 98, normalized size = 2.88

$$\frac{b^6 x^{12}}{384} + \frac{1}{96} b^5 c x^{13} + \frac{5}{288} b^4 c^2 x^{14} + \frac{5}{324} b^3 c^3 x^{15} + \frac{5}{648} b^2 c^4 x^{16} + \frac{1}{486} b c^5 x^{17} + \frac{bx^2}{2} + \frac{c^6 x^{18}}{4374} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (c^6*x^18)/4374

fricas [B] time = 0.68, size = 80, normalized size = 2.35

$$\frac{1}{4374} x^{18} c^6 + \frac{1}{486} x^{17} c^5 b + \frac{5}{648} x^{16} c^4 b^2 + \frac{5}{324} x^{15} c^3 b^3 + \frac{5}{288} x^{14} c^2 b^4 + \frac{1}{96} x^{13} c b^5 + \frac{1}{384} x^{12} b^6 + \frac{1}{3} x^3 c + \frac{1}{2} x^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5), x, algorithm="fricas")

[Out] $1/4374*x^{18}*c^6 + 1/486*x^{17}*c^5*b + 5/648*x^{16}*c^4*b^2 + 5/324*x^{15}*c^3*b^3 + 5/288*x^{14}*c^2*b^4 + 1/96*x^{13}*c*b^5 + 1/384*x^{12}*b^6 + 1/3*x^3*c + 1/2*x^2*b$

giac [A] time = 0.29, size = 30, normalized size = 0.88

$$\frac{1}{279936} (2cx^3 + 3bx^2)^6 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")`

[Out] $1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/3*c*x^3 + 1/2*b*x^2$

maple [B] time = 0.00, size = 81, normalized size = 2.38

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x)`

[Out] $1/4374*c^6*x^{18} + 1/486*b*c^5*x^{17} + 5/648*b^2*c^4*x^{16} + 5/324*b^3*c^3*x^{15} + 5/288*b^4*c^2*x^{14} + 1/96*b^5*c*x^{13} + 1/384*b^6*x^{12} + 1/3*c*x^3 + 1/2*b*x^2$

maxima [B] time = 0.44, size = 80, normalized size = 2.35

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")`

[Out] $1/4374*c^6*x^{18} + 1/486*b*c^5*x^{17} + 5/648*b^2*c^4*x^{16} + 5/324*b^3*c^3*x^{15} + 5/288*b^4*c^2*x^{14} + 1/96*b^5*c*x^{13} + 1/384*b^6*x^{12} + 1/3*c*x^3 + 1/2*b*x^2$

mupad [B] time = 0.07, size = 80, normalized size = 2.35

$$\frac{b^6x^{12}}{384} + \frac{b^5cx^{13}}{96} + \frac{5b^4c^2x^{14}}{288} + \frac{5b^3c^3x^{15}}{324} + \frac{5b^2c^4x^{16}}{648} + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x + c*x^2)*((b*x^2)/2 + (c*x^3)/3)^5 + 1),x)`

[Out] $(b*x^2)/2 + (c*x^3)/3 + (b^6*x^{12})/384 + (c^6*x^{18})/4374 + (b^5*c*x^{13})/96 + (b*c^5*x^{17})/486 + (5*b^4*c^2*x^{14})/288 + (5*b^3*c^3*x^{15})/324 + (5*b^2*c^4*x^{16})/648$

sympy [B] time = 0.10, size = 90, normalized size = 2.65

$$\frac{b^6x^{12}}{384} + \frac{b^5cx^{13}}{96} + \frac{5b^4c^2x^{14}}{288} + \frac{5b^3c^3x^{15}}{324} + \frac{5b^2c^4x^{16}}{648} + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)`

[Out] $b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**15/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/4374 + c*x**3/3$

$$3.213 \quad \int (bx + cx^2) \left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=41

$$\frac{1}{6} \left(\frac{bx^2}{2} + \frac{cx^3}{3} + d \right)^6 + \frac{bx^2}{2} + \frac{cx^3}{3}$$

[Out] 1/2*b*x^2+1/3*c*x^3+1/6*(d+1/2*b*x^2+1/3*c*x^3)^6

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1591}

$$\frac{1}{6} \left(\frac{bx^2}{2} + \frac{cx^3}{3} + d \right)^6 + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (bx + cx^2) \left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, d + \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{6} \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.05, size = 146, normalized size = 3.56

$$\frac{x^2(3b + 2cx) \left(243b^5x^{10} + 810b^4cx^{11} + 1080b^3c^2x^{12} + 720b^2c^3x^{13} + 240bc^4x^{14} + 19440d^4x^2(3b + 2cx) + 4320d^3x^3 \right)}{279936}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*d^4*x^2*(3*b + 2*c*x) + 4320*d^3*x^3*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b + 2*c*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936

fricas [B] time = 0.93, size = 298, normalized size = 7.27

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{486}x^{17}c^5b + \frac{5}{648}x^{16}c^4b^2 + \frac{1}{243}x^{15}dc^5 + \frac{5}{324}x^{15}c^3b^3 + \frac{5}{162}x^{14}dc^4b + \frac{5}{288}x^{14}c^2b^4 + \frac{5}{54}x^{13}dc^3b^2 + \frac{1}{96}x^{13}cb^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")

[Out] $\frac{1}{4374}x^{18}c^6 + \frac{1}{486}x^{17}c^5b + \frac{5}{648}x^{16}c^4b^2 + \frac{1}{243}x^{15}d^5c^5 + \frac{5}{324}x^{15}c^3b^3 + \frac{5}{162}x^{14}d^4c^4b + \frac{5}{288}x^{14}c^2b^4 + \frac{5}{54}x^{13}d^3c^3b^2 + \frac{1}{96}x^{13}c^2b^5 + \frac{5}{162}x^{12}d^2c^4 + \frac{5}{36}x^{12}d^2c^2b^3 + \frac{1}{384}x^{12}b^6 + \frac{5}{27}x^{11}d^2c^3b + \frac{5}{48}x^{11}d^2c^2b^4 + \frac{5}{12}x^{10}d^2c^2b^2 + \frac{1}{32}x^{10}d^2b^5 + \frac{10}{81}x^9d^3c^3 + \frac{5}{12}x^9d^2c^2b^3 + \frac{5}{9}x^8d^3c^2b + \frac{5}{32}x^8d^2b^4 + \frac{5}{6}x^7d^3c^2b^2 + \frac{5}{18}x^6d^4c^2 + \frac{5}{12}x^6d^3b^3 + \frac{5}{6}x^5d^4c^2b + \frac{5}{8}x^4d^4b^2 + \frac{1}{3}x^3d^5c + \frac{1}{2}x^2d^5b + \frac{1}{3}x^3c + \frac{1}{2}x^2b$

giac [B] time = 0.31, size = 126, normalized size = 3.07

$$\frac{1}{279936} (2cx^3 + 3bx^2)^6 + \frac{1}{7776} (2cx^3 + 3bx^2)^5 d + \frac{5}{2592} (2cx^3 + 3bx^2)^4 d^2 + \frac{5}{324} (2cx^3 + 3bx^2)^3 d^3 + \frac{5}{72} (2cx^3 + 3bx^2)^2 d^4 + \frac{1}{6} (2cx^3 + 3bx^2) d^5 + \frac{1}{3} c x^3 + \frac{1}{2} b x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")

[Out] $\frac{1}{279936}(2cx^3 + 3bx^2)^6 + \frac{1}{7776}(2cx^3 + 3bx^2)^5d + \frac{5}{2592}(2cx^3 + 3bx^2)^4d^2 + \frac{5}{324}(2cx^3 + 3bx^2)^3d^3 + \frac{5}{72}(2cx^3 + 3bx^2)^2d^4 + \frac{1}{6}(2cx^3 + 3bx^2)d^5 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$

maple [B] time = 0.00, size = 646, normalized size = 15.76

$$\frac{c^6x^{18}}{4374} + \frac{bc^5x^{17}}{486} + \frac{5b^2c^4x^{16}}{648} + \frac{\left(\frac{5b^3c^3}{54} + \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3}\right)c\right)x^{15}}{15} + \frac{\left(\left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3}\right)b + \left(\frac{2bc^3d}{27} + \frac{1}{3}c\right)\right)x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x)

[Out] $\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{15} \left(\frac{5}{54}b^3c^3 + c \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) \right) x^{15} + \frac{1}{14} \left(b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \left(\frac{2bc^3d}{27} + \frac{1}{3}c \right) \right) x^{14} + \frac{1}{13} \left(b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{2}b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{3}c \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) \right) x^{13} + \frac{1}{12} \left(b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{2}b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{3}c \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) \right) x^{12} + \frac{1}{11} \left(b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{2}b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{3}c \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) \right) x^{11} + \frac{1}{10} \left(b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{2}b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{3}c \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) \right) x^{10} + \frac{1}{9} \left(\frac{5}{2}b^3c^3d^2 + c \left(\frac{2}{3}c^2d^2 + \frac{1}{2}d^2b^3 \right) + \frac{3}{4}b^3d^2 + \frac{4}{9}c^2d^3 \right) x^9 + \frac{1}{8} \left(b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{2}b \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) + \frac{1}{3}c \left(\frac{b^3c^2}{12} + \frac{c^4d}{81} + \frac{\left(\frac{1}{6}b^3c + \frac{4}{27}c^3d\right)c}{3} \right) \right) x^8 + \frac{5}{6}b^2c^2d^3x^7 + \frac{1}{6} \left(\frac{5}{2}b^3d^3 + \frac{5}{3}c^2d^4 \right) x^6 + \frac{5}{6}b^2c^2d^4x^5 + \frac{5}{8}b^2d^4x^4 + \frac{1}{3} \left(d^5 + 1 \right) c x^3 + \frac{1}{2}b \left(d^5 + 1 \right) x^2$

maxima [B] time = 0.45, size = 289, normalized size = 7.05

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{972}(15b^3c^3 + 4c^5d)x^{15} + \frac{5}{2592}(9b^4c^2 + 16bc^4d)x^{14} + \frac{1}{864}(9b^5c + 80b^2c^3d)x^{13} + \frac{5}{6}b^2c^2d^3x^7 + \frac{1}{10368}(27b^6 + 1440b^3c^2d + 80b^2c^3d)x^{13} + \frac{5}{6}b^2c^2d^3x^7 + \frac{1}{10368}(27b^6 + 1440b^3c^2d + 80b^2c^3d)x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")

[Out] $\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{972}(15b^3c^3 + 4c^5d)x^{15} + \frac{5}{2592}(9b^4c^2 + 16bc^4d)x^{14} + \frac{1}{864}(9b^5c + 80b^2c^3d)x^{13} + \frac{5}{6}b^2c^2d^3x^7 + \frac{1}{10368}(27b^6 + 1440b^3c^2d + 80b^2c^3d)x^{13}$

$$320*c^4*d^2)*x^{12} + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^{11} + 5/6*b*c*d^4*x^5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^{10} + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2$$

mupad [B] time = 2.28, size = 273, normalized size = 6.66

$$x^{13} \left(\frac{b^5 c}{96} + \frac{5 d b^2 c^3}{54} \right) + x^{14} \left(\frac{5 b^4 c^2}{288} + \frac{5 d b c^4}{162} \right) + x^{12} \left(\frac{b^6}{384} + \frac{5 b^3 c^2 d}{36} + \frac{5 c^4 d^2}{162} \right) + \frac{c^6 x^{18}}{4374} + x^{15} \left(\frac{5 b^3 c^3}{324} + \frac{d c^5}{243} \right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)*((d + (b*x^2)/2 + (c*x^3)/3)^5 + 1), x)

[Out] x^13*((b^5*c)/96 + (5*b^2*c^3*d)/54) + x^14*((5*b^4*c^2)/288 + (5*b*c^4*d)/162) + x^12*(b^6/384 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36) + (c^6*x^18)/4374 + x^15*((c^5*d)/243 + (5*b^3*c^3)/324) + (5*d^3*x^6*(2*c^2*d + 3*b^3))/36 + (b*c^5*x^17)/486 + (5*b^2*c^4*x^16)/648 + (b*x^2*(d^5 + 1))/2 + (5*b^2*d^4*x^4)/8 + (c*x^3*(d^5 + 1))/3 + (5*b^2*c*d^3*x^7)/6 + (5*b*d^2*x^8*(32*c^2*d + 9*b^3))/288 + (b^2*d*x^10*(40*c^2*d + 3*b^3))/96 + (5*c*d^2*x^9*(8*c^2*d + 27*b^3))/324 + (5*b*c*d^4*x^5)/6 + (5*b*c*d*x^11*(16*c^2*d + 9*b^3))/432

sympy [B] time = 0.15, size = 321, normalized size = 7.83

$$\frac{5b^2c^4x^{16}}{648} + \frac{5b^2cd^3x^7}{6} + \frac{5b^2d^4x^4}{8} + \frac{bc^5x^{17}}{486} + \frac{5bcd^4x^5}{6} + \frac{c^6x^{18}}{4374} + x^{15} \left(\frac{5b^3c^3}{324} + \frac{c^5d}{243} \right) + x^{14} \left(\frac{5b^4c^2}{288} + \frac{5bc^4d}{162} \right) + x^{13} \left(\frac{b^5c}{96} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5), x)

[Out] 5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8 + b*c**5*x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15*(5*b**3*c**3/324 + c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(b**5*c/96 + 5*b**2*c**3*d/54) + x**12*(b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3*(c*d**5/3 + c/3) + x**2*(b*d**5/2 + b/2)

$$3.214 \quad \int \left(a + bx + cx^2 \right) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=46

$$\frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(a*x+1/2*b*x^2+1/3*c*x^3)^6

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1591}

$$\frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.06, size = 244, normalized size = 5.30

$$\frac{a^6 x^6}{6} + \frac{1}{6} a^5 x^7 (3b + 2cx) + \frac{5}{72} a^4 x^8 (3b + 2cx)^2 + \frac{5}{324} a^3 x^9 (3b + 2cx)^3 + \frac{5a^2 x^{10} (3b + 2cx)^4}{2592} + a \left(\frac{b^5 x^{11}}{32} + \frac{5}{48} b^4 c x^{12} + \frac{5}{36} b^3 c^2 x^{13} + \frac{5}{54} b^2 c^3 x^{14} + \frac{5}{162} b c^4 x^{15} + \frac{c^5 x^{16}}{243} \right) + \frac{x^2 (729 b^6 x^{10} + 2916 b^5 c x^{11} + 4860 b^4 c^2 x^{12} + 4320 b^3 c^3 x^{13} + 2160 b^2 c^4 x^{14} + 576 b c^4 x^{15} + 64 c^5 x^{16})}{279936}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 + (5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(x + (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x^14)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916*b^5*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 + 576*b*c^4*x^15 + 64*c^5*x^16))/279936

fricas [B] time = 0.68, size = 309, normalized size = 6.72

$$\frac{1}{4374} x^{18} c^6 + \frac{1}{486} x^{17} c^5 b + \frac{5}{648} x^{16} c^4 b^2 + \frac{1}{243} x^{16} c^5 a + \frac{5}{324} x^{15} c^3 b^3 + \frac{5}{162} x^{15} c^4 b a + \frac{5}{288} x^{14} c^2 b^4 + \frac{5}{54} x^{14} c^3 b^2 a + \frac{5}{162} x^{14} c^4 b a^2 + \frac{5}{162} x^{14} c^5 a^2 b + \frac{5}{162} x^{14} c^6 a^3 + \frac{5}{162} x^{14} c^7 a^4 + \frac{5}{162} x^{14} c^8 a^5 + \frac{5}{162} x^{14} c^9 a^6 + \frac{5}{162} x^{14} c^{10} a^7 + \frac{5}{162} x^{14} c^{11} a^8 + \frac{5}{162} x^{14} c^{12} a^9 + \frac{5}{162} x^{14} c^{13} a^{10} + \frac{5}{162} x^{14} c^{14} a^{11} + \frac{5}{162} x^{14} c^{15} a^{12} + \frac{5}{162} x^{14} c^{16} a^{13} + \frac{5}{162} x^{14} c^{17} a^{14} + \frac{5}{162} x^{14} c^{18} a^{15} + \frac{5}{162} x^{14} c^{19} a^{16} + \frac{5}{162} x^{14} c^{20} a^{17} + \frac{5}{162} x^{14} c^{21} a^{18} + \frac{5}{162} x^{14} c^{22} a^{19} + \frac{5}{162} x^{14} c^{23} a^{20} + \frac{5}{162} x^{14} c^{24} a^{21} + \frac{5}{162} x^{14} c^{25} a^{22} + \frac{5}{162} x^{14} c^{26} a^{23} + \frac{5}{162} x^{14} c^{27} a^{24} + \frac{5}{162} x^{14} c^{28} a^{25} + \frac{5}{162} x^{14} c^{29} a^{26} + \frac{5}{162} x^{14} c^{30} a^{27} + \frac{5}{162} x^{14} c^{31} a^{28} + \frac{5}{162} x^{14} c^{32} a^{29} + \frac{5}{162} x^{14} c^{33} a^{30} + \frac{5}{162} x^{14} c^{34} a^{31} + \frac{5}{162} x^{14} c^{35} a^{32} + \frac{5}{162} x^{14} c^{36} a^{33} + \frac{5}{162} x^{14} c^{37} a^{34} + \frac{5}{162} x^{14} c^{38} a^{35} + \frac{5}{162} x^{14} c^{39} a^{36} + \frac{5}{162} x^{14} c^{40} a^{37} + \frac{5}{162} x^{14} c^{41} a^{38} + \frac{5}{162} x^{14} c^{42} a^{39} + \frac{5}{162} x^{14} c^{43} a^{40} + \frac{5}{162} x^{14} c^{44} a^{41} + \frac{5}{162} x^{14} c^{45} a^{42} + \frac{5}{162} x^{14} c^{46} a^{43} + \frac{5}{162} x^{14} c^{47} a^{44} + \frac{5}{162} x^{14} c^{48} a^{45} + \frac{5}{162} x^{14} c^{49} a^{46} + \frac{5}{162} x^{14} c^{50} a^{47} + \frac{5}{162} x^{14} c^{51} a^{48} + \frac{5}{162} x^{14} c^{52} a^{49} + \frac{5}{162} x^{14} c^{53} a^{50} + \frac{5}{162} x^{14} c^{54} a^{51} + \frac{5}{162} x^{14} c^{55} a^{52} + \frac{5}{162} x^{14} c^{56} a^{53} + \frac{5}{162} x^{14} c^{57} a^{54} + \frac{5}{162} x^{14} c^{58} a^{55} + \frac{5}{162} x^{14} c^{59} a^{56} + \frac{5}{162} x^{14} c^{60} a^{57} + \frac{5}{162} x^{14} c^{61} a^{58} + \frac{5}{162} x^{14} c^{62} a^{59} + \frac{5}{162} x^{14} c^{63} a^{60} + \frac{5}{162} x^{14} c^{64} a^{61} + \frac{5}{162} x^{14} c^{65} a^{62} + \frac{5}{162} x^{14} c^{66} a^{63} + \frac{5}{162} x^{14} c^{67} a^{64} + \frac{5}{162} x^{14} c^{68} a^{65} + \frac{5}{162} x^{14} c^{69} a^{66} + \frac{5}{162} x^{14} c^{70} a^{67} + \frac{5}{162} x^{14} c^{71} a^{68} + \frac{5}{162} x^{14} c^{72} a^{69} + \frac{5}{162} x^{14} c^{73} a^{70} + \frac{5}{162} x^{14} c^{74} a^{71} + \frac{5}{162} x^{14} c^{75} a^{72} + \frac{5}{162} x^{14} c^{76} a^{73} + \frac{5}{162} x^{14} c^{77} a^{74} + \frac{5}{162} x^{14} c^{78} a^{75} + \frac{5}{162} x^{14} c^{79} a^{76} + \frac{5}{162} x^{14} c^{80} a^{77} + \frac{5}{162} x^{14} c^{81} a^{78} + \frac{5}{162} x^{14} c^{82} a^{79} + \frac{5}{162} x^{14} c^{83} a^{80} + \frac{5}{162} x^{14} c^{84} a^{81} + \frac{5}{162} x^{14} c^{85} a^{82} + \frac{5}{162} x^{14} c^{86} a^{83} + \frac{5}{162} x^{14} c^{87} a^{84} + \frac{5}{162} x^{14} c^{88} a^{85} + \frac{5}{162} x^{14} c^{89} a^{86} + \frac{5}{162} x^{14} c^{90} a^{87} + \frac{5}{162} x^{14} c^{91} a^{88} + \frac{5}{162} x^{14} c^{92} a^{89} + \frac{5}{162} x^{14} c^{93} a^{90} + \frac{5}{162} x^{14} c^{94} a^{91} + \frac{5}{162} x^{14} c^{95} a^{92} + \frac{5}{162} x^{14} c^{96} a^{93} + \frac{5}{162} x^{14} c^{97} a^{94} + \frac{5}{162} x^{14} c^{98} a^{95} + \frac{5}{162} x^{14} c^{99} a^{96} + \frac{5}{162} x^{14} c^{100} a^{97} + \frac{5}{162} x^{14} c^{101} a^{98} + \frac{5}{162} x^{14} c^{102} a^{99} + \frac{5}{162} x^{14} c^{103} a^{100} + \frac{5}{162} x^{14} c^{104} a^{101} + \frac{5}{162} x^{14} c^{105} a^{102} + \frac{5}{162} x^{14} c^{106} a^{103} + \frac{5}{162} x^{14} c^{107} a^{104} + \frac{5}{162} x^{14} c^{108} a^{105} + \frac{5}{162} x^{14} c^{109} a^{106} + \frac{5}{162} x^{14} c^{110} a^{107} + \frac{5}{162} x^{14} c^{111} a^{108} + \frac{5}{162} x^{14} c^{112} a^{109} + \frac{5}{162} x^{14} c^{113} a^{110} + \frac{5}{162} x^{14} c^{114} a^{111} + \frac{5}{162} x^{14} c^{115} a^{112} + \frac{5}{162} x^{14} c^{116} a^{113} + \frac{5}{162} x^{14} c^{117} a^{114} + \frac{5}{162} x^{14} c^{118} a^{115} + \frac{5}{162} x^{14} c^{119} a^{116} + \frac{5}{162} x^{14} c^{120} a^{117} + \frac{5}{162} x^{14} c^{121} a^{118} + \frac{5}{162} x^{14} c^{122} a^{119} + \frac{5}{162} x^{14} c^{123} a^{120} + \frac{5}{162} x^{14} c^{124} a^{121} + \frac{5}{162} x^{14} c^{125} a^{122} + \frac{5}{162} x^{14} c^{126} a^{123} + \frac{5}{162} x^{14} c^{127} a^{124} + \frac{5}{162} x^{14} c^{128} a^{125} + \frac{5}{162} x^{14} c^{129} a^{126} + \frac{5}{162} x^{14} c^{130} a^{127} + \frac{5}{162} x^{14} c^{131} a^{128} + \frac{5}{162} x^{14} c^{132} a^{129} + \frac{5}{162} x^{14} c^{133} a^{130} + \frac{5}{162} x^{14} c^{134} a^{131} + \frac{5}{162} x^{14} c^{135} a^{132} + \frac{5}{162} x^{14} c^{136} a^{133} + \frac{5}{162} x^{14} c^{137} a^{134} + \frac{5}{162} x^{14} c^{138} a^{135} + \frac{5}{162} x^{14} c^{139} a^{136} + \frac{5}{162} x^{14} c^{140} a^{137} + \frac{5}{162} x^{14} c^{141} a^{138} + \frac{5}{162} x^{14} c^{142} a^{139} + \frac{5}{162} x^{14} c^{143} a^{140} + \frac{5}{162} x^{14} c^{144} a^{141} + \frac{5}{162} x^{14} c^{145} a^{142} + \frac{5}{162} x^{14} c^{146} a^{143} + \frac{5}{162} x^{14} c^{147} a^{144} + \frac{5}{162} x^{14} c^{148} a^{145} + \frac{5}{162} x^{14} c^{149} a^{146} + \frac{5}{162} x^{14} c^{150} a^{147} + \frac{5}{162} x^{14} c^{151} a^{148} + \frac{5}{162} x^{14} c^{152} a^{149} + \frac{5}{162} x^{14} c^{153} a^{150} + \frac{5}{162} x^{14} c^{154} a^{151} + \frac{5}{162} x^{14} c^{155} a^{152} + \frac{5}{162} x^{14} c^{156} a^{153} + \frac{5}{162} x^{14} c^{157} a^{154} + \frac{5}{162} x^{14} c^{158} a^{155} + \frac{5}{162} x^{14} c^{159} a^{156} + \frac{5}{162} x^{14} c^{160} a^{157} + \frac{5}{162} x^{14} c^{161} a^{158} + \frac{5}{162} x^{14} c^{162} a^{159} + \frac{5}{162} x^{14} c^{163} a^{160} + \frac{5}{162} x^{14} c^{164} a^{161} + \frac{5}{162} x^{14} c^{165} a^{162} + \frac{5}{162} x^{14} c^{166} a^{163} + \frac{5}{162} x^{14} c^{167} a^{164} + \frac{5}{162} x^{14} c^{168} a^{165} + \frac{5}{162} x^{14} c^{169} a^{166} + \frac{5}{162} x^{14} c^{170} a^{167} + \frac{5}{162} x^{14} c^{171} a^{168} + \frac{5}{162} x^{14} c^{172} a^{169} + \frac{5}{162} x^{14} c^{173} a^{170} + \frac{5}{162} x^{14} c^{174} a^{171} + \frac{5}{162} x^{14} c^{175} a^{172} + \frac{5}{162} x^{14} c^{176} a^{173} + \frac{5}{162} x^{14} c^{177} a^{174} + \frac{5}{162} x^{14} c^{178} a^{175} + \frac{5}{162} x^{14} c^{179} a^{176} + \frac{5}{162} x^{14} c^{180} a^{177} + \frac{5}{162} x^{14} c^{181} a^{178} + \frac{5}{162} x^{14} c^{182} a^{179} + \frac{5}{162} x^{14} c^{183} a^{180} + \frac{5}{162} x^{14} c^{184} a^{181} + \frac{5}{162} x^{14} c^{185} a^{182} + \frac{5}{162} x^{14} c^{186} a^{183} + \frac{5}{162} x^{14} c^{187} a^{184} + \frac{5}{162} x^{14} c^{188} a^{185} + \frac{5}{162} x^{14} c^{189} a^{186} + \frac{5}{162} x^{14} c^{190} a^{187} + \frac{5}{162} x^{14} c^{191} a^{188} + \frac{5}{162} x^{14} c^{192} a^{189} + \frac{5}{162} x^{14} c^{193} a^{190} + \frac{5}{162} x^{14} c^{194} a^{191} + \frac{5}{162} x^{14} c^{195} a^{192} + \frac{5}{162} x^{14} c^{196} a^{193} + \frac{5}{162} x^{14} c^{197} a^{194} + \frac{5}{162} x^{14} c^{198} a^{195} + \frac{5}{162} x^{14} c^{199} a^{196} + \frac{5}{162} x^{14} c^{200} a^{197} + \frac{5}{162} x^{14} c^{201} a^{198} + \frac{5}{162} x^{14} c^{202} a^{199} + \frac{5}{162} x^{14} c^{203} a^{200} + \frac{5}{162} x^{14} c^{204} a^{201} + \frac{5}{162} x^{14} c^{205} a^{202} + \frac{5}{162} x^{14} c^{206} a^{203} + \frac{5}{162} x^{14} c^{207} a^{204} + \frac{5}{162} x^{14} c^{208} a^{205} + \frac{5}{162} x^{14} c^{209} a^{206} + \frac{5}{162} x^{14} c^{210} a^{207} + \frac{5}{162} x^{14} c^{211} a^{208} + \frac{5}{162} x^{14} c^{212} a^{209} + \frac{5}{162} x^{14} c^{213} a^{210} + \frac{5}{162} x^{14} c^{214} a^{211} + \frac{5}{162} x^{14} c^{215} a^{212} + \frac{5}{162} x^{14} c^{216} a^{213} + \frac{5}{162} x^{14} c^{217} a^{214} + \frac{5}{162} x^{14} c^{218} a^{215} + \frac{5}{162} x^{14} c^{219} a^{216} + \frac{5}{162} x^{14} c^{220} a^{217} + \frac{5}{162} x^{14} c^{221} a^{218} + \frac{5}{162} x^{14} c^{222} a^{219} + \frac{5}{162} x^{14} c^{223} a^{220} + \frac{5}{162} x^{14} c^{224} a^{221} + \frac{5}{162} x^{14} c^{225} a^{222} + \frac{5}{162} x^{14} c^{226} a^{223} + \frac{5}{162} x^{14} c^{227} a^{224} + \frac{5}{162} x^{14} c^{228} a^{225} + \frac{5}{162} x^{14} c^{229} a^{226} + \frac{5}{162} x^{14} c^{230} a^{227} + \frac{5}{162} x^{14} c^{231} a^{228} + \frac{5}{162} x^{14} c^{232} a^{229} + \frac{5}{162} x^{14} c^{233} a^{230} + \frac{5}{162} x^{14} c^{234} a^{231} + \frac{5}{162} x^{14} c^{235} a^{232} + \frac{5}{162} x^{14} c^{236} a^{233} + \frac{5}{162} x^{14} c^{237} a^{234} + \frac{5}{162} x^{14} c^{238} a^{235} + \frac{5}{162} x^{14} c^{239} a^{236} + \frac{5}{162} x^{14} c^{240} a^{237} + \frac{5}{162} x^{14} c^{241} a^{238} + \frac{5}{162} x^{14} c^{242} a^{239} + \frac{5}{162} x^{14} c^{243} a^{240} + \frac{5}{162} x^{14} c^{244} a^{241} + \frac{5}{162} x^{14} c^{245} a^{242} + \frac{5}{162} x^{14} c^{246} a^{243} + \frac{5}{162} x^{14} c^{247} a^{244} + \frac{5}{162} x^{14} c^{248} a^{245} + \frac{5}{162} x^{14} c^{249} a^{246} + \frac{5}{162} x^{14} c^{250} a^{247} + \frac{5}{162} x^{14} c^{251} a^{248} + \frac{5}{162} x^{14} c^{252} a^{249} + \frac{5}{162} x^{14} c^{253} a^{250} + \frac{5}{162} x^{14} c^{254} a^{251} + \frac{5}{162} x^{14} c^{255} a^{252} + \frac{5}{162} x^{14} c^{256} a^{253} + \frac{5}{162} x^{14} c^{257} a^{254} + \frac{5}{162} x^{14} c^{258} a^{255} + \frac{5}{162} x^{14} c^{259} a^{256} + \frac{5}{162} x^{14} c^{260} a^{257} + \frac{5}{162} x^{14} c^{261} a^{258} + \frac{5}{162} x^{14} c^{262} a^{259} + \frac{5}{162} x^{14} c^{263} a^{260} + \frac{5}{162} x^{14} c^{264} a^{261} + \frac{5}{162} x^{14} c^{265} a^{262} + \frac{5}{162} x^{14} c^{266} a^{263} + \frac{5}{162} x^{14} c^{267} a^{264} + \frac{5}{162} x^{14} c^{268} a^{265} + \frac{5}{162} x^{14} c^{269} a^{266} + \frac{5}{162} x^{14} c^{270} a^{267} + \frac{5}{162} x^{14} c^{271} a^{268} + \frac{5}{162} x^{14} c^{272} a^{269} + \frac{5}{162} x^{14} c^{273} a^{270} + \frac{5}{162} x^{14} c^{274} a^{271} + \frac{5}{162} x^{14} c^{275} a^{272} + \frac{5}{162} x^{14} c^{276} a^{273} + \frac{5}{162} x^{14} c^{277} a^{274} + \frac{5}{162} x^{14} c^{278} a^{275} + \frac{5}{162} x^{14} c^{279} a^{276} + \frac{5}{162} x^{14} c^{280} a^{277} + \frac{5}{162} x^{14} c^{281} a^{278} + \frac{5}{162} x^{14} c^{282} a^{279} + \frac{5}{162} x^{14} c^{283} a^{280} + \frac{5}{162} x^{14} c^{284} a^{281} + \frac{5}{162} x^{14} c^{285} a^{282} + \frac{5}{162} x^{14} c^{286} a^{283} + \frac{5}{162} x^{14} c^{287} a^{284} + \frac{5}{162} x^{14} c^{288} a^{285} + \frac{5}{162} x^{14} c^{289} a^{286} + \frac{5}{162} x^{14} c^{290} a^{287} + \frac{5}{162} x^{14} c^{291} a^{288} + \frac{5}{162} x^{14} c^{292} a^{289} + \frac{5}{162} x^{14} c^{293} a^{290} + \frac{5}{162} x^{14} c^{294} a^{291} + \frac{5}{162} x^{14} c^{295} a^{292} + \frac{5}{162} x^{14} c^{296} a^{293} + \frac{5}{162} x^{14} c^{297} a^{294} + \frac{5}{162} x^{14} c^{298} a^{295} + \frac{5}{162} x^{14} c^{299} a^{296} + \frac{5}{162} x^{14} c^{300} a^{297} + \frac{5}{162} x^{14} c^{301} a^{298} + \frac{5}{162} x^{14} c^{302} a^{299} + \frac{5}{162} x^{14} c^{303} a^{300} + \frac{5}{162} x^{14} c^{304} a^{301} + \frac{5}{162} x^{14} c^{305} a^{302} + \frac{5}{162} x^{14} c^{306} a^{303} + \frac{5}{162} x^{14} c^{307} a^{304} + \frac{5}{162} x^{14} c^{308} a^{305} + \frac{5}{162} x^{14} c^{309} a^{306} + \frac{5}{162} x^{14} c^{310} a^{307} + \frac{5}{162} x^{14} c^{311} a^{308} + \frac{5}{162} x^{14} c^{312} a^{309} + \frac{5}{162} x^{14} c^{313} a^{310} + \frac{5}{162} x^{14} c^{314} a^{311} + \frac{5}{162} x^{14} c^{315} a^{312} + \frac{5}{162} x^{14} c^{316} a^{313} + \frac{5}{162} x^{14} c^{317} a^{314} + \frac{5}{162} x^{14} c^{318} a^{315} + \frac{5}{162} x^{14} c^{319} a^{316} + \frac{5}{162} x^{14} c^{320} a^{317} + \frac{5}{162} x^{14} c^{321} a^{318} + \frac{5}{162} x^{14} c^{322} a^{319} + \frac{5}{162} x^{14} c^{323} a^{320} + \frac{5}{162} x^{14} c^{324} a^{321} + \frac{5}{162} x^{14} c^{325} a^{322} + \frac{5}{162} x^{14} c^{326} a^{323} + \frac{5}{162} x^{14} c^{327} a^{324} + \frac{5}{162} x^{14} c^{328} a^{325} + \frac{5}{162} x^{14} c^{329} a^{326} + \frac{5}{162} x^{14} c^{330} a^{327} + \frac{5}{162} x^{14} c^{331} a^{328} + \frac{5}{162} x^{14} c^{332} a^{329} + \frac{5}{162} x^{14} c^{333} a^{330} + \frac{5}{162} x^{14} c^{334} a^{331} + \frac{5}{162} x^{14} c^{335} a^{332} + \frac{5}{162} x^{14} c^{336} a^{333} + \frac{5}{162} x^{14} c^{337} a^{334} + \frac{5}{162} x^{14} c^{338} a^{335} + \frac{5}{162} x^{14} c^{339} a^{336} + \frac{5}{162} x^{14} c^{340} a^{337} + \frac{5}{162} x^{14} c^{341} a^{338} + \frac{5}{162} x^{14} c^{342} a^{339} + \frac{5}{162} x^{14} c^{343} a^{340} + \frac{5}{162} x^{14} c^{344} a^{341} + \frac{5}{162} x^{14} c^{345} a^{342} + \frac{5}{162} x^{14} c^{346} a^{343} + \frac{5}{162} x^{14} c^{347} a^{344} + \frac{5}{162} x^{14} c^{348} a^{345} + \frac{5}{162} x^{14} c^{349} a^{346} + \frac{5}{162} x^{14} c^{350} a^{347} + \frac{5}{162} x^{14} c^{351} a^{348} + \frac{5}{162} x^{14} c^{352} a^{349} + \frac{5}{162} x^{14} c^{353} a^{350} + \frac{5}{162} x^{14} c^{354} a^{351} + \frac{5}{162} x^{14} c^{355} a^{352} + \frac{5}{162} x^{14} c^{356} a^{353} + \frac{5}{162} x^{14} c^{357} a^{354} + \frac{5}{162} x^{14} c^{358} a^{355} + \frac{5}{162} x^{14} c^{359} a^{356} + \frac{5}{162} x^{14} c^{360} a^{357} + \frac{5}{162} x^{14} c^{361} a^{358} + \frac{5}{162} x^{14} c^{362} a^{359} + \frac{5}{162} x^{14} c^{363} a^{360} + \frac{5}{162} x^{14} c^{364} a^{361} + \frac{5}{162} x^{14} c^{365} a^{362} + \frac{5}{162} x^{14} c^{366} a^{363} + \frac{5}{162} x^{14} c^{367} a^{364} + \frac{5}{162} x^{14} c^{368} a^{365} + \frac{5}{162} x^{14} c^{369} a^{366} + \frac{5}{162} x^{14} c^{370} a^{367} + \frac{5}{162} x^{14} c^{371} a^{368} + \frac{5}{162} x^{14} c^{372} a^{369} + \frac{5}{162} x^{14} c^{373} a^{370} + \frac{5}{162} x^{14} c^{374} a^{371} + \frac{5}{162} x^{14} c^{375} a^{372} + \frac{5}{162} x^{14} c^{376} a^{373} + \frac{5}{162} x^{14} c^{377} a^{374}$$

$\wedge 2)) + 1/2 * b * (2 * a^2 * (2/3 * a * c + 1/4 * b^2) + a^2 * b^2) + 2/3 * c * a^3 * b) + c * (a * (2 * a^2 * (2/3 * a * c + 1/4 * b^2) + a^2 * b^2) + b^2 * a^3 + 1/3 * c * a^4)) * x^{10} + 1/9 * (a * (a * (2/3 * a^2 * b * c + 2 * a * b * (2/3 * a * c + 1/4 * b^2)) + 1/2 * b * (2 * a^2 * (2/3 * a * c + 1/4 * b^2) + a^2 * b^2) + 2/3 * c * a^3 * b) + b * (a * (2 * a^2 * (2/3 * a * c + 1/4 * b^2) + a^2 * b^2) + b^2 * a^3 + 1/3 * c * a^4) + 5/2 * c * a^4 * b) * x^9 + 1/8 * (a * (a * (2 * a^2 * (2/3 * a * c + 1/4 * b^2) + a^2 * b^2) + b^2 * a^3 + 1/3 * c * a^4) + 5/2 * b^2 * a^4 + c * a^5) * x^8 + 1/2 * a^5 * b * x^7 + 1/6 * a^6 * x^6 + 1/3 * c * x^3 + 1/2 * b * x^2 + a * x$

maxima [B] time = 0.45, size = 289, normalized size = 6.28

$$\frac{1}{4374} c^6 x^{18} + \frac{1}{486} b c^5 x^{17} + \frac{1}{1944} (15 b^2 c^4 + 8 a c^5) x^{16} + \frac{5}{324} (b^3 c^3 + 2 a b c^4) x^{15} + \frac{5}{2592} (9 b^4 c^2 + 48 a b^2 c^3 + 16 a^2 c^4) x^{14} + \frac{1}{864} (9 b^5 c + 120 a b^3 c^2 + 160 a^2 b^2 c^3) x^{13} + \frac{1}{10368} (27 b^6 + 1080 a b^4 c + 4320 a^2 b^2 c^2 + 1280 a^3 c^3) x^{12} + \frac{1}{6} a^6 x^6 + \frac{1}{288} (9 a^2 b^5 + 120 a^2 b^3 c + 160 a^3 b^2 c^2) x^{11} + \frac{5}{288} (9 a^2 b^4 + 48 a^3 b^2 c + 16 a^4 c^2) x^{10} + \frac{5}{12} (a^3 b^3 + 2 a^4 b^2 c) x^9 + \frac{1}{24} (15 a^4 b^2 + 8 a^5 c) x^8 + \frac{1}{3} c x^3 + \frac{1}{2} b x^2 + a x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")

[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b^2*c^3)*x^13 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*x^12 + 1/6*a^6*x^6 + 1/288*(9*a^2*b^5 + 120*a^2*b^3*c + 160*a^3*b^2*c^2)*x^11 + 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4*b^2*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x

mupad [B] time = 2.24, size = 270, normalized size = 5.87

$$x^{12} \left(\frac{10 a^3 c^3}{81} + \frac{5 a^2 b^2 c^2}{12} + \frac{5 a b^4 c}{48} + \frac{b^6}{384} \right) + a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{a^6 x^6}{6} + \frac{c^6 x^{18}}{4374} + \frac{5 a^2 x^{10} (16 a^2 c^2 + 48 a b^2 c + 9 b^4 c^2)}{288}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)

[Out] x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48) + a*x + (b*x^2)/2 + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (5*a^2*x^10*(9*b^4 + 16*a^2*c^2 + 48*a*b^2*c))/288 + (5*c^2*x^14*(9*b^4 + 16*a^2*c^2 + 48*a*b^2*c))/2592 + (a^5*b*x^7)/2 + (b*c^5*x^17)/486 + (a^4*x^8*(8*a*c + 15*b^2))/24 + (c^4*x^16*(8*a*c + 15*b^2))/1944 + (a*b*x^11*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/288 + (b*c*x^13*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/864 + (5*a^3*b*x^9*(2*a*c + b^2))/12 + (5*b*c^3*x^15*(2*a*c + b^2))/324

sympy [B] time = 0.16, size = 323, normalized size = 7.02

$$\frac{a^6 x^6}{6} + \frac{a^5 b x^7}{2} + a x + \frac{b c^5 x^{17}}{486} + \frac{b x^2}{2} + \frac{c^6 x^{18}}{4374} + \frac{c x^3}{3} + x^{16} \left(\frac{a c^5}{243} + \frac{5 b^2 c^4}{648} \right) + x^{15} \left(\frac{5 a b c^4}{162} + \frac{5 b^3 c^3}{324} \right) + x^{14} \left(\frac{5 a^2 c^4}{162} + \frac{5 a b^2 c^3}{54} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)

[Out] a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384) + x**11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*c**2/18 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12) + x**8*(a**5*c/3 + 5*a**4*b**2/8)

$$3.215 \quad \int (a + bx + cx^2) \left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=47

$$\frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} + d \right)^6 + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(d+a*x+1/2*b*x^2+1/3*c*x^3)^6

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1591}

$$\frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} + d \right)^6 + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2) \left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{6} \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.12, size = 248, normalized size = 5.28

$$x(6a + x(3b + 2cx)) \left(7776a^5x^5 + 6480a^4x^6(3b + 2cx) + 2160a^3x^7(3b + 2cx)^2 + 360a^2x^8(3b + 2cx)^3 + 19440d^4x(6a + x(3b + 2cx)) \right) / 279936$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2 + 360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6*a + x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x^3*(6*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936

fricas [B] time = 0.63, size = 928, normalized size = 19.74

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{486}x^{17}c^5b + \frac{5}{648}x^{16}c^4b^2 + \frac{1}{243}x^{16}c^5a + \frac{1}{243}x^{15}dc^5 + \frac{5}{324}x^{15}c^3b^3 + \frac{5}{162}x^{15}c^4ba + \frac{5}{162}x^{14}dc^4b + \frac{5}{288}x^{14}c^2b^2$$


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*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^2*(a^2+b*d)+4
*a^2*d^2))*x^8+1/7*(a*(d*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4
*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(
2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^
2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d)))+b*(d*(2/3*d^2*b*c+4*a*d*(2/
3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*
(2/3*c*d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*
c*(2*d^2*(a^2+b*d)+4*a^2*d^2))+c*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d
+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^
2+b*d)+4*a^2*d^2)+4/3*a*c*d^3))*x^7+1/6*(a*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1
/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*
d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^
2*(a^2+b*d)+4*a^2*d^2))+b*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(
a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^2+b*d)+
4*a^2*d^2)+4/3*a*c*d^3))+c*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2
*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4))*x^6+1/5*(a*(d*(2*d^2*(2/3*a*c+1
/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+
b*d))+1/2*b*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4/3*a*c*d^3)+b*(d*(2*d^2*(2/3*c*d+a
*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4))+c*(
d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4))*x^5+1/4*(a*(d*(2*d^2*(2
/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*
d^4)+b*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*a*c*d^4))*x^4+1
/3*(a*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*b*d^4*a+(d^5+1)
*c))*x^3+1/2*(5*a^2*d^4+b*(d^5+1))*x^2+(d^5+1)*a*x

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maxima [B] time = 0.48, size = 773, normalized size = 16.45

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{1}{1944}(15b^2c^4 + 8ac^5)x^{16} + \frac{1}{972}(15b^3c^3 + 30abc^4 + 4c^5d)x^{15} + \frac{5}{2592}(9b^4c^2 + 48ab^3c + 12a^2b^2c + 16a^2c^4 + 16b^2c^4d)x^{14} + \frac{1}{2592}(27b^5c + 360a^2b^3c^2 + 480a^2b^2c^3 + 80(3b^2c^3 + 2a^2c^4)d)x^{13} + \frac{1}{10368}(27b^6 + 1080a^2b^4c + 4320a^2b^2c^2 + 1280a^3c^3 + 320c^4d^2 + 480(3b^3c^2 + 8a^2b^2c^3)d)x^{12} + \frac{1}{864}(27a^2b^5 + 360a^2b^3c + 480a^3b^2c^2 + 160b^2c^3d^2 + 10(9b^4c + 72a^2b^2c^2 + 32a^2c^3)d)x^{11} + \frac{1}{864}(135a^2b^4 + 720a^3b^2c + 240a^4c^2 + 40(9b^2c^2 + 8a^2c^3)d^2 + 9(3b^5 + 80a^2b^3c + 160a^2b^2c^2)d)x^{10} + \frac{5}{1296}(108a^3b^3 + 216a^4b^2c + 32c^3d^3 + 108(b^3c + 4a^2b^2c^2)d^2 + 9(9a^2b^4 + 72a^2b^2c + 32a^3c^2)d)x^9 + \frac{1}{288}(180a^4b^2 + 96a^5c + 160b^2c^2d^3 + 15(3b^4 + 48a^2b^2c + 32a^2c^2)d^2 + 120(3a^2b^3 + 8a^3b^2c)d)x^8 + \frac{1}{36}(18a^5b + 10(3b^2c + 4a^2c^2)d^3 + 45(a^2b^3 + 4a^2b^2c)d^2 + 30(3a^3b^2 + 2a^4c)d)x^7 + \frac{1}{36}(6a^6 + 90a^4b^2d + 10c^2d^4 + 15(b^3 + 8a^2b^2c)d^3 + 15(9a^2b^2 + 8a^3c)d^2)x^6 + \frac{1}{6}(6a^5d + 30a^3b^2d^2 + 5b^2c^2d^4 + 5(3a^2b^2 + 4a^2c)d^3)x^5 + \frac{5}{24}(12a^4d^2 + 24a^2b^2d^3 + (3b^2 + 8a^2c)d^4)x^4 + \frac{1}{6}(20a^3d^3 + 15a^2b^2d^4 + 2c^2d^5 + 2c)x^3 + \frac{1}{2}(5a^2d^4 + b^2d^5 + b)x^2 + (a^2d^5 + a)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")

[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2*c^4 + 16*b^2*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a^2*b^3*c^2 + 480*a^2*b^2*c^3 + 80*(3*b^2*c^3 + 2*a^2*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a^2*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8*a^2*b^2*c^3)*d)*x^12 + 1/864*(27*a^2*b^5 + 360*a^2*b^3*c + 480*a^3*b^2*c^2 + 160*b^2*c^3*d^2 + 10*(9*b^4*c + 72*a^2*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(135*a^2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a^2*c^3)*d^2 + 9*(3*b^5 + 80*a^2*b^3*c + 160*a^2*b^2*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 216*a^4*b^2*c + 32*c^3*d^3 + 108*(b^3*c + 4*a^2*b^2*c^2)*d^2 + 9*(9*a^2*b^4 + 72*a^2*b^2*c + 32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b^2*c^2*d^3 + 15*(3*b^4 + 48*a^2*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b^2*c)*d)*x^8 + 1/36*(18*a^5*b + 10*(3*b^2*c + 4*a^2*c^2)*d^3 + 45*(a^2*b^3 + 4*a^2*b^2*c)*d^2 + 30*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b^2*d + 10*c^2*d^4 + 15*(b^3 + 8*a^2*b^2*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 30*a^3*b^2*d^2 + 5*b^2*c^2*d^4 + 5*(3*a^2*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2 + 24*a^2*b^2*d^3 + (3*b^2 + 8*a^2*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a^2*b^2*d^4 + 2*c^2*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b^2*d^5 + b)*x^2 + (a^2*d^5 + a)*x

mupad [B] time = 2.45, size = 753, normalized size = 16.02

$$x^{10} \left(\frac{5a^4c^2}{18} + \frac{5a^3b^2c}{6} + \frac{5a^2b^4}{32} + \frac{5a^2bc^2d}{3} + \frac{5ab^3cd}{6} + \frac{10ac^3d^2}{27} + \frac{b^5d}{32} + \frac{5b^2c^2d^2}{12} \right) + x^8 \left(\frac{a^5c}{3} + \frac{5a^4b^2}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)

[Out] x^10*((b^5*d)/32 + (5*a^2*b^4)/32 + (5*a^4*c^2)/18 + (5*a^3*b^2*c)/6 + (10*a*c^3*d^2)/27 + (5*b^2*c^2*d^2)/12 + (5*a*b^3*c*d)/6 + (5*a^2*b*c^2*d)/3) + x^8*((a^5*c)/3 + (5*a^4*b^2)/8 + (5*b^4*d^2)/32 + (5*a^2*b^3*d)/4 + (5*b*c^2*d^3)/9 + (5*a^2*c^2*d^2)/3 + (10*a^3*b*c*d)/3 + (5*a*b^2*c*d^2)/2) + x^9*((5*a^3*b^3)/12 + (10*c^3*d^3)/81 + (10*a^3*c^2*d)/9 + (5*b^3*c*d^2)/12 + (5*a^4*b*c)/6 + (5*a*b^4*d)/16 + (5*a*b*c^2*d^2)/3 + (5*a^2*b^2*c*d)/2) + x^14*((5*a^2*c^4)/162 + (5*b^4*c^2)/288 + (5*a*b^2*c^3)/54 + (5*b*c^4*d)/162) + x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48 + (10*a*b*c^3*d)/27) + x^6*(a^6/6 + (5*b^3*d^3)/12 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3 + (15*a^2*b^2*d^2)/4 + (5*a^4*b*d)/2 + (10*a*b*c*d^3)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3 + (5*a*b*d^4)/2) + x^11*((a*b^5)/32 + (5*a^2*b^3*c)/12 + (5*a^3*b*c^2)/9 + (10*a^2*c^3*d)/27 + (5*b*c^3*d^2)/27 + (5*b^4*c*d)/48 + (5*a*b^2*c^2*d)/6) + x^7*((a^5*b)/2 + (5*a*b^3*d^2)/4 + (5*a^3*b^2*d)/2 + (10*a*c^2*d^3)/9 + (5*b^2*c*d^3)/6 + (5*a^4*c*d)/3 + 5*a^2*b*c*d^2) + x^2*(b/2 + (b*d^5)/2 + (5*a^2*d^4)/2) + x^13*((b^5*c)/96 + (5*a*b^3*c^2)/36 + (5*a^2*b*c^3)/27 + (5*b^2*c^3*d)/54 + (5*a*c^4*d)/81) + x^5*(a^5*d + (5*a*b^2*d^3)/2 + 5*a^3*b*d^2 + (10*a^2*c*d^3)/3 + (5*b*c*d^4)/6) + (c^6*x^18)/4374 + (5*d^2*x^4*(12*a^4 + 3*b^2*d^2 + 24*a^2*b*d + 8*a*c*d^2))/24 + a*x*(d^5 + 1) + (b*c^5*x^17)/486 + (c^3*x^15*(4*c^2*d + 15*b^3 + 30*a*b*c))/972 + (c^4*x^16*(8*a*c + 15*b^2))/1944

sympy [B] time = 0.27, size = 930, normalized size = 19.79

$$\frac{bc^5x^{17}}{486} + \frac{c^6x^{18}}{4374} + x^{16}\left(\frac{ac^5}{243} + \frac{5b^2c^4}{648}\right) + x^{15}\left(\frac{5abc^4}{162} + \frac{5b^3c^3}{324} + \frac{c^5d}{243}\right) + x^{14}\left(\frac{5a^2c^4}{162} + \frac{5ab^2c^3}{54} + \frac{5b^4c^2}{288} + \frac{5bc^4d}{162}\right) + x^{13}\left(\frac{5a^3b^3}{12} + \frac{10c^3d^3}{81} + \frac{10a^3c^2d}{9} + \frac{5b^3cd^2}{12} + \frac{5a^4bc}{6} + \frac{5ab^4d}{16} + \frac{5abc^2d^2}{3} + \frac{5a^2b^2cd}{2}\right) + x^{12}\left(\frac{b^6}{384} + \frac{10a^3c^3}{81} + \frac{5c^4d^2}{162} + \frac{5b^3c^2d}{36} + \frac{5a^2b^2c^2}{12} + \frac{5ab^4c}{48} + \frac{10ab^3cd}{27}\right) + x^{11}\left(\frac{ab^5}{32} + \frac{5a^2b^3c}{12} + \frac{5a^3b^2c^2}{9} + \frac{10a^2c^3d}{27} + \frac{5b^4cd}{48} + \frac{5ab^2c^2d}{6}\right) + x^{10}\left(\frac{5a^3b^3}{12} + \frac{10c^3d^3}{81} + \frac{10a^3c^2d}{9} + \frac{5b^3cd^2}{12} + \frac{5a^4bc}{6} + \frac{5ab^4d}{16} + \frac{5abc^2d^2}{3} + \frac{5a^2b^2cd}{2}\right) + x^9\left(\frac{5a^2c^4}{162} + \frac{5b^4c^2}{288} + \frac{5ab^2c^3}{54} + \frac{5b^4cd}{162}\right) + x^8\left(\frac{a^5c}{3} + \frac{5a^4b^2}{8} + \frac{10a^3b^2cd}{3} + \frac{5a^2b^3d}{4} + \frac{5a^2c^2d^2}{3} + \frac{5ab^2c^2d^2}{2} + \frac{5b^4d^2}{32} + \frac{5b^3cd^2}{9}\right) + x^7\left(\frac{a^5b}{2} + \frac{5a^4cd}{3} + \frac{5a^3b^2d}{2} + \frac{5a^2b^3cd^2}{2} + \frac{5ab^3d^2}{4} + \frac{10a^3cd^2}{3} + \frac{15a^2b^2d^2}{4} + \frac{10ab^3cd^3}{3} + \frac{5b^3d^3}{12} + \frac{5c^2d^4}{18}\right) + x^6\left(\frac{a^6}{6} + \frac{5a^4bd}{2} + \frac{10a^3cd^2}{3} + \frac{15a^2b^2d^2}{4} + \frac{10ab^3cd^3}{3} + \frac{5b^3d^3}{12} + \frac{5c^2d^4}{18}\right) + x^5\left(a^5d + \frac{5a^3bd^2}{2} + \frac{10a^2cd^3}{3} + \frac{5ab^2d^3}{2} + \frac{5b^2cd^4}{6}\right) + x^4\left(\frac{5a^4d^2}{2} + \frac{5a^2bd^3}{2} + \frac{5ac^2d^4}{3} + \frac{5b^2d^4}{8}\right) + x^3\left(\frac{10a^3d^3}{3} + \frac{5ab^3d^4}{2} + \frac{c^2d^5}{3} + \frac{c}{3}\right) + x^2\left(\frac{5a^2d^4}{2} + \frac{bd^5}{2} + \frac{b}{2}\right) + x(a^5d^5 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)

[Out] b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3*d/54) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10*a*b*c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b**2*c**2*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10*a*c**3*d**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*a*b*c**2*d**2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**4*b**2/8 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*a*b**2*c*d**2/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a**4*c*d/3 + 5*a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**2*d**3/9 + 5*b**2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2/3 + 15*a**2*b**2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/18) + x**5*(a**5*d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 + 5*b*c*d**4/6) + x**4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**2*d**4/8) + x**3*(10*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**2*(5*a**2*d**4/2 + b*d**5/2 + b/2) + x*(a*d**5 + a)

$$3.216 \quad \int \left(a + cx^2 \right) \left(1 + \left(ax + \frac{cx^3}{3} \right)^n \right) dx$$

Optimal. Leaf size=34

$$\frac{\left(ax + \frac{cx^3}{3} \right)^{n+1}}{n+1} + ax + \frac{cx^3}{3}$$

[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1591}

$$\frac{\left(ax + \frac{cx^3}{3} \right)^{n+1}}{n+1} + ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]

[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int \left(a + cx^2 \right) \left(1 + \left(ax + \frac{cx^3}{3} \right)^n \right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, ax + \frac{cx^3}{3} \right) \\ &= ax + \frac{cx^3}{3} + \frac{\left(ax + \frac{cx^3}{3} \right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 1.06

$$\frac{x \left(3a + cx^2 \right) \left(\left(ax + \frac{cx^3}{3} \right)^n + n + 1 \right)}{3(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]

[Out] (x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))

fricas [A] time = 1.02, size = 48, normalized size = 1.41

$$\frac{(cn + c)x^3 + (cx^3 + 3ax) \left(\frac{1}{3} cx^3 + ax \right)^n + 3(an + a)x}{3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="fricas")

[Out] 1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/(n + 1)

giac [A] time = 0.42, size = 30, normalized size = 0.88

$$\frac{1}{3}cx^3 + ax + \frac{\left(\frac{1}{3}cx^3 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="giac")

[Out] 1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{cx^3}{3} + ax + \frac{\left(\frac{1}{3}cx^3 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x)

[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(n+1)/(n+1)

maxima [A] time = 1.14, size = 54, normalized size = 1.59

$$\frac{1}{3}cx^3 + ax + \frac{(cx^3 + 3ax)e^{(n\log(cx^2+3a)+n\log(x))}}{3^{n+1}n + 3^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")

[Out] 1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x))/(3^(n + 1)*n + 3^(n + 1))

mupad [B] time = 2.11, size = 33, normalized size = 0.97

$$\frac{x(c x^2 + 3 a) \left(n + \left(\frac{c x^3}{3} + a x \right)^n + 1 \right)}{3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^n + 1),x)

[Out] (x*(3*a + c*x^2)*(n + (a*x + (c*x^3)/3)^n + 1))/(3*(n + 1))

sympy [B] time = 112.00, size = 201, normalized size = 5.91

$$\begin{cases} \frac{3 \cdot 3^n a n x}{3 \cdot 3^{n+3} 3^n} + \frac{3 \cdot 3^n a x}{3 \cdot 3^{n+3} 3^n} + \frac{3^n c n x^3}{3 \cdot 3^{n+3} 3^n} + \frac{3^n c x^3}{3 \cdot 3^{n+3} 3^n} + \frac{3 a x (3 a x + c x^3)^n}{3 \cdot 3^{n+3} 3^n} + \frac{c x^3 (3 a x + c x^3)^n}{3 \cdot 3^{n+3} 3^n} & \text{for } n \neq -1 \\ a x + \frac{c x^3}{3} + \log(x) + \log\left(-\sqrt{3} i \sqrt{a} \sqrt{\frac{1}{c}} + x\right) + \log\left(\sqrt{3} i \sqrt{a} \sqrt{\frac{1}{c}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)


```
[Out] Piecewise((3**n*a*n*x/(3**n*n + 3**n) + 3**n*a*x/(3**n*n + 3**n)
) + 3**n*c*n*x**3/(3**n*n + 3**n) + 3**n*c*x**3/(3**n*n + 3**n) + 3
*a*x*(3*a*x + c*x**3)**n/(3**n*n + 3**n) + c*x**3*(3*a*x + c*x**3)**n/(
3**n*n + 3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(-sqrt(3)*I*s
qrt(a)*sqrt(1/c) + x) + log(sqrt(3)*I*sqrt(a)*sqrt(1/c) + x), True))
```

$$3.217 \quad \int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx$$

Optimal. Leaf size=44

$$\frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{n+1}}{n+1} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

[Out] $1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {1591}

$$\frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{n+1}}{n+1} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n), x]

[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 0.95

$$\frac{x^2(3b + 2cx) \left(\left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^n + n + 1 \right)}{6(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n), x]

[Out] (x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n)/(6*(1 + n))

fricas [A] time = 0.64, size = 57, normalized size = 1.30

$$\frac{2(cn + c)x^3 + 3(bn + b)x^2 + (2cx^3 + 3bx^2) \left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 \right)^n}{6(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 + 1/2*b*x^2)^n)/(n + 1)$

giac [A] time = 0.39, size = 36, normalized size = 0.82

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + \frac{\left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")

[Out] $\frac{1}{3}c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^{(n + 1)}/(n + 1)$

maple [A] time = 0.00, size = 37, normalized size = 0.84

$$\frac{cx^3}{3} + \frac{bx^2}{2} + \frac{\left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x)

[Out] $\frac{1}{2}b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^{(n+1)}/(n+1)$

maxima [A] time = 1.17, size = 71, normalized size = 1.61

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + \frac{(2cx^3 + 3bx^2)e^{(n \log(2cx+3b)+2n \log(x))}}{3^{n+1}2^{n+1}n + 3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")

[Out] $\frac{1}{3}c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^{(n*\log(2*c*x + 3*b) + 2*n*\log(x))}/(3^{(n + 1)}*2^{(n + 1)}*n + 3^{(n + 1)}*2^{(n + 1)})$

mupad [B] time = 2.21, size = 37, normalized size = 0.84

$$\frac{x^2(3b + 2cx) \left(n + \left(\frac{cx^3}{3} + \frac{bx^2}{2} \right)^n + 1 \right)}{6(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^n + 1),x)

[Out] $\frac{(x^2*(3*b + 2*c*x)*(n + ((b*x^2)/2 + (c*x^3)/3)^n + 1))/(6*(n + 1))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)

[Out] Timed out

$$3.218 \quad \int \left(a + bx + cx^2 \right) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^n \right) dx$$

Optimal. Leaf size=50

$$\frac{\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

[Out] $a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^{(1+n)}/(1+n)$

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1591}

$$\frac{\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n), x]$

[Out] $a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^{(1+n)}/(1+n)$

Rule 1591

$\text{Int}[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n), x]$ \rightarrow $\text{With}[\{q = \text{Expon}[Pq, x], r = \text{Expon}[Qr, x]\}, \text{Dist}[\text{Coeff}[Qr, x, r]/(q*\text{Coeff}[Pq, x, q]), \text{Subst}[\text{Int}[(a + b*x^n)^p, x], x, Pq], x] /; \text{EqQ}[r, q - 1] \&\& \text{EqQ}[\text{Coeff}[Qr, x, r]*D[Pq, x], q*\text{Coeff}[Pq, x, q]*Qr] /; \text{FreeQ}[\{a, b, n, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{PolyQ}[Qr, x]$

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^n \right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.19, size = 49, normalized size = 0.98

$$\frac{x(6a + x(3b + 2cx)) \left(\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^n + n + 1 \right)}{6(n+1)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n), x]$

[Out] $(x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n)/(6*(1 + n))$

fricas [A] time = 0.83, size = 72, normalized size = 1.44

$$\frac{2(cn + c)x^3 + 3(bn + b)x^2 + (2cx^3 + 3bx^2 + 6ax) \left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax \right)^n + 6(an + a)x}{6(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")

[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*c*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)

giac [A] time = 0.39, size = 42, normalized size = 0.84

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax + \frac{\left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 43, normalized size = 0.86

$$\frac{cx^3}{3} + \frac{bx^2}{2} + ax + \frac{\left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x)

[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(n+1)/(n+1)

maxima [A] time = 1.22, size = 83, normalized size = 1.66

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax + \frac{(2cx^3 + 3bx^2 + 6ax)e^{(n \log(2cx^2 + 3bx + 6a) + n \log(x))}}{3^{n+1}2^{n+1}n + 3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2 + 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))

mupad [B] time = 2.17, size = 73, normalized size = 1.46

$$ax + \left(\frac{3bx^2}{6n+6} + \frac{2cx^3}{6n+6} + \frac{6ax}{6n+6}\right) \left(\frac{cx^3}{3} + \frac{bx^2}{2} + ax\right)^n + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^n + 1)*(a + b*x + c*x^2),x)

[Out] a*x + ((3*b*x^2)/(6*n + 6) + (2*c*x^3)/(6*n + 6) + (6*a*x)/(6*n + 6))*(a*x + (b*x^2)/2 + (c*x^3)/3)^n + (b*x^2)/2 + (c*x^3)/3

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)

[Out] Timed out

$$3.219 \quad \int (-4 + 4x + x^2)(5 - 12x + 6x^2 + x^3) dx$$

Optimal. Leaf size=19

$$\frac{1}{6}(x^3 + 6x^2 - 12x + 5)^2$$

[Out] 1/6*(x^3+6*x^2-12*x+5)^2

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1588}

$$\frac{1}{6}(x^3 + 6x^2 - 12x + 5)^2$$

Antiderivative was successfully verified.

[In] Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]

[Out] (5 - 12*x + 6*x^2 + x^3)^2/6

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (-4 + 4x + x^2)(5 - 12x + 6x^2 + x^3) dx = \frac{1}{6}(5 - 12x + 6x^2 + x^3)^2$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.74

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]

[Out] -20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6

fricas [A] time = 0.77, size = 29, normalized size = 1.53

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="fricas")

[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x

giac [A] time = 0.35, size = 30, normalized size = 1.58

$$\frac{5}{3}x^3 + \frac{1}{6}(x^3 + 6x^2 - 12x)^2 + 10x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="giac")

[Out] 5/3*x^3 + 1/6*(x^3 + 6*x^2 - 12*x)^2 + 10*x^2 - 20*x

maple [A] time = 0.00, size = 30, normalized size = 1.58

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x)

[Out] 1/6*x^6+2*x^5+2*x^4-67/3*x^3+34*x^2-20*x

maxima [A] time = 0.43, size = 17, normalized size = 0.89

$$\frac{1}{6}(x^3 + 6x^2 - 12x + 5)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")

[Out] 1/6*(x^3 + 6*x^2 - 12*x + 5)^2

mupad [B] time = 0.03, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + x^2 - 4)*(6*x^2 - 12*x + x^3 + 5),x)

[Out] 34*x^2 - 20*x - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6

sympy [A] time = 0.06, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)

[Out] x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x

$$3.220 \quad \int (2x + x^3)(1 + 4x^2 + x^4) dx$$

Optimal. Leaf size=16

$$\frac{1}{8}(x^4 + 4x^2 + 1)^2$$

[Out] 1/8*(x^4+4*x^2+1)^2

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1588}

$$\frac{1}{8}(x^4 + 4x^2 + 1)^2$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] (1 + 4*x^2 + x^4)^2/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2x + x^3)(1 + 4x^2 + x^4) dx = \frac{1}{8}(1 + 4x^2 + x^4)^2$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.31

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8

fricas [A] time = 0.82, size = 17, normalized size = 1.06

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="fricas")

[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2

giac [A] time = 0.23, size = 22, normalized size = 1.38

$$\frac{1}{4}x^4 + \frac{1}{8}(x^4 + 4x^2)^2 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="giac")

[Out] 1/4*x^4 + 1/8*(x^4 + 4*x^2)^2 + x^2

maple [A] time = 0.00, size = 18, normalized size = 1.12

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+2*x)*(x^4+4*x^2+1),x)

[Out] 1/8*x^8+x^6+9/4*x^4+x^2

maxima [A] time = 0.44, size = 14, normalized size = 0.88

$$\frac{1}{8}(x^4 + 4x^2 + 1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")

[Out] 1/8*(x^4 + 4*x^2 + 1)^2

mupad [B] time = 0.03, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^3)*(4*x^2 + x^4 + 1),x)

[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8

sympy [A] time = 0.06, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+2*x)*(x**4+4*x**2+1),x)

[Out] x**8/8 + x**6 + 9*x**4/4 + x**2

$$3.221 \quad \int (1 + 2x) (x + x^2)^3 \left(-18 + 7(x + x^2)^3 \right)^2 dx$$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10} - 36x^7(x+1)^7 + 81x^4(x+1)^4$$

[Out] 81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10

Rubi [B] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1593, 1612}

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]

[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x^m*(c + d*x^n*(e + f*x)^p), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int (1 + 2x) (x + x^2)^3 \left(-18 + 7(x + x^2)^3 \right)^2 dx &= \int x^3(1 + x)^3(1 + 2x) \left(-18 + 7(x + x^2)^3 \right)^2 dx \\ &= \int (324x^3 + 1620x^4 + 2916x^5 + 2016x^6 - 1368x^7 - 6804x^8 - 12551x^9 - 756x^{10} - 171x^{11} - \frac{12551x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10}) dx \end{aligned}$$

Mathematica [B] time = 0.01, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]

[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10

fricas [B] time = 0.72, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")

[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4

giac [A] time = 0.29, size = 28, normalized size = 0.85

$$\frac{49}{10}(x^2+x)^{10}-36(x^2+x)^7+81(x^2+x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")

[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4

maple [B] time = 0.00, size = 87, normalized size = 2.64

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x)

[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*x^14+336*x^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8+288*x^7+486*x^6+324*x^5+81*x^4

maxima [B] time = 0.44, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")

[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4

mupad [B] time = 0.22, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)*(x + x^2)^3*(7*(x + x^2)^3 - 18)^2,x)

[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10

sympy [B] time = 0.08, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)

[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4

$$3.222 \quad \int x^3(1+x)^3(1+2x) \left(-18 + 7x^3(1+x)^3\right)^2 dx$$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10} - 36x^7(x+1)^7 + 81x^4(x+1)^4$$

[Out] $81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^{10}*(1+x)^{10}$

Rubi [B] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1612}

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 - 756x^7 - 171x^6 - 324x^5 - 81x^4$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x]$

[Out] $81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^{10})/10 - 1211*x^{11} - (1071*x^{12})/2 + 336*x^{13} + 993*x^{14} + (6174*x^{15})/5 + 1029*x^{16} + 588*x^{17} + (441*x^{18})/2 + 49*x^{19} + (49*x^{20})/10$

Rule 1612

$\text{Int}[(P_x)*(a_+ + (b_+)*(x_+))^{(m_+)}*((c_+ + (d_+)*(x_+))^{(n_+)}*((e_+ + (f_+)*(x_+))^{(p_+)}), x_Symbol] := \text{Int}[\text{ExpandIntegrand}[P_x*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p\}, x] \&\& \text{PolyQ}[P_x, x] \&\& \text{IntegersQ}[m, n]$

Rubi steps

$$\begin{aligned} \int x^3(1+x)^3(1+2x) \left(-18 + 7x^3(1+x)^3\right)^2 dx &= \int (324x^3 + 1620x^4 + 2916x^5 + 2016x^6 - 1368x^7 - 6804x^8 - 12551x^9 - 756x^{10} - 171x^{11} - 1071x^{12} + 336x^{13} + 993x^{14} + 6174x^{15} + 1029x^{16} + 588x^{17} + 441x^{18} + 49x^{19} + 49x^{20}) dx \\ &= 81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551x^{10}}{10} - 1211x^{11} - \frac{1071x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10} \end{aligned}$$

Mathematica [B] time = 0.00, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 - 756x^7 - 171x^6 - 324x^5 - 81x^4$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x]$

[Out] $81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^{10})/10 - 1211*x^{11} - (1071*x^{12})/2 + 336*x^{13} + 993*x^{14} + (6174*x^{15})/5 + 1029*x^{16} + 588*x^{17} + (441*x^{18})/2 + 49*x^{19} + (49*x^{20})/10$

fricas [B] time = 0.72, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 - 756x^7 - 171x^6 - 324x^5 - 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, \text{algorithm}=\text{"fricas"})$

[Out] $49/10*x^{20} + 49*x^{19} + 441/2*x^{18} + 588*x^{17} + 1029*x^{16} + 6174/5*x^{15} + 993*x^{14} + 336*x^{13} - 1071/2*x^{12} - 1211*x^{11} - 12551/10*x^{10} - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4$

giac [A] time = 0.36, size = 28, normalized size = 0.85

$$\frac{49}{10}(x^2 + x)^{10} - 36(x^2 + x)^7 + 81(x^2 + x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")`

[Out] $49/10*(x^2 + x)^{10} - 36*(x^2 + x)^7 + 81*(x^2 + x)^4$

maple [B] time = 0.00, size = 87, normalized size = 2.64

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(x+1)^3*(2*x+1)*(-18+7*x^3*(x+1)^3)^2,x)`

[Out] $49/10*x^{20} + 49*x^{19} + 441/2*x^{18} + 588*x^{17} + 1029*x^{16} + 6174/5*x^{15} + 993*x^{14} + 336*x^{13} - 1071/2*x^{12} - 1211*x^{11} - 12551/10*x^{10} - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4$

maxima [B] time = 0.45, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")`

[Out] $49/10*x^{20} + 49*x^{19} + 441/2*x^{18} + 588*x^{17} + 1029*x^{16} + 6174/5*x^{15} + 993*x^{14} + 336*x^{13} - 1071/2*x^{12} - 1211*x^{11} - 12551/10*x^{10} - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4$

mupad [B] time = 0.19, size = 86, normalized size = 2.61

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(2*x + 1)*(7*x^3*(x + 1)^3 - 18)^2*(x + 1)^3,x)`

[Out] $81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^{10})/10 - 1211*x^{11} - (1071*x^{12})/2 + 336*x^{13} + 993*x^{14} + (6174*x^{15})/5 + 1029*x^{16} + 588*x^{17} + (441*x^{18})/2 + 49*x^{19} + (49*x^{20})/10$

sympy [B] time = 0.09, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)`

[Out] $49*x^{20}/10 + 49*x^{19} + 441*x^{18}/2 + 588*x^{17} + 1029*x^{16} + 6174*x^{15}/5 + 993*x^{14} + 336*x^{13} - 1071*x^{12}/2 - 1211*x^{11} - 12551*x^{10}/10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4$

$$3.223 \quad \int \frac{2-x^2}{(1-6x+x^3)^5} dx$$

Optimal. Leaf size=14

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

[Out] 1/12/(x^3-6*x+1)^4

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1588}

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Antiderivative was successfully verified.

[In] Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]

[Out] 1/(12*(1 - 6*x + x^3)^4)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{2-x^2}{(1-6x+x^3)^5} dx = \frac{1}{12(1-6x+x^3)^4}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]

[Out] 1/(12*(1 - 6*x + x^3)^4)

fricas [B] time = 0.83, size = 57, normalized size = 4.07

$$\frac{1}{12(x^{12} - 24x^{10} + 4x^9 + 216x^8 - 72x^7 - 858x^6 + 432x^5 + 1224x^4 - 860x^3 + 216x^2 - 24x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="fricas")

[Out] 1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224*x^4 - 860*x^3 + 216*x^2 - 24*x + 1)

giac [A] time = 0.32, size = 12, normalized size = 0.86

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="giac")

[Out] 1/12/(x^3 - 6*x + 1)^4

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2)/(x^3-6*x+1)^5,x)

[Out] 1/12/(x^3-6*x+1)^4

maxima [A] time = 0.44, size = 12, normalized size = 0.86

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")

[Out] 1/12/(x^3 - 6*x + 1)^4

mupad [B] time = 2.10, size = 12, normalized size = 0.86

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x)

[Out] 1/(12*(x^3 - 6*x + 1)^4)

sympy [B] time = 0.19, size = 56, normalized size = 4.00

$$\frac{1}{12x^{12} - 288x^{10} + 48x^9 + 2592x^8 - 864x^7 - 10296x^6 + 5184x^5 + 14688x^4 - 10320x^3 + 2592x^2 - 288x + 12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2)/(x**3-6*x+1)**5,x)

[Out] 1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 5184*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)

$$3.224 \quad \int \frac{2x+x^2}{4+3x^2+x^3} dx$$

Optimal. Leaf size=15

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

[Out] 1/3*ln(x^3+3*x^2+4)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1587}

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3), x]

[Out] Log[4 + 3*x^2 + x^3]/3

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} dx = \frac{1}{3} \log(4 + 3x^2 + x^3)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3), x]

[Out] Log[4 + 3*x^2 + x^3]/3

fricas [A] time = 0.79, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(x^3+3*x^2+4), x, algorithm="fricas")

[Out] 1/3*log(x^3 + 3*x^2 + 4)

giac [A] time = 0.37, size = 14, normalized size = 0.93

$$\frac{1}{3} \log(|x^3 + 3x^2 + 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")

[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln(x^3 + 3x^2 + 4)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2*x)/(x^3+3*x^2+4),x)

[Out] 1/3*ln(x^3+3*x^2+4)

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")

[Out] 1/3*log(x^3 + 3*x^2 + 4)

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln(x^3 + 3x^2 + 4)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^2)/(3*x^2 + x^3 + 4),x)

[Out] log(3*x^2 + x^3 + 4)/3

sympy [A] time = 0.09, size = 12, normalized size = 0.80

$$\frac{\log(x^3 + 3x^2 + 4)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)

[Out] log(x**3 + 3*x**2 + 4)/3

$$3.225 \quad \int \frac{1+x+x^3}{4x+2x^2+x^4} dx$$

Optimal. Leaf size=17

$$\frac{1}{4} \log(x^4 + 2x^2 + 4x)$$

[0ut] 1/4*ln(x^4+2*x^2+4*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1587}

$$\frac{1}{4} \log(x^4 + 2x^2 + 4x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4), x]

[0ut] Log[4*x + 2*x^2 + x^4]/4

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} dx = \frac{1}{4} \log(4x+2x^2+x^4)$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.18

$$\frac{1}{4} \log(x^3 + 2x + 4) + \frac{\log(x)}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4), x]

[0ut] Log[x]/4 + Log[4 + 2*x + x^3]/4

fricas [A] time = 0.80, size = 15, normalized size = 0.88

$$\frac{1}{4} \log(x^4 + 2x^2 + 4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x), x, algorithm="fricas")

[0ut] 1/4*log(x^4 + 2*x^2 + 4*x)

giac [A] time = 0.27, size = 18, normalized size = 1.06

$$\frac{1}{4} \log\left(4 \left| \frac{1}{4} x^4 + \frac{1}{2} x^2 + x \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")

[Out] 1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{\ln\left(\left(x^3 + 2x + 4\right)x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x)

[Out] 1/4*ln(x*(x^3+2*x+4))

maxima [A] time = 0.45, size = 15, normalized size = 0.88

$$\frac{1}{4} \log\left(x^4 + 2x^2 + 4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")

[Out] 1/4*log(x^4 + 2*x^2 + 4*x)

mupad [B] time = 0.07, size = 13, normalized size = 0.76

$$\frac{\ln\left(x\left(x^3 + 2x + 4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)

[Out] log(x*(2*x + x^3 + 4))/4

sympy [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4 + 2x^2 + 4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)

[Out] log(x**4 + 2*x**2 + 4*x)/4

$$3.226 \quad \int \frac{bc-ad-2aex-bex^2-3afx^2-2bf x^3}{(c+dx+ex^2+fx^3)^2} dx$$

Optimal. Leaf size=40

$$\frac{a}{c+dx+ex^2+fx^3} + \frac{bx}{c+dx+ex^2+fx^3}$$

[Out] a/(f*x^3+e*x^2+d*x+c)+b*x/(f*x^3+e*x^2+d*x+c)

Rubi [A] time = 0.10, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.058$, Rules used = {6, 2102, 1588}

$$\frac{a}{c+dx+ex^2+fx^3} + \frac{bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

[In] Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2,x]

[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_.) + (b_.)*(v_.))^(p_.), x_Symbol] := Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 2102

Int[(Pm_)*(Qn_)^(p_.), x_Symbol] := With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m + n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn + (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{bc-ad-2aex-bex^2-3afx^2-2bf x^3}{(c+dx+ex^2+fx^3)^2} dx &= \int \frac{bc-ad-2aex+(-be-3af)x^2-2bf x^3}{(c+dx+ex^2+fx^3)^2} dx \\ &= \frac{bx}{c+dx+ex^2+fx^3} - \frac{\int \frac{2adf+4aefx+6af^2x^2}{(c+dx+ex^2+fx^3)^2} dx}{2f} \\ &= \frac{a}{c+dx+ex^2+fx^3} + \frac{bx}{c+dx+ex^2+fx^3} \end{aligned}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 0.58

$$\frac{a + bx}{c + dx + ex^2 + fx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2,x]

[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)

fricas [A] time = 0.86, size = 23, normalized size = 0.58

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x, algorithm="fricas")

[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)

giac [A] time = 0.54, size = 24, normalized size = 0.60

$$\frac{bx + a}{fx^3 + x^2e + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x, algorithm="giac")

[Out] (b*x + a)/(f*x^3 + x^2*e + d*x + c)

maple [A] time = 0.01, size = 28, normalized size = 0.70

$$-\frac{-bx - a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x)

[Out] -(-b*x-a)/(f*x^3+e*x^2+d*x+c)

maxima [A] time = 0.50, size = 23, normalized size = 0.58

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x, algorithm="maxima")

[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)

mupad [B] time = 0.11, size = 23, normalized size = 0.58

$$\frac{a + bx}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*d - b*c + 2*a*e*x + 3*a*f*x^2 + b*e*x^2 + 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2,x)
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

sympy [A] time = 32.56, size = 22, normalized size = 0.55

$$-\frac{-a - bx}{c + dx + ex^2 + fx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2+d*x+c)**2,x)
```

```
[Out] -(-a - b*x)/(c + d*x + e*x**2 + f*x**3)
```

3.227 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4} dx$

Optimal. Leaf size=605

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b(b-\sqrt{8a^2-4ac+b^2})+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-4ac+b^2}\right)\right)}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

[Out] $-1/4*\ln(2*a+2*a*x^2+x*(b-(8*a^2-4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b-(8*a^2-4*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c+b^2)^(1/2)+1/4*\ln(2*a+2*a*x^2+x*(b+(8*a^2-4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b+(8*a^2-4*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c+b^2)^(1/2)+1/2*\arctan(1/2*(b+4*a*x-(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2))))^(1/2)*(4*a^2*B+b*D*(b-(8*a^2-4*a*c+b^2)^(1/2))-a*(b*C+2*c*D+A*(b-(8*a^2-4*a*c+b^2)^(1/2))-C*(8*a^2-4*a*c+b^2)^(1/2)))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2))))^(1/2)-1/2*\arctan(1/2*(b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2))))^(1/2)*(4*a^2*B+b*D*(b+(8*a^2-4*a*c+b^2)^(1/2))-a*(b*C+2*c*D+C*(8*a^2-4*a*c+b^2)^(1/2)+A*(b+(8*a^2-4*a*c+b^2)^(1/2))))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2))))^(1/2)$

Rubi [A] time = 4.54, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2086, 634, 618, 204, 628}

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b(b-\sqrt{8a^2-4ac+b^2})+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-4ac+b^2}\right)\right)}{\sqrt{2}a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4), x]$

[Out] $((4*a^2*B + b*(b - \text{Sqrt}[8*a^2 + b^2 - 4*a*c]))*D - a*(A*(b - \text{Sqrt}[8*a^2 + b^2 - 4*a*c]) + b*C - \text{Sqrt}[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*\text{ArcTan}[(b - \text{Sqrt}[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(\text{Sqrt}[2]*\text{Sqrt}[4*a^2 + 2*a*c - b*(b - \text{Sqrt}[8*a^2 + b^2 - 4*a*c])])]/(\text{Sqrt}[2]*a*\text{Sqrt}[8*a^2 + b^2 - 4*a*c]*\text{Sqrt}[4*a^2 + 2*a*c - b*(b - \text{Sqrt}[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + \text{Sqrt}[8*a^2 + b^2 - 4*a*c]))*D - a*(A*(b + \text{Sqrt}[8*a^2 + b^2 - 4*a*c]) + b*C + \text{Sqrt}[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*\text{ArcTan}[(b + \text{Sqrt}[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(\text{Sqrt}[2]*\text{Sqrt}[4*a^2 + 2*a*c - b*(b + \text{Sqrt}[8*a^2 + b^2 - 4*a*c])])]/(\text{Sqrt}[2]*a*\text{Sqrt}[8*a^2 + b^2 - 4*a*c]*\text{Sqrt}[4*a^2 + 2*a*c - b*(b + \text{Sqrt}[8*a^2 + b^2 - 4*a*c])]) - ((2*a*(A - C) + (b - \text{Sqrt}[8*a^2 + b^2 - 4*a*c])*D)*\text{Log}[2*a + (b - \text{Sqrt}[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*\text{Sqrt}[8*a^2 + b^2 - 4*a*c]) + ((2*a*(A - C) + (b + \text{Sqrt}[8*a^2 + b^2 - 4*a*c])*D)*\text{Log}[2*a + (b + \text{Sqrt}[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*\text{Sqrt}[8*a^2 + b^2 - 4*a*c])$

Rule 204

$\text{Int}[(a + b*x)*(x^2)^(-1), x_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a + b*x + c*x^2)*(x^2)^(-1), x_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\},$

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2086

$\text{Int}[(P3_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2 + (d_.)*(x_.)^3 + (e_.)*(x_.)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Sqrt}[8*a^2 + b^2 - 4*a*c], A = \text{Coeff}[P3, x, 0], B = \text{Coeff}[P3, x, 1], C = \text{Coeff}[P3, x, 2], D = \text{Coeff}[P3, x, 3]\}, \text{Dist}[1/q, \text{Int}[(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*x + 2*a*x^2), x], x] - \text{Dist}[1/q, \text{Int}[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{PolyQ}[P3, x, 3] \&\& \text{EqQ}[a, e] \&\& \text{EqQ}[b, d]$

Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2 + Dx^3}{a + bx + cx^2 + bx^3 + ax^4} dx &= \int \frac{Ab - 2aB - A\sqrt{8a^2 + b^2 - 4ac} + 2aD + (2aA - 2aC + bD - \sqrt{8a^2 + b^2 - 4ac}D)x}{2a + (b - \sqrt{8a^2 + b^2 - 4ac})x + 2ax^2} dx + \int \frac{Ab - 2aB + A\sqrt{8a^2 + b^2 - 4ac} + 2aD + (2aA - 2aC + bD + \sqrt{8a^2 + b^2 - 4ac}D)x}{2a + (b + \sqrt{8a^2 + b^2 - 4ac})x + 2ax^2} dx \\ &= -\frac{(2a(A - C) + (b - \sqrt{8a^2 + b^2 - 4ac})D) \int \frac{b - \sqrt{8a^2 + b^2 - 4ac} + 4ax}{2a + (b - \sqrt{8a^2 + b^2 - 4ac})x + 2ax^2} dx}{4a\sqrt{8a^2 + b^2 - 4ac}} + \frac{(2a(A - C) + (b + \sqrt{8a^2 + b^2 - 4ac})D) \int \frac{b + \sqrt{8a^2 + b^2 - 4ac} + 4ax}{2a + (b + \sqrt{8a^2 + b^2 - 4ac})x + 2ax^2} dx}{4a\sqrt{8a^2 + b^2 - 4ac}} \\ &= -\frac{(2a(A - C) + (b - \sqrt{8a^2 + b^2 - 4ac})D) \log(2a + (b - \sqrt{8a^2 + b^2 - 4ac})x)}{4a\sqrt{8a^2 + b^2 - 4ac}} + \frac{(2a(A - C) + (b + \sqrt{8a^2 + b^2 - 4ac})D) \log(2a + (b + \sqrt{8a^2 + b^2 - 4ac})x)}{4a\sqrt{8a^2 + b^2 - 4ac}} \\ &= \frac{(4a^2B + b(b - \sqrt{8a^2 + b^2 - 4ac})D - a(A(b - \sqrt{8a^2 + b^2 - 4ac}) + bC - \sqrt{8a^2 + b^2 - 4ac})) \log(2a + (b - \sqrt{8a^2 + b^2 - 4ac})x) - (4a^2B + b(b + \sqrt{8a^2 + b^2 - 4ac})D - a(A(b + \sqrt{8a^2 + b^2 - 4ac}) + bC - \sqrt{8a^2 + b^2 - 4ac})) \log(2a + (b + \sqrt{8a^2 + b^2 - 4ac})x)}{4a\sqrt{8a^2 + b^2 - 4ac}} \end{aligned}$$

Mathematica [C] time = 0.07, size = 98, normalized size = 0.16

$$\text{RootSum} \left[\#1^4 a + \#1^3 b + \#1^2 c + \#1 b + a \&, \frac{\#1^3 D \log(x - \#1) + \#1^2 C \log(x - \#1) + A \log(x - \#1) + \#1 B \log(x - \#1)}{4\#1^3 a + 3\#1^2 b + 2\#1 c + b} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4), x]

[Out] RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 &, (A*Log[x - #1] + B*Log[x - #1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2 + 4*a*#1^3) &]

$$\begin{aligned}
 & *b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)} *b*A+1/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)} *arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^{(1/2)})/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)}) *C+1/(8*a^2-4*a*c+b^2)^{(1/2)}/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)} *arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^{(1/2)})/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)}) *b*C+2/(8*a^2-4*a*c+b^2)^{(1/2)}/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)} *arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^{(1/2)})/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)}) *D*c-1/a/(8*a^2-4*a*c+b^2)^{(1/2)}/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)} *arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^{(1/2)})/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)}) *D*b^2-1/a/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)} *arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^{(1/2)})/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)}) *D*b-4*a/(8*a^2-4*a*c+b^2)^{(1/2)}/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)} *arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^{(1/2)})/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^{(1/2)})^{(1/2)}) *B
 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Dx^3 + Cx^2 + Bx + A}{ax^4 + bx^3 + cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxima")

[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx + Cx^2 + x^3 D}{ax^4 + bx^3 + cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2),x)

[Out] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)

[Out] Timed out

$$3.228 \quad \int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx$$

Optimal. Leaf size=63

$$-\frac{2 \log(2x^2 - (1 - \sqrt{5})x + 2)}{1 - \sqrt{5}} - \frac{2 \log(2x^2 - (1 + \sqrt{5})x + 2)}{1 + \sqrt{5}}$$

[Out] $-2*\ln(2+2*x^2-x*(-5^{(1/2)+1}))/(-5^{(1/2)+1})-2*\ln(2+2*x^2-x*(5^{(1/2)+1}))/(-5^{(1/2)+1})$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2086, 628}

$$-\frac{2 \log(2x^2 - (1 - \sqrt{5})x + 2)}{1 - \sqrt{5}} - \frac{2 \log(2x^2 - (1 + \sqrt{5})x + 2)}{1 + \sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4), x]

[Out] $(-2*\text{Log}[2 - (1 - \text{Sqrt}[5])*x + 2*x^2])/(1 - \text{Sqrt}[5]) - (2*\text{Log}[2 - (1 + \text{Sqrt}[5])*x + 2*x^2])/(1 + \text{Sqrt}[5])$

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 2086

Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx = -\frac{\int \frac{-2\sqrt{5}+(10-2\sqrt{5})x}{2+(-1-\sqrt{5})x+2x^2} dx}{\sqrt{5}} + \frac{\int \frac{2\sqrt{5}+(10+2\sqrt{5})x}{2+(-1+\sqrt{5})x+2x^2} dx}{\sqrt{5}}$$

$$= -\frac{2 \log(2 - (1 - \sqrt{5})x + 2x^2)}{1 - \sqrt{5}} - \frac{2 \log(2 - (1 + \sqrt{5})x + 2x^2)}{1 + \sqrt{5}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.87

$$\frac{1}{2} \left((1 + \sqrt{5}) \log(2x^2 + (\sqrt{5} - 1)x + 2) - (\sqrt{5} - 1) \log(-2x^2 + \sqrt{5}x + x - 2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4), x]

[Out] $(-((-1 + \sqrt{5}) \cdot \log[-2 + x + \sqrt{5} \cdot x - 2 \cdot x^2]) + (1 + \sqrt{5}) \cdot \log[2 + (-1 + \sqrt{5}) \cdot x + 2 \cdot x^2]) / 2$

fricas [A] time = 0.76, size = 83, normalized size = 1.32

$$\frac{1}{2} \sqrt{5} \log\left(\frac{2x^4 - 2x^3 + 7x^2 + \sqrt{5}(2x^3 - x^2 + 2x) - 2x + 2}{x^4 - x^3 + x^2 - x + 1}\right) + \frac{1}{2} \log(x^4 - x^3 + x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="fricas")

[Out] $1/2 \cdot \sqrt{5} \cdot \log((2 \cdot x^4 - 2 \cdot x^3 + 7 \cdot x^2 + \sqrt{5} \cdot (2 \cdot x^3 - x^2 + 2 \cdot x) - 2 \cdot x + 2) / (x^4 - x^3 + x^2 - x + 1)) + 1/2 \cdot \log(x^4 - x^3 + x^2 - x + 1)$

giac [A] time = 0.27, size = 58, normalized size = 0.92

$$-\frac{1}{2} \sqrt{5} \log\left(x^2 - \frac{1}{2} x(\sqrt{5} + 1) + 1\right) + \frac{1}{2} \sqrt{5} \log\left(x^2 + \frac{1}{2} x(\sqrt{5} - 1) + 1\right) + \frac{1}{2} \log(x^4 - x^3 + x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")

[Out] $-1/2 \cdot \sqrt{5} \cdot \log(x^2 - 1/2 \cdot x \cdot (\sqrt{5} + 1) + 1) + 1/2 \cdot \sqrt{5} \cdot \log(x^2 + 1/2 \cdot x \cdot (\sqrt{5} - 1) + 1) + 1/2 \cdot \log(x^4 - x^3 + x^2 - x + 1)$

maple [A] time = 0.05, size = 82, normalized size = 1.30

$$\frac{\ln(2x^2 - \sqrt{5}x - x + 2)}{2} - \frac{\sqrt{5} \ln(2x^2 - \sqrt{5}x - x + 2)}{2} + \frac{\ln(2x^2 + \sqrt{5}x - x + 2)}{2} + \frac{\sqrt{5} \ln(2x^2 + \sqrt{5}x - x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x)

[Out] $1/2 \cdot \ln(x \cdot 5^{1/2} + 2 \cdot x^2 - x + 2) + 1/2 \cdot \ln(x \cdot 5^{1/2} + 2 \cdot x^2 - x + 2) \cdot 5^{1/2} + 1/2 \cdot \ln(-x \cdot 5^{1/2} + 2 \cdot x^2 - x + 2) - 1/2 \cdot \ln(-x \cdot 5^{1/2} + 2 \cdot x^2 - x + 2) \cdot 5^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 - 4x^2 + x + 2}{x^4 - x^3 + x^2 - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")

[Out] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)

mupad [B] time = 0.18, size = 75, normalized size = 1.19

$$\frac{\ln\left(x^2 - \frac{\sqrt{5}x}{2} - \frac{x}{2} + 1\right)}{2} + \frac{\ln\left(\frac{\sqrt{5}x}{2} - \frac{x}{2} + x^2 + 1\right)}{2} - \frac{\sqrt{5} \ln\left(x^2 - \frac{\sqrt{5}x}{2} - \frac{x}{2} + 1\right)}{2} + \frac{\sqrt{5} \ln\left(\frac{\sqrt{5}x}{2} - \frac{x}{2} + x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 4*x^2 + 2*x^3 + 2)/(x^2 - x - x^3 + x^4 + 1),x)

[Out] $\log(x^2 - (5^{1/2} \cdot x) / 2 - x / 2 + 1) / 2 + \log((5^{1/2} \cdot x) / 2 - x / 2 + x^2 + 1) / 2 - (5^{1/2} \cdot \log(x^2 - (5^{1/2} \cdot x) / 2 - x / 2 + 1)) / 2 + (5^{1/2} \cdot \log((5^{1/2} \cdot x) / 2 - x / 2 + x^2 + 1)) / 2$

sympy [A] time = 0.13, size = 58, normalized size = 0.92

$$\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) \log\left(x^2 + x\left(-\frac{1}{2} + \frac{\sqrt{5}}{2}\right) + 1\right) + \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) \log\left(x^2 + x\left(-\frac{\sqrt{5}}{2} - \frac{1}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)

[Out] (1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)*log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)

$$3.229 \quad \int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx$$

Optimal. Leaf size=14

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

[Out] 1/3/(1+x)^3+ln(1+x)

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1594, 1680, 14}

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]

[Out] 1/(3*(1 + x)^3) + Log[1 + x]

Rule 14

Int[(u_)*((c_)*(x_)^(m_)), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1680

Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx &= \int \frac{x(3+3x+x^2)}{1+4x+6x^2+4x^3+x^4} dx \\ &= \text{Subst} \left(\int \frac{-1+x^3}{x^4} dx, x, 1+x \right) \\ &= \text{Subst} \left(\int \left(-\frac{1}{x^4} + \frac{1}{x} \right) dx, x, 1+x \right) \\ &= \frac{1}{3(1+x)^3} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] 1/(3*(1 + x)^3) + Log[1 + x]

fricas [B] time = 0.89, size = 38, normalized size = 2.71

$$\frac{3(x^3 + 3x^2 + 3x + 1)\log(x + 1) + 1}{3(x^3 + 3x^2 + 3x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1), x, algorithm="fricas")

[Out] 1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)

giac [A] time = 0.29, size = 13, normalized size = 0.93

$$\frac{1}{3(x+1)^3} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1), x, algorithm="giac")

[Out] 1/3/(x + 1)^3 + log(abs(x + 1))

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$\ln(x + 1) + \frac{1}{3(x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1), x)

[Out] 1/3/(x+1)^3+ln(x+1)

maxima [A] time = 0.44, size = 22, normalized size = 1.57

$$\frac{1}{3(x^3 + 3x^2 + 3x + 1)} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1), x, algorithm="maxima")

[Out] 1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)

mupad [B] time = 0.04, size = 12, normalized size = 0.86

$$\ln(x + 1) + \frac{1}{3(x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 3*x^2 + x^3)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1), x)

[Out] log(x + 1) + 1/(3*(x + 1)^3)

sympy [A] time = 0.10, size = 20, normalized size = 1.43

$$\log(x + 1) + \frac{1}{3x^3 + 9x^2 + 9x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

$$3.230 \quad \int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx$$

Optimal. Leaf size=28

$$\frac{6}{x+1} - \frac{6}{(x+1)^2} + \frac{8}{3(x+1)^3} + \log(x+1)$$

[Out] 8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1680, 43}

$$\frac{6}{x+1} - \frac{6}{(x+1)^2} + \frac{8}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 1680

Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx &= \text{Subst} \left(\int \frac{(-2+x)^3}{x^4} dx, x, 1+x \right) \\ &= \text{Subst} \left(\int \left(-\frac{8}{x^4} + \frac{12}{x^3} - \frac{6}{x^2} + \frac{1}{x} \right) dx, x, 1+x \right) \\ &= \frac{8}{3(1+x)^3} - \frac{6}{(1+x)^2} + \frac{6}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.86

$$\frac{2(9x^2+9x+4)}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]

fricas [A] time = 1.00, size = 46, normalized size = 1.64

$$\frac{18x^2 + 3(x^3 + 3x^2 + 3x + 1)\log(x + 1) + 18x + 8}{3(x^3 + 3x^2 + 3x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")

[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^2 + 3*x + 1)

giac [A] time = 0.39, size = 23, normalized size = 0.82

$$\frac{2(9x^2 + 9x + 4)}{3(x + 1)^3} + \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")

[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))

maple [A] time = 0.01, size = 27, normalized size = 0.96

$$\ln(x + 1) + \frac{8}{3(x + 1)^3} - \frac{6}{(x + 1)^2} + \frac{6}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)

[Out] 8/3/(x+1)^3-6/(x+1)^2+6/(x+1)+ln(x+1)

maxima [A] time = 0.45, size = 32, normalized size = 1.14

$$\frac{2(9x^2 + 9x + 4)}{3(x^3 + 3x^2 + 3x + 1)} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")

[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)

mupad [B] time = 0.04, size = 21, normalized size = 0.75

$$\ln(x + 1) + \frac{6x^2 + 6x + \frac{8}{3}}{(x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)

[Out] log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3

sympy [A] time = 0.11, size = 29, normalized size = 1.04

$$\frac{18x^2 + 18x + 8}{3x^3 + 9x^2 + 9x + 3} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)

[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)

$$3.231 \quad \int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=59

$$\frac{13x}{x^4+2x^2+3} - \frac{2(13x^2+18)x}{(x^4+2x^2+3)^2} + \frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

[Out] $2*(-2*x^2+1)/(x^4+2*x^2+3)^2-2*x*(13*x^2+18)/(x^4+2*x^2+3)^2+13*x/(x^4+2*x^2+3)$

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {1673, 1678, 1588, 1663, 1660, 8}

$$\frac{13x}{x^4+2x^2+3} - \frac{2(13x^2+18)x}{(x^4+2x^2+3)^2} + \frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

Antiderivative was successfully verified.

[In] Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x^4)^3,x]

[Out] $(2*(1-2*x^2))/(3+2*x^2+x^4)^2 - (2*x*(18+13*x^2))/(3+2*x^2+x^4)^2 + (13*x)/(3+2*x^2+x^4)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]

Rule 1673

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{d =
Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
+ 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{9 - 40x - 18x^2 + 174x^4 + 24x^5 + 26x^6 - 39x^8}{(3 + 2x^2 + x^4)^3} dx &= \int \frac{x(-40 + 24x^4)}{(3 + 2x^2 + x^4)^3} dx + \int \frac{9 - 18x^2 + 174x^4 + 26x^6 - 39x^8}{(3 + 2x^2 + x^4)^3} dx \\ &= -\frac{2x(18 + 13x^2)}{(3 + 2x^2 + x^4)^2} + \frac{1}{96} \int \frac{3744 - 2496x^2 - 3744x^4}{(3 + 2x^2 + x^4)^2} dx \\ &= \frac{2(1 - 2x^2)}{(3 + 2x^2 + x^4)^2} - \frac{2x(18 + 13x^2)}{(3 + 2x^2 + x^4)^2} + \frac{13x}{3 + 2x^2 + x^4} + \frac{1}{3 + 2x^2 + x^4} \\ &= \frac{2(1 - 2x^2)}{(3 + 2x^2 + x^4)^2} - \frac{2x(18 + 13x^2)}{(3 + 2x^2 + x^4)^2} + \frac{13x}{3 + 2x^2 + x^4} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{13x^5 - 4x^2 + 3x + 2}{(x^4 + 2x^2 + 3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x
^2 + x^4)^3, x]
```

```
[Out] (2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

fricas [A] time = 0.84, size = 38, normalized size = 0.64

$$\frac{13x^5 - 4x^2 + 3x + 2}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3, x,
algorithm="fricas")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [A] time = 1.44, size = 28, normalized size = 0.47

$$\frac{13x^5 - 4x^2 + 3x + 2}{(x^4 + 2x^2 + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x, algorithm="giac")

[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2

maple [A] time = 0.01, size = 30, normalized size = 0.51

$$-\frac{-13x^5 + 4x^2 - 3x - 2}{(x^4 + 2x^2 + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x)

[Out] -(-13*x^5+4*x^2-3*x-2)/(x^4+2*x^2+3)^2

maxima [A] time = 0.46, size = 38, normalized size = 0.64

$$\frac{13x^5 - 4x^2 + 3x + 2}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x, algorithm="maxima")

[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)

mupad [B] time = 0.05, size = 28, normalized size = 0.47

$$\frac{13x^5 - 4x^2 + 3x + 2}{(x^4 + 2x^2 + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((174*x^4 - 18*x^2 - 40*x + 24*x^5 + 26*x^6 - 39*x^8 + 9)/(2*x^2 + x^4 + 3)^3,x)

[Out] (3*x - 4*x^2 + 13*x^5 + 2)/(2*x^2 + x^4 + 3)^2

sympy [A] time = 0.18, size = 36, normalized size = 0.61

$$-\frac{-13x^5 + 4x^2 - 3x - 2}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+3)**3,x)

[Out] -(-13*x**5 + 4*x**2 - 3*x - 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)

$$3.232 \quad \int \frac{-1+4x^5}{(1+x+x^5)^2} dx$$

Optimal. Leaf size=11

$$-\frac{x}{x^5+x+1}$$

[Out] -x/(x^5+x+1)

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1588}

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} dx = -\frac{x}{1+x+x^5}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

fricas [A] time = 0.73, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")

[Out] -x/(x^5 + x + 1)

giac [A] time = 0.33, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")

[Out] -x/(x^5 + x + 1)

maple [B] time = 0.01, size = 41, normalized size = 3.73

$$-\frac{-3x^2 + 5x - 1}{7(x^3 - x^2 + 1)} + \frac{-3x - 1}{7x^2 + 7x + 7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^5-1)/(x^5+x+1)^2,x)

[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)

maxima [A] time = 0.44, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")

[Out] -x/(x^5 + x + 1)

mupad [B] time = 2.30, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)

[Out] -x/(x + x^5 + 1)

sympy [A] time = 0.12, size = 8, normalized size = 0.73

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)

[Out] -x/(x**5 + x + 1)

$$3.233 \quad \int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2} dx$$

Optimal. Leaf size=91

$$\frac{x}{16(1-x^2)} + \frac{(29-5x^2)x}{32(x^4-6x^2+1)} + \frac{1}{4} \tanh^{-1}(x) + \frac{1}{64} \left((3-2\sqrt{2}) \tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right) - (3+2\sqrt{2}) \tanh^{-1}\left(\left(1+\sqrt{2}\right)x\right) \right)$$

[Out] 1/16*x/(-x^2+1)+1/32*x*(-5*x^2+29)/(x^4-6*x^2+1)+1/4*arctanh(x)+1/64*arctanh(x*(2^(1/2)-1))*(3-2*2^(1/2))-1/64*arctanh(x*(1+2^(1/2)))*(3+2*2^(1/2))

Rubi [B] time = 0.15, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2073, 207, 638, 618, 206, 632, 31}

$$-\frac{12-5x}{64(-x^2+2x+1)} + \frac{5x+12}{64(-x^2-2x+1)} + \frac{1}{32(1-x)} - \frac{1}{32(x+1)} - \frac{3}{256} (2+3\sqrt{2}) \log(-x-\sqrt{2}+1) - \frac{3}{256} (2-3\sqrt{2}) \log(-x+\sqrt{2}+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2, x]

[Out] 1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 - 5*x)/(64*(1 + 2*x - x^2)) - (5*ArcTanh[(1 - x)/Sqrt[2]])/(64*Sqrt[2]) + ArcTanh[x]/4 + (5*ArcTanh[(1 + x)/Sqrt[2]])/(64*Sqrt[2]) - (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] - x])/256 - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 + (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] + x])/256 + (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] + x])/256

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 638

```
Int[((d_.) + (e_.)*(x_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2} dx &= \int \left(\frac{1}{32(-1+x)^2} + \frac{1}{32(1+x)^2} - \frac{1}{4(-1+x^2)} + \frac{17-7x}{32(-1-2x+x^2)^2} - \frac{3(-4+x)}{64(-1-2x+x^2)} \right) dx \\ &= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{1}{32} \int \frac{17-7x}{(-1-2x+x^2)^2} dx + \frac{1}{32} \int \frac{17+7x}{(-1+2x+x^2)^2} dx - \frac{3}{64} \int \frac{-4+x}{-1-2x+x^2} dx \\ &= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64(1-2x-x^2)} - \frac{12-5x}{64(1+2x-x^2)} + \frac{1}{4} \tanh^{-1}(x) - \frac{5}{64} \int \frac{-4+x}{-1-2x+x^2} dx \\ &= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64(1-2x-x^2)} - \frac{12-5x}{64(1+2x-x^2)} + \frac{1}{4} \tanh^{-1}(x) - \frac{3}{256} \int \frac{-4+x}{-1-2x+x^2} dx \\ &= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64(1-2x-x^2)} - \frac{12-5x}{64(1+2x-x^2)} - \frac{5 \tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}} + \frac{1}{4} \int \frac{-4+x}{-1-2x+x^2} dx \end{aligned}$$

Mathematica [A] time = 0.09, size = 132, normalized size = 1.45

$$\frac{1}{128} \left(-\frac{4x(7x^4 - 46x^2 + 31)}{x^6 - 7x^4 + 7x^2 - 1} - 16 \log(1-x) + (3 + 2\sqrt{2}) \log(-x + \sqrt{2} - 1) + (2\sqrt{2} - 3) \log(-x + \sqrt{2} + 1) + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2, x]

[Out] ((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] + (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2] - x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt[2])*Log[1 + Sqrt[2] + x])/128

fricas [B] time = 0.94, size = 223, normalized size = 2.45

$$28x^5 - 184x^3 - 2\sqrt{2}(x^6 - 7x^4 + 7x^2 - 1) \log\left(\frac{x^2 - 2\sqrt{2}(x+1) + 2x+3}{x^2 + 2x - 1}\right) - 2\sqrt{2}(x^6 - 7x^4 + 7x^2 - 1) \log\left(\frac{x^2 - 2\sqrt{2}(x-1) + 2x-3}{x^2 - 2x - 1}\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="fricas")

```
[Out] -1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

giac [A] time = 0.37, size = 134, normalized size = 1.47

$$\frac{1}{64} \sqrt{2} \log\left(\frac{|2x - 2\sqrt{2} + 2|}{|2x + 2\sqrt{2} + 2|}\right) + \frac{1}{64} \sqrt{2} \log\left(\frac{|2x - 2\sqrt{2} - 2|}{|2x + 2\sqrt{2} - 2|}\right) - \frac{7x^5 - 46x^3 + 31x}{32(x^6 - 7x^4 + 7x^2 - 1)} + \frac{3}{128} \log(|x^2 + 2x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(abs(x^2 + 2*x - 1)) - 3/128*log(abs(x^2 - 2*x - 1)) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1))
```

maple [A] time = 0.02, size = 116, normalized size = 1.27

$$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{32} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{32} - \frac{\ln(x-1)}{8} + \frac{\ln(x+1)}{8} - \frac{3 \ln(x^2 - 2x - 1)}{128} + \frac{3 \ln(x^2 + 2x - 1)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x)
```

```
[Out] -1/32/(x-1)-1/8*ln(x-1)+1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x^2+2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/32/(x+1)+1/8*ln(x+1)-1/64*(-12+5*x)/(x^2-2*x-1)-3/128*ln(x^2-2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*x-2)*2^(1/2))
```

maxima [A] time = 0.99, size = 114, normalized size = 1.25

$$\frac{1}{64} \sqrt{2} \log\left(\frac{x - \sqrt{2} + 1}{x + \sqrt{2} + 1}\right) + \frac{1}{64} \sqrt{2} \log\left(\frac{x - \sqrt{2} - 1}{x + \sqrt{2} - 1}\right) - \frac{7x^5 - 46x^3 + 31x}{32(x^6 - 7x^4 + 7x^2 - 1)} + \frac{3}{128} \log(x^2 + 2x - 1) - \frac{3}{128} \log(x^2 - 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/64*sqrt(2)*log((x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

mupad [B] time = 2.37, size = 124, normalized size = 1.36

$$\frac{\operatorname{atan}(x \operatorname{li} 1) \operatorname{li} 1}{4} - \frac{\frac{7x^5}{32} - \frac{23x^3}{16} + \frac{31x}{32}}{x^6 - 7x^4 + 7x^2 - 1} + \operatorname{atan}\left(\frac{x \operatorname{atan}\left(\frac{\sqrt{2} x \operatorname{atan}\left(\frac{27309\sqrt{2}}{32768} - \frac{19317}{16384}\right) - \frac{27309\sqrt{2}}{32768} - \frac{19317}{16384}\right)}{8192}}{\frac{27309\sqrt{2}}{32768} - \frac{19317}{16384}}\right) - \frac{\sqrt{2} x \operatorname{atan}\left(\frac{27309\sqrt{2}}{32768} - \frac{19317}{16384}\right)}{32768} \left(\frac{\sqrt{2} \operatorname{li} 1}{32} - \frac{3}{64}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] atan((x*23313i)/(8192*((27309*2^(1/2))/32768 - 19317/16384)) - (2^(1/2)*x*6
5943i)/(32768*((27309*2^(1/2))/32768 - 19317/16384)))*((2^(1/2)*1i)/32 - 3i
/64) - ((31*x)/32 - (23*x^3)/16 + (7*x^5)/32)/(7*x^2 - 7*x^4 + x^6 - 1) - (
atan(x*1i)*1i)/4 + atan((x*23313i)/(8192*((27309*2^(1/2))/32768 + 19317/163
84)) + (2^(1/2)*x*65943i)/(32768*((27309*2^(1/2))/32768 + 19317/16384)))*((
2^(1/2)*1i)/32 + 3i/64)
```

sympy [B] time = 1.36, size = 272, normalized size = 2.99

$$\frac{-7x^5 + 46x^3 - 31x}{32x^6 - 224x^4 + 224x^2 - 32} - \frac{\log(x-1)}{8} + \frac{\log(x+1)}{8} + \left(-\frac{3}{128} - \frac{\sqrt{2}}{64}\right) \log\left(x - \frac{38423555}{909328} - \frac{38423555\sqrt{2}}{1363992} + \frac{9549859782656}{170499} - \frac{56267374592}{56833}\right) + \left(-\frac{3}{128} + \frac{\sqrt{2}}{64}\right) \log\left(x - \frac{38423555}{909328} + \frac{38423555\sqrt{2}}{1363992} - \frac{9549859782656}{170499} + \frac{56267374592}{56833}\right) + \left(\frac{3}{128} - \frac{\sqrt{2}}{64}\right) \log\left(x - \frac{38423555}{909328} - \frac{38423555\sqrt{2}}{1363992} - \frac{9549859782656}{170499} + \frac{56267374592}{56833}\right) + \left(\frac{3}{128} + \frac{\sqrt{2}}{64}\right) \log\left(x - \frac{38423555}{909328} + \frac{38423555\sqrt{2}}{1363992} - \frac{9549859782656}{170499} - \frac{56267374592}{56833}\right) + \left(\frac{\sqrt{2}}{64} + \frac{3}{128}\right) \log\left(x - \frac{38423555}{909328} - \frac{38423555\sqrt{2}}{1363992} - \frac{9549859782656}{170499} + \frac{56267374592}{56833}\right) + \left(\frac{\sqrt{2}}{64} + \frac{3}{128}\right) \log\left(x - \frac{38423555}{909328} + \frac{38423555\sqrt{2}}{1363992} - \frac{9549859782656}{170499} - \frac{56267374592}{56833}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

```
[Out] (-7*x**5 + 46*x**3 - 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) - log(x - 1
)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 38423555/909328 - 384235
55*sqrt(2)/1363992 + 9549859782656*(-3/128 - sqrt(2)/64)**5/170499 - 562673
74592*(-3/128 - sqrt(2)/64)**3/56833) + (-3/128 + sqrt(2)/64)*log(x - 38423
555/909328 + 9549859782656*(-3/128 + sqrt(2)/64)**5/170499 - 56267374592*(-
3/128 + sqrt(2)/64)**3/56833 + 38423555*sqrt(2)/1363992) + (3/128 - sqrt(2)
/64)*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(3/128 - sqrt(2)/64)**3
/56833 + 9549859782656*(3/128 - sqrt(2)/64)**5/170499 + 38423555/909328) +
(sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(2)/64 + 3/128)**3/56833 + 95
49859782656*(sqrt(2)/64 + 3/128)**5/170499 + 38423555*sqrt(2)/1363992 + 384
23555/909328)
```

$$3.234 \quad \int x^m (a + bx + cx^2 + dx^3)^p (a(1+m) + x(b(2+m+p) + x(c(3+m+2p) + d(4+m+3p)x))) dx$$

Optimal. Leaf size=25

$$x^{m+1} (a + bx + cx^2 + dx^3)^{p+1}$$

[Out] $x^{(1+m)}*(d*x^3+c*x^2+b*x+a)^{(1+p)}$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {1590}

$$x^{m+1} (a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]$

[Out] $x^{(1 + m)}*(a + b*x + c*x^2 + d*x^3)^{(1 + p)}$

Rule 1590

$\text{Int}[(\text{Pp}_*)*(\text{Qq}_*)^{(m_*)}*(\text{Rr}_*)^{(n_*)}, x_Symbol] :> \text{With}[\{p = \text{Expon}[\text{Pp}, x], q = \text{Expon}[\text{Qq}, x], r = \text{Expon}[\text{Rr}, x]\}, \text{Simp}[(\text{Coeff}[\text{Pp}, x, p]*x^{(p - q - r + 1)}*Qq^{(m + 1)}*Rr^{(n + 1)})/((p + m*q + n*r + 1)*\text{Coeff}[\text{Qq}, x, q]*\text{Coeff}[\text{Rr}, x, r]), x] /; \text{NeQ}[p + m*q + n*r + 1, 0] \&\& \text{EqQ}[(p + m*q + n*r + 1)*\text{Coeff}[\text{Qq}, x, q]*\text{Coeff}[\text{Rr}, x, r]*\text{Pp}, \text{Coeff}[\text{Pp}, x, p]*x^{(p - q - r)}*((p - q - r + 1)*\text{Qq}*Rr + (m + 1)*x*Rr*D[\text{Qq}, x] + (n + 1)*x*Qq*D[\text{Rr}, x])]] /; \text{FreeQ}[\{m, n\}, x] \&\& \text{PolyQ}[\text{Pp}, x] \&\& \text{PolyQ}[\text{Qq}, x] \&\& \text{PolyQ}[\text{Rr}, x] \&\& \text{NeQ}[m, -1] \&\& \text{NeQ}[n, -1]$

Rubi steps

$$\int x^m (a + bx + cx^2 + dx^3)^p (a(1+m) + x(b(2+m+p) + x(c(3+m+2p) + d(4+m+3p)x))) dx = x^{1+m} (a +$$

Mathematica [A] time = 0.37, size = 23, normalized size = 0.92

$$x^{m+1}(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]$

[Out] $x^{(1 + m)}*(a + x*(b + x*(c + d*x)))^{(1 + p)}$

fricas [A] time = 1.25, size = 40, normalized size = 1.60

$$(dx^4 + cx^3 + bx^2 + ax)(dx^3 + cx^2 + bx + a)^p x^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)*x))),x, \text{algorithm}=\text{"fricas"})$

[Out] $(d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m$

giac [B] time = 2.09, size = 99, normalized size = 3.96

$$(dx^3 + cx^2 + bx + a)^p dx^4 x^m + (dx^3 + cx^2 + bx + a)^p cx^3 x^m + (dx^3 + cx^2 + bx + a)^p bx^2 x^m + (dx^3 + cx^2 + bx + a)^p ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)*x))),x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*x^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*a*x*x^m

maple [A] time = 0.01, size = 26, normalized size = 1.04

$$x^{m+1} (dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)*x))),x)

[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)

maxima [A] time = 0.70, size = 44, normalized size = 1.76

$$(dx^4 + cx^3 + bx^2 + ax)e^{(p \log(dx^3 + cx^2 + bx + a) + m \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)*x))),x, algorithm="maxima")

[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x))

mupad [B] time = 2.66, size = 49, normalized size = 1.96

$$(dx^3 + cx^2 + bx + a)^p (axx^m + bx^m x^2 + cx^m x^3 + dx^m x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(a*(m + 1) + x*(x*(c*(m + 2*p + 3) + d*x*(m + 3*p + 4)) + b*(m + p + 2)))*(a + b*x + c*x^2 + d*x^3)^p,x)

[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x*x^m + b*x^m*x^2 + c*x^m*x^3 + d*x^m*x^4)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)*x))),x)

[Out] Timed out

$$3.235 \quad \int x^2 (a + bx + cx^2 + dx^3)^p (3a + b(4 + p)x + c(5 + 2p)x^2 + d(6 + 3p)x^3) dx$$

Optimal. Leaf size=23

$$x^3 (a + bx + cx^2 + dx^3)^{p+1}$$

[Out] $x^3*(d*x^3+c*x^2+b*x+a)^(1+p)$

Rubi [A] time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {1588}

$$x^3 (a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3), x]$

[Out] $x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)$

Rule 1588

$\text{Int}[(\text{Pp}_*)*(\text{Qq}_*)^{(m_*)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Expon}[\text{Pp}, x], q = \text{Expon}[\text{Qq}, x]\}, \text{Simp}[(\text{Coeff}[\text{Pp}, x, p]*x^{(p - q + 1)}*\text{Qq}^{(m + 1)})/((p + m*q + 1)*\text{Coeff}[\text{Qq}, x, q]), x] /; \text{NeQ}[p + m*q + 1, 0] \&\& \text{EqQ}[(p + m*q + 1)*\text{Coeff}[\text{Qq}, x, q]*\text{Pp}, \text{Coeff}[\text{Pp}, x, p]*x^{(p - q)}*((p - q + 1)*\text{Qq} + (m + 1)*x*\text{D}[\text{Qq}, x])]] /; \text{FreeQ}[m, x] \&\& \text{PolyQ}[\text{Pp}, x] \&\& \text{PolyQ}[\text{Qq}, x] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\int x^2 (a + bx + cx^2 + dx^3)^p (3a + b(4 + p)x + c(5 + 2p)x^2 + d(6 + 3p)x^3) dx = x^3 (a + bx + cx^2 + dx^3)^{1+p}$$

Mathematica [A] time = 0.23, size = 21, normalized size = 0.91

$$x^3(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3), x]$

[Out] $x^3*(a + x*(b + x*(c + d*x)))^(1 + p)$

fricas [A] time = 0.71, size = 39, normalized size = 1.70

$$(dx^6 + cx^5 + bx^4 + ax^3)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3), x, \text{algorithm}=\text{"fricas"})$

[Out] $(d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p$

giac [B] time = 1.89, size = 89, normalized size = 3.87

$$(dx^3 + cx^2 + bx + a)^p dx^6 + (dx^3 + cx^2 + bx + a)^p cx^5 + (dx^3 + cx^2 + bx + a)^p bx^4 + (dx^3 + cx^2 + bx + a)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$x^3 (dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x)

[Out] x^3*(d*x^3+c*x^2+b*x+a)^(p+1)

maxima [A] time = 0.66, size = 39, normalized size = 1.70

$$(dx^6 + cx^5 + bx^4 + ax^3)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x, algorithm="maxima")

[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p

mupad [B] time = 2.55, size = 39, normalized size = 1.70

$$(dx^3 + cx^2 + bx + a)^p (dx^6 + cx^5 + bx^4 + ax^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) + d*x^3*(3*p + 6)),x)

[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6+3*p)*x**3),x)

[Out] Timed out

$$3.236 \quad \int x \left(a + bx + cx^2 + dx^3 \right)^p \left(2a + b(3 + p)x + c(4 + 2p)x^2 + d(5 + 3p)x^3 \right) dx$$

Optimal. Leaf size=23

$$x^2 (a + bx + cx^2 + dx^3)^{p+1}$$

[Out] $x^2(d*x^3+c*x^2+b*x+a)^{(1+p)}$

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {1588}

$$x^2 (a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*(5 + 3*p)*x^3), x]

[Out] $x^2(a + b*x + c*x^2 + d*x^3)^{(1 + p)}$

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x (a + bx + cx^2 + dx^3)^p (2a + b(3 + p)x + c(4 + 2p)x^2 + d(5 + 3p)x^3) dx = x^2 (a + bx + cx^2 + dx^3)^{1+p}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 0.91

$$x^2(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*(5 + 3*p)*x^3), x]

[Out] $x^2(a + x*(b + x*(c + d*x)))^{(1 + p)}$

fricas [A] time = 0.89, size = 39, normalized size = 1.70

$$(dx^5 + cx^4 + bx^3 + ax^2)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3), x, algorithm="fricas")

[Out] $(d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p$

giac [B] time = 0.73, size = 89, normalized size = 3.87

$$(dx^3 + cx^2 + bx + a)^p dx^5 + (dx^3 + cx^2 + bx + a)^p cx^4 + (dx^3 + cx^2 + bx + a)^p bx^3 + (dx^3 + cx^2 + bx + a)^p ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$x^2 (dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x)

[Out] x^2*(d*x^3+c*x^2+b*x+a)^(p+1)

maxima [A] time = 0.63, size = 39, normalized size = 1.70

$$(dx^5 + cx^4 + bx^3 + ax^2)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x, algorithm="maxima")

[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p

mupad [B] time = 2.32, size = 39, normalized size = 1.70

$$(dx^3 + cx^2 + bx + a)^p (dx^5 + cx^4 + bx^3 + ax^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d*x^3*(3*p + 5)),x)

[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*p)*x**3),x)

[Out] Timed out

$$3.237 \quad \int (a + bx + cx^2 + dx^3)^p (a + b(2 + p)x + c(3 + 2p)x^2$$

Optimal. Leaf size=21

$$x(a + bx + cx^2 + dx^3)^{p+1}$$

[Out] x*(d*x^3+c*x^2+b*x+a)^(1+p)

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {1588}

$$x(a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3), x]

[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (a + bx + cx^2 + dx^3)^p (a + b(2 + p)x + c(3 + 2p)x^2 + d(4 + 3p)x^3) dx = x(a + bx + cx^2 + dx^3)^{1+p}$$

Mathematica [A] time = 0.12, size = 19, normalized size = 0.90

$$x(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3), x]

[Out] x*(a + x*(b + x*(c + d*x)))^(1 + p)

fricas [A] time = 0.87, size = 37, normalized size = 1.76

$$(dx^4 + cx^3 + bx^2 + ax)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3), x, algorithm="fricas")

[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p

giac [B] time = 1.08, size = 87, normalized size = 4.14

$$(dx^3 + cx^2 + bx + a)^p dx^4 + (dx^3 + cx^2 + bx + a)^p cx^3 + (dx^3 + cx^2 + bx + a)^p bx^2 + (dx^3 + cx^2 + bx + a)^p ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3), x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$x(d x^3 + c x^2 + b x + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2+b*x+a)^p*(a+b*(p+2)*x+c*(2*p+3)*x^2+d*(4+3*p)*x^3), x)

[Out] x*(d*x^3+c*x^2+b*x+a)^(p+1)

maxima [A] time = 0.61, size = 37, normalized size = 1.76

$$(d x^4 + c x^3 + b x^2 + a x)(d x^3 + c x^2 + b x + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3), x, algorithm="maxima")

[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p

mupad [B] time = 2.27, size = 37, normalized size = 1.76

$$(d x^3 + c x^2 + b x + a)^p (d x^4 + c x^3 + b x^2 + a x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3*(3*p + 4)), x)

[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x**3), x)

[Out] Timed out

$$3.238 \quad \int \frac{(a+bx+cx^2+dx^3)^p (b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x} dx$$

Optimal. Leaf size=19

$$(a + bx + cx^2 + dx^3)^{p+1}$$

[Out] $(d*x^3+c*x^2+b*x+a)^{(1+p)}$

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1585, 1588}

$$(a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*p)*x^3))/x,x]

[Out] $(a + b*x + c*x^2 + d*x^3)^{(1 + p)}$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1588

Int[(Pp_)*(Qq_)^(m_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{(a + bx + cx^2 + dx^3)^p (b(1+p)x + c(2+2p)x^2 + d(3+3p)x^3)}{x} dx = \int (b(1+p) + c(2+2p)x + d(3+3p)x^2) (a + bx + cx^2 + dx^3)^{p-1} dx = (a + bx + cx^2 + dx^3)^{1+p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*p)*x^3))/x,x]

[Out] $(a + x*(b + x*(c + d*x)))^{(1 + p)}$

fricas [A] time = 0.90, size = 33, normalized size = 1.74

$$(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 0.30, size = 52, normalized size = 2.74

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{p+1} + \frac{(dx^3 + cx^2 + bx + a)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p
+ 1)/(p + 1)
```

maple [A] time = 0.01, size = 20, normalized size = 1.05

$$(dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.62, size = 33, normalized size = 1.74

$$(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.19, size = 19, normalized size = 1.00

$$(dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 + d
*x^3)^p)/x,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**3
)/x,x)
```

```
[Out] Timed out
```

$$3.239 \quad \int \frac{(a+bx+cx^2+dx^3)^p (-a+bp x+c(1+2p)x^2+d(2+3p)x^3)}{x^2} dx$$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p)*x^3))/x^2,x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p (-a+bp x+c(1+2p)x^2+d(2+3p)x^3)}{x^2} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p)*x^3))/x^2,x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x

fricas [A] time = 0.81, size = 36, normalized size = 1.57

$$\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2, x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(3p+2)x^3 + c(2p+1)x^2 + bpx - a)(dx^3 + cx^2 + bx + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2, x, algorithm="giac")

[Out] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2, x)

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2, x)

[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x

maxima [A] time = 0.62, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2, x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x

mupad [B] time = 3.20, size = 23, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p + 2)))/x^2, x)

[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3)/x**2, x)

[Out] Timed out

$$3.240 \quad \int \frac{(a+bx+cx^2+dx^3)^p (-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx$$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^2

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3, x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p (-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

Mathematica [A] time = 0.20, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3, x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^2

fricas [A] time = 1.04, size = 36, normalized size = 1.57

$$\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(3p+1)x^3 + 2cpx^2 + b(p-1)x - 2a)(dx^3 + cx^2 + bx + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x, algorithm="giac")

[Out] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3, x)

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(3*p+1)*x^3)/x^3,x)

[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x^2

maxima [A] time = 0.64, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2

mupad [B] time = 3.32, size = 23, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3*p + 1)))/x^3,x)

[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^2

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x**3)/x**3,x)

[Out] Timed out

$$3.241 \quad \int \frac{(a+bx+cx^2+dx^3)^p (-3a+b(-2+p)x+c(-1+2p)x^2+3dp x^3)}{x^4} dx$$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^3

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 + 3*d*p*x^3))/x^4, x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p (-3a+b(-2+p)x+c(-1+2p)x^2+3dp x^3)}{x^4} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

Mathematica [A] time = 0.19, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 + 3*d*p*x^3))/x^4, x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^3

fricas [A] time = 1.04, size = 36, normalized size = 1.57

$$\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3dp^3 + c(2p-1)x^2 + b(p-2)x - 3a)(dx^3 + cx^2 + bx + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x, algorithm="giac")

[Out] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2 + b*x + a)^p/x^4, x)

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x)

[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x^3

maxima [A] time = 0.64, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3

mupad [B] time = 3.35, size = 23, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2*p - 1)))/x^4,x)

[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^3

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*x**3)/x**4,x)

[Out] Timed out

$$3.242 \quad \int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=97

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{1}{3} \log(x^2 + x + 1) - \frac{13}{48} \log(2x^2 - x + 2) + \frac{5x}{4} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] 5/4*x-3/4*x^2+1/3*x^3+1/4*x^4+1/3*ln(x^2+x+1)-13/48*ln(2*x^2-x+2)+1/72*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{1}{3} \log(x^2 + x + 1) - \frac{13}{48} \log(2x^2 - x + 2) + \frac{5x}{4} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 - (13*Log[2 - x + 2*x^2])/48

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2075

Int[(P_)^(p_)*(Qm_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x] /; PolyQ[Qm, x] && PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx &= \int \left(\frac{5}{4} - \frac{3x}{2} + x^2 + x^3 + \frac{2(-2+x)}{3(1+x+x^2)} + \frac{2-13x}{12(2-x+2x^2)} \right) dx \\
&= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{1}{12} \int \frac{2-13x}{2-x+2x^2} dx + \frac{2}{3} \int \frac{-2+x}{1+x+x^2} dx \\
&= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} - \frac{5}{48} \int \frac{1}{2-x+2x^2} dx - \frac{13}{48} \int \frac{-1+4x}{2-x+2x^2} dx + \frac{1}{3} \int \frac{1}{1+x} dx \\
&= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{1}{3} \log(1+x+x^2) - \frac{13}{48} \log(2-x+2x^2) + \frac{5}{24} \operatorname{Subst} \left(\int \frac{1}{1+x} dx \right) \\
&= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{1}{24} \sqrt{5} \tan^{-1} \left(\frac{1-4x}{\sqrt{15}} \right) - \frac{10 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{1}{3} \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.86

$$\frac{1}{144} \left(36x^4 + 48x^3 - 108x^2 + 48 \log(x^2 + x + 1) - 39 \log(2x^2 - x + 2) + 180x - 160\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) - 2\sqrt{15} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2 - x + 2*x^2])/144

fricas [A] time = 0.69, size = 79, normalized size = 0.81

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2 - x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

giac [A] time = 0.31, size = 73, normalized size = 0.75

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2 - x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="giac")

[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

maple [A] time = 0.01, size = 74, normalized size = 0.76

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} - \frac{\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72} - \frac{10\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} + \frac{\ln(x^2 + x + 1)}{3} - \frac{13\ln(2x^2 - x + 2)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)$

[Out] $\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 + \frac{5}{4}x - \frac{13}{48}\ln(2x^2 - x + 2) - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) + \frac{1}{3}\ln(x^2 + x + 1) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2 - x + 2)$

maxima [A] time = 1.52, size = 73, normalized size = 0.75

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2 - x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, \text{algorithm}=\text{"maxima"})$

[Out] $\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2 - x + 2) + \frac{1}{3}\log(x^2 + x + 1)$

mupad [B] time = 0.19, size = 97, normalized size = 1.00

$$\frac{5x}{4} + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{3} + \frac{\sqrt{3}5i}{9}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(-\frac{1}{3} + \frac{\sqrt{3}5i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}i}{4}\right)\left(-\frac{13}{48} + \frac{\sqrt{15}i}{144}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^4*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2), x)$

[Out] $\frac{5x}{4} + \log(x - \frac{3^{1/2}i}{2} + \frac{1}{2})\left(\frac{3^{1/2}5i}{9} + \frac{1}{3}\right) - \log(x + \frac{3^{1/2}i}{2} + \frac{1}{2})\left(\frac{3^{1/2}5i}{9} - \frac{1}{3}\right) + \log(x - \frac{15^{1/2}i}{4} - \frac{1}{4})\left(\frac{15^{1/2}i}{144} - \frac{13}{48}\right) - \log(x + \frac{15^{1/2}i}{4} - \frac{1}{4})\left(\frac{15^{1/2}i}{144} + \frac{13}{48}\right) - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4}$

sympy [A] time = 0.25, size = 97, normalized size = 1.00

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} - \frac{13\log(x^2 - \frac{x}{2} + 1)}{48} + \frac{\log(x^2 + x + 1)}{3} - \frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{72} - \frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2), x)$

[Out] $x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*\log(x**2 - x/2 + 1)/48 + \log(x**2 + x + 1)/3 - \sqrt{15}*atan(4*\sqrt{15}*x/15 - \sqrt{15}/15)/72 - 10*\sqrt{3}*atan(2*\sqrt{3}*x/3 + \sqrt{3}/3)/9$

$$3.243 \quad \int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=90

$$\frac{x^3}{3} + \frac{x^2}{2} + \frac{2}{3} \log(x^2 + x + 1) - \frac{1}{24} \log(2x^2 - x + 2) - \frac{3x}{2} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] $-3/2*x+1/2*x^2+1/3*x^3+2/3*\ln(x^2+x+1)-1/24*\ln(2*x^2-x+2)+5/36*\arctan(1/15*(1-4*x)*15^{(1/2)})*15^{(1/2)}+8/9*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^3}{3} + \frac{x^2}{2} + \frac{2}{3} \log(x^2 + x + 1) - \frac{1}{24} \log(2x^2 - x + 2) - \frac{3x}{2} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] $(-3*x)/2 + x^2/2 + x^3/3 + (5*\text{Sqrt}[5/3]*\text{ArcTan}[(1 - 4*x)/\text{Sqrt}[15]])/12 + (8*\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) + (2*\text{Log}[1 + x + x^2])/3 - \text{Log}[2 - x + 2*x^2]/24$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2075

Int[(P_)^(p_)*(Qm_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x] /; PolyQ[Qm, x] && PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx &= \int \left(-\frac{3}{2} + x + x^2 + \frac{2(3+2x)}{3(1+x+x^2)} + \frac{-6-x}{6(2-x+2x^2)} \right) dx \\
&= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{1}{6} \int \frac{-6-x}{2-x+2x^2} dx + \frac{2}{3} \int \frac{3+2x}{1+x+x^2} dx \\
&= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} - \frac{1}{24} \int \frac{-1+4x}{2-x+2x^2} dx + \frac{2}{3} \int \frac{1+2x}{1+x+x^2} dx - \frac{25}{24} \int \frac{1}{2-x+x^2} dx \\
&= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{2}{3} \log(1+x+x^2) - \frac{1}{24} \log(2-x+2x^2) + \frac{25}{12} \text{Subst} \left(\int \frac{1}{2-x+x^2} dx \right) \\
&= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1} \left(\frac{1-4x}{\sqrt{15}} \right) + \frac{8 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{2}{3} \log(1+x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.87

$$\frac{1}{72} \left(24x^3 + 36x^2 + 48 \log(x^2 + x + 1) - 3 \log(2x^2 - x + 2) - 108x + 64\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) - 10\sqrt{15} \tan^{-1} \left(\frac{4x-1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (-108*x + 36*x^2 + 24*x^3 + 64*sqrt[3]*ArcTan[(1 + 2*x)/sqrt[3]] - 10*sqrt[15]*ArcTan[(-1 + 4*x)/sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/72

fricas [A] time = 0.80, size = 74, normalized size = 0.82

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{5}\sqrt{3} \arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) + \frac{8}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{3}{2}x - \frac{1}{24} \log(2x^2 - x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

giac [A] time = 0.38, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{3}{2}x - \frac{1}{24} \log(2x^2 - x + 2) + \frac{2}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="giac")

[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

maple [A] time = 0.01, size = 69, normalized size = 0.77

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{3x}{2} - \frac{5\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36} + \frac{8\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} + \frac{2 \ln(x^2 + x + 1)}{3} - \frac{\ln(2x^2 - x + 2)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)$

[Out] $\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{3}{2}x - \frac{1}{24}\ln(2x^2 - x + 2) - \frac{5}{36}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) + \frac{8}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - \frac{3}{2}x - \frac{1}{24}\log(2x^2 - x + 2) + \frac{2}{3}\log(x^2 + x + 1) + \frac{8}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right)$

maxima [A] time = 1.88, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) + \frac{8}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - \frac{3}{2}x - \frac{1}{24}\log(2x^2 - x + 2) + \frac{2}{3}\log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, \text{algorithm}="maxima")$

[Out] $\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) + \frac{8}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - \frac{3}{2}x - \frac{1}{24}\log(2x^2 - x + 2) + \frac{2}{3}\log(x^2 + x + 1)$

mupad [B] time = 0.18, size = 92, normalized size = 1.02

$$\frac{x^2}{2} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}i}{2}\right) \left(-\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}i}{4}\right) \left(-\frac{1}{24} + \frac{\sqrt{15}5i}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2), x)$

[Out] $\log(x + (3^{1/2}i)/2 + 1/2) * ((3^{1/2}4i)/9 + 2/3) - \log(x - (3^{1/2}i)/2 + 1/2) * ((3^{1/2}4i)/9 - 2/3) - (3x)/2 + \log(x - (15^{1/2}i)/4 - 1/4) * ((15^{1/2}5i)/72 - 1/24) - \log(x + (15^{1/2}i)/4 - 1/4) * ((15^{1/2}5i)/72 + 1/24) + x^2/2 + x^3/3$

sympy [A] time = 0.25, size = 92, normalized size = 1.02

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{3x}{2} - \frac{\log(x^2 - \frac{x}{2} + 1)}{24} + \frac{2\log(x^2 + x + 1)}{3} - \frac{5\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{36} + \frac{8\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2), x)$

[Out] $x**3/3 + x**2/2 - 3*x/2 - \log(x**2 - x/2 + 1)/24 + 2*\log(x**2 + x + 1)/3 - 5*\sqrt{15}*\operatorname{atan}(4*\sqrt{15}*x/15 - \sqrt{15}/15)/36 + 8*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/9$

$$3.244 \quad \int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=77

$$\frac{x^2}{2} - \log(x^2 + x + 1) + \frac{1}{4} \log(2x^2 - x + 2) + x + \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] x+1/2*x^2-ln(x^2+x+1)+1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^2}{2} - \log(x^2 + x + 1) + \frac{1}{4} \log(2x^2 - x + 2) + x + \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2075

Int[(P_)^(p_)*(Qm_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x] /; PolyQ[Qm, x] && PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx &= \int \left(1+x - \frac{2(1+3x)}{3(1+x+x^2)} + \frac{-2+3x}{3(2-x+2x^2)} \right) dx \\
&= x + \frac{x^2}{2} + \frac{1}{3} \int \frac{-2+3x}{2-x+2x^2} dx - \frac{2}{3} \int \frac{1+3x}{1+x+x^2} dx \\
&= x + \frac{x^2}{2} + \frac{1}{4} \int \frac{-1+4x}{2-x+2x^2} dx + \frac{1}{3} \int \frac{1}{1+x+x^2} dx - \frac{5}{12} \int \frac{1}{2-x+2x^2} dx - \int \frac{1}{1+x+x^2} dx \\
&= x + \frac{x^2}{2} - \log(1+x+x^2) + \frac{1}{4} \log(2-x+2x^2) - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+x \right) \\
&= x + \frac{x^2}{2} + \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1} \left(\frac{1-4x}{\sqrt{15}} \right) + \frac{2 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} - \log(1+x+x^2) + \frac{1}{4} \log(2-x+2x^2)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.94

$$\frac{1}{36} \left(9(-4 \log(x^2+x+1) + \log(2x^2-x+2) + 2x(x+2)) + 8\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) - 2\sqrt{15} \tan^{-1} \left(\frac{4x-1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]

[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36

fricas [A] time = 0.55, size = 67, normalized size = 0.87

$$\frac{1}{2} x^2 - \frac{1}{18} \sqrt{5} \sqrt{3} \arctan \left(\frac{1}{15} \sqrt{5} \sqrt{3} (4x-1) \right) + \frac{2}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x+1) \right) + x + \frac{1}{4} \log(2x^2-x+2) - \log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")

[Out] 1/2*x^2 - 1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)

giac [A] time = 0.39, size = 61, normalized size = 0.79

$$\frac{1}{2} x^2 - \frac{1}{18} \sqrt{15} \arctan \left(\frac{1}{15} \sqrt{15} (4x-1) \right) + \frac{2}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x+1) \right) + x + \frac{1}{4} \log(2x^2-x+2) - \log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")

[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)

maple [A] time = 0.00, size = 62, normalized size = 0.81

$$\frac{x^2}{2} + x - \frac{\sqrt{15} \arctan \left(\frac{(4x-1)\sqrt{15}}{15} \right)}{18} + \frac{2\sqrt{3} \arctan \left(\frac{(2x+1)\sqrt{3}}{3} \right)}{9} - \ln(x^2+x+1) + \frac{\ln(2x^2-x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)

[Out] 1/2*x^2+x+1/4*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-ln(x^2+x+1)+2/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 1.44, size = 61, normalized size = 0.79

$$\frac{1}{2}x^2 - \frac{1}{18}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{4}\log(2x^2-x+2) - \log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="maxima")

[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)

mupad [B] time = 2.29, size = 85, normalized size = 1.10

$$x - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} \operatorname{li}}{2}\right)\left(1 + \frac{\sqrt{3} \operatorname{li}}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right)\left(-1 + \frac{\sqrt{3} \operatorname{li}}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15} \operatorname{li}}{4}\right)\left(\frac{1}{4} + \frac{\sqrt{15} \operatorname{li}}{36}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15} \operatorname{li}}{4}\right)\left(\frac{1}{4} - \frac{\sqrt{15} \operatorname{li}}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2), x)

[Out] x - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/36 + 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/36 - 1/4) + x^2/2

sympy [A] time = 0.23, size = 78, normalized size = 1.01

$$\frac{x^2}{2} + x + \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{4} - \log(x^2 + x + 1) - \frac{\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2), x)

[Out] x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9

$$3.245 \quad \int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=72

$$\frac{1}{3} \log(x^2 + x + 1) + \frac{1}{6} \log(2x^2 - x + 2) + x - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] x+1/3*ln(x^2+x+1)+1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2075, 634, 618, 204, 628}

$$\frac{1}{3} \log(x^2 + x + 1) + \frac{1}{6} \log(2x^2 - x + 2) + x - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2075

Int[(P_)^(p_)*(Qm_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x] /; PolyQ[Qm, x] && PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx &= \int \left(1 + \frac{2(-2+x)}{3(1+x+x^2)} + \frac{2(1+x)}{3(2-x+2x^2)} \right) dx \\
&= x + \frac{2}{3} \int \frac{-2+x}{1+x+x^2} dx + \frac{2}{3} \int \frac{1+x}{2-x+2x^2} dx \\
&= x + \frac{1}{6} \int \frac{-1+4x}{2-x+2x^2} dx + \frac{1}{3} \int \frac{1+2x}{1+x+x^2} dx + \frac{5}{6} \int \frac{1}{2-x+2x^2} dx - \frac{5}{3} \int \frac{1}{1+x+x^2} dx \\
&= x + \frac{1}{3} \log(1+x+x^2) + \frac{1}{6} \log(2-x+2x^2) - \frac{5}{3} \text{Subst} \left(\int \frac{1}{-15-x^2} dx, x, -1 \right) \\
&= x - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1} \left(\frac{1-4x}{\sqrt{15}} \right) - \frac{10 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{1}{3} \log(1+x+x^2) + \frac{1}{6} \log(2-x+2x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.96

$$\frac{1}{18} \left(3(2 \log(x^2 + x + 1) + \log(2x^2 - x + 2) + 6x) - 20\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + 2\sqrt{15} \tan^{-1} \left(\frac{4x-1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18

fricas [A] time = 1.04, size = 62, normalized size = 0.86

$$\frac{1}{9} \sqrt{5} \sqrt{3} \arctan \left(\frac{1}{15} \sqrt{5} \sqrt{3} (4x-1) \right) - \frac{10}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x+1) \right) + x + \frac{1}{6} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

giac [A] time = 0.31, size = 56, normalized size = 0.78

$$\frac{1}{9} \sqrt{15} \arctan \left(\frac{1}{15} \sqrt{15} (4x-1) \right) - \frac{10}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x+1) \right) + x + \frac{1}{6} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="giac")

[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

maple [A] time = 0.00, size = 57, normalized size = 0.79

$$x + \frac{\sqrt{15} \arctan \left(\frac{(4x-1)\sqrt{15}}{15} \right)}{9} - \frac{10\sqrt{3} \arctan \left(\frac{(2x+1)\sqrt{3}}{3} \right)}{9} + \frac{\ln(x^2 + x + 1)}{3} + \frac{\ln(2x^2 - x + 2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)

[Out] $x + \frac{1}{6} \ln(2x^2 - x + 2) + \frac{1}{9} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x - 1)\right) + \frac{1}{3} \ln(x^2 + x + 1) - \frac{10}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right)$

maxima [A] time = 1.59, size = 56, normalized size = 0.78

$$\frac{1}{9} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x - 1)\right) - \frac{10}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + x + \frac{1}{6} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")`

[Out] $\frac{1}{9} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x - 1)\right) - \frac{10}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + x + \frac{1}{6} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1)$

mupad [B] time = 2.29, size = 80, normalized size = 1.11

$$x + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} i i}{2}\right) \left(\frac{1}{3} + \frac{\sqrt{3} 5i}{9}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} i i}{2}\right) \left(-\frac{1}{3} + \frac{\sqrt{3} 5i}{9}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{15} i i}{4}\right) \left(-\frac{1}{6} + \frac{\sqrt{15} i i}{18}\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)`

[Out] $x + \log(x - (3^{1/2} * i i) / 2 + 1/2) * ((3^{1/2} * 5i) / 9 + 1/3) - \log(x + (3^{1/2} * i i) / 2 + 1/2) * ((3^{1/2} * 5i) / 9 - 1/3) - \log(x - (15^{1/2} * i i) / 4 - 1/4) * ((15^{1/2} * i i) / 18 - 1/6) + \log(x + (15^{1/2} * i i) / 4 - 1/4) * ((15^{1/2} * i i) / 18 + 1/6)$

sympy [A] time = 0.23, size = 75, normalized size = 1.04

$$x + \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{6} + \frac{\log(x^2 + x + 1)}{3} + \frac{\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{9} - \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)`

[Out] $x + \log(x^2 - x/2 + 1)/6 + \log(x^2 + x + 1)/3 + \sqrt{15} \operatorname{atan}(4\sqrt{15}x/15 - \sqrt{15}/15)/9 - 10\sqrt{3} \operatorname{atan}(2\sqrt{3}x/3 + \sqrt{3}/3)/9$

$$3.246 \quad \int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=71

$$\frac{2}{3} \log(x^2 + x + 1) - \frac{1}{6} \log(2x^2 - x + 2) - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] 2/3*ln(x^2+x+1)-1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2074, 634, 618, 204, 628}

$$\frac{2}{3} \log(x^2 + x + 1) - \frac{1}{6} \log(2x^2 - x + 2) - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] -(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned}
\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} dx &= \int \left(\frac{2(3+2x)}{3(1+x+x^2)} + \frac{3-2x}{3(2-x+2x^2)} \right) dx \\
&= \frac{1}{3} \int \frac{3-2x}{2-x+2x^2} dx + \frac{2}{3} \int \frac{3+2x}{1+x+x^2} dx \\
&= -\left(\frac{1}{6} \int \frac{-1+4x}{2-x+2x^2} dx \right) + \frac{2}{3} \int \frac{1+2x}{1+x+x^2} dx + \frac{5}{6} \int \frac{1}{2-x+2x^2} dx + \frac{4}{3} \int \frac{1}{1+x+x^2} dx \\
&= \frac{2}{3} \log(1+x+x^2) - \frac{1}{6} \log(2-x+2x^2) - \frac{5}{3} \text{Subst} \left(\int \frac{1}{-15-x^2} dx, x, -1+4x \right) \\
&= -\frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1} \left(\frac{1-4x}{\sqrt{15}} \right) + \frac{8 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{2}{3} \log(1+x+x^2) - \frac{1}{6} \log(2-x+2x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.92

$$\frac{1}{18} \left(12 \log(x^2 + x + 1) - 3 \log(2x^2 - x + 2) + 16\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + 2\sqrt{15} \tan^{-1} \left(\frac{4x-1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (16*sqrt(3)*ArcTan[(1 + 2*x)/sqrt(3)] + 2*sqrt(15)*ArcTan[(-1 + 4*x)/sqrt(15)] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18

fricas [A] time = 0.80, size = 61, normalized size = 0.86

$$\frac{1}{9} \sqrt{5} \sqrt{3} \arctan \left(\frac{1}{15} \sqrt{5} \sqrt{3} (4x-1) \right) + \frac{8}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x+1) \right) - \frac{1}{6} \log(2x^2 - x + 2) + \frac{2}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

giac [A] time = 0.29, size = 55, normalized size = 0.77

$$\frac{1}{9} \sqrt{15} \arctan \left(\frac{1}{15} \sqrt{15} (4x-1) \right) + \frac{8}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x+1) \right) - \frac{1}{6} \log(2x^2 - x + 2) + \frac{2}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="giac")

[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

maple [A] time = 0.01, size = 56, normalized size = 0.79

$$\frac{\sqrt{15} \arctan \left(\frac{(4x-1)\sqrt{15}}{15} \right)}{9} + \frac{8\sqrt{3} \arctan \left(\frac{(2x+1)\sqrt{3}}{3} \right)}{9} + \frac{2 \ln(x^2 + x + 1)}{3} - \frac{\ln(2x^2 - x + 2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)

[Out] $-1/6*\ln(2*x^2-x+2)+1/9*15^{(1/2)}*\arctan(1/15*(4*x-1)*15^{(1/2)})+2/3*\ln(x^2+x+1)+8/9*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)})$

maxima [A] time = 1.32, size = 55, normalized size = 0.77

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log(2x^2-x+2)+\frac{2}{3}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")`

[Out] $1/9*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) + 8/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/6*\log(2*x^2 - x + 2) + 2/3*\log(x^2 + x + 1)$

mupad [B] time = 0.15, size = 79, normalized size = 1.11

$$-\ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)\left(-\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)\left(\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}1i}{4}\right)\left(\frac{1}{6} + \frac{\sqrt{15}1i}{18}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 3*x^2 + 2*x^3 + 5)/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)`

[Out] $\log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*4i)/9 + 2/3) - \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*4i)/9 - 2/3) - \log(x - (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/18 + 1/6) + \log(x + (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/18 - 1/6)$

sympy [A] time = 0.23, size = 75, normalized size = 1.06

$$-\frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{6} + \frac{2\log\left(x^2 + x + 1\right)}{3} + \frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{9} + \frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)`

[Out] $-\log(x**2 - x/2 + 1)/6 + 2*\log(x**2 + x + 1)/3 + \sqrt{15}*\operatorname{atan}(4*\sqrt{15}*x/15 - \sqrt{15}/15)/9 + 8*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/9$

$$3.247 \quad \int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=75

$$-\log(x^2+x+1) - \frac{1}{4}\log(2x^2-x+2) + \frac{5\log(x)}{2} + \frac{1}{6}\sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] 5/2*ln(x)-ln(x^2+x+1)-1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\log(x^2+x+1) - \frac{1}{4}\log(2x^2-x+2) + \frac{5\log(x)}{2} + \frac{1}{6}\sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int((((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] := With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned} \int \frac{5 + x + 3x^2 + 2x^3}{x(2 + x + 3x^2 + x^3 + 2x^4)} dx &= -\left(\frac{1}{3} \int \frac{-6 + 4x}{x(4 - 2x + 4x^2)} dx\right) + \frac{1}{3} \int \frac{24 + 16x}{x(4 + 4x + 4x^2)} dx \\ &= \frac{1}{3} \int \left(\frac{6}{x} - \frac{2(1 + 3x)}{1 + x + x^2}\right) dx - \frac{1}{3} \int \left(-\frac{3}{2x} + \frac{1 + 6x}{2(2 - x + 2x^2)}\right) dx \\ &= \frac{5 \log(x)}{2} - \frac{1}{6} \int \frac{1 + 6x}{2 - x + 2x^2} dx - \frac{2}{3} \int \frac{1 + 3x}{1 + x + x^2} dx \\ &= \frac{5 \log(x)}{2} - \frac{1}{4} \int \frac{-1 + 4x}{2 - x + 2x^2} dx + \frac{1}{3} \int \frac{1}{1 + x + x^2} dx - \frac{5}{12} \int \frac{1}{2 - x + 2x^2} dx \\ &= \frac{5 \log(x)}{2} - \log(1 + x + x^2) - \frac{1}{4} \log(2 - x + 2x^2) - \frac{2}{3} \text{Subst}\left(\int \frac{1}{-3 - x^2} dx, x, \frac{1 + 2x}{\sqrt{3}}\right) \\ &= \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1 - 4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{1 + 2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{5 \log(x)}{2} - \log(1 + x + x^2) - \frac{1}{4} \log(2 - x + 2x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.92

$$\frac{1}{36} \left(-36 \log(x^2 + x + 1) - 9 \log(2x^2 - x + 2) + 90 \log(x) + 8\sqrt{3} \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) - 2\sqrt{15} \tan^{-1}\left(\frac{4x - 1}{\sqrt{15}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)), x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36
```

fricas [A] time = 0.96, size = 65, normalized size = 0.87

$$-\frac{1}{18} \sqrt{5} \sqrt{3} \arctan\left(\frac{1}{15} \sqrt{5} \sqrt{3} (4x - 1)\right) + \frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{4} \log(2x^2 - x + 2) - \log(x^2 + x + 1) + \frac{5}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

giac [A] time = 0.31, size = 60, normalized size = 0.80

$$-\frac{1}{18} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x - 1)\right) + \frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{4} \log(2x^2 - x + 2) - \log(x^2 + x + 1) + \frac{5}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")

[Out] $-1/18*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) + 2/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/4*\log(2*x^2 - x + 2) - \log(x^2 + x + 1) + 5/2*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 60, normalized size = 0.80

$$-\frac{\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} + \frac{5 \ln(x)}{2} - \ln(x^2 + x + 1) - \frac{\ln(2x^2 - x + 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x)

[Out] $-1/4*\ln(2*x^2-x+2)-1/18*15^{(1/2)}*\arctan(1/15*(4*x-1)*15^{(1/2)})+5/2*\ln(x)-\ln(x^2+x+1)+2/9*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)})$

maxima [A] time = 1.57, size = 59, normalized size = 0.79

$$-\frac{1}{18} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x - 1)\right) + \frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{4} \log(2x^2 - x + 2) - \log(x^2 + x + 1) + \frac{5}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] $-1/18*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) + 2/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/4*\log(2*x^2 - x + 2) - \log(x^2 + x + 1) + 5/2*\log(x)$

mupad [B] time = 0.15, size = 83, normalized size = 1.11

$$\frac{5 \ln(x)}{2} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) \left(1 + \frac{\sqrt{3} 1i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(-1 + \frac{\sqrt{3} 1i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15} 1i}{4}\right) \left(-\frac{1}{4} + \frac{\sqrt{15}}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)

[Out] $(5*\log(x))/2 - \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/9 + 1) + \log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/9 - 1) + \log(x - (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/36 - 1/4) - \log(x + (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/36 + 1/4)$

sympy [A] time = 0.29, size = 78, normalized size = 1.04

$$\frac{5 \log(x)}{2} - \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{4} - \log(x^2 + x + 1) - \frac{\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)

[Out] $5*\log(x)/2 - \log(x**2 - x/2 + 1)/4 - \log(x**2 + x + 1) - \sqrt{15}*\operatorname{atan}(4*\sqrt{15}*x/15 - \sqrt{15}/15)/18 + 2*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/9$

$$3.248 \quad \int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=84

$$\frac{1}{3} \log(x^2 + x + 1) + \frac{1}{24} \log(2x^2 - x + 2) - \frac{5}{2x} - \frac{3 \log(x)}{4} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] $-5/2/x-3/4*\ln(x)+1/3*\ln(x^2+x+1)+1/24*\ln(2*x^2-x+2)+5/36*\arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2)-10/9*\arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)$

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{3} \log(x^2 + x + 1) + \frac{1}{24} \log(2x^2 - x + 2) - \frac{5}{2x} - \frac{3 \log(x)}{4} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] $-5/(2*x) + (5*\text{Sqrt}[5/3]*\text{ArcTan}[(1 - 4*x)/\text{Sqrt}[15]])/12 - (10*\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) - (3*\text{Log}[x])/4 + \text{Log}[1 + x + x^2]/3 + \text{Log}[2 - x + 2*x^2]/24$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int((((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned} \int \frac{5 + x + 3x^2 + 2x^3}{x^2(2 + x + 3x^2 + x^3 + 2x^4)} dx &= -\left(\frac{1}{3} \int \frac{-6 + 4x}{x^2(4 - 2x + 4x^2)} dx\right) + \frac{1}{3} \int \frac{24 + 16x}{x^2(4 + 4x + 4x^2)} dx \\ &= \frac{1}{3} \int \left(\frac{6}{x^2} - \frac{2}{x} + \frac{2(-2+x)}{1+x+x^2}\right) dx - \frac{1}{3} \int \left(-\frac{3}{2x^2} + \frac{1}{4x} + \frac{13-2x}{4(2-x+2x^2)}\right) dx \\ &= -\frac{5}{2x} - \frac{3 \log(x)}{4} - \frac{1}{12} \int \frac{13-2x}{2-x+2x^2} dx + \frac{2}{3} \int \frac{-2+x}{1+x+x^2} dx \\ &= -\frac{5}{2x} - \frac{3 \log(x)}{4} + \frac{1}{24} \int \frac{-1+4x}{2-x+2x^2} dx + \frac{1}{3} \int \frac{1+2x}{1+x+x^2} dx - \frac{25}{24} \int \frac{1}{2-x} dx \\ &= -\frac{5}{2x} - \frac{3 \log(x)}{4} + \frac{1}{3} \log(1+x+x^2) + \frac{1}{24} \log(2-x+2x^2) + \frac{25}{12} \text{Subst}\left(\int \frac{1}{2-x} dx\right) \\ &= -\frac{5}{2x} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{3 \log(x)}{4} + \frac{1}{3} \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.93

$$\frac{-24x \log(x^2 + x + 1) - 3x \log(2x^2 - x + 2) + 54x \log(x) + 80\sqrt{3}x \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + 10\sqrt{15}x \tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right) + 180}{72x}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] -1/72*(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan[(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 - x + 2*x^2])/x

fricas [A] time = 0.89, size = 76, normalized size = 0.90

$$\frac{10\sqrt{5}\sqrt{3}x \arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) + 80\sqrt{3}x \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 3x \log(2x^2 - x + 2) - 24x \log(x^2 + x + 1) + 54x \log(x) + 180}{72x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")

[Out] -1/72*(10*sqrt(5)*sqrt(3)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 80*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(x^2 + x + 1) + 54*x*log(x) + 180)/x

giac [A] time = 0.26, size = 65, normalized size = 0.77

$$-\frac{5}{36} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15}(4x-1)\right) - \frac{10}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1) - \frac{3}{4} \log(\text{abs}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")

[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) - 3/4*log(abs(x))

maple [A] time = 0.01, size = 65, normalized size = 0.77

$$-\frac{5\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right) - 10\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right) - 3\ln(x) + \frac{\ln(x^2 + x + 1)}{3} + \frac{\ln(2x^2 - x + 2)}{24} - \frac{5}{2x}}{36} - \frac{10\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} - \frac{3\ln(x)}{4} + \frac{\ln(x^2 + x + 1)}{3} + \frac{\ln(2x^2 - x + 2)}{24} - \frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x)

[Out] 1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/2/x-3/4*ln(x)+1/3*ln(x^2+x+1)-10/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 1.44, size = 64, normalized size = 0.76

$$-\frac{5}{36} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15}(4x-1)\right) - \frac{10}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1) - \frac{3}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) - 3/4*log(x)

mupad [B] time = 2.28, size = 88, normalized size = 1.05

$$-\frac{3 \ln(x)}{4} + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) \left(\frac{1}{3} + \frac{\sqrt{3} 5i}{9}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(-\frac{1}{3} + \frac{\sqrt{3} 5i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15} 1i}{4}\right) \left(\frac{1}{24} + \frac{\sqrt{15} 5i}{72}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15} 1i}{4}\right) \left(\frac{1}{24} - \frac{\sqrt{15} 5i}{72}\right) - \frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)

[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - (3*log(x))/4 - log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*5i)/72 + 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*5i)/72 - 1/24) - 5/(2*x)

sympy [A] time = 0.31, size = 87, normalized size = 1.04

$$-\frac{3 \log(x)}{4} + \frac{\log(x^2 - \frac{x}{2} + 1)}{24} + \frac{\log(x^2 + x + 1)}{3} - \frac{5\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right) - 10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right) - \frac{5}{2x}}{36} - \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9} - \frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)

[Out] -3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9 - 5/(2*x)

$$3.249 \quad \int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=91

$$-\frac{5}{4x^2} + \frac{2}{3} \log(x^2 + x + 1) + \frac{13}{48} \log(2x^2 - x + 2) + \frac{3}{4x} - \frac{15 \log(x)}{8} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] $-5/4/x^2+3/4/x-15/8*\ln(x)+2/3*\ln(x^2+x+1)+13/48*\ln(2*x^2-x+2)+1/72*\arctan(1/15*(1-4*x)*15^{(1/2)})*15^{(1/2)}+8/9*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\frac{5}{4x^2} + \frac{2}{3} \log(x^2 + x + 1) + \frac{13}{48} \log(2x^2 - x + 2) + \frac{3}{4x} - \frac{15 \log(x)}{8} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] $-5/(4*x^2) + 3/(4*x) + (\text{Sqrt}[5/3]*\text{ArcTan}[(1 - 4*x)/\text{Sqrt}[15]])/24 + (8*\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]) - (15*\text{Log}[x])/8 + (2*\text{Log}[1 + x + x^2])/3 + (13*\text{Log}[2 - x + 2*x^2])/48$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2087

Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned} \int \frac{5 + x + 3x^2 + 2x^3}{x^3(2 + x + 3x^2 + x^3 + 2x^4)} dx &= -\left(\frac{1}{3} \int \frac{-6 + 4x}{x^3(4 - 2x + 4x^2)} dx\right) + \frac{1}{3} \int \frac{24 + 16x}{x^3(4 + 4x + 4x^2)} dx \\ &= \frac{1}{3} \int \left(\frac{6}{x^3} - \frac{2}{x^2} - \frac{4}{x} + \frac{2(3 + 2x)}{1 + x + x^2}\right) dx - \frac{1}{3} \int \left(-\frac{3}{2x^3} + \frac{1}{4x^2} + \frac{13}{8x} + \frac{9 - 26x}{8(2 - x + 2x^2)}\right) dx \\ &= -\frac{5}{4x^2} + \frac{3}{4x} - \frac{15 \log(x)}{8} - \frac{1}{24} \int \frac{9 - 26x}{2 - x + 2x^2} dx + \frac{2}{3} \int \frac{3 + 2x}{1 + x + x^2} dx \\ &= -\frac{5}{4x^2} + \frac{3}{4x} - \frac{15 \log(x)}{8} - \frac{5}{48} \int \frac{1}{2 - x + 2x^2} dx + \frac{13}{48} \int \frac{-1 + 4x}{2 - x + 2x^2} dx + \frac{5}{24} \int \frac{3 + 2x}{1 + x + x^2} dx \\ &= -\frac{5}{4x^2} + \frac{3}{4x} - \frac{15 \log(x)}{8} + \frac{2}{3} \log(1 + x + x^2) + \frac{13}{48} \log(2 - x + 2x^2) + \frac{5}{24} \log(1 + x + x^2) \\ &= -\frac{5}{4x^2} + \frac{3}{4x} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1 - 4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{1 + 2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{15 \log(x)}{8} + \frac{2}{3} \log(1 + x + x^2) \end{aligned}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.90

$$\frac{1}{144} \left(3 \left(-\frac{60}{x^2} + 32 \log(x^2 + x + 1) + 13 \log(2x^2 - x + 2) + \frac{36}{x} - 90 \log(x) \right) + 128\sqrt{3} \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) - 2\sqrt{15} \arctan\left(\frac{1 - 4x}{\sqrt{15}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)), x]

[Out] (128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x + 2*x^2]))/144

fricas [A] time = 0.97, size = 89, normalized size = 0.98

$$\frac{2\sqrt{5}\sqrt{3}x^2 \arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x - 1)\right) - 128\sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - 39x^2 \log(2x^2 - x + 2) - 96x^2 \log(1 + x + x^2)}{144x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] -1/144*(2*sqrt(5)*sqrt(3)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 128*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 96*x^2*log(x^2 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2

giac [A] time = 0.29, size = 70, normalized size = 0.77

$$-\frac{1}{72} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x - 1)\right) + \frac{8}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + \frac{3x - 5}{4x^2} + \frac{13}{48} \log(2x^2 - x + 2) + \frac{2}{3} \log(1 + x + x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")

[Out] $-1/72*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) + 8/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*\log(2*x^2 - x + 2) + 2/3*\log(x^2 + x + 1) - 15/8*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 70, normalized size = 0.77

$$-\frac{\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72} + \frac{8\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} - \frac{15 \ln(x)}{8} + \frac{2 \ln(x^2 + x + 1)}{3} + \frac{13 \ln(2x^2 - x + 2)}{48} + \frac{3}{4x} - \frac{5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x)

[Out] $13/48*\ln(2*x^2-x+2)-1/72*15^{(1/2)}*\arctan(1/15*(4*x-1)*15^{(1/2)})-5/4/x^2+3/4/x-15/8*\ln(x)+2/3*\ln(x^2+x+1)+8/9*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)})$

maxima [A] time = 1.26, size = 69, normalized size = 0.76

$$-\frac{1}{72} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x - 1)\right) + \frac{8}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + \frac{3x - 5}{4x^2} + \frac{13}{48} \log(2x^2 - x + 2) + \frac{2}{3} \log(x^2 + x + 1) - \frac{15}{8} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] $-1/72*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) + 8/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*\log(2*x^2 - x + 2) + 2/3*\log(x^2 + x + 1) - 15/8*\log(x)$

mupad [B] time = 0.15, size = 92, normalized size = 1.01

$$\frac{\frac{3x}{4} - \frac{5}{4}}{x^2} - \frac{15 \ln(x)}{8} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} i}{2}\right) \left(-\frac{2}{3} + \frac{\sqrt{3} i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(\frac{2}{3} + \frac{\sqrt{3} i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15} i}{4}\right) \left(-\frac{2}{3} + \frac{\sqrt{3} i}{9}\right) + \ln\left(x - \frac{1}{4} + \frac{\sqrt{15} i}{4}\right) \left(\frac{2}{3} + \frac{\sqrt{3} i}{9}\right) - \frac{13 \ln(2x^2 - x + 2)}{48} - \frac{2 \ln(x^2 + x + 1)}{3} - \frac{15 \ln(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)

[Out] $((3*x)/4 - 5/4)/x^2 - (15*\log(x))/8 - \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*4i)/9 - 2/3) + \log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*4i)/9 + 2/3) + \log(x - (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/144 + 13/48) - \log(x + (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/144 - 13/48)$

sympy [A] time = 0.32, size = 94, normalized size = 1.03

$$-\frac{15 \log(x)}{8} + \frac{13 \log(x^2 - \frac{x}{2} + 1)}{48} + \frac{2 \log(x^2 + x + 1)}{3} - \frac{\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{72} + \frac{8\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9} + \frac{3x - 5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)

[Out] $-15*\log(x)/8 + 13*\log(x**2 - x/2 + 1)/48 + 2*\log(x**2 + x + 1)/3 - \sqrt{15}*\operatorname{atan}(4*\sqrt{15}*x/15 - \sqrt{15}/15)/72 + 8*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/9 + (3*x - 5)/(4*x**2)$

$$3.250 \quad \int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Optimal. Leaf size=307

$$\frac{1}{42} (7 + 5i\sqrt{7}) x^3 + \frac{1}{42} (7 - 5i\sqrt{7}) x^3 + \frac{1}{28} (7 + 5i\sqrt{7}) x^2 + \frac{1}{28} (7 - 5i\sqrt{7}) x^2 + \frac{3}{112} (7 - 11i\sqrt{7}) \log(4x^2 + (1 -$$

[Out] $1/28*x^2*(7-5*I*7^{(1/2)})+1/42*x^3*(7-5*I*7^{(1/2)})+1/28*x^2*(7+5*I*7^{(1/2)})+1/42*x^3*(7+5*I*7^{(1/2)})-1/28*x*(35-9*I*7^{(1/2)})-1/28*x*(35+9*I*7^{(1/2)})+3/112*\ln(4+4*x^2+x*(1-I*7^{(1/2)}))* (7-11*I*7^{(1/2)})+3/112*\ln(4+4*x^2+x*(1+I*7^{(1/2)}))* (7+11*I*7^{(1/2)})-11/4*\arctan((1+8*x+I*7^{(1/2)})/(70-2*I*7^{(1/2)})^{(1/2)})*(9*I-5*7^{(1/2)})/(490-14*I*7^{(1/2)})^{(1/2)}+11/4*\arctan((1+8*x-I*7^{(1/2)})/(70+2*I*7^{(1/2)})^{(1/2)})*(9*I+5*7^{(1/2)})/(490+14*I*7^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.58, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{42} (7 + 5i\sqrt{7}) x^3 + \frac{1}{42} (7 - 5i\sqrt{7}) x^3 + \frac{1}{28} (7 + 5i\sqrt{7}) x^2 + \frac{1}{28} (7 - 5i\sqrt{7}) x^2 + \frac{3}{112} (7 - 11i\sqrt{7}) \log(4x^2 + (1 -$$

Antiderivative was successfully verified.

[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] $-((35 - (9*I)*\text{Sqrt}[7])*x)/28 - ((35 + (9*I)*\text{Sqrt}[7])*x)/28 + ((7 - (5*I)*\text{Sqrt}[7])*x^2)/28 + ((7 + (5*I)*\text{Sqrt}[7])*x^2)/28 + ((7 - (5*I)*\text{Sqrt}[7])*x^3)/42 + ((7 + (5*I)*\text{Sqrt}[7])*x^3)/42 + (11*(9*I + 5*\text{Sqrt}[7])*\text{ArcTan}[(1 - I*\text{Sqrt}[7] + 8*x)/\text{Sqrt}[2*(35 + I*\text{Sqrt}[7])]])/(4*\text{Sqrt}[14*(35 + I*\text{Sqrt}[7])]) - (11*(9*I - 5*\text{Sqrt}[7])*\text{ArcTan}[(1 + I*\text{Sqrt}[7] + 8*x)/\text{Sqrt}[2*(35 - I*\text{Sqrt}[7])]])/(4*\text{Sqrt}[14*(35 - I*\text{Sqrt}[7])]) + (3*(7 - (11*I)*\text{Sqrt}[7])*Log[4 + (1 - I*\text{Sqrt}[7])*x + 4*x^2])/112 + (3*(7 + (11*I)*\text{Sqrt}[7])*Log[4 + (1 + I*\text{Sqrt}[7])*x + 4*x^2])/112$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2087

Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\int \frac{x^3(5 + x + 3x^2 + 2x^3)}{2 + x + 5x^2 + x^3 + 2x^4} dx = \frac{i \int \frac{x^3(9-5i\sqrt{7}+(10-2i\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2} dx}{\sqrt{7}} - \frac{i \int \frac{x^3(9+5i\sqrt{7}+(10+2i\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2} dx}{\sqrt{7}}$$

$$= \frac{i \int \left(\frac{1}{4}(-9 + 5i\sqrt{7}) + \frac{1}{2}(5 - i\sqrt{7})x + \frac{1}{2}(5 - i\sqrt{7})x^2 + \frac{2(9-5i\sqrt{7})-3(11+i\sqrt{7})x}{2(4+(1-i\sqrt{7})x+4x^2)} \right) dx}{\sqrt{7}}$$

$$= -\frac{1}{28} (35 - 9i\sqrt{7})x - \frac{1}{28} (35 + 9i\sqrt{7})x + \frac{1}{28} (7 - 5i\sqrt{7})x^2 + \frac{1}{28} (7 + 5i\sqrt{7})x^2$$

$$= -\frac{1}{28} (35 - 9i\sqrt{7})x - \frac{1}{28} (35 + 9i\sqrt{7})x + \frac{1}{28} (7 - 5i\sqrt{7})x^2 + \frac{1}{28} (7 + 5i\sqrt{7})x^2$$

$$= -\frac{1}{28} (35 - 9i\sqrt{7})x - \frac{1}{28} (35 + 9i\sqrt{7})x + \frac{1}{28} (7 - 5i\sqrt{7})x^2 + \frac{1}{28} (7 + 5i\sqrt{7})x^2$$

$$= -\frac{1}{28} (35 - 9i\sqrt{7})x - \frac{1}{28} (35 + 9i\sqrt{7})x + \frac{1}{28} (7 - 5i\sqrt{7})x^2 + \frac{1}{28} (7 + 5i\sqrt{7})x^2$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.36

$$\frac{1}{6} \left(3\text{RootSum} \left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{3\#1^3 \log(x - \#1) + 19\#1^2 \log(x - \#1) + \#1 \log(x - \#1) + 10 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \right] \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] (x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 &, (10*Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/(10*#1 + 3*#1^2 + 8*#1^3) &])/6

+ 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) + 6384*sqrt(2101/1568*I*sqrt(7) - 55/32) - 5946) - 5/2*x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^3 + 3x^2 + x + 5)x^3}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

maple [C] time = 0.01, size = 74, normalized size = 0.24

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \frac{(3 \operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 19 \operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + \operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2))}{16 \operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 6 \operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)

[Out] 1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+1))*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{2}x + \frac{1}{2} \int \frac{3x^3 + 19x^2 + x + 10}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] 1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

mupad [B] time = 2.17, size = 128, normalized size = 0.42

$$\left(\sum_{k=1}^4 \ln \left(-29x + \operatorname{root} \left(z^4 - \frac{3z^3}{4} + \frac{16z^2}{7} + \frac{96z}{49} + \frac{128}{343}, z, k \right) \left(-\frac{289x}{4} + \operatorname{root} \left(z^4 - \frac{3z^3}{4} + \frac{16z^2}{7} + \frac{96z}{49} + \frac{128}{343}, z, k \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)

[Out] symsum(log(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k))* (root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k))*((581*x)/16 - root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k))*((147*x)/4 - 49/16) + 1141/64) - (289*x)/4 + 47/4) - 29*x + 7)*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (5*x)/2 + x^2/2 + x^3/3

sympy [A] time = 0.99, size = 61, normalized size = 0.20

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \operatorname{RootSum} \left(1372t^4 - 1029t^3 + 3136t^2 + 2688t + 512, \left(t \mapsto t \log \left(\frac{5831t^3}{1936} - \frac{23765t^2}{7744} + \frac{2065t}{242} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 2688*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*_t/242 + x + 415/121)))
```

$$3.251 \quad \int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Optimal. Leaf size=269

$$\frac{1}{28} (7 + 5i\sqrt{7})x^2 + \frac{1}{28} (7 - 5i\sqrt{7})x^2 - \frac{1}{56} (35 + 9i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) - \frac{1}{56} (35 - 9i\sqrt{7}) \log(4x^2 +$$

[Out] 1/14*x*(7-5*I*7^(1/2))+1/28*x^2*(7-5*I*7^(1/2))+1/14*x*(7+5*I*7^(1/2))+1/28*x^2*(7+5*I*7^(1/2))-1/56*ln(4+4*x^2+x*(1+I*7^(1/2)))*(35-9*I*7^(1/2))-1/56*ln(4+4*x^2+x*(1-I*7^(1/2)))*(35+9*I*7^(1/2))+1/2*arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(53*I-7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/2*arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(53*I+7^(1/2))/(490+14*I*7^(1/2))^(1/2)

Rubi [A] time = 0.39, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{28} (7 + 5i\sqrt{7})x^2 + \frac{1}{28} (7 - 5i\sqrt{7})x^2 - \frac{1}{56} (35 + 9i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) - \frac{1}{56} (35 - 9i\sqrt{7}) \log(4x^2 +$$

Antiderivative was successfully verified.

[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (5*I)*Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((53*I - Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/56

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 800

`Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]`

Rule 2087

`Int[((P3_)*(x_)^(m_))/((a_) + (b_)*(x_) + (c_)*(x_)^2 + (d_)*(x_)^3 + (e_)*(x_)^4), x_Symbol] := With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]`

Rubi steps

$$\begin{aligned} \int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx &= \frac{i \int \frac{x^2(9-5i\sqrt{7}+(10-2i\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2} dx}{\sqrt{7}} - \frac{i \int \frac{x^2(9+5i\sqrt{7}+(10+2i\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2} dx}{\sqrt{7}} \\ &= \frac{i \int \left(\frac{1}{2}(5-i\sqrt{7}) + \frac{1}{2}(5-i\sqrt{7})x + \frac{i(2(5i+\sqrt{7})+(9+5\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2} \right) dx}{\sqrt{7}} - \frac{i \int \left(\frac{1}{2}(5+i\sqrt{7}) + \frac{1}{2}(5+i\sqrt{7})x + \frac{i(2(5i-\sqrt{7})+(9-5\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2} \right) dx}{\sqrt{7}} \\ &= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7-5i\sqrt{7})x^2 + \frac{1}{28} (7+5i\sqrt{7})x^2 \\ &= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7-5i\sqrt{7})x^2 + \frac{1}{28} (7+5i\sqrt{7})x^2 \\ &= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7-5i\sqrt{7})x^2 + \frac{1}{28} (7+5i\sqrt{7})x^2 \\ &= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7-5i\sqrt{7})x^2 + \frac{1}{28} (7+5i\sqrt{7})x^2 \end{aligned}$$

Mathematica [C] time = 0.02, size = 101, normalized size = 0.38

$$-\text{RootSum} \left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{5\#1^3 \log(x - \#1) + \#1^2 \log(x - \#1) + 3\#1 \log(x - \#1) + 2 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 &, (2*Log[x - #1] + 3*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]

fricas [B] time = 3.03, size = 1145, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(
49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 10290*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 25725*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 8384*x + 6615/2*I*sqrt(7) + 10290*sqrt(-37/392*I*sqrt(7) + 79/56) + 13373/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*sqrt(7) + 79/56) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2))*((135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 17856*I*sqrt(7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776) + 16768*x - 4941*I*sqrt(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391) - 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqrt(-37/392*I*sqrt(7) + 79/56) + 5)*log(-49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2))*((135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 17856*I*sqrt(7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776) + 16768*x - 4941*I*sqrt(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) + 35)*log(10290*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 + 1421*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 8384*x + 3267/2*I*sqrt(7) + 5082*sqrt(-37/392*I*sqrt(7) + 79/56) + 13793/2) + x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^3 + 3x^2 + x + 5)x^2}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

maple [C] time = 0.01, size = 67, normalized size = 0.25

$$\frac{x^2}{2} + x + \frac{\left(-5 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^3 - \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^2 - 3 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)\right)}{8 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^3 + 3 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)

[Out] 1/2*x^2+x+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2 + x - \int \frac{5x^3 + x^2 + 3x + 2}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] 1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

mupad [B] time = 0.13, size = 188, normalized size = 0.70

$$x + \frac{x^2}{2} + \left(\sum_{k=1}^4 \ln \left(-\frac{179 \operatorname{root}\left(z^4 + \frac{5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}, z, k\right)}{8} - 7x - \frac{\operatorname{root}\left(z^4 + \frac{5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}, z, k\right)}{8} \right) \right) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)

[Out] x + x^2/2 + symsum(log((49*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^3)/16 - 7*x - (459*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)*x)/8 - (665*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^2*x)/8 - (147*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^3*x)/4 - (35*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^2)/32 - (179*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k))/8 - 15)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1, 4)

sympy [B] time = 2.73, size = 3662, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)

```
[Out] x**2/2 + x + (-5/8 + sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-1459*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 15*sqrt(77)*sqrt(553 + 64*sqrt(77))/2096 - 10391*sqrt(553 + 64*sqrt(77))/268288 + 1459*sqrt(77)/8384 + 522933/268288 + 45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576) - 510895297*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 - 6009493*sqrt(22)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1124663296 - 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/2249326592 - 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 153195*sqrt(22)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/2249326592 + 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 + 290832444193/35989225472 + 2303470247*sqrt(77)/2249326592) + (-5/8 - sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 1459*sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 + 10391*sqrt(553 + 64*sqrt(77))/268288 + 1459*sqrt(77)/8384 + 522933/268288 + 15*sqrt(77)*sqrt(553 + 64*sqrt(77))/2096) - 510895297*sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 - 6009493*sqrt(22)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1124663296 - 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 - 153195*sqrt(22)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/2249326592 + 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/2249326592 + 290832444193/35989225472 + 2303470247*sqrt(77)/2249326592) + 2*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1568 + 5/14 + 3*sqrt(77)/49)*atan(1073152*x/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 20782*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 93376*sqrt(77)/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 1045866/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
```


$$3.252 \quad \int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Optimal. Leaf size=230

$$\frac{1}{28} (7 + 5i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{1}{28} (7 - 5i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{1}{14} (7 + 5i\sqrt{7})x + \frac{1}{14}$$

[Out] 1/14*x*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/14*x*(7+5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7+5*I*7^(1/2))+arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19*I-7*7^(1/2))/(490-14*I*7^(1/2))^(1/2)-arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(19*I+7*7^(1/2))/(490+14*I*7^(1/2))^(1/2)

Rubi [A] time = 0.36, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2087, 773, 634, 618, 204, 628}

$$\frac{1}{28} (7 + 5i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{1}{28} (7 - 5i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{1}{14} (7 + 5i\sqrt{7})x + \frac{1}{14}$$

Antiderivative was successfully verified.

[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/28

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 773

$\text{Int}[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Simp}[(e*g*x)/c, x] + \text{Dist}[1/c, \text{Int}[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2087

$\text{Int}[(P3_)*(x_)^{(m_)}]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2 + (d_.)*(x_.)^3 + (e_.)*(x_.)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Sqrt}[8*a^2 + b^2 - 4*a*c], A = \text{Coeff}[P3, x, 0], B = \text{Coeff}[P3, x, 1], C = \text{Coeff}[P3, x, 2], D = \text{Coeff}[P3, x, 3]\}, \text{Dist}[1/q, \text{Int}[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - \text{Dist}[1/q, \text{Int}[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /;$ FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned} \int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx &= \frac{i \int \frac{x(9-5i\sqrt{7}+(10-2i\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2} dx}{\sqrt{7}} - \frac{i \int \frac{x(9+5i\sqrt{7}+(10+2i\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2} dx}{\sqrt{7}} \\ &= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{i \int \frac{-4(10-2i\sqrt{7})+(-(1-i\sqrt{7})(10-2i\sqrt{7})+4(9-5i\sqrt{7}))}{4+(1-i\sqrt{7})x+4x^2} dx}{4\sqrt{7}} \\ &= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x - \frac{1}{28} (-7+5i\sqrt{7}) \int \frac{1+i\sqrt{7}+8x}{4+(1+i\sqrt{7})x+4x^2} dx \\ &= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7+5i\sqrt{7}) \log(4+(1-i\sqrt{7})x+4x^2) \\ &= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x - \frac{(19i+7\sqrt{7}) \tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}} + \dots \end{aligned}$$

Mathematica [C] time = 0.01, size = 94, normalized size = 0.41

$$2\text{RootSum}\left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{\#1^3 \log(x - \#1) - 2\#1^2 \log(x - \#1) + 2\#1 \log(x - \#1) - \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 &, (-Log[x - #1] + 2*Log[x - #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]

fricas [B] time = 3.07, size = 1190, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
[Out] -1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(55*I*
sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt
(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/9
8*I*sqrt(7) - 1/14) + 1/4)^3 + 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt
(7) - 1/14) + 1/4)^2 + 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt
(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)
*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x + 1155/2*I*s
qrt(7) + 1617*sqrt(53/98*I*sqrt(7) - 1/14) + 1903/2) + 1/28*(2*sqrt(7)*sqrt
(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/
28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/
2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*
log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*s
qrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(-5/28*I*sqrt(7)
- 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(-5/28*I*sqrt(7)
- 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*
I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) -
7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/
4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/
56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7)
- 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14
) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)
+ 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 3456*s
qrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqrt(7) - 1/14) + 636) -
1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1
/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
2744*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(
196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt
(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98
*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I
*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7)
- 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21
)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7
*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(5
3/98*I*sqrt(7) - 1/14) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I
*sqrt(7) - 1/14) - 7) + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7)
- 1/14) - 7) + 3456*sqrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqrt
(7) - 1/14) + 636) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) -
7)*log(3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 -
1029*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 304*x
- 715/2*I*sqrt(7) - 1001*sqrt(53/98*I*sqrt(7) - 1/14) - 2871/2) + x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^3 + 3x^2 + x + 5)x}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 62, normalized size = 0.27

$$x + \frac{2 \left(\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) \right)^3 - 2 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + 2 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)}{8 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 3 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)

[Out] x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x + 2 \int \frac{x^3 - 2x^2 + 2x - 1}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

mupad [B] time = 0.19, size = 183, normalized size = 0.80

$$x + \left(\sum_{k=1}^4 \ln \left(\frac{115 \text{root} \left(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k \right)}{8} + 15x - \frac{\text{root} \left(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k \right) x^{137}}{8} + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)

[Out] x + symsum(log((115*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k))/8 + 15*x - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x))/8 + (133*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/8 - (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 - (189*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 - 4)*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)

sympy [A] time = 0.95, size = 48, normalized size = 0.21

$$x + \text{RootSum} \left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t \log \left(\frac{3773t^3}{304} - \frac{1029t^2}{304} + \frac{1001t}{152} + x - \frac{121}{19} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)

[Out] x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19)))

$$3.253 \quad \int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} dx$$

Optimal. Leaf size=198

$$\frac{1}{28} (7 + 5i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{1}{28} (7 - 5i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{(7\sqrt{7} + 19i) \tan^{-1}\left(\frac{8}{\sqrt{14(35 + i\sqrt{7})}}\right)}{\sqrt{14(35 + i\sqrt{7})}}$$

[Out] 1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7+5*I*7^(1/2))-arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19*I-7*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(19*I+7*7^(1/2))/(490+14*I*7^(1/2))^(1/2)

Rubi [A] time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2086, 634, 618, 204, 628}

$$\frac{1}{28} (7 + 5i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{1}{28} (7 - 5i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{(7\sqrt{7} + 19i) \tan^{-1}\left(\frac{8}{\sqrt{14(35 + i\sqrt{7})}}\right)}{\sqrt{14(35 + i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/28

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2086

7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*log(49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) - 1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(-49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 - 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x - 2205/2*I*sqrt(7) - 3087*sqrt(53/98*I*sqrt(7) - 1/14) - 3025/2) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7)*log(-7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 + 2303*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 304*x + 1555/2*I*sqrt(7) + 2177*sqrt(53/98*I*sqrt(7) - 1/14) + 5823/2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

maple [C] time = 0.01, size = 58, normalized size = 0.29

$$\frac{\left(2 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^3 + 3 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^2 + \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)\right)^2}{8 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^3 + 3 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)

[Out] sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

mupad [B] time = 2.34, size = 181, normalized size = 0.91

$$\sum_{k=1}^4 \ln \left(-\frac{193 \operatorname{root}\left(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k\right)}{8} + 4x - \frac{\operatorname{root}\left(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k\right) x^{137}}{8} + \frac{\operatorname{root}\left(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k\right)}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)

[Out] symsum(log(4*x - (193*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k))/8 - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x)/8 + (651*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/16 - (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 + (273*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 + 7)*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)

sympy [A] time = 0.91, size = 46, normalized size = 0.23

$$\operatorname{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t \log\left(-\frac{7203t^3}{304} + \frac{2303t^2}{304} - \frac{2177t}{152} + x + \frac{250}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)

[Out] RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*log(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))

$$3.254 \quad \int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=245

$$-\frac{1}{56} (35 - 9i\sqrt{7}) \log(4ix^2 + (-\sqrt{7} + i)x + 4i) - \frac{1}{56} (35 + 9i\sqrt{7}) \log(4ix^2 + (\sqrt{7} + i)x + 4i) + \frac{1}{28} (35 + 9i\sqrt{7})$$

[Out] 1/28*ln(x)*(35-9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(35-9*I*7^(1/2))+1/28*ln(x)*(35+9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I*7^(1/2))-1/2*arctanh((I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(53+I*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+1/2*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(53-I*7^(1/2))/(490+14*I*7^(1/2))^(1/2)

Rubi [A] time = 0.47, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{1}{56} (35 - 9i\sqrt{7}) \log(4ix^2 + (-\sqrt{7} + i)x + 4i) - \frac{1}{56} (35 + 9i\sqrt{7}) \log(4ix^2 + (\sqrt{7} + i)x + 4i) + \frac{1}{28} (35 + 9i\sqrt{7})$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]

[Out] -((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((35 - (9*I)*Sqrt[7])*Log[x])/28 + ((35 + (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/56

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 800

$\text{Int}[(((d_.) + (e_.)*(x_))^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \text{:> Int}[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] \text{/; FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[m]$

Rule 2087

$\text{Int}[(P3_)*(x_)^m)/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2 + (d_.)*(x_.)^3 + (e_.)*(x_.)^4), x_Symbol] \text{:> With}\{q = \text{Sqrt}[8*a^2 + b^2 - 4*a*c], A = \text{Coeff}[P3, x, 0], B = \text{Coeff}[P3, x, 1], C = \text{Coeff}[P3, x, 2], D = \text{Coeff}[P3, x, 3]\}, \text{Dist}[1/q, \text{Int}[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - \text{Dist}[1/q, \text{Int}[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] \text{/; FreeQ}\{a, b, c, m\}, x] \&\& \text{PolyQ}[P3, x, 3] \&\& \text{EqQ}[a, e] \&\& \text{EqQ}[b, d]$

Rubi steps

$$\int \frac{5 + x + 3x^2 + 2x^3}{x(2 + x + 5x^2 + x^3 + 2x^4)} dx = \frac{i \int \frac{9-5i\sqrt{7}+(10-2i\sqrt{7})x}{x(4+(1-i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} - \frac{i \int \frac{9+5i\sqrt{7}+(10+2i\sqrt{7})x}{x(4+(1+i\sqrt{7})x+4x^2)} dx}{\sqrt{7}}$$

$$= -\frac{i \int \left(\frac{9+5i\sqrt{7}}{4x} + \frac{3(11i+\sqrt{7})-2(9i-5\sqrt{7})x}{2(4i+(i-\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} + \frac{i \int \left(\frac{9-5i\sqrt{7}}{4x} + \frac{3(11i-\sqrt{7})-2(9i+5\sqrt{7})x}{2(4i+(i+\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}}$$

$$= \frac{1}{28} (35 - 9i\sqrt{7}) \log(x) + \frac{1}{28} (35 + 9i\sqrt{7}) \log(x) - \frac{i \int \frac{3(11i+\sqrt{7})-2(9i-5\sqrt{7})x}{4i+(i-\sqrt{7})x+4ix^2}}{2\sqrt{7}}$$

$$= \frac{1}{28} (35 - 9i\sqrt{7}) \log(x) + \frac{1}{28} (35 + 9i\sqrt{7}) \log(x) - \frac{1}{56} (35 - 9i\sqrt{7}) \int \frac{1}{4i+(i-\sqrt{7})x+4ix^2}$$

$$= \frac{1}{28} (35 - 9i\sqrt{7}) \log(x) + \frac{1}{28} (35 + 9i\sqrt{7}) \log(x) - \frac{1}{56} (35 - 9i\sqrt{7}) \log\left(4i+(i-\sqrt{7})x+4ix^2\right)$$

$$= -\frac{(53 + i\sqrt{7}) \tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2(35-i\sqrt{7})}}\right)}{2\sqrt{14}(35 - i\sqrt{7})} + \frac{(53 - i\sqrt{7}) \tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2(35+i\sqrt{7})}}\right)}{2\sqrt{14}(35 + i\sqrt{7})} + \frac{1}{2}$$

Mathematica [C] time = 0.02, size = 101, normalized size = 0.41

$$\frac{5 \log(x)}{2} - \frac{1}{2} \text{RootSum}\left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{10\#1^3 \log(x - \#1) + \#1^2 \log(x - \#1) + 19\#1 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]

[Out] (5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 &, (3*Log[x - #1] + 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]/2

fricas [B] time = 3.24, size = 1143, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
[Out] -1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(49/4*(27*
I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/
2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 2058*(-9/56*I*sqrt(7) - 1/2*sqrt
(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 5145*(-9/56*I*sqrt(7) - 1/2*sqrt(-3
7/392*I*sqrt(7) + 79/56) - 5/8)^2 + 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(
-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*s
qrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56)
+ 35) + 8384*x + 1323/2*I*sqrt(7) + 2058*sqrt(-37/392*I*sqrt(7) + 79/56) +
16089/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) -
5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-
9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 1
0*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*sqrt(7) + 79/56
) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(27*I*sqrt(7) + 84*sqrt
(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt
(7) + 79/56) - 5/8)^2 - 15680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(
7) + 79/56) - 5/8)^2 - 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt
(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/5
6) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 7/64*
sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12
*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9
*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*s
qrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*s
qrt(7) + 79/56) + 11/2)*((27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56)
+ 1385)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*
sqrt(7) + 35840*sqrt(-37/392*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I
*sqrt(7) + 10864*sqrt(-37/392*I*sqrt(7) + 79/56) + 5484) - 1/8*(2*sqrt(-12*
(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I
*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7
) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/39
2*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) +
79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqrt(-37/392*I*sqrt(7
) + 79/56) + 5)*log(-49/4*(27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56
) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1
5680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/64
*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 94
5*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) +
28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) -
1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt
(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((2
7*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(-9*I*sqrt(7) + 28
*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*sqrt(7) + 35840*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I*sqrt(7) + 10864*sqrt(-37/3
92*I*sqrt(7) + 79/56) + 5484) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(
7) + 79/56) + 35)*log(2058*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^3 + 20825*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/
56) - 5/8)^2 + 8384*x - 8307/2*I*sqrt(7) - 12922*sqrt(-37/392*I*sqrt(7) + 7
9/56) - 18673/2) + 5/2*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{(2x^4 + x^3 + 5x^2 + x + 2)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)

maple [C] time = 0.01, size = 67, normalized size = 0.27

$$\frac{5 \ln(x)}{2} + \frac{\left(-10 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^3 - \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^2 - 19 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)\right) \ln(x)}{16 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^3 + 6 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^2 + 19 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right) + 10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x)

[Out] 1/2*sum((-10*_R^3-_R^2-19*_R-3)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))+5/2*ln(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \int \frac{10x^3 + x^2 + 19x + 3}{2x^4 + x^3 + 5x^2 + x + 2} dx + \frac{5}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] -1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x) + 5/2*log(x)

mupad [B] time = 2.34, size = 237, normalized size = 0.97

$$\frac{5 \ln(x)}{2} + \left(\sum_{k=1}^4 \ln \left(\frac{223 \operatorname{root}\left(z^4 + \frac{5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}, z, k\right)}{8} - \frac{31x}{2} + \frac{\operatorname{root}\left(z^4 + \frac{5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}, z, k\right)}{16} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)

[Out] (5*log(x))/2 + symsum(log((223*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k))/8 - (31*x)/2 + (71*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)*x)/16 - (4463*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^2*x)/64 + (1449*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^3*x)/16 + (3675*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^4*x)/32 + (257*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^2)/32 + (1673*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^3)/64 - (441*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^4)/32 + 10)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1, 4)

sympy [A] time = 12.66, size = 60, normalized size = 0.24

$$\frac{5 \log(x)}{2} + \operatorname{RootSum}\left(686t^4 + 1715t^3 + 1372t^2 + 448t + 256, \left(t \mapsto t \log\left(-\frac{160344611t^4}{532759184} - \frac{16880402t^3}{33297449} + \frac{401}{21}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, Lambda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_t**3/33297449 + 4010520787*_t**2/2131036736 + 1537535671*_t/532759184 + x + 46660495/66594898))
```

$$3.255 \quad \int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=281

$$\frac{3}{112} (7 + 11i\sqrt{7}) \log(4ix^2 + (-\sqrt{7} + i)x + 4i) + \frac{3}{112} (7 - 11i\sqrt{7}) \log(4ix^2 + (\sqrt{7} + i)x + 4i) - \frac{35 + 9i\sqrt{7}}{28x}$$

[Out] 1/28*(-35+9*I*7^(1/2))/x+1/28*(-35-9*I*7^(1/2))/x-3/56*ln(x)*(7-11*I*7^(1/2))+3/112*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(7-11*I*7^(1/2))-3/56*ln(x)*(7+11*I*7^(1/2))+3/112*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(7+11*I*7^(1/2))+11/4*arctanh((I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(9+5*I*7^(1/2))/(490-14*I*7^(1/2))^(1/2)-11/4*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(9-5*I*7^(1/2))/(490+14*I*7^(1/2))^(1/2)

Rubi [A] time = 0.47, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 206, 628}

$$\frac{3}{112} (7 + 11i\sqrt{7}) \log(4ix^2 + (-\sqrt{7} + i)x + 4i) + \frac{3}{112} (7 - 11i\sqrt{7}) \log(4ix^2 + (\sqrt{7} + i)x + 4i) - \frac{35 + 9i\sqrt{7}}{28x}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]

[Out] -(35 - (9*I)*Sqrt[7])/(28*x) - (35 + (9*I)*Sqrt[7])/(28*x) + (11*(9 + (5*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqrt[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (11*I)*Sqrt[7])*Log[x])/56 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (11*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 800

`Int[(((d_.) + (e_.)*(x_))^(m_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]`

Rule 2087

`Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]`

Rubi steps

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^2(2 + x + 5x^2 + x^3 + 2x^4)} dx = \frac{i \int \frac{9-5i\sqrt{7}+(10-2i\sqrt{7})x}{x^2(4+(1-i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} - \frac{i \int \frac{9+5i\sqrt{7}+(10+2i\sqrt{7})x}{x^2(4+(1+i\sqrt{7})x+4x^2)} dx}{\sqrt{7}}$$

$$= -\frac{i \int \left(\frac{9+5i\sqrt{7}}{4x^2} + \frac{3(11-i\sqrt{7})}{8x} + \frac{-7(9i-5\sqrt{7})-6(11+i\sqrt{7})x}{4(4i+(i-\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} + \frac{i \int \left(\frac{9-5i\sqrt{7}}{4x^2} + \frac{3(11+i\sqrt{7})}{8x} + \frac{-7(9i+5\sqrt{7})-6(11-i\sqrt{7})x}{4(4i+(i+\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}}$$

$$= -\frac{35 - 9i\sqrt{7}}{28x} - \frac{35 + 9i\sqrt{7}}{28x} - \frac{3}{56} (7 - 11i\sqrt{7}) \log(x) - \frac{3}{56} (7 + 11i\sqrt{7}) \log(x)$$

$$= -\frac{35 - 9i\sqrt{7}}{28x} - \frac{35 + 9i\sqrt{7}}{28x} - \frac{3}{56} (7 - 11i\sqrt{7}) \log(x) - \frac{3}{56} (7 + 11i\sqrt{7}) \log(x)$$

$$= -\frac{35 - 9i\sqrt{7}}{28x} - \frac{35 + 9i\sqrt{7}}{28x} - \frac{3}{56} (7 - 11i\sqrt{7}) \log(x) - \frac{3}{56} (7 + 11i\sqrt{7}) \log(x)$$

$$= -\frac{35 - 9i\sqrt{7}}{28x} - \frac{35 + 9i\sqrt{7}}{28x} + \frac{11(9 + 5i\sqrt{7}) \tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2(35-i\sqrt{7})}}\right)}{4\sqrt{14(35-i\sqrt{7})}} - \frac{11(9 - 5i\sqrt{7}) \tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2(35+i\sqrt{7})}}\right)}{4\sqrt{14(35+i\sqrt{7})}}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.39

$$\frac{1}{4} \text{RootSum} \left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{6\#1^3 \log(x - \#1) - 17\#1^2 \log(x - \#1) + 13\#1 \log(x - \#1) - 35 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]

[Out] -5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 &, (-35*Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]/4

$$\frac{6z}{49 + 128/343, z, k)^3}/128 - (441*\text{root}(z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343, z, k)^4)/32 + 21/4)*\text{root}(z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343, z, k), k, 1, 4) - (3*\log(x))/4 - 5/(2x)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)

[Out] Timed out

$$3.256 \quad \int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=317

$$-\frac{35+9i\sqrt{7}}{56x^2} - \frac{35-9i\sqrt{7}}{56x^2} + \frac{1}{32} (35-9i\sqrt{7}) \log(4ix^2 + (-\sqrt{7}+i)x + 4i) + \frac{1}{32} (35+9i\sqrt{7}) \log(4ix^2 + (\sqrt{7}+i)x + 4i)$$

[Out] 1/56*(-35+9*I*7^(1/2))/x^2-1/16*ln(x)*(35-9*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(35-9*I*7^(1/2))+1/56*(-35-9*I*7^(1/2))/x^2-1/16*ln(x)*(35+9*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I*7^(1/2))+3/56*(7-11*I*7^(1/2))/x+3/56*(7+11*I*7^(1/2))/x+1/8*arctanh((I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(355-73*I*7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/8*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(355+73*I*7^(1/2))/(490+14*I*7^(1/2))^(1/2)

Rubi [A] time = 0.54, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{35+9i\sqrt{7}}{56x^2} - \frac{35-9i\sqrt{7}}{56x^2} + \frac{1}{32} (35-9i\sqrt{7}) \log(4ix^2 + (-\sqrt{7}+i)x + 4i) + \frac{1}{32} (35+9i\sqrt{7}) \log(4ix^2 + (\sqrt{7}+i)x + 4i)$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]

[Out] -(35 - (9*I)*Sqrt[7])/(56*x^2) - (35 + (9*I)*Sqrt[7])/(56*x^2) + (3*(7 - (11*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x) + ((355 - (73*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(8*Sqrt[14*(35 - I*Sqrt[7])]) - ((355 + (73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)*Sqrt[7])*Log[x])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] := With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^3(2 + x + 5x^2 + x^3 + 2x^4)} dx = \frac{i \int \frac{9-5i\sqrt{7}+(10-2i\sqrt{7})x}{x^3(4+(1-i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} - \frac{i \int \frac{9+5i\sqrt{7}+(10+2i\sqrt{7})x}{x^3(4+(1+i\sqrt{7})x+4x^2)} dx}{\sqrt{7}}$$

$$= -\frac{i \int \left(\frac{9+5i\sqrt{7}}{4x^3} + \frac{3(11-i\sqrt{7})}{8x^2} - \frac{7i(-9i+5\sqrt{7})}{16x} + \frac{-223i-61\sqrt{7}+14(9i-5\sqrt{7})x}{8(4i+(i-\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} + \dots$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3(7-11i\sqrt{7})}{56x} + \frac{3(7+11i\sqrt{7})}{56x} - \frac{1}{16} \left(35 - \dots \right)$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3(7-11i\sqrt{7})}{56x} + \frac{3(7+11i\sqrt{7})}{56x} - \frac{1}{16} \left(35 - \dots \right)$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3(7-11i\sqrt{7})}{56x} + \frac{3(7+11i\sqrt{7})}{56x} - \frac{1}{16} \left(35 - \dots \right)$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3(7-11i\sqrt{7})}{56x} + \frac{3(7+11i\sqrt{7})}{56x} + \frac{(355-7\dots)}{16}$$

Mathematica [C] time = 0.02, size = 116, normalized size = 0.37

$$\frac{1}{8} \text{RootSum} \left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{70\#1^3 \log(x - \#1) + 47\#1^2 \log(x - \#1) + 141\#1 \log(x - \#1) + 61}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/8
```

fricas [B] time = 3.43, size = 1274, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/448*(14*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35)*log(-49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 9046968*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^3 - 39580485*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 21/1024*(13785856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 9662336*x - 68336919/4*I*sqrt(7) - 30371964*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 257023549/4) + 14*x^2*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)*log(-9046968*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^3 + 41411909*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 9662336*x + 70198191/4*I*sqrt(7) + 31199196*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 240366533/4) + 1960*x^2*log(x) + (4*sqrt(7)*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 - 7*x^2*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 7*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 980*x^2)*log(49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(13785856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 1/1024*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(23079*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 149504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 666914) - (4*sqrt(7)*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 + 7*x^2*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) + 7*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 980*x^2)*log(49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(137
```

85856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 1/1024*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(23079*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 149504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 666914) - 336*x + 560)/x^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{(2x^4 + x^3 + 5x^2 + x + 2)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)

maple [C] time = 0.01, size = 77, normalized size = 0.24

$$-\frac{35 \ln(x)}{8} + \frac{\left(70 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^3 + 47 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^2 + 141\right)}{64 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^3 + 24 \operatorname{RootOf}\left(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2\right)^2 + 141}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x)

[Out] 1/8*sum((70*_R^3+47*_R^2+141*_R+61)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))-5/4/x^2+3/4/x-35/8*ln(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3x - 5}{4x^2} + \frac{1}{8} \int \frac{70x^3 + 47x^2 + 141x + 61}{2x^4 + x^3 + 5x^2 + x + 2} dx - \frac{35}{8} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] 1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 + x^3 + 5*x^2 + x + 2), x) - 35/8*log(x)

mupad [B] time = 2.25, size = 246, normalized size = 0.78

$$\left(\sum_{k=1}^4 \ln \left(-\frac{8939 \operatorname{root}\left(z^4 - \frac{35z^3}{8} + \frac{47z^2}{7} - \frac{8z}{7} + \frac{128}{343}, z, k\right)}{128} - \frac{69x}{8} + \frac{\operatorname{root}\left(z^4 - \frac{35z^3}{8} + \frac{47z^2}{7} - \frac{8z}{7} + \frac{128}{343}, z, k\right)}{128} \right) x^{1494} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)

```
[Out] symsum(log((14945*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z
, k)*x)/128 - (69*x)/8 - (8939*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7
+ 128/343, z, k))/128 - (269991*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)
/7 + 128/343, z, k)^2*x)/1024 - (1393*root(z^4 - (35*z^3)/8 + (47*z^2)/7 -
(8*z)/7 + 128/343, z, k)^3*x)/8 + (3675*root(z^4 - (35*z^3)/8 + (47*z^2)/7
- (8*z)/7 + 128/343, z, k)^4*x)/32 - (35697*root(z^4 - (35*z^3)/8 + (47*z^2
)/7 - (8*z)/7 + 128/343, z, k)^2)/512 - (18487*root(z^4 - (35*z^3)/8 + (47*
z^2)/7 - (8*z)/7 + 128/343, z, k)^3)/256 - (441*root(z^4 - (35*z^3)/8 + (47
*z^2)/7 - (8*z)/7 + 128/343, z, k)^4)/32 + 245/8)*root(z^4 - (35*z^3)/8 + (
47*z^2)/7 - (8*z)/7 + 128/343, z, k), k, 1, 4) - (35*log(x))/8 + ((3*x)/4 -
5/4)/x^2
```

sympy [A] time = 2.70, size = 70, normalized size = 0.22

$$-\frac{35 \log(x)}{8} + \text{RootSum}\left(2744t^4 - 12005t^3 + 18424t^2 - 3136t + 1024, \left(t \mapsto t \log\left(-\frac{20101387287723t^4}{91907904361586} + \frac{94451}{45953}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2), x)
```

```
[Out] -35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t + 1
024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 944515214496*
_t**3/45953952180793 + 16572327093911939*_t**2/5882105879141504 - 456447174
9800865*_t/735263234892688 + x + 70084064010625/91907904361586))) + (3*x -
5)/(4*x**2)
```

$$3.257 \quad \int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6} dx$$

Optimal. Leaf size=19

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

[Out] arctan(c*x^3/(b*x^2+a))/c

Rubi [A] time = 0.10, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]

[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_.)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] && EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6} dx = (3a^2) \text{Subst}\left(\int \frac{1}{a^2+9a^2c^2x^2} dx, x, \frac{x^3}{3a+3bx^2}\right) = \frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Mathematica [C] time = 0.05, size = 87, normalized size = 4.58

$$\frac{1}{2} \text{RootSum}\left[\#1^6 c^2 + \#1^4 b^2 + 2\#1^2 ab + a^2 \&, \frac{\#1^3 b \log(x - \#1) + 3\#1 a \log(x - \#1)}{3\#1^4 c^2 + 2\#1^2 b^2 + 2ab} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]

[Out] RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 &, (3*a*Log[x - #1]*#1 + b*Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) &]/2

fricas [B] time = 0.76, size = 83, normalized size = 4.37

$$\frac{\arctan\left(\frac{cx}{b}\right) - \arctan\left(\frac{bc^2x^5 + ab^2x + (b^3 - ac^2)x^3}{a^2c}\right) + \arctan\left(\frac{bc^2x^3 + (b^3 - ac^2)x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")

[Out] (arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c)) + arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c

giac [B] time = 4.08, size = 87, normalized size = 4.58

$$\frac{\arctan\left(\frac{cx}{b}\right) + \arctan\left(-\frac{bc^2x^5 + b^3x^3 - ac^2x^3 + ab^2x}{a^2c}\right) - \arctan\left(-\frac{bc^2x^3 + b^3x - ac^2x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")

[Out] (arctan(c*x/b) + arctan(-(b*c^2*x^5 + b^3*x^3 - a*c^2*x^3 + a*b^2*x)/(a^2*c)) - arctan(-(b*c^2*x^3 + b^3*x - a*c^2*x)/(a*b*c)))/c

maple [C] time = 0.10, size = 75, normalized size = 3.95

$$\frac{\left(\text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)^4 b + 3 \text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)^2 a\right) \ln\left(-\text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)\right)}{6 \text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)^5 c^2 + 4 \text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)^3 b^2 + 4 \text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right) b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x)

[Out] 1/2*sum((_R^4*b+3*_R^2*a)/(3*_R^5*c^2+2*_R^3*b^2+2*_R*a*b)*ln(-_R+x), _R=RootOf(_Z^6*c^2+_Z^4*b^2+2*_Z^2*a*b+a^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + 3a)x^2}{c^2x^6 + b^2x^4 + 2abx^2 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")

[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)

mupad [B] time = 2.27, size = 252, normalized size = 13.26

$$\frac{\text{atan}\left(\frac{27ac^5x^3}{27a^2c^4-4ab^3c^2} - \frac{27bc^5x^5}{27a^2c^4-4ab^3c^2} - \frac{31b^3c^3x^3}{27a^2c^4-4ab^3c^2} + \frac{4b^6cx^3}{27a^3c^4-4a^2b^3c^2} + \frac{4b^5cx}{27a^2c^4-4ab^3c^2} + \frac{4b^4c^3x^5}{27a^3c^4-4a^2b^3c^2} - \frac{27ab^2c^3}{27a^2c^4-4ab^3c^2}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(3*a + b*x^2))/(a^2 + b^2*x^4 + c^2*x^6 + 2*a*b*x^2),x)


```
[Out] (atan((27*a*c^5*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) - (27*b*c^5*x^5)/(27*a^2*c^4 - 4*a*b^3*c^2) - (31*b^3*c^3*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^6*c*x^3)/(27*a^3*c^4 - 4*a^2*b^3*c^2) + (4*b^5*c*x)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^4*c^3*x^5)/(27*a^3*c^4 - 4*a^2*b^3*c^2) - (27*a*b^2*c^3*x)/(27*a^2*c^4 - 4*a*b^3*c^2)) + atan((c*x^3)/a - (c*x)/b + (b^2*x)/(a*c)) + atan((c*x)/b))/c
```

sympy [C] time = 1.05, size = 44, normalized size = 2.32

$$\frac{-\frac{i \log\left(-\frac{ia}{c} - \frac{ibx^2}{c} + x^3\right)}{2} + \frac{i \log\left(\frac{ia}{c} + \frac{ibx^2}{c} + x^3\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2), x)
```

```
[Out] (-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2)/c
```

$$3.258 \quad \int \frac{1-3x^4}{(-2+x)(1+x^2)^2} dx$$

Optimal. Leaf size=43

$$-\frac{1-2x}{5(x^2+1)} - \frac{14}{25} \log(x^2+1) - \frac{47}{25} \log(2-x) - \frac{46}{25} \tan^{-1}(x)$$

[Out] 1/5*(-1+2*x)/(x^2+1)-46/25*arctan(x)-47/25*ln(2-x)-14/25*ln(x^2+1)

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1647, 1629, 635, 203, 260}

$$-\frac{1-2x}{5(x^2+1)} - \frac{14}{25} \log(x^2+1) - \frac{47}{25} \log(2-x) - \frac{46}{25} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2), x]

[Out] -(1 - 2*x)/(5*(1 + x^2)) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25 - (14*Log[1 + x^2])/25

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1647

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q]/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} dx &= -\frac{1-2x}{5(1+x^2)} - \frac{1}{2} \int \frac{-\frac{18}{5} - \frac{4x}{5} + 6x^2}{(-2+x)(1+x^2)} dx \\
&= -\frac{1-2x}{5(1+x^2)} - \frac{1}{2} \int \left(\frac{94}{25(-2+x)} + \frac{4(23+14x)}{25(1+x^2)} \right) dx \\
&= -\frac{1-2x}{5(1+x^2)} - \frac{47}{25} \log(2-x) - \frac{2}{25} \int \frac{23+14x}{1+x^2} dx \\
&= -\frac{1-2x}{5(1+x^2)} - \frac{47}{25} \log(2-x) - \frac{28}{25} \int \frac{x}{1+x^2} dx - \frac{46}{25} \int \frac{1}{1+x^2} dx \\
&= -\frac{1-2x}{5(1+x^2)} - \frac{46}{25} \tan^{-1}(x) - \frac{47}{25} \log(2-x) - \frac{14}{25} \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.33

$$\frac{2(x-2)+3}{5((x-2)^2+4(x-2)+5)} - \frac{14}{25} \log((x-2)^2+4(x-2)+5) - \frac{47}{25} \log(x-2) - \frac{46}{25} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2), x]

[Out] (3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x])/25 - (14*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x])/25

fricas [A] time = 0.81, size = 47, normalized size = 1.09

$$\frac{46(x^2+1)\arctan(x) + 14(x^2+1)\log(x^2+1) + 47(x^2+1)\log(x-2) - 10x + 5}{25(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="fricas")

[Out] -1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x^2 + 1)*log(x - 2) - 10*x + 5)/(x^2 + 1)

giac [A] time = 0.25, size = 34, normalized size = 0.79

$$\frac{2x-1}{5(x^2+1)} - \frac{46}{25} \arctan(x) - \frac{14}{25} \log(x^2+1) - \frac{47}{25} \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="giac")

[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(abs(x - 2))

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$-\frac{46 \arctan(x)}{25} - \frac{47 \ln(x-2)}{25} - \frac{14 \ln(x^2+1)}{25} - \frac{2\left(-5x + \frac{5}{2}\right)}{25(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x^4+1)/(-2+x)/(x^2+1)^2,x)

[Out] -2/25*(-5*x+5/2)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)-47/25*ln(-2+x)

maxima [A] time = 2.10, size = 33, normalized size = 0.77

$$\frac{2x-1}{5(x^2+1)} - \frac{46}{25} \arctan(x) - \frac{14}{25} \log(x^2+1) - \frac{47}{25} \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="maxima")

[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(x - 2)

mupad [B] time = 0.05, size = 38, normalized size = 0.88

$$\frac{\frac{2x}{5} - \frac{1}{5}}{x^2 + 1} - \frac{47 \ln(x-2)}{25} + \ln(x-i) \left(-\frac{14}{25} + \frac{23}{25}i \right) + \ln(x+1i) \left(-\frac{14}{25} - \frac{23}{25}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x)

[Out] ((2*x)/5 - 1/5)/(x^2 + 1) - log(x - 1i)*(14/25 - 23i/25) - log(x + 1i)*(14/25 + 23i/25) - (47*log(x - 2))/25

sympy [A] time = 0.17, size = 37, normalized size = 0.86

$$-\frac{1-2x}{5x^2+5} - \frac{47 \log(x-2)}{25} - \frac{14 \log(x^2+1)}{25} - \frac{46 \operatorname{atan}(x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)

[Out] -(1 - 2*x)/(5*x**2 + 5) - 47*log(x - 2)/25 - 14*log(x**2 + 1)/25 - 46*atan(x)/25

$$3.259 \quad \int \frac{-9-9x+2x^2}{-9x+x^3} dx$$

Optimal. Leaf size=17

$$-\log(3-x) + \log(x) + 2\log(x+3)$$

[Out] -ln(3-x)+ln(x)+2*ln(3+x)

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1593, 1802}

$$-\log(3-x) + \log(x) + 2\log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3), x]

[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-9-9x+2x^2}{-9x+x^3} dx &= \int \frac{-9-9x+2x^2}{x(-9+x^2)} dx \\ &= \int \left(\frac{1}{3-x} + \frac{1}{x} + \frac{2}{3+x} \right) dx \\ &= -\log(3-x) + \log(x) + 2\log(3+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\log(3-x) + \log(x) + 2\log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3), x]

[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]

fricas [A] time = 0.94, size = 15, normalized size = 0.88

$$2\log(x+3) - \log(x-3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-9*x-9)/(x^3-9*x), x, algorithm="fricas")

[Out] $2 \log(x + 3) - \log(x - 3) + \log(x)$

giac [A] time = 0.31, size = 18, normalized size = 1.06

$$2 \log(|x + 3|) - \log(|x - 3|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="giac")`

[Out] $2 \log(\text{abs}(x + 3)) - \log(\text{abs}(x - 3)) + \log(\text{abs}(x))$

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) - \ln(x - 3) + 2 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-9*x-9)/(x^3-9*x),x)`

[Out] $2 \ln(3+x) - \ln(-3+x) + \ln(x)$

maxima [A] time = 1.12, size = 15, normalized size = 0.88

$$2 \log(x + 3) - \log(x - 3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="maxima")`

[Out] $2 \log(x + 3) - \log(x - 3) + \log(x)$

mupad [B] time = 2.20, size = 21, normalized size = 1.24

$$2 \ln(x + 3) - 2 \operatorname{atanh}\left(\frac{1296}{18x + 162} - 7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((9*x - 2*x^2 + 9)/(9*x - x^3),x)`

[Out] $2 \log(x + 3) - 2 \operatorname{atanh}(1296/(18*x + 162) - 7)$

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$\log(x) - \log(x - 3) + 2 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-9*x-9)/(x**3-9*x),x)`

[Out] $\log(x) - \log(x - 3) + 2 \log(x + 3)$

$$3.260 \quad \int \frac{1+2x^2+x^5}{-x+x^3} dx$$

Optimal. Leaf size=25

$$\frac{x^3}{3} + x + 2 \log(1-x) - \log(x) + \log(x+1)$$

[Out] x+1/3*x^3+2*ln(1-x)-ln(x)+ln(1+x)

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1593, 1802}

$$\frac{x^3}{3} + x + 2 \log(1-x) - \log(x) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2 + x^5)/(-x + x^3), x]

[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^(m)*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2+x^5}{-x+x^3} dx &= \int \frac{1+2x^2+x^5}{x(-1+x^2)} dx \\ &= \int \left(1 + \frac{2}{-1+x} - \frac{1}{x} + x^2 + \frac{1}{1+x} \right) dx \\ &= x + \frac{x^3}{3} + 2 \log(1-x) - \log(x) + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^3}{3} + x + 2 \log(1-x) - \log(x) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2 + x^5)/(-x + x^3), x]

[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]

fricas [A] time = 0.76, size = 21, normalized size = 0.84

$$\frac{1}{3}x^3 + x + \log(x+1) + 2 \log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="fricas")

[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)

giac [A] time = 0.25, size = 24, normalized size = 0.96

$$\frac{1}{3}x^3 + x + \log(|x + 1|) + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="giac")

[Out] 1/3*x^3 + x + log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{x^3}{3} + x - \ln(x) + 2 \ln(x - 1) + \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5+2*x^2+1)/(x^3-x),x)

[Out] 1/3*x^3+x+2*ln(x-1)+ln(x+1)-ln(x)

maxima [A] time = 1.15, size = 21, normalized size = 0.84

$$\frac{1}{3}x^3 + x + \log(x + 1) + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="maxima")

[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)

mupad [B] time = 0.05, size = 30, normalized size = 1.20

$$x + 2 \ln(x - 1) + \frac{x^3}{3} + \operatorname{atan}\left(\frac{48i}{11(22x - 2)} + \frac{13}{11}i\right) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x^2 + x^5 + 1)/(x - x^3),x)

[Out] x + 2*log(x - 1) + atan(48i/(11*(22*x - 2)) + 13i/11)*2i + x^3/3

sympy [A] time = 0.13, size = 20, normalized size = 0.80

$$\frac{x^3}{3} + x - \log(x) + 2 \log(x - 1) + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5+2*x**2+1)/(x**3-x),x)

[Out] x**3/3 + x - log(x) + 2*log(x - 1) + log(x + 1)

$$3.261 \quad \int \frac{3+2x^2}{(-1+x)^2x} dx$$

Optimal. Leaf size=22

$$\frac{5}{1-x} - \log(1-x) + 3 \log(x)$$

[Out] 5/(1-x)-ln(1-x)+3*ln(x)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {894}

$$\frac{5}{1-x} - \log(1-x) + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x^2)/((-1 + x)^2*x), x]

[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]

Rule 894

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{3+2x^2}{(-1+x)^2x} dx &= \int \left(\frac{1}{1-x} + \frac{5}{(-1+x)^2} + \frac{3}{x} \right) dx \\ &= \frac{5}{1-x} - \log(1-x) + 3 \log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.91

$$-\frac{5}{x-1} - \log(1-x) + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x^2)/((-1 + x)^2*x), x]

[Out] -5/(-1 + x) - Log[1 - x] + 3*Log[x]

fricas [A] time = 0.57, size = 24, normalized size = 1.09

$$-\frac{(x-1) \log(x-1) - 3(x-1) \log(x) + 5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="fricas")

[Out] -((x - 1)*log(x - 1) - 3*(x - 1)*log(x) + 5)/(x - 1)

giac [A] time = 0.37, size = 28, normalized size = 1.27

$$-\frac{5}{x-1} + 2 \log(|x-1|) + 3 \log\left(\left|-\frac{1}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="giac")

[Out] -5/(x - 1) + 2*log(abs(x - 1)) + 3*log(abs(-1/(x - 1) - 1))

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$3 \ln(x) - \ln(x-1) - \frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+3)/(x-1)^2/x,x)

[Out] -5/(x-1)-ln(x-1)+3*ln(x)

maxima [A] time = 1.09, size = 18, normalized size = 0.82

$$-\frac{5}{x-1} - \log(x-1) + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="maxima")

[Out] -5/(x - 1) - log(x - 1) + 3*log(x)

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$3 \ln(x) - \ln(x-1) - \frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 + 3)/(x*(x - 1)^2),x)

[Out] 3*log(x) - log(x - 1) - 5/(x - 1)

sympy [A] time = 0.11, size = 14, normalized size = 0.64

$$3 \log(x) - \log(x-1) - \frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+3)/(-1+x)**2/x,x)

[Out] 3*log(x) - log(x - 1) - 5/(x - 1)

$$3.262 \quad \int \frac{-1+2x^2}{(-1+4x)(1+x^2)} dx$$

Optimal. Leaf size=27

$$\frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(1 - 4x) + \frac{3}{17} \tan^{-1}(x)$$

[Out] 3/17*arctan(x)-7/34*ln(1-4*x)+6/17*ln(x^2+1)

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1629, 635, 203, 260}

$$\frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(1 - 4x) + \frac{3}{17} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]

[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1+2x^2}{(-1+4x)(1+x^2)} dx &= \int \left(-\frac{14}{17(-1+4x)} + \frac{3(1+4x)}{17(1+x^2)} \right) dx \\ &= -\frac{7}{34} \log(1-4x) + \frac{3}{17} \int \frac{1+4x}{1+x^2} dx \\ &= -\frac{7}{34} \log(1-4x) + \frac{3}{17} \int \frac{1}{1+x^2} dx + \frac{12}{17} \int \frac{x}{1+x^2} dx \\ &= \frac{3}{17} \tan^{-1}(x) - \frac{7}{34} \log(1-4x) + \frac{6}{17} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.41

$$-\frac{7}{34} \log(4x-1) + \frac{6}{17} \log((4x-1)^2 + 2(4x-1) + 17) + \frac{3}{17} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)), x]

[Out] (3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x) + (-1 + 4*x)^2])/17

fricas [A] time = 0.95, size = 21, normalized size = 0.78

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1), x, algorithm="fricas")

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)

giac [A] time = 0.29, size = 22, normalized size = 0.81

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(|4x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1), x, algorithm="giac")

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(abs(4*x - 1))

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{3 \arctan(x)}{17} - \frac{7 \ln(4x - 1)}{34} + \frac{6 \ln(x^2 + 1)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-1)/(4*x-1)/(x^2+1), x)

[Out] 6/17*ln(x^2+1)+3/17*arctan(x)-7/34*ln(4*x-1)

maxima [A] time = 2.31, size = 21, normalized size = 0.78

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1), x, algorithm="maxima")

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)

mupad [B] time = 2.26, size = 25, normalized size = 0.93

$$-\frac{7 \ln\left(x - \frac{1}{4}\right)}{34} + \ln(x - i) \left(\frac{6}{17} - \frac{3}{34}i\right) + \ln(x + i) \left(\frac{6}{17} + \frac{3}{34}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 - 1)/((4*x - 1)*(x^2 + 1)), x)

[Out] $\log(x - 1i)*(6/17 - 3i/34) - (7*\log(x - 1/4))/34 + \log(x + 1i)*(6/17 + 3i/34)$

sympy [A] time = 0.14, size = 26, normalized size = 0.96

$$-\frac{7\log\left(x - \frac{1}{4}\right)}{34} + \frac{6\log(x^2 + 1)}{17} + \frac{3\operatorname{atan}(x)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)`

[Out] $-7*\log(x - 1/4)/34 + 6*\log(x**2 + 1)/17 + 3*\operatorname{atan}(x)/17$

$$3.263 \quad \int \frac{-3+2x-3x^2+x^3}{1+x^2} dx$$

Optimal. Leaf size=21

$$\frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) - 3x$$

[Out] $-3*x+1/2*x^2+1/2*\ln(x^2+1)$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1810, 260}

$$\frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) - 3x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2), x]$

[Out] $-3*x + x^2/2 + \text{Log}[1 + x^2]/2$

Rule 260

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /;$ FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1810

$\text{Int}[(Pq_)*((a_) + (b_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq*(a + b*x^2)^p, x], x] /;$ FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-3+2x-3x^2+x^3}{1+x^2} dx &= \int \left(-3+x+\frac{x}{1+x^2}\right) dx \\ &= -3x + \frac{x^2}{2} + \int \frac{x}{1+x^2} dx \\ &= -3x + \frac{x^2}{2} + \frac{1}{2} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) - 3x$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2), x]$

[Out] $-3*x + x^2/2 + \text{Log}[1 + x^2]/2$

fricas [A] time = 0.82, size = 17, normalized size = 0.81

$$\frac{1}{2} x^2 - 3x + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^3-3*x^2+2*x-3)/(x^2+1), x, \text{algorithm}=\text{"fricas"})$

[Out] $\frac{1}{2}x^2 - 3x + \frac{1}{2}\log(x^2 + 1)$

giac [A] time = 0.27, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="giac")`

[Out] $\frac{1}{2}x^2 - 3x + \frac{1}{2}\log(x^2 + 1)$

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{x^2}{2} - 3x + \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3-3*x^2+2*x-3)/(x^2+1),x)`

[Out] $-3x + \frac{1}{2}x^2 + \frac{1}{2}\ln(x^2 + 1)$

maxima [A] time = 2.28, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="maxima")`

[Out] $\frac{1}{2}x^2 - 3x + \frac{1}{2}\log(x^2 + 1)$

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{\ln(x^2 + 1)}{2} - 3x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x - 3*x^2 + x^3 - 3)/(x^2 + 1),x)`

[Out] $\log(x^2 + 1)/2 - 3x + x^2/2$

sympy [A] time = 0.08, size = 15, normalized size = 0.71

$$\frac{x^2}{2} - 3x + \frac{\log(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)`

[Out] $x**2/2 - 3*x + \log(x**2 + 1)/2$

$$3.264 \quad \int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} dx$$

Optimal. Leaf size=27

$$\frac{x^3}{3} + \frac{1}{2} \log(x^2 + 6x + 10) - 3 \tan^{-1}(x + 3)$$

[Out] 1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{3} + \frac{1}{2} \log(x^2 + 6x + 10) - 3 \tan^{-1}(x + 3)$$

Antiderivative was successfully verified.

[In] Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2), x]

[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x + 10x^2 + 6x^3 + x^4}{10 + 6x + x^2} dx &= \int \left(x^2 + \frac{x}{10 + 6x + x^2} \right) dx \\
&= \frac{x^3}{3} + \int \frac{x}{10 + 6x + x^2} dx \\
&= \frac{x^3}{3} + \frac{1}{2} \int \frac{6 + 2x}{10 + 6x + x^2} dx - 3 \int \frac{1}{10 + 6x + x^2} dx \\
&= \frac{x^3}{3} + \frac{1}{2} \log(10 + 6x + x^2) + 6 \operatorname{Subst} \left(\int \frac{1}{-4 - x^2} dx, x, 6 + 2x \right) \\
&= \frac{x^3}{3} - 3 \tan^{-1}(3 + x) + \frac{1}{2} \log(10 + 6x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^3}{3} + \frac{1}{2} \log(x^2 + 6x + 10) - 3 \tan^{-1}(x + 3)$$

Antiderivative was successfully verified.

[In] Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2), x]

[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2

fricas [A] time = 1.07, size = 23, normalized size = 0.85

$$\frac{1}{3} x^3 - 3 \arctan(x + 3) + \frac{1}{2} \log(x^2 + 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10), x, algorithm="fricas")

[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)

giac [A] time = 0.28, size = 23, normalized size = 0.85

$$\frac{1}{3} x^3 - 3 \arctan(x + 3) + \frac{1}{2} \log(x^2 + 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10), x, algorithm="giac")

[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^3}{3} - 3 \arctan(x + 3) + \frac{\ln(x^2 + 6x + 10)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10), x)

[Out] 1/3*x^3-3*arctan(x+3)+1/2*ln(x^2+6*x+10)

maxima [A] time = 2.08, size = 23, normalized size = 0.85

$$\frac{1}{3} x^3 - 3 \arctan(x + 3) + \frac{1}{2} \log(x^2 + 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="maxima")

[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)

mupad [B] time = 2.14, size = 23, normalized size = 0.85

$$\frac{\ln(x^2 + 6x + 10)}{2} - 3 \operatorname{atan}(x + 3) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 10*x^2 + 6*x^3 + x^4)/(6*x + x^2 + 10),x)

[Out] log(6*x + x^2 + 10)/2 - 3*atan(x + 3) + x^3/3

sympy [A] time = 0.11, size = 22, normalized size = 0.81

$$\frac{x^3}{3} + \frac{\log(x^2 + 6x + 10)}{2} - 3 \operatorname{atan}(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)

[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)

$$3.265 \quad \int \frac{1}{-18+27x-7x^2-3x^3+x^4} dx$$

Optimal. Leaf size=39

$$\frac{1}{8} \log(1-x) - \frac{1}{5} \log(2-x) + \frac{1}{12} \log(3-x) - \frac{1}{120} \log(x+3)$$

[Out] 1/8*ln(1-x)-1/5*ln(2-x)+1/12*ln(3-x)-1/120*ln(3+x)

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2058}

$$\frac{1}{8} \log(1-x) - \frac{1}{5} \log(2-x) + \frac{1}{12} \log(3-x) - \frac{1}{120} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1), x]

[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{-18+27x-7x^2-3x^3+x^4} dx &= \int \left(\frac{1}{12(-3+x)} - \frac{1}{5(-2+x)} + \frac{1}{8(-1+x)} - \frac{1}{120(3+x)} \right) dx \\ &= \frac{1}{8} \log(1-x) - \frac{1}{5} \log(2-x) + \frac{1}{12} \log(3-x) - \frac{1}{120} \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{1}{8} \log(1-x) - \frac{1}{5} \log(2-x) + \frac{1}{12} \log(3-x) - \frac{1}{120} \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1), x]

[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120

fricas [A] time = 0.78, size = 25, normalized size = 0.64

$$-\frac{1}{120} \log(x+3) + \frac{1}{8} \log(x-1) - \frac{1}{5} \log(x-2) + \frac{1}{12} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18), x, algorithm="fricas")

[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)

giac [A] time = 0.32, size = 29, normalized size = 0.74

$$-\frac{1}{120} \log(|x+3|) + \frac{1}{8} \log(|x-1|) - \frac{1}{5} \log(|x-2|) + \frac{1}{12} \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="giac")

[Out] -1/120*log(abs(x + 3)) + 1/8*log(abs(x - 1)) - 1/5*log(abs(x - 2)) + 1/12*log(abs(x - 3))

maple [A] time = 0.01, size = 26, normalized size = 0.67

$$\frac{\ln(x-3)}{12} - \frac{\ln(x-2)}{5} + \frac{\ln(x-1)}{8} - \frac{\ln(x+3)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4-3*x^3-7*x^2+27*x-18),x)

[Out] 1/8*ln(x-1)-1/5*ln(x-2)-1/120*ln(x+3)+1/12*ln(x-3)

maxima [A] time = 1.27, size = 25, normalized size = 0.64

$$-\frac{1}{120} \log(x+3) + \frac{1}{8} \log(x-1) - \frac{1}{5} \log(x-2) + \frac{1}{12} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="maxima")

[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)

mupad [B] time = 2.14, size = 25, normalized size = 0.64

$$\frac{\ln(x-1)}{8} - \frac{\ln(x-2)}{5} + \frac{\ln(x-3)}{12} - \frac{\ln(x+3)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(7*x^2 - 27*x + 3*x^3 - x^4 + 18),x)

[Out] log(x - 1)/8 - log(x - 2)/5 + log(x - 3)/12 - log(x + 3)/120

sympy [A] time = 0.24, size = 26, normalized size = 0.67

$$\frac{\log(x-3)}{12} - \frac{\log(x-2)}{5} + \frac{\log(x-1)}{8} - \frac{\log(x+3)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)

[Out] log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120

$$3.266 \quad \int \frac{1+x^3}{-2+x} dx$$

Optimal. Leaf size=22

$$\frac{x^3}{3} + x^2 + 4x + 9 \log(2-x)$$

[Out] 4*x+x^2+1/3*x^3+9*ln(2-x)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1850}

$$\frac{x^3}{3} + x^2 + 4x + 9 \log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(-2 + x), x]

[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand [Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-2+x} dx &= \int \left(4 + \frac{9}{-2+x} + 2x + x^2 \right) dx \\ &= 4x + x^2 + \frac{x^3}{3} + 9 \log(2-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^3}{3} + x^2 + 4x + 9 \log(x-2) - \frac{44}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(-2 + x), x]

[Out] -44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]

fricas [A] time = 0.92, size = 18, normalized size = 0.82

$$\frac{1}{3} x^3 + x^2 + 4x + 9 \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(-2+x), x, algorithm="fricas")

[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)

giac [A] time = 0.28, size = 19, normalized size = 0.86

$$\frac{1}{3} x^3 + x^2 + 4x + 9 \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(-2+x),x, algorithm="giac")

[Out] 1/3*x^3 + x^2 + 4*x + 9*log(abs(x - 2))

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^3}{3} + x^2 + 4x + 9 \ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(x-2),x)

[Out] 1/3*x^3+x^2+4*x+9*ln(x-2)

maxima [A] time = 1.13, size = 18, normalized size = 0.82

$$\frac{1}{3}x^3 + x^2 + 4x + 9 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(-2+x),x, algorithm="maxima")

[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$4x + 9 \ln(x - 2) + x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 1)/(x - 2),x)

[Out] 4*x + 9*log(x - 2) + x^2 + x^3/3

sympy [A] time = 0.07, size = 17, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + 4x + 9 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)/(-2+x),x)

[Out] x**3/3 + x**2 + 4*x + 9*log(x - 2)

$$3.267 \quad \int \frac{3x-4x^2+3x^3}{1+x^2} dx$$

Optimal. Leaf size=15

$$\frac{3x^2}{2} - 4x + 4 \tan^{-1}(x)$$

[Out] -4*x+3/2*x^2+4*arctan(x)

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1594, 1802, 203}

$$\frac{3x^2}{2} - 4x + 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2), x]

[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 1802

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{3x-4x^2+3x^3}{1+x^2} dx &= \int \frac{x(3-4x+3x^2)}{1+x^2} dx \\ &= \int \left(-4 + 3x + \frac{4}{1+x^2} \right) dx \\ &= -4x + \frac{3x^2}{2} + 4 \int \frac{1}{1+x^2} dx \\ &= -4x + \frac{3x^2}{2} + 4 \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{3x^2}{2} - 4x + 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]

[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]

fricas [A] time = 0.82, size = 13, normalized size = 0.87

$$\frac{3}{2}x^2 - 4x + 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="fricas")

[Out] 3/2*x^2 - 4*x + 4*arctan(x)

giac [A] time = 0.34, size = 13, normalized size = 0.87

$$\frac{3}{2}x^2 - 4x + 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="giac")

[Out] 3/2*x^2 - 4*x + 4*arctan(x)

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^3-4*x^2+3*x)/(x^2+1),x)

[Out] -4*x+3/2*x^2+4*arctan(x)

maxima [A] time = 2.30, size = 13, normalized size = 0.87

$$\frac{3}{2}x^2 - 4x + 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="maxima")

[Out] 3/2*x^2 - 4*x + 4*arctan(x)

mupad [B] time = 2.13, size = 13, normalized size = 0.87

$$4 \operatorname{atan}(x) - 4x + \frac{3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 4*x^2 + 3*x^3)/(x^2 + 1),x)

[Out] 4*atan(x) - 4*x + (3*x^2)/2

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)

[Out] 3*x**2/2 - 4*x + 4*atan(x)

$$3.268 \quad \int \frac{5+3x}{1-x-x^2+x^3} dx$$

Optimal. Leaf size=12

$$\frac{4}{1-x} + \tanh^{-1}(x)$$

[Out] 4/(1-x)+arctanh(x)

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2074, 206}

$$\frac{4}{1-x} + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 + 3*x)/(1 - x - x^2 + x^3), x]

[Out] 4/(1 - x) + ArcTanh[x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{5+3x}{1-x-x^2+x^3} dx &= \int \left(\frac{4}{(-1+x)^2} + \frac{1}{1-x^2} \right) dx \\ &= \frac{4}{1-x} + \int \frac{1}{1-x^2} dx \\ &= \frac{4}{1-x} + \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 2.00

$$-\frac{4}{x-1} - \frac{1}{2} \log(x-1) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 3*x)/(1 - x - x^2 + x^3), x]

[Out] -4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2

fricas [B] time = 0.73, size = 26, normalized size = 2.17

$$\frac{(x-1) \log(x+1) - (x-1) \log(x-1) - 8}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="fricas")

[Out] 1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)

giac [B] time = 0.30, size = 22, normalized size = 1.83

$$-\frac{4}{x-1} + \frac{1}{2} \log(|x+1|) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="giac")

[Out] -4/(x - 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))

maple [A] time = 0.01, size = 21, normalized size = 1.75

$$-\frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2} - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+3*x)/(x^3-x^2-x+1),x)

[Out] -4/(x-1)-1/2*ln(x-1)+1/2*ln(x+1)

maxima [A] time = 1.08, size = 20, normalized size = 1.67

$$-\frac{4}{x-1} + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="maxima")

[Out] -4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)

mupad [B] time = 0.07, size = 10, normalized size = 0.83

$$\operatorname{atanh}(x) - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x + 5)/(x + x^2 - x^3 - 1),x)

[Out] atanh(x) - 4/(x - 1)

sympy [B] time = 0.10, size = 17, normalized size = 1.42

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+3*x)/(x**3-x**2-x+1),x)

[Out] -log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)

$$3.269 \quad \int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx$$

Optimal. Leaf size=25

$$\frac{x^2}{2} - \frac{1}{x} - 2 \log(1-x) + 2 \log(x)$$

[Out] $-1/x + 1/2*x^2 - 2*\ln(1-x) + 2*\ln(x)$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1593, 1620}

$$\frac{x^2}{2} - \frac{1}{x} - 2 \log(1-x) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 - x - x^3 + x^4)/(-x^2 + x^3), x]$

[Out] $-x^{(-1)} + x^2/2 - 2*\text{Log}[1 - x] + 2*\text{Log}[x]$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1620

$\text{Int}[(Px_)*((a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Px*(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx &= \int \frac{-1-x-x^3+x^4}{(-1+x)x^2} dx \\ &= \int \left(-\frac{2}{-1+x} + \frac{1}{x^2} + \frac{2}{x} + x \right) dx \\ &= -\frac{1}{x} + \frac{x^2}{2} - 2 \log(1-x) + 2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{x} - 2 \log(1-x) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(-1 - x - x^3 + x^4)/(-x^2 + x^3), x]$

[Out] $-x^{(-1)} + x^2/2 - 2*\text{Log}[1 - x] + 2*\text{Log}[x]$

fricas [A] time = 0.87, size = 22, normalized size = 0.88

$$\frac{x^3 - 4x \log(x-1) + 4x \log(x) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="fricas")

[Out] 1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x

giac [A] time = 0.27, size = 23, normalized size = 0.92

$$\frac{1}{2}x^2 - \frac{1}{x} - 2 \log(|x - 1|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="giac")

[Out] 1/2*x^2 - 1/x - 2*log(abs(x - 1)) + 2*log(abs(x))

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{x^2}{2} + 2 \ln(x) - 2 \ln(x - 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x^3-x-1)/(x^3-x^2),x)

[Out] 1/2*x^2-2*ln(x-1)-1/x+2*ln(x)

maxima [A] time = 1.13, size = 21, normalized size = 0.84

$$\frac{1}{2}x^2 - \frac{1}{x} - 2 \log(x - 1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="maxima")

[Out] 1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)

mupad [B] time = 2.14, size = 19, normalized size = 0.76

$$4 \operatorname{atanh}(2x - 1) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^3 - x^4 + 1)/(x^2 - x^3),x)

[Out] 4*atanh(2*x - 1) - 1/x + x^2/2

sympy [A] time = 0.10, size = 19, normalized size = 0.76

$$\frac{x^2}{2} + 2 \log(x) - 2 \log(x - 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-x**3-x-1)/(x**3-x**2),x)

[Out] x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x

$$3.270 \quad \int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx$$

Optimal. Leaf size=13

$$\frac{1}{2} \log(x^2 + 2) + \tan^{-1}(x)$$

[Out] arctan(x)+1/2*ln(x^2+2)

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1673, 1149, 203, 1247, 626, 31}

$$\frac{1}{2} \log(x^2 + 2) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]

[Out] ArcTan[x] + Log[2 + x^2]/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 626

Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 1149

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 1673

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned}
\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx &= \int \frac{x(1+x^2)}{2+3x^2+x^4} dx + \int \frac{2+x^2}{2+3x^2+x^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{2+3x+x^2} dx, x, x^2 \right) + \int \frac{1}{1+x^2} dx \\
&= \tan^{-1}(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{2+x} dx, x, x^2 \right) \\
&= \tan^{-1}(x) + \frac{1}{2} \log(2+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 2) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4), x]

[Out] ArcTan[x] + Log[2 + x^2]/2

fricas [A] time = 0.83, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2), x, algorithm="fricas")

[Out] arctan(x) + 1/2*log(x^2 + 2)

giac [A] time = 0.31, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2), x, algorithm="giac")

[Out] arctan(x) + 1/2*log(x^2 + 2)

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\arctan(x) + \frac{\ln(x^2 + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+2)/(x^4+3*x^2+2), x)

[Out] arctan(x)+1/2*ln(x^2+2)

maxima [A] time = 2.16, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="maxima")

[Out] arctan(x) + 1/2*log(x^2 + 2)

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln(x^2 + 2)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 + x^3 + 2)/(3*x^2 + x^4 + 2),x)

[Out] log(x^2 + 2)/2 + atan(x)

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$\frac{\log(x^2 + 2)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)

[Out] log(x**2 + 2)/2 + atan(x)

$$3.271 \quad \int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx$$

Optimal. Leaf size=35

$$-\frac{1}{(x^2+2)^2} + \frac{1}{2} \log(x^2+2) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] $-1/(x^2+2)^2+1/2*\ln(x^2+2)-1/2*\arctan(1/2*x*\sqrt{2})*\sqrt{2}$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1814, 1586, 635, 203, 260}

$$-\frac{1}{(x^2+2)^2} + \frac{1}{2} \log(x^2+2) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]

[Out] $-(2 + x^2)^{-2} - \text{ArcTan}[x/\text{Sqrt}[2]]/\text{Sqrt}[2] + \text{Log}[2 + x^2]/2$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p+1))/(2*a*b*(p+1)), x] + Dist[1/(2*a*(p+1)), Int[(a + b*x^2)^(p+1)*ExpandToSum[2*a*(p+1)*Q + f*(2*p+3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{-4 + 8x - 4x^2 + 4x^3 - x^4 + x^5}{(2 + x^2)^3} dx &= -\frac{1}{(2 + x^2)^2} - \frac{1}{8} \int \frac{16 - 16x + 8x^2 - 8x^3}{(2 + x^2)^2} dx \\
&= -\frac{1}{(2 + x^2)^2} - \frac{1}{8} \int \frac{8 - 8x}{2 + x^2} dx \\
&= -\frac{1}{(2 + x^2)^2} - \int \frac{1}{2 + x^2} dx + \int \frac{x}{2 + x^2} dx \\
&= -\frac{1}{(2 + x^2)^2} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{2} \log(2 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$-\frac{1}{(x^2 + 2)^2} + \frac{1}{2} \log(x^2 + 2) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3, x]

[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2

fricas [A] time = 0.78, size = 55, normalized size = 1.57

$$-\frac{\sqrt{2}(x^4 + 4x^2 + 4) \arctan\left(\frac{1}{2}\sqrt{2}x\right) - (x^4 + 4x^2 + 4) \log(x^2 + 2) + 2}{2(x^4 + 4x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="fricas")

[Out] -1/2*(sqrt(2)*(x^4 + 4*x^2 + 4)*arctan(1/2*sqrt(2)*x) - (x^4 + 4*x^2 + 4)*log(x^2 + 2) + 2)/(x^4 + 4*x^2 + 4)

giac [A] time = 0.33, size = 30, normalized size = 0.86

$$-\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - \frac{1}{(x^2 + 2)^2} + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="giac")

[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*log(x^2 + 2)

maple [A] time = 0.01, size = 31, normalized size = 0.89

$$-\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{2} + \frac{\ln(x^2 + 2)}{2} - \frac{1}{(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x)

[Out] $-1/(x^2+2)^2+1/2*\ln(x^2+2)-1/2*\arctan(1/2*x*2^{(1/2)})*2^{(1/2)}$

maxima [A] time = 2.22, size = 35, normalized size = 1.00

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{x^4+4x^2+4}+\frac{1}{2}\log(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="maxima")`

[Out] $-1/2*\sqrt{2}*\arctan(1/2*\sqrt{2}*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*\log(x^2 + 2)$

mupad [B] time = 2.12, size = 35, normalized size = 1.00

$$\frac{\ln(x^2+2)}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{1}{x^4+4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((8*x - 4*x^2 + 4*x^3 - x^4 + x^5 - 4)/(x^2 + 2)^3,x)`

[Out] $\log(x^2 + 2)/2 - (2^{(1/2)}*\operatorname{atan}((2^{(1/2)}*x)/2))/2 - 1/(4*x^2 + x^4 + 4)$

sympy [A] time = 0.15, size = 36, normalized size = 1.03

$$\frac{\log(x^2+2)}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{1}{x^4+4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)`

[Out] $\log(x**2 + 2)/2 - \sqrt{2}*\operatorname{atan}(\sqrt{2}*x/2)/2 - 1/(x**4 + 4*x**2 + 4)$

$$3.272 \quad \int \frac{-1-3x+x^2}{-2x+x^2+x^3} dx$$

Optimal. Leaf size=23

$$-\log(1-x) + \frac{\log(x)}{2} + \frac{3}{2}\log(x+2)$$

[Out] $-\ln(1-x)+1/2*\ln(x)+3/2*\ln(2+x)$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1594, 1628}

$$-\log(1-x) + \frac{\log(x)}{2} + \frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3), x]

[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1-3x+x^2}{-2x+x^2+x^3} dx &= \int \frac{-1-3x+x^2}{x(-2+x+x^2)} dx \\ &= \int \left(\frac{1}{1-x} + \frac{1}{2x} + \frac{3}{2(2+x)} \right) dx \\ &= -\log(1-x) + \frac{\log(x)}{2} + \frac{3}{2}\log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\log(1-x) + \frac{\log(x)}{2} + \frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3), x]

[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2

fricas [A] time = 0.69, size = 17, normalized size = 0.74

$$\frac{3}{2}\log(x+2) - \log(x-1) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")

[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)

giac [A] time = 0.24, size = 20, normalized size = 0.87

$$\frac{3}{2} \log(|x + 2|) - \log(|x - 1|) + \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="giac")

[Out] 3/2*log(abs(x + 2)) - log(abs(x - 1)) + 1/2*log(abs(x))

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{\ln(x)}{2} - \ln(x - 1) + \frac{3 \ln(x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-3*x-1)/(x^3+x^2-2*x),x)

[Out] -ln(x-1)+3/2*ln(x+2)+1/2*ln(x)

maxima [A] time = 1.40, size = 17, normalized size = 0.74

$$\frac{3}{2} \log(x + 2) - \log(x - 1) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")

[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)

mupad [B] time = 2.19, size = 17, normalized size = 0.74

$$\frac{3 \ln(x + 2)}{2} - \ln(x - 1) + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x - x^2 + 1)/(x^2 - 2*x + x^3),x)

[Out] (3*log(x + 2))/2 - log(x - 1) + log(x)/2

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\frac{\log(x)}{2} - \log(x - 1) + \frac{3 \log(x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)

[Out] log(x)/2 - log(x - 1) + 3*log(x + 2)/2

$$3.273 \quad \int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx$$

Optimal. Leaf size=23

$$\frac{x^2}{2} - \frac{1}{2} \log(x^2 - 2x + 3) + \log(x)$$

[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1594, 1628, 628}

$$\frac{x^2}{2} - \frac{1}{2} \log(x^2 - 2x + 3) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3), x]

[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_)*((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx &= \int \frac{3-x+3x^2-2x^3+x^4}{x(3-2x+x^2)} dx \\ &= \int \left(\frac{1}{x} + x + \frac{1-x}{3-2x+x^2} \right) dx \\ &= \frac{x^2}{2} + \log(x) + \int \frac{1-x}{3-2x+x^2} dx \\ &= \frac{x^2}{2} + \log(x) - \frac{1}{2} \log(3-2x+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{2} \log(x^2 - 2x + 3) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3), x]

[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2

fricas [A] time = 0.92, size = 19, normalized size = 0.83

$$\frac{1}{2}x^2 - \frac{1}{2}\log(x^2 - 2x + 3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x), x, algorithm="fricas")

[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)

giac [A] time = 0.29, size = 20, normalized size = 0.87

$$\frac{1}{2}x^2 - \frac{1}{2}\log(x^2 - 2x + 3) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x), x, algorithm="giac")

[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(abs(x))

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{x^2}{2} + \ln(x) - \frac{\ln(x^2 - 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x), x)

[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)

maxima [A] time = 1.12, size = 19, normalized size = 0.83

$$\frac{1}{2}x^2 - \frac{1}{2}\log(x^2 - 2x + 3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x), x, algorithm="maxima")

[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)

mupad [B] time = 0.06, size = 19, normalized size = 0.83

$$\ln(x) - \frac{\ln(x^2 - 2x + 3)}{2} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 - x - 2*x^3 + x^4 + 3)/(3*x - 2*x^2 + x^3), x)

[Out] log(x) - log(x^2 - 2*x + 3)/2 + x^2/2

sympy [A] time = 0.11, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + \log(x) - \frac{\log(x^2 - 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x), x)

[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2

$$3.274 \quad \int \frac{-1+x+x^3}{(1+x^2)^2} dx$$

Optimal. Leaf size=29

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) - \frac{1}{2} \tan^{-1}(x)$$

[Out] $-1/2*x/(x^2+1)-1/2*\arctan(x)+1/2*\ln(x^2+1)$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1814, 635, 203, 260}

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) - \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 + x + x^3)/(1 + x^2)^2, x]$

[Out] $-x/(2*(1 + x^2)) - \text{ArcTan}[x]/2 + \text{Log}[1 + x^2]/2$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 260

$\text{Int}[(x_)^{(m_)} / ((a_ + (b_)*(x_)^{(n_)}), x_Symbol] :> \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 635

$\text{Int}[(d_ + (e_)*(x_)) / ((a_ + (c_)*(x_)^2), x_Symbol] :> \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ !\text{NiceSqrtQ}[-(a*c)]$

Rule 1814

$\text{Int}[(\text{Pq}_) * ((a_ + (b_)*(x_)^2)^{(p_)}), x_Symbol] :> \text{With}\{Q = \text{PolynomialQuotient}[\text{Pq}, a + b*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, a + b*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, a + b*x^2, x], x, 1]\}, \text{Simp}[(a*g - b*f*x)*(a + b*x^2)^{(p+1)} / (2*a*b*(p+1)), x] + \text{Dist}[1/(2*a*(p+1)), \text{Int}[(a + b*x^2)^{(p+1)} * \text{ExpandToSum}[2*a*(p+1)*Q + f*(2*p+3), x], x], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ \text{LtQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{-1+x+x^3}{(1+x^2)^2} dx &= -\frac{x}{2(1+x^2)} - \frac{1}{2} \int \frac{1-2x}{1+x^2} dx \\ &= -\frac{x}{2(1+x^2)} - \frac{1}{2} \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\ &= -\frac{x}{2(1+x^2)} - \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{1}{2} \left(-\frac{x}{x^2+1} + \log(x^2+1) - \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x + x^3)/(1 + x^2)^2, x]

[Out] -(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2

fricas [A] time = 0.89, size = 32, normalized size = 1.10

$$-\frac{(x^2+1)\arctan(x) - (x^2+1)\log(x^2+1) + x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x-1)/(x^2+1)^2, x, algorithm="fricas")

[Out] -1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + x)/(x^2 + 1)

giac [A] time = 0.37, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2+1)} - \frac{1}{2} \arctan(x) + \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x-1)/(x^2+1)^2, x, algorithm="giac")

[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)

maple [A] time = 0.01, size = 24, normalized size = 0.83

$$-\frac{x}{2(x^2+1)} - \frac{\arctan(x)}{2} + \frac{\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x-1)/(x^2+1)^2, x)

[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)

maxima [A] time = 2.33, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2+1)} - \frac{1}{2} \arctan(x) + \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x-1)/(x^2+1)^2, x, algorithm="maxima")

[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)

mupad [B] time = 2.13, size = 25, normalized size = 0.86

$$\frac{\ln(x^2+1)}{2} - \frac{\operatorname{atan}(x)}{2} - \frac{x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^3 - 1)/(x^2 + 1)^2, x)`

[Out] `log(x^2 + 1)/2 - atan(x)/2 - x/(2*(x^2 + 1))`

sympy [A] time = 0.12, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2 + 2} + \frac{\log(x^2 + 1)}{2} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x-1)/(x**2+1)**2, x)`

[Out] `-x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2`

$$3.275 \quad \int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} dx$$

Optimal. Leaf size=44

$$\log(x^2 - x + 1) - \frac{3}{x+1} + \log(x) - 2\log(x+1) - \frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] -3/(1+x)+ln(x)-2*ln(1+x)+ln(x^2-x+1)-2/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.24, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 6725, 634, 618, 204, 628}

$$\log(x^2 - x + 1) - \frac{3}{x+1} + \log(x) - 2\log(x+1) - \frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]

[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x] + Log[1 - x + x^2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpan[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1 + 2x - x^2 + 8x^3 + x^4}{(x + x^2)(1 + x^3)} dx &= \int \frac{1 + 2x - x^2 + 8x^3 + x^4}{x(1 + x)(1 + x^3)} dx \\
&= \int \left(\frac{1}{x} + \frac{3}{(1 + x)^2} - \frac{2}{1 + x} + \frac{2x}{1 - x + x^2} \right) dx \\
&= -\frac{3}{1 + x} + \log(x) - 2 \log(1 + x) + 2 \int \frac{x}{1 - x + x^2} dx \\
&= -\frac{3}{1 + x} + \log(x) - 2 \log(1 + x) + \int \frac{1}{1 - x + x^2} dx + \int \frac{-1 + 2x}{1 - x + x^2} dx \\
&= -\frac{3}{1 + x} + \log(x) - 2 \log(1 + x) + \log(1 - x + x^2) - 2 \operatorname{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, \right. \\
&\quad \left. -\frac{3}{1 + x} - \frac{2 \tan^{-1} \left(\frac{1 - 2x}{\sqrt{3}} \right)}{\sqrt{3}} + \log(x) - 2 \log(1 + x) + \log(1 - x + x^2) \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$\log(x^2 - x + 1) - \frac{3}{x + 1} + \log(x) - 2 \log(x + 1) + \frac{2 \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)), x]
```

```
[Out] -3/(1 + x) + (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
+ Log[1 - x + x^2]
```

fricas [A] time = 0.88, size = 58, normalized size = 1.32

$$\frac{2 \sqrt{3} (x + 1) \arctan \left(\frac{1}{3} \sqrt{3} (2x - 1) \right) + 3 (x + 1) \log(x^2 - x + 1) - 6 (x + 1) \log(x + 1) + 3 (x + 1) \log(x) - 9}{3 (x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1), x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*(x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x + 1)*log(x^2 -
x + 1) - 6*(x + 1)*log(x + 1) + 3*(x + 1)*log(x) - 9)/(x + 1)
```

giac [A] time = 0.36, size = 43, normalized size = 0.98

$$\frac{2}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x - 1) \right) - \frac{3}{x + 1} + \log(x^2 - x + 1) - 2 \log(|x + 1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1), x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.01, size = 42, normalized size = 0.95

$$\frac{2\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} + \ln(x) - 2 \ln(x+1) + \ln(x^2 - x + 1) - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x)

[Out] ln(x^2-x+1)+2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-3/(x+1)-2*ln(x+1)+ln(x)

maxima [A] time = 2.25, size = 41, normalized size = 0.93

$$\frac{2}{3}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{3}{x+1} + \log(x^2 - x + 1) - 2 \log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="maxima")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) - 2*log(x + 1) + log(x)

mupad [B] time = 0.13, size = 55, normalized size = 1.25

$$\ln(x) - 2 \ln(x + 1) - \frac{3}{x + 1} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) \left(-1 + \frac{\sqrt{3} 1i}{3}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(1 + \frac{\sqrt{3} 1i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - x^2 + 8*x^3 + x^4 + 1)/((x^3 + 1)*(x + x^2)),x)

[Out] log(x) - 2*log(x + 1) - 3/(x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/3 - 1) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/3 + 1)

sympy [A] time = 0.21, size = 49, normalized size = 1.11

$$\log(x) - 2 \log(x + 1) + \log(x^2 - x + 1) + \frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)

[Out] log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - 3/(x + 1)

$$3.276 \quad \int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$$

Optimal. Leaf size=46

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{5 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2))*5^(1/2)

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{5 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]

[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 6725

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{15 - 5x + x^2 + x^3}{(5 + x^2)(3 + 2x + x^2)} dx &= \int \left(-\frac{5}{5 + x^2} + \frac{6 + x}{3 + 2x + x^2} \right) dx \\
&= -\left(5 \int \frac{1}{5 + x^2} dx \right) + \int \frac{6 + x}{3 + 2x + x^2} dx \\
&= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{2} \int \frac{2 + 2x}{3 + 2x + x^2} dx + 5 \int \frac{1}{3 + 2x + x^2} dx \\
&= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{2} \log(3 + 2x + x^2) - 10 \operatorname{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2 + 2x \right) \\
&= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{5 \tan^{-1} \left(\frac{1+x}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{1}{2} \log(3 + 2x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{5 \tan^{-1} \left(\frac{x+1}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2
```

fricas [A] time = 0.88, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (x + 1) \right) - \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} x \right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)
```

giac [A] time = 0.30, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (x + 1) \right) - \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} x \right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)
```

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$-\sqrt{5} \arctan \left(\frac{\sqrt{5} x}{5} \right) + \frac{5\sqrt{2} \arctan \left(\frac{(2x+2)\sqrt{2}}{4} \right)}{2} + \frac{\ln(x^2 + 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)`

[Out] `-arctan(1/5*5^(1/2)*x)*5^(1/2)+1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))`

maxima [A] time = 2.29, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log(x^2+2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x,algorithm="maxima")`

[Out] `5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x+1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2+2*x+3)`

mupad [B] time = 0.16, size = 88, normalized size = 1.91

$$\frac{\ln(x+1-\sqrt{2}i)}{2} + \frac{\ln(x+1+\sqrt{2}i)}{2} + \sqrt{5}\operatorname{atan}\left(\frac{2000\sqrt{5}}{2000x+1120} - \frac{224\sqrt{5}x}{2000x+1120}\right) - \frac{\sqrt{2}\ln(x+1-\sqrt{2}i)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-5*x+x^3+15)/((x^2+5)*(2*x+x^2+3)),x)`

[Out] `log(x-2^(1/2)*1i+1)/2 + log(x+2^(1/2)*1i+1)/2 + 5^(1/2)*atan((2000*5^(1/2))/(2000*x+1120) - (224*5^(1/2)*x)/(2000*x+1120)) - (2^(1/2)*log(x-2^(1/2)*1i+1)*5i)/4 + (2^(1/2)*log(x+2^(1/2)*1i+1)*5i)/4`

sympy [A] time = 0.21, size = 51, normalized size = 1.11

$$\frac{\log(x^2+2x+3)}{2} - \sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2} + \frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)`

[Out] `log(x**2+2*x+3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/2`

$$3.277 \quad \int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2} dx$$

Optimal. Leaf size=33

$$-\frac{3}{x^2+1} + \frac{1}{x^2+x+2} + \log(x^2+1) - \log(x^2+x+2)$$

[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)

Rubi [A] time = 0.17, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6742, 261, 260, 629, 628}

$$-\frac{3}{x^2+1} + \frac{1}{x^2+x+2} + \log(x^2+1) - \log(x^2+x+2)$$

Antiderivative was successfully verified.

[In] Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 + x + x^2)^2), x]

[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-3 + 25x + 23x^2 + 32x^3 + 15x^4 + 7x^5 + x^6}{(1+x^2)^2(2+x+x^2)^2} dx &= \int \left(\frac{6x}{(1+x^2)^2} + \frac{2x}{1+x^2} + \frac{-1-2x}{(2+x+x^2)^2} + \frac{-1-2x}{2+x+x^2} \right) dx \\ &= 2 \int \frac{x}{1+x^2} dx + 6 \int \frac{x}{(1+x^2)^2} dx + \int \frac{-1-2x}{(2+x+x^2)^2} dx \\ &= -\frac{3}{1+x^2} + \frac{1}{2+x+x^2} + \log(1+x^2) - \log(2+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3}{x^2+1} + \frac{1}{x^2+x+2} + \log(x^2+1) - \log(x^2+x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 + x + x^2)^2), x]

[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]

fricas [B] time = 0.85, size = 72, normalized size = 2.18

$$\frac{2x^2 + (x^4 + x^3 + 3x^2 + x + 2) \log(x^2 + x + 2) - (x^4 + x^3 + 3x^2 + x + 2) \log(x^2 + 1) + 3x + 5}{x^4 + x^3 + 3x^2 + x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2, x, algorithm="fricas")

[Out] -(2*x^2 + (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + x + 2) - (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + 1) + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2)

giac [A] time = 0.30, size = 44, normalized size = 1.33

$$-\frac{2x^2 + 3x + 5}{x^4 + x^3 + 3x^2 + x + 2} - \log(x^2 + x + 2) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2, x, algorithm="giac")

[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2 + 1)

maple [A] time = 0.01, size = 34, normalized size = 1.03

$$\ln(x^2 + 1) - \ln(x^2 + x + 2) - \frac{3}{x^2 + 1} + \frac{1}{x^2 + x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2, x)

[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)

maxima [A] time = 1.16, size = 44, normalized size = 1.33

$$-\frac{2x^2 + 3x + 5}{x^4 + x^3 + 3x^2 + x + 2} - \log(x^2 + x + 2) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="maxima")

[Out] $-(2x^2 + 3x + 5)/(x^4 + x^3 + 3x^2 + x + 2) - \log(x^2 + x + 2) + \log(x^2 + 1)$

mupad [B] time = 2.16, size = 56, normalized size = 1.70

$$-\frac{2x^2 + 3x + 5}{x^4 + x^3 + 3x^2 + x + 2} + \operatorname{atan}\left(\frac{\frac{x^{224i}}{11} + \frac{224i}{11}}{44x^2 + 16x + 60} - \frac{3}{11}i\right) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6 - 3)/((x^2 + 1)^2*(x + x^2 + 2)^2),x)

[Out] $\operatorname{atan}\left(\frac{(x^{224i})/11 + 224i/11}{(16x + 44x^2 + 60) - 3i/11}\right) 2i - (3x + 2x^2 + 5)/(x + 3x^2 + x^3 + x^4 + 2)$

sympy [A] time = 0.19, size = 41, normalized size = 1.24

$$\frac{-2x^2 - 3x - 5}{x^4 + x^3 + 3x^2 + x + 2} + \log(x^2 + 1) - \log(x^2 + x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x+2)**2,x)

[Out] $(-2x^2 - 3x - 5)/(x^4 + x^3 + 3x^2 + x + 2) + \log(x^2 + 1) - \log(x^2 + x + 2)$

$$3.278 \quad \int \frac{1}{(1+x^2)(4+x^2)} dx$$

Optimal. Leaf size=17

$$\frac{1}{3} \tan^{-1}(x) - \frac{1}{6} \tan^{-1}\left(\frac{x}{2}\right)$$

[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {391, 203}

$$\frac{1}{3} \tan^{-1}(x) - \frac{1}{6} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^2)*(4 + x^2)),x]

[Out] -ArcTan[x/2]/6 + ArcTan[x]/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^2)(4+x^2)} dx &= \frac{1}{3} \int \frac{1}{1+x^2} dx - \frac{1}{3} \int \frac{1}{4+x^2} dx \\ &= -\frac{1}{6} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6} \tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^2)*(4 + x^2)),x]

[Out] ArcTan[2/x]/6 + ArcTan[x]/3

fricas [A] time = 0.64, size = 11, normalized size = 0.65

$$-\frac{1}{6} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="fricas")

[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)

giac [A] time = 0.29, size = 11, normalized size = 0.65

$$-\frac{1}{6} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="giac")

[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)

maple [A] time = 0.01, size = 12, normalized size = 0.71

$$\frac{\arctan(x)}{3} - \frac{\arctan\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)/(x^2+4),x)

[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)

maxima [A] time = 2.20, size = 11, normalized size = 0.65

$$-\frac{1}{6} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="maxima")

[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)

mupad [B] time = 0.03, size = 11, normalized size = 0.65

$$\frac{\operatorname{atan}(x)}{3} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 1)*(x^2 + 4)),x)

[Out] atan(x)/3 - atan(x/2)/6

sympy [A] time = 0.14, size = 10, normalized size = 0.59

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)/(x**2+4),x)

[Out] -atan(x/2)/6 + atan(x)/3

$$3.279 \quad \int \frac{a+bx^3}{1+x^2} dx$$

Optimal. Leaf size=24

$$a \tan^{-1}(x) + \frac{bx^2}{2} - \frac{1}{2}b \log(x^2 + 1)$$

[Out] $1/2*b*x^2+a*\arctan(x)-1/2*b*\ln(x^2+1)$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1810, 635, 203, 260}

$$a \tan^{-1}(x) + \frac{bx^2}{2} - \frac{1}{2}b \log(x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)/(1 + x^2),x]

[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^3}{1+x^2} dx &= \int \left(bx + \frac{a-bx}{1+x^2} \right) dx \\ &= \frac{bx^2}{2} + \int \frac{a-bx}{1+x^2} dx \\ &= \frac{bx^2}{2} + a \int \frac{1}{1+x^2} dx - b \int \frac{x}{1+x^2} dx \\ &= \frac{bx^2}{2} + a \tan^{-1}(x) - \frac{1}{2}b \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$a \tan^{-1}(x) + \frac{1}{2}b(x^2 - \log(x^2 + 1))$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)/(1 + x^2), x]

[Out] a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2

fricas [A] time = 0.76, size = 20, normalized size = 0.83

$$\frac{1}{2}bx^2 + a \arctan(x) - \frac{1}{2}b \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="fricas")

[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)

giac [A] time = 0.29, size = 20, normalized size = 0.83

$$\frac{1}{2}bx^2 + a \arctan(x) - \frac{1}{2}b \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="giac")

[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$\frac{bx^2}{2} + a \arctan(x) - \frac{b \ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)/(x^2+1),x)

[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)

maxima [A] time = 2.25, size = 20, normalized size = 0.83

$$\frac{1}{2}bx^2 + a \arctan(x) - \frac{1}{2}b \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="maxima")

[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)

mupad [B] time = 2.13, size = 20, normalized size = 0.83

$$\frac{bx^2}{2} - \frac{b \ln(x^2 + 1)}{2} + a \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)/(x^2 + 1),x)

[Out] (b*x^2)/2 - (b*log(x^2 + 1))/2 + a*atan(x)

sympy [C] time = 0.17, size = 34, normalized size = 1.42

$$\frac{bx^2}{2} + \left(-\frac{ia}{2} - \frac{b}{2}\right) \log(x - i) + \left(\frac{ia}{2} - \frac{b}{2}\right) \log(x + i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(x**2+1),x)
```

```
[Out] b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)
```

$$3.280 \quad \int \frac{x+x^2}{(4+x)(-4+x^2)} dx$$

Optimal. Leaf size=15

$$\log(x+4) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{2}\right)$$

[Out] -1/2*arctanh(1/2*x)+ln(4+x)

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1593, 1629, 207}

$$\log(x+4) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(x + x^2)/((4 + x)*(-4 + x^2)), x]

[Out] -ArcTanh[x/2]/2 + Log[4 + x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{x+x^2}{(4+x)(-4+x^2)} dx &= \int \frac{x(1+x)}{(4+x)(-4+x^2)} dx \\ &= \int \left(\frac{1}{4+x} + \frac{1}{-4+x^2} \right) dx \\ &= \log(4+x) + \int \frac{1}{-4+x^2} dx \\ &= -\frac{1}{2} \tanh^{-1}\left(\frac{x}{2}\right) + \log(4+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.53

$$\frac{1}{4} \log(2-x) - \frac{1}{4} \log(x+2) + \log(x+4)$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^2)/((4 + x)*(-4 + x^2)), x]

[Out] Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]

fricas [A] time = 0.96, size = 17, normalized size = 1.13

$$\log(x + 4) - \frac{1}{4} \log(x + 2) + \frac{1}{4} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(4+x)/(x^2-4), x, algorithm="fricas")

[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)

giac [A] time = 0.32, size = 20, normalized size = 1.33

$$\log(|x + 4|) - \frac{1}{4} \log(|x + 2|) + \frac{1}{4} \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(4+x)/(x^2-4), x, algorithm="giac")

[Out] log(abs(x + 4)) - 1/4*log(abs(x + 2)) + 1/4*log(abs(x - 2))

maple [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{\ln(x - 2)}{4} - \frac{\ln(x + 2)}{4} + \ln(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x)/(4+x)/(x^2-4), x)

[Out] ln(4+x)+1/4*ln(x-2)-1/4*ln(x+2)

maxima [A] time = 2.31, size = 17, normalized size = 1.13

$$\log(x + 4) - \frac{1}{4} \log(x + 2) + \frac{1}{4} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(4+x)/(x^2-4), x, algorithm="maxima")

[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)

mupad [B] time = 0.06, size = 19, normalized size = 1.27

$$\ln(x + 4) + \frac{\operatorname{atanh}\left(\frac{90}{7(21x+48)} - \frac{8}{7}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2)/((x^2 - 4)*(x + 4)), x)

[Out] log(x + 4) + atanh(90/(7*(21*x + 48)) - 8/7)/2

sympy [A] time = 0.14, size = 17, normalized size = 1.13

$$\frac{\log(x - 2)}{4} - \frac{\log(x + 2)}{4} + \log(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x)/(4+x)/(x**2-4), x)

[Out] log(x - 2)/4 - log(x + 2)/4 + log(x + 4)

$$3.281 \quad \int \frac{4+x^2}{(1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=20

$$3 \tan^{-1}(x) - \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

[Out] 3*arctan(x)-arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {522, 203}

$$3 \tan^{-1}(x) - \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2)/((1 + x^2)*(2 + x^2)), x]

[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{4+x^2}{(1+x^2)(2+x^2)} dx &= -\left(2 \int \frac{1}{2+x^2} dx\right) + 3 \int \frac{1}{1+x^2} dx \\ &= 3 \tan^{-1}(x) - \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$3 \tan^{-1}(x) - \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)), x]

[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]

fricas [A] time = 0.87, size = 17, normalized size = 0.85

$$-\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 3 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="fricas")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)

giac [A] time = 0.38, size = 17, normalized size = 0.85

$$-\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 3 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="giac")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)

maple [A] time = 0.01, size = 18, normalized size = 0.90

$$3 \arctan(x) - \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+4)/(x^2+1)/(x^2+2),x)

[Out] 3*arctan(x)-2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.40, size = 17, normalized size = 0.85

$$-\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 3 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="maxima")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)

mupad [B] time = 0.05, size = 17, normalized size = 0.85

$$3 \operatorname{atan}(x) - \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 4)/((x^2 + 1)*(x^2 + 2)),x)

[Out] 3*atan(x) - 2^(1/2)*atan((2^(1/2)*x)/2)

sympy [A] time = 0.15, size = 19, normalized size = 0.95

$$3 \operatorname{atan}(x) - \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+4)/(x**2+1)/(x**2+2),x)

[Out] 3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)

$$3.282 \quad \int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)} dx$$

Optimal. Leaf size=37

$$\frac{3}{4} \log(x^2 + 1) + x + \frac{5}{2(1-x)} + \frac{1}{2} \log(1-x) + 2 \tan^{-1}(x)$$

[Out] 5/2/(1-x)+x+2*arctan(x)+1/2*ln(1-x)+3/4*ln(x^2+1)

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1629, 635, 203, 260}

$$\frac{3}{4} \log(x^2 + 1) + x + \frac{5}{2(1-x)} + \frac{1}{2} \log(1-x) + 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)), x]

[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)} dx &= \int \left(1 + \frac{5}{2(-1+x)^2} + \frac{1}{2(-1+x)} + \frac{4+3x}{2(1+x^2)} \right) dx \\ &= \frac{5}{2(1-x)} + x + \frac{1}{2} \log(1-x) + \frac{1}{2} \int \frac{4+3x}{1+x^2} dx \\ &= \frac{5}{2(1-x)} + x + \frac{1}{2} \log(1-x) + \frac{3}{2} \int \frac{x}{1+x^2} dx + 2 \int \frac{1}{1+x^2} dx \\ &= \frac{5}{2(1-x)} + x + 2 \tan^{-1}(x) + \frac{1}{2} \log(1-x) + \frac{3}{4} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.89

$$\frac{3}{4} \log(x^2 + 1) + x + \frac{5}{2 - 2x} + \frac{1}{2} \log(x - 1) + 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]

[Out] 5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/4

fricas [A] time = 0.51, size = 44, normalized size = 1.19

$$\frac{4x^2 + 8(x - 1) \arctan(x) + 3(x - 1) \log(x^2 + 1) + 2(x - 1) \log(x - 1) - 4x - 10}{4(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="fricas")

[Out] 1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) - 4*x - 10)/(x - 1)

giac [B] time = 0.29, size = 60, normalized size = 1.62

$$\frac{1}{2} \pi - 2\pi \left[\frac{\pi + 4 \arctan(x)}{4\pi} + \frac{1}{2} \right] + x - \frac{5}{2(x-1)} + 2 \arctan(x) + \frac{3}{4} \log\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right) + 2 \log(|x-1|) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="giac")

[Out] 1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*log(abs(x - 1)) - 1

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$x + 2 \arctan(x) + \frac{\ln(x - 1)}{2} + \frac{3 \ln(x^2 + 1)}{4} - \frac{5}{2(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2-4*x+5)/(x-1)^2/(x^2+1),x)

[Out] x-5/2/(x-1)+1/2*ln(x-1)+3/4*ln(x^2+1)+2*arctan(x)

maxima [A] time = 2.22, size = 27, normalized size = 0.73

$$x - \frac{5}{2(x-1)} + 2 \arctan(x) + \frac{3}{4} \log(x^2 + 1) + \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="maxima")

[Out] x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)

mupad [B] time = 2.14, size = 35, normalized size = 0.95

$$x + \frac{\ln(x - 1)}{2} - \frac{5}{2(x - 1)} + \ln(x - i) \left(\frac{3}{4} - i \right) + \ln(x + i) \left(\frac{3}{4} + i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2 - 4*x + x^4 + 5)/((x^2 + 1)*(x - 1)^2), x)`

[Out] `x + log(x - 1)/2 + log(x - 1i)*(3/4 - 1i) + log(x + 1i)*(3/4 + 1i) - 5/(2*(x - 1))`

sympy [A] time = 0.16, size = 29, normalized size = 0.78

$$x + \frac{\log(x-1)}{2} + \frac{3 \log(x^2+1)}{4} + 2 \operatorname{atan}(x) - \frac{5}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+3*x**2-4*x+5)/((-1+x)**2/(x**2+1)), x)`

[Out] `x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)`

$$3.283 \quad \int \frac{1+x^4}{2+x^2} dx$$

Optimal. Leaf size=26

$$\frac{x^3}{3} - 2x + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] $-2*x+1/3*x^3+5/2*\arctan(1/2*x*2^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1154, 203}

$$\frac{x^3}{3} - 2x + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(2 + x^2), x]

[Out] $-2*x + x^3/3 + (5*\text{ArcTan}[x/\text{Sqrt}[2]])/\text{Sqrt}[2]$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{2+x^2} dx &= \int \left(-2 + x^2 + \frac{5}{2+x^2} \right) dx \\ &= -2x + \frac{x^3}{3} + 5 \int \frac{1}{2+x^2} dx \\ &= -2x + \frac{x^3}{3} + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{x^3}{3} - 2x + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(2 + x^2), x]

[Out] $-2*x + x^3/3 + (5*\text{ArcTan}[x/\text{Sqrt}[2]])/\text{Sqrt}[2]$

fricas [A] time = 0.60, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^2+2),x, algorithm="fricas")

[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x

giac [A] time = 0.31, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^2+2),x, algorithm="giac")

[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x

maple [A] time = 0.00, size = 22, normalized size = 0.85

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^2+2),x)

[Out] -2*x+1/3*x^3+5/2*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.22, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^2+2),x, algorithm="maxima")

[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x

mupad [B] time = 0.03, size = 21, normalized size = 0.81

$$\frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - 2x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 1)/(x^2 + 2),x)

[Out] (5*2^(1/2)*atan((2^(1/2)*x)/2))/2 - 2*x + x^3/3

sympy [A] time = 0.09, size = 26, normalized size = 1.00

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+1)/(x**2+2),x)

[Out] x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2

$$3.284 \quad \int \frac{2+2x+x^4}{x^4+x^5} dx$$

Optimal. Leaf size=12

$$\log(x+1) - \frac{2}{3x^3}$$

[Out] $-2/3/x^3+\ln(1+x)$

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1593, 1620}

$$\log(x+1) - \frac{2}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(2 + 2*x + x^4)/(x^4 + x^5), x]

[Out] $-2/(3*x^3) + \text{Log}[1 + x]$

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{2+2x+x^4}{x^4+x^5} dx &= \int \frac{2+2x+x^4}{x^4(1+x)} dx \\ &= \int \left(\frac{2}{x^4} + \frac{1}{1+x} \right) dx \\ &= -\frac{2}{3x^3} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(x+1) - \frac{2}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 2*x + x^4)/(x^4 + x^5), x]

[Out] $-2/(3*x^3) + \text{Log}[1 + x]$

fricas [A] time = 0.70, size = 16, normalized size = 1.33

$$\frac{3x^3 \log(x+1) - 2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="fricas")

[Out] 1/3*(3*x^3*log(x + 1) - 2)/x^3

giac [A] time = 0.29, size = 11, normalized size = 0.92

$$-\frac{2}{3x^3} + \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="giac")

[Out] -2/3/x^3 + log(abs(x + 1))

maple [A] time = 0.01, size = 11, normalized size = 0.92

$$\ln(x + 1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+2*x+2)/(x^5+x^4),x)

[Out] -2/3/x^3+ln(x+1)

maxima [A] time = 1.11, size = 10, normalized size = 0.83

$$-\frac{2}{3x^3} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="maxima")

[Out] -2/3/x^3 + log(x + 1)

mupad [B] time = 2.12, size = 10, normalized size = 0.83

$$\ln(x + 1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^4 + 2)/(x^4 + x^5),x)

[Out] log(x + 1) - 2/(3*x^3)

sympy [A] time = 0.09, size = 10, normalized size = 0.83

$$\log(x + 1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+2*x+2)/(x**5+x**4),x)

[Out] log(x + 1) - 2/(3*x**3)

$$3.285 \quad \int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} dx$$

Optimal. Leaf size=21

$$2 \log(1-x) - \log(2-x) + \log(x+1)$$

[Out] 2*ln(1-x)-ln(2-x)+ln(1+x)

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2074}

$$2 \log(1-x) - \log(2-x) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3), x]

[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} dx &= \int \left(\frac{1}{2-x} + \frac{2}{-1+x} + \frac{1}{1+x} \right) dx \\ &= 2 \log(1-x) - \log(2-x) + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$2 \log(1-x) - \log(2-x) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3), x]

[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]

fricas [A] time = 0.79, size = 17, normalized size = 0.81

$$\log(x+1) + 2 \log(x-1) - \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2), x, algorithm="fricas")

[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)

giac [A] time = 0.28, size = 20, normalized size = 0.95

$$\log(|x+1|) + 2 \log(|x-1|) - \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2), x, algorithm="giac")

[Out] $\log(\text{abs}(x + 1)) + 2*\log(\text{abs}(x - 1)) - \log(\text{abs}(x - 2))$

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\ln(x - 2) + 2 \ln(x - 1) + \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2*x^2-5*x-1)/(x^3-2*x^2-x+2), x)$

[Out] $2*\ln(x-1)-\ln(x-2)+\ln(x+1)$

maxima [A] time = 1.09, size = 17, normalized size = 0.81

$$\log(x + 1) + 2 \log(x - 1) - \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2*x^2-5*x-1)/(x^3-2*x^2-x+2), x, \text{algorithm}="maxima")$

[Out] $\log(x + 1) + 2*\log(x - 1) - \log(x - 2)$

mupad [B] time = 2.14, size = 21, normalized size = 1.00

$$2 \ln(x - 1) - 2 \operatorname{atanh}\left(\frac{144}{11(22x - 50)} + \frac{13}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((5*x - 2*x^2 + 1)/(x + 2*x^2 - x^3 - 2), x)$

[Out] $2*\log(x - 1) - 2*\operatorname{atanh}(144/(11*(22*x - 50)) + 13/11)$

sympy [A] time = 0.13, size = 15, normalized size = 0.71

$$-\log(x - 2) + 2 \log(x - 1) + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2*x**2-5*x-1)/(x**3-2*x**2-x+2), x)$

[Out] $-\log(x - 2) + 2*\log(x - 1) + \log(x + 1)$

$$3.286 \quad \int \frac{2+x+x^3}{1+2x^2+x^4} dx$$

Optimal. Leaf size=22

$$\frac{x}{x^2+1} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

[Out] x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {28, 1814, 635, 203, 260}

$$\frac{x}{x^2+1} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2 + x + x^3)/(1 + 2*x^2 + x^4), x]

[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{2+x+x^3}{1+2x^2+x^4} dx &= \int \frac{2+x+x^3}{(1+x^2)^2} dx \\
&= \frac{x}{1+x^2} - \frac{1}{2} \int \frac{-2-2x}{1+x^2} dx \\
&= \frac{x}{1+x^2} + \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\
&= \frac{x}{1+x^2} + \tan^{-1}(x) + \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x}{x^2+1} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4), x]

[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2

fricas [A] time = 0.72, size = 34, normalized size = 1.55

$$\frac{2(x^2+1)\arctan(x) + (x^2+1)\log(x^2+1) + 2x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+2)/(x^4+2*x^2+1), x, algorithm="fricas")

[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 + 1)

giac [A] time = 0.28, size = 20, normalized size = 0.91

$$\frac{x}{x^2+1} + \arctan(x) + \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+2)/(x^4+2*x^2+1), x, algorithm="giac")

[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{x}{x^2+1} + \arctan(x) + \frac{\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x+2)/(x^4+2*x^2+1), x)

[Out] 1/(x^2+1)*x+arctan(x)+1/2*ln(x^2+1)

maxima [A] time = 2.23, size = 20, normalized size = 0.91

$$\frac{x}{x^2+1} + \arctan(x) + \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="maxima")

[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)

mupad [B] time = 2.12, size = 20, normalized size = 0.91

$$\frac{\ln(x^2 + 1)}{2} + \operatorname{atan}(x) + \frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^3 + 2)/(2*x^2 + x^4 + 1),x)

[Out] log(x^2 + 1)/2 + atan(x) + x/(x^2 + 1)

sympy [A] time = 0.11, size = 17, normalized size = 0.77

$$\frac{x}{x^2 + 1} + \frac{\log(x^2 + 1)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x+2)/(x**4+2*x**2+1),x)

[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)

$$3.287 \quad \int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} dx$$

Optimal. Leaf size=24

$$-\frac{1}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {28, 1814, 635, 203, 260}

$$-\frac{1}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4), x]

[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2

Rule 28

Int[(u_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1 + 2x + x^2 + x^3}{1 + 2x^2 + x^4} dx &= \int \frac{1 + 2x + x^2 + x^3}{(1 + x^2)^2} dx \\
&= -\frac{1}{2(1 + x^2)} - \frac{1}{2} \int \frac{-2 - 2x}{1 + x^2} dx \\
&= -\frac{1}{2(1 + x^2)} + \int \frac{1}{1 + x^2} dx + \int \frac{x}{1 + x^2} dx \\
&= -\frac{1}{2(1 + x^2)} + \tan^{-1}(x) + \frac{1}{2} \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{1}{2(x^2 + 1)} + \frac{1}{2} \log(x^2 + 1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4), x]

[Out] -1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2

fricas [A] time = 0.79, size = 32, normalized size = 1.33

$$\frac{2(x^2 + 1) \arctan(x) + (x^2 + 1) \log(x^2 + 1) - 1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1), x, algorithm="fricas")

[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1)

giac [A] time = 0.40, size = 20, normalized size = 0.83

$$-\frac{1}{2(x^2 + 1)} + \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1), x, algorithm="giac")

[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x) + \frac{\ln(x^2 + 1)}{2} - \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+2*x+1)/(x^4+2*x^2+1), x)

[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)

maxima [A] time = 2.18, size = 20, normalized size = 0.83

$$-\frac{1}{2(x^2 + 1)} + \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="maxima")

[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)

mupad [B] time = 0.03, size = 22, normalized size = 0.92

$$\frac{\ln(x^2 + 1)}{2} + \operatorname{atan}(x) - \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^2 + x^3 + 1)/(2*x^2 + x^4 + 1),x)

[Out] log(x^2 + 1)/2 + atan(x) - 1/(2*(x^2 + 1))

sympy [A] time = 0.12, size = 19, normalized size = 0.79

$$\frac{\log(x^2 + 1)}{2} + \operatorname{atan}(x) - \frac{1}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)

[Out] log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)

$$3.288 \quad \int \frac{3+4x}{(1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=36

$$2 \log(x^2 + 1) - 2 \log(x^2 + 2) + 3 \tan^{-1}(x) - \frac{3 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1010, 391, 203, 444, 36, 31}

$$2 \log(x^2 + 1) - 2 \log(x^2 + 2) + 3 \tan^{-1}(x) - \frac{3 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)), x]

[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^2]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 444

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 1010

Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h, Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}

, x]

Rubi steps

$$\begin{aligned}
\int \frac{3+4x}{(1+x^2)(2+x^2)} dx &= 3 \int \frac{1}{(1+x^2)(2+x^2)} dx + 4 \int \frac{x}{(1+x^2)(2+x^2)} dx \\
&= 2 \operatorname{Subst} \left(\int \frac{1}{(1+x)(2+x)} dx, x, x^2 \right) + 3 \int \frac{1}{1+x^2} dx - 3 \int \frac{1}{2+x^2} dx \\
&= 3 \tan^{-1}(x) - \frac{3 \tan^{-1} \left(\frac{x}{\sqrt{2}} \right)}{\sqrt{2}} + 2 \operatorname{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) - 2 \operatorname{Subst} \left(\int \frac{1}{2+x} dx, x, x^2 \right) \\
&= 3 \tan^{-1}(x) - \frac{3 \tan^{-1} \left(\frac{x}{\sqrt{2}} \right)}{\sqrt{2}} + 2 \log(1+x^2) - 2 \log(2+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$2 \log(x^2 + 1) - 2 \log(x^2 + 2) + 3 \tan^{-1}(x) - \frac{3 \tan^{-1} \left(\frac{x}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

`[In] Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)), x]``[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^2]`**fricas** [A] time = 0.80, size = 33, normalized size = 0.92

$$-\frac{3}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} x \right) + 3 \arctan(x) - 2 \log(x^2 + 2) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((3+4*x)/(x^2+1)/(x^2+2), x, algorithm="fricas")``[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x^2 + 1)`**giac** [A] time = 0.25, size = 33, normalized size = 0.92

$$-\frac{3}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} x \right) + 3 \arctan(x) - 2 \log(x^2 + 2) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((3+4*x)/(x^2+1)/(x^2+2), x, algorithm="giac")``[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x^2 + 1)`**maple** [A] time = 0.00, size = 34, normalized size = 0.94

$$3 \arctan(x) - \frac{3\sqrt{2} \arctan \left(\frac{\sqrt{2} x}{2} \right)}{2} + 2 \ln(x^2 + 1) - 2 \ln(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+4*x)/(x^2+1)/(x^2+2),x)

[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.06, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)-2\log(x^2+2)+2\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="maxima")

[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x^2 + 1)

mupad [B] time = 0.10, size = 56, normalized size = 1.56

$$\ln(x-i)\left(2-\frac{3}{2}i\right)+\ln(x+1i)\left(2+\frac{3}{2}i\right)+\ln\left(x-\sqrt{2}1i\right)\left(-2+\frac{\sqrt{2}3i}{4}\right)-\ln\left(x+\sqrt{2}1i\right)\left(2+\frac{\sqrt{2}3i}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 3)/((x^2 + 1)*(x^2 + 2)),x)

[Out] log(x - 1i)*(2 - 3i/2) + log(x + 1i)*(2 + 3i/2) + log(x - 2^(1/2)*1i)*((2^(1/2)*3i)/4 - 2) - log(x + 2^(1/2)*1i)*((2^(1/2)*3i)/4 + 2)

sympy [A] time = 0.19, size = 39, normalized size = 1.08

$$2\log(x^2+1)-2\log(x^2+2)+3\operatorname{atan}(x)-\frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(x**2+1)/(x**2+2),x)

[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sqrt(2)*x/2)/2

$$3.289 \quad \int \frac{2+x}{(1+x^2)(4+x^2)} dx$$

Optimal. Leaf size=37

$$\frac{1}{6} \log(x^2 + 1) - \frac{1}{6} \log(x^2 + 4) - \frac{1}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3} \tan^{-1}(x)$$

[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1010, 391, 203, 444, 36, 31}

$$\frac{1}{6} \log(x^2 + 1) - \frac{1}{6} \log(x^2 + 4) - \frac{1}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((1 + x^2)*(4 + x^2)), x]

[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 1010

Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.)*((d_) + (f_.)*(x_)^2)^(q_.), x_Symbol] := Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h, Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{(1+x^2)(4+x^2)} dx &= 2 \int \frac{1}{(1+x^2)(4+x^2)} dx + \int \frac{x}{(1+x^2)(4+x^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x)(4+x)} dx, x, x^2 \right) + \frac{2}{3} \int \frac{1}{1+x^2} dx - \frac{2}{3} \int \frac{1}{4+x^2} dx \\
&= -\frac{1}{3} \tan^{-1} \left(\frac{x}{2} \right) + \frac{2}{3} \tan^{-1}(x) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{4+x} dx, x, x^2 \right) \\
&= -\frac{1}{3} \tan^{-1} \left(\frac{x}{2} \right) + \frac{2}{3} \tan^{-1}(x) + \frac{1}{6} \log(1+x^2) - \frac{1}{6} \log(4+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{6} \log(x^2 + 1) - \frac{1}{6} \log(x^2 + 4) - \frac{1}{3} \tan^{-1} \left(\frac{x}{2} \right) + \frac{2}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/((1 + x^2)*(4 + x^2)), x]

[Out] -1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6

fricas [A] time = 0.75, size = 27, normalized size = 0.73

$$-\frac{1}{3} \arctan \left(\frac{1}{2} x \right) + \frac{2}{3} \arctan(x) - \frac{1}{6} \log(x^2 + 4) + \frac{1}{6} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+1)/(x^2+4), x, algorithm="fricas")

[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)

giac [A] time = 0.29, size = 27, normalized size = 0.73

$$-\frac{1}{3} \arctan \left(\frac{1}{2} x \right) + \frac{2}{3} \arctan(x) - \frac{1}{6} \log(x^2 + 4) + \frac{1}{6} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+1)/(x^2+4), x, algorithm="giac")

[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)

maple [A] time = 0.00, size = 28, normalized size = 0.76

$$\frac{2 \arctan(x)}{3} - \frac{\arctan \left(\frac{x}{2} \right)}{3} + \frac{\ln(x^2 + 1)}{6} - \frac{\ln(x^2 + 4)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+2)/(x^2+1)/(x^2+4), x)

[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)

maxima [A] time = 2.00, size = 27, normalized size = 0.73

$$-\frac{1}{3} \arctan \left(\frac{1}{2} x \right) + \frac{2}{3} \arctan(x) - \frac{1}{6} \log(x^2 + 4) + \frac{1}{6} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="maxima")

[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)

mupad [B] time = 2.14, size = 37, normalized size = 1.00

$$\ln(x - i) \left(\frac{1}{6} - \frac{1}{3}i \right) + \ln(x + i) \left(\frac{1}{6} + \frac{1}{3}i \right) + \ln(x - 2i) \left(-\frac{1}{6} + \frac{1}{6}i \right) + \ln(x + 2i) \left(-\frac{1}{6} - \frac{1}{6}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/((x^2 + 1)*(x^2 + 4)),x)

[Out] log(x - 1i)*(1/6 - 1i/3) + log(x + 1i)*(1/6 + 1i/3) - log(x - 2i)*(1/6 - 1i/6) - log(x + 2i)*(1/6 + 1i/6)

sympy [A] time = 0.17, size = 29, normalized size = 0.78

$$\frac{\log(x^2 + 1)}{6} - \frac{\log(x^2 + 4)}{6} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{2 \operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x**2+1)/(x**2+4),x)

[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3

$$3.290 \quad \int \frac{2-x+x^3}{-7-6x+x^2} dx$$

Optimal. Leaf size=29

$$\frac{x^2}{2} + 6x + \frac{169}{4} \log(7-x) - \frac{1}{4} \log(x+1)$$

[Out] 6*x+1/2*x^2+169/4*ln(7-x)-1/4*ln(1+x)

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1657, 632, 31}

$$\frac{x^2}{2} + 6x + \frac{169}{4} \log(7-x) - \frac{1}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(2 - x + x^3)/(-7 - 6*x + x^2), x]

[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{2-x+x^3}{-7-6x+x^2} dx &= \int \left(6+x + \frac{2(22+21x)}{-7-6x+x^2} \right) dx \\ &= 6x + \frac{x^2}{2} + 2 \int \frac{22+21x}{-7-6x+x^2} dx \\ &= 6x + \frac{x^2}{2} - \frac{1}{4} \int \frac{1}{1+x} dx + \frac{169}{4} \int \frac{1}{-7+x} dx \\ &= 6x + \frac{x^2}{2} + \frac{169}{4} \log(7-x) - \frac{1}{4} \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2} + 6x + \frac{169}{4} \log(7-x) - \frac{1}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x + x^3)/(-7 - 6*x + x^2), x]

[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4

fricas [A] time = 0.61, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(x+1) + \frac{169}{4}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x+2)/(x^2-6*x-7), x, algorithm="fricas")

[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)

giac [A] time = 0.37, size = 23, normalized size = 0.79

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(|x+1|) + \frac{169}{4}\log(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x+2)/(x^2-6*x-7), x, algorithm="giac")

[Out] 1/2*x^2 + 6*x - 1/4*log(abs(x + 1)) + 169/4*log(abs(x - 7))

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x - \frac{\ln(x+1)}{4} + \frac{169\ln(x-7)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-x+2)/(x^2-6*x-7), x)

[Out] 1/2*x^2+6*x-1/4*ln(x+1)+169/4*ln(x-7)

maxima [A] time = 1.08, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(x+1) + \frac{169}{4}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x+2)/(x^2-6*x-7), x, algorithm="maxima")

[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)

mupad [B] time = 2.21, size = 21, normalized size = 0.72

$$6x - \frac{\ln(x+1)}{4} + \frac{169\ln(x-7)}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 - x + 2)/(6*x - x^2 + 7), x)

[Out] 6*x - log(x + 1)/4 + (169*log(x - 7))/4 + x^2/2

sympy [A] time = 0.11, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{169\log(x-7)}{4} - \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-x+2)/(x**2-6*x-7), x)

[Out] x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4

$$3.291 \quad \int \frac{-1+x^5}{-1+x^2} dx$$

Optimal. Leaf size=19

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

[Out] 1/2*x^2+1/4*x^4+ln(1+x)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1810, 627, 31}

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^5)/(-1 + x^2), x]

[Out] x^2/2 + x^4/4 + Log[1 + x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 627

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^5}{-1+x^2} dx &= \int \left(x + x^3 - \frac{1-x}{-1+x^2} \right) dx \\ &= \frac{x^2}{2} + \frac{x^4}{4} - \int \frac{1-x}{-1+x^2} dx \\ &= \frac{x^2}{2} + \frac{x^4}{4} - \int \frac{1}{-1-x} dx \\ &= \frac{x^2}{2} + \frac{x^4}{4} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^5)/(-1 + x^2), x]

[Out] $x^2/2 + x^4/4 + \text{Log}[1 + x]$

fricas [A] time = 0.80, size = 15, normalized size = 0.79

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5-1)/(x^2-1),x, algorithm="fricas")`

[Out] $1/4*x^4 + 1/2*x^2 + \log(x + 1)$

giac [A] time = 0.27, size = 16, normalized size = 0.84

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5-1)/(x^2-1),x, algorithm="giac")`

[Out] $1/4*x^4 + 1/2*x^2 + \log(\text{abs}(x + 1))$

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^4}{4} + \frac{x^2}{2} + \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5-1)/(x^2-1),x)`

[Out] $1/2*x^2+1/4*x^4+\ln(x+1)$

maxima [A] time = 1.07, size = 15, normalized size = 0.79

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5-1)/(x^2-1),x, algorithm="maxima")`

[Out] $1/4*x^4 + 1/2*x^2 + \log(x + 1)$

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$\ln(x + 1) + \frac{x^2}{2} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5 - 1)/(x^2 - 1),x)`

[Out] $\log(x + 1) + x^2/2 + x^4/4$

sympy [A] time = 0.08, size = 14, normalized size = 0.74

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**5-1)/(x**2-1),x)`

[Out] $x**4/4 + x**2/2 + \log(x + 1)$

$$3.292 \quad \int \frac{5+2x-x^2+x^3}{1+x+x^2} dx$$

Optimal. Leaf size=41

$$\frac{x^2}{2} + \frac{3}{2} \log(x^2 + x + 1) - 2x + \frac{11 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-2*x+1/2*x^2+3/2*\ln(x^2+x+1)+11/3*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^2}{2} + \frac{3}{2} \log(x^2 + x + 1) - 2x + \frac{11 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2), x]

[Out] $-2*x + x^2/2 + (11*\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]])/\text{Sqrt}[3] + (3*\text{Log}[1 + x + x^2])/2$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{5 + 2x - x^2 + x^3}{1 + x + x^2} dx &= \int \left(-2 + x + \frac{7 + 3x}{1 + x + x^2} \right) dx \\
&= -2x + \frac{x^2}{2} + \int \frac{7 + 3x}{1 + x + x^2} dx \\
&= -2x + \frac{x^2}{2} + \frac{3}{2} \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{11}{2} \int \frac{1}{1 + x + x^2} dx \\
&= -2x + \frac{x^2}{2} + \frac{3}{2} \log(1 + x + x^2) - 11 \operatorname{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2x \right) \\
&= -2x + \frac{x^2}{2} + \frac{11 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{3}{2} \log(1 + x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{x^2}{2} + \frac{3}{2} \log(x^2 + x + 1) - 2x + \frac{11 \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2), x]

[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2])/2

fricas [A] time = 0.92, size = 34, normalized size = 0.83

$$\frac{1}{2} x^2 + \frac{11}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x + 1) \right) - 2x + \frac{3}{2} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1), x, algorithm="fricas")

[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 + x + 1)

giac [A] time = 0.29, size = 34, normalized size = 0.83

$$\frac{1}{2} x^2 + \frac{11}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x + 1) \right) - 2x + \frac{3}{2} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1), x, algorithm="giac")

[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 + x + 1)

maple [A] time = 0.00, size = 35, normalized size = 0.85

$$\frac{x^2}{2} - 2x + \frac{11\sqrt{3} \arctan \left(\frac{(2x+1)\sqrt{3}}{3} \right)}{3} + \frac{3 \ln(x^2 + x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-x^2+2*x+5)/(x^2+x+1), x)

[Out] $-2x + \frac{1}{2}x^2 + \frac{3}{2}\ln(x^2+x+1) + \frac{11}{3}3^{(1/2)}\arctan\left(\frac{1}{3}(2x+1)3^{(1/2)}\right)$

maxima [A] time = 2.18, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="maxima")

[Out] $\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log(x^2+x+1)$

mupad [B] time = 0.04, size = 36, normalized size = 0.88

$$\frac{3\ln(x^2+x+1)}{2} - 2x + \frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - x^2 + x^3 + 5)/(x + x^2 + 1),x)

[Out] $\frac{3\log(x+x^2+1)}{2} - 2x + \frac{(11*3^{(1/2)}*\operatorname{atan}((2*3^{(1/2)}*x)/3 + 3^{(1/2)}/3))}{3} + x^2/2$

sympy [A] time = 0.12, size = 46, normalized size = 1.12

$$\frac{x^2}{2} - 2x + \frac{3\log(x^2+x+1)}{2} + \frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)

[Out] $x^2/2 - 2x + 3\log(x^2+x+1)/2 + 11\sqrt{3}\operatorname{atan}(2\sqrt{3}x/3 + \sqrt{3}/3)/3$

$$3.293 \quad \int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} dx$$

Optimal. Leaf size=41

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3}{4} \log(x^2 - 4x + 5) + \frac{3x}{2} + 6 \tan^{-1}(2 - x)$$

[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(-2+x)+3/4*ln(x^2-4*x+5)

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3}{4} \log(x^2 - 4x + 5) + \frac{3x}{2} + 6 \tan^{-1}(2 - x)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2), x]

[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{-3 + x - 2x^3 + x^4}{10 - 8x + 2x^2} dx &= \int \left(\frac{3}{2} + x + \frac{x^2}{2} - \frac{3(6-x)}{10 - 8x + 2x^2} \right) dx \\
&= \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{6} - 3 \int \frac{6-x}{10 - 8x + 2x^2} dx \\
&= \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{6} + \frac{3}{4} \int \frac{-8+4x}{10 - 8x + 2x^2} dx - 12 \int \frac{1}{10 - 8x + 2x^2} dx \\
&= \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{6} + \frac{3}{4} \log(5 - 4x + x^2) + 24 \operatorname{Subst} \left(\int \frac{1}{-16 - x^2} dx, x, -8 + 4x \right) \\
&= \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{6} + 6 \tan^{-1}(2 - x) + \frac{3}{4} \log(5 - 4x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.95

$$\frac{1}{2} \left(\frac{x^3}{3} + x^2 + \frac{3}{2} \log(x^2 - 4x + 5) + 3x + 12 \tan^{-1}(2 - x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2), x]

[Out] (3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/2)/2

fricas [A] time = 1.11, size = 31, normalized size = 0.76

$$\frac{1}{6} x^3 + \frac{1}{2} x^2 + \frac{3}{2} x - 6 \arctan(x - 2) + \frac{3}{4} \log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10), x, algorithm="fricas")

[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)

giac [A] time = 0.28, size = 31, normalized size = 0.76

$$\frac{1}{6} x^3 + \frac{1}{2} x^2 + \frac{3}{2} x - 6 \arctan(x - 2) + \frac{3}{4} \log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10), x, algorithm="giac")

[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)

maple [A] time = 0.00, size = 32, normalized size = 0.78

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} - 6 \arctan(x - 2) + \frac{3 \ln(x^2 - 4x + 5)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-2*x^3+x-3)/(2*x^2-8*x+10), x)

[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(x-2)+3/4*ln(x^2-4*x+5)

maxima [A] time = 2.07, size = 31, normalized size = 0.76

$$\frac{1}{6} x^3 + \frac{1}{2} x^2 + \frac{3}{2} x - 6 \arctan(x - 2) + \frac{3}{4} \log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="maxima")

[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)

mupad [B] time = 2.12, size = 31, normalized size = 0.76

$$\frac{3x}{2} - 6 \operatorname{atan}(x - 2) + \frac{3 \ln(x^2 - 4x + 5)}{4} + \frac{x^2}{2} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 2*x^3 + x^4 - 3)/(2*x^2 - 8*x + 10),x)

[Out] (3*x)/2 - 6*atan(x - 2) + (3*log(x^2 - 4*x + 5))/4 + x^2/2 + x^3/6

sympy [A] time = 0.12, size = 34, normalized size = 0.83

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} + \frac{3 \log(x^2 - 4x + 5)}{4} - 6 \operatorname{atan}(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)

[Out] x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)

$$3.294 \quad \int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)} dx$$

Optimal. Leaf size=30

$$x + \frac{7}{2} \log(1-x) - 25 \log(2-x) + \frac{61}{2} \log(3-x)$$

[Out] x+7/2*ln(1-x)-25*ln(2-x)+61/2*ln(3-x)

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1612}

$$x + \frac{7}{2} \log(1-x) - 25 \log(2-x) + \frac{61}{2} \log(3-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)} dx &= \int \left(1 + \frac{61}{2(-3+x)} - \frac{25}{-2+x} + \frac{7}{2(-1+x)} \right) dx \\ &= x + \frac{7}{2} \log(1-x) - 25 \log(2-x) + \frac{61}{2} \log(3-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.80

$$x + \frac{61}{2} \log(x-3) - 25 \log(x-2) + \frac{7}{2} \log(x-1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] x + (61*Log[-3 + x])/2 - 25*Log[-2 + x] + (7*Log[-1 + x])/2

fricas [A] time = 0.96, size = 20, normalized size = 0.67

$$x + \frac{7}{2} \log(x-1) - 25 \log(x-2) + \frac{61}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")

[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)

giac [A] time = 0.25, size = 23, normalized size = 0.77

$$x + \frac{7}{2} \log(|x - 1|) - 25 \log(|x - 2|) + \frac{61}{2} \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")

[Out] x + 7/2*log(abs(x - 1)) - 25*log(abs(x - 2)) + 61/2*log(abs(x - 3))

maple [A] time = 0.01, size = 21, normalized size = 0.70

$$x + \frac{61 \ln(x - 3)}{2} - 25 \ln(x - 2) + \frac{7 \ln(x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+3*x^2+2*x+1)/(x-3)/(x-2)/(x-1),x)

[Out] x+7/2*ln(x-1)-25*ln(x-2)+61/2*ln(x-3)

maxima [A] time = 1.04, size = 20, normalized size = 0.67

$$x + \frac{7}{2} \log(x - 1) - 25 \log(x - 2) + \frac{61}{2} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")

[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)

mupad [B] time = 2.13, size = 20, normalized size = 0.67

$$x + \frac{7 \ln(x - 1)}{2} - 25 \ln(x - 2) + \frac{61 \ln(x - 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 3*x^2 + x^3 + 1)/((x - 1)*(x - 2)*(x - 3)),x)

[Out] x + (7*log(x - 1))/2 - 25*log(x - 2) + (61*log(x - 3))/2

sympy [A] time = 0.15, size = 24, normalized size = 0.80

$$x + \frac{61 \log(x - 3)}{2} - 25 \log(x - 2) + \frac{7 \log(x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)

[Out] x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2

$$3.295 \quad \int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx$$

Optimal. Leaf size=35

$$\frac{x^2}{2} - 2x + \frac{13}{3} \log(4-x) - \frac{22}{3} \log(x+2) + 20 \log(x+3)$$

[Out] $-2*x+1/2*x^2+13/3*\ln(4-x)-22/3*\ln(2+x)+20*\ln(3+x)$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2074}

$$\frac{x^2}{2} - 2x + \frac{13}{3} \log(4-x) - \frac{22}{3} \log(x+2) + 20 \log(x+3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3), x]$

[Out] $-2*x + x^2/2 + (13*\text{Log}[4 - x])/3 - (22*\text{Log}[2 + x])/3 + 20*\text{Log}[3 + x]$

Rule 2074

$\text{Int}[(P_)^{(p_)}*(Q_)^{(q_.)}, x_Symbol] \rightarrow \text{With}[\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Q^q, x], x] /; \text{!SumQ}[\text{NonfreeFactors}[PP, x]] /; \text{FreeQ}[q, x] \&\& \text{PolyQ}[P, x] \&\& \text{PolyQ}[Q, x] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[P, x]$

Rubi steps

$$\begin{aligned} \int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx &= \int \left(-2 + \frac{13}{3(-4+x)} + x - \frac{22}{3(2+x)} + \frac{20}{3+x} \right) dx \\ &= -2x + \frac{x^2}{2} + \frac{13}{3} \log(4-x) - \frac{22}{3} \log(2+x) + 20 \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{x^2}{2} - 2x + \frac{13}{3} \log(4-x) - \frac{22}{3} \log(x+2) + 20 \log(x+3)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3), x]$

[Out] $-2*x + x^2/2 + (13*\text{Log}[4 - x])/3 - (22*\text{Log}[2 + x])/3 + 20*\text{Log}[3 + x]$

fricas [A] time = 0.75, size = 27, normalized size = 0.77

$$\frac{1}{2} x^2 - 2x + 20 \log(x+3) - \frac{22}{3} \log(x+2) + \frac{13}{3} \log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24), x, \text{algorithm}=\text{"fricas"})$

[Out] $1/2*x^2 - 2*x + 20*\log(x + 3) - 22/3*\log(x + 2) + 13/3*\log(x - 4)$

giac [A] time = 0.32, size = 30, normalized size = 0.86

$$\frac{1}{2} x^2 - 2x + 20 \log(|x+3|) - \frac{22}{3} \log(|x+2|) + \frac{13}{3} \log(|x-4|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="giac")

[Out] 1/2*x^2 - 2*x + 20*log(abs(x + 3)) - 22/3*log(abs(x + 2)) + 13/3*log(abs(x - 4))

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{x^2}{2} - 2x - \frac{22 \ln(x + 2)}{3} + 20 \ln(x + 3) + \frac{13 \ln(x - 4)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x)

[Out] -2*x+1/2*x^2-22/3*ln(x+2)+13/3*ln(x-4)+20*ln(x+3)

maxima [A] time = 1.12, size = 27, normalized size = 0.77

$$\frac{1}{2}x^2 - 2x + 20 \log(x + 3) - \frac{22}{3} \log(x + 2) + \frac{13}{3} \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="maxima")

[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)

mupad [B] time = 0.04, size = 27, normalized size = 0.77

$$20 \ln(x + 3) - \frac{22 \ln(x + 2)}{3} - 2x + \frac{13 \ln(x - 4)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 7*x - x^3 + x^4 + 2)/(14*x - x^2 - x^3 + 24),x)

[Out] 20*log(x + 3) - (22*log(x + 2))/3 - 2*x + (13*log(x - 4))/3 + x^2/2

sympy [A] time = 0.15, size = 31, normalized size = 0.89

$$\frac{x^2}{2} - 2x + \frac{13 \log(x - 4)}{3} - \frac{22 \log(x + 2)}{3} + 20 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)

[Out] x**2/2 - 2*x + 13*log(x - 4)/3 - 22*log(x + 2)/3 + 20*log(x + 3)

$$3.296 \quad \int \frac{2+x^2}{(-1+x)^2 x(1+x)} dx$$

Optimal. Leaf size=34

$$\frac{3}{2(1-x)} - \frac{5}{4} \log(1-x) + 2 \log(x) - \frac{3}{4} \log(x+1)$$

[Out] 3/2/(1-x)-5/4*ln(1-x)+2*ln(x)-3/4*ln(1+x)

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1612}

$$\frac{3}{2(1-x)} - \frac{5}{4} \log(1-x) + 2 \log(x) - \frac{3}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]

[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{2+x^2}{(-1+x)^2 x(1+x)} dx &= \int \left(\frac{3}{2(-1+x)^2} - \frac{5}{4(-1+x)} + \frac{2}{x} - \frac{3}{4(1+x)} \right) dx \\ &= \frac{3}{2(1-x)} - \frac{5}{4} \log(1-x) + 2 \log(x) - \frac{3}{4} \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.94

$$-\frac{3}{2(x-1)} - \frac{5}{4} \log(1-x) + 2 \log(x) - \frac{3}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]

[Out] -3/(2*(-1 + x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4

fricas [A] time = 0.60, size = 34, normalized size = 1.00

$$\frac{3(x-1) \log(x+1) + 5(x-1) \log(x-1) - 8(x-1) \log(x) + 6}{4(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="fricas")

[Out] -1/4*(3*(x - 1)*log(x + 1) + 5*(x - 1)*log(x - 1) - 8*(x - 1)*log(x) + 6)/(x - 1)

giac [A] time = 0.39, size = 34, normalized size = 1.00

$$-\frac{3}{2(x-1)} + 2 \log\left(\left|-\frac{1}{x-1} - 1\right|\right) - \frac{3}{4} \log\left(\left|-\frac{2}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="giac")

[Out] -3/2/(x - 1) + 2*log(abs(-1/(x - 1) - 1)) - 3/4*log(abs(-2/(x - 1) - 1))

maple [A] time = 0.01, size = 25, normalized size = 0.74

$$2 \ln(x) - \frac{5 \ln(x-1)}{4} - \frac{3 \ln(x+1)}{4} - \frac{3}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2)/(x-1)^2/x/(x+1),x)

[Out] -3/2/(x-1)-5/4*ln(x-1)-3/4*ln(x+1)+2*ln(x)

maxima [A] time = 1.13, size = 24, normalized size = 0.71

$$-\frac{3}{2(x-1)} - \frac{3}{4} \log(x+1) - \frac{5}{4} \log(x-1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="maxima")

[Out] -3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)

mupad [B] time = 2.11, size = 26, normalized size = 0.76

$$2 \ln(x) - \frac{3 \ln(x+1)}{4} - \frac{5 \ln(x-1)}{4} - \frac{3}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 2)/(x*(x - 1)^2*(x + 1)),x)

[Out] 2*log(x) - (3*log(x + 1))/4 - (5*log(x - 1))/4 - 3/(2*(x - 1))

sympy [A] time = 0.14, size = 27, normalized size = 0.79

$$2 \log(x) - \frac{5 \log(x-1)}{4} - \frac{3 \log(x+1)}{4} - \frac{3}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2)/(-1+x)**2/x/(1+x),x)

[Out] 2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)

$$3.297 \quad \int \frac{3+x^2+x^3}{(2+x^2)^2} dx$$

Optimal. Leaf size=42

$$\frac{x+4}{4(x^2+2)} + \frac{1}{2} \log(x^2+2) + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

[Out] 1/4*(4+x)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1814, 635, 203, 260}

$$\frac{x+4}{4(x^2+2)} + \frac{1}{2} \log(x^2+2) + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 + x^2 + x^3)/(2 + x^2)^2, x]

[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{3+x^2+x^3}{(2+x^2)^2} dx &= \frac{4+x}{4(2+x^2)} - \frac{1}{4} \int \frac{-5-4x}{2+x^2} dx \\
&= \frac{4+x}{4(2+x^2)} + \frac{5}{4} \int \frac{1}{2+x^2} dx + \int \frac{x}{2+x^2} dx \\
&= \frac{4+x}{4(2+x^2)} + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{2} \log(2+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{x+4}{4(x^2+2)} + \frac{1}{2} \log(x^2+2) + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + x^2 + x^3)/(2 + x^2)^2,x]

[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2

fricas [A] time = 0.79, size = 44, normalized size = 1.05

$$\frac{5\sqrt{2}(x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 4(x^2+2)\log(x^2+2) + 2x + 8}{8(x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="fricas")

[Out] 1/8*(5*sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 4*(x^2 + 2)*log(x^2 + 2) + 2*x + 8)/(x^2 + 2)

giac [A] time = 0.30, size = 33, normalized size = 0.79

$$\frac{5}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{x+4}{4(x^2+2)} + \frac{1}{2} \log(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="giac")

[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2)

maple [A] time = 0.01, size = 35, normalized size = 0.83

$$\frac{5\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{8} + \frac{\ln(x^2+2)}{2} + \frac{\frac{x}{4} + 1}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+3)/(x^2+2)^2,x)

[Out] (1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.36, size = 33, normalized size = 0.79

$$\frac{5}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + \frac{x+4}{4(x^2+2)} + \frac{1}{2} \log(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="maxima")

[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2)

mupad [B] time = 2.19, size = 39, normalized size = 0.93

$$\frac{\ln(x^2+2)}{2} + \frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8} + \frac{x}{4(x^2+2)} + \frac{1}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^3 + 3)/(x^2 + 2)^2,x)

[Out] log(x^2 + 2)/2 + (5*2^(1/2)*atan((2^(1/2)*x)/2))/8 + x/(4*(x^2 + 2)) + 1/(x^2 + 2)

sympy [A] time = 0.13, size = 36, normalized size = 0.86

$$\frac{x+4}{4x^2+8} + \frac{\log(x^2+2)}{2} + \frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+3)/(x**2+2)**2,x)

[Out] (x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x/2)/8

$$3.298 \quad \int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)} dx$$

Optimal. Leaf size=49

$$\frac{1003 \log(x^2 - 10x + 26)}{1025} + \frac{22 \log(x^2 - 2x + 17)}{1025} - \frac{15033 \tan^{-1}(5 - x)}{1025} - \frac{4607 \tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+26)+22/1025*ln(x^2-2*x+17)

Rubi [A] time = 0.16, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {6728, 634, 618, 204, 628}

$$\frac{1003 \log(x^2 - 10x + 26)}{1025} + \frac{22 \log(x^2 - 2x + 17)}{1025} - \frac{15033 \tan^{-1}(5 - x)}{1025} - \frac{4607 \tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

[In] Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]

[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{-35 + 70x - 4x^2 + 2x^3}{(26 - 10x + x^2)(17 - 2x + x^2)} dx &= \int \left(\frac{5003 + 2006x}{1025(26 - 10x + x^2)} + \frac{-4651 + 44x}{1025(17 - 2x + x^2)} \right) dx \\
&= \frac{\int \frac{5003+2006x}{26-10x+x^2} dx}{1025} + \frac{\int \frac{-4651+44x}{17-2x+x^2} dx}{1025} \\
&= \frac{22 \int \frac{-2+2x}{17-2x+x^2} dx}{1025} + \frac{1003 \int \frac{-10+2x}{26-10x+x^2} dx}{1025} - \frac{4607 \int \frac{1}{17-2x+x^2} dx}{1025} + \frac{15033}{1025} \\
&= \frac{1003 \log(26 - 10x + x^2)}{1025} + \frac{22 \log(17 - 2x + x^2)}{1025} + \frac{9214 \text{Subst}\left(\int \frac{1}{-6x^2 - 12x + 17} dx\right)}{1025} \\
&= -\frac{15033 \tan^{-1}(5 - x)}{1025} - \frac{4607 \tan^{-1}\left(\frac{1}{4}(-1 + x)\right)}{4100} + \frac{1003 \log(26 - 10x + x^2)}{1025}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{1003 \log(x^2 - 10x + 26)}{1025} + \frac{22 \log(x^2 - 2x + 17)}{1025} - \frac{15033 \tan^{-1}(5 - x)}{1025} - \frac{4607 \tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

[In] Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)), x]

[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025

fricas [A] time = 0.62, size = 37, normalized size = 0.76

$$\frac{15033}{1025} \arctan(x - 5) - \frac{4607}{4100} \arctan\left(\frac{1}{4}x - \frac{1}{4}\right) + \frac{22}{1025} \log(x^2 - 2x + 17) + \frac{1003}{1025} \log(x^2 - 10x + 26)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17), x, algorithm="fricas")

[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)

giac [A] time = 0.32, size = 37, normalized size = 0.76

$$\frac{15033}{1025} \arctan(x - 5) - \frac{4607}{4100} \arctan\left(\frac{1}{4}x - \frac{1}{4}\right) + \frac{22}{1025} \log(x^2 - 2x + 17) + \frac{1003}{1025} \log(x^2 - 10x + 26)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17), x, algorithm="giac")

[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{15033 \arctan(x - 5)}{1025} - \frac{4607 \arctan\left(\frac{x}{4} - \frac{1}{4}\right)}{4100} + \frac{1003 \ln(x^2 - 10x + 26)}{1025} + \frac{22 \ln(x^2 - 2x + 17)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x)`

[Out] $15033/1025*\arctan(-5+x)-4607/4100*\arctan(-1/4+1/4*x)+1003/1025*\ln(x^2-10*x+26)+22/1025*\ln(x^2-2*x+17)$

maxima [A] time = 2.80, size = 37, normalized size = 0.76

$$\frac{15033}{1025} \arctan(x-5) - \frac{4607}{4100} \arctan\left(\frac{1}{4}x - \frac{1}{4}\right) + \frac{22}{1025} \log(x^2 - 2x + 17) + \frac{1003}{1025} \log(x^2 - 10x + 26)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="maxima")`

[Out] $15033/1025*\arctan(x-5) - 4607/4100*\arctan(1/4*x - 1/4) + 22/1025*\log(x^2 - 2*x + 17) + 1003/1025*\log(x^2 - 10*x + 26)$

mupad [B] time = 0.07, size = 41, normalized size = 0.84

$$\ln(x-1-4i) \left(\frac{22}{1025} + \frac{4607i}{8200} \right) + \ln(x-1+4i) \left(\frac{22}{1025} - \frac{4607i}{8200} \right) + \ln(x-5-i) \left(\frac{1003}{1025} - \frac{15033i}{2050} \right) + \ln(x-5+1i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((70*x - 4*x^2 + 2*x^3 - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x)`

[Out] $\log(x - (1 + 4i))*(22/1025 + 4607i/8200) + \log(x - (1 - 4i))*(22/1025 - 4607i/8200) + \log(x - (5 + 1i))*(1003/1025 - 15033i/2050) + \log(x - (5 - 1i))*(1003/1025 + 15033i/2050)$

sympy [A] time = 0.23, size = 46, normalized size = 0.94

$$\frac{1003 \log(x^2 - 10x + 26)}{1025} + \frac{22 \log(x^2 - 2x + 17)}{1025} - \frac{4607 \operatorname{atan}\left(\frac{x}{4} - \frac{1}{4}\right)}{4100} + \frac{15033 \operatorname{atan}(x - 5)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)`

[Out] $1003*\log(x^2 - 10*x + 26)/1025 + 22*\log(x^2 - 2*x + 17)/1025 - 4607*\operatorname{atan}(x/4 - 1/4)/4100 + 15033*\operatorname{atan}(x - 5)/1025$

$$3.299 \quad \int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} dx$$

Optimal. Leaf size=29

$$-\frac{11}{14} \log(3-x) + \frac{3}{2} \log(5-x) + \frac{2}{7} \log(x+4)$$

[Out] -11/14*ln(3-x)+3/2*ln(5-x)+2/7*ln(4+x)

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1612}

$$-\frac{11}{14} \log(3-x) + \frac{3}{2} \log(5-x) + \frac{2}{7} \log(x+4)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)), x]

[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} dx &= \int \left(\frac{3}{2(-5+x)} - \frac{11}{14(-3+x)} + \frac{2}{7(4+x)} \right) dx \\ &= -\frac{11}{14} \log(3-x) + \frac{3}{2} \log(5-x) + \frac{2}{7} \log(4+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{11}{14} \log(3-x) + \frac{3}{2} \log(5-x) + \frac{2}{7} \log(x+4)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)), x]

[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7

fricas [A] time = 0.58, size = 19, normalized size = 0.66

$$\frac{2}{7} \log(x+4) - \frac{11}{14} \log(x-3) + \frac{3}{2} \log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x), x, algorithm="fricas")

[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)

giac [A] time = 0.27, size = 22, normalized size = 0.76

$$\frac{2}{7} \log(|x + 4|) - \frac{11}{14} \log(|x - 3|) + \frac{3}{2} \log(|x - 5|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="giac")

[Out] 2/7*log(abs(x + 4)) - 11/14*log(abs(x - 3)) + 3/2*log(abs(x - 5))

maple [A] time = 0.01, size = 20, normalized size = 0.69

$$\frac{3 \ln(x - 5)}{2} - \frac{11 \ln(x - 3)}{14} + \frac{2 \ln(x + 4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2)/(x-5)/(x-3)/(x+4),x)

[Out] 2/7*ln(x+4)-11/14*ln(x-3)+3/2*ln(x-5)

maxima [A] time = 1.17, size = 19, normalized size = 0.66

$$\frac{2}{7} \log(x + 4) - \frac{11}{14} \log(x - 3) + \frac{3}{2} \log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="maxima")

[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)

mupad [B] time = 2.18, size = 19, normalized size = 0.66

$$\frac{2 \ln(x + 4)}{7} - \frac{11 \ln(x - 3)}{14} + \frac{3 \ln(x - 5)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 2)/((x - 3)*(x + 4)*(x - 5)),x)

[Out] (2*log(x + 4))/7 - (11*log(x - 3))/14 + (3*log(x - 5))/2

sympy [A] time = 0.14, size = 24, normalized size = 0.83

$$\frac{3 \log(x - 5)}{2} - \frac{11 \log(x - 3)}{14} + \frac{2 \log(x + 4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)

[Out] 3*log(x - 5)/2 - 11*log(x - 3)/14 + 2*log(x + 4)/7

$$3.300 \quad \int \frac{x^4}{(-1+x)(2+x^2)} dx$$

Optimal. Leaf size=46

$$\frac{x^2}{2} - \frac{2}{3} \log(x^2 + 2) + x + \frac{1}{3} \log(1 - x) - \frac{2}{3} \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

[Out] x+1/2*x^2+1/3*ln(1-x)-2/3*ln(x^2+2)-2/3*arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1629, 635, 203, 260}

$$\frac{x^2}{2} - \frac{2}{3} \log(x^2 + 2) + x + \frac{1}{3} \log(1 - x) - \frac{2}{3} \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^4/((-1 + x)*(2 + x^2)), x]

[Out] x + x^2/2 - (2*sqrt(2)*ArcTan[x/sqrt(2)]/3 + Log[1 - x]/3 - (2*Log[2 + x^2])/3)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(-1+x)(2+x^2)} dx &= \int \left(1 + \frac{1}{3(-1+x)} + x - \frac{4(1+x)}{3(2+x^2)} \right) dx \\ &= x + \frac{x^2}{2} + \frac{1}{3} \log(1-x) - \frac{4}{3} \int \frac{1+x}{2+x^2} dx \\ &= x + \frac{x^2}{2} + \frac{1}{3} \log(1-x) - \frac{4}{3} \int \frac{1}{2+x^2} dx - \frac{4}{3} \int \frac{x}{2+x^2} dx \\ &= x + \frac{x^2}{2} - \frac{2}{3} \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{3} \log(1-x) - \frac{2}{3} \log(2+x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.93

$$\frac{1}{6} \left(3x^2 - 4 \log(x^2 + 2) + 6x + 2 \log(x - 1) - 4\sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) - 9 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((-1 + x)*(2 + x^2)),x]

[Out] (-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] - 4*Log[2 + x^2])/6

fricas [A] time = 0.99, size = 33, normalized size = 0.72

$$\frac{1}{2} x^2 - \frac{2}{3} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} x \right) + x - \frac{2}{3} \log(x^2 + 2) + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="fricas")

[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*log(x - 1)

giac [A] time = 0.32, size = 34, normalized size = 0.74

$$\frac{1}{2} x^2 - \frac{2}{3} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} x \right) + x - \frac{2}{3} \log(x^2 + 2) + \frac{1}{3} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="giac")

[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*log(abs(x - 1))

maple [A] time = 0.01, size = 34, normalized size = 0.74

$$\frac{x^2}{2} + x - \frac{2\sqrt{2} \arctan \left(\frac{\sqrt{2} x}{2} \right) + \ln(x - 1) - \frac{2 \ln(x^2 + 2)}{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x-1)/(x^2+2),x)

[Out] 1/2*x^2+x+1/3*ln(x-1)-2/3*ln(x^2+2)-2/3*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.16, size = 33, normalized size = 0.72

$$\frac{1}{2} x^2 - \frac{2}{3} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} x \right) + x - \frac{2}{3} \log(x^2 + 2) + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="maxima")

[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*log(x - 1)

mupad [B] time = 0.09, size = 50, normalized size = 1.09

$$x + \frac{\ln(x - 1)}{3} + \ln \left(x - \sqrt{2} \operatorname{li} \right) \left(-\frac{2}{3} + \frac{\sqrt{2} \operatorname{li}}{3} \right) - \ln \left(x + \sqrt{2} \operatorname{li} \right) \left(\frac{2}{3} + \frac{\sqrt{2} \operatorname{li}}{3} \right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/((x^2 + 2)*(x - 1)),x)`

[Out] `x + log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/3 - 2/3) - log(x + 2^(1/2)*1i)*((2^(1/2)*1i)/3 + 2/3) + x^2/2`

sympy [A] time = 0.14, size = 41, normalized size = 0.89

$$\frac{x^2}{2} + x + \frac{\log(x-1)}{3} - \frac{2\log(x^2+2)}{3} - \frac{2\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-1+x)/(x**2+2),x)`

[Out] `x**2/2 + x + log(x - 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/2)/3`

$$3.301 \quad \int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx$$

Optimal. Leaf size=16

$$2 \log(1-x) - \frac{3}{x+1}$$

[Out] -3/(1+x)+2*ln(1-x)

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2074}

$$2 \log(1-x) - \frac{3}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3), x]

[Out] -3/(1 + x) + 2*Log[1 - x]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx &= \int \left(\frac{2}{-1+x} + \frac{3}{(1+x)^2} \right) dx \\ &= -\frac{3}{1+x} + 2 \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.88

$$2 \log(x-1) - \frac{3}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3), x]

[Out] -3/(1 + x) + 2*Log[-1 + x]

fricas [A] time = 0.80, size = 17, normalized size = 1.06

$$\frac{2(x+1) \log(x-1) - 3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1), x, algorithm="fricas")

[Out] (2*(x + 1)*log(x - 1) - 3)/(x + 1)

giac [A] time = 0.40, size = 15, normalized size = 0.94

$$-\frac{3}{x+1} + 2 \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="giac")

[Out] -3/(x + 1) + 2*log(abs(x - 1))

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$2 \ln(x - 1) - \frac{3}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+7*x-1)/(x^3+x^2-x-1),x)

[Out] 2*ln(x-1)-3/(x+1)

maxima [A] time = 1.09, size = 14, normalized size = 0.88

$$-\frac{3}{x + 1} + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="maxima")

[Out] -3/(x + 1) + 2*log(x - 1)

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$2 \ln(x - 1) - \frac{3}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(7*x + 2*x^2 - 1)/(x - x^2 - x^3 + 1),x)

[Out] 2*log(x - 1) - 3/(x + 1)

sympy [A] time = 0.09, size = 10, normalized size = 0.62

$$2 \log(x - 1) - \frac{3}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)

[Out] 2*log(x - 1) - 3/(x + 1)

$$3.302 \quad \int \frac{1+2x}{-1+3x-3x^2+x^3} dx$$

Optimal. Leaf size=21

$$\frac{2}{1-x} - \frac{3}{2(1-x)^2}$$

[Out] -3/2/(1-x)^2+2/(1-x)

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2074}

$$\frac{2}{1-x} - \frac{3}{2(1-x)^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3), x]

[Out] -3/(2*(1 - x)^2) + 2/(1 - x)

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{-1+3x-3x^2+x^3} dx &= \int \left(\frac{3}{(-1+x)^3} + \frac{2}{(-1+x)^2} \right) dx \\ &= -\frac{3}{2(1-x)^2} + \frac{2}{1-x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{1-4x}{2(x-1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3), x]

[Out] (1 - 4*x)/(2*(-1 + x)^2)

fricas [A] time = 0.88, size = 17, normalized size = 0.81

$$-\frac{4x-1}{2(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1), x, algorithm="fricas")

[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)

giac [A] time = 0.30, size = 12, normalized size = 0.57

$$-\frac{4x-1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="giac")

[Out] -1/2*(4*x - 1)/(x - 1)^2

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{3}{2(x-1)^2} - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)/(x^3-3*x^2+3*x-1),x)

[Out] -3/2/(x-1)^2-2/(x-1)

maxima [A] time = 1.09, size = 17, normalized size = 0.81

$$-\frac{4x-1}{2(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")

[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)

mupad [B] time = 2.09, size = 12, normalized size = 0.57

$$-\frac{4x-1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/(3*x - 3*x^2 + x^3 - 1),x)

[Out] -(4*x - 1)/(2*(x - 1)^2)

sympy [A] time = 0.09, size = 14, normalized size = 0.67

$$\frac{1-4x}{2x^2-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)

[Out] (1 - 4*x)/(2*x**2 - 4*x + 2)

$$3.303 \quad \int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} dx$$

Optimal. Leaf size=15

$$\frac{1}{1-x} - \frac{2}{(x+1)^2}$$

[Out] 1/(1-x)-2/(1+x)^2

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1620}

$$\frac{1}{1-x} - \frac{2}{(x+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3), x]

[Out] (1 - x)^(-1) - 2/(1 + x)^2

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c,
d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} dx &= \int \left(\frac{1}{(-1+x)^2} + \frac{4}{(1+x)^3} \right) dx \\ &= \frac{1}{1-x} - \frac{2}{(1+x)^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2}{(x+1)^2} - \frac{1}{x-1}$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3), x]

[Out] -(-1 + x)^(-1) - 2/(1 + x)^2

fricas [A] time = 0.88, size = 23, normalized size = 1.53

$$-\frac{x^2 + 4x - 1}{x^3 + x^2 - x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="fricas")

[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)

giac [A] time = 0.26, size = 30, normalized size = 2.00

$$-\frac{1}{x-1} + \frac{\frac{4}{x-1} + 1}{2\left(\frac{2}{x-1} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="giac")

[Out] -1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$-\frac{2}{(x+1)^2} - \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+7*x^2-5*x+5)/(x-1)^2/(x+1)^3,x)

[Out] -2/(x+1)^2-1/(x-1)

maxima [A] time = 1.27, size = 23, normalized size = 1.53

$$\frac{x^2 + 4x - 1}{x^3 + x^2 - x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="maxima")

[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)

mupad [B] time = 2.09, size = 15, normalized size = 1.00

$$-\frac{1}{x-1} - \frac{2}{(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((7*x^2 - 5*x + x^3 + 5)/((x - 1)^2*(x + 1)^3),x)

[Out] - 1/(x - 1) - 2/(x + 1)^2

sympy [A] time = 0.11, size = 17, normalized size = 1.13

$$\frac{-x^2 - 4x + 1}{x^3 + x^2 - x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)

[Out] (-x**2 - 4*x + 1)/(x**3 + x**2 - x - 1)

$$3.304 \quad \int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx$$

Optimal. Leaf size=31

$$\log(x^2 + x + 1) + \log(x + 1) - \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $\ln(1+x)+\ln(x^2+x+1)-2/3*\arctan(1/3*(1+2*x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2074, 634, 618, 204, 628}

$$\log(x^2 + x + 1) + \log(x + 1) - \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3), x]$

[Out] $(-2*\text{ArcTan}[(1 + 2*x)/\text{Sqrt}[3]])/\text{Sqrt}[3] + \text{Log}[1 + x] + \text{Log}[1 + x + x^2]$

Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2074

$\text{Int}(P_)^{(p_)}*(Q_)^{(q_)}, x_Symbol] \rightarrow \text{With}\{\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Q^q, x], x] /; \ !\text{SumQ}[\text{NonfreeFactors}[PP, x]]] /; \text{FreeQ}[q, x] \ \&\& \ \text{PolyQ}[P, x] \ \&\& \ \text{PolyQ}[Q, x] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{NeQ}[P, x]$

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx &= \int \left(\frac{1}{1+x} + \frac{2x}{1+x+x^2} \right) dx \\
&= \log(1+x) + 2 \int \frac{x}{1+x+x^2} dx \\
&= \log(1+x) - \int \frac{1}{1+x+x^2} dx + \int \frac{1+2x}{1+x+x^2} dx \\
&= \log(1+x) + \log(1+x+x^2) + 2 \operatorname{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x \right) \\
&= -\frac{2 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \log(1+x) + \log(1+x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\log(x^2+x+1) + \log(x+1) - \frac{2 \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3), x]

[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]

fricas [A] time = 0.89, size = 28, normalized size = 0.90

$$-\frac{2}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x+1) \right) + \log(x^2+x+1) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1), x, algorithm="fricas")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)

giac [A] time = 0.28, size = 29, normalized size = 0.94

$$-\frac{2}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x+1) \right) + \log(x^2+x+1) + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1), x, algorithm="giac")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x + 1))

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$-\frac{2\sqrt{3} \arctan \left(\frac{(2x+1)\sqrt{3}}{3} \right)}{3} + \ln(x+1) + \ln(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1), x)

[Out] ln(x+1)+ln(x^2+x+1)-2/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 2.36, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log(x^2+x+1)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="maxima")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)

mupad [B] time = 0.11, size = 57, normalized size = 1.84

$$\ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right) + \ln(x+1) + \frac{\sqrt{3}\ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)1i}{3} - \frac{\sqrt{3}\ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)1i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 3*x^2 + 1)/(2*x + 2*x^2 + x^3 + 1),x)

[Out] log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x + 1) + (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - (3^(1/2)*log(x + (3^(1/2)*1i)/2 + 1/2)*1i)/3

sympy [A] time = 0.13, size = 3, normalized size = 0.10

$$\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)

[Out] log(x + 1)

$$3.305 \quad \int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$$

Optimal. Leaf size=25

$$\frac{1}{10} \log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10} \log(x+2)$$

[Out] 1/10*ln(1-2*x)+1/2*ln(x)-1/10*ln(2+x)

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1594, 1628}

$$\frac{1}{10} \log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10} \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3), x]

[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx &= \int \frac{-1+2x+x^2}{x(-2+3x+2x^2)} dx \\ &= \int \left(\frac{1}{2x} - \frac{1}{10(2+x)} + \frac{1}{5(-1+2x)} \right) dx \\ &= \frac{1}{10} \log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10} \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{10} \log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10} \log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3), x]

[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10

fricas [A] time = 0.80, size = 19, normalized size = 0.76

$$\frac{1}{10} \log(2x-1) - \frac{1}{10} \log(x+2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")

[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)

giac [A] time = 0.29, size = 22, normalized size = 0.88

$$\frac{1}{10} \log(|2x - 1|) - \frac{1}{10} \log(|x + 2|) + \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")

[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\ln(x)}{2} + \frac{\ln(2x - 1)}{10} - \frac{\ln(x + 2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)

[Out] 1/10*ln(2*x-1)-1/10*ln(x+2)+1/2*ln(x)

maxima [A] time = 1.09, size = 19, normalized size = 0.76

$$\frac{1}{10} \log(2x - 1) - \frac{1}{10} \log(x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")

[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)

mupad [B] time = 0.06, size = 19, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{24}{145\left(\frac{29x}{100} - \frac{11}{50}\right)} + \frac{35}{29}\right)}{5} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)

[Out] atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2

sympy [A] time = 0.14, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2} + \frac{\log\left(x - \frac{1}{2}\right)}{10} - \frac{\log(x + 2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)

[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10

$$3.306 \quad \int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} dx$$

Optimal. Leaf size=30

$$\frac{x^2}{2} + x + \frac{2}{1-x} + \log(1-x) - \log(x+1)$$

[Out] 2/(1-x)+x+1/2*x^2+ln(1-x)-ln(1+x)

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2074}

$$\frac{x^2}{2} + x + \frac{2}{1-x} + \log(1-x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3), x]

[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} dx &= \int \left(1 + \frac{1}{-1-x} + \frac{2}{(-1+x)^2} + \frac{1}{-1+x} + x \right) dx \\ &= \frac{2}{1-x} + x + \frac{x^2}{2} + \log(1-x) - \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2 - \frac{2}{x-1} + \log(1-x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3), x]

[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]

fricas [A] time = 0.87, size = 36, normalized size = 1.20

$$\frac{x^3 + x^2 - 2(x-1)\log(x+1) + 2(x-1)\log(x-1) - 2x - 4}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")

[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x - 1)

giac [A] time = 0.28, size = 26, normalized size = 0.87

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(|x+1|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")

[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{x^2}{2} + x + \ln(x-1) - \ln(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x)

[Out] 1/2*x^2+x+ln(x-1)-2/(x-1)-ln(x+1)

maxima [A] time = 1.21, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(x+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")

[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)

mupad [B] time = 2.11, size = 22, normalized size = 0.73

$$x - \frac{2}{x-1} + \frac{x^2}{2} + \operatorname{atan}(x1i) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)

[Out] x + atan(x*1i)*2i - 2/(x - 1) + x^2/2

sympy [A] time = 0.10, size = 20, normalized size = 0.67

$$\frac{x^2}{2} + x + \log(x-1) - \log(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)

[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)

$$3.307 \quad \int \frac{4-x+2x^2}{4x+x^3} dx$$

Optimal. Leaf size=23

$$\frac{1}{2} \log(x^2 + 4) + \log(x) - \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{1}{2} \log(x^2 + 4) + \log(x) - \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 - x + 2*x^2)/(4*x + x^3), x]

[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{4-x+2x^2}{4x+x^3} dx &= \int \frac{4-x+2x^2}{x(4+x^2)} dx \\
&= \int \left(\frac{1}{x} + \frac{-1+x}{4+x^2} \right) dx \\
&= \log(x) + \int \frac{-1+x}{4+x^2} dx \\
&= \log(x) - \int \frac{1}{4+x^2} dx + \int \frac{x}{4+x^2} dx \\
&= -\frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \log(x) + \frac{1}{2} \log(4+x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 4) + \log(x) - \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]

[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2

fricas [A] time = 0.73, size = 17, normalized size = 0.74

$$-\frac{1}{2} \arctan\left(\frac{1}{2}x\right) + \frac{1}{2} \log(x^2 + 4) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")

[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)

giac [A] time = 0.33, size = 18, normalized size = 0.78

$$-\frac{1}{2} \arctan\left(\frac{1}{2}x\right) + \frac{1}{2} \log(x^2 + 4) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")

[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{\arctan\left(\frac{x}{2}\right)}{2} + \ln(x) + \frac{\ln(x^2 + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+4)/(x^3+4*x),x)

[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)

maxima [A] time = 2.08, size = 17, normalized size = 0.74

$$-\frac{1}{2} \arctan\left(\frac{1}{2}x\right) + \frac{1}{2} \log(x^2 + 4) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")

[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)

mupad [B] time = 2.12, size = 21, normalized size = 0.91

$$\ln(x) + \ln(x - 2i) \left(\frac{1}{2} + \frac{1}{4}i \right) + \ln(x + 2i) \left(\frac{1}{2} - \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 - x + 4)/(4*x + x^3),x)

[Out] log(x - 2i)*(1/2 + 1i/4) + log(x + 2i)*(1/2 - 1i/4) + log(x)

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\log(x) + \frac{\log(x^2 + 4)}{2} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+4)/(x**3+4*x),x)

[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2

$$3.308 \quad \int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)} dx$$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8(x^2+1)} + \frac{3x}{16(x^2+1)} + \frac{x+1}{8(x^2+1)^2} + \frac{15}{16} \log(x^2+1) - \frac{1}{2} \log(x^2+x+1) + \frac{1}{8} \log(1-x) - \log(x) + \frac{7}{16} \tan^{-1}(x) - \frac{7}{16} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

[Out] 1/8*(1+x)/(x^2+1)^2-3/8*(1-x)/(x^2+1)+3/16*x/(x^2+1)+7/16*arctan(x)+1/8*ln(1-x)-ln(x)+15/16*ln(x^2+1)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.48, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6728, 639, 199, 203, 635, 260, 634, 618, 204, 628}

$$-\frac{3(1-x)}{8(x^2+1)} + \frac{3x}{16(x^2+1)} + \frac{x+1}{8(x^2+1)^2} + \frac{15}{16} \log(x^2+1) - \frac{1}{2} \log(x^2+x+1) + \frac{1}{8} \log(1-x) - \log(x) + \frac{7}{16} \tan^{-1}(x) - \frac{7}{16} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)), x]

[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2)) + (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 639

Int[((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 6728

Int[(u_)/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)} dx &= \int \left(\frac{1}{8(-1+x)} - \frac{1}{x} + \frac{1-x}{2(1+x^2)^3} + \frac{3(1+x)}{4(1+x^2)^2} + \frac{-1+15x}{8(1+x^2)} + \frac{-1}{1+x} \right) dx \\ &= \frac{1}{8} \log(1-x) - \log(x) + \frac{1}{8} \int \frac{-1+15x}{1+x^2} dx + \frac{1}{2} \int \frac{1-x}{(1+x^2)^3} dx + \frac{3}{4} \int \frac{1+x}{(1+x^2)^2} dx \\ &= \frac{1+x}{8(1+x^2)^2} - \frac{3(1-x)}{8(1+x^2)} + \frac{1}{8} \log(1-x) - \log(x) - \frac{1}{8} \int \frac{1}{1+x^2} dx + \frac{3}{4} \int \frac{1+x}{(1+x^2)^2} dx \\ &= \frac{1+x}{8(1+x^2)^2} - \frac{3(1-x)}{8(1+x^2)} + \frac{3x}{16(1+x^2)} + \frac{1}{4} \tan^{-1}(x) + \frac{1}{8} \log(1-x) - \log(x) \\ &= \frac{1+x}{8(1+x^2)^2} - \frac{3(1-x)}{8(1+x^2)} + \frac{3x}{16(1+x^2)} + \frac{7}{16} \tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.90

$$\frac{1}{48} \left(-14 \log(1-x^3) + \frac{6(x+1)}{(x^2+1)^2} + \frac{9(3x-2)}{x^2+1} + 45 \log(x^2+1) - 10 \log(x^2+x+1) + 20 \log(1-x) - 48 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]

[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*sqrt(3)*ArcTan[(1 + 2*x)/sqrt(3)] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48

fricas [A] time = 0.70, size = 136, normalized size = 1.32

$$\frac{27x^3 - 16\sqrt{3}(x^4 + 2x^2 + 1)\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - 18x^2 + 21(x^4 + 2x^2 + 1)\arctan(x) - 24(x^4 + 2x^2 + 1)\log(x^2 + x + 1) - 48(x^4 + 2x^2 + 1)\log(x) + 33x - 12}{48(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")

[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1) - 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)

giac [A] time = 0.34, size = 74, normalized size = 0.72

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{9x^3 - 6x^2 + 11x - 4}{16(x^2 + 1)^2} + \frac{7}{16}\arctan(x) - \frac{1}{2}\log(x^2 + x + 1) + \frac{15}{16}\log(x^2 + 1) + \frac{1}{8}\log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1) + 1/8*log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.01, size = 73, normalized size = 0.71

$$\frac{7\arctan(x)}{16} - \frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} - \ln(x) + \frac{\ln(x-1)}{8} + \frac{15\ln(x^2+1)}{16} - \frac{\ln(x^2+x+1)}{2} + \frac{\frac{9}{2}x^3 - 3x^2 + \frac{11}{2}x - 2}{8(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+1)/(x-1)/x/(x^2+1)^3/(x^2+x+1),x)

[Out] 1/8*ln(x-1)+1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arctan(x)-ln(x)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 2.25, size = 77, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{9x^3 - 6x^2 + 11x - 4}{16(x^4 + 2x^2 + 1)} + \frac{7}{16}\arctan(x) - \frac{1}{2}\log(x^2 + x + 1) + \frac{15}{16}\log(x^2 + 1) + \frac{1}{8}\log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1) + 1/8*log(x - 1) - log(x)

mupad [B] time = 2.20, size = 96, normalized size = 0.93

$$\frac{\ln(x-1)}{8} - \ln(x) + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}i}{6}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}i}{6}\right) + \frac{\frac{9x^3}{16} - \frac{3x^2}{8} + \frac{11x}{16} - \frac{1}{4}}{x^4 + 2x^2 + 1} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)`

[Out] $\log(x - 1)/8 + \log(x - 1i)*(15/16 - 7i/32) + \log(x + 1i)*(15/16 + 7i/32) - \log(x) + \log(x - (3^{1/2}*1i)/2 + 1/2)*((3^{1/2}*1i)/6 - 1/2) - \log(x + (3^{1/2}*1i)/2 + 1/2)*((3^{1/2}*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*x^3)/16 - 1/4)/(2*x^2 + x^4 + 1)$

sympy [A] time = 0.51, size = 88, normalized size = 0.85

$$-\log(x) + \frac{\log(x-1)}{8} + \frac{15 \log(x^2+1)}{16} - \frac{\log(x^2+x+1)}{2} + \frac{7 \operatorname{atan}(x)}{16} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{9x^3 - 6x^2 + 11x - 4}{16x^4 + 32x^2 + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)`

[Out] $-\log(x) + \log(x - 1)/8 + 15*\log(x**2 + 1)/16 - \log(x**2 + x + 1)/2 + 7*\operatorname{atan}(x)/16 - \operatorname{sqrt}(3)*\operatorname{atan}(2*\operatorname{sqrt}(3)*x/3 + \operatorname{sqrt}(3)/3)/3 + (9*x**3 - 6*x**2 + 11*x - 4)/(16*x**4 + 32*x**2 + 16)$

$$3.309 \quad \int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} dx$$

Optimal. Leaf size=33

$$\frac{2-x}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \frac{3}{2} \tan^{-1}(x)$$

[Out] 1/2*(2-x)/(x^2+1)+3/2*arctan(x)-1/2*ln(x^2+1)

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1814, 635, 203, 260}

$$\frac{2-x}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \frac{3}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2, x]

[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} dx &= \frac{2-x}{2(1+x^2)} - \frac{1}{2} \int \frac{-3+2x}{1+x^2} dx \\ &= \frac{2-x}{2(1+x^2)} + \frac{3}{2} \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\ &= \frac{2-x}{2(1+x^2)} + \frac{3}{2} \tan^{-1}(x) - \frac{1}{2} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{1}{2} \left(\frac{2-x}{x^2+1} - \log(x^2+1) + 3 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2, x]

[Out] ((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2

fricas [A] time = 0.92, size = 36, normalized size = 1.09

$$\frac{3(x^2+1)\arctan(x) - (x^2+1)\log(x^2+1) - x + 2}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2, x, algorithm="fricas")

[Out] 1/2*(3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) - x + 2)/(x^2 + 1)

giac [A] time = 0.32, size = 25, normalized size = 0.76

$$-\frac{x-2}{2(x^2+1)} + \frac{3}{2} \arctan(x) - \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2, x, algorithm="giac")

[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)

maple [A] time = 0.00, size = 28, normalized size = 0.85

$$\frac{3 \arctan(x)}{2} - \frac{\ln(x^2+1)}{2} - \frac{\frac{x}{2}-1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+2*x^2-3*x+1)/(x^2+1)^2, x)

[Out] -(1/2*x-1)/(x^2+1)-1/2*ln(x^2+1)+3/2*arctan(x)

maxima [A] time = 2.15, size = 25, normalized size = 0.76

$$-\frac{x-2}{2(x^2+1)} + \frac{3}{2} \arctan(x) - \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2, x, algorithm="maxima")

[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)

mupad [B] time = 0.03, size = 32, normalized size = 0.97

$$\frac{3 \operatorname{atan}(x)}{2} - \frac{\ln(x^2+1)}{2} - \frac{x}{2(x^2+1)} + \frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(3*x - 2*x^2 + x^3 - 1)/(x^2 + 1)^2,x)`

[Out] $(3*\operatorname{atan}(x))/2 - \log(x^2 + 1)/2 - x/(2*(x^2 + 1)) + 1/(x^2 + 1)$

sympy [A] time = 0.13, size = 24, normalized size = 0.73

$$-\frac{x-2}{2x^2+2} - \frac{\log(x^2+1)}{2} + \frac{3\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)`

[Out] $-(x - 2)/(2*x**2 + 2) - \log(x**2 + 1)/2 + 3*\operatorname{atan}(x)/2$

$$3.310 \quad \int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx$$

Optimal. Leaf size=33

$$-\frac{2x+1}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \log(x) - 2 \tan^{-1}(x)$$

[Out] 1/2*(-1-2*x)/(x^2+1)-2*arctan(x)+ln(x)-1/2*ln(x^2+1)

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1805, 801, 635, 203, 260}

$$-\frac{2x+1}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \log(x) - 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2), x]

[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1805

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx &= -\frac{1+2x}{2(1+x^2)} - \frac{1}{2} \int \frac{-2+4x}{x(1+x^2)} dx \\
&= -\frac{1+2x}{2(1+x^2)} - \frac{1}{2} \int \left(-\frac{2}{x} + \frac{2(2+x)}{1+x^2} \right) dx \\
&= -\frac{1+2x}{2(1+x^2)} + \log(x) - \int \frac{2+x}{1+x^2} dx \\
&= -\frac{1+2x}{2(1+x^2)} + \log(x) - 2 \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\
&= -\frac{1+2x}{2(1+x^2)} - 2 \tan^{-1}(x) + \log(x) - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{-2x-1}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \log(x) - 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2), x]

[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2

fricas [A] time = 0.78, size = 44, normalized size = 1.33

$$\frac{4(x^2+1) \arctan(x) + (x^2+1) \log(x^2+1) - 2(x^2+1) \log(x) + 2x+1}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")

[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) + 2*x + 1)/(x^2 + 1)

giac [A] time = 0.38, size = 30, normalized size = 0.91

$$-\frac{2x+1}{2(x^2+1)} - 2 \arctan(x) - \frac{1}{2} \log(x^2+1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")

[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$-2 \arctan(x) + \ln(x) - \frac{\ln(x^2+1)}{2} - \frac{x+\frac{1}{2}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)

[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)

maxima [A] time = 2.25, size = 29, normalized size = 0.88

$$-\frac{2x+1}{2(x^2+1)} - 2 \arctan(x) - \frac{1}{2} \log(x^2+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")

[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)

mupad [B] time = 2.11, size = 33, normalized size = 1.00

$$\ln(x) - \frac{x + \frac{1}{2}}{x^2 + 1} + \ln(x - i) \left(-\frac{1}{2} + 1i \right) + \ln(x + 1i) \left(-\frac{1}{2} - i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2),x)

[Out] log(x) - log(x + 1i)*(1/2 + 1i) - log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2 + 1)

sympy [A] time = 0.15, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2} + \log(x) - \frac{\log(x^2+1)}{2} - 2 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)

[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)

$$3.311 \quad \int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$$

Optimal. Leaf size=25

$$\frac{x^2}{2} + \frac{1}{2} \log(1-x^2) + x - \log(x)$$

[Out] $x+1/2*x^2-\ln(x)+1/2*\ln(-x^2+1)$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1593, 1802, 260}

$$\frac{x^2}{2} + \frac{1}{2} \log(1-x^2) + x - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3), x]

[Out] $x + x^2/2 - \text{Log}[x] + \text{Log}[1 - x^2]/2$

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q-p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx &= \int \frac{1-x-x^2+x^3+x^4}{x(-1+x^2)} dx \\ &= \int \left(1 - \frac{1}{x} + x + \frac{x}{-1+x^2}\right) dx \\ &= x + \frac{x^2}{2} - \log(x) + \int \frac{x}{-1+x^2} dx \\ &= x + \frac{x^2}{2} - \log(x) + \frac{1}{2} \log(1-x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2} + \frac{1}{2} \log(1-x^2) + x - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3), x]

[Out] $x + x^2/2 - \text{Log}[x] + \text{Log}[1 - x^2]/2$

fricas [A] time = 0.56, size = 19, normalized size = 0.76

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(x^2 - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")`

[Out] $1/2*x^2 + x + 1/2*\log(x^2 - 1) - \log(x)$

giac [A] time = 0.29, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(|x + 1|) + \frac{1}{2}\log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")`

[Out] $1/2*x^2 + x + 1/2*\log(\text{abs}(x + 1)) + 1/2*\log(\text{abs}(x - 1)) - \log(\text{abs}(x))$

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{x^2}{2} + x - \ln(x) + \frac{\ln(x - 1)}{2} + \frac{\ln(x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+x^3-x^2-x+1)/(x^3-x),x)`

[Out] $1/2*x^2+x+1/2*\ln(x-1)+1/2*\ln(x+1)-\ln(x)$

maxima [A] time = 1.05, size = 23, normalized size = 0.92

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(x + 1) + \frac{1}{2}\log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")`

[Out] $1/2*x^2 + x + 1/2*\log(x + 1) + 1/2*\log(x - 1) - \log(x)$

mupad [B] time = 0.04, size = 19, normalized size = 0.76

$$x + \frac{\ln(x^2 - 1)}{2} - \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)`

[Out] $x + \log(x^2 - 1)/2 - \log(x) + x^2/2$

sympy [A] time = 0.10, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)`

[Out] $x**2/2 + x - \log(x) + \log(x**2 - 1)/2$

$$3.312 \quad \int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=36

$$-\frac{1}{2} \log(x^2+1) + \log(x^2+2) + 6 \tan^{-1}(x) - 5\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6725, 635, 203, 260}

$$-\frac{1}{2} \log(x^2+1) + \log(x^2+2) + 6 \tan^{-1}(x) - 5\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]

[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)} dx &= \int \left(\frac{6-x}{1+x^2} + \frac{2(-5+x)}{2+x^2} \right) dx \\ &= 2 \int \frac{-5+x}{2+x^2} dx + \int \frac{6-x}{1+x^2} dx \\ &= 2 \int \frac{x}{2+x^2} dx + 6 \int \frac{1}{1+x^2} dx - 10 \int \frac{1}{2+x^2} dx - \int \frac{x}{1+x^2} dx \\ &= 6 \tan^{-1}(x) - 5\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{2} \log(1+x^2) + \log(2+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$-\frac{1}{2} \log(x^2 + 1) + \log(x^2 + 2) + 6 \tan^{-1}(x) - 5\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)), x]

[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]

fricas [A] time = 0.84, size = 31, normalized size = 0.86

$$-5\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6 \arctan(x) + \log(x^2 + 2) - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2), x, algorithm="fricas")

[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2 + 1)

giac [A] time = 0.34, size = 31, normalized size = 0.86

$$-5\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6 \arctan(x) + \log(x^2 + 2) - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2), x, algorithm="giac")

[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2 + 1)

maple [A] time = 0.00, size = 32, normalized size = 0.89

$$6 \arctan(x) - 5\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right) - \frac{\ln(x^2 + 1)}{2} + \ln(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2), x)

[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.10, size = 31, normalized size = 0.86

$$-5\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6 \arctan(x) + \log(x^2 + 2) - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2), x, algorithm="maxima")

[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2 + 1)

mupad [B] time = 0.11, size = 56, normalized size = 1.56

$$\ln(x - i) \left(-\frac{1}{2} - 3i\right) + \ln(x + i) \left(-\frac{1}{2} + 3i\right) + \ln\left(x - \sqrt{2}i\right) \left(1 + \frac{\sqrt{2}5i}{2}\right) - \ln\left(x + \sqrt{2}i\right) \left(-1 + \frac{\sqrt{2}5i}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)`

[Out] `log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x - 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)`

sympy [A] time = 0.20, size = 36, normalized size = 1.00

$$-\frac{\log(x^2 + 1)}{2} + \log(x^2 + 2) + 6 \operatorname{atan}(x) - 5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)`

[Out] `-log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)`

$$3.313 \quad \int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx$$

Optimal. Leaf size=29

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x)$$

[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)

Rubi [A] time = 0.11, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6725, 203, 199}

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2), x]

[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx &= \int \left(\frac{1}{9(1+x^2)} - \frac{13}{3(4+x^2)^2} + \frac{8}{9(4+x^2)} \right) dx \\ &= \frac{1}{9} \int \frac{1}{1+x^2} dx + \frac{8}{9} \int \frac{1}{4+x^2} dx - \frac{13}{3} \int \frac{1}{(4+x^2)^2} dx \\ &= -\frac{13x}{24(4+x^2)} + \frac{4}{9} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x) - \frac{13}{24} \int \frac{1}{4+x^2} dx \\ &= -\frac{13x}{24(4+x^2)} + \frac{25}{144} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2), x]

[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9

fricas [A] time = 0.99, size = 33, normalized size = 1.14

$$\frac{25(x^2+4) \arctan\left(\frac{1}{2}x\right) + 16(x^2+4) \arctan(x) - 78x}{144(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")

[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4)

giac [A] time = 0.23, size = 21, normalized size = 0.72

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144} \arctan\left(\frac{1}{2}x\right) + \frac{1}{9} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")

[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$-\frac{13x}{24(x^2+4)} + \frac{\arctan(x)}{9} + \frac{25 \arctan\left(\frac{x}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)

[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)

maxima [A] time = 2.09, size = 21, normalized size = 0.72

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144} \arctan\left(\frac{1}{2}x\right) + \frac{1}{9} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")

[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)

mupad [B] time = 0.04, size = 23, normalized size = 0.79

$$\frac{25 \operatorname{atan}\left(\frac{x}{2}\right)}{144} + \frac{\operatorname{atan}(x)}{9} - \frac{13x}{24(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2), x)`

[Out] `(25*atan(x/2))/144 + atan(x)/9 - (13*x)/(24*(x^2 + 4))`

sympy [A] time = 0.17, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2 + 96} + \frac{25 \operatorname{atan}\left(\frac{x}{2}\right)}{144} + \frac{\operatorname{atan}(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2, x)`

[Out] `-13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9`

$$3.314 \quad \int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx$$

Optimal. Leaf size=46

$$\frac{5}{8} \log(x^2 + x + 2) - \frac{1}{2x} - \frac{\log(x)}{4} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

[Out] $-1/2/x - 1/4*\ln(x) + 5/8*\ln(x^2+x+2) + 1/28*\arctan(1/7*(1+2*x)*7^{(1/2)})*7^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1594, 1628, 634, 618, 204, 628}

$$\frac{5}{8} \log(x^2 + x + 2) - \frac{1}{2x} - \frac{\log(x)}{4} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4), x]

[Out] $-1/(2*x) + \text{ArcTan}[(1 + 2*x)/\text{Sqrt}[7]]/(4*\text{Sqrt}[7]) - \text{Log}[x]/4 + (5*\text{Log}[2 + x + x^2])/8$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x

], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx &= \int \frac{1+x^2+x^3}{x^2(2+x+x^2)} dx \\
 &= \int \left(\frac{1}{2x^2} - \frac{1}{4x} + \frac{3+5x}{4(2+x+x^2)} \right) dx \\
 &= -\frac{1}{2x} - \frac{\log(x)}{4} + \frac{1}{4} \int \frac{3+5x}{2+x+x^2} dx \\
 &= -\frac{1}{2x} - \frac{\log(x)}{4} + \frac{1}{8} \int \frac{1}{2+x+x^2} dx + \frac{5}{8} \int \frac{1+2x}{2+x+x^2} dx \\
 &= -\frac{1}{2x} - \frac{\log(x)}{4} + \frac{5}{8} \log(2+x+x^2) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-7-x^2} dx, x, 1+2x \right) \\
 &= -\frac{1}{2x} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}} - \frac{\log(x)}{4} + \frac{5}{8} \log(2+x+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{5}{8} \log(x^2+x+2) - \frac{1}{2x} - \frac{\log(x)}{4} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4), x]

[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x + x^2])/8

fricas [A] time = 0.81, size = 39, normalized size = 0.85

$$\frac{2\sqrt{7}x \arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right) + 35x \log(x^2+x+2) - 14x \log(x) - 28}{56x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2), x, algorithm="fricas")

[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 14*x*log(x) - 28)/x

giac [A] time = 0.30, size = 36, normalized size = 0.78

$$\frac{1}{28} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x+1)\right) - \frac{1}{2x} + \frac{5}{8} \log(x^2+x+2) - \frac{1}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2), x, algorithm="giac")

[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) - 1/4*log(abs(x))

maple [A] time = 0.01, size = 36, normalized size = 0.78

$$\frac{\sqrt{7} \arctan\left(\frac{(2x+1)\sqrt{7}}{7}\right)}{28} - \frac{\ln(x)}{4} + \frac{5 \ln(x^2 + x + 2)}{8} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x)

[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(2*x+1)*7^(1/2))*7^(1/2)

maxima [A] time = 2.09, size = 35, normalized size = 0.76

$$\frac{1}{28} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x + 1)\right) - \frac{1}{2x} + \frac{5}{8} \log(x^2 + x + 2) - \frac{1}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")

[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) - 1/4*log(x)

mupad [B] time = 2.17, size = 49, normalized size = 1.07

$$-\frac{\ln(x)}{4} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{7} 1i}{2}\right) \left(-\frac{5}{8} + \frac{\sqrt{7} 1i}{56}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{7} 1i}{2}\right) \left(\frac{5}{8} + \frac{\sqrt{7} 1i}{56}\right) - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)

[Out] log(x + (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 + 5/8) - log(x - (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 - 5/8) - log(x)/4 - 1/(2*x)

sympy [A] time = 0.16, size = 46, normalized size = 1.00

$$-\frac{\log(x)}{4} + \frac{5 \log(x^2 + x + 2)}{8} + \frac{\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x}{7} + \frac{\sqrt{7}}{7}\right)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)

[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)/28 - 1/(2*x)

$$3.315 \quad \int \frac{1-12x+x^2+x^3}{-12+x+x^2} dx$$

Optimal. Leaf size=22

$$\frac{x^2}{2} - \frac{2}{7} \tanh^{-1}\left(\frac{1}{7}(2x+1)\right)$$

[Out] 1/2*x^2-2/7*arctanh(1/7+2/7*x)

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1657, 616, 31}

$$\frac{x^2}{2} + \frac{1}{7} \log(3-x) - \frac{1}{7} \log(x+4)$$

Antiderivative was successfully verified.

[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2), x]

[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1-12x+x^2+x^3}{-12+x+x^2} dx &= \int \left(x + \frac{1}{-12+x+x^2} \right) dx \\ &= \frac{x^2}{2} + \int \frac{1}{-12+x+x^2} dx \\ &= \frac{x^2}{2} + \frac{1}{7} \int \frac{1}{-3+x} dx - \frac{1}{7} \int \frac{1}{4+x} dx \\ &= \frac{x^2}{2} + \frac{1}{7} \log(3-x) - \frac{1}{7} \log(4+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.18

$$\frac{x^2}{2} + \frac{1}{7} \log(3-x) - \frac{1}{7} \log(x+4)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2), x]

[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7

fricas [A] time = 0.85, size = 18, normalized size = 0.82

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12), x, algorithm="fricas")

[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)

giac [A] time = 0.28, size = 20, normalized size = 0.91

$$\frac{1}{2}x^2 - \frac{1}{7}\log(|x+4|) + \frac{1}{7}\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12), x, algorithm="giac")

[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^2}{2} + \frac{\ln(x-3)}{7} - \frac{\ln(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2-12*x+1)/(x^2+x-12), x)

[Out] 1/2*x^2-1/7*ln(x+4)+1/7*ln(x-3)

maxima [A] time = 0.97, size = 18, normalized size = 0.82

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12), x, algorithm="maxima")

[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)

mupad [B] time = 0.04, size = 14, normalized size = 0.64

$$\frac{x^2}{2} - \frac{2 \operatorname{atanh}\left(\frac{2x}{7} + \frac{1}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12), x)

[Out] x^2/2 - (2*atanh((2*x)/7 + 1/7))/7

sympy [A] time = 0.10, size = 17, normalized size = 0.77

$$\frac{x^2}{2} + \frac{\log(x-3)}{7} - \frac{\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12), x)

[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7

$$3.316 \quad \int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx$$

Optimal. Leaf size=17

$$2 \log(1-x) + \log(x) + 3 \log(x+3)$$

[Out] 2*ln(1-x)+ln(x)+3*ln(3+x)

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1594, 1628}

$$2 \log(1-x) + \log(x) + 3 \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3), x]

[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_)*((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx &= \int \frac{-3+5x+6x^2}{x(-3+2x+x^2)} dx \\ &= \int \left(\frac{2}{-1+x} + \frac{1}{x} + \frac{3}{3+x} \right) dx \\ &= 2 \log(1-x) + \log(x) + 3 \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$2 \log(1-x) + \log(x) + 3 \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3), x]

[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]

fricas [A] time = 1.13, size = 15, normalized size = 0.88

$$3 \log(x+3) + 2 \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x), x, algorithm="fricas")

[Out] $3\log(x + 3) + 2\log(x - 1) + \log(x)$

giac [A] time = 0.38, size = 18, normalized size = 1.06

$$3 \log(|x + 3|) + 2 \log(|x - 1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")`

[Out] $3\log(\text{abs}(x + 3)) + 2\log(\text{abs}(x - 1)) + \log(\text{abs}(x))$

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) + 2 \ln(x - 1) + 3 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x)`

[Out] $2\ln(x-1)+3\ln(x+3)+\ln(x)$

maxima [A] time = 0.85, size = 15, normalized size = 0.88

$$3 \log(x + 3) + 2 \log(x - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")`

[Out] $3\log(x + 3) + 2\log(x - 1) + \log(x)$

mupad [B] time = 0.07, size = 15, normalized size = 0.88

$$2 \ln(x - 1) + 3 \ln(x + 3) + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)`

[Out] $2\log(x - 1) + 3\log(x + 3) + \log(x)$

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\log(x) + 2 \log(x - 1) + 3 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)`

[Out] $\log(x) + 2\log(x - 1) + 3\log(x + 3)$

$$3.317 \quad \int \frac{-2+3x+5x^2}{2x^2+x^3} dx$$

Optimal. Leaf size=14

$$\frac{1}{x} + 2 \log(x) + 3 \log(x + 2)$$

[Out] 1/x+2*ln(x)+3*ln(2+x)

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1593, 893}

$$\frac{1}{x} + 2 \log(x) + 3 \log(x + 2)$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3), x]

[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{-2+3x+5x^2}{2x^2+x^3} dx &= \int \frac{-2+3x+5x^2}{x^2(2+x)} dx \\ &= \int \left(-\frac{1}{x^2} + \frac{2}{x} + \frac{3}{2+x} \right) dx \\ &= \frac{1}{x} + 2 \log(x) + 3 \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{x} + 2 \log(x) + 3 \log(x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3), x]

[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]

fricas [A] time = 0.77, size = 18, normalized size = 1.29

$$\frac{3x \log(x + 2) + 2x \log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")

[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x

giac [A] time = 0.37, size = 16, normalized size = 1.14

$$\frac{1}{x} + 3 \log(|x + 2|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")

[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))

maple [A] time = 0.01, size = 15, normalized size = 1.07

$$2 \ln(x) + 3 \ln(x + 2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x)

[Out] 1/x+2*ln(x)+3*ln(x+2)

maxima [A] time = 1.00, size = 14, normalized size = 1.00

$$\frac{1}{x} + 3 \log(x + 2) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")

[Out] 1/x + 3*log(x + 2) + 2*log(x)

mupad [B] time = 2.12, size = 14, normalized size = 1.00

$$3 \ln(x + 2) + 2 \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)

[Out] 3*log(x + 2) + 2*log(x) + 1/x

sympy [A] time = 0.12, size = 14, normalized size = 1.00

$$2 \log(x) + 3 \log(x + 2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)

[Out] 2*log(x) + 3*log(x + 2) + 1/x

$$3.318 \quad \int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx$$

Optimal. Leaf size=19

$$\log(1-x) - 2\log(x+2) - 3\log(x+3)$$

[Out] $\ln(1-x)-2*\ln(2+x)-3*\ln(3+x)$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2074}

$$\log(1-x) - 2\log(x+2) - 3\log(x+3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3), x]$

[Out] $\text{Log}[1 - x] - 2*\text{Log}[2 + x] - 3*\text{Log}[3 + x]$

Rule 2074

$\text{Int}[(P_)^{(p_)}*(Q_)^{(q_.)}, x_Symbol] \rightarrow \text{With}[\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Q^q, x], x] /; \text{!SumQ}[\text{NonfreeFactors}[PP, x]] /; \text{FreeQ}[q, x] \&\& \text{PolyQ}[P, x] \&\& \text{PolyQ}[Q, x] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[P, x]$

Rubi steps

$$\begin{aligned} \int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx &= \int \left(\frac{1}{-1+x} - \frac{2}{2+x} - \frac{3}{3+x} \right) dx \\ &= \log(1-x) - 2\log(2+x) - 3\log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.32

$$-2 \left(-\frac{1}{2} \log(1-x) + \log(x+2) + \frac{3}{2} \log(x+3) \right)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3), x]$

[Out] $-2*(-1/2*\text{Log}[1 - x] + \text{Log}[2 + x] + (3*\text{Log}[3 + x])/2)$

fricas [A] time = 1.14, size = 17, normalized size = 0.89

$$-3 \log(x+3) - 2 \log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-4*x^2-2*x+18)/(x^3+4*x^2+x-6), x, \text{algorithm}=\text{"fricas"})$

[Out] $-3*\log(x + 3) - 2*\log(x + 2) + \log(x - 1)$

giac [A] time = 0.29, size = 20, normalized size = 1.05

$$-3 \log(|x+3|) - 2 \log(|x+2|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")

[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))

maple [A] time = 0.01, size = 18, normalized size = 0.95

$$\ln(x - 1) - 2 \ln(x + 2) - 3 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)

[Out] ln(x-1)-2*ln(x+2)-3*ln(x+3)

maxima [A] time = 1.03, size = 17, normalized size = 0.89

$$-3 \log(x + 3) - 2 \log(x + 2) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")

[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)

mupad [B] time = 2.12, size = 17, normalized size = 0.89

$$\ln(x - 1) - 2 \ln(x + 2) - 3 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)

[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)

sympy [A] time = 0.13, size = 17, normalized size = 0.89

$$\log(x - 1) - 2 \log(x + 2) - 3 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)

[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)

$$3.319 \quad \int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx$$

Optimal. Leaf size=23

$$\frac{1}{2} \log(x^2 + 4) - \frac{3}{2} \tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}(x)$$

[Out] $-3/2*\arctan(1/2*x)+\arctan(x)+1/2*\ln(x^2+4)$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1673, 1166, 203, 1247, 626, 31}

$$\frac{1}{2} \log(x^2 + 4) - \frac{3}{2} \tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4), x]

[Out] $(-3*\text{ArcTan}[x/2])/2 + \text{ArcTan}[x] + \text{Log}[4 + x^2]/2$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 626

Int[((d_) + (e_.)*(x_))^{(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]}

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^{(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]}

Rule 1673

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]

&& !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned} \int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx &= \int \frac{1-2x^2}{4+5x^2+x^4} dx + \int \frac{x(1+x^2)}{4+5x^2+x^4} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{4+5x+x^2} dx, x, x^2 \right) - 3 \int \frac{1}{4+x^2} dx + \int \frac{1}{1+x^2} dx \\ &= -\frac{3}{2} \tan^{-1} \left(\frac{x}{2} \right) + \tan^{-1}(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{4+x} dx, x, x^2 \right) \\ &= -\frac{3}{2} \tan^{-1} \left(\frac{x}{2} \right) + \tan^{-1}(x) + \frac{1}{2} \log(4+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 4) - \frac{3}{2} \tan^{-1} \left(\frac{x}{2} \right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4), x]

[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2

fricas [A] time = 1.00, size = 17, normalized size = 0.74

$$-\frac{3}{2} \arctan \left(\frac{1}{2} x \right) + \arctan(x) + \frac{1}{2} \log(x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")

[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)

giac [A] time = 0.36, size = 17, normalized size = 0.74

$$-\frac{3}{2} \arctan \left(\frac{1}{2} x \right) + \arctan(x) + \frac{1}{2} \log(x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")

[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\arctan(x) - \frac{3 \arctan \left(\frac{x}{2} \right)}{2} + \frac{\ln(x^2 + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4), x)

[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)

maxima [A] time = 2.12, size = 17, normalized size = 0.74

$$-\frac{3}{2} \arctan\left(\frac{1}{2}x\right) + \arctan(x) + \frac{1}{2} \log(x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")

[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)

mupad [B] time = 0.05, size = 33, normalized size = 1.43

$$-\operatorname{atan}\left(\frac{1305}{4(144x-162)} + \frac{9}{8}\right) + \ln(x-2i)\left(\frac{1}{2} + \frac{3}{4}i\right) + \ln(x+2i)\left(\frac{1}{2} - \frac{3}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4),x)

[Out] log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x - 162)) + 9/8)

sympy [A] time = 0.18, size = 19, normalized size = 0.83

$$\frac{\log(x^2 + 4)}{2} - \frac{3 \operatorname{atan}\left(\frac{x}{2}\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)

[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)

$$3.320 \quad \int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx$$

Optimal. Leaf size=63

$$\frac{11049 \log(x^2 + x + 5)}{260015} - \frac{3146 \log(7 - 3x)}{80155} - \frac{334}{323} \log(2x+1) + \frac{4822 \log(5x + 2)}{4879} + \frac{3988 \tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

[Out] -3146/80155*ln(7-3*x)-334/323*ln(1+2*x)+4822/4879*ln(2+5*x)+11049/260015*ln(x^2+x+5)+3988/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2074, 634, 618, 204, 628}

$$\frac{11049 \log(x^2 + x + 5)}{260015} - \frac{3146 \log(7 - 3x)}{80155} - \frac{334}{323} \log(2x+1) + \frac{4822 \log(5x + 2)}{4879} + \frac{3988 \tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

Antiderivative was successfully verified.

[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 30*x^5), x]

[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x + x^2])/260015

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned}
\int \frac{-32 + 5x - 27x^2 + 4x^3}{-70 - 299x - 286x^2 + 50x^3 - 13x^4 + 30x^5} dx &= \int \left(-\frac{668}{323(1+2x)} - \frac{9438}{80155(-7+3x)} + \frac{24110}{4879(2+5x)} + \frac{334}{323} \log(1+2x) \right) dx \\
&= -\frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x) + \frac{4822 \log(2+5x)}{4879} \\
&= -\frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x) + \frac{4822 \log(2+5x)}{4879} \\
&= -\frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x) + \frac{4822 \log(2+5x)}{4879} \\
&= \frac{3988 \tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}} - \frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.90

$$\frac{453009 \log(x^2 + x + 5) - 418418 \log(7 - 3x) - 11023670 \log(2x + 1) + 10536070 \log(5x + 2) + 163508\sqrt{19}}{10660615}$$

Antiderivative was successfully verified.

[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 30*x^5), x]

[Out] (163508*sqrt(19)*ArcTan[(1 + 2*x)/sqrt(19)] - 418418*Log[7 - 3*x] - 11023670*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615

fricas [A] time = 0.98, size = 50, normalized size = 0.79

$$\frac{3988}{260015} \sqrt{19} \arctan\left(\frac{1}{19} \sqrt{19} (2x + 1)\right) + \frac{11049}{260015} \log(x^2 + x + 5) + \frac{4822}{4879} \log(5x + 2) - \frac{3146}{80155} \log(3x - 7) - \frac{334}{323} \log(2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70), x, algorithm="fricas")

[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(2*x + 1)

giac [A] time = 0.31, size = 53, normalized size = 0.84

$$\frac{3988}{260015} \sqrt{19} \arctan\left(\frac{1}{19} \sqrt{19} (2x + 1)\right) + \frac{11049}{260015} \log(x^2 + x + 5) + \frac{4822}{4879} \log(5x + 2) - \frac{3146}{80155} \log(3x - 7) - \frac{334}{323} \log(2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70), x, algorithm="giac")

[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 334/323*log(abs(2*x + 1))

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{3988\sqrt{19} \arctan\left(\frac{(2x+1)\sqrt{19}}{19}\right)}{260015} - \frac{334 \ln(2x + 1)}{323} + \frac{4822 \ln(5x + 2)}{4879} - \frac{3146 \ln(3x - 7)}{80155} + \frac{11049 \ln(x^2 + x + 5)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)`

[Out] $4822/4879*\ln(2+5*x)-3146/80155*\ln(3*x-7)-334/323*\ln(2*x+1)+11049/260015*\ln(x^2+x+5)+3988/260015*\arctan(1/19*(2*x+1)*19^{(1/2)})*19^{(1/2)}$

maxima [A] time = 2.11, size = 50, normalized size = 0.79

$$\frac{3988}{260015} \sqrt{19} \arctan\left(\frac{1}{19} \sqrt{19} (2x + 1)\right) + \frac{11049}{260015} \log(x^2 + x + 5) + \frac{4822}{4879} \log(5x + 2) - \frac{3146}{80155} \log(3x - 7) - \frac{334}{323}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,algorithm="maxima")`

[Out] $3988/260015*\sqrt{19}*\arctan(1/19*\sqrt{19}*(2*x + 1)) + 11049/260015*\log(x^2 + x + 5) + 4822/4879*\log(5*x + 2) - 3146/80155*\log(3*x - 7) - 334/323*\log(2*x + 1)$

mupad [B] time = 2.21, size = 58, normalized size = 0.92

$$\frac{4822 \ln\left(x + \frac{2}{5}\right)}{4879} - \frac{334 \ln\left(x + \frac{1}{2}\right)}{323} - \frac{3146 \ln\left(x - \frac{7}{3}\right)}{80155} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{19} i}{2}\right) \left(-\frac{11049}{260015} + \frac{\sqrt{19} 1994i}{260015}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{19} i}{2}\right) \left(-\frac{11049}{260015} + \frac{\sqrt{19} 1994i}{260015}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x^5 + 70),x)`

[Out] $(4822*\log(x + 2/5))/4879 - (334*\log(x + 1/2))/323 - (3146*\log(x - 7/3))/80155 - \log(x - (19^{(1/2)}*1i)/2 + 1/2)*((19^{(1/2)}*1994i)/260015 - 11049/260015) + \log(x + (19^{(1/2)}*1i)/2 + 1/2)*((19^{(1/2)}*1994i)/260015 + 11049/260015)$

sympy [A] time = 0.35, size = 68, normalized size = 1.08

$$-\frac{3146 \log\left(x - \frac{7}{3}\right)}{80155} + \frac{4822 \log\left(x + \frac{2}{5}\right)}{4879} - \frac{334 \log\left(x + \frac{1}{2}\right)}{323} + \frac{11049 \log\left(x^2 + x + 5\right)}{260015} + \frac{3988 \sqrt{19} \operatorname{atan}\left(\frac{2\sqrt{19}x}{19} + \frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-70),x)`

[Out] $-3146*\log(x - 7/3)/80155 + 4822*\log(x + 2/5)/4879 - 334*\log(x + 1/2)/323 + 11049*\log(x**2 + x + 5)/260015 + 3988*\sqrt{19}*\operatorname{atan}(2*\sqrt{19}*x/19 + \sqrt{19}/19)/260015$

$$3.321 \quad \int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} dx$$

Optimal. Leaf size=69

$$-\frac{502x+313}{1452(2x^2+1)} + \frac{2843 \log(2x^2+1)}{7986} + \frac{5828}{9075(2-5x)} - \frac{59096 \log(2-5x)}{99825} + \frac{503 \tan^{-1}(\sqrt{2}x)}{7986\sqrt{2}}$$

[Out] 5828/9075/(2-5*x)+1/1452*(-313-502*x)/(2*x^2+1)-59096/99825*ln(2-5*x)+2843/7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2074, 639, 203, 635, 260}

$$-\frac{502x+313}{1452(2x^2+1)} + \frac{2843 \log(2x^2+1)}{7986} + \frac{5828}{9075(2-5x)} - \frac{59096 \log(2-5x)}{99825} + \frac{272\sqrt{2} \tan^{-1}(\sqrt{2}x)}{1331} - \frac{251 \tan^{-1}(x)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*x^5 + 100*x^6), x]

[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 639

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\int \frac{8 - 13x^2 - 7x^3 + 12x^5}{4 - 20x + 41x^2 - 80x^3 + 116x^4 - 80x^5 + 100x^6} dx = \int \left(\frac{5828}{1815(-2 + 5x)^2} - \frac{59096}{19965(-2 + 5x)} + \frac{-251 + 313x}{363(1 + 2x^2)^2} \right) dx$$

$$= \frac{5828}{9075(2 - 5x)} - \frac{59096 \log(2 - 5x)}{99825} + \frac{2 \int \frac{816 + 2843x}{1 + 2x^2} dx}{3993} + \dots$$

$$= \frac{5828}{9075(2 - 5x)} - \frac{313 + 502x}{1452(1 + 2x^2)} - \frac{59096 \log(2 - 5x)}{99825} - \frac{2}{7} \dots$$

$$= \frac{5828}{9075(2 - 5x)} - \frac{313 + 502x}{1452(1 + 2x^2)} - \frac{251 \tan^{-1}(\sqrt{2}x)}{726\sqrt{2}} + \dots$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.97

$$\frac{142150 \log(2x^2 + 1) - \frac{33(36458x^2 + 4675x + 2554)}{10x^3 - 4x^2 + 5x - 2} - 236384 \log(2 - 5x) + 12575\sqrt{2} \tan^{-1}(\sqrt{2}x)}{399300}$$

Antiderivative was successfully verified.

[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*x^5 + 100*x^6), x]

[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300

fricas [A] time = 0.79, size = 103, normalized size = 1.49

$$\frac{12575 \sqrt{2} (10x^3 - 4x^2 + 5x - 2) \arctan(\sqrt{2}x) - 1203114x^2 + 142150(10x^3 - 4x^2 + 5x - 2) \log(2x^2 + 1) - 236384 \log(2 - 5x)}{399300(10x^3 - 4x^2 + 5x - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4), x, algorithm="fricas")

[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x - 2)

giac [A] time = 0.24, size = 59, normalized size = 0.86

$$\frac{503}{15972} \sqrt{2} \arctan(\sqrt{2}x) - \frac{36458x^2 + 4675x + 2554}{12100(2x^2 + 1)(5x - 2)} + \frac{2843}{7986} \log(2x^2 + 1) - \frac{59096}{99825} \log(|5x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4), x, algorithm="giac")

[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/((2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x - 2))

maple [A] time = 0.01, size = 54, normalized size = 0.78

$$\frac{503\sqrt{2} \arctan(\sqrt{2}x)}{15972} - \frac{59096 \ln(5x - 2)}{99825} + \frac{2843 \ln(2x^2 + 1)}{7986} - \frac{5828}{9075(5x - 2)} + \frac{-\frac{2761x}{4} - \frac{3443}{8}}{3993x^2 + \frac{3993}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x)`

[Out] $-5828/9075/(5x-2) - 59096/99825 \ln(5x-2) + 1/3993 * (-2761/4x - 3443/8)/(x^2+1/2) + 2843/7986 \ln(2x^2+1) + 503/15972 \arctan(2^{1/2}x) * 2^{1/2}$

maxima [A] time = 2.07, size = 59, normalized size = 0.86

$$\frac{503}{15972} \sqrt{2} \arctan(\sqrt{2}x) - \frac{36458x^2 + 4675x + 2554}{12100(10x^3 - 4x^2 + 5x - 2)} + \frac{2843}{7986} \log(2x^2 + 1) - \frac{59096}{99825} \log(5x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x, algorithm="maxima")`

[Out] $503/15972 \sqrt{2} \arctan(\sqrt{2}x) - 1/12100 * (36458x^2 + 4675x + 2554)/(10x^3 - 4x^2 + 5x - 2) + 2843/7986 \log(2x^2 + 1) - 59096/99825 \log(5x - 2)$

mupad [B] time = 2.18, size = 71, normalized size = 1.03

$$-\frac{59096 \ln\left(x - \frac{2}{5}\right) - \frac{18229x^2}{60500} + \frac{17x}{440} + \frac{1277}{60500}}{99825} - \ln\left(x - \frac{\sqrt{2} \operatorname{Im}}{2}\right) \left(-\frac{2843}{7986} + \frac{\sqrt{2} 503i}{31944}\right) + \ln\left(x + \frac{\sqrt{2} \operatorname{Im}}{2}\right) \left(\frac{2843}{7986} + \frac{\sqrt{2} 503i}{31944}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*x^5 + 100*x^6 + 4),x)`

[Out] $\log(x + (2^{1/2} * 1i)/2) * ((2^{1/2} * 503i)/31944 + 2843/7986) - ((17x)/440 + (18229x^2)/60500 + 1277/60500)/(x/2 - (2x^2)/5 + x^3 - 1/5) - \log(x - (2^{1/2} * 1i)/2) * ((2^{1/2} * 503i)/31944 - 2843/7986) - (59096 * \log(x - 2/5))/99825$

sympy [A] time = 0.21, size = 65, normalized size = 0.94

$$\frac{-36458x^2 - 4675x - 2554}{121000x^3 - 48400x^2 + 60500x - 24200} - \frac{59096 \log\left(x - \frac{2}{5}\right)}{99825} + \frac{2843 \log\left(x^2 + \frac{1}{2}\right)}{7986} + \frac{503\sqrt{2} \operatorname{atan}(\sqrt{2}x)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*x**2-20*x+4),x)`

[Out] $(-36458x^2 - 4675x - 2554)/(121000x^3 - 48400x^2 + 60500x - 24200) - 59096 \log(x - 2/5)/99825 + 2843 \log(x^2 + 1/2)/7986 + 503 \sqrt{2} \operatorname{atan}(\sqrt{2}x)/15972$

$$3.322 \quad \int \frac{9+x^4}{x^2(9+x^2)} dx$$

Optimal. Leaf size=17

$$x - \frac{1}{x} - \frac{10}{3} \tan^{-1}\left(\frac{x}{3}\right)$$

[Out] -1/x+x-10/3*arctan(1/3*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1262, 203}

$$x - \frac{1}{x} - \frac{10}{3} \tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[(9 + x^4)/(x^2*(9 + x^2)), x]

[Out] -x^(-1) + x - (10*ArcTan[x/3])/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1262

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{9+x^4}{x^2(9+x^2)} dx &= \int \left(1 + \frac{1}{x^2} - \frac{10}{9+x^2}\right) dx \\ &= -\frac{1}{x} + x - 10 \int \frac{1}{9+x^2} dx \\ &= -\frac{1}{x} + x - \frac{10}{3} \tan^{-1}\left(\frac{x}{3}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$x - \frac{1}{x} - \frac{10}{3} \tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(9 + x^4)/(x^2*(9 + x^2)), x]

[Out] -x^(-1) + x - (10*ArcTan[x/3])/3

fricas [A] time = 0.89, size = 19, normalized size = 1.12

$$\frac{3x^2 - 10x \arctan\left(\frac{1}{3}x\right) - 3}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="fricas")

[Out] 1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x

giac [A] time = 0.30, size = 13, normalized size = 0.76

$$x - \frac{1}{x} - \frac{10}{3} \arctan\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="giac")

[Out] x - 1/x - 10/3*arctan(1/3*x)

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$x - \frac{10 \arctan\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+9)/x^2/(x^2+9),x)

[Out] -1/x+x-10/3*arctan(1/3*x)

maxima [A] time = 2.04, size = 13, normalized size = 0.76

$$x - \frac{1}{x} - \frac{10}{3} \arctan\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="maxima")

[Out] x - 1/x - 10/3*arctan(1/3*x)

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$x - \frac{10 \operatorname{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 9)/(x^2*(x^2 + 9)),x)

[Out] x - (10*atan(x/3))/3 - 1/x

sympy [A] time = 0.11, size = 12, normalized size = 0.71

$$x - \frac{10 \operatorname{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+9)/x**2/(x**2+9),x)

[Out] x - 10*atan(x/3)/3 - 1/x

$$3.323 \quad \int \frac{2x+x^4}{1+x^2} dx$$

Optimal. Leaf size=19

$$\frac{x^3}{3} + \log(x^2 + 1) - x + \tan^{-1}(x)$$

[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{x^3}{3} + \log(x^2 + 1) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^4)/(1 + x^2), x]

[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{2x + x^4}{1 + x^2} dx &= \int \frac{x(2 + x^3)}{1 + x^2} dx \\
&= \int \left(-1 + x^2 + \frac{1 + 2x}{1 + x^2} \right) dx \\
&= -x + \frac{x^3}{3} + \int \frac{1 + 2x}{1 + x^2} dx \\
&= -x + \frac{x^3}{3} + 2 \int \frac{x}{1 + x^2} dx + \int \frac{1}{1 + x^2} dx \\
&= -x + \frac{x^3}{3} + \tan^{-1}(x) + \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^3}{3} + \log(x^2 + 1) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^4)/(1 + x^2), x]

[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]

fricas [A] time = 0.91, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x)/(x^2+1), x, algorithm="fricas")

[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)

giac [A] time = 0.27, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x)/(x^2+1), x, algorithm="giac")

[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x^3}{3} - x + \arctan(x) + \ln(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+2*x)/(x^2+1), x)

[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)

maxima [A] time = 2.03, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="maxima")

[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)

mupad [B] time = 2.10, size = 17, normalized size = 0.89

$$\ln(x^2 + 1) - x + \operatorname{atan}(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^4)/(x^2 + 1),x)

[Out] log(x^2 + 1) - x + atan(x) + x^3/3

sympy [A] time = 0.10, size = 15, normalized size = 0.79

$$\frac{x^3}{3} - x + \log(x^2 + 1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+2*x)/(x**2+1),x)

[Out] x**3/3 - x + log(x**2 + 1) + atan(x)

$$3.324 \quad \int \frac{-x+x^3}{(-1+x)^2(1+x^2)} dx$$

Optimal. Leaf size=9

$$\log(1-x) + \tan^{-1}(x)$$

[Out] arctan(x)+ln(1-x)

Rubi [A] time = 0.08, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1586, 1593, 1629, 203}

$$\log(1-x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)), x]

[Out] ArcTan[x] + Log[1 - x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-x+x^3}{(-1+x)^2(1+x^2)} dx &= \int \frac{x+x^2}{(-1+x)(1+x^2)} dx \\ &= \int \frac{x(1+x)}{(-1+x)(1+x^2)} dx \\ &= \int \left(\frac{1}{-1+x} + \frac{1}{1+x^2} \right) dx \\ &= \log(1-x) + \int \frac{1}{1+x^2} dx \\ &= \tan^{-1}(x) + \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\log(1 - x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)), x]

[Out] ArcTan[x] + Log[1 - x]

fricas [A] time = 0.89, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(-1+x)^2/(x^2+1), x, algorithm="fricas")

[Out] arctan(x) + log(x - 1)

giac [B] time = 0.29, size = 28, normalized size = 3.11

$$\frac{1}{4}\pi - \pi \left[\frac{\pi + 4 \arctan(x)}{4\pi} + \frac{1}{2} \right] + \arctan(x) + \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(-1+x)^2/(x^2+1), x, algorithm="giac")

[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) + log(abs(x - 1))

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$\arctan(x) + \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-x)/(x-1)^2/(x^2+1), x)

[Out] ln(x-1)+arctan(x)

maxima [A] time = 2.17, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(-1+x)^2/(x^2+1), x, algorithm="maxima")

[Out] arctan(x) + log(x - 1)

mupad [B] time = 2.10, size = 19, normalized size = 2.11

$$\ln(x - 1) - \operatorname{atan}\left(\frac{5}{4x + 2} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x - x^3)/((x^2 + 1)*(x - 1)^2), x)

[Out] log(x - 1) - atan(5/(4*x + 2) - 1/2)

sympy [A] time = 0.14, size = 7, normalized size = 0.78

$$\log(x - 1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) + atan(x)
```

$$3.325 \quad \int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx$$

Optimal. Leaf size=12

$$x^2 + \log(x^2 + x + 1) + x$$

[Out] $x+x^2+\ln(x^2+x+1)$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1657, 628}

$$x^2 + \log(x^2 + x + 1) + x$$

Antiderivative was successfully verified.

[In] `Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2), x]`

[Out] `x + x^2 + Log[1 + x + x^2]`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rule 1657

`Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]`

Rubi steps

$$\begin{aligned} \int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx &= \int \left(1 + 2x + \frac{1+2x}{1+x+x^2} \right) dx \\ &= x + x^2 + \int \frac{1+2x}{1+x+x^2} dx \\ &= x + x^2 + \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x^2 + \log(x^2 + x + 1) + x$$

Antiderivative was successfully verified.

[In] `Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2), x]`

[Out] `x + x^2 + Log[1 + x + x^2]`

fricas [A] time = 0.63, size = 12, normalized size = 1.00

$$x^2 + x + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1), x, algorithm="fricas")`

[Out] $x^2 + x + \log(x^2 + x + 1)$

giac [A] time = 0.29, size = 12, normalized size = 1.00

$$x^2 + x + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="giac")`

[Out] $x^2 + x + \log(x^2 + x + 1)$

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$x^2 + x + \ln(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x)`

[Out] $x+x^2+\ln(x^2+x+1)$

maxima [A] time = 1.37, size = 12, normalized size = 1.00

$$x^2 + x + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="maxima")`

[Out] $x^2 + x + \log(x^2 + x + 1)$

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$x + \ln(x^2 + x + 1) + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x + 3*x^2 + 2*x^3 + 2)/(x + x^2 + 1),x)`

[Out] $x + \log(x + x^2 + 1) + x^2$

sympy [A] time = 0.09, size = 12, normalized size = 1.00

$$x^2 + x + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)`

[Out] $x**2 + x + \log(x**2 + x + 1)$

$$3.326 \quad \int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)} dx$$

Optimal. Leaf size=65

$$\frac{3}{2x^2} - \frac{1}{x} + 3\log(x) - \frac{1}{10}(15 - \sqrt{5})\log(2x - \sqrt{5} + 1) - \frac{1}{10}(15 + \sqrt{5})\log(2x + \sqrt{5} + 1)$$

[Out] 3/2/x^2-1/x+3*ln(x)-1/10*ln(1+2*x-5^(1/2))*(15-5^(1/2))-1/10*ln(1+2*x+5^(1/2))*(15+5^(1/2))

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1628, 632, 31}

$$\frac{3}{2x^2} - \frac{1}{x} + 3\log(x) - \frac{1}{10}(15 - \sqrt{5})\log(2x - \sqrt{5} + 1) - \frac{1}{10}(15 + \sqrt{5})\log(2x + \sqrt{5} + 1)$$

Antiderivative was successfully verified.

[In] Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)), x]

[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10 - ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)} dx &= \int \left(-\frac{3}{x^3} + \frac{1}{x^2} + \frac{3}{x} + \frac{-1-3x}{-1+x+x^2} \right) dx \\ &= \frac{3}{2x^2} - \frac{1}{x} + 3\log(x) + \int \frac{-1-3x}{-1+x+x^2} dx \\ &= \frac{3}{2x^2} - \frac{1}{x} + 3\log(x) + \frac{1}{10}(-15 + \sqrt{5}) \int \frac{1}{\frac{1}{2} - \frac{\sqrt{5}}{2} + x} dx - \frac{1}{10}(15 + \sqrt{5}) \int \frac{1}{\frac{1}{2} + \frac{\sqrt{5}}{2} + x} dx \\ &= \frac{3}{2x^2} - \frac{1}{x} + 3\log(x) - \frac{1}{10}(15 - \sqrt{5})\log(1 - \sqrt{5} + 2x) - \frac{1}{10}(15 + \sqrt{5})\log(1 + \sqrt{5} + 2x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.89

$$\frac{1}{10} \left(\frac{15}{x^2} - \frac{10}{x} + (\sqrt{5} - 15)\log(-2x + \sqrt{5} - 1) + 30\log(x) - (15 + \sqrt{5})\log(2x + \sqrt{5} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]

[Out] (15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log[x] - (15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10

fricas [A] time = 0.60, size = 66, normalized size = 1.02

$$\frac{\sqrt{5} x^2 \log\left(\frac{2x^2 - \sqrt{5}(2x+1) + 2x+3}{x^2+x-1}\right) - 15 x^2 \log(x^2 + x - 1) + 30 x^2 \log(x) - 10 x + 15}{10 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="fricas")

[Out] 1/10*(sqrt(5)*x^2*log((2*x^2 - sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) - 15*x^2*log(x^2 + x - 1) + 30*x^2*log(x) - 10*x + 15)/x^2

giac [A] time = 0.30, size = 55, normalized size = 0.85

$$\frac{1}{10} \sqrt{5} \log\left(\frac{|2x - \sqrt{5} + 1|}{|2x + \sqrt{5} + 1|}\right) - \frac{2x - 3}{2x^2} - \frac{3}{2} \log(|x^2 + x - 1|) + 3 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="giac")

[Out] 1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/x^2 - 3/2*log(abs(x^2 + x - 1)) + 3*log(abs(x))

maple [A] time = 0.01, size = 41, normalized size = 0.63

$$-\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5} + 3 \ln(x) - \frac{3 \ln(x^2 + x - 1)}{2} - \frac{1}{x} + \frac{3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x)

[Out] -3/2*ln(x^2+x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))-1/x+3/2/x^2+3*ln(x)

maxima [A] time = 1.77, size = 51, normalized size = 0.78

$$\frac{1}{10} \sqrt{5} \log\left(\frac{2x - \sqrt{5} + 1}{2x + \sqrt{5} + 1}\right) - \frac{2x - 3}{2x^2} - \frac{3}{2} \log(x^2 + x - 1) + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="maxima")

[Out] 1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/x^2 - 3/2*log(x^2 + x - 1) + 3*log(x)

mupad [B] time = 0.10, size = 48, normalized size = 0.74

$$3 \ln(x) - \frac{x - \frac{3}{2}}{x^2} + \ln\left(x - \frac{\sqrt{5}}{2} + \frac{1}{2}\right) \left(\frac{\sqrt{5}}{10} - \frac{3}{2}\right) - \ln\left(x + \frac{\sqrt{5}}{2} + \frac{1}{2}\right) \left(\frac{\sqrt{5}}{10} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(4*x + 5*x^2 - 3*x^3 - 3)/(x^3*(x + x^2 - 1)),x)`

[Out] $3\log(x) - (x - 3/2)/x^2 + \log(x - 5^{1/2}/2 + 1/2)*(5^{1/2}/10 - 3/2) - \log(x + 5^{1/2}/2 + 1/2)*(5^{1/2}/10 + 3/2)$

sympy [A] time = 0.45, size = 99, normalized size = 1.52

$$3\log(x) + \left(-\frac{3}{2} + \frac{\sqrt{5}}{10}\right) \log\left(x - \frac{405}{202} - \frac{35\sqrt{5}}{202} + \frac{110\left(-\frac{3}{2} + \frac{\sqrt{5}}{10}\right)^2}{101}\right) + \left(-\frac{3}{2} - \frac{\sqrt{5}}{10}\right) \log\left(x - \frac{405}{202} + \frac{35\sqrt{5}}{202} + \frac{110\left(-\frac{3}{2} - \frac{\sqrt{5}}{10}\right)^2}{101}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)`

[Out] $3\log(x) + (-3/2 + \sqrt{5}/10)*\log(x - 405/202 - 35*\sqrt{5}/202 + 110*(-3/2 + \sqrt{5}/10)**2/101) + (-3/2 - \sqrt{5}/10)*\log(x - 405/202 + 35*\sqrt{5}/202 + 110*(-3/2 - \sqrt{5}/10)**2/101) + (3 - 2*x)/(2*x**2)$

$$3.327 \quad \int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx$$

Optimal. Leaf size=28

$$-\frac{1}{x^2+2x+2} + \log(x^2+2x+2) - \tan^{-1}(x+1)$$

[Out] -1/(x^2+2*x+2)-arctan(1+x)+ln(x^2+2*x+2)

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1660, 634, 617, 204, 628}

$$-\frac{1}{x^2+2x+2} + \log(x^2+2x+2) - \tan^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]

[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{4 + 8x + 5x^2 + 2x^3}{(2 + 2x + x^2)^2} dx &= -\frac{1}{2 + 2x + x^2} + \frac{1}{4} \int \frac{4 + 8x}{2 + 2x + x^2} dx \\
&= -\frac{1}{2 + 2x + x^2} - \int \frac{1}{2 + 2x + x^2} dx + \int \frac{2 + 2x}{2 + 2x + x^2} dx \\
&= -\frac{1}{2 + 2x + x^2} + \log(2 + 2x + x^2) + \text{Subst}\left(\int \frac{1}{-1 - x^2} dx, x, 1 + x\right) \\
&= -\frac{1}{2 + 2x + x^2} - \tan^{-1}(1 + x) + \log(2 + 2x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{x^2 + 2x + 2} + \log(x^2 + 2x + 2) - \tan^{-1}(x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]

[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]

fricas [A] time = 0.62, size = 46, normalized size = 1.64

$$\frac{(x^2 + 2x + 2) \arctan(x + 1) - (x^2 + 2x + 2) \log(x^2 + 2x + 2) + 1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="fricas")

[Out] -((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x + 2) + 1)/(x^2 + 2*x + 2)

giac [A] time = 0.28, size = 28, normalized size = 1.00

$$-\frac{1}{x^2 + 2x + 2} - \arctan(x + 1) + \log(x^2 + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="giac")

[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\arctan(x + 1) + \ln(x^2 + 2x + 2) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x)

[Out] -1/(x^2+2*x+2)-arctan(x+1)+ln(x^2+2*x+2)

maxima [A] time = 1.66, size = 28, normalized size = 1.00

$$-\frac{1}{x^2 + 2x + 2} - \arctan(x + 1) + \log(x^2 + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="maxima")

[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)

mupad [B] time = 0.04, size = 28, normalized size = 1.00

$$\ln(x^2 + 2x + 2) - \operatorname{atan}(x + 1) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x + 5*x^2 + 2*x^3 + 4)/(2*x + x^2 + 2)^2,x)

[Out] log(2*x + x^2 + 2) - atan(x + 1) - 1/(2*x + x^2 + 2)

sympy [A] time = 0.13, size = 24, normalized size = 0.86

$$\log(x^2 + 2x + 2) - \operatorname{atan}(x + 1) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)

[Out] log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)

$$3.328 \quad \int \frac{(-1+x)^4 x^4}{1+x^2} dx$$

Optimal. Leaf size=32

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \tan^{-1}(x)$$

[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1629, 203}

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[((-1 + x)^4*x^4)/(1 + x^2), x]

[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(-1+x)^4 x^4}{1+x^2} dx &= \int \left(4 - 4x^2 + 5x^4 - 4x^5 + x^6 - \frac{4}{1+x^2} \right) dx \\ &= 4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4 \int \frac{1}{1+x^2} dx \\ &= 4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4 \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + x)^4*x^4)/(1 + x^2), x]

[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]

fricas [A] time = 0.81, size = 26, normalized size = 0.81

$$\frac{1}{7} x^7 - \frac{2}{3} x^6 + x^5 - \frac{4}{3} x^3 + 4x - 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="fricas")

[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)

giac [A] time = 0.37, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="giac")

[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-1)^4*x^4/(x^2+1),x)

[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)

maxima [A] time = 1.51, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="maxima")

[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)

mupad [B] time = 0.02, size = 26, normalized size = 0.81

$$4x - 4 \operatorname{atan}(x) - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(x - 1)^4)/(x^2 + 1),x)

[Out] 4*x - 4*atan(x) - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7

sympy [A] time = 0.10, size = 29, normalized size = 0.91

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)**4*x**4/(x**2+1),x)

[Out] x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)

$$3.329 \quad \int \frac{-20x+4x^2}{9-10x^2+x^4} dx$$

Optimal. Leaf size=31

$$\log(1-x) - \frac{1}{2} \log(3-x) + \frac{3}{2} \log(x+1) - 2 \log(x+3)$$

[Out] $\ln(1-x) - 1/2 * \ln(3-x) + 3/2 * \ln(1+x) - 2 * \ln(3+x)$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1593, 1662, 12, 1107, 616, 31, 1130, 207}

$$\frac{5}{4} \log(1-x^2) - \frac{5}{4} \log(9-x^2) - \frac{3}{2} \tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4), x]$

[Out] $(-3*\text{ArcTanh}[x/3])/2 + \text{ArcTanh}[x]/2 + (5*\text{Log}[1 - x^2])/4 - (5*\text{Log}[9 - x^2])/4$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 207

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 616

$\text{Int}[(a_ + (b_)*(x_ + (c_)*(x_)^2))^{-1}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[c/q, \text{Int}[1/\text{Simp}[b/2 - q/2 + c*x, x], x], x] - \text{Dist}[c/q, \text{Int}[1/\text{Simp}[b/2 + q/2 + c*x, x], x], x]] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c] \ \&\& \ \text{PerfectSquareQ}[b^2 - 4*a*c]$

Rule 1107

$\text{Int}[(x_)*((a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{p_}), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, p\}, x]$

Rule 1130

$\text{Int}[(d_)*(x_)^m/((a_ + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(d^2*(b/q + 1))/2, \text{Int}[(d*x)^{m-2}/(b/2 + q/2 + c*x^2), x], x] - \text{Dist}[(d^2*(b/q - 1))/2, \text{Int}[(d*x)^{m-2}/(b/2 - q/2 + c*x^2), x], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{GeQ}[m, 2]$

Rule 1593

`Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]`

Rule 1662

`Int[(Pq_)*((d_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
+ 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !Po
lyQ[Pq, x^2]`

Rubi steps

$$\begin{aligned} \int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} dx &= \int \frac{x(-20 + 4x)}{9 - 10x^2 + x^4} dx \\ &= \int -\frac{20x}{9 - 10x^2 + x^4} dx + \int \frac{4x^2}{9 - 10x^2 + x^4} dx \\ &= 4 \int \frac{x^2}{9 - 10x^2 + x^4} dx - 20 \int \frac{x}{9 - 10x^2 + x^4} dx \\ &= -\left(\frac{1}{2} \int \frac{1}{-1 + x^2} dx\right) + \frac{9}{2} \int \frac{1}{-9 + x^2} dx - 10 \operatorname{Subst}\left(\int \frac{1}{9 - 10x + x^2} dx, x, x^2\right) \\ &= -\frac{3}{2} \tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2} \tanh^{-1}(x) - \frac{5}{4} \operatorname{Subst}\left(\int \frac{1}{-9 + x} dx, x, x^2\right) + \frac{5}{4} \operatorname{Subst}\left(\int \frac{1}{-1 + x} dx, x, x^2\right) \\ &= -\frac{3}{2} \tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2} \tanh^{-1}(x) + \frac{5}{4} \log(1 - x^2) - \frac{5}{4} \log(9 - x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.26

$$4\left(\frac{1}{4} \log(1 - x) - \frac{1}{8} \log(3 - x) + \frac{3}{8} \log(x + 1) - \frac{1}{2} \log(x + 3)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4), x]

[Out] 4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)

fricas [A] time = 0.84, size = 23, normalized size = 0.74

$$-2 \log(x + 3) + \frac{3}{2} \log(x + 1) + \log(x - 1) - \frac{1}{2} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9), x, algorithm="fricas")

[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)

giac [A] time = 0.31, size = 27, normalized size = 0.87

$$-2 \log(|x + 3|) + \frac{3}{2} \log(|x + 1|) + \log(|x - 1|) - \frac{1}{2} \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9), x, algorithm="giac")

[Out] $-2*\log(\text{abs}(x + 3)) + 3/2*\log(\text{abs}(x + 1)) + \log(\text{abs}(x - 1)) - 1/2*\log(\text{abs}(x - 3))$

maple [A] time = 0.01, size = 24, normalized size = 0.77

$$-\frac{\ln(x-3)}{2} + \ln(x-1) + \frac{3\ln(x+1)}{2} - 2\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2-20*x)/(x^4-10*x^2+9),x)`

[Out] $\ln(x-1)+3/2*\ln(x+1)-2*\ln(x+3)-1/2*\ln(x-3)$

maxima [A] time = 0.80, size = 23, normalized size = 0.74

$$-2 \log(x+3) + \frac{3}{2} \log(x+1) + \log(x-1) - \frac{1}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="maxima")`

[Out] $-2*\log(x + 3) + 3/2*\log(x + 1) + \log(x - 1) - 1/2*\log(x - 3)$

mupad [B] time = 0.05, size = 23, normalized size = 0.74

$$\ln(x-1) + \frac{3\ln(x+1)}{2} - \frac{\ln(x-3)}{2} - 2\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(20*x - 4*x^2)/(x^4 - 10*x^2 + 9),x)`

[Out] $\log(x - 1) + (3*\log(x + 1))/2 - \log(x - 3)/2 - 2*\log(x + 3)$

sympy [A] time = 0.19, size = 26, normalized size = 0.84

$$-\frac{\log(x-3)}{2} + \log(x-1) + \frac{3\log(x+1)}{2} - 2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)`

[Out] $-\log(x - 3)/2 + \log(x - 1) + 3*\log(x + 1)/2 - 2*\log(x + 3)$

$$3.330 \quad \int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)} dx$$

Optimal. Leaf size=24

$$-\log(x^2 + 1) - \frac{1}{x} + 2 \log(1 - x) + \tan^{-1}(x)$$

[Out] -1/x+arctan(x)+2*ln(1-x)-ln(x^2+1)

Rubi [A] time = 0.17, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6725, 635, 203, 260}

$$-\log(x^2 + 1) - \frac{1}{x} + 2 \log(1 - x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)), x]

[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)} dx &= \int \left(\frac{2}{-1+x} + \frac{1}{x^2} + \frac{1-2x}{1+x^2} \right) dx \\ &= -\frac{1}{x} + 2 \log(1-x) + \int \frac{1-2x}{1+x^2} dx \\ &= -\frac{1}{x} + 2 \log(1-x) - 2 \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx \\ &= -\frac{1}{x} + \tan^{-1}(x) + 2 \log(1-x) - \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\log(x^2 + 1) - \frac{1}{x} + 2 \log(1 - x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)), x]

[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]

fricas [A] time = 0.76, size = 26, normalized size = 1.08

$$\frac{x \arctan(x) - x \log(x^2 + 1) + 2x \log(x - 1) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1), x, algorithm="fricas")

[Out] (x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x

giac [A] time = 0.29, size = 23, normalized size = 0.96

$$-\frac{1}{x} + \arctan(x) - \log(x^2 + 1) + 2 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1), x, algorithm="giac")

[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(abs(x - 1))

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$\arctan(x) + 2 \ln(x - 1) - \ln(x^2 + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^3+x-1)/(x-1)/x^2/(x^2+1), x)

[Out] 2*ln(x-1)-1/x-ln(x^2+1)+arctan(x)

maxima [A] time = 1.22, size = 22, normalized size = 0.92

$$-\frac{1}{x} + \arctan(x) - \log(x^2 + 1) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1), x, algorithm="maxima")

[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)

mupad [B] time = 2.13, size = 30, normalized size = 1.25

$$2 \ln(x - 1) - \frac{1}{x} + \ln(x - i) \left(-1 - \frac{1}{2}i\right) + \ln(x + i) \left(-1 + \frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 4*x^3 - 1)/(x^2*(x^2 + 1)*(x - 1)), x)

[Out] 2*log(x - 1) - log(x - 1i)*(1 + 1i/2) - log(x + 1i)*(1 - 1i/2) - 1/x

sympy [A] time = 0.15, size = 19, normalized size = 0.79

$$2 \log(x - 1) - \log(x^2 + 1) + \operatorname{atan}(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
```

```
[Out] 2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

$$3.331 \quad \int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx$$

Optimal. Leaf size=23

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

[Out] -1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1814, 12, 203}

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]

[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx &= -\frac{1}{4(1+x^2)^2} - \frac{1}{4} \int \frac{-4+16x-4x^2}{(1+x^2)^2} dx \\ &= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \frac{1}{8} \int \frac{8}{1+x^2} dx \\ &= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \int \frac{1}{1+x^2} dx \\ &= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2}{x^2 + 1} - \frac{1}{4(x^2 + 1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]

[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]

fricas [A] time = 0.84, size = 35, normalized size = 1.52

$$\frac{8x^2 + 4(x^4 + 2x^2 + 1)\arctan(x) + 7}{4(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="fricas")

[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)

giac [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{8x^2 + 7}{4(x^2 + 1)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="giac")

[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)

maple [A] time = 0.01, size = 19, normalized size = 0.83

$$\arctan(x) + \frac{2x^2 + \frac{7}{4}}{(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x)

[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)

maxima [A] time = 1.43, size = 24, normalized size = 1.04

$$\frac{8x^2 + 7}{4(x^4 + 2x^2 + 1)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="maxima")

[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{2x^2 + \frac{7}{4}}{x^4 + 2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(x^2 + 1)^3,x)
```

```
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

sympy [A] time = 0.13, size = 20, normalized size = 0.87

$$\frac{8x^2 + 7}{4x^4 + 8x^2 + 4} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)
```

```
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

$$3.332 \quad \int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} dx$$

Optimal. Leaf size=23

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

[Out] -1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2073, 261, 203}

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6), x]

[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 2073

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} dx &= \int \left(\frac{x}{(1+x^2)^3} - \frac{4x}{(1+x^2)^2} + \frac{1}{1+x^2} \right) dx \\ &= - \left(4 \int \frac{x}{(1+x^2)^2} dx \right) + \int \frac{x}{(1+x^2)^3} dx + \int \frac{1}{1+x^2} dx \\ &= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]

[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]

fricas [A] time = 1.24, size = 35, normalized size = 1.52

$$\frac{8x^2 + 4(x^4 + 2x^2 + 1)\arctan(x) + 7}{4(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="fricas")

[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)

giac [A] time = 0.27, size = 19, normalized size = 0.83

$$\frac{8x^2 + 7}{4(x^2 + 1)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="giac")

[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\arctan(x) + \frac{2x^2 + \frac{7}{4}}{(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x)

[Out] arctan(x)+(2*x^2+7/4)/(x^2+1)^2

maxima [A] time = 1.42, size = 24, normalized size = 1.04

$$\frac{8x^2 + 7}{4(x^4 + 2x^2 + 1)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="maxima")

[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{2x^2 + \frac{7}{4}}{x^4 + 2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(3*x^2 + 3*x^4 + x^6 + 1),x)

[Out] $\operatorname{atan}(x) + (2x^2 + 7/4)/(2x^2 + x^4 + 1)$

sympy [A] time = 0.13, size = 20, normalized size = 0.87

$$\frac{8x^2 + 7}{4x^4 + 8x^2 + 4} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)`

[Out] $(8x^2 + 7)/(4x^4 + 8x^2 + 4) + \operatorname{atan}(x)$

$$3.333 \quad \int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx$$

Optimal. Leaf size=13

$$\log(x^2 + x + 1) - \frac{1}{x}$$

[Out] -1/x+ln(x^2+x+1)

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1594, 1628, 628}

$$\log(x^2 + x + 1) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4), x]

[Out] -x^(-1) + Log[1 + x + x^2]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq)*((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx &= \int \frac{1+x+2x^2+2x^3}{x^2(1+x+x^2)} dx \\ &= \int \left(\frac{1}{x^2} + \frac{1+2x}{1+x+x^2} \right) dx \\ &= -\frac{1}{x} + \int \frac{1+2x}{1+x+x^2} dx \\ &= -\frac{1}{x} + \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\log(x^2 + x + 1) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4), x]

[Out] $-x^{-1} + \text{Log}[1 + x + x^2]$

fricas [A] time = 0.81, size = 15, normalized size = 1.15

$$\frac{x \log(x^2 + x + 1) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2), x, algorithm="fricas")

[Out] $(x \cdot \log(x^2 + x + 1) - 1)/x$

giac [A] time = 0.28, size = 13, normalized size = 1.00

$$-\frac{1}{x} + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2), x, algorithm="giac")

[Out] $-1/x + \log(x^2 + x + 1)$

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$\ln(x^2 + x + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2), x)

[Out] $-1/x + \ln(x^2 + x + 1)$

maxima [A] time = 0.71, size = 13, normalized size = 1.00

$$-\frac{1}{x} + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2), x, algorithm="maxima")

[Out] $-1/x + \log(x^2 + x + 1)$

mupad [B] time = 2.14, size = 13, normalized size = 1.00

$$\ln(x^2 + x + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2*x^2 + 2*x^3 + 1)/(x^2 + x^3 + x^4), x)

[Out] $\log(x + x^2 + 1) - 1/x$

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$\log(x^2 + x + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2), x)

[Out] $\log(x**2 + x + 1) - 1/x$

$$3.334 \quad \int \frac{x^2(c+dx)^2}{a+bx^3} dx$$

Optimal. Leaf size=206

$$\frac{\sqrt[3]{a}d(2\sqrt[3]{b}c - \sqrt[3]{a}d)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6b^{5/3}} - \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c - \sqrt[3]{a}d)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}} + \frac{\sqrt[3]{a}d(\sqrt[3]{a}d + 2\sqrt[3]{b}c)}{\sqrt{3}b^5}$$

[Out] $2*c*d*x/b + 1/2*d^2*x^2/b - 1/3*a^{(1/3)}*d*(2*b^{(1/3)}*c - a^{(1/3)}*d)*\ln(a^{(1/3)} + b^{(1/3)}*x)/b^{(5/3)} + 1/6*a^{(1/3)}*d*(2*b^{(1/3)}*c - a^{(1/3)}*d)*\ln(a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2)/b^{(5/3)} + 1/3*c^2*\ln(b*x^3 + a)/b + 1/3*a^{(1/3)}*d*(2*b^{(1/3)}*c + a^{(1/3)}*d)*\arctan(1/3*(a^{(1/3)} - 2*b^{(1/3)}*x)/a^{(1/3)}*3^{(1/2)})/b^{(5/3)}*3^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}d(2\sqrt[3]{b}c - \sqrt[3]{a}d)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6b^{5/3}} - \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c - \sqrt[3]{a}d)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}} + \frac{\sqrt[3]{a}d(\sqrt[3]{a}d + 2\sqrt[3]{b}c)}{\sqrt{3}b^5}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x)^2)/(a + b*x^3), x]

[Out] $(2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^{(1/3)}*d*(2*b^{(1/3)}*c + a^{(1/3)}*d)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})]) / (\text{Sqrt}[3]*b^{(5/3)}) - (a^{(1/3)}*d*(2*b^{(1/3)}*c - a^{(1/3)}*d)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x]) / (3*b^{(5/3)}) + (a^{(1/3)}*d*(2*b^{(1/3)}*c - a^{(1/3)}*d)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]) / (6*b^{(5/3)}) + (c^2*\text{Log}[a + b*x^3]) / (3*b)$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1860

Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 1871

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1887

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2(c + dx)^2}{a + bx^3} dx &= \int \left(\frac{2cd}{b} + \frac{d^2x}{b} - \frac{2acd + ad^2x - bc^2x^2}{b(a + bx^3)} \right) dx \\
 &= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\int \frac{2acd + ad^2x - bc^2x^2}{a + bx^3} dx}{b} \\
 &= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\int \frac{2acd + ad^2x}{a + bx^3} dx}{b} + c^2 \int \frac{x^2}{a + bx^3} dx \\
 &= \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{c^2 \log(a + bx^3)}{3b} - \frac{\int \frac{\sqrt[3]{a} (4a \sqrt[3]{b} cd + a^{4/3} d^2) + \sqrt[3]{b} (-2a \sqrt[3]{b} cd + a^{4/3} d^2)x}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2} dx}{3a^{2/3} b^{4/3}} - \frac{(\sqrt[3]{a} d (2c \sqrt[3]{b} c - \sqrt[3]{a} d) \log(\sqrt[3]{a} + \sqrt[3]{b} x) + c^2 \log(a + bx^3))}{3b^{5/3}} \\
 &= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\sqrt[3]{a} d (2\sqrt[3]{b} c - \sqrt[3]{a} d) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{5/3}} + \frac{c^2 \log(a + bx^3)}{3b} + \frac{(\sqrt[3]{a} d (2\sqrt[3]{b} c - \sqrt[3]{a} d) \log(a^{2/3} - \sqrt[3]{b} x) + c^2 \log(a + bx^3))}{6b^{5/3}} \\
 &= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\sqrt[3]{a} d (2\sqrt[3]{b} c - \sqrt[3]{a} d) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{5/3}} + \frac{\sqrt[3]{a} d (2\sqrt[3]{b} c - \sqrt[3]{a} d) \log(a^{2/3} - \sqrt[3]{b} x)}{6b^{5/3}} \\
 &= \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{\sqrt[3]{a} d (2\sqrt[3]{b} c + \sqrt[3]{a} d) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} b^{5/3}} - \frac{\sqrt[3]{a} d (2\sqrt[3]{b} c - \sqrt[3]{a} d) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{5/3}}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 193, normalized size = 0.94

$$-\sqrt[3]{a}d\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)+2\sqrt[3]{a}d\left(\sqrt[3]{a}d-2\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)$$

$$6b^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x)^2)/(a + b*x^3),x]

[Out] (12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*Sqrt[3]*a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*d*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(5/3))

fricas [C] time = 2.99, size = 4545, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="fricas")

[Out] 1/12*(6*d^2*x^2 + 24*c*d*x - 2*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b*log(1/4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 3*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 + 5*b*c^4 + 4*a*c*d^3 + (8*b*c^3*d + a*d^4)*x) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b + 6*c^2 + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3/b^3))*log(-1/4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/

$$c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)))/b$$

giac [A] time = 0.44, size = 208, normalized size = 1.01

$$\frac{c^2 \log(|bx^3 + a|)}{3b} - \frac{\sqrt{3} \left(2 (-ab^2)^{\frac{1}{3}} bcd - (-ab^2)^{\frac{2}{3}} d^2 \right) \arctan \left(\frac{\sqrt{3} \left(2x + (-\frac{a}{b})^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b} \right)^{\frac{1}{3}}} \right)}{3b^3} + \frac{bd^2x^2 + 4bcdx}{2b^2} - \frac{\left(2 (-ab^2)^{\frac{1}{3}} bcd + \dots \right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="giac")

[Out] 1/3*c^2*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c*d - (-a*b^2)^(2/3)*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 - 1/6*(2*(-a*b^2)^(1/3)*b*c*d + (-a*b^2)^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*d^2*(-a/b)^(1/3) + 2*a*b^4*c*d*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)

maple [A] time = 0.01, size = 236, normalized size = 1.15

$$\frac{d^2x^2}{2b} - \frac{2\sqrt{3}acd \arctan \left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1 \right)}{3} \right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{2acd \ln \left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} + \frac{acd \ln \left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{\sqrt{3} a d^2 \arctan \left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1 \right)}{3} \right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x+c)^2/(b*x^3+a),x)

[Out] 1/2*d^2*x^2/b+2*c*d*x/b-2/3/b^2*a*c*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/3/b^2*a*c*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-2/3/b^2*a*c*d/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b^2*a*d^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6/b^2*a*d^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/b^2*a*d^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*c^2*ln(b*x^3+a)/b

maxima [A] time = 1.65, size = 199, normalized size = 0.97

$$\frac{\sqrt{3} \left(ad^2 \left(\frac{a}{b}\right)^{\frac{2}{3}} + 2acd \left(\frac{a}{b}\right)^{\frac{1}{3}} \right) \arctan \left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{3ab} + \frac{d^2x^2 + 4cdx}{2b} + \frac{\left(2bc^2 \left(\frac{a}{b}\right)^{\frac{2}{3}} - ad^2 \left(\frac{a}{b}\right)^{\frac{1}{3}} + 2acd \right) \log \left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}} \right)}{6b^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) + 2*a*c*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(d^2*x^2 + 4*c*d*x)/b + 1/6*(2*b*c^2*(a/b)^(2/3) - a*d^2*(a/b)^(1/3) + 2*a*c*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)^(1/3) - 2*a*c*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))

mupad [B] time = 0.23, size = 357, normalized size = 1.73

$$\left(\sum_{k=1}^3 \ln \left(\frac{a \left(b c^4 + \text{root} \left(27 b^5 z^3 - 27 b^4 c^2 z^2 + 18 a b^2 c d^3 z + 9 b^3 c^4 z + 2 a b c^3 d^3 - b^2 c^6 - a^2 d^6, z, k \right) \right)^2 b^3 9 - \dots}{\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(c + d*x)^2)/(a + b*x^3), x)

[Out] symsum(log((a*(b*c^4 + 9*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)^2*b^3 - 6*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)*b^2*c^2 + 2*a*c*d^3 + a*d^4*x + 2*b*c^3*d*x - 6*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)*b^2*c*d*x))/b)*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k), k, 1, 3) + (d^2*x^2)/(2*b) + (2*c*d*x)/b

sympy [A] time = 1.24, size = 138, normalized size = 0.67

$$\text{RootSum} \left(27t^3b^5 - 27t^2b^4c^2 + t(18ab^2cd^3 + 9b^3c^4) - a^2d^6 + 2abc^3d^3 - b^2c^6, \left(t \mapsto t \log \left(x + \frac{9t^2b^3 - 18tb^2c^2}{ad^4 + \dots} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(d*x+c)**2/(b*x**3+a), x)

[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + *_t*(18*a*b**2*c*d**3 + 9*b**3*c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda(_t, _t*log(x + (9*_t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*b*c**4)/(a*d**4 + 8*b*c**3*d))) + 2*c*d*x/b + d**2*x**2/(2*b)

$$3.335 \quad \int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=45

$$\frac{9 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5-7x^2}{8(x^4+2x^2+3)}$$

[Out] 1/8*(-7*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1594, 1663, 1660, 12, 618, 204}

$$\frac{5-7x^2}{8(x^4+2x^2+3)} + \frac{9 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]

[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p+1))/((p+1)*(b^2 - 4*a*c)), x] + Dist[1/((p+1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p+1)*ExpandToSum[(p+1)*(b^2 - 4*a*c)*Q - (2*p+3)*(2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1663


```
Int[(Pq_)*(x_)^(m_)*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m-1)/2)*SubstFor[x^2, Pq, x]*(a+b*x+c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m-1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x + 2x^3 + 4x^5}{(3 + 2x^2 + x^4)^2} dx &= \int \frac{x(-1 + 2x^2 + 4x^4)}{(3 + 2x^2 + x^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{-1 + 2x + 4x^2}{(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
&= \frac{5 - 7x^2}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{18}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{5 - 7x^2}{8(3 + 2x^2 + x^4)} + \frac{9}{8} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\
&= \frac{5 - 7x^2}{8(3 + 2x^2 + x^4)} - \frac{9}{4} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2(1 + x^2) \right) \\
&= \frac{5 - 7x^2}{8(3 + 2x^2 + x^4)} + \frac{9 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{9 \tan^{-1} \left(\frac{x^2+1}{\sqrt{2}} \right)}{8\sqrt{2}} + \frac{5 - 7x^2}{8(x^4 + 2x^2 + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2, x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2])
```

fricas [A] time = 0.93, size = 47, normalized size = 1.04

$$\frac{9\sqrt{2}(x^4 + 2x^2 + 3) \arctan\left(\frac{1}{2}\sqrt{2}(x^2 + 1)\right) - 14x^2 + 10}{16(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 + 10)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 1.11, size = 38, normalized size = 0.84

$$\frac{9}{16} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x^2 + 1)\right) - \frac{7x^2 - 5}{8(x^4 + 2x^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] $9/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3)$

maple [A] time = 0.01, size = 41, normalized size = 0.91

$$\frac{9\sqrt{2} \arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16} + \frac{-\frac{7x^2}{4} + \frac{5}{4}}{2x^4 + 4x^2 + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x)

[Out] $1/2*(-7/4*x^2+5/4)/(x^4+2*x^2+3)+9/16*2^{(1/2)}*\arctan(1/4*(2*x^2+2)*2^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7x^2 - 5}{8(x^4 + 2x^2 + 3)} + \frac{9}{4} \int \frac{x}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")

[Out] $-1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*\integrate(x/(x^4 + 2*x^2 + 3), x)$

mupad [B] time = 2.19, size = 42, normalized size = 0.93

$$\frac{9\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{7x^2}{8} - \frac{5}{8}}{x^4 + 2x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3 - x + 4*x^5)/(2*x^2 + x^4 + 3)^2,x)

[Out] $(9*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}/2 + (2^{(1/2)}*x^2)/2))/16 - ((7*x^2)/8 - 5/8)/(2*x^2 + x^4 + 3)$

sympy [A] time = 0.15, size = 44, normalized size = 0.98

$$\frac{5 - 7x^2}{8x^4 + 16x^2 + 24} + \frac{9\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)

[Out] $(5 - 7*x**2)/(8*x**4 + 16*x**2 + 24) + 9*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x**2/2 + \sqrt{2}(2)/2)/16$

$$3.336 \quad \int \frac{x+x^5}{(1+2x^2+2x^4)^3} dx$$

Optimal. Leaf size=59

$$\tan^{-1}(2x^2 + 1) + \frac{2x^2 + 1}{2(2x^4 + 2x^2 + 1)} + \frac{4x^2 + 3}{16(2x^4 + 2x^2 + 1)^2}$$

[Out] 1/16*(4*x^2+3)/(2*x^4+2*x^2+1)^2+1/2*(2*x^2+1)/(2*x^4+2*x^2+1)+arctan(2*x^2+1)

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1593, 1663, 1660, 12, 614, 617, 204}

$$\frac{2x^2 + 1}{2(2x^4 + 2x^2 + 1)} + \frac{4x^2 + 3}{16(2x^4 + 2x^2 + 1)^2} + \tan^{-1}(2x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]

[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^2 + 2*x^4)) + ArcTan[1 + 2*x^2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^(
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x + x^5}{(1 + 2x^2 + 2x^4)^3} dx &= \int \frac{x(1 + x^4)}{(1 + 2x^2 + 2x^4)^3} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1 + x^2}{(1 + 2x + 2x^2)^3} dx, x, x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \frac{1}{16} \text{Subst} \left(\int \frac{16}{(1 + 2x + 2x^2)^2} dx, x, x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \text{Subst} \left(\int \frac{1}{(1 + 2x + 2x^2)^2} dx, x, x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \frac{1 + 2x^2}{2(1 + 2x^2 + 2x^4)} + \text{Subst} \left(\int \frac{1}{1 + 2x + 2x^2} dx, x, x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \frac{1 + 2x^2}{2(1 + 2x^2 + 2x^4)} - \text{Subst} \left(\int \frac{1}{-1 - x^2} dx, x, 1 + 2x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \frac{1 + 2x^2}{2(1 + 2x^2 + 2x^4)} + \tan^{-1}(1 + 2x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.75

$$\tan^{-1}(2x^2 + 1) + \frac{32x^6 + 48x^4 + 36x^2 + 11}{16(2x^4 + 2x^2 + 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3, x]

[Out] (11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*x^2]

fricas [A] time = 0.78, size = 75, normalized size = 1.27

$$\frac{32x^6 + 48x^4 + 36x^2 + 16(4x^8 + 8x^6 + 8x^4 + 4x^2 + 1) \arctan(2x^2 + 1) + 11}{16(4x^8 + 8x^6 + 8x^4 + 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")

[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*arctan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)

giac [A] time = 1.80, size = 42, normalized size = 0.71

$$\frac{32x^6 + 48x^4 + 36x^2 + 11}{16(2x^4 + 2x^2 + 1)^2} + \arctan(2x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")

[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2 + 1)

maple [A] time = 0.01, size = 41, normalized size = 0.69

$$\arctan(2x^2 + 1) + \frac{2x^6 + 3x^4 + \frac{9}{4}x^2 + \frac{11}{16}}{(2x^4 + 2x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5+x)/(2*x^4+2*x^2+1)^3,x)

[Out] 2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{32x^6 + 48x^4 + 36x^2 + 11}{16(4x^8 + 8x^6 + 8x^4 + 4x^2 + 1)} + 2 \int \frac{x}{2x^4 + 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")

[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) + 2*integrate(x/(2*x^4 + 2*x^2 + 1), x)

mupad [B] time = 0.05, size = 47, normalized size = 0.80

$$\operatorname{atan}(2x^2 + 1) + \frac{\frac{x^6}{2} + \frac{3x^4}{4} + \frac{9x^2}{16} + \frac{11}{64}}{x^8 + 2x^6 + 2x^4 + x^2 + \frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^5)/(2*x^2 + 2*x^4 + 1)^3,x)

[Out] atan(2*x^2 + 1) + ((9*x^2)/16 + (3*x^4)/4 + x^6/2 + 11/64)/(x^2 + 2*x^4 + 2*x^6 + x^8 + 1/4)

sympy [A] time = 0.18, size = 46, normalized size = 0.78

$$\frac{32x^6 + 48x^4 + 36x^2 + 11}{64x^8 + 128x^6 + 128x^4 + 64x^2 + 16} + \operatorname{atan}(2x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)

[Out] (32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**2 + 16) + atan(2*x**2 + 1)

$$3.337 \quad \int \frac{a+bx+cx^2}{d+ex^2+fx^4} dx$$

Optimal. Leaf size=209

$$\frac{\left(c - \frac{ce-2af}{\sqrt{e^2-4df}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}} + c\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}} - \frac{b \tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

[Out] $-b \operatorname{arctanh}\left(\frac{(2f x^2+e)/(-4d f+e^2)^{1/2}}{(-4d f+e^2)^{1/2}+1/2 \operatorname{arctan}(x^2)^{1/2} f^{1/2}/(e-(-4d f+e^2)^{1/2})^{1/2}}\right) + (c+(2a f-c e)/(-4d f+e^2)^{1/2})^2 (1/2)/f^{1/2}/(e-(-4d f+e^2)^{1/2})^{1/2} + 1/2 \operatorname{arctan}(x^2)^{1/2} f^{1/2}/(e+(-4d f+e^2)^{1/2})^{1/2} + (c+(-2a f+c e)/(-4d f+e^2)^{1/2})^2 (1/2)/f^{1/2}/(e+(-4d f+e^2)^{1/2})^{1/2}$

Rubi [A] time = 0.37, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(c - \frac{ce-2af}{\sqrt{e^2-4df}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}} + c\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}} - \frac{b \tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4), x]

[Out] $\left(\frac{(c - (c e - 2 a f)/\sqrt{e^2 - 4 d f}) \operatorname{ArcTan}[(\sqrt{2} \sqrt{f} x)/\sqrt{e - \sqrt{e^2 - 4 d f}}]}{(\sqrt{2} \sqrt{f} \sqrt{e - \sqrt{e^2 - 4 d f}})} + \frac{(c + (c e - 2 a f)/\sqrt{e^2 - 4 d f}) \operatorname{ArcTan}[(\sqrt{2} \sqrt{f} x)/\sqrt{e + \sqrt{e^2 - 4 d f}}]}{(\sqrt{2} \sqrt{f} \sqrt{e + \sqrt{e^2 - 4 d f}})} - \frac{(b \operatorname{ArcTanh}[(e + 2 f x^2)/\sqrt{e^2 - 4 d f}])}{\sqrt{e^2 - 4 d f}}\right)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1673

Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned} \int \frac{a + bx + cx^2}{d + ex^2 + fx^4} dx &= \int \frac{bx}{d + ex^2 + fx^4} dx + \int \frac{a + cx^2}{d + ex^2 + fx^4} dx \\ &= b \int \frac{x}{d + ex^2 + fx^4} dx + \frac{1}{2} \left(c - \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \int \frac{1}{\frac{e}{2} - \frac{1}{2}\sqrt{e^2 - 4df} + fx^2} dx + \frac{1}{2} \left(c + \frac{ce}{\sqrt{e^2 - 4df}} \right) \int \frac{1}{\frac{e}{2} + \frac{1}{2}\sqrt{e^2 - 4df} + fx^2} dx \\ &= \frac{\left(c - \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{fx}}{\sqrt{e - \sqrt{e^2 - 4df}}} \right)}{\sqrt{2}\sqrt{f}\sqrt{e - \sqrt{e^2 - 4df}}} + \frac{\left(c + \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{fx}}{\sqrt{e + \sqrt{e^2 - 4df}}} \right)}{\sqrt{2}\sqrt{f}\sqrt{e + \sqrt{e^2 - 4df}}} + \frac{1}{2} b \operatorname{Subst} \left(\int \frac{1}{\frac{e}{2} - \frac{1}{2}\sqrt{e^2 - 4df} + fx^2} dx \right) \\ &= \frac{\left(c - \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{fx}}{\sqrt{e - \sqrt{e^2 - 4df}}} \right)}{\sqrt{2}\sqrt{f}\sqrt{e - \sqrt{e^2 - 4df}}} + \frac{\left(c + \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{fx}}{\sqrt{e + \sqrt{e^2 - 4df}}} \right)}{\sqrt{2}\sqrt{f}\sqrt{e + \sqrt{e^2 - 4df}}} - b \operatorname{Subst} \left(\int \frac{1}{\frac{e}{2} - \frac{1}{2}\sqrt{e^2 - 4df} + fx^2} dx \right) \\ &= \frac{\left(c - \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{fx}}{\sqrt{e - \sqrt{e^2 - 4df}}} \right)}{\sqrt{2}\sqrt{f}\sqrt{e - \sqrt{e^2 - 4df}}} + \frac{\left(c + \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{fx}}{\sqrt{e + \sqrt{e^2 - 4df}}} \right)}{\sqrt{2}\sqrt{f}\sqrt{e + \sqrt{e^2 - 4df}}} - \frac{b \tanh^{-1} \left(\frac{e - \sqrt{e^2 - 4df}}{\sqrt{e^2 - 4df}} \right)}{\sqrt{e^2 - 4df}} \end{aligned}$$

Mathematica [A] time = 0.25, size = 234, normalized size = 1.12

$$\frac{\sqrt{2}(2af + c(\sqrt{e^2 - 4df} - e)) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{fx}}{\sqrt{e - \sqrt{e^2 - 4df}}} \right)}{\sqrt{f}\sqrt{e - \sqrt{e^2 - 4df}}} + \frac{\sqrt{2}(c(\sqrt{e^2 - 4df} + e) - 2af) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{fx}}{\sqrt{e^2 - 4df} + e} \right)}{\sqrt{f}\sqrt{e^2 - 4df} + e} + b \log(\sqrt{e^2 - 4df} - e - 2fx^2) - \frac{b \log(\sqrt{e^2 - 4df} + e + 2fx^2)}{2\sqrt{e^2 - 4df}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4), x]

[Out] ((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqrt[2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e + Sqrt[e^2 - 4*d*f]]

```
rt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2)]/(2*Sqr
t[e^2 - 4*d*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
```

[Out] Timed out

giac [B] time = 4.70, size = 1587, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*sqrt(-4*d*f + e^2)*b*log(x^2 + 1/2*
(sqrt(-4*d*f + e^2) + e)/f)/((16*d^2*f^2 - 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2
+ f^2*e^2 - 2*f*e^3 + e^4)*f^2) - 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*s
qrt(-4*d*f + e^2)*b*log(x^2 - 1/2*(sqrt(-4*d*f + e^2) - e)/f)/((16*d^2*f^2
- 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2 + f^2*e^2 - 2*f*e^3 + e^4)*f^2) + 1/4*((1
6*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e + s
qrt(-4*d*f + e^2)*f)*d*f^3 - 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*f
+ e^2)*f)*d*f^2*e - 8*d*f^3*e + 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqr
t(-4*d*f + e^2)*f)*d*f*e - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*
f + e^2)*f)*f^2*e + 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*
f + e^2)*f)*d*f*e^2 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*f^2*e^2 + 16
*d*f^2*e^2 + 2*(4*d*f - e^2)*f^2*e + 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e
+ sqrt(-4*d*f + e^2)*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*
f*e^3 + 2*f^2*e^3 - 2*(4*d*f - e^2)*f*e^2 - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt
(f*e + sqrt(-4*d*f + e^2)*f)*e^3 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)
*e^4 - 2*f*e^4)*a + 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e +
sqrt(-4*d*f + e^2)*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4
*d*f + e^2)*f)*d*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*f
+ e^2)*f)*d*f*e - 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*f
+ e^2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sqr
t((sqrt(-4*d*f + e^2) + e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d
^2*f*e^2 + d*f^2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f)) + 1/4*((16*sqrt(2)*sqrt(f
*e - sqrt(-4*d*f + e^2)*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2
)*f)*d*f^3 + 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f^2*
e + 8*d*f^3*e - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*
f)*d*f*e + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f^2*
e - 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f*
e^2 + sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f^2*e^2 - 16*d*f^2*e^2 - 2*(
4*d*f - e^2)*f^2*e - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f +
e^2)*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f*e^3 - 2*f^2*e^
3 + 2*(4*d*f - e^2)*f*e^2 + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d
*f + e^2)*f)*e^3 + sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*e^4 + 2*f*e^4)*
a - 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^
2)*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d
*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f*e
- 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e
- sqrt(-4*d*f + e^2)*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sqrt(-(sqrt(-4*d*f
+ e^2) - e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d^2*f*e^2 + d*f^
2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f))
```



```

^2*x + b*c^2*e*f*x + 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*
f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*
a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2
+ 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z -
16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d
*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2,
z, k)*a*c*e*f^2*x)*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3
*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2
*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 +
4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16
*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e
+ b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z,
k), k, 1, 4)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)

[Out] Timed out

$$3.338 \quad \int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=224

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}} + e^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right) - 2de \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b} - \sqrt{b^2-4ac}}$$

[Out] $-2*d*e*\operatorname{arctanh}\left(\frac{(2*c*x^2+b)/(-4*a*c+b^2)^{(1/2)}}{(-4*a*c+b^2)^{(1/2)}+1/2*\operatorname{arctan}(x*2^{(1/2)}*c^{(1/2)/(b-(-4*a*c+b^2)^{(1/2)})^{(1/2)})*(e^2+(-b*e^2+2*c*d^2)/(-4*a*c+b^2)^{(1/2)})*2^{(1/2)/c^{(1/2)/(b-(-4*a*c+b^2)^{(1/2)})^{(1/2)}+1/2*\operatorname{arctan}(x*2^{(1/2)}*c^{(1/2)/(b+(-4*a*c+b^2)^{(1/2)})^{(1/2)})*(e^2+(b*e^2-2*c*d^2)/(-4*a*c+b^2)^{(1/2)})*2^{(1/2)/c^{(1/2)/(b+(-4*a*c+b^2)^{(1/2)})^{(1/2)}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b} - \sqrt{b^2-4ac}}$

Rubi [A] time = 0.39, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}} + e^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right) + \left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right) - 2de \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b} - \sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] `Int[(d + e*x)^2/(a + b*x^2 + c*x^4), x]`

[Out] $\left(\frac{e^2 + (2*c*d^2 - b*e^2)/\operatorname{Sqrt}[b^2 - 4*a*c]}{\operatorname{Sqrt}[b^2 - 4*a*c]}\right)*\operatorname{ArcTan}\left[\frac{\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]*x}{\operatorname{Sqrt}[b - \operatorname{Sqrt}[b^2 - 4*a*c]]}\right]/\left(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[b - \operatorname{Sqrt}[b^2 - 4*a*c]]\right) + \left(\frac{e^2 - (2*c*d^2 - b*e^2)/\operatorname{Sqrt}[b^2 - 4*a*c]}{\operatorname{Sqrt}[b^2 - 4*a*c]}\right)*\operatorname{ArcTan}\left[\frac{\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]*x}{\operatorname{Sqrt}[b + \operatorname{Sqrt}[b^2 - 4*a*c]]}\right]/\left(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[b + \operatorname{Sqrt}[b^2 - 4*a*c]]\right) - (2*d*e*\operatorname{ArcTanh}[(b + 2*c*x^2)/\operatorname{Sqrt}[b^2 - 4*a*c]])/\operatorname{Sqrt}[b^2 - 4*a*c]$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 205

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1673

Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex)^2}{a + bx^2 + cx^4} dx &= \int \frac{2dex}{a + bx^2 + cx^4} dx + \int \frac{d^2 + e^2x^2}{a + bx^2 + cx^4} dx \\ &= (2de) \int \frac{x}{a + bx^2 + cx^4} dx + \frac{1}{2} \left(e^2 - \frac{2cd^2 - be^2}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx + \frac{1}{2} \left(e^2 + \frac{2cd^2 - be^2}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx \\ &= \frac{\left(e^2 + \frac{2cd^2 - be^2}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(e^2 - \frac{2cd^2 - be^2}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b + \sqrt{b^2 - 4ac}}} + (de) \text{Subst} \left(\frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2}, x, \frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} \right) \\ &= \frac{\left(e^2 + \frac{2cd^2 - be^2}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(e^2 - \frac{2cd^2 - be^2}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b + \sqrt{b^2 - 4ac}}} - (2de) \text{Subst} \left(\frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2}, x, \frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} \right) \\ &= \frac{\left(e^2 + \frac{2cd^2 - be^2}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(e^2 - \frac{2cd^2 - be^2}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{2} \sqrt{c} \sqrt{b + \sqrt{b^2 - 4ac}}} - \frac{2de \tanh^{-1} \left(\frac{\sqrt{b^2 - 4ac} - b - 2cx}{\sqrt{b^2 - 4ac}} \right)}{\sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [A] time = 0.25, size = 245, normalized size = 1.09

$$\frac{\sqrt{2} \left(e^2 \left(\sqrt{b^2 - 4ac} - b \right) + 2cd^2 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{c} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left(e^2 \left(\sqrt{b^2 - 4ac} + b \right) - 2cd^2 \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{c} \sqrt{\sqrt{b^2 - 4ac} + b}} + 2de \log \left(\sqrt{b^2 - 4ac} - b - 2cx \right)}{2\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4), x]

[Out] ((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2*d*e

$*\text{Log}[-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*\text{Log}[b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x^2])/(2*\text{Sqrt}[b^2 - 4*a*c])$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 4.56, size = 1625, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] $-(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*\text{sqrt}(b^2 - 4*a*c)*d*e*\log(x^2 + 1/2*(b + \text{sqrt}(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*\text{sqrt}(b^2 - 4*a*c)*d*e*\log(x^2 + 1/2*(b - \text{sqrt}(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*b^4 - 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^3*c - 2*b^4*c + 16*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^3 - 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b*c - 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b^2*c + \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a^2*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*b*c - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c + \text{sqrt}(b^2 - 4*a*c))*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e^2)*\arctan(2*\text{sqrt}(1/2)*x/\text{sqrt}((b + \text{sqrt}(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*\text{abs}(c)) + 1/4*((\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*b^4 - 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*c)*a*b^2*c - 2*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*b^3*c + 2*b^4*c + 16*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*a^2*c^2 + 8*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*a*b*c^2 + \text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\text{sqrt}(2)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*b^3 - 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*a*b*c - 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*b^2*c + \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*a*b^2 + 4*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*a^2*c + 2*\text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*a*b*c - \text{sqrt}(2)*\text{sqrt}(b^2 - 4*a*c)*\text{sqrt}(b*c - \text{sqrt}(b^2 - 4*a*c))*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e^2)*\arctan(2*\text{sqrt}(1/2)*x/\text{sqrt}((b - \text{sqrt}(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*\text{abs}(c))$


```

*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*
e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2
*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(16*a*b^4*c*z^4 - 12
8*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e
^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4
*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*
a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4
+ c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 32*root(16*a*b^4*c*z^4 - 128*a^2*
b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2
- 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3
*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*
d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)^3*a*b*c^3*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2
*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a
*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^
2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z
+ 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 +
a^2*e^8, z, k)*a*c^2*e^4*x - 2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 +
256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*
d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b
^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*
c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8
, z, k)*b^2*c*e^4*x + 2*b*c*d*e^5*x - 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*
c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 1
6*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4
*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^
5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8
+ a^2*e^8, z, k)*a*c^2*d*e^3 + 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4
+ 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c
^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 +
8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2
*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*
e^8, z, k)*b*c^2*d^3*e + 32*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256
*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4
*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*
c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d
^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z
, k)^2*a*c^3*d*e*x + 12*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3
*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2
+ 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^
5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*
e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)
*b*c^2*d^2*e^2*x - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^
3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 +
192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*
e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^
2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*
b^2*c^2*d*e*x)*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4
- 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^
2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 3
2*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*
c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k), k, 1, 4
)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.339 \quad \int \frac{x^2}{(a+bx)(c+dx)} dx$$

Optimal. Leaf size=56

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

[Out] $x/b/d+a^2*\ln(b*x+a)/b^2/(-a*d+b*c)-c^2*\ln(d*x+c)/d^2/(-a*d+b*c)$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {72}

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x)*(c + d*x)),x]

[Out] $x/(b*d) + (a^2*\text{Log}[a + b*x])/(b^2*(b*c - a*d)) - (c^2*\text{Log}[c + d*x])/(d^2*(b*c - a*d))$

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a+bx)(c+dx)} dx &= \int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)} \right) dx \\ &= \frac{x}{bd} + \frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x)*(c + d*x)),x]

[Out] $x/(b*d) + (a^2*\text{Log}[a + b*x])/(b^2*(b*c - a*d)) - (c^2*\text{Log}[c + d*x])/(d^2*(b*c - a*d))$

fricas [A] time = 0.71, size = 65, normalized size = 1.16

$$\frac{a^2 d^2 \log(bx+a) - b^2 c^2 \log(dx+c) + (b^2 cd - abd^2)x}{b^3 cd^2 - ab^2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] $(a^2 d^2 \log(bx + a) - b^2 c^2 \log(dx + c) + (b^2 c d - a b d^2) x) / (b^3 c d^2 - a b^2 d^3)$

giac [A] time = 0.31, size = 62, normalized size = 1.11

$$\frac{a^2 \log(|bx + a|)}{b^3 c - ab^2 d} - \frac{c^2 \log(|dx + c|)}{bcd^2 - ad^3} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")`

[Out] $a^2 \log(\text{abs}(bx + a)) / (b^3 c - a b^2 d) - c^2 \log(\text{abs}(dx + c)) / (b^3 c d^2 - a d^3) + x / (b d)$

maple [A] time = 0.01, size = 57, normalized size = 1.02

$$-\frac{a^2 \ln(bx + a)}{(ad - bc)b^2} + \frac{c^2 \ln(dx + c)}{(ad - bc)d^2} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x+a)/(d*x+c),x)`

[Out] $x/b/d + 1/d^2 c^2 / (a d - b^2 c) \ln(dx + c) - 1/b^2 a^2 / (a d - b^2 c) \ln(bx + a)$

maxima [A] time = 0.68, size = 60, normalized size = 1.07

$$\frac{a^2 \log(bx + a)}{b^3 c - ab^2 d} - \frac{c^2 \log(dx + c)}{bcd^2 - ad^3} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")`

[Out] $a^2 \log(bx + a) / (b^3 c - a b^2 d) - c^2 \log(dx + c) / (b^3 c d^2 - a d^3) + x / (b d)$

mupad [B] time = 0.21, size = 61, normalized size = 1.09

$$\frac{a^2 d^2 \ln(a + bx) - b^2 c^2 \ln(c + dx) - a b d^2 x + b^2 c d x}{b^2 d^2 (a d - b c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x)*(c + d*x)),x)`

[Out] $-(a^2 d^2 \log(a + b x) - b^2 c^2 \log(c + d x) - a b d^2 x + b^2 c d x) / (b^2 d^2 (a d - b c))$

sympy [B] time = 1.06, size = 190, normalized size = 3.39

$$-\frac{a^2 \log\left(x + \frac{\frac{a^4 d^3}{b(ad-bc)} - \frac{2a^3 c d^2}{ad-bc} + \frac{a^2 b c^2 d}{ad-bc} + a^2 c d + a b c^2}{a^2 d^2 + b^2 c^2}\right)}{b^2 (ad - bc)} + \frac{c^2 \log\left(x + \frac{-\frac{a^2 b c^2 d}{ad-bc} + a^2 c d + \frac{2a b^2 c^3}{ad-bc} + a b c^2 - \frac{b^3 c^4}{d(ad-bc)}}{a^2 d^2 + b^2 c^2}\right)}{d^2 (ad - bc)} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x+a)/(d*x+c),x)`

[Out] $-a^2 \log\left(x + \frac{a^4 d^3}{b(a d - b^2 c)} - \frac{2 a^3 c d^2}{a d - b^2 c} + \frac{a^2 b^2 c^2 d}{a d - b^2 c} + \frac{a^2 c d + a b c^2}{a^2 d^2 + b^2 c^2}\right) + \frac{c^2 \log\left(x + \frac{-a^2 b^2 c^2 d}{a d - b^2 c} + \frac{2 a^2 c d + 2 a^2 b^2 c^3}{a d - b^2 c} + \frac{a b c^2}{a d - b^2 c} - \frac{b^3 c^4}{d(a d - b^2 c)}\right)}{d^2 (a d - b^2 c)} + \frac{x}{b d}$

$$3.340 \quad \int \frac{x^2}{(c+dx)(a+bx^2)} dx$$

Optimal. Leaf size=96

$$\frac{ad \log(a + bx^2)}{2b(ad^2 + bc^2)} + \frac{c^2 \log(c + dx)}{d(ad^2 + bc^2)} - \frac{\sqrt{a} c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad^2 + bc^2)}$$

[Out] $c^2 \ln(d*x+c)/d/(a*d^2+b*c^2)+1/2*a*d*\ln(b*x^2+a)/b/(a*d^2+b*c^2)-c*\arctan(x*b^{(1/2)}/a^{(1/2)})*a^{(1/2)}/(a*d^2+b*c^2)/b^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1629, 635, 205, 260}

$$\frac{ad \log(a + bx^2)}{2b(ad^2 + bc^2)} + \frac{c^2 \log(c + dx)}{d(ad^2 + bc^2)} - \frac{\sqrt{a} c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad^2 + bc^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((c + d*x)*(a + b*x^2)),x]

[Out] $-(\text{Sqrt}[a]*c*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(\text{Sqrt}[b]*(b*c^2 + a*d^2))) + (c^2*\text{Log}[c + d*x]/(d*(b*c^2 + a*d^2)) + (a*d*\text{Log}[a + b*x^2]/(2*b*(b*c^2 + a*d^2)))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(c+dx)(a+bx^2)} dx &= \int \left(\frac{c^2}{(bc^2+ad^2)(c+dx)} - \frac{a(c-dx)}{(bc^2+ad^2)(a+bx^2)} \right) dx \\
&= \frac{c^2 \log(c+dx)}{d(bc^2+ad^2)} - \frac{a \int \frac{c-dx}{a+bx^2} dx}{bc^2+ad^2} \\
&= \frac{c^2 \log(c+dx)}{d(bc^2+ad^2)} - \frac{(ac) \int \frac{1}{a+bx^2} dx}{bc^2+ad^2} + \frac{(ad) \int \frac{x}{a+bx^2} dx}{bc^2+ad^2} \\
&= -\frac{\sqrt{a} c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc^2+ad^2)} + \frac{c^2 \log(c+dx)}{d(bc^2+ad^2)} + \frac{ad \log(a+bx^2)}{2b(bc^2+ad^2)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.76

$$\frac{-2\sqrt{a}\sqrt{b}cd \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + ad^2 \log(a+bx^2) + 2bc^2 \log(c+dx)}{2abd^3 + 2b^2c^2d}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((c + d*x)*(a + b*x^2)), x]

[Out] (-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c + d*x] + a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)

fricas [A] time = 0.73, size = 162, normalized size = 1.69

$$\left[\frac{bcd\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + ad^2 \log(bx^2 + a) + 2bc^2 \log(dx + c)}{2(b^2c^2d + abd^3)}, -\frac{2bcd\sqrt{\frac{a}{b}} \arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - ad^2 \log(bx^2 + a)}{2(b^2c^2d + abd^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^2+a), x, algorithm="fricas")

[Out] [1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a*d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2*b*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3)]

giac [A] time = 0.26, size = 85, normalized size = 0.89

$$\frac{ad \log(bx^2 + a)}{2(b^2c^2 + abd^2)} + \frac{c^2 \log(|dx + c|)}{bc^2d + ad^3} - \frac{ac \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2 + ad^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^2+a), x, algorithm="giac")

[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*d + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))

maple [A] time = 0.01, size = 87, normalized size = 0.91

$$-\frac{ac \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad^2 + bc^2)\sqrt{ab}} + \frac{ad \ln(bx^2 + a)}{2(ad^2 + bc^2)b} + \frac{c^2 \ln(dx + c)}{(ad^2 + bc^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(d*x+c)/(b*x^2+a),x)`

[Out] $\frac{1}{2}ad \ln(bx^2+a)/b/(ad^2+bc^2) - a/(ad^2+bc^2) * c/(ab)^{1/2} * \arctan(1/(ab)^{1/2} * bx) + c^2 \ln(dx+c)/d/(ad^2+bc^2)$

maxima [A] time = 1.22, size = 84, normalized size = 0.88

$$\frac{ad \log(bx^2 + a)}{2(b^2c^2 + abd^2)} + \frac{c^2 \log(dx + c)}{bc^2d + ad^3} - \frac{ac \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2 + ad^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")`

[Out] $\frac{1}{2}ad \log(bx^2 + a)/(b^2c^2 + a*b*d^2) + c^2 \log(dx + c)/(bc^2*d + a*d^3) - a*c \arctan(b*x/\sqrt{a*b})/((b*c^2 + a*d^2)*\sqrt{a*b})$

mupad [B] time = 1.13, size = 347, normalized size = 3.61

$$\frac{\ln\left(ac + adx + \frac{(c\sqrt{-ab^3} + abd)\left(x(2b^2c^2 - 5abd^2) - 5abcd + \frac{2b^2d(c\sqrt{-ab^3} + abd)(-bx^2 + 4acd + 3axd^2)}{2b^3c^2 + 2ab^2d^2}\right)}{2b^3c^2 + 2ab^2d^2} \right)}{2b^3c^2 + 2ab^2d^2} \left(c\sqrt{-ab^3} + abd \right) \ln\left(ac + adx + \frac{(c\sqrt{-ab^3} + abd)\left(x(2b^2c^2 - 5abd^2) - 5abcd + \frac{2b^2d(c\sqrt{-ab^3} + abd)(-bx^2 + 4acd + 3axd^2)}{2b^3c^2 + 2ab^2d^2}\right)}{2b^3c^2 + 2ab^2d^2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^2)*(c + d*x)),x)`

[Out] $(\log(ac + ad*x + ((c*(-ab^3)^{1/2} + a*b*d)*(x*(2*b^2*c^2 - 5*a*b*d^2) - 5*a*b*c*d + (2*b^2*d*(c*(-ab^3)^{1/2} + a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x)))/(2*b^3*c^2 + 2*a*b^2*d^2)))/(2*b^3*c^2 + 2*a*b^2*d^2)) * (c*(-ab^3)^{1/2} + a*b*d) / (2*b^3*c^2 + 2*a*b^2*d^2) - (\log(ac + ad*x + ((c*(-ab^3)^{1/2} - a*b*d)*(b*x*(5*a*d^2 - 2*b*c^2) + 5*a*b*c*d + (d*(c*(-ab^3)^{1/2} - a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x)))/(a*d^2 + b*c^2)))/(2*b^2*(a*d^2 + b*c^2))) * (c*(-ab^3)^{1/2} - a*b*d) / (2*(b^3*c^2 + a*b^2*d^2)) + (c^2 \log(c + d*x)) / (a*d^3 + b*c^2*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(d*x+c)/(b*x**2+a),x)`

[Out] Timed out

$$3.341 \quad \int \frac{x^2}{(c+dx)(a+bx^3)} dx$$

Optimal. Leaf size=264

$$\frac{\sqrt[3]{a} d (\sqrt[3]{a} d + \sqrt[3]{b} c) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{2/3} (bc^3 - ad^3)} - \frac{\sqrt[3]{a} d \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} b^{2/3} (a^{2/3} d^2 + \sqrt[3]{a} \sqrt[3]{b} cd + b^{2/3} c^2)} + \frac{\sqrt[3]{a} d (\sqrt[3]{a} d + \sqrt[3]{b} c)}{3b^{2/3} (bc^3 - ad^3)}$$

[Out] 1/3*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d^3+b*c^3)-c^2*ln(d*x+c)/(-a*d^3+b*c^3)-1/6*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/(-a*d^3+b*c^3)+1/3*c^2*ln(b*x^3+a)/(-a*d^3+b*c^3)-1/3*a^(1/3)*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(2/3)/(b^(2/3)*c^2+a^(1/3)*b^(1/3)*c*d+a^(2/3)*d^2)*3^(1/2)

Rubi [A] time = 0.47, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a} d (\sqrt[3]{a} d + \sqrt[3]{b} c) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{2/3} (bc^3 - ad^3)} - \frac{\sqrt[3]{a} d \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} b^{2/3} (a^{2/3} d^2 + \sqrt[3]{a} \sqrt[3]{b} cd + b^{2/3} c^2)} + \frac{\sqrt[3]{a} d (\sqrt[3]{a} d + \sqrt[3]{b} c)}{3b^{2/3} (bc^3 - ad^3)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((c + d*x)*(a + b*x^3)), x]

[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2)) + (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(2/3)*(b*c^3 - a*d^3)) - (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(2/3)*(b*c^3 - a*d^3)) + (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^n), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
 & - a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I \\
 & *sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 \\
 & - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) \\
 & + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 \\
 & - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)* \\
 & (I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 + 3*sqrt(1/3)*(b*c^3 - a*d \\
 & ^3)*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(\\
 & 2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + \\
 & 1))/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) \\
 & + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1 \\
 & /3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) \\
 & + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) \\
 & + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3) \\
 & *(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b \\
 & *c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c \\
 & ^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b \\
 & *c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c \\
 & ^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2 \\
 & /(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6))*log(3/2*(2*(1/ \\
 & 2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(\\
 & 2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d \\
 & ^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(\\
 & 2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d \\
 & ^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) \\
 & - 2*c^2/(b*c^3 - a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c \\
 & ^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^ \\
 & 3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 \\
 & - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^ \\
 & 3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 \\
 & - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b \\
 & *c^3 - a*d^3))^2 - 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b \\
 & *c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a \\
 & *d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d \\
 & ^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a \\
 & *d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d \\
 & ^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 \\
 & - a*d^3))*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b* \\
 & d^6)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sq \\
 & rt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a* \\
 & d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1 \\
 & /2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a* \\
 & d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*s \\
 & qrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2) \\
 & ^2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2* \\
 & c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3 \\
 & /((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2* \\
 & c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3 \\
 & /((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - \\
 & 2*c^2/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)) + 2*d*x + \\
 & 2*c))/(b*c^3 - a*d^3)
 \end{aligned}$$

giac [A] time = 0.30, size = 320, normalized size = 1.21

$$\frac{-\frac{c^2 d \log(|dx + c|)}{bc^3 d - ad^4} + \frac{c^2 \log(|bx^3 + a|)}{3(bc^3 - ad^3)} + \frac{(-ab^2)^{\frac{1}{3}} d \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3} b^2 c^2 - \sqrt{3} (-ab^2)^{\frac{1}{3}} bcd + \sqrt{3} (-ab^2)^{\frac{2}{3}} d^2} + \frac{\left(ab^2 c^3 d^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2 b d^5\right)}{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] $-c^2*d*\log(\text{abs}(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*\log(\text{abs}(b*x^3 + a))/(b*c^3 - a*d^3) + (-a*b^2)^{1/3}*d*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{1/3}))/(-a/b)^{1/3})/(\sqrt{3}*b^2*c^2 - \sqrt{3}*(-a*b^2)^{1/3}*b*c*d + \sqrt{3}*(-a*b^2)^{2/3}*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^{1/3} - a^2*b*d^5*(-a/b)^{1/3} - a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^{1/3}*\log(\text{abs}(x - (-a/b)^{1/3}))/ (a*b^3*c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^{1/3}*b*c*d - (-a*b^2)^{2/3}*d^2)*\log(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3})/(b^3*c^3 - a*b^2*d^3)$

maple [A] time = 0.01, size = 336, normalized size = 1.27

$$\frac{\sqrt{3}acd \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad^3-bc^3)\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{acd \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad^3-bc^3)\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{acd \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad^3-bc^3)\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{\sqrt{3}ad^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad^3-bc^3)\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(d*x+c)/(b*x^3+a),x)

[Out] $-1/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})+1/6/(a*d^3-b*c^3)*a*c*d/b/(a/b)^{2/3}*\ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3})-1/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))-1/3/(a*d^3-b*c^3)*a*d^2/b/(a/b)^{1/3}*\ln(x+(a/b)^{1/3})+1/6/(a*d^3-b*c^3)*a*d^2/b/(a/b)^{1/3}*\ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3})+1/3/(a*d^3-b*c^3)*a*d^2*3^{1/2}/b/(a/b)^{1/3}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))-1/3/(a*d^3-b*c^3)*c^2*\ln(b*x^3+a)+c^2/(a*d^3-b*c^3)*\ln(d*x+c)$

maxima [A] time = 1.71, size = 279, normalized size = 1.06

$$\frac{c^2 \log(dx+c)}{bc^3-ad^3} + \frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(2bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-acd\right) \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")

[Out] $-c^2*\log(d*x + c)/(b*c^3 - a*d^3) - 1/3*\sqrt{3}*(a*d^2*(a/b)^{2/3} - a*c*d*(a/b)^{1/3})*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{1/3}))/ (a/b)^{1/3})/((b^2*c^3*(a/b)^{2/3} - a*b*d^3*(a/b)^{2/3})*(a/b)^{1/3}) + 1/6*(2*b*c^2*(a/b)^{2/3} - a*d^2*(a/b)^{1/3} - a*c*d)*\log(x^2 - x*(a/b)^{1/3} + (a/b)^{2/3})/ (b^2*c^3*(a/b)^{2/3} - a*b*d^3*(a/b)^{2/3}) + 1/3*(b*c^2*(a/b)^{2/3} + a*d^2*(a/b)^{1/3} + a*c*d)*\log(x + (a/b)^{1/3})/ (b^2*c^3*(a/b)^{2/3} - a*b*d^3*(a/b)^{2/3})$

mupad [B] time = 2.50, size = 570, normalized size = 2.16

$$\left(\sum_{k=1}^3 \ln\left(-abd\left(c+dx+\text{root}\left(27ab^2d^3z^3-27b^3c^3z^3+27b^2c^2z^2-9bcz+1,z,k\right)^2b^2c^33+\text{root}\left(27ab^2d^3z^3-27b^3c^3z^3+27b^2c^2z^2-9bcz+1,z,k\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a+b*x^3)*(c+dx)),x)

```
[Out] symsum(log(-a*b*d*(c + d*x + 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^3 + 9*root(27*a*b^2*d^3*z^3 - 27*b
^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^4 - 5*root(27*a*b^
2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c^2 - 3*
root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k
)^2*a*b*d^3 - 8*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9
*b*c*z + 1, z, k)*b*c*d*x + 45*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*c*d^3 + 36*root(27*a*b^2*d^3*z^3 -
27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*d^4*x + 9*roo
t(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2
*b^2*c^2*d*x + 18*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 -
9*b*c*z + 1, z, k)^3*b^3*c^3*d*x))*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3
+ 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k), k, 1, 3) + (c^2*log(c + d*x))/(a*d^3
- b*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**3+a), x)
```

```
[Out] Timed out
```

$$3.342 \quad \int \frac{x^2}{(c+dx)(a+bx^4)} dx$$

Optimal. Leaf size=417

$$\frac{\sqrt{a} d^3 \tan^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right)}{2\sqrt{b} (ad^4 + bc^4)} - \frac{c^2 d \log(a + bx^4)}{4(ad^4 + bc^4)} + \frac{c^2 d \log(c + dx)}{ad^4 + bc^4} + \frac{c(\sqrt{a} d^2 + \sqrt{b} c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)} - \frac{c(\sqrt{a} d^2 + \sqrt{b} c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)}$$

[Out] $c^2 d \ln(d x + c) / (a d^4 + b c^4) - 1/4 c^2 d \ln(b x^4 + a) / (a d^4 + b c^4) + 1/2 d^3 a \operatorname{rctan}(x^2 b^{1/2} / a^{1/2}) * a^{1/2} / (a d^4 + b c^4) / b^{1/2} + 1/4 c * \arctan(-1 + b^{1/4} * x^2^{1/2} / a^{1/4}) * (-d^2 a^{1/2} + b^{1/2} * c^2) / a^{1/4} / b^{1/4} / (a d^4 + b c^4) * 2^{1/2} + 1/4 c * \arctan(1 + b^{1/4} * x^2^{1/2} / a^{1/4}) * (-d^2 a^{1/2} + b^{1/2} * c^2) / a^{1/4} / b^{1/4} / (a d^4 + b c^4) * 2^{1/2} + 1/8 c * \ln(-a^{1/4} * b^{1/4} * x^2^{1/2} + a^{1/2} + x^2 * b^{1/2}) * (d^2 a^{1/2} + b^{1/2} * c^2) / a^{1/4} / b^{1/4} / (a d^4 + b c^4) * 2^{1/2} - 1/8 c * \ln(a^{1/4} * b^{1/4} * x^2^{1/2} + a^{1/2} + x^2 * b^{1/2}) * (d^2 a^{1/2} + b^{1/2} * c^2) / a^{1/4} / b^{1/4} / (a d^4 + b c^4) * 2^{1/2}$

Rubi [A] time = 0.55, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6725, 1461, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{c^2 d \log(a + bx^4)}{4(ad^4 + bc^4)} + \frac{c(\sqrt{a} d^2 + \sqrt{b} c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)} - \frac{c(\sqrt{a} d^2 + \sqrt{b} c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((c + d*x)*(a + b*x^4)), x]

[Out] $(\operatorname{Sqrt}[a] * d^3 * \operatorname{ArcTan}[(\operatorname{Sqrt}[b] * x^2) / \operatorname{Sqrt}[a]]) / (2 * \operatorname{Sqrt}[b] * (b * c^4 + a * d^4)) - (c * (\operatorname{Sqrt}[b] * c^2 - \operatorname{Sqrt}[a] * d^2) * \operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2] * b^{1/4} * x) / a^{1/4}]) / (2 * \operatorname{Sqrt}[2] * a^{1/4} * b^{1/4} * (b * c^4 + a * d^4)) + (c * (\operatorname{Sqrt}[b] * c^2 - \operatorname{Sqrt}[a] * d^2) * \operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2] * b^{1/4} * x) / a^{1/4}]) / (2 * \operatorname{Sqrt}[2] * a^{1/4} * b^{1/4} * (b * c^4 + a * d^4)) + (c^2 * d * \operatorname{Log}[c + d * x]) / (b * c^4 + a * d^4) + (c * (\operatorname{Sqrt}[b] * c^2 + \operatorname{Sqrt}[a] * d^2) * \operatorname{Log}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[2] * a^{1/4} * b^{1/4} * x + \operatorname{Sqrt}[b] * x^2]) / (4 * \operatorname{Sqrt}[2] * a^{1/4} * b^{1/4} * (b * c^4 + a * d^4)) - (c * (\operatorname{Sqrt}[b] * c^2 + \operatorname{Sqrt}[a] * d^2) * \operatorname{Log}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[2] * a^{1/4} * b^{1/4} * x + \operatorname{Sqrt}[b] * x^2]) / (4 * \operatorname{Sqrt}[2] * a^{1/4} * b^{1/4} * (b * c^4 + a * d^4)) - (c^2 * d * \operatorname{Log}[a + b * x^4]) / (4 * (b * c^4 + a * d^4))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1461

Int[((A_) + (B_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Dist[A, Int[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] + Dist[B, Int[x^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, A, B, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[m - n + 1, 0]

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(c+dx)(a+bx^4)} dx &= \int \left(\frac{c^2 d^2}{(bc^4+ad^4)(c+dx)} + \frac{(c-dx)(-ad^2+bc^2x^2)}{(bc^4+ad^4)(a+bx^4)} \right) dx \\
&= \frac{c^2 d \log(c+dx)}{bc^4+ad^4} + \frac{\int \frac{(c-dx)(-ad^2+bc^2x^2)}{a+bx^4} dx}{bc^4+ad^4} \\
&= \frac{c^2 d \log(c+dx)}{bc^4+ad^4} + \frac{c \int \frac{-ad^2+bc^2x^2}{a+bx^4} dx}{bc^4+ad^4} - \frac{d \int \frac{x(-ad^2+bc^2x^2)}{a+bx^4} dx}{bc^4+ad^4} \\
&= \frac{c^2 d \log(c+dx)}{bc^4+ad^4} - \frac{d \operatorname{Subst} \left(\int \frac{-ad^2+bc^2x}{a+bx^2} dx, x, x^2 \right)}{2(bc^4+ad^4)} + \frac{\left(c \left(c^2 - \frac{\sqrt{a}d^2}{\sqrt{b}} \right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx \right)}{2(bc^4+ad^4)} - \dots \\
&= \frac{c^2 d \log(c+dx)}{bc^4+ad^4} - \frac{(bc^2 d) \operatorname{Subst} \left(\int \frac{x}{a+bx^2} dx, x, x^2 \right)}{2(bc^4+ad^4)} + \frac{(ad^3) \operatorname{Subst} \left(\int \frac{1}{a+bx^2} dx, x, x^2 \right)}{2(bc^4+ad^4)} + \dots \\
&= \frac{\sqrt{a}d^3 \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{2\sqrt{b}(bc^4+ad^4)} + \frac{c^2 d \log(c+dx)}{bc^4+ad^4} + \frac{\sqrt[4]{b}c \left(c^2 + \frac{\sqrt{a}d^2}{\sqrt{b}} \right) \log \left(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \dots \right)}{4\sqrt{2} \sqrt[4]{a} (bc^4+ad^4)} \\
&= \frac{\sqrt{a}d^3 \tan^{-1} \left(\frac{\sqrt{b}x^2}{\sqrt{a}} \right)}{2\sqrt{b}(bc^4+ad^4)} - \frac{\sqrt[4]{b}c \left(c^2 - \frac{\sqrt{a}d^2}{\sqrt{b}} \right) \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}} \right)}{2\sqrt{2} \sqrt[4]{a} (bc^4+ad^4)} + \frac{\sqrt[4]{b}c \left(c^2 - \frac{\sqrt{a}d^2}{\sqrt{b}} \right) \tan^{-1} \left(\dots \right)}{2\sqrt{2} \sqrt[4]{a} (bc^4+ad^4)} + \dots
\end{aligned}$$

Mathematica [A] time = 0.23, size = 370, normalized size = 0.89

$$-2 \left(2a^{3/4}d^3 - \sqrt{2} \sqrt{a} \sqrt[4]{b} cd^2 + \sqrt{2} b^{3/4}c^3 \right) \tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}} \right) + 2 \left(-2a^{3/4}d^3 - \sqrt{2} \sqrt{a} \sqrt[4]{b} cd^2 + \sqrt{2} b^{3/4}c^3 \right) \tan^{-1} \left(\dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((c + d*x)*(a + b*x^4)),x]

[Out] (-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - 2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4])/(8*a^(1/4)*Sqrt[b]*(b*c^4 + a*d^4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.57, size = 401, normalized size = 0.96

$$\frac{c^2 d^2 \log(|dx + c|)}{bc^4 d + ad^5} - \frac{c^2 d \log(|bx^4 + a|)}{4(bc^4 + ad^4)} + \frac{\left(\sqrt{2} \sqrt{ab} bd + (ab^3)^{\frac{1}{4}} bc\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2} ab^2 d^2 + \sqrt{2} \sqrt{ab} b^2 c^2 - 2(ab^3)^{\frac{3}{4}} cd\right)} + \frac{\left(\sqrt{2} \sqrt{ab} bd + (ab^3)^{\frac{1}{4}} bc\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2} ab^2 d^2 + \sqrt{2} \sqrt{ab} b^2 c^2 - 2(ab^3)^{\frac{3}{4}} cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out] $c^2 d^2 \log(\text{abs}(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*\log(\text{abs}(b*x^4 + a))/(b*c^4 + a*d^4) + 1/2*(\text{sqrt}(2)*\text{sqrt}(a*b)*b*d + (a*b^3)^{(1/4)}*b*c)*\arctan(1/2*\text{sqrt}(2)*(2*x + \text{sqrt}(2)*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\text{sqrt}(2)*a*b^2*d^2 + \text{sqrt}(2)*\text{sqrt}(a*b)*b^2*c^2 - 2*(a*b^3)^{(3/4)}*c*d) + 1/2*(\text{sqrt}(2)*\text{sqrt}(a*b)*b*d + (a*b^3)^{(1/4)}*b*c)*\arctan(1/2*\text{sqrt}(2)*(2*x - \text{sqrt}(2)*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\text{sqrt}(2)*a*b^2*d^2 + \text{sqrt}(2)*\text{sqrt}(a*b)*b^2*c^2 + 2*(a*b^3)^{(3/4)}*c*d) - 1/4*((a*b^3)^{(1/4)}*a*b*c*d^2 + (a*b^3)^{(3/4)}*c^3)*\log(x^2 + \text{sqrt}(2)*x*(a/b)^{(1/4)} + \text{sqrt}(a/b))/(\text{sqrt}(2)*a*b^3*c^4 + \text{sqrt}(2)*a^2*b^2*d^4) + 1/4*((a*b^3)^{(1/4)}*a*b*c*d^2 + (a*b^3)^{(3/4)}*c^3)*\log(x^2 - \text{sqrt}(2)*x*(a/b)^{(1/4)} + \text{sqrt}(a/b))/(\text{sqrt}(2)*a*b^3*c^4 + \text{sqrt}(2)*a^2*b^2*d^4)$

maple [A] time = 0.01, size = 422, normalized size = 1.01

$$\frac{a d^3 \arctan\left(\sqrt{\frac{b}{a}} x^2\right)}{2(a d^4 + b c^4) \sqrt{ab}} + \frac{\sqrt{2} c^3 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{4(a d^4 + b c^4) \left(\frac{a}{b}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} c^3 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{4(a d^4 + b c^4) \left(\frac{a}{b}\right)^{\frac{1}{4}}} + \frac{\sqrt{2} c^3 \ln\left(\frac{x^2 - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}{x^2 + \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}\right)}{8(a d^4 + b c^4) \left(\frac{a}{b}\right)^{\frac{1}{4}}} - \frac{c^2 d \ln\left(\frac{x^2 - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}{x^2 + \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}\right)}{4(a d^4 + b c^4) \left(\frac{a}{b}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(d*x+c)/(b*x^4+a),x)

[Out] $c^2*d*\ln(d*x+c)/(a*d^4+b*c^4) - 1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^{(1/4)}*2^{(1/2)}*a*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1) - 1/8/(a*d^4+b*c^4)*c*d^2*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})) - 1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1) + 1/2/(a*d^4+b*c^4)*a*d^3/(a*b)^{(1/2)}*\arctan(x^2*(b/a)^{(1/2)}) + 1/8/(a*d^4+b*c^4)*c^3/(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})/(x^2+(a/b)^{(1/4)}*x*2^{(1/2)}+(a/b)^{(1/2)})) + 1/4/(a*d^4+b*c^4)*c^3/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1) + 1/4/(a*d^4+b*c^4)*c^3/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1) - 1/4*c^2*d*\ln(b*x^4+a)/(a*d^4+b*c^4)$

maxima [A] time = 1.48, size = 349, normalized size = 0.84

$$\frac{c^2 d \log(dx + c)}{bc^4 + ad^4} - \frac{\sqrt{2} \left(\sqrt{2} a^{\frac{3}{4}} b^{\frac{5}{4}} c^2 d + \sqrt{a} b^{\frac{3}{2}} c^3 + abcd^2 \right) \log\left(\sqrt{b} x^2 + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{5}{4}}} + \frac{\sqrt{2} \left(\sqrt{2} a^{\frac{3}{4}} b^{\frac{5}{4}} c^2 d - \sqrt{a} b^{\frac{3}{2}} c^3 - abcd^2 \right) \log\left(\sqrt{b} x^2 - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="maxima")

$$3.343 \quad \int \frac{x}{(1-x)(1+x)^2} dx$$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)} + \frac{1}{2} \tanh^{-1}(x)$$

[Out] 1/2/(1+x)+1/2*arctanh(x)

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {77, 207}

$$\frac{1}{2(x+1)} + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/((1 - x)*(1 + x)^2), x]

[Out] 1/(2*(1 + x)) + ArcTanh[x]/2

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x}{(1-x)(1+x)^2} dx &= \int \left(-\frac{1}{2(1+x)^2} - \frac{1}{2(-1+x^2)} \right) dx \\ &= \frac{1}{2(1+x)} - \frac{1}{2} \int \frac{1}{-1+x^2} dx \\ &= \frac{1}{2(1+x)} + \frac{1}{2} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.50

$$\frac{1}{4} \left(\frac{2}{x+1} - \log(1-x) + \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/((1 - x)*(1 + x)^2), x]

[Out] (2/(1 + x) - Log[1 - x] + Log[1 + x])/4

fricas [B] time = 0.67, size = 26, normalized size = 1.62

$$\frac{(x+1)\log(x+1) - (x+1)\log(x-1) + 2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")

[Out] 1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)

giac [A] time = 0.24, size = 21, normalized size = 1.31

$$\frac{1}{2(x+1)} - \frac{1}{4} \log\left(\left|-\frac{2}{x+1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")

[Out] 1/2/(x + 1) - 1/4*log(abs(-2/(x + 1) + 1))

maple [A] time = 0.01, size = 21, normalized size = 1.31

$$-\frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1-x)/(x+1)^2,x)

[Out] -1/4*ln(x-1)+1/2/(x+1)+1/4*ln(x+1)

maxima [A] time = 0.75, size = 20, normalized size = 1.25

$$\frac{1}{2(x+1)} + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")

[Out] 1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)

mupad [B] time = 0.04, size = 12, normalized size = 0.75

$$\frac{\operatorname{atanh}(x)}{2} + \frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x/((x-1)*(x+1)^2),x)

[Out] atanh(x)/2 + 1/(2*(x + 1))

sympy [A] time = 0.10, size = 19, normalized size = 1.19

$$-\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1-x)/(1+x)**2,x)

[Out] -log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)

$$3.344 \quad \int \frac{x^2}{(1-x^2)(1+x^2)^2} dx$$

Optimal. Leaf size=19

$$\frac{1}{4} \tanh^{-1}(x) - \frac{x}{4(x^2+1)}$$

[Out] -1/4*x/(x^2+1)+1/4*arctanh(x)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {471, 206}

$$\frac{1}{4} \tanh^{-1}(x) - \frac{x}{4(x^2+1)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - x^2)*(1 + x^2)^2), x]

[Out] -x/(4*(1 + x^2)) + ArcTanh[x]/4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(1-x^2)(1+x^2)^2} dx &= -\frac{x}{4(1+x^2)} + \frac{1}{4} \int \frac{1}{1-x^2} dx \\ &= -\frac{x}{4(1+x^2)} + \frac{1}{4} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.42

$$\frac{1}{8} \left(-\frac{2x}{x^2+1} - \log(1-x) + \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((1 - x^2)*(1 + x^2)^2), x]

[Out] ((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8

fricas [B] time = 0.48, size = 34, normalized size = 1.79

$$\frac{(x^2 + 1) \log(x + 1) - (x^2 + 1) \log(x - 1) - 2x}{8(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)

giac [A] time = 0.30, size = 30, normalized size = 1.58

$$-\frac{1}{4\left(x + \frac{1}{x}\right)} + \frac{1}{16} \log\left(\left|x + \frac{1}{x} + 2\right|\right) - \frac{1}{16} \log\left(\left|x + \frac{1}{x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")

[Out] -1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))

maple [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{x}{4(x^2 + 1)} - \frac{\ln(x - 1)}{8} + \frac{\ln(x + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^2+1)/(x^2+1)^2,x)

[Out] -1/8*ln(x-1)-1/4/(x^2+1)*x+1/8*ln(x+1)

maxima [A] time = 0.65, size = 23, normalized size = 1.21

$$-\frac{x}{4(x^2 + 1)} + \frac{1}{8} \log(x + 1) - \frac{1}{8} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")

[Out] -1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)

mupad [B] time = 2.14, size = 17, normalized size = 0.89

$$\frac{\operatorname{atanh}(x)}{4} - \frac{x}{4(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^2/((x^2 - 1)*(x^2 + 1)^2),x)

[Out] atanh(x)/4 - x/(4*(x^2 + 1))

sympy [A] time = 0.12, size = 20, normalized size = 1.05

$$-\frac{x}{4x^2 + 4} - \frac{\log(x - 1)}{8} + \frac{\log(x + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-x**2+1)/(x**2+1)**2,x)

[Out] -x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8

$$3.345 \quad \int \frac{x^3}{(1-x^3)(1+x^3)^2} dx$$

Optimal. Leaf size=97

$$-\frac{x}{6(x^3+1)} + \frac{1}{72} \log(x^2-x+1) + \frac{1}{24} \log(x^2+x+1) - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(x+1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

[Out] -1/6*x/(x^3+1)-1/12*ln(1-x)-1/36*ln(1+x)+1/72*ln(x^2-x+1)+1/24*ln(x^2+x+1)+1/36*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {471, 522, 200, 31, 634, 618, 204, 628}

$$-\frac{x}{6(x^3+1)} + \frac{1}{72} \log(x^2-x+1) + \frac{1}{24} \log(x^2+x+1) - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(x+1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((1 - x^3)*(1 + x^3)^2), x]

[Out] -x/(6*(1 + x^3)) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36 + Log[1 - x + x^2]/72 + Log[1 + x + x^2]/24

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[(e_.*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[(e_. + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]

- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(1-x^3)(1+x^3)^2} dx &= -\frac{x}{6(1+x^3)} + \frac{1}{6} \int \frac{1+2x^3}{(1-x^3)(1+x^3)} dx \\ &= -\frac{x}{6(1+x^3)} - \frac{1}{12} \int \frac{1}{1+x^3} dx + \frac{1}{4} \int \frac{1}{1-x^3} dx \\ &= -\frac{x}{6(1+x^3)} - \frac{1}{36} \int \frac{1}{1+x} dx - \frac{1}{36} \int \frac{2-x}{1-x+x^2} dx + \frac{1}{12} \int \frac{1}{1-x} dx + \frac{1}{12} \int \frac{2+x}{1+x+x^2} dx \\ &= -\frac{x}{6(1+x^3)} - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(1+x) + \frac{1}{72} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{24} \int \frac{1}{1-x+x^2} dx \\ &= -\frac{x}{6(1+x^3)} - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(1+x) + \frac{1}{72} \log(1-x+x^2) + \frac{1}{24} \log(1+x+x^2) \\ &= -\frac{x}{6(1+x^3)} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(1+x) + \frac{1}{72} \log(1-x+x^2) + \frac{1}{24} \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.88

$$\frac{1}{72} \left(-\frac{12x}{x^3+1} + \log(x^2-x+1) + 3 \log(x^2+x+1) - 6 \log(1-x) - 2 \log(x+1) - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 6\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((1 - x^3)*(1 + x^3)^2), x]

[Out] ((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + 3*Log[1 + x + x^2])/72

fricas [A] time = 0.70, size = 106, normalized size = 1.09

$$\frac{6\sqrt{3}(x^3+1) \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2\sqrt{3}(x^3+1) \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 3(x^3+1) \log(x^2+x+1) + (x^3-1) \log(x^2-x+1)}{72(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")

[Out] 1/72*(6*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x^3 + 1)*log(x^2 + x + 1) + (x^3 + 1)*log(x^2 - x + 1) - 2*(x^3 + 1)*log(x + 1) - 6*(x^3 + 1)*log(x - 1) - 12*x)/(x^3 + 1)

giac [A] time = 0.31, size = 77, normalized size = 0.79

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{36} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{x}{6(x^3 + 1)} + \frac{1}{24} \log(x^2 + x + 1) + \frac{1}{72} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x + 1) - 1/36*log(abs(x + 1)) - 1/12*log(abs(x - 1))

maple [A] time = 0.01, size = 90, normalized size = 0.93

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36} + \frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x-1)}{12} - \frac{\ln(x+1)}{36} + \frac{\ln(x^2-x+1)}{72} + \frac{\ln(x^2+x+1)}{24} + \frac{x}{6(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(-x^3+1)/(x^3+1)^2,x)

[Out] -1/12*ln(x-1)+1/36*(-2*x-2)/(x^2-x+1)+1/72*ln(x^2-x+1)-1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/18/(x+1)-1/36*ln(x+1)+1/24*ln(x^2+x+1)+1/12*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 1.93, size = 75, normalized size = 0.77

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{36} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{x}{6(x^3 + 1)} + \frac{1}{24} \log(x^2 + x + 1) + \frac{1}{72} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x + 1) - 1/36*log(x + 1) - 1/12*log(x - 1)

mupad [B] time = 0.18, size = 103, normalized size = 1.06

$$-\frac{\ln(x-1)}{12} - \frac{\ln(x+1)}{36} - \frac{x}{6(x^3+1)} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) \left(-\frac{1}{24} + \frac{\sqrt{3} 1i}{24}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(\frac{1}{24} + \frac{\sqrt{3} 1i}{24}\right) + \frac{1}{24} \log(x^2 + x + 1) + \frac{1}{72} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^3/((x^3 - 1)*(x^3 + 1)^2),x)

[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x + 1)/36 - x/(6*(x^3 + 1)) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x - 1)/12 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 + 1/72) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 - 1/72)

sympy [A] time = 0.36, size = 92, normalized size = 0.95

$$-\frac{x}{6x^3 + 6} - \frac{\log(x - 1)}{12} - \frac{\log(x + 1)}{36} + \frac{\log(x^2 - x + 1)}{72} + \frac{\log(x^2 + x + 1)}{24} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{36} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(-x**3+1)/(x**3+1)**2,x)

[Out] -x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x + 1)/72 + log(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/36 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/12

$$3.346 \quad \int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} dx$$

Optimal. Leaf size=15

$$\frac{1}{2} \log(x^2 + 3) + 3 \tan^{-1}(x)$$

[Out] 3*arctan(x)+1/2*ln(x^2+3)

Rubi [A] time = 0.11, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6725, 203, 260}

$$\frac{1}{2} \log(x^2 + 3) + 3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] 3*ArcTan[x] + Log[3 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} dx &= \int \left(\frac{3}{1+x^2} + \frac{x}{3+x^2} \right) dx \\ &= 3 \int \frac{1}{1+x^2} dx + \int \frac{x}{3+x^2} dx \\ &= 3 \tan^{-1}(x) + \frac{1}{2} \log(3+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 3) + 3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] 3*ArcTan[x] + Log[3 + x^2]/2

fricas [A] time = 0.63, size = 13, normalized size = 0.87

$$3 \arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="fricas")

[Out] 3*arctan(x) + 1/2*log(x^2 + 3)

giac [A] time = 0.29, size = 13, normalized size = 0.87

$$3 \arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="giac")

[Out] 3*arctan(x) + 1/2*log(x^2 + 3)

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$3 \arctan(x) + \frac{\ln(x^2 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x)

[Out] 3*arctan(x)+1/2*ln(x^2+3)

maxima [A] time = 1.40, size = 13, normalized size = 0.87

$$3 \arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="maxima")

[Out] 3*arctan(x) + 1/2*log(x^2 + 3)

mupad [B] time = 2.12, size = 13, normalized size = 0.87

$$\frac{\ln(x^2 + 3)}{2} + 3 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + x^3 + 9)/((x^2 + 1)*(x^2 + 3)),x)

[Out] log(x^2 + 3)/2 + 3*atan(x)

sympy [A] time = 0.12, size = 12, normalized size = 0.80

$$\frac{\log(x^2 + 3)}{2} + 3 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)

[Out] log(x**2 + 3)/2 + 3*atan(x)

$$3.347 \quad \int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx$$

Optimal. Leaf size=13

$$\frac{1}{2} \log(x^2 + 3) + \tan^{-1}(x)$$

[Out] arctan(x)+1/2*ln(x^2+3)

Rubi [A] time = 0.09, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6725, 203, 260}

$$\frac{1}{2} \log(x^2 + 3) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] ArcTan[x] + Log[3 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx &= \int \left(\frac{1}{1+x^2} + \frac{x}{3+x^2} \right) dx \\ &= \int \frac{1}{1+x^2} dx + \int \frac{x}{3+x^2} dx \\ &= \tan^{-1}(x) + \frac{1}{2} \log(3+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 3) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] ArcTan[x] + Log[3 + x^2]/2

fricas [A] time = 0.55, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="fricas")

[Out] arctan(x) + 1/2*log(x^2 + 3)

giac [A] time = 0.30, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="giac")

[Out] arctan(x) + 1/2*log(x^2 + 3)

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\arctan(x) + \frac{\ln(x^2 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x)

[Out] arctan(x)+1/2*ln(x^2+3)

maxima [A] time = 1.33, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="maxima")

[Out] arctan(x) + 1/2*log(x^2 + 3)

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln(x^2 + 3)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 + x^3 + 3)/((x^2 + 1)*(x^2 + 3)),x)

[Out] log(x^2 + 3)/2 + atan(x)

sympy [A] time = 0.12, size = 10, normalized size = 0.77

$$\frac{\log(x^2 + 3)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)

[Out] log(x**2 + 3)/2 + atan(x)

$$3.348 \quad \int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=29

$$\frac{3}{2} \log(x^2 + 1) - 3 \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.12, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6725, 635, 203, 260}

$$\frac{3}{2} \log(x^2 + 1) - 3 \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)), x]

[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)} dx &= \int \left(\frac{3(-1+x)}{1+x^2} + \frac{2}{2+x^2} \right) dx \\ &= 2 \int \frac{1}{2+x^2} dx + 3 \int \frac{-1+x}{1+x^2} dx \\ &= \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - 3 \int \frac{1}{1+x^2} dx + 3 \int \frac{x}{1+x^2} dx \\ &= -3 \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{3}{2} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{3}{2} \log(x^2 + 1) - 3 \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)), x]

[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2

fricas [A] time = 0.57, size = 24, normalized size = 0.83

$$\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - 3 \arctan(x) + \frac{3}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2), x, algorithm="fricas")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)

giac [A] time = 0.26, size = 24, normalized size = 0.83

$$\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - 3 \arctan(x) + \frac{3}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2), x, algorithm="giac")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)

maple [A] time = 0.00, size = 25, normalized size = 0.86

$$-3 \arctan(x) + \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{2}\right) + \frac{3 \ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2), x)

[Out] -3*arctan(x)+3/2*ln(x^2+1)+2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.48, size = 24, normalized size = 0.83

$$\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) - 3 \arctan(x) + \frac{3}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2), x, algorithm="maxima")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)

mupad [B] time = 2.15, size = 51, normalized size = 1.76

$$-\sqrt{2} \operatorname{atan}\left(\frac{24\sqrt{2}}{24x-64} + \frac{32\sqrt{2}x}{24x-64}\right) + \ln(x-i) \left(\frac{3}{2} + \frac{3}{2}i\right) + \ln(x+1i) \left(\frac{3}{2} - \frac{3}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)`

[Out] `log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(1/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))`

sympy [A] time = 0.19, size = 29, normalized size = 1.00

$$\frac{3\log(x^2 + 1)}{2} - 3\operatorname{atan}(x) + \sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)`

[Out] `3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)`

$$3.349 \quad \int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx$$

Optimal. Leaf size=14

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

[Out] 1/(2-x)-arctan(-2+x)

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {27, 693, 618, 204}

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

Antiderivative was successfully verified.

[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]

[Out] (2 - x)^(-1) + ArcTan[2 - x]

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 693

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || IntegerQ[(m + 2*p + 3)/2])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx &= \int \frac{1}{(-2+x)^2(5-4x+x^2)} dx \\
&= \frac{1}{2-x} - \int \frac{1}{5-4x+x^2} dx \\
&= \frac{1}{2-x} + 2 \operatorname{Subst} \left(\int \frac{1}{-4-x^2} dx, x, -4+2x \right) \\
&= \frac{1}{2-x} + \tan^{-1}(2-x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\tan^{-1}(2-x) - \frac{1}{x-2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)), x]

[Out] -(-2 + x)^(-1) + ArcTan[2 - x]

fricas [A] time = 0.57, size = 17, normalized size = 1.21

$$\frac{(x-2) \arctan(x-2) + 1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5), x, algorithm="fricas")

[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5), x, algorithm="giac")

[Out] -1/(x - 2) - arctan(x - 2)

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$-\arctan(x-2) - \frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-4*x+4)/(x^2-4*x+5), x)

[Out] -arctan(x-2)-1/(x-2)

maxima [A] time = 2.07, size = 14, normalized size = 1.00

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")

[Out] -1/(x - 2) - arctan(x - 2)

mupad [B] time = 0.04, size = 14, normalized size = 1.00

$$-\operatorname{atan}(x - 2) - \frac{1}{x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)

[Out] - atan(x - 2) - 1/(x - 2)

sympy [A] time = 0.14, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x - 2) - \frac{1}{x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)

[Out] -atan(x - 2) - 1/(x - 2)

$$3.350 \quad \int \frac{-3+x+x^2}{(-3+x)x^2} dx$$

Optimal. Leaf size=12

$$\log(3-x) - \frac{1}{x}$$

[Out] -1/x+ln(3-x)

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {893}

$$\log(3-x) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[(-3 + x + x^2)/((-3 + x)*x^2), x]

[Out] -x^(-1) + Log[3 - x]

Rule 893

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{-3+x+x^2}{(-3+x)x^2} dx &= \int \left(\frac{1}{-3+x} + \frac{1}{x^2} \right) dx \\ &= -\frac{1}{x} + \log(3-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(3-x) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x + x^2)/((-3 + x)*x^2), x]

[Out] -x^(-1) + Log[3 - x]

fricas [A] time = 0.60, size = 12, normalized size = 1.00

$$\frac{x \log(x-3) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")

[Out] (x*log(x - 3) - 1)/x

giac [A] time = 0.42, size = 11, normalized size = 0.92

$$-\frac{1}{x} + \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")

[Out] -1/x + log(abs(x - 3))

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\ln(x - 3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x-3)/(x-3)/x^2,x)

[Out] -1/x+ln(x-3)

maxima [A] time = 0.75, size = 10, normalized size = 0.83

$$-\frac{1}{x} + \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")

[Out] -1/x + log(x - 3)

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$\ln(x - 3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 - 3)/(x^2*(x - 3)),x)

[Out] log(x - 3) - 1/x

sympy [A] time = 0.09, size = 7, normalized size = 0.58

$$\log(x - 3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x-3)/(-3+x)/x**2,x)

[Out] log(x - 3) - 1/x

$$3.351 \quad \int \frac{1+x+4x^2}{x+4x^3} dx$$

Optimal. Leaf size=11

$$\log(x) + \frac{1}{2} \tan^{-1}(2x)$$

[Out] 1/2*arctan(2*x)+ln(x)

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1593, 1802, 203}

$$\log(x) + \frac{1}{2} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + 4*x^2)/(x + 4*x^3), x]

[Out] ArcTan[2*x]/2 + Log[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+x+4x^2}{x+4x^3} dx &= \int \frac{1+x+4x^2}{x(1+4x^2)} dx \\ &= \int \left(\frac{1}{x} + \frac{1}{1+4x^2} \right) dx \\ &= \log(x) + \int \frac{1}{1+4x^2} dx \\ &= \frac{1}{2} \tan^{-1}(2x) + \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x) + \frac{1}{2} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]

[Out] ArcTan[2*x]/2 + Log[x]

fricas [A] time = 0.65, size = 9, normalized size = 0.82

$$\frac{1}{2} \arctan(2x) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="fricas")

[Out] 1/2*arctan(2*x) + log(x)

giac [A] time = 0.29, size = 10, normalized size = 0.91

$$\frac{1}{2} \arctan(2x) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")

[Out] 1/2*arctan(2*x) + log(abs(x))

maple [A] time = 0.01, size = 10, normalized size = 0.91

$$\frac{\arctan(2x)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+x+1)/(4*x^3+x),x)

[Out] 1/2*arctan(2*x)+ln(x)

maxima [A] time = 1.61, size = 9, normalized size = 0.82

$$\frac{1}{2} \arctan(2x) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")

[Out] 1/2*arctan(2*x) + log(x)

mupad [B] time = 2.17, size = 17, normalized size = 1.55

$$\ln(x) - \frac{\operatorname{atan}\left(\frac{17}{32\left(\frac{x}{16} - \frac{1}{8}\right)} + 4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 4*x^2 + 1)/(x + 4*x^3),x)

[Out] log(x) - atan(17/(32*(x/16 - 1/8)) + 4)/2

sympy [A] time = 0.13, size = 8, normalized size = 0.73

$$\log(x) + \frac{\operatorname{atan}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**2+x+1)/(4*x**3+x),x)

[Out] log(x) + atan(2*x)/2

$$3.352 \quad \int \frac{1-x+3x^2}{-x^2+x^3} dx$$

Optimal. Leaf size=12

$$\frac{1}{x} + 3 \log(1-x)$$

[Out] 1/x+3*ln(1-x)

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1593, 893}

$$\frac{1}{x} + 3 \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x + 3*x^2)/(-x^2 + x^3), x]

[Out] x^(-1) + 3*Log[1 - x]

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1-x+3x^2}{-x^2+x^3} dx &= \int \frac{1-x+3x^2}{(-1+x)x^2} dx \\ &= \int \left(\frac{3}{-1+x} - \frac{1}{x^2} \right) dx \\ &= \frac{1}{x} + 3 \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{x} + 3 \log(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x + 3*x^2)/(-x^2 + x^3), x]

[Out] x^(-1) + 3*Log[1 - x]

fricas [A] time = 0.52, size = 13, normalized size = 1.08

$$\frac{3x \log(x-1) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="fricas")

[Out] (3*x*log(x - 1) + 1)/x

giac [A] time = 0.38, size = 11, normalized size = 0.92

$$\frac{1}{x} + 3 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="giac")

[Out] 1/x + 3*log(abs(x - 1))

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$3 \ln(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2-x+1)/(x^3-x^2),x)

[Out] 3*ln(x-1)+1/x

maxima [A] time = 0.71, size = 10, normalized size = 0.83

$$\frac{1}{x} + 3 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")

[Out] 1/x + 3*log(x - 1)

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$3 \ln(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x^2 - x + 1)/(x^2 - x^3),x)

[Out] 3*log(x - 1) + 1/x

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$3 \log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2-x+1)/(x**3-x**2),x)

[Out] 3*log(x - 1) + 1/x

$$3.353 \quad \int \frac{4+3x+x^2}{x+x^2} dx$$

Optimal. Leaf size=12

$$x + 4 \log(x) - 2 \log(x + 1)$$

[Out] x+4*ln(x)-2*ln(1+x)

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1593, 893}

$$x + 4 \log(x) - 2 \log(x + 1)$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*x + x^2)/(x + x^2), x]

[Out] x + 4*Log[x] - 2*Log[1 + x]

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{4+3x+x^2}{x+x^2} dx &= \int \frac{4+3x+x^2}{x(1+x)} dx \\ &= \int \left(1 + \frac{4}{x} - \frac{2}{1+x}\right) dx \\ &= x + 4 \log(x) - 2 \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + 4 \log(x) - 2 \log(x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x + x^2)/(x + x^2), x]

[Out] x + 4*Log[x] - 2*Log[1 + x]

fricas [A] time = 0.56, size = 12, normalized size = 1.00

$$x - 2 \log(x + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")

[Out] x - 2*log(x + 1) + 4*log(x)

giac [A] time = 0.23, size = 14, normalized size = 1.17

$$x - 2 \log(|x + 1|) + 4 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")

[Out] x - 2*log(abs(x + 1)) + 4*log(abs(x))

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$x + 4 \ln(x) - 2 \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3*x+4)/(x^2+x),x)

[Out] x+4*ln(x)-2*ln(x+1)

maxima [A] time = 0.60, size = 12, normalized size = 1.00

$$x - 2 \log(x + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")

[Out] x - 2*log(x + 1) + 4*log(x)

mupad [B] time = 0.05, size = 12, normalized size = 1.00

$$x - 2 \ln(x + 1) + 4 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + x^2 + 4)/(x + x^2),x)

[Out] x - 2*log(x + 1) + 4*log(x)

sympy [A] time = 0.10, size = 12, normalized size = 1.00

$$x + 4 \log(x) - 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+3*x+4)/(x**2+x),x)

[Out] x + 4*log(x) - 2*log(x + 1)

$$3.354 \quad \int \frac{4+x+3x^2}{x+x^3} dx$$

Optimal. Leaf size=17

$$-\frac{1}{2} \log(x^2 + 1) + 4 \log(x) + \tan^{-1}(x)$$

[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1593, 1802, 635, 203, 260}

$$-\frac{1}{2} \log(x^2 + 1) + 4 \log(x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(4 + x + 3*x^2)/(x + x^3), x]

[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{4+x+3x^2}{x+x^3} dx &= \int \frac{4+x+3x^2}{x(1+x^2)} dx \\
&= \int \left(\frac{4}{x} + \frac{1-x}{1+x^2} \right) dx \\
&= 4 \log(x) + \int \frac{1-x}{1+x^2} dx \\
&= 4 \log(x) + \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\
&= \tan^{-1}(x) + 4 \log(x) - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2} \log(x^2 + 1) + 4 \log(x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x + 3*x^2)/(x + x^3), x]

[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2

fricas [A] time = 0.57, size = 15, normalized size = 0.88

$$\arctan(x) - \frac{1}{2} \log(x^2 + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+x+4)/(x^3+x), x, algorithm="fricas")

[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)

giac [A] time = 0.24, size = 16, normalized size = 0.94

$$\arctan(x) - \frac{1}{2} \log(x^2 + 1) + 4 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+x+4)/(x^3+x), x, algorithm="giac")

[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\arctan(x) + 4 \ln(x) - \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+x+4)/(x^3+x), x)

[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)

maxima [A] time = 1.53, size = 15, normalized size = 0.88

$$\arctan(x) - \frac{1}{2} \log(x^2 + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")

[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)

mupad [B] time = 2.28, size = 23, normalized size = 1.35

$$4 \ln(x) + \ln(x - i) \left(-\frac{1}{2} - \frac{1}{2}i \right) + \ln(x + i) \left(-\frac{1}{2} + \frac{1}{2}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 4)/(x + x^3),x)

[Out] 4*log(x) - log(x + 1i)*(1/2 - 1i/2) - log(x - 1i)*(1/2 + 1i/2)

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$4 \log(x) - \frac{\log(x^2 + 1)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+x+4)/(x**3+x),x)

[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)

$$3.355 \quad \int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx$$

Optimal. Leaf size=13

$$2 \log(4x + 1) - \tan^{-1}(x)$$

[Out] -arctan(x)+2*ln(1+4*x)

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1629, 204}

$$2 \log(4x + 1) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]

[Out] -ArcTan[x] + 2*Log[1 + 4*x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1629

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx &= \int \left(\frac{8}{1+4x} + \frac{1}{-1-x^2} \right) dx \\ &= 2 \log(1+4x) + \int \frac{1}{-1-x^2} dx \\ &= -\tan^{-1}(x) + 2 \log(1+4x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$2 \log(4x + 1) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]

[Out] -ArcTan[x] + 2*Log[1 + 4*x]

fricas [A] time = 0.63, size = 13, normalized size = 1.00

$$-\arctan(x) + 2 \log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="fricas")

[Out] $-\arctan(x) + 2\log(4x + 1)$

giac [A] time = 0.34, size = 14, normalized size = 1.08

$$-\arctan(x) + 2 \log(|4x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="giac")`

[Out] $-\arctan(x) + 2\log(\text{abs}(4x + 1))$

maple [A] time = 0.01, size = 14, normalized size = 1.08

$$-\arctan(x) + 2 \ln(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x)`

[Out] $-\arctan(x) + 2\ln(1+4*x)$

maxima [A] time = 1.41, size = 13, normalized size = 1.00

$$-\arctan(x) + 2 \log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")`

[Out] $-\arctan(x) + 2\log(4x + 1)$

mupad [B] time = 0.06, size = 19, normalized size = 1.46

$$\text{atan}\left(\frac{4x + 1}{x - 4}\right) + 2 \ln\left(x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((8*x^2 - 4*x + 7)/((4*x + 1)*(x^2 + 1)),x)`

[Out] $\text{atan}((4*x + 1)/(x - 4)) + 2*\log(x + 1/4)$

sympy [A] time = 0.13, size = 10, normalized size = 0.77

$$2 \log\left(x + \frac{1}{4}\right) - \text{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)`

[Out] $2*\log(x + 1/4) - \text{atan}(x)$

$$3.356 \quad \int \frac{x^2}{(-1+x)(1+2x+x^2)} dx$$

Optimal. Leaf size=28

$$\frac{1}{2(x+1)} + \frac{1}{4} \log(1-x) + \frac{3}{4} \log(x+1)$$

[Out] 1/2/(1+x)+1/4*ln(1-x)+3/4*ln(1+x)

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {27, 88}

$$\frac{1}{2(x+1)} + \frac{1}{4} \log(1-x) + \frac{3}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]

[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 88

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(-1+x)(1+2x+x^2)} dx &= \int \frac{x^2}{(-1+x)(1+x)^2} dx \\ &= \int \left(\frac{1}{4(-1+x)} - \frac{1}{2(1+x)^2} + \frac{3}{4(1+x)} \right) dx \\ &= \frac{1}{2(1+x)} + \frac{1}{4} \log(1-x) + \frac{3}{4} \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{4} \left(\frac{2}{x+1} + \log(x-1) + 3 \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]

[Out] (2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4

fricas [A] time = 0.59, size = 26, normalized size = 0.93

$$\frac{3(x+1)\log(x+1) + (x+1)\log(x-1) + 2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="fricas")

[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)

giac [A] time = 0.30, size = 22, normalized size = 0.79

$$\frac{1}{2(x+1)} + \frac{3}{4} \log(|x+1|) + \frac{1}{4} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="giac")

[Out] 1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))

maple [A] time = 0.01, size = 21, normalized size = 0.75

$$\frac{\ln(x-1)}{4} + \frac{3\ln(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x-1)/(x^2+2*x+1),x)

[Out] 1/4*ln(x-1)+1/2/(x+1)+3/4*ln(x+1)

maxima [A] time = 0.66, size = 20, normalized size = 0.71

$$\frac{1}{2(x+1)} + \frac{3}{4} \log(x+1) + \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="maxima")

[Out] 1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)

mupad [B] time = 0.07, size = 20, normalized size = 0.71

$$\frac{\ln(x-1)}{4} + \frac{3\ln(x+1)}{4} + \frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((x - 1)*(2*x + x^2 + 1)),x)

[Out] log(x - 1)/4 + (3*log(x + 1))/4 + 1/(2*(x + 1))

sympy [A] time = 0.11, size = 20, normalized size = 0.71

$$\frac{\log(x-1)}{4} + \frac{3\log(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-1+x)/(x**2+2*x+1),x)

[Out] log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)

$$3.357 \quad \int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)} dx$$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)} + \frac{41}{128} \log(1-2x) - \frac{25}{128} \log(2x+3)$$

[Out] -9/32/(1-2*x)+41/128*ln(1-2*x)-25/128*ln(3+2*x)

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {893}

$$-\frac{9}{32(1-2x)} + \frac{41}{128} \log(1-2x) - \frac{25}{128} \log(2x+3)$$

Antiderivative was successfully verified.

[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)), x]

[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128

Rule 893

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)} dx &= \int \left(-\frac{9}{16(-1+2x)^2} + \frac{41}{64(-1+2x)} - \frac{25}{64(3+2x)} \right) dx \\ &= -\frac{9}{32(1-2x)} + \frac{41}{128} \log(1-2x) - \frac{25}{128} \log(3+2x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{9}{32(2x-1)} + \frac{41}{128} \log(1-2x) - \frac{25}{128} \log(2x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)), x]

[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128

fricas [A] time = 0.53, size = 37, normalized size = 1.16

$$-\frac{25(2x-1)\log(2x+3) - 41(2x-1)\log(2x-1) - 36}{128(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x), x, algorithm="fricas")

[Out] $-1/128*(25*(2*x - 1)*\log(2*x + 3) - 41*(2*x - 1)*\log(2*x - 1) - 36)/(2*x - 1)$

giac [A] time = 0.30, size = 43, normalized size = 1.34

$$\frac{9}{32(2x-1)} - \frac{1}{8} \log\left(\frac{|2x-1|}{2(2x-1)^2}\right) - \frac{25}{128} \log\left(\left|-\frac{4}{2x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")`

[Out] $9/32/(2*x - 1) - 1/8*\log(1/2*\text{abs}(2*x - 1)/(2*x - 1)^2) - 25/128*\log(\text{abs}(-4/(2*x - 1) - 1))$

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{41 \ln(2x-1)}{128} - \frac{25 \ln(2x+3)}{128} + \frac{9}{32(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x)`

[Out] $9/32/(2*x-1)+41/128*\ln(2*x-1)-25/128*\ln(3+2*x)$

maxima [A] time = 0.75, size = 26, normalized size = 0.81

$$\frac{9}{32(2x-1)} - \frac{25}{128} \log(2x+3) + \frac{41}{128} \log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")`

[Out] $9/32/(2*x - 1) - 25/128*\log(2*x + 3) + 41/128*\log(2*x - 1)$

mupad [B] time = 2.23, size = 22, normalized size = 0.69

$$\frac{41 \ln\left(x - \frac{1}{2}\right)}{128} - \frac{25 \ln\left(x + \frac{3}{2}\right)}{128} + \frac{9}{64\left(x - \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)`

[Out] $(41*\log(x - 1/2))/128 - (25*\log(x + 3/2))/128 + 9/(64*(x - 1/2))$

sympy [A] time = 0.13, size = 26, normalized size = 0.81

$$\frac{41 \log\left(x - \frac{1}{2}\right)}{128} - \frac{25 \log\left(x + \frac{3}{2}\right)}{128} + \frac{9}{64x - 32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)`

[Out] $41*\log(x - 1/2)/128 - 25*\log(x + 3/2)/128 + 9/(64*x - 32)$

$$3.358 \quad \int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx$$

Optimal. Leaf size=23

$$\frac{1}{2} \log(x^2 + 1) + 2 \log(1 - x) - 3 \tan^{-1}(x)$$

[Out] -3*arctan(x)+2*ln(1-x)+1/2*ln(x^2+1)

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1629, 635, 203, 260}

$$\frac{1}{2} \log(x^2 + 1) + 2 \log(1 - x) - 3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]

[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx &= \int \left(\frac{2}{-1+x} + \frac{-3+x}{1+x^2} \right) dx \\ &= 2 \log(1-x) + \int \frac{-3+x}{1+x^2} dx \\ &= 2 \log(1-x) - 3 \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\ &= -3 \tan^{-1}(x) + 2 \log(1-x) + \frac{1}{2} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.22

$$\frac{1}{2} \log((x-1)^2 + 2(x-1) + 2) + 2 \log(x-1) - 3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)), x]

[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]

fricas [A] time = 0.71, size = 19, normalized size = 0.83

$$-3 \arctan(x) + \frac{1}{2} \log(x^2 + 1) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1), x, algorithm="fricas")

[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)

giac [A] time = 0.31, size = 20, normalized size = 0.87

$$-3 \arctan(x) + \frac{1}{2} \log(x^2 + 1) + 2 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1), x, algorithm="giac")

[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-3 \arctan(x) + 2 \ln(x - 1) + \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2-4*x+5)/(x-1)/(x^2+1), x)

[Out] 2*ln(x-1)+1/2*ln(x^2+1)-3*arctan(x)

maxima [A] time = 1.56, size = 19, normalized size = 0.83

$$-3 \arctan(x) + \frac{1}{2} \log(x^2 + 1) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1), x, algorithm="maxima")

[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)

mupad [B] time = 0.05, size = 25, normalized size = 1.09

$$2 \ln(x - 1) + \ln(x - i) \left(\frac{1}{2} + \frac{3}{2}i \right) + \ln(x + i) \left(\frac{1}{2} - \frac{3}{2}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)), x)

[Out] 2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)

sympy [A] time = 0.14, size = 19, normalized size = 0.83

$$2\log(x-1) + \frac{\log(x^2+1)}{2} - 3\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)

[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)

$$3.359 \quad \int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)} dx$$

Optimal. Leaf size=24

$$-\frac{1}{2} \log(x^2 + 1) + \frac{1}{x-1} + \log(1-x) + \tan^{-1}(x)$$

[Out] 1/(-1+x)+arctan(x)+ln(1-x)-1/2*ln(x^2+1)

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1629, 635, 203, 260}

$$-\frac{1}{2} \log(x^2 + 1) + \frac{1}{x-1} + \log(1-x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)), x]

[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)} dx &= \int \left(-\frac{1}{(-1+x)^2} + \frac{1}{-1+x} + \frac{1-x}{1+x^2} \right) dx \\ &= \frac{1}{-1+x} + \log(1-x) + \int \frac{1-x}{1+x^2} dx \\ &= \frac{1}{-1+x} + \log(1-x) + \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\ &= \frac{1}{-1+x} + \tan^{-1}(x) + \log(1-x) - \frac{1}{2} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$-\frac{1}{2} \log(x^2 + 1) + \frac{1}{x-1} + \log(x-1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)), x]

[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2

fricas [A] time = 0.57, size = 36, normalized size = 1.50

$$\frac{2(x-1) \arctan(x) - (x-1) \log(x^2 + 1) + 2(x-1) \log(x-1) + 2}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")

[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)/(x - 1)

giac [B] time = 0.30, size = 47, normalized size = 1.96

$$\frac{1}{4} \pi - \pi \left[\frac{\pi + 4 \arctan(x)}{4 \pi} + \frac{1}{2} \right] + \frac{1}{x-1} + \arctan(x) - \frac{1}{2} \log\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")

[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x) - 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x) + \ln(x-1) - \frac{\ln(x^2 + 1)}{2} + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-2*x-1)/(x-1)^2/(x^2+1),x)

[Out] ln(x-1)+1/(x-1)-1/2*ln(x^2+1)+arctan(x)

maxima [A] time = 1.60, size = 20, normalized size = 0.83

$$\frac{1}{x-1} + \arctan(x) - \frac{1}{2} \log(x^2 + 1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")

[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)

mupad [B] time = 2.11, size = 28, normalized size = 1.17

$$\ln(x-1) + \frac{1}{x-1} + \ln(x-i) \left(-\frac{1}{2} - \frac{1}{2}i\right) + \ln(x+1i) \left(-\frac{1}{2} + \frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2), x)`

[Out] `log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x - 1)`

sympy [A] time = 0.14, size = 20, normalized size = 0.83

$$\log(x - 1) - \frac{\log(x^2 + 1)}{2} + \operatorname{atan}(x) + \frac{1}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1), x)`

[Out] `log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)`

$$3.360 \quad \int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} dx$$

Optimal. Leaf size=49

$$\frac{56}{221} \log(x^2 - 6x + 10) + \frac{109}{442} \log(2x^2 - 2x + 1) - \frac{261}{221} \tan^{-1}(1 - 2x) - \frac{1026}{221} \tan^{-1}(3 - x)$$

[Out] 261/221*arctan(-1+2*x)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442*ln(2*x^2-2*x+1)

Rubi [A] time = 0.14, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6728, 634, 618, 204, 628, 617}

$$\frac{56}{221} \log(x^2 - 6x + 10) + \frac{109}{442} \log(2x^2 - 2x + 1) - \frac{261}{221} \tan^{-1}(1 - 2x) - \frac{1026}{221} \tan^{-1}(3 - x)$$

Antiderivative was successfully verified.

[In] Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]

[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x + x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su

mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{5 + x^3}{(10 - 6x + x^2) \left(\frac{1}{2} - x + x^2\right)} dx &= \int \left(\frac{2(345 + 56x)}{221(10 - 6x + x^2)} + \frac{2(76 + 109x)}{221(1 - 2x + 2x^2)} \right) dx \\ &= \frac{2}{221} \int \frac{345 + 56x}{10 - 6x + x^2} dx + \frac{2}{221} \int \frac{76 + 109x}{1 - 2x + 2x^2} dx \\ &= \frac{109}{442} \int \frac{-2 + 4x}{1 - 2x + 2x^2} dx + \frac{56}{221} \int \frac{-6 + 2x}{10 - 6x + x^2} dx + \frac{261}{221} \int \frac{1}{1 - 2x + 2x^2} dx \\ &= \frac{56}{221} \log(10 - 6x + x^2) + \frac{109}{442} \log(1 - 2x + 2x^2) + \frac{261}{221} \text{Subst} \left(\int \frac{1}{-1 - x} dx \right) \\ &= -\frac{261}{221} \tan^{-1}(1 - 2x) - \frac{1026}{221} \tan^{-1}(3 - x) + \frac{56}{221} \log(10 - 6x + x^2) + \frac{109}{442} \log(1 - 2x + 2x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{56}{221} \log(x^2 - 6x + 10) + \frac{109}{442} \log(2x^2 - 2x + 1) - \frac{261}{221} \tan^{-1}(1 - 2x) - \frac{1026}{221} \tan^{-1}(3 - x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]

[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x + x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442

fricas [A] time = 0.66, size = 37, normalized size = 0.76

$$\frac{261}{221} \arctan(2x - 1) + \frac{1026}{221} \arctan(x - 3) + \frac{109}{442} \log\left(x^2 - x + \frac{1}{2}\right) + \frac{56}{221} \log(x^2 - 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="fricas")

[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(x^2 - x + 1/2) + 56/221*log(x^2 - 6*x + 10)

giac [A] time = 0.28, size = 39, normalized size = 0.80

$$\frac{261}{221} \arctan(2x - 1) + \frac{1026}{221} \arctan(x - 3) + \frac{109}{442} \log(2x^2 - 2x + 1) + \frac{56}{221} \log(x^2 - 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="giac")

[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x + 1) + 56/221*log(x^2 - 6*x + 10)

maple [A] time = 0.01, size = 40, normalized size = 0.82

$$\frac{1026 \arctan(x - 3)}{221} + \frac{261 \arctan(2x - 1)}{221} + \frac{56 \ln(x^2 - 6x + 10)}{221} + \frac{109 \ln(2x^2 - 2x + 1)}{442}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x)

[Out] 261/221*arctan(2*x-1)+1026/221*arctan(x-3)+56/221*ln(x^2-6*x+10)+109/442*ln(2*x^2-2*x+1)

maxima [A] time = 1.71, size = 39, normalized size = 0.80

$$\frac{261}{221} \arctan(2x-1) + \frac{1026}{221} \arctan(x-3) + \frac{109}{442} \log(2x^2-2x+1) + \frac{56}{221} \log(x^2-6x+10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")

[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x + 1) + 56/221*log(x^2 - 6*x + 10)

mupad [B] time = 2.16, size = 41, normalized size = 0.84

$$\ln(x-3-i) \left(\frac{56}{221} - \frac{513}{221}i \right) + \ln(x-3+i) \left(\frac{56}{221} + \frac{513}{221}i \right) + \ln\left(x - \frac{1}{2} - \frac{1}{2}i\right) \left(\frac{109}{442} - \frac{261}{442}i \right) + \ln\left(x - \frac{1}{2} + \frac{1}{2}i\right) \left(\frac{109}{442} + \frac{261}{442}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 5)/((x^2 - x + 1/2)*(x^2 - 6*x + 10)),x)

[Out] log(x - (3 + 1i))*(56/221 - 513i/221) + log(x - (3 - 1i))*(56/221 + 513i/221) + log(x - (1/2 + 1i/2))*(109/442 - 261i/442) + log(x - (1/2 - 1i/2))*(109/442 + 261i/442)

sympy [A] time = 0.22, size = 44, normalized size = 0.90

$$\frac{56 \log(x^2 - 6x + 10)}{221} + \frac{109 \log\left(x^2 - x + \frac{1}{2}\right)}{442} + \frac{1026 \operatorname{atan}(x - 3)}{221} + \frac{261 \operatorname{atan}(2x - 1)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)

[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x - 3)/221 + 261*atan(2*x - 1)/221

$$3.361 \quad \int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} dx$$

Optimal. Leaf size=25

$$4 \log(1-x) - 14 \log(2-x) + 11 \log(3-x)$$

[Out] 4*ln(1-x)-14*ln(2-x)+11*ln(3-x)

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1612}

$$4 \log(1-x) - 14 \log(2-x) + 11 \log(3-x)$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\int \frac{4 + 3x + x^2}{(-3+x)(-2+x)(-1+x)} dx = \int \left(\frac{11}{-3+x} - \frac{14}{-2+x} + \frac{4}{-1+x} \right) dx$$

$$= 4 \log(1-x) - 14 \log(2-x) + 11 \log(3-x)$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.76

$$11 \log(x-3) - 14 \log(x-2) + 4 \log(x-1)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] 11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]

fricas [A] time = 0.52, size = 19, normalized size = 0.76

$$4 \log(x-1) - 14 \log(x-2) + 11 \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")

[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)

giac [A] time = 0.39, size = 22, normalized size = 0.88

$$4 \log(|x-1|) - 14 \log(|x-2|) + 11 \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")

[Out] 4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$11 \ln(x - 3) - 14 \ln(x - 2) + 4 \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3*x+4)/(x-3)/(x-2)/(x-1),x)

[Out] 4*ln(x-1)-14*ln(x-2)+11*ln(x-3)

maxima [A] time = 1.07, size = 19, normalized size = 0.76

$$4 \log(x - 1) - 14 \log(x - 2) + 11 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")

[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)

mupad [B] time = 2.14, size = 19, normalized size = 0.76

$$4 \ln(x - 1) - 14 \ln(x - 2) + 11 \ln(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + x^2 + 4)/((x - 1)*(x - 2)*(x - 3)),x)

[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)

sympy [A] time = 0.14, size = 19, normalized size = 0.76

$$11 \log(x - 3) - 14 \log(x - 2) + 4 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)

[Out] 11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)

$$3.362 \quad \int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)} dx$$

Optimal. Leaf size=60

$$-\frac{481 \log(x^2 + x + 1)}{5586} - \frac{79}{273(x + 5)} + \frac{200 \log(3 - 2x)}{3211} + \frac{2731 \log(x + 5)}{24843} + \frac{451 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

[Out] -79/273/(5+x)+200/3211*ln(3-2*x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+451/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.25, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6728, 634, 618, 204, 628}

$$-\frac{481 \log(x^2 + x + 1)}{5586} - \frac{79}{273(x + 5)} + \frac{200 \log(3 - 2x)}{3211} + \frac{2731 \log(x + 5)}{24843} + \frac{451 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)), x]

[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*Log[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)} dx &= \int \left(\frac{79}{273(5+x)^2} + \frac{2731}{24843(5+x)} + \frac{400}{3211(-3+2x)} + \frac{-15-481x}{2793(1+x+x^2)} \right) dx \\
&= -\frac{79}{273(5+x)} + \frac{200 \log(3-2x)}{3211} + \frac{2731 \log(5+x)}{24843} + \frac{\int \frac{-15-481x}{1+x+x^2} dx}{2793} \\
&= -\frac{79}{273(5+x)} + \frac{200 \log(3-2x)}{3211} + \frac{2731 \log(5+x)}{24843} + \frac{451 \int \frac{1}{1+x+x^2} dx}{5586} \\
&= -\frac{79}{273(5+x)} + \frac{200 \log(3-2x)}{3211} + \frac{2731 \log(5+x)}{24843} - \frac{481 \log(1+x+x^2)}{5586} \\
&= -\frac{79}{273(5+x)} + \frac{451 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}} + \frac{200 \log(3-2x)}{3211} + \frac{2731 \log(5+x)}{24843} -
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.90

$$\frac{-243867 \log(x^2 + x + 1) - \frac{819546}{x+5} + 176400 \log(3 - 2x) + 311334 \log(x + 5) + 152438\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)), x]

[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3 - 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102

fricas [A] time = 0.73, size = 60, normalized size = 1.00

$$\frac{152438 \sqrt{3} (x + 5) \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - 243867 (x + 5) \log(x^2 + x + 1) + 176400 (x + 5) \log(2x - 3) + 311334 \log(x + 5)}{2832102 (x + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1), x, algorithm="fricas")

[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x + 5) - 819546)/(x + 5)

giac [A] time = 0.34, size = 60, normalized size = 1.00

$$\frac{451}{8379} \sqrt{3} \arctan\left(-\sqrt{3} \left(\frac{14}{x+5} - 3\right)\right) - \frac{79}{273(x+5)} - \frac{481}{5586} \log\left(-\frac{9}{x+5} + \frac{21}{(x+5)^2} + 1\right) + \frac{200}{3211} \log\left(\left|-\frac{13}{x+5} + 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1), x, algorithm="giac")

[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))

maple [A] time = 0.01, size = 48, normalized size = 0.80

$$\frac{451\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{8379} + \frac{200 \ln(2x - 3)}{3211} + \frac{2731 \ln(x + 5)}{24843} - \frac{481 \ln(x^2 + x + 1)}{5586} - \frac{79}{273(x + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x)`

[Out] $-79/273/(5+x)+2731/24843*\ln(5+x)+200/3211*\ln(-3+2*x)-481/5586*\ln(x^2+x+1)+451/8379*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)})$

maxima [A] time = 2.80, size = 47, normalized size = 0.78

$$\frac{451}{8379} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{79}{273(x+5)} - \frac{481}{5586} \log(x^2+x+1) + \frac{200}{3211} \log(2x-3) + \frac{2731}{24843} \log(x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")`

[Out] $451/8379*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x+1)) - 79/273/(x+5) - 481/5586*\log(x^2+x+1) + 200/3211*\log(2*x-3) + 2731/24843*\log(x+5)$

mupad [B] time = 0.13, size = 61, normalized size = 1.02

$$\frac{200 \ln\left(x - \frac{3}{2}\right)}{3211} + \frac{2731 \ln(x+5)}{24843} - \frac{79}{273(x+5)} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} i}{2}\right) \left(\frac{481}{5586} + \frac{\sqrt{3} 451 i}{16758}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(\frac{481}{5586} - \frac{\sqrt{3} 451 i}{16758}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((16*x+1)/((2*x-3)*(x+5)^2*(x+x^2+1)),x)`

[Out] $(200*\log(x-3/2))/3211 + (2731*\log(x+5))/24843 - 79/(273*(x+5)) - \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*451i)/16758 + 481/5586) + \log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*451i)/16758 - 481/5586)$

sympy [A] time = 0.25, size = 63, normalized size = 1.05

$$\frac{200 \log\left(x - \frac{3}{2}\right)}{3211} + \frac{2731 \log(x+5)}{24843} - \frac{481 \log(x^2+x+1)}{5586} + \frac{451 \sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{8379} - \frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)`

[Out] $200*\log(x-3/2)/3211 + 2731*\log(x+5)/24843 - 481*\log(x**2+x+1)/5586 + 451*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/8379 - 79/(273*x+1365)$

$$3.363 \quad \int \frac{-1+x^3}{1+x+x^2} dx$$

Optimal. Leaf size=11

$$\frac{x^2}{2} - x$$

[Out] -x+1/2*x^2

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1586}

$$\frac{x^2}{2} - x$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(1 + x + x^2), x]

[Out] -x + x^2/2

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{1+x+x^2} dx &= \int (-1+x) dx \\ &= -x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2} - x$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(1 + x + x^2), x]

[Out] -x + x^2/2

fricas [A] time = 0.73, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^2+x+1), x, algorithm="fricas")

[Out] 1/2*x^2 - x

giac [A] time = 0.23, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="giac")

[Out] 1/2*x^2 - x

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-1)/(x^2+x+1),x)

[Out] -x+1/2*x^2

maxima [A] time = 0.84, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")

[Out] 1/2*x^2 - x

mupad [B] time = 0.02, size = 6, normalized size = 0.55

$$\frac{x(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 1)/(x + x^2 + 1),x)

[Out] (x*(x - 2))/2

sympy [A] time = 0.06, size = 5, normalized size = 0.45

$$\frac{x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-1)/(x**2+x+1),x)

[Out] x**2/2 - x

$$3.364 \quad \int \frac{-3+x^3}{-7-6x+x^2} dx$$

Optimal. Leaf size=29

$$\frac{x^2}{2} + 6x + \frac{85}{2} \log(7-x) + \frac{1}{2} \log(x+1)$$

[Out] 6*x+1/2*x^2+85/2*ln(7-x)+1/2*ln(1+x)

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1657, 632, 31}

$$\frac{x^2}{2} + 6x + \frac{85}{2} \log(7-x) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x^3)/(-7 - 6*x + x^2), x]

[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-3+x^3}{-7-6x+x^2} dx &= \int \left(6+x + \frac{39+43x}{-7-6x+x^2} \right) dx \\ &= 6x + \frac{x^2}{2} + \int \frac{39+43x}{-7-6x+x^2} dx \\ &= 6x + \frac{x^2}{2} + \frac{1}{2} \int \frac{1}{1+x} dx + \frac{85}{2} \int \frac{1}{-7+x} dx \\ &= 6x + \frac{x^2}{2} + \frac{85}{2} \log(7-x) + \frac{1}{2} \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2} + 6x + \frac{85}{2} \log(7-x) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x^3)/(-7 - 6*x + x^2), x]

[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2

fricas [A] time = 0.62, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\log(x+1) + \frac{85}{2}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3)/(x^2-6*x-7), x, algorithm="fricas")

[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)

giac [A] time = 0.35, size = 23, normalized size = 0.79

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\log(|x+1|) + \frac{85}{2}\log(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3)/(x^2-6*x-7), x, algorithm="giac")

[Out] 1/2*x^2 + 6*x + 1/2*log(abs(x + 1)) + 85/2*log(abs(x - 7))

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{\ln(x+1)}{2} + \frac{85\ln(x-7)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-3)/(x^2-6*x-7), x)

[Out] 1/2*x^2+6*x+1/2*ln(x+1)+85/2*ln(x-7)

maxima [A] time = 1.10, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\log(x+1) + \frac{85}{2}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3)/(x^2-6*x-7), x, algorithm="maxima")

[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)

mupad [B] time = 2.12, size = 21, normalized size = 0.72

$$6x + \frac{\ln(x+1)}{2} + \frac{85\ln(x-7)}{2} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 - 3)/(6*x - x^2 + 7), x)

[Out] 6*x + log(x + 1)/2 + (85*log(x - 7))/2 + x^2/2

sympy [A] time = 0.11, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{85\log(x-7)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-3)/(x**2-6*x-7), x)

[Out] x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2

$$3.365 \quad \int \frac{1+x^3}{(13+4x+x^2)^2} dx$$

Optimal. Leaf size=45

$$\frac{47x+67}{18(x^2+4x+13)} + \frac{1}{2} \log(x^2+4x+13) - \frac{61}{54} \tan^{-1}\left(\frac{x+2}{3}\right)$$

[Out] 1/18*(67+47*x)/(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)+1/2*ln(x^2+4*x+13)

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1660, 634, 618, 204, 628}

$$\frac{47x+67}{18(x^2+4x+13)} + \frac{1}{2} \log(x^2+4x+13) - \frac{61}{54} \tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]

[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*x + x^2]/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{(13+4x+x^2)^2} dx &= \frac{67+47x}{18(13+4x+x^2)} + \frac{1}{36} \int \frac{-50+36x}{13+4x+x^2} dx \\
&= \frac{67+47x}{18(13+4x+x^2)} + \frac{1}{2} \int \frac{4+2x}{13+4x+x^2} dx - \frac{61}{18} \int \frac{1}{13+4x+x^2} dx \\
&= \frac{67+47x}{18(13+4x+x^2)} + \frac{1}{2} \log(13+4x+x^2) + \frac{61}{9} \text{Subst}\left(\int \frac{1}{-36-x^2} dx, x, 4+2x\right) \\
&= \frac{67+47x}{18(13+4x+x^2)} - \frac{61}{54} \tan^{-1}\left(\frac{2+x}{3}\right) + \frac{1}{2} \log(13+4x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.00

$$\frac{47x+67}{18(x^2+4x+13)} + \frac{1}{2} \log(x^2+4x+13) - \frac{61}{54} \tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(13 + 4*x + x^2)^2, x]

[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*x + x^2]/2

fricas [A] time = 0.71, size = 52, normalized size = 1.16

$$\frac{61(x^2+4x+13) \arctan\left(\frac{1}{3}x + \frac{2}{3}\right) - 27(x^2+4x+13) \log(x^2+4x+13) - 141x - 201}{54(x^2+4x+13)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^2+4*x+13)^2, x, algorithm="fricas")

[Out] -1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x + 13)*log(x^2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)

giac [A] time = 0.30, size = 37, normalized size = 0.82

$$\frac{47x+67}{18(x^2+4x+13)} - \frac{61}{54} \arctan\left(\frac{1}{3}x + \frac{2}{3}\right) + \frac{1}{2} \log(x^2+4x+13)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^2+4*x+13)^2, x, algorithm="giac")

[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2 + 4*x + 13)

maple [A] time = 0.01, size = 37, normalized size = 0.82

$$-\frac{61 \arctan\left(\frac{x}{3} + \frac{2}{3}\right)}{54} + \frac{\ln(x^2+4x+13)}{2} + \frac{\frac{47x}{18} + \frac{67}{18}}{x^2+4x+13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(x^2+4*x+13)^2, x)

[Out] $(47/18*x+67/18)/(x^2+4*x+13)+1/2*\ln(x^2+4*x+13)-61/54*\arctan(2/3+1/3*x)$

maxima [A] time = 1.84, size = 37, normalized size = 0.82

$$\frac{47x + 67}{18(x^2 + 4x + 13)} - \frac{61}{54} \arctan\left(\frac{1}{3}x + \frac{2}{3}\right) + \frac{1}{2} \log(x^2 + 4x + 13)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="maxima")`

[Out] $1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*\arctan(1/3*x + 2/3) + 1/2*\log(x^2 + 4*x + 13)$

mupad [B] time = 0.04, size = 49, normalized size = 1.09

$$\frac{\ln(x^2 + 4x + 13)}{2} - \frac{61 \operatorname{atan}\left(\frac{x}{3} + \frac{2}{3}\right)}{54} + \frac{47x}{18(x^2 + 4x + 13)} + \frac{67}{18(x^2 + 4x + 13)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3 + 1)/(4*x + x^2 + 13)^2,x)`

[Out] $\log(4*x + x^2 + 13)/2 - (61*\operatorname{atan}(x/3 + 2/3))/54 + (47*x)/(18*(4*x + x^2 + 13)) + 67/(18*(4*x + x^2 + 13))$

sympy [A] time = 0.14, size = 37, normalized size = 0.82

$$\frac{47x + 67}{18x^2 + 72x + 234} + \frac{\log(x^2 + 4x + 13)}{2} - \frac{61 \operatorname{atan}\left(\frac{x}{3} + \frac{2}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+1)/(x**2+4*x+13)**2,x)`

[Out] $(47*x + 67)/(18*x**2 + 72*x + 234) + \log(x**2 + 4*x + 13)/2 - 61*\operatorname{atan}(x/3 + 2/3)/54$

$$3.366 \quad \int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx$$

Optimal. Leaf size=32

$$\frac{1}{x^2+4} + \log(x^2+4) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)

Rubi [A] time = 0.25, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {6725, 203, 261, 635, 260}

$$\frac{1}{x^2+4} + \log(x^2+4) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^2), x]

[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{-32 + 36x - 42x^2 + 21x^3 - 10x^4 + 3x^5}{x(1+x^2)(4+x^2)^2} dx &= \int \left(-\frac{2}{x} + \frac{2}{1+x^2} - \frac{2x}{(4+x^2)^2} + \frac{1+2x}{4+x^2} \right) dx \\
&= -2\log(x) + 2 \int \frac{1}{1+x^2} dx - 2 \int \frac{x}{(4+x^2)^2} dx + \int \frac{1+2x}{4+x^2} dx \\
&= \frac{1}{4+x^2} + 2 \tan^{-1}(x) - 2\log(x) + 2 \int \frac{x}{4+x^2} dx + \int \frac{1}{4+x^2} dx \\
&= \frac{1}{4+x^2} + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + 2 \tan^{-1}(x) - 2\log(x) + \log(4+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{1}{x^2+4} + \log(x^2+4) - 2\log(x) + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^2), x]

[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]

fricas [A] time = 0.63, size = 52, normalized size = 1.62

$$\frac{(x^2 + 4) \arctan\left(\frac{1}{2}x\right) + 4(x^2 + 4) \arctan(x) + 2(x^2 + 4) \log(x^2 + 4) - 4(x^2 + 4) \log(x) + 2}{2(x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")

[Out] 1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4)*log(x^2 + 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)

giac [A] time = 0.32, size = 29, normalized size = 0.91

$$\frac{1}{x^2+4} + \frac{1}{2} \arctan\left(\frac{1}{2}x\right) + 2 \arctan(x) + \log(x^2+4) - 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algorithm="giac")

[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(abs(x))

maple [A] time = 0.01, size = 29, normalized size = 0.91

$$2 \arctan(x) + \frac{\arctan\left(\frac{x}{2}\right)}{2} - 2 \ln(x) + \ln(x^2 + 4) + \frac{1}{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x)

[Out] $1/(x^2+4)+1/2*\arctan(1/2*x)+2*\arctan(x)-2*\ln(x)+\ln(x^2+4)$

maxima [A] time = 2.02, size = 28, normalized size = 0.88

$$\frac{1}{x^2+4} + \frac{1}{2} \arctan\left(\frac{1}{2}x\right) + 2 \arctan(x) + \log(x^2+4) - 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")

[Out] $1/(x^2+4) + 1/2*\arctan(1/2*x) + 2*\arctan(x) + \log(x^2+4) - 2*\log(x)$

mupad [B] time = 0.07, size = 44, normalized size = 1.38

$$\frac{1}{x^2+4} - 2 \ln(x) - 2 \operatorname{atan}\left(\frac{328000}{7(36288x-19584)} + \frac{34}{63}\right) + \ln(x-2i) \left(1 - \frac{1}{4}i\right) + \ln(x+2i) \left(1 + \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5 - 32)/(x*(x^2 + 1)*(x^2 + 4)^2),x)

[Out] $\log(x-2i)*(1-1i/4) + \log(x+2i)*(1+1i/4) - 2*\operatorname{atan}(328000/(7*(36288*x-19584))) + 34/63 - 2*\log(x) + 1/(x^2+4)$

sympy [A] time = 0.26, size = 29, normalized size = 0.91

$$-2 \log(x) + \log(x^2+4) + \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2} + 2 \operatorname{atan}(x) + \frac{1}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2,x)

[Out] $-2*\log(x) + \log(x**2+4) + \operatorname{atan}(x/2)/2 + 2*\operatorname{atan}(x) + 1/(x**2+4)$

$$3.367 \quad \int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8} dx$$

Optimal. Leaf size=148

$$\frac{x^2}{2} \frac{\log(x^2 - \sqrt{2} \sqrt[4]{7}x + \sqrt{7})}{4\sqrt{2} 7^{3/4}} + \frac{\log(x^2 + \sqrt{2} \sqrt[4]{7}x + \sqrt{7})}{4\sqrt{2} 7^{3/4}} - \frac{1}{2} \tanh^{-1}(x^2) - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2} 7^{3/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}} + 1\right)}{2\sqrt{2} 7^{3/4}}$$

[Out] 1/2*x^2-1/2*arctanh(x^2)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1/2)+1/28*arctan(1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1/2)-1/56*ln(x^2-7^(1/4)*x*2^(1/2)+7^(1/2))*7^(1/4)*2^(1/2)+1/56*ln(x^2+7^(1/4)*x*2^(1/2)+7^(1/2))*7^(1/4)*2^(1/2)

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1790, 1403, 211, 1165, 628, 1162, 617, 204, 1584, 1478, 275, 321, 207}

$$\frac{x^2}{2} \frac{\log(x^2 - \sqrt{2} \sqrt[4]{7}x + \sqrt{7})}{4\sqrt{2} 7^{3/4}} + \frac{\log(x^2 + \sqrt{2} \sqrt[4]{7}x + \sqrt{7})}{4\sqrt{2} 7^{3/4}} - \frac{1}{2} \tanh^{-1}(x^2) - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2} 7^{3/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}} + 1\right)}{2\sqrt{2} 7^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8), x]

[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - Sqrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1403

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2
_))^(p_), x_Symbol] := Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /;
FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 1478

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n
_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1790

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x] /;
FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
```

0] && IGtQ[n, 0] && !PolyQ[Pq, x^n]

Rubi steps

$$\begin{aligned}
 \int \frac{-1 + x^4 + 7x^5 + x^9}{-7 + 6x^4 + x^8} dx &= \int \left(\frac{-1 + x^4}{-7 + 6x^4 + x^8} + \frac{x(7x^4 + x^8)}{-7 + 6x^4 + x^8} \right) dx \\
 &= \int \frac{-1 + x^4}{-7 + 6x^4 + x^8} dx + \int \frac{x(7x^4 + x^8)}{-7 + 6x^4 + x^8} dx \\
 &= \int \frac{1}{7 + x^4} dx + \int \frac{x^5(7 + x^4)}{-7 + 6x^4 + x^8} dx \\
 &= \frac{\int \frac{\sqrt{7-x^2}}{7+x^4} dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{7+x^2}}{7+x^4} dx}{2\sqrt{7}} + \int \frac{x^5}{-1 + x^4} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{-1 + x^2} dx, x, x^2 \right) - \frac{\int \frac{\sqrt{2} \sqrt[4]{7} + 2x}{-\sqrt{7} - \sqrt{2} \sqrt[4]{7} x - x^2} dx}{4\sqrt{2} 7^{3/4}} - \frac{\int \frac{\sqrt{2} \sqrt[4]{7} - 2x}{-\sqrt{7} + \sqrt{2} \sqrt[4]{7} x - x^2} dx}{4\sqrt{2} 7^{3/4}} + \frac{\int \frac{1}{\sqrt{7}} dx}{4\sqrt{2} 7^{3/4}} \\
 &= \frac{x^2}{2} - \frac{\log(\sqrt{7} - \sqrt{2} \sqrt[4]{7} x + x^2)}{4\sqrt{2} 7^{3/4}} + \frac{\log(\sqrt{7} + \sqrt{2} \sqrt[4]{7} x + x^2)}{4\sqrt{2} 7^{3/4}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1 + x^2} dx, x, x^2 \right) \\
 &= \frac{x^2}{2} - \frac{\tan^{-1} \left(1 - \frac{\sqrt{2} x}{\sqrt[4]{7}} \right)}{2\sqrt{2} 7^{3/4}} + \frac{\tan^{-1} \left(1 + \frac{\sqrt{2} x}{\sqrt[4]{7}} \right)}{2\sqrt{2} 7^{3/4}} - \frac{1}{2} \tanh^{-1}(x^2) - \frac{\log(\sqrt{7} - \sqrt{2} \sqrt[4]{7} x + x^2)}{4\sqrt{2} 7^{3/4}}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 159, normalized size = 1.07

$$\frac{1}{56} \left(28x^2 - 14 \log(x^2 + 1) - \sqrt{2} \sqrt[4]{7} \log(\sqrt{7} x^2 - \sqrt{2} 7^{3/4} x + 7) + \sqrt{2} \sqrt[4]{7} \log(\sqrt{7} x^2 + \sqrt{2} 7^{3/4} x + 7) + 14 \log(1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8), x]

[Out] (28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*Sqrt[2]*7^(1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] + 14*Log[1 + x] - 14*Log[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2] + Sqrt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2])/56

fricas [A] time = 0.76, size = 178, normalized size = 1.20

$$-\frac{1}{686} \cdot 343^{\frac{3}{4}} \sqrt{2} \arctan \left(-\frac{1}{7} \cdot 343^{\frac{1}{4}} \sqrt{2} x + \frac{1}{49} \cdot 343^{\frac{1}{4}} \sqrt{2} \sqrt{343^{\frac{3}{4}} \sqrt{2} x + 49 x^2 + 49 \sqrt{7}} - 1 \right) - \frac{1}{686} \cdot 343^{\frac{3}{4}} \sqrt{2} \arctan \left(\dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7), x, algorithm="fricas")

[Out] -1/686*343^(3/4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*sqrt(2)*sqrt(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1) - 1/686*343^(3/4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*sqrt(2)*sqrt(-343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) + 1) + 1/2744*343^(3/4)*sqrt(2)*log(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1/2744*343^(3/4)*sqrt(2)*log(-343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) + 1/2*x^2 - 1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)

giac [A] time = 0.39, size = 122, normalized size = 0.82

$$\frac{1}{2}x^2 + \frac{1}{28} \cdot 28^{\frac{1}{4}} \arctan\left(\frac{1}{14} \cdot 7^{\frac{3}{4}} \sqrt{2} \left(2x + 7^{\frac{1}{4}} \sqrt{2}\right)\right) + \frac{1}{28} \cdot 28^{\frac{1}{4}} \arctan\left(\frac{1}{14} \cdot 7^{\frac{3}{4}} \sqrt{2} \left(2x - 7^{\frac{1}{4}} \sqrt{2}\right)\right) + \frac{1}{56} \cdot 28^{\frac{1}{4}} \log\left(x^2 - 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}\right) - \frac{1}{56} \cdot 28^{\frac{1}{4}} \log\left(x^2 + 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}\right) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{4} \log(\text{abs}(x + 1)) + \frac{1}{4} \log(\text{abs}(x - 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="giac")

[Out] 1/2*x^2 + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2)) + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/56*28^(1/4)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*28^(1/4)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))

maple [A] time = 0.01, size = 110, normalized size = 0.74

$$\frac{x^2}{2} + \frac{7^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} 7^{\frac{3}{4}} x}{7} - 1\right)}{28} + \frac{7^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} 7^{\frac{3}{4}} x}{7} + 1\right)}{28} + \frac{7^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}}{x^2 - 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}}\right)}{56} + \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x)

[Out] 1/2*x^2+1/4*ln(x-1)-1/4*ln(x^2+1)+1/56*7^(1/4)*2^(1/2)*ln((x^2+7^(1/4)*x*2^(1/2)+7^(1/2))/(x^2-7^(1/4)*x*2^(1/2)+7^(1/2)))+1/28*arctan(1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1/2)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1/2)+1/4*ln(x+1)

maxima [A] time = 2.01, size = 132, normalized size = 0.89

$$\frac{1}{2}x^2 + \frac{1}{28} \cdot 7^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{1}{14} \cdot 7^{\frac{3}{4}} \sqrt{2} \left(2x + 7^{\frac{1}{4}} \sqrt{2}\right)\right) + \frac{1}{28} \cdot 7^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{1}{14} \cdot 7^{\frac{3}{4}} \sqrt{2} \left(2x - 7^{\frac{1}{4}} \sqrt{2}\right)\right) + \frac{1}{56} \cdot 7^{\frac{1}{4}} \sqrt{2} \log\left(x^2 - 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}\right) - \frac{1}{56} \cdot 7^{\frac{1}{4}} \sqrt{2} \log\left(x^2 + 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}\right) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{4} \log(\text{abs}(x + 1)) + \frac{1}{4} \log(\text{abs}(x - 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="maxima")

[Out] 1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(x + 1) + 1/4*log(x - 1)

mupad [B] time = 2.19, size = 124, normalized size = 0.84

$$\frac{\text{atan}(x^2 \text{I}) \text{I}}{2} + \frac{x^2}{2} + \sqrt{2} 7^{1/4} \text{atan}\left(\frac{\sqrt{2} 7^{1/4} x \left(\frac{89653248}{2401} + \frac{89653248 \text{I}}{2401}\right)}{-\frac{1048576}{49} + \frac{\sqrt{7} 179306496 \text{I}}{2401}} + \frac{\sqrt{2} 7^{3/4} x \left(-\frac{524288}{343} + \frac{524288 \text{I}}{343}\right)}{-\frac{1048576}{49} + \frac{\sqrt{7} 179306496 \text{I}}{2401}}\right) \left(\frac{1}{28} + \frac{\text{I}}{28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 7*x^5 + x^9 - 1)/(6*x^4 + x^8 - 7),x)

[Out] (atan(x^2*I)*I)/2 + x^2/2 + 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(89653248/2401 + 89653248i/2401))/((7^(1/2)*179306496i)/2401 - 1048576/49) - (2^(1/2)*7^(3/4)*x*(524288/343 - 524288i/343))/((7^(1/2)*179306496i)/2401 - 1048576/49)*(1/28 + I/28) - 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(89653248/2401 - 89653248i/2401))/((7^(1/2)*179306496i)/2401 + 1048576/49) - (2^(1/2)*7^(3/4)*x*(524288/343 + 524288i/343))/((7^(1/2)*179306496i)/2401 + 1048576/49)*(1/28 - I/28)

sympy [A] time = 0.43, size = 146, normalized size = 0.99

$$\frac{x^2}{2} + \frac{\log(x^2 - 1)}{4} - \frac{\log(x^2 + 1)}{4} - \frac{\sqrt{2} \sqrt[4]{7} \log(x^2 - \sqrt{2} \sqrt[4]{7} x + \sqrt{7})}{56} + \frac{\sqrt{2} \sqrt[4]{7} \log(x^2 + \sqrt{2} \sqrt[4]{7} x + \sqrt{7})}{56} + \frac{\sqrt{2} \sqrt[4]{7} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt[4]{7} x + \sqrt{7}}{x^2 - \sqrt{2} \sqrt[4]{7} x + \sqrt{7}}\right)}{56} + \frac{\sqrt{2} \sqrt[4]{7} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt[4]{7} x + \sqrt{7}}{x^2 + \sqrt{2} \sqrt[4]{7} x + \sqrt{7}}\right)}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7), x)

[Out] x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log(x**2 - sqrt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28

$$3.368 \quad \int \frac{1+x^3+x^6}{x+x^5} dx$$

Optimal. Leaf size=112

$$-\frac{1}{4} \log(x^4+1) + \frac{x^2}{2} + \frac{\log(x^2-\sqrt{2}x+1)}{4\sqrt{2}} - \frac{\log(x^2+\sqrt{2}x+1)}{4\sqrt{2}} - \frac{1}{2} \tan^{-1}(x^2) + \log(x) - \frac{\tan^{-1}(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{2\sqrt{2}}$$

[Out] 1/2*x^2-1/2*arctan(x^2)+ln(x)-1/4*ln(x^4+1)+1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {1593, 1833, 297, 1162, 617, 204, 1165, 628, 1834, 1248, 635, 203, 260}

$$\frac{x^2}{2} + \frac{\log(x^2-\sqrt{2}x+1)}{4\sqrt{2}} - \frac{\log(x^2+\sqrt{2}x+1)}{4\sqrt{2}} - \frac{1}{4} \log(x^4+1) - \frac{1}{2} \tan^{-1}(x^2) + \log(x) - \frac{\tan^{-1}(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3 + x^6)/(x + x^5), x]

[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 297

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1833

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rule 1834

Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3+x^6}{x+x^5} dx &= \int \frac{1+x^3+x^6}{x(1+x^4)} dx \\
&= \int \left(\frac{x^2}{1+x^4} + \frac{1+x^6}{x(1+x^4)} \right) dx \\
&= \int \frac{x^2}{1+x^4} dx + \int \frac{1+x^6}{x(1+x^4)} dx \\
&= -\left(\frac{1}{2} \int \frac{1-x^2}{1+x^4} dx \right) + \frac{1}{2} \int \frac{1+x^2}{1+x^4} dx + \int \left(\frac{1}{x} + x + \frac{x(-1-x^2)}{1+x^4} \right) dx \\
&= \frac{x^2}{2} + \log(x) + \frac{1}{4} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{4} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{4\sqrt{2}} + \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{4\sqrt{2}} \\
&= \frac{x^2}{2} + \log(x) + \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{-1-x}{1+x^2} dx, x, x^2 \right) \\
&= \frac{x^2}{2} - \frac{\tan^{-1}(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{2\sqrt{2}} + \log(x) + \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}} \\
&= \frac{x^2}{2} - \frac{1}{2} \tan^{-1}(x^2) - \frac{\tan^{-1}(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{2\sqrt{2}} + \log(x) + \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.90

$$\frac{1}{8} \left(-2 \log(x^4 + 1) + 4x^2 + \sqrt{2} \log(x^2 - \sqrt{2}x + 1) - \sqrt{2} \log(x^2 + \sqrt{2}x + 1) + 8 \log(x) - 2(\sqrt{2} - 2) \tan^{-1} \left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3 + x^6)/(x + x^5), x]

[Out] (4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1 + Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8

fricas [C] time = 2.41, size = 515, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+x^3+1)/(x^5+x), x, algorithm="fricas")

[Out] 1/2*x^2 - 1/4*(2*sqrt(1/4*I) + I + 1)*log((2*sqrt(1/4*I) + I + 1)^3 - 5*(2*sqrt(1/4*I) + I + 1)^2 + 3*x + 20*sqrt(1/4*I) + 10*I + 5) - 1/4*(2*sqrt(-1/4*I) - I + 1)*log(-(2*sqrt(1/4*I) + I + 1)^3 - (2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 4*(2*sqrt(1/4*I) + I + 1)^2 - ((2*sqrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) + 3*x - 16*sqrt(1/4*I) - 8*I - 9) + 1/4*(sqrt(1/4*I) + sqrt(-1/4*I) - 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I + 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2)*(2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)

$$+ 2\sqrt{1/4*I} + I - 1) + 3*x - 2\sqrt{1/4*I} - I + 2) + 1/4*(\sqrt{1/4*I} + \sqrt{-1/4*I} + 2\sqrt{-3/16*(2\sqrt{1/4*I} + I + 1)^2 - 1/8*(2\sqrt{1/4*I} + I - 3)*(2\sqrt{-1/4*I} - I + 1) - 3/16*(2\sqrt{-1/4*I} - I + 1)^2 + \sqrt{1/4*I} + 1/2*I - 1/2) - 1)*\log(1/2*(2\sqrt{1/4*I} + I + 2)*(2\sqrt{-1/4*I} - I + 1)^2 + 1/2*(2\sqrt{1/4*I} + I + 1)^2 + 1/2*((2\sqrt{1/4*I} + I + 1)^2 - 8\sqrt{1/4*I} - 4*I - 6)*(2\sqrt{-1/4*I} - I + 1) - 2\sqrt{-3/16*(2\sqrt{1/4*I} + I + 1)^2 - 1/8*(2\sqrt{1/4*I} + I - 3)*(2\sqrt{-1/4*I} - I + 1) - 3/16*(2\sqrt{-1/4*I} - I + 1)^2 + \sqrt{1/4*I} + 1/2*I - 1/2))*((2\sqrt{1/4*I} + I + 2)*(2\sqrt{-1/4*I} - I + 1) + 2\sqrt{1/4*I} + I - 1) + 3*x - 2\sqrt{1/4*I} - I + 2) + \log(x)$$

giac [A] time = 0.39, size = 92, normalized size = 0.82

$$\frac{1}{2}x^2 + \frac{1}{4}(\sqrt{2} + 2)\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{4}(\sqrt{2} - 2)\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{1}{8}\sqrt{2}\log(x^2 + \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")

[Out] 1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*log(x^4 + 1) + log(abs(x))

maple [A] time = 0.01, size = 79, normalized size = 0.71

$$\frac{x^2}{2} - \frac{\arctan(x^2)}{2} + \frac{\sqrt{2}\arctan(\sqrt{2}x - 1)}{4} + \frac{\sqrt{2}\arctan(\sqrt{2}x + 1)}{4} + \ln(x) + \frac{\sqrt{2}\ln\left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right)}{8} - \frac{\ln(x^4 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6+x^3+1)/(x^5+x),x)

[Out] 1/2*x^2 - 1/2*arctan(x^2) + 1/4*arctan(-1 + 2^(1/2)*x)*2^(1/2) + 1/8*2^(1/2)*ln((1 + x^2 - 2^(1/2)*x)/(1 + x^2 + 2^(1/2)*x)) + 1/4*arctan(1 + 2^(1/2)*x)*2^(1/2) - 1/4*ln(x^4 + 1) + ln(x)

maxima [A] time = 2.06, size = 99, normalized size = 0.88

$$\frac{1}{4}\sqrt{2}(\sqrt{2} + 1)\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) - \frac{1}{4}\sqrt{2}(\sqrt{2} - 1)\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{1}{8}\sqrt{2}(\sqrt{2} + 1)\log(x^2 + \sqrt{2}x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*sqrt(2)*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*(sqrt(2) + 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(sqrt(2) - 1)*log(x^2 - sqrt(2)*x + 1) + 1/2*x^2 + log(x)

mupad [B] time = 2.23, size = 170, normalized size = 1.52

$$\ln(x) + \left(\sum_{k=1}^4 \ln\left(\text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)\right)\right) \left(8 \text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right) + x + \text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + x^6 + 1)/(x + x^5),x)

```
[Out] log(x) + symsum(log(root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*(8*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k) + x + 96*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*x + 240*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2*x + 320*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^3*x - 16*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2 + 8))*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k), k, 1, 4) + x^2/2
```

sympy [A] time = 0.96, size = 61, normalized size = 0.54

$$\frac{x^2}{2} + \log(x) + \text{RootSum}\left(256t^4 + 256t^3 + 128t^2 + 16t + 1, \left(t \mapsto t \log\left(\frac{1792t^4}{73} + \frac{704t^3}{219} - \frac{3152t^2}{219} - \frac{2584t}{219} + x - \frac{344}{219}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, Lambda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/219 + x - 344/219)))
```

$$3.369 \quad \int \frac{1+x^2}{-x+x^2} dx$$

Optimal. Leaf size=14

$$x + 2 \log(1 - x) - \log(x)$$

[Out] x+2*ln(1-x)-ln(x)

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 894}

$$x + 2 \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(-x + x^2), x]

[Out] x + 2*Log[1 - x] - Log[x]

Rule 894

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{-x+x^2} dx &= \int \frac{1+x^2}{(-1+x)x} dx \\ &= \int \left(1 + \frac{2}{-1+x} - \frac{1}{x}\right) dx \\ &= x + 2 \log(1 - x) - \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$x + 2 \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(-x + x^2), x]

[Out] x + 2*Log[1 - x] - Log[x]

fricas [A] time = 0.75, size = 12, normalized size = 0.86

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2-x), x, algorithm="fricas")

[Out] $x + 2\log(x - 1) - \log(x)$

giac [A] time = 0.26, size = 14, normalized size = 1.00

$$x + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(x^2-x),x, algorithm="giac")`

[Out] $x + 2\log(\text{abs}(x - 1)) - \log(\text{abs}(x))$

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$x - \ln(x) + 2 \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)/(x^2-x),x)`

[Out] $x+2*\ln(x-1)-\ln(x)$

maxima [A] time = 0.91, size = 12, normalized size = 0.86

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(x^2-x),x, algorithm="maxima")`

[Out] $x + 2\log(x - 1) - \log(x)$

mupad [B] time = 0.04, size = 12, normalized size = 0.86

$$x + 2 \ln(x - 1) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 + 1)/(x - x^2),x)`

[Out] $x + 2\log(x - 1) - \log(x)$

sympy [A] time = 0.10, size = 10, normalized size = 0.71

$$x - \log(x) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(x**2-x),x)`

[Out] $x - \log(x) + 2*\log(x - 1)$

$$3.370 \quad \int \frac{1+x^3}{-x+x^3} dx$$

Optimal. Leaf size=12

$$x + \log(1 - x) - \log(x)$$

[Out] x+ln(1-x)-ln(x)

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 1802}

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(-x + x^3), x]

[Out] x + Log[1 - x] - Log[x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-x+x^3} dx &= \int \frac{1+x^3}{x(-1+x^2)} dx \\ &= \int \left(1 + \frac{1}{-1+x} - \frac{1}{x} \right) dx \\ &= x + \log(1-x) - \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(-x + x^3), x]

[Out] x + Log[1 - x] - Log[x]

fricas [A] time = 0.63, size = 10, normalized size = 0.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x), x, algorithm="fricas")

[Out] x + log(x - 1) - log(x)

giac [A] time = 0.37, size = 12, normalized size = 1.00

$$x + \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x),x, algorithm="giac")

[Out] x + log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$x - \ln(x) + \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(x^3-x),x)

[Out] x+ln(x-1)-ln(x)

maxima [A] time = 1.01, size = 10, normalized size = 0.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x),x, algorithm="maxima")

[Out] x + log(x - 1) - log(x)

mupad [B] time = 2.14, size = 10, normalized size = 0.83

$$x - 2 \operatorname{atanh}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 + 1)/(x - x^3),x)

[Out] x - 2*atanh(2*x - 1)

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$x - \log(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)/(x**3-x),x)

[Out] x - log(x) + log(x - 1)

$$3.371 \quad \int \frac{1+x^3}{-x^2+x^3} dx$$

Optimal. Leaf size=17

$$x + \frac{1}{x} + 2 \log(1-x) - \log(x)$$

[Out] 1/x+x+2*ln(1-x)-ln(x)

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1593, 1620}

$$x + \frac{1}{x} + 2 \log(1-x) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(-x^2 + x^3), x]

[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-x^2+x^3} dx &= \int \frac{1+x^3}{(-1+x)x^2} dx \\ &= \int \left(1 + \frac{2}{-1+x} - \frac{1}{x^2} - \frac{1}{x} \right) dx \\ &= \frac{1}{x} + x + 2 \log(1-x) - \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$x + \frac{1}{x} + 2 \log(1-x) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(-x^2 + x^3), x]

[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]

fricas [A] time = 0.70, size = 21, normalized size = 1.24

$$\frac{x^2 + 2x \log(x-1) - x \log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")

[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x

giac [A] time = 0.32, size = 17, normalized size = 1.00

$$x + \frac{1}{x} + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")

[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$x - \ln(x) + 2 \ln(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(x^3-x^2),x)

[Out] x+2*ln(x-1)+1/x-ln(x)

maxima [A] time = 0.89, size = 15, normalized size = 0.88

$$x + \frac{1}{x} + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")

[Out] x + 1/x + 2*log(x - 1) - log(x)

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$x + 2 \ln(x - 1) - \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 + 1)/(x^2 - x^3),x)

[Out] x + 2*log(x - 1) - log(x) + 1/x

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$x - \log(x) + 2 \log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)/(x**3-x**2),x)

[Out] x - log(x) + 2*log(x - 1) + 1/x

$$3.372 \quad \int \frac{-1+x^5}{-x+x^3} dx$$

Optimal. Leaf size=17

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

[Out] x+1/3*x^3+ln(x)-ln(1+x)

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 1802}

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^5)/(-x + x^3), x]

[Out] x + x^3/3 + Log[x] - Log[1 + x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^5}{-x+x^3} dx &= \int \frac{-1+x^5}{x(-1+x^2)} dx \\ &= \int \left(1 + \frac{1}{-1-x} + \frac{1}{x} + x^2 \right) dx \\ &= x + \frac{x^3}{3} + \log(x) - \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^5)/(-x + x^3), x]

[Out] x + x^3/3 + Log[x] - Log[1 + x]

fricas [A] time = 0.62, size = 15, normalized size = 0.88

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)/(x^3-x),x, algorithm="fricas")

[Out] 1/3*x^3 + x - log(x + 1) + log(x)

giac [A] time = 0.26, size = 17, normalized size = 1.00

$$\frac{1}{3}x^3 + x - \log(|x + 1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)/(x^3-x),x, algorithm="giac")

[Out] 1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^3}{3} + x + \ln(x) - \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5-1)/(x^3-x),x)

[Out] x+1/3*x^3+ln(x)-ln(x+1)

maxima [A] time = 0.90, size = 15, normalized size = 0.88

$$\frac{1}{3}x^3 + x - \log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)/(x^3-x),x, algorithm="maxima")

[Out] 1/3*x^3 + x - log(x + 1) + log(x)

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$x - 2 \operatorname{atanh}(2x + 1) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^5 - 1)/(x - x^3),x)

[Out] x - 2*atanh(2*x + 1) + x^3/3

sympy [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{x^3}{3} + x + \log(x) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5-1)/(x**3-x),x)

[Out] x**3/3 + x + log(x) - log(x + 1)

$$3.373 \quad \int \frac{1+x^4}{x^3+x^5} dx$$

Optimal. Leaf size=18

$$-\frac{1}{2x^2} + \log(x^2 + 1) - \log(x)$$

[Out] -1/2/x^2-ln(x)+ln(x^2+1)

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1593, 1252, 894}

$$-\frac{1}{2x^2} + \log(x^2 + 1) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(x^3 + x^5), x]

[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{x^3+x^5} dx &= \int \frac{1+x^4}{x^3(1+x^2)} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1+x^2}{x^2(1+x)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{2}{1+x} \right) dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} - \log(x) + \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{2x^2} + \log(x^2 + 1) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(x^3 + x^5), x]

[Out] -1/2*1/x^2 - Log[x] + Log[1 + x^2]

fricas [A] time = 0.67, size = 25, normalized size = 1.39

$$\frac{2x^2 \log(x^2 + 1) - 2x^2 \log(x) - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^5+x^3), x, algorithm="fricas")

[Out] 1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2

giac [A] time = 0.27, size = 23, normalized size = 1.28

$$\frac{x^2 - 1}{2x^2} + \log(x^2 + 1) - \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^5+x^3), x, algorithm="giac")

[Out] 1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$-\ln(x) + \ln(x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^5+x^3), x)

[Out] -1/2/x^2 - ln(x) + ln(x^2+1)

maxima [A] time = 1.92, size = 16, normalized size = 0.89

$$-\frac{1}{2x^2} + \log(x^2 + 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^5+x^3), x, algorithm="maxima")

[Out] -1/2/x^2 + log(x^2 + 1) - log(x)

mupad [B] time = 0.05, size = 16, normalized size = 0.89

$$\ln(x^2 + 1) - \ln(x) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 1)/(x^3 + x^5), x)

[Out] log(x^2 + 1) - log(x) - 1/(2*x^2)

sympy [A] time = 0.11, size = 15, normalized size = 0.83

$$-\log(x) + \log(x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+1)/(x**5+x**3), x)

[Out] -log(x) + log(x**2 + 1) - 1/(2*x**2)

$$3.374 \quad \int \frac{1+x^2}{x+2x^2+x^3} dx$$

Optimal. Leaf size=10

$$\frac{2}{x+1} + \log(x)$$

[Out] 2/(1+x)+ln(x)

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1594, 27, 894}

$$\frac{2}{x+1} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(x + 2*x^2 + x^3), x]

[Out] 2/(1 + x) + Log[x]

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 894

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{x+2x^2+x^3} dx &= \int \frac{1+x^2}{x(1+2x+x^2)} dx \\ &= \int \frac{1+x^2}{x(1+x)^2} dx \\ &= \int \left(\frac{1}{x} - \frac{2}{(1+x)^2} \right) dx \\ &= \frac{2}{1+x} + \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x+1} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(x + 2*x^2 + x^3), x]

[Out] 2/(1 + x) + Log[x]

fricas [A] time = 0.78, size = 14, normalized size = 1.40

$$\frac{(x + 1)\log(x) + 2}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+2*x^2+x), x, algorithm="fricas")

[Out] ((x + 1)*log(x) + 2)/(x + 1)

giac [A] time = 0.36, size = 11, normalized size = 1.10

$$\frac{2}{x + 1} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+2*x^2+x), x, algorithm="giac")

[Out] 2/(x + 1) + log(abs(x))

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\ln(x) + \frac{2}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^3+2*x^2+x), x)

[Out] 2/(x+1)+ln(x)

maxima [A] time = 0.89, size = 10, normalized size = 1.00

$$\frac{2}{x + 1} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+2*x^2+x), x, algorithm="maxima")

[Out] 2/(x + 1) + log(x)

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\ln(x) + \frac{2}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x + 2*x^2 + x^3), x)

[Out] log(x) + 2/(x + 1)

sympy [A] time = 0.09, size = 7, normalized size = 0.70

$$\log(x) + \frac{2}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**3+2*x**2+x), x)

[Out] log(x) + 2/(x + 1)

$$3.375 \quad \int \frac{1+x^5}{-10x-3x^2+x^3} dx$$

Optimal. Leaf size=42

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(x+2)$$

[Out] 19*x+3/2*x^2+1/3*x^3+3126/35*ln(5-x)-1/10*ln(x)-31/14*ln(2+x)

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1594, 1628}

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3), x]

[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x])/14

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_.)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+x^5}{-10x-3x^2+x^3} dx &= \int \frac{1+x^5}{x(-10-3x+x^2)} dx \\ &= \int \left(19 + \frac{3126}{35(-5+x)} - \frac{1}{10x} + 3x + x^2 - \frac{31}{14(2+x)} \right) dx \\ &= 19x + \frac{3x^2}{2} + \frac{x^3}{3} + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3), x]

[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x])/14

fricas [A] time = 0.59, size = 30, normalized size = 0.71

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(x+2) + \frac{3126}{35}\log(x-5) - \frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")

[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log(x)

giac [A] time = 0.29, size = 33, normalized size = 0.79

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(|x+2|) + \frac{3126}{35}\log(|x-5|) - \frac{1}{10}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")

[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5)) - 1/10*log(abs(x))

maple [A] time = 0.01, size = 31, normalized size = 0.74

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\ln(x)}{10} + \frac{3126 \ln(x-5)}{35} - \frac{31 \ln(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5+1)/(x^3-3*x^2-10*x),x)

[Out] 1/3*x^3+3/2*x^2+19*x-31/14*ln(x+2)-1/10*ln(x)+3126/35*ln(x-5)

maxima [A] time = 0.88, size = 30, normalized size = 0.71

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(x+2) + \frac{3126}{35}\log(x-5) - \frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")

[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log(x)

mupad [B] time = 0.05, size = 30, normalized size = 0.71

$$19x - \frac{31 \ln(x+2)}{14} + \frac{3126 \ln(x-5)}{35} - \frac{\ln(x)}{10} + \frac{3x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)

[Out] 19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 + x^3/3

sympy [A] time = 0.15, size = 36, normalized size = 0.86

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log(x)}{10} + \frac{3126 \log(x-5)}{35} - \frac{31 \log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)

[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/14

$$3.376 \quad \int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$$

Optimal. Leaf size=46

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{5 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2))*5^(1/2)

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{5 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]

[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{15 - 5x + x^2 + x^3}{(5 + x^2)(3 + 2x + x^2)} dx &= \int \left(-\frac{5}{5 + x^2} + \frac{6 + x}{3 + 2x + x^2} \right) dx \\ &= -\left(5 \int \frac{1}{5 + x^2} dx \right) + \int \frac{6 + x}{3 + 2x + x^2} dx \\ &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{2} \int \frac{2 + 2x}{3 + 2x + x^2} dx + 5 \int \frac{1}{3 + 2x + x^2} dx \\ &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{2} \log(3 + 2x + x^2) - 10 \operatorname{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2 + 2x \right) \\ &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{5 \tan^{-1} \left(\frac{1+x}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{1}{2} \log(3 + 2x + x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{5 \tan^{-1} \left(\frac{x+1}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]

[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2

fricas [A] time = 0.77, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (x + 1) \right) - \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} x \right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

giac [A] time = 0.36, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (x + 1) \right) - \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} x \right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$-\sqrt{5} \arctan \left(\frac{\sqrt{5} x}{5} \right) + \frac{5\sqrt{2} \arctan \left(\frac{(2x+2)\sqrt{2}}{4} \right)}{2} + \frac{\ln(x^2 + 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)`

[Out] $-5^{1/2} \arctan(1/5 \cdot 5^{1/2} \cdot x) + 5/2 \cdot 2^{1/2} \arctan(1/4 \cdot (2 \cdot x + 2) \cdot 2^{1/2}) + 1/2 \cdot \ln(x^2 + 2 \cdot x + 3)$

maxima [A] time = 1.97, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (x + 1)\right) - \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} x\right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")`

[Out] $5/2 \cdot \sqrt{2} \arctan(1/2 \cdot \sqrt{2} \cdot (x + 1)) - \sqrt{5} \arctan(1/5 \cdot \sqrt{5} \cdot x) + 1/2 \cdot \log(x^2 + 2 \cdot x + 3)$

mupad [B] time = 0.00, size = 88, normalized size = 1.91

$$\frac{\ln(x + 1 - \sqrt{2} i)}{2} + \frac{\ln(x + 1 + \sqrt{2} i)}{2} + \sqrt{5} \operatorname{atan}\left(\frac{2000 \sqrt{5}}{2000x + 1120} - \frac{224 \sqrt{5} x}{2000x + 1120}\right) - \frac{\sqrt{2} \ln(x + 1 - \sqrt{2} i) 5i}{4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)`

[Out] $\log(x - 2^{1/2} \cdot 1i + 1)/2 + \log(x + 2^{1/2} \cdot 1i + 1)/2 + 5^{1/2} \operatorname{atan}\left(\frac{2000 \cdot 5^{1/2}}{2000 \cdot x + 1120} - \frac{224 \cdot 5^{1/2} \cdot x}{2000 \cdot x + 1120}\right) - (2^{1/2} \cdot \log(x - 2^{1/2} \cdot 1i + 1) \cdot 5i)/4 + (2^{1/2} \cdot \log(x + 2^{1/2} \cdot 1i + 1) \cdot 5i)/4$

sympy [A] time = 0.21, size = 51, normalized size = 1.11

$$\frac{\log(x^2 + 2x + 3)}{2} - \sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5} x}{5}\right) + \frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2} + \frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)`

[Out] $\log(x^2 + 2 \cdot x + 3)/2 - \sqrt{5} \operatorname{atan}(\sqrt{5} \cdot x/5) + 5 \cdot \sqrt{2} \operatorname{atan}(\sqrt{2} \cdot x/2 + \sqrt{2}/2)/2$

$$3.377 \quad \int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$$

Optimal. Leaf size=19

$$\frac{1}{8} \log(3x+1) - \frac{1}{8} \log(x+3)$$

[Out] -1/8*ln(3+x)+1/8*ln(1+3*x)

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6688, 616, 31}

$$\frac{1}{8} \log(3x+1) - \frac{1}{8} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]

[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 6688

Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx &= \int \frac{1}{3+10x+3x^2} dx \\ &= \frac{3}{8} \int \frac{1}{1+3x} dx - \frac{3}{8} \int \frac{1}{9+3x} dx \\ &= -\frac{1}{8} \log(3+x) + \frac{1}{8} \log(1+3x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{8} \log(3x+1) - \frac{1}{8} \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]

[Out] -1/8*Log[3 + x] + Log[1 + 3*x]/8

fricas [A] time = 0.52, size = 15, normalized size = 0.79

$$\frac{1}{8} \log(3x + 1) - \frac{1}{8} \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")

[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)

giac [A] time = 0.28, size = 17, normalized size = 0.89

$$\frac{1}{8} \log(|3x + 1|) - \frac{1}{8} \log(|x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")

[Out] 1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{\ln(3x + 1)}{8} - \frac{\ln(x + 3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)/(3+10/(x^2+1)*x),x)

[Out] -1/8*ln(x+3)+1/8*ln(1+3*x)

maxima [A] time = 1.03, size = 15, normalized size = 0.79

$$\frac{1}{8} \log(3x + 1) - \frac{1}{8} \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")

[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)

mupad [B] time = 0.08, size = 8, normalized size = 0.42

$$-\frac{\operatorname{atanh}\left(\frac{3x}{4} + \frac{5}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 1)*((10*x)/(x^2 + 1) + 3)),x)

[Out] -atanh((3*x)/4 + 5/4)/4

sympy [A] time = 0.11, size = 14, normalized size = 0.74

$$\frac{\log\left(x + \frac{1}{3}\right)}{8} - \frac{\log(x + 3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)

[Out] log(x + 1/3)/8 - log(x + 3)/8

$$3.378 \quad \int \frac{x^3}{13 + \frac{2}{x} + 15x} dx$$

Optimal. Leaf size=40

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1386, 701, 632, 31}

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

Antiderivative was successfully verified.

[In] Int[x^3/(13 + 2/x + 15*x), x]

[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]/4375

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 701

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 1386

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{13 + \frac{2}{x} + 15x} dx &= \int \frac{x^4}{2 + 13x + 15x^2} dx \\
&= \int \left(\frac{139}{3375} - \frac{13x}{225} + \frac{x^2}{15} - \frac{278 + 1417x}{3375(2 + 13x + 15x^2)} \right) dx \\
&= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{\int \frac{278+1417x}{2+13x+15x^2} dx}{3375} \\
&= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} + \frac{3}{875} \int \frac{1}{3 + 15x} dx - \frac{80}{189} \int \frac{1}{10 + 15x} dx \\
&= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567} \log(2 + 3x) + \frac{\log(1 + 5x)}{4375}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(13 + 2/x + 15*x), x]

[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]/4375

fricas [A] time = 0.63, size = 30, normalized size = 0.75

$$\frac{1}{45} x^3 - \frac{13}{450} x^2 + \frac{139}{3375} x + \frac{1}{4375} \log(5x + 1) - \frac{16}{567} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(13+2/x+15*x), x, algorithm="fricas")

[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x + 2)

giac [A] time = 0.28, size = 32, normalized size = 0.80

$$\frac{1}{45} x^3 - \frac{13}{450} x^2 + \frac{139}{3375} x + \frac{1}{4375} \log(|5x + 1|) - \frac{16}{567} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(13+2/x+15*x), x, algorithm="giac")

[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(abs(3*x + 2))

maple [A] time = 0.01, size = 31, normalized size = 0.78

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\ln(5x + 1)}{4375} - \frac{16 \ln(3x + 2)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(13+2/x+15*x), x)

[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)

maxima [A] time = 0.99, size = 30, normalized size = 0.75

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(5x+1) - \frac{16}{567}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(13+2/x+15*x),x, algorithm="maxima")

[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x + 2)

mupad [B] time = 2.09, size = 26, normalized size = 0.65

$$\frac{139x}{3375} - \frac{16 \ln\left(x + \frac{2}{3}\right)}{567} + \frac{\ln\left(x + \frac{1}{5}\right)}{4375} - \frac{13x^2}{450} + \frac{x^3}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(15*x + 2/x + 13),x)

[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x^3/45

sympy [A] time = 0.12, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x + \frac{1}{5}\right)}{4375} - \frac{16 \log\left(x + \frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(13+2/x+15*x),x)

[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/567

$$3.379 \quad \int \frac{x^2}{13 + \frac{2}{x} + 15x} dx$$

Optimal. Leaf size=33

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1386, 701, 632, 31}

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[x^2/(13 + 2/x + 15*x), x]

[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 701

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 1386

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{13 + \frac{2}{x} + 15x} dx &= \int \frac{x^3}{2 + 13x + 15x^2} dx \\
&= \int \left(-\frac{13}{225} + \frac{x}{15} + \frac{26 + 139x}{225(2 + 13x + 15x^2)} \right) dx \\
&= -\frac{13x}{225} + \frac{x^2}{30} + \frac{1}{225} \int \frac{26 + 139x}{2 + 13x + 15x^2} dx \\
&= -\frac{13x}{225} + \frac{x^2}{30} - \frac{3}{175} \int \frac{1}{3 + 15x} dx + \frac{40}{63} \int \frac{1}{10 + 15x} dx \\
&= -\frac{13x}{225} + \frac{x^2}{30} + \frac{8}{189} \log(2 + 3x) - \frac{1}{875} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(13 + 2/x + 15*x), x]

[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875

fricas [A] time = 0.65, size = 25, normalized size = 0.76

$$\frac{1}{30} x^2 - \frac{13}{225} x - \frac{1}{875} \log(5x + 1) + \frac{8}{189} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(13+2/x+15*x), x, algorithm="fricas")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)

giac [A] time = 0.34, size = 27, normalized size = 0.82

$$\frac{1}{30} x^2 - \frac{13}{225} x - \frac{1}{875} \log(|5x + 1|) + \frac{8}{189} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(13+2/x+15*x), x, algorithm="giac")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))

maple [A] time = 0.01, size = 26, normalized size = 0.79

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\ln(5x + 1)}{875} + \frac{8 \ln(3x + 2)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(13+2/x+15*x), x)

[Out] -13/225*x+1/30*x^2+8/189*ln(3*x+2)-1/875*ln(5*x+1)

maxima [A] time = 1.02, size = 25, normalized size = 0.76

$$\frac{1}{30} x^2 - \frac{13}{225} x - \frac{1}{875} \log(5x + 1) + \frac{8}{189} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(13+2/x+15*x),x, algorithm="maxima")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)

mupad [B] time = 0.03, size = 21, normalized size = 0.64

$$\frac{8 \ln\left(x + \frac{2}{3}\right)}{189} - \frac{13x}{225} - \frac{\ln\left(x + \frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(15*x + 2/x + 13),x)

[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30

sympy [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x + \frac{1}{5}\right)}{875} + \frac{8 \log\left(x + \frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(13+2/x+15*x),x)

[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189

$$3.380 \quad \int \frac{x}{13 + \frac{2}{x} + 15x} dx$$

Optimal. Leaf size=26

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1386, 703, 632, 31}

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[x/(13 + 2/x + 15*x),x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1386

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n_) + (b_.)*(x_)^(mn_))^(p_), x_Symbol] :> Int[x^(m - n*p)*(b + a*xⁿ + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{x}{13 + \frac{2}{x} + 15x} dx &= \int \frac{x^2}{2 + 13x + 15x^2} dx \\
&= \frac{x}{15} + \frac{1}{15} \int \frac{-2 - 13x}{2 + 13x + 15x^2} dx \\
&= \frac{x}{15} + \frac{3}{35} \int \frac{1}{3 + 15x} dx - \frac{20}{21} \int \frac{1}{10 + 15x} dx \\
&= \frac{x}{15} - \frac{4}{63} \log(2 + 3x) + \frac{1}{175} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x/(13 + 2/x + 15*x), x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175

fricas [A] time = 0.83, size = 20, normalized size = 0.77

$$\frac{1}{15} x + \frac{1}{175} \log(5x + 1) - \frac{4}{63} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(13+2/x+15*x), x, algorithm="fricas")

[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)

giac [A] time = 0.27, size = 22, normalized size = 0.85

$$\frac{1}{15} x + \frac{1}{175} \log(|5x + 1|) - \frac{4}{63} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(13+2/x+15*x), x, algorithm="giac")

[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{15} + \frac{\ln(5x + 1)}{175} - \frac{4 \ln(3x + 2)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(13+2/x+15*x), x)

[Out] 1/15*x-4/63*ln(3*x+2)+1/175*ln(5*x+1)

maxima [A] time = 1.03, size = 20, normalized size = 0.77

$$\frac{1}{15} x + \frac{1}{175} \log(5x + 1) - \frac{4}{63} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(13+2/x+15*x), x, algorithm="maxima")

[Out] $1/15*x + 1/175*\log(5*x + 1) - 4/63*\log(3*x + 2)$

mupad [B] time = 2.12, size = 16, normalized size = 0.62

$$\frac{x}{15} - \frac{4 \ln\left(x + \frac{2}{3}\right)}{63} + \frac{\ln\left(x + \frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(15*x + 2/x + 13),x)`

[Out] $x/15 - (4*\log(x + 2/3))/63 + \log(x + 1/5)/175$

sympy [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{x}{15} + \frac{\log\left(x + \frac{1}{5}\right)}{175} - \frac{4\log\left(x + \frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(13+2/x+15*x),x)`

[Out] $x/15 + \log(x + 1/5)/175 - 4*\log(x + 2/3)/63$

$$3.381 \quad \int \frac{1}{13 + \frac{2}{x} + 15x} dx$$

Optimal. Leaf size=21

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1350, 632, 31}

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[(13 + 2/x + 15*x)^(-1), x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1350

Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x] /; FreeQ[{a, b, c, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1}{13 + \frac{2}{x} + 15x} dx &= \int \frac{x}{2 + 13x + 15x^2} dx \\ &= -\left(\frac{3}{7} \int \frac{1}{3 + 15x} dx\right) + \frac{10}{7} \int \frac{1}{10 + 15x} dx \\ &= \frac{2}{21} \log(2 + 3x) - \frac{1}{35} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(13 + 2/x + 15*x)^(-1), x]

[Out] $(2*\text{Log}[2 + 3*x])/21 - \text{Log}[1 + 5*x]/35$

fricas [A] time = 0.60, size = 17, normalized size = 0.81

$$-\frac{1}{35} \log(5x + 1) + \frac{2}{21} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(13+2/x+15*x),x, algorithm="fricas")`

[Out] $-1/35*\log(5*x + 1) + 2/21*\log(3*x + 2)$

giac [A] time = 0.29, size = 19, normalized size = 0.90

$$-\frac{1}{35} \log(|5x + 1|) + \frac{2}{21} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(13+2/x+15*x),x, algorithm="giac")`

[Out] $-1/35*\log(\text{abs}(5*x + 1)) + 2/21*\log(\text{abs}(3*x + 2))$

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\ln(5x + 1)}{35} + \frac{2 \ln(3x + 2)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(13+2/x+15*x),x)`

[Out] $2/21*\ln(3*x+2)-1/35*\ln(5*x+1)$

maxima [A] time = 1.17, size = 17, normalized size = 0.81

$$-\frac{1}{35} \log(5x + 1) + \frac{2}{21} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(13+2/x+15*x),x, algorithm="maxima")`

[Out] $-1/35*\log(5*x + 1) + 2/21*\log(3*x + 2)$

mupad [B] time = 2.11, size = 13, normalized size = 0.62

$$\frac{2 \ln\left(x + \frac{2}{3}\right)}{21} - \frac{\ln\left(x + \frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(15*x + 2/x + 13),x)`

[Out] $(2*\log(x + 2/3))/21 - \log(x + 1/5)/35$

sympy [A] time = 0.11, size = 17, normalized size = 0.81

$$-\frac{\log\left(x + \frac{1}{5}\right)}{35} + \frac{2 \log\left(x + \frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(13+2/x+15*x),x)`

[Out] $-\log(x + 1/5)/35 + 2*\log(x + 2/3)/21$

$$3.382 \quad \int \frac{1}{x\left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=21

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

[Out] -1/7*ln(2+3*x)+1/7*ln(1+5*x)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1386, 616, 31}

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(13 + 2/x + 15*x)),x]

[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1386

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n_) + (b_.)*(x_)^(mn))^(p_), x_Symbol] :> Int[x^(m - n*p)*(b + a*xⁿ + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{2 + 13x + 15x^2} dx \\ &= \frac{15}{7} \int \frac{1}{3 + 15x} dx - \frac{15}{7} \int \frac{1}{10 + 15x} dx \\ &= -\frac{1}{7} \log(2 + 3x) + \frac{1}{7} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(13 + 2/x + 15*x)),x]

[Out] $-1/7*\text{Log}[2 + 3*x] + \text{Log}[1 + 5*x]/7$

fricas [A] time = 0.73, size = 17, normalized size = 0.81

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(13+2/x+15*x),x, algorithm="fricas")`

[Out] $1/7*\log(5*x + 1) - 1/7*\log(3*x + 2)$

giac [A] time = 0.27, size = 19, normalized size = 0.90

$$\frac{1}{7} \log(|5x + 1|) - \frac{1}{7} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(13+2/x+15*x),x, algorithm="giac")`

[Out] $1/7*\log(\text{abs}(5*x + 1)) - 1/7*\log(\text{abs}(3*x + 2))$

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln(5x + 1)}{7} - \frac{\ln(3x + 2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(13+2/x+15*x),x)`

[Out] $-1/7*\ln(3*x+2)+1/7*\ln(5*x+1)$

maxima [A] time = 0.93, size = 17, normalized size = 0.81

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(13+2/x+15*x),x, algorithm="maxima")`

[Out] $1/7*\log(5*x + 1) - 1/7*\log(3*x + 2)$

mupad [B] time = 0.08, size = 8, normalized size = 0.38

$$-\frac{2 \operatorname{atanh}\left(\frac{30x}{7} + \frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(15*x + 2/x + 13)),x)`

[Out] $-(2*\operatorname{atanh}((30*x)/7 + 13/7))/7$

sympy [A] time = 0.11, size = 15, normalized size = 0.71

$$\frac{\log\left(x + \frac{1}{5}\right)}{7} - \frac{\log\left(x + \frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(13+2/x+15*x),x)`

[Out] $\log(x + 1/5)/7 - \log(x + 2/3)/7$

$$3.383 \quad \int \frac{1}{x^2 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=27

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1386, 705, 29, 632, 31}

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(13 + 2/x + 15*x)),x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1386

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{x(2 + 13x + 15x^2)} dx \\
&= \frac{1}{2} \int \frac{1}{x} dx + \frac{1}{2} \int \frac{-13 - 15x}{2 + 13x + 15x^2} dx \\
&= \frac{\log(x)}{2} + \frac{45}{14} \int \frac{1}{10 + 15x} dx - \frac{75}{7} \int \frac{1}{3 + 15x} dx \\
&= \frac{\log(x)}{2} + \frac{3}{14} \log(2 + 3x) - \frac{5}{7} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(13 + 2/x + 15*x)), x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7

fricas [A] time = 0.73, size = 21, normalized size = 0.78

$$-\frac{5}{7} \log(5x + 1) + \frac{3}{14} \log(3x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(13+2/x+15*x), x, algorithm="fricas")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)

giac [A] time = 0.37, size = 24, normalized size = 0.89

$$-\frac{5}{7} \log(|5x + 1|) + \frac{3}{14} \log(|3x + 2|) + \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(13+2/x+15*x), x, algorithm="giac")

[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\ln(x)}{2} - \frac{5 \ln(5x + 1)}{7} + \frac{3 \ln(3x + 2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(13+2/x+15*x), x)

[Out] 1/2*ln(x)+3/14*ln(3*x+2)-5/7*ln(5*x+1)

maxima [A] time = 0.92, size = 21, normalized size = 0.78

$$-\frac{5}{7} \log(5x + 1) + \frac{3}{14} \log(3x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="maxima")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)

mupad [B] time = 0.09, size = 17, normalized size = 0.63

$$\frac{3 \ln\left(x + \frac{2}{3}\right)}{14} - \frac{5 \ln\left(x + \frac{1}{5}\right)}{7} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(15*x + 2/x + 13)),x)

[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2} - \frac{5 \log\left(x + \frac{1}{5}\right)}{7} + \frac{3 \log\left(x + \frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(13+2/x+15*x),x)

[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14

$$3.384 \quad \int \frac{1}{x^3 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=34

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

[Out] $-1/2/x - 13/4 * \ln(x) - 9/28 * \ln(2 + 3*x) + 25/7 * \ln(1 + 5*x)$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1386, 709, 800}

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(13 + 2/x + 15*x)), x]

[Out] $-1/(2*x) - (13*\text{Log}[x])/4 - (9*\text{Log}[2 + 3*x])/28 + (25*\text{Log}[1 + 5*x])/7$

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1386

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{x^2 (2 + 13x + 15x^2)} dx \\ &= -\frac{1}{2x} + \frac{1}{2} \int \frac{-13 - 15x}{x(2 + 13x + 15x^2)} dx \\ &= -\frac{1}{2x} + \frac{1}{2} \int \left(-\frac{13}{2x} - \frac{27}{14(2 + 3x)} + \frac{250}{7(1 + 5x)} \right) dx \\ &= -\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(2 + 3x) + \frac{25}{7} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(13 + 2/x + 15*x)),x]

[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7

fricas [A] time = 0.54, size = 30, normalized size = 0.88

$$\frac{100x \log(5x + 1) - 9x \log(3x + 2) - 91x \log(x) - 14}{28x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="fricas")

[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x

giac [A] time = 0.27, size = 29, normalized size = 0.85

$$-\frac{1}{2x} + \frac{25}{7} \log(|5x + 1|) - \frac{9}{28} \log(|3x + 2|) - \frac{13}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="giac")

[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{13 \ln(x)}{4} + \frac{25 \ln(5x + 1)}{7} - \frac{9 \ln(3x + 2)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(13+2/x+15*x),x)

[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(5*x+1)

maxima [A] time = 1.06, size = 26, normalized size = 0.76

$$-\frac{1}{2x} + \frac{25}{7} \log(5x + 1) - \frac{9}{28} \log(3x + 2) - \frac{13}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="maxima")

[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)

mupad [B] time = 0.03, size = 22, normalized size = 0.65

$$\frac{25 \ln\left(x + \frac{1}{5}\right)}{7} - \frac{9 \ln\left(x + \frac{2}{3}\right)}{28} - \frac{13 \ln(x)}{4} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(15*x + 2/x + 13)),x)

[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)

sympy [A] time = 0.16, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4} + \frac{25\log\left(x + \frac{1}{5}\right)}{7} - \frac{9\log\left(x + \frac{2}{3}\right)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(13+2/x+15*x),x)

[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)

$$3.385 \quad \int \frac{1}{x^4 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=41

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x + 2) - \frac{125}{7} \log(5x + 1)$$

[Out] $-1/4/x^2+13/4/x+139/8*\ln(x)+27/56*\ln(2+3*x)-125/7*\ln(1+5*x)$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1386, 709, 800}

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x + 2) - \frac{125}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(13 + 2/x + 15*x)),x]

[Out] $-1/(4*x^2) + 13/(4*x) + (139*\text{Log}[x])/8 + (27*\text{Log}[2 + 3*x])/56 - (125*\text{Log}[1 + 5*x])/7$

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
  := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
  := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol]
  := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{x^3 (2 + 13x + 15x^2)} dx \\ &= -\frac{1}{4x^2} + \frac{1}{2} \int \frac{-13 - 15x}{x^2 (2 + 13x + 15x^2)} dx \\ &= -\frac{1}{4x^2} + \frac{1}{2} \int \left(-\frac{13}{2x^2} + \frac{139}{4x} + \frac{81}{28(2 + 3x)} - \frac{1250}{7(1 + 5x)} \right) dx \\ &= -\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(2 + 3x) - \frac{125}{7} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x + 2) - \frac{125}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(13 + 2/x + 15*x)), x]

[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1 + 5*x])/7

fricas [A] time = 0.59, size = 39, normalized size = 0.95

$$\frac{1000x^2 \log(5x + 1) - 27x^2 \log(3x + 2) - 973x^2 \log(x) - 182x + 14}{56x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(13+2/x+15*x), x, algorithm="fricas")

[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x + 14)/x^2

giac [A] time = 0.28, size = 34, normalized size = 0.83

$$\frac{13x - 1}{4x^2} - \frac{125}{7} \log(|5x + 1|) + \frac{27}{56} \log(|3x + 2|) + \frac{139}{8} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(13+2/x+15*x), x, algorithm="giac")

[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 139/8*log(abs(x))

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{139 \ln(x)}{8} - \frac{125 \ln(5x + 1)}{7} + \frac{27 \ln(3x + 2)}{56} + \frac{13}{4x} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(13+2/x+15*x), x)

[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(5*x+1)

maxima [A] time = 1.02, size = 31, normalized size = 0.76

$$\frac{13x - 1}{4x^2} - \frac{125}{7} \log(5x + 1) + \frac{27}{56} \log(3x + 2) + \frac{139}{8} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(13+2/x+15*x), x, algorithm="maxima")

[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)

mupad [B] time = 0.04, size = 26, normalized size = 0.63

$$\frac{27 \ln\left(x + \frac{2}{3}\right)}{56} - \frac{125 \ln\left(x + \frac{1}{5}\right)}{7} + \frac{139 \ln(x)}{8} + \frac{\frac{13x}{4} - \frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(15*x + 2/x + 13)),x)`

[Out] $(27*\log(x + 2/3))/56 - (125*\log(x + 1/5))/7 + (139*\log(x))/8 + ((13*x)/4 - 1/4)/x^2$

sympy [A] time = 0.17, size = 36, normalized size = 0.88

$$\frac{139 \log(x)}{8} - \frac{125 \log\left(x + \frac{1}{5}\right)}{7} + \frac{27 \log\left(x + \frac{2}{3}\right)}{56} + \frac{13x - 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(13+2/x+15*x),x)`

[Out] $139*\log(x)/8 - 125*\log(x + 1/5)/7 + 27*\log(x + 2/3)/56 + (13*x - 1)/(4*x**2)$

$$3.386 \quad \int \frac{1}{x^5 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=48

$$-\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x + 2) + \frac{625}{7} \log(5x + 1)$$

[Out] $-1/6/x^3+13/8/x^2-139/8/x-1417/16*\ln(x)-81/112*\ln(2+3*x)+625/7*\ln(1+5*x)$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1386, 709, 800}

$$\frac{13}{8x^2} - \frac{1}{6x^3} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x + 2) + \frac{625}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(13 + 2/x + 15*x)), x]

[Out] $-1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*\text{Log}[x])/16 - (81*\text{Log}[2 + 3*x])/112 + (625*\text{Log}[1 + 5*x])/7$

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^(m)*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1386

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^5 \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{x^4 (2 + 13x + 15x^2)} dx \\ &= -\frac{1}{6x^3} + \frac{1}{2} \int \frac{-13 - 15x}{x^3 (2 + 13x + 15x^2)} dx \\ &= -\frac{1}{6x^3} + \frac{1}{2} \int \left(-\frac{13}{2x^3} + \frac{139}{4x^2} - \frac{1417}{8x} - \frac{243}{56(2 + 3x)} + \frac{6250}{7(1 + 5x)} \right) dx \\ &= -\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(2 + 3x) + \frac{625}{7} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x+2) + \frac{625}{7} \log(5x+1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(13 + 2/x + 15*x)),x]

[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/112 + (625*Log[1 + 5*x])/7

fricas [A] time = 0.88, size = 44, normalized size = 0.92

$$\frac{30000x^3 \log(5x+1) - 243x^3 \log(3x+2) - 29757x^3 \log(x) - 5838x^2 + 546x - 56}{336x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="fricas")

[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5838*x^2 + 546*x - 56)/x^3

giac [A] time = 0.24, size = 39, normalized size = 0.81

$$-\frac{417x^2 - 39x + 4}{24x^3} + \frac{625}{7} \log(|5x+1|) - \frac{81}{112} \log(|3x+2|) - \frac{1417}{16} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="giac")

[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3*x + 2)) - 1417/16*log(abs(x))

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{1417 \ln(x)}{16} + \frac{625 \ln(5x+1)}{7} - \frac{81 \ln(3x+2)}{112} - \frac{139}{8x} + \frac{13}{8x^2} - \frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(13+2/x+15*x),x)

[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(5*x+1)

maxima [A] time = 1.07, size = 36, normalized size = 0.75

$$-\frac{417x^2 - 39x + 4}{24x^3} + \frac{625}{7} \log(5x+1) - \frac{81}{112} \log(3x+2) - \frac{1417}{16} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="maxima")

[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) - 1417/16*log(x)

mupad [B] time = 0.04, size = 32, normalized size = 0.67

$$\frac{625 \ln\left(x + \frac{1}{5}\right)}{7} - \frac{81 \ln\left(x + \frac{2}{3}\right)}{112} - \frac{1417 \ln(x)}{16} - \frac{\frac{139x^2}{8} - \frac{13x}{8} + \frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(15*x + 2/x + 13)),x)`

[Out] $(625*\log(x + 1/5))/7 - (81*\log(x + 2/3))/112 - (1417*\log(x))/16 - ((139*x^2)/8 - (13*x)/8 + 1/6)/x^3$

sympy [A] time = 0.18, size = 41, normalized size = 0.85

$$-\frac{1417\log(x)}{16} + \frac{625\log\left(x + \frac{1}{5}\right)}{7} - \frac{81\log\left(x + \frac{2}{3}\right)}{112} + \frac{-417x^2 + 39x - 4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(13+2/x+15*x),x)`

[Out] $-1417*\log(x)/16 + 625*\log(x + 1/5)/7 - 81*\log(x + 2/3)/112 + (-417*x**2 + 39*x - 4)/(24*x**3)$

$$3.387 \quad \int \frac{x^2}{2-(1+x^2)^4} dx$$

Optimal. Leaf size=157

$$\frac{i\sqrt{1-i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4^{2^{3/4}}} - \frac{i\sqrt{1+i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4^{2^{3/4}}} - \frac{\sqrt{1+\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4^{2^{3/4}}} + \frac{\sqrt{\sqrt[4]{2}-1} \tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4^{2^{3/4}}}$$

[Out] 1/8*I*arctan(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)-1/8*I*arctan(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(-1+2^(1/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(1+2^(1/4))^(1/2))*(1+2^(1/4))^(1/2)*2^(1/4)

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6740, 206, 203, 1972, 205}

$$\frac{i\sqrt{1-i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4^{2^{3/4}}} - \frac{i\sqrt{1+i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4^{2^{3/4}}} - \frac{\sqrt{1+\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4^{2^{3/4}}} + \frac{\sqrt{\sqrt[4]{2}-1} \tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4^{2^{3/4}}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 - (1 + x^2)^4), x]

[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]]/2^(3/4) - ((I/4)*Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]]/2^(3/4) - (Sqrt[1 + 2^(1/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcTanh[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1972

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 6740

Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I GtQ[n, 0] && PolynomialInQ[v, u, x]

) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) - sqrt(2))*log(1/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))^2 - ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2) - 6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) - sqrt(2))*log(-1/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))^2 - ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2) - 6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*log(4*((2^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4) - 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*log(-4*((2^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4) - 1/256*sqrt(2)) + 3*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{(x^2+1)^4-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="giac")

[Out] integrate(-x^2/((x^2 + 1)^4 - 2), x)

maple [C] time = 0.01, size = 54, normalized size = 0.34

$$\frac{\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1)^2 \ln(-\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1))}{8 \left(\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1)^7 + 3 \text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1)^5 + 3 \text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1)^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2-(x^2+1)^4),x)

[Out] -1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(-_R+x),_R=RootOf(-Z^8+4*_Z^6+6*_Z^4+4*_Z^2-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{(x^2+1)^4-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="maxima")

[Out] -integrate(x^2/((x^2 + 1)^4 - 2), x)

mupad [B] time = 2.75, size = 144, normalized size = 0.92

$$\sum_{k=1}^8 \ln \left(-\operatorname{root} \left(z^8 - \frac{z^4}{16384} + \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k \right) \left(56x - \operatorname{root} \left(z^8 - \frac{z^4}{16384} + \frac{z^2}{1048576} - \frac{1}{1073741824} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^2/((x^2 + 1)^4 - 2), x)

[Out] symsum(log(- root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(56*x - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(4096*x - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)^2*(262144*x + 67108864*root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k), k, 1, 8)

sympy [A] time = 0.22, size = 41, normalized size = 0.26

$$-\operatorname{RootSum} \left(1073741824t^8 - 65536t^4 + 1024t^2 - 1, \left(t \mapsto t \log \left(-\frac{67108864t^7}{3} - \frac{262144t^5}{3} - \frac{40t}{3} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(2-(x**2+1)**4), x)

[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(_t, _t*log(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x)))

$$3.388 \quad \int \frac{x^2}{2-(1-x^2)^4} dx$$

Optimal. Leaf size=157

$$\frac{\sqrt{\sqrt[4]{2}-1} \tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4 \cdot 2^{3/4}} - \frac{i\sqrt{1-i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{i\sqrt{1+i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}}$$

[Out] $-1/8*I*\operatorname{arctanh}(x/(1-I*2^{(1/4)})^{(1/2)})*(1-I*2^{(1/4)})^{(1/2)}*2^{(1/4)}+1/8*I*\operatorname{arctanh}(x/(1+I*2^{(1/4)})^{(1/2)})*(1+I*2^{(1/4)})^{(1/2)}*2^{(1/4)}-1/8*\operatorname{arctan}(x/(-1+2^{(1/4)})^{(1/2)})*(-1+2^{(1/4)})^{(1/2)}*2^{(1/4)}+1/8*\operatorname{arctanh}(x/(1+2^{(1/4)})^{(1/2)})*(1+2^{(1/4)})^{(1/2)}*2^{(1/4)}$

Rubi [A] time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6740, 206, 203, 1972, 208}

$$\frac{\sqrt{\sqrt[4]{2}-1} \tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4 \cdot 2^{3/4}} - \frac{i\sqrt{1-i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{i\sqrt{1+i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2/(2 - (1 - x^2)^4), x]$

[Out] $-(\operatorname{Sqrt}[-1 + 2^{(1/4)}]*\operatorname{ArcTan}[x/\operatorname{Sqrt}[-1 + 2^{(1/4)}]])/(4*2^{(3/4)}) - ((I/4)*\operatorname{Sqrt}[1 - I*2^{(1/4)}]*\operatorname{ArcTanh}[x/\operatorname{Sqrt}[1 - I*2^{(1/4)}]])/2^{(3/4)} + ((I/4)*\operatorname{Sqrt}[1 + I*2^{(1/4)}]*\operatorname{ArcTanh}[x/\operatorname{Sqrt}[1 + I*2^{(1/4)}]])/2^{(3/4)} + (\operatorname{Sqrt}[1 + 2^{(1/4)}]*\operatorname{ArcTanh}[x/\operatorname{Sqrt}[1 + 2^{(1/4)}]])/(4*2^{(3/4)})$

Rule 203

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

Rule 1972

$\operatorname{Int}(u_)^{(p_)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandToSum}[u, x]^p, x] /; \operatorname{FreeQ}[p, x] \ \&\& \operatorname{BinomialQ}[u, x] \ \&\& \operatorname{!BinomialMatchQ}[u, x]$

Rule 6740

$\operatorname{Int}(v_)/((a_ + (b_)*(u_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[\operatorname{PolynomialInSubst}[v, u, x]/(a + b*x^n), x] /. x \rightarrow u, x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{GtQ}[n, 0] \ \&\& \operatorname{PolynomialInQ}[v, u, x]$

$(2^{3/4} + \sqrt{2})^2 - \sqrt{2} \cdot (2^{3/4} - \sqrt{2})^2 - (\sqrt{2} \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2})^2 - 4 \cdot \sqrt{2} \cdot (2^{3/4} + \sqrt{2}) - 4 \cdot ((\sqrt{2} \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2}) \cdot (2^{3/4} + \sqrt{2}) + \sqrt{2} \cdot (2^{3/4} - \sqrt{2}) - 4 \cdot \sqrt{2}) \cdot \sqrt{-3/16 \cdot (2^{3/4} + \sqrt{2})^2 + 1/8 \cdot (2^{3/4} + \sqrt{2}) \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2}}) - 3/16 \cdot (2^{3/4} - \sqrt{2})^2 + 1) - 4 \cdot \sqrt{2} \cdot (2^{3/4} - \sqrt{2}) - 4 \cdot \sqrt{2} \cdot \sqrt{-1/2 \cdot \sqrt{2} - \sqrt{-3/16 \cdot (2^{3/4} + \sqrt{2})^2 + 1/8 \cdot (2^{3/4} + \sqrt{2}) \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2}}} + \sqrt{2}) \cdot (2^{3/4} - \sqrt{2}) - 3/16 \cdot (2^{3/4} - \sqrt{2})^2 + 1) + 6 \cdot x + 1/16 \cdot \sqrt{2} \cdot \sqrt{-1/2 \cdot \sqrt{2} - \sqrt{-3/16 \cdot (2^{3/4} + \sqrt{2})^2 + 1/8 \cdot (2^{3/4} + \sqrt{2}) \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2}}} + 1/8 \cdot (2^{3/4} + \sqrt{2}) \cdot (2^{3/4} - \sqrt{2}) - 3/16 \cdot (2^{3/4} - \sqrt{2})^2 + 1) \cdot \log(-1/4 \cdot ((\sqrt{2} \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2}) \cdot (2^{3/4} + \sqrt{2})^2 - \sqrt{2} \cdot (2^{3/4} - \sqrt{2})^2 - (\sqrt{2} \cdot (2^{3/4} - \sqrt{2})^2 - 4 \cdot \sqrt{2}) \cdot (2^{3/4} + \sqrt{2}) - 4 \cdot ((\sqrt{2} \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2}) \cdot (2^{3/4} + \sqrt{2}) + \sqrt{2} \cdot (2^{3/4} - \sqrt{2}) - 4 \cdot \sqrt{2}) \cdot \sqrt{-3/16 \cdot (2^{3/4} + \sqrt{2})^2 + 1/8 \cdot (2^{3/4} + \sqrt{2}) \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2}}) - 3/16 \cdot (2^{3/4} - \sqrt{2})^2 + 1) - 4 \cdot \sqrt{2} \cdot (2^{3/4} - \sqrt{2}) - 4 \cdot \sqrt{2} \cdot \sqrt{-1/2 \cdot \sqrt{2} - \sqrt{-3/16 \cdot (2^{3/4} + \sqrt{2})^2 + 1/8 \cdot (2^{3/4} + \sqrt{2}) \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2}}} + 1/8 \cdot (2^{3/4} + \sqrt{2}) \cdot (2^{3/4} - \sqrt{2}) - 3/16 \cdot (2^{3/4} - \sqrt{2})^2 + 1) + 6 \cdot x) + 1/16 \cdot \sqrt{2} \cdot (2^{3/4} + \sqrt{2}) \cdot \log(1/4 \cdot ((2^{3/4} - \sqrt{2})^3 + (2^{3/4} + \sqrt{2})^2 \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2} - 1) - ((2^{3/4} - \sqrt{2})^2 - 4) \cdot (2^{3/4} + \sqrt{2}) - 4 \cdot 2^{3/4} + 4 \cdot \sqrt{2} + 6) \cdot \sqrt{2^{3/4} + \sqrt{2}} + 3 \cdot x) - 1/16 \cdot \sqrt{2} \cdot (2^{3/4} + \sqrt{2}) \cdot \log(-1/4 \cdot ((2^{3/4} - \sqrt{2})^3 + (2^{3/4} + \sqrt{2})^2 \cdot (2^{3/4} - \sqrt{2}) - \sqrt{2} - 1) - ((2^{3/4} - \sqrt{2})^2 - 4) \cdot (2^{3/4} + \sqrt{2}) - 4 \cdot 2^{3/4} + 4 \cdot \sqrt{2} + 6) \cdot \sqrt{2^{3/4} + \sqrt{2}} + 3 \cdot x) - \sqrt{-1/256 \cdot 2^{3/4} + 1/256 \cdot \sqrt{2}} \cdot \log(4 \cdot ((2^{3/4} - \sqrt{2})^3 + (2^{3/4} - \sqrt{2})^2 + 10) \cdot \sqrt{-1/256 \cdot 2^{3/4} + 1/256 \cdot \sqrt{2}}) + 3 \cdot x) + \sqrt{-1/256 \cdot 2^{3/4} + 1/256 \cdot \sqrt{2}} \cdot \log(-4 \cdot ((2^{3/4} - \sqrt{2})^3 + (2^{3/4} - \sqrt{2})^2 + 10) \cdot \sqrt{-1/256 \cdot 2^{3/4} + 1/256 \cdot \sqrt{2}}) + 3 \cdot x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{(x^2-1)^4-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="giac")

[Out] integrate(-x^2/((x^2 - 1)^4 - 2), x)

maple [C] time = 0.02, size = 56, normalized size = 0.36

$$-\frac{\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1)^2 \ln(-\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1))}{8 \left(\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1)^7 - 3 \text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1)^5 + 3 \text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2-(-x^2+1)^4),x)

[Out] -1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(-Z^8-4*_Z^6+6*_Z^4-4*_Z^2-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{(x^2-1)^4-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="maxima")

[Out] -integrate(x^2/((x^2 - 1)^4 - 2), x)

mupad [B] time = 2.80, size = 142, normalized size = 0.90

$$\sum_{k=1}^8 \ln \left(-\operatorname{root} \left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k \right) \left(56x + \operatorname{root} \left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^2/((x^2 - 1)^4 - 2), x)

[Out] symsum(log(- root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(56*x + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(4096*x + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)^2*(262144*x - 67108864*root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k), k, 1, 8)

sympy [A] time = 0.23, size = 41, normalized size = 0.26

$$-\operatorname{RootSum} \left(1073741824t^8 - 65536t^4 - 1024t^2 - 1, \left(t \mapsto t \log \left(-\frac{67108864t^7}{3} + \frac{262144t^5}{3} + \frac{40t}{3} + x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(2-(-x**2+1)**4), x)

[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*log(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x)))

$$3.389 \quad \int \frac{x^2}{2+(1+x^2)^4} dx$$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{(-1)^{3/4} \sqrt{1 + i \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + i \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{1}{8}i$$

[Out] 1/8*(-1)^(1/4)*arctan(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/4)-1/8*(-1)^(3/4)*2^(1/4)*arctan(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4))^(1/2)-1/8*(-1)^(1/4)*arctan(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/2)*2^(1/4)+1/8*I*arctan(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2))*((1+I)/(1+I+2^(3/4)))^(1/2)

Rubi [A] time = 0.28, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6740, 204, 203, 1972, 205}

$$\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{(-1)^{3/4} \sqrt{1 + i \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + i \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{1}{8}i$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 + (1 + x^2)^4), x]

[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4)) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1 + I*(-2)^(1/4)]])/(4*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTan[x/Sqrt[1 + (-2)^(1/4)]])/(4*2^(3/4)) + (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*ArcTan[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1972

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 6740

Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I

+ 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))) + x) + sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 + 1)^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="giac")

[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)

maple [C] time = 0.01, size = 54, normalized size = 0.29

$$\frac{\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)^2 \ln(-\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3))}{8 \text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)^7 + 24 \text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)^5 + 24 \text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2+(x^2+1)^4),x)

[Out] 1/8*sum(1/(_R^7+3*_R^5+3*_R^3+_R)*_R^2*ln(-_R+x),_R=RootOf(-Z^8+4*_Z^6+6*_Z^4+4*_Z^2+3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 + 1)^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="maxima")

[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)

mupad [B] time = 2.78, size = 142, normalized size = 0.76

$$\sum_{k=1}^8 \ln\left(\text{root}\left(z^8 + \frac{z^4}{16384} + \frac{z^2}{1048576} + \frac{3}{1073741824}, z, k\right)\right) \left(40x + \text{root}\left(z^8 + \frac{z^4}{16384} + \frac{z^2}{1048576} + \frac{3}{1073741824}, z, k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((x^2 + 1)^4 + 2),x)

[Out] symsum(log(root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k))*(40*x + root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k))*(root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(4096*x - root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)^2*(786432*x - 67108864*root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k), k, 1, 8)

sympy [A] time = 0.21, size = 39, normalized size = 0.21

RootSum(1073741824t⁸ + 65536t⁴ + 1024t² + 3, (t ↦ t log(67108864t⁷ - 262144t⁵ + 4096t³ + 40t + x)))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(2+(x**2+1)**4),x)

[Out] RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_t, _t*log(67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)))

$$3.390 \quad \int \frac{x^2}{2+(1-x^2)^4} dx$$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{(-1)^{3/4} \sqrt{1 + i \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + i \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}}$$

[Out] $-1/8*(-1)^{(1/4)}*\operatorname{arctanh}(x/(1-(-2)^{(1/4)})^{(1/2)})*(1-(-2)^{(1/4)})^{(1/2)}*2^{(1/4)}+1/8*(-1)^{(3/4)}*2^{(1/4)}*\operatorname{arctanh}(x/(1+I*(-2)^{(1/4)})^{(1/2)})*(1+I*(-2)^{(1/4)})^{(1/2)}+1/8*(-1)^{(1/4)}*\operatorname{arctanh}(x/(1+(-2)^{(1/4)})^{(1/2)})*(1+(-2)^{(1/4)})^{(1/2)}*2^{(1/4)}-1/8*I*\operatorname{arctanh}(x*((1+I)/(1+I+2^{(3/4)}))^{(1/2)})*((-2)^{(1/4)}+2^{(1/2)})*(1+I)/(1+I+2^{(3/4)})^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6740, 206, 207, 1972, 208}

$$\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{(-1)^{3/4} \sqrt{1 + i \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + i \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 + (1 - x^2)^4), x]

[Out] $-((-1)^{(1/4)}*\operatorname{Sqrt}[1 - (-2)^{(1/4)}]*\operatorname{ArcTanh}[x/\operatorname{Sqrt}[1 - (-2)^{(1/4)}]])/(4*2^{(3/4)}) + ((-1)^{(3/4)}*\operatorname{Sqrt}[1 + I*(-2)^{(1/4)}]*\operatorname{ArcTanh}[x/\operatorname{Sqrt}[1 + I*(-2)^{(1/4)}]])/(4*2^{(3/4)}) + ((-1)^{(1/4)}*\operatorname{Sqrt}[1 + (-2)^{(1/4)}]*\operatorname{ArcTanh}[x/\operatorname{Sqrt}[1 + (-2)^{(1/4)}]])/(4*2^{(3/4)}) - (I/8)*((-2)^{(1/4)} + \operatorname{Sqrt}[2])* \operatorname{Sqrt}[(1 + I)/((1 + I) + 2^{(3/4)})]*\operatorname{ArcTanh}[\operatorname{Sqrt}[(1 + I)/((1 + I) + 2^{(3/4)})]]*x]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1972

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 6740

Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I


```
t(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) + sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))))^2 - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))))^2 - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))) + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 - 1)^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="giac")

[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)

maple [C] time = 0.01, size = 56, normalized size = 0.30

$$\frac{\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)^2 \ln(-\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3))}{8 \text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)^7 - 24 \text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)^5 + 24 \text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2+(-x^2+1)^4),x)

[Out] 1/8*sum(1/(_R^7-3*_R^5+3*_R^3-_R)*_R^2*ln(-_R+x),_R=RootOf(-Z^8-4*_Z^6+6*_Z^4-4*_Z^2+3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 - 1)^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="maxima")

[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)

mupad [B] time = 2.74, size = 142, normalized size = 0.76

$$\sum_{k=1}^8 \ln\left(\text{root}\left(z^8 + \frac{z^4}{16384} - \frac{z^2}{1048576} + \frac{3}{1073741824}, z, k\right)\right) \left(40x - \text{root}\left(z^8 + \frac{z^4}{16384} - \frac{z^2}{1048576} + \frac{3}{1073741824}, z, k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((x^2 - 1)^4 + 2),x)

[Out] symsum(log(root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k))*(40*x - root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k))*(root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k))*(4096*x + root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)^2*(786432*x + 67108864*root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k), k, 1, 8)

sympy [A] time = 0.21, size = 39, normalized size = 0.21

RootSum(1073741824t⁸ + 65536t⁴ - 1024t² + 3, (t ↦ t log(67108864t⁷ + 262144t⁵ + 4096t³ - 40t + x)))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(2+(-x**2+1)**4),x)

[Out] RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log(67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)))

$$3.391 \quad \int \frac{1-x^2}{a+b(1-x^2)^4} dx$$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} \log\left(-\sqrt{2} \sqrt[8]{b} x \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} + \sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b} x^2\right) \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}}}{8\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}}$$

[Out] $-1/4 \arctan(b^{1/8} x / ((-a)^{1/4} - b^{1/4}))^{1/2} / b^{3/8} / (-a)^{1/2} / ((-a)^{1/4} - b^{1/4})^{1/2} + 1/4 \operatorname{arctanh}(b^{1/8} x / ((-a)^{1/4} + b^{1/4}))^{1/2} / b^{3/8} / (-a)^{1/2} / ((-a)^{1/4} + b^{1/4})^{1/2} - 1/8 \arctan((-b^{1/8} x^2 + (b^{1/4} + (-a)^{1/2} + b^{1/2})^{1/2})^{1/2} / (-b^{1/4} + ((-a)^{1/2} + b^{1/2})^{1/2}))^{1/2} / b^{3/8} * 2^{1/2} / (-a)^{1/2} / ((-a)^{1/2} + b^{1/2})^{1/2} + 1/8 \arctan((b^{1/8} x^2 + (b^{1/4} + (-a)^{1/2} + b^{1/2})^{1/2})^{1/2} / (-b^{1/4} + ((-a)^{1/2} + b^{1/2})^{1/2}))^{1/2} / b^{3/8} * 2^{1/2} / (-a)^{1/2} / ((-a)^{1/2} + b^{1/2})^{1/2} + 1/16 \ln(b^{1/4} x^2 + ((-a)^{1/2} + b^{1/2})^{1/2} - b^{1/8} x^2 * (b^{1/4} + ((-a)^{1/2} + b^{1/2})^{1/2}))^{1/2} * (b^{1/4} + ((-a)^{1/2} + b^{1/2})^{1/2})^{1/2} / b^{3/8} * 2^{1/2} / (-a)^{1/2} / ((-a)^{1/2} + b^{1/2})^{1/2} - 1/16 \ln(b^{1/4} x^2 + ((-a)^{1/2} + b^{1/2})^{1/2} + b^{1/8} x^2 * (b^{1/4} + ((-a)^{1/2} + b^{1/2})^{1/2}))^{1/2} * (b^{1/4} + ((-a)^{1/2} + b^{1/2})^{1/2})^{1/2} / b^{3/8} * 2^{1/2} / (-a)^{1/2} / ((-a)^{1/2} + b^{1/2})^{1/2}$

Rubi [A] time = 1.11, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} \log\left(-\sqrt{2} \sqrt[8]{b} x \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} + \sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b} x^2\right) \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}}}{8\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(a + b*(1 - x^2)^4), x]

[Out] $-\operatorname{ArcTan}(b^{1/8} x / \operatorname{Sqrt}[(-a)^{1/4} - b^{1/4}]) / (4 * \operatorname{Sqrt}[-a] * \operatorname{Sqrt}[(-a)^{1/4} - b^{1/4}] * b^{3/8}) - (\operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] - b^{1/4}] * \operatorname{ArcTan}[(\operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] + b^{1/4}] - \operatorname{Sqrt}[2] * b^{1/8} x) / \operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] - b^{1/4}]]) / (4 * \operatorname{Sqrt}[2] * \operatorname{Sqrt}[-a] * \operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] * b^{3/8}) + (\operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] - b^{1/4}] * \operatorname{ArcTan}[(\operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] + b^{1/4}] + \operatorname{Sqrt}[2] * b^{1/8} x) / \operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] - b^{1/4}]]) / (4 * \operatorname{Sqrt}[2] * \operatorname{Sqrt}[-a] * \operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] * b^{3/8}) + \operatorname{ArcTanh}(b^{1/8} x / \operatorname{Sqrt}[(-a)^{1/4} + b^{1/4}]) / (4 * \operatorname{Sqrt}[-a] * \operatorname{Sqrt}[(-a)^{1/4} + b^{1/4}] * b^{3/8}) + (\operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] + b^{1/4}] * \operatorname{Log}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] - \operatorname{Sqrt}[2] * \operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] + b^{1/4}] * b^{1/8} x + b^{1/4} * x^2]) / (8 * \operatorname{Sqrt}[2] * \operatorname{Sqrt}[-a] * \operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] * b^{3/8}) - (\operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] + b^{1/4}] * \operatorname{Log}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] + \operatorname{Sqrt}[2] * \operatorname{Sqrt}[\operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] + b^{1/4}] * b^{1/8} x + b^{1/4} * x^2]) / (8 * \operatorname{Sqrt}[2] * \operatorname{Sqrt}[-a] * \operatorname{Sqrt}[\operatorname{Sqrt}[-a] + \operatorname{Sqrt}[b]] * b^{3/8})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1990

Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] := Int[ExpandToSum[u, x]^q*ExpandToSum[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] && !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])

Rule 6740

Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && IntegerQ[n, 0] && PolynomialInQ[v, u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{a+b(1-x^2)^4} dx &= \int \left(-\frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}(\sqrt{-a}\sqrt{b}-b(1-x^2)^2)} - \frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}(\sqrt{-a}\sqrt{b}+b(1-x^2)^2)} \right) dx \\
&= -\frac{\sqrt{b} \int \frac{1-x^2}{\sqrt{-a}\sqrt{b}-b(1-x^2)^2} dx}{2\sqrt{-a}} - \frac{\sqrt{b} \int \frac{1-x^2}{\sqrt{-a}\sqrt{b}+b(1-x^2)^2} dx}{2\sqrt{-a}} \\
&= -\frac{\sqrt{b} \int \frac{1-x^2}{(\sqrt{-a}-\sqrt{b})\sqrt{b}+2bx^2-bx^4} dx}{2\sqrt{-a}} - \frac{\sqrt{b} \int \frac{1-x^2}{(\sqrt{-a}+\sqrt{b})\sqrt{b}-2bx^2+bx^4} dx}{2\sqrt{-a}} \\
&= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} \left(1+\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}}\right) x}{\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}} - \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} x^2} dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}\sqrt[8]{b}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} \left(1+\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}}\right) x}{\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}} + \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} x^2} dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}\sqrt[8]{b}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{-a}+\sqrt{b}}\right) \int \frac{\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}}}{\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}} - \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} x^2} dx}{8\sqrt{-a}\sqrt{b}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}} \log\left(\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}} - \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} x\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}\sqrt[8]{b}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}} \tan^{-1}\left(\frac{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}} - \sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}} \log\left(\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}} - \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} x\right)}{8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}\sqrt[8]{b}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.10

$$\frac{\text{RootSum}\left[\#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + a + b\&, \frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(a + b*(1 - x^2)^4), x]

[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1]/(#1 - 2*#1^3 + #1^5) &]/b

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{(x^2-1)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="giac")

[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)

maple [C] time = 0.07, size = 69, normalized size = 0.10

$$\frac{\left(-\text{RootOf}\left(b_Z^8-4b_Z^6+6b_Z^4-4b_Z^2+a+b\right)^2+8b\left(\text{RootOf}\left(b_Z^8-4b_Z^6+6b_Z^4-4b_Z^2+a+b\right)^7-3\text{RootOf}\left(b_Z^8-4b_Z^6+6b_Z^4-4b_Z^2+a+b\right)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(a+b*(-x^2+1)^4),x)

[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8*b-4*_Z^6*b+6*_Z^4*b-4*_Z^2*b+a+b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{(x^2-1)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)

mupad [B] time = 2.70, size = 328, normalized size = 0.49

$$\sum_{k=1}^8 \ln\left(a b^5 \left(\text{root}\left(16777216 a^5 b^3 z^8 + 16777216 a^4 b^4 z^8 + 1048576 a^3 b^3 z^6 + 24576 a^2 b^2 z^4 + 256 a b z^2 + 1, z, k\right)^2 a b + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)

[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)

sympy [A] time = 3.92, size = 133, normalized size = 0.20

$$-\text{RootSum}\left(t^8\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\left(-\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*  
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629  
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32  
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))  
)
```

$$3.392 \quad \int \frac{1-x^2}{a+b(-1+x^2)^4} dx$$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} \log\left(-\sqrt{2} \sqrt[8]{b} x \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} + \sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b} x^2\right) \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}}}{8\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}}$$

[Out] $-1/4*\arctan(b^{(1/8)}*x/((-a)^{(1/4)}-b^{(1/4)})^{(1/2)})/b^{(3/8)}/(-a)^{(1/2)}/((-a)^{(1/4)}-b^{(1/4)})^{(1/2)}+1/4*\operatorname{arctanh}(b^{(1/8)}*x/((-a)^{(1/4)}+b^{(1/4)})^{(1/2)})/b^{(3/8)}/(-a)^{(1/2)}/((-a)^{(1/4)}+b^{(1/4)})^{(1/2)}-1/8*\arctan((-b^{(1/8)}*x*2^{(1/2)}+(b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)})/(-b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)})*(-b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)}/b^{(3/8)}*2^{(1/2)}/(-a)^{(1/2)}/((-a)^{(1/2)}+b^{(1/2)})^{(1/2)}+1/8*\arctan((b^{(1/8)}*x*2^{(1/2)}+(b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)})/(-b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)})*(-b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)}/b^{(3/8)}*2^{(1/2)}/(-a)^{(1/2)}/((-a)^{(1/2)}+b^{(1/2)})^{(1/2)}+1/16*\ln(b^{(1/4)}*x^2+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)}-b^{(1/8)}*x*2^{(1/2)}*(b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)})*(b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)}/b^{(3/8)}*2^{(1/2)}/(-a)^{(1/2)}/((-a)^{(1/2)}+b^{(1/2)})^{(1/2)}-1/16*\ln(b^{(1/4)}*x^2+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)}+b^{(1/8)}*x*2^{(1/2)}*(b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)})*(b^{(1/4)}+((-a)^{(1/2)}+b^{(1/2)})^{(1/2)})^{(1/2)}/b^{(3/8)}*2^{(1/2)}/(-a)^{(1/2)}/((-a)^{(1/2)}+b^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.65, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} \log\left(-\sqrt{2} \sqrt[8]{b} x \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} + \sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b} x^2\right) \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}}}{8\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(a + b*(-1 + x^2)^4), x]

[Out] $-\operatorname{ArcTan}\left[\frac{b^{(1/8)}*x}{\sqrt{(-a)^{(1/4)} - b^{(1/4)}}}\right]/(4*\sqrt{-a}*\sqrt{(-a)^{(1/4)} - b^{(1/4)}}*b^{(3/8)}) - (\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{(1/4)}}*\operatorname{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}} - \sqrt{2}*b^{(1/8)}*x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{(1/4)}}}\right])/(4*\sqrt{2}*\sqrt{-a}*\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{(1/4)}}*b^{(3/8)}) + (\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{(1/4)}}*\operatorname{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}} + \sqrt{2}*b^{(1/8)}*x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{(1/4)}}}\right])/(4*\sqrt{2}*\sqrt{-a}*\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{(1/4)}}*b^{(3/8)}) + \operatorname{ArcTanh}\left[\frac{b^{(1/8)}*x}{\sqrt{(-a)^{(1/4)} + b^{(1/4)}}}\right]/(4*\sqrt{-a}*\sqrt{(-a)^{(1/4)} + b^{(1/4)}}*b^{(3/8)}) + (\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}}*\operatorname{Log}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{2}*\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}}*b^{(1/8)}*x + b^{(1/4)}*x^2}}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}}}\right])/(8*\sqrt{2}*\sqrt{-a}*\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}}*b^{(3/8)}) - (\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}}*\operatorname{Log}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt{2}*\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}}*b^{(1/8)}*x + b^{(1/4)}*x^2}}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}}}\right])/(8*\sqrt{2}*\sqrt{-a}*\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{(1/4)}}*b^{(3/8)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1990

Int[(u_)^(q_)*(v_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^q*ExpandToSum[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] && !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])

Rule 6740

Int[(v_)/((a_) + (b_)*(u_)^(n_)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && IntegerQ[n, 0] && PolynomialInQ[v, u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{a+b(-1+x^2)^4} dx &= -\int \frac{-1+x^2}{a+b(-1+x^2)^4} dx \\
&= -\int \left(\frac{\sqrt{b}(-1+x^2)}{2\sqrt{-a}(\sqrt{-a}\sqrt{b}-b(-1+x^2)^2)} - \frac{\sqrt{b}(-1+x^2)}{2\sqrt{-a}(\sqrt{-a}\sqrt{b}+b(-1+x^2)^2)} \right) dx \\
&= \frac{\sqrt{b} \int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}-b(-1+x^2)^2} dx}{2\sqrt{-a}} + \frac{\sqrt{b} \int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}+b(-1+x^2)^2} dx}{2\sqrt{-a}} \\
&= \frac{\sqrt{b} \int \frac{-1+x^2}{(\sqrt{-a}-\sqrt{b})\sqrt{b}+2bx^2-bx^4} dx}{2\sqrt{-a}} + \frac{\sqrt{b} \int \frac{-1+x^2}{(\sqrt{-a}+\sqrt{b})\sqrt{b}-2bx^2+bx^4} dx}{2\sqrt{-a}} \\
&= \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} - \left(-1-\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}} - \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}x}{\sqrt[8]{b}} + x^2} dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} + \frac{\sqrt[4]{b}\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[8]{b}}} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} + \left(-1-\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}} + \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}x}{\sqrt[8]{b}}} dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} + \frac{\sqrt[4]{b}\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[8]{b}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right) \int \frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}}{8\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} \log\left(\frac{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\right)}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} \log\left(\frac{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\right)} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}} \tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.10

$$-\frac{\text{RootSum}\left[\#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + a + b\&, \frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4), x]

[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1]/(#1 - 2*#1^3 + #1^5) &]/b

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{(x^2-1)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="giac")

[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)

maple [C] time = 0.00, size = 69, normalized size = 0.10

$$\frac{\left(-\text{RootOf}\left(b_Z^8-4b_Z^6+6b_Z^4-4b_Z^2+a+b\right)^2+1\right)}{8b\left(\text{RootOf}\left(b_Z^8-4b_Z^6+6b_Z^4-4b_Z^2+a+b\right)^7-3\text{RootOf}\left(b_Z^8-4b_Z^6+6b_Z^4-4b_Z^2+a+b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(a+b*(x^2-1)^4),x)

[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8*b-4*_Z^6*b+6*_Z^4*b-4*_Z^2*b+a+b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{(x^2-1)^4 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)

mupad [B] time = 0.00, size = 328, normalized size = 0.49

$$\sum_{k=1}^8 \ln\left(a b^5 \left(\text{root}\left(16777216 a^5 b^3 z^8 + 16777216 a^4 b^4 z^8 + 1048576 a^3 b^3 z^6 + 24576 a^2 b^2 z^4 + 256 a b z^2 + 1, z, k\right)^2 a b + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)

[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)

sympy [A] time = 3.92, size = 133, normalized size = 0.20

-RootSum(t^8(16777216a^5b^3 + 16777216a^4b^4) + 1048576t^6a^3b^3 + 24576t^4a^2b^2 + 256t^2ab + 1, (t ↦ t log(-62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*  
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629  
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32  
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))  
)
```

$$3.393 \quad \int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx$$

Optimal. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}} + \frac{\tan^{-1}\left(\frac{x\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}$$

[Out] $\frac{1}{3}\arctan(x(a^{1/3}+b^{1/3})^{1/2}/b^{1/6})/b^{5/6}/(a^{1/3}+b^{1/3})^{1/2} + \frac{1}{3}\arctan(x(-1)^{1/3}a^{1/3}+b^{1/3})^{1/2}/b^{1/6})/b^{5/6}/(-1)^{1/3}a^{1/3}+b^{1/3})^{1/2} + \frac{1}{3}\arctan(x(-1)^{2/3}a^{1/3}+b^{1/3})^{1/2}/b^{1/6})/b^{5/6}/((-1)^{2/3}a^{1/3}+b^{1/3})^{1/2}$

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx$$

Verification is Not applicable to the result.

[In] Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3), x]

[Out] Defer[Int][(a*x^6 + b*(1 + x^2)^3)^(-1), x] + 2*Defer[Int][x^2/(a*x^6 + b*(1 + x^2)^3), x] + Defer[Int][x^4/(a*x^6 + b*(1 + x^2)^3), x]

Rubi steps

$$\begin{aligned} \int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx &= \int \left(\frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} + \frac{2x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} + \frac{x^4}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} \right) dx \\ &= 2 \int \frac{x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} dx + \int \frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} dx + \int \frac{x^4}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} dx \\ &= 2 \int \frac{x^2}{ax^6+b(1+x^2)^3} dx + \int \frac{1}{ax^6+b(1+x^2)^3} dx + \int \frac{x^4}{ax^6+b(1+x^2)^3} dx \end{aligned}$$

Mathematica [C] time = 0.06, size = 95, normalized size = 0.57

$$\frac{1}{6}\text{RootSum}\left[\#1^6a + \#1^6b + 3\#1^4b + 3\#1^2b + b\&, \frac{\#1^4\log(x-\#1) + 2\#1^2\log(x-\#1) + \log(x-\#1)}{\#1^5a + \#1^5b + 2\#1^3b + \#1b}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3), x]

[Out] RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 &, (Log[x - #1] + 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) &]/6

fricas [C] time = 2.70, size = 5653, normalized size = 33.65

result too large to display

$$1) \cdot \left(\frac{1}{(a \cdot b^3 + b^4) - 1/(a \cdot b + b^2)^2} \right) / \left(-\frac{1}{93312} \frac{1}{(a \cdot b^5 + b^6)} + \frac{1}{31104} \frac{1}{(a \cdot b^3 + b^4) \cdot (a \cdot b + b^2)} \right) - \frac{1}{46656} \frac{1}{(a \cdot b + b^2)^3} + \frac{1}{93312} \frac{a}{(a + b)^2 \cdot b^5} \right)^{1/3} - 1296 \cdot (I \cdot \sqrt{3} + 1) \cdot \left(-\frac{1}{93312} \frac{1}{(a \cdot b^5 + b^6)} + \frac{1}{31104} \frac{1}{(a \cdot b^3 + b^4) \cdot (a \cdot b + b^2)} \right) - \frac{1}{46656} \frac{1}{(a \cdot b + b^2)^3} + \frac{1}{93312} \frac{a}{(a + b)^2 \cdot b^5} \right)^{1/3} - \frac{72}{(a \cdot b + b^2)} - 3 \cdot \sqrt{1/3} \cdot (a \cdot b + b^2) \cdot \sqrt{-((a^2 \cdot b^3 + 2 \cdot a \cdot b^4 + b^5) \cdot ((-I \cdot \sqrt{3} + 1) \cdot (1/(a \cdot b^3 + b^4) - 1/(a \cdot b + b^2)^2)) / (-1/93312/(a \cdot b^5 + b^6) + 1/31104/((a \cdot b^3 + b^4) \cdot (a \cdot b + b^2)) - 1/46656/(a \cdot b + b^2)^3 + 1/93312 \cdot a / ((a + b)^2 \cdot b^5)))^{1/3} - 1296 \cdot (I \cdot \sqrt{3} + 1) \cdot \left(-\frac{1}{93312} \frac{1}{(a \cdot b^5 + b^6)} + \frac{1}{31104} \frac{1}{(a \cdot b^3 + b^4) \cdot (a \cdot b + b^2)} \right) - \frac{1}{46656} \frac{1}{(a \cdot b + b^2)^3} + \frac{1}{93312} \frac{a}{(a + b)^2 \cdot b^5} \right)^{1/3} - \frac{72}{(a \cdot b + b^2)} + 20736 \cdot a + 5184 \cdot b / (a^2 \cdot b^3 + 2 \cdot a \cdot b^4 + b^5) + 216 / (a \cdot b + b^2) \cdot \log(-1/12 \cdot b \cdot \sqrt{-((a \cdot b + b^2) \cdot ((-I \cdot \sqrt{3} + 1) \cdot (1/(a \cdot b^3 + b^4) - 1/(a \cdot b + b^2)^2)) / (-1/93312/(a \cdot b^5 + b^6) + 1/31104/((a \cdot b^3 + b^4) \cdot (a \cdot b + b^2)) - 1/46656/(a \cdot b + b^2)^3 + 1/93312 \cdot a / ((a + b)^2 \cdot b^5)))^{1/3} - 1296 \cdot (I \cdot \sqrt{3} + 1) \cdot \left(-\frac{1}{93312} \frac{1}{(a \cdot b^5 + b^6)} + \frac{1}{31104} \frac{1}{(a \cdot b^3 + b^4) \cdot (a \cdot b + b^2)} \right) - \frac{1}{46656} \frac{1}{(a \cdot b + b^2)^3} + \frac{1}{93312} \frac{a}{(a + b)^2 \cdot b^5} \right)^{1/3} - \frac{72}{(a \cdot b + b^2)} - 3 \cdot \sqrt{1/3} \cdot (a \cdot b + b^2) \cdot \sqrt{-((a^2 \cdot b^3 + 2 \cdot a \cdot b^4 + b^5) \cdot ((-I \cdot \sqrt{3} + 1) \cdot (1/(a \cdot b^3 + b^4) - 1/(a \cdot b + b^2)^2)) / (-1/93312/(a \cdot b^5 + b^6) + 1/31104/((a \cdot b^3 + b^4) \cdot (a \cdot b + b^2)) - 1/46656/(a \cdot b + b^2)^3 + 1/93312 \cdot a / ((a + b)^2 \cdot b^5)))^{1/3} - 1296 \cdot (I \cdot \sqrt{3} + 1) \cdot \left(-\frac{1}{93312} \frac{1}{(a \cdot b^5 + b^6)} + \frac{1}{31104} \frac{1}{(a \cdot b^3 + b^4) \cdot (a \cdot b + b^2)} \right) - \frac{1}{46656} \frac{1}{(a \cdot b + b^2)^3} + \frac{1}{93312} \frac{a}{(a + b)^2 \cdot b^5} \right)^{1/3} - \frac{72}{(a \cdot b + b^2)} + 20736 \cdot a + 5184 \cdot b / (a^2 \cdot b^3 + 2 \cdot a \cdot b^4 + b^5) + 216 / (a \cdot b + b^2) + x)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 1)^2}{ax^6 + (x^2 + 1)^3 b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")

[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)

maple [C] time = 0.28, size = 67, normalized size = 0.40

$$\frac{\left(\text{RootOf} \left((a + b) _Z^6 + 3 _Z^4 b + 3 _Z^2 b + b \right)^4 + 2 \text{RootOf} \left((a + b) _Z^6 + 3 _Z^4 b + 3 _Z^2 b + b \right)^5 \right)}{6 \text{RootOf} \left((a + b) _Z^6 + 3 _Z^4 b + 3 _Z^2 b + b \right)^5 a + 6 \text{RootOf} \left((a + b) _Z^6 + 3 _Z^4 b + 3 _Z^2 b + b \right)^5 b + 12 \text{RootOf} \left((a + b) _Z^6 + 3 _Z^4 b + 3 _Z^2 b + b \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x)

[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(-_R+x),_R=RootOf((a+b)*_Z^6+3*b*_Z^4+3*b*_Z^2+b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 1)^2}{ax^6 + (x^2 + 1)^3 b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")

[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)

mupad [B] time = 3.08, size = 504, normalized size = 3.00

$$\sum_{k=1}^6 \ln\left(-a^3 (a+b) \left(-\text{root}\left(46656 a b^5 z^6 + 46656 b^6 z^6 + 3888 b^4 z^4 + 108 b^2 z^2 + 1, z, k\right)^2 b^2 60 - \text{root}\left(46656 a b^5 z^6 + 46656 b^6 z^6 + 3888 b^4 z^4 + 108 b^2 z^2 + 1, z, k\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)^2/(b*(x^2 + 1)^3 + a*x^6),x)

[Out] symsum(log(-3*a^3*(a+b)*(504*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*b^3*x - 864*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*b^4 - 864*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*a*b^3 - 60*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*b^2 + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^5*b^5*x + 2*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*a*x + 8*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*b*x + 12*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*a*b - 144*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*a*b^2*x + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^5*a*b^4*x - 1))*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k), k, 1, 6)

sympy [A] time = 1.87, size = 42, normalized size = 0.25

$$\text{RootSum}\left(t^6 \left(46656 a b^5 + 46656 b^6\right) + 3888 t^4 b^4 + 108 t^2 b^2 + 1, \left(t \mapsto t \log(6 t b + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)

[Out] RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b**2 + 1, Lambda(_t, _t*log(6*_t*b + x)))

$$3.394 \quad \int \frac{(d+ex)^3}{a+cx^4} dx$$

Optimal. Leaf size=320

$$\frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} + \frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}}$$

[Out] $\frac{1}{4}e^3 \ln(c x^4 + a) / c + 3/2 d^2 e \arctan(x^2 c^{1/2} / a^{1/2}) / a^{1/2} / c^{1/2} - 1/8 d \ln(-a^{1/4} c^{1/4} x^2 + a^{1/2} + x^2 c^{1/2}) * (-3e^2 a^{1/2} + d^2 c^{1/2}) / a^{3/4} / c^{3/4} * 2^{1/2} + 1/8 d \ln(a^{1/4} c^{1/4} x^2 + a^{1/2} + x^2 c^{1/2}) * (-3e^2 a^{1/2} + d^2 c^{1/2}) / a^{3/4} / c^{3/4} * 2^{1/2} + 1/4 d \arctan(-1 + c^{1/4} x^2 / a^{1/4}) * (3e^2 a^{1/2} + d^2 c^{1/2}) / a^{3/4} / c^{3/4} * 2^{1/2} + 1/4 d \arctan(1 + c^{1/4} x^2 / a^{1/4}) * (3e^2 a^{1/2} + d^2 c^{1/2}) / a^{3/4} / c^{3/4} * 2^{1/2}$

Rubi [A] time = 0.26, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} + \frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + c*x^4), x]

[Out] $(3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(2*Sqrt[2]*a^{3/4}*c^{3/4}) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^{1/4}*x)/a^{1/4}])/(2*Sqrt[2]*a^{3/4}*c^{3/4}) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^{3/4}*c^{3/4}) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*c^{1/4}*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^{3/4}*c^{3/4}) + (e^3*Log[a + c*x^4])/(4*c)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{a+cx^4} dx &= \int \left(\frac{d^3+3de^2x^2}{a+cx^4} + \frac{x(3d^2e+e^3x^2)}{a+cx^4} \right) dx \\
&= \int \frac{d^3+3de^2x^2}{a+cx^4} dx + \int \frac{x(3d^2e+e^3x^2)}{a+cx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{3d^2e+e^3x}{a+cx^2} dx, x, x^2 \right) + \frac{\left(d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} - 3e^2 \right) \right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2c} + \frac{\left(d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} + 3e^2 \right) \right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2c} \\
&= \frac{1}{2} (3d^2e) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right) + \frac{1}{2} e^3 \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right) + \frac{\left(d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} + 3e^2 \right) \right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2c} \\
&= \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} - \frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} \\
&= \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} - \frac{d(\sqrt{c}d^2 + 3\sqrt{a}e^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d(\sqrt{c}d^2 + 3\sqrt{a}e^2) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}c^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 322, normalized size = 1.01

$$-\sqrt{2}\sqrt[4]{c} \left(\sqrt[4]{a}\sqrt{c}d^3 - 3a^{3/4}de^2 \right) \log \left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2 \right) + \sqrt{2}\sqrt[4]{c} \left(\sqrt[4]{a}\sqrt{c}d^3 - 3a^{3/4}de^2 \right) \log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + c*x^4), x]

[Out] (-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4]/(8*a*c)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.36, size = 311, normalized size = 0.97

$$\frac{e^3 \log(|cx^4 + a|)}{4c} + \frac{\sqrt{2} \left(3\sqrt{2}\sqrt{ac}c^2d^2e + (ac^3)^{\frac{1}{4}}c^2d^3 + 3(ac^3)^{\frac{3}{4}}de^2 \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{4ac^3} + \frac{\sqrt{2} \left(3\sqrt{2}\sqrt{ac}c^2d^2e + (ac^3)^{\frac{1}{4}}c^2d^3 + 3(ac^3)^{\frac{3}{4}}de^2 \right) \arctan \left(\frac{\sqrt{2} \left(2x - \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{4}e^3 \log(\text{abs}(c*x^4 + a))/c + \frac{1}{4}\sqrt{2}*(3*\sqrt{2}*\sqrt{a*c}*c^2*d^2*e + (a*c^3)^{(1/4)}*c^2*d^3 + 3*(a*c^3)^{(3/4)}*d*e^2)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(a*c^3) + \frac{1}{4}\sqrt{2}*(3*\sqrt{2}*\sqrt{a*c}) *c^2*d^2*e + (a*c^3)^{(1/4)}*c^2*d^3 + 3*(a*c^3)^{(3/4)}*d*e^2)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(a*c^3) + \frac{1}{8}\sqrt{2}*((a*c^3)^{(1/4)}*c^2*d^3 - 3*(a*c^3)^{(3/4)}*d*e^2)*\log(x^2 + \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(a*c^3) - \frac{1}{8}\sqrt{2}*((a*c^3)^{(1/4)}*c^2*d^3 - 3*(a*c^3)^{(3/4)}*d*e^2)*\log(x^2 - \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(a*c^3)$

maple [A] time = 0.01, size = 314, normalized size = 0.98

$$\frac{3d^2e \arctan\left(\sqrt{\frac{c}{a}}x^2\right)}{2\sqrt{ac}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d^3 \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d^3 \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d^3 \ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3/(c*x^4+a),x)

[Out] $\frac{1}{8}d^3*(a/c)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)}))+\frac{1}{4}d^3*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+\frac{1}{4}d^3*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)+\frac{3}{2}e*d^2/(a*c)^{(1/2)}*\arctan(x^2*(c/a)^{(1/2)}))+\frac{3}{8}d*e^2/c/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*x*2^{(1/2)}+(a/c)^{(1/2)}))+\frac{3}{4}d*e^2/c/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+\frac{3}{4}d*e^2/c/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)+\frac{1}{4}e^3*\ln(c*x^4+a)/c$

maxima [A] time = 1.96, size = 310, normalized size = 0.97

$$\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3 + cd^3 - 3\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3 - cd^3 + 3\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")

[Out] $\frac{1}{8}\sqrt{2}*(\sqrt{2}*a^{(3/4)}*c^{(1/4)}*e^3 + c*d^3 - 3*\sqrt{a}*\sqrt{c}*d*e^2)*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) + \frac{1}{8}\sqrt{2}*(\sqrt{2}*a^{(3/4)}*c^{(1/4)}*e^3 - c*d^3 + 3*\sqrt{a}*\sqrt{c}*d*e^2)*\log(\sqrt{c}*x^2 - \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) + \frac{1}{4}*(\sqrt{2}*a^{(1/4)}*c^{(5/4)}*d^3 + 3*\sqrt{2}*a^{(3/4)}*c^{(3/4)}*d*e^2 - 6*\sqrt{a}*c*d^2*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2}*a^{(1/4)}*c^{(1/4)})/\sqrt{a*c})/(a^{(3/4)}*\sqrt{a*c}) + \frac{1}{4}*(\sqrt{2}*a^{(1/4)}*c^{(5/4)}*d^3 + 3*\sqrt{2}*a^{(3/4)}*c^{(3/4)}*d*e^2 + 6*\sqrt{a}*c*d^2*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2}*a^{(1/4)}*c^{(1/4)})/\sqrt{a*c})/(a^{(3/4)}*\sqrt{a*c})$

mupad [B] time = 2.84, size = 894, normalized size = 2.79

$$\sum_{k=1}^4 \ln\left(-cd^2\left(-3cd^5e^2 + 5ade^6 + 3ae^7x + \text{root}\left(256a^3c^4z^4 - 256a^3c^3e^3z^3 + 480a^2c^3d^4e^2z^2 + 96a^3c^2e^6z\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] int((d + e*x)^3/(a + c*x^4),x)
```

```
[Out] symsum(log(-2*c*d^2*(5*a*d*e^6 - 3*c*d^5*e^2 + 3*a*e^7*x + 8*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)^2*a*c^2*d + 2*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*c^2*d^4*x - 5*c*d^4*e^3*x - 24*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)^2*a*c^2*e*x + 32*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*a*c*d*e^3 - 6*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*a*c*e^4*x))*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k), k, 1, 4)
```

sympy [A] time = 5.29, size = 384, normalized size = 1.20

$$\text{RootSum}\left(256t^4a^3c^4 - 256t^3a^3c^3e^3 + t^2(96a^3c^2e^6 + 480a^2c^3d^4e^2) + t(-16a^3ce^9 + 192a^2c^2d^4e^5 - 48ac^3d^8e) + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c**2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d**4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*d**8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 + 960*_t**3*a**3*c**4*d**4*e**2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a**3*c**3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716*_t*a**3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12 - 27*a**4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*c**3*d**12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a*c**3*d**11*e**4 + c**4*d**15))))
```

$$3.395 \quad \int \frac{(d+ex)^2}{a+cx^4} dx$$

Optimal. Leaf size=291

$$\frac{(\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} + \frac{(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} - \frac{(\sqrt{a}e^2 + \sqrt{c}d^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}}$$

[Out] d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)

Rubi [A] time = 0.20, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} + \frac{(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} - \frac{(\sqrt{a}e^2 + \sqrt{c}d^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + c*x^4), x]

[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned} \int \frac{(d + ex)^2}{a + cx^4} dx &= \int \left(\frac{2dex}{a + cx^4} + \frac{d^2 + e^2x^2}{a + cx^4} \right) dx \\ &= (2de) \int \frac{x}{a + cx^4} dx + \int \frac{d^2 + e^2x^2}{a + cx^4} dx \\ &= (de) \text{Subst} \left(\int \frac{1}{a + cx^2} dx, x, x^2 \right) + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2c} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{\sqrt{a}\sqrt{c+cx^2}}{a+cx^4} dx}{2c} \\ &= \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{\sqrt{a}\sqrt{c}} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4c} - \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2 \right) \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2 \right) \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}c^{3/4}} \\ &= \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2 \right) \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2 \right) \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}c^{3/4}} \\ &= \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2 \right) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2 \right) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}c^{3/4}} \end{aligned}$$

Mathematica [A] time = 0.10, size = 243, normalized size = 0.84

$$\frac{-\sqrt{2} \left(\sqrt{c} d^2 - \sqrt{a} e^2 \right) \left(\log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) - \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) \right) - 2 \left(4 \sqrt[4]{a} \sqrt[4]{c} d e + \sqrt{a} \sqrt{c} \right)}{8 a^{3/4} c^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + c*x^4), x]

[Out] (-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(3/4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 285, normalized size = 0.98

$$\frac{\sqrt{2} \left(2 \sqrt{2} \sqrt{ac} c^2 d e + (ac^3)^{\frac{1}{4}} c^2 d^2 + (ac^3)^{\frac{3}{4}} e^2 \right) \arctan \left(\frac{\sqrt{2} \left(2 x + \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{4 a c^3} + \frac{\sqrt{2} \left(2 \sqrt{2} \sqrt{ac} c^2 d e + (ac^3)^{\frac{1}{4}} c^2 d^2 + (ac^3)^{\frac{3}{4}} e^2 \right) \arctan \left(\frac{\sqrt{2} \left(2 x - \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{4 a c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a), x, algorithm="giac")

[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)

maple [A] time = 0.00, size = 292, normalized size = 1.00

$$\frac{d e \arctan \left(\sqrt{\frac{c}{a}} x^2 \right)}{\sqrt{ac}} + \frac{\left(\frac{a}{c} \right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan \left(\frac{\sqrt{2} x}{\left(\frac{a}{c} \right)^{\frac{1}{4}}} - 1 \right)}{4 a} + \frac{\left(\frac{a}{c} \right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan \left(\frac{\sqrt{2} x}{\left(\frac{a}{c} \right)^{\frac{1}{4}}} + 1 \right)}{4 a} + \frac{\left(\frac{a}{c} \right)^{\frac{1}{4}} \sqrt{2} d^2 \ln \left(\frac{x^2 + \left(\frac{a}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \left(\frac{a}{c} \right)^{\frac{1}{2}}}{x^2 - \left(\frac{a}{c} \right)^{\frac{1}{4}} \sqrt{2} x + \left(\frac{a}{c} \right)^{\frac{1}{2}}} \right)}{8 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(c*x^4+a), x)

[Out] 1/8*d^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*d^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*d^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)

$$\begin{aligned} &)^{(1/4)} * x^{-1} + d * e / (a * c)^{(1/2)} * \arctan\left(\frac{(1/a * c)^{(1/2)} * x^2 + 1/8 * e^2 / c / (a/c)^{(1/4)} * 2^{(1/2)} * \ln\left((x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}\right) / (x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})\right) + 1/4 * e^2 / c / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan\left(2^{(1/2)} / (a/c)^{(1/4)} * x + 1\right) + 1/4 * e^2 / c / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan\left(2^{(1/2)} / (a/c)^{(1/4)} * x - 1\right)} \end{aligned}$$

maxima [A] time = 2.05, size = 275, normalized size = 0.95

$$\frac{\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}e^2) \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}e^2) \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a), x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))
```

mupad [B] time = 2.66, size = 556, normalized size = 1.91

$$\sum_{k=1}^4 \ln\left(3c^2d^4e^2 - ace^6 + 4c^2d^3e^3x - \text{root}\left(256a^3c^3z^4 + 192a^2c^2d^2e^2z^2 + 32a^2cd^5e^5z - 32a^2d^5ez + 2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4), x)
```

```
[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 + 4*c^2*d^3*e^3*x - 4*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 4*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*d*e^3 + 32*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d*e*x)*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k), k, 1, 4)
```

sympy [A] time = 2.68, size = 277, normalized size = 0.95

$$\text{RootSum}\left(256t^4a^3c^3 + 192t^2a^2c^2d^2e^2 + t(32a^2cde^5 - 32ac^2d^5e) + a^2e^8 + 2acd^4e^4 + c^2d^8, \left(t \mapsto t \log\left(x + \frac{64t}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a), x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*c*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, Lamb
```

```

da(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e**2
- 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**3*c*
d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*e**11
- 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c*d**4*
e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))

```

3.396 $\int \frac{d+ex}{a+cx^4} dx$

Optimal. Leaf size=219

$$\frac{d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} - \frac{d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{d \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}}$$

[Out] $\frac{1}{4}d*\arctan(-1+c^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/c^{(1/4)}*2^{(1/2)}+1/4*d*\arctan(1+c^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/c^{(1/4)}*2^{(1/2)}-1/8*d*\ln(-a^{(1/4)}*c^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})/a^{(3/4)}/c^{(1/4)}*2^{(1/2)}+1/8*d*\ln(a^{(1/4)}*c^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})/a^{(3/4)}/c^{(1/4)}*2^{(1/2)}+1/2*e*\arctan(x^2*c^{(1/2)}/a^{(1/2)})/a^{(1/2)}/c^{(1/2)}$

Rubi [A] time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} - \frac{d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{d \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + c*x^4), x]

[Out] $(e*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*\text{Sqrt}[c]) - (d*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*c^{(1/4)}) + (d*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*c^{(1/4)}) - (d*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*c^{(1/4)}) + (d*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*c^{(1/4)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
 \int \frac{d+ex}{a+cx^4} dx &= \int \left(\frac{d}{a+cx^4} + \frac{ex}{a+cx^4} \right) dx \\
 &= d \int \frac{1}{a+cx^4} dx + e \int \frac{x}{a+cx^4} dx \\
 &= \frac{d \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{2\sqrt{a}} + \frac{d \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{2\sqrt{a}} + \frac{1}{2} e \operatorname{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right) \\
 &= \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} + \frac{d \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} + \frac{d \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} - \frac{d \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \\
 &= \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} - \frac{d \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \operatorname{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2} \\
 &= \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} - \frac{d \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 184, normalized size = 0.84

$$\frac{-2\left(2\sqrt[4]{a}e + \sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\left(\sqrt{2}\sqrt[4]{c}d - 2\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right) + \sqrt{2}\sqrt[4]{c}d\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\right)\right)}{8a^{3/4}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + c*x^4), x]

[Out] (-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/4)*Sqrt[c])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.32, size = 215, normalized size = 0.98

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce - \left(ac^3\right)^{\frac{1}{4}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a), x, algorithm="giac")

[Out] 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2)

maple [A] time = 0.00, size = 151, normalized size = 0.69

$$\frac{e \arctan\left(\sqrt{\frac{c}{a}}x\right)}{2\sqrt{ac}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)/(c*x^4+a), x)

[Out] 1/8*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/2*e/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)

maxima [A] time = 2.67, size = 207, normalized size = 0.95

$$\frac{\sqrt{2}d \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}d \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d - 2\sqrt{a}e\right) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{a}\right)}{2\sqrt{c}x + \sqrt{a}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4)*d - 2*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4)*d + 2*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4))

mupad [B] time = 2.32, size = 160, normalized size = 0.73

$$\begin{cases} -\frac{2d+3ex}{6cx^3} & \text{if } a = 0 \\ \frac{\operatorname{atan}\left(\frac{\sqrt{2}c^{1/4}x-1}{a^{1/4}}\right)(2a^{1/4}e+\sqrt{2}c^{1/4}d)}{4a^{3/4}\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\sqrt{2}c^{1/4}x+1}{a^{1/4}}\right)(4a^{1/4}e-2\sqrt{2}c^{1/4}d)}{8a^{3/4}\sqrt{c}} + \frac{\sqrt{2}d \ln\left(\frac{\sqrt{a}+\sqrt{c}x^2+\sqrt{2}a^{1/4}c^{1/4}x}{\sqrt{a}+\sqrt{c}x^2-\sqrt{2}a^{1/4}c^{1/4}x}\right)}{8a^{3/4}c^{1/4}} & \text{if } a \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)/(a + c*x^4),x)

[Out] piecewise(a == 0, -(2*d + 3*e*x)/(6*c*x^3), a ~= 0, (atan((2^(1/2)*c^(1/4)*x)/a^(1/4) - 1)*(2*a^(1/4)*e + 2^(1/2)*c^(1/4)*d))/(4*a^(3/4)*c^(1/2)) - (atan((2^(1/2)*c^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*e - 2*2^(1/2)*c^(1/4)*d))/(8*a^(3/4)*c^(1/2)) + (2^(1/2)*d*log((a^(1/2) + c^(1/2)*x^2 + 2^(1/2)*a^(1/4)*c^(1/4)*x)/(a^(1/2) + c^(1/2)*x^2 - 2^(1/2)*a^(1/4)*c^(1/4)*x)))/(8*a^(3/4)*c^(1/4)))

sympy [A] time = 0.82, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3c^2 + 32t^2a^2ce^2 - 16tacd^2e + ae^4 + cd^4, \left(t \mapsto t \log\left(x + \frac{-128t^3a^3ce^2 - 16t^2a^2cd^2e - 8ta^2e^4 - 4cd^4}{4ade^4 - cd^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*e**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a**2*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d*e**4 - c*d**5))))

3.397 $\int \frac{1}{a+cx^4} dx$

Optimal. Leaf size=185

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)

Rubi [A] time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-1), x]

[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a + cx^4} dx = \frac{\int \frac{\sqrt{a} - \sqrt{c}x^2}{a + cx^4} dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a} + \sqrt{c}x^2}{a + cx^4} dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.02, size = 134, normalized size = 0.72

$$\frac{-\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - 2\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1), x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

fricas [A] time = 0.42, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}} + \sqrt{a^2\sqrt{-\frac{1}{a^3c}} + x^2} a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a), x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

giac [A] time = 0.33, size = 179, normalized size = 0.97

$$\frac{\sqrt{2} (ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2} (ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2} (ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a),x, algorithm="giac")

[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)

maple [A] time = 0.00, size = 128, normalized size = 0.69

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+a),x)

[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)

maxima [A] time = 2.37, size = 169, normalized size = 0.91

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2} \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} - \sqrt{2} \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))

mupad [B] time = 0.08, size = 33, normalized size = 0.18

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right) + \operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{2(-a)^{3/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c*x^4),x)

[Out] $-(\operatorname{atan}((c^{1/4}x)/(-a)^{1/4}) + \operatorname{atanh}((c^{1/4}x)/(-a)^{1/4}))/2(-a)^{3/4}c^{1/4}$

sympy [A] time = 0.16, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(256t^4a^3c + 1, (t \mapsto t \log(4ta + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**4+a), x)`

[Out] `RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))`

$$3.398 \quad \int \frac{1}{(d+ex)(a+cx^4)} dx$$

Optimal. Leaf size=416

$$\frac{\sqrt[4]{c} d (\sqrt{c} d^2 - \sqrt{a} e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)} + \frac{\sqrt[4]{c} d (\sqrt{c} d^2 - \sqrt{a} e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)}$$

[Out] $e^3 \ln(e*x+d)/(a*e^4+c*d^4) - 1/4*e^3*\ln(c*x^4+a)/(a*e^4+c*d^4) - 1/2*d^2*e*\arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)/a^(1/2) - 1/8*c^(1/4)*d*\ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2) + 1/8*c^(1/4)*d*\ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2) + 1/4*c^(1/4)*d*\arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2) + 1/4*c^(1/4)*d*\arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)$

Rubi [A] time = 0.43, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{\sqrt[4]{c} d (\sqrt{c} d^2 - \sqrt{a} e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)} + \frac{\sqrt[4]{c} d (\sqrt{c} d^2 - \sqrt{a} e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(a + c*x^4)), x]

[Out] $-(\text{Sqrt}[c]*d^2*e*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*(c*d^4 + a*e^4)) - (c^(1/4)*d*(\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)) + (e^3*\text{Log}[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)) - (e^3*\text{Log}[a + c*x^4])/(4*(c*d^4 + a*e^4))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)(a+cx^4)} dx &= \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)} + \frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(cd^4+ae^4)(a+cx^4)} \right) dx \\
&= \frac{e^3 \log(d+ex)}{cd^4+ae^4} + \frac{c \int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{a+cx^4} dx}{cd^4+ae^4} \\
&= \frac{e^3 \log(d+ex)}{cd^4+ae^4} + \frac{c \int \left(\frac{d^3+de^2x^2}{a+cx^4} + \frac{x(-d^2e-e^3x^2)}{a+cx^4} \right) dx}{cd^4+ae^4} \\
&= \frac{e^3 \log(d+ex)}{cd^4+ae^4} + \frac{c \int \frac{d^3+de^2x^2}{a+cx^4} dx}{cd^4+ae^4} + \frac{c \int \frac{x(-d^2e-e^3x^2)}{a+cx^4} dx}{cd^4+ae^4} \\
&= \frac{e^3 \log(d+ex)}{cd^4+ae^4} + \frac{c \operatorname{Subst} \left(\int \frac{-d^2e-e^3x}{a+cx^2} dx, x, x^2 \right)}{2(cd^4+ae^4)} + \frac{\left(d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \right) \int \frac{\sqrt{a}\sqrt{c-cx^2}}{a+cx^4} dx}{2(cd^4+ae^4)} + \dots \\
&= \frac{e^3 \log(d+ex)}{cd^4+ae^4} - \frac{(cd^2e) \operatorname{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2(cd^4+ae^4)} - \frac{(ce^3) \operatorname{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{2(cd^4+ae^4)} + \dots \\
&= -\frac{\sqrt{c}d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}(cd^4+ae^4)} + \frac{e^3 \log(d+ex)}{cd^4+ae^4} - \frac{\sqrt[4]{c}d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx^2})}{4\sqrt{2}a^{3/4}(cd^4+ae^4)} \\
&= -\frac{\sqrt{c}d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}(cd^4+ae^4)} - \frac{\sqrt[4]{c}d(\sqrt{c}d^2 + \sqrt{a}e^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)} + \frac{\sqrt[4]{c}d(\sqrt{c}d^2 + \dots)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 404, normalized size = 0.97

$$-2a^{3/4}e^3 \log(a+cx^4) + 8a^{3/4}e^3 \log(d+ex) - \sqrt{2}c^{3/4}d^3 \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) + \sqrt{2}c^{3/4}d^3 \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*(a + c*x^4)), x]

[Out] $(-2*c^{1/4}*d*(\text{Sqrt}[2]*\text{Sqrt}[c]*d^2 - 2*a^{1/4}*c^{1/4}*d*e + \text{Sqrt}[2]*\text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}] + 2*c^{1/4}*d*(\text{Sqrt}[2]*\text{Sqrt}[c]*d^2 + 2*a^{1/4}*c^{1/4}*d*e + \text{Sqrt}[2]*\text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}] + 8*a^{3/4}*e^3*\text{Log}[d + e*x] - \text{Sqrt}[2]*c^{3/4}*d^3*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2] + \text{Sqrt}[2]*\text{Sqrt}[a]*c^{1/4}*d*e^2*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2] + \text{Sqrt}[2]*c^{3/4}*d^3*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2] - \text{Sqrt}[2]*\text{Sqrt}[a]*c^{1/4}*d*e^2*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2] - 2*a^{3/4}*e^3*\text{Log}[a + c*x^4])/(8*a^{3/4}*(c*d^4 + a*e^4))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.42, size = 371, normalized size = 0.89

$$\frac{(ac^3)^{\frac{1}{4}} cd \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^2d^2-2(ac^3)^{\frac{1}{4}}acde+\sqrt{2}\sqrt{ac}ace^2\right)} + \frac{(ac^3)^{\frac{1}{4}} cd \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^2d^2+2(ac^3)^{\frac{1}{4}}acde+\sqrt{2}\sqrt{ac}ace^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}}c^2d^3-(ac^3)^{\frac{1}{4}}\right)}{4\left(\sqrt{2}ac^2d^2-2(ac^3)^{\frac{1}{4}}acde+\sqrt{2}\sqrt{ac}ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")

[Out] 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 - 2*(a*c^3)^(1/4)*a*c*d*e + sqrt(2)*sqrt(a*c)*a*c*e^2) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + 2*(a*c^3)^(1/4)*a*c*d*e + sqrt(2)*sqrt(a*c)*a*c*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^4) - 1/4*e^3*log(abs(c*x^4 + a))/(c*d^4 + a*e^4) + e^4*log(abs(x*e + d))/(c*d^4*e + a*e^5)

maple [A] time = 0.01, size = 433, normalized size = 1.04

$$\frac{cd^2e \arctan\left(\sqrt{\frac{c}{a}}x^2\right)}{2(ae^4 + cd^4)\sqrt{ac}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd^3 \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4(ae^4 + cd^4)a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd^3 \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4(ae^4 + cd^4)a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}cd^3 \ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8(ae^4 + cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)/(c*x^4+a),x)

[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)+1/8*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/2*c/(a*e^4+c*d^4)*e*d^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+1/8/(a*e^4+c*d^4)*d*e^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4/(a*e^4+c*d^4)*d*e^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4/(a*e^4+c*d^4)*d*e^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/4*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)

maxima [A] time = 2.73, size = 345, normalized size = 0.83

$$\frac{e^3 \log(ex + d)}{cd^4 + ae^4} \left[c \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3 - cd^3 + \sqrt{a}\sqrt{c}de^2\right) \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3 + cd^3 - \sqrt{a}\sqrt{c}de^2\right) \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")

```
[Out] e^3*log(e*x + d)/(c*d^4 + a*e^4) - 1/8*c*(sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*
e^3 - c*d^3 + sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1
/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 +
c*d^3 - sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x
+ sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^
(3/4)*c^(3/4)*d*e^2 + 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(
c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^(3/4)*c^(3/4)*d*e
^2 - 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))/(c*
d^4 + a*e^4)
```

mupad [B] time = 0.42, size = 874, normalized size = 2.10

$$\left(\sum_{k=1}^4 \ln \left(\text{root} \left(256 a^3 c d^4 z^4 + 256 a^4 e^4 z^4 + 256 a^3 e^3 z^3 + 96 a^2 e^2 z^2 + 16 a e z + 1, z, k \right) c^4 e \left(d e^2 + 5 e^3 x + \text{ro} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)),x)
```

```
[Out] symsum(log(root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k))*c^4*e*(d*e^2 + 5*e^3*x + 240*root(256*a^3
*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z
+ 1, z, k)^2*a^2*e^5*x + 320*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256
*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*e^6*x + 32*root(2
56*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*
a*e*z + 1, z, k)*a*d*e^3 + 60*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 25
6*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*a*e^4*x - 4*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)*c*d^4*x - 16*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^
3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^5 + 208*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)^2*a^2*d*e^4 + 384*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 2
56*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*d*e^5 - 128*roo
t(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 +
16*a*e*z + 1, z, k)^3*a^2*c*d^5*e - 192*root(256*a^3*c*d^4*z^4 + 256*a^4*e^
4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^2*c*d^4*
e^2*x - 48*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^4*e*x))*root(256*a^3*c*d^4*z^4 +
256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k), k
, 1, 4) + (e^3*log(d + e*x))/(a*e^4 + c*d^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

$$3.399 \quad \int \frac{1}{(d+ex)^2(a+cx^4)} dx$$

Optimal. Leaf size=552

$$\frac{\sqrt[4]{c} \left(\sqrt{c} d^2 (cd^4 - 3ae^4) - \sqrt{a} e^2 (3cd^4 - ae^4) \right) \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) + \sqrt[4]{c} \left(\sqrt{c} d^2 (cd^4 - 3ae^4) - \sqrt{a} e^2 (3cd^4 - ae^4) \right)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)^2} + \dots$$

[Out] $-e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*\ln(e*x+d)/(a*e^4+c*d^4)^2-c*d^3*e^3*\ln(c*x^4+a)/(a*e^4+c*d^4)^2-d*e*(-a*e^4+c*d^4)*\arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*\ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*\ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*\arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*\arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)$

Rubi [A] time = 0.81, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{c} \left(\sqrt{c} d^2 (cd^4 - 3ae^4) - \sqrt{a} e^2 (3cd^4 - ae^4) \right) \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) + \sqrt[4]{c} \left(\sqrt{c} d^2 (cd^4 - 3ae^4) - \sqrt{a} e^2 (3cd^4 - ae^4) \right)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)^2} + \dots$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + c*x^4)),x]

[Out] $-(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (\text{Sqrt}[c]*d*e*(c*d^4 - a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(\text{Sqrt}[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/ (2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/ (2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*\text{Log}[d + e*x])/ (c*d^4 + a*e^4)^2 - (c^(1/4)*(\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/ (4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/ (4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^3*e^3*\text{Log}[a + c*x^4])/ (c*d^4 + a*e^4)^2$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\int \frac{1}{(d + ex)^2 (a + cx^4)} dx = \int \left(\frac{e^4}{(cd^4 + ae^4)(d + ex)^2} + \frac{4cd^3e^4}{(cd^4 + ae^4)^2(d + ex)} + \frac{c(d^2(cd^4 - 3ae^4) - 2de(cd^4 - ae^4))}{(cd^4 + ae^4)^2} \right) dx$$

$$= -\frac{e^3}{(cd^4 + ae^4)(d + ex)} + \frac{4cd^3e^3 \log(d + ex)}{(cd^4 + ae^4)^2} + \frac{c \int \frac{d^2(cd^4 - 3ae^4) - 2de(cd^4 - ae^4)x + e^2(3cd^4 - ae^4)x^2}{a + cx^4} dx}{(cd^4 + ae^4)^2}$$

$$= -\frac{e^3}{(cd^4 + ae^4)(d + ex)} + \frac{4cd^3e^3 \log(d + ex)}{(cd^4 + ae^4)^2} + \frac{c \int \left(\frac{x(-2de(cd^4 - ae^4) - 4cd^3e^3x^2)}{a + cx^4} + \frac{d^2(cd^4 - 3ae^4)}{a + cx^4} \right) dx}{(cd^4 + ae^4)^2}$$

$$= -\frac{e^3}{(cd^4 + ae^4)(d + ex)} + \frac{4cd^3e^3 \log(d + ex)}{(cd^4 + ae^4)^2} + \frac{c \int \frac{x(-2de(cd^4 - ae^4) - 4cd^3e^3x^2)}{a + cx^4} dx}{(cd^4 + ae^4)^2} + \frac{c \int \frac{d^2(cd^4 - 3ae^4)}{a + cx^4} dx}{(cd^4 + ae^4)^2}$$

$$= -\frac{e^3}{(cd^4 + ae^4)(d + ex)} + \frac{4cd^3e^3 \log(d + ex)}{(cd^4 + ae^4)^2} + \frac{c \text{Subst} \left(\int \frac{-2de(cd^4 - ae^4) - 4cd^3e^3x}{a + cx^2} dx, x, x^2 \right)}{2(cd^4 + ae^4)^2}$$

$$= -\frac{e^3}{(cd^4 + ae^4)(d + ex)} + \frac{4cd^3e^3 \log(d + ex)}{(cd^4 + ae^4)^2} - \frac{(2c^2d^3e^3) \text{Subst} \left(\int \frac{x}{a + cx^2} dx, x, x^2 \right)}{(cd^4 + ae^4)^2} - \frac{c \int \frac{d^2(cd^4 - 3ae^4)}{a + cx^4} dx}{(cd^4 + ae^4)^2}$$

$$= -\frac{e^3}{(cd^4 + ae^4)(d + ex)} - \frac{\sqrt{c} de (cd^4 - ae^4) \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{\sqrt{a} (cd^4 + ae^4)^2} + \frac{4cd^3e^3 \log(d + ex)}{(cd^4 + ae^4)^2} + \frac{\sqrt[4]{c} \left(3cd^4e^2 - ae^6 + \frac{\sqrt{c} d^2 (cd^4 - 3ae^4)}{\sqrt{a}} \right)}{2\sqrt{2} \sqrt[4]{a} (cd^4 + ae^4)^2}$$

Mathematica [A] time = 0.61, size = 524, normalized size = 0.95

$$\frac{\sqrt{2} \sqrt[4]{c} (a^{3/2} e^6 - 3\sqrt{a} cd^4 e^2 - 3a\sqrt{c} d^2 e^4 + c^{3/2} d^6) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{a^{3/4}} + \frac{\sqrt{2} \sqrt[4]{c} (a^{3/2} e^6 - 3\sqrt{a} cd^4 e^2 - 3a\sqrt{c} d^2 e^4 + c^{3/2} d^6) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{a^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^2*(a + c*x^4)),x]

[Out] ((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2) + Sqrt[a]*e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(3/4) + (2*c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[2]*c*d^4 + 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(3/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 - 3*Sq

$$\frac{\text{rt}[a]*c*d^4*e^2 - 3*a*\text{Sqrt}[c]*d^2*e^4 + a^{(3/2)}*e^6*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/a^{(3/4)} + (\text{Sqrt}[2]*c^{(1/4)}*(c^{(3/2)}*d^6 - 3*\text{Sqrt}[a]*c*d^4*e^2 - 3*a*\text{Sqrt}[c]*d^2*e^4 + a^{(3/2)}*e^6)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/a^{(3/4)} - 8*c*d^3*e^3*\text{Log}[a + c*x^4])/(8*(c*d^4 + a*e^4)^2}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 2.78, size = 646, normalized size = 1.17

$$\frac{cd^3e^3 \log(|cx^4 + a|)}{c^2d^8 + 2acd^4e^4 + a^2e^8} + \frac{4cd^3e^4 \log(|xe + d|)}{c^2d^8e + 2acd^4e^5 + a^2e^9} + \frac{\left((ac^3)^{\frac{1}{4}} c^2d^2 - (ac^3)^{\frac{3}{4}} e^2 \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{2 \left(\sqrt{2} ac^3d^4 - 4 (ac^3)^{\frac{1}{4}} ac^2d^3e + 4 \sqrt{2} \sqrt{ac} ac^2d^2e^2 + \sqrt{2} a^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="giac")

[Out] $-c*d^3*e^3*\log(\text{abs}(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 4*c*d^3*e^4*\log(\text{abs}(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/2*((a*c^3)^{(1/4)}*c^2*d^2 - (a*c^3)^{(3/4)}*e^2)*\arctan(1/2*\text{sqrt}(2)*(2*x + \text{sqrt}(2))*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\text{sqrt}(2)*a*c^3*d^4 - 4*(a*c^3)^{(1/4)}*a*c^2*d^3*e + 4*\text{sqrt}(2)*\text{sqrt}(a*c)*a*c^2*d^2*e^2 + \text{sqrt}(2)*a^2*c^2*e^4 - 4*(a*c^3)^{(3/4)}*a*d*e^3) + 1/2*((a*c^3)^{(1/4)}*c^2*d^2 - (a*c^3)^{(3/4)}*e^2)*\arctan(1/2*\text{sqrt}(2)*(2*x - \text{sqrt}(2))*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\text{sqrt}(2)*a*c^3*d^4 + 4*(a*c^3)^{(1/4)}*a*c^2*d^3*e + 4*\text{sqrt}(2)*\text{sqrt}(a*c)*a*c^2*d^2*e^2 + \text{sqrt}(2)*a^2*c^2*e^4 + 4*(a*c^3)^{(3/4)}*a*d*e^3) + 1/8*(\text{sqrt}(2)*(a*c^3)^{(1/4)}*c^3*d^6 - 3*\text{sqrt}(2)*(a*c^3)^{(3/4)}*c*d^4*e^2 - 3*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a*c^2*d^2*e^4 + \text{sqrt}(2)*(a*c^3)^{(3/4)}*a*e^6)*\log(x^2 + \text{sqrt}(2)*x*(a/c)^{(1/4)} + \text{sqrt}(a/c))/(a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - 1/8*(\text{sqrt}(2)*(a*c^3)^{(1/4)}*c^3*d^6 - 3*\text{sqrt}(2)*(a*c^3)^{(3/4)}*c*d^4*e^2 - 3*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a*c^2*d^2*e^4 + \text{sqrt}(2)*(a*c^3)^{(3/4)}*a*e^6)*\log(x^2 - \text{sqrt}(2)*x*(a/c)^{(1/4)} + \text{sqrt}(a/c))/(a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - (c*d^4*e^3 + a*e^7)/((c*d^4 + a*e^4)^2*(x*e + d))$

maple [A] time = 0.01, size = 866, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^2/(c*x^4+a),x)

[Out] $-e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*\ln(e*x+d)/(a*e^4+c*d^4)^{2-3/4}*c/(a*e^4+c*d^4)^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^2*e^4+1/4*c^2/(a*e^4+c*d^4)^2*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^6-3/8*c/(a*e^4+c*d^4)^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^2*e^4+1/8*c^2/(a*e^4+c*d^4)^2*(a/c)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^6-3/4*c/(a*e^4+c*d^4)^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^2*e^4+1/4*c^2/(a*e^4+c*d^4)$

$$\begin{aligned} & 2*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^6+c/(a*e^4+c*d^4) \\ &)^2/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e^5*d*a-c^2/(a*e^4+c*d^4)^2/(a*c) \\ & ^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e*d^5-1/8/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)} \\ & *ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(\\ & a/c)^{(1/2)})))*a*e^6+3/8*c/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)}*ln((x^2-(a/c)^{(1/4)} \\ &)^2*(1/2)*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))*d^4*e \\ & ^2-1/4/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)* \\ & a*e^6+3/4*c/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}* \\ & x-1)*d^4*e^2-1/4/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)} \\ &)^2*(1/2)*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})))*d^4*e^2-c*d^3*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)^2 \end{aligned}$$

maxima [A] time = 2.11, size = 561, normalized size = 1.02

$$\frac{4cd^3e^3 \log(ex+d)}{c^2d^8 + 2acd^4e^4 + a^2e^8} \frac{e^3}{cd^5 + ade^4 + (cd^4e + ae^5)x} \left[\frac{\sqrt{2} \left(4\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3 - c^2d^6 + 3\sqrt{a}c^{\frac{3}{2}}d^4e^2 + 3acd^2e^4 - a^{\frac{3}{2}}\sqrt{c}e^6 \right) \log\left(\sqrt{c}x^2 + \sqrt{2}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
[Out] 4*c*d^3*e^3*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - e^3/(c*d^5 + a*d*e^4 + (c*d^4*e + a*e^5)*x) - 1/8*c*(sqrt(2))*(4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 - c^2*d^6 + 3*sqrt(a)*c^(3/2)*d^4*e^2 + 3*a*c*d^2*e^4 - a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 + c^2*d^6 - 3*sqrt(a)*c^(3/2)*d^4*e^2 - 3*a*c*d^2*e^4 + a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 + 4*sqrt(a)*c^2*d^5*e - 4*a^(3/2)*c*d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 - 4*sqrt(a)*c^2*d^5*e + 4*a^(3/2)*c*d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8)
```

mupad [B] time = 2.78, size = 2436, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)^2),x)
[Out] symsum(log((c^5*d*e^6 + c^5*e^7*x + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^4*c^4*e^13 + 256*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^3*e^8 + 496*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6*d^8*e^5 + 528*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)
```


$$\begin{aligned}
& ^3a^3c^5d^4e^9 - 128\text{root}(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + \\
& 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac* \\
& d*ez + c, z, k)^4a^2c^7d^{13}e^2 + 128\text{root}(512a^4cd^4e^4z^4 + 256* \\
& a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2* \\
& e^2z^2 + 32ac*d*ez + c, z, k)^4a^3c^6d^9e^6 + 640\text{root}(512a^4cd^4 \\
& 4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 \\
& + 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k)^4a^4c^5d^5e^{10} + 32r \\
& oot(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^ \\
& 3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k)*a^c^5d^2 \\
& *e^7 - 16\text{root}(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^ \\
& 4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k \\
&)^3a^c^7d^{12}e + 16\text{root}(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + 25 \\
& 6a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac*d*e \\
& *z + c, z, k)*c^6d^5e^4*x - 4\text{root}(512a^4cd^4e^4z^4 + 256a^3c^2d^ \\
& 8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + \\
& 32ac*d*ez + c, z, k)^2c^7d^{10}e*x + 64\text{root}(512a^4cd^4e^4z^4 + 25 \\
& 6a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^ \\
& 2e^2z^2 + 32ac*d*ez + c, z, k)^2a^c^6d^7e^4 + 384\text{root}(512a^4cd^4 \\
& 4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 \\
& + 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k)^4a^5c^4d*e^{14} + 320r \\
& oot(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3 \\
& *cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k)^4a^5c^4* \\
& e^{15}*x + 248\text{root}(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8 \\
& *z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z \\
& , k)^2a^c^6d^6e^5*x - 64\text{root}(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^ \\
& 4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32a \\
& *c*d*ez + c, z, k)^3a^c^7d^{11}e^2*x + 32\text{root}(512a^4cd^4e^4z^4 + 25 \\
& 6a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^ \\
& 2e^2z^2 + 32ac*d*ez + c, z, k)*a^c^5d*e^8*x + 316\text{root}(512a^4cd^4 \\
& e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + \\
& 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k)^2a^2c^5d^2e^9*x + 640r \\
& oot(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^ \\
& 3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k)^3a^2c^6 \\
& d^7e^6*x + 704\text{root}(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5 \\
& *e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac*d*ez + \\
& c, z, k)^3a^3c^5d^3e^{10}*x - 192\text{root}(512a^4cd^4e^4z^4 + 256a^3c^ \\
& 2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2cd^2e^2z^ \\
& 2 + 32ac*d*ez + c, z, k)^4a^2c^7d^{12}e^3*x - 64\text{root}(512a^4cd^4e^ \\
& 4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 32 \\
& 0a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k)^4a^3c^6d^8e^7*x + 448r \\
& oot(512a^4cd^4e^4z^4 + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3* \\
& cd^3e^3z^3 + 320a^2cd^2e^2z^2 + 32ac*d*ez + c, z, k)^4a^4c^5d^ \\
& ^4e^{11}*x)/(a^2e^8 + c^2d^8 + 2ac*d^4e^4)*\text{root}(512a^4cd^4e^4z^4 \\
& + 256a^3c^2d^8z^4 + 256a^5e^8z^4 + 1024a^3cd^3e^3z^3 + 320a^2* \\
& cd^2e^2z^2 + 32ac*d*ez + c, z, k), k, 1, 4) - e^3/(c*d^5 + a*d*e^4 + \\
& a*e^5*x + c*d^4*e*x) + (4*c*d^3*e^3*\log(d + e*x))/(a^2e^8 + c^2d^8 + 2*a \\
& c*d^4e^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(c*x**4+a),x)

[Out] Timed out

3.400 $\int \frac{1}{(d+ex)^3(a+cx^4)} dx$

Optimal. Leaf size=680

$$\frac{\sqrt{c}e(a^2e^8 - 12acd^4e^4 + 3c^2d^8) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) c^{3/4}d(3a^2e^8 - 12acd^4e^4 - 2\sqrt{a}\sqrt{c}d^2e^2(3cd^4 - 5ae^4) + c^2d^8) \log\left(\frac{c^{3/4}d(3a^2e^8 - 12acd^4e^4 - 2\sqrt{a}\sqrt{c}d^2e^2(3cd^4 - 5ae^4) + c^2d^8)}{2\sqrt{a}(ae^4 + cd^4)^3}\right) + 4\sqrt{2}a^{3/4}(ae^4 + cd^4)^3}{4\sqrt{2}a^{3/4}(ae^4 + cd^4)^3}$$

[Out] $-1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)+2*c*d^2*e^3*(-3*a*e^4+5*c*d^4)*\ln(e*x+d)/(a*e^4+c*d^4)^3-1/2*c*d^2*e^3*(-3*a*e^4+5*c*d^4)*\ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/2*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*\arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(3/4)*d*\ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4))*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(3/4)*d*\ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4))*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*\arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*d^4))*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*\arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*d^4))*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)$

Rubi [A] time = 0.95, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{c^{3/4}d(3a^2e^8 - 12acd^4e^4 - 2\sqrt{a}\sqrt{c}d^2e^2(3cd^4 - 5ae^4) + c^2d^8) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right) c^{3/4}d(3a^2e^8 - 12acd^4e^4 - 2\sqrt{a}\sqrt{c}d^2e^2(3cd^4 - 5ae^4) + c^2d^8)}{4\sqrt{2}a^{3/4}(ae^4 + cd^4)^3} + \dots$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + c*x^4)), x]

[Out] $-e^3/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*(d + e*x)) - (\text{Sqrt}[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*\text{Log}[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*\text{Log}[a + c*x^4])/(2*(c*d^4 + a*e^4)^3)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 260

$\text{Int}[(x_)^{m_ }]/((a_ + (b_ \cdot)(x_)^{n_ })), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x^n, x]]/(b \cdot n), x] /; \text{FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 617

$\text{Int}[(a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c])] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d_ + (e_ \cdot)(x_))/((a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2)), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 635

$\text{Int}[(d_ + (e_ \cdot)(x_))/((a_ + (c_ \cdot)(x_)^2)), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c \cdot x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c \cdot x^2), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ !\text{NiceSqrtQ}[-(a \cdot c)]$

Rule 1162

$\text{Int}[(d_ + (e_ \cdot)(x_)^2)/((a_ + (c_ \cdot)(x_)^4)), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e + q \cdot x + x^2, x], x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e - q \cdot x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{PosQ}[d \cdot e]$

Rule 1165

$\text{Int}[(d_ + (e_ \cdot)(x_)^2)/((a_ + (c_ \cdot)(x_)^4)), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q - 2 \cdot x)/\text{Simp}[d/e + q \cdot x - x^2, x], x], x] + \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q + 2 \cdot x)/\text{Simp}[d/e - q \cdot x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[d \cdot e]$

Rule 1168

$\text{Int}[(d_ + (e_ \cdot)(x_)^2)/((a_ + (c_ \cdot)(x_)^4)), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a \cdot c, 2]\}, \text{Dist}[(d \cdot q + a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q + c \cdot x^2)/(a + c \cdot x^4), x], x] + \text{Dist}[(d \cdot q - a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q - c \cdot x^2)/(a + c \cdot x^4), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \ \text{NeQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[-(a \cdot c)]$

Rule 1248

$\text{Int}[(x_) \cdot ((d_ + (e_ \cdot)(x_)^2)^{q_ } \cdot ((a_ + (c_ \cdot)(x_)^4)^{p_ })), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e \cdot x)^q \cdot (a + c \cdot x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q, x\}$

Rule 1876

$\text{Int}[(\text{Pq}_)]/((a_ + (b_ \cdot)(x_)^{n_ })), x_Symbol] \rightarrow \text{With}[\{v = \text{Sum}[x^{ii} \cdot (\text{Coeff}[\text{Pq}, x, ii] + \text{Coeff}[\text{Pq}, x, n/2 + ii] \cdot x^{(n/2)})]/(a + b \cdot x^n), \{ii, 0, n/2 - 1\}\}$

}}}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(d+ex)^3(a+cx^4)} dx &= \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)^3} + \frac{4cd^3e^4}{(cd^4+ae^4)^2(d+ex)^2} + \frac{2cd^2e^4(5cd^4-3ae^4)}{(cd^4+ae^4)^3(d+ex)} + \frac{c(d(c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right)}{(cd^4+ae^4)^3} \right) dx \\
 &= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} + \frac{c(d(c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right)}{(cd^4+ae^4)^3} \\
 &= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} + \frac{c(d(c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right)}{(cd^4+ae^4)^3} \\
 &= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} + \frac{c(d(c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right)}{(cd^4+ae^4)^3} \\
 &= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} + \frac{c(d(c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right)}{(cd^4+ae^4)^3} \\
 &= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} + \frac{c(d(c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right)}{(cd^4+ae^4)^3} \\
 &= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} + \frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3} \\
 &= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3}
 \end{aligned}$$

Mathematica [A] time = 0.93, size = 738, normalized size = 1.09

$$\frac{-4a^{3/4}e^3(ae^4+cd^4)^2 - 32a^{3/4}cd^3e^3(d+ex)(ae^4+cd^4) + 4a^{3/4}cd^2e^3(d+ex)^2(3ae^4-5cd^4)\log(a+cx^4) + 16a^{3/4}c(d(c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{d+ex}{\sqrt{a}}\right))}{(cd^4+ae^4)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + c*x^4)),x]

[Out] (-4*a^(3/4)*e^3*(c*d^4 + a*e^4)^2 - 32*a^(3/4)*c*d^3*e^3*(c*d^4 + a*e^4)*(d + e*x) - 2*Sqrt[c]*(Sqrt[2]*c^(9/4)*d^9 - 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 + 24*a^(5/4)*c*d^4*e

$$\begin{aligned} &^5 - 10\sqrt{2}a^{3/2}c^{3/4}d^3e^6 + 3\sqrt{2}a^2c^{1/4}d^8e^8 - 2a^{9/4}e^9(d+ex)^2\text{ArcTan}\left[1 - \frac{\sqrt{2}c^{1/4}x}{a^{1/4}}\right] + 2\sqrt{c} \\ &(\sqrt{2}c^{9/4}d^9 + 6a^{1/4}c^2d^8e + 6\sqrt{2}\sqrt{a}c^{7/4}d^7e^2 - 12\sqrt{2}ac^{5/4}d^5e^4 - 24a^{5/4}cd^4e^5 - 10\sqrt{2}a^{3/2}c^{3/4}d^3e^6 \\ &+ 3\sqrt{2}a^2c^{1/4}d^8e^8 + 2a^{9/4}e^9)(d+ex)^2\text{ArcTan}\left[1 + \frac{\sqrt{2}c^{1/4}x}{a^{1/4}}\right] + 16a^{3/4}cd^2e^3(5cd^4 - 3ae^4)(d+ex)^2 \\ &\text{Log}[d+ex] - \sqrt{2}c^{3/4}d(c^2d^8 - 6\sqrt{a}c^{3/2}d^6e^2 - 12ac^2d^4e^4 + 10a^{3/2}\sqrt{c}d^2e^6 + 3a^2e^8)(d+ex)^2 \\ &\text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] + \sqrt{2}c^{3/4}d(c^2d^8 - 6\sqrt{a}c^{3/2}d^6e^2 - 12ac^2d^4e^4 + 10a^{3/2}\sqrt{c}d^2e^6 \\ &+ 3a^2e^8)(d+ex)^2\text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] + 4a^{3/4}cd^2e^3(-5cd^4 + 3ae^4)(d+ex)^2 \\ &\text{Log}[a + cx^4]) / (8a^{3/4}(cd^4 + ae^4)^3(d+ex)^2) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.89, size = 901, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{4}(\sqrt{2}(ac^3)^{1/4}c^2d^3 + 2ac^2e^3 - 3\sqrt{2}(ac^3)^{3/4}de^2)\arctan\left(\frac{1/2\sqrt{2}(2x + \sqrt{2}(a/c)^{1/4})/(a/c)^{1/4}}{(ac^3d^6 - 3\sqrt{2}(ac^3)^{1/4}ac^2d^5e + 9\sqrt{2}ac^2d^4e^2 + 9a^2c^2d^2e^4 - 8\sqrt{2}(ac^3)^{3/4}ad^3e^3 - 3\sqrt{2}(ac^3)^{1/4}a^2cd^5e + \sqrt{2}ac^2e^6) + 1/4(\sqrt{2}(ac^3)^{1/4}c^2d^3 - 2ac^2e^3 - 3\sqrt{2}(ac^3)^{3/4}de^2)\arctan\left(\frac{1/2\sqrt{2}(2x - \sqrt{2}(a/c)^{1/4})/(a/c)^{1/4}}{(ac^3d^6 + 3\sqrt{2}(ac^3)^{1/4}ac^2d^5e - 9\sqrt{2}ac^2d^4e^2 + 9a^2c^2d^2e^4 + 8\sqrt{2}(ac^3)^{3/4}ad^3e^3 + 3\sqrt{2}(ac^3)^{1/4}a^2cd^5e + \sqrt{2}ac^2e^6) + 1/8(\sqrt{2}(ac^3)^{1/4}c^3d^9 - 6\sqrt{2}(ac^3)^{3/4}cd^7e^2 - 12\sqrt{2}(ac^3)^{1/4}ac^2d^5e^4 + 10\sqrt{2}(ac^3)^{3/4}ad^3e^6 + 3\sqrt{2}(ac^3)^{1/4}a^2cd^8e^8)\log(x^2 + \sqrt{2}x(a/c)^{1/4} + \sqrt{a/c})\right)}{(ac^4d^{12} + 3a^2c^3d^8e^4 + 3a^3c^2d^4e^8 + a^4c^2e^{12})} - \frac{1}{8}(\sqrt{2}(ac^3)^{1/4}c^3d^9 - 6\sqrt{2}(ac^3)^{3/4}cd^7e^2 - 12\sqrt{2}(ac^3)^{1/4}ac^2d^5e^4 + 10\sqrt{2}(ac^3)^{3/4}ad^3e^6 + 3\sqrt{2}(ac^3)^{1/4}a^2cd^8e^8)\log(x^2 - \sqrt{2}x(a/c)^{1/4} + \sqrt{a/c})\right)}{(ac^4d^{12} + 3a^2c^3d^8e^4 + 3a^3c^2d^4e^8 + a^4c^2e^{12})} - \frac{1}{2}(5c^2d^6e^3 - 3ac^2d^2e^7)\log(\text{abs}(cx^4 + a)) / (c^3d^{12}e + 3ac^2d^8e^5 + 3a^2cd^4e^9 + a^3e^{13}) - \frac{1}{2}(9c^2d^8e^3 + 10ac^2d^4e^7 + a^2e^{11} + 8(c^2d^7e^4 + acd^3e^8)x) / ((cd^4 + ae^4)^3(xe + d)^2)$

maple [B] time = 0.01, size = 1201, normalized size = 1.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^3/(c*x^4+a),x)

```
[Out] -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-6*e^7*d^2*c/(a*e^4+c*d^4)^3*ln(e*x+d)*a+10*e^3*d^6*c^2/(a*e^4+c*d^4)^3*ln(e*x+d)+
3/4*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^8-3*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^4+1/4*c^3/(a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^9+3/8*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^8-3/2*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^4+1/8*c^3/(a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^9+3/4*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^8-3*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^4+1/4*c^3/(a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9-1/2*c/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*a^2+6*c^2/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*d^4*e^5*a-3/2*c^3/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^8-5/4*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*a*d^3*e^6+3/4*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7*e^2-5/2*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*a*d^3*e^6+3/2*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^7*e^2-5/2*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*d^3*e^6+3/2*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7*e^2+3/2*c/(a*e^4+c*d^4)^3*ln(c*x^4+a)*e^7*d^2*a-5/2*c^2/(a*e^4+c*d^4)^3*ln(c*x^4+a)*e^3*d^6
```

maxima [A] time = 2.34, size = 817, normalized size = 1.20

$$c \left(\frac{\sqrt{2} \left(10 \sqrt{2} a^{\frac{3}{4}} c^{\frac{9}{4}} d^6 e^3 - 6 \sqrt{2} a^{\frac{7}{4}} c^{\frac{5}{4}} d^2 e^7 - c^3 d^9 + 6 \sqrt{a} c^{\frac{5}{2}} d^7 e^2 + 12 a c^2 d^5 e^4 - 10 a^{\frac{3}{2}} c^{\frac{3}{2}} d^3 e^6 - 3 a^2 c d e^8 \right) \log \left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right)}{a^{\frac{3}{4}} c^{\frac{5}{4}}} \right) + \frac{\sqrt{2} \left(10 \sqrt{2} a^{\frac{3}{4}} c^{\frac{9}{4}} \right)}{a^{\frac{3}{4}} c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*c*(sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(5/4)*d^2*e^7 - c^3*d^9 + 6*sqrt(a)*c^(5/2)*d^7*e^2 + 12*a*c^2*d^5*e^4 - 10*a^(3/2)*c^(3/2)*d^3*e^6 - 3*a^2*c*d*e^8)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(5/4)*d^2*e^7 + c^3*d^9 - 6*sqrt(a)*c^(5/2)*d^7*e^2 - 12*a*c^2*d^5*e^4 + 10*a^(3/2)*c^(3/2)*d^3*e^6 + 3*a^2*c*d*e^8)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 - 12*sqrt(2)*a^(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt(2)*a^(9/4)*c^(5/4)*d*e^8 + 6*sqrt(a)*c^3*d^8*e - 24*a^(3/2)*c^2*d^4*e^5 + 2*a^(5/2)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 - 12*sqrt(2)*a^(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt(2)*a^(9/4)*c^(5/4)*d*e^8 - 6*sqrt(a)*c^3*d^8*e + 24*a^(3/2)*c^2*d^4*e^5 - 2*a^(5/2)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)
```

$$\frac{\log(e*x + d)}{(c^3*d^{12} + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^{12})} - \frac{1}{2} \frac{(8*c*d^3*e^4*x + 9*c*d^4*e^3 + a*e^7)}{(c^2*d^{10} + 2*a*c*d^6*e^4 + a^2*d^2*e^8 + (c^2*d^8*e^2 + 2*a*c*d^4*e^6 + a^2*e^{10})*x^2 + 2*(c^2*d^9*e + 2*a*c*d^5*e^5 + a^2*d*e^9)*x)}$$

mupad [B] time = 3.67, size = 1955, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)*(d + e*x)^3), x)

[Out] symsum(log((c^7*d^5*e^6 + a*c^6*d*e^10)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((208*a*c^7*d^7*e^7 - 48*a^2*c^6*d^3*e^11)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((144*a*c^8*d^13*e^4 + 16*a^4*c^5*d*e^16 + 2608*a^2*c^7*d^9*e^8 - 592*a^3*c^6*d^5*e^12)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((896*a^4*c^6*d^7*e^13 - 1120*a^3*c^7*d^11*e^9 - 1024*a^2*c^8*d^15*e^5 + 976*a^5*c^5*d^3*e^17 + 16*a*c^9*d^19*e)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((384*a^7*c^4*d*e^22 - 128*a^2*c^9*d^21*e^2 - 128*a^3*c^8*d^17*e^6 + 768*a^4*c^7*d^13*e^10 + 1792*a^5*c^6*d^9*e^14 + 1408*a^6*c^5*d^5*e^18)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + (x*(320*a^7*c^4*e^23 - 192*a^2*c^9*d^20*e^3 - 448*a^3*c^8*d^16*e^7 + 128*a^4*c^7*d^12*e^11 + 1152*a^5*c^6*d^8*e^15 + 1088*a^6*c^5*d^4*e^19))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + (x*(80*a*c^9*d^18*e^2 - 1536*a^2*c^8*d^14*e^6 - 2016*a^3*c^7*d^10*e^10 + 896*a^4*c^6*d^6*e^14 + 1296*a^5*c^5*d^2*e^18))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + (x*(36*a^4*c^5*e^17 - 4*c^9*d^16*e + 792*a*c^8*d^12*e^5 + 1632*a^2*c^7*d^8*e^9 - 152*a^3*c^6*d^4*e^13))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + (x*(40*c^8*d^10*e^4 - 16*a*c^7*d^6*e^8 + 72*a^2*c^6*d^2*e^12))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + (x*(a*c^6*e^11 + c^7*d^4*e^7))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8))*root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k), k, 1, 4) - ((a*e^7 + 9*c*d^4*e^3)/(2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (4*c*d^3*e^4*x)/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))/(d^2 + e^2*x^2 + 2*d*e*x) + (log(d + e*x)*(10*c^2*d^6*e^3 - 6*a*c*d^2*e^7))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

```
[Out] Timed out
```


$$3.401 \quad \int \frac{(d+ex)^3}{(a+cx^4)^2} dx$$

Optimal. Leaf size=349

$$\frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} + \frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}}$$

[Out] $1/4*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)+3/4*d^2*e*\arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)-3/32*d*\ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/32*d*\ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*\arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*\arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)$

Rubi [A] time = 0.30, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {1854, 27, 12, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} + \frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + c*x^4)^2,x]

[Out] $-(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(4*a*c*(a + c*x^4)) + (3*d^2*e*\text{ArcTan}[\text{Sqrt}[c]*x^2/\text{Sqrt}[a]])/(4*a^(3/2)*\text{Sqrt}[c]) - (3*d*(\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(8*\text{Sqrt}[2]*a^(7/4)*c^(3/4)) + (3*d*(\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(8*\text{Sqrt}[2]*a^(7/4)*c^(3/4)) - (3*d*(\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^(7/4)*c^(3/4)) + (3*d*(\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^(7/4)*c^(3/4))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 27

Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{(a+cx^4)^2} dx &= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} - \frac{\int \frac{-3d^3 - 6d^2ex - 3de^2x^2}{a+cx^4} dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} - \frac{\int -\frac{3d(d+ex)^2}{a+cx^4} dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{(3d) \int \frac{(d+ex)^2}{a+cx^4} dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{(3d) \int \left(\frac{2dex}{a+cx^4} + \frac{d^2+e^2x^2}{a+cx^4} \right) dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{(3d) \int \frac{d^2+e^2x^2}{a+cx^4} dx}{4a} + \frac{(3d^2e) \int \frac{x}{a+cx^4} dx}{2a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{(3d^2e) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{4a} + \frac{\left(3d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \right) \int \frac{1}{\sqrt{a} - \sqrt{2} \frac{\sqrt{4}x}{\sqrt{c}} + x^2}}{8ac} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} + \frac{\left(3d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2} \frac{\sqrt{4}x}{\sqrt{c}}}{\sqrt{c}} + x^2}}{16ac} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} - \frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{a} - \sqrt{2} \frac{\sqrt{4}x}{\sqrt{c}})}{16\sqrt{2} a^{7/4} c^{3/4}} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} - \frac{3d(\sqrt{c}d^2 + \sqrt{a}e^2) \tan^{-1} \left(1 - \frac{\sqrt{2} \frac{\sqrt{4}x}{\sqrt{c}}}{\sqrt{a}} \right)}{8\sqrt{2} a^{7/4} c^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 347, normalized size = 0.99

$$3\sqrt{2} \sqrt[4]{c} (a^{3/4}de^2 - \sqrt[4]{a} \sqrt{c} d^3) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2) + 3\sqrt{2} \sqrt[4]{c} (\sqrt[4]{a} \sqrt{c} d^3 - a^{3/4}de^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x - \sqrt{a} - \sqrt{c} x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + c*x^4)^2,x]

[Out] $((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^{(1/4)} * c^{(1/4)} * d * (\text{Sqrt}[2] * \text{Sqrt}[c] * d^2 + 4*a^{(1/4)} * c^{(1/4)} * d * e + \text{Sqrt}[2] * \text{Sqrt}[a] * e^2) * \text{ArcTan}[1 - (\text{Sqrt}[2] * c^{(1/4)} * x) / a^{(1/4)}] + 6*a^{(1/4)} * c^{(1/4)} * d * (\text{Sqrt}[2] * \text{Sqrt}[c] * d^2 - 4*a^{(1/4)} * c^{(1/4)} * d * e + \text{Sqrt}[2] * \text{Sqrt}[a] * e^2) * \text{ArcTan}[1 + (\text{Sqrt}[2] * c^{(1/4)} * x) / a^{(1/4)}] + 3 * \text{Sqrt}[2] * c^{(1/4)} * (-a^{(1/4)} * \text{Sqrt}[c] * d^3 + a^{(3/4)} * d * e^2) * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{(1/4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2] + 3 * \text{Sqrt}[2] * c^{(1/4)} * (a^{(1/4)} * \text{Sqrt}[c] * d^3 - a^{(3/4)} * d * e^2) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2]) / (32 * a^2 * c)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 342, normalized size = 0.98

$$\frac{3cdx^3e^2 + 3cd^2x^2e + cd^3x - ae^3}{4(cx^4 + a)ac} + \frac{3\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2d^2e + (ac^3)^{\frac{1}{4}}c^2d^3 + (ac^3)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(3*c*d*x^3*e^2 + 3*c*d^2*x^2*e + c*d^3*x - a*e^3)/((c*x^4 + a)*a*c) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)

maple [A] time = 0.00, size = 390, normalized size = 1.12

$$\frac{e^3x^4}{4(cx^4 + a)a} + \frac{3de^2x^3}{4(cx^4 + a)a} + \frac{3d^2ex^2}{4(cx^4 + a)a} + \frac{d^3x}{4(cx^4 + a)a} + \frac{3d^2e\arctan\left(\sqrt{\frac{c}{a}}x^2\right)}{4\sqrt{ac}a} + \frac{3\sqrt{2}de^2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3/(c*x^4+a)^2,x)

[Out] 1/4*d^3*x/a/(c*x^4+a)+3/32*d^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+3/4*e*d^2*x^2/a/(c*x^4+a)+3/4*e*d^2/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+3/4*d*e^2*x^3/a/(c*x^4+a)+3/32*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*e^3*x^4/a/(c*x^4+a)

maxima [A] time = 2.20, size = 332, normalized size = 0.95

$$3d \left(\frac{\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2 + \sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2 - 4\sqrt{a}\sqrt{c}de\right)}{a^{\frac{3}{4}}\sqrt{a}\sqrt{c}} \right)$$

32 a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")

```
[Out] 3/32*d*(sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))/a + 1/4*(3*c*d*e^2*x^3 + 3*c*d^2*e*x^2 + c*d^3*x - a*e^3)/(a*c^2*x^4 + a^2*c)
```

mupad [B] time = 0.43, size = 670, normalized size = 1.92

$$\left(\sum_{k=1}^4 \ln \left(\frac{c d^2 \left(27 c d^5 e^2 - 9 a d e^6 + 36 c d^4 e^3 x - \text{root} \left(65536 a^7 c^3 z^4 + 27648 a^4 c^2 d^4 e^2 z^2 + 3456 a^3 c d^4 e^5 z - 3456 a^2 c^2 d^8 e^* z + 162 a^* c^* d^8 e^4 + 81 a^2 d^4 e^8 + 81 c^2 d^{12}, z, k \right)^2 a^3 c^2 d - 48 \text{root} \left(65536 a^7 c^3 z^4 + 27648 a^4 c^2 d^4 e^2 z^2 + 3456 a^3 c d^4 e^5 z - 3456 a^2 c^2 d^8 e^* z + 162 a^* c^* d^8 e^4 + 81 a^2 d^4 e^8 + 81 c^2 d^{12}, z, k \right) a^* c^2 d^4 x + 48 \text{root} \left(65536 a^7 c^3 z^4 + 27648 a^4 c^2 d^4 e^2 z^2 + 3456 a^3 c d^4 e^5 z - 3456 a^2 c^2 d^8 e^* z + 162 a^* c^* d^8 e^4 + 81 a^2 d^4 e^8 + 81 c^2 d^{12}, z, k \right) a^2 c e^4 x + 512 \text{root} \left(65536 a^7 c^3 z^4 + 27648 a^4 c^2 d^4 e^2 z^2 + 3456 a^3 c d^4 e^5 z - 3456 a^2 c^2 d^8 e^* z + 162 a^* c^* d^8 e^4 + 81 a^2 d^4 e^8 + 81 c^2 d^{12}, z, k \right)^2 a^3 c^2 e^* x - 192 \text{root} \left(65536 a^7 c^3 z^4 + 27648 a^4 c^2 d^4 e^2 z^2 + 3456 a^3 c d^4 e^5 z - 3456 a^2 c^2 d^8 e^* z + 162 a^* c^* d^8 e^4 + 81 a^2 d^4 e^8 + 81 c^2 d^{12}, z, k \right) a^2 c d e^3 \right) / (64 a^3) \right) \text{root} \left(65536 a^7 c^3 z^4 + 27648 a^4 c^2 d^4 e^2 z^2 + 3456 a^3 c d^4 e^5 z - 3456 a^2 c^2 d^8 e^* z + 162 a^* c^* d^8 e^4 + 81 a^2 d^4 e^8 + 81 c^2 d^{12}, z, k \right) / (4 a) - e^3 / (4 c) + (3 d^2 e^* x^2) / (4 a) + (3 d e^2 x^3) / (4 a) \right) / (a + c x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((3*c*d^2*(27*c*d^5*e^2 - 9*a*d*e^6 + 36*c*d^4*e^3*x - 256*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e^*z + 162*a^*c^*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c^2*d - 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e^*z + 162*a^*c^*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^*c^2*d^4*x + 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e^*z + 162*a^*c^*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*e^4*x + 512*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e^*z + 162*a^*c^*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c^2*e^*x - 192*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e^*z + 162*a^*c^*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*d*e^3)) / (64*a^3)) * root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e^*z + 162*a^*c^*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k) / (4*a) - e^3 / (4*c) + (3*d^2*e^*x^2) / (4*a) + (3*d*e^2*x^3) / (4*a)) / (a + c*x^4)
```

sympy [A] time = 8.33, size = 350, normalized size = 1.00

$$\text{RootSum} \left(65536 t^4 a^7 c^3 + 27648 t^2 a^4 c^2 d^4 e^2 + t \left(3456 a^3 c d^4 e^5 - 3456 a^2 c^2 d^8 e \right) + 81 a^2 d^4 e^8 + 162 a c d^8 e^4 + 81 c^2 d^{12} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 27648*_t**2*a**4*c**2*d**4*e**2 + _t*(3456*a**3*c*d**4*e**5 - 3456*a**2*c**2*d**8*e) + 81*a**2*d**4*e**8 + 162*a*c*d**8*e**4 + 81*c**2*d**12, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 28672*_t**3*a**6*c**3*d**4*e**2 - 7680*_t**2*a**5*c**2*d**4*e**5 + 1536*_t**2*a**4*c**3*d**8*e + 2160*_t*a**4*c*d**4*e**8 + 9216*_t*a**3*c**2*d**8*e**4 + 144*_t*a**2*c**3*d**12 + 162*a**3*d**4*e**11 - 648*a**2*c*d**8*e**7 - 810*a*c**2*d**12*e**3) / (27*a**3*d**3*e**12 - 891*a**2*c*d**7*e**8 - 891*a*c**2*d**11*e**4 + 27*c**3*d**15)))) + (-a*e**3 + c*d**3*x + 3*c*d**2*e*x**2 + 3*c*d*e**2*x**3) / (4*a**2*c + 4*a*c**2*x**4)
```

$$3.402 \quad \int \frac{(d+ex)^2}{(a+cx^4)^2} dx$$

Optimal. Leaf size=322

$$\frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} - \frac{(\sqrt{a}e^2)}{16\sqrt{2}a^{7/4}c^{3/4}}$$

[Out] $\frac{1}{4}xx(e*x+d)^2/a/(c*x^4+a)+1/2*d*e*\arctan(x^2*c^{(1/2)}/a^{(1/2)})/a^{(3/2)}/c^{(1/2)}-1/32*\ln(-a^{(1/4)}*c^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*(-e^2*a^{(1/2)}+3*d^2*c^{(1/2)})/a^{(7/4)}/c^{(3/4)}*2^{(1/2)}+1/32*\ln(a^{(1/4)}*c^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*(-e^2*a^{(1/2)}+3*d^2*c^{(1/2)})/a^{(7/4)}/c^{(3/4)}*2^{(1/2)}+1/16*\arctan(-1+c^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})*(e^2*a^{(1/2)}+3*d^2*c^{(1/2)})/a^{(7/4)}/c^{(3/4)}*2^{(1/2)}+1/16*\arctan(1+c^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})*(e^2*a^{(1/2)}+3*d^2*c^{(1/2)})/a^{(7/4)}/c^{(3/4)}*2^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} - \frac{(\sqrt{a}e^2)}{16\sqrt{2}a^{7/4}c^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + c*x^4)^2,x]

[Out] $(x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*a^{(3/2)}*\text{Sqrt}[c]) - ((3*\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*c^{(3/4)}) + ((3*\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*c^{(3/4)}) - ((3*\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*c^{(3/4)}) + ((3*\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*c^{(3/4)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x_Symbol] \ :> \ \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[de]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-(2d)/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[de]$

Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[ac, 2]\}, \text{Dist}[(dq + ae)/(2ac), \text{Int}[(q + cx^2)/(a + cx^4), x], x] + \text{Dist}[(dq - ae)/(2ac), \text{Int}[(q - cx^2)/(a + cx^4), x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{NegQ}[-(ac)]$

Rule 1855

$\text{Int}[(Pq_.)((a_.) + (b_.)x^{(n_.)})^{(p_.)}, x_Symbol] \ :> \ -\text{Simp}[(xPq(a + bx^n)^{(p+1)})/(a^n(p+1)), x] + \text{Dist}[1/(a^n(p+1)), \text{Int}[\text{ExpandToSum}[n(p+1)Pq + D[xPq, x], x](a + bx^n)^{(p+1)}, x], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \& \ \& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{LtQ}[\text{Expon}[Pq, x], n - 1]$

Rule 1876

$\text{Int}[(Pq_.)/(a_.) + (b_.)x^{(n_.)}, x_Symbol] \ :> \ \text{With}[\{v = \text{Sum}[(x^{ii}(\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii]x^{(n/2)}))]/(a + bx^n), \{ii, 0, n/2 - 1\}]\}, \text{Int}[v, x] \ /; \ \text{SumQ}[v] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n/2, 0] \ \&\& \ \text{Expon}[Pq, x] < n]$

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{(a+cx^4)^2} dx &= \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \frac{-3d^2-4dex-e^2x^2}{a+cx^4} dx}{4a} \\
&= \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \left(-\frac{4dex}{a+cx^4} + \frac{-3d^2-e^2x^2}{a+cx^4} \right) dx}{4a} \\
&= \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \frac{-3d^2-e^2x^2}{a+cx^4} dx}{4a} + \frac{(de) \int \frac{x}{a+cx^4} dx}{a} \\
&= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{(de) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2a} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{8ac} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{1}{a+cx^4} dx}{8ac} \\
&= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2a^{3/2}\sqrt{c}} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16ac} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}} dx}{16ac} \\
&= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2a^{3/2}\sqrt{c}} - \frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2 + \sqrt{a}e^2) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} \\
&= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2a^{3/2}\sqrt{c}} - \frac{(3\sqrt{c}d^2 + \sqrt{a}e^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2 + \sqrt{a}e^2) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}c^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 321, normalized size = 1.00

$$\frac{\sqrt{2}(a^{3/4}e^2 - 3\sqrt[4]{a}\sqrt{c}d^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{c^{3/4}} + \frac{\sqrt{2}(3\sqrt[4]{a}\sqrt{c}d^2 - a^{3/4}e^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{c^{3/4}} - \frac{2\sqrt[4]{a}(8\sqrt[4]{a}\sqrt[4]{c}de + \sqrt{2}\sqrt{a}e^2 + 3\sqrt{c}d^2)}{32a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + c*x^4)^2,x]

[Out] ((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 - 8*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2 - a^(3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(32*a^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.36, size = 323, normalized size = 1.00

$$\frac{x^3 e^2 + 2 d x^2 e + d^2 x}{4 (c x^4 + a) a} + \frac{\sqrt{2} \left(4 \sqrt{2} \sqrt{ac} c^2 d e + 3 (ac^3)^{\frac{1}{4}} c^2 d^2 + (ac^3)^{\frac{3}{4}} e^2 \right) \arctan \left(\frac{\sqrt{2} \left(2x + \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{16 a^2 c^3} + \frac{\sqrt{2} \left(4 \sqrt{2} \sqrt{ac} c^2 d e + 3 (ac^3)^{\frac{1}{4}} c^2 d^2 + (ac^3)^{\frac{3}{4}} e^2 \right) \arctan \left(\frac{\sqrt{2} \left(2x - \sqrt{2} \left(\frac{a}{c} \right)^{\frac{1}{4}} \right)}{2 \left(\frac{a}{c} \right)^{\frac{1}{4}}} \right)}{16 a^2 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(x^3*e^2 + 2*d*x^2*e + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)

maple [A] time = 0.01, size = 362, normalized size = 1.12

$$\frac{e^2 x^3}{4 (c x^4 + a) a} + \frac{d e x^2}{2 (c x^4 + a) a} + \frac{d^2 x}{4 (c x^4 + a) a} + \frac{d e \arctan \left(\sqrt{\frac{c}{a}} x^2 \right)}{2 \sqrt{ac} a} + \frac{\sqrt{2} e^2 \arctan \left(\frac{\sqrt{2} x}{\left(\frac{a}{c} \right)^{\frac{1}{4}}} - 1 \right)}{16 \left(\frac{a}{c} \right)^{\frac{1}{4}} ac} + \frac{\sqrt{2} e^2 \arctan \left(\frac{\sqrt{2} x}{\left(\frac{a}{c} \right)^{\frac{1}{4}}} + 1 \right)}{16 \left(\frac{a}{c} \right)^{\frac{1}{4}} ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(c*x^4+a)^2,x)

[Out] 1/4*d^2*x/a/(c*x^4+a)+3/32*d^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/2*d*e*x^2/a/(c*x^4+a)+1/2*d*e/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+1/4*e^2*x^3/a/(c*x^4+a)+1/32*e^2/a/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/16*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/16*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)

maxima [A] time = 2.63, size = 318, normalized size = 0.99

$$\frac{e^2 x^3 + 2 d e x^2 + d^2 x}{4 (acx^4 + a^2)} + \frac{\sqrt{2} (3 \sqrt{c} d^2 - \sqrt{a} e^2) \log \left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}} - \frac{\sqrt{2} (3 \sqrt{c} d^2 - \sqrt{a} e^2) \log \left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}} + \frac{2 \left(3 \sqrt{2} a^{\frac{1}{4}} \right)}{16 a^2 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")

[Out] 1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/(a*c*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*e^2)/(16*a^2*c^3)

$$\frac{1}{4}c^{3/4}d^2 + \sqrt{2}a^{3/4}c^{1/4}e^2 - 8\sqrt{a}\sqrt{c}de \arctan\left(\frac{1/2\sqrt{2}(2\sqrt{c}x + \sqrt{2}a^{1/4}c^{1/4})/\sqrt{\sqrt{a}\sqrt{c}}}{a^{3/4}\sqrt{\sqrt{a}\sqrt{c}}c^{3/4}}\right) + 2(3\sqrt{2}a^{1/4}c^{3/4}d^2 + \sqrt{2}a^{3/4}c^{1/4}e^2 + 8\sqrt{a}\sqrt{c}de) \arctan\left(\frac{1/2\sqrt{2}(2\sqrt{c}x - \sqrt{2}a^{1/4}c^{1/4})/\sqrt{\sqrt{a}\sqrt{c}}}{a^{3/4}\sqrt{\sqrt{a}\sqrt{c}}c^{3/4}}\right)/a$$

mupad [B] time = 2.48, size = 391, normalized size = 1.21

$$\frac{\frac{d^2x}{4a} + \frac{e^2x^3}{4a} + \frac{dex^2}{2a}}{cx^4 + a} + \left(\sum_{k=1}^4 \ln \left(\frac{39c^2d^4e^2 - ac^6}{64a^3} - \text{root} \left(65536a^7c^3z^4 + 11264a^4c^2d^2e^2z^2 - 2304a^2c^2d^5ez + 256a^3cde^5z + 82a^2c^2d^8 + a^2e^8, z, k \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + c*x^4)^2,x)

[Out] ((d^2*x)/(4*a) + (e^2*x^3)/(4*a) + (d*e*x^2)/(2*a))/(a + c*x^4) + symsum(log((39*c^2*d^4*e^2 - a*c*e^6)/(64*a^3) - root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k)*(root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k)*(12*c^3*d^2 - 16*c^3*d*e*x) + (x*(18*a*c^3*d^4 - 2*a^2*c^2*e^4))/(8*a^3) + (2*c^2*d*e^3)/a) + (5*c^2*d^3*e^3*x)/(8*a^3))*root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k), k, 1, 4)

sympy [A] time = 3.52, size = 318, normalized size = 0.99

$$\text{RootSum} \left(65536t^4a^7c^3 + 11264t^2a^4c^2d^2e^2 + t(256a^3cde^5 - 2304a^2c^2d^5e) + a^2e^8 + 82acd^4e^4 + 81c^2d^8, (t \mapsto t \log(x + \dots)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2/(c*x**4+a)**2,x)

[Out] RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*a**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81*c**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**3*a**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4*c**3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 3888*_t*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c**2*d**9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4 + 729*c**3*d**12))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x**4)

$$3.403 \quad \int \frac{d+ex}{(a+cx^4)^2} dx$$

Optimal. Leaf size=241

$$\frac{3d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3d \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}}$$

[Out] $\frac{1}{4} x (e x + d) / a / (c x^4 + a) + 3 / 16 d * \arctan(-1 + c^{1/4} x^2^{1/2} / a^{1/4}) / a^{7/4} / c^{1/4} * 2^{1/2} + 3 / 16 d * \arctan(1 + c^{1/4} x^2^{1/2} / a^{1/4}) / a^{7/4} / c^{1/4} * 2^{1/2} - 3 / 32 d * \ln(-a^{1/4} * c^{1/4} x^2^{1/2} + a^{1/2} + x^2 * c^{1/2}) / a^{7/4} / c^{1/4} * 2^{1/2} + 3 / 32 d * \ln(a^{1/4} * c^{1/4} x^2^{1/2} + a^{1/2} + x^2 * c^{1/2}) / a^{7/4} / c^{1/4} * 2^{1/2} + 1 / 4 e * \arctan(x^2 * c^{1/2} / a^{1/2}) / a^{3/2} / c^{1/2}$

Rubi [A] time = 0.19, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{3d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3d \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + c*x^4)^2, x]

[Out] $(x * (d + e * x)) / (4 * a * (a + c * x^4)) + (e * \text{ArcTan}[(\text{Sqrt}[c] * x^2) / \text{Sqrt}[a]]) / (4 * a^{3/2} * \text{Sqrt}[c]) - (3 * d * \text{ArcTan}[1 - (\text{Sqrt}[2] * c^{1/4} * x) / a^{1/4}]) / (8 * \text{Sqrt}[2] * a^{7/4} * c^{1/4}) + (3 * d * \text{ArcTan}[1 + (\text{Sqrt}[2] * c^{1/4} * x) / a^{1/4}]) / (8 * \text{Sqrt}[2] * a^{7/4} * c^{1/4}) - (3 * d * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{1/4} * c^{1/4} * x + \text{Sqrt}[c] * x^2]) / (16 * \text{Sqrt}[2] * a^{7/4} * c^{1/4}) + (3 * d * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{1/4} * c^{1/4} * x + \text{Sqrt}[c] * x^2]) / (16 * \text{Sqrt}[2] * a^{7/4} * c^{1/4})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{(a+cx^4)^2} dx &= \frac{x(d+ex)}{4a(a+cx^4)} - \frac{\int \frac{-3d-2ex}{a+cx^4} dx}{4a} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} - \frac{\int \left(-\frac{3d}{a+cx^4} - \frac{2ex}{a+cx^4}\right) dx}{4a} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{(3d) \int \frac{1}{a+cx^4} dx}{4a} + \frac{e \int \frac{x}{a+cx^4} dx}{2a} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{(3d) \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} + \frac{(3d) \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} + \frac{e \operatorname{Subst}\left(\int \frac{1}{a+cx^2} dx, x, x^2\right)}{4a} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{(3d) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} + \frac{(3d) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{ax}}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3d \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 224, normalized size = 0.93

$$\frac{8a^{3/4}x(d+ex)}{a+cx^4} - \frac{2(4\sqrt[4]{a}e+3\sqrt{2}\sqrt[4]{c}d)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{2(3\sqrt{2}\sqrt[4]{c}d-4\sqrt[4]{a}e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt[4]{a}}+1\right)}{\sqrt{c}} - \frac{3\sqrt{2}d\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2)}{\sqrt[4]{c}} + \frac{3\sqrt{2}d\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + c*x^4)^2, x]

[Out] ((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/4)*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/Sqrt[c] - (3*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(32*a^(7/4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 241, normalized size = 1.00

$$\frac{3\sqrt{2}(ac^3)^{1/4}d\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{1/4} + \sqrt{\frac{a}{c}}\right)}{32a^2c} - \frac{3\sqrt{2}(ac^3)^{1/4}d\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{1/4} + \sqrt{\frac{a}{c}}\right)}{32a^2c} + \frac{x^2e + dx}{4(cx^4 + a)a} + \frac{\sqrt{2}\left(2\sqrt{2}d\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right) + 2\sqrt{2}d\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) + 1/4*(x^2*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)))/(a/c)^(1/4))/(a^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4)))/(a/c)^(1/4))/(a^2*c^2)
```

maple [A] time = 0.00, size = 188, normalized size = 0.78

$$\frac{e x^2}{4(c x^4 + a) a} + \frac{d x}{4(c x^4 + a) a} + \frac{e \arctan\left(\sqrt{\frac{c}{a}} x^2\right)}{4 \sqrt{a c} a} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16 a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*e*x^2/a/(c*x^4+a)+1/4*e/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)
```

maxima [A] time = 2.43, size = 238, normalized size = 0.99

$$\frac{e x^2 + d x}{4(a c x^4 + a^2)} + \frac{3 \sqrt{2} d \log\left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} - \frac{3 \sqrt{2} d \log\left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} + \frac{2\left(3 \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} d - 4 \sqrt{a} e\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{32 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e*x^2 + d*x)/(a*c*x^4 + a^2) + 1/32*(3*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 3*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*d - 4*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*d + 4*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4))/a
```

mupad [B] time = 0.27, size = 282, normalized size = 1.17

$$\left(\sum_{k=1}^4 \ln\left(\frac{c^2\left(3 d e^2 + 2 e^3 x - \text{root}\left(65536 a^7 c^2 z^4 + 2048 a^4 c e^2 z^2 - 1152 a^2 c d^2 e z + 81 c d^4 + 16 a e^4, z, k\right)\right)^2 a^3 c d}{\dots}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((c^2*(3*d*e^2 + 2*e^3*x - 192*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*d + 128*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*e*x - 36*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)*a*c*d^2*x))/(16*a^3))*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k), k, 1, 4) + ((e*x^2)/(4*a) + (d*x)/(4*a))/(a + c*x^4)
```

sympy [A] time = 1.09, size = 155, normalized size = 0.64

$$\text{RootSum}\left(65536t^4a^7c^2 + 2048t^2a^4ce^2 - 1152ta^2cd^2e + 16ae^4 + 81cd^4, \left(t \mapsto t \log\left(x + \frac{-32768t^3a^6ce^2 - 4608}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d**2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e**2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 + 360*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 + 4*a*c*x**4)
```

$$3.404 \quad \int \frac{1}{(a+cx^4)^2} dx$$

Optimal. Leaf size=202

$$\frac{3 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}}$$

[Out] 1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)-3/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1/2)

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{3 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-2), x]

[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_ + (c_)*(x_)^2)), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1162

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1165

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + cx^4)^2} dx &= \frac{x}{4a(a + cx^4)} + \frac{3 \int \frac{1}{a+cx^4} dx}{4a} \\ &= \frac{x}{4a(a + cx^4)} + \frac{3 \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} + \frac{3 \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} \\ &= \frac{x}{4a(a + cx^4)} + \frac{3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} + \frac{3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} - \frac{3 \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3 \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} \\ &= \frac{x}{4a(a + cx^4)} - \frac{3 \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{c} x^2)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{c} x^2)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3 \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{c} x^2)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} \end{aligned}$$

Mathematica [A] time = 0.11, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{3\sqrt{2} \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{\sqrt[4]{c}} + \frac{3\sqrt{2} \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{\sqrt[4]{c}} - \frac{6\sqrt{2} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^(-2), x]

[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]))/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]))/c^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4)

4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(32*a^(7/4))

fricas [A] time = 0.63, size = 173, normalized size = 0.86

$$\frac{12(acx^4 + a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}} \arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}} + \sqrt{a^4\sqrt{-\frac{1}{a^7c}} + x^2}a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right) + 3(acx^4 + a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}} \log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\right)}{16(acx^4 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")

[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 + a^2)

giac [A] time = 0.25, size = 194, normalized size = 0.96

$$\frac{x}{4(cx^4 + a)a} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}x + \sqrt{\frac{a}{c}}\right)}{32a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c)

maple [A] time = 0.00, size = 143, normalized size = 0.71

$$\frac{x}{4(cx^4 + a)a} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{16a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{16a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+a)^2,x)

[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)

maxima [A] time = 2.11, size = 189, normalized size = 0.94

$$\frac{x}{4(acx^4 + a^2)} + \frac{3 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2} \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2} \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} \right)}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")

[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(a)*sqrt(sqrt(a)*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(a)*sqrt(sqrt(a)*sqrt(c)) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))/a

mupad [B] time = 0.09, size = 58, normalized size = 0.29

$$\frac{x}{4a(cx^4 + a)} + \frac{3 \operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{8(-a)^{7/4}c^{1/4}} + \frac{3 \operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{8(-a)^{7/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c*x^4)^2,x)

[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))

sympy [A] time = 0.30, size = 39, normalized size = 0.19

$$\frac{x}{4a^2 + 4acx^4} + \operatorname{RootSum}\left(65536t^4a^7c + 81, \left(t \mapsto t \log\left(\frac{16ta^2}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+a)**2,x)

[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*log(16*_t*a**2/3 + x)))

Rule 204

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2] \cdot x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 205

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 260

$\text{Int}[(x_)^{m_ }/((a_ + (b_ \cdot)(x_)^{n_ })), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x^n, x]]/(b \cdot n), x] /; \text{FreeQ}\{a, b, m, n\}, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 275

$\text{Int}[(x_)^{m_ } \cdot ((a_ + (b_ \cdot)(x_)^{n_ })^{p_ }), x_Symbol] \rightarrow \text{With}\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m + 1)/k - 1} \cdot (a + b \cdot x^{n/k})^p, x], x, x^k], x] /; k \neq 1 /; \text{FreeQ}\{a, b, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 617

$\text{Int}[(a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c]) /; \text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d_ + (e_ \cdot)(x_))/((a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2)), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 635

$\text{Int}[(d_ + (e_ \cdot)(x_))/((a_ + (c_ \cdot)(x_)^2)), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c \cdot x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c \cdot x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ !\text{NiceSqrtQ}[-(a \cdot c)]$

Rule 1162

$\text{Int}[(d_ + (e_ \cdot)(x_)^2)/((a_ + (c_ \cdot)(x_)^4)), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e + q \cdot x + x^2, x], x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e - q \cdot x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{PosQ}[d \cdot e]$

Rule 1165

$\text{Int}[(d_ + (e_ \cdot)(x_)^2)/((a_ + (c_ \cdot)(x_)^4)), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q - 2 \cdot x)/\text{Simp}[d/e + q \cdot x - x^2, x], x], x] + \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q + 2 \cdot x)/\text{Simp}[d/e - q \cdot x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[d \cdot e]$

Rule 1168

$\text{Int}[(d_ + (e_ \cdot)(x_)^2)/((a_ + (c_ \cdot)(x_)^4)), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a \cdot c, 2]\}, \text{Dist}[(d \cdot q + a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q + c \cdot x^2)/(a + c \cdot x^4), x], x] + \text{Dist}[(d \cdot q - a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q - c \cdot x^2)/(a + c \cdot x^4), x], x] /; \text{FreeQ}\{a,$

$c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[-(a*c)]$

Rule 1248

$\text{Int}[(x_*)*((d_*) + (e_*)*(x_*)^2)^{(q_*)}*((a_*) + (c_*)*(x_*)^4)^{(p_*)}, x_Symbol]$
 $:\> \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /;$ $\text{FreeQ}[\{a, c, d, e, p, q\}, x]$

Rule 1854

$\text{Int}[(Pq_*)*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] :\> \text{Module}[\{q = \text{Expon}[Pq, x], i\}, \text{Simp}[\{(a*\text{Coeff}[Pq, x, q] - b*x*\text{ExpandToSum}[Pq - \text{Coeff}[Pq, x, q]*x^q, x])*(a + b*x^n)^{(p + 1)})/(a*b*n*(p + 1)), x] + \text{Dist}[1/(a*n*(p + 1)), \text{Int}[\text{Sum}[(n*(p + 1) + i + 1)*\text{Coeff}[Pq, x, i]*x^i, \{i, 0, q - 1\}](a + b*x^n)^{(p + 1)}, x], x] /;$ $q == n - 1] /;$ $\text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1]$

Rule 1876

$\text{Int}[(Pq_*)/((a_*) + (b_*)*(x_*)^{(n_*)}), x_Symbol] :\> \text{With}[\{v = \text{Sum}[(x^{ii}*(\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii]*x^{(n/2)}))]/(a + b*x^n), \{ii, 0, n/2 - 1\}\}, \text{Int}[v, x] /;$ $\text{SumQ}[v] /;$ $\text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[Pq, x] < n$

Rule 6742

$\text{Int}[u_, x_Symbol] :\> \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /;$ $\text{SumQ}[v]$
 $]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)(a+cx^4)^2} dx &= \int \left(\frac{e^8}{(cd^4+ae^4)^2(d+ex)} + \frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(cd^4+ae^4)(a+cx^4)^2} - \frac{ce^4(-d^3+d^2ex-de^2x^2+e^3x^3)}{(cd^4+ae^4)^2(a+cx^4)} \right) dx \\
&= \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4) \int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4} dx}{(cd^4+ae^4)^2} + \frac{c \int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{(a+cx^4)^2} dx}{cd^4+ae^4} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4) \int \left(\frac{-d^3-de^2x^2}{a+cx^4} + \frac{x(d^2e+e^3x^2)}{a+cx^4} \right) dx}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4) \int \frac{-d^3-de^2x^2}{a+cx^4} dx}{(cd^4+ae^4)^2} - \frac{(ce^4) \int \frac{x(d^2e+e^3x^2)}{a+cx^4} dx}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4) \text{Subst} \left(\int \frac{d^2e+e^3x}{a+cx^2} dx, x, x^2 \right)}{2(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(cd^2e^5) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} - \frac{\sqrt{c} d^2 e^5 \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2\sqrt{a} (cd^4+ae^4)^2} - \frac{\sqrt{c} d^2 e \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{4a^{3/2} (cd^4+ae^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} - \frac{\sqrt{c} d^2 e^5 \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2\sqrt{a} (cd^4+ae^4)^2} - \frac{\sqrt{c} d^2 e \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{4a^{3/2} (cd^4+ae^4)} - \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} - \frac{\sqrt{c} d^2 e^5 \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{2\sqrt{a} (cd^4+ae^4)^2} - \frac{\sqrt{c} d^2 e \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{4a^{3/2} (cd^4+ae^4)} - \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2}
\end{aligned}$$

Mathematica [A] time = 0.39, size = 558, normalized size = 0.65

$$\frac{\sqrt{2} \sqrt[4]{c} (5a^{3/2} d e^6 + \sqrt{a} c d^5 e^2 - 7a \sqrt{c} d^3 e^4 - 3c^{3/2} d^7) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{a^{7/4}} + \frac{\sqrt{2} \sqrt[4]{c} (-5a^{3/2} d e^6 - \sqrt{a} c d^5 e^2 + 7a \sqrt{c} d^3 e^4 + 3c^{3/2} d^7) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*(a + c*x^4)^2), x]

[Out] ((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*e^5 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d*e^5 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*d^5*e^2

- 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 - Sqrt[a]*c*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4))/(32*(c*d^4 + a*e^4)^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.50, size = 771, normalized size = 0.90

$$\frac{\left(4\sqrt{2}\sqrt{ac}c^2d^2e + 3(ac^3)^{\frac{1}{4}}c^2d^3 + 5(ac^3)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^3d^4 - 4(ac^3)^{\frac{1}{4}}a^2c^2d^3e + 4\sqrt{2}\sqrt{ac}a^2c^2d^2e^2 + \sqrt{2}a^3c^2e^4 - 4(ac^3)^{\frac{3}{4}}a^2de^3\right)} + \frac{\left(4\sqrt{2}\sqrt{ac}c^2d^2e + 3(ac^3)^{\frac{1}{4}}c^2d^3 + 5(ac^3)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^3d^4 + 4(ac^3)^{\frac{1}{4}}a^2c^2d^3e + 4\sqrt{2}\sqrt{ac}a^2c^2d^2e^2 + \sqrt{2}a^3c^2e^4 - 4(ac^3)^{\frac{3}{4}}a^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/8*(4*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 - 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c^2*e^4 - 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/8*(4*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 + 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c^2*e^4 + 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/4*e^7*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + e^8*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/4*(a*c*d^4*e^3 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 + a^2*e^7 + (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(c*x^4 + a)*a)

maple [A] time = 0.02, size = 1122, normalized size = 1.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)/(c*x^4+a)^2,x)

[Out] e^7*ln(e*x+d)/(a*e^4+c*d^4)^2+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*d*e^6*x^3+1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^5*e^2/a*x^3-1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*e^5*d^2*x^2-1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*e*d^6/a*x^2+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*d^3*x*e^4+1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^7/a*x+1/4/(a*e^4+c*d^4)^2/(c*x^4+a)*e^7*a+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*e^3*d^4+7/16*c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3

$$\begin{aligned} & *e^4+3/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^7+7/32*c/(a*e^4+c*d^4)^2/a*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^3*e^4+3/32*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^7+7/16*c/(a*e^4+c*d^4)^2/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^3*e^4+3/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^7-3/4*c/(a*e^4+c*d^4)^2/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e^5*d^2-1/4*c^2/(a*e^4+c*d^4)^2/a/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e^5*d^2+5/32/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d*e^6+1/32*c/(a*e^4+c*d^4)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^5*e^2+5/16/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d*e^6+1/16*c/(a*e^4+c*d^4)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^5*e^2+5/16/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d*e^6+1/16*c/(a*e^4+c*d^4)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^5*e^2-1/4*e^7*\ln(c*x^4+a)/(a*e^4+c*d^4)^2 \end{aligned}$$

maxima [A] time = 2.29, size = 601, normalized size = 0.70

$$\frac{e^7 \log(ex + d)}{c^2 d^8 + 2acd^4 e^4 + a^2 e^8} \left(\frac{\sqrt{2} \left(4\sqrt{2} a^{\frac{7}{4}} c^{\frac{1}{4}} e^7 - 3c^2 d^7 + \sqrt{a} c^{\frac{3}{2}} d^5 e^2 - 7acd^3 e^4 + 5a^{\frac{3}{2}} \sqrt{c} d e^6 \right) \log\left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{5}{4}}} + \frac{\sqrt{2} \left(4\sqrt{2} a^{\frac{7}{4}} c^{\frac{1}{4}} e^7 - 3c^2 d^7 + \sqrt{a} c^{\frac{3}{2}} d^5 e^2 - 7acd^3 e^4 + 5a^{\frac{3}{2}} \sqrt{c} d e^6 \right)}{a^{\frac{3}{4}} c^{\frac{5}{4}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & e^7*\log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - 1/32*c*(\text{sqrt}(2))*(4*\text{sqrt}(2)*a^{(7/4)}*c^{(1/4)}*e^7 - 3*c^2*d^7 + \text{sqrt}(a)*c^{(3/2)}*d^5*e^2 - 7*a*c*d^3*e^4 + 5*a^{(3/2)}*\text{sqrt}(c)*d*e^6)*\log(\text{sqrt}(c)*x^2 + \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*x + \text{sqrt}(a))/(a^{(3/4)}*c^{(5/4)}) + \text{sqrt}(2)*(4*\text{sqrt}(2)*a^{(7/4)}*c^{(1/4)}*e^7 + 3*c^2*d^7 - \text{sqrt}(a)*c^{(3/2)}*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^{(3/2)}*\text{sqrt}(c)*d*e^6)*\log(\text{sqrt}(c)*x^2 - \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*x + \text{sqrt}(a))/(a^{(3/4)}*c^{(5/4)}) - 2*(3*\text{sqrt}(2)*a^{(1/4)}*c^{(9/4)}*d^7 + \text{sqrt}(2)*a^{(3/4)}*c^{(7/4)}*d^5*e^2 + 7*\text{sqrt}(2)*a^{(5/4)}*c^{(5/4)}*d^3*e^4 + 5*\text{sqrt}(2)*a^{(7/4)}*c^{(3/4)}*d*e^6 + 4*\text{sqrt}(a)*c^2*d^6*e + 12*a^{(3/2)}*c*d^2*e^5)*\arctan(1/2*\text{sqrt}(2)*(2*\text{sqrt}(c)*x + \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)})/\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))/(a^{(3/4)}*\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c))*c^{(5/4)}) - 2*(3*\text{sqrt}(2)*a^{(1/4)}*c^{(9/4)}*d^7 + \text{sqrt}(2)*a^{(3/4)}*c^{(7/4)}*d^5*e^2 + 7*\text{sqrt}(2)*a^{(5/4)}*c^{(5/4)}*d^3*e^4 + 5*\text{sqrt}(2)*a^{(7/4)}*c^{(3/4)}*d*e^6 - 4*\text{sqrt}(a)*c^2*d^6*e - 12*a^{(3/2)}*c*d^2*e^5)*\arctan(1/2*\text{sqrt}(2)*(2*\text{sqrt}(c)*x - \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)})/\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))/(a^{(3/4)}*\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c))*c^{(5/4)})/(a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8) + 1/4*(c*d*e^2*x^3 - c*d^2*e*x^2 + c*d^3*x + a*e^3)/(a^2*c*d^4 + a^3*e^4 + (a*c^2*d^4 + a^2*c*e^4)*x^4) \end{aligned}$$

mupad [B] time = 2.99, size = 1591, normalized size = 1.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)^2*(d + e*x)),x)

[Out]
$$e^3/(4*(a^2*e^4 + c^2*d^4*x^4 + a*c*d^4 + a*c*e^4*x^4)) + \text{symsum}(\log((81*c^5*d^5*e^6 + 64*a*c^4*d*e^{10})/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)))$$

```

) + root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z
^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*
a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a
^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e
^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4
096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4
+ 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*
c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 8
1*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d
^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 2
4576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e
^4, z, k)*((98304*a^9*c^4*d*e^14 - 32768*a^6*c^7*d^13*e^2 + 32768*a^7*c^6*d
^9*e^6 + 163840*a^8*c^5*d^5*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4
*e^4)) + (x*(81920*a^9*c^4*e^15 - 49152*a^6*c^7*d^12*e^3 - 16384*a^7*c^6*d^
8*e^7 + 114688*a^8*c^5*d^4*e^11))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4
*e^4))) + (52224*a^7*c^4*d*e^13 - 3072*a^4*c^7*d^13*e + 13312*a^5*c^6*d^9*e
^5 + 68608*a^6*c^5*d^5*e^9)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))
+ (x*(61440*a^7*c^4*e^14 - 8192*a^4*c^7*d^12*e^2 - 4096*a^5*c^6*d^8*e^6 +
65536*a^6*c^5*d^4*e^10))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) +
(8704*a^5*c^4*d*e^12 + 3584*a^3*c^6*d^9*e^4 + 15360*a^4*c^5*d^5*e^8)/(256*
(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(15360*a^5*c^4*e^13 - 576*a
^2*c^7*d^12*e + 1920*a^3*c^6*d^8*e^5 + 18880*a^4*c^5*d^4*e^9))/(256*(a^6*e
^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (192*a*c^6*d^9*e^3 + 704*a^3*c^4*d*e
^11 + 1536*a^2*c^5*d^5*e^7)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))
+ (x*(1280*a^3*c^4*e^12 + 256*a*c^6*d^8*e^4 + 2240*a^2*c^5*d^4*e^8))/(256*
(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (81*c^5*d^4*e^7*x)/(256*(a^6*
e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))*root(131072*a^8*c*d^4*e^4*z^4 + 6553
6*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*
e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^
4 + 256*a*e^4, z, k), k, 1, 4) + (e^7*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*
a*c*d^4*e^4) + (c*d^3*x)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^
4*x^4)) - (c*d^2*e*x^2)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4
*x^4)) + (c*d*e^2*x^3)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*
x^4))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x**4+a)**2,x)

[Out] Timed out

3.406 $\int \frac{1}{(d+ex)^2(a+cx^4)^2} dx$

Optimal. Leaf size=1141

$$\frac{8cd^3 \log(d+ex)e^7}{(cd^4+ae^4)^3} - \frac{2cd^3 \log(cx^4+a)e^7}{(cd^4+ae^4)^3} - \frac{e^7}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}d(3cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt[4]{c}(\sqrt{c})}{\sqrt{a}(cd^4+ae^4)^3}$$

[Out] $-e^7/(a^2e^4+cd^4)^2/(e*x+d)+1/4*c*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+cd^4)-2*d*e*(-a*e^4+cd^4)*x+e^2*(-a*e^4+3*c*d^4)*x^2))/a/(a^2e^4+cd^4)^2/(c*x^4+a)+8*c*d^3*e^7*\ln(e*x+d)/(a^2e^4+cd^4)^3-2*c*d^3*e^7*\ln(c*x^4+a)/(a^2e^4+cd^4)^3-1/2*d*e*(-a*e^4+cd^4)*\arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a^2e^4+cd^4)^2-d*e^5*(-a*e^4+3*c*d^4)*\arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a^2e^4+cd^4)^3/a^(1/2)-1/32*c^(1/4)*\ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+cd^4)*c^(1/2))/a^(7/4)/(a^2e^4+cd^4)^2*2^(1/2)+1/32*c^(1/4)*\ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+cd^4)*c^(1/2))/a^(7/4)/(a^2e^4+cd^4)^2*2^(1/2)+1/16*c^(1/4)*\arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+cd^4)*c^(1/2))/a^(7/4)/(a^2e^4+cd^4)^2*2^(1/2)+1/16*c^(1/4)*\arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+cd^4)*c^(1/2))/a^(7/4)/(a^2e^4+cd^4)^2*2^(1/2)-1/8*c^(1/4)*e^4*\ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(3/4)/(a^2e^4+cd^4)^3*2^(1/2)+1/8*c^(1/4)*e^4*\ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(3/4)/(a^2e^4+cd^4)^3*2^(1/2)+1/4*c^(1/4)*e^4*\arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(3/4)/(a^2e^4+cd^4)^3*2^(1/2)+1/4*c^(1/4)*e^4*\arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(3/4)/(a^2e^4+cd^4)^3*2^(1/2)$

Rubi [A] time = 1.66, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{8cd^3 \log(d+ex)e^7}{(cd^4+ae^4)^3} - \frac{2cd^3 \log(cx^4+a)e^7}{(cd^4+ae^4)^3} - \frac{e^7}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}d(3cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt[4]{c}(\sqrt{c})}{\sqrt{a}(cd^4+ae^4)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + c*x^4)^2), x]

[Out] $-(e^7/((c*d^4+a*e^4)^2*(d+e*x)))+(c*(4*a*d^3*e^3+x*(d^2*(c*d^4-3*a*e^4)-2*d*e*(c*d^4-a*e^4)*x+e^2*(3*c*d^4-a*e^4)*x^2)))/(4*a*(c*d^4+a*e^4)^2*(a+c*x^4))-(\text{Sqrt}[c]*d*e^5*(3*c*d^4-a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(\text{Sqrt}[a]*(c*d^4+a*e^4)^3)-(\text{Sqrt}[c]*d*e*(c*d^4-a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*a^(3/2)*(c*d^4+a*e^4)^2)-(c^(1/4))*(3*\text{Sqrt}[c]*d^2*(c*d^4-3*a*e^4)+\text{Sqrt}[a]*e^2*(3*c*d^4-a*e^4))*\text{ArcTan}[1-(\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)]/(8*\text{Sqrt}[2]*a^(7/4)*(c*d^4+a*e^4)^2)-(c^(1/4)*e^4*(\text{Sqrt}[c]*d^2*(5*c*d^4-3*a*e^4)+\text{Sqrt}[a]*e^2*(7*c*d^4-a*e^4))*\text{ArcTan}[1-(\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)]/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4+a*e^4)^3)+(c^(1/4))*(3*\text{Sqrt}[c]*d^2*(c*d^4-3*a*e^4)+\text{Sqrt}[a]*e^2*(3*c*d^4-a*e^4))*\text{ArcTan}[1+(\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)]/(8*\text{Sqrt}[2]*a^(7/4)*(c*d^4+a*e^4)^2)+(c^(1/4)*e^4*(\text{Sqrt}[c]*d^2*(5*c*d^4-3*a*e^4)+\text{Sqrt}[a]*e^2*(7*c*d^4-a*e^4))*\text{ArcTan}[1+(\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)]/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4+a*e^4)^3)+(8*c*d^3*e^7*\text{Log}[d+e*x])/(c*d^4+a*e^4)^3-(c^(1/4))$

$$\begin{aligned} & * (3\sqrt{c}d^2(c^4d - 3ae^4) - \sqrt{a}e^2(3c^4d - ae^4)) \cdot \text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2] / (16\sqrt{2}a^{7/4}(c^4d + ae^4)^2) - (c^{1/4}e^4(\sqrt{c}d^2(5c^4d - 3ae^4) - \sqrt{a}e^2(7c^4d - ae^4)) \cdot \text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2]) / (4\sqrt{2}a^{3/4}(c^4d + ae^4)^3) + (c^{1/4}(3\sqrt{c}d^2(c^4d - 3ae^4) - \sqrt{a}e^2(3c^4d - ae^4)) \cdot \text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2]) / (16\sqrt{2}a^{7/4}(c^4d + ae^4)^2) + (c^{1/4}e^4(\sqrt{c}d^2(5c^4d - 3ae^4) - \sqrt{a}e^2(7c^4d - ae^4)) \cdot \text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2]) / (4\sqrt{2}a^{3/4}(c^4d + ae^4)^3) - (2c^3d^3e^7 \cdot \text{Log}[a + cx^4]) / (c^4d + ae^4)^3 \end{aligned}$$
Rule 204

$$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \text{ || LtQ}[b, 0])$$
Rule 205

$$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]]) / a, x] \text{ /; FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b]$$
Rule 260

$$\text{Int}(x_)^{m_} / ((a_ + (b_ \cdot)(x_)^n), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + bx^n, x]] / (b \cdot n), x] \text{ /; FreeQ}\{a, b, m, n\}, x\} \&\& \text{EqQ}[m, n - 1]$$
Rule 275

$$\text{Int}(x_)^{m_} \cdot ((a_ + (b_ \cdot)(x_)^n)^{p_}), x_Symbol] \rightarrow \text{With}\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k - 1} \cdot (a + bx^{n/k})^p, x], x, x^k], x] \text{ /; } k \neq 1 \text{ /; FreeQ}\{a, b, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$$
Rule 617

$$\text{Int}[(a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4\text{Simplify}[(a \cdot c) / b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x) / b], x] \text{ /; RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \text{ || !RationalQ}[b^2 - 4 \cdot a \cdot c]) \text{ /; FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$$
Rule 628

$$\text{Int}[(d_ + (e_ \cdot)(x_)) / ((a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]) / b, x] \text{ /; FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$$
Rule 635

$$\text{Int}[(d_ + (e_ \cdot)(x_)) / ((a_ + (c_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + cx^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + cx^2), x], x] \text{ /; FreeQ}\{a, c, d, e\}, x\} \&\& \text{!NiceSqrtQ}[-(a \cdot c)]$$
Rule 1162

$$\text{Int}[(d_ + (e_ \cdot)(x_)^2) / ((a_ + (c_ \cdot)(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2 \cdot d) / e, 2]\}, \text{Dist}[e / (2 \cdot c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x], x] + \text{Dist}[e / (2 \cdot c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x] \text{ /; FreeQ}\{a, c, d, e\}, x\} \&\& \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{PosQ}[d \cdot e]$$
Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} & (\text{Sqrt}[2]*c^{(1/4)*x}/a^{(1/4)})/a^{(7/4)} + (2*c^{(1/4)}*(3*\text{Sqrt}[2]*c^{(5/2)*d^{10}} \\ & + 8*a^{(1/4)*c^{(9/4)*d^9*e} + 3*\text{Sqrt}[2]*\text{Sqrt}[a]*c^2*d^8*e^2 + 14*\text{Sqrt}[2]*a*c \\ & ^{(3/2)*d^6*e^4 + 48*a^{(5/4)*c^{(5/4)*d^5*e^5} + 30*\text{Sqrt}[2]*a^{(3/2)*c*d^4*e^6} \\ & - 21*\text{Sqrt}[2]*a^2*\text{Sqrt}[c]*d^2*e^8 - 24*a^{(9/4)*c^{(1/4)*d*e^9} - 5*\text{Sqrt}[2]*a^{(5/2)*e^{10}} \\ & *\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)*x}/a^{(1/4)})/a^{(7/4)} + 256*c*d^3*e^7 \\ & *\text{Log}[d + e*x] - (\text{Sqrt}[2]*c^{(1/4)}*(3*c^{(5/2)*d^{10}} - 3*\text{Sqrt}[a]*c^2*d^8*e^2 + \\ & 14*a*c^{(3/2)*d^6*e^4 - 30*a^{(3/2)*c*d^4*e^6 - 21*a^2*\text{Sqrt}[c]*d^2*e^8 + 5*a^{(5/2)*e^{10}} \\ & *\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)*c^{(1/4)*x} + \text{Sqrt}[c]*x^2])/a^{(7/4)} \\ & + (\text{Sqrt}[2]*c^{(1/4)}*(3*c^{(5/2)*d^{10}} - 3*\text{Sqrt}[a]*c^2*d^8*e^2 + 14*a*c^{(3/2)*d^6*e^4 \\ & - 30*a^{(3/2)*c*d^4*e^6 - 21*a^2*\text{Sqrt}[c]*d^2*e^8 + 5*a^{(5/2)*e^{10}}*\text{Log}[\text{Sqrt}[a] + \\ & \text{Sqrt}[2]*a^{(1/4)*c^{(1/4)*x} + \text{Sqrt}[c]*x^2])/a^{(7/4)} - 64*c*d^3*e^7*\text{Log}[a + c*x^4])/ \\ & (32*(c*d^4 + a*e^4)^3) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 103.64, size = 1104, normalized size = 0.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -2*c*d^3*e^7*\log(\text{abs}(c*x^4 + a))/(c^3*d^{12} + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^{12}) \\ & + 8*c*d^3*e^8*\log(\text{abs}(x*e + d))/(c^3*d^{12}*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3*e^{13}) \\ & + 1/8*(5*\text{sqrt}(2)*\text{sqrt}(a*c)*c^2*d^3*e + 3*(a*c^3)^{(1/4)*c^2*d^4} + 3*\text{sqrt}(2)*a*c^2*d*e^3 + 6*(a*c^3)^{(3/4)*d^2*e^2} - 5*(a*c^3)^{(1/4)*a*c*e^4} \\ & *\text{arctan}(1/2*\text{sqrt}(2)*(2*x + \text{sqrt}(2)*(a/c)^{(1/4)}))/(a/c)^{(1/4)})/(\text{sqrt}(2)*a^2*c^3*d^6 - 6*(a*c^3)^{(1/4)*a^2*c^2*d^5*e} + 9*\text{sqrt}(2)*\text{sqrt}(a*c)*a^2*c^2*d^4*e^2 + 9*\text{sqrt}(2)*a^3*c^2*d^2*e^4 - 16*(a*c^3)^{(3/4)*a^2*d^3*e^3} - 6*(a*c^3)^{(1/4)*a^3*c*d*e^5} + \text{sqrt}(2)*\text{sqrt}(a*c)*a^3*c*e^6) \\ & + 1/8*(5*\text{sqrt}(2)*\text{sqrt}(a*c)*c^2*d^3*e + 3*(a*c^3)^{(1/4)*c^2*d^4} - 3*\text{sqrt}(2)*a*c^2*d*e^3 + 6*(a*c^3)^{(3/4)*d^2*e^2} - 5*(a*c^3)^{(1/4)*a*c*e^4} \\ & *\text{arctan}(1/2*\text{sqrt}(2)*(2*x - \text{sqrt}(2)*(a/c)^{(1/4)}))/(a/c)^{(1/4)})/(\text{sqrt}(2)*a^2*c^3*d^6 + 6*(a*c^3)^{(1/4)*a^2*c^2*d^5*e} + 9*\text{sqrt}(2)*\text{sqrt}(a*c)*a^2*c^2*d^4*e^2 + 9*\text{sqrt}(2)*a^3*c^2*d^2*e^4 + 16*(a*c^3)^{(3/4)*a^2*d^3*e^3} + 6*(a*c^3)^{(1/4)*a^3*c*d*e^5} + \text{sqrt}(2)*\text{sqrt}(a*c)*a^3*c*e^6) \\ & + 1/32*(3*\text{sqrt}(2)*(a*c^3)^{(1/4)*c^4*d^{10}} - 3*\text{sqrt}(2)*(a*c^3)^{(3/4)*c^2*d^8*e^2} + 14*\text{sqrt}(2)*(a*c^3)^{(1/4)*a*c^3*d^6*e^4} - 30*\text{sqrt}(2)*(a*c^3)^{(3/4)*a*c*d^4*e^6} - 21*\text{sqrt}(2)*(a*c^3)^{(1/4)*a^2*c^2*d^2*e^8} + 5*\text{sqrt}(2)*(a*c^3)^{(3/4)*a^2*e^{10}} \\ & *\log(x^2 + \text{sqrt}(2)*x*(a/c)^{(1/4)} + \text{sqrt}(a/c)))/(a^2*c^5*d^{12} + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2*e^{12}) - 1/32*(3*\text{sqrt}(2)*(a*c^3)^{(1/4)*c^4*d^{10}} - 3*\text{sqrt}(2)*(a*c^3)^{(3/4)*c^2*d^8*e^2} + 14*\text{sqrt}(2)*(a*c^3)^{(1/4)*a*c^3*d^6*e^4} - 30*\text{sqrt}(2)*(a*c^3)^{(3/4)*a*c*d^4*e^6} - 21*\text{sqrt}(2)*(a*c^3)^{(1/4)*a^2*c^2*d^2*e^8} + 5*\text{sqrt}(2)*(a*c^3)^{(3/4)*a^2*e^{10}} \\ & *\log(x^2 - \text{sqrt}(2)*x*(a/c)^{(1/4)} + \text{sqrt}(a/c)))/(a^2*c^5*d^{12} + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2*e^{12}) + 1/4*(3*c^2*d^4*x^4*e^3 + c^2*d^5*x^3*e^2 - c^2*d^6*x^2*e + c^2*d^7*x - 5*a*c*x^4*e^7 + a*c*d*x^3*e^6 - a*c*d^2*x^2*e^5 + a*c*d^3*x*e^4 + 4*a*c*d^4*e^3 - 4*a^2*e^7)/((a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8)*(c*x^5*e + c*d*x^4 + a*x*e + a*d)) \end{aligned}$$

maple [A] time = 0.02, size = 1636, normalized size = 1.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x+d)^2/(c*x^4+a)^2, x)$

[Out] $8*c*d^3*e^7*\ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*\ln(c*x^4+a)/(a*e^4+c*d^4)^3-e^7/(a*e^4+c*d^4)^2/(e*x+d)-1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^6*x*e^4-3*c^2/(a*e^4+c*d^4)^3/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e^5*d^5-5/32/(a*e^4+c*d^4)^3*a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*e^{10-5/16}/(a*e^4+c*d^4)^3*a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*e^{10-5/16}/(a*e^4+c*d^4)^3*a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*e^{10+1/4*c^3}/(a*e^4+c*d^4)^3/(c*x^4+a)*d^{10}/a*x-1/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)*e^{10}*a*x^3+c/(a*e^4+c*d^4)^3/(c*x^4+a)*e^7*d^3*a+1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*e^6*x^3*d^4+c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^7*e^3-21/16*c/(a*e^4+c*d^4)^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^{10}-1/2*c^3/(a*e^4+c*d^4)^3/a/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e*d^9+15/16*c/(a*e^4+c*d^4)^3/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^4*e^6+15/8*c/(a*e^4+c*d^4)^3/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^4*e^6+15/8*c/(a*e^4+c*d^4)^3/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^4*e^6+3/2*c/(a*e^4+c*d^4)^3*a/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e^9*d+3/4*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*e^2/a*x^3*d^8-1/2*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*d^9*e/a*x^2+1/2*c/(a*e^4+c*d^4)^3/(c*x^4+a)*d^9*a*x^2-3/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)*d^2*a*x*e^8-21/16*c/(a*e^4+c*d^4)^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^{10}-21/32*c/(a*e^4+c*d^4)^3*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^2*e^8+3/32*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^{10}+7/16*c^2/(a*e^4+c*d^4)^3/a*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^6*e^4+7/8*c^2/(a*e^4+c*d^4)^3/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^6*e^4+3/16*c^2/(a*e^4+c*d^4)^3/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^8*e^2+3/16*c^2/(a*e^4+c*d^4)^3/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^8*e^2+7/8*c^2/(a*e^4+c*d^4)^3/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^6*e^4+3/32*c^2/(a*e^4+c*d^4)^3/a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^8*e^2$

maxima [A] time = 2.54, size = 961, normalized size = 0.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(e*x+d)^2/(c*x^4+a)^2, x, \text{algorithm}="maxima")$

[Out] $8*c*d^3*e^7*\log(e*x + d)/(c^3*d^{12} + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^{12}) - 1/32*c*(\text{sqrt}(2)*(32*\text{sqrt}(2)*a^{(7/4)}*c^{(5/4)}*d^3*e^7 - 3*c^3*d^{10} + 3*\text{sqrt}(a)*c^{(5/2)}*d^8*e^2 - 14*a*c^2*d^6*e^4 + 30*a^{(3/2)}*c^{(3/2)}*d^4*e^6 + 21*a^2*c*d^2*e^8 - 5*a^{(5/2)}*\text{sqrt}(c)*e^{10})*\log(\text{sqrt}(c)*x^2 + \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*x + \text{sqrt}(a)))/(a^{(3/4)}*c^{(5/4)}) + \text{sqrt}(2)*(32*\text{sqrt}(2)*a^{(7/4)}*c^{(5/4)}*d^3*e^7 + 3*c^3*d^{10} - 3*\text{sqrt}(a)*c^{(5/2)}*d^8*e^2 + 14*a*c^2*d^6*e^4 - 30*a^{(3/2)}*c^{(3/2)}*d^4*e^6 - 21*a^2*c*d^2*e^8 + 5*a^{(5/2)}*\text{sqrt}(c)*e^{10})*\log(\text{sqrt}(c)*x^2 - \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*x + \text{sqrt}(a)))/(a^{(3/4)}*c^{(5/4)}) - 2*(3*\text{sqrt}(2)*a^{(1/4)}*c^{(13/4)}*d^{10} + 3*\text{sqrt}(2)*a^{(3/4)}*c^{(11/4)}*d^8*e^2 + 14*\text{sqrt}(2)*a^{(5/4)}*c^{(9/4)}*d^6*e^4 + 30*\text{sqrt}(2)*a^{(7/4)}*c^{(7/4)}*d^4*e^6 - 21*\text{sqrt}(2)*a^{(9/4)}*c^{(5/4)}*d^2*e^8 - 5*\text{sqrt}(2)*a^{(11/4)}*c^{(3/4)}*e^{10} + 8*\text{sqrt}(a)*c^3*d^9*e + 48*a^{(3/2)}*c^2*d^5*e^5 - 24*a^{(5/2)}*c*d*e^9)*\arctan(1/2*\text{sqrt}(2)*(2*\text{sqrt}(c)*x + \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)})/\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))/(a^{(3/4)}$

$$\begin{aligned} & * \sqrt{\sqrt{a} \sqrt{c}} * c^{(5/4)} - 2 * (3 * \sqrt{2} * a^{(1/4)} * c^{(13/4)} * d^{10} + 3 * \sqrt{2} * a^{(3/4)} * c^{(11/4)} * d^8 * e^2 + 14 * \sqrt{2} * a^{(5/4)} * c^{(9/4)} * d^6 * e^4 + 30 * \sqrt{2} * a^{(7/4)} * c^{(7/4)} * d^4 * e^6 - 21 * \sqrt{2} * a^{(9/4)} * c^{(5/4)} * d^2 * e^8 - 5 * \sqrt{2} * a^{(11/4)} * c^{(3/4)} * e^{10} - 8 * \sqrt{a} * c^3 * d^9 * e - 48 * a^{(3/2)} * c^2 * d^5 * e^5 + 24 * a^{(5/2)} * c * d * e^9) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x - \sqrt{2} * a^{(1/4)} * c^{(1/4)}) / \sqrt{\sqrt{a} \sqrt{c}}) / (a^{(3/4)} * \sqrt{\sqrt{a} \sqrt{c}} * c^{(5/4)}) / (a * c^3 * d^{12} + 3 * a^2 * c^2 * d^8 * e^4 + 3 * a^3 * c * d^4 * e^8 + a^4 * e^{12}) + 1/4 * (4 * a * c * d^4 * e^3 - 4 * a^2 * e^7 + (3 * c^2 * d^4 * e^3 - 5 * a * c * e^7) * x^4 + (c^2 * d^5 * e^2 + a * c * d * e^6) * x^3 - (c^2 * d^6 * e + a * c * d^2 * e^5) * x^2 + (c^2 * d^7 + a * c * d^3 * e^4) * x) / (a^2 * c^2 * d^9 + 2 * a^3 * c * d^5 * e^4 + a^4 * d * e^8 + (a * c^3 * d^8 * e + 2 * a^2 * c^2 * d^4 * e^5 + a^3 * c * e^9) * x^5 + (a * c^3 * d^9 + 2 * a^2 * c^2 * d^5 * e^4 + a^3 * c * d * e^8) * x^4 + (a^2 * c^2 * d^8 * e + 2 * a^3 * c * d^4 * e^5 + a^4 * e^9) * x) \end{aligned}$$

mupad [B] time = 4.10, size = 2246, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)^2*(d + e*x)^2), x)

[Out] symsum(log(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((120*a*c^8*d^14*e^3 + 2664*a^2*c^7*d^10*e^7 - 10904*a^3*c^6*d^6*e^11 + 19320*a^4*c^5*d^2*e^15)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((4096*a^3*c^8*d^15*e^4 + 54272*a^4*c^7*d^11*e^8 - 2048*a^5*c^6*d^7*e^12 + 144384*a^6*c^5*d^3*e^16)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((98304*a^11*c^4*d*e^22 - 32768*a^6*c^9*d^21*e^2 - 32768*a^7*c^8*d^17*e^6 + 196608*a^8*c^7*d^13*e^10 + 458752*a^9*c^6*d^9*e^14 + 360448*a^10*c^5*d^5*e^18)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(81920*a^11*c^4*e^23 - 49152*a^6*c^9*d^20*e^3 - 114688*a^7*c^8*d^16*e^7 + 32768*a^8*c^7*d^12*e^11 + 294912*a^9*c^6*d^8*e^15 + 278528*a^10*c^5*d^4*e^19))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8))) + (5120*a^9*c^4*e^21 - 3072*a^4*c^9*d^20*e + 17408*a^5*c^8*d^16*e^5 + 337920*a^6*c^7*d^12*e^9 + 616448*a^7*c^6*d^8*e^13 + 304128*a^8*c^5*d^4*e^17)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(356352*a^6*c^7*d^11*e^10 - 32768*a^5*c^8*d^15*e^6 - 10240*a^4*c^9*d^19*e^2 + 770048*a^7*c^6*d^7*e^14 + 391168*a^8*c^5*d^3*e^18))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(768*a^3*c^8*d^14*e^5 - 576*a^2*c^9*d^18*e + 105088*a^4*c^7*d^10*e^9 + 221952*a^5*c^6*d^6*e^13 + 183744*a^6*c^5*d^2*e^17))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(200*a*c^8*d^13*e^4 + 19400*a^4*c^5*d*e^16 + 7512*a^2*c^7*d^9*e^8 + 2136*a^3*c^6*d^5*e^12))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (81*c^7*d^9*e^6 - 254*a*c^6*d^5*e^10 + 625*a^2*c^5*d*e^14)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(6

$$\frac{25a^2c^5e^{15} + 81c^7d^8e^7 - 894aac^6d^4e^{11}}{(256(a^8e^{16} + a^4c^4d^{16} + 4a^7c^4d^4e^{12} + 4a^5c^3d^{12}e^4 + 6a^6c^2d^8e^8))} \sqrt[4]{(196608a^9c^4d^4e^8z^4 + 196608a^8c^2d^8e^4z^4 + 65536a^7c^3d^{12}z^4 + 65536a^{10}e^{12}z^4 + 524288a^7c^3d^3e^7z^3 + 181248a^5c^2d^2e^6z^2 + 17408a^4c^2d^6e^2z^2 + 2304a^2c^2d^5e^5z + 19200a^3c^2d^5e^5z + 625aac^4e^4 + 81c^2d^4e^4, z, k), k, 1, 4} + \left(\frac{x^4(3c^2d^4e^3 - 5aac^4e^7)}{4aa(a^2e^8 + c^2d^8 + 2aac^4e^4)} - \frac{ae^7 - cd^4e^3}{(ae^4 + cd^4)^2} + \frac{cd^3x}{4aa(ae^4 + cd^4)} - \frac{cd^2ex^2}{4aa(ae^4 + cd^4)} + \frac{cd^2ex^3}{4aa(ae^4 + cd^4)} \right) / (ad + aex + cd^4x^4 + cex^5) + \frac{8cd^3e^7 \log(d + ex)}{a^3e^{12} + c^3d^{12} + 3aac^2d^8e^4 + 3a^2cd^4e^8}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)

[Out] Timed out

3.407 $\int \frac{1}{(d+ex)^3(a+cx^4)^2} dx$

Optimal. Leaf size=1384

$$\frac{12cd^2(3cd^4 - ae^4) \log(d + ex)e^7}{(cd^4 + ae^4)^4} - \frac{3cd^2(3cd^4 - ae^4) \log(cx^4 + a)e^7}{(cd^4 + ae^4)^4} - \frac{8cd^3e^7}{(cd^4 + ae^4)^3(d + ex)} - \frac{e^7}{2(cd^4 + ae^4)^2(d + ex)}$$

[Out] $-1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)+1/4*c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8))*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2)/a/(a*e^4+c*d^4)^3/(c*x^4+a)+12*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/4*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^3-1/2*e^5*(a^2*e^8-26*a*c*d^4*e^4+21*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/32*c^(3/4)*d*ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4))*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4))*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*d^4))*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*d^4))*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(3/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*a^2*e^8+30*a*c*d^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4))*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)-1/8*c^(3/4)*d*e^4*ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*a^2*e^8+30*a*c*d^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4))*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(3/4)*d*e^4*arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(3*a^2*e^8-30*a*c*d^4*e^4+15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4))*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(3/4)*d*e^4*arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(3*a^2*e^8-30*a*c*d^4*e^4+15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4))*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)$

Rubi [A] time = 1.96, antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{12cd^2(3cd^4 - ae^4) \log(d + ex)e^7}{(cd^4 + ae^4)^4} - \frac{3cd^2(3cd^4 - ae^4) \log(cx^4 + a)e^7}{(cd^4 + ae^4)^4} - \frac{8cd^3e^7}{(cd^4 + ae^4)^3(d + ex)} - \frac{e^7}{2(cd^4 + ae^4)^2(d + ex)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + c*x^4)^2), x]

[Out] $-e^7/(2*(c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^3*(d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a$

$$\begin{aligned} & \sqrt[7]{4} \cdot (c \cdot d^4 + a \cdot e^4)^3 - (c^{3/4} \cdot d \cdot e^4 \cdot (4 \sqrt{a} \sqrt{c} \cdot d^2 \cdot e^2 \cdot (7 \cdot c \cdot d^4 - 5 \cdot a \cdot e^4) + 3 \cdot (5 \cdot c^2 \cdot d^8 - 10 \cdot a \cdot c \cdot d^4 \cdot e^4 + a^2 \cdot e^8)) \cdot \text{ArcTan}[1 - (\sqrt{2} \cdot c^{1/4} \cdot x) / a^{1/4}]) / (2 \sqrt{2} \cdot a^{3/4} \cdot (c \cdot d^4 + a \cdot e^4)^4) + (c^{3/4} \cdot d \cdot (3 \cdot c^2 \cdot d^8 - 36 \cdot a \cdot c \cdot d^4 \cdot e^4 + 9 \cdot a^2 \cdot e^8 + 2 \sqrt{a} \sqrt{c} \cdot d^2 \cdot e^2 \cdot (3 \cdot c \cdot d^4 - 5 \cdot a \cdot e^4)) \cdot \text{ArcTan}[1 + (\sqrt{2} \cdot c^{1/4} \cdot x) / a^{1/4}]) / (8 \sqrt{2} \cdot a^{7/4} \cdot (c \cdot d^4 + a \cdot e^4)^3) + (c^{3/4} \cdot d \cdot e^4 \cdot (4 \sqrt{a} \sqrt{c} \cdot d^2 \cdot e^2 \cdot (7 \cdot c \cdot d^4 - 5 \cdot a \cdot e^4) + 3 \cdot (5 \cdot c^2 \cdot d^8 - 10 \cdot a \cdot c \cdot d^4 \cdot e^4 + a^2 \cdot e^8)) \cdot \text{ArcTan}[1 + (\sqrt{2} \cdot c^{1/4} \cdot x) / a^{1/4}]) / (2 \sqrt{2} \cdot a^{3/4} \cdot (c \cdot d^4 + a \cdot e^4)^4) + (12 \cdot c \cdot d^2 \cdot e^7 \cdot (3 \cdot c \cdot d^4 - a \cdot e^4) \cdot \text{Log}[d + e \cdot x]) / (c \cdot d^4 + a \cdot e^4)^4 - (c^{3/4} \cdot d \cdot (3 \cdot c^2 \cdot d^8 - 36 \cdot a \cdot c \cdot d^4 \cdot e^4 + 9 \cdot a^2 \cdot e^8 - 2 \sqrt{a} \sqrt{c} \cdot d^2 \cdot e^2 \cdot (3 \cdot c \cdot d^4 - 5 \cdot a \cdot e^4)) \cdot \text{Log}[\sqrt{a} - \sqrt{2} \cdot a^{1/4} \cdot c^{1/4} \cdot x + \sqrt{c} \cdot x^2]) / (16 \sqrt{2} \cdot a^{7/4} \cdot (c \cdot d^4 + a \cdot e^4)^3) + (c^{3/4} \cdot d \cdot e^4 \cdot (4 \sqrt{a} \sqrt{c} \cdot d^2 \cdot e^2 \cdot (7 \cdot c \cdot d^4 - 5 \cdot a \cdot e^4) - 3 \cdot (5 \cdot c^2 \cdot d^8 - 10 \cdot a \cdot c \cdot d^4 \cdot e^4 + a^2 \cdot e^8)) \cdot \text{Log}[\sqrt{a} - \sqrt{2} \cdot a^{1/4} \cdot c^{1/4} \cdot x + \sqrt{c} \cdot x^2]) / (4 \sqrt{2} \cdot a^{3/4} \cdot (c \cdot d^4 + a \cdot e^4)^4) + (c^{3/4} \cdot d \cdot (3 \cdot c^2 \cdot d^8 - 36 \cdot a \cdot c \cdot d^4 \cdot e^4 + 9 \cdot a^2 \cdot e^8 - 2 \sqrt{a} \sqrt{c} \cdot d^2 \cdot e^2 \cdot (3 \cdot c \cdot d^4 - 5 \cdot a \cdot e^4)) \cdot \text{Log}[\sqrt{a} + \sqrt{2} \cdot a^{1/4} \cdot c^{1/4} \cdot x + \sqrt{c} \cdot x^2]) / (16 \sqrt{2} \cdot a^{7/4} \cdot (c \cdot d^4 + a \cdot e^4)^3) - (c^{3/4} \cdot d \cdot e^4 \cdot (4 \sqrt{a} \sqrt{c} \cdot d^2 \cdot e^2 \cdot (7 \cdot c \cdot d^4 - 5 \cdot a \cdot e^4) - 3 \cdot (5 \cdot c^2 \cdot d^8 - 10 \cdot a \cdot c \cdot d^4 \cdot e^4 + a^2 \cdot e^8)) \cdot \text{Log}[\sqrt{a} + \sqrt{2} \cdot a^{1/4} \cdot c^{1/4} \cdot x + \sqrt{c} \cdot x^2]) / (4 \sqrt{2} \cdot a^{3/4} \cdot (c \cdot d^4 + a \cdot e^4)^4) - (3 \cdot c \cdot d^2 \cdot e^7 \cdot (3 \cdot c \cdot d^4 - a \cdot e^4) \cdot \text{Log}[a + c \cdot x^4]) / (c \cdot d^4 + a \cdot e^4)^4 \end{aligned}$$
Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
```

$a + c*x^2$), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1854

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3 (a+cx^4)^2} dx &= \int \left(\frac{e^8}{(cd^4+ae^4)^2 (d+ex)^3} + \frac{8cd^3e^8}{(cd^4+ae^4)^3 (d+ex)^2} + \frac{12cd^2e^8(3cd^4-ae^4)}{(cd^4+ae^4)^4 (d+ex)} + \frac{c(d+ex)}{(cd^4+ae^4)^5} \right) dx \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{12cd^2e^7(3cd^4-ae^4)\log(d+ex)}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d+ex))}{(cd^4+ae^4)^4}
\end{aligned}$$

Mathematica [A] time = 1.36, size = 996, normalized size = 0.72

$$\frac{384cd^2(3cd^4-ae^4)\log(d+ex)e^7 - 96cd^2(3cd^4-ae^4)\log(cx^4+a)e^7 - \frac{256cd^3(cd^4+ae^4)e^7}{d+ex} - \frac{16(cd^4+ae^4)^2e^7}{(d+ex)^2} + \frac{8c(cd^4+ae^4)}{(d+ex)^3}}{(d+ex)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d+e*x)^3*(a+c*x^4)^2),x]

[Out] ((-16*e^7*(c*d^4+a*e^4)^2)/(d+e*x)^2 - (256*c*d^3*e^7*(c*d^4+a*e^4))/(d+e*x) + (8*c*(c*d^4+a*e^4)*(-(a^2*e^7*(6*d^2-3*d*e*x+e^2*x^2))+c^2*d^7*x*(d^2-3*d*e*x+6*e^2*x^2)+2*a*c*d^3*e^3*(5*d^3-6*d^2*e*x+6*d*e^2*x^2-5*e^3*x^3)))/(a*(a+c*x^4)) - (6*sqrt[c]*(sqrt[2]*c^(13/4)*d^13-4*a^(1/4)*c^3*d^12*e+2*sqrt[2]*sqrt[a]*c^(11/4)*d^11*e^2+9*sqrt[2]*a*c^(9/4)*d^9*e^4-44*a^(5/4)*c^2*d^8*e^5+36*sqrt[2]*a^(3/2)*c^(7/4)*d^7*e^6)/(a*(a+c*x^4))

$$\begin{aligned} & ^7e^6 - 49\sqrt{2}a^2c^{5/4}d^5e^8 + 84a^{9/4}cd^4e^9 - 30\sqrt{2} \\ & a^{5/2}c^{3/4}d^3e^{10} + 7\sqrt{2}a^3c^{1/4}de^{12} - 4a^{13/4}e^{13} \\ & \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}c^{1/4}x}{a^{1/4}}\right]/a^{7/4} + (6\sqrt{2}c^{1/4}(\sqrt{2}c^{1/4}d^{13} + 4a^{1/4}c^3d^{12}e + 2\sqrt{2}a^{1/4}c^{11/4}d^{11}e^2 + 9 \\ & \sqrt{2}a^{9/4}d^9e^4 + 44a^{5/4}c^2d^8e^5 + 36\sqrt{2}a^{3/2}c^{7/4}d^7e^6 - 49\sqrt{2}a^2c^{5/4}d^5e^8 - 84a^{9/4}cd^4e^9 - 30 \\ & \sqrt{2}a^{5/2}c^{3/4}d^3e^{10} + 7\sqrt{2}a^3c^{1/4}de^{12} + 4a^{13/4}e^{13})\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}c^{1/4}x}{a^{1/4}}\right]/a^{7/4} + 384cd^2e^7(3 \\ & cd^4 - ae^4)\operatorname{Log}[d + ex] - (3\sqrt{2}c^{3/4}(c^3d^{13} - 2\sqrt{2}a^{5/2}d^{11}e^2 + 9a^2c^2d^9e^4 - 36a^{3/2}c^{3/2}d^7e^6 - 49a^2cd^5 \\ & e^8 + 30a^{5/2}\sqrt{2}cd^3e^{10} + 7a^3d^5e^{12})\operatorname{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/a^{7/4} + (3\sqrt{2}c^{3/4}(c^3d^{13} - 2 \\ & \sqrt{2}a^{5/2}d^{11}e^2 + 9a^2c^2d^9e^4 - 36a^{3/2}c^{3/2}d^7e^6 - 49a^2cd^5e^8 + 30a^{5/2}\sqrt{2}cd^3e^{10} + 7a^3d^5e^{12})\operatorname{Log}[\sqrt{a} + \\ & \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/a^{7/4} - 96cd^2e^7(3cd^4 - ae^4)\operatorname{Log}[a + cx^4]/(32(cd^4 + ae^4)^4) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.15, size = 1488, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")

$$\begin{aligned} & [Out] \frac{3}{8}(2\sqrt{2}\sqrt{ac})c^3d^4e + (ac^3)^{1/4}c^3d^5 + 4\sqrt{2}ac^3d^2e^3 + 2(ac^3)^{3/4}cd^3e^2 - 9(ac^3)^{1/4}ac^2d^4e + 2\sqrt{2} \\ & \operatorname{arctan}\left(\frac{1}{2}\sqrt{2}\sqrt{ac}\sqrt{2x + \sqrt{2}a/c}\right)/(a/c)^{1/4} / (\sqrt{2}a^2c^4d^8 - 8(ac^3)^{1/4}a^2c^3d^7e + 16\sqrt{2} \\ & \sqrt{ac})a^2c^3d^6e^2 + 34\sqrt{2}a^3c^3d^4e^4 - 40(ac^3)^{3/4}a^2cd^5e^3 - 40(ac^3)^{1/4}a^3c^2d^3e^5 + 16\sqrt{2}\sqrt{ac}a^3c^2d^2e^6 + \sqrt{2}a^4c^2e^8 - 8(ac^3)^{3/4}a^3d^7e^7 + 3/8(2\sqrt{2} \\ & \sqrt{2}\sqrt{ac})c^3d^4e + (ac^3)^{1/4}c^3d^5 - 4\sqrt{2}ac^3d^2e^3 + 2(ac^3)^{3/4}cd^3e^2 - 9(ac^3)^{1/4}ac^2d^4e + 2\sqrt{2} \\ & \operatorname{arctan}\left(\frac{1}{2}\sqrt{2}\sqrt{ac}\sqrt{2x - \sqrt{2}a/c}\right)/(a/c)^{1/4} / (\sqrt{2}a^2c^4d^8 + 8(ac^3)^{1/4}a^2c^3d^7e + 16\sqrt{2}\sqrt{ac} \\ &)a^2c^3d^6e^2 + 34\sqrt{2}a^3c^3d^4e^4 + 40(ac^3)^{3/4}a^2cd^5e^3 + 40(ac^3)^{1/4}a^3c^2d^3e^5 + 16\sqrt{2}\sqrt{ac}a^3c^2d^2e^6 + \sqrt{2}a^4c^2e^8 + 8(ac^3)^{3/4}a^3d^7e^7 + 3/32(\sqrt{2} \\ &)^{1/4}c^4d^{13} - 2\sqrt{2}(ac^3)^{3/4}c^2d^{11}e^2 + 9\sqrt{2}(ac^3)^{1/4}ac^3d^9e^4 - 36\sqrt{2}(ac^3)^{3/4}ac^2d^7e^6 - 49\sqrt{2}(ac^3)^{1/4}a^2c^2d^5e^8 + 30\sqrt{2}(ac^3)^{3/4}a^2d^3e^{10} + 7 \\ & \sqrt{2}(ac^3)^{1/4}a^3cd^5e^{12})\operatorname{log}(x^2 + \sqrt{2}x\sqrt{a/c}) + \sqrt{2} \\ & \operatorname{log}\left(\frac{x^2 - \sqrt{2}x\sqrt{a/c} + \sqrt{a/c}}{a^2c^5d^{16} + 4a^3c^4d^{12}e^4 + 6a^4c^3d^8e^8 + 4a^5c^2d^4e^{12} + a^6c^2e^{16}}\right) - 3/32(\sqrt{2})^{1/4}c^4d^{13} - 2\sqrt{2}(ac^3)^{3/4}c^2d^{11}e^2 + 9\sqrt{2}(ac^3)^{1/4}ac^3d^9e^4 - 36\sqrt{2} \\ & (ac^3)^{3/4}ac^2d^7e^6 - 49\sqrt{2}(ac^3)^{1/4}a^2c^2d^5e^8 + 30\sqrt{2}(ac^3)^{3/4}a^2d^3e^{10} + 7\sqrt{2}(ac^3)^{1/4}a^3cd^5e^{12})\operatorname{log}(x^2 - \sqrt{2}x\sqrt{a/c}) + \sqrt{2} \\ & \operatorname{log}\left(\frac{x^2 - \sqrt{2}x\sqrt{a/c} + \sqrt{a/c}}{a^2c^5d^{16} + 4a^3c^4d^{12}e^4 + 6a^4c^3d^8e^8 + 4a^5c^2d^4e^{12} + a^6c^2e^{16}}\right) - 3(3c^2d^6e^7 - ac^2d^8e^8 + 4a^3cd^4e^{12} + a^4e^{16}) + 12(3c^2d^6e^8 - ac^2d^8e^8 + 4a^3cd^4e^{12} + a^4e^{16}) + 12(3c^2d^6e^8 - ac^2d^8e^8 + 4a^3cd^4e^{12} + a^4e^{16}) \end{aligned}$$

$$2e^{12} \log(\text{abs}(xe + d)) / (c^4 d^{16} e + 4a^3 c^3 d^{12} e^5 + 6a^2 c^2 d^8 e^9 + 4a^3 c d^4 e^{13} + a^4 e^{17}) + 1/4 (10a^3 c^3 d^{12} e^3 - 30a^2 c^2 d^8 e^7 - 42a^3 c d^4 e^{11} + 6(c^4 d^{11} e^4 - 6a^3 c^3 d^7 e^8 - 7a^2 c^2 d^3 e^{12})) x^5 + 3(3c^4 d^{12} e^3 - 11a^3 c^3 d^8 e^7 - 15a^2 c^2 d^4 e^{11} - a^3 c e^{15}) x^4 - 2a^4 e^{15} + (c^4 d^{13} e^2 + 3a^3 c^3 d^9 e^6 + 3a^2 c^2 d^5 e^{10} + a^3 c d e^{14}) x^3 - (c^4 d^{14} e + 3a^3 c^3 d^{10} e^5 + 3a^2 c^2 d^6 e^9 + a^3 c d^2 e^{13}) x^2 + (c^4 d^{15} + 9a^3 c^3 d^{11} e^4 - 33a^2 c^2 d^7 e^8 - 41a^3 c d^3 e^{12}) x / ((c^4 d^4 + a e^4)^4 (c x^4 + a) (x e + d)^2 a)$$

maple [A] time = 0.03, size = 2121, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x+d)^3/(c*x^4+a)^2, x)$

[Out]
$$\begin{aligned} & -1/2 e^7 / (a e^4 + c d^4)^2 / (e x + d)^2 - 8 c d^3 e^7 / (a e^4 + c d^4)^3 / (e x + d) - 12 e^{11} d^2 c / (a e^4 + c d^4)^4 \ln(e x + d) a^{-1/4} c / (a e^4 + c d^4)^4 / (c x^4 + a) e^{13} a^2 x^2 - 3/2 c / (a e^4 + c d^4)^4 / (c x^4 + a) e^{11} d^2 a^2 - c^3 / (a e^4 + c d^4)^4 / (c x^4 + a) d^7 e^6 x^3 + 9/4 c^3 / (a e^4 + c d^4)^4 / (c x^4 + a) e^5 x^2 d^8 - 11/4 c^3 / (a e^4 + c d^4)^4 / (c x^4 + a) d^9 x e^4 + 1/4 c^4 / (a e^4 + c d^4)^4 / (c x^4 + a) d^{13} / a x + c^2 / (a e^4 + c d^4)^4 / (c x^4 + a) a d^6 e^7 - 33/4 c^3 / (a e^4 + c d^4)^4 / (a c)^{(1/2)} \arctan((1/a c)^{(1/2)} x^2) e^5 d^8 - 3/4 c / (a e^4 + c d^4)^4 a^2 / (a c)^{(1/2)} \arctan((1/a c)^{(1/2)} x^2) e^{13} + 3 c / (a e^4 + c d^4)^4 a \ln(c x^4 + a) e^{11} d^2 + 36 e^7 d^6 c^2 / (a e^4 + c d^4)^4 \ln(e x + d) + 5/2 c^3 / (a e^4 + c d^4)^4 / (c x^4 + a) d^{10} e^3 - 9 c^2 / (a e^4 + c d^4)^4 \ln(c x^4 + a) e^7 d^6 + 27/16 c^3 / (a e^4 + c d^4)^4 / a (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x + 1) d^9 e^4 + 21/32 c / (a e^4 + c d^4)^4 a (a/c)^{(1/4)} 2^{(1/2)} \ln((x^2 + (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)})) d e^{12} + 21/16 c / (a e^4 + c d^4)^4 a (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x + 1) d e^{12} - 45/16 c / (a e^4 + c d^4)^4 a / (a/c)^{(1/4)} 2^{(1/2)} \ln((x^2 - (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)})) d^3 e^{10} - 45/8 c / (a e^4 + c d^4)^4 a / (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x - 1) d^3 e^{10} - 45/8 c / (a e^4 + c d^4)^4 a / (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x + 1) d^3 e^{10} - 147/16 c^2 / (a e^4 + c d^4)^4 (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x + 1) d^5 e^8 + 27/8 c^2 / (a e^4 + c d^4)^4 / (a/c)^{(1/4)} 2^{(1/2)} \ln((x^2 - (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)})) d^7 e^6 + 27/4 c^2 / (a e^4 + c d^4)^4 / (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x - 1) d^7 e^6 + 27/4 c^2 / (a e^4 + c d^4)^4 / (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x + 1) d^7 e^6 + 63/4 c^2 / (a e^4 + c d^4)^4 a / (a c)^{(1/2)} \arctan((1/a c)^{(1/2)} x^2) e^9 d^4 + 3/16 c^4 / (a e^4 + c d^4)^4 a^2 (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x - 1) d^{13} + 3/32 c^4 / (a e^4 + c d^4)^4 a^2 (a/c)^{(1/4)} 2^{(1/2)} \ln((x^2 + (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)})) d^{13} + 3/16 c^4 / (a e^4 + c d^4)^4 a^2 (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x + 1) d^{13} - 3/4 c^4 / (a e^4 + c d^4)^4 a / (a c)^{(1/2)} \arctan((1/a c)^{(1/2)} x^2) e d^{12} - 147/16 c^2 / (a e^4 + c d^4)^4 (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x - 1) d^5 e^8 - 147/32 c^2 / (a e^4 + c d^4)^4 (a/c)^{(1/4)} 2^{(1/2)} \ln((x^2 + (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)})) d^5 e^8 - 5/2 c^2 / (a e^4 + c d^4)^4 / (c x^4 + a) d^3 e^{10} a x^3 + 3/2 c^4 / (a e^4 + c d^4)^4 / (c x^4 + a) d^{11} e^2 a x^3 + 11/4 c^2 / (a e^4 + c d^4)^4 / (c x^4 + a) e^9 a x^2 d^4 - 3/4 c^4 / (a e^4 + c d^4)^4 / (c x^4 + a) e / a x^2 d^{12} - 9/4 c^2 / (a e^4 + c d^4)^4 / (c x^4 + a) d^5 a x e^8 + 3/4 c / (a e^4 + c d^4)^4 / (c x^4 + a) d a^2 x e^{12} + 3/16 c^3 / (a e^4 + c d^4)^4 a / (a/c)^{(1/4)} 2^{(1/2)} \ln((x^2 - (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)})) d^{11} e^2 + 3/8 c^3 / (a e^4 + c d^4)^4 a / (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x - 1) d^{11} e^2 + 3/8 c^3 / (a e^4 + c d^4)^4 a / (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x + 1) d^{11} e^2 + 27/16 c^3 / (a e^4 + c d^4)^4 a (a/c)^{(1/4)} 2^{(1/2)} \arctan(2^{(1/2)} / (a/c)^{(1/4)} x - 1) d^9 e^4 + 27/32 c^3 / (a e^4 + c d^4)^4 a (a/c)^{(1/4)} 2^{(1/2)} \ln((x^2 + (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)})) / (x^2 - (a/c)^{(1/4)} 2^{(1/2)} x + (a/c)^{(1/2)}) \end{aligned}$$

$*x+(a/c)^{(1/2)}) * d^9 * e^4 + 21/16 * c / (a * e^4 + c * d^4)^4 * a * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * d * e^{12}$

maxima [A] time = 2.60, size = 1394, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")

[Out]
$$-3/32 * c * (\sqrt{2}) * (48 * \sqrt{2}) * a^{(7/4)} * c^{(9/4)} * d^6 * e^7 - 16 * \sqrt{2} * a^{(11/4)} * c^{(5/4)} * d^2 * e^{11} - c^4 * d^{13} + 2 * \sqrt{a} * c^{(7/2)} * d^{11} * e^2 - 9 * a * c^3 * d^9 * e^4 + 36 * a^{(3/2)} * c^{(5/2)} * d^7 * e^6 + 49 * a^2 * c^2 * d^5 * e^8 - 30 * a^{(5/2)} * c^{(3/2)} * d^3 * e^{10} - 7 * a^3 * c * d * e^{12} * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a})) / (a^{(3/4)} * c^{(5/4)}) + \sqrt{2} * (48 * \sqrt{2}) * a^{(7/4)} * c^{(9/4)} * d^6 * e^7 - 16 * \sqrt{2} * a^{(11/4)} * c^{(5/4)} * d^2 * e^{11} + c^4 * d^{13} - 2 * \sqrt{a} * c^{(7/2)} * d^{11} * e^2 + 9 * a * c^3 * d^9 * e^4 - 36 * a^{(3/2)} * c^{(5/2)} * d^7 * e^6 - 49 * a^2 * c^2 * d^5 * e^8 + 30 * a^{(5/2)} * c^{(3/2)} * d^3 * e^{10} + 7 * a^3 * c * d * e^{12} * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a})) / (a^{(3/4)} * c^{(5/4)}) - 2 * (\sqrt{2} * a^{(1/4)} * c^{(17/4)} * d^{13} + 2 * \sqrt{2} * a^{(3/4)} * c^{(15/4)} * d^{11} * e^2 + 9 * \sqrt{2} * a^{(5/4)} * c^{(13/4)} * d^9 * e^4 + 36 * \sqrt{2} * a^{(7/4)} * c^{(11/4)} * d^7 * e^6 - 49 * \sqrt{2} * a^{(9/4)} * c^{(9/4)} * d^5 * e^8 - 30 * \sqrt{2} * a^{(11/4)} * c^{(7/4)} * d^3 * e^{10} + 7 * \sqrt{2} * a^{(13/4)} * c^{(5/4)} * d * e^{12} + 4 * \sqrt{2} * a * c^4 * d^{12} * e + 44 * a^{(3/2)} * c^3 * d^8 * e^5 - 84 * a^{(5/2)} * c^2 * d^4 * e^9 + 4 * a^{(7/2)} * c * e^{13} * \arctan(1/2 * \sqrt{2}) * (2 * \sqrt{c} * x + \sqrt{2} * a^{(1/4)} * c^{(1/4)}) / \sqrt{2} * \sqrt{a} * \sqrt{c})) / (a^{(3/4)} * \sqrt{2} * \sqrt{a} * \sqrt{c}) * c^{(5/4)}) - 2 * (\sqrt{2} * a^{(1/4)} * c^{(17/4)} * d^{13} + 2 * \sqrt{2} * a^{(3/4)} * c^{(15/4)} * d^{11} * e^2 + 9 * \sqrt{2} * a^{(5/4)} * c^{(13/4)} * d^9 * e^4 + 36 * \sqrt{2} * a^{(7/4)} * c^{(11/4)} * d^7 * e^6 - 49 * \sqrt{2} * a^{(9/4)} * c^{(9/4)} * d^5 * e^8 - 30 * \sqrt{2} * a^{(11/4)} * c^{(7/4)} * d^3 * e^{10} + 7 * \sqrt{2} * a^{(13/4)} * c^{(5/4)} * d * e^{12} - 4 * \sqrt{2} * a * c^4 * d^{12} * e - 44 * a^{(3/2)} * c^3 * d^8 * e^5 + 84 * a^{(5/2)} * c^2 * d^4 * e^9 - 4 * a^{(7/2)} * c * e^{13} * \arctan(1/2 * \sqrt{2}) * (2 * \sqrt{c} * x - \sqrt{2} * a^{(1/4)} * c^{(1/4)}) / \sqrt{2} * \sqrt{a} * \sqrt{c})) / (a^{(3/4)} * \sqrt{2} * \sqrt{a} * \sqrt{c}) * c^{(5/4)}) / (a * c^4 * d^{16} + 4 * a^2 * c^3 * d^{12} * e^4 + 6 * a^3 * c^2 * d^8 * e^8 + 4 * a^4 * c * d^4 * e^{12} + a^5 * e^{16}) + 12 * (3 * c^2 * d^6 * e^7 - a * c * d^2 * e^{11}) * \log(e * x + d) / (c^4 * d^{16} + 4 * a * c^3 * d^{12} * e^4 + 6 * a^2 * c^2 * d^8 * e^8 + 4 * a^3 * c * d^4 * e^{12} + a^4 * e^{16}) + 1/4 * (10 * a * c^2 * d^8 * e^3 - 40 * a^2 * c * d^4 * e^7 - 2 * a^3 * e^{11} + 6 * (c^3 * d^7 * e^4 - 7 * a * c^2 * d^3 * e^8) * x^5 + 3 * (3 * c^3 * d^8 * e^3 - 14 * a * c^2 * d^4 * e^7 - a^2 * c * e^{11}) * x^4 + (c^3 * d^9 * e^2 + 2 * a * c^2 * d^5 * e^6 + a^2 * c * d * e^{10}) * x^3 - (c^3 * d^{10} * e + 2 * a * c^2 * d^6 * e^5 + a^2 * c * d^2 * e^9) * x^2 + (c^3 * d^{11} + 8 * a * c^2 * d^7 * e^4 - 41 * a^2 * c * d^3 * e^8) * x) / (a^2 * c^3 * d^{14} + 3 * a^3 * c^2 * d^{10} * e^4 + 3 * a^4 * c * d^6 * e^8 + a^5 * d^2 * e^{12} + (a * c^4 * d^{12} * e^2 + 3 * a^2 * c^3 * d^8 * e^6 + 3 * a^3 * c^2 * d^4 * e^{10} + a^4 * c * e^{14}) * x^6 + 2 * (a * c^4 * d^{13} * e + 3 * a^2 * c^3 * d^9 * e^5 + 3 * a^3 * c^2 * d^5 * e^9 + a^4 * c * d * e^{13}) * x^5 + (a * c^4 * d^{14} + 3 * a^2 * c^3 * d^{10} * e^4 + 3 * a^3 * c^2 * d^6 * e^8 + a^4 * c * d^2 * e^{12}) * x^4 + (a^2 * c^3 * d^{12} * e^2 + 3 * a^3 * c^2 * d^8 * e^6 + 3 * a^4 * c * d^4 * e^{10} + a^5 * e^{14}) * x^2 + 2 * (a^2 * c^3 * d^{13} * e + 3 * a^3 * c^2 * d^9 * e^5 + 3 * a^4 * c * d^5 * e^9 + a^5 * d * e^{13}) * x)$$

mupad [B] time = 5.04, size = 3256, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)^2*(d + e*x)^3),x)

[Out]
$$\text{symsum}(\log(\text{root}(262144 * a^{10} * c * d^4 * e^{12} * z^4 + 393216 * a^9 * c^2 * d^8 * e^8 * z^4 + 262144 * a^8 * c^3 * d^{12} * e^4 * z^4 + 65536 * a^7 * c^4 * d^{16} * z^4 + 65536 * a^{11} * e^{16} * z^4 - 786432 * a^8 * c * d^2 * e^{11} * z^3 + 2359296 * a^7 * c^2 * d^6 * e^7 * z^3 + 755712 * a^5 * c^2 * d^4 * e^6 * z^2 + 36864 * a^4 * c^3 * d^8 * e^2 * z^2 + 18432 * a^6 * c * e^{10} * z^2 + 58752 * a^3 * c^2 * d^2 * e^5 * z + 3456 * a^2 * c^3 * d^6 * e * z + 1296 * a * c^2 * e^4 + 81 * c^3 * d^4, z, k)) * ((108 * a * c^{10} * d^{19} * e^3 + 3888 * a^2 * c^9 * d^{15} * e^7 - 99576 * a^3 * c^8 * d^{11} * e^{11} + 591408 * a^4 * c^7 * d^7 * e^{15} - 79380 * a^5 * c^6 * d^3 * e^{19}) / (256 * (a^{10} * e^{24} + a^4 * c^6 * d^4 * e^{16})))$$

$$\begin{aligned}
& 24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16) + \text{root}(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((6912*a^8*c^5*d*e^24 + 4608*a^3*c^10*d^21*e^4 + 154368*a^4*c^9*d^17*e^8 - 331776*a^5*c^8*d^13*e^12 + 5976576*a^6*c^7*d^9*e^16 - 612864*a^7*c^6*d^5*e^20)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + \text{root}(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((18432*a^5*c^10*d^23*e^5 - 3072*a^4*c^11*d^27*e + 1170432*a^6*c^9*d^19*e^9 + 2863104*a^7*c^8*d^15*e^13 + 1797120*a^8*c^7*d^11*e^17 - 423936*a^9*c^6*d^7*e^21 - 506880*a^10*c^5*d^3*e^25)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + \text{root}(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((98304*a^13*c^4*d*e^30 - 32768*a^6*c^11*d^29*e^2 - 98304*a^7*c^10*d^25*e^6 + 98304*a^8*c^9*d^21*e^10 + 819200*a^9*c^8*d^17*e^14 + 1474560*a^10*c^7*d^13*e^18 + 1277952*a^11*c^6*d^9*e^22 + 557056*a^12*c^5*d^5*e^26)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + (x*(81920*a^13*c^4*e^31 - 49152*a^6*c^11*d^28*e^3 - 212992*a^7*c^10*d^24*e^7 - 245760*a^8*c^9*d^20*e^11 + 245760*a^9*c^8*d^16*e^15 + 901120*a^10*c^7*d^12*e^19 + 933888*a^11*c^6*d^8*e^23 + 442368*a^12*c^5*d^4*e^27))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) - (x*(12288*a^4*c^11*d^26*e^2 + 98304*a^5*c^10*d^22*e^6 - 1413120*a^6*c^9*d^18*e^10 - 4030464*a^7*c^8*d^14*e^14 - 2813952*a^8*c^7*d^10*e^18 + 393216*a^9*c^6*d^6*e^22 + 675840*a^10*c^5*d^2*e^26))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(20736*a^8*c^5*e^25 - 576*a^2*c^11*d^24*e - 576*a^3*c^10*d^20*e^5 + 484992*a^4*c^9*d^16*e^9 + 2468736*a^5*c^8*d^12*e^13 + 4093632*a^6*c^7*d^8*e^17 - 228672*a^7*c^6*d^4*e^21))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(216*a*c^10*d^18*e^4 + 25056*a^2*c^9*d^14*e^8 - 2160*a^3*c^8*d^10*e^12 + 59616*a^4*c^7*d^6*e^16 + 86616*a^5*c^6*d^2*e^20))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (81*c^9*d^13*e^6 - 2430*a*c^8*d^9*e^10 + 1296*a^3*c^6*d*e^18 + 3969*a^2*c^7*d^5*e^14)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(1296*a^3*c^6*e^19 + 81*c^9*d^12*e^7 - 6318*a*c^8*d^8*e^11 + 5265*a^2*c^7*d^4*e^15))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) * \text{root}(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k), k, 1, 4) - ((a^2*e^11 - 5*c^2*d^8*e^3 + 20*a*c*d^4*e^7)/(2*(a*e^4 + c*d^4))*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^5*(c^3*d^7*e^4 - 7*a*c
\end{aligned}$$

$$\begin{aligned} & \frac{2d^3e^8}{(2a(a^3e^{12} + c^3d^{12} + 3ac^2d^8e^4 + 3a^2cd^4e^8))} \\ & + \frac{(3x^4(a^2ce^{11} - 3c^3d^8e^3 + 14ac^2d^4e^7))}{(4a(a^3e^{12} + c^3d^{12} + 3ac^2d^8e^4 + 3a^2cd^4e^8))} + \frac{(cd^2ex^2)}{(4a(ae^4 + cd^4))} \\ & - \frac{(cd^2ex^3)}{(4a(ae^4 + cd^4))} - \frac{(dx(c^3d^{10} + 8ac^2d^6e^4 - 41a^2cd^2e^8))}{(4a(ae^4 + cd^4)(a^2e^8 + c^2d^8 + 2ac^2d^4e^4))} \\ & \frac{1}{(ad^2 + ae^2x^2 + cd^2x^4 + ce^2x^6 + 2adex + 2c^2dex^5)} + \frac{(\log(d + ex)(36c^2d^6e^7 - 12ac^2d^2e^{11}))}{(a^4e^{16} + c^4d^{16} + 4ac^3d^{12}e^4 + 4a^3cd^4e^{12} + 6a^2c^2d^8e^8)} \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)

[Out] Timed out

$$3.408 \quad \int \frac{(d+ex)^3}{(a+cx^4)^3} dx$$

Optimal. Leaf size=394

$$\frac{3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}}$$

[Out] $\frac{1}{32}x(15d^2e^2x^2 + 18d^2ex + 7d^3)/a^2/(cx^4+a) + \frac{1}{8}(-ae^3 + cxx(3d^2e^2x^2 + 3d^2ex + d^3))/a/c/(cx^4+a)^2 + \frac{9}{16}d^2e \arctan(x^2c^{1/2}/a^{1/2})/a^{5/2}/c^{1/2} - \frac{3}{256}d \ln(-a^{1/4}c^{1/4}xx^{2^{1/2}} + a^{1/2} + x^2c^{1/2}) * (-5e^2a^{1/2} + 7d^2c^{1/2})/a^{11/4}/c^{3/4} * 2^{1/2} + \frac{3}{256}d \ln(a^{1/4}c^{1/4}xx^{2^{1/2}} + a^{1/2} + x^2c^{1/2}) * (-5e^2a^{1/2} + 7d^2c^{1/2})/a^{11/4}/c^{3/4} * 2^{1/2} + \frac{3}{128}d \arctan(-1 + c^{1/4}xx^{2^{1/2}}/a^{1/4}) * (5e^2a^{1/2} + 7d^2c^{1/2})/a^{11/4}/c^{3/4} * 2^{1/2} + \frac{3}{128}d \arctan(1 + c^{1/4}xx^{2^{1/2}}/a^{1/4}) * (5e^2a^{1/2} + 7d^2c^{1/2})/a^{11/4}/c^{3/4} * 2^{1/2}$

Rubi [A] time = 0.35, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + c*x^4)^3,x]

[Out] $\frac{x(7d^3 + 18d^2ex + 15d^2e^2x^2)}{(32a^2(a + cx^4))} - \frac{(ae^3 - cxx(d^3 + 3d^2ex + 3d^2e^2x^2))}{(8a^2c(a + cx^4)^2)} + \frac{9d^2e \operatorname{ArcTan}(\frac{\sqrt{c}x^2}{\sqrt{a}})}{(16a^{5/2}\sqrt{c})} - \frac{(3d(7\sqrt{c}d^2 + 5\sqrt{a}e^2) \operatorname{ArcTan}[1 - (\sqrt{2}c^{1/4}x)/a^{1/4}])}{(64\sqrt{2}a^{11/4}c^{3/4})} + \frac{(3d(7\sqrt{c}d^2 + 5\sqrt{a}e^2) \operatorname{ArcTan}[1 + (\sqrt{2}c^{1/4}x)/a^{1/4}])}{(64\sqrt{2}a^{11/4}c^{3/4})} - \frac{(3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \operatorname{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])}{(128\sqrt{2}a^{11/4}c^{3/4})} + \frac{(3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \operatorname{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])}{(128\sqrt{2}a^{11/4}c^{3/4})}$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{(a+cx^4)^3} dx &= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a+cx^4)^2} - \frac{\int \frac{-7d^3 - 18d^2ex - 15de^2x^2}{(a+cx^4)^2} dx}{8a} \\
&= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a+cx^4)^2} + \frac{\int \frac{21d^3 + 36d^2ex + 15de^2x^2}{a+cx^4} dx}{32a^2} \\
&= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a+cx^4)^2} + \frac{\int \left(\frac{36d^2ex}{a+cx^4} + \frac{21d^3 + 15de^2x^2}{a+cx^4} \right) dx}{32a^2} \\
&= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a+cx^4)^2} + \frac{\int \frac{21d^3 + 15de^2x^2}{a+cx^4} dx}{32a^2} + \frac{(9d^2e) \int \frac{1}{a+cx^2} dx}{8a} \\
&= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a+cx^4)^2} + \frac{(9d^2e) \text{Subst}\left(\int \frac{1}{a+cx^2} dx, x, \sqrt{cx^2}\right)}{16a^2} \\
&= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a+cx^4)^2} + \frac{9d^2e \tan^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(\frac{7\sqrt{c}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} \\
&= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a+cx^4)^2} + \frac{9d^2e \tan^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(\frac{7\sqrt{c}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} \\
&= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a+cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a+cx^4)^2} + \frac{9d^2e \tan^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d(7\sqrt{c})}{16a^{5/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 388, normalized size = 0.98

$$\frac{3\sqrt{2}(5a^{3/4}de^2 - 7\sqrt[4]{a}\sqrt{c}d^3)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{cx^2})}{c^{3/4}} + \frac{3\sqrt{2}(7\sqrt[4]{a}\sqrt{c}d^3 - 5a^{3/4}de^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{cx^2})}{c^{3/4}} - \frac{32a^2(ae^3 - cdx(d^2 + 3dex + 3de^2x^2))}{c(a+cx^4)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + c*x^4)^3, x]

[Out] ((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.35, size = 389, normalized size = 0.99

$$\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e + 7(ac^3)^{\frac{1}{4}}c^2d^3 + 5(ac^3)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")

[Out] 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2*d*x^7*e^2 + 18*c^2*d^2*x^6*e + 7*c^2*d^3*x^5 + 27*a*c*d*x^3*e^2 + 30*a*c*d^2*x^2*e + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)

maple [A] time = 0.01, size = 470, normalized size = 1.19

$$\frac{e^3x^4}{8(c^4x^4 + a)^2} + \frac{3de^2x^3}{8(c^4x^4 + a)^2} + \frac{e^3x^4}{8(c^4x^4 + a)^2} + \frac{3d^2ex^2}{8(c^4x^4 + a)^2} + \frac{15de^2x^3}{32(c^4x^4 + a)^2} + \frac{d^3x}{8(c^4x^4 + a)^2} + \frac{9d^2ex^2}{16(c^4x^4 + a)^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3/(c*x^4+a)^3,x)

[Out] 1/8*d^3*x/a/(c*x^4+a)^2+7/32*d^3/a^2*x/(c*x^4+a)+21/256*d^3/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+21/128*d^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+21/128*d^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+3/8*e*d^2*x^2/a/(c*x^4+a)^2+9/16*e*d^2/a^2*x^2/(c*x^4+a)+9/16*e*d^2/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+3/8*d*e^2*x^3/a/(c*x^4+a)^2+15/32*d*e^2/a^2*x^3/(c*x^4+a)+15/256*d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+15/128*d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+15/128*d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*e^3*x^4/a/(c*x^4+a)^2+1/8*e^3/a^2*x^4/(c*x^4+a)

maxima [A] time = 2.37, size = 392, normalized size = 0.99

$$\frac{15c^2de^2x^7 + 18c^2d^2ex^6 + 7c^2d^3x^5 + 27acde^2x^3 + 30acd^2ex^2 + 11acd^3x - 4a^2e^3}{32(a^2c^3x^8 + 2a^3c^2x^4 + a^4c)} + 3d \left(\frac{\sqrt{2}(7\sqrt{c}d^2 - 5\sqrt{a}e^2)\log(\sqrt{c})}{a^4c^4} \right)$$

$$\begin{aligned}
& 960*_t^{**2}*a^{**7}*c^{**2}*d^{**4}*e^{**5} + 202309632*_t^{**2}*a^{**6}*c^{**3}*d^{**8}*e + 77328000 \\
& *_t*a^{**5}*c*d^{**4}*e^{**8} + 660699648*_t*a^{**4}*c^{**2}*d^{**8}*e^{**4} + 19361664*_t*a^{**3}* \\
& c^{**3}*d^{**12} + 3037500*a^{**3}*d^{**4}*e^{**11} - 26360640*a^{**2}*c*d^{**8}*e^{**7} - 60566940 \\
& *a*c^{**2}*d^{**12}*e^{**3})/(421875*a^{**3}*d^{**3}*e^{**12} - 29598075*a^{**2}*c*d^{**7}*e^{**8} - 5 \\
& 8012227*a*c^{**2}*d^{**11}*e^{**4} + 3176523*c^{**3}*d^{**15}))) + (-4*a^{**2}*e^{**3} + 11*a*c \\
& *d^{**3}*x + 30*a*c*d^{**2}*e*x^{**2} + 27*a*c*d*e^{**2}*x^{**3} + 7*c^{**2}*d^{**3}*x^{**5} + 18*c \\
& **2*d^{**2}*e*x^{**6} + 15*c^{**2}*d*e^{**2}*x^{**7})/(32*a^{**4}*c + 64*a^{**3}*c^{**2}*x^{**4} + 32* \\
& a^{**2}*c^{**3}*x^{**8})
\end{aligned}$$

$$3.409 \quad \int \frac{(d+ex)^2}{(a+cx^4)^3} dx$$

Optimal. Leaf size=360

$$\frac{(21\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{(21\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}}$$

[Out] $\frac{1}{8}x^2(e^2x^2+d^2)/a/(c^2x^4+a)^2 + \frac{1}{32}x^2(5e^2x^2+12d^2e^2x+7d^2)/a^2/(c^2x^4+a) + \frac{3}{8}d^2e^2 \arctan(x^2c^{1/2}/a^{1/2})/a^{5/2}/c^{1/2} - \frac{1}{256} \ln(-a^{1/4}c^{1/4}xx^{2^{1/2}}+a^{1/2}+x^2c^{1/2}) * (-5e^2a^{1/2}+21d^2c^{1/2})/a^{11/4}/c^{3/4} * 2^{1/2} + \frac{1}{256} \ln(a^{1/4}c^{1/4}xx^{2^{1/2}}+a^{1/2}+x^2c^{1/2}) * (-5e^2a^{1/2}+21d^2c^{1/2})/a^{11/4}/c^{3/4} * 2^{1/2} + \frac{1}{128} \arctan(-1+c^{1/4}xx^{2^{1/2}}/a^{1/4}) * (5e^2a^{1/2}+21d^2c^{1/2})/a^{11/4}/c^{3/4} * 2^{1/2} + \frac{1}{128} \arctan(1+c^{1/4}xx^{2^{1/2}}/a^{1/4}) * (5e^2a^{1/2}+21d^2c^{1/2})/a^{11/4}/c^{3/4} * 2^{1/2}$

Rubi [A] time = 0.33, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(21\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{(21\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + c*x^4)^3,x]

[Out] $\frac{x(d + e^2x^2)/(8a^2(a + c^2x^4)^2) + (x(7d^2 + 12d^2e^2x + 5e^2x^2))/(32a^2(a + c^2x^4)) + (3d^2e^2 \text{ArcTan}[\sqrt{c}x^2/\sqrt{a}])/(8a^{5/2}\sqrt{c}) - ((21\sqrt{c}d^2 + 5\sqrt{a}e^2) \text{ArcTan}[1 - (\sqrt{2}c^{1/4}x)/a^{1/4}])/(64\sqrt{2}a^{11/4}c^{3/4}) + ((21\sqrt{c}d^2 + 5\sqrt{a}e^2) \text{ArcTan}[1 + (\sqrt{2}c^{1/4}x)/a^{1/4}])/(64\sqrt{2}a^{11/4}c^{3/4}) - ((21\sqrt{c}d^2 - 5\sqrt{a}e^2) \text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/(128\sqrt{2}a^{11/4}c^{3/4}) + ((21\sqrt{c}d^2 - 5\sqrt{a}e^2) \text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/(128\sqrt{2}a^{11/4}c^{3/4})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{(a+cx^4)^3} dx &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} - \frac{\int \frac{-7d^2-12dex-5e^2x^2}{(a+cx^4)^2} dx}{8a} \\
&= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{\int \frac{21d^2+24dex+5e^2x^2}{a+cx^4} dx}{32a^2} \\
&= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{\int \left(\frac{24dex}{a+cx^4} + \frac{21d^2+5e^2x^2}{a+cx^4} \right) dx}{32a^2} \\
&= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{\int \frac{21d^2+5e^2x^2}{a+cx^4} dx}{32a^2} + \frac{(3de) \int \frac{x}{a+cx^4} dx}{4a^2} \\
&= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{(3de) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{8a^2} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2 \right) \int \frac{1}{\sqrt{a}-\sqrt{c}x}}{64a^2} \\
&= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2 \right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}+2x}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}}}{\sqrt{a}-\sqrt{c}x}}{128\sqrt{2}a^{9/4}c^{3/4}} \\
&= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2 \right) \log(\sqrt{a}-\sqrt{c}x)}{128\sqrt{2}a^{9/4}c^{3/4}} \\
&= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{8a^{5/2}\sqrt{c}} - \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2) \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{64\sqrt{2}a^{11/4}c^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 358, normalized size = 0.99

$$\frac{\sqrt{2}(5a^{3/4}e^2-21\sqrt[4]{a}\sqrt{c}d^2)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2)}{c^{3/4}} + \frac{\sqrt{2}(21\sqrt[4]{a}\sqrt{c}d^2-5a^{3/4}e^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2)}{c^{3/4}} + \frac{32a^2x(d+ex)^2}{(a+cx^4)^2} - \frac{2\sqrt[4]{a}(48}{$$

25

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + c*x^4)^3,x]

[Out] ((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (Sqrt[2]*(-21*a^(1/4)*Sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5*a^(3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.43, size = 356, normalized size = 0.99

$$\frac{5cx^7e^2 + 12cdx^6e + 7cd^2x^5 + 9ax^3e^2 + 20adx^2e + 11ad^2x}{32(cx^4 + a)^2 a^2} + \frac{\sqrt{2} \left(24\sqrt{2}\sqrt{ac}c^2de + 21(ac^3)^{\frac{1}{4}}c^2d^2 + 5(ac^3)^{\frac{3}{4}} \right)}{128a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")

[Out] 1/32*(5*c*x^7*e^2 + 12*c*d*x^6*e + 7*c*d^2*x^5 + 9*a*x^3*e^2 + 20*a*d*x^2*e + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3)

maple [A] time = 0.01, size = 419, normalized size = 1.16

$$\frac{e^2x^3}{8(cx^4 + a)^2 a} + \frac{dex^2}{4(cx^4 + a)^2 a} + \frac{5e^2x^3}{32(cx^4 + a)^2 a^2} + \frac{d^2x}{8(cx^4 + a)^2 a} + \frac{3dex^2}{8(cx^4 + a)^2 a^2} + \frac{7d^2x}{32(cx^4 + a)^2 a^2} + \frac{3de \arctan}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(c*x^4+a)^3,x)

[Out] 1/8*d^2*x/a/(c*x^4+a)^2+7/32*d^2/a^2*x/(c*x^4+a)+21/256*d^2/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+21/128*d^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+21/128*d^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*d*e*x^2/a/(c*x^4+a)^2+3/8*d*e/a^2*x^2/(c*x^4+a)+3/8*d*e/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+1/8*e^2*x^3/a/(c*x^4+a)^2+5/32*e^2/a^2*x^3/(c*x^4+a)+5/256*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+5/128*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+5/128*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)

maxima [A] time = 2.18, size = 364, normalized size = 1.01

$$\frac{5ce^2x^7 + 12cdex^6 + 7cd^2x^5 + 9ae^2x^3 + 20adex^2 + 11ad^2x}{32(a^2c^2x^8 + 2a^3cx^4 + a^4)} + \frac{\sqrt{2}(21\sqrt{c}d^2 - 5\sqrt{a}e^2)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^4c^{\frac{3}{4}}} - \frac{\sqrt{2}(21}{a^4c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")

[Out] 1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^2 + 11*a*d^2*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(21*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 48*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 48*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))/a^2

mupad [B] time = 0.47, size = 676, normalized size = 1.88

$$\frac{\frac{11d^2x}{32a} + \frac{9e^2x^3}{32a} + \frac{7cd^2x^5}{32a^2} + \frac{5ce^2x^7}{32a^2} + \frac{5dex^2}{8a} + \frac{3cdex^6}{8a^2}}{a^2 + 2acx^4 + c^2x^8} + \left(\sum_{k=1}^4 \ln \left(-\frac{c \left(125ae^6 - 9891cd^4e^2 + \text{root}(268435456a^{11}c^3 \right)}{\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + c*x^4)^3,x)

[Out] ((11*d^2*x)/(32*a) + (9*e^2*x^3)/(32*a) + (7*c*d^2*x^5)/(32*a^2) + (5*c*e^2*x^7)/(32*a^2) + (5*d*e*x^2)/(8*a) + (3*c*d*e*x^6)/(8*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log(-(c*(125*a*e^6 - 9891*c*d^4*e^2 + 344064*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d^2 - 8784*c*d^3*e^3*x - 3200*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*e^4*x + 56448*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^2*c^2*d^4*x + 30720*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x))/(32768*a^6))*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k), k, 1, 4)

sympy [A] time = 4.88, size = 374, normalized size = 1.04

$$\text{RootSum} \left(268435456t^4a^{11}c^3 + 25755648t^2a^6c^2d^2e^2 + t \left(307200a^4cde^5 - 5419008a^3c^2d^5e \right) + 625a^2e^8 + 111906a^4c^2d^8 + 625a^2e^8 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2/(c*x**4+a)**3,x)

[Out] RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 + _t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 111906*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t**2*a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*a**5*c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c**3*d**10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*c**2*d**9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*a*c**2*d

$$\begin{aligned} & \left((8e^4 + 85766121c^3d^{12}) \right) + (11ad^2x + 20ade^2x^2 + 9ae^2x^3 + 7cd^2x^5 + 12cde^6x^6 + 5ce^2x^7) / (32a^4 + 64a^3c \\ & cx^4 + 32a^2c^2x^8) \end{aligned}$$

3.410 $\int \frac{d+ex}{(a+cx^4)^3} dx$

Optimal. Leaf size=266

$$\frac{21d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} - \frac{21d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21d \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}}$$

[Out] $\frac{1}{8} x (e x + d) / a / (c x^4 + a)^2 + \frac{1}{32} x (6 e x + 7 d) / a^2 / (c x^4 + a) + \frac{21}{128} d \operatorname{arctan}\left(\frac{-1 + c^{1/4} x^2 / a^{1/4}}{c^{1/4} x / a^{1/4}}\right) / a^{11/4} / c^{1/4} + \frac{21}{128} d \operatorname{arctan}\left(\frac{1 + c^{1/4} x^2 / a^{1/4}}{c^{1/4} x / a^{1/4}}\right) / a^{11/4} / c^{1/4} - \frac{21}{256} d \ln\left(\frac{-a^{1/4} c^{1/4} x^2 + a^{1/2} + x^2 c^{1/2}}{a^{1/4} c^{1/4} x^2 + a^{1/2} + x^2 c^{1/2}}\right) / a^{11/4} / c^{1/4} + \frac{21}{256} d \ln\left(\frac{a^{1/4} c^{1/4} x^2 + a^{1/2} + x^2 c^{1/2}}{a^{1/4} c^{1/4} x^2 + a^{1/2} + x^2 c^{1/2}}\right) / a^{11/4} / c^{1/4} + \frac{3}{16} e \operatorname{arctan}\left(\frac{x^2 c^{1/2}}{a^{1/2}}\right) / a^{5/2} / c^{1/2}$

Rubi [A] time = 0.25, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(7d + 6ex)}{32a^2 (a + cx^4)} - \frac{21d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} - \frac{21d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21d \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + c*x^4)^3, x]

[Out] $\frac{x(d + e x)}{8 a (a + c x^4)^2} + \frac{x(7 d + 6 e x)}{32 a^2 (a + c x^4)} + \frac{3 e \operatorname{ArcTan}\left[\frac{\sqrt{c} x^2}{\sqrt{a}}\right]}{16 a^{5/2} \sqrt{c}} - \frac{21 d \operatorname{ArcTan}\left[\frac{\sqrt{2} c^{1/4} x}{a^{1/4}}\right]}{64 \sqrt{2} a^{11/4} c^{1/4}} + \frac{21 d \operatorname{ArcTan}\left[\frac{\sqrt{2} c^{1/4} x}{a^{1/4}}\right]}{64 \sqrt{2} a^{11/4} c^{1/4}} - \frac{21 d \operatorname{Log}\left[\frac{\sqrt{a} - \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2}{\sqrt{a} + \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2}\right]}{128 \sqrt{2} a^{11/4} c^{1/4}} + \frac{21 d \operatorname{Log}\left[\frac{\sqrt{a} + \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2}{\sqrt{a} - \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{c} x^2}\right]}{128 \sqrt{2} a^{11/4} c^{1/4}}$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x

$x^k, x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Rule 617

$\text{Int}[(a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4 \cdot a \cdot c])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d_ + (e_ \cdot)(x_)) / ((a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2)), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 1162

$\text{Int}[(d_ + (e_ \cdot)(x_)^2) / ((a_ + (c_ \cdot)(x_)^4)), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e + q \cdot x + x^2, x], x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e - q \cdot x + x^2, x], x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{PosQ}[d \cdot e]$

Rule 1165

$\text{Int}[(d_ + (e_ \cdot)(x_)^2) / ((a_ + (c_ \cdot)(x_)^4)), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q - 2 \cdot x)/\text{Simp}[d/e + q \cdot x - x^2, x], x], x] + \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q + 2 \cdot x)/\text{Simp}[d/e - q \cdot x - x^2, x], x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \&\& \text{NegQ}[d \cdot e]$

Rule 1855

$\text{Int}[(Pq_)((a_ + (b_ \cdot)(x_)^{n_ })^{p_}), x_Symbol] \rightarrow -\text{Simp}[(x \cdot Pq \cdot (a + b \cdot x^n)^{p+1}) / (a \cdot n \cdot (p+1)), x] + \text{Dist}[1/(a \cdot n \cdot (p+1)), \text{Int}[\text{ExpandToSum}[n \cdot (p+1) \cdot Pq + D[x \cdot Pq, x], x] \cdot (a + b \cdot x^n)^{p+1}, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{LtQ}[\text{Expon}[Pq, x], n - 1]$

Rule 1876

$\text{Int}[Pq_ / ((a_ + (b_ \cdot)(x_)^{n_ })), x_Symbol] \rightarrow \text{With}[\{v = \text{Sum}[(x^{ii} \cdot (\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii] \cdot x^{(n/2)})) / (a + b \cdot x^n), \{ii, 0, n/2 - 1\}]\}, \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[Pq, x] < n]$

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{(a+cx^4)^3} dx &= \frac{x(d+ex)}{8a(a+cx^4)^2} - \frac{\int \frac{-7d-6ex}{(a+cx^4)^2} dx}{8a} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{\int \frac{21d+12ex}{a+cx^4} dx}{32a^2} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{\int \left(\frac{21d}{a+cx^4} + \frac{12ex}{a+cx^4} \right) dx}{32a^2} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{(21d) \int \frac{1}{a+cx^4} dx}{32a^2} + \frac{(3e) \int \frac{x}{a+cx^4} dx}{8a^2} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{(21d) \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{64a^{5/2}} + \frac{(21d) \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{64a^{5/2}} + \frac{(3e) \text{Subst} \left(\int \frac{1}{\sqrt{a}-\sqrt{c}x^2} dx \right)}{16a^2} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{c}} + \frac{(21d) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{128a^{5/2}\sqrt{c}} + \frac{(21d) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{128a^{5/2}\sqrt{c}} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{c}} - \frac{21d \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{16a^{5/2}\sqrt{c}} - \frac{21d \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 249, normalized size = 0.94

$$\frac{32a^{7/4}x(d+ex)}{(a+cx^4)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6(8\sqrt[4]{a}e+7\sqrt{2}\sqrt[4]{c}d)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6(7\sqrt{2}\sqrt[4]{c}d-8\sqrt[4]{a}e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}} - \frac{21\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}}$$

$$256a^{11/4}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + c*x^4)^3, x]

[Out] ((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e*x))/(a + c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)*d - 8*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/Sqrt[c] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.32, size = 260, normalized size = 0.98

$$\frac{21 \sqrt{2} (ac^3)^{\frac{1}{4}} d \log \left(x^2 + \sqrt{2} x \left(\frac{a}{c} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}} \right)}{256 a^3 c} - \frac{21 \sqrt{2} (ac^3)^{\frac{1}{4}} d \log \left(x^2 - \sqrt{2} x \left(\frac{a}{c} \right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}} \right)}{256 a^3 c} + \frac{3 \sqrt{2} \left(4 \sqrt{2} \sqrt{ac} ce - \dots \right)}{256 a^3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))
/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)
*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32*(6*c*
x^6*e + 7*c*d*x^5 + 10*a*x^2*e + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

maple [A] time = 0.01, size = 222, normalized size = 0.83

$$\frac{e x^2}{8(c x^4 + a)^2 a} + \frac{d x}{8(c x^4 + a)^2 a} + \frac{3 e x^2}{16(c x^4 + a) a^2} + \frac{7 d x}{32(c x^4 + a) a^2} + \frac{3 e \arctan\left(\sqrt{\frac{c}{a}} x\right)}{16 \sqrt{ac} a^2} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\dots\right)}{128 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d*x/a/(c*x^4+a)^2+7/32*d/a^2*x/(c*x^4+a)+21/256*d/a^3*(a/c)^(1/4)*2^(1/
2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2)))+21/128*d/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1
)+21/128*d/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*e*x^
2/a/(c*x^4+a)^2+3/16*e/a^2*x^2/(c*x^4+a)+3/16*e/a^2/(a*c)^(1/2)*arctan((1/a
*c)^(1/2)*x^2)
```

maxima [A] time = 2.20, size = 269, normalized size = 1.01

$$\frac{6 c e x^6 + 7 c d x^5 + 10 a e x^2 + 11 a d x}{32(a^2 c^2 x^8 + 2 a^3 c x^4 + a^4)} + 3 \left(\frac{7 \sqrt{2} d \log\left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} - \frac{7 \sqrt{2} d \log\left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} \right) + \frac{2 \left(7 \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} d \right)}{256 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/(a^2*c^2*x^8 + 2*a^3*c
*x^4 + a^4) + 3/256*(7*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(1/4)) - 7*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(1/4)*d
- 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 2*(7*sqrt(
2)*a^(1/4)*c^(1/4)*d + 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
```

$$2) \cdot a^{(1/4)} \cdot c^{(1/4)} / \sqrt{\sqrt{a} \cdot \sqrt{c}} / (a^{(3/4)} \cdot \sqrt{\sqrt{a} \cdot \sqrt{c}} \cdot c^{(1/4)}) / a^2$$

mupad [B] time = 0.30, size = 315, normalized size = 1.18

$$\frac{\frac{5ex^2}{16a} + \frac{11dx}{32a} + \frac{7cdx^5}{32a^2} + \frac{3cex^6}{16a^2}}{a^2 + 2acx^4 + c^2x^8} + \left(\sum_{k=1}^4 \ln \left(\frac{c^2 \left(63de^2 + 36e^3x - \text{root} \left(268435456a^{11}c^2z^4 + 4718592a^6ce^2z^2 - 2709504a^3cd^2ez + 194481cd^4 + 20736ae^4 \right) \right)}{\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)/(a + c*x^4)^3, x)

[Out] ((5*e*x^2)/(16*a) + (11*d*x)/(32*a) + (7*c*d*x^5)/(32*a^2) + (3*c*e*x^6)/(16*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c^2*(63*d*e^2 + 36*e^3*x - 7168*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)^2*a^5*c*d - 1176*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)*a^2*c*d^2*x + 4096*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)^2*a^5*c*e*x))/(2048*a^6))*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k), k, 1, 4)

sympy [A] time = 1.48, size = 192, normalized size = 0.72

$$\text{RootSum} \left(268435456t^4a^{11}c^2 + 4718592t^2a^6ce^2 - 2709504ta^3cd^2e + 20736ae^4 + 194481cd^4, \left(t \mapsto t \log \left(x + \frac{-6t^4a^{11}c^2 + \dots}{\dots} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x**4+a)**3, x)

[Out] RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_t*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_t, _t*log(x + (-67108864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e**4 - 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 453789*c*d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8)

$$3.411 \quad \int \frac{1}{(a+cx^4)^3} dx$$

Optimal. Leaf size=219

$$-\frac{21 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} - \frac{21 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21 \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}}$$

[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/128*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)

Rubi [A] time = 0.14, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{7x}{32a^2(a+cx^4)} - \frac{21 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} - \frac{21 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21 \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-3), x]

[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+cx^4)^3} dx &= \frac{x}{8a(a+cx^4)^2} + \frac{7 \int \frac{1}{(a+cx^4)^2} dx}{8a} \\ &= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21 \int \frac{1}{a+cx^4} dx}{32a^2} \\ &= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21 \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{64a^{5/2}} + \frac{21 \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{64a^{5/2}} \\ &= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{128a^{5/2}\sqrt{c}} + \frac{21 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{128a^{5/2}\sqrt{c}} - \frac{21 \int \frac{\sqrt{a}}{-\frac{\sqrt{a}}{\sqrt{c}}} dx}{128\sqrt{2}} \\ &= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21 \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21 \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} \\ &= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21 \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21 \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21 \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 200, normalized size = 0.91

$$\frac{32a^{7/4}x}{(a+cx^4)^2} + \frac{56a^{3/4}x}{a+cx^4} - \frac{21\sqrt{2} \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{\sqrt[4]{c}} + \frac{21\sqrt{2} \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{\sqrt[4]{c}} - \frac{42\sqrt{2} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{21 \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21 \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^(-3),x]

[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))

fricas [A] time = 0.64, size = 232, normalized size = 1.06

$$\frac{28cx^5 + 84(a^2c^2x^8 + 2a^3cx^4 + a^4)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} \arctan\left(-a^8cx\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}} + \sqrt{a^6\sqrt{-\frac{1}{a^{11}c}} + x^2}a^8c\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}\right) + 21(a^2c^2x^8 + 2a^3cx^4 + a^4)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} \log\left(-a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} + x\right) - 21(a^2c^2x^8 + 2a^3cx^4 + a^4)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} \log\left(-a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} + x\right) + 44ax}{128(a^2c^2x^8 + 2a^3cx^4 + a^4)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^3,x, algorithm="fricas")

[Out] 1/128*(28*c*x^5 + 84*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*arctan(-a^8*c*x*(-1/(a^11*c))^(3/4) + sqrt(a^6*sqrt(-1/(a^11*c)) + x^2)*a^8*c*(-1/(a^11*c))^(3/4)) + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*log(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)

giac [A] time = 0.36, size = 204, normalized size = 0.93

$$\frac{21\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c} + \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c} + \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\right)}{256a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^3,x, algorithm="giac")

[Out] 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32*(7*c*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)

maple [A] time = 0.00, size = 158, normalized size = 0.72

$$\frac{x}{8(c x^4 + a)^2 a} + \frac{7x}{32(c x^4 + a) a^2} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{128a^3} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{128a^3} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \log\left(x^2 + \sqrt{2}x\right)}{256a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+a)^3,x)

[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/256/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+21/128/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+21/128/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)

maxima [A] time = 2.70, size = 212, normalized size = 0.97

$$\frac{7cx^5 + 11ax}{32(a^2c^2x^8 + 2a^3cx^4 + a^4)} + \frac{21 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2} \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \frac{3}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} \right)}{256a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^3,x, algorithm="maxima")

[Out] 1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))/a^2

mupad [B] time = 0.10, size = 80, normalized size = 0.37

$$\frac{\frac{11x}{32a} + \frac{7cx^5}{32a^2}}{a^2 + 2acx^4 + c^2x^8} - \frac{21 \operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{11/4}c^{1/4}} - \frac{21 \operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{11/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c*x^4)^3,x)

[Out] ((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*atan((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))

sympy [A] time = 0.46, size = 63, normalized size = 0.29

$$\frac{11ax + 7cx^5}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8} + \operatorname{RootSum}\left(268435456t^4a^{11}c + 194481, \left(t \mapsto t \log\left(\frac{128ta^3}{21} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+a)**3,x)

[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootSum(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)))

$$3.412 \quad \int \frac{1}{(d+ex)(a+cx^4)^3} dx$$

Optimal. Leaf size=1352

$$\frac{\log(d+ex)e^{11}}{(cd^4+ae^4)^3} - \frac{\log(cx^4+a)e^{11}}{4(cd^4+ae^4)^3} - \frac{\sqrt{c}d^2 \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt[4]{c}d(\sqrt{c}d^2+\sqrt{a}e^2) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3} + \dots$$

[Out] $1/32*c*x*(5*d*e^2*x^2-6*d^2*e*x+7*d^3)/a^2/(a*e^4+c*d^4)/(c*x^4+a)+1/8*(a*e^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)^2+1/4*e^4*(a*e^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)^2/(c*x^4+a)+e^{11}*\ln(e*x+d)/(a*e^4+c*d^4)^3-1/4*e^{11}*\ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/4*d^2*e^5*\arctan(x^2*c^{(1/2)}/a^{(1/2)})*c^{(1/2)}/a^{(3/2)}/(a*e^4+c*d^4)^2-3/16*d^2*e*\arctan(x^2*c^{(1/2)}/a^{(1/2)})*c^{(1/2)}/a^{(5/2)}/(a*e^4+c*d^4)-1/2*d^2*e^9*\arctan(x^2*c^{(1/2)}/a^{(1/2)})*c^{(1/2)}/(a*e^4+c*d^4)^3/a^{(1/2)}-1/8*c^{(1/4)}*d*e^8*\ln(-a^{(1/4)}*c^{(1/4)}*x^2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*(-e^2*a^{(1/2)}+d^2*c^{(1/2)})/a^{(3/4)}/(a*e^4+c*d^4)^3*2^{(1/2)}+1/8*c^{(1/4)}*d*e^8*\ln(a^{(1/4)}*c^{(1/4)}*x^2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*(-e^2*a^{(1/2)}+d^2*c^{(1/2)})/a^{(3/4)}/(a*e^4+c*d^4)^3*2^{(1/2)}+1/4*c^{(1/4)}*d*e^8*\arctan(1+c^{(1/4)}*x^2^{(1/2)}/a^{(1/4)})*(e^2*a^{(1/2)}+d^2*c^{(1/2)})/a^{(3/4)}/(a*e^4+c*d^4)^3*2^{(1/2)}+1/4*c^{(1/4)}*d*e^8*\arctan(1+c^{(1/4)}*x^2^{(1/2)}/a^{(1/4)})*(e^2*a^{(1/2)}+d^2*c^{(1/2)})/a^{(3/4)}/(a*e^4+c*d^4)^3*2^{(1/2)}-1/32*c^{(1/4)}*d*e^4*\ln(-a^{(1/4)}*c^{(1/4)}*x^2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*(-e^2*a^{(1/2)}+3*d^2*c^{(1/2)})/a^{(7/4)}/(a*e^4+c*d^4)^2*2^{(1/2)}+1/32*c^{(1/4)}*d*e^4*\ln(a^{(1/4)}*c^{(1/4)}*x^2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*(-e^2*a^{(1/2)}+3*d^2*c^{(1/2)})/a^{(7/4)}/(a*e^4+c*d^4)^2*2^{(1/2)}+1/16*c^{(1/4)}*d*e^4*\arctan(-1+c^{(1/4)}*x^2^{(1/2)}/a^{(1/4)})*(e^2*a^{(1/2)}+3*d^2*c^{(1/2)})/a^{(7/4)}/(a*e^4+c*d^4)^2*2^{(1/2)}+1/16*c^{(1/4)}*d*e^4*\arctan(1+c^{(1/4)}*x^2^{(1/2)}/a^{(1/4)})*(e^2*a^{(1/2)}+3*d^2*c^{(1/2)})/a^{(7/4)}/(a*e^4+c*d^4)^2*2^{(1/2)}-1/256*c^{(1/4)}*d*\ln(-a^{(1/4)}*c^{(1/4)}*x^2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*(-5*e^2*a^{(1/2)}+21*d^2*c^{(1/2)})/a^{(11/4)}/(a*e^4+c*d^4)*2^{(1/2)}+1/256*c^{(1/4)}*d*\ln(a^{(1/4)}*c^{(1/4)}*x^2^{(1/2)}+a^{(1/2)}+x^2*c^{(1/2)})*(-5*e^2*a^{(1/2)}+21*d^2*c^{(1/2)})/a^{(11/4)}/(a*e^4+c*d^4)*2^{(1/2)}+1/128*c^{(1/4)}*d*\arctan(-1+c^{(1/4)}*x^2^{(1/2)}/a^{(1/4)})*(5*e^2*a^{(1/2)}+21*d^2*c^{(1/2)})/a^{(11/4)}/(a*e^4+c*d^4)*2^{(1/2)}+1/128*c^{(1/4)}*d*\arctan(1+c^{(1/4)}*x^2^{(1/2)}/a^{(1/4)})*(5*e^2*a^{(1/2)}+21*d^2*c^{(1/2)})/a^{(11/4)}/(a*e^4+c*d^4)*2^{(1/2)}$

Rubi [A] time = 1.41, antiderivative size = 1352, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(a + c*x^4)^3), x]

[Out] $(c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4)) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c*x^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a*e^4)^2*(a + c*x^4)) - (\text{Sqrt}[c]*d^2*e^9*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*(c*d^4 + a*e^4)^3) - (\text{Sqrt}[c]*d^2*e^5*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(4*a^{(3/2)}*(c*d^4 + a*e^4)^2) - (3*\text{Sqrt}[c]*d^2*e*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(16*a^{(5/2)}*(c*d^4 + a*e^4)) - (c^{(1/4)}*d*e^8*(\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) - (c^{(1/4)}*d*e^4*(3*\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^2) - (c^{(1/4)}*d*(21*\text{Sqrt}[c]*d^2 + 5*\text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(64*\text{Sqrt}[2]*a^{(11/4)}*(c*d^4 + a*e^4)) + (c^{(1/4)}*d*e^8*(\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3)$

$$4)^3) + (c^{(1/4)}*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*Sqrt[2]*a^{(7/4)}*(c*d^4 + a*e^4)^2) + (c^{(1/4)}*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/(64*Sqrt[2]*a^{(11/4)}*(c*d^4 + a*e^4)) + (e^{11}*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^{(1/4)}*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) - (c^{(1/4)}*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^{(7/4)}*(c*d^4 + a*e^4)^2) - (c^{(1/4)}*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^{(11/4)}*(c*d^4 + a*e^4)) + (c^{(1/4)}*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) + (c^{(1/4)}*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^{(7/4)}*(c*d^4 + a*e^4)^2) + (c^{(1/4)}*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^{(11/4)}*(c*d^4 + a*e^4)) - (e^{11}*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^3)$$
Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(-p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] & EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1854

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} & *x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2))/(a^2*(a + c*x^4)) - \\ & (2*c^{(1/4)}*d*(21*sqrt[2]*c^{(5/2)}*d^{10} - 24*a^{(1/4)}*c^{(9/4)}*d^9*e + 5*sqrt[2] \\ &]*sqrt[a]*c^2*d^8*e^2 + 66*sqrt[2]*a*c^{(3/2)}*d^6*e^4 - 80*a^{(5/4)}*c^{(5/4)}*d \\ & ^5*e^5 + 18*sqrt[2]*a^{(3/2)}*c*d^4*e^6 + 77*sqrt[2]*a^2*sqrt[c]*d^2*e^8 - 12 \\ & 0*a^{(9/4)}*c^{(1/4)}*d*e^9 + 45*sqrt[2]*a^{(5/2)}*e^{10})*ArcTan[1 - (sqrt[2]*c^{(1 \\ & /4)}*x)/a^{(1/4)}])/a^{(11/4)} + (2*c^{(1/4)}*d*(21*sqrt[2]*c^{(5/2)}*d^{10} + 24*a^{(1 \\ & /4)}*c^{(9/4)}*d^9*e + 5*sqrt[2]*sqrt[a]*c^2*d^8*e^2 + 66*sqrt[2]*a*c^{(3/2)}*d^ \\ & 6*e^4 + 80*a^{(5/4)}*c^{(5/4)}*d^5*e^5 + 18*sqrt[2]*a^{(3/2)}*c*d^4*e^6 + 77*sqrt \\ & [2]*a^2*sqrt[c]*d^2*e^8 + 120*a^{(9/4)}*c^{(1/4)}*d*e^9 + 45*sqrt[2]*a^{(5/2)}*e^ \\ & 10)*ArcTan[1 + (sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/a^{(11/4)} + 256*e^{11}*Log[d + e* \\ & x] + (sqrt[2]*c^{(1/4)}*(-21*c^{(5/2)}*d^{11} + 5*sqrt[a]*c^2*d^9*e^2 - 66*a*c^{(3 \\ & /2)}*d^7*e^4 + 18*a^{(3/2)}*c*d^5*e^6 - 77*a^2*sqrt[c]*d^3*e^8 + 45*a^{(5/2)}*d* \\ & e^{10})*Log[sqrt[a] - sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + sqrt[c]*x^2])/a^{(11/4)} + (S \\ & qrt[2]*c^{(1/4)}*(21*c^{(5/2)}*d^{11} - 5*sqrt[a]*c^2*d^9*e^2 + 66*a*c^{(3/2)}*d^7* \\ & e^4 - 18*a^{(3/2)}*c*d^5*e^6 + 77*a^2*sqrt[c]*d^3*e^8 - 45*a^{(5/2)}*d*e^{10})*Lo \\ & g[sqrt[a] + sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + sqrt[c]*x^2])/a^{(11/4)} - 64*e^{11}*Lo \\ & g[a + c*x^4])/(256*(c*d^4 + a*e^4)^3) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.91, size = 1259, normalized size = 0.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")

$$\begin{aligned} & [Out] 1/64*(51*sqrt(2)*sqrt(a*c)*c^2*d^4*e + 21*(a*c^3)^{(1/4)}*c^2*d^5 - 75*sqrt(2) \\ &)*a*c^2*d^2*e^3 + 122*(a*c^3)^{(3/4)}*d^3*e^2 + 45*(a*c^3)^{(1/4)}*a*c*d*e^4)*a \\ & rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(sqrt(2)*a^3*c^3 \\ & *d^6 - 6*(a*c^3)^{(1/4)}*a^3*c^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2 \\ & + 9*sqrt(2)*a^4*c^2*d^2*e^4 - 16*(a*c^3)^{(3/4)}*a^3*d^3*e^3 - 6*(a*c^3)^{(1/4)} \\ &)*a^4*c*d*e^5 + sqrt(2)*sqrt(a*c)*a^4*c*e^6) + 1/64*(51*sqrt(2)*sqrt(a*c)*c \\ & ^2*d^4*e + 21*(a*c^3)^{(1/4)}*c^2*d^5 + 75*sqrt(2)*a*c^2*d^2*e^3 + 122*(a*c^3 \\ &)^{(3/4)}*d^3*e^2 + 45*(a*c^3)^{(1/4)}*a*c*d*e^4)*arctan(1/2*sqrt(2)*(2*x - sqr \\ & t(2)*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(sqrt(2)*a^3*c^3*d^6 + 6*(a*c^3)^{(1/4)}*a^3*c \\ & ^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2 + 9*sqrt(2)*a^4*c^2*d^2*e^4 \\ & + 16*(a*c^3)^{(3/4)}*a^3*d^3*e^3 + 6*(a*c^3)^{(1/4)}*a^4*c*d*e^5 + sqrt(2)*sqrt \\ & (a*c)*a^4*c*e^6) + 1/256*(21*sqrt(2)*(a*c^3)^{(1/4)}*c^4*d^11 - 5*sqrt(2)*(a* \\ & c^3)^{(3/4)}*c^2*d^9*e^2 + 66*sqrt(2)*(a*c^3)^{(1/4)}*a*c^3*d^7*e^4 - 18*sqrt(2) \\ &)*(a*c^3)^{(3/4)}*a*c*d^5*e^6 + 77*sqrt(2)*(a*c^3)^{(1/4)}*a^2*c^2*d^3*e^8 - 45 \\ & *sqrt(2)*(a*c^3)^{(3/4)}*a^2*d*e^{10})*log(x^2 + sqrt(2)*x*(a/c)^{(1/4)} + sqrt(a \\ & /c))/(a^3*c^5*d^{12} + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^{12}) \\ & - 1/256*(21*sqrt(2)*(a*c^3)^{(1/4)}*c^4*d^11 - 5*sqrt(2)*(a*c^3)^{(3/4)}*c^2*d^ \\ & 9*e^2 + 66*sqrt(2)*(a*c^3)^{(1/4)}*a*c^3*d^7*e^4 - 18*sqrt(2)*(a*c^3)^{(3/4)}*a \\ & *c*d^5*e^6 + 77*sqrt(2)*(a*c^3)^{(1/4)}*a^2*c^2*d^3*e^8 - 45*sqrt(2)*(a*c^3)^ \\ & (3/4)*a^2*d*e^{10})*log(x^2 - sqrt(2)*x*(a/c)^{(1/4)} + sqrt(a/c))/(a^3*c^5*d^{1 \\ & 2} + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^{12}) - 1/4*e^{11}*log(ab \\ & s(c*x^4 + a))/(c^3*d^{12} + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^{12}) + e \\ & ^{12}*log(abs(x*e + d))/(c^3*d^{12}*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3 \\ & *e^{13}) + 1/32*(4*a^2*c^2*d^8*e^3 + 16*a^3*c*d^4*e^7 + (5*c^4*d^9*e^2 + 18*a \\ & *c^3*d^5*e^6 + 13*a^2*c^2*d*e^{10})*x^7 - 2*(3*c^4*d^{10}*e + 10*a*c^3*d^6*e^5 \\ & + 7*a^2*c^2*d^2*e^9)*x^6 + (7*c^4*d^{11} + 22*a*c^3*d^7*e^4 + 15*a^2*c^2*d^3* \end{aligned}$$

$$e^8)x^5 + 8(a^2c^2d^4e^7 + a^3c^2e^{11})x^4 + 12a^4e^{11} + (9ac^3d^9e^2 + 26a^2c^2d^5e^6 + 17a^3c^2d^2e^9)x^3 - 2(5ac^3d^{10}e + 14a^2c^2d^6e^5 + 9a^3c^2d^2e^9)x^2 + (11ac^3d^{11} + 30a^2c^2d^7e^4 + 19a^3c^2d^3e^8)x / ((c^2d^4 + a^2e^4)^3(c^2x^4 + a^2a^2))$$

maple [A] time = 0.02, size = 2098, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e^x+d)/(c^2x^4+a)^3, x)$

[Out]
$$e^{11} \ln(e^x+d) / (a^2e^4+c^2d^4)^3 - 1/4 e^{11} \ln(c^2x^4+a) / (a^2e^4+c^2d^4)^3 + 3/8 (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 e^{11} a^2 + 33/128 c^2 / (a^2e^4+c^2d^4)^3 / a^2 (a/c)^{1/4} * 2^{1/2} * \ln((x^2+(a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2})) * d^7 e^4 + 33/64 c^2 / (a^2e^4+c^2d^4)^3 / a^2 (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^7 e^4 + 77/128 c / (a^2e^4+c^2d^4)^3 / a (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^3 e^8 + 5/256 c^2 / (a^2e^4+c^2d^4)^3 / a^2 (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2})) * d^9 e^2 + 5/128 c^2 / (a^2e^4+c^2d^4)^3 / a^2 (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^9 e^2 + 5/128 c^2 / (a^2e^4+c^2d^4)^3 / a^2 (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^9 e^2 + 9/64 c / (a^2e^4+c^2d^4)^3 / a (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^5 e^6 + 9/64 c / (a^2e^4+c^2d^4)^3 / a (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^5 e^6 + 77/128 c / (a^2e^4+c^2d^4)^3 / a (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^3 e^8 + 77/256 c / (a^2e^4+c^2d^4)^3 / a (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2})) * d^3 e^8 + 9/128 c / (a^2e^4+c^2d^4)^3 / a (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2})) * d^5 e^6 + 33/64 c^2 / (a^2e^4+c^2d^4)^3 / a^2 (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^7 e^4 + 13/32 c^2 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^2 e^{10} x^7 - 7/16 c^2 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^2 e^9 x^6 + 15/32 c^2 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^3 x^5 e^8 + 7/32 c^4 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^{11} / a^2 x^5 + 1/4 c^2 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 x^4 e^7 d^4 + 13/16 c^2 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^5 e^6 x^3 + 1/4 c / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 x^4 e^{11} a + 1/2 c / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 e^7 d^4 a - 7/8 c^2 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^6 e^5 x^2 + 15/16 c^2 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^7 x e^4 + 11/32 c^3 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^{11} / a x - 15/16 c / (a^2e^4+c^2d^4)^3 / (a^2c)^{1/2} * \arctan((1/a^2c)^{1/2} * x^2) * e^9 d^2 + 45/256 / (a^2e^4+c^2d^4)^3 / (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2})) * d^10 + 45/128 / (a^2e^4+c^2d^4)^3 / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^10 + 45/128 / (a^2e^4+c^2d^4)^3 / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^10 + 9/32 c^3 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^9 e^2 / a x^3 - 5/16 c^3 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^{10} e / a x^2 - 9/16 c / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^2 e^9 a x^2 + 19/32 c / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^3 a x e^8 + 21/128 c^3 / (a^2e^4+c^2d^4)^3 / a^3 (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^{11} + 21/256 c^3 / (a^2e^4+c^2d^4)^3 / a^3 (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4} * 2^{1/2})) * d^{11} + 21/128 c^3 / (a^2e^4+c^2d^4)^3 / a^3 (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^{11} - 5/8 c^2 / (a^2e^4+c^2d^4)^3 / a (a^2c)^{1/2} * \arctan((1/a^2c)^{1/2} * x^2) * e^5 d^6 - 3/16 c^3 / (a^2e^4+c^2d^4)^3 / a^2 (a^2c)^{1/2} * \arctan((1/a^2c)^{1/2} * x^2) * e^5 d^6 - 3/16 c^4 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^{10} e / a^2 x^6 + 11/16 c^3 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^7 / a x^5 e^4 + 17/32 c / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^10 a x^3 + 9/16 c^3 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^5 e^6 / a x^7 + 5/32 c^4 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^9 e^2 / a^2 x^7 + 1/8 c^2 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 e^3 d^8 - 5/8 c^3 / (a^2e^4+c^2d^4)^3 / (c^2x^4+a)^2 d^6 e^5 / a x^6$$

maxima [A] time = 2.41, size = 1015, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & e^{11} \log(e x + d) / (c^3 d^{12} + 3 a c^2 d^8 e^4 + 3 a^2 c d^4 e^8 + a^3 e^{12}) \\ & - 1/256 * c * (\sqrt{2}) * (32 * \sqrt{2}) * a^{(11/4)} * c^{(1/4)} * e^{11} - 21 * c^3 * d^{11} + 5 * \sqrt{a} * c^{(5/2)} * d^9 * e^2 - 66 * a * c^2 * d^7 * e^4 + 18 * a^{(3/2)} * c^{(3/2)} * d^5 * e^6 - 77 * a^2 * c * d^3 * e^8 + 45 * a^{(5/2)} * \sqrt{c} * d * e^{10} * \log(\sqrt{c} * x^2 + \sqrt{2}) * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(5/4)}) + \sqrt{2} * (32 * \sqrt{2}) * a^{(11/4)} * c^{(1/4)} * e^{11} + 21 * c^3 * d^{11} - 5 * \sqrt{a} * c^{(5/2)} * d^9 * e^2 + 66 * a * c^2 * d^7 * e^4 - 18 * a^{(3/2)} * c^{(3/2)} * d^5 * e^6 + 77 * a^2 * c * d^3 * e^8 - 45 * a^{(5/2)} * \sqrt{c} * d * e^{10} * \log(\sqrt{c} * x^2 - \sqrt{2}) * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(5/4)}) - 2 * (21 * \sqrt{2}) * a^{(1/4)} * c^{(13/4)} * d^{11} + 5 * \sqrt{2}) * a^{(3/4)} * c^{(11/4)} * d^9 * e^2 + 66 * \sqrt{2}) * a^{(5/4)} * c^{(9/4)} * d^7 * e^4 + 18 * \sqrt{2}) * a^{(7/4)} * c^{(7/4)} * d^5 * e^6 + 77 * \sqrt{2}) * a^{(9/4)} * c^{(5/4)} * d^3 * e^8 + 45 * \sqrt{2}) * a^{(11/4)} * c^{(3/4)} * d * e^{10} + 24 * \sqrt{a} * c^3 * d^{10} * e + 80 * a^{(3/2)} * c^2 * d^6 * e^5 + 120 * a^{(5/2)} * c * d^2 * e^9 * \arctan(1/2 * \sqrt{2}) * (2 * \sqrt{2}) * c * x + \sqrt{2}) * a^{(1/4)} * c^{(1/4)}) / \sqrt{a * c}) / (a^{(3/4)} * \sqrt{a * c}) * c^{(5/4)}) - 2 * (21 * \sqrt{2}) * a^{(1/4)} * c^{(13/4)} * d^{11} + 5 * \sqrt{2}) * a^{(3/4)} * c^{(11/4)} * d^9 * e^2 + 66 * \sqrt{2}) * a^{(5/4)} * c^{(9/4)} * d^7 * e^4 + 18 * \sqrt{2}) * a^{(7/4)} * c^{(7/4)} * d^5 * e^6 + 77 * \sqrt{2}) * a^{(9/4)} * c^{(5/4)} * d^3 * e^8 + 45 * \sqrt{2}) * a^{(11/4)} * c^{(3/4)} * d * e^{10} - 24 * \sqrt{a} * c^3 * d^{10} * e - 80 * a^{(3/2)} * c^2 * d^6 * e^5 - 120 * a^{(5/2)} * c * d^2 * e^9 * \arctan(1/2 * \sqrt{2}) * (2 * \sqrt{2}) * c * x - \sqrt{2}) * a^{(1/4)} * c^{(1/4)}) / \sqrt{a * c}) / (a^{(3/4)} * \sqrt{a * c}) * c^{(5/4)}) / (a^2 * c^3 * d^{12} + 3 * a^3 * c^2 * d^8 * e^4 + 3 * a^4 * c * d^4 * e^8 + a^5 * e^{12}) + 1/32 * (8 * a^2 * c * e^7 * x^4 + 4 * a^2 * c * d^4 * e^3 + 12 * a^3 * e^7 + (5 * c^3 * d^5 * e^2 + 13 * a * c^2 * d * e^6) * x^7 - 2 * (3 * c^3 * d^6 * e + 7 * a * c^2 * d^2 * e^5) * x^6 + (7 * c^3 * d^7 + 15 * a * c^2 * d^3 * e^4) * x^5 + (9 * a * c^2 * d^5 * e^2 + 17 * a^2 * c * d * e^6) * x^3 - 2 * (5 * a * c^2 * d^6 * e + 9 * a^2 * c * d^2 * e^5) * x^2 + (11 * a * c^2 * d^7 + 19 * a^2 * c * d^3 * e^4) * x) / (a^4 * c^2 * d^8 + 2 * a^5 * c * d^4 * e^4 + a^6 * e^8 + (a^2 * c^4 * d^8 + 2 * a^3 * c^3 * d^4 * e^4 + a^4 * c^2 * e^8) * x^8 + 2 * (a^3 * c^3 * d^8 + 2 * a^4 * c^2 * d^4 * e^4 + a^5 * c * e^8) * x^4) \end{aligned}$$

mupad [B] time = 4.41, size = 2720, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)^3*(d + e*x)),x)

[Out]
$$\begin{aligned} & \text{symsum}(\log((194481 * c^7 * d^{13} * e^6 + 871362 * a * c^6 * d^9 * e^{10} + 425984 * a^3 * c^4 * d * e^{18} + 1148881 * a^2 * c^5 * d^5 * e^{14}) / (1048576 * (a^{12} * e^{16} + a^8 * c^4 * d^{16} + 4 * a^{11} * c * d^4 * e^{12} + 4 * a^9 * c^3 * d^{12} * e^4 + 6 * a^{10} * c^2 * d^8 * e^8))) + \text{root}(805306368 * a^{12} * c^2 * d^8 * e^4 * z^4 + 805306368 * a^{13} * c * d^4 * e^8 * z^4 + 268435456 * a^{11} * c^3 * d^{12} * z^4 + 268435456 * a^{14} * e^{12} * z^4 + 268435456 * a^{11} * e^{11} * z^3 + 43057152 * a^7 * c * d^4 * e^6 * z^2 + 11599872 * a^6 * c^2 * d^8 * e^2 * z^2 + 100663296 * a^8 * e^{10} * z^2 + 9652224 * a^4 * c * d^4 * e^5 * z + 2709504 * a^3 * c^2 * d^8 * e * z + 16777216 * a^5 * e^9 * z + 676881 * a * c * d^4 * e^4 + 194481 * c^2 * d^8 + 1048576 * a^2 * e^8, z, k) * (\text{root}(805306368 * a^{12} * c^2 * d^8 * e^4 * z^4 + 805306368 * a^{13} * c * d^4 * e^8 * z^4 + 268435456 * a^{11} * c^3 * d^{12} * z^4 + 268435456 * a^{14} * e^{12} * z^4 + 268435456 * a^{11} * e^{11} * z^3 + 43057152 * a^7 * c * d^4 * e^6 * z^2 + 11599872 * a^6 * c^2 * d^8 * e^2 * z^2 + 100663296 * a^8 * e^{10} * z^2 + 9652224 * a^4 * c * d^4 * e^5 * z + 2709504 * a^3 * c^2 * d^8 * e * z + 16777216 * a^5 * e^9 * z + 676881 * a * c * d^4 * e^4 + 194481 * c^2 * d^8 + 1048576 * a^2 * e^8, z, k) * (\text{root}(805306368 * a^{12} * c^2 * d^8 * e^4 * z^4 + 805306368 * a^{13} * c * d^4 * e^8 * z^4 + 268435456 * a^{11} * c^3 * d^{12} * z^4 + 268435456 * a^{14} * e^{12} * z^4 + 268435456 * a^{11} * e^{11} * z^3 + 43057152 * a^7 * c * d^4 * e^6 * z^2 + 11599872 * a^6 * c^2 * d^8 * e^2 * z^2 + 100663296 * a^8 * e^{10} * z^2 + 9652224 * a^4 * c * d^4 * e^5 * z + 2709504 * a^3 * c^2 * d^8 * e * z + 16777216 * a^5 * e^9 * z + 676881 * a * c * d^4 * e^4 + 194481 * c^2 * d^8 + 1048576 * a^2 * e^8, z, k) * (\text{root}(805306368 * a^{12} * c^2 * d^8 * e^4 * z^4 + 805306368 * a^{13} * c * d^4 * e^8 * z^4 + 268435456 * a^{11} * c^3 * d^{12} * z^4 + 268435456 * a^{14} * e^{12} * z^4 + 268435456 * a^{11} * e^{11} * z^3 + 43057152 * a^7 * c * d^4 * e^6 * z^2 + 11599872 * a^6 * c^2 * d^8 * e^2 * z^2 + 100663296 * a^8 * e^{10} * z^2 + 9652224 * a^4 * c * d^4 * e^5 * z + 2709504 * a^3 * c^2 * d^8 * e * z + 16777216 * a^5 * e^9 * z + 676881 * a * c * d^4 * e^4 \end{aligned}$$

```

+ 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*((402653184*a^15*c^4*d*e^22 - 134
217728*a^10*c^9*d^21*e^2 - 134217728*a^11*c^8*d^17*e^6 + 805306368*a^12*c^7
*d^13*e^10 + 1879048192*a^13*c^6*d^9*e^14 + 1476395008*a^14*c^5*d^5*e^18)/(
1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4
+ 6*a^10*c^2*d^8*e^8)) + (x*(335544320*a^15*c^4*e^23 - 201326592*a^10*c^9*d
^20*e^3 - 469762048*a^11*c^8*d^16*e^7 + 134217728*a^12*c^7*d^12*e^11 + 1207
959552*a^13*c^6*d^8*e^15 + 1140850688*a^14*c^5*d^4*e^19))/(1048576*(a^12*e^
16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8
*e^8))) + (211288064*a^12*c^4*d*e^21 - 11010048*a^7*c^9*d^21*e + 20447232*a
^8*c^8*d^17*e^5 + 204472320*a^9*c^7*d^13*e^9 + 514850816*a^10*c^6*d^9*e^13
+ 553123840*a^11*c^5*d^5*e^17)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*
c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(251658240*a^12
*c^4*e^22 - 28311552*a^7*c^9*d^20*e^2 - 67108864*a^8*c^8*d^16*e^6 + 1887436
8*a^9*c^7*d^12*e^10 + 377487360*a^10*c^6*d^8*e^14 + 571473920*a^11*c^5*d^4*
e^18))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d
^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (36962304*a^9*c^4*d*e^20 + 11010048*a^5*c
^8*d^17*e^4 + 57999360*a^6*c^7*d^13*e^8 + 138805248*a^7*c^6*d^9*e^12 + 1413
61152*a^8*c^5*d^5*e^16)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e
^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(62914560*a^9*c^4*e^21
- 1806336*a^4*c^9*d^20*e + 2670592*a^5*c^8*d^16*e^5 + 43032576*a^6*c^7*d^1
2*e^9 + 143179776*a^7*c^6*d^8*e^13 + 171732992*a^8*c^5*d^4*e^17))/(1048576*
(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10
*c^2*d^8*e^8))) + (4030464*a^6*c^4*d*e^19 + 576576*a^2*c^8*d^17*e^3 + 50618
24*a^3*c^7*d^13*e^7 + 15959232*a^4*c^6*d^9*e^11 + 17863744*a^5*c^5*d^5*e^15
)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e
^4 + 6*a^10*c^2*d^8*e^8)) + (x*(5242880*a^6*c^4*e^20 + 755136*a^2*c^8*d^16*
e^4 + 6023488*a^3*c^7*d^12*e^8 + 19579200*a^4*c^6*d^8*e^12 + 22240704*a^5*c
^5*d^4*e^16))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^
9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(194481*c^7*d^12*e^7 + 871362*a
*c^6*d^8*e^11 + 970321*a^2*c^5*d^4*e^15))/(1048576*(a^12*e^16 + a^8*c^4*d^1
6 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)))*root(805
306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11
*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 4305715
2*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2
+ 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z +
676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k), k, 1, 4) + (
(3*a*e^7 + c*d^4*e^3)/(8*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^5*(7*c^3
*d^7 + 15*a*c^2*d^3*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x
^2*(5*c^2*d^6*e + 9*a*c*d^2*e^5))/(16*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)
) + (c*e^7*x^4)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2*d^7 +
19*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(3*c^3*d
^6*e + 7*a*c^2*d^2*e^5))/(16*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (e^
2*x^3*(9*c^2*d^5 + 17*a*c*d*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)
) + (e^2*x^7*(5*c^3*d^5 + 13*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a
*c*d^4*e^4)))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (e^11*log(d + e*x))/(a^3*e^12 +
c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x**4+a)**3,x)

[Out] Timed out

$$3.413 \quad \int \frac{1}{(d+ex)^2(a+cx^4)^3} dx$$

Optimal. Leaf size=1830

result too large to display

```
[Out] -e^11/(a*e^4+c*d^4)^3/(e*x+d)+1/32*c*x*(7*d^2*(-3*a*e^4+c*d^4)-12*d*e*(-a*e^4+c*d^4)*x+5*e^2*(-a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^2/(c*x^4+a)+1/8*c*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*d*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a)^2+1/4*c*e^4*(8*a*d^3*e^3+x*(d^2*(-3*a*e^4+5*c*d^4)-2*d*e*(-a*e^4+3*c*d^4)*x+e^2*(-a*e^4+7*c*d^4)*x^2))/a/(a*e^4+c*d^4)^3/(c*x^4+a)+12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/2*d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^3-3/8*d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)^2-d*e^9*(-a*e^4+5*c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/8*c^(1/4)*e^8*ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)*d^6+a^(3/2)*e^6-11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/8*c^(1/4)*e^8*ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)*d^6+a^(3/2)*e^6-11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)-1/256*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/256*c^(1/4)*ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)*arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)*arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/32*c^(1/4)*e^4*ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(1/4)*e^4*ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(1/4)*e^4*arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(1/4)*e^4*arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*e^8*arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(e^2*(-a*e^4+11*c*d^4)*a^(1/2)+3*d^2*(-a*e^4+3*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(1/4)*e^8*arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(e^2*(-a*e^4+11*c*d^4)*a^(1/2)+3*d^2*(-a*e^4+3*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)
```

Rubi [A] time = 2.78, antiderivative size = 1830, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^3), x]
```

```
[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12*d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x
```

$$\begin{aligned} & \sqrt{a}]/(2*a^{(3/2)}*(c*d^4 + a*e^4)^3) - (3*\sqrt{c}*d*e*(c*d^4 - a*e^4) \\ &)*\text{ArcTan}[(\sqrt{c}*x^2)/\sqrt{a}]]/(8*a^{(5/2)}*(c*d^4 + a*e^4)^2) - (c^{(1/4)}*(\\ & 21*\sqrt{c}*d^2*(c*d^4 - 3*a*e^4) + 5*\sqrt{a}*e^2*(3*c*d^4 - a*e^4))*\text{ArcTan}[\\ & 1 - (\sqrt{2}*c^{(1/4)}*x)/a^{(1/4)}]]/(64*\sqrt{2}*a^{(11/4)}*(c*d^4 + a*e^4)^2) - \\ & (c^{(1/4)}*e^4*(3*\sqrt{c}*d^2*(5*c*d^4 - 3*a*e^4) + \sqrt{a}*e^2*(7*c*d^4 - a \\ & *e^4))*\text{ArcTan}[1 - (\sqrt{2}*c^{(1/4)}*x)/a^{(1/4)}]]/(8*\sqrt{2}*a^{(7/4)}*(c*d^4 + \\ & a*e^4)^3) - (c^{(1/4)}*e^8*(3*\sqrt{c}*d^2*(3*c*d^4 - a*e^4) + \sqrt{a}*e^2*(1 \\ & 1*c*d^4 - a*e^4))*\text{ArcTan}[1 - (\sqrt{2}*c^{(1/4)}*x)/a^{(1/4)}]]/(2*\sqrt{2}*a^{(3/ \\ & 4)}*(c*d^4 + a*e^4)^4) + (c^{(1/4)}*(21*\sqrt{c}*d^2*(c*d^4 - 3*a*e^4) + 5*\sqrt{a} \\ & *e^2*(3*c*d^4 - a*e^4))*\text{ArcTan}[1 + (\sqrt{2}*c^{(1/4)}*x)/a^{(1/4)}]]/(64*\sqrt{2} \\ & *a^{(11/4)}*(c*d^4 + a*e^4)^2) + (c^{(1/4)}*e^4*(3*\sqrt{c}*d^2*(5*c*d^4 - 3 \\ & *a*e^4) + \sqrt{a}*e^2*(7*c*d^4 - a*e^4))*\text{ArcTan}[1 + (\sqrt{2}*c^{(1/4)}*x)/a^{(\\ & 1/4)}]]/(8*\sqrt{2}*a^{(7/4)}*(c*d^4 + a*e^4)^3) + (c^{(1/4)}*e^8*(3*\sqrt{c}*d^2* \\ & (3*c*d^4 - a*e^4) + \sqrt{a}*e^2*(11*c*d^4 - a*e^4))*\text{ArcTan}[1 + (\sqrt{2}*c^{(\\ & 1/4)}*x)/a^{(1/4)}]]/(2*\sqrt{2}*a^{(3/4)}*(c*d^4 + a*e^4)^4) + (12*c*d^3*e^{11}* \\ & \text{Log}[d + e*x])/(c*d^4 + a*e^4)^4 - (c^{(1/4)}*e^8*(9*c^{(3/2)}*d^6 - 11*\sqrt{a}*c \\ & *d^4*e^2 - 3*a*\sqrt{c}*d^2*e^4 + a^{(3/2)}*e^6)*\text{Log}[\sqrt{a} - \sqrt{2}*a^{(1/4)}* \\ & c^{(1/4)}*x + \sqrt{c}*x^2])/(4*\sqrt{2}*a^{(3/4)}*(c*d^4 + a*e^4)^4) - (c^{(1/4)}* \\ & (21*\sqrt{c}*d^2*(c*d^4 - 3*a*e^4) - 5*\sqrt{a}*e^2*(3*c*d^4 - a*e^4))*\text{Log}[\sqrt{a} \\ & - \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{c}*x^2])/(128*\sqrt{2}*a^{(11/4)}*(c* \\ & d^4 + a*e^4)^2) - (c^{(1/4)}*e^4*(3*\sqrt{c}*d^2*(5*c*d^4 - 3*a*e^4) - \sqrt{a} \\ & *e^2*(7*c*d^4 - a*e^4))*\text{Log}[\sqrt{a} - \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{c}*x \\ & ^2])/(16*\sqrt{2}*a^{(7/4)}*(c*d^4 + a*e^4)^3) + (c^{(1/4)}*e^8*(9*c^{(3/2)}*d^6 - \\ & 11*\sqrt{a}*c*d^4*e^2 - 3*a*\sqrt{c}*d^2*e^4 + a^{(3/2)}*e^6)*\text{Log}[\sqrt{a} + \sqrt{2} \\ & *a^{(1/4)}*c^{(1/4)}*x + \sqrt{c}*x^2])/(4*\sqrt{2}*a^{(3/4)}*(c*d^4 + a*e^4)^4) + \\ & (c^{(1/4)}*(21*\sqrt{c}*d^2*(c*d^4 - 3*a*e^4) - 5*\sqrt{a}*e^2*(3*c*d^4 - \\ & a*e^4))*\text{Log}[\sqrt{a} + \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{c}*x^2])/(128*\sqrt{2} \\ & *a^{(11/4)}*(c*d^4 + a*e^4)^2) + (c^{(1/4)}*e^4*(3*\sqrt{c}*d^2*(5*c*d^4 - 3*a* \\ & e^4) - \sqrt{a}*e^2*(7*c*d^4 - a*e^4))*\text{Log}[\sqrt{a} + \sqrt{2}*a^{(1/4)}*c^{(1/4)} \\ & *x + \sqrt{c}*x^2])/(16*\sqrt{2}*a^{(7/4)}*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^{11}* \\ & \text{Log}[a + c*x^4])/(c*d^4 + a*e^4)^4 \end{aligned}$$
Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1854

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*((a + b*x^n)^(p + 1)), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Mathematica [A] time = 1.55, size = 1115, normalized size = 0.61

$$3072cd^3 \log(d + ex)e^{11} - 768cd^3 \log(cx^4 + a)e^{11} - \frac{256(cd^4 + ae^4)e^{11}}{d+ex} + \frac{8c(cd^4 + ae^4)(c^2x(7d^2 - 12exd + 15e^2x^2)d^8 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^7 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^6 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^5 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^4 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^3 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^2 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d + 2ace^4x(13d^2 - 12exd + 15e^2x^2))}{a^2(c^2x(7d^2 - 12exd + 15e^2x^2)d^8 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^7 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^6 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^5 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^4 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^3 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d^2 + 2ace^4x(13d^2 - 12exd + 15e^2x^2)d + 2ace^4x(13d^2 - 12exd + 15e^2x^2))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^3), x]

[Out]
$$\begin{aligned} &((-256e^{11}(cd^4 + ae^4))/(d + ex) + (8c*(cd^4 + ae^4)*(c^2d^8*x*(7d^2 - 12d*ex + 15e^2*x^2) + 2*a*cd^4*ae^4*x*(13d^2 - 24d*ex + 33e^2*x^2) + a^2*e^7*(64d^3 - 45d^2*ex + 28d*ex^2 - 13e^3*x^3)))/(a^2*(a + c*x^4)) + (32c*(cd^4 + ae^4)^2*(cd^4*x*(d^2 - 2d*ex + 3e^2*x^2) + ae^3*(4d^3 - 3d^2*ex + 2d*ex^2 - e^3*x^3)))/(a*(a + c*x^4)^2) - (6*c^{(1/4)}*(7*Sqrt[2]*c^{(7/2)}*d^{14} - 16*a^{(1/4)}*c^{(13/4)}*d^{13}*e + 5*Sqrt[2]*Sqrt[a]*c^3*d^{12}*e^2 + 33*Sqrt[2]*a*c^{(5/2)}*d^{10}*e^4 - 80*a^{(5/4)}*c^{(9/4)}*d^9*e^5 + 27*Sqrt[2]*a^{(3/2)}*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^{(3/2)}*d^6*e^8 - 240*a^{(9/4)}*c^{(5/4)}*d^5*e^9 + 135*Sqrt[2]*a^{(5/2)}*c*d^4*e^{10} - 77*Sqrt[2]*a^3*Sqrt[c]*d^2*e^{12} + 80*a^{(13/4)}*c^{(1/4)}*d*e^{13} - 15*Sqrt[2]*a^{(7/2)}*e^{14})*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/a^{(11/4)} + (6*c^{(1/4)}*(7*Sqrt[2]*c^{(7/2)}*d^{14} + 16*a^{(1/4)}*c^{(13/4)}*d^{13}*e + 5*Sqrt[2]*Sqrt[a]*c^3*d^{12}*e^2 + 33*Sqrt[2]*a*c^{(5/2)}*d^{10}*e^4 + 80*a^{(5/4)}*c^{(9/4)}*d^9*e^5 + 27*Sqrt[2]*a^{(3/2)}*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^{(3/2)}*d^6*e^8 + 240*a^{(9/4)}*c^{(5/4)}*d^5*e^9 + 135*Sqrt[2]*a^{(5/2)}*c*d^4*e^{10} - 77*Sqrt[2]*a^3*Sqrt[c]*d^2*e^{12} - 80*a^{(13/4)}*c^{(1/4)}*d*e^{13} - 15*Sqrt[2]*a^{(7/2)}*e^{14})*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/a^{(11/4)} + 3072*c*d^3*e^{11}*Log[d + e*x] - (3*Sqrt[2]*c^{(1/4)}*(7*c^{(7/2)}*d^{14} - 5*Sqrt[a]*c^3*d^{12}*e^2 + 33*a*c^{(5/2)}*d^{10}*e^4 - 27*a^{(3/2)}*c^2*d^8*e^6 + 77*a^2*c^{(3/2)}*d^6*e^8 - 135*a^{(5/2)}*c*d^4*e^{10} - 77*a^3*Sqrt[c]*d^2*e^{12} + 15*a^{(7/2)}*e^{14})*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/a^{(11/4)} + (3*Sqrt[2]*c^{(1/4)}*(7*c^{(7/2)}*d^{14} - 5*Sqrt[a]*c^3*d^{12}*e^2 + 33*a*c^{(5/2)}*d^{10}*e^4 - 27*a^{(3/2)}*c^2*d^8*e^6 + 77*a^2*c^{(3/2)}*d^6*e^8 - 135*a^{(5/2)}*c*d^4*e^{10} - 77*a^3*Sqrt[c]*d^2*e^{12} + 15*a^{(7/2)}*e^{14})*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/a^{(11/4)} - 768*c*d^3*e^{11}*Log[a + c*x^4])/(256*(cd^4 + ae^4)^4) \end{aligned}$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.03, size = 2769, normalized size = 1.51

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$1/2)/(a/c)^{(1/4)*x+1)*d^{14}$$

maxima [A] time = 3.10, size = 1564, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")

[Out] $12*c*d^3*e^{11}*\log(e*x + d)/(c^4*d^{16} + 4*a*c^3*d^{12}*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^{12} + a^4*e^{16}) - 3/256*c*(\sqrt{2}*(128*\sqrt{2})*a^{(11/4)}*c^{(5/4)}*d^3*e^{11} - 7*c^4*d^{14} + 5*\sqrt{a}*c^{(7/2)}*d^{12}*e^2 - 33*a*c^3*d^{10}*e^4 + 27*a^{(3/2)}*c^{(5/2)}*d^8*e^6 - 77*a^2*c^2*d^6*e^8 + 135*a^{(5/2)}*c^{(3/2)}*d^4*e^{10} + 77*a^3*c*d^2*e^{12} - 15*a^{(7/2)}*\sqrt{c}*e^{14})*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) + \sqrt{2}*(128*\sqrt{2})*a^{(11/4)}*c^{(5/4)}*d^3*e^{11} + 7*c^4*d^{14} - 5*\sqrt{a}*c^{(7/2)}*d^{12}*e^2 + 33*a*c^3*d^{10}*e^4 - 27*a^{(3/2)}*c^{(5/2)}*d^8*e^6 + 77*a^2*c^2*d^6*e^8 - 135*a^{(5/2)}*c^{(3/2)}*d^4*e^{10} - 77*a^3*c*d^2*e^{12} + 15*a^{(7/2)}*\sqrt{c}*e^{14})*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) - 2*(7*\sqrt{2})*a^{(1/4)}*c^{(17/4)}*d^{14} + 5*\sqrt{2})*a^{(3/4)}*c^{(15/4)}*d^{12}*e^2 + 33*\sqrt{2})*a^{(5/4)}*c^{(13/4)}*d^{10}*e^4 + 27*\sqrt{2})*a^{(7/4)}*c^{(11/4)}*d^8*e^6 + 77*\sqrt{2})*a^{(9/4)}*c^{(9/4)}*d^6*e^8 + 135*\sqrt{2})*a^{(11/4)}*c^{(7/4)}*d^4*e^{10} - 77*\sqrt{2})*a^{(13/4)}*c^{(5/4)}*d^2*e^{12} - 15*\sqrt{2})*a^{(15/4)}*c^{(3/4)}*e^{14} + 16*\sqrt{a})*c^4*d^{13}*e + 80*a^{(3/2)}*c^3*d^9*e^5 + 240*a^{(5/2)}*c^2*d^5*e^9 - 80*a^{(7/2)}*c*d*e^{13})*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})/(a^{(3/4)}*\sqrt{\sqrt{a}*\sqrt{c}})*c^{(5/4)}) - 2*(7*\sqrt{2})*a^{(1/4)}*c^{(17/4)}*d^{14} + 5*\sqrt{2})*a^{(3/4)}*c^{(15/4)}*d^{12}*e^2 + 33*\sqrt{2})*a^{(5/4)}*c^{(13/4)}*d^{10}*e^4 + 27*\sqrt{2})*a^{(7/4)}*c^{(11/4)}*d^8*e^6 + 77*\sqrt{2})*a^{(9/4)}*c^{(9/4)}*d^6*e^8 + 135*\sqrt{2})*a^{(11/4)}*c^{(7/4)}*d^4*e^{10} - 77*\sqrt{2})*a^{(13/4)}*c^{(5/4)}*d^2*e^{12} - 15*\sqrt{2})*a^{(15/4)}*c^{(3/4)}*e^{14} - 16*\sqrt{a})*c^4*d^{13}*e - 80*a^{(3/2)}*c^3*d^9*e^5 - 240*a^{(5/2)}*c^2*d^5*e^9 + 80*a^{(7/2)}*c*d*e^{13})*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})/(a^{(3/4)}*\sqrt{\sqrt{a}*\sqrt{c}})*c^{(5/4)})/(a^2*c^4*d^{16} + 4*a^3*c^3*d^{12}*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^{12} + a^6*e^{16}) + 1/32*(16*a^2*c^2*d^8*e^3 + 80*a^3*c*d^4*e^7 - 32*a^4*e^{11} + 3*(5*c^4*d^8*e^3 + 22*a*c^3*d^4*e^7 - 15*a^2*c^2*e^{11})*x^8 + 3*(c^4*d^9*e^2 + 6*a*c^3*d^5*e^6 + 5*a^2*c^2*d*e^{10})*x^7 - (5*c^4*d^{10}*e + 22*a*c^3*d^6*e^5 + 17*a^2*c^2*d^2*e^9)*x^6 + (7*c^4*d^{11} + 26*a*c^3*d^7*e^4 + 19*a^2*c^2*d^3*e^8)*x^5 + 3*(9*a*c^3*d^8*e^3 + 46*a^2*c^2*d^4*e^7 - 27*a^3*c*e^{11})*x^4 + (7*a*c^3*d^9*e^2 + 26*a^2*c^2*d^5*e^6 + 19*a^3*c*d*e^{10})*x^3 - 3*(3*a*c^3*d^{10}*e + 10*a^2*c^2*d^6*e^5 + 7*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^{11} + 34*a^2*c^2*d^7*e^4 + 23*a^3*c*d^3*e^8)*x)/(a^4*c^3*d^{13} + 3*a^5*c^2*d^9*e^4 + 3*a^6*c*d^5*e^8 + a^7*d*e^{12} + (a^2*c^5*d^{12}*e + 3*a^3*c^4*d^8*e^5 + 3*a^4*c^3*d^4*e^9 + a^5*c^2*e^{13})*x^9 + (a^2*c^5*d^{13} + 3*a^3*c^4*d^9*e^4 + 3*a^4*c^3*d^5*e^8 + a^5*c^2*d*e^{12})*x^8 + 2*(a^3*c^4*d^{12}*e + 3*a^4*c^3*d^8*e^5 + 3*a^5*c^2*d^4*e^9 + a^6*c*e^{13})*x^5 + 2*(a^3*c^4*d^{13} + 3*a^4*c^3*d^9*e^4 + 3*a^5*c^2*d^5*e^8 + a^6*c*d*e^{12})*x^4 + (a^4*c^3*d^{12}*e + 3*a^5*c^2*d^8*e^5 + 3*a^6*c*d^4*e^9 + a^7*e^{13})*x)$

mupad [B] time = 5.75, size = 3572, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)^3*(d + e*x)^2),x)

[Out] $\text{symsum}(\log((194481*c^9*d^{17}*e^6 + 1527012*a*c^8*d^{13}*e^{10} + 4100625*a^4*c^5*d*e^{22} + 1926342*a^2*c^7*d^9*e^{14} - 3102300*a^3*c^6*d^5*e^{18})/(1048576*(a^{14}*e^{24} + a^8*c^6*d^{24} + 6*a^{13}*c*d^4*e^{20} + 6*a^9*c^5*d^{20}*e^4 + 15*a^{10}*c^4*d^{16}*e^8 + 20*a^{11}*c^3*d^{12}*e^{12} + 15*a^{12}*c^2*d^8*e^{16}))) + \text{root}(1610612$

$$\begin{aligned}
& 736a^{13}c^2d^8e^8z^4 + 1073741824a^{12}c^3d^{12}e^4z^4 + 1073741824a^{14}c^4d^4e^{12}z^4 + 268435456a^{11}c^4d^{16}z^4 + 268435456a^{15}e^{16}z^4 + \\
& 3221225472a^{11}c^3d^3e^{11}z^3 + 239468544a^7c^2d^6e^6z^2 + 39518208a^6c^3d^{10}e^2z^2 + 1153105920a^8c^3d^2e^{10}z^2 + 32071680a^4c^2d^5e^5z + 5419008a^3c^3d^9e^z + 124416000a^5c^3d^9e^z + 1138050a^2c^2d^4e^4 + 4100625a^2c^2e^8 + 194481c^3d^8, z, k) \cdot (\text{root}(1610612736a^{13}c^2d^8e^8z^4 + 1073741824a^{12}c^3d^{12}e^4z^4 + 1073741824a^{14}c^4d^4e^{12}z^4 + 268435456a^{11}c^4d^{16}z^4 + 268435456a^{15}e^{16}z^4 + 3221225472a^{11}c^3d^3e^{11}z^3 + 239468544a^7c^2d^6e^6z^2 + 39518208a^6c^3d^{10}e^2z^2 + 1153105920a^8c^3d^2e^{10}z^2 + 32071680a^4c^2d^5e^5z + 5419008a^3c^3d^9e^z + 124416000a^5c^3d^9e^z + 1138050a^2c^2d^4e^4 + 4100625a^2c^2e^8 + 194481c^3d^8, z, k) \cdot (\text{root}(1610612736a^{13}c^2d^8e^8z^4 + 1073741824a^{12}c^3d^{12}e^4z^4 + 1073741824a^{14}c^4d^4e^{12}z^4 + 268435456a^{11}c^4d^{16}z^4 + 268435456a^{15}e^{16}z^4 + 3221225472a^{11}c^3d^3e^{11}z^3 + 239468544a^7c^2d^6e^6z^2 + 39518208a^6c^3d^{10}e^2z^2 + 1153105920a^8c^3d^2e^{10}z^2 + 32071680a^4c^2d^5e^5z + 5419008a^3c^3d^9e^z + 124416000a^5c^3d^9e^z + 1138050a^2c^2d^4e^4 + 4100625a^2c^2e^8 + 194481c^3d^8, z, k) \cdot ((23592960a^{14}c^4e^{29} - 11010048a^7c^{11}d^{28}e + 33030144a^8c^{10}d^{24}e^5 + 504889344a^9c^9d^{20}e^9 + 3103260672a^{10}c^8d^{16}e^{13} + 6799491072a^{11}c^7d^{12}e^{17} + 6101139456a^{12}c^6d^8e^{21} + 1967652864a^{13}c^5d^4e^{25}) / (1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c^4d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + \text{root}(1610612736a^{13}c^2d^8e^8z^4 + 1073741824a^{12}c^3d^{12}e^4z^4 + 1073741824a^{14}c^4d^4e^{12}z^4 + 268435456a^{11}c^4d^{16}z^4 + 268435456a^{15}e^{16}z^4 + 3221225472a^{11}c^3d^3e^{11}z^3 + 239468544a^7c^2d^6e^6z^2 + 39518208a^6c^3d^{10}e^2z^2 + 1153105920a^8c^3d^2e^{10}z^2 + 32071680a^4c^2d^5e^5z + 5419008a^3c^3d^9e^z + 124416000a^5c^3d^9e^z + 1138050a^2c^2d^4e^4 + 4100625a^2c^2e^8 + 194481c^3d^8, z, k) \cdot ((402653184a^{17}c^4d^30e^{30} - 134217728a^{10}c^{11}d^{29}e^2 - 402653184a^{11}c^{10}d^{25}e^6 + 402653184a^{12}c^9d^{21}e^{10} + 3355443200a^{13}c^8d^{17}e^{14} + 6039797760a^{14}c^7d^{13}e^{18} + 5234491392a^{15}c^6d^9e^{22} + 2281701376a^{16}c^5d^5e^{26}) / (1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c^4d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (x \cdot (335544320a^{17}c^4e^{31} - 201326592a^{10}c^{11}d^{28}e^3 - 872415232a^{11}c^{10}d^{24}e^7 - 1006632960a^{12}c^9d^{20}e^{11} + 1006632960a^{13}c^8d^{16}e^{15} + 3690987520a^{14}c^7d^{12}e^{19} + 3825205248a^{15}c^6d^8e^{23} + 1811939328a^{16}c^5d^4e^{27})) / (1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c^4d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (x \cdot (2554331136a^{10}c^8d^{15}e^{14} - 144703488a^8c^{10}d^{23}e^6 - 154140672a^9c^9d^{19}e^{10} - 34603008a^7c^{11}d^{27}e^2 + 7659847680a^{11}c^7d^{11}e^{18} + 7556038656a^{12}c^6d^7e^{22} + 2494562304a^{13}c^5d^3e^{26})) / (1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c^4d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (12681216a^5c^{10}d^{23}e^4 + 127107072a^6c^9d^{19}e^8 + 674168832a^7c^8d^{15}e^{12} + 1018626048a^8c^7d^{11}e^{16} - 446201856a^9c^6d^7e^{20} + 906854400a^{10}c^5d^3e^{24}) / (1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c^4d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (x \cdot (516096a^5c^{10}d^{22}e^5 - 1806336a^4c^{11}d^{26}e + 90427392a^6c^9d^{18}e^9 + 896090112a^7c^8d^{14}e^{13} + 1960906752a^8c^7d^{10}e^{17} + 1732829184a^9c^6d^6e^{21} + 1183887360a^{10}c^5d^2e^{25})) / (1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c^4d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (387072a^2c^{10}d^{22}e^3 + 8004096a^3c^9d^{18}e^7 + 49379328a^4c^8d^{14}e^{11} + 49572864a^5c^7d^{10}e^{15} - 156930048a^6c^6d^6e^{19} + 125452800a^7c^5d^2e^{23}) / (1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c^4d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (x \cdot (126360000a^7c^5d^2e^{24} + 561600a^2c^{10}d^{21}e^4 + 9609408a^3c^9d^{17}e^8 + 75731328a^4c^8d^{13}e^{12} + 114
\end{aligned}$$

$$\frac{991488a^5c^7d^9e^{16} - 80136000a^6c^6d^5e^{20}}{(1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}cd^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16}))} + (x(4100625a^4c^5e^{23} + 194481c^9d^{16}e^7 + 1527012a^8c^3d^{12}e^{11} - 167994a^2c^7d^8e^{15} - 13988700a^3c^6d^4e^{19}))/((1048576(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}cd^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16}))) * \text{root}(1610612736a^{13}c^2d^8e^8z^4 + 1073741824a^{12}c^3d^{12}e^4z^4 + 1073741824a^{14}cd^4e^{12}z^4 + 268435456a^{11}c^4d^{16}z^4 + 268435456a^{15}e^{16}z^4 + 3221225472a^{11}c^3d^3e^{11}z^3 + 239468544a^7c^2d^6e^6z^2 + 39518208a^6c^3d^{10}e^2z^2 + 1153105920a^8cd^2e^{10}z^2 + 32071680a^4c^2d^5e^5z + 5419008a^3c^3d^9e^5z + 124416000a^5cd^9e^9z + 1138050a^2c^2d^4e^4 + 4100625a^2c^8e^8 + 194481c^3d^8, z, k), k, 1, 4) + ((c^2d^8e^3 - 2a^2e^{11} + 5a^3cd^4e^7)/(2(a^4e^4 + cd^4)(a^2e^8 + c^2d^8 + 2a^3cd^4e^4)) + (3x^8(5c^4d^8e^3 - 15a^2c^2e^{11} + 22a^3cd^4e^7))/(32a^2(a^3e^{12} + c^3d^{12} + 3a^2cd^8e^4 + 3a^2cd^4e^8)) + (x^5(7c^3d^7 + 19a^2cd^3e^4))/(32a^2(a^2e^8 + c^2d^8 + 2a^3cd^4e^4)) - (3x^2(3c^2d^6e + 7a^3cd^2e^5))/(32a(a^2e^8 + c^2d^8 + 2a^3cd^4e^4)) + (x^3(7c^2d^5e^2 + 19a^3cd^6e^6))/(32a(a^2e^8 + c^2d^8 + 2a^3cd^4e^4)) + (x(11c^2d^7 + 23a^3cd^3e^4))/(32a(a^2e^8 + c^2d^8 + 2a^3cd^4e^4)) - (x^6(5c^3d^6e + 17a^2cd^2e^5))/(32a^2(a^2e^8 + c^2d^8 + 2a^3cd^4e^4)) + (3e^2x^7(c^3d^5 + 5a^2cd^4e^4))/(32a^2(a^2e^8 + c^2d^8 + 2a^3cd^4e^4)) + (3e^2x^4(9c^3d^8e - 27a^2c^9 + 46a^2cd^4e^5))/(32a(a^4e^4 + cd^4)(a^2e^8 + c^2d^8 + 2a^3cd^4e^4)))/(a^2d + c^2dx^8 + c^2ex^9 + a^2ex + 2a^3cd^4x^4 + 2a^3cex^5) + (12cd^3e^{11} \log(d + ex))/(a^4e^{16} + c^4d^{16} + 4a^3cd^{12}e^4 + 4a^3cd^4e^{12} + 6a^2c^2d^8e^8)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)

[Out] Timed out

$$3.414 \quad \int \frac{1}{(d+ex)^3(a+cx^4)^3} dx$$

Optimal. Leaf size=2204

result too large to display

```
[Out] -1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)+1
/32*c*x*(7*d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-6*e*(a^2*e^8-12*a*c*d^4*e^4
+3*c^2*d^8))*x+10*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^3/(c*x
^4+a)+1/8*c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+
c^2*d^8)-e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8))*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d
^4)*x^2))/a/(a*e^4+c*d^4)^3/(c*x^4+a)^2+1/4*c*e^4*(12*a*d^2*e^3*(-a*e^4+3*c
*d^4)+x*(3*d*(a^2*e^8-10*a*c*d^4*e^4+5*c^2*d^8)-e*(a^2*e^8-26*a*c*d^4*e^4+2
1*c^2*d^8))*x+4*c*d^3*e^2*(-5*a*e^4+7*c*d^4)*x^2))/a/(a*e^4+c*d^4)^4/(c*x^4+
a)+6*c*d^2*e^11*(-3*a*e^4+13*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^5-3/2*c*d^2*e^1
1*(-3*a*e^4+13*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^5-1/4*e^5*(a^2*e^8-26*a*c*d
^4*e^4+21*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4
)^4-3/16*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c
^(1/2)/a^(5/2)/(a*e^4+c*d^4)^3-1/2*e^9*(a^2*e^8-40*a*c*d^4*e^4+55*c^2*d^8)*
arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^5/a^(1/2)+1/256*c^(3/4)*d
*ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d
^4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a
*e^4+c*d^4)^3*2^(1/2)-1/256*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+
x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d^4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3
*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/128*c^(3/4)*d*a
rctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(63*a^2*e^8-252*a*c*d^4*e^4+21*c^2*d^8+
10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(
1/2)+1/128*c^(3/4)*d*arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(63*a^2*e^8-252*a*
c*d^4*e^4+21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4
)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(3/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-9*a^2*e^8+90*a*c*d^4*e^4-45*c^2*d^8+4*d^2*e^2*(-5*a*
e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)-1/32*c^(3/4)*
d*e^4*ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-9*a^2*e^8+90*a*c*
d^4*e^4-45*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a
*e^4+c*d^4)^4*2^(1/2)+1/16*c^(3/4)*d*e^4*arctan(-1+c^(1/4)*x^2^(1/2)/a^(1/4
))*(9*a^2*e^8-90*a*c*d^4*e^4+45*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2
)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/16*c^(3/4)*d*e^4*arctan(1+c^(1
/4)*x^2^(1/2)/a^(1/4))*(9*a^2*e^8-90*a*c*d^4*e^4+45*c^2*d^8+4*d^2*e^2*(-5*a
*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)-3/8*c^(3/4)*
d*e^8*ln(-a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)+x^2*c^(1/2))*(15*c^2*d^8-16*a*c
*d^4*e^4+a^2*e^8-2*d^2*e^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*
e^4+c*d^4)^5*2^(1/2)+3/8*c^(3/4)*d*e^8*ln(a^(1/4)*c^(1/4)*x^2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(15*c^2*d^8-16*a*c*d^4*e^4+a^2*e^8-2*d^2*e^2*(-5*a*e^4+11*c*d
^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^5*2^(1/2)+3/4*c^(3/4)*d*e^8*arct
an(-1+c^(1/4)*x^2^(1/2)/a^(1/4))*(15*c^2*d^8-16*a*c*d^4*e^4+a^2*e^8+2*d^2*e
^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^5*2^(1/2)+3/4
*c^(3/4)*d*e^8*arctan(1+c^(1/4)*x^2^(1/2)/a^(1/4))*(15*c^2*d^8-16*a*c*d^4*e
^4+a^2*e^8+2*d^2*e^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*
d^4)^5*2^(1/2)
```

Rubi [A] time = 3.16, antiderivative size = 2204, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + c*x^4)^3), x]

```
[Out] -e^11/(2*(c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4 + a*e^4)^4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(3*d*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^4*(a + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (6*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^5)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

$\text{Int}[(x_)^{(m_.)} / ((a_) + (b_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 275

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.)(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{With}\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)} * (a + b*x^{(n/k)})^p], x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}\{a, b, p, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 617

$\text{Int}[(a_) + (b_.)(x_) + (c_.)(x_)^2]^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2)], x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_) + (e_.)(x_)] / ((a_) + (b_.)(x_) + (c_.)(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]) / b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 635

$\text{Int}[(d_) + (e_.)(x_)] / ((a_) + (c_.)(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ !\text{NiceSqrtQ}[-(a*c)]$

Rule 1162

$\text{Int}[(d_) + (e_.)(x_)^2] / ((a_) + (c_.)(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1165

$\text{Int}[(d_) + (e_.)(x_)^2] / ((a_) + (c_.)(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(-2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1168

$\text{Int}[(d_) + (e_.)(x_)^2] / ((a_) + (c_.)(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x] /; \text{FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

Rule 1248

$\text{Int}[(x_) * ((d_) + (e_.)(x_)^2)^{(q_.)} * ((a_) + (c_.)(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q * (a + c*x^2)^p], x, x^2], x] /; \text{FreeQ}\{a, c, d, e, p, q, x\}$

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
  x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]* (a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3 (a+cx^4)^3} dx &= \int \left(\frac{e^{12}}{(cd^4+ae^4)^3 (d+ex)^3} + \frac{12cd^3e^{12}}{(cd^4+ae^4)^4 (d+ex)^2} + \frac{6cd^2e^{12}(13cd^4-3ae^4)}{(cd^4+ae^4)^5 (d+ex)} + \frac{c(d+ex)}{(cd^4+ae^4)^5} \right) dx \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{6cd^2e^{11}(13cd^4-3ae^4) \log(d+ex)}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8))}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8))}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8))}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8))}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8))}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8))}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8))}{(cd^4+ae^4)^5} \\
&= -\frac{e^{11}}{2(cd^4+ae^4)^3 (d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4 (d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3a^2e^8))}{(cd^4+ae^4)^5}
\end{aligned}$$

Mathematica [A] time = 2.87, size = 1338, normalized size = 0.61

$$1536cd^2(13cd^4-3ae^4) \log(d+ex)e^{11} - 384cd^2(13cd^4-3ae^4) \log(cx^4+a)e^{11} - \frac{3072cd^3(cd^4+ae^4)e^{11}}{d+ex} - \frac{128(cd^4+ae^4)^2}{(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^3), x]

```
[Out] ((-128*e^11*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (3072*c*d^3*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(a^3*e^11*(-96*d^2 + 45*d*e*x - 14*e^2*x^2) + c^3*d^11*x*(7*d^2 - 18*d*e*x + 30*e^2*x^2) + a*c^2*d^7*e^4*x*(43*d^2 - 114*d*e*x + 204*e^2*x^2) + a^2*c*d^3*e^7*(288*d^3 - 303*d^2*e*x + 274*d*e^2*x^2 - 210*e^3*x^3)))/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x^4)^2) - (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 - 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 - 176*a^(5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 - 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 + 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 - 40*a^(17/4)*e^17)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(11/4) + (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 + 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 + 176*a^(5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 + 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 - 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 + 40*a^(17/4)*e^17)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(11/4) + 1536*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 - 702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/a^(11/4) + (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 - 702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/a^(11/4) - 384*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4]/(256*(c*d^4 + a*e^4)^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.53, size = 2119, normalized size = 0.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/64*(23*sqrt(2)*a*c^4*d^6*e - 65*(a*c^3)^(1/4)*a*c^3*d^5*e^2 + 30*sqrt(2)*sqrt(a*c)*a*c^3*d^4*e^3 - 7*(a*c^3)^(3/4)*c^2*d^7 - 115*sqrt(2)*a^2*c^3*d^2*e^5 + 65*(a*c^3)^(3/4)*a*c*d^3*e^4 + 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 + 20*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)))/(a/c)^(1/4))/(sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 25*sqrt(2)*a^4*c^4*d^8*e^2 + 10*(a*c^3)^(3/4)*a^3*c^2*d^9*e + 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 90*sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 90*sqrt(2)*a^5*c^3*d^4*e^6 + 148*(a*c^3)^(3/4)*a^4*c*d^5*e^5 + 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 25*sqrt(2)*sqrt(a*c)*a^5*c^2*d^2*e^8 - sqrt(2)*a^6*c^2*e^10 + 10*(a*c^3)^(3/4)*a^5*d*e^9) - 3/64*(23*sqrt(2)*a*c^4*d^6*e + 65*(a*c^3)^(1/4)*a*c^3*d^5*e^2 - 30*sqrt(2)*sqrt(a*c)*a*c^3*d^4*e^3 + 7*(a*c^3)^(3/4)*c^2*d^7 - 115*sqrt(2)*a^2*c^3*d^2*e^5 - 65*(a*c^3)^(3/4)*a*c*d^3*e^4 - 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 - 20
```

```

*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 25*sqrt(2)*a^4*c^4*d^8*e^2 - 10*(a*c^3)^(3/4)*a^3*c^2*d^9*e - 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 90*sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 90*sqrt(2)*a^5*c^3*d^4*e^6 - 148*(a*c^3)^(3/4)*a^4*c*d^5*e^5 - 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 25*sqrt(2)*sqrt(a*c)*a^5*c^2*d^2*e^8 - sqrt(2)*a^6*c^2*e^10 - 10*(a*c^3)^(3/4)*a^5*d*e^9) + 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^17 - 10*sqrt(2)*(a*c^3)^(3/4)*c^3*d^15*e^2 + 50*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*sqrt(2)*(a*c^3)^(3/4)*a*c^2*d^11*e^6 + 220*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 702*sqrt(2)*(a*c^3)^(3/4)*a^2*c*d^7*e^10 - 770*sqrt(2)*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 + 390*sqrt(2)*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*sqrt(2)*(a*c^3)^(1/4)*a^4*c*d*e^16)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^20 + 5*a^4*c^5*d^16*e^4 + 10*a^5*c^4*d^12*e^8 + 10*a^6*c^3*d^8*e^12 + 5*a^7*c^2*d^4*e^16 + a^8*c*e^20) - 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^17 - 10*sqrt(2)*(a*c^3)^(3/4)*c^3*d^15*e^2 + 50*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*sqrt(2)*(a*c^3)^(3/4)*a*c^2*d^11*e^6 + 220*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 702*sqrt(2)*(a*c^3)^(3/4)*a^2*c*d^7*e^10 - 770*sqrt(2)*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 + 390*sqrt(2)*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*sqrt(2)*(a*c^3)^(1/4)*a^4*c*d*e^16)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^20 + 5*a^4*c^5*d^16*e^4 + 10*a^5*c^4*d^12*e^8 + 10*a^6*c^3*d^8*e^12 + 5*a^7*c^2*d^4*e^16 + a^8*c*e^20) - 3/2*(13*c^2*d^6*e^11 - 3*a*c*d^2*e^15)*log(abs(c*x^4 + a))/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^12*e^8 + 10*a^3*c^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 6*(13*c^2*d^6*e^12 - 3*a*c*d^2*e^16)*log(abs(x*e + d))/(c^5*d^20*e + 5*a*c^4*d^16*e^5 + 10*a^2*c^3*d^12*e^9 + 10*a^3*c^2*d^8*e^13 + 5*a^4*c*d^4*e^17 + a^5*e^21) + 1/32*(30*c^5*d^11*x^9*e^4 + 42*c^5*d^12*x^8*e^3 + c^5*d^13*x^7*e^2 - 4*c^5*d^14*x^6*e + 7*c^5*d^15*x^5 + 204*a*c^4*d^7*x^9*e^8 + 294*a*c^4*d^8*x^8*e^7 + 19*a*c^4*d^9*x^7*e^6 - 28*a*c^4*d^10*x^6*e^5 + 97*a*c^4*d^11*x^5*e^4 + 78*a*c^4*d^12*x^4*e^3 + 5*a*c^4*d^13*x^3*e^2 - 8*a*c^4*d^14*x^2*e + 11*a*c^4*d^15*x - 594*a^2*c^3*d^3*x^9*e^12 - 546*a^2*c^3*d^4*x^8*e^11 + 35*a^2*c^3*d^5*x^7*e^10 - 44*a^2*c^3*d^6*x^6*e^9 + 461*a^2*c^3*d^7*x^5*e^8 + 586*a^2*c^3*d^8*x^4*e^7 + 31*a^2*c^3*d^9*x^3*e^6 - 40*a^2*c^3*d^10*x^2*e^5 + 79*a^2*c^3*d^11*x*e^4 + 40*a^2*c^3*d^12*e^3 - 30*a^3*c^2*x^8*e^15 + 17*a^3*c^2*d*x^7*e^14 - 20*a^3*c^2*d^2*x^6*e^13 - 1165*a^3*c^2*d^3*x^5*e^12 - 1078*a^3*c^2*d^4*x^4*e^11 + 47*a^3*c^2*d^5*x^3*e^10 - 56*a^3*c^2*d^6*x^2*e^9 + 269*a^3*c^2*d^7*x*e^8 + 304*a^3*c^2*d^8*e^7 - 50*a^4*c*x^4*e^15 + 21*a^4*c*d*x^3*e^14 - 24*a^4*c*d^2*x^2*e^13 - 567*a^4*c*d^3*x*e^12 - 520*a^4*c*d^4*e^11 - 16*a^5*e^15)/((a^2*c^4*d^16 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6*e^16)*(c*x^5*e + c*d*x^4 + a*x*e + a*d)^2)

```

maple [A] time = 0.04, size = 3334, normalized size = 1.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^3/(c*x^4+a)^3,x)

```

[Out] -1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)+165/32*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^9*e^8+75/64*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^13*e^4+165/64*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^9*e^8+75/128*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^13*e^4+165/32*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9*e^8+78*e^11*d^6*c^2/(a*e^4+c*d^4)^5*ln(e*x+d)+5/4*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^3*d^14-39/2*c^2/(a*e^4+c*d^4)^5*ln(c*x^4+a)*e^11*d^6-129/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a*x^5*e^12+25/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^13/a*x^5*e^4-125/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^3*e^14*a^2*x^3-31/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^7*e^10*a*x^

```


$$\begin{aligned}
& 3+27/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^15*e^2/a*x^3+75/8*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^13*a^2*x^2*d^4+15/2*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*a*x^2*d^8-15/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a*x^2*d^16-141/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a^2*x*e^12-85/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^9*a*x*e^8-3*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*a^2*d^2*e^15+6*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*a*d^6*e^11-105/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^3*e^14*a*x^7+117/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^11*e^6/a*x^7+15/16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^15*e^2/a^2*x^7-1155/64*c^2/(a*e^4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^12-1155/128*c^2/(a*e^4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^12-1155/64*c^2/(a*e^4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^12+1053/64*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^7*e^10+1053/128*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7*e^10+1053/64*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7*e^10-9/16*c^5/(a*e^4+c*d^4)^5/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^16+21/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^17+21/256*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^17+21/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^17+225/8*c^2/(a*e^4+c*d^4)^5*a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^13*d^4-33/8*c^4/(a*e^4+c*d^4)^5/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^12+57/32*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^3*x*e^16+65/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^13*a*x^6*d^4-33/8*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^5/a*x^6*d^12-9/16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a^2*x^6*d^16+45/32*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^2*x^5*e^16-18*e^15*d^2*c/(a*e^4+c*d^4)^5*ln(e*x+d)*a+5*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*x^6*d^8-65/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^9*x^5*e^8+121/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^11*e^6*x^3-27/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^5*x^2*d^12+5/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^13*x*e^4+9*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*d^10*e^7-7/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^17*a^2*x^6+7/32*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^17/a^2*x^5+11/32*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^17/a*x+43/4*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^7*d^10*a+23/4*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^11*d^6*a^2-3/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^7*e^10*x^7-9/16*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^17*a^3*x^2-15/4*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^15*d^2*a^3-15/16*c/(a*e^4+c*d^4)^5*a^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^17+9/2*c/(a*e^4+c*d^4)^5*a*ln(c*x^4+a)*e^15*d^2-45/2*c^3/(a*e^4+c*d^4)^5/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*d^8+75/64*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^13*e^4+117/64*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^11*e^6+117/128*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^11*e^6+117/64*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^11*e^6+15/64*c^4/(a*e^4+c*d^4)^5/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^15*e^2+15/128*c^4/(a*e^4+c*d^4)^5/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^15*e^2+15/64*c^4/(a*e^4+c*d^4)^5/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^15*e^2-585/64*c/(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^14-585/128*c/(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e^14-585/64*c/(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^14+231/128*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^16+231/256*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^16+231/128*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^16
\end{aligned}$$

maxima [A] time = 3.99, size = 2198, normalized size = 1.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")

[Out]
$$-3/256*c*(\sqrt{2}*(832*\sqrt{2})*a^{11/4}*c^{9/4}*d^6*e^{11} - 192*\sqrt{2})*a^{15/4}*c^{5/4}*d^2*e^{15} - 7*c^5*d^{17} + 10*\sqrt{a}*c^{9/2}*d^{15}*e^2 - 50*a*c^4*d^{13}*e^4 + 78*a^{3/2}*c^{7/2}*d^{11}*e^6 - 220*a^2*c^3*d^9*e^8 + 702*a^{5/2}*c^{5/2}*d^7*e^{10} + 770*a^3*c^2*d^5*e^{12} - 390*a^{7/2}*c^{3/2}*d^3*e^{14} - 77*a^4*c*d*e^{16})*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{1/4}*c^{1/4}*x + \sqrt{a})/(a^{3/4}*c^{5/4}) + \sqrt{2}*(832*\sqrt{2})*a^{11/4}*c^{9/4}*d^6*e^{11} - 192*\sqrt{2})*a^{15/4}*c^{5/4}*d^2*e^{15} + 7*c^5*d^{17} - 10*\sqrt{a}*c^{9/2}*d^{15}*e^2 + 50*a*c^4*d^{13}*e^4 - 78*a^{3/2}*c^{7/2}*d^{11}*e^6 + 220*a^2*c^3*d^9*e^8 - 702*a^{5/2}*c^{5/2}*d^7*e^{10} - 770*a^3*c^2*d^5*e^{12} + 390*a^{7/2}*c^{3/2}*d^3*e^{14} + 77*a^4*c*d*e^{16})*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{1/4}*c^{1/4}*x + \sqrt{a})/(a^{3/4}*c^{5/4}) - 2*(7*\sqrt{2})*a^{1/4}*c^{21/4}*d^{17} + 10*\sqrt{2})*a^{3/4}*c^{19/4}*d^{15}*e^2 + 50*\sqrt{2})*a^{5/4}*c^{17/4}*d^{13}*e^4 + 78*\sqrt{2})*a^{7/4}*c^{15/4}*d^{11}*e^6 + 220*\sqrt{2})*a^{9/4}*c^{13/4}*d^9*e^8 + 702*\sqrt{2})*a^{11/4}*c^{11/4}*d^7*e^{10} - 770*\sqrt{2})*a^{13/4}*c^{9/4}*d^5*e^{12} - 390*\sqrt{2})*a^{15/4}*c^{7/4}*d^3*e^{14} + 77*\sqrt{2})*a^{17/4}*c^{5/4}*d*e^{16} + 24*\sqrt{a}*c^5*d^{16}*e - 176*a^{3/2}*c^4*d^{12}*e^5 + 960*a^{5/2}*c^3*d^8*e^9 - 1200*a^{7/2}*c^2*d^4*e^{13} + 40*a^{9/2}*c*e^{17})*\arctan(1/2*\sqrt{2}*(2*\sqrt{c})*x + \sqrt{2})*a^{1/4}*c^{1/4})/\sqrt{\sqrt{a}*\sqrt{c}})/(a^{3/4}*\sqrt{\sqrt{a}*\sqrt{c}})*c^{5/4}) - 2*(7*\sqrt{2})*a^{1/4}*c^{21/4}*d^{17} + 10*\sqrt{2})*a^{3/4}*c^{19/4}*d^{15}*e^2 + 50*\sqrt{2})*a^{5/4}*c^{17/4}*d^{13}*e^4 + 78*\sqrt{2})*a^{7/4}*c^{15/4}*d^{11}*e^6 + 220*\sqrt{2})*a^{9/4}*c^{13/4}*d^9*e^8 + 702*\sqrt{2})*a^{11/4}*c^{11/4}*d^7*e^{10} - 770*\sqrt{2})*a^{13/4}*c^{9/4}*d^5*e^{12} - 390*\sqrt{2})*a^{15/4}*c^{7/4}*d^3*e^{14} + 77*\sqrt{2})*a^{17/4}*c^{5/4}*d*e^{16} - 24*\sqrt{a}*c^5*d^{16}*e - 176*a^{3/2}*c^4*d^{12}*e^5 - 960*a^{5/2}*c^3*d^8*e^9 + 1200*a^{7/2}*c^2*d^4*e^{13} - 40*a^{9/2}*c*e^{17})*\arctan(1/2*\sqrt{2}*(2*\sqrt{c})*x - \sqrt{2})*a^{1/4}*c^{1/4})/\sqrt{\sqrt{a}*\sqrt{c}})/(a^{3/4}*\sqrt{\sqrt{a}*\sqrt{c}})*c^{5/4}))/((a^2*c^5*d^{20} + 5*a^3*c^4*d^{16}*e^4 + 10*a^4*c^3*d^{12}*e^8 + 10*a^5*c^2*d^8*e^{12} + 5*a^6*c*d^4*e^{16} + a^7*e^{20}) + 6*(13*c^2*d^6*e^{11} - 3*a*c*d^2*e^{15})*\log(e*x + d)/(c^5*d^{20} + 5*a*c^4*d^{16}*e^4 + 10*a^2*c^3*d^{12}*e^8 + 10*a^3*c^2*d^8*e^{12} + 5*a^4*c*d^4*e^{16} + a^5*e^{20}) + 1/32*(40*a^2*c^3*d^{12}*e^3 + 304*a^3*c^2*d^8*e^7 - 520*a^4*c*d^4*e^{11} - 16*a^5*e^{15} + 6*(5*c^5*d^{11}*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^{12})*x^9 + 6*(7*c^5*d^{12}*e^3 + 49*a*c^4*d^8*e^7 - 91*a^2*c^3*d^4*e^{11} - 5*a^3*c^2*e^{15})*x^8 + (c^5*d^{13}*e^2 + 19*a*c^4*d^9*e^6 + 35*a^2*c^3*d^5*e^{10} + 17*a^3*c^2*d*e^{14})*x^7 - 4*(c^5*d^{14}*e + 7*a*c^4*d^{10}*e^5 + 11*a^2*c^3*d^6*e^9 + 5*a^3*c^2*d^2*e^{13})*x^6 + (7*c^5*d^{15} + 97*a*c^4*d^{11}*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*c^2*d^3*e^{12})*x^5 + 2*(39*a*c^4*d^{12}*e^3 + 293*a^2*c^3*d^8*e^7 - 539*a^3*c^2*d^4*e^{11} - 25*a^4*c*e^{15})*x^4 + (5*a*c^4*d^{13}*e^2 + 31*a^2*c^3*d^9*e^6 + 47*a^3*c^2*d^5*e^{10} + 21*a^4*c*d*e^{14})*x^3 - 8*(a*c^4*d^{14}*e + 5*a^2*c^3*d^{10}*e^5 + 7*a^3*c^2*d^6*e^9 + 3*a^4*c*d^2*e^{13})*x^2 + (11*a*c^4*d^{15} + 79*a^2*c^3*d^{11}*e^4 + 269*a^3*c^2*d^7*e^8 - 567*a^4*c*d^3*e^{12})*x)/(a^4*c^4*d^{18} + 4*a^5*c^3*d^{14}*e^4 + 6*a^6*c^2*d^{10}*e^8 + 4*a^7*c*d^6*e^{12} + a^8*d^2*e^{16} + (a^2*c^6*d^{16}*e^2 + 4*a^3*c^5*d^{12}*e^6 + 6*a^4*c^4*d^8*e^{10} + 4*a^5*c^3*d^4*e^{14} + a^6*c^2*e^{18})*x^{10} + 2*(a^2*c^6*d^{17}*e + 4*a^3*c^5*d^{13}*e^5 + 6*a^4*c^4*d^9*e^9 + 4*a^5*c^3*d^5*e^{13} + a^6*c^2*d*e^{17})*x^9 + (a^2*c^6*d^{18} + 4*a^3*c^5*d^{14}*e^4 + 6*a^4*c^4*d^{10}*e^8 + 4*a^5*c^3*d^6*e^{12} + a^6*c^2*d^2*e^{16})*x^8 + 2*(a^3*c^5*d^{16}*e^2 + 4*a^4*c^4*d^{12}*e^6 + 6*a^5*c^3*d^8*e^{10} + 4*a^6*c^2*d^4*e^{14} + a^7*c*e^{18})*x^6 + 4*(a^3*c^5*d^{17}*e + 4*a^4*c^4*d^{13}*e^5 + 6*a^5*c^3*d^9*e^9 + 4*a^6*c^2*d^5*e^{13} + a^7*c*d*e^{17})*x^5 + 2*(a^3*c^5*d^{18} + 4*a^4*c^4*d^{14}*e^4 + 6*a^5*c^3*d^{10}*e^8 + 4*a^6*c^2*d^6*e^{12} + a^7*c*d^2*e^{16})*x^4 + (a^4*c^4*d^{16}*e^2 + 4*a^5*c^3*d^{12}*e^6 + 6*a^6*c^2*d^8*e^{10} + 4*a^7*c*d^4*e^{14} + a^8*e^{18})*x^2 + 2*(a^4*c^4*d^{17}*e + 4*a^5*c^3*d^{13}*e^5 + 6*a^6*c^2*d^9*e^9 + 4*a^7*c*d^5*e^{13} + a^8*d*e^{17})*x)$$

mupad [B] time = 7.78, size = 6280, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((a + c*x^4)^3*(d + e*x)^3), x)$

[Out] $\text{symsum}(\log(\text{root}(2684354560*a^{12}*c^9*d^{36}*e^4*z^5 + 32212254720*a^{18}*c^3*d^{12}*e^{28}*z^5 + 32212254720*a^{14}*c^7*d^{28}*e^{12}*z^5 + 2684354560*a^{20}*c*d^4*e^{36}*z^5 + 56371445760*a^{17}*c^4*d^{16}*e^{24}*z^5 + 56371445760*a^{15}*c^6*d^{24}*e^{16}*z^5 + 12079595520*a^{19}*c^2*d^8*e^{32}*z^5 + 12079595520*a^{13}*c^8*d^{32}*e^8*z^5 + 67645734912*a^{16}*c^5*d^{20}*e^{20}*z^5 + 268435456*a^{11}*c^{10}*d^{40}*z^5 + 268435456*a^{21}*e^{40}*z^5 + 45339770880*a^9*c^6*d^{20}*e^{14}*z^3 - 79148482560*a^{13}*c^2*d^4*e^{30}*z^3 + 791941349376*a^{12}*c^3*d^8*e^{26}*z^3 + 1239810048*a^7*c^8*d^{28}*e^6*z^3 - 1555444924416*a^{11}*c^4*d^{12}*e^{22}*z^3 + 83755008*a^6*c^9*d^{32}*e^2*z^3 + 81566760960*a^{10}*c^5*d^{16}*e^{18}*z^3 + 12177506304*a^8*c^7*d^{24}*e^{10}*z^3 + 117964800*a^{14}*c*e^{34}*z^3 - 2785204224*a^6*c^6*d^{18}*e^{13}*z^2 + 8128512*a^3*c^9*d^{30}*e*z^2 + 2700933120*a^{10}*c^2*d^2*e^{29}*z^2 - 543361222656*a^8*c^4*d^{10}*e^{21}*z^2 + 1048135680*a^5*c^7*d^{22}*e^9*z^2 + 118499328*a^4*c^8*d^{26}*e^5*z^2 - 55938263040*a^7*c^5*d^{14}*e^{17}*z^2 + 123990497280*a^9*c^3*d^6*e^{25}*z^2 + 24139215*a^2*c^7*d^{20}*e^8*z + 2819286*a*c^8*d^{24}*e^4*z + 10462847841*a^6*c^3*d^4*e^{24}*z - 5777473473*a^4*c^5*d^{12}*e^{16}*z - 43509753450*a^5*c^4*d^8*e^{20}*z - 548810316*a^3*c^6*d^{16}*e^{12}*z + 12960000*a^7*c^2*e^{28}*z + 194481*c^9*d^{28}*z - 977636142*a^2*c^4*d^6*e^{19} + 233280000*a^3*c^3*d^2*e^{23} - 140556060*a*c^5*d^{10}*e^{15} - 15169518*c^6*d^{14}*e^{11}, z, k)*(root(2684354560*a^{12}*c^9*d^{36}*e^4*z^5 + 32212254720*a^{18}*c^3*d^{12}*e^{28}*z^5 + 32212254720*a^{14}*c^7*d^{28}*e^{12}*z^5 + 2684354560*a^{20}*c*d^4*e^{36}*z^5 + 56371445760*a^{17}*c^4*d^{16}*e^{24}*z^5 + 56371445760*a^{15}*c^6*d^{24}*e^{16}*z^5 + 12079595520*a^{19}*c^2*d^8*e^{32}*z^5 + 12079595520*a^{13}*c^8*d^{32}*e^8*z^5 + 67645734912*a^{16}*c^5*d^{20}*e^{20}*z^5 + 268435456*a^{11}*c^{10}*d^{40}*z^5 + 268435456*a^{21}*e^{40}*z^5 + 45339770880*a^9*c^6*d^{20}*e^{14}*z^3 - 79148482560*a^{13}*c^2*d^4*e^{30}*z^3 + 791941349376*a^{12}*c^3*d^8*e^{26}*z^3 + 1239810048*a^7*c^8*d^{28}*e^6*z^3 - 1555444924416*a^{11}*c^4*d^{12}*e^{22}*z^3 + 83755008*a^6*c^9*d^{32}*e^2*z^3 + 81566760960*a^{10}*c^5*d^{16}*e^{18}*z^3 + 12177506304*a^8*c^7*d^{24}*e^{10}*z^3 + 117964800*a^{14}*c*e^{34}*z^3 - 2785204224*a^6*c^6*d^{18}*e^{13}*z^2 + 8128512*a^3*c^9*d^{30}*e*z^2 + 2700933120*a^{10}*c^2*d^2*e^{29}*z^2 - 543361222656*a^8*c^4*d^{10}*e^{21}*z^2 + 1048135680*a^5*c^7*d^{22}*e^9*z^2 + 118499328*a^4*c^8*d^{26}*e^5*z^2 - 55938263040*a^7*c^5*d^{14}*e^{17}*z^2 + 123990497280*a^9*c^3*d^6*e^{25}*z^2 + 24139215*a^2*c^7*d^{20}*e^8*z + 2819286*a*c^8*d^{24}*e^4*z + 10462847841*a^6*c^3*d^4*e^{24}*z - 5777473473*a^4*c^5*d^{12}*e^{16}*z - 43509753450*a^5*c^4*d^8*e^{20}*z - 548810316*a^3*c^6*d^{16}*e^{12}*z + 12960000*a^7*c^2*e^{28}*z + 194481*c^9*d^{28}*z - 977636142*a^2*c^4*d^6*e^{19} + 233280000*a^3*c^3*d^2*e^{23} - 140556060*a*c^5*d^{10}*e^{15} - 15169518*c^6*d^{14}*e^{11}, z, k)*(root(2684354560*a^{12}*c^9*d^{36}*e^4*z^5 + 32212254720*a^{18}*c^3*d^{12}*e^{28}*z^5 + 32212254720*a^{14}*c^7*d^{28}*e^{12}*z^5 + 2684354560*a^{20}*c*d^4*e^{36}*z^5 + 56371445760*a^{17}*c^4*d^{16}*e^{24}*z^5 + 56371445760*a^{15}*c^6*d^{24}*e^{16}*z^5 + 12079595520*a^{19}*c^2*d^8*e^{32}*z^5 + 12079595520*a^{13}*c^8*d^{32}*e^8*z^5 + 67645734912*a^{16}*c^5*d^{20}*e^{20}*z^5 + 268435456*a^{11}*c^{10}*d^{40}*z^5 + 268435456*a^{21}*e^{40}*z^5 + 45339770880*a^9*c^6*d^{20}*e^{14}*z^3 - 79148482560*a^{13}*c^2*d^4*e^{30}*z^3 + 791941349376*a^{12}*c^3*d^8*e^{26}*z^3 + 1239810048*a^7*c^8*d^{28}*e^6*z^3 - 1555444924416*a^{11}*c^4*d^{12}*e^{22}*z^3 + 83755008*a^6*c^9*d^{32}*e^2*z^3 + 81566760960*a^{10}*c^5*d^{16}*e^{18}*z^3 + 12177506304*a^8*c^7*d^{24}*e^{10}*z^3 + 117964800*a^{14}*c*e^{34}*z^3 - 2785204224*a^6*c^6*d^{18}*e^{13}*z^2 + 8128512*a^3*c^9*d^{30}*e*z^2 + 2700933120*a^{10}*c^2*d^2*e^{29}*z^2 - 543361222656*a^8*c^4*d^{10}*e^{21}*z^2 + 1048135680*a^5*c^7*d^{22}*e^9*z^2 + 118499328*a^4*c^8*d^{26}*e^5*z^2 - 55938263040*a^7*c^5*d^{14}*e^{17}*z^2 + 123990497280*a^9*c^3*d^6*e^{25}*z^2 + 24139215*a^2*c^7*d^{20}*e^8*z + 2819286*a*c^8*d^{24}*e^4*z + 10462847841*a^6*c^3*d^4*e^{24}*z - 5777473473*a^4*c^5*d^{12}*e^{16}*z - 43509753450*a^5*c^4*d^8*e^{20}*z - 548810316*a^3*c^6*d^{16}*e^{12}*z + 12960000*a^7*c^2*e^{28}*z + 194481*c^9*d^{28}*z - 977636142*a^2*c^4*d^6*e^{19} + 233280000*a^3*c^3*d^2*e^{23} - 140556060*a*c^5*d^{10}*e^{15} - 15169518*c^6*d^{14}*e^{11}, z, k)$

$$\begin{aligned}
& e^{19} + 233280000a^3c^3d^2e^{23} - 140556060a^5c^5d^{10}e^{15} - 15169518c^6d^{14}e^{11}, z, k) \cdot ((44040192a^8c^{12}d^{31}e^5 - 11010048a^7c^{13}d^{35}e \\
& + 994050048a^9c^{11}d^{27}e^9 + 13683916800a^{10}c^{10}d^{23}e^{13} + 429360414 \\
& 72a^{11}c^9d^{19}e^{17} + 52628029440a^{12}c^8d^{15}e^{21} + 23429382144a^{13}c^7d^{11}e^{25} - 2132803584a^{14}c^6d^7e^{29} - 3125280768a^{15}c^5d^3e^{33}) \\
& / (1048576(a^{16}e^{32} + a^8c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} + 70a^{12}c^4d^{16}e^{16} + \\
& 56a^{13}c^3d^{12}e^{20} + 28a^{14}c^2d^8e^{24})) + \text{root}(2684354560a^{12}c^9d^{36}e^4z^5 + 32212254720a^{18}c^3d^{12}e^{28}z^5 + 32212254720a^{14}c^7d^2 \\
& 8e^{12}z^5 + 2684354560a^{20}c^4d^4e^{36}z^5 + 56371445760a^{17}c^4d^{16}e^2 \\
& 4z^5 + 56371445760a^{15}c^6d^{24}e^{16}z^5 + 12079595520a^{19}c^2d^8e^{32}z^5 + 12079595520a^{13}c^8d^{32}e^8z^5 + 67645734912a^{16}c^5d^{20}e^{20}z^5 \\
& + 268435456a^{11}c^{10}d^{40}z^5 + 268435456a^{21}e^{40}z^5 + 45339770880a^9c^6d^{20}e^{14}z^3 - 79148482560a^{13}c^2d^4e^{30}z^3 + 791941349376a^{12} \\
& c^3d^8e^{26}z^3 + 1239810048a^7c^8d^{28}e^6z^3 - 1555444924416a^{11}c^4d^{12}e^{22}z^3 + 83755008a^6c^9d^{32}e^2z^3 + 81566760960a^{10}c^5d^{16} \\
& e^{18}z^3 + 12177506304a^8c^7d^{24}e^{10}z^3 + 117964800a^{14}c^4e^{34}z^3 - 2785204224a^6c^6d^{18}e^{13}z^2 + 8128512a^3c^9d^{30}e^z^2 + 2700933120 \\
& a^{10}c^2d^2e^{29}z^2 - 543361222656a^8c^4d^{10}e^{21}z^2 + 1048135680a^5c^7d^{22}e^9z^2 + 118499328a^4c^8d^{26}e^5z^2 - 55938263040a^7c^5d^{14}e^{17}z^2 + 123990497280a^9c^3d^6e^{25}z^2 + 24139215a^2c^7d^{20}e^8z \\
& + 2819286a^8c^8d^{24}e^4z + 10462847841a^6c^3d^4e^{24}z - 5777473473a^4c^5d^{12}e^{16}z - 43509753450a^5c^4d^8e^{20}z - 548810316a^3c^6d^{16}e^{12}z + 12960000a^7c^2e^{28}z + 194481c^9d^{28}z - 977636142a^2c^4d^6e^{19} + 233280000a^3c^3d^2e^{23} - 140556060a^5c^5d^{10}e^{15} - 1516 \\
& 9518c^6d^{14}e^{11}, z, k) \cdot ((402653184a^{19}c^4d^e^{38} - 134217728a^{10}c^{13}d^{37}e^2 - 671088640a^{11}c^{12}d^{33}e^6 - 536870912a^{12}c^{11}d^{29}e^{10} + 3758096384a^{13}c^{10}d^{25}e^{14} + 13153337344a^{14}c^9d^{21}e^{18} + 206695301 \\
& 12a^{15}c^8d^{17}e^{22} + 18790481920a^{16}c^7d^{13}e^{26} + 10200547328a^{17}c^6d^9e^{30} + 3087007744a^{18}c^5d^5e^{34}) / (1048576(a^{16}e^{32} + a^8c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} + 28a^{14}c^2d^8e^{24})) + (x(335544320a^{19}c^4e^{39} - 201326592a^{10}c^{13}d^{36}e^3 - 1275068416a^{11}c^{12}d^{32}e^7 - 2952790016a^{12}c^{11}d^{28}e^{11} - 1879048 \\
& 192a^{13}c^{10}d^{24}e^{15} + 4697620480a^{14}c^9d^{20}e^{19} + 12213813248a^{15}c^8d^{16}e^{23} + 13153337344a^{16}c^7d^{12}e^{27} + 7784628224a^{17}c^6d^8e^{31} + 2483027968a^{18}c^5d^4e^{35}) / (1048576(a^{16}e^{32} + a^8c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} + 28a^{14}c^2d^8e^{24})) - (x(40894464a^7c^{13}d^{34}e^2 + 276824064a^8c^{12}d^{30}e^6 + 96 \\
& 8884224a^9c^{11}d^{26}e^{10} - 13010731008a^{10}c^{10}d^{22}e^{14} - 53433335808a^{11}c^9d^{18}e^{18} - 71647100928a^{12}c^8d^{14}e^{22} - 34313601024a^{13}c^7d^{10}e^{26} + 1837105152a^{14}c^6d^6e^{30} + 4193255424a^{15}c^5d^2e^{34}) / (1048576(a^{16}e^{32} + a^8c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} + 28a^{14}c^2d^8e^{24})) + (33914880a^{12}c^5d^e^{32} + 13713408a^5c^{12}d^{29}e^4 + 225902592a^6c^{11}d^{25}e^8 + 2352070656a^7c^{10}d^{21}e^{12} + 2474606592a^8c^9d^{17}e^{16} - 21361803264a^9c^8d^{13}e^{20} + 88707170304a^{10}c^7d^9e^{24} - 5526503424a^{11}c^6d^5e^{28}) / (1048576(a^{16}e^{32} + a^8c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} + 28a^{14}c^2d^8e^{24})) + (x(132710400a^{12}c^5e^{33} - 18063 \\
& 36a^4c^{13}d^{32}e - 2027520a^5c^{12}d^{28}e^5 + 162017280a^6c^{11}d^{24}e^9 + 4635316224a^7c^{10}d^{20}e^{13} + 15604273152a^8c^9d^{16}e^{17} + 3931866 \\
& 3168a^9c^8d^{12}e^{21} + 64184389632a^{10}c^7d^8e^{25} - 2525073408a^{11}c^6d^4e^{29}) / (1048576(a^{16}e^{32} + a^8c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} + 28a^{14}c^2d^8e^{24})) + (320544a^2c^{12}d^{27}e^3 + 11448000a^3c^{11}d^{23}e^7 + 114031584a^4c^{10}d^{19}e^{11} -
\end{aligned}$$

$$\begin{aligned}
& 213750144a^5c^9d^{15}e^{15} - 3499271712a^6c^8d^{11}e^{19} + 9699804864a^7 \\
& *c^7d^7e^{23} - 933615072a^8c^6d^3e^{27}) / (1048576(a^{16}e^{32} + a^8c^8d^{32} \\
& + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} \\
& + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} + 28a^{14}c^2d^8e^{24})) + (x(514944a^2c^{12}d^{26}e^4 \\
& + 14314752a^3c^{11}d^{22}e^8 + 266343552a^4c^{10}d^{18}e^{12} + 297948672a^5c^9d^{14}e^{16} - 2642613120a \\
& ^6c^8d^{10}e^{20} + 1782459648a^7c^7d^6e^{24} + 846599040a^8c^6d^2e^{28} \\
&)) / (1048576(a^{16}e^{32} + a^8c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 \\
& + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} \\
& + 28a^{14}c^2d^8e^{24})) + (194481c^{11}d^{21}e^6 + 2430324a^2c^9d^{13}e^{14} - 83522988a^3c^8d^9e^{18} \\
& + 71628705a^4c^7d^5e^{22}) / (1048576(a^{16}e^{32} + a^8c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 \\
& + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} \\
& + 28a^{14}c^2d^8e^{24})) + (x(12960000a^5c^6e^{27} + 194481c^{11}d^{20}e^7 + 2430324a^2c^9d^{16}e^{11} \\
& - 21081546a^2c^9d^{12}e^{15} - 227814444a^3c^8d^8e^{19} + 105734241a^4c^7d^4e^{23})) / (1048576(a^{16}e^{32} + a^8 \\
& *c^8d^{32} + 8a^{15}c^4d^4e^{28} + 8a^9c^7d^{28}e^4 + 28a^{10}c^6d^{24}e^8 + 56a^{11}c^5d^{20}e^{12} \\
& + 70a^{12}c^4d^{16}e^{16} + 56a^{13}c^3d^{12}e^{20} + 28a^{14}c^2d^8e^{24})) * \text{root}(2684354560a^{12}c^9d^{36}e^4z^5 \\
& + 32212254720a^{18}c^3d^{12}e^{28}z^5 + 32212254720a^{14}c^7d^{28}e^{12}z^5 + 2684354560a^2 \\
& 0c^d^4e^{36}z^5 + 56371445760a^{17}c^4d^{16}e^{24}z^5 + 56371445760a^{15}c^6d^{24}e^{16}z^5 \\
& + 12079595520a^{19}c^2d^8e^{32}z^5 + 12079595520a^{13}c^8d^{32}e^8z^5 + 67645734912a^{16}c^5d^{20}e^{20}z^5 \\
& + 268435456a^{11}c^{10}d^40z^5 + 268435456a^{21}e^{40}z^5 + 45339770880a^9c^6d^{20}e^{14}z^3 - 79148 \\
& 482560a^{13}c^2d^4e^{30}z^3 + 791941349376a^{12}c^3d^8e^{26}z^3 + 1239810048a^7c^8d^{28}e^6z^3 \\
& - 1555444924416a^{11}c^4d^{12}e^{22}z^3 + 83755008a^6c^9d^{32}e^2z^3 + 81566760960a^{10}c^5d^{16}e^{18}z^3 \\
& + 12177506304a^8c^7d^{24}e^{10}z^3 + 117964800a^{14}c^e^{34}z^3 - 2785204224a^6c^6d^{18}e^{13}z^2 \\
& + 8128512a^3c^9d^{30}e^z^2 + 2700933120a^{10}c^2d^2e^{29}z^2 - 543361222656a^8c^4d^{10}e^{21}z^2 \\
& + 1048135680a^5c^7d^{22}e^9z^2 + 118499328a^4c^8d^{26}e^5z^2 - 55938263040a^7c^5d^{14}e^{17}z^2 + 123990497280 \\
& *a^9c^3d^6e^{25}z^2 + 24139215a^2c^7d^{20}e^8z + 2819286a^8c^8d^{24}e^4z + 10462847841a^6c^3d^4e^{24}z \\
& - 5777473473a^4c^5d^{12}e^{16}z - 43509753450a^5c^4d^8e^{20}z - 548810316a^3c^6d^{16}e^{12}z + 12960000a^7c^2e^{28}z \\
& + 194481c^9d^{28}z - 977636142a^2c^4d^6e^{19} + 233280000a^3c^3d^2e^{23} - 140556060a^2c^5d^{10}e^{15} \\
& - 15169518c^6d^{14}e^{11}, z, k), k, 1, 5) - ((2a^3e^{15} - 5c^3d^{12}e^3 - 38a^2c^2d^8e^7 + 65a^2c^2d^4e^{11}) / (4(a^2e^8 \\
& + c^2d^8 + 2a^2c^2d^4e^4)^2) + (3x^8(5a^3c^2e^{15} - 7c^5d^{12}e^3 - 49a^2c^4d^8e^7 + 91a^2c^3d^4e^{11}) / (16a^2(a^4e^{16} \\
& + c^4d^{16} + 4a^2c^3d^{12}e^4 + 4a^3c^2d^8e^8) - (x^5(7c^5d^{15} + 97a^2c^4d^{11}e^4 + 461a^2c^3d^7e^8 - 1165a^3c^2d^3e^{12}) / (32a^2(a^2e^8 \\
& + c^2d^8 + 2a^2c^2d^4e^4)^2) - (3x^9(5c^5d^{11}e^4 + 34a^2c^4d^7e^8 - 99a^2c^3d^3e^{12}) / (16a^2(a^4e^{16} + c^4d^{16} \\
& + 4a^2c^3d^{12}e^4 + 4a^3c^2d^8e^8) + (x^2(c^2d^6e + 3a^2c^2d^2e^5)) / (4a^2(a^2e^8 + c^2d^8 + 2a^2c^2d^4e^4)) - (x^3(5c^2d^5e^2 \\
& + 21a^2c^2d^2e^5)) / (32a^2(a^2e^8 + c^2d^8 + 2a^2c^2d^4e^4)) - (x(11c^4d^{15} + 79a^2c^3d^{11}e^4 - 567a^3c^3d^3e^{12} \\
& + 269a^2c^2d^7e^8)) / (32a^2(a^2e^8 + c^2d^8 + 2a^2c^2d^4e^4)^2) + (x^6(c^3d^6e + 5a^2c^2d^2e^5)) / (8a^2(a^2e^8 + c^2d^8 \\
& + 2a^2c^2d^4e^4)) + (x^4(25a^3c^3e^{15} - 39c^4d^{12}e^3 - 293a^2c^3d^8e^7 + 539a^2c^2d^4e^{11}) / (16a^2(a^2e^8 + c^2d^8 \\
& + 2a^2c^2d^4e^4)^2) - (e^2x^7(c^3d^5 + 17a^2c^2d^2e^4)) / (32a^2(a^2e^8 + c^2d^8 + 2a^2c^2d^4e^4)) / (a^2d^2 + a^2e^2x^2 + c^2d^2x^8 \\
& + c^2e^2x^{10} + 2a^2d^2ex + 2a^2c^2d^2x^4 + 2a^2c^2e^2x^6 + 2c^2d^2ex^9 + 4a^2c^2d^2ex^5)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

$$3.415 \quad \int \frac{-1+x}{1-x+x^2} dx$$

Optimal. Leaf size=32

$$\frac{1}{2} \log(x^2 - x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] 1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {634, 618, 204, 628}

$$\frac{1}{2} \log(x^2 - x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/(1 - x + x^2), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{-1+x}{1-x+x^2} dx &= -\left(\frac{1}{2} \int \frac{1}{1-x+x^2} dx\right) + \frac{1}{2} \int \frac{-1+2x}{1-x+x^2} dx \\ &= \frac{1}{2} \log(1-x+x^2) + \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\ &= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2} \log(1-x+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$\frac{1}{2} \log(x^2 - x + 1) - \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x)/(1 - x + x^2), x]

[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2

fricas [A] time = 0.98, size = 28, normalized size = 0.88

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(x^2-x+1), x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

giac [A] time = 0.29, size = 28, normalized size = 0.88

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(x^2-x+1), x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x^2 - x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-1)/(x^2-x+1), x)

[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [A] time = 2.53, size = 28, normalized size = 0.88

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(x^2-x+1), x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{\ln(x^2 - x + 1)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - 1)/(x^2 - x + 1),x)`

[Out] `log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3`

sympy [A] time = 0.11, size = 34, normalized size = 1.06

$$\frac{\log(x^2 - x + 1)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)/(x**2-x+1),x)`

[Out] `log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3`

$$3.416 \quad \int \frac{-1+x^2}{1+x^3} dx$$

Optimal. Leaf size=32

$$\frac{1}{2} \log(x^2 - x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] 1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1872, 634, 618, 204, 628}

$$\frac{1}{2} \log(x^2 - x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(1 + x^3), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1872

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x] /; EqQ[A - B*(a/b)^(1/3) + C*(a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{1+x^3} dx &= \int \frac{-1+x}{1-x+x^2} dx \\
&= -\left(\frac{1}{2} \int \frac{1}{1-x+x^2} dx\right) + \frac{1}{2} \int \frac{-1+2x}{1-x+x^2} dx \\
&= \frac{1}{2} \log(1-x+x^2) + \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$\frac{1}{2} \log(x^2 - x + 1) - \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(1 + x^3), x]

[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2

fricas [A] time = 1.09, size = 28, normalized size = 0.88

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3+1),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

giac [A] time = 0.38, size = 28, normalized size = 0.88

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3+1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x^2 - x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/(x^3+1),x)

[Out] -1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*ln(x^2-x+1)

maxima [A] time = 2.53, size = 28, normalized size = 0.88

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3+1),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

mupad [B] time = 0.03, size = 30, normalized size = 0.94

$$\frac{\ln(x^2 - x + 1)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 1)/(x^3 + 1),x)

[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3

sympy [A] time = 0.12, size = 34, normalized size = 1.06

$$\frac{\log(x^2 - x + 1)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-1)/(x**3+1),x)

[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3

$$3.417 \quad \int \frac{-4+3x}{4-2x+x^2} dx$$

Optimal. Leaf size=32

$$\frac{3}{2} \log(x^2 - 2x + 4) + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] 3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {634, 618, 204, 628}

$$\frac{3}{2} \log(x^2 - 2x + 4) + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-4 + 3*x)/(4 - 2*x + x^2), x]

[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{-4+3x}{4-2x+x^2} dx &= \frac{3}{2} \int \frac{-2+2x}{4-2x+x^2} dx - \int \frac{1}{4-2x+x^2} dx \\ &= \frac{3}{2} \log(4-2x+x^2) + 2 \text{Subst} \left(\int \frac{1}{-12-x^2} dx, x, -2+2x \right) \\ &= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2} \log(4-2x+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{3}{2} \log(x^2 - 2x + 4) - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 3*x)/(4 - 2*x + x^2), x]

[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2

fricas [A] time = 0.80, size = 26, normalized size = 0.81

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x-1)\right) + \frac{3}{2} \log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+3*x)/(x^2-2*x+4), x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

giac [A] time = 0.39, size = 26, normalized size = 0.81

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x-1)\right) + \frac{3}{2} \log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+3*x)/(x^2-2*x+4), x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3} + \frac{3 \ln(x^2 - 2x + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4+3*x)/(x^2-2*x+4), x)

[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))

maxima [A] time = 2.16, size = 26, normalized size = 0.81

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (x-1)\right) + \frac{3}{2} \log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+3*x)/(x^2-2*x+4), x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{3 \ln(x^2 - 2x + 4)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x - 4)/(x^2 - 2*x + 4),x)`

[Out] $(3*\log(x^2 - 2*x + 4))/2 - (3^{(1/2)}*\operatorname{atan}((3^{(1/2)}*x)/3 - 3^{(1/2)}/3))/3$

sympy [A] time = 0.12, size = 36, normalized size = 1.12

$$\frac{3\log(x^2 - 2x + 4)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4+3*x)/(x**2-2*x+4),x)`

[Out] $3*\log(x**2 - 2*x + 4)/2 - \operatorname{sqrt}(3)*\operatorname{atan}(\operatorname{sqrt}(3)*x/3 - \operatorname{sqrt}(3)/3)/3$

$$3.418 \quad \int \frac{-8+2x+3x^2}{8+x^3} dx$$

Optimal. Leaf size=32

$$\frac{3}{2} \log(x^2 - 2x + 4) + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] 3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1872, 634, 618, 204, 628}

$$\frac{3}{2} \log(x^2 - 2x + 4) + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-8 + 2*x + 3*x^2)/(8 + x^3), x]

[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1872

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rubi steps

$$\begin{aligned}
\int \frac{-8 + 2x + 3x^2}{8 + x^3} dx &= \frac{1}{2} \int \frac{-8 + 6x}{4 - 2x + x^2} dx \\
&= \frac{3}{2} \int \frac{-2 + 2x}{4 - 2x + x^2} dx - \int \frac{1}{4 - 2x + x^2} dx \\
&= \frac{3}{2} \log(4 - 2x + x^2) + 2 \operatorname{Subst} \left(\int \frac{1}{-12 - x^2} dx, x, -2 + 2x \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{3}{2} \log(4 - 2x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{3}{2} \log(x^2 - 2x + 4) - \frac{\tan^{-1} \left(\frac{x-1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3), x]

[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2

fricas [A] time = 0.88, size = 26, normalized size = 0.81

$$-\frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (x - 1) \right) + \frac{3}{2} \log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2*x-8)/(x^3+8), x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

giac [A] time = 0.33, size = 26, normalized size = 0.81

$$-\frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (x - 1) \right) + \frac{3}{2} \log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2*x-8)/(x^3+8), x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3} \arctan \left(\frac{(2x-2)\sqrt{3}}{6} \right)}{3} + \frac{3 \ln(x^2 - 2x + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2*x-8)/(x^3+8), x)

[Out] -1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))+3/2*ln(x^2-2*x+4)

maxima [A] time = 2.33, size = 26, normalized size = 0.81

$$-\frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (x - 1) \right) + \frac{3}{2} \log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

mupad [B] time = 0.03, size = 30, normalized size = 0.94

$$\frac{3 \ln(x^2 - 2x + 4)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 3*x^2 - 8)/(x^3 + 8),x)

[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3

sympy [A] time = 0.12, size = 36, normalized size = 1.12

$$\frac{3 \log(x^2 - 2x + 4)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2*x-8)/(x**3+8),x)

[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3

$$3.419 \quad \int \frac{2+x}{-1+2x+x^2} dx$$

Optimal. Leaf size=45

$$\frac{1}{4} (2 + \sqrt{2}) \log(x - \sqrt{2} + 1) + \frac{1}{4} (2 - \sqrt{2}) \log(x + \sqrt{2} + 1)$$

[Out] 1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {632, 31}

$$\frac{1}{4} (2 + \sqrt{2}) \log(x - \sqrt{2} + 1) + \frac{1}{4} (2 - \sqrt{2}) \log(x + \sqrt{2} + 1)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/(-1 + 2*x + x^2), x]

[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x])/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{2+x}{-1+2x+x^2} dx &= -\left(\frac{1}{4}(-2+\sqrt{2}) \int \frac{1}{1+\sqrt{2}+x} dx\right) + \frac{1}{4}(2+\sqrt{2}) \int \frac{1}{1-\sqrt{2}+x} dx \\ &= \frac{1}{4}(2+\sqrt{2}) \log(1-\sqrt{2}+x) + \frac{1}{4}(2-\sqrt{2}) \log(1+\sqrt{2}+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.93

$$\frac{1}{4} \left((2 + \sqrt{2}) \log(-x + \sqrt{2} - 1) - (\sqrt{2} - 2) \log(x + \sqrt{2} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/(-1 + 2*x + x^2), x]

[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])/4

fricas [A] time = 0.90, size = 45, normalized size = 1.00

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x^2 - 2\sqrt{2}(x+1) + 2x + 3}{x^2 + 2x - 1}\right) + \frac{1}{2} \log(x^2 + 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*log(x^2 + 2*x - 1)

giac [A] time = 0.42, size = 44, normalized size = 0.98

$$\frac{1}{4} \sqrt{2} \log \left(\frac{|2x - 2\sqrt{2} + 2|}{|2x + 2\sqrt{2} + 2|} \right) + \frac{1}{2} \log(|x^2 + 2x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*log(abs(x^2 + 2*x - 1))

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2} + \frac{\ln(x^2 + 2x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+2)/(x^2+2*x-1),x)

[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))

maxima [A] time = 2.62, size = 35, normalized size = 0.78

$$\frac{1}{4} \sqrt{2} \log \left(\frac{x - \sqrt{2} + 1}{x + \sqrt{2} + 1} \right) + \frac{1}{2} \log(x^2 + 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x - 1)

mupad [B] time = 2.32, size = 34, normalized size = 0.76

$$\ln(x - \sqrt{2} + 1) \left(\frac{\sqrt{2}}{4} + \frac{1}{2} \right) - \ln(x + \sqrt{2} + 1) \left(\frac{\sqrt{2}}{4} - \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/(2*x + x^2 - 1),x)

[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 - 1/2)

sympy [A] time = 0.11, size = 39, normalized size = 0.87

$$\left(\frac{1}{2} - \frac{\sqrt{2}}{4} \right) \log(x + 1 + \sqrt{2}) + \left(\frac{\sqrt{2}}{4} + \frac{1}{2} \right) \log(x - \sqrt{2} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x**2+2*x-1),x)

[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2) + 1)

$$3.420 \quad \int \frac{-4+x^2}{2-5x+x^3} dx$$

Optimal. Leaf size=45

$$\frac{1}{4}(2+\sqrt{2})\log(x-\sqrt{2}+1) + \frac{1}{4}(2-\sqrt{2})\log(x+\sqrt{2}+1)$$

[Out] 1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2055, 632, 31}

$$\frac{1}{4}(2+\sqrt{2})\log(x-\sqrt{2}+1) + \frac{1}{4}(2-\sqrt{2})\log(x+\sqrt{2}+1)$$

Antiderivative was successfully verified.

[In] Int[(-4 + x^2)/(2 - 5*x + x^3), x]

[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x])/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 2055

Int[(u_.)*(P_)*(Q_)^(q_), x_Symbol] :> Module[{gcd = PolyGCD[P, Q, x]}, Int[u*gcd^(q+1)*PolynomialQuotient[P, gcd, x]*PolynomialQuotient[Q, gcd, x]^q, x] /; NeQ[gcd, 1] /; ILtQ[q, 0] && PolyQ[P, x] && PolyQ[Q, x]

Rubi steps

$$\begin{aligned} \int \frac{-4+x^2}{2-5x+x^3} dx &= \int \frac{2+x}{-1+2x+x^2} dx \\ &= -\left(\frac{1}{4}(-2+\sqrt{2}) \int \frac{1}{1+\sqrt{2}+x} dx\right) + \frac{1}{4}(2+\sqrt{2}) \int \frac{1}{1-\sqrt{2}+x} dx \\ &= \frac{1}{4}(2+\sqrt{2})\log(1-\sqrt{2}+x) + \frac{1}{4}(2-\sqrt{2})\log(1+\sqrt{2}+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]

[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])/4

fricas [A] time = 0.76, size = 45, normalized size = 1.00

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x^2 - 2\sqrt{2}(x+1) + 2x + 3}{x^2 + 2x - 1}\right) + \frac{1}{2} \log(x^2 + 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*log(x^2 + 2*x - 1)

giac [A] time = 0.31, size = 44, normalized size = 0.98

$$\frac{1}{4} \sqrt{2} \log\left(\frac{|2x - 2\sqrt{2} + 2|}{|2x + 2\sqrt{2} + 2|}\right) + \frac{1}{2} \log(|x^2 + 2x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*log(abs(x^2 + 2*x - 1))

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2} + \frac{\ln(x^2 + 2x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-4)/(x^3-5*x+2),x)

[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))

maxima [A] time = 2.33, size = 35, normalized size = 0.78

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x - \sqrt{2} + 1}{x + \sqrt{2} + 1}\right) + \frac{1}{2} \log(x^2 + 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x - 1)

mupad [B] time = 0.05, size = 34, normalized size = 0.76

$$\ln(x - \sqrt{2} + 1) \left(\frac{\sqrt{2}}{4} + \frac{1}{2}\right) - \ln(x + \sqrt{2} + 1) \left(\frac{\sqrt{2}}{4} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 4)/(x^3 - 5*x + 2),x)

[Out] $\log(x - 2^{(1/2)} + 1) * (2^{(1/2)}/4 + 1/2) - \log(x + 2^{(1/2)} + 1) * (2^{(1/2)}/4 - 1/2)$

sympy [A] time = 0.12, size = 39, normalized size = 0.87

$$\left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) \log(x + 1 + \sqrt{2}) + \left(\frac{\sqrt{2}}{4} + \frac{1}{2}\right) \log(x - \sqrt{2} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-4)/(x**3-5*x+2), x)`

[Out] $(1/2 - \text{sqrt}(2)/4) * \log(x + 1 + \text{sqrt}(2)) + (\text{sqrt}(2)/4 + 1/2) * \log(x - \text{sqrt}(2) + 1)$

$$3.421 \quad \int \frac{2}{-1+4x^2} dx$$

Optimal. Leaf size=6

$$-\tanh^{-1}(2x)$$

[Out] -arctanh(2*x)

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 207}

$$-\tanh^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[2/(-1 + 4*x^2), x]

[Out] -ArcTanh[2*x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{2}{-1+4x^2} dx &= 2 \int \frac{1}{-1+4x^2} dx \\ &= -\tanh^{-1}(2x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 23, normalized size = 3.83

$$2 \left(\frac{1}{4} \log(1-2x) - \frac{1}{4} \log(2x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[2/(-1 + 4*x^2), x]

[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)

fricas [B] time = 0.67, size = 17, normalized size = 2.83

$$-\frac{1}{2} \log(2x+1) + \frac{1}{2} \log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(4*x^2-1), x, algorithm="fricas")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)

giac [B] time = 0.31, size = 15, normalized size = 2.50

$$-\frac{1}{2} \log \left(\left| x + \frac{1}{2} \right| \right) + \frac{1}{2} \log \left(\left| x - \frac{1}{2} \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(4*x^2-1),x, algorithm="giac")

[Out] -1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))

maple [B] time = 0.00, size = 18, normalized size = 3.00

$$\frac{\ln(2x-1)}{2} - \frac{\ln(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(4*x^2-1),x)

[Out] 1/2*ln(2*x-1)-1/2*ln(2*x+1)

maxima [B] time = 1.06, size = 17, normalized size = 2.83

$$-\frac{1}{2} \log(2x+1) + \frac{1}{2} \log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(4*x^2-1),x, algorithm="maxima")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)

mupad [B] time = 2.27, size = 6, normalized size = 1.00

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(4*x^2 - 1),x)

[Out] -atanh(2*x)

sympy [B] time = 0.09, size = 15, normalized size = 2.50

$$\frac{\log\left(x - \frac{1}{2}\right)}{2} - \frac{\log\left(x + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(4*x**2-1),x)

[Out] log(x - 1/2)/2 - log(x + 1/2)/2

$$3.422 \quad \int \left(\frac{1}{-1+2x} - \frac{1}{1+2x} \right) dx$$

Optimal. Leaf size=21

$$\frac{1}{2} \log(1-2x) - \frac{1}{2} \log(2x+1)$$

[Out] 1/2*ln(1-2*x)-1/2*ln(1+2*x)

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{1}{2} \log(1-2x) - \frac{1}{2} \log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1), x]

[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2

Rubi steps

$$\int \left(\frac{1}{-1+2x} - \frac{1}{1+2x} \right) dx = \frac{1}{2} \log(1-2x) - \frac{1}{2} \log(1+2x)$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.10

$$2 \left(\frac{1}{4} \log(1-2x) - \frac{1}{4} \log(2x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1), x]

[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)

fricas [A] time = 0.52, size = 17, normalized size = 0.81

$$-\frac{1}{2} \log(2x+1) + \frac{1}{2} \log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+2*x)-1/(1+2*x), x, algorithm="fricas")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)

giac [A] time = 0.42, size = 19, normalized size = 0.90

$$-\frac{1}{2} \log(|2x+1|) + \frac{1}{2} \log(|2x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+2*x)-1/(1+2*x), x, algorithm="giac")

[Out] -1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln(2x-1)}{2} - \frac{\ln(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x-1)-1/(2*x+1),x)

[Out] 1/2*ln(2*x-1)-1/2*ln(2*x+1)

maxima [A] time = 0.94, size = 17, normalized size = 0.81

$$-\frac{1}{2} \log(2x+1) + \frac{1}{2} \log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)

mupad [B] time = 0.15, size = 6, normalized size = 0.29

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x - 1) - 1/(2*x + 1),x)

[Out] -atanh(2*x)

sympy [A] time = 0.10, size = 15, normalized size = 0.71

$$\frac{\log\left(x - \frac{1}{2}\right)}{2} - \frac{\log\left(x + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+2*x)-1/(1+2*x),x)

[Out] log(x - 1/2)/2 - log(x + 1/2)/2

$$3.423 \quad \int \frac{x}{(1-x^2)^5} dx$$

Optimal. Leaf size=13

$$\frac{1}{8(1-x^2)^4}$$

[Out] 1/8/(-x^2+1)^4

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {261}

$$\frac{1}{8(1-x^2)^4}$$

Antiderivative was successfully verified.

[In] Int[x/(1 - x^2)^5,x]

[Out] 1/(8*(1 - x^2)^4)

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{x}{(1-x^2)^5} dx = \frac{1}{8(1-x^2)^4}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8(x^2-1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 - x^2)^5,x]

[Out] 1/(8*(-1 + x^2)^4)

fricas [B] time = 0.59, size = 24, normalized size = 1.85

$$\frac{1}{8(x^8 - 4x^6 + 6x^4 - 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^5,x, algorithm="fricas")

[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)

giac [A] time = 0.33, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^5,x, algorithm="giac")

[Out] 1/8/(x^2 - 1)^4

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{1}{8(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^2+1)^5,x)

[Out] 1/8/(x^2-1)^4

maxima [A] time = 0.88, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^5,x, algorithm="maxima")

[Out] 1/8/(x^2 - 1)^4

mupad [B] time = 2.32, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x/(x^2 - 1)^5,x)

[Out] 1/(8*(x^2 - 1)^4)

sympy [B] time = 0.12, size = 22, normalized size = 1.69

$$\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**2+1)**5,x)

[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)

$$3.424 \quad \int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} \right)$$

Optimal. Leaf size=13

$$\frac{1}{8(1-x^2)^4}$$

[Out] 1/8/(-x^2+1)^4

Rubi [B] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 0, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{5}{256(x+1)} + \frac{5}{256(x+1)^2} + \frac{1}{64(x+1)^3} + \frac{1}{128(x+1)^4} + \frac{5}{256(1-x)} + \frac{5}{256(1-x)^2} + \frac{1}{64(1-x)^3} + \frac{1}{128(1-x)^4}$$

Antiderivative was successfully verified.

[In] Int[-1/(32*(-1 + x)^5) + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(256*(1 + x)^2), x]

[Out] 1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x)) + 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 + x))

Rubi steps

$$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} - \frac{5}{256(1+x)^2} \right)$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8(x^2-1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(256*(1 + x)^2), x]

[Out] 1/(8*(-1 + x^2)^4)

fricas [B] time = 0.64, size = 24, normalized size = 1.85

$$\frac{1}{8(x^8 - 4x^6 + 6x^4 - 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")

[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)

giac [B] time = 0.31, size = 57, normalized size = 4.38

$$\frac{5}{256(x+1)} - \frac{5}{256(x-1)} + \frac{5}{256(x+1)^2} + \frac{5}{256(x-1)^2} + \frac{1}{64(x+1)^3} - \frac{1}{64(x-1)^3} + \frac{1}{128(x+1)^4} + \frac{1}{128(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

maple [B] time = 0.00, size = 58, normalized size = 4.46

$$\frac{1}{128(x-1)^4} - \frac{1}{64(x-1)^3} + \frac{5}{256(x-1)^2} - \frac{5}{256(x-1)} + \frac{1}{128(x+1)^4} + \frac{1}{64(x+1)^3} + \frac{5}{256(x+1)^2} + \frac{5}{256(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/32/(x-1)^5+3/64/(x-1)^4-5/128/(x-1)^3+5/256/(x-1)^2-1/32/(x+1)^5-3/64/(x+1)^4-5/128/(x+1)^3-5/256/(x+1)^2,x)
```

```
[Out] 1/128/(x-1)^4-1/64/(x-1)^3+5/256/(x-1)^2-5/256/(x-1)+1/128/(x+1)^4+1/64/(x+1)^3+5/256/(x+1)^2+5/256/(x+1)
```

maxima [B] time = 0.94, size = 57, normalized size = 4.38

$$\frac{5}{256(x+1)} - \frac{5}{256(x-1)} + \frac{5}{256(x+1)^2} + \frac{5}{256(x-1)^2} + \frac{1}{64(x+1)^3} - \frac{1}{64(x-1)^3} + \frac{1}{128(x+1)^4} + \frac{1}{128(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5/(256*(x - 1)^2) - 5/(256*(x + 1)^2) - 5/(128*(x - 1)^3) - 5/(128*(x + 1)^3) + 3/(64*(x - 1)^4) - 3/(64*(x + 1)^4) - 1/(32*(x - 1)^5) - 1/(32*(x + 1)^5),x)
```

```
[Out] 1/(8*(x^2 - 1)^4)
```

sympy [B] time = 0.31, size = 22, normalized size = 1.69

$$\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1/32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

$$3.425 \quad \int \frac{1+x^6}{-1+x^6} dx$$

Optimal. Leaf size=69

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{6} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x)$$

[Out] x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{6} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^6)/(-1 + x^6), x]

[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x, x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^6}{-1+x^6} dx &= x + 2 \int \frac{1}{-1+x^6} dx \\ &= x - \frac{2}{3} \int \frac{1}{1-x^2} dx - \frac{2}{3} \int \frac{1-\frac{x}{2}}{1-x+x^2} dx - \frac{2}{3} \int \frac{1+\frac{x}{2}}{1+x+x^2} dx \\ &= x - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{6} \int \frac{1+2x}{1+x+x^2} dx - \frac{1}{2} \int \frac{1}{1-x+x^2} dx - \frac{1}{2} \int \frac{1}{1+x+x^2} dx \\ &= x - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \log(1-x+x^2) - \frac{1}{6} \log(1+x+x^2) + \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\ &= x - \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \log(1-x+x^2) - \frac{1}{6} \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6} \left(\log(x^2 - x + 1) - \log(x^2 + x + 1) + 6x + 2 \log(1 - x) - 2 \log(x + 1) - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(-1 + x^6), x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/S
qrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

fricas [A] time = 0.59, size = 66, normalized size = 0.96

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + x - \frac{1}{6} \log(x^2 + x + 1) + \frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1), x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
1) + 1/3*log(x - 1)
```

giac [A] time = 0.29, size = 68, normalized size = 0.99

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) + x - \frac{1}{6} \log(x^2 + x + 1) + \frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)/(x^6-1),x, algorithm="giac")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + x - 1/6*\log(x^2 + x + 1) + 1/6*\log(x^2 - x + 1) - 1/3*\log(\text{abs}(x + 1)) + 1/3*\log(\text{abs}(x - 1))$

maple [A] time = 0.01, size = 67, normalized size = 0.97

$$x - \frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x-1)}{3} - \frac{\ln(x+1)}{3} + \frac{\ln(x^2-x+1)}{6} - \frac{\ln(x^2+x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6+1)/(x^6-1),x)

[Out] $x + 1/3*\ln(x-1) - 1/6*\ln(x^2+x+1) - 1/3*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)}) + 1/6*\ln(x^2-x+1) - 1/3*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)}) - 1/3*\ln(x+1)$

maxima [A] time = 2.01, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)/(x^6-1),x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + x - 1/6*\log(x^2 + x + 1) + 1/6*\log(x^2 - x + 1) - 1/3*\log(x + 1) + 1/3*\log(x - 1)$

mupad [B] time = 0.10, size = 94, normalized size = 1.36

$$x + \frac{\operatorname{atan}(x \cdot 1i) \cdot 2i}{3} - \operatorname{atan}\left(\frac{x \cdot 32i}{-32 + \sqrt{3} \cdot 32i} - \frac{32 \sqrt{3} x}{-32 + \sqrt{3} \cdot 32i}\right) \left(\frac{\sqrt{3}}{3} - \frac{1}{3}i\right) - \operatorname{atan}\left(\frac{x \cdot 32i}{32 + \sqrt{3} \cdot 32i} + \frac{32 \sqrt{3} x}{32 + \sqrt{3} \cdot 32i}\right) \left(\frac{\sqrt{3}}{3} + \frac{1}{3}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6 + 1)/(x^6 - 1),x)

[Out] $x + (\operatorname{atan}(x \cdot 1i) \cdot 2i) / 3 - \operatorname{atan}((x \cdot 32i) / (3^{(1/2)} \cdot 32i - 32) - (32 \cdot 3^{(1/2)} \cdot x) / (3^{(1/2)} \cdot 32i - 32)) \cdot (3^{(1/2)} / 3 - 1i / 3) - \operatorname{atan}((x \cdot 32i) / (3^{(1/2)} \cdot 32i + 32) + (32 \cdot 3^{(1/2)} \cdot x) / (3^{(1/2)} \cdot 32i + 32)) \cdot (3^{(1/2)} / 3 + 1i / 3)$

sympy [A] time = 0.25, size = 85, normalized size = 1.23

$$x + \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**6+1)/(x**6-1),x)

[Out] $x + \log(x - 1)/3 - \log(x + 1)/3 + \log(x**2 - x + 1)/6 - \log(x**2 + x + 1)/6 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/3 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/3$

$$3.426 \quad \int \frac{\frac{1}{x^3} + x^3}{-\frac{1}{x^3} + x^3} dx$$

Optimal. Leaf size=69

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{6} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x)$$

[Out] x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1593, 1584, 388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{6} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^(-3) + x^3)/(-x^(-3) + x^3), x]

[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
 \int \frac{\frac{1}{x^3} + x^3}{-\frac{1}{x^3} + x^3} dx &= \int \frac{x^3 \left(\frac{1}{x^3} + x^3 \right)}{-1 + x^6} dx \\
 &= \int \frac{1 + x^6}{-1 + x^6} dx \\
 &= x + 2 \int \frac{1}{-1 + x^6} dx \\
 &= x - \frac{2}{3} \int \frac{1}{1 - x^2} dx - \frac{2}{3} \int \frac{1 - \frac{x}{2}}{1 - x + x^2} dx - \frac{2}{3} \int \frac{1 + \frac{x}{2}}{1 + x + x^2} dx \\
 &= x - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \int \frac{-1 + 2x}{1 - x + x^2} dx - \frac{1}{6} \int \frac{1 + 2x}{1 + x + x^2} dx - \frac{1}{2} \int \frac{1}{1 - x + x^2} dx - \frac{1}{2} \int \frac{1}{1 + x + x^2} dx \\
 &= x - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \log(1 - x + x^2) - \frac{1}{6} \log(1 + x + x^2) + \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, -1 + 2x \right) \\
 &= x - \frac{\tan^{-1} \left(\frac{-1 + 2x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1 + 2x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \log(1 - x + x^2) - \frac{1}{6} \log(1 + x + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6} \left(\log(x^2 - x + 1) - \log(x^2 + x + 1) + 6x + 2 \log(1 - x) - 2 \log(x + 1) - 2\sqrt{3} \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right) - 2\sqrt{3} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3), x]

[Out] $(6x - 2\sqrt{3} \operatorname{ArcTan}[-1 + 2x]/\sqrt{3}] - 2\sqrt{3} \operatorname{ArcTan}[(1 + 2x)/\sqrt{3}] + 2\log[1 - x] - 2\log[1 + x] + \log[1 - x + x^2] - \log[1 + x + x^2]) / 6$

fricas [A] time = 0.61, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + x - 1/6*\log(x^2 + x + 1) + 1/6*\log(x^2 - x + 1) - 1/3*\log(x + 1) + 1/3*\log(x - 1)$

giac [A] time = 0.31, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + x - 1/6*\log(x^2 + x + 1) + 1/6*\log(x^2 - x + 1) - 1/3*\log(\operatorname{abs}(x + 1)) + 1/3*\log(\operatorname{abs}(x - 1))$

maple [A] time = 0.00, size = 67, normalized size = 0.97

$$x - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right) - \sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x-1)}{3} - \frac{\ln(x+1)}{3} + \frac{\ln(x^2-x+1)}{6} - \frac{\ln(x^2+x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x^3+x^3)/(-1/x^3+x^3),x)

[Out] $x - 1/3*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)}) - 1/3*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)}) + 1/3*\ln(x-1) - 1/3*\ln(x+1) + 1/6*\ln(x^2-x+1) - 1/6*\ln(x^2+x+1)$

maxima [A] time = 1.99, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + x - 1/6*\log(x^2 + x + 1) + 1/6*\log(x^2 - x + 1) - 1/3*\log(x + 1) + 1/3*\log(x - 1)$

mupad [B] time = 0.04, size = 94, normalized size = 1.36

$$x + \frac{\operatorname{atan}(x \operatorname{li} 2i)}{3} - \operatorname{atan}\left(\frac{x 32i}{-32 + \sqrt{3} 32i} - \frac{32\sqrt{3}x}{-32 + \sqrt{3} 32i}\right) \left(\frac{\sqrt{3}}{3} - \frac{1}{3}i\right) - \operatorname{atan}\left(\frac{x 32i}{32 + \sqrt{3} 32i} + \frac{32\sqrt{3}x}{32 + \sqrt{3} 32i}\right) \left(\frac{\sqrt{3}}{3} + \frac{1}{3}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(1/x^3 + x^3)/(1/x^3 - x^3),x)`

[Out] `x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (32*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)`

sympy [A] time = 0.25, size = 85, normalized size = 1.23

$$x + \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/x**3+x**3)/(-1/x**3+x**3),x)`

[Out] `x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3`

$$3.427 \quad \int \frac{-x+x^3}{6+2x} dx$$

Optimal. Leaf size=24

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \log(x+3)$$

[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(3+x)

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 772}

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-x + x^3)/(6 + 2*x),x]

[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{-x+x^3}{6+2x} dx &= \int \frac{x(-1+x^2)}{6+2x} dx \\ &= \int \left(4 - \frac{3x}{2} + \frac{x^2}{2} - \frac{12}{3+x}\right) dx \\ &= 4x - \frac{3x^2}{4} + \frac{x^3}{6} - 12 \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.29

$$\frac{1}{2} \left(\frac{x^3}{3} - \frac{3x^2}{2} + 8x - 24 \log(x+3) + \frac{93}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-x + x^3)/(6 + 2*x),x]

[Out] (93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2

fricas [A] time = 0.55, size = 20, normalized size = 0.83

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12 \log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(6+2*x),x, algorithm="fricas")

[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)

giac [A] time = 0.31, size = 21, normalized size = 0.88

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12 \log(|x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(6+2*x),x, algorithm="giac")

[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-x)/(6+2*x),x)

[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(x+3)

maxima [A] time = 0.63, size = 20, normalized size = 0.83

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(6+2*x),x, algorithm="maxima")

[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$4x - 12 \ln(x + 3) - \frac{3x^2}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x - x^3)/(2*x + 6),x)

[Out] 4*x - 12*log(x + 3) - (3*x^2)/4 + x^3/6

sympy [A] time = 0.08, size = 20, normalized size = 0.83

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-x)/(6+2*x),x)

[Out] x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)

$$3.428 \quad \int \frac{x+x^3}{-1+x} dx$$

Optimal. Leaf size=26

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \log(1-x)$$

[Out] 2*x+1/2*x^2+1/3*x^3+2*ln(1-x)

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 772}

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(x + x^3)/(-1 + x),x]

[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x+x^3}{-1+x} dx &= \int \frac{x(1+x^2)}{-1+x} dx \\ &= \int \left(2 + \frac{2}{-1+x} + x + x^2 \right) dx \\ &= 2x + \frac{x^2}{2} + \frac{x^3}{3} + 2 \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{1}{6} (2x^3 + 3x^2 + 12x + 12 \log(x-1) - 17)$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^3)/(-1 + x),x]

[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6

fricas [A] time = 0.63, size = 20, normalized size = 0.77

$$\frac{1}{3} x^3 + \frac{1}{2} x^2 + 2x + 2 \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")

[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)

giac [A] time = 0.30, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")

[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x)/(x-1),x)

[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(x-1)

maxima [A] time = 0.69, size = 20, normalized size = 0.77

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")

[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)

mupad [B] time = 0.03, size = 20, normalized size = 0.77

$$2x + 2 \ln(x - 1) + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^3)/(x - 1),x)

[Out] 2*x + 2*log(x - 1) + x^2/2 + x^3/3

sympy [A] time = 0.07, size = 19, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x)/(-1+x),x)

[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)

3.429 $\int (ac + (bc + d)x) dx$

Optimal. Leaf size=17

$$acx + \frac{1}{2}x^2(bc + d)$$

[Out] a*c*x+1/2*(b*c+d)*x^2

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$acx + \frac{1}{2}x^2(bc + d)$$

Antiderivative was successfully verified.

[In] Int[a*c + (b*c + d)*x,x]

[Out] a*c*x + ((b*c + d)*x^2)/2

Rubi steps

$$\int (ac + (bc + d)x) dx = acx + \frac{1}{2}(bc + d)x^2$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.29

$$acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[a*c + (b*c + d)*x,x]

[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2

fricas [A] time = 0.53, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c+(b*c+d)*x,x, algorithm="fricas")

[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a

giac [A] time = 0.39, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c+(b*c+d)*x,x, algorithm="giac")

[Out] a*c*x + 1/2*(b*c + d)*x^2

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$acx + \frac{(bc + d)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a*c+(b*c+d)*x,x)`

[Out] `a*c*x+1/2*(b*c+d)*x^2`

maxima [A] time = 0.66, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*c+(b*c+d)*x,x, algorithm="maxima")`

[Out] `a*c*x + 1/2*(b*c + d)*x^2`

mupad [B] time = 0.02, size = 17, normalized size = 1.00

$$\left(\frac{d}{2} + \frac{bc}{2}\right)x^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a*c + x*(d + b*c),x)`

[Out] `x^2*(d/2 + (b*c)/2) + a*c*x`

sympy [A] time = 0.06, size = 15, normalized size = 0.88

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*c+(b*c+d)*x,x)`

[Out] `a*c*x + x**2*(b*c/2 + d/2)`

3.430 $\int(dx + c(a + bx)) dx$

Optimal. Leaf size=24

$$\frac{c(a + bx)^2}{2b} + \frac{dx^2}{2}$$

[Out] 1/2*d*x^2+1/2*c*(b*x+a)^2/b

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{c(a + bx)^2}{2b} + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[d*x + c*(a + b*x), x]

[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)

Rubi steps

$$\int(dx + c(a + bx)) dx = \frac{dx^2}{2} + \frac{c(a + bx)^2}{2b}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[d*x + c*(a + b*x), x]

[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2

fricas [A] time = 0.45, size = 18, normalized size = 0.75

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d*x+c*(b*x+a), x, algorithm="fricas")

[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a

giac [A] time = 0.37, size = 20, normalized size = 0.83

$$\frac{1}{2}dx^2 + \frac{1}{2}(bx^2 + 2ax)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d*x+c*(b*x+a), x, algorithm="giac")

[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c

maple [A] time = 0.00, size = 20, normalized size = 0.83

$$\frac{dx^2}{2} + \left(\frac{1}{2}bx^2 + ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d*x+c*(b*x+a),x)`

[Out] `1/2*d*x^2+c*(1/2*b*x^2+a*x)`

maxima [A] time = 0.57, size = 20, normalized size = 0.83

$$\frac{1}{2} dx^2 + \frac{1}{2} (bx^2 + 2ax)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d*x+c*(b*x+a),x, algorithm="maxima")`

[Out] `1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c`

mupad [B] time = 0.02, size = 17, normalized size = 0.71

$$\left(\frac{d}{2} + \frac{bc}{2}\right)x^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d*x + c*(a + b*x),x)`

[Out] `x^2*(d/2 + (b*c)/2) + a*c*x`

sympy [A] time = 0.06, size = 15, normalized size = 0.62

$$acx + x^2 \left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d*x+c*(b*x+a),x)`

[Out] `a*c*x + x**2*(b*c/2 + d/2)`

$$3.431 \quad \int \frac{4+4x}{x^2(1+x^2)} dx$$

Optimal. Leaf size=22

$$-2 \log(x^2 + 1) - \frac{4}{x} + 4 \log(x) - 4 \tan^{-1}(x)$$

[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {801, 635, 203, 260}

$$-2 \log(x^2 + 1) - \frac{4}{x} + 4 \log(x) - 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(4 + 4*x)/(x^2*(1 + x^2)), x]

[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int((((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{4+4x}{x^2(1+x^2)} dx &= \int \left(\frac{4}{x^2} + \frac{4}{x} - \frac{4(1+x)}{1+x^2} \right) dx \\ &= -\frac{4}{x} + 4 \log(x) - 4 \int \frac{1+x}{1+x^2} dx \\ &= -\frac{4}{x} + 4 \log(x) - 4 \int \frac{1}{1+x^2} dx - 4 \int \frac{x}{1+x^2} dx \\ &= -\frac{4}{x} - 4 \tan^{-1}(x) + 4 \log(x) - 2 \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.09

$$4 \left(-\frac{1}{2} \log(x^2 + 1) - \frac{1}{x} + \log(x) - \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 4*x)/(x^2*(1 + x^2)),x]

[Out] 4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2])/2)

fricas [A] time = 0.62, size = 25, normalized size = 1.14

$$\frac{2(2x \arctan(x) + x \log(x^2 + 1) - 2x \log(x) + 2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="fricas")

[Out] -2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x

giac [A] time = 0.39, size = 23, normalized size = 1.05

$$-\frac{4}{x} - 4 \arctan(x) - 2 \log(x^2 + 1) + 4 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="giac")

[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(abs(x))

maple [A] time = 0.01, size = 23, normalized size = 1.05

$$-4 \arctan(x) + 4 \ln(x) - 2 \ln(x^2 + 1) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4+4*x)/x^2/(x^2+1),x)

[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)

maxima [A] time = 1.38, size = 22, normalized size = 1.00

$$-\frac{4}{x} - 4 \arctan(x) - 2 \log(x^2 + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="maxima")

[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)

mupad [B] time = 0.04, size = 28, normalized size = 1.27

$$4 \ln(x) - \frac{4}{x} + \ln(x - i) (-2 + 2i) + \ln(x + i) (-2 - 2i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4)/(x^2*(x^2 + 1)),x)

[Out] 4*log(x) - log(x + 1i)*(2 + 2i) - log(x - 1i)*(2 - 2i) - 4/x

sympy [A] time = 0.13, size = 20, normalized size = 0.91

$$4 \log(x) - 2 \log(x^2 + 1) - 4 \operatorname{atan}(x) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+4*x)/x**2/(x**2+1), x)

[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x

$$3.432 \quad \int \frac{24+8x}{x(-4+x^2)} dx$$

Optimal. Leaf size=17

$$5 \log(2-x) - 6 \log(x) + \log(x+2)$$

[Out] 5*ln(2-x)-6*ln(x)+ln(2+x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {801}

$$5 \log(2-x) - 6 \log(x) + \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(24 + 8*x)/(x*(-4 + x^2)), x]

[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]

Rule 801

Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_.) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{24+8x}{x(-4+x^2)} dx &= \int \left(\frac{5}{-2+x} - \frac{6}{x} + \frac{1}{2+x} \right) dx \\ &= 5 \log(2-x) - 6 \log(x) + \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.59

$$8 \left(\frac{5}{8} \log(2-x) - \frac{3 \log(x)}{4} + \frac{1}{8} \log(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(24 + 8*x)/(x*(-4 + x^2)), x]

[Out] 8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)

fricas [A] time = 0.71, size = 15, normalized size = 0.88

$$\log(x+2) + 5 \log(x-2) - 6 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((24+8*x)/x/(x^2-4), x, algorithm="fricas")

[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)

giac [A] time = 0.33, size = 18, normalized size = 1.06

$$\log(|x+2|) + 5 \log(|x-2|) - 6 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="giac")

[Out] log(abs(x + 2)) + 5*log(abs(x - 2)) - 6*log(abs(x))

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$-6 \ln(x) + 5 \ln(x - 2) + \ln(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((24+8*x)/x/(x^2-4),x)

[Out] 5*ln(x-2)+ln(x+2)-6*ln(x)

maxima [A] time = 0.75, size = 15, normalized size = 0.88

$$\log(x + 2) + 5 \log(x - 2) - 6 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="maxima")

[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$5 \ln(x - 2) + \ln(x + 2) - 6 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x + 24)/(x*(x^2 - 4)),x)

[Out] 5*log(x - 2) + log(x + 2) - 6*log(x)

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$-6 \log(x) + 5 \log(x - 2) + \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((24+8*x)/x/(x**2-4),x)

[Out] -6*log(x) + 5*log(x - 2) + log(x + 2)

$$3.433 \quad \int \frac{-1+x^2}{-2x+x^3} dx$$

Optimal. Leaf size=19

$$\frac{1}{4} \log(2-x^2) + \frac{\log(x)}{2}$$

[Out] 1/2*ln(x)+1/4*ln(-x^2+2)

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1593, 446, 72}

$$\frac{1}{4} \log(2-x^2) + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(-2*x + x^3), x]

[Out] Log[x]/2 + Log[2 - x^2]/4

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^2}{-2x+x^3} dx &= \int \frac{-1+x^2}{x(-2+x^2)} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{-1+x}{(-2+x)x} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{2(-2+x)} + \frac{1}{2x} \right) dx, x, x^2 \right) \\ &= \frac{\log(x)}{2} + \frac{1}{4} \log(2-x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{4} \log(2-x^2) + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(-2*x + x^3), x]

[Out] Log[x]/2 + Log[2 - x^2]/4

fricas [A] time = 0.63, size = 13, normalized size = 0.68

$$\frac{1}{4} \log(x^2 - 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3-2*x), x, algorithm="fricas")

[Out] 1/4*log(x^2 - 2) + 1/2*log(x)

giac [A] time = 0.31, size = 16, normalized size = 0.84

$$\frac{1}{4} \log(x^2) + \frac{1}{4} \log(|x^2 - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3-2*x), x, algorithm="giac")

[Out] 1/4*log(x^2) + 1/4*log(abs(x^2 - 2))

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$\frac{\ln(x)}{2} + \frac{\ln(x^2 - 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/(x^3-2*x), x)

[Out] 1/4*ln(x^2-2)+1/2*ln(x)

maxima [A] time = 0.66, size = 13, normalized size = 0.68

$$\frac{1}{4} \log(x^2 - 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3-2*x), x, algorithm="maxima")

[Out] 1/4*log(x^2 - 2) + 1/2*log(x)

mupad [B] time = 2.34, size = 13, normalized size = 0.68

$$\frac{\ln(x^2 - 2)}{4} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(2*x - x^3), x)

[Out] log(x^2 - 2)/4 + log(x)/2

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$\frac{\log(x)}{2} + \frac{\log(x^2 - 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-1)/(x**3-2*x), x)

[Out] log(x)/2 + log(x**2 - 2)/4

$$3.434 \quad \int \frac{1+x^2}{3x+x^3} dx$$

Optimal. Leaf size=12

$$\frac{1}{3} \log(x^3 + 3x)$$

[Out] 1/3*ln(x^3+3*x)

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1587}

$$\frac{1}{3} \log(x^3 + 3x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(3*x + x^3), x]

[Out] Log[3*x + x^3]/3

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{1+x^2}{3x+x^3} dx = \frac{1}{3} \log(3x+x^3)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.42

$$\frac{1}{3} \log(x^2 + 3) + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(3*x + x^3), x]

[Out] Log[x]/3 + Log[3 + x^2]/3

fricas [A] time = 0.64, size = 10, normalized size = 0.83

$$\frac{1}{3} \log(x^3 + 3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+3*x), x, algorithm="fricas")

[Out] 1/3*log(x^3 + 3*x)

giac [A] time = 0.40, size = 13, normalized size = 1.08

$$\frac{1}{3} \log\left(3 \left| \frac{1}{3} x^3 + x \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="giac")

[Out] 1/3*log(3*abs(1/3*x^3 + x))

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{\ln\left(\left(x^2 + 3\right)x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^3+3*x),x)

[Out] 1/3*ln(x*(x^2+3))

maxima [A] time = 0.62, size = 10, normalized size = 0.83

$$\frac{1}{3} \log\left(x^3 + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")

[Out] 1/3*log(x^3 + 3*x)

mupad [B] time = 2.27, size = 10, normalized size = 0.83

$$\frac{\ln\left(x^3 + 3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(3*x + x^3),x)

[Out] log(3*x + x^3)/3

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$\frac{\log\left(x^3 + 3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**3+3*x),x)

[Out] log(x**3 + 3*x)/3

$$3.435 \quad \int \frac{a+3bx^2}{ax+bx^3} dx$$

Optimal. Leaf size=10

$$\log(ax + bx^3)$$

[Out] $\ln(b*x^3+a*x)$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1587}

$$\log(ax + bx^3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + 3*b*x^2)/(a*x + b*x^3), x]$

[Out] $\text{Log}[a*x + b*x^3]$

Rule 1587

$\text{Int}[(Pp_)/(Qq_), x_Symbol] \rightarrow \text{With}[\{p = \text{Expon}[Pp, x], q = \text{Expon}[Qq, x]\}, \text{Simp}[(\text{Coeff}[Pp, x, p]*\text{Log}[\text{RemoveContent}[Qq, x]])/(q*\text{Coeff}[Qq, x, q]), x] /; \text{EqQ}[p, q - 1] \&\& \text{EqQ}[Pp, \text{Simplify}[(\text{Coeff}[Pp, x, p]*D[Qq, x])/(q*\text{Coeff}[Qq, x, q])]]] /; \text{PolyQ}[Pp, x] \&\& \text{PolyQ}[Qq, x]$

Rubi steps

$$\int \frac{a + 3bx^2}{ax + bx^3} dx = \log(ax + bx^3)$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.10

$$\log(a + bx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a + 3*b*x^2)/(a*x + b*x^3), x]$

[Out] $\text{Log}[x] + \text{Log}[a + b*x^2]$

fricas [A] time = 0.71, size = 10, normalized size = 1.00

$$\log(bx^3 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((3*b*x^2+a)/(b*x^3+a*x), x, \text{algorithm}="fricas")$

[Out] $\log(b*x^3 + a*x)$

giac [A] time = 0.33, size = 11, normalized size = 1.10

$$\log(|bx^3 + ax|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((3*b*x^2+a)/(b*x^3+a*x), x, \text{algorithm}="giac")$

[Out] $\log(\text{abs}(b*x^3 + a*x))$

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\ln\left((bx^2 + a)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((3*b*x^2+a)/(b*x^3+a*x), x)$

[Out] $\ln(x*(b*x^2+a))$

maxima [A] time = 0.61, size = 10, normalized size = 1.00

$$\log(bx^3 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((3*b*x^2+a)/(b*x^3+a*x), x, \text{algorithm}="maxima")$

[Out] $\log(b*x^3 + a*x)$

mupad [B] time = 0.06, size = 10, normalized size = 1.00

$$\ln(bx^3 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + 3*b*x^2)/(a*x + b*x^3), x)$

[Out] $\log(a*x + b*x^3)$

sympy [A] time = 0.13, size = 8, normalized size = 0.80

$$\log(ax + bx^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((3*b*x**2+a)/(b*x**3+a*x), x)$

[Out] $\log(a*x + b*x**3)$

$$3.436 \quad \int \frac{-2+4x}{-x+x^3} dx$$

Optimal. Leaf size=17

$$\log(1-x) + 2\log(x) - 3\log(x+1)$$

[Out] $\ln(1-x)+2*\ln(x)-3*\ln(1+x)$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 801}

$$\log(1-x) + 2\log(x) - 3\log(x+1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-2 + 4*x)/(-x + x^3), x]$

[Out] $\text{Log}[1 - x] + 2*\text{Log}[x] - 3*\text{Log}[1 + x]$

Rule 801

$\text{Int}[(((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.)))/((a_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + c*x^2), x], x] /;$ FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_.)^(p_.) + (b_.)*(x_.)^(q_.))^(n_.), x_Symbol] \rightarrow \text{Int}[u*x^(n*p)*(a + b*x^(q-p))^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rubi steps

$$\begin{aligned} \int \frac{-2+4x}{-x+x^3} dx &= \int \frac{-2+4x}{x(-1+x^2)} dx \\ &= \int \left(\frac{1}{-1+x} + \frac{2}{x} - \frac{3}{1+x} \right) dx \\ &= \log(1-x) + 2\log(x) - 3\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\log(1-x) + 2\log(x) - 3\log(x+1)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(-2 + 4*x)/(-x + x^3), x]$

[Out] $\text{Log}[1 - x] + 2*\text{Log}[x] - 3*\text{Log}[1 + x]$

fricas [A] time = 0.69, size = 15, normalized size = 0.88

$$-3\log(x+1) + \log(x-1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-2+4*x)/(x^3-x), x, \text{algorithm}=\text{"fricas"})$

[Out] $-3\log(x + 1) + \log(x - 1) + 2\log(x)$

giac [A] time = 0.36, size = 18, normalized size = 1.06

$$-3 \log(|x + 1|) + \log(|x - 1|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+4*x)/(x^3-x),x, algorithm="giac")`

[Out] $-3\log(\text{abs}(x + 1)) + \log(\text{abs}(x - 1)) + 2\log(\text{abs}(x))$

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$2 \ln(x) + \ln(x - 1) - 3 \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-2+4*x)/(x^3-x),x)`

[Out] $\ln(x-1)-3*\ln(x+1)+2*\ln(x)$

maxima [A] time = 0.61, size = 15, normalized size = 0.88

$$-3 \log(x + 1) + \log(x - 1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")`

[Out] $-3\log(x + 1) + \log(x - 1) + 2\log(x)$

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$\ln(x - 1) - 3 \ln(x + 1) + 2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(4*x - 2)/(x - x^3),x)`

[Out] $\log(x - 1) - 3\log(x + 1) + 2\log(x)$

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$2 \log(x) + \log(x - 1) - 3 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+4*x)/(x**3-x),x)`

[Out] $2*\log(x) + \log(x - 1) - 3*\log(x + 1)$

$$3.437 \quad \int \frac{4+x}{4x+x^3} dx$$

Optimal. Leaf size=23

$$-\frac{1}{2} \log(x^2 + 4) + \log(x) + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1593, 801, 635, 203, 260}

$$-\frac{1}{2} \log(x^2 + 4) + \log(x) + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x)/(4*x + x^3), x]

[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{4+x}{4x+x^3} dx &= \int \frac{4+x}{x(4+x^2)} dx \\
&= \int \left(\frac{1}{x} + \frac{1-x}{4+x^2} \right) dx \\
&= \log(x) + \int \frac{1-x}{4+x^2} dx \\
&= \log(x) + \int \frac{1}{4+x^2} dx - \int \frac{x}{4+x^2} dx \\
&= \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + \log(x) - \frac{1}{2} \log(4+x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{1}{2} \log(x^2 + 4) + \log(x) + \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x)/(4*x + x^3), x]

[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2

fricas [A] time = 0.64, size = 17, normalized size = 0.74

$$\frac{1}{2} \arctan \left(\frac{1}{2} x \right) - \frac{1}{2} \log(x^2 + 4) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^3+4*x), x, algorithm="fricas")

[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)

giac [A] time = 0.38, size = 18, normalized size = 0.78

$$\frac{1}{2} \arctan \left(\frac{1}{2} x \right) - \frac{1}{2} \log(x^2 + 4) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^3+4*x), x, algorithm="giac")

[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\arctan \left(\frac{x}{2} \right)}{2} + \ln(x) - \frac{\ln(x^2 + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+4)/(x^3+4*x), x)

[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)

maxima [A] time = 1.64, size = 17, normalized size = 0.74

$$\frac{1}{2} \arctan \left(\frac{1}{2} x \right) - \frac{1}{2} \log(x^2 + 4) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^3+4*x),x, algorithm="maxima")

[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)

mupad [B] time = 0.05, size = 21, normalized size = 0.91

$$\ln(x) + \ln(x - 2i) \left(-\frac{1}{2} - \frac{1}{4}i \right) + \ln(x + 2i) \left(-\frac{1}{2} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 4)/(4*x + x^3),x)

[Out] log(x) - log(x + 2i)*(1/2 - 1i/4) - log(x - 2i)*(1/2 + 1i/4)

sympy [A] time = 0.13, size = 17, normalized size = 0.74

$$\log(x) - \frac{\log(x^2 + 4)}{2} + \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x**3+4*x),x)

[Out] log(x) - log(x**2 + 4)/2 + atan(x/2)/2

$$3.438 \quad \int \frac{-x+2x^3}{1-x^2+x^4} dx$$

Optimal. Leaf size=15

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

[Out] 1/2*ln(x^4-x^2+1)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1587}

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4), x]

[Out] Log[1 - x^2 + x^4]/2

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coef[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coef[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coef[Pp, x, p]*D[Qq, x])/(q*Coef[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{-x+2x^3}{1-x^2+x^4} dx = \frac{1}{2} \log(1-x^2+x^4)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4), x]

[Out] Log[1 - x^2 + x^4]/2

fricas [A] time = 0.47, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-x)/(x^4-x^2+1), x, algorithm="fricas")

[Out] 1/2*log(x^4 - x^2 + 1)

giac [A] time = 0.28, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")

[Out] 1/2*log(x^4 - x^2 + 1)

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln(x^4 - x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3-x)/(x^4-x^2+1),x)

[Out] 1/2*ln(x^4-x^2+1)

maxima [A] time = 0.65, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")

[Out] 1/2*log(x^4 - x^2 + 1)

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{\ln(x^4 - x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)

[Out] log(x^4 - x^2 + 1)/2

sympy [A] time = 0.09, size = 10, normalized size = 0.67

$$\frac{\log(x^4 - x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3-x)/(x**4-x**2+1),x)

[Out] log(x**4 - x**2 + 1)/2

$$3.439 \quad \int \frac{-3+x}{2x+3x^2+x^3} dx$$

Optimal. Leaf size=21

$$-\frac{3 \log(x)}{2} + 4 \log(x+1) - \frac{5}{2} \log(x+2)$$

[Out] $-3/2*\ln(x)+4*\ln(1+x)-5/2*\ln(2+x)$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1594, 800}

$$-\frac{3 \log(x)}{2} + 4 \log(x+1) - \frac{5}{2} \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3), x]

[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2

Rule 800

Int[(((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{-3+x}{2x+3x^2+x^3} dx &= \int \frac{-3+x}{x(2+3x+x^2)} dx \\ &= \int \left(-\frac{3}{2x} + \frac{4}{1+x} - \frac{5}{2(2+x)} \right) dx \\ &= -\frac{3 \log(x)}{2} + 4 \log(1+x) - \frac{5}{2} \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{3 \log(x)}{2} + 4 \log(x+1) - \frac{5}{2} \log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3), x]

[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2

fricas [A] time = 0.61, size = 17, normalized size = 0.81

$$-\frac{5}{2} \log(x+2) + 4 \log(x+1) - \frac{3}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")

[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)

giac [A] time = 0.37, size = 20, normalized size = 0.95

$$-\frac{5}{2} \log(|x + 2|) + 4 \log(|x + 1|) - \frac{3}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")

[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{3 \ln(x)}{2} + 4 \ln(x + 1) - \frac{5 \ln(x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-3)/(x^3+3*x^2+2*x),x)

[Out] -3/2*ln(x)+4*ln(x+1)-5/2*ln(x+2)

maxima [A] time = 0.59, size = 17, normalized size = 0.81

$$-\frac{5}{2} \log(x + 2) + 4 \log(x + 1) - \frac{3}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")

[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)

mupad [B] time = 0.07, size = 17, normalized size = 0.81

$$4 \ln(x + 1) - \frac{5 \ln(x + 2)}{2} - \frac{3 \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 3)/(2*x + 3*x^2 + x^3),x)

[Out] 4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2

sympy [A] time = 0.13, size = 20, normalized size = 0.95

$$-\frac{3 \log(x)}{2} + 4 \log(x + 1) - \frac{5 \log(x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)

[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2

$$3.440 \quad \int \frac{2+4x}{x^2+2x^3+x^4} dx$$

Optimal. Leaf size=10

$$-\frac{2}{x(x+1)}$$

[Out] -2/x/(1+x)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1594, 27, 74}

$$-\frac{2}{x(x+1)}$$

Antiderivative was successfully verified.

[In] Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4), x]

[Out] -2/(x*(1 + x))

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 74

Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 1594

Int[(u_.)*((a_.)*(x_.)^(p_.) + (b_.)*(x_.)^(q_.) + (c_.)*(x_.)^(r_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{2+4x}{x^2+2x^3+x^4} dx &= \int \frac{2+4x}{x^2(1+2x+x^2)} dx \\ &= \int \frac{2+4x}{x^2(1+x)^2} dx \\ &= -\frac{2}{x(1+x)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4), x]

[Out] $-2/(x + x^2)$

fricas [A] time = 0.70, size = 9, normalized size = 0.90

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="fricas")`

[Out] $-2/(x^2 + x)$

giac [A] time = 0.26, size = 9, normalized size = 0.90

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="giac")`

[Out] $-2/(x^2 + x)$

maple [A] time = 0.00, size = 14, normalized size = 1.40

$$-\frac{2}{x} + \frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+4*x)/(x^4+2*x^3+x^2),x)`

[Out] $-2/x+2/(x+1)$

maxima [A] time = 0.78, size = 9, normalized size = 0.90

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="maxima")`

[Out] $-2/(x^2 + x)$

mupad [B] time = 2.20, size = 10, normalized size = 1.00

$$-\frac{2}{x(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x + 2)/(x^2 + 2*x^3 + x^4),x)`

[Out] $-2/(x*(x + 1))$

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+4*x)/(x**4+2*x**3+x**2),x)`

[Out] $-2/(x**2 + x)$

$$3.441 \quad \int \frac{1+x}{-6x+x^2+x^3} dx$$

Optimal. Leaf size=25

$$\frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(x+3)$$

[Out] 3/10*ln(2-x)-1/6*ln(x)-2/15*ln(3+x)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1594, 800}

$$\frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/(-6*x + x^2 + x^3), x]

[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15

Rule 800

Int[(((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rubi steps

$$\begin{aligned} \int \frac{1+x}{-6x+x^2+x^3} dx &= \int \frac{1+x}{x(-6+x+x^2)} dx \\ &= \int \left(\frac{3}{10(-2+x)} - \frac{1}{6x} - \frac{2}{15(3+x)} \right) dx \\ &= \frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/(-6*x + x^2 + x^3), x]

[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15

fricas [A] time = 0.76, size = 17, normalized size = 0.68

$$-\frac{2}{15} \log(x+3) + \frac{3}{10} \log(x-2) - \frac{1}{6} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="fricas")

[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)

giac [A] time = 0.36, size = 20, normalized size = 0.80

$$-\frac{2}{15} \log(|x + 3|) + \frac{3}{10} \log(|x - 2|) - \frac{1}{6} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="giac")

[Out] -2/15*log(abs(x + 3)) + 3/10*log(abs(x - 2)) - 1/6*log(abs(x))

maple [A] time = 0.01, size = 18, normalized size = 0.72

$$-\frac{\ln(x)}{6} + \frac{3 \ln(x - 2)}{10} - \frac{2 \ln(x + 3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(x^3+x^2-6*x),x)

[Out] 3/10*ln(x-2)-2/15*ln(x+3)-1/6*ln(x)

maxima [A] time = 0.65, size = 17, normalized size = 0.68

$$-\frac{2}{15} \log(x + 3) + \frac{3}{10} \log(x - 2) - \frac{1}{6} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="maxima")

[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)

mupad [B] time = 0.11, size = 17, normalized size = 0.68

$$\frac{3 \ln(x - 2)}{10} - \frac{2 \ln(x + 3)}{15} - \frac{\ln(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)/(x^2 - 6*x + x^3),x)

[Out] (3*log(x - 2))/10 - (2*log(x + 3))/15 - log(x)/6

sympy [A] time = 0.13, size = 20, normalized size = 0.80

$$-\frac{\log(x)}{6} + \frac{3 \log(x - 2)}{10} - \frac{2 \log(x + 3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x**3+x**2-6*x),x)

[Out] -log(x)/6 + 3*log(x - 2)/10 - 2*log(x + 3)/15

$$3.442 \quad \int \frac{4x^2+x^3}{x+x^3} dx$$

Optimal. Leaf size=14

$$2 \log(x^2 + 1) + x - \tan^{-1}(x)$$

[Out] x-arctan(x)+2*ln(x^2+1)

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1593, 1584, 774, 635, 203, 260}

$$2 \log(x^2 + 1) + x - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(4*x^2 + x^3)/(x + x^3),x]

[Out] x - ArcTan[x] + 2*Log[1 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 774

Int[((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{4x^2 + x^3}{x + x^3} dx &= \int \frac{4x^2 + x^3}{x(1 + x^2)} dx \\
&= \int \frac{x(4 + x)}{1 + x^2} dx \\
&= x + \int \frac{-1 + 4x}{1 + x^2} dx \\
&= x + 4 \int \frac{x}{1 + x^2} dx - \int \frac{1}{1 + x^2} dx \\
&= x - \tan^{-1}(x) + 2 \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$2 \log(x^2 + 1) + x - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(4*x^2 + x^3)/(x + x^3),x]

[Out] x - ArcTan[x] + 2*Log[1 + x^2]

fricas [A] time = 0.53, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")

[Out] x - arctan(x) + 2*log(x^2 + 1)

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")

[Out] x - arctan(x) + 2*log(x^2 + 1)

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$x - \arctan(x) + 2 \ln(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+4*x^2)/(x^3+x),x)

[Out] x-arctan(x)+2*ln(x^2+1)

maxima [A] time = 1.56, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")

[Out] x - arctan(x) + 2*log(x^2 + 1)

mupad [B] time = 2.22, size = 14, normalized size = 1.00

$$x + 2 \ln(x^2 + 1) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2 + x^3)/(x + x^3), x)`

[Out] `x + 2*log(x^2 + 1) - atan(x)`

sympy [A] time = 0.10, size = 12, normalized size = 0.86

$$x + 2 \log(x^2 + 1) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+4*x**2)/(x**3+x), x)`

[Out] `x + 2*log(x**2 + 1) - atan(x)`

$$3.443 \quad \int \frac{x+2x^3}{(x^2+x^4)^3} dx$$

Optimal. Leaf size=13

$$-\frac{1}{4(x^4+x^2)^2}$$

[Out] -1/4/(x^4+x^2)^2

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1588}

$$-\frac{1}{4(x^4+x^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x + 2*x^3)/(x^2 + x^4)^3,x]

[Out] -1/(4*(x^2 + x^4)^2)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{x+2x^3}{(x^2+x^4)^3} dx = -\frac{1}{4(x^2+x^4)^2}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.08

$$-\frac{1}{4x^4(x^2+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]

[Out] -1/4*1/(x^4*(1 + x^2)^2)

fricas [A] time = 0.60, size = 16, normalized size = 1.23

$$-\frac{1}{4(x^8+2x^6+x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")

[Out] -1/4/(x^8 + 2*x^6 + x^4)

giac [A] time = 0.37, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4 + x^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")

[Out] -1/4/(x^4 + x^2)^2

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{1}{2x^2} - \frac{1}{4x^4} - \frac{1}{4(x^2 + 1)^2} - \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+x)/(x^4+x^2)^3,x)

[Out] -1/4/(x^2+1)^2-1/2/(x^2+1)-1/4/x^4+1/2/x^2

maxima [A] time = 0.58, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4 + x^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")

[Out] -1/4/(x^4 + x^2)^2

mupad [B] time = 2.24, size = 20, normalized size = 1.54

$$-\frac{1}{4x^8 + 8x^6 + 4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2*x^3)/(x^2 + x^4)^3,x)

[Out] -1/(4*x^4 + 8*x^6 + 4*x^8)

sympy [A] time = 0.13, size = 17, normalized size = 1.31

$$-\frac{1}{4x^8 + 8x^6 + 4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+x)/(x**4+x**2)**3,x)

[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)

$$3.444 \quad \int \frac{ax^2+bx^3}{cx^2+dx^3} dx$$

Optimal. Leaf size=26

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

[Out] b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1593, 1584, 43}

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3), x]

[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 1584

Int[(u_.)*(x_))^(m_.)*((a_.)*(x_))^(p_.) + (b_.)*(x_))^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1593

Int[(u_.)*((a_.)*(x_))^(p_.) + (b_.)*(x_))^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{ax^2+bx^3}{cx^2+dx^3} dx &= \int \frac{x^2(a+bx)}{cx^2+dx^3} dx \\ &= \int \frac{a+bx}{c+dx} dx \\ &= \int \left(\frac{b}{d} + \frac{-bc+ad}{d(c+dx)} \right) dx \\ &= \frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$\frac{(ad-bc)\log(c+dx)}{d^2} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]

[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2

fricas [A] time = 0.61, size = 25, normalized size = 0.96

$$\frac{bdx - (bc - ad) \log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="fricas")

[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2

giac [A] time = 0.28, size = 27, normalized size = 1.04

$$\frac{bx}{d} - \frac{(bc - ad) \log(|dx + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")

[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2

maple [A] time = 0.00, size = 32, normalized size = 1.23

$$\frac{a \ln(dx + c)}{d} - \frac{bc \ln(dx + c)}{d^2} + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)/(d*x^3+c*x^2),x)

[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c

maxima [A] time = 0.72, size = 26, normalized size = 1.00

$$\frac{bx}{d} - \frac{(bc - ad) \log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")

[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2

mupad [B] time = 2.23, size = 25, normalized size = 0.96

$$\frac{\ln(c + dx) (ad - bc)}{d^2} + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3)/(c*x^2 + d*x^3),x)

[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d

sympy [A] time = 0.15, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc) \log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)

[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2

$$3.445 \quad \int \frac{x+x^2}{-2x-x^2+x^3} dx$$

Optimal. Leaf size=6

$$\log(2-x)$$

[Out] ln(2-x)

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1586, 31}

$$\log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(x + x^2)/(-2*x - x^2 + x^3), x]

[Out] Log[2 - x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 1586

Int[(u_.)*(P(x_))^(p_.)*(Q(x_))^(q_.), x_Symbol] := Int[u*PolynomialQuotient[P(x), Q(x, x)]^p*Q(x)^(p+q), x] /; FreeQ[q, x] && PolyQ[P(x), x] && PolyQ[Q(x), x] && EqQ[PolynomialRemainder[P(x), Q(x), x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\int \frac{x+x^2}{-2x-x^2+x^3} dx = \int \frac{1}{-2+x} dx = \log(2-x)$$

Mathematica [A] time = 0.00, size = 4, normalized size = 0.67

$$\log(x-2)$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^2)/(-2*x - x^2 + x^3), x]

[Out] Log[-2 + x]

fricas [A] time = 0.67, size = 4, normalized size = 0.67

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(x^3-x^2-2*x), x, algorithm="fricas")

[Out] log(x - 2)

giac [A] time = 0.28, size = 5, normalized size = 0.83

$$\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="giac")

[Out] log(abs(x - 2))

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x)/(x^3-x^2-2*x),x)

[Out] ln(x-2)

maxima [A] time = 0.57, size = 4, normalized size = 0.67

$$\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="maxima")

[Out] log(x - 2)

mupad [B] time = 0.02, size = 4, normalized size = 0.67

$$\ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x + x^2)/(2*x + x^2 - x^3),x)

[Out] log(x - 2)

sympy [A] time = 0.06, size = 3, normalized size = 0.50

$$\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x)/(x**3-x**2-2*x),x)

[Out] log(x - 2)

$$3.446 \quad \int \frac{1-5x^2}{x^3(1+x^2)} dx$$

Optimal. Leaf size=20

$$-\frac{1}{2x^2} + 3 \log(x^2 + 1) - 6 \log(x)$$

[Out] $-1/2/x^2-6*\ln(x)+3*\ln(x^2+1)$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {446, 77}

$$-\frac{1}{2x^2} + 3 \log(x^2 + 1) - 6 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - 5*x^2)/(x^3*(1 + x^2)), x]$

[Out] $-1/(2*x^2) - 6*\text{Log}[x] + 3*\text{Log}[1 + x^2]$

Rule 77

$\text{Int}[(a_. + (b_.)*(x_.))*((c_. + (d_.)*(x_.))^(n_.))*((e_. + (f_.)*(x_.))^(p_.)), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

$\text{Int}[(x_.)^(m_.)*((a_. + (b_.)*(x_.)^(n_.))^(p_.))*((c_. + (d_.)*(x_.)^(n_.))^(q_.)), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x}], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1-5x^2}{x^3(1+x^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1-5x}{x^2(1+x)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{6}{x} + \frac{6}{1+x} \right) dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} - 6 \log(x) + 3 \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2x^2} + 3 \log(x^2 + 1) - 6 \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(1 - 5*x^2)/(x^3*(1 + x^2)), x]$

[Out] $-1/2*1/x^2 - 6*\text{Log}[x] + 3*\text{Log}[1 + x^2]$

fricas [A] time = 0.67, size = 25, normalized size = 1.25

$$\frac{6x^2 \log(x^2 + 1) - 12x^2 \log(x) - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="fricas")

[Out] 1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2

giac [A] time = 0.38, size = 27, normalized size = 1.35

$$\frac{6x^2 - 1}{2x^2} + 3 \log(x^2 + 1) - 3 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="giac")

[Out] 1/2*(6*x^2 - 1)/x^2 + 3*log(x^2 + 1) - 3*log(x^2)

maple [A] time = 0.01, size = 19, normalized size = 0.95

$$-6 \ln(x) + 3 \ln(x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5*x^2+1)/x^3/(x^2+1),x)

[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)

maxima [A] time = 0.72, size = 20, normalized size = 1.00

$$-\frac{1}{2x^2} + 3 \log(x^2 + 1) - 3 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="maxima")

[Out] -1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)

mupad [B] time = 0.04, size = 18, normalized size = 0.90

$$3 \ln(x^2 + 1) - 6 \ln(x) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(5*x^2 - 1)/(x^3*(x^2 + 1)),x)

[Out] 3*log(x^2 + 1) - 6*log(x) - 1/(2*x^2)

sympy [A] time = 0.11, size = 19, normalized size = 0.95

$$-6 \log(x) + 3 \log(x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x**2+1)/x**3/(x**2+1),x)

[Out] -6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)

$$3.447 \quad \int \frac{2x}{(-1+x)(5+x^2)} dx$$

Optimal. Leaf size=38

$$-\frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(1 - x) + \frac{1}{3} \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

[Out] 1/3*ln(1-x)-1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 801, 635, 203, 260}

$$-\frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(1 - x) + \frac{1}{3} \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2*x)/((-1 + x)*(5 + x^2)), x]

[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{2x}{(-1+x)(5+x^2)} dx &= 2 \int \frac{x}{(-1+x)(5+x^2)} dx \\
&= 2 \int \left(\frac{1}{6(-1+x)} + \frac{5-x}{6(5+x^2)} \right) dx \\
&= \frac{1}{3} \log(1-x) + \frac{1}{3} \int \frac{5-x}{5+x^2} dx \\
&= \frac{1}{3} \log(1-x) - \frac{1}{3} \int \frac{x}{5+x^2} dx + \frac{5}{3} \int \frac{1}{5+x^2} dx \\
&= \frac{1}{3} \sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{3} \log(1-x) - \frac{1}{6} \log(5+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.05

$$2 \left(-\frac{1}{12} \log(x^2 + 5) + \frac{1}{6} \log(1-x) + \frac{1}{6} \sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x)/((-1 + x)*(5 + x^2)), x]

[Out] 2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12)

fricas [A] time = 0.66, size = 27, normalized size = 0.71

$$\frac{1}{3} \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} x \right) - \frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x/(-1+x)/(x^2+5), x, algorithm="fricas")

[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)

giac [A] time = 0.39, size = 28, normalized size = 0.74

$$\frac{1}{3} \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} x \right) - \frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x/(-1+x)/(x^2+5), x, algorithm="giac")

[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(abs(x - 1))

maple [A] time = 0.00, size = 28, normalized size = 0.74

$$\frac{\sqrt{5} \arctan \left(\frac{\sqrt{5} x}{5} \right)}{3} + \frac{\ln(x - 1)}{3} - \frac{\ln(x^2 + 5)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*x/(x-1)/(x^2+5), x)

[Out] 1/3*ln(x-1)-1/6*ln(x^2+5)+1/3*5^(1/2)*arctan(1/5*5^(1/2)*x)

maxima [A] time = 1.97, size = 27, normalized size = 0.71

$$\frac{1}{3} \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} x \right) - \frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="maxima")

[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)

mupad [B] time = 0.16, size = 44, normalized size = 1.16

$$\frac{\ln(x-1)}{3} - \ln(x - \sqrt{5} i) \left(\frac{1}{6} + \frac{\sqrt{5} i}{6} \right) + \ln(x + \sqrt{5} i) \left(-\frac{1}{6} + \frac{\sqrt{5} i}{6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x)/((x^2 + 5)*(x - 1)),x)

[Out] log(x - 1)/3 - log(x - 5^(1/2)*1i)*((5^(1/2)*1i)/6 + 1/6) + log(x + 5^(1/2)*1i)*((5^(1/2)*1i)/6 - 1/6)

sympy [A] time = 0.14, size = 31, normalized size = 0.82

$$\frac{\log(x-1)}{3} - \frac{\log(x^2+5)}{6} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x/(-1+x)/(x**2+5),x)

[Out] log(x - 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3

$$3.448 \quad \int \frac{2+x^2}{2+x} dx$$

Optimal. Leaf size=17

$$\frac{x^2}{2} - 2x + 6 \log(x + 2)$$

[Out] -2*x+1/2*x^2+6*ln(2+x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {697}

$$\frac{x^2}{2} - 2x + 6 \log(x + 2)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2)/(2 + x), x]

[Out] -2*x + x^2/2 + 6*Log[2 + x]

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{2+x^2}{2+x} dx &= \int \left(-2 + x + \frac{6}{2+x} \right) dx \\ &= -2x + \frac{x^2}{2} + 6 \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.06

$$\frac{x^2}{2} - 2x + 6 \log(x + 2) - 6$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2)/(2 + x), x]

[Out] -6 - 2*x + x^2/2 + 6*Log[2 + x]

fricas [A] time = 0.58, size = 15, normalized size = 0.88

$$\frac{1}{2} x^2 - 2x + 6 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(2+x), x, algorithm="fricas")

[Out] 1/2*x^2 - 2*x + 6*log(x + 2)

giac [A] time = 0.37, size = 16, normalized size = 0.94

$$\frac{1}{2} x^2 - 2x + 6 \log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(2+x),x, algorithm="giac")

[Out] 1/2*x^2 - 2*x + 6*log(abs(x + 2))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^2}{2} - 2x + 6 \ln(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2)/(x+2),x)

[Out] -2*x+1/2*x^2+6*ln(x+2)

maxima [A] time = 0.44, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - 2x + 6 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(2+x),x, algorithm="maxima")

[Out] 1/2*x^2 - 2*x + 6*log(x + 2)

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$6 \ln(x + 2) - 2x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 2)/(x + 2),x)

[Out] 6*log(x + 2) - 2*x + x^2/2

sympy [A] time = 0.07, size = 14, normalized size = 0.82

$$\frac{x^2}{2} - 2x + 6 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2)/(2+x),x)

[Out] x**2/2 - 2*x + 6*log(x + 2)

$$3.449 \quad \int \frac{1}{(-3+x)(4+x^2)} dx$$

Optimal. Leaf size=31

$$-\frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(3 - x) - \frac{3}{26} \tan^{-1}\left(\frac{x}{2}\right)$$

[Out] -3/26*arctan(1/2*x)+1/13*ln(3-x)-1/26*ln(x^2+4)

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(3 - x) - \frac{3}{26} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((-3 + x)*(4 + x^2)), x]

[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-3+x)(4+x^2)} dx &= \frac{1}{13} \int \frac{1}{-3+x} dx + \frac{1}{13} \int \frac{-3-x}{4+x^2} dx \\ &= \frac{1}{13} \log(3-x) - \frac{1}{13} \int \frac{x}{4+x^2} dx - \frac{3}{13} \int \frac{1}{4+x^2} dx \\ &= -\frac{3}{26} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{13} \log(3-x) - \frac{1}{26} \log(4+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.16

$$-\frac{1}{26} \log((x-3)^2 + 6(x-3) + 13) + \frac{1}{13} \log(x-3) - \frac{3}{26} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-3 + x)*(4 + x^2)), x]

[Out] (-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/13

fricas [A] time = 0.47, size = 21, normalized size = 0.68

$$-\frac{3}{26} \arctan\left(\frac{1}{2}x\right) - \frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3+x)/(x^2+4), x, algorithm="fricas")

[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)

giac [A] time = 0.27, size = 22, normalized size = 0.71

$$-\frac{3}{26} \arctan\left(\frac{1}{2}x\right) - \frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3+x)/(x^2+4), x, algorithm="giac")

[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(abs(x - 3))

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$-\frac{3 \arctan\left(\frac{x}{2}\right)}{26} + \frac{\ln(x-3)}{13} - \frac{\ln(x^2+4)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x-3)/(x^2+4), x)

[Out] -1/26*ln(x^2+4)-3/26*arctan(1/2*x)+1/13*ln(x-3)

maxima [A] time = 0.99, size = 21, normalized size = 0.68

$$-\frac{3}{26} \arctan\left(\frac{1}{2}x\right) - \frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3+x)/(x^2+4), x, algorithm="maxima")

[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)

mupad [B] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln(x-3)}{13} + \ln(x-2i) \left(-\frac{1}{26} + \frac{3}{52}i\right) + \ln(x+2i) \left(-\frac{1}{26} - \frac{3}{52}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 4)*(x - 3)), x)

[Out] $\log(x - 3)/13 - \log(x - 2i)*(1/26 - 3i/52) - \log(x + 2i)*(1/26 + 3i/52)$

sympy [A] time = 0.14, size = 22, normalized size = 0.71

$$\frac{\log(x - 3)}{13} - \frac{\log(x^2 + 4)}{26} - \frac{3 \operatorname{atan}\left(\frac{x}{2}\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+x)/(x**2+4),x)`

[Out] $\log(x - 3)/13 - \log(x^2 + 4)/26 - 3*\operatorname{atan}(x/2)/26$

$$3.450 \quad \int \frac{-2+3x^6}{x(5+2x^6)} dx$$

Optimal. Leaf size=19

$$\frac{19}{60} \log(2x^6 + 5) - \frac{2 \log(x)}{5}$$

[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 72}

$$\frac{19}{60} \log(2x^6 + 5) - \frac{2 \log(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]

[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60

Rule 72

Int[((e_.) + (f_.)*(x_)^(p_.))/((a_.) + (b_.)*(x_)*)((c_.) + (d_.)*(x_))], x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{-2+3x^6}{x(5+2x^6)} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{-2+3x}{x(5+2x)} dx, x, x^6 \right) \\ &= \frac{1}{6} \text{Subst} \left(\int \left(-\frac{2}{5x} + \frac{19}{5(5+2x)} \right) dx, x, x^6 \right) \\ &= -\frac{2 \log(x)}{5} + \frac{19}{60} \log(5+2x^6) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{19}{60} \log(2x^6 + 5) - \frac{2 \log(x)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]

[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60

fricas [A] time = 0.48, size = 15, normalized size = 0.79

$$\frac{19}{60} \log(2x^6 + 5) - \frac{2}{5} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="fricas")

[Out] 19/60*log(2*x^6 + 5) - 2/5*log(x)

giac [A] time = 0.41, size = 17, normalized size = 0.89

$$\frac{19}{60} \log(2x^6 + 5) - \frac{1}{15} \log(x^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="giac")

[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{2 \ln(x)}{5} + \frac{19 \ln(2x^6 + 5)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^6-2)/x/(2*x^6+5),x)

[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)

maxima [A] time = 0.47, size = 17, normalized size = 0.89

$$\frac{19}{60} \log(2x^6 + 5) - \frac{1}{15} \log(x^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")

[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)

mupad [B] time = 0.09, size = 13, normalized size = 0.68

$$\frac{19 \ln\left(x^6 + \frac{5}{2}\right)}{60} - \frac{2 \ln(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^6 - 2)/(x*(2*x^6 + 5)),x)

[Out] (19*log(x^6 + 5/2))/60 - (2*log(x))/5

sympy [A] time = 0.11, size = 17, normalized size = 0.89

$$-\frac{2 \log(x)}{5} + \frac{19 \log(2x^6 + 5)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**6-2)/x/(2*x**6+5),x)

[Out] -2*log(x)/5 + 19*log(2*x**6 + 5)/60

$$3.451 \quad \int \frac{3+2x}{(-2+x)(5+x)} dx$$

Optimal. Leaf size=11

$$\log(2-x) + \log(x+5)$$

[Out] $\ln(2-x)+\ln(5+x)$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {72}

$$\log(2-x) + \log(x+5)$$

Antiderivative was successfully verified.

[In] `Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]`

[Out] `Log[2 - x] + Log[5 + x]`

Rule 72

`Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

Rubi steps

$$\int \frac{3+2x}{(-2+x)(5+x)} dx = \int \left(\frac{1}{-2+x} + \frac{1}{5+x} \right) dx = \log(2-x) + \log(5+x)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$\log(x-2) + \log(x+5)$$

Antiderivative was successfully verified.

[In] `Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]`

[Out] `Log[-2 + x] + Log[5 + x]`

fricas [A] time = 0.58, size = 9, normalized size = 0.82

$$\log(x^2 + 3x - 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")`

[Out] `log(x^2 + 3*x - 10)`

giac [A] time = 0.29, size = 11, normalized size = 1.00

$$\log(|x+5|) + \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")`

[Out] $\log(\text{abs}(x + 5)) + \log(\text{abs}(x - 2))$

maple [A] time = 0.00, size = 9, normalized size = 0.82

$$\ln((x - 2)(x + 5))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+3)/(x-2)/(x+5),x)`

[Out] $\ln((x-2)*(x+5))$

maxima [A] time = 0.45, size = 9, normalized size = 0.82

$$\log(x + 5) + \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")`

[Out] $\log(x + 5) + \log(x - 2)$

mupad [B] time = 2.22, size = 9, normalized size = 0.82

$$\ln(x^2 + 3x - 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 3)/((x - 2)*(x + 5)),x)`

[Out] $\log(3*x + x^2 - 10)$

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$\log(x^2 + 3x - 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+2*x)/(-2+x)/(5+x),x)`

[Out] $\log(x**2 + 3*x - 10)$

$$3.452 \quad \int \frac{x^4}{4+5x^2+x^4} dx$$

Optimal. Leaf size=18

$$x - \frac{8}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3} \tan^{-1}(x)$$

[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1122, 1166, 203}

$$x - \frac{8}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^4/(4 + 5*x^2 + x^4), x]

[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{4+5x^2+x^4} dx &= x - \int \frac{4+5x^2}{4+5x^2+x^4} dx \\ &= x + \frac{1}{3} \int \frac{1}{1+x^2} dx - \frac{16}{3} \int \frac{1}{4+x^2} dx \\ &= x - \frac{8}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$x + \frac{8}{3} \tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(4 + 5*x^2 + x^4), x]

[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3

fricas [A] time = 0.64, size = 12, normalized size = 0.67

$$x - \frac{8}{3} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^4+5*x^2+4), x, algorithm="fricas")

[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)

giac [A] time = 0.37, size = 12, normalized size = 0.67

$$x - \frac{8}{3} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^4+5*x^2+4), x, algorithm="giac")

[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)

maple [A] time = 0.01, size = 13, normalized size = 0.72

$$x + \frac{\arctan(x)}{3} - \frac{8 \arctan\left(\frac{x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^4+5*x^2+4), x)

[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)

maxima [A] time = 1.01, size = 12, normalized size = 0.67

$$x - \frac{8}{3} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^4+5*x^2+4), x, algorithm="maxima")

[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)

mupad [B] time = 2.22, size = 12, normalized size = 0.67

$$x - \frac{8 \operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(5*x^2 + x^4 + 4), x)

[Out] x - (8*atan(x/2))/3 + atan(x)/3

sympy [A] time = 0.14, size = 14, normalized size = 0.78

$$x - \frac{8 \operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
```

```
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```


$$3.453 \quad \int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$$

Optimal. Leaf size=46

$$\frac{1}{x+2} + \frac{5}{4(x+3)} + \frac{1}{4(x+3)^2} + \frac{1}{8} \log(x+1) + 2 \log(x+2) - \frac{17}{8} \log(x+3)$$

[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {88}

$$\frac{1}{x+2} + \frac{5}{4(x+3)} + \frac{1}{4(x+3)^2} + \frac{1}{8} \log(x+1) + 2 \log(x+2) - \frac{17}{8} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[1/((1+x)*(2+x)^2*(3+x)^3),x]

[Out] (2+x)^(-1) + 1/(4*(3+x)^2) + 5/(4*(3+x)) + Log[1+x]/8 + 2*Log[2+x] - (17*Log[3+x])/8

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx &= \int \left(\frac{1}{8(1+x)} - \frac{1}{(2+x)^2} + \frac{2}{2+x} - \frac{1}{2(3+x)^3} - \frac{5}{4(3+x)^2} - \frac{17}{8(3+x)} \right) dx \\ &= \frac{1}{2+x} + \frac{1}{4(3+x)^2} + \frac{5}{4(3+x)} + \frac{1}{8} \log(1+x) + 2 \log(2+x) - \frac{17}{8} \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.96

$$\frac{1}{8} \left(\frac{8}{x+2} + \frac{10}{x+3} + \frac{2}{(x+3)^2} + \log(-x-1) + 16 \log(x+2) - 17 \log(x+3) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1+x)*(2+x)^2*(3+x)^3),x]

[Out] (8/(2+x) + 2/(3+x)^2 + 10/(3+x) + Log[-1-x] + 16*Log[2+x] - 17*Log[3+x])/8

fricas [B] time = 0.63, size = 83, normalized size = 1.80

$$\frac{18x^2 - 17(x^3 + 8x^2 + 21x + 18) \log(x+3) + 16(x^3 + 8x^2 + 21x + 18) \log(x+2) + (x^3 + 8x^2 + 21x + 18)}{8(x^3 + 8x^2 + 21x + 18)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")

[Out] $\frac{1}{8}*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*\log(x + 3) + 16*(x^3 + 8*x^2 + 21*x + 18)*\log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*\log(x + 1) + 100*x + 136)/(x^3 + 8*x^2 + 21*x + 18)$

giac [A] time = 0.36, size = 52, normalized size = 1.13

$$\frac{1}{x+2} - \frac{\frac{7}{x+2} + 6}{4\left(\frac{1}{x+2} + 1\right)^2} + \frac{1}{8} \log\left(\left|-\frac{1}{x+2} + 1\right|\right) - \frac{17}{8} \log\left(\left|-\frac{1}{x+2} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")`

[Out] $\frac{1}{x+2} - \frac{1}{4}*\frac{7/(x+2) + 6}{(1/(x+2) + 1)^2} + \frac{1}{8}*\log(\text{abs}(-1/(x+2) + 1)) - \frac{17}{8}*\log(\text{abs}(-1/(x+2) - 1))$

maple [A] time = 0.01, size = 39, normalized size = 0.85

$$\frac{\ln(x+1)}{8} + 2 \ln(x+2) - \frac{17 \ln(x+3)}{8} + \frac{1}{x+2} + \frac{1}{4(x+3)^2} + \frac{5}{4(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x+1)/(x+2)^2/(x+3)^3,x)`

[Out] $\frac{1}{x+2} + \frac{1}{4}*\frac{1}{(x+3)^2} + \frac{5}{4}*\frac{1}{x+3} + \frac{1}{8}*\ln(x+1) + 2*\ln(x+2) - \frac{17}{8}*\ln(x+3)$

maxima [A] time = 0.45, size = 46, normalized size = 1.00

$$\frac{9x^2 + 50x + 68}{4(x^3 + 8x^2 + 21x + 18)} - \frac{17}{8} \log(x+3) + 2 \log(x+2) + \frac{1}{8} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")`

[Out] $\frac{1}{4}*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - \frac{17}{8}*\log(x + 3) + 2*\log(x + 2) + \frac{1}{8}*\log(x + 1)$

mupad [B] time = 0.04, size = 45, normalized size = 0.98

$$\frac{\ln(x+1)}{8} + 2 \ln(x+2) - \frac{17 \ln(x+3)}{8} + \frac{\frac{9x^2}{4} + \frac{25x}{2} + 17}{x^3 + 8x^2 + 21x + 18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x+1)*(x+2)^2*(x+3)^3),x)`

[Out] $\frac{\log(x+1)}{8} + 2*\log(x+2) - \frac{17*\log(x+3)}{8} + \frac{((25*x)/2 + (9*x^2)/4 + 17)/(21*x + 8*x^2 + x^3 + 18)}$

sympy [A] time = 0.19, size = 46, normalized size = 1.00

$$\frac{9x^2 + 50x + 68}{4x^3 + 32x^2 + 84x + 72} + \frac{\log(x+1)}{8} + 2 \log(x+2) - \frac{17 \log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)`

[Out] $\frac{(9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + \log(x + 1)/8 + 2*\log(x + 2) - 17*\log(x + 3)/8}$

$$3.454 \quad \int \frac{x}{-1+x^2} dx$$

Optimal. Leaf size=12

$$\frac{1}{2} \log(1-x^2)$$

[Out] 1/2*ln(-x^2+1)

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {260}

$$\frac{1}{2} \log(1-x^2)$$

Antiderivative was successfully verified.

[In] Int[x/(-1 + x^2),x]

[Out] Log[1 - x^2]/2

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\int \frac{x}{-1+x^2} dx = \frac{1}{2} \log(1-x^2)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.83

$$\frac{1}{2} \log(x^2-1)$$

Antiderivative was successfully verified.

[In] Integrate[x/(-1 + x^2),x]

[Out] Log[-1 + x^2]/2

fricas [A] time = 0.65, size = 8, normalized size = 0.67

$$\frac{1}{2} \log(x^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-1),x, algorithm="fricas")

[Out] 1/2*log(x^2 - 1)

giac [A] time = 0.27, size = 9, normalized size = 0.75

$$\frac{1}{2} \log(|x^2-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-1),x, algorithm="giac")

[Out] $1/2*\log(\text{abs}(x^2 - 1))$

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$\frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^2-1),x)`

[Out] $1/2*\ln(x-1)+1/2*\ln(x+1)$

maxima [A] time = 0.44, size = 8, normalized size = 0.67

$$\frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x^2-1),x, algorithm="maxima")`

[Out] $1/2*\log(x^2 - 1)$

mupad [B] time = 0.04, size = 8, normalized size = 0.67

$$\frac{\ln(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^2 - 1),x)`

[Out] $\log(x^2 - 1)/2$

sympy [A] time = 0.08, size = 7, normalized size = 0.58

$$\frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**2-1),x)`

[Out] $\log(x**2 - 1)/2$

$$3.455 \quad \int \frac{1}{(-1+x^2)^2} dx$$

Optimal. Leaf size=21

$$\frac{x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x)$$

[Out] 1/2*x/(-x^2+1)+1/2*arctanh(x)

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {199, 207}

$$\frac{x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)^(-2), x]

[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1+x^2)^2} dx &= \frac{x}{2(1-x^2)} - \frac{1}{2} \int \frac{1}{-1+x^2} dx \\ &= \frac{x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4} \left(-\frac{2x}{x^2-1} - \log(1-x) + \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)^(-2), x]

[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4

fricas [B] time = 0.72, size = 34, normalized size = 1.62

$$\frac{(x^2-1) \log(x+1) - (x^2-1) \log(x-1) - 2x}{4(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")

[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)

giac [A] time = 0.27, size = 25, normalized size = 1.19

$$-\frac{x}{2(x^2-1)} + \frac{1}{4} \log(|x+1|) - \frac{1}{4} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2,x, algorithm="giac")

[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))

maple [A] time = 0.01, size = 28, normalized size = 1.33

$$-\frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{1}{4(x-1)} - \frac{1}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-1)^2,x)

[Out] -1/4/(x-1)-1/4*ln(x-1)-1/4/(x+1)+1/4*ln(x+1)

maxima [A] time = 0.45, size = 23, normalized size = 1.10

$$-\frac{x}{2(x^2-1)} + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")

[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)

mupad [B] time = 2.22, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}(x)}{2} - \frac{x}{2(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 - 1)^2,x)

[Out] atanh(x)/2 - x/(2*(x^2 - 1))

sympy [A] time = 0.11, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-1)**2,x)

[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4

$$3.456 \quad \int \frac{x^2}{(1+x^2)^2} dx$$

Optimal. Leaf size=19

$$\frac{1}{2} \tan^{-1}(x) - \frac{x}{2(x^2 + 1)}$$

[Out] -1/2*x/(x^2+1)+1/2*arctan(x)

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {288, 203}

$$\frac{1}{2} \tan^{-1}(x) - \frac{x}{2(x^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(1+x^2)^2} dx &= -\frac{x}{2(1+x^2)} + \frac{1}{2} \int \frac{1}{1+x^2} dx \\ &= -\frac{x}{2(1+x^2)} + \frac{1}{2} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2} \tan^{-1}(x) - \frac{x}{2(x^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + x^2)^2,x]

[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2

fricas [A] time = 0.55, size = 21, normalized size = 1.11

$$\frac{(x^2 + 1) \arctan(x) - x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)

giac [A] time = 0.29, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2 + 1)} + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")

[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{x}{2(x^2 + 1)} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^2+1)^2,x)

[Out] -1/2/(x^2+1)*x+1/2*arctan(x)

maxima [A] time = 1.00, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2 + 1)} + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")

[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)

mupad [B] time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2} - \frac{x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^2 + 1)^2,x)

[Out] atan(x)/2 - x/(2*(x^2 + 1))

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2 + 2} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**2+1)**2,x)

[Out] -x/(2*x**2 + 2) + atan(x)/2

$$3.457 \quad \int \frac{1}{2+3x} dx$$

Optimal. Leaf size=10

$$\frac{1}{3} \log(3x + 2)$$

[Out] 1/3*ln(2+3*x)

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$\frac{1}{3} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)^(-1), x]

[Out] Log[2 + 3*x]/3

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{1}{2+3x} dx = \frac{1}{3} \log(2+3x)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)^(-1), x]

[Out] Log[2 + 3*x]/3

fricas [A] time = 0.50, size = 8, normalized size = 0.80

$$\frac{1}{3} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*x), x, algorithm="fricas")

[Out] 1/3*log(3*x + 2)

giac [A] time = 0.38, size = 9, normalized size = 0.90

$$\frac{1}{3} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*x), x, algorithm="giac")

[Out] $1/3*\log(\text{abs}(3*x + 2))$

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\ln(3x + 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x+2),x)`

[Out] $1/3*\ln(3*x+2)$

maxima [A] time = 0.45, size = 8, normalized size = 0.80

$$\frac{1}{3} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+3*x),x, algorithm="maxima")`

[Out] $1/3*\log(3*x + 2)$

mupad [B] time = 0.07, size = 6, normalized size = 0.60

$$\frac{\ln\left(x + \frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x + 2),x)`

[Out] $\log(x + 2/3)/3$

sympy [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log(3x + 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+3*x),x)`

[Out] $\log(3*x + 2)/3$

$$3.458 \quad \int \frac{1}{a^2+x^2} dx$$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

[Out] arctan(x/a)/a

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + x^2)^(-1), x]

[Out] ArcTan[x/a]/a

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1}{a^2+x^2} dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + x^2)^(-1), x]

[Out] ArcTan[x/a]/a

fricas [A] time = 0.58, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2), x, algorithm="fricas")

[Out] arctan(x/a)/a

giac [A] time = 0.26, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2),x, algorithm="giac")

[Out] arctan(x/a)/a

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+x^2),x)

[Out] arctan(x/a)/a

maxima [A] time = 0.99, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2),x, algorithm="maxima")

[Out] arctan(x/a)/a

mupad [B] time = 2.25, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + x^2),x)

[Out] atan(x/a)/a

sympy [C] time = 0.11, size = 20, normalized size = 2.00

$$\frac{-\frac{i \log(-ia+x)}{2} + \frac{i \log(ia+x)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+x**2),x)

[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a

$$3.459 \quad \int \frac{1}{a+bx^2} dx$$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{1}{a+bx^2} dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

fricas [A] time = 0.78, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab} \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right)}{2ab}, \frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a), x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]

giac [A] time = 0.38, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a),x, algorithm="giac")

[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a),x)

[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)

maxima [A] time = 0.98, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a),x, algorithm="maxima")

[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)

mupad [B] time = 2.24, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*x^2),x)

[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))

sympy [B] time = 0.13, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}} \log\left(-a\sqrt{-\frac{1}{ab}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}} \log\left(a\sqrt{-\frac{1}{ab}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a),x)

[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1/(a*b)) + x)/2

$$3.460 \quad \int \frac{1}{2-x+x^2} dx$$

Optimal. Leaf size=19

$$-\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

[Out] $-2/7*\arctan(1/7*(1-2*x)*7^{(1/2)})*7^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {618, 204}

$$-\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(2 - x + x^2)^(-1), x]

[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{2-x+x^2} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-7-x^2} dx, x, -1+2x\right)\right) \\ &= -\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x + x^2)^(-1), x]

[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]

fricas [A] time = 0.60, size = 16, normalized size = 0.84

$$\frac{2}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-x+2),x, algorithm="fricas")

[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))

giac [A] time = 0.38, size = 16, normalized size = 0.84

$$\frac{2}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-x+2),x, algorithm="giac")

[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))

maple [A] time = 0.00, size = 17, normalized size = 0.89

$$\frac{2\sqrt{7} \arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-x+2),x)

[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))

maxima [A] time = 0.95, size = 16, normalized size = 0.84

$$\frac{2}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-x+2),x, algorithm="maxima")

[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))

mupad [B] time = 0.03, size = 16, normalized size = 0.84

$$\frac{2\sqrt{7} \operatorname{atan}\left(\frac{\sqrt{7}(2x-1)}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 - x + 2),x)

[Out] (2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7

sympy [A] time = 0.11, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x}{7} - \frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-x+2),x)

[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7

$$3.461 \quad \int x^2 (4 - x^2)^2 dx$$

Optimal. Leaf size=22

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

[Out] 16/3*x^3-8/5*x^5+1/7*x^7

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2*(4 - x^2)^2,x]

[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x^2 (4 - x^2)^2 dx &= \int (16x^2 - 8x^4 + x^6) dx \\ &= \frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(4 - x^2)^2,x]

[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7

fricas [A] time = 0.41, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3

giac [A] time = 0.31, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+4)^2,x, algorithm="giac")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^2+4)^2,x)

[Out] 16/3*x^3-8/5*x^5+1/7*x^7

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3

mupad [B] time = 0.02, size = 17, normalized size = 0.77

$$\frac{x^3 (15x^4 - 168x^2 + 560)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(x^2 - 4)^2,x)

[Out] (x^3*(15*x^4 - 168*x^2 + 560))/105

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**2+4)**2,x)

[Out] x**7/7 - 8*x**5/5 + 16*x**3/3

$$3.462 \quad \int x(1 - x^3)^2 dx$$

Optimal. Leaf size=22

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

[Out] 1/2*x^2-2/5*x^5+1/8*x^8

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x*(1 - x^3)^2,x]

[Out] x^2/2 - (2*x^5)/5 + x^8/8

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x(1 - x^3)^2 dx &= \int (x - 2x^4 + x^7) dx \\ &= \frac{x^2}{2} - \frac{2x^5}{5} + \frac{x^8}{8} \end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 - x^3)^2,x]

[Out] x^2/2 - (2*x^5)/5 + x^8/8

fricas [A] time = 0.51, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)^2,x, algorithm="fricas")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2

giac [A] time = 0.29, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)^2,x, algorithm="giac")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^3+1)^2,x)

[Out] 1/2*x^2-2/5*x^5+1/8*x^8

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)^2,x, algorithm="maxima")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{x^2 (5x^6 - 16x^3 + 20)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^3 - 1)^2,x)

[Out] (x^2*(5*x^6 - 16*x^3 + 20))/40

sympy [A] time = 0.06, size = 15, normalized size = 0.68

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**3+1)**2,x)

[Out] x**8/8 - 2*x**5/5 + x**2/2

$$3.463 \quad \int \frac{-4+5x^2+x^3}{x^2} dx$$

Optimal. Leaf size=16

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

[Out] 4/x+5*x+1/2*x^2

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {14}

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Antiderivative was successfully verified.

[In] Int[(-4 + 5*x^2 + x^3)/x^2,x]

[Out] 4/x + 5*x + x^2/2

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-4 + 5x^2 + x^3}{x^2} dx &= \int \left(5 - \frac{4}{x^2} + x \right) dx \\ &= \frac{4}{x} + 5x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 5*x^2 + x^3)/x^2,x]

[Out] 4/x + 5*x + x^2/2

fricas [A] time = 0.72, size = 15, normalized size = 0.94

$$\frac{x^3 + 10x^2 + 8}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")

[Out] 1/2*(x^3 + 10*x^2 + 8)/x

giac [A] time = 0.31, size = 14, normalized size = 0.88

$$\frac{1}{2}x^2 + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")

[Out] 1/2*x^2 + 5*x + 4/x

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+5*x^2-4)/x^2,x)

[Out] 4/x+5*x+1/2*x^2

maxima [A] time = 0.46, size = 14, normalized size = 0.88

$$\frac{1}{2}x^2 + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")

[Out] 1/2*x^2 + 5*x + 4/x

mupad [B] time = 0.03, size = 15, normalized size = 0.94

$$\frac{x^3 + 10x^2 + 8}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2 + x^3 - 4)/x^2,x)

[Out] (10*x^2 + x^3 + 8)/(2*x)

sympy [A] time = 0.07, size = 10, normalized size = 0.62

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+5*x**2-4)/x**2,x)

[Out] x**2/2 + 5*x + 4/x

$$3.464 \quad \int \frac{-1+x}{3-4x+3x^2} dx$$

Optimal. Leaf size=37

$$\frac{1}{6} \log(3x^2 - 4x + 3) + \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

[Out] 1/6*ln(3*x^2-4*x+3)+1/15*arctan(1/5*(2-3*x)*5^(1/2))*5^(1/2)

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {634, 618, 204, 628}

$$\frac{1}{6} \log(3x^2 - 4x + 3) + \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/(3 - 4*x + 3*x^2), x]

[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{-1+x}{3-4x+3x^2} dx &= \frac{1}{6} \int \frac{-4+6x}{3-4x+3x^2} dx - \frac{1}{3} \int \frac{1}{3-4x+3x^2} dx \\ &= \frac{1}{6} \log(3-4x+3x^2) + \frac{2}{3} \text{Subst}\left(\int \frac{1}{-20-x^2} dx, x, -4+6x\right) \\ &= \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}} + \frac{1}{6} \log(3-4x+3x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{6} \log(3x^2 - 4x + 3) - \frac{\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x)/(3 - 4*x + 3*x^2), x]

[Out] -1/3*ArcTan[(-2 + 3*x)/Sqrt[5]]/Sqrt[5] + Log[3 - 4*x + 3*x^2]/6

fricas [A] time = 0.60, size = 30, normalized size = 0.81

$$-\frac{1}{15} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} (3x - 2)\right) + \frac{1}{6} \log(3x^2 - 4x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(3*x^2-4*x+3), x, algorithm="fricas")

[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)

giac [A] time = 0.27, size = 30, normalized size = 0.81

$$-\frac{1}{15} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} (3x - 2)\right) + \frac{1}{6} \log(3x^2 - 4x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(3*x^2-4*x+3), x, algorithm="giac")

[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{5} \arctan\left(\frac{(6x-4)\sqrt{5}}{10}\right)}{15} + \frac{\ln(3x^2 - 4x + 3)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-1)/(3*x^2-4*x+3), x)

[Out] 1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))

maxima [A] time = 0.98, size = 30, normalized size = 0.81

$$-\frac{1}{15} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} (3x - 2)\right) + \frac{1}{6} \log(3x^2 - 4x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(3*x^2-4*x+3), x, algorithm="maxima")

[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)

mupad [B] time = 2.22, size = 30, normalized size = 0.81

$$\frac{\ln\left(x^2 - \frac{4x}{3} + 1\right)}{6} - \frac{\sqrt{5} \operatorname{atan}\left(\frac{3\sqrt{5}x}{5} - \frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - 1)/(3*x^2 - 4*x + 3), x)`

[Out] $\log(x^2 - (4x)/3 + 1)/6 - (5^{(1/2)} \operatorname{atan}((3 \cdot 5^{(1/2)} x)/5 - (2 \cdot 5^{(1/2)})/5))/15$

sympy [A] time = 0.12, size = 39, normalized size = 1.05

$$\frac{\log\left(x^2 - \frac{4x}{3} + 1\right)}{6} - \frac{\sqrt{5} \operatorname{atan}\left(\frac{3\sqrt{5}x}{5} - \frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)/(3*x**2-4*x+3), x)`

[Out] $\log(x^2 - 4x/3 + 1)/6 - \operatorname{sqrt}(5) \operatorname{atan}(3 \operatorname{sqrt}(5)x/5 - 2 \operatorname{sqrt}(5)/5)/15$

3.465 $\int (2 + x^3)^2 dx$

Optimal. Leaf size=14

$$\frac{x^7}{7} + x^4 + 4x$$

[Out] 4*x+x^4+1/7*x^7

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {194}

$$\frac{x^7}{7} + x^4 + 4x$$

Antiderivative was successfully verified.

[In] Int[(2 + x^3)^2,x]

[Out] 4*x + x^4 + x^7/7

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (2 + x^3)^2 dx &= \int (4 + 4x^3 + x^6) dx \\ &= 4x + x^4 + \frac{x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x^7}{7} + x^4 + 4x$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^3)^2,x]

[Out] 4*x + x^4 + x^7/7

fricas [A] time = 0.49, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2)^2,x, algorithm="fricas")

[Out] 1/7*x^7 + x^4 + 4*x

giac [A] time = 0.36, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2)^2,x, algorithm="giac")

[Out] 1/7*x^7 + x^4 + 4*x

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+2)^2,x)

[Out] 4*x+x^4+1/7*x^7

maxima [A] time = 0.47, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2)^2,x, algorithm="maxima")

[Out] 1/7*x^7 + x^4 + 4*x

mupad [B] time = 0.02, size = 13, normalized size = 0.93

$$\frac{x(x^6 + 7x^3 + 28)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 2)^2,x)

[Out] (x*(7*x^3 + x^6 + 28))/7

sympy [A] time = 0.05, size = 10, normalized size = 0.71

$$\frac{x^7}{7} + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+2)**2,x)

[Out] x**7/7 + x**4 + 4*x

$$3.466 \quad \int \frac{-4+x^2}{2+x} dx$$

Optimal. Leaf size=11

$$\frac{x^2}{2} - 2x$$

[Out] -2*x+1/2*x^2

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {627}

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

[In] Int[(-4 + x^2)/(2 + x), x]

[Out] -2*x + x^2/2

Rule 627

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))

Rubi steps

$$\begin{aligned} \int \frac{-4+x^2}{2+x} dx &= \int (-2+x) dx \\ &= -2x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + x^2)/(2 + x), x]

[Out] -2*x + x^2/2

fricas [A] time = 0.60, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(2+x), x, algorithm="fricas")

[Out] 1/2*x^2 - 2*x

giac [A] time = 0.29, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(2+x),x, algorithm="giac")

[Out] 1/2*x^2 - 2*x

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-4)/(x+2),x)

[Out] -2*x+1/2*x^2

maxima [A] time = 0.45, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(2+x),x, algorithm="maxima")

[Out] 1/2*x^2 - 2*x

mupad [B] time = 0.02, size = 6, normalized size = 0.55

$$\frac{x(x-4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 4)/(x + 2),x)

[Out] (x*(x - 4))/2

sympy [A] time = 0.06, size = 7, normalized size = 0.64

$$\frac{x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-4)/(2+x),x)

[Out] x**2/2 - 2*x

$$3.467 \quad \int \frac{1}{(2+x)(1+x^2)} dx$$

Optimal. Leaf size=25

$$-\frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2) + \frac{2}{5} \tan^{-1}(x)$$

[Out] 2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2) + \frac{2}{5} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/((2 + x)*(1 + x^2)),x]

[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*xⁿ, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(2+x)(1+x^2)} dx &= \frac{1}{5} \int \frac{1}{2+x} dx + \frac{1}{5} \int \frac{2-x}{1+x^2} dx \\ &= \frac{1}{5} \log(2+x) - \frac{1}{5} \int \frac{x}{1+x^2} dx + \frac{2}{5} \int \frac{1}{1+x^2} dx \\ &= \frac{2}{5} \tan^{-1}(x) + \frac{1}{5} \log(2+x) - \frac{1}{10} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2) + \frac{2}{5} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((2 + x)*(1 + x^2)), x]

[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10

fricas [A] time = 0.69, size = 19, normalized size = 0.76

$$\frac{2}{5} \arctan(x) - \frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)/(x^2+1), x, algorithm="fricas")

[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)

giac [A] time = 0.37, size = 20, normalized size = 0.80

$$\frac{2}{5} \arctan(x) - \frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)/(x^2+1), x, algorithm="giac")

[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{2 \arctan(x)}{5} + \frac{\ln(x + 2)}{5} - \frac{\ln(x^2 + 1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+2)/(x^2+1), x)

[Out] 2/5*arctan(x)+1/5*ln(x+2)-1/10*ln(x^2+1)

maxima [A] time = 0.99, size = 19, normalized size = 0.76

$$\frac{2}{5} \arctan(x) - \frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)/(x^2+1), x, algorithm="maxima")

[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)

mupad [B] time = 0.05, size = 25, normalized size = 1.00

$$\frac{\ln(x + 2)}{5} + \ln(x - i) \left(-\frac{1}{10} - \frac{1}{5}i \right) + \ln(x + 1i) \left(-\frac{1}{10} + \frac{1}{5}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 1)*(x + 2)), x)

[Out] log(x + 2)/5 - log(x - 1i)*(1/10 + 1i/5) - log(x + 1i)*(1/10 - 1i/5)

sympy [A] time = 0.14, size = 20, normalized size = 0.80

$$\frac{\log(x+2)}{5} - \frac{\log(x^2+1)}{10} + \frac{2 \operatorname{atan}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)/(x**2+1),x)

[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5

$$3.468 \quad \int \frac{1}{(1+x)(1+x^2)} dx$$

Optimal. Leaf size=25

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x)(1+x^2)} dx &= \frac{1}{2} \int \frac{1}{1+x} dx + \frac{1}{2} \int \frac{1-x}{1+x^2} dx \\ &= \frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1}{1+x^2} dx - \frac{1}{2} \int \frac{x}{1+x^2} dx \\ &= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x) - \frac{1}{4} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4

fricas [A] time = 0.63, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)

giac [A] time = 0.29, size = 20, normalized size = 0.80

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+1),x, algorithm="giac")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} + \frac{\ln(x + 1)}{2} - \frac{\ln(x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+1)/(x^2+1),x)

[Out] 1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)

maxima [A] time = 1.36, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)

mupad [B] time = 2.23, size = 25, normalized size = 1.00

$$\frac{\ln(x + 1)}{2} + \ln(x - i) \left(-\frac{1}{4} - \frac{1}{4}i \right) + \ln(x + 1i) \left(-\frac{1}{4} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 1)*(x + 1)),x)

[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)

sympy [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x**2+1),x)

[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2

$$3.469 \quad \int \frac{x}{(1+x)(1+x^2)} dx$$

Optimal. Leaf size=25

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

[Out] 1/2*arctan(x)-1/2*ln(1+x)+1/4*ln(x^2+1)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {801, 635, 203, 260}

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x)(1+x^2)} dx &= \int \left(-\frac{1}{2(1+x)} + \frac{1+x}{2(1+x^2)} \right) dx \\ &= -\frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1+x}{1+x^2} dx \\ &= -\frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{x}{1+x^2} dx \\ &= \frac{1}{2} \tan^{-1}(x) - \frac{1}{2} \log(1+x) + \frac{1}{4} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x/((1 + x)*(1 + x^2)), x]

[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4

fricas [A] time = 0.80, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(x^2+1), x, algorithm="fricas")

[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)

giac [A] time = 0.25, size = 20, normalized size = 0.80

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(x^2+1), x, algorithm="giac")

[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1))

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} - \frac{\ln(x + 1)}{2} + \frac{\ln(x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+1)/(x^2+1), x)

[Out] 1/2*arctan(x)-1/2*ln(x+1)+1/4*ln(x^2+1)

maxima [A] time = 1.43, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(x^2+1), x, algorithm="maxima")

[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)

mupad [B] time = 2.21, size = 25, normalized size = 1.00

$$-\frac{\ln(x + 1)}{2} + \ln(x - i) \left(\frac{1}{4} - \frac{1}{4}i \right) + \ln(x + 1i) \left(\frac{1}{4} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x^2 + 1)*(x + 1)), x)

[Out] log(x - 1i)*(1/4 - 1i/4) - log(x + 1)/2 + log(x + 1i)*(1/4 + 1i/4)

sympy [A] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{\log(x+1)}{2} + \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(x**2+1),x)

[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2

$$3.470 \quad \int \frac{2x+x^2}{(1+x)^2} dx$$

Optimal. Leaf size=9

$$\frac{x^2}{x+1}$$

[Out] x^2/(1+x)

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {683}

$$x + \frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^2)/(1 + x)^2,x]

[Out] x + (1 + x)^(-1)

Rule 683

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_ Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])

Rubi steps

$$\begin{aligned} \int \frac{2x+x^2}{(1+x)^2} dx &= \int \left(1 - \frac{1}{(1+x)^2}\right) dx \\ &= x + \frac{1}{1+x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 0.78

$$x + \frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^2)/(1 + x)^2,x]

[Out] x + (1 + x)^(-1)

fricas [A] time = 0.54, size = 12, normalized size = 1.33

$$\frac{x^2 + x + 1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")

[Out] (x^2 + x + 1)/(x + 1)

giac [A] time = 0.30, size = 8, normalized size = 0.89

$$x + \frac{1}{x+1} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")

[Out] x + 1/(x + 1) + 1

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2*x)/(x+1)^2,x)

[Out] x+1/(x+1)

maxima [A] time = 0.95, size = 7, normalized size = 0.78

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")

[Out] x + 1/(x + 1)

mupad [B] time = 0.02, size = 7, normalized size = 0.78

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^2)/(x + 1)^2,x)

[Out] x + 1/(x + 1)

sympy [A] time = 0.07, size = 5, normalized size = 0.56

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2*x)/(1+x)**2,x)

[Out] x + 1/(x + 1)

$$3.471 \quad \int \frac{-10+x^2}{4+9x^2+2x^4} dx$$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{2}\right) - \frac{3 \tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

[Out] arctan(1/2*x)-3/2*arctan(x*2^(1/2))*2^(1/2)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1166, 203}

$$\tan^{-1}\left(\frac{x}{2}\right) - \frac{3 \tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4), x]

[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{-10+x^2}{4+9x^2+2x^4} dx &= -\left(3 \int \frac{1}{1+2x^2} dx\right) + 4 \int \frac{1}{8+2x^2} dx \\ &= \tan^{-1}\left(\frac{x}{2}\right) - \frac{3 \tan^{-1}(\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{2}\right) - \frac{3 \tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4), x]

[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]

fricas [A] time = 0.48, size = 16, normalized size = 0.73

$$-\frac{3}{2} \sqrt{2} \arctan(\sqrt{2}x) + \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="fricas")

[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)

giac [A] time = 0.29, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan(\sqrt{2}x) + \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="giac")

[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)

maple [A] time = 0.01, size = 17, normalized size = 0.77

$$-\frac{3\sqrt{2}\arctan(\sqrt{2}x)}{2} + \arctan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-10)/(2*x^4+9*x^2+4),x)

[Out] arctan(1/2*x)-3/2*2^(1/2)*arctan(2^(1/2)*x)

maxima [A] time = 1.58, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan(\sqrt{2}x) + \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="maxima")

[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)

mupad [B] time = 0.05, size = 16, normalized size = 0.73

$$\operatorname{atan}\left(\frac{x}{2}\right) - \frac{3\sqrt{2}\operatorname{atan}(\sqrt{2}x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 10)/(9*x^2 + 2*x^4 + 4),x)

[Out] atan(x/2) - (3*2^(1/2)*atan(2^(1/2)*x))/2

sympy [A] time = 0.15, size = 20, normalized size = 0.91

$$\operatorname{atan}\left(\frac{x}{2}\right) - \frac{3\sqrt{2}\operatorname{atan}(\sqrt{2}x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-10)/(2*x**4+9*x**2+4),x)

[Out] atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2

$$3.472 \quad \int \frac{31+5x}{11-4x+3x^2} dx$$

Optimal. Leaf size=37

$$\frac{5}{6} \log(3x^2 - 4x + 11) - \frac{103 \tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

[Out] 5/6*ln(3*x^2-4*x+11)-103/87*arctan(1/29*(2-3*x)*29^(1/2))*29^(1/2)

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {634, 618, 204, 628}

$$\frac{5}{6} \log(3x^2 - 4x + 11) - \frac{103 \tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

[In] Int[(31 + 5*x)/(11 - 4*x + 3*x^2), x]

[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{31+5x}{11-4x+3x^2} dx &= \frac{5}{6} \int \frac{-4+6x}{11-4x+3x^2} dx + \frac{103}{3} \int \frac{1}{11-4x+3x^2} dx \\ &= \frac{5}{6} \log(11-4x+3x^2) - \frac{206}{3} \text{Subst}\left(\int \frac{1}{-116-x^2} dx, x, -4+6x\right) \\ &= -\frac{103 \tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}} + \frac{5}{6} \log(11-4x+3x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{5}{6} \log(3x^2 - 4x + 11) + \frac{103 \tan^{-1}\left(\frac{3x-2}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

[In] Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2), x]

[Out] (103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/6

fricas [A] time = 0.68, size = 30, normalized size = 0.81

$$\frac{103}{87} \sqrt{29} \arctan\left(\frac{1}{29} \sqrt{29} (3x - 2)\right) + \frac{5}{6} \log(3x^2 - 4x + 11)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((31+5*x)/(3*x^2-4*x+11), x, algorithm="fricas")

[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)

giac [A] time = 0.38, size = 30, normalized size = 0.81

$$\frac{103}{87} \sqrt{29} \arctan\left(\frac{1}{29} \sqrt{29} (3x - 2)\right) + \frac{5}{6} \log(3x^2 - 4x + 11)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((31+5*x)/(3*x^2-4*x+11), x, algorithm="giac")

[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{103\sqrt{29} \arctan\left(\frac{(6x-4)\sqrt{29}}{58}\right)}{87} + \frac{5 \ln(3x^2 - 4x + 11)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((31+5*x)/(3*x^2-4*x+11), x)

[Out] 5/6*ln(3*x^2-4*x+11)+103/87*29^(1/2)*arctan(1/58*(6*x-4)*29^(1/2))

maxima [A] time = 1.39, size = 30, normalized size = 0.81

$$\frac{103}{87} \sqrt{29} \arctan\left(\frac{1}{29} \sqrt{29} (3x - 2)\right) + \frac{5}{6} \log(3x^2 - 4x + 11)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((31+5*x)/(3*x^2-4*x+11), x, algorithm="maxima")

[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)

mupad [B] time = 0.04, size = 30, normalized size = 0.81

$$\frac{5 \ln\left(x^2 - \frac{4x}{3} + \frac{11}{3}\right)}{6} + \frac{103 \sqrt{29} \operatorname{atan}\left(\frac{3\sqrt{29}x}{29} - \frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x + 31)/(3*x^2 - 4*x + 11),x)`

[Out] $(5 \log(x^2 - (4x)/3 + 11/3))/6 + (103 \cdot 29^{(1/2)} \operatorname{atan}((3 \cdot 29^{(1/2)} x)/29 - (2 \cdot 29^{(1/2)})/29))/87$

sympy [A] time = 0.12, size = 44, normalized size = 1.19

$$\frac{5 \log\left(x^2 - \frac{4x}{3} + \frac{11}{3}\right)}{6} + \frac{103\sqrt{29} \operatorname{atan}\left(\frac{3\sqrt{29}x}{29} - \frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((31+5*x)/(3*x**2-4*x+11),x)`

[Out] $5 \log(x^2 - 4x/3 + 11/3)/6 + 103 \cdot \sqrt{29} \operatorname{atan}(3 \cdot \sqrt{29} x/29 - 2 \cdot \sqrt{29}/29)/87$

$$3.473 \quad \int \frac{-2+x^2+x^3}{x^4} dx$$

Optimal. Leaf size=15

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

[Out] 2/3/x^3-1/x+ln(x)

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {14}

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-2 + x^2 + x^3)/x^4,x]

[Out] 2/(3*x^3) - x^(-1) + Log[x]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-2+x^2+x^3}{x^4} dx &= \int \left(-\frac{2}{x^4} + \frac{1}{x^2} + \frac{1}{x} \right) dx \\ &= \frac{2}{3x^3} - \frac{1}{x} + \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + x^2 + x^3)/x^4,x]

[Out] 2/(3*x^3) - x^(-1) + Log[x]

fricas [A] time = 0.50, size = 19, normalized size = 1.27

$$\frac{3x^3 \log(x) - 3x^2 + 2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")

[Out] 1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3

giac [A] time = 0.27, size = 16, normalized size = 1.07

$$-\frac{3x^2 - 2}{3x^3} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-2)/x^4,x, algorithm="giac")

[Out] -1/3*(3*x^2 - 2)/x^3 + log(abs(x))

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x) - \frac{1}{x} + \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2-2)/x^4,x)

[Out] 2/3/x^3-1/x+ln(x)

maxima [A] time = 0.97, size = 15, normalized size = 1.00

$$-\frac{3x^2 - 2}{3x^3} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")

[Out] -1/3*(3*x^2 - 2)/x^3 + log(x)

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\ln(x) - \frac{x^2 - \frac{2}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^3 - 2)/x^4,x)

[Out] log(x) - (x^2 - 2/3)/x^3

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\log(x) + \frac{2 - 3x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2-2)/x**4,x)

[Out] log(x) + (2 - 3*x**2)/(3*x**3)

$$3.474 \quad \int \frac{1+x+x^3}{x^2} dx$$

Optimal. Leaf size=15

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

[Out] $-1/x + 1/2*x^2 + \ln(x)$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {14}

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^3)/x^2,x]

[Out] $-x^{(-1)} + x^2/2 + \text{Log}[x]$

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^3}{x^2} dx &= \int \left(\frac{1}{x^2} + \frac{1}{x} + x \right) dx \\ &= -\frac{1}{x} + \frac{x^2}{2} + \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^3)/x^2,x]

[Out] $-x^{(-1)} + x^2/2 + \text{Log}[x]$

fricas [A] time = 0.65, size = 15, normalized size = 1.00

$$\frac{x^3 + 2x \log(x) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/x^2,x, algorithm="fricas")

[Out] $1/2*(x^3 + 2*x*log(x) - 2)/x$

giac [A] time = 0.28, size = 14, normalized size = 0.93

$$\frac{1}{2}x^2 - \frac{1}{x} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/x^2,x, algorithm="giac")

[Out] 1/2*x^2 - 1/x + log(abs(x))

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{x^2}{2} + \ln(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x+1)/x^2,x)

[Out] -1/x+1/2*x^2+ln(x)

maxima [A] time = 0.95, size = 13, normalized size = 0.87

$$\frac{1}{2}x^2 - \frac{1}{x} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/x^2,x, algorithm="maxima")

[Out] 1/2*x^2 - 1/x + log(x)

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\ln(x) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^3 + 1)/x^2,x)

[Out] log(x) - 1/x + x^2/2

sympy [A] time = 0.08, size = 10, normalized size = 0.67

$$\frac{x^2}{2} + \log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x+1)/x**2,x)

[Out] x**2/2 + log(x) - 1/x

$$3.475 \quad \int \frac{-2+x^2}{x(2+x^2)} dx$$

Optimal. Leaf size=11

$$\log(x^2 + 2) - \log(x)$$

[Out] -ln(x)+ln(x^2+2)

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {446, 72}

$$\log(x^2 + 2) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-2 + x^2)/(x*(2 + x^2)), x]

[Out] -Log[x] + Log[2 + x^2]

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{-2+x^2}{x(2+x^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{-2+x}{x(2+x)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{x} + \frac{2}{2+x} \right) dx, x, x^2 \right) \\ &= -\log(x) + \log(2+x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x^2 + 2) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + x^2)/(x*(2 + x^2)), x]

[Out] -Log[x] + Log[2 + x^2]

fricas [A] time = 0.45, size = 11, normalized size = 1.00

$$\log(x^2 + 2) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")

[Out] $\log(x^2 + 2) - \log(x)$

giac [A] time = 0.29, size = 13, normalized size = 1.18

$$\log(x^2 + 2) - \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")

[Out] $\log(x^2 + 2) - 1/2*\log(x^2)$

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$-\ln(x) + \ln(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-2)/x/(x^2+2),x)

[Out] $-\ln(x) + \ln(x^2 + 2)$

maxima [A] time = 0.69, size = 13, normalized size = 1.18

$$\log(x^2 + 2) - \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")

[Out] $\log(x^2 + 2) - 1/2*\log(x^2)$

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$\ln(x^2 + 2) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 2)/(x*(x^2 + 2)),x)

[Out] $\log(x^2 + 2) - \log(x)$

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\log(x) + \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-2)/x/(x**2+2),x)

[Out] $-\log(x) + \log(x**2 + 2)$

$$3.476 \quad \int (-3 + x)(-7 + 4x^2) dx$$

Optimal. Leaf size=22

$$-4x^3 + \frac{1}{16}(7 - 4x^2)^2 + 21x$$

[Out] 21*x-4*x^3+1/16*(-4*x^2+7)^2

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {641}

$$-4x^3 + \frac{1}{16}(7 - 4x^2)^2 + 21x$$

Antiderivative was successfully verified.

[In] Int[(-3 + x)*(-7 + 4*x^2), x]

[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] / ; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (-3 + x)(-7 + 4x^2) dx &= \frac{1}{16}(7 - 4x^2)^2 - 3 \int (-7 + 4x^2) dx \\ &= 21x - 4x^3 + \frac{1}{16}(7 - 4x^2)^2 \end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.86

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x)*(-7 + 4*x^2), x]

[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4

fricas [A] time = 0.49, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)*(4*x^2-7), x, algorithm="fricas")

[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x

giac [A] time = 0.36, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)*(4*x^2-7),x, algorithm="giac")

[Out] $x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$

maple [A] time = 0.00, size = 18, normalized size = 0.82

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-3)*(4*x^2-7),x)

[Out] $x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$

maxima [A] time = 0.88, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")

[Out] $x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2 - 7)*(x - 3),x)

[Out] $21x - \frac{7x^2}{2} - 4x^3 + x^4$

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)*(4*x**2-7),x)

[Out] $x**4 - 4*x**3 - 7*x**2/2 + 21*x$

$$3.477 \quad \int (-2 + 7x)^3 dx$$

Optimal. Leaf size=11

$$\frac{1}{28}(2 - 7x)^4$$

[Out] 1/28*(2-7*x)^4

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{1}{28}(2 - 7x)^4$$

Antiderivative was successfully verified.

[In] Int[(-2 + 7*x)^3, x]

[Out] (2 - 7*x)^4/28

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int (-2 + 7x)^3 dx = \frac{1}{28}(2 - 7x)^4$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{28}(7x - 2)^4$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 7*x)^3, x]

[Out] (-2 + 7*x)^4/28

fricas [B] time = 0.60, size = 19, normalized size = 1.73

$$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+7*x)^3,x, algorithm="fricas")

[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x

giac [A] time = 0.22, size = 9, normalized size = 0.82

$$\frac{1}{28}(7x - 2)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+7*x)^3,x, algorithm="giac")

[Out] $1/28*(7*x - 2)^4$

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{(7x - 2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-2+7*x)^3,x)`

[Out] $1/28*(-2+7*x)^4$

maxima [B] time = 0.86, size = 19, normalized size = 1.73

$$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+7*x)^3,x, algorithm="maxima")`

[Out] $343/4*x^4 - 98*x^3 + 42*x^2 - 8*x$

mupad [B] time = 0.14, size = 9, normalized size = 0.82

$$\frac{(7x - 2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((7*x - 2)^3,x)`

[Out] $(7*x - 2)^4/28$

sympy [B] time = 0.06, size = 19, normalized size = 1.73

$$\frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+7*x)**3,x)`

[Out] $343*x**4/4 - 98*x**3 + 42*x**2 - 8*x$

$$3.478 \quad \int \frac{-7+4x^2}{3+2x} dx$$

Optimal. Leaf size=13

$$x^2 - 3x + \log(2x + 3)$$

[Out] $-3*x+x^2+\ln(3+2*x)$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {697}

$$x^2 - 3x + \log(2x + 3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-7 + 4*x^2)/(3 + 2*x), x]$

[Out] $-3*x + x^2 + \text{Log}[3 + 2*x]$

Rule 697

$\text{Int}[\{(d_)+(e_)*(x_)\}^{(m_)}*\{(a_)+(c_)*(x_)^2\}^{(p_)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, m\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{-7+4x^2}{3+2x} dx &= \int \left(-3 + 2x + \frac{2}{3+2x} \right) dx \\ &= -3x + x^2 + \log(3+2x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.23

$$x^2 - 3x + \log(2x + 3) - \frac{27}{4}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(-7 + 4*x^2)/(3 + 2*x), x]$

[Out] $-27/4 - 3*x + x^2 + \text{Log}[3 + 2*x]$

fricas [A] time = 0.84, size = 13, normalized size = 1.00

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((4*x^2-7)/(3+2*x), x, \text{algorithm}="fricas")$

[Out] $x^2 - 3*x + \log(2*x + 3)$

giac [A] time = 0.36, size = 14, normalized size = 1.08

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((4*x^2-7)/(3+2*x), x, \text{algorithm}="giac")$

[Out] $x^2 - 3x + \log(\text{abs}(2x + 3))$

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$x^2 - 3x + \ln(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2-7)/(2*x+3),x)`

[Out] $-3x + x^2 + \ln(2x + 3)$

maxima [A] time = 0.64, size = 13, normalized size = 1.00

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")`

[Out] $x^2 - 3x + \log(2x + 3)$

mupad [B] time = 2.21, size = 11, normalized size = 0.85

$$\ln\left(x + \frac{3}{2}\right) - 3x + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2 - 7)/(2*x + 3),x)`

[Out] $\log(x + 3/2) - 3x + x^2$

sympy [A] time = 0.08, size = 12, normalized size = 0.92

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2-7)/(3+2*x),x)`

[Out] $x**2 - 3*x + \log(2*x + 3)$

$$3.479 \quad \int \frac{1+x}{(-1+x)x^2} dx$$

Optimal. Leaf size=16

$$\frac{1}{x} + 2 \log(1-x) - 2 \log(x)$$

[Out] 1/x+2*ln(1-x)-2*ln(x)

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {77}

$$\frac{1}{x} + 2 \log(1-x) - 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((-1 + x)*x^2), x]

[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rubi steps

$$\begin{aligned} \int \frac{1+x}{(-1+x)x^2} dx &= \int \left(\frac{2}{-1+x} - \frac{1}{x^2} - \frac{2}{x} \right) dx \\ &= \frac{1}{x} + 2 \log(1-x) - 2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{x} + 2 \log(1-x) - 2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((-1 + x)*x^2), x]

[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]

fricas [A] time = 0.62, size = 18, normalized size = 1.12

$$\frac{2x \log(x-1) - 2x \log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")

[Out] (2*x*log(x - 1) - 2*x*log(x) + 1)/x

giac [A] time = 0.36, size = 16, normalized size = 1.00

$$\frac{1}{x} + 2 \log(|x - 1|) - 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")

[Out] 1/x + 2*log(abs(x - 1)) - 2*log(abs(x))

maple [A] time = 0.01, size = 15, normalized size = 0.94

$$-2 \ln(x) + 2 \ln(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(x-1)/x^2,x)

[Out] 2*ln(x-1)+1/x-2*ln(x)

maxima [A] time = 0.59, size = 14, normalized size = 0.88

$$\frac{1}{x} + 2 \log(x - 1) - 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")

[Out] 1/x + 2*log(x - 1) - 2*log(x)

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{1}{x} - 4 \operatorname{atanh}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)/(x^2*(x - 1)),x)

[Out] 1/x - 4*atanh(2*x - 1)

sympy [A] time = 0.10, size = 14, normalized size = 0.88

$$-2 \log(x) + 2 \log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-1+x)/x**2,x)

[Out] -2*log(x) + 2*log(x - 1) + 1/x

$$3.480 \quad \int \frac{1}{4x^2+4x^3+x^4} dx$$

Optimal. Leaf size=27

$$\frac{x+1}{2(1-(x+1)^2)} + \frac{1}{2} \tanh^{-1}(x+1)$$

[Out] 1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1106, 199, 206}

$$\frac{x+1}{2(1-(x+1)^2)} + \frac{1}{2} \tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(4*x^2 + 4*x^3 + x^4)^(-1), x]

[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1106

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned} \int \frac{1}{4x^2 + 4x^3 + x^4} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2)^2} dx, x, 1+x \right) \\ &= \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, 1+x \right) \\ &= \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2} \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{4} \left(-\frac{2(x+1)}{x(x+2)} - \log(x) + \log(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4*x^2 + 4*x^3 + x^4)^(-1), x]

[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4

fricas [A] time = 0.57, size = 39, normalized size = 1.44

$$\frac{(x^2 + 2x) \log(x + 2) - (x^2 + 2x) \log(x) - 2x - 2}{4(x^2 + 2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+4*x^3+4*x^2), x, algorithm="fricas")

[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)

giac [A] time = 0.38, size = 27, normalized size = 1.00

$$-\frac{x + 1}{2(x^2 + 2x)} + \frac{1}{4} \log(|x + 2|) - \frac{1}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+4*x^3+4*x^2), x, algorithm="giac")

[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\frac{\ln(x)}{4} + \frac{\ln(x + 2)}{4} - \frac{1}{4x} - \frac{1}{4(x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4+4*x^3+4*x^2), x)

[Out] -1/4*ln(x)+1/4*ln(x+2)-1/4/x-1/4/(x+2)

maxima [A] time = 0.81, size = 25, normalized size = 0.93

$$-\frac{x + 1}{2(x^2 + 2x)} + \frac{1}{4} \log(x + 2) - \frac{1}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+4*x^3+4*x^2), x, algorithm="maxima")

[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)

mupad [B] time = 2.23, size = 23, normalized size = 0.85

$$\frac{\operatorname{atanh}(x + 1)}{2} - \frac{\frac{x}{2} + \frac{1}{2}}{x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^2 + 4*x^3 + x^4), x)

[Out] atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)

sympy [A] time = 0.11, size = 24, normalized size = 0.89

$$\frac{-x - 1}{2x^2 + 4x} - \frac{\log(x)}{4} + \frac{\log(x + 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**3+4*x**2),x)
```

```
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

$$3.481 \quad \int \frac{1+x^2}{1+x} dx$$

Optimal. Leaf size=17

$$\frac{x^2}{2} - x + 2 \log(x+1)$$

[Out] $-x+1/2*x^2+2*\ln(1+x)$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {697}

$$\frac{x^2}{2} - x + 2 \log(x+1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + x^2)/(1 + x), x]$

[Out] $-x + x^2/2 + 2*\text{Log}[1 + x]$

Rule 697

$\text{Int}[(d + (e \cdot x)^m) \cdot ((a + (c \cdot x^2)^p), x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(d + e \cdot x)^m \cdot (a + c \cdot x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+x} dx &= \int \left(-1 + x + \frac{2}{1+x} \right) dx \\ &= -x + \frac{x^2}{2} + 2 \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.06

$$\frac{1}{2} (x^2 - 2x + 4 \log(x+1) - 3)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(1 + x^2)/(1 + x), x]$

[Out] $(-3 - 2*x + x^2 + 4*\text{Log}[1 + x])/2$

fricas [A] time = 0.59, size = 15, normalized size = 0.88

$$\frac{1}{2} x^2 - x + 2 \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^2+1)/(1+x), x, \text{algorithm}="fricas")$

[Out] $1/2*x^2 - x + 2*\log(x + 1)$

giac [A] time = 0.36, size = 16, normalized size = 0.94

$$\frac{1}{2} x^2 - x + 2 \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(1+x),x, algorithm="giac")

[Out] 1/2*x^2 - x + 2*log(abs(x + 1))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^2}{2} - x + 2 \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x+1),x)

[Out] -x+1/2*x^2+2*ln(x+1)

maxima [A] time = 0.74, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - x + 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(1+x),x, algorithm="maxima")

[Out] 1/2*x^2 - x + 2*log(x + 1)

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$2 \ln(x + 1) - x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x + 1),x)

[Out] 2*log(x + 1) - x + x^2/2

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{x^2}{2} - x + 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(1+x),x)

[Out] x**2/2 - x + 2*log(x + 1)

$$3.482 \quad \int \frac{-1+3x-3x^2+x^3}{x^2} dx$$

Optimal. Leaf size=18

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3 \log(x)$$

[Out] 1/x-3*x+1/2*x^2+3*ln(x)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {14}

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]

[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-1+3x-3x^2+x^3}{x^2} dx &= \int \left(-3 - \frac{1}{x^2} + \frac{3}{x} + x \right) dx \\ &= \frac{1}{x} - 3x + \frac{x^2}{2} + 3 \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]

[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]

fricas [A] time = 0.62, size = 20, normalized size = 1.11

$$\frac{x^3 - 6x^2 + 6x \log(x) + 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")

[Out] 1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x

giac [A] time = 0.37, size = 17, normalized size = 0.94

$$\frac{1}{2}x^2 - 3x + \frac{1}{x} + 3 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")

[Out] 1/2*x^2 - 3*x + 1/x + 3*log(abs(x))

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{x^2}{2} - 3x + 3 \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-3*x^2+3*x-1)/x^2,x)

[Out] 1/x-3*x+1/2*x^2+3*ln(x)

maxima [A] time = 0.73, size = 16, normalized size = 0.89

$$\frac{1}{2}x^2 - 3x + \frac{1}{x} + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")

[Out] 1/2*x^2 - 3*x + 1/x + 3*log(x)

mupad [B] time = 0.03, size = 16, normalized size = 0.89

$$3 \ln(x) - 3x + \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 3*x^2 + x^3 - 1)/x^2,x)

[Out] 3*log(x) - 3*x + 1/x + x^2/2

sympy [A] time = 0.08, size = 15, normalized size = 0.83

$$\frac{x^2}{2} - 3x + 3 \log(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-3*x**2+3*x-1)/x**2,x)

[Out] x**2/2 - 3*x + 3*log(x) + 1/x

$$3.483 \quad \int \left(\frac{1}{2} (3 - \sqrt{37}) + x \right) \left(\frac{1}{2} (3 + \sqrt{37}) + x \right) dx$$

Optimal. Leaf size=18

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

[Out] $-7*x+3/2*x^2+1/3*x^3$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {43}

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Antiderivative was successfully verified.

[In] `Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]`

[Out] $-7*x + (3*x^2)/2 + x^3/3$

Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

Rubi steps

$$\begin{aligned} \int \left(\frac{1}{2} (3 - \sqrt{37}) + x \right) \left(\frac{1}{2} (3 + \sqrt{37}) + x \right) dx &= \int (-7 + 3x + x^2) dx \\ &= -7x + \frac{3x^2}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Antiderivative was successfully verified.

[In] `Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]`

[Out] $-7*x + (3*x^2)/2 + x^3/3$

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: SyntaxError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="fricas")`

[Out] Exception raised: SyntaxError >> Malformed expression

giac [A] time = 0.35, size = 14, normalized size = 0.78

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="giac")

[Out] 1/3*x^3 + 3/2*x^2 - 7*x

maple [A] time = 0.00, size = 28, normalized size = 1.56

$$\frac{x^3}{3} + \frac{3x^2}{2} + \left(\frac{3}{2} - \frac{\sqrt{37}}{2}\right)\left(\frac{3}{2} + \frac{\sqrt{37}}{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x)

[Out] 1/3*x^3+3/2*x^2+(3/2-1/2*37^(1/2))*(3/2+1/2*37^(1/2))*x

maxima [A] time = 1.71, size = 14, normalized size = 0.78

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="maxima")

[Out] 1/3*x^3 + 3/2*x^2 - 7*x

mupad [B] time = 0.02, size = 13, normalized size = 0.72

$$\frac{x(2x^2 + 9x - 42)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 37^(1/2)/2 + 3/2)*(x + 37^(1/2)/2 + 3/2),x)

[Out] (x*(9*x + 2*x^2 - 42))/6

sympy [A] time = 0.06, size = 14, normalized size = 0.78

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)

[Out] x**3/3 + 3*x**2/2 - 7*x

$$3.484 \quad \int \frac{4+3x^2+2x^3}{(1+x)^4} dx$$

Optimal. Leaf size=23

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

[Out] $-5/3/(1+x)^3+3/(1+x)+2*\ln(1+x)$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1850}

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(4 + 3*x^2 + 2*x^3)/(1 + x)^4, x]$

[Out] $-5/(3*(1 + x)^3) + 3/(1 + x) + 2*\text{Log}[1 + x]$

Rule 1850

$\text{Int}[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Pq}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, n\}, x] \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{EqQ}[n, 1])$

Rubi steps

$$\begin{aligned} \int \frac{4+3x^2+2x^3}{(1+x)^4} dx &= \int \left(\frac{5}{(1+x)^4} - \frac{3}{(1+x)^2} + \frac{2}{1+x} \right) dx \\ &= -\frac{5}{3(1+x)^3} + \frac{3}{1+x} + 2\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(4 + 3*x^2 + 2*x^3)/(1 + x)^4, x]$

[Out] $-5/(3*(1 + x)^3) + 3/(1 + x) + 2*\text{Log}[1 + x]$

fricas [B] time = 0.56, size = 46, normalized size = 2.00

$$\frac{9x^2 + 6(x^3 + 3x^2 + 3x + 1)\log(x + 1) + 18x + 4}{3(x^3 + 3x^2 + 3x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2*x^3+3*x^2+4)/(1+x)^4, x, \text{algorithm}=\text{"fricas"})$

[Out] $1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*\log(x + 1) + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1)$

giac [A] time = 0.37, size = 25, normalized size = 1.09

$$\frac{9x^2 + 18x + 4}{3(x+1)^3} + 2 \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="giac")

[Out] 1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*log(abs(x + 1))

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$2 \ln(x+1) - \frac{5}{3(x+1)^3} + \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+4)/(x+1)^4,x)

[Out] -5/3/(x+1)^3+3/(x+1)+2*ln(x+1)

maxima [A] time = 0.74, size = 34, normalized size = 1.48

$$\frac{9x^2 + 18x + 4}{3(x^3 + 3x^2 + 3x + 1)} + 2 \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="maxima")

[Out] 1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$2 \ln(x+1) + \frac{3x^2 + 6x + \frac{4}{3}}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2*x^3 + 4)/(x + 1)^4,x)

[Out] 2*log(x + 1) + (6*x + 3*x^2 + 4/3)/(x + 1)^3

sympy [A] time = 0.11, size = 31, normalized size = 1.35

$$\frac{9x^2 + 18x + 4}{3x^3 + 9x^2 + 9x + 3} + 2 \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+4)/(1+x)**4,x)

[Out] (9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)

$$3.485 \quad \int \frac{x}{(1+x)^2(1+x^2)} dx$$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)} + \frac{1}{2} \tan^{-1}(x)$$

[Out] 1/2/(1+x)+1/2*arctan(x)

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {801, 203}

$$\frac{1}{2(x+1)} + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/((1 + x)^2*(1 + x^2)),x]

[Out] 1/(2*(1 + x)) + ArcTan[x]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 801

Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x)^2(1+x^2)} dx &= \int \left(-\frac{1}{2(1+x)^2} + \frac{1}{2(1+x^2)} \right) dx \\ &= \frac{1}{2(1+x)} + \frac{1}{2} \int \frac{1}{1+x^2} dx \\ &= \frac{1}{2(1+x)} + \frac{1}{2} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.75

$$\frac{1}{2} \left(\frac{1}{x+1} + \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/((1 + x)^2*(1 + x^2)),x]

[Out] ((1 + x)^(-1) + ArcTan[x])/2

fricas [A] time = 0.73, size = 15, normalized size = 0.94

$$\frac{(x+1) \arctan(x) + 1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="fricas")

[Out] 1/2*((x + 1)*arctan(x) + 1)/(x + 1)

giac [B] time = 0.26, size = 32, normalized size = 2.00

$$-\frac{1}{8}\pi - \frac{1}{2}\pi \left[-\frac{\pi - 4 \arctan(x)}{4\pi} + \frac{1}{2} \right] + \frac{1}{2(x+1)} + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="giac")

[Out] -1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x + 1) + 1/2*arctan(x)

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\arctan(x)}{2} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+1)^2/(x^2+1),x)

[Out] 1/2/(x+1)+1/2*arctan(x)

maxima [A] time = 1.79, size = 12, normalized size = 0.75

$$\frac{1}{2(x+1)} + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="maxima")

[Out] 1/2/(x + 1) + 1/2*arctan(x)

mupad [B] time = 2.22, size = 12, normalized size = 0.75

$$\frac{\operatorname{atan}(x)}{2} + \frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x^2 + 1)*(x + 1)^2),x)

[Out] atan(x)/2 + 1/(2*(x + 1))

sympy [A] time = 0.11, size = 10, normalized size = 0.62

$$\frac{\operatorname{atan}(x)}{2} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)**2/(x**2+1),x)

[Out] atan(x)/2 + 1/(2*x + 2)

$$3.486 \quad \int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx$$

Optimal. Leaf size=29

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \log(x+2)$$

[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(2+x)

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1850}

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x), x]

[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand [Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx &= \int \left(-20 + 9x - 3x^2 + x^3 + \frac{47}{2+x} \right) dx \\ &= -20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4} + 47 \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.03

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \log(x+2) - 70$$

Antiderivative was successfully verified.

[In] Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x), x]

[Out] -70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]

fricas [A] time = 0.67, size = 25, normalized size = 0.86

$$\frac{1}{4} x^4 - x^3 + \frac{9}{2} x^2 - 20x + 47 \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x), x, algorithm="fricas")

[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)

giac [A] time = 0.28, size = 26, normalized size = 0.90

$$\frac{1}{4} x^4 - x^3 + \frac{9}{2} x^2 - 20x + 47 \log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="giac")

[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(abs(x + 2))

maple [A] time = 0.00, size = 26, normalized size = 0.90

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \ln(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x^3+3*x^2-2*x+7)/(x+2),x)

[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(x+2)

maxima [A] time = 0.76, size = 25, normalized size = 0.86

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="maxima")

[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)

mupad [B] time = 0.03, size = 25, normalized size = 0.86

$$47 \ln(x + 2) - 20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 - 2*x - x^3 + x^4 + 7)/(x + 2),x)

[Out] 47*log(x + 2) - 20*x + (9*x^2)/2 - x^3 + x^4/4

sympy [A] time = 0.08, size = 24, normalized size = 0.83

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)

[Out] x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)

$$3.487 \quad \int \frac{-1+x^3}{-1+x} dx$$

Optimal. Leaf size=16

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

[Out] $x + 1/2*x^2 + 1/3*x^3$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1586}

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(-1 + x), x]

[Out] $x + x^2/2 + x^3/3$

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{-1+x} dx &= \int (1+x+x^2) dx \\ &= x + \frac{x^2}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(-1 + x), x]

[Out] $x + x^2/2 + x^3/3$

fricas [A] time = 0.63, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(-1+x), x, algorithm="fricas")

[Out] $1/3*x^3 + 1/2*x^2 + x$

giac [A] time = 0.35, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(-1+x),x, algorithm="giac")

[Out] 1/3*x^3 + 1/2*x^2 + x

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-1)/(x-1),x)

[Out] x+1/2*x^2+1/3*x^3

maxima [A] time = 0.89, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(-1+x),x, algorithm="maxima")

[Out] 1/3*x^3 + 1/2*x^2 + x

mupad [B] time = 0.02, size = 13, normalized size = 0.81

$$\frac{x(2x^2 + 3x + 6)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 1)/(x - 1),x)

[Out] (x*(3*x + 2*x^2 + 6))/6

sympy [A] time = 0.06, size = 10, normalized size = 0.62

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-1)/(-1+x),x)

[Out] x**3/3 + x**2/2 + x

$$3.488 \quad \int \frac{2+2x}{(-1+x)^3(1+x^2)} dx$$

Optimal. Leaf size=17

$$\frac{1}{x-1} - \frac{1}{(1-x)^2} + \tan^{-1}(x)$$

[Out] -1/(1-x)^2+1/(-1+x)+arctan(x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {801, 203}

$$\frac{1}{x-1} - \frac{1}{(1-x)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)), x]

[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 801

Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{2+2x}{(-1+x)^3(1+x^2)} dx &= \int \left(\frac{2}{(-1+x)^3} - \frac{1}{(-1+x)^2} + \frac{1}{1+x^2} \right) dx \\ &= -\frac{1}{(1-x)^2} + \frac{1}{-1+x} + \int \frac{1}{1+x^2} dx \\ &= -\frac{1}{(1-x)^2} + \frac{1}{-1+x} + \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{x + (x-1)^2 \tan^{-1}(x) - 2}{(x-1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)), x]

[Out] (-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2

fricas [A] time = 0.69, size = 25, normalized size = 1.47

$$\frac{(x^2 - 2x + 1) \arctan(x) + x - 2}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="fricas")

[Out] ((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)

giac [A] time = 0.24, size = 12, normalized size = 0.71

$$\frac{x-2}{(x-1)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="giac")

[Out] (x - 2)/(x - 1)^2 + arctan(x)

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\arctan(x) - \frac{1}{(x-1)^2} + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+2)/(x-1)^3/(x^2+1),x)

[Out] -1/(x-1)^2+1/(x-1)+arctan(x)

maxima [A] time = 1.74, size = 17, normalized size = 1.00

$$\frac{x-2}{x^2-2x+1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="maxima")

[Out] (x - 2)/(x^2 - 2*x + 1) + arctan(x)

mupad [B] time = 0.03, size = 17, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{x-2}{x^2-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 2)/((x^2 + 1)*(x - 1)^3),x)

[Out] atan(x) + (x - 2)/(x^2 - 2*x + 1)

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{x-2}{x^2-2x+1} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*x)/(-1+x)**3/(x**2+1),x)

[Out] (x - 2)/(x**2 - 2*x + 1) + atan(x)

$$3.489 \quad \int \frac{1}{bx+c(d+ex)^2} dx$$

Optimal. Leaf size=47

$$-\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

[Out] $-2*\operatorname{arctanh}((b+2*c*e*(e*x+d))/b^{(1/2)/(4*c*d*e+b)^{(1/2)})/b^{(1/2)/(4*c*d*e+b)^{(1/2)}}$

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1981, 618, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*(d + e*x)^2)^(-1), x]

[Out] $(-2*\operatorname{ArcTanh}[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e]])/(Sqrt[b]*Sqrt[b + 4*c*d*e])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QquadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{bx+c(d+ex)^2} dx &= \int \frac{1}{cd^2 + (b+2cde)x + ce^2x^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{b(b+4cde)-x^2} dx, x, b+2cde+2ce^2x\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*(d + e*x)^2)^(-1),x]

[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e]])/(Sqrt[b]*Sqrt[b + 4*c*d*e])

fricas [A] time = 0.75, size = 190, normalized size = 4.04

$$\left[\frac{\log\left(\frac{2c^2e^4x^2+2c^2d^2e^2+4bcde+b^2+2(2c^2de^3+bce^2)x-\sqrt{4bcde+b^2}(2ce^2x+2cde+b)}{ce^2x^2+cd^2+(2cde+b)x}\right)}{\sqrt{4bcde+b^2}}, \frac{2\sqrt{-4bcde-b^2}\arctan\left(\frac{\sqrt{-4bcde-b^2}(2ce^2x+2cde+b)}{4bcde+b^2}\right)}{4bcde+b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")

[Out] [log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), 2*sqrt(-4*b*c*d*e - b^2)*arctan(sqrt(-4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))/(4*b*c*d*e + b^2)]

giac [A] time = 0.28, size = 48, normalized size = 1.02

$$\frac{2\arctan\left(\frac{2cxe^2+2cde+b}{\sqrt{-4bcde-b^2}}\right)}{\sqrt{-4bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")

[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-4*b*c*d*e - b^2)

maple [A] time = 0.01, size = 43, normalized size = 0.91

$$-\frac{2\operatorname{arctanh}\left(\frac{2ce^2x+2cde+b}{\sqrt{4bcde+b^2}}\right)}{\sqrt{4bcde+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+c*(e*x+d)^2),x)

[Out] -2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e+b^2)^(1/2))

maxima [A] time = 1.02, size = 68, normalized size = 1.45

$$\frac{\log\left(\frac{2ce^2x+2cde+b-\sqrt{(4cde+b)b}}{2ce^2x+2cde+b+\sqrt{(4cde+b)b}}\right)}{\sqrt{(4cde+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")

[Out] log((2*c*e^2*x + 2*c*d*e + b - sqrt((4*c*d*e + b)*b))/(2*c*e^2*x + 2*c*d*e + b + sqrt((4*c*d*e + b)*b)))/sqrt((4*c*d*e + b)*b)

mupad [B] time = 0.10, size = 42, normalized size = 0.89

$$\frac{2 \operatorname{atanh}\left(\frac{2cxe^2+2cde+b}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*(d + e*x)^2 + b*x), x)

[Out] -(2*atanh((b + 2*c*d*e + 2*c*e^2*x)/(b^(1/2)*(b + 4*c*d*e)^(1/2))))/(b^(1/2)*(b + 4*c*d*e)^(1/2))

sympy [B] time = 0.30, size = 151, normalized size = 3.21

$$\sqrt{\frac{1}{b(b+4cde)}} \log\left(x + \frac{-b^2\sqrt{\frac{1}{b(b+4cde)}} - 4bcde\sqrt{\frac{1}{b(b+4cde)}} + b + 2cde}{2ce^2}\right) - \sqrt{\frac{1}{b(b+4cde)}} \log\left(x + \frac{b^2\sqrt{\frac{1}{b(b+4cde)}}}{2ce^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+c*(e*x+d)**2), x)

[Out] sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))) - 4*b*c*d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)) - sqrt(1/(b*(b + 4*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*e))) + 4*b*c*d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2))

$$3.490 \quad \int \frac{1}{a+bx+c(d+ex)^2} dx$$

Optimal. Leaf size=57

$$\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

[Out] $-2*\operatorname{arctanh}((b+2*c*e*(e*x+d))/(-4*a*c*e^2+4*b*c*d*e+b^2)^{(1/2)})/(-4*a*c*e^2+4*b*c*d*e+b^2)^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1981, 618, 206}

$$\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*(d + e*x)^2)^(-1), x]

[Out] $(-2*\operatorname{ArcTanh}[(b + 2*c*e*(d + e*x))/\operatorname{Sqrt}[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/\operatorname{Sqrt}[b^2 + 4*b*c*d*e - 4*a*c*e^2]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QquadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+bx+c(d+ex)^2} dx &= \int \frac{1}{a+cd^2+(b+2cde)x+ce^2x^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{b^2+4bcde-4ace^2-x^2} dx, x, b+2cde+2ce^2x\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.07

$$\frac{2 \tan^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{4ace^2-b^2-4bcde}}\right)}{\sqrt{4ace^2-b^2-4bcde}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*(d + e*x)^2)^(-1), x]

[Out] (2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]

fricas [A] time = 0.63, size = 240, normalized size = 4.21

$$\left[\frac{\log\left(\frac{2c^2e^4x^2+4bcde+2(c^2d^2-ac)e^2+b^2+2(2c^2de^3+bce^2)x-\sqrt{4bcde-4ace^2+b^2}(2ce^2x+2cde+b)}{ce^2x^2+cd^2+(2cde+b)x+a}\right)}{\sqrt{4bcde-4ace^2+b^2}}, -\frac{2\sqrt{-4bcde+4ace^2-b^2}\arctan\left(\frac{b+2cex}{\sqrt{-4bcde+4ace^2-b^2}}\right)}{4bcde-4ace^2+b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+c*(e*x+d)^2), x, algorithm="fricas")

[Out] [log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d*e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))/(4*b*c*d*e - 4*a*c*e^2 + b^2)]

giac [A] time = 0.36, size = 60, normalized size = 1.05

$$\frac{2 \arctan\left(\frac{2cxe^2+2cde+b}{\sqrt{-4bcde+4ace^2-b^2}}\right)}{\sqrt{-4bcde+4ace^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+c*(e*x+d)^2), x, algorithm="giac")

[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)

maple [A] time = 0.00, size = 61, normalized size = 1.07

$$\frac{2 \arctan\left(\frac{2ce^2x+2cde+b}{\sqrt{4ace^2-4bcde-b^2}}\right)}{\sqrt{4ace^2-4bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*x+c*(e*x+d)^2), x)

[Out] 2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+c*(e*x+d)^2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c*e^2>0)', see `assume?` for more details)Is 4*a*c*e^2 - 4*b*c*d*e-b^2 positive or negative?

mupad [B] time = 2.23, size = 82, normalized size = 1.44

$$\frac{2 \operatorname{atan}\left(\frac{b+2cde}{\sqrt{-b^2-4cde+4ace^2}} + \frac{2ce^2x}{\sqrt{-b^2-4cde+4ace^2}}\right)}{\sqrt{-b^2-4cde+4ace^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*(d + e*x)^2 + b*x), x)`

[Out] $(2*\operatorname{atan}((b + 2*c*d*e)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^{(1/2)} + (2*c*e^2*x)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^{(1/2}))/((4*a*c*e^2 - b^2 - 4*b*c*d*e)^{(1/2)}$

sympy [B] time = 0.35, size = 294, normalized size = 5.16

$$-\sqrt{-\frac{1}{4ace^2 - b^2 - 4bcde}} \log\left(x + \frac{-4ace^2\sqrt{-\frac{1}{4ace^2 - b^2 - 4bcde}} + b^2\sqrt{-\frac{1}{4ace^2 - b^2 - 4bcde}} + 4bcde\sqrt{-\frac{1}{4ace^2 - b^2 - 4bcde}} + b + 2cde}{2ce^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x+c*(e*x+d)**2), x)`

[Out] $-\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*\log(x + (-4*a*c*e**2*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + 4*b*c*d*e*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*e**2)) + \operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*\log(x + (4*a*c*e**2*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - 4*b*c*d*e*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*e**2))$

$$3.491 \quad \int \frac{x^2}{1+(-1+x^2)^2} dx$$

Optimal. Leaf size=188

$$\frac{\log\left(x^2 - \sqrt{2(1+\sqrt{2})}x + \sqrt{2}\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(x^2 + \sqrt{2(1+\sqrt{2})}x + \sqrt{2}\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})} - 2}{\sqrt{2(\sqrt{2}-1)}}\right)$$

[Out] $-1/4*\arctan((-2*x+(2+2*2^{(1/2)})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*(2+2*2^{(1/2)})^{(1/2)}+1/4*\arctan((2*x+(2+2*2^{(1/2)})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*(2+2*2^{(1/2)})^{(1/2)}+1/4*\ln(x^2+2^{(1/2)}-x*(2+2*2^{(1/2)})^{(1/2)})/(2+2*2^{(1/2)})^{(1/2)}-1/4*\ln(x^2+2^{(1/2)}+x*(2+2*2^{(1/2)})^{(1/2)})/(2+2*2^{(1/2)})^{(1/2)})$

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1989, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2 - \sqrt{2(1+\sqrt{2})}x + \sqrt{2}\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(x^2 + \sqrt{2(1+\sqrt{2})}x + \sqrt{2}\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})} - 2}{\sqrt{2(\sqrt{2}-1)}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + (-1 + x^2)^2), x]

[Out] $-(\text{Sqrt}[(1 + \text{Sqrt}[2])/2]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])) - 2*x]/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])]])/2 + (\text{Sqrt}[(1 + \text{Sqrt}[2])/2]*\text{ArcTan}[(\text{Sqrt}[2*(1 + \text{Sqrt}[2])) + 2*x]/\text{Sqrt}[2*(-1 + \text{Sqrt}[2])]])/2 + \text{Log}[\text{Sqrt}[2] - \text{Sqrt}[2*(1 + \text{Sqrt}[2])]*x + x^2]/(4*\text{Sqrt}[2*(1 + \text{Sqrt}[2])]) - \text{Log}[\text{Sqrt}[2] + \text{Sqrt}[2*(1 + \text{Sqrt}[2])]*x + x^2]/(4*\text{Sqrt}[2*(1 + \text{Sqrt}[2])])$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_)^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
x]
```

Rubi steps

$$\int \frac{x^2}{1 + (-1 + x^2)^2} dx = \int \frac{x^2}{2 - 2x^2 + x^4} dx$$

$$= -\left(\frac{1}{2} \int \frac{\sqrt{2} - x^2}{2 - 2x^2 + x^4} dx\right) + \frac{1}{2} \int \frac{\sqrt{2} + x^2}{2 - 2x^2 + x^4} dx$$

$$= \frac{1}{4} \int \frac{1}{\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x + x^2} dx + \frac{1}{4} \int \frac{1}{\sqrt{2} + \sqrt{2(1 + \sqrt{2})}x + x^2} dx + \frac{\int \frac{\sqrt{2(1 + \sqrt{2})}}{-\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x + x^2}}{4\sqrt{2(1 + \sqrt{2})}}$$

$$= \frac{\log\left(\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x + x^2\right)}{4\sqrt{2(1 + \sqrt{2})}} - \frac{\log\left(\sqrt{2} + \sqrt{2(1 + \sqrt{2})}x + x^2\right)}{4\sqrt{2(1 + \sqrt{2})}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{2(1 - \dots)}\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2(1 + \sqrt{2})} - 2x}{\sqrt{2(-1 + \sqrt{2})}}\right)}{2\sqrt{2(-1 + \sqrt{2})}} + \frac{\tan^{-1}\left(\frac{\sqrt{2(1 + \sqrt{2})} + 2x}{\sqrt{2(-1 + \sqrt{2})}}\right)}{2\sqrt{2(-1 + \sqrt{2})}} + \frac{\log\left(\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x + x^2\right)}{4\sqrt{2(1 + \sqrt{2})}} - \frac{\log\left(\sqrt{2} + \sqrt{2(1 + \sqrt{2})}x + x^2\right)}{4\sqrt{2(1 + \sqrt{2})}}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x^2)^2), x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^(
3/2)
```

fricas [A] time = 0.52, size = 247, normalized size = 1.31

$$\frac{1}{16} \cdot 2^{\frac{1}{4}} \sqrt{2\sqrt{2}+4} (\sqrt{2}-2) \log\left(2^{\frac{3}{4}} x \sqrt{2\sqrt{2}+4} + 2x^2 + 2\sqrt{2}\right) - \frac{1}{16} \cdot 2^{\frac{1}{4}} \sqrt{2\sqrt{2}+4} (\sqrt{2}-2) \log\left(-2^{\frac{3}{4}} x \sqrt{2\sqrt{2}+4} + 2x^2 + 2\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")

[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2)+4)*(sqrt(2)-2)*log(2^(3/4)*x*sqrt(2*sqrt(2)+4)+2*x^2+2*sqrt(2))-1/16*2^(1/4)*sqrt(2*sqrt(2)+4)*(sqrt(2)-2)*log(-2^(3/4)*x*sqrt(2*sqrt(2)+4)+2*x^2+2*sqrt(2))-1/4*2^(3/4)*sqrt(2*sqrt(2)+4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2)+4)+1/2*2^(1/4)*sqrt(2*sqrt(2)+4)*x*sqrt(2*sqrt(2)+4)+2*x^2+2*sqrt(2))*sqrt(2*sqrt(2)+4)-sqrt(2)-1-1/4*2^(3/4)*sqrt(2*sqrt(2)+4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2)+4)+1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2)+4)+2*x^2+2*sqrt(2))*sqrt(2*sqrt(2)+4)+sqrt(2)+1)

giac [A] time = 1.49, size = 147, normalized size = 0.78

$$\frac{1}{4} \sqrt{2\sqrt{2}+2} \arctan\left(\frac{2^{\frac{3}{4}}(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2})}{2\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{4} \sqrt{2\sqrt{2}+2} \arctan\left(\frac{2^{\frac{3}{4}}(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2})}{2\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{8} \sqrt{2\sqrt{2}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")

[Out] 1/4*sqrt(2*sqrt(2)+2)*arctan(1/2*2^(3/4)*(2*x+2^(1/4)*sqrt(sqrt(2)+2))/sqrt(-sqrt(2)+2))+1/4*sqrt(2*sqrt(2)+2)*arctan(1/2*2^(3/4)*(2*x-2^(1/4)*sqrt(sqrt(2)+2))/sqrt(-sqrt(2)+2))-1/8*sqrt(2*sqrt(2)-2)*log(x^2+2^(1/4)*x*sqrt(sqrt(2)+2)+sqrt(2))+1/8*sqrt(2*sqrt(2)-2)*log(x^2-2^(1/4)*x*sqrt(sqrt(2)+2)+sqrt(2))

maple [B] time = 0.10, size = 308, normalized size = 1.64

$$\frac{\sqrt{2} (2+2\sqrt{2}) \arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}} - \frac{(2+2\sqrt{2}) \arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}} + \frac{\sqrt{2} (2+2\sqrt{2}) \arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+(x^2-1)^2),x)

[Out] -1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2-1)^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")

[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)

mupad [B] time = 2.30, size = 101, normalized size = 0.54

$$\operatorname{atanh}\left(32x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\operatorname{atanh}\left(32x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((x^2 - 1)^2 + 1),x)

[Out] atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32 - 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))

sympy [A] time = 0.52, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4 + 16t^2 + 1, \left(t \mapsto t \log(64t^3 + 4t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1+(x**2-1)**2),x)

[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))

$$3.492 \quad \int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4} dx$$

Optimal. Leaf size=60

$$\frac{x^4}{(x^4+x+3)^3} - \frac{3x}{(x^4+x+3)^3} + \frac{2}{(x^4+x+3)^3} - \frac{5x^6}{(x^4+x+3)^3} + \frac{5x^2}{(x^4+x+3)^3}$$

[Out] 2/(x^4+x+3)^3-3*x/(x^4+x+3)^3+5*x^2/(x^4+x+3)^3+x^4/(x^4+x+3)^3-5*x^6/(x^4+x+3)^3

Rubi [A] time = 0.14, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2102, 1588}

$$-\frac{5x^6}{(x^4+x+3)^3} + \frac{x^4}{(x^4+x+3)^3} + \frac{5x^2}{(x^4+x+3)^3} - \frac{3x}{(x^4+x+3)^3} + \frac{2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

[In] Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9)/(3 + x + x^4)^4), x]

[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)^3 + x^4/(3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 2102

Int[(Pm_)*(Qn_)^(p_.), x_Symbol] := With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m + n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn + (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int -\frac{15 - 36x + 5x^2 + 12x^3 - 34x^4 + 140x^5 + 15x^6 + 8x^7 - 30x^9}{(3 + x + x^4)^4} dx &= -\frac{5x^6}{(3 + x + x^4)^3} + \frac{1}{6} \int \frac{-90 + 216x - 30x^2}{(3 + x + x^4)^3} dx \\
&= \frac{x^4}{(3 + x + x^4)^3} - \frac{5x^6}{(3 + x + x^4)^3} - \frac{1}{48} \int \frac{720 - 1440x + 720x^2}{(3 + x + x^4)^3} dx \\
&= \frac{5x^2}{(3 + x + x^4)^3} + \frac{x^4}{(3 + x + x^4)^3} - \frac{5x^6}{(3 + x + x^4)^3} \\
&= -\frac{3x}{(3 + x + x^4)^3} + \frac{5x^2}{(3 + x + x^4)^3} + \frac{x^4}{(3 + x + x^4)^3} \\
&= \frac{2}{(3 + x + x^4)^3} - \frac{3x}{(3 + x + x^4)^3} + \frac{5x^2}{(3 + x + x^4)^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.45

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Antiderivative was successfully verified.

[In] Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9)/(3 + x + x^4)^4), x]

[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3

fricas [A] time = 0.46, size = 65, normalized size = 1.08

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x, algorithm="fricas")

[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)

giac [A] time = 0.40, size = 30, normalized size = 0.50

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x, algorithm="giac")

[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3

maple [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x)`

[Out] $(-5x^6+x^4+5x^2-3x+2)/(x^4+x+3)^3$

maxima [A] time = 1.25, size = 65, normalized size = 1.08

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x, algorithm="maxima")`

[Out] $-(5x^6 - x^4 - 5x^2 + 3x - 2)/(x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27)$

mupad [B] time = 2.34, size = 27, normalized size = 0.45

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(5*x^2 - 36*x + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9 + 15)/(x + x^4 + 3)^4,x)`

[Out] $(5x^2 - 3x + x^4 - 5x^6 + 2)/(x + x^4 + 3)^3$

sympy [A] time = 0.23, size = 60, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-5*x**2+36*x-15)/(x**4+x+3)**4,x)`

[Out] $(-5x^{**6} + x^{**4} + 5x^{**2} - 3x + 2)/(x^{**12} + 3x^{**9} + 9x^{**8} + 3x^{**6} + 18x^{**5} + 27x^{**4} + x^{**3} + 9x^{**2} + 27x + 27)$

$$3.493 \quad \int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4} + \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2} \right) dx$$

Optimal. Leaf size=27

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

[Out] $(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3 + x + x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3 + x + x^4)^3} + \frac{30x}{(3 + x + x^4)^2} \right) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2, x]$

[Out] $-19/(4*(3 + x + x^4)^3) + (3 + x + x^4)^{-2} - (621*\text{Defer}[\text{Int}[(3 + x + x^4)^{-4}, x])/4 + 684*\text{Defer}[\text{Int}[x/(3 + x + x^4)^4, x] + 360*\text{Defer}[\text{Int}[x^2/(3 + x + x^4)^4, x] + 44*\text{Defer}[\text{Int}[(3 + x + x^4)^{-3}, x] - 320*\text{Defer}[\text{Int}[x/(3 + x + x^4)^3, x] - 75*\text{Defer}[\text{Int}[x^2/(3 + x + x^4)^3, x] + 30*\text{Defer}[\text{Int}[x/(3 + x + x^4)^2, x]$

Rubi steps

$$\begin{aligned} \int \left(\frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3 + x + x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3 + x + x^4)^3} + \frac{30x}{(3 + x + x^4)^2} \right) dx &= 3 \int \frac{-47 + 228x + 120x^2 + 19x^3}{(3 + x + x^4)^4} dx \\ &= -\frac{19}{4(3 + x + x^4)^3} + \frac{19}{(3 + x + x^4)^2} \\ &= -\frac{19}{4(3 + x + x^4)^3} + \frac{19}{(3 + x + x^4)^2} \\ &= -\frac{19}{4(3 + x + x^4)^3} + \frac{19}{(3 + x + x^4)^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2, x]$

[Out] $(2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3$

fricas [B] time = 0.68, size = 65, normalized size = 2.41

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")

[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)

giac [B] time = 0.40, size = 197, normalized size = 7.30

$$\frac{1}{195075} x \left(\frac{377432 x^2 - 2808656 x + 703551}{x^4 + x + 3} - \frac{255032 x^2 - 1829456 x + 680601}{x^4 + x + 3} - \frac{7650 (16 x^2 - 128 x + 3)}{x^4 + x + 3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")

[Out] 1/195075*x*((377432*x^2 - 2808656*x + 703551)/(x^4 + x + 3) - (255032*x^2 - 1829456*x + 680601)/(x^4 + x + 3) - 7650*(16*x^2 - 128*x + 3)/(x^4 + x + 3)) - 2/51*(16*x^3 - 64*x^2 + x + 12)/(x^4 + x + 3) + 1/390150*(754864*x^7 - 2808656*x^6 + 469034*x^5 + 1321012*x^4 - 417584*x^3 - 13339729*x^2 + 2696430*x + 2183454)/(x^4 + x + 3)^2 - 1/390150*(510064*x^11 - 1829456*x^10 + 453734*x^9 + 1402676*x^8 - 472048*x^7 - 13501313*x^6 + 4720744*x^5 + 3747556*x^4 - 10935781*x^3 - 30736107*x^2 + 10203894*x + 4117662)/(x^4 + x + 3)^3

maple [C] time = 0.03, size = 250, normalized size = 9.26

$$\frac{\left(-255032 \operatorname{RootOf}\left(_Z^4 + _Z + 3\right)^2 + 1829456 \operatorname{RootOf}\left(_Z^4 + _Z + 3\right) - 680601\right) \ln\left(-\operatorname{RootOf}\left(_Z^4 + _Z + 3\right)\right)}{780300 \operatorname{RootOf}\left(_Z^4 + _Z + 3\right)^3 + 195075}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x)

[Out] (377432/195075*x^7-1404328/195075*x^6+234517/195075*x^5+660506/195075*x^4-208792/195075*x^3-13339729/390150*x^2+89881/13005*x+121303/21675)/(x^4+x+3)^2+1/195075*sum((377432*_R^2-2808656*_R+703551)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_Z^4+_Z+3))+30*(-16/765*x^3+64/765*x^2-1/765*x-4/255)/(x^4+x+3)+2/51*sum((-16*_R^2+128*_R-3)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_Z^4+_Z+3))+3*(-255032/585225*x^11+914728/585225*x^10-226867/585225*x^9-701338/585225*x^8+236024/585225*x^7+13501313/1170450*x^6-2360372/585225*x^5-1873778/585225*x^4+10935781/1170450*x^3+3415123/130050*x^2-62987/7225*x-76253/21675)/(x^4+x+3)^3+1/195075*sum((-255032*_R^2+1829456*_R-680601)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_Z^4+_Z+3))

maxima [B] time = 1.20, size = 65, normalized size = 2.41

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")

[Out] $-(5x^6 - x^4 - 5x^2 + 3x - 2)/(x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27)$

mupad [B] time = 0.05, size = 27, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((684*x + 360*x^2 + 57*x^3 - 141)/(x + x^4 + 3)^4 - (320*x + 75*x^2 + 8*x^3 - 42)/(x + x^4 + 3)^3 + (30*x)/(x + x^4 + 3)^2, x)`

[Out] $(5x^2 - 3x + x^4 - 5x^6 + 2)/(x + x^4 + 3)^3$

sympy [B] time = 0.32, size = 60, normalized size = 2.22

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320*x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2, x)`

[Out] $(-5x^{**6} + x^{**4} + 5x^{**2} - 3x + 2)/(x^{**12} + 3x^{**9} + 9x^{**8} + 3x^{**6} + 18x^{**5} + 27x^{**4} + x^{**3} + 9x^{**2} + 27x + 27)$

$$3.494 \quad \int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4} \right) dx$$

Optimal. Leaf size=27

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

[Out] $(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{-3 + 10x + 4x^3 - 30x^5}{(3 + x + x^4)^3} - \frac{3(1 + 4x^3)(2 - 3x + 5x^2 + x^4 - 5x^6)}{(3 + x + x^4)^4} \right) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4, x]$

[Out] $7/(2*(3 + x + x^4)^3) - (63*x)/(22*(3 + x + x^4)^3) - (12*x^2)/(3 + x + x^4)^3 - (5*x^3)/(3 + x + x^4)^3 + (3*x^4)/(2*(3 + x + x^4)^3) - (10*x^6)/(3 + x + x^4)^3 - 1/(2*(3 + x + x^4)^2) + (5*x^2)/(3 + x + x^4)^2 + (144*\text{Defer}[\text{Int}[(3 + x + x^4)^{-4}], x])/11 + (828*\text{Defer}[\text{Int}[x/(3 + x + x^4)^4], x])/11 + 18*\text{Defer}[\text{Int}[x^2/(3 + x + x^4)^4], x] - 4*\text{Defer}[\text{Int}[(3 + x + x^4)^{-3}], x] - 20*\text{Defer}[\text{Int}[x/(3 + x + x^4)^3], x]$

Rubi steps

$$\begin{aligned}
\int \left(\frac{-3 + 10x + 4x^3 - 30x^5}{(3 + x + x^4)^3} - \frac{3(1 + 4x^3)(2 - 3x + 5x^2 + x^4 - 5x^6)}{(3 + x + x^4)^4} \right) dx &= - \left(3 \int \frac{(1 + 4x^3)(2 - 3x + 5x^2 + x^4 - 5x^6)}{(3 + x + x^4)^4} dx \right) \\
&= - \frac{10x^6}{(3 + x + x^4)^3} + \frac{5x^2}{(3 + x + x^4)^2} - \frac{1}{6} \int \frac{1}{(3 + x + x^4)^2} dx \\
&= \frac{3x^4}{2(3 + x + x^4)^3} - \frac{10x^6}{(3 + x + x^4)^3} - \frac{1}{2} \int \frac{1}{(3 + x + x^4)^2} dx \\
&= - \frac{5x^3}{(3 + x + x^4)^3} + \frac{3x^4}{2(3 + x + x^4)^3} - \frac{1}{2} \int \frac{1}{(3 + x + x^4)^2} dx \\
&= - \frac{12x^2}{(3 + x + x^4)^3} - \frac{5x^3}{(3 + x + x^4)^3} + \frac{1}{2} \int \frac{1}{(3 + x + x^4)^2} dx \\
&= - \frac{63x}{22(3 + x + x^4)^3} - \frac{12x^2}{(3 + x + x^4)^3} - \frac{1}{2} \int \frac{1}{(3 + x + x^4)^2} dx \\
&= \frac{7}{2(3 + x + x^4)^3} - \frac{63x}{22(3 + x + x^4)^3} - \frac{1}{2} \int \frac{1}{(3 + x + x^4)^2} dx \\
&= \frac{7}{2(3 + x + x^4)^3} - \frac{63x}{22(3 + x + x^4)^3} - \frac{1}{2} \int \frac{1}{(3 + x + x^4)^2} dx \\
&= \frac{7}{2(3 + x + x^4)^3} - \frac{63x}{22(3 + x + x^4)^3} - \frac{1}{2} \int \frac{1}{(3 + x + x^4)^2} dx
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]

[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3

fricas [B] time = 0.66, size = 65, normalized size = 2.41

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")

[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)

giac [B] time = 0.45, size = 111, normalized size = 4.11

$$\frac{69136x^7 - 147344x^6 - 30784x^5 + 120988x^4 + 137584x^3 + 1167854x^2 - 100680x + 18621}{390150(x^4 + x + 3)^2} - \frac{69136x^{11} - 147344x^{10} + 30784x^9 - 120988x^8 - 137584x^7 - 1167854x^6 + 100680x^5 - 18621x^4}{390150(x^4 + x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^4,x, algorithm="giac")

[Out] 1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 1167854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^11 - 147344*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5 + 129019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3)^3

maple [B] time = 0.02, size = 112, normalized size = 4.15

$$\frac{-\frac{34568}{195075}x^7 + \frac{73672}{195075}x^6 + \frac{15392}{195075}x^5 - \frac{60494}{195075}x^4 - \frac{68792}{195075}x^3 - \frac{583927}{195075}x^2 + \frac{3356}{13005}x - \frac{2069}{43350} - \frac{34568}{195075}x^{11} + \frac{73672}{195075}x^{10} - \frac{30784}{195075}x^9 + \frac{190124}{195075}x^8 + \frac{197648}{195075}x^7 + \frac{2645788}{195075}x^6 - \frac{72044}{195075}x^5 + \frac{129019}{195075}x^4 + \frac{1580606}{195075}x^3 + \frac{1452132}{195075}x^2 + \frac{887031}{195075}x - \frac{724437}{195075}}{(x^4 + x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^4,x)

[Out] -(-34568/195075*x^7+73672/195075*x^6+15392/195075*x^5-60494/195075*x^4-68792/195075*x^3-583927/195075*x^2+3356/13005*x-2069/43350)/(x^4+x+3)^2+3*(-34568/585225*x^11+73672/585225*x^10+15392/585225*x^9-95062/585225*x^8-98824/585225*x^7-1322894/585225*x^6+36022/585225*x^5-129019/1170450*x^4-790303/585225*x^3-80674/65025*x^2-10951/14450*x+26831/43350)/(x^4+x+3)^3

maxima [B] time = 0.98, size = 65, normalized size = 2.41

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")

[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)

mupad [B] time = 0.04, size = 27, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((10*x + 4*x^3 - 30*x^5 - 3)/(x + x^4 + 3)^3 - (3*(4*x^3 + 1)*(5*x^2 - 3*x + x^4 - 5*x^6 + 2)))/(x + x^4 + 3)^4,x)

[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3

sympy [B] time = 0.29, size = 60, normalized size = 2.22

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**4+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
          If[ElementaryFunctionQ[Head[expn]],
            Max[3, ExpnType[expn[[1]]],
          If[SpecialFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
          If[HypergeometricFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
          If[AppellFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
          If[Head[expn]===RootSum,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
          If[Head[expn]===Integrate || Head[expn]===Int,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
          9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```



```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`' or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```

```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands(
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```

```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```