Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.1-Rational-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [494]. This is test number [51].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.39 (491)	% 0.61 (3)
Mathematica	% 100.00 (494)	% 0.00 (0)
Maple	% 98.99 (489)	% 1.01 (5)
Maxima	% 82.79 (409)	% 17.21 (85)
Fricas	% 87.25 (431)	% 12.75 (63)
Sympy	% 87.25 (431)	% 12.75 (63)
Giac	% 85.22 (421)	% 14.78 (73)
Mupad	% 98.18 (485)	% 1.82 (9)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

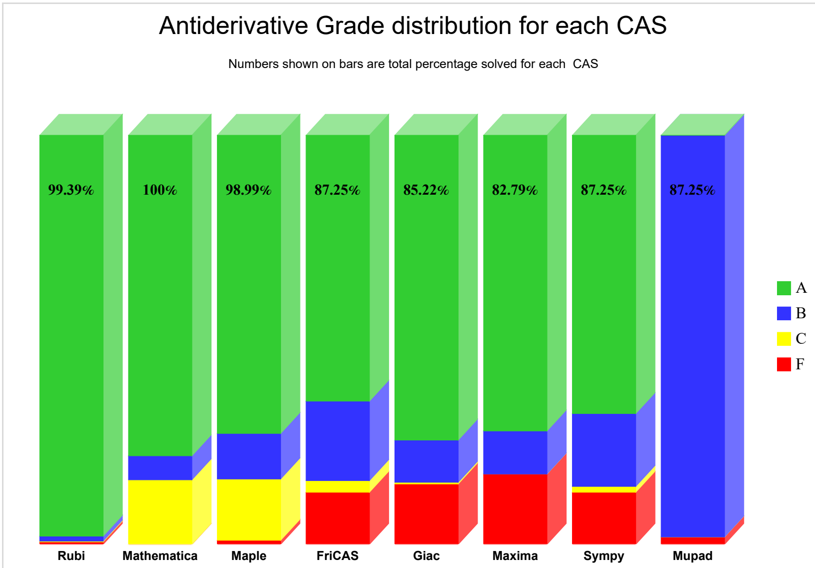
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

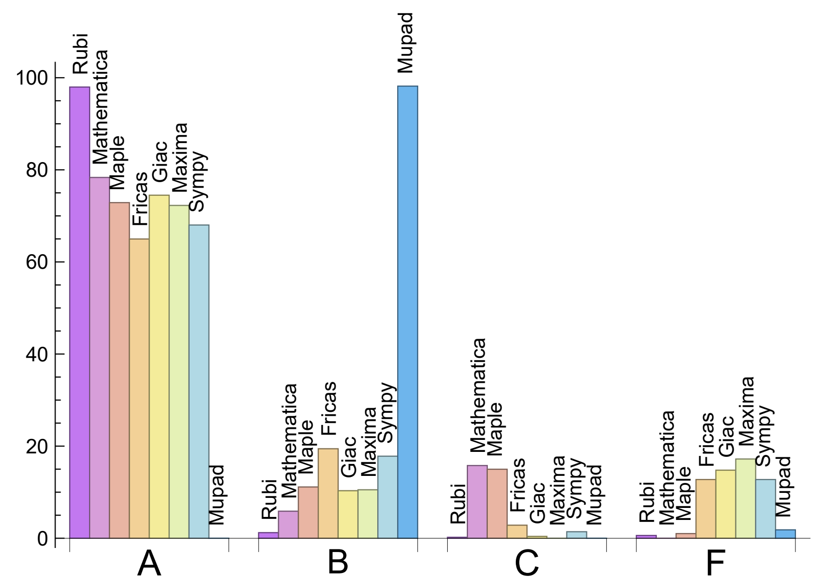
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.98	1.21	0.20	0.61
Mathematica	78.34	5.87	15.79	0.00
Maple	72.87	11.13	14.98	1.01
Maxima	72.27	10.53	0.00	17.21
Fricas	64.98	19.43	2.83	12.75
Sympy	68.02	17.81	1.42	12.75
Giac	74.49	10.32	0.40	14.78
Mupad	0.00	98.18	0.00	1.82

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	5	100.00 %	0.00 %	0.00 %
Maxima	85	94.12 %	3.53 %	2.35 %
Fricas	63	6.35 %	90.48 %	3.17 %
Sympy	63	12.70 %	87.30 %	0.00 %
Giac	73	91.78 %	2.74 %	5.48 %
Mupad	9	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

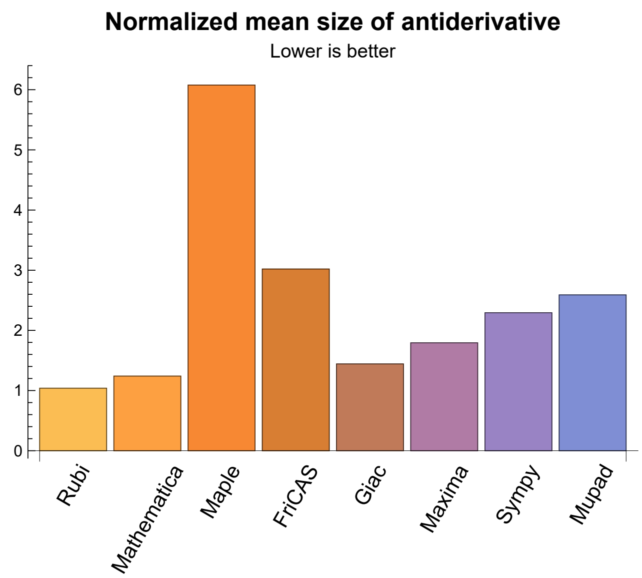
1.3 Performance

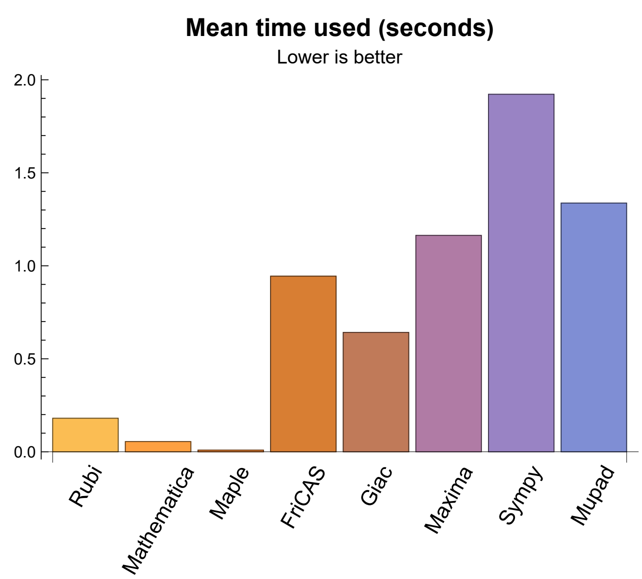
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	119.38	1.04	33.00	1.00
Mathematica	0.05	80.70	1.24	37.00	1.00
Maple	0.01	208.48	6.08	34.00	0.91
Maxima	1.16	115.91	1.79	28.00	0.88
Fricas	0.94	297.22	3.02	36.00	1.10
Sympy	1.92	141.91	2.29	39.00	0.91
Giac	0.64	110.16	1.44	28.00	0.92
Mupad	1.34	450.36	2.59	41.00	0.95

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {31, 32, 151, 152, 153, 154, 155, 156, 157}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

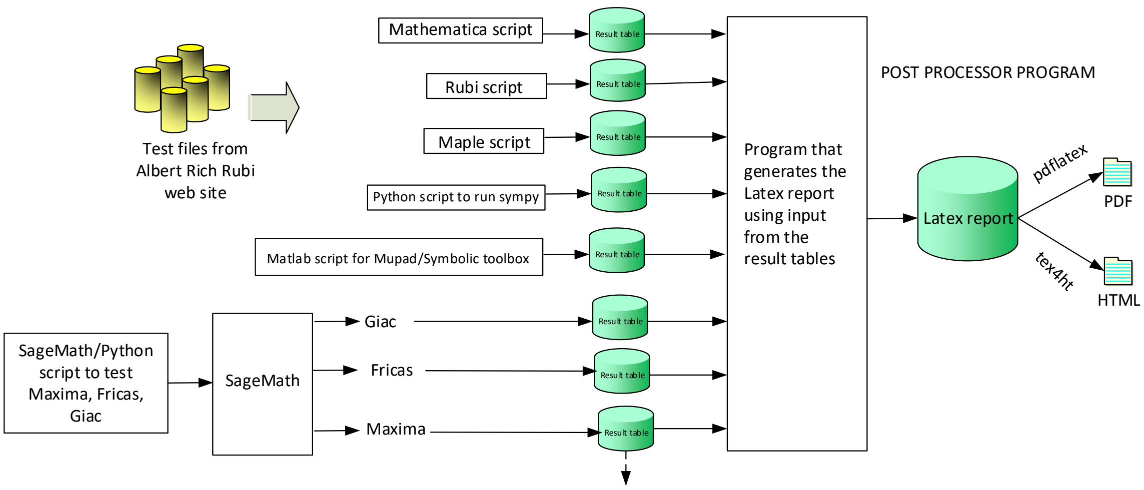
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 65, 77, 221, 222, 233, 424 }

C grade: { 174 }

F grade: { 393, 493, 494 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 193, 194, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494 }

B grade: { 65, 92, 95, 160, 161, 162, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421 }

C grade: { 12, 13, 14, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 173, 174, 175, 176, 184, 185, 227, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 491 }

F grade: { }

2.1.3 Maple

A grade: { 2, 5, 6, 7, 8, 10, 11, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492 }

B grade: { 3, 4, 9, 15, 20, 63, 64, 65, 92, 95, 101, 102, 116, 160, 161, 162, 163, 168, 169, 170, 171, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 227, 232, 337, 338, 400, 421, 424, 443, 491, 494 }

C grade: { 1, 12, 13, 14, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 174, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 493 }

F grade: { 29, 30, 31, 32, 176 }

2.1.4 Maxima

A grade: { 2, 5, 6, 10, 11, 15, 16, 17, 18, 21, 22, 23, 24, 25, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 93, 96, 97, 98, 99, 100, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 164, 165, 166, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492 }

B grade: { 3, 4, 7, 8, 9, 19, 20, 63, 64, 65, 66, 67, 68, 74, 87, 88, 91, 92, 94, 95, 101, 102, 161, 162, 163, 167, 169, 170, 171, 194, 197, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421, 424, 477, 493, 494 }

C grade: { }

F grade: { 1, 12, 13, 14, 26, 29, 30, 31, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 227, 228, 250, 251, 252, 253, 254, 255, 256, 257, 335, 336, 337, 338, 387, 388, 389, 390, 391, 392, 393, 490, 491 }

2.1.5 FriCAS

A grade: { 1, 2, 5, 6, 10, 11, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 96, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 224, 225, 226, 228, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 397, 404, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 3, 4, 7, 8, 9, 12, 13, 14, 15, 37, 38, 43, 44, 63, 64, 65, 66, 67, 68, 74, 75, 76, 77, 87, 91, 92, 93, 94, 95, 97, 116, 120, 121, 122, 146, 154, 160, 161, 162, 163, 168, 169, 170, 171, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 223, 229, 233, 250, 251, 252, 253, 254, 255, 256, 257, 268, 277, 343, 344, 387, 388, 389, 390, 421, 423, 424, 453, 455, 477, 484, 493, 494 }

C grade: { 49, 50, 55, 56, 103, 104, 105, 107, 108, 109, 334, 341, 368, 393 }

F grade: { 19, 20, 29, 30, 31, 32, 61, 62, 110, 111, 112, 114, 115, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 227, 337, 338, 342, 391, 392, 394, 395, 396, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 412, 413, 414, 483 }

2.1.6 Sympy

A grade: { 1, 5, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 27, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 57, 58, 59, 60, 61, 69, 70, 71, 72, 73, 75, 76, 93, 96, 97, 98, 99, 103, 104, 105, 106, 110, 111, 112, 113, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 130, 131, 132, 133, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 172, 177, 178, 187, 188, 189, 191, 208, 219, 220, 224, 225, 226, 228, 229, 230, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 403, 404, 408, 409, 410, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492 }

B grade: { 3, 4, 6, 7, 8, 9, 15, 26, 50, 55, 56, 62, 63, 64, 65, 66, 67, 68, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 94, 95, 121, 122, 128, 129, 134, 135, 160, 161, 162, 168, 169, 170, 179, 184, 185, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 233, 251, 268, 339, 421, 423, 424, 459, 477, 489, 490, 493, 494 }

C grade: { 32, 89, 90, 91, 257, 279, 458 }

F grade: { 2, 18, 19, 20, 28, 29, 30, 31, 44, 100, 101, 102, 107, 108, 109, 114, 115, 136, 137, 138, 139, 140, 141, 142, 163, 171, 173, 174, 175, 176, 180, 181, 182, 183, 186, 190, 209, 217, 218, 227, 234, 235, 236, 237, 238, 239, 240, 241, 255, 337, 338, 340, 341, 342, 398, 399, 400, 405, 406, 407, 412, 413, 414 }

2.1.7 Giac

A grade: { 1, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 160, 164, 165, 166, 168, 169, 170, 172, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 189, 190, 191, 193, 194, 196, 197, 199, 200, 201, 206, 208, 209, 210, 212, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 2, 3, 4, 9, 15, 19, 20, 63, 64, 65, 74, 92, 95, 116, 159, 161, 162, 163, 173, 174, 175, 182, 188, 192, 195, 198, 202, 203, 204, 205, 207, 211, 213, 215, 234, 235, 236, 237, 238, 257, 268, 282, 324, 337, 338, 359, 421, 424, 485, 493, 494 }

C grade: { 55, 56 }

F grade: { 29, 30, 31, 32, 43, 44, 49, 50, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 167, 171, 176, 227, 239, 240, 241, 250, 251, 252, 253, 254, 255, 256, 387, 388, 389, 390, 391, 392, 393, 413 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494 }

C grade: { }

F grade: { 31, 100, 101, 102, 167, 171, 174, 176, 227 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	143	43	0	76	60	55	51
normalized size	1	1.00	1.86	0.56	0.00	0.99	0.78	0.71	0.66
time (sec)	N/A	0.134	0.074	0.054	0.000	0.586	0.334	0.221	0.196

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	23	46	25	43	0	73	52
normalized size	1	1.00	0.77	1.53	0.83	1.43	0.00	2.43	1.73
time (sec)	N/A	0.018	0.105	0.012	0.489	0.864	0.000	0.196	2.132

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	98	216	97	107	97	97
normalized size	1	1.00	1.00	7.00	15.43	6.93	7.64	6.93	6.93
time (sec)	N/A	0.007	0.001	0.003	0.686	0.458	0.088	0.254	2.073

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	99	64	66	64	64
normalized size	1	1.00	1.00	4.64	7.07	4.57	4.71	4.57	4.57
time (sec)	N/A	0.007	0.001	0.001	0.657	0.425	0.079	0.375	0.028

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
normalized size	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.007	0.000	0.001	0.735	0.518	0.066	0.283	0.037

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	24	24	26	12	26
normalized size	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.86
time (sec)	N/A	0.009	0.005	0.023	0.649	0.590	0.183	0.264	0.034

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	57	57	61	12	59
normalized size	1	1.00	1.00	0.93	4.07	4.07	4.36	0.86	4.21
time (sec)	N/A	0.009	0.004	0.003	0.714	0.733	0.349	0.295	2.048

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	90	90	97	12	92
normalized size	1	1.00	1.00	0.93	6.43	6.43	6.93	0.86	6.57
time (sec)	N/A	0.008	0.003	0.005	0.538	0.830	0.543	0.392	2.069

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	159	295	204	166	175	166	149
normalized size	1	1.00	1.89	3.51	2.43	1.98	2.08	1.98	1.77
time (sec)	N/A	0.125	0.025	0.002	0.676	0.516	0.101	0.273	2.077

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	82	84	93	83	87	83	79
normalized size	1	1.00	1.46	1.50	1.66	1.48	1.55	1.48	1.41
time (sec)	N/A	0.068	0.010	0.002	0.636	0.693	0.083	0.344	0.039

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
normalized size	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.006	0.000	0.001	0.550	0.601	0.064	0.270	0.038

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	63	57	0	387	53	212	174
normalized size	1	1.00	0.34	0.30	0.00	2.06	0.28	1.13	0.93
time (sec)	N/A	0.312	0.021	0.005	0.000	0.860	0.406	0.358	0.494

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	112	136	0	704	192	289	247
normalized size	1	1.00	0.46	0.56	0.00	2.87	0.78	1.18	1.01
time (sec)	N/A	0.249	0.066	0.020	0.000	0.821	1.225	0.357	2.645

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	149	276	0	1268	474	366	483
normalized size	1	1.00	0.49	0.90	0.00	4.16	1.55	1.20	1.58
time (sec)	N/A	0.302	0.090	0.018	0.000	0.779	2.542	0.397	2.983

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	653	861	461	987	1018	971	787
normalized size	1	1.00	1.81	2.39	1.28	2.73	2.82	2.69	2.18
time (sec)	N/A	0.658	0.224	0.002	0.680	0.668	0.246	0.268	2.226

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	241	188	180	346	345	346	270
normalized size	1	1.00	1.25	0.97	0.93	1.79	1.79	1.79	1.40
time (sec)	N/A	0.231	0.086	0.002	0.625	0.562	0.135	0.415	0.084

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	76	51	50	62	63	54	54
normalized size	1	1.00	1.36	0.91	0.89	1.11	1.12	0.96	0.96
time (sec)	N/A	0.016	0.000	0.000	0.827	0.696	0.073	0.232	0.045

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	87	112	112	0	137	106
normalized size	1	1.00	0.93	1.01	1.30	1.30	0.00	1.59	1.23
time (sec)	N/A	0.073	0.057	0.011	0.647	10.239	0.000	0.307	2.335

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	232	398	2096	0	0	1414	1940
normalized size	1	1.00	0.99	1.70	8.96	0.00	0.00	6.04	8.29
time (sec)	N/A	0.405	0.552	0.026	1.539	0.000	0.000	0.392	8.176

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	490	1076	11005	0	0	6908	82532
normalized size	1	1.00	0.99	2.17	22.23	0.00	0.00	13.96	166.73
time (sec)	N/A	1.462	1.192	0.034	5.723	0.000	0.000	1.103	20.456

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.015	0.009	0.013	1.453	0.615	0.131	0.305	2.201

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	24	26	25
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.77	0.84	0.81
time (sec)	N/A	0.020	0.007	0.007	1.280	0.753	0.146	0.352	0.053

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.001	0.000	0.001	0.626	0.698	0.064	0.274	0.034

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
normalized size	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.015	0.006	0.009	0.587	0.890	0.180	0.362	0.060

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	18	15	24	18
normalized size	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.012	0.007	0.005	0.793	0.776	0.205	0.296	2.134

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	211	564	62	213
normalized size	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.055	0.082	0.009	0.000	0.864	4.193	0.294	0.465

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	98	299	20	112	99
normalized size	1	1.00	0.77	0.79	0.85	2.60	0.17	0.97	0.86
time (sec)	N/A	0.063	0.036	0.008	1.281	0.832	0.157	0.264	0.233

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	16	0	16	16
normalized size	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00
time (sec)	N/A	0.004	0.002	0.002	0.574	0.846	0.000	0.309	2.500

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0	56
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.02
time (sec)	N/A	0.019	0.012	0.089	0.000	0.461	0.000	0.000	2.206

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	61	0	0	0	0	0	56
normalized size	1	1.11	1.15	0.00	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.023	0.016	0.092	0.000	0.454	0.000	0.000	2.217

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	157	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.360	0.024	0.000	0.453	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	196	0	0	0	34	0	41
normalized size	1	1.26	5.60	0.00	0.00	0.00	0.97	0.00	1.17
time (sec)	N/A	0.009	0.195	0.085	0.000	0.445	10.544	0.000	2.176

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	285	392	372	277	299	277	261
normalized size	1	1.00	1.06	1.45	1.38	1.03	1.11	1.03	0.97
time (sec)	N/A	0.539	0.040	0.002	0.628	0.386	0.127	0.235	2.298

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	231	205	166	180	166	160
normalized size	1	1.00	1.00	1.35	1.20	0.97	1.05	0.97	0.94
time (sec)	N/A	0.092	0.028	0.002	0.802	0.548	0.102	0.243	2.160

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	84	94	83	95	83	82
normalized size	1	1.00	1.00	0.91	1.02	0.90	1.03	0.90	0.89
time (sec)	N/A	0.043	0.021	0.001	0.570	0.399	0.085	0.252	0.036

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
normalized size	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.006	0.000	0.002	0.651	0.385	0.067	0.268	0.037

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	71	64	0	905	88	603	1551
normalized size	1	1.00	0.13	0.12	0.00	1.71	0.17	1.14	2.93
time (sec)	N/A	0.896	0.027	0.059	0.000	0.491	1.149	0.336	4.541

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	182	232	0	3222	427	1057	5844
normalized size	1	1.00	0.24	0.31	0.00	4.32	0.57	1.42	7.83
time (sec)	N/A	1.328	0.116	0.018	0.000	0.545	109.971	0.367	4.304

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	345	500	383	353	366	323	331
normalized size	1	1.00	1.17	1.69	1.30	1.20	1.24	1.09	1.12
time (sec)	N/A	0.532	0.055	0.002	0.745	0.387	0.137	0.308	0.268

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	207	288	214	205	218	187	201
normalized size	1	1.00	1.02	1.42	1.05	1.01	1.07	0.92	0.99
time (sec)	N/A	0.123	0.029	0.001	0.598	0.377	0.109	0.238	2.243

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	109	100	101	99	112	90	98
normalized size	1	1.00	1.02	0.93	0.94	0.93	1.05	0.84	0.92
time (sec)	N/A	0.051	0.015	0.000	0.607	0.391	0.087	0.293	0.044

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	33	33	36	30	33
normalized size	1	1.00	1.00	0.92	0.89	0.89	0.97	0.81	0.89
time (sec)	N/A	0.007	0.000	0.000	0.667	0.377	0.068	0.380	0.043

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	71	67	0	1115	122	0	1264
normalized size	1	1.00	0.46	0.44	0.00	7.29	0.80	0.00	8.26
time (sec)	N/A	0.254	0.033	0.066	0.000	0.481	1.835	0.000	3.731

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	234	288	0	4285	0	0	10351
normalized size	1	1.00	0.68	0.84	0.00	12.53	0.00	0.00	30.27
time (sec)	N/A	0.532	0.193	0.023	0.000	0.734	0.000	0.000	7.109

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	85	84	84	94	84	84
normalized size	1	1.00	1.00	0.89	0.88	0.88	0.98	0.88	0.88
time (sec)	N/A	0.030	0.003	0.002	1.248	0.349	0.075	0.307	0.185

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	65	64	64	71	64	64
normalized size	1	1.00	1.00	0.88	0.86	0.86	0.96	0.86	0.86
time (sec)	N/A	0.023	0.001	0.001	0.939	0.347	0.070	0.334	0.076

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	44	44
normalized size	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.017	0.001	0.000	0.884	0.377	0.062	0.332	0.032

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	19	19
normalized size	1	1.00	1.00	0.87	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.003	0.000	0.000	0.911	0.346	0.057	0.315	0.031

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	45	41	0	1015	41	0	123
normalized size	1	1.00	0.17	0.15	0.00	3.79	0.15	0.00	0.46
time (sec)	N/A	0.396	0.010	0.015	0.000	1.731	0.955	0.000	2.445

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	113	83	0	1201	3834	0	176
normalized size	1	1.00	0.32	0.23	0.00	3.36	10.74	0.00	0.49
time (sec)	N/A	0.397	0.018	0.013	0.000	2.233	3.224	0.000	0.207

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	78	77	77	94	77	77
normalized size	1	1.00	1.00	0.80	0.79	0.79	0.97	0.79	0.79
time (sec)	N/A	0.028	0.002	0.003	0.570	0.343	0.071	0.364	0.150

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.020	0.001	0.001	0.455	0.357	0.066	0.362	0.063

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	37	37	42	37	37
normalized size	1	1.00	1.00	0.84	0.82	0.82	0.93	0.82	0.82
time (sec)	N/A	0.015	0.001	0.001	0.568	0.337	0.062	0.255	0.026

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	19	17	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.90	0.81	0.81
time (sec)	N/A	0.003	0.000	0.000	0.569	0.344	0.056	0.361	0.028

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	47	41	0	499	3432	265	87
normalized size	1	1.00	0.20	0.18	0.00	2.13	14.67	1.13	0.37
time (sec)	N/A	0.315	0.016	0.007	0.000	1.330	2.572	0.520	2.359

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	108	79	0	704	3834	315	174
normalized size	1	1.00	0.34	0.25	0.00	2.22	12.09	0.99	0.55
time (sec)	N/A	0.335	0.024	0.013	0.000	1.345	3.665	0.724	2.207

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	85	84	84	100	84	84
normalized size	1	1.00	1.00	0.82	0.81	0.81	0.96	0.81	0.81
time (sec)	N/A	0.034	0.002	0.002	0.661	0.368	0.081	0.364	2.230

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	65	64	64	73	64	64
normalized size	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84
time (sec)	N/A	0.024	0.001	0.000	0.646	0.344	0.076	0.384	0.078

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	44	44	49	44	44
normalized size	1	1.00	1.00	0.87	0.85	0.85	0.94	0.85	0.85
time (sec)	N/A	0.016	0.001	0.002	0.610	0.343	0.066	0.357	0.033

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	27	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.90	0.80	0.80
time (sec)	N/A	0.004	0.000	0.001	0.462	0.342	0.058	0.300	0.019

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	55	49	0	0	41	0	123
normalized size	1	1.00	0.21	0.19	0.00	0.00	0.16	0.00	0.47
time (sec)	N/A	0.493	0.015	0.008	0.000	0.000	2.372	0.000	0.411

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	128	96	0	0	3839	0	181
normalized size	1	1.00	0.35	0.26	0.00	0.00	10.49	0.00	0.49
time (sec)	N/A	0.508	0.029	0.011	0.000	0.000	3.946	0.000	0.205

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	164	592	163	185	163	163
normalized size	1	1.00	1.00	11.71	42.29	11.64	13.21	11.64	11.64
time (sec)	N/A	0.017	0.002	0.003	0.683	0.707	0.114	0.283	0.167

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	109	228	108	114	108	108
normalized size	1	1.00	1.00	7.79	16.29	7.71	8.14	7.71	7.71
time (sec)	N/A	0.018	0.001	0.002	0.491	0.684	0.101	0.242	0.061

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	61	61	54	53	53	60	53	53
normalized size	1	4.36	4.36	3.86	3.79	3.79	4.29	3.79	3.79
time (sec)	N/A	0.014	0.000	0.001	0.458	0.563	0.074	0.292	0.024

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	46	46	49	12	48
normalized size	1	1.00	1.00	0.93	3.29	3.29	3.50	0.86	3.43
time (sec)	N/A	0.018	0.004	0.006	0.690	0.831	0.288	0.354	0.047

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	101	101	109	12	103
normalized size	1	1.00	1.00	0.93	7.21	7.21	7.79	0.86	7.36
time (sec)	N/A	0.017	0.004	0.005	0.602	0.616	0.593	0.276	2.105

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	156	156	168	12	158
normalized size	1	1.00	1.00	0.93	11.14	11.14	12.00	0.86	11.29
time (sec)	N/A	0.018	0.005	0.005	0.544	0.786	0.936	0.355	3.033

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	30	30	29	31	36
normalized size	1	1.00	1.00	0.82	0.79	0.79	0.76	0.82	0.95
time (sec)	N/A	0.025	0.008	0.009	1.113	0.793	0.154	0.295	2.165

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	65	64	64	80	64	64
normalized size	1	1.00	1.00	0.77	0.76	0.76	0.95	0.76	0.76
time (sec)	N/A	0.086	0.002	0.000	0.488	0.488	0.077	0.380	2.166

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	50	49	49	60	49	49
normalized size	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.076	0.002	0.002	0.727	0.776	0.070	0.365	0.053

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.069	0.001	0.003	0.749	0.650	0.064	0.362	0.024

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	22	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.003	0.000	0.002	0.639	0.716	0.057	0.279	0.032

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	62	42	54	56	63	62	27
normalized size	1	1.00	2.00	1.35	1.74	1.81	2.03	2.00	0.87
time (sec)	N/A	0.023	0.018	0.011	1.055	0.878	0.154	0.282	0.066

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	103	84	89	177	104	97	64
normalized size	1	1.00	1.16	0.94	1.00	1.99	1.17	1.09	0.72
time (sec)	N/A	0.061	0.063	0.023	1.289	0.819	1.359	0.278	0.084

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	137	126	119	282	134	112	93
normalized size	1	1.00	0.85	0.78	0.74	1.75	0.83	0.70	0.58
time (sec)	N/A	0.119	0.092	0.023	1.320	0.884	1.453	0.308	0.089

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	114	223	296	134	126
normalized size	1	2.25	1.45	1.27	1.25	2.45	3.25	1.47	1.38
time (sec)	N/A	0.130	0.103	0.022	1.271	0.872	1.426	0.435	2.184

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	127	81	198	209	77	87
normalized size	1	1.00	0.94	1.63	1.04	2.54	2.68	0.99	1.12
time (sec)	N/A	0.063	0.067	0.012	1.458	0.869	0.694	0.341	2.269

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	54	89	61	157	153	54	206
normalized size	1	1.00	1.08	1.78	1.22	3.14	3.06	1.08	4.12
time (sec)	N/A	0.039	0.033	0.007	1.648	0.803	0.462	0.308	0.090

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	54	50	136	124	43	46
normalized size	1	1.00	0.93	1.32	1.22	3.32	3.02	1.05	1.12
time (sec)	N/A	0.021	0.016	0.003	1.362	0.822	0.250	0.356	2.076

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	24	83	54	17	17
normalized size	1	1.00	1.00	1.33	1.14	3.95	2.57	0.81	0.81
time (sec)	N/A	0.008	0.004	0.003	1.520	0.659	0.186	0.377	0.043

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	72	68	154	738	62	173
normalized size	1	1.00	0.81	1.22	1.15	2.61	12.51	1.05	2.93
time (sec)	N/A	0.035	0.041	0.007	1.518	0.841	3.403	0.377	2.588

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	81	123	123	229	1620	117	425
normalized size	1	1.00	1.03	1.56	1.56	2.90	20.51	1.48	5.38
time (sec)	N/A	0.086	0.053	0.011	1.591	0.920	11.116	0.367	2.583

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	198	197	371	3284	195	573
normalized size	1	1.00	0.88	1.64	1.63	3.07	27.14	1.61	4.74
time (sec)	N/A	0.125	0.166	0.010	1.612	0.846	38.258	0.415	2.771

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	30	109	61	24	27
normalized size	1	1.00	1.00	1.10	0.97	3.52	1.97	0.77	0.87
time (sec)	N/A	0.024	0.012	0.006	1.561	0.723	0.207	0.453	0.057

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	86	75	253	117	65	76
normalized size	1	1.00	0.95	1.37	1.19	4.02	1.86	1.03	1.21
time (sec)	N/A	0.033	0.035	0.004	1.560	0.562	0.580	0.359	0.098

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	147	184	595	257	103	181
normalized size	1	1.00	0.82	1.62	2.02	6.54	2.82	1.13	1.99
time (sec)	N/A	0.048	0.071	0.005	1.585	1.008	1.248	0.409	2.221

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	42	66	279	92	30	31
normalized size	1	1.00	1.00	1.20	1.89	7.97	2.63	0.86	0.89
time (sec)	N/A	0.035	0.019	0.010	1.404	0.873	0.218	0.446	0.103

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	18	10	24	10	10
normalized size	1	1.00	1.00	1.10	1.80	1.00	2.40	1.00	1.00
time (sec)	N/A	0.003	0.005	0.003	1.569	0.850	0.168	0.318	0.040

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	59	51	55	56	41	42
normalized size	1	1.00	0.84	1.59	1.38	1.49	1.51	1.11	1.14
time (sec)	N/A	0.010	0.014	0.006	1.315	0.815	0.437	0.377	2.066

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	94	115	153	146	73	111
normalized size	1	1.00	0.87	1.57	1.92	2.55	2.43	1.22	1.85
time (sec)	N/A	0.016	0.020	0.006	1.505	0.839	0.931	0.304	0.121

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	32	26	25	22	22	27	10
normalized size	1	1.00	3.20	2.60	2.50	2.20	2.20	2.70	1.00
time (sec)	N/A	0.003	0.006	0.010	0.490	0.792	0.177	0.384	2.050

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	45	52	56	85	54	56	43
normalized size	1	1.00	1.15	1.33	1.44	2.18	1.38	1.44	1.10
time (sec)	N/A	0.013	0.026	0.010	0.624	0.831	0.469	0.351	2.064

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	78	122	220	141	88	114
normalized size	1	1.00	1.02	1.22	1.91	3.44	2.20	1.38	1.78
time (sec)	N/A	0.022	0.032	0.010	0.641	0.845	1.030	0.357	2.118

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	15	12	11	11	10	13	4
normalized size	1	1.00	3.75	3.00	2.75	2.75	2.50	3.25	1.00
time (sec)	N/A	0.002	0.002	0.005	0.571	0.888	0.099	0.355	0.149

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
normalized size	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.007	0.018	0.008	0.649	0.792	0.117	0.325	0.066

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	37	36	44	71	44	39	36
normalized size	1	1.00	0.82	0.80	0.98	1.58	0.98	0.87	0.80
time (sec)	N/A	0.012	0.018	0.010	0.644	0.828	0.144	0.339	2.090

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	64	64	54	54	58	62	55
normalized size	1	1.00	1.08	1.08	0.92	0.92	0.98	1.05	0.93
time (sec)	N/A	0.055	0.028	0.004	0.542	0.844	0.171	0.310	0.050

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	12	11	11	10	11	11
normalized size	1	1.00	1.10	1.20	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.011	0.007	0.001	0.678	0.677	0.086	0.353	0.032

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	35	36	35	0	23	-1
normalized size	1	1.00	1.16	0.80	0.82	0.80	0.00	0.52	-0.02
time (sec)	N/A	0.026	0.032	0.006	1.289	0.869	0.000	0.386	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	152	139	92	0	55	-1
normalized size	1	1.00	0.82	2.27	2.07	1.37	0.00	0.82	-0.01
time (sec)	N/A	0.049	0.086	0.020	1.462	0.934	0.000	0.486	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	146	135	70	0	70	-1
normalized size	1	1.00	0.81	2.32	2.14	1.11	0.00	1.11	-0.02
time (sec)	N/A	0.040	0.088	0.011	0.587	0.849	0.000	0.417	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	132	108	0	6315	238	0	374
normalized size	1	1.00	0.56	0.46	0.00	26.99	1.02	0.00	1.60
time (sec)	N/A	0.372	0.055	0.006	0.000	30.343	2.901	0.000	2.460

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	81	74	0	4759	158	0	437
normalized size	1	1.00	0.39	0.35	0.00	22.66	0.75	0.00	2.08
time (sec)	N/A	0.228	0.035	0.004	0.000	2.967	0.992	0.000	2.300

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	79	72	0	1950	83	0	145
normalized size	1	1.00	0.44	0.40	0.00	10.83	0.46	0.00	0.81
time (sec)	N/A	0.156	0.022	0.003	0.000	2.568	0.700	0.000	0.257

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	116	71	0	442	26	160	144
normalized size	1	1.00	0.83	0.51	0.00	3.16	0.19	1.14	1.03
time (sec)	N/A	0.107	0.033	0.002	0.000	0.924	0.261	0.452	2.311

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	238	119	105	0	4370	0	0	553
normalized size	1	1.06	0.53	0.47	0.00	19.51	0.00	0.00	2.47
time (sec)	N/A	0.482	0.055	0.010	0.000	2.819	0.000	0.000	0.123

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	312	173	144	0	8919	0	0	1588
normalized size	1	0.99	0.55	0.46	0.00	28.40	0.00	0.00	5.06
time (sec)	N/A	0.545	0.105	0.010	0.000	4.139	0.000	0.000	2.329

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	244	217	0	14765	0	0	1328
normalized size	1	1.00	0.62	0.55	0.00	37.57	0.00	0.00	3.38
time (sec)	N/A	0.604	0.170	0.012	0.000	11.496	0.000	0.000	2.489

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	106	97	0	0	374	0	1003
normalized size	1	1.00	0.30	0.27	0.00	0.00	1.05	0.00	2.82
time (sec)	N/A	0.426	0.055	0.016	0.000	0.000	3.736	0.000	2.692

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	106	97	0	0	274	0	625
normalized size	1	1.00	0.33	0.31	0.00	0.00	0.86	0.00	1.97
time (sec)	N/A	0.312	0.036	0.005	0.000	0.000	2.671	0.000	2.588

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	104	95	0	0	131	0	205
normalized size	1	1.00	0.40	0.36	0.00	0.00	0.50	0.00	0.79
time (sec)	N/A	0.264	0.034	0.007	0.000	0.000	0.879	0.000	2.357

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	161	94	0	189	26	103	60
normalized size	1	1.00	0.73	0.43	0.00	0.86	0.12	0.47	0.27
time (sec)	N/A	0.185	0.099	0.005	0.000	0.492	0.301	0.501	0.116

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	163	139	0	0	0	0	882
normalized size	1	1.00	0.41	0.35	0.00	0.00	0.00	0.00	2.24
time (sec)	N/A	0.466	0.073	0.008	0.000	0.000	0.000	0.000	2.183

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	238	188	0	0	0	0	2440
normalized size	1	1.00	0.48	0.38	0.00	0.00	0.00	0.00	4.92
time (sec)	N/A	0.894	0.132	0.013	0.000	0.000	0.000	0.000	2.479

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	195	264	192	219	199	219	175
normalized size	1	1.00	1.59	2.15	1.56	1.78	1.62	1.78	1.42
time (sec)	N/A	0.240	0.037	0.003	0.692	0.365	0.118	0.328	0.215

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	114	138	119	128	114	128	108
normalized size	1	1.00	0.95	1.15	0.99	1.07	0.95	1.07	0.90
time (sec)	N/A	0.063	0.020	0.000	0.667	0.364	0.090	0.296	0.095

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	63	65	65	65	65	61
normalized size	1	1.00	0.92	0.88	0.90	0.90	0.90	0.90	0.85
time (sec)	N/A	0.030	0.008	0.000	0.707	0.371	0.076	0.313	0.039

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	22	22	22
normalized size	1	1.00	1.00	0.88	0.85	0.85	0.85	0.85	0.85
time (sec)	N/A	0.004	0.000	0.000	0.627	0.354	0.061	0.303	0.018

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	49	0	457	66	0	571
normalized size	1	1.00	0.64	0.55	0.00	5.13	0.74	0.00	6.42
time (sec)	N/A	0.087	0.023	0.016	0.000	0.452	0.933	0.000	2.576

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	158	0	1948	294	0	4591
normalized size	1	1.00	0.89	0.93	0.00	11.53	1.74	0.00	27.17
time (sec)	N/A	0.291	0.075	0.013	0.000	0.437	6.317	0.000	5.352

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	254	398	0	3971	697	0	8242
normalized size	1	1.00	1.01	1.58	0.00	15.76	2.77	0.00	32.71
time (sec)	N/A	0.532	0.150	0.023	0.000	0.517	15.616	0.000	6.405

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	182	222	212	222	178
normalized size	1	1.00	0.97	1.27	0.87	1.06	1.01	1.06	0.85
time (sec)	N/A	0.226	0.035	0.003	0.621	0.360	0.123	0.390	0.217

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	130	143	113	133	128	133	113
normalized size	1	1.00	0.97	1.07	0.84	0.99	0.96	0.99	0.84
time (sec)	N/A	0.140	0.020	0.002	0.624	0.348	0.098	0.351	2.120

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	66	59	68	70	68	64
normalized size	1	1.00	0.95	0.84	0.75	0.86	0.89	0.86	0.81
time (sec)	N/A	0.076	0.008	0.000	0.616	0.386	0.077	0.348	0.038

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.010	0.002	0.000	0.611	0.397	0.061	0.360	0.022

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	59	50	0	0	155	0	275
normalized size	1	1.00	0.51	0.43	0.00	0.00	1.34	0.00	2.37
time (sec)	N/A	0.083	0.017	0.004	0.000	0.000	4.430	0.000	2.582

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	166	162	0	0	539	0	1167
normalized size	1	1.00	0.72	0.70	0.00	0.00	2.33	0.00	5.05
time (sec)	N/A	0.241	0.072	0.012	0.000	0.000	31.420	0.000	2.817

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	284	405	0	0	1102	0	2200
normalized size	1	1.00	0.81	1.16	0.00	0.00	3.16	0.00	6.30
time (sec)	N/A	0.369	0.164	0.023	0.000	0.000	88.090	0.000	3.394

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	182	222	219	222	178
normalized size	1	1.00	0.97	1.27	0.87	1.06	1.04	1.06	0.85
time (sec)	N/A	0.164	0.036	0.001	0.639	0.357	0.145	0.243	2.294

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	132	143	113	133	134	133	113
normalized size	1	1.00	0.96	1.04	0.82	0.96	0.97	0.96	0.82
time (sec)	N/A	0.121	0.019	0.003	0.563	0.342	0.097	0.285	0.092

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	66	59	68	73	68	64
normalized size	1	1.00	0.92	0.84	0.75	0.86	0.92	0.86	0.81
time (sec)	N/A	0.077	0.010	0.001	0.546	0.357	0.077	0.287	0.042

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.009	0.004	0.000	0.609	0.345	0.062	0.313	0.021

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	61	52	0	0	172	0	878
normalized size	1	1.00	0.62	0.53	0.00	0.00	1.74	0.00	8.87
time (sec)	N/A	0.087	0.016	0.004	0.000	0.000	7.607	0.000	2.783

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	182	160	0	0	561	0	1218
normalized size	1	1.00	0.81	0.71	0.00	0.00	2.49	0.00	5.41
time (sec)	N/A	0.213	0.076	0.014	0.000	0.000	43.733	0.000	2.846

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	99	93	0	0	0	0	1563
normalized size	1	1.00	0.18	0.17	0.00	0.00	0.00	0.00	2.87
time (sec)	N/A	1.476	0.069	0.015	0.000	0.000	0.000	0.000	3.175

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	99	93	0	0	0	0	1354
normalized size	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.78
time (sec)	N/A	0.756	0.049	0.003	0.000	0.000	0.000	0.000	3.104

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	97	93	0	0	0	0	825
normalized size	1	1.00	0.29	0.28	0.00	0.00	0.00	0.00	2.47
time (sec)	N/A	0.470	0.059	0.005	0.000	0.000	0.000	0.000	3.342

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	95	91	0	0	0	0	1057
normalized size	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.25
time (sec)	N/A	0.684	0.047	0.005	0.000	0.000	0.000	0.000	2.903

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	99	90	0	0	0	0	1394
normalized size	1	1.00	0.19	0.17	0.00	0.00	0.00	0.00	2.67
time (sec)	N/A	0.861	0.066	0.005	0.000	0.000	0.000	0.000	0.711

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	157	134	0	0	0	0	4002
normalized size	1	1.00	0.28	0.24	0.00	0.00	0.00	0.00	7.11
time (sec)	N/A	1.157	0.115	0.009	0.000	0.000	0.000	0.000	2.546

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	640	163	133	0	0	0	0	2663
normalized size	1	0.99	0.25	0.21	0.00	0.00	0.00	0.00	4.13
time (sec)	N/A	1.380	0.139	0.008	0.000	0.000	0.000	0.000	2.721

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	61	56	0	0	70	0	427
normalized size	1	1.00	0.15	0.14	0.00	0.00	0.18	0.00	1.08
time (sec)	N/A	1.438	0.023	0.008	0.000	0.000	0.257	0.000	0.651

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	61	56	0	0	65	0	390
normalized size	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	1.03
time (sec)	N/A	0.910	0.014	0.009	0.000	0.000	0.275	0.000	2.704

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	61	56	0	0	61	0	276
normalized size	1	1.00	0.17	0.16	0.00	0.00	0.17	0.00	0.76
time (sec)	N/A	0.515	0.015	0.009	0.000	0.000	0.247	0.000	0.522

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	59	56	0	1277	48	0	247
normalized size	1	1.00	0.24	0.23	0.00	5.15	0.19	0.00	1.00
time (sec)	N/A	0.323	0.013	0.008	0.000	2.004	0.205	0.000	2.679

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	57	54	0	0	61	0	176
normalized size	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	0.49
time (sec)	N/A	0.548	0.016	0.008	0.000	0.000	0.257	0.000	2.423

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	62	53	0	0	65	0	306
normalized size	1	1.00	0.16	0.14	0.00	0.00	0.17	0.00	0.81
time (sec)	N/A	0.721	0.013	0.007	0.000	0.000	0.274	0.000	2.675

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	103	75	0	0	82	0	432
normalized size	1	1.00	0.25	0.18	0.00	0.00	0.20	0.00	1.04
time (sec)	N/A	0.902	0.019	0.013	0.000	0.000	0.417	0.000	2.328

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	109	74	0	0	70	0	340
normalized size	1	1.00	0.24	0.17	0.00	0.00	0.16	0.00	0.76
time (sec)	N/A	1.102	0.021	0.011	0.000	0.000	0.319	0.000	0.292

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	167	122	0	0	112	0	388
normalized size	1	1.00	0.16	0.11	0.00	0.00	0.11	0.00	0.36
time (sec)	N/A	2.504	0.040	0.015	0.000	0.000	0.397	0.000	2.341

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1005	1005	167	122	0	0	112	0	387
normalized size	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	2.403	0.030	0.016	0.000	0.000	0.403	0.000	2.304

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	677	677	167	122	0	0	112	0	388
normalized size	1	1.00	0.25	0.18	0.00	0.00	0.17	0.00	0.57
time (sec)	N/A	1.553	0.045	0.015	0.000	0.000	0.378	0.000	2.328

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	682	682	167	122	0	1445	104	0	299
normalized size	1	1.00	0.24	0.18	0.00	2.12	0.15	0.00	0.44
time (sec)	N/A	1.235	0.026	0.012	0.000	2.972	0.307	0.000	0.314

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	850	850	167	122	0	0	112	0	388
normalized size	1	1.00	0.20	0.14	0.00	0.00	0.13	0.00	0.46
time (sec)	N/A	1.470	0.036	0.013	0.000	0.000	0.388	0.000	2.420

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	873	873	167	122	0	0	112	0	387
normalized size	1	1.00	0.19	0.14	0.00	0.00	0.13	0.00	0.44
time (sec)	N/A	1.916	0.028	0.013	0.000	0.000	0.397	0.000	2.416

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	986	986	167	122	0	0	112	0	388
normalized size	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	1.927	0.035	0.014	0.000	0.000	0.387	0.000	2.478

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.055	0.002	0.000	0.443	0.708	0.094	0.370	0.037

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	114	105	105	88	365	106
normalized size	1	1.00	0.84	1.21	1.12	1.12	0.94	3.88	1.13
time (sec)	N/A	0.127	0.037	0.005	0.439	0.963	0.308	0.263	0.059

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	155	13	154	175	13	154
normalized size	1	1.00	11.47	10.33	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.018	0.006	0.003	0.436	0.655	0.131	0.300	2.224

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	156	156
normalized size	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.055	0.006	0.002	0.608	0.816	0.136	0.313	0.143

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	156	156
normalized size	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.050	0.006	0.002	0.552	0.684	0.139	0.332	2.171

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	262	0	189	229
normalized size	1	1.00	1.00	10.95	10.90	12.48	0.00	9.00	10.90
time (sec)	N/A	0.031	0.118	0.052	0.645	1.042	0.000	1.857	4.019

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
normalized size	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.004	0.004	0.000	0.555	0.776	0.127	0.230	0.050

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	18	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.87
time (sec)	N/A	0.026	0.006	0.004	0.644	0.854	0.179	0.334	2.081

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.027	0.005	0.006	0.525	1.003	0.197	0.314	0.060

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	47	23	29	0	-1
normalized size	1	1.00	1.00	1.20	3.13	1.53	1.93	0.00	-0.07
time (sec)	N/A	0.025	0.011	0.018	0.577	0.801	1.483	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	13	81	87	13	12
normalized size	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.004	0.021	0.018	0.528	0.743	0.933	0.270	4.377

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.023	0.029	0.019	0.670	0.821	1.428	0.406	2.225

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.023	0.036	0.016	0.800	0.815	2.061	0.304	7.220

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	143	0	0	-1
normalized size	1	1.00	1.00	9.67	29.14	6.81	0.00	0.00	-0.05
time (sec)	N/A	0.032	0.178	0.064	1.034	1.011	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	24	19	26	46	19	23
normalized size	1	1.00	0.89	1.26	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.005	0.009	0.005	0.559	0.785	0.710	0.374	2.095

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	97	31	35	32	0	54	45
normalized size	1	1.00	3.59	1.15	1.30	1.19	0.00	2.00	1.67
time (sec)	N/A	0.023	0.075	0.004	0.990	1.213	0.000	0.334	2.213

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	116	97	142	35	33	0	54	-1
normalized size	1	4.30	3.59	5.26	1.30	1.22	0.00	2.00	-0.04
time (sec)	N/A	0.102	0.027	0.259	1.073	0.748	0.000	0.494	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	99	33	35	34	0	58	49
normalized size	1	1.00	3.41	1.14	1.21	1.17	0.00	2.00	1.69
time (sec)	N/A	0.024	0.077	0.005	1.013	1.007	0.000	0.346	2.205

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	108	0	39	42	0	0	-1
normalized size	1	1.00	3.00	0.00	1.08	1.17	0.00	0.00	-0.03
time (sec)	N/A	0.086	0.166	0.135	1.072	0.872	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.034	0.002	0.001	0.781	0.845	0.095	0.313	2.142

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.049	0.001	0.001	0.472	0.820	0.095	0.396	0.019

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	32
normalized size	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.76
time (sec)	N/A	0.057	0.015	0.005	1.489	1.012	0.297	0.404	2.130

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	25	38	0	25	54
normalized size	1	1.00	0.92	1.04	1.00	1.52	0.00	1.00	2.16
time (sec)	N/A	0.024	0.015	0.003	0.672	0.836	0.000	0.294	2.191

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	21	34	24	36	0	24	46
normalized size	1	1.00	0.88	1.42	1.00	1.50	0.00	1.00	1.92
time (sec)	N/A	0.018	0.047	0.005	0.629	0.868	0.000	0.307	2.124

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	26	39	35	0	65	51
normalized size	1	1.00	0.96	1.04	1.56	1.40	0.00	2.60	2.04
time (sec)	N/A	0.021	0.022	0.006	0.862	1.016	0.000	0.395	2.155

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	28	0	20	39
normalized size	1	1.00	1.00	1.05	1.00	1.40	0.00	1.00	1.95
time (sec)	N/A	0.010	0.008	0.004	0.651	0.831	0.000	0.282	2.143

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	106	26	19	26	73	19	25
normalized size	1	1.00	5.58	1.37	1.00	1.37	3.84	1.00	1.32
time (sec)	N/A	0.009	0.066	0.005	0.534	0.913	11.344	0.341	2.127

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	108	23	31	27	76	39	26
normalized size	1	1.00	4.91	1.05	1.41	1.23	3.45	1.77	1.18
time (sec)	N/A	0.008	0.036	0.003	1.209	1.072	52.838	0.338	2.161

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	0	22	43
normalized size	1	1.00	0.95	1.05	1.00	1.45	0.00	1.00	1.95
time (sec)	N/A	0.018	0.012	0.004	0.661	0.767	0.000	0.345	2.138

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	28	21	30	53	21	27
normalized size	1	1.00	0.90	1.33	1.00	1.43	2.52	1.00	1.29
time (sec)	N/A	0.012	0.011	0.004	0.594	0.805	1.160	0.310	2.162

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	28	32	31	56	51	28
normalized size	1	1.00	0.92	1.17	1.33	1.29	2.33	2.12	1.17
time (sec)	N/A	0.023	0.014	0.003	0.826	0.914	6.170	0.460	2.199

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	32	29	53	41	26
normalized size	1	1.00	1.00	1.05	1.45	1.32	2.41	1.86	1.18
time (sec)	N/A	0.009	0.010	0.005	0.895	0.981	6.154	0.314	2.188

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	32	32	0	22	43
normalized size	1	1.00	0.95	1.05	1.45	1.45	0.00	1.00	1.95
time (sec)	N/A	0.013	0.006	0.004	0.751	1.228	0.000	0.323	2.142

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	28	32	30	53	21	27
normalized size	1	1.00	0.90	1.33	1.52	1.43	2.52	1.00	1.29
time (sec)	N/A	0.012	0.005	0.003	0.930	0.757	1.142	0.398	2.148

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	143	25686	19	1956	1771	160	1576
normalized size	1	1.00	6.81	1223.14	0.90	93.14	84.33	7.62	75.05
time (sec)	N/A	0.125	0.151	0.004	0.740	0.558	0.420	0.299	2.979

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	5596	18	496	469	18	418
normalized size	1	1.00	0.90	279.80	0.90	24.80	23.45	0.90	20.90
time (sec)	N/A	0.052	0.036	0.002	0.633	0.498	0.182	0.314	2.320

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	5596	441	496	469	18	418
normalized size	1	1.00	0.95	294.53	23.21	26.11	24.68	0.95	22.00
time (sec)	N/A	0.069	0.017	0.002	0.601	0.524	0.170	0.297	2.267

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	127	2185	14	486	483	120	438
normalized size	1	1.00	7.94	136.56	0.88	30.38	30.19	7.50	27.38
time (sec)	N/A	0.024	0.056	0.001	0.549	0.529	0.170	0.413	2.628

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	98	89	13	88	97	13	88
normalized size	1	1.00	6.53	5.93	0.87	5.87	6.47	0.87	5.87
time (sec)	N/A	0.013	0.003	0.000	0.511	0.647	0.096	0.301	0.050

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	98	89	88	88	97	13	88
normalized size	1	1.00	6.12	5.56	5.50	5.50	6.06	0.81	5.50
time (sec)	N/A	0.028	0.003	0.000	0.645	0.774	0.088	0.268	0.040

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	115	2205	16	488	484	136	440
normalized size	1	1.00	6.39	122.50	0.89	27.11	26.89	7.56	24.44
time (sec)	N/A	0.044	0.053	0.003	0.623	0.725	0.173	0.430	2.632

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	98	89	15	88	97	15	88
normalized size	1	1.00	5.76	5.24	0.88	5.18	5.71	0.88	5.18
time (sec)	N/A	0.025	0.003	0.001	0.661	0.571	0.098	0.373	2.072

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	15	88
normalized size	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.227	0.003	0.001	0.525	0.728	0.095	0.234	0.047

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	15	88
normalized size	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.011	0.003	0.002	0.659	0.625	0.088	0.256	0.041

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	115	2205	458	488	484	488	440
normalized size	1	1.00	6.39	122.50	25.44	27.11	26.89	27.11	24.44
time (sec)	N/A	0.060	0.011	0.002	0.595	1.061	0.169	0.317	0.568

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	88	88
normalized size	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.022	0.003	0.001	0.598	0.711	0.094	0.306	0.040

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	98	89	88	88	97	88	88
normalized size	1	1.00	5.44	4.94	4.89	4.89	5.39	4.89	4.89
time (sec)	N/A	0.019	0.003	0.002	0.570	0.559	0.096	0.294	0.040

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	88	88
normalized size	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.002	0.002	0.000	0.670	0.794	0.088	0.243	0.039

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	80	67	66	66	70	24	66
normalized size	1	1.00	2.86	2.39	2.36	2.36	2.50	0.86	2.36
time (sec)	N/A	0.027	0.005	0.002	0.610	0.713	0.086	0.366	0.051

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	108	325	187	208	194	88	180
normalized size	1	1.00	3.48	10.48	6.03	6.71	6.26	2.84	5.81
time (sec)	N/A	0.031	0.035	0.001	0.438	0.554	0.114	0.366	0.105

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	52	48	230	30	31
normalized size	1	1.00	1.00	0.91	1.53	1.41	6.76	0.88	0.91
time (sec)	N/A	0.008	0.059	0.004	1.133	0.793	50.746	0.403	2.121

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	54	52	0	32	58
normalized size	1	1.00	1.00	0.94	1.54	1.49	0.00	0.91	1.66
time (sec)	N/A	0.009	0.056	0.003	1.159	0.947	0.000	0.251	2.110

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	93	78	77	77	87	24	77
normalized size	1	1.00	3.10	2.60	2.57	2.57	2.90	0.80	2.57
time (sec)	N/A	0.020	0.006	0.002	0.438	0.759	0.095	0.220	0.055

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	140	618	280	291	314	105	266
normalized size	1	1.00	4.52	19.94	9.03	9.39	10.13	3.39	8.58
time (sec)	N/A	0.038	0.046	0.002	0.455	0.993	0.143	0.336	2.266

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	98	81	80	80	90	30	80
normalized size	1	1.00	2.88	2.38	2.35	2.35	2.65	0.88	2.35
time (sec)	N/A	0.033	0.008	0.002	0.439	0.682	0.098	0.294	0.065

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	146	646	289	298	321	126	273
normalized size	1	1.00	3.56	15.76	7.05	7.27	7.83	3.07	6.66
time (sec)	N/A	0.045	0.051	0.001	0.446	0.931	0.150	0.312	2.283

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	244	1523	289	309	323	37	270
normalized size	1	1.00	5.30	33.11	6.28	6.72	7.02	0.80	5.87
time (sec)	N/A	0.047	0.063	0.002	0.453	0.682	0.157	0.307	2.237

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	248	4284	773	928	930	153	753
normalized size	1	1.00	5.28	91.15	16.45	19.74	19.79	3.26	16.02
time (sec)	N/A	0.091	0.123	0.002	0.480	0.627	0.269	0.442	2.450

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	31	54	48	201	30	33
normalized size	1	1.00	1.06	0.91	1.59	1.41	5.91	0.88	0.97
time (sec)	N/A	0.009	0.068	0.004	1.143	1.017	112.002	0.417	2.111

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	71	57	0	36	37
normalized size	1	1.00	0.95	0.84	1.61	1.30	0.00	0.82	0.84
time (sec)	N/A	0.010	0.086	0.003	1.173	0.643	0.000	0.392	2.210

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	43	83	72	0	42	73
normalized size	1	1.00	0.98	0.86	1.66	1.44	0.00	0.84	1.46
time (sec)	N/A	0.009	0.193	0.003	1.218	0.831	0.000	0.391	2.170

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	30	17	29	29	30	29
normalized size	1	1.00	1.74	1.58	0.89	1.53	1.53	1.58	1.53
time (sec)	N/A	0.009	0.002	0.001	0.434	0.773	0.061	0.346	0.025

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	18	14	17	17	22	17
normalized size	1	1.00	1.31	1.12	0.88	1.06	1.06	1.38	1.06
time (sec)	N/A	0.007	0.002	0.000	0.439	0.816	0.057	0.233	0.034

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	86	86	94	28	86
normalized size	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.198	0.006	0.001	0.437	0.718	0.082	0.288	0.218

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	86	86	94	28	86
normalized size	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.154	0.005	0.001	0.449	0.723	0.091	0.360	0.191

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	57	56	12	12
normalized size	1	1.00	1.00	0.93	0.86	4.07	4.00	0.86	0.86
time (sec)	N/A	0.008	0.006	0.001	0.436	0.829	0.189	0.316	2.100

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87
time (sec)	N/A	0.009	0.004	0.002	0.442	0.786	0.089	0.371	0.049

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	15	15	14	18	13
normalized size	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76
time (sec)	N/A	0.010	0.006	0.003	0.448	0.799	0.099	0.270	0.066

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	28	23	23	22	24	23
normalized size	1	1.00	0.58	0.70	0.58	0.58	0.55	0.60	0.58
time (sec)	N/A	0.097	0.059	0.013	0.503	0.856	32.564	0.536	0.108

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	98	2105	0	0	0	0	-1
normalized size	1	1.00	0.16	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.535	0.069	0.088	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	82	0	83	58	58	75
normalized size	1	1.00	0.87	1.30	0.00	1.32	0.92	0.92	1.19
time (sec)	N/A	0.066	0.029	0.049	0.000	0.756	0.126	0.274	0.185

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	38	20	13	12
normalized size	1	1.00	1.00	0.93	1.57	2.71	1.43	0.93	0.86
time (sec)	N/A	0.050	0.008	0.007	0.436	0.892	0.096	0.289	0.040

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	27	32	46	29	23	21
normalized size	1	1.00	0.86	0.96	1.14	1.64	1.04	0.82	0.75
time (sec)	N/A	0.028	0.013	0.006	0.446	1.000	0.109	0.392	0.037

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	28	30	38	38	36	28	28
normalized size	1	1.00	0.47	0.51	0.64	0.64	0.61	0.47	0.47
time (sec)	N/A	0.091	0.012	0.006	0.463	0.838	0.181	1.436	0.054

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	11	11	8	11	11
normalized size	1	1.00	1.00	3.73	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.007	0.007	0.009	0.438	0.733	0.117	0.331	2.301

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	114	223	272	134	124
normalized size	1	2.25	1.45	1.27	1.25	2.45	2.99	1.47	1.36
time (sec)	N/A	0.149	0.090	0.019	0.987	0.941	1.362	0.372	2.369

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	44	40	0	99	49
normalized size	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.96
time (sec)	N/A	0.025	0.375	0.010	0.699	1.247	0.000	2.090	2.656

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
normalized size	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.082	0.233	0.010	0.656	0.707	0.000	1.887	2.549

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
normalized size	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.059	0.177	0.008	0.629	0.892	0.000	0.728	2.319

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	37	37	0	87	37
normalized size	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.76
time (sec)	N/A	0.044	0.122	0.005	0.608	0.871	0.000	1.083	2.273

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	33	33	0	52	19
normalized size	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.00
time (sec)	N/A	0.043	0.010	0.006	0.621	0.898	0.000	0.296	2.193

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
normalized size	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.034	0.176	0.009	0.619	0.811	0.000	0.000	3.202

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
normalized size	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.035	0.199	0.008	0.636	1.041	0.000	0.000	3.319

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
normalized size	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.036	0.188	0.008	0.642	1.035	0.000	0.000	3.355

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	73	79	97	73	97
normalized size	1	1.00	0.86	0.76	0.75	0.81	1.00	0.75	1.00
time (sec)	N/A	0.138	0.036	0.007	1.520	0.687	0.245	0.309	0.194

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	69	68	74	92	68	92
normalized size	1	1.00	0.87	0.77	0.76	0.82	1.02	0.76	1.02
time (sec)	N/A	0.123	0.023	0.006	1.880	0.800	0.247	0.378	0.178

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	62	61	67	78	61	85
normalized size	1	1.00	0.94	0.81	0.79	0.87	1.01	0.79	1.10
time (sec)	N/A	0.121	0.031	0.005	1.441	0.545	0.232	0.391	2.290

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	57	56	62	75	56	80
normalized size	1	1.00	0.96	0.79	0.78	0.86	1.04	0.78	1.11
time (sec)	N/A	0.093	0.023	0.005	1.586	1.037	0.232	0.307	2.292

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	56	55	61	75	55	79
normalized size	1	1.00	0.92	0.79	0.77	0.86	1.06	0.77	1.11
time (sec)	N/A	0.078	0.017	0.006	1.316	0.800	0.225	0.287	0.147

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	60	59	65	78	60	83
normalized size	1	1.00	0.92	0.80	0.79	0.87	1.04	0.80	1.11
time (sec)	N/A	0.135	0.019	0.007	1.571	0.965	0.288	0.307	0.149

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	65	64	76	87	65	88
normalized size	1	1.00	0.93	0.77	0.76	0.90	1.04	0.77	1.05
time (sec)	N/A	0.152	0.033	0.009	1.441	0.885	0.315	0.257	2.280

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	69	89	94	70	92
normalized size	1	1.00	0.90	0.77	0.76	0.98	1.03	0.77	1.01
time (sec)	N/A	0.157	0.055	0.010	1.264	0.966	0.321	0.286	0.152

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	109	74	0	1202	61	0	128
normalized size	1	1.00	0.36	0.24	0.00	3.92	0.20	0.00	0.42
time (sec)	N/A	0.578	0.021	0.008	0.000	2.942	0.985	0.000	2.169

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	101	67	0	1145	3662	0	188
normalized size	1	1.00	0.38	0.25	0.00	4.26	13.61	0.00	0.70
time (sec)	N/A	0.392	0.016	0.007	0.000	3.031	2.732	0.000	0.134

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	94	62	0	1190	48	0	183
normalized size	1	1.00	0.41	0.27	0.00	5.17	0.21	0.00	0.80
time (sec)	N/A	0.357	0.015	0.008	0.000	3.070	0.950	0.000	0.193

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	90	58	0	1189	46	0	181
normalized size	1	1.00	0.45	0.29	0.00	6.01	0.23	0.00	0.91
time (sec)	N/A	0.194	0.013	0.007	0.000	3.226	0.914	0.000	2.342

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	101	67	0	1143	60	0	237
normalized size	1	1.00	0.41	0.27	0.00	4.67	0.24	0.00	0.97
time (sec)	N/A	0.473	0.020	0.011	0.000	3.241	12.657	0.000	2.342

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	109	72	0	1245	0	0	242
normalized size	1	1.00	0.39	0.26	0.00	4.43	0.00	0.00	0.86
time (sec)	N/A	0.467	0.018	0.013	0.000	3.268	0.000	0.000	2.302

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	116	77	0	1274	70	0	246
normalized size	1	1.00	0.37	0.24	0.00	4.02	0.22	0.00	0.78
time (sec)	N/A	0.539	0.019	0.012	0.000	3.434	2.705	0.000	2.248

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	87	75	0	83	44	87	252
normalized size	1	1.00	4.58	3.95	0.00	4.37	2.32	4.58	13.26
time (sec)	N/A	0.105	0.047	0.095	0.000	0.756	1.045	4.080	2.270

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	34	33	47	37	34	38
normalized size	1	1.00	1.33	0.79	0.77	1.09	0.86	0.79	0.88
time (sec)	N/A	0.067	0.023	0.009	2.104	0.808	0.170	0.254	0.052

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	18	21
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	1.06	1.24
time (sec)	N/A	0.037	0.005	0.008	1.116	0.936	0.131	0.311	2.200

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	20	24	30
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.80	0.96	1.20
time (sec)	N/A	0.037	0.006	0.006	1.147	0.761	0.132	0.254	0.054

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	19	18	24	14	28	18
normalized size	1	1.00	0.91	0.86	0.82	1.09	0.64	1.27	0.82
time (sec)	N/A	0.013	0.010	0.007	1.095	0.567	0.111	0.366	0.036

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	38	22	21	21	26	22	25
normalized size	1	1.00	1.41	0.81	0.78	0.78	0.96	0.81	0.93
time (sec)	N/A	0.039	0.011	0.007	2.309	0.949	0.144	0.286	2.260

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.015	0.004	0.001	2.281	0.819	0.082	0.270	0.031

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	22	23	23
normalized size	1	1.00	1.00	0.89	0.85	0.85	0.81	0.85	0.85
time (sec)	N/A	0.026	0.007	0.005	2.077	1.073	0.109	0.278	2.140

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	26	25	25	26	29	25
normalized size	1	1.00	1.00	0.67	0.64	0.64	0.67	0.74	0.64
time (sec)	N/A	0.020	0.018	0.009	1.273	0.781	0.241	0.317	2.141

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	19	18	18	17	19	18
normalized size	1	1.00	1.05	0.86	0.82	0.82	0.77	0.86	0.82
time (sec)	N/A	0.015	0.004	0.002	1.133	0.916	0.075	0.281	0.027

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.029	0.005	0.003	2.301	0.823	0.091	0.341	2.131

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	24	21	20	26	17	22	10
normalized size	1	1.00	2.00	1.75	1.67	2.17	1.42	1.83	0.83
time (sec)	N/A	0.021	0.011	0.008	1.078	0.726	0.098	0.297	0.072

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	22	19	23	19
normalized size	1	1.00	1.00	0.88	0.84	0.88	0.76	0.92	0.76
time (sec)	N/A	0.036	0.006	0.007	1.129	0.872	0.102	0.269	2.139

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.028	0.006	0.005	2.157	0.828	0.114	0.309	0.038

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	31	35	55	36	30	35
normalized size	1	1.00	1.00	0.89	1.00	1.57	1.03	0.86	1.00
time (sec)	N/A	0.034	0.017	0.007	2.223	0.778	0.147	0.329	2.125

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	17
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.74
time (sec)	N/A	0.037	0.005	0.009	1.405	0.688	0.138	0.239	2.188

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	20	19
normalized size	1	1.00	1.00	0.87	0.83	0.83	0.83	0.87	0.83
time (sec)	N/A	0.054	0.007	0.006	1.115	0.922	0.109	0.295	0.055

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	23	32	20	23	25
normalized size	1	1.00	0.86	0.83	0.79	1.10	0.69	0.79	0.86
time (sec)	N/A	0.014	0.010	0.006	2.335	0.893	0.119	0.374	2.125

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	42	41	58	49	43	55
normalized size	1	1.00	1.00	0.95	0.93	1.32	1.11	0.98	1.25
time (sec)	N/A	0.243	0.024	0.010	2.248	0.875	0.212	0.363	0.129

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	38	38	51	38	88
normalized size	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.133	0.020	0.005	2.287	0.883	0.213	0.304	0.160

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	44	72	41	44	56
normalized size	1	1.00	1.00	1.03	1.33	2.18	1.24	1.33	1.70
time (sec)	N/A	0.166	0.022	0.012	1.156	0.852	0.186	0.301	2.162

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	10	11	11
normalized size	1	1.00	1.00	0.71	0.65	0.65	0.59	0.65	0.65
time (sec)	N/A	0.007	0.006	0.007	2.200	0.643	0.141	0.288	0.032

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	20	34	20	20
normalized size	1	1.00	0.92	0.88	0.83	0.83	1.42	0.83	0.83
time (sec)	N/A	0.018	0.009	0.003	2.255	0.757	0.166	0.288	2.134

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	23	18	17	17	17	20	19
normalized size	1	1.00	1.53	1.20	1.13	1.13	1.13	1.33	1.27
time (sec)	N/A	0.060	0.006	0.008	2.313	0.959	0.137	0.319	0.056

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	17	17	19	17	17
normalized size	1	1.00	1.00	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.012	0.009	0.007	2.398	0.872	0.148	0.375	0.050

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	27	44	29	60	35
normalized size	1	1.00	0.89	0.76	0.73	1.19	0.78	1.62	0.95
time (sec)	N/A	0.040	0.023	0.007	2.224	0.507	0.164	0.287	2.144

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	21	21	26	21	21
normalized size	1	1.00	1.00	0.85	0.81	0.81	1.00	0.81	0.81
time (sec)	N/A	0.011	0.008	0.002	2.216	0.595	0.094	0.309	0.030

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	16	10	11	10
normalized size	1	1.00	1.00	0.92	0.83	1.33	0.83	0.92	0.83
time (sec)	N/A	0.032	0.004	0.006	1.105	0.700	0.095	0.287	2.125

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	20	21
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.95	1.00
time (sec)	N/A	0.030	0.007	0.008	1.092	0.793	0.132	0.282	2.144

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	34	17	20	20
normalized size	1	1.00	1.00	0.95	0.91	1.55	0.77	0.91	0.91
time (sec)	N/A	0.015	0.008	0.005	2.231	0.722	0.112	0.282	2.124

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	32	19	20	22
normalized size	1	1.00	1.00	0.88	0.83	1.33	0.79	0.83	0.92
time (sec)	N/A	0.016	0.011	0.007	2.176	0.789	0.116	0.405	0.029

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	33	33	39	33	56
normalized size	1	1.00	1.00	0.94	0.92	0.92	1.08	0.92	1.56
time (sec)	N/A	0.026	0.015	0.005	2.057	0.802	0.188	0.245	0.103

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	28	27	27	29	27	37
normalized size	1	1.00	1.00	0.76	0.73	0.73	0.78	0.73	1.00
time (sec)	N/A	0.024	0.008	0.003	2.003	0.752	0.174	0.285	2.136

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
normalized size	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.017	0.006	0.006	1.083	0.612	0.109	0.367	2.214

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	16	15
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.74	0.84	0.79
time (sec)	N/A	0.013	0.004	0.002	1.066	0.803	0.081	0.266	0.034

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	34	34	46	34	36
normalized size	1	1.00	1.00	0.85	0.83	0.83	1.12	0.83	0.88
time (sec)	N/A	0.028	0.014	0.005	2.178	0.916	0.118	0.288	0.043

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	32	31	31	34	31	31
normalized size	1	1.00	0.95	0.78	0.76	0.76	0.83	0.76	0.76
time (sec)	N/A	0.028	0.007	0.004	2.074	1.114	0.118	0.278	2.116

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	24	21	20	20	24	23	20
normalized size	1	1.00	0.80	0.70	0.67	0.67	0.80	0.77	0.67
time (sec)	N/A	0.058	0.011	0.007	1.038	0.962	0.152	0.250	2.125

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	31	30	27
normalized size	1	1.00	1.00	0.80	0.77	0.77	0.89	0.86	0.77
time (sec)	N/A	0.040	0.008	0.008	1.118	0.746	0.149	0.324	0.044

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	25	24	34	27	34	26
normalized size	1	1.00	0.94	0.74	0.71	1.00	0.79	1.00	0.76
time (sec)	N/A	0.057	0.014	0.008	1.133	0.602	0.145	0.387	2.109

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	33	44	36	33	39
normalized size	1	1.00	1.00	0.83	0.79	1.05	0.86	0.79	0.93
time (sec)	N/A	0.021	0.021	0.006	2.355	0.792	0.131	0.297	2.187

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	38	37	37	46	37	41
normalized size	1	1.00	1.00	0.78	0.76	0.76	0.94	0.76	0.84
time (sec)	N/A	0.157	0.013	0.006	2.803	0.624	0.227	0.315	0.072

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	19	19	24	22	19
normalized size	1	1.00	1.00	0.69	0.66	0.66	0.83	0.76	0.66
time (sec)	N/A	0.053	0.007	0.008	1.166	0.582	0.142	0.275	2.182

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	34	33	33	41	34	50
normalized size	1	1.00	0.93	0.74	0.72	0.72	0.89	0.74	1.09
time (sec)	N/A	0.044	0.016	0.006	2.161	0.992	0.144	0.317	0.086

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	17	10	15	14
normalized size	1	1.00	0.88	0.94	0.88	1.06	0.62	0.94	0.88
time (sec)	N/A	0.025	0.007	0.004	1.085	0.800	0.093	0.397	0.043

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	16	17	17	14	12	12
normalized size	1	1.00	0.67	0.76	0.81	0.81	0.67	0.57	0.57
time (sec)	N/A	0.019	0.003	0.004	1.088	0.882	0.087	0.301	2.085

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	23	23	17	30	15
normalized size	1	1.00	1.00	1.07	1.53	1.53	1.13	2.00	1.00
time (sec)	N/A	0.027	0.011	0.005	1.268	0.881	0.114	0.262	2.089

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	28	28	3	29	57
normalized size	1	1.00	1.00	0.94	0.90	0.90	0.10	0.94	1.84
time (sec)	N/A	0.042	0.012	0.005	2.357	0.889	0.130	0.278	0.112

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	22	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.042	0.006	0.007	1.088	0.804	0.141	0.287	0.060

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	24	36	20	26	22
normalized size	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73
time (sec)	N/A	0.030	0.015	0.006	1.212	0.874	0.098	0.276	2.108

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.038	0.005	0.005	2.075	0.733	0.137	0.331	2.121

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	73	77	136	88	74	96
normalized size	1	1.00	0.90	0.71	0.75	1.32	0.85	0.72	0.93
time (sec)	N/A	0.476	0.043	0.013	2.248	0.702	0.515	0.344	2.200

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	25	36	24	25	32
normalized size	1	1.00	0.91	0.85	0.76	1.09	0.73	0.76	0.97
time (sec)	N/A	0.017	0.011	0.005	2.149	0.916	0.127	0.317	0.035

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	29	44	27	30	33
normalized size	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00
time (sec)	N/A	0.041	0.019	0.008	2.254	0.778	0.149	0.379	2.113

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	19	17	26	19
normalized size	1	1.00	1.00	0.96	0.92	0.76	0.68	1.04	0.76
time (sec)	N/A	0.041	0.005	0.007	1.046	0.564	0.096	0.291	0.043

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	31	36	31	56
normalized size	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56
time (sec)	N/A	0.115	0.015	0.004	2.096	0.842	0.202	0.336	0.106

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	33	22	21	23
normalized size	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79
time (sec)	N/A	0.107	0.017	0.009	2.089	0.990	0.174	0.227	0.042

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	39	46	36	49
normalized size	1	1.00	1.00	0.78	0.76	0.85	1.00	0.78	1.07
time (sec)	N/A	0.058	0.026	0.008	2.093	0.806	0.162	0.296	2.169

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	26	19	18	18	17	20	14
normalized size	1	1.18	1.18	0.86	0.82	0.82	0.77	0.91	0.64
time (sec)	N/A	0.015	0.006	0.005	0.974	0.853	0.098	0.284	0.039

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.039	0.006	0.007	0.850	1.135	0.136	0.381	0.065

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	14	16	14
normalized size	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00
time (sec)	N/A	0.025	0.004	0.007	0.999	0.773	0.115	0.373	2.120

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	17	17	17	20	17
normalized size	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89
time (sec)	N/A	0.027	0.007	0.007	1.029	1.139	0.131	0.292	2.116

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	19	17	33
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43
time (sec)	N/A	0.029	0.009	0.006	2.122	1.002	0.183	0.364	0.054

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	50	50	68	53	58
normalized size	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92
time (sec)	N/A	0.088	0.027	0.010	2.112	0.980	0.346	0.306	2.213

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	86	67	54	59	103	65	59	71
normalized size	1	1.25	0.97	0.78	0.86	1.49	0.94	0.86	1.03
time (sec)	N/A	0.099	0.047	0.014	2.070	0.786	0.213	0.236	2.177

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	19	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.12	0.71	0.76	0.76
time (sec)	N/A	0.013	0.006	0.006	2.043	0.889	0.106	0.302	0.033

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	15	17	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	0.79	0.89	0.89
time (sec)	N/A	0.027	0.005	0.004	2.030	0.908	0.101	0.269	2.098

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	28	19
normalized size	1	1.00	1.00	0.89	0.78	0.78	0.78	3.11	2.11
time (sec)	N/A	0.075	0.006	0.005	2.166	0.888	0.138	0.292	2.104

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	12	12
normalized size	1	1.00	1.00	1.08	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.016	0.005	0.002	1.375	0.635	0.087	0.295	0.028

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	41	51	66	99	55	48
normalized size	1	1.00	0.89	0.63	0.78	1.02	1.52	0.85	0.74
time (sec)	N/A	0.062	0.036	0.008	1.771	0.601	0.448	0.295	0.099

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	46	24	28	28
normalized size	1	1.00	1.00	1.04	1.00	1.64	0.86	1.00	1.00
time (sec)	N/A	0.022	0.011	0.006	1.656	0.625	0.131	0.285	0.038

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.040	0.023	0.004	1.515	0.809	0.103	0.374	0.025

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	41	39	24	23	23	26	27	23
normalized size	1	1.32	1.26	0.77	0.74	0.74	0.84	0.87	0.74
time (sec)	N/A	0.049	0.007	0.008	0.797	0.845	0.194	0.308	0.050

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	26	19	23	30
normalized size	1	1.00	1.00	0.96	0.92	1.08	0.79	0.96	1.25
time (sec)	N/A	0.167	0.009	0.006	1.223	0.761	0.155	0.292	2.129

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
normalized size	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.025	0.011	0.006	1.431	0.845	0.130	0.391	0.033

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
normalized size	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.039	0.006	0.005	1.416	1.244	0.127	0.274	0.030

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	10	13	13
normalized size	1	1.00	1.00	1.08	1.00	1.15	0.77	1.00	1.00
time (sec)	N/A	0.049	0.005	0.005	0.708	0.811	0.100	0.283	2.144

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	193	236	199	4545	138	208	357
normalized size	1	1.00	0.94	1.15	0.97	22.06	0.67	1.01	1.73
time (sec)	N/A	0.274	0.107	0.008	1.652	2.993	1.238	0.442	0.231

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	0	47	44	38	42
normalized size	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.93
time (sec)	N/A	0.067	0.025	0.009	0.000	0.931	0.154	1.109	2.191

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	0	75	46	42	47
normalized size	1	1.00	0.75	0.69	0.00	1.27	0.78	0.71	0.80
time (sec)	N/A	0.064	0.021	0.010	0.000	0.776	0.184	1.803	0.049

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	234	616	0	0	0	1587	3942
normalized size	1	1.00	1.12	2.95	0.00	0.00	0.00	7.59	18.86
time (sec)	N/A	0.371	0.255	0.061	0.000	0.000	0.000	4.697	3.437

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	245	633	0	0	0	1625	3046
normalized size	1	1.00	1.09	2.83	0.00	0.00	0.00	7.25	13.60
time (sec)	N/A	0.389	0.248	0.060	0.000	0.000	0.000	4.563	3.220

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	60	65	190	62	61
normalized size	1	1.00	1.00	1.02	1.07	1.16	3.39	1.11	1.09
time (sec)	N/A	0.047	0.027	0.007	0.685	0.709	1.065	0.313	0.213

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	73	87	84	162	0	85	347
normalized size	1	1.00	0.76	0.91	0.88	1.69	0.00	0.89	3.61
time (sec)	N/A	0.108	0.035	0.008	1.222	0.735	0.000	0.257	1.131

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	228	336	279	5975	0	320	570
normalized size	1	1.00	0.86	1.27	1.06	22.63	0.00	1.21	2.16
time (sec)	N/A	0.472	0.090	0.007	1.706	2.990	0.000	0.302	2.496

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	370	422	349	0	0	401	823
normalized size	1	1.00	0.89	1.01	0.84	0.00	0.00	0.96	1.97
time (sec)	N/A	0.547	0.230	0.014	1.481	0.000	0.000	0.567	2.449

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	24	21	20	26	19	21	12
normalized size	1	1.00	1.50	1.31	1.25	1.62	1.19	1.31	0.75
time (sec)	N/A	0.008	0.009	0.007	0.753	0.674	0.105	0.239	0.039

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	23	34	20	30	17
normalized size	1	1.00	1.42	1.26	1.21	1.79	1.05	1.58	0.89
time (sec)	N/A	0.013	0.011	0.009	0.651	0.484	0.118	0.303	2.138

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	90	75	106	92	77	103
normalized size	1	1.00	0.88	0.93	0.77	1.09	0.95	0.79	1.06
time (sec)	N/A	0.071	0.050	0.012	1.934	0.697	0.361	0.314	0.177

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.106	0.008	0.005	1.400	0.633	0.120	0.289	2.120

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.094	0.007	0.004	1.327	0.547	0.117	0.298	0.035

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	24	24	29	24	51
normalized size	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76
time (sec)	N/A	0.118	0.014	0.004	1.479	0.574	0.193	0.259	2.153

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	14	14
normalized size	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.011	0.008	0.004	2.074	0.571	0.138	0.253	0.042

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	7	11	10
normalized size	1	1.00	1.00	0.92	0.83	1.00	0.58	0.92	0.83
time (sec)	N/A	0.011	0.003	0.004	0.752	0.597	0.091	0.418	0.037

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	17
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	1.55
time (sec)	N/A	0.034	0.005	0.006	1.608	0.646	0.131	0.292	2.166

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	13	8	11	10
normalized size	1	1.00	1.00	0.92	0.83	1.08	0.67	0.92	0.83
time (sec)	N/A	0.021	0.003	0.005	0.714	0.516	0.091	0.384	0.039

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	14	12
normalized size	1	1.00	1.00	1.08	1.00	1.00	1.00	1.17	1.00
time (sec)	N/A	0.018	0.003	0.005	0.596	0.564	0.105	0.229	0.047

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	16	23
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	1.35
time (sec)	N/A	0.037	0.005	0.005	1.532	0.570	0.133	0.243	2.284

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	14	19
normalized size	1	1.00	1.00	1.08	1.00	1.00	0.77	1.08	1.46
time (sec)	N/A	0.035	0.008	0.006	1.410	0.631	0.132	0.343	0.060

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	21	20	26	20	22	20
normalized size	1	1.00	0.79	0.75	0.71	0.93	0.71	0.79	0.71
time (sec)	N/A	0.011	0.015	0.007	0.662	0.593	0.112	0.297	0.069

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	37	26	43	22
normalized size	1	1.00	1.00	0.84	0.81	1.16	0.81	1.34	0.69
time (sec)	N/A	0.026	0.017	0.009	0.746	0.530	0.134	0.298	2.226

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	19	19	19	20	25
normalized size	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09
time (sec)	N/A	0.034	0.008	0.005	1.558	0.712	0.136	0.310	0.049

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	36	20	47	28
normalized size	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17
time (sec)	N/A	0.035	0.014	0.007	1.600	0.566	0.141	0.303	2.111

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	37	44	39	41
normalized size	1	1.00	1.00	0.82	0.80	0.76	0.90	0.80	0.84
time (sec)	N/A	0.142	0.014	0.007	1.708	0.657	0.224	0.283	2.156

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	19	20	19	19	19	22	19
normalized size	1	1.00	0.76	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.055	0.008	0.006	1.067	0.520	0.145	0.392	2.135

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	47	60	63	60	61
normalized size	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02
time (sec)	N/A	0.246	0.053	0.009	2.798	0.732	0.252	0.343	0.130

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	5	9	6
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.45	0.82	0.55
time (sec)	N/A	0.007	0.000	0.000	0.845	0.731	0.060	0.227	0.016

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
normalized size	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.016	0.005	0.005	1.096	0.615	0.108	0.355	2.124

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	37	37	52	37	37	49
normalized size	1	1.00	1.00	0.82	0.82	1.16	0.82	0.82	1.09
time (sec)	N/A	0.025	0.013	0.007	1.839	0.709	0.137	0.299	0.041

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	52	29	29	44
normalized size	1	1.00	1.00	0.91	0.88	1.62	0.91	0.91	1.38
time (sec)	N/A	0.254	0.019	0.010	2.025	0.626	0.263	0.325	0.069

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	159	110	132	178	146	122	124
normalized size	1	1.00	1.07	0.74	0.89	1.20	0.99	0.82	0.84
time (sec)	N/A	0.136	0.068	0.011	2.009	0.758	0.429	0.388	2.189

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	101	79	99	515	61	92	170
normalized size	1	1.00	0.90	0.71	0.88	4.60	0.54	0.82	1.52
time (sec)	N/A	0.114	0.049	0.007	2.058	2.414	0.965	0.390	2.228

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	14	12
normalized size	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.016	0.003	0.005	0.907	0.751	0.103	0.262	0.044

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	12	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83
time (sec)	N/A	0.029	0.005	0.004	1.005	0.628	0.098	0.372	2.138

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	21	14	17	15
normalized size	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88
time (sec)	N/A	0.030	0.004	0.007	0.892	0.697	0.108	0.319	0.027

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	17	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	1.00	0.88
time (sec)	N/A	0.029	0.004	0.005	0.900	0.620	0.100	0.260	0.033

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	25	15	23	16
normalized size	1	1.00	1.00	0.94	0.89	1.39	0.83	1.28	0.89
time (sec)	N/A	0.030	0.004	0.006	1.921	0.666	0.105	0.267	0.050

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	14	7	11	10
normalized size	1	1.00	1.00	1.10	1.00	1.40	0.70	1.10	1.00
time (sec)	N/A	0.019	0.005	0.005	0.892	0.784	0.086	0.361	0.034

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	36	33	30
normalized size	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71
time (sec)	N/A	0.040	0.007	0.008	0.878	0.592	0.151	0.287	0.053

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	38	38	51	38	88
normalized size	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.125	0.017	0.002	1.972	0.769	0.212	0.355	0.002

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	17	8
normalized size	1	1.00	1.00	0.84	0.79	0.79	0.74	0.89	0.42
time (sec)	N/A	0.062	0.003	0.007	1.026	0.522	0.109	0.281	0.084

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	30	30	34	32	26
normalized size	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.021	0.005	0.007	0.995	0.627	0.119	0.284	2.093

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	25	27	27	21
normalized size	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.020	0.004	0.006	1.020	0.646	0.117	0.343	0.033

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	20	22	16
normalized size	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.015	0.005	0.006	1.027	0.832	0.117	0.270	2.117

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	19	13
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.007	0.003	0.003	1.173	0.599	0.109	0.286	2.110

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	19	8
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.38
time (sec)	N/A	0.010	0.003	0.005	0.926	0.731	0.110	0.275	0.078

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	24	24	17
normalized size	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.016	0.004	0.008	0.916	0.732	0.148	0.373	0.095

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	26	30	31	29	22
normalized size	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.030	0.004	0.008	1.062	0.537	0.162	0.266	0.035

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	39	36	34	26
normalized size	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.034	0.005	0.008	1.015	0.591	0.170	0.276	0.036

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	36	44	41	39	32
normalized size	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.041	0.005	0.009	1.071	0.883	0.182	0.238	0.037

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	61	54	0	1506	41	0	144
normalized size	1	1.00	0.39	0.34	0.00	9.59	0.26	0.00	0.92
time (sec)	N/A	0.168	0.016	0.014	0.000	2.684	0.225	0.000	2.745

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	61	56	0	1546	41	0	142
normalized size	1	1.00	0.39	0.36	0.00	9.85	0.26	0.00	0.90
time (sec)	N/A	0.112	0.017	0.015	0.000	2.520	0.227	0.000	2.805

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	61	54	0	2271	39	0	142
normalized size	1	1.00	0.32	0.29	0.00	12.08	0.21	0.00	0.76
time (sec)	N/A	0.276	0.013	0.008	0.000	2.944	0.211	0.000	2.780

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	61	56	0	2259	39	0	142
normalized size	1	1.00	0.32	0.30	0.00	12.02	0.21	0.00	0.76
time (sec)	N/A	0.156	0.013	0.010	0.000	2.704	0.212	0.000	2.740

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	0	133	0	328
normalized size	1	1.00	0.10	0.10	0.00	0.00	0.20	0.00	0.49
time (sec)	N/A	1.108	0.037	0.074	0.000	0.000	3.924	0.000	2.699

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	0	133	0	328
normalized size	1	1.00	0.10	0.10	0.00	0.00	0.20	0.00	0.49
time (sec)	N/A	0.648	0.031	0.002	0.000	0.000	3.920	0.000	0.002

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	C	A	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	95	67	0	5653	42	0	504
normalized size	1	0.00	0.57	0.40	0.00	33.65	0.25	0.00	3.00
time (sec)	N/A	0.380	0.061	0.278	0.000	2.699	1.866	0.000	3.082

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	322	314	310	0	384	311	894
normalized size	1	1.00	1.01	0.98	0.97	0.00	1.20	0.97	2.79
time (sec)	N/A	0.256	0.249	0.011	1.956	0.000	5.289	0.361	2.843

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	243	292	275	0	277	285	556
normalized size	1	1.00	0.84	1.00	0.95	0.00	0.95	0.98	1.91
time (sec)	N/A	0.198	0.100	0.003	2.047	0.000	2.678	0.336	2.657

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	184	151	207	0	124	215	160
normalized size	1	1.00	0.84	0.69	0.95	0.00	0.57	0.98	0.73
time (sec)	N/A	0.174	0.059	0.002	2.666	0.000	0.823	0.316	2.323

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	169	121	20	179	33
normalized size	1	1.00	0.72	0.69	0.91	0.65	0.11	0.97	0.18
time (sec)	N/A	0.098	0.018	0.003	2.367	0.421	0.161	0.325	0.083

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	404	433	345	0	0	371	874
normalized size	1	1.00	0.97	1.04	0.83	0.00	0.00	0.89	2.10
time (sec)	N/A	0.428	0.149	0.007	2.727	0.000	0.000	0.422	0.418

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	524	866	561	0	0	646	2436
normalized size	1	1.00	0.95	1.57	1.02	0.00	0.00	1.17	4.41
time (sec)	N/A	0.808	0.606	0.010	2.109	0.000	0.000	2.780	2.779

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	738	1201	817	0	0	901	1955
normalized size	1	1.00	1.09	1.77	1.20	0.00	0.00	1.32	2.88
time (sec)	N/A	0.950	0.929	0.012	2.342	0.000	0.000	0.892	3.674

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	347	390	332	0	350	342	670
normalized size	1	1.00	0.99	1.12	0.95	0.00	1.00	0.98	1.92
time (sec)	N/A	0.298	0.374	0.004	2.200	0.000	8.334	0.342	0.433

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	321	362	318	0	318	323	391
normalized size	1	1.00	1.00	1.12	0.99	0.00	0.99	1.00	1.21
time (sec)	N/A	0.269	0.316	0.007	2.630	0.000	3.521	0.364	2.483

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	224	188	238	0	155	241	282
normalized size	1	1.00	0.93	0.78	0.99	0.00	0.64	1.00	1.17
time (sec)	N/A	0.189	0.189	0.005	2.433	0.000	1.092	0.339	0.274

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	183	143	189	173	39	194	58
normalized size	1	1.00	0.91	0.71	0.94	0.86	0.19	0.96	0.29
time (sec)	N/A	0.116	0.107	0.005	2.108	0.632	0.297	0.246	0.089

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	855	855	558	1122	601	0	0	771	1591
normalized size	1	1.00	0.65	1.31	0.70	0.00	0.00	0.90	1.86
time (sec)	N/A	0.849	0.391	0.019	2.291	0.000	0.000	0.504	2.995

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1141	807	1636	961	0	0	1104	2246
normalized size	1	1.00	0.71	1.43	0.84	0.00	0.00	0.97	1.97
time (sec)	N/A	1.659	0.865	0.023	2.541	0.000	0.000	103.636	4.104

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1384	1384	996	2121	1394	0	0	1488	3256
normalized size	1	1.00	0.72	1.53	1.01	0.00	0.00	1.08	2.35
time (sec)	N/A	1.963	1.357	0.026	2.602	0.000	0.000	1.148	5.038

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	388	470	392	0	413	389	721
normalized size	1	1.00	0.98	1.19	0.99	0.00	1.05	0.99	1.83
time (sec)	N/A	0.346	0.351	0.007	2.373	0.000	8.013	0.354	0.478

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	358	419	364	0	374	356	676
normalized size	1	1.00	0.99	1.16	1.01	0.00	1.04	0.99	1.88
time (sec)	N/A	0.326	0.294	0.007	2.181	0.000	4.878	0.433	0.471

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	249	222	269	0	192	260	315
normalized size	1	1.00	0.94	0.83	1.01	0.00	0.72	0.98	1.18
time (sec)	N/A	0.249	0.206	0.007	2.204	0.000	1.483	0.320	0.303

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	200	158	212	232	63	204	80
normalized size	1	1.00	0.91	0.72	0.97	1.06	0.29	0.93	0.37
time (sec)	N/A	0.142	0.083	0.005	2.704	0.641	0.461	0.358	0.096

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1352	1352	835	2098	1015	0	0	1259	2720
normalized size	1	1.00	0.62	1.55	0.75	0.00	0.00	0.93	2.01
time (sec)	N/A	1.412	0.738	0.025	2.408	0.000	0.000	0.910	4.406

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1830	1830	1115	2769	1564	0	0	0	3572
normalized size	1	1.00	0.61	1.51	0.85	0.00	0.00	0.00	1.95
time (sec)	N/A	2.781	1.551	0.033	3.097	0.000	0.000	0.000	5.754

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2204	2204	1338	3334	2198	0	0	2119	6280
normalized size	1	1.00	0.61	1.51	1.00	0.00	0.00	0.96	2.85
time (sec)	N/A	3.165	2.873	0.038	3.988	0.000	0.000	1.534	7.785

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	28	28	34	28	30
normalized size	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.019	0.010	0.002	2.532	0.980	0.112	0.295	0.041

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	28	28	34	28	30
normalized size	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.029	0.006	0.002	2.530	1.088	0.115	0.380	0.035

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	26	26	36	26	30
normalized size	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.017	0.009	0.003	2.156	0.800	0.116	0.388	0.043

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	26	26	36	26	30
normalized size	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.035	0.005	0.003	2.329	0.877	0.120	0.335	0.032

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
normalized size	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.012	0.022	0.003	2.621	0.901	0.111	0.415	2.319

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
normalized size	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.021	0.005	0.003	2.332	0.755	0.118	0.315	0.047

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	23	18	17	17	15	15	6
normalized size	1	1.00	3.83	3.00	2.83	2.83	2.50	2.50	1.00
time (sec)	N/A	0.002	0.003	0.005	1.061	0.673	0.094	0.310	2.270

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	18	17	17	15	19	6
normalized size	1	1.00	1.10	0.86	0.81	0.81	0.71	0.90	0.29
time (sec)	N/A	0.004	0.003	0.002	0.942	0.516	0.098	0.416	0.146

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	24	22	9	9
normalized size	1	1.00	0.85	0.77	0.69	1.85	1.69	0.69	0.69
time (sec)	N/A	0.002	0.003	0.001	0.882	0.593	0.125	0.335	2.319

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	81	11	58	57	24	22	57	9
normalized size	1	6.23	0.85	4.46	4.38	1.85	1.69	4.38	0.69
time (sec)	N/A	0.012	0.002	0.002	0.937	0.641	0.314	0.312	0.025

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	66	66	85	68	94
normalized size	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.111	0.014	0.009	2.014	0.593	0.250	0.293	0.102

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	66	66	85	68	94
normalized size	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.124	0.006	0.003	1.990	0.614	0.254	0.307	0.039

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	31	21	20	20	20	21	20
normalized size	1	1.00	1.29	0.88	0.83	0.83	0.83	0.88	0.83
time (sec)	N/A	0.019	0.006	0.003	0.631	0.546	0.078	0.307	0.037

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	20	20	19	21	20
normalized size	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77
time (sec)	N/A	0.016	0.004	0.002	0.686	0.634	0.073	0.295	0.033

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	22	16	15	18	15	15	17
normalized size	1	1.00	1.29	0.94	0.88	1.06	0.88	0.88	1.00
time (sec)	N/A	0.006	0.000	0.000	0.664	0.530	0.057	0.388	0.024

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	20	20	18	15	20	17
normalized size	1	1.00	0.92	0.83	0.83	0.75	0.62	0.83	0.71
time (sec)	N/A	0.005	0.001	0.001	0.568	0.450	0.058	0.368	0.021

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	23	22	25	20	23	28
normalized size	1	1.00	1.09	1.05	1.00	1.14	0.91	1.05	1.27
time (sec)	N/A	0.019	0.006	0.007	1.378	0.624	0.132	0.393	0.045

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	16	15	15	15	18	15
normalized size	1	1.00	1.59	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.013	0.004	0.007	0.755	0.710	0.130	0.327	0.057

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	16	13
normalized size	1	1.00	1.00	0.74	0.68	0.68	0.63	0.84	0.68
time (sec)	N/A	0.021	0.004	0.006	0.659	0.632	0.096	0.305	2.336

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	17	11	10	10	8	13	10
normalized size	1	1.00	1.42	0.92	0.83	0.83	0.67	1.08	0.83
time (sec)	N/A	0.006	0.004	0.001	0.621	0.639	0.089	0.398	2.269

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	11	10	10	8	11	10
normalized size	1	1.00	1.10	1.10	1.00	1.00	0.80	1.10	1.00
time (sec)	N/A	0.009	0.006	0.002	0.610	0.711	0.132	0.332	0.056

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.018	0.006	0.005	0.614	0.693	0.125	0.356	0.056

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.021	0.004	0.005	1.640	0.637	0.128	0.376	0.049

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.009	0.004	0.000	0.655	0.467	0.092	0.278	0.042

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.021	0.005	0.006	0.586	0.610	0.131	0.369	0.067

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	14	9	9	7	9	10
normalized size	1	1.00	0.90	1.40	0.90	0.90	0.70	0.90	1.00
time (sec)	N/A	0.010	0.005	0.003	0.781	0.700	0.083	0.261	2.204

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	20	20	17
normalized size	1	1.00	1.00	0.72	0.68	0.68	0.80	0.80	0.68
time (sec)	N/A	0.021	0.005	0.006	0.646	0.764	0.132	0.362	0.107

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	14	14
normalized size	1	1.00	1.00	1.07	1.00	1.00	0.86	1.00	1.00
time (sec)	N/A	0.025	0.004	0.001	1.557	0.525	0.105	0.252	2.219

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	14	30	11	16	17	11	20
normalized size	1	1.00	1.08	2.31	0.85	1.23	1.31	0.85	1.54
time (sec)	N/A	0.007	0.007	0.010	0.582	0.604	0.130	0.371	2.245

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	32	26	25	20	27	25
normalized size	1	1.00	0.96	1.23	1.00	0.96	0.77	1.04	0.96
time (sec)	N/A	0.045	0.008	0.003	0.717	0.611	0.154	0.283	2.234

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	4	5	4	4	3	5	4
normalized size	1	1.00	0.67	0.83	0.67	0.67	0.50	0.83	0.67
time (sec)	N/A	0.008	0.001	0.001	0.570	0.665	0.064	0.280	0.018

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	20	25	19	27	18
normalized size	1	1.00	1.00	0.95	1.00	1.25	0.95	1.35	0.90
time (sec)	N/A	0.015	0.005	0.007	0.718	0.671	0.105	0.381	0.039

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	28	27	27	31	28	44
normalized size	1	1.00	1.05	0.74	0.71	0.71	0.82	0.74	1.16
time (sec)	N/A	0.025	0.010	0.005	1.973	0.655	0.141	0.389	0.158

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	14	16	15
normalized size	1	1.00	1.06	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.008	0.004	0.002	0.440	0.579	0.073	0.367	0.031

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	36	22	21	21	22	22	25
normalized size	1	1.00	1.16	0.71	0.68	0.68	0.71	0.71	0.81
time (sec)	N/A	0.012	0.005	0.005	0.995	0.466	0.141	0.274	0.053

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	17	15	17	17	13
normalized size	1	1.00	1.00	0.84	0.89	0.79	0.89	0.89	0.68
time (sec)	N/A	0.017	0.006	0.006	0.474	0.482	0.114	0.408	0.090

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
normalized size	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.005	0.004	0.000	0.447	0.583	0.088	0.287	2.217

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
normalized size	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.014	0.009	0.006	1.008	0.642	0.143	0.371	2.216

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
normalized size	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.023	0.019	0.010	0.448	0.631	0.195	0.363	0.042

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	14	8	8	7	9	8
normalized size	1	1.00	0.83	1.17	0.67	0.67	0.58	0.75	0.67
time (sec)	N/A	0.002	0.001	0.002	0.438	0.650	0.076	0.275	0.042

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	23	34	20	25	17
normalized size	1	1.00	1.29	1.33	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.003	0.008	0.008	0.451	0.721	0.108	0.271	2.216

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
normalized size	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.004	0.008	0.006	0.996	0.551	0.101	0.290	0.027

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	9	6
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.001	0.001	0.001	0.453	0.495	0.058	0.382	0.068

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.003	0.002	0.003	0.995	0.579	0.112	0.261	2.245

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
normalized size	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.007	0.004	0.002	0.982	0.780	0.134	0.379	2.243

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
normalized size	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.011	0.006	0.002	0.955	0.598	0.108	0.380	0.032

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	16	17
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.77
time (sec)	N/A	0.007	0.001	0.000	0.460	0.409	0.057	0.310	0.024

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	17
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.77
time (sec)	N/A	0.005	0.001	0.000	0.456	0.514	0.056	0.287	0.029

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	10	14	15
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.62	0.88	0.94
time (sec)	N/A	0.005	0.001	0.003	0.456	0.720	0.069	0.306	0.028

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	39	30	30
normalized size	1	1.00	1.00	0.84	0.81	0.81	1.05	0.81	0.81
time (sec)	N/A	0.021	0.010	0.005	0.976	0.599	0.118	0.266	2.219

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	13
normalized size	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.93
time (sec)	N/A	0.004	0.000	0.001	0.466	0.494	0.054	0.364	0.024

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	7	9	6
normalized size	1	1.00	1.00	0.91	0.82	0.82	0.64	0.82	0.55
time (sec)	N/A	0.004	0.000	0.001	0.450	0.603	0.058	0.286	0.016

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	20	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.80	0.80	1.00
time (sec)	N/A	0.010	0.006	0.005	0.994	0.690	0.136	0.365	0.049

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.010	0.005	0.003	1.359	0.630	0.129	0.292	2.228

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
normalized size	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.017	0.006	0.005	1.432	0.801	0.123	0.249	2.209

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	7	7	8	7	12	5	8	7
normalized size	1	0.78	0.78	0.89	0.78	1.33	0.56	0.89	0.78
time (sec)	N/A	0.006	0.003	0.003	0.951	0.543	0.073	0.298	0.023

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.011	0.013	0.007	1.579	0.482	0.149	0.292	0.055

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	44	30	30
normalized size	1	1.00	1.00	0.84	0.81	0.81	1.19	0.81	0.81
time (sec)	N/A	0.022	0.013	0.003	1.393	0.681	0.122	0.384	0.045

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	14	16	13
normalized size	1	1.00	1.00	0.93	1.00	1.27	0.93	1.07	0.87
time (sec)	N/A	0.004	0.002	0.004	0.969	0.495	0.086	0.269	0.026

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	15	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	1.00	0.67	0.93	0.87
time (sec)	N/A	0.004	0.001	0.005	0.948	0.646	0.076	0.280	0.023

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	13	11	8	13	11
normalized size	1	1.00	1.00	1.09	1.18	1.00	0.73	1.18	1.00
time (sec)	N/A	0.011	0.003	0.005	0.692	0.455	0.095	0.287	0.056

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	18	17	17	17	17	17
normalized size	1	1.00	0.86	0.82	0.77	0.77	0.77	0.77	0.77
time (sec)	N/A	0.004	0.001	0.000	0.876	0.491	0.056	0.356	0.030

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	9
normalized size	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	0.82
time (sec)	N/A	0.001	0.001	0.000	0.862	0.602	0.057	0.216	0.142

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	16	14	13	13	12	14	11
normalized size	1	1.00	1.23	1.08	1.00	1.00	0.92	1.08	0.85
time (sec)	N/A	0.010	0.004	0.002	0.643	0.843	0.076	0.361	2.211

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	14	16	12
normalized size	1	1.00	1.00	0.94	0.88	1.12	0.88	1.00	0.75
time (sec)	N/A	0.006	0.003	0.007	0.588	0.620	0.100	0.360	0.034

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
normalized size	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.011	0.016	0.008	0.815	0.566	0.111	0.379	2.228

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	12	16	15
normalized size	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88
time (sec)	N/A	0.008	0.003	0.002	0.740	0.585	0.073	0.357	0.026

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	15	17	16
normalized size	1	1.00	1.00	0.94	0.89	1.11	0.83	0.94	0.89
time (sec)	N/A	0.006	0.001	0.005	0.728	0.619	0.078	0.371	0.029

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	14	0	14	14	13
normalized size	1	1.00	1.00	1.56	0.78	0.00	0.78	0.78	0.72
time (sec)	N/A	0.010	0.001	0.001	1.708	0.000	0.057	0.351	0.024

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	34	46	31	25	23
normalized size	1	1.00	1.00	0.96	1.48	2.00	1.35	1.09	1.00
time (sec)	N/A	0.018	0.010	0.006	0.738	0.556	0.109	0.369	0.032

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	15	10	32	12
normalized size	1	1.00	0.75	0.81	0.75	0.94	0.62	2.00	0.75
time (sec)	N/A	0.012	0.006	0.004	1.789	0.734	0.107	0.262	2.219

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	26	25	25	24	26	25
normalized size	1	1.00	1.03	0.90	0.86	0.86	0.83	0.90	0.86
time (sec)	N/A	0.020	0.008	0.001	0.765	0.667	0.080	0.283	0.026

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81
time (sec)	N/A	0.006	0.000	0.000	0.891	0.628	0.059	0.348	0.021

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	17	25	14	12	17
normalized size	1	1.00	1.00	0.94	1.00	1.47	0.82	0.71	1.00
time (sec)	N/A	0.015	0.012	0.006	1.745	0.688	0.124	0.237	0.031

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	68	190	151	48	42
normalized size	1	1.00	1.00	0.91	1.45	4.04	3.21	1.02	0.89
time (sec)	N/A	0.066	0.025	0.006	1.021	0.753	0.296	0.281	0.097

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	61	0	240	294	60	82
normalized size	1	1.00	1.07	1.07	0.00	4.21	5.16	1.05	1.44
time (sec)	N/A	0.082	0.027	0.004	0.000	0.630	0.350	0.358	2.227

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	39	308	0	247	24	147	101
normalized size	1	1.00	0.21	1.64	0.00	1.31	0.13	0.78	0.54
time (sec)	N/A	0.189	0.031	0.102	0.000	0.521	0.517	1.488	2.300

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	27	28	65	65	60	30	27
normalized size	1	1.00	0.45	0.47	1.08	1.08	1.00	0.50	0.45
time (sec)	N/A	0.136	0.015	0.007	1.249	0.459	0.229	0.397	2.337

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	B	B	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	250	65	65	60	197	27
normalized size	1	0.00	1.00	9.26	2.41	2.41	2.22	7.30	1.00
time (sec)	N/A	0.311	0.010	0.030	1.199	0.679	0.324	0.397	0.045

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	B	B	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	112	65	65	60	111	27
normalized size	1	0.00	1.00	4.15	2.41	2.41	2.22	4.11	1.00
time (sec)	N/A	0.433	0.010	0.019	0.975	0.660	0.288	0.447	0.043

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [412] had the largest ratio of [.8824]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	24	0.083
2	A	3	3	1.00	29	0.103
3	A	2	2	1.00	29	0.069
4	A	2	2	1.00	29	0.069
5	A	1	0	1.00	27	0.000
6	A	2	2	1.00	29	0.069
7	A	2	2	1.00	29	0.069
8	A	2	2	1.00	29	0.069
9	A	3	2	1.00	27	0.074
10	A	3	2	1.00	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	1	0	1.00	25	0.000
12	A	7	7	1.00	27	0.259
13	A	8	8	1.00	27	0.296
14	A	9	8	1.00	27	0.296
15	A	3	2	1.00	46	0.043
16	A	3	2	1.00	46	0.043
17	A	1	0	1.00	44	0.000
18	A	2	1	1.00	46	0.022
19	A	2	1	1.00	46	0.022
20	A	2	1	1.00	46	0.022
21	A	5	4	1.00	11	0.364
22	A	5	4	1.00	17	0.235
23	A	2	2	1.00	7	0.286
24	A	3	2	1.00	13	0.154
25	A	5	5	1.00	11	0.454
26	A	7	7	1.00	16	0.438
27	A	6	6	1.00	9	0.667
28	A	2	2	1.00	7	0.286
29	A	3	3	1.00	13	0.231
30	A	3	3	1.11	11	0.273
31	A	3	3	1.00	16	0.188
32	A	2	2	1.26	9	0.222
33	A	3	2	1.00	29	0.069
34	A	2	1	1.00	29	0.034
35	A	2	1	1.00	29	0.034
36	A	1	0	1.00	27	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
37	A	10	6	1.00	29	0.207
38	A	11	7	1.00	29	0.241
39	A	3	2	1.00	32	0.062
40	A	2	1	1.00	32	0.031
41	A	2	1	1.00	32	0.031
42	A	1	0	1.00	30	0.000
43	A	4	3	1.00	32	0.094
44	A	5	4	1.00	32	0.125
45	A	2	1	1.00	17	0.059
46	A	2	1	1.00	17	0.059
47	A	2	1	1.00	17	0.059
48	A	1	0	1.00	15	0.000
49	A	16	9	1.00	17	0.529
50	A	18	11	1.00	17	0.647
51	A	2	1	1.00	17	0.059
52	A	2	1	1.00	17	0.059
53	A	2	1	1.00	17	0.059
54	A	1	0	1.00	15	0.000
55	A	15	9	1.00	17	0.529
56	A	17	11	1.00	17	0.647
57	A	2	1	1.00	22	0.045
58	A	2	1	1.00	22	0.045
59	A	2	1	1.00	22	0.045
60	A	1	0	1.00	20	0.000
61	A	16	9	1.00	22	0.409
62	A	18	11	1.00	22	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
63	A	2	2	1.00	51	0.039
64	A	2	2	1.00	51	0.039
65	B	1	0	4.36	49	0.000
66	A	2	2	1.00	51	0.039
67	A	2	2	1.00	51	0.039
68	A	2	2	1.00	51	0.039
69	A	6	5	1.00	13	0.385
70	A	5	3	1.00	19	0.158
71	A	5	3	1.00	19	0.158
72	A	5	3	1.00	19	0.158
73	A	1	0	1.00	17	0.000
74	A	5	2	1.00	19	0.105
75	A	7	3	1.00	19	0.158
76	A	10	3	1.00	19	0.158
77	B	15	7	2.25	17	0.412
78	A	6	5	1.00	15	0.333
79	A	6	5	1.00	15	0.333
80	A	4	4	1.00	13	0.308
81	A	2	2	1.00	11	0.182
82	A	6	6	1.00	15	0.400
83	A	7	6	1.00	15	0.400
84	A	7	6	1.00	15	0.400
85	A	2	2	1.00	13	0.154
86	A	3	3	1.00	13	0.231
87	A	4	3	1.00	13	0.231
88	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	2	2	1.00	11	0.182
90	A	3	3	1.00	11	0.273
91	A	4	3	1.00	11	0.273
92	A	2	2	1.00	13	0.154
93	A	3	3	1.00	13	0.231
94	A	4	3	1.00	13	0.231
95	A	2	2	1.00	11	0.182
96	A	3	3	1.00	11	0.273
97	A	4	3	1.00	11	0.273
98	A	3	2	1.00	15	0.133
99	A	4	3	1.00	13	0.231
100	A	4	4	1.00	17	0.235
101	A	4	4	1.00	19	0.210
102	A	4	4	1.00	17	0.235
103	A	11	10	1.00	17	0.588
104	A	9	9	1.00	17	0.529
105	A	7	7	1.00	15	0.467
106	A	7	7	1.00	13	0.538
107	A	11	10	1.06	17	0.588
108	A	11	10	0.99	17	0.588
109	A	11	10	1.00	17	0.588
110	A	16	12	1.00	17	0.706
111	A	14	10	1.00	17	0.588
112	A	14	10	1.00	15	0.667
113	A	10	7	1.00	13	0.538
114	A	18	13	1.00	17	0.765

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
115	A	18	13	1.00	17	0.765
116	A	3	2	1.00	22	0.091
117	A	2	1	1.00	22	0.045
118	A	2	1	1.00	22	0.045
119	A	1	0	1.00	20	0.000
120	A	4	3	1.00	22	0.136
121	A	5	4	1.00	22	0.182
122	A	6	5	1.00	22	0.227
123	A	2	1	1.00	24	0.042
124	A	2	1	1.00	24	0.042
125	A	2	1	1.00	24	0.042
126	A	2	1	1.00	22	0.045
127	A	8	7	1.00	24	0.292
128	A	10	9	1.00	24	0.375
129	A	12	10	1.00	24	0.417
130	A	2	1	1.00	26	0.038
131	A	2	1	1.00	26	0.038
132	A	2	1	1.00	26	0.038
133	A	2	1	1.00	24	0.042
134	A	9	8	1.00	26	0.308
135	A	11	10	1.00	26	0.385
136	A	14	5	1.00	46	0.109
137	A	14	5	1.00	46	0.109
138	A	8	3	1.00	46	0.065
139	A	14	5	1.00	44	0.114
140	A	14	5	1.00	42	0.119

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
141	A	14	5	1.00	46	0.109
142	A	14	5	0.99	46	0.109
143	A	14	6	1.00	26	0.231
144	A	14	6	1.00	26	0.231
145	A	14	6	1.00	26	0.231
146	A	8	4	1.00	26	0.154
147	A	14	6	1.00	24	0.250
148	A	14	6	1.00	22	0.273
149	A	14	6	1.00	26	0.231
150	A	14	6	1.00	26	0.231
151	A	23	7	1.00	26	0.269
152	A	23	7	1.00	26	0.269
153	A	14	6	1.00	26	0.231
154	A	17	5	1.00	26	0.192
155	A	23	7	1.00	26	0.269
156	A	23	7	1.00	26	0.269
157	A	23	7	1.00	26	0.269
158	A	2	1	1.00	52	0.019
159	A	4	3	1.00	52	0.058
160	A	1	1	1.00	18	0.056
161	A	3	3	1.00	23	0.130
162	A	3	3	1.00	23	0.130
163	A	3	3	1.00	29	0.103
164	A	1	1	1.00	18	0.056
165	A	4	3	1.00	20	0.150
166	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
167	A	4	3	1.00	22	0.136
168	A	1	1	1.00	18	0.056
169	A	3	3	1.00	23	0.130
170	A	3	3	1.00	23	0.130
171	A	3	3	1.00	29	0.103
172	A	1	1	1.00	18	0.056
173	A	1	1	1.00	25	0.040
174	C	7	3	4.30	38	0.079
175	A	1	1	1.00	27	0.037
176	A	1	1	1.00	31	0.032
177	A	2	1	1.00	54	0.019
178	A	3	1	1.00	54	0.019
179	A	5	4	1.00	54	0.074
180	A	1	1	1.00	30	0.033
181	A	1	1	1.00	29	0.034
182	A	1	1	1.00	28	0.036
183	A	1	1	1.00	21	0.048
184	A	1	1	1.00	20	0.050
185	A	1	1	1.00	21	0.048
186	A	1	1	1.00	26	0.038
187	A	1	1	1.00	25	0.040
188	A	2	2	1.00	26	0.077
189	A	1	1	1.00	24	0.042
190	A	1	1	1.00	24	0.042
191	A	1	1	1.00	23	0.043
192	A	1	1	1.00	30	0.033

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	1	1	1.00	29	0.034
194	A	1	1	1.00	28	0.036
195	A	1	1	1.00	21	0.048
196	A	1	1	1.00	20	0.050
197	A	2	2	1.00	21	0.095
198	A	1	1	1.00	26	0.038
199	A	1	1	1.00	25	0.040
200	A	2	2	1.00	26	0.077
201	A	1	1	1.00	22	0.045
202	A	1	1	1.00	24	0.042
203	A	2	2	1.00	23	0.087
204	A	1	1	1.00	23	0.043
205	A	1	1	1.00	19	0.053
206	A	2	1	1.00	22	0.045
207	A	2	1	1.00	23	0.043
208	A	2	1	1.00	22	0.045
209	A	2	1	1.00	23	0.043
210	A	2	1	1.00	24	0.042
211	A	2	1	1.00	25	0.040
212	A	2	1	1.00	31	0.032
213	A	2	1	1.00	32	0.031
214	A	2	1	1.00	35	0.029
215	A	2	1	1.00	36	0.028
216	A	2	1	1.00	24	0.042
217	A	2	1	1.00	31	0.032
218	A	2	1	1.00	35	0.029

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
219	A	1	1	1.00	22	0.045
220	A	1	1	1.00	18	0.056
221	B	3	2	2.91	26	0.077
222	B	2	1	2.91	28	0.036
223	A	1	1	1.00	18	0.056
224	A	1	1	1.00	20	0.050
225	A	1	1	1.00	21	0.048
226	A	3	3	1.00	52	0.058
227	A	9	5	1.00	38	0.132
228	A	3	2	1.00	32	0.062
229	A	4	3	1.00	33	0.091
230	A	3	2	1.00	34	0.059
231	A	6	6	1.00	43	0.140
232	A	1	1	1.00	16	0.062
233	B	15	7	2.25	25	0.280
234	A	1	1	1.00	56	0.018
235	A	1	1	1.00	51	0.020
236	A	1	1	1.00	49	0.020
237	A	1	1	1.00	46	0.022
238	A	2	2	1.00	48	0.042
239	A	1	1	1.00	49	0.020
240	A	1	1	1.00	48	0.021
241	A	1	1	1.00	48	0.021
242	A	10	5	1.00	35	0.143
243	A	10	5	1.00	35	0.143
244	A	10	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	10	5	1.00	33	0.152
246	A	10	5	1.00	32	0.156
247	A	13	6	1.00	35	0.171
248	A	13	6	1.00	35	0.171
249	A	13	6	1.00	35	0.171
250	A	13	6	1.00	35	0.171
251	A	13	6	1.00	35	0.171
252	A	11	6	1.00	33	0.182
253	A	9	5	1.00	32	0.156
254	A	13	6	1.00	35	0.171
255	A	13	6	1.00	35	0.171
256	A	13	6	1.00	35	0.171
257	A	2	2	1.00	40	0.050
258	A	6	5	1.00	20	0.250
259	A	3	2	1.00	20	0.100
260	A	3	2	1.00	20	0.100
261	A	2	1	1.00	16	0.062
262	A	5	4	1.00	22	0.182
263	A	3	2	1.00	21	0.095
264	A	6	5	1.00	26	0.192
265	A	2	1	1.00	20	0.050
266	A	2	1	1.00	11	0.091
267	A	4	3	1.00	22	0.136
268	A	3	2	1.00	21	0.095
269	A	3	2	1.00	25	0.080
270	A	6	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
271	A	5	5	1.00	31	0.161
272	A	3	2	1.00	21	0.095
273	A	4	3	1.00	33	0.091
274	A	4	4	1.00	14	0.286
275	A	7	6	1.00	33	0.182
276	A	7	6	1.00	29	0.207
277	A	6	5	1.00	44	0.114
278	A	3	2	1.00	15	0.133
279	A	5	4	1.00	15	0.267
280	A	4	3	1.00	18	0.167
281	A	3	2	1.00	20	0.100
282	A	5	4	1.00	26	0.154
283	A	3	2	1.00	13	0.154
284	A	3	2	1.00	18	0.111
285	A	2	1	1.00	26	0.038
286	A	5	5	1.00	19	0.263
287	A	5	5	1.00	24	0.208
288	A	8	6	1.00	20	0.300
289	A	8	6	1.00	18	0.333
290	A	5	3	1.00	19	0.158
291	A	4	3	1.00	13	0.231
292	A	6	5	1.00	22	0.227
293	A	6	5	1.00	24	0.208
294	A	2	1	1.00	29	0.034
295	A	2	1	1.00	30	0.033
296	A	2	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	4	4	1.00	16	0.250
298	A	10	5	1.00	36	0.139
299	A	2	1	1.00	21	0.048
300	A	5	4	1.00	16	0.250
301	A	2	1	1.00	24	0.042
302	A	2	1	1.00	21	0.048
303	A	2	1	1.00	24	0.042
304	A	6	5	1.00	26	0.192
305	A	3	2	1.00	25	0.080
306	A	2	1	1.00	29	0.034
307	A	6	5	1.00	20	0.250
308	A	14	10	1.00	32	0.312
309	A	4	4	1.00	23	0.174
310	A	6	5	1.00	26	0.192
311	A	4	3	1.00	26	0.115
312	A	8	4	1.00	25	0.160
313	A	6	3	1.00	23	0.130
314	A	7	6	1.00	23	0.261
315	A	5	3	1.18	20	0.150
316	A	3	2	1.00	25	0.080
317	A	3	2	1.00	22	0.091
318	A	2	1	1.00	24	0.042
319	A	7	6	1.00	24	0.250
320	A	6	5	1.00	43	0.116
321	A	7	5	1.25	50	0.100
322	A	3	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	6	5	1.00	15	0.333
324	A	5	4	1.00	20	0.200
325	A	3	2	1.00	24	0.083
326	A	5	3	1.00	27	0.111
327	A	5	5	1.00	26	0.192
328	A	3	2	1.00	16	0.125
329	A	11	8	1.32	22	0.364
330	A	5	4	1.00	24	0.167
331	A	4	3	1.00	26	0.115
332	A	5	3	1.00	36	0.083
333	A	4	3	1.00	26	0.115
334	A	10	9	1.00	20	0.450
335	A	6	6	1.00	27	0.222
336	A	7	7	1.00	20	0.350
337	A	8	7	1.00	25	0.280
338	A	8	7	1.00	22	0.318
339	A	2	1	1.00	18	0.056
340	A	5	4	1.00	20	0.200
341	A	10	9	1.00	20	0.450
342	A	16	12	1.00	20	0.600
343	A	3	2	1.00	14	0.143
344	A	2	2	1.00	20	0.100
345	A	14	8	1.00	20	0.400
346	A	4	3	1.00	26	0.115
347	A	4	3	1.00	24	0.125
348	A	6	4	1.00	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
349	A	4	4	1.00	21	0.190
350	A	2	1	1.00	15	0.067
351	A	4	3	1.00	18	0.167
352	A	3	2	1.00	22	0.091
353	A	3	2	1.00	16	0.125
354	A	6	5	1.00	16	0.312
355	A	3	2	1.00	25	0.080
356	A	3	2	1.00	19	0.105
357	A	2	1	1.00	23	0.043
358	A	5	4	1.00	23	0.174
359	A	5	4	1.00	21	0.190
360	A	10	6	1.00	28	0.214
361	A	2	1	1.00	24	0.042
362	A	6	5	1.00	26	0.192
363	A	2	1	1.00	14	0.071
364	A	5	3	1.00	16	0.188
365	A	5	5	1.00	16	0.312
366	A	7	5	1.00	43	0.116
367	A	17	13	1.00	26	0.500
368	A	18	13	1.00	16	0.812
369	A	3	2	1.00	15	0.133
370	A	3	2	1.00	15	0.133
371	A	3	2	1.00	17	0.118
372	A	3	2	1.00	15	0.133
373	A	4	3	1.00	15	0.200
374	A	4	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
375	A	3	2	1.00	20	0.100
376	A	7	6	1.00	29	0.207
377	A	4	3	1.00	22	0.136
378	A	6	4	1.00	16	0.250
379	A	6	4	1.00	16	0.250
380	A	5	4	1.00	14	0.286
381	A	4	3	1.00	12	0.250
382	A	4	3	1.00	16	0.188
383	A	6	5	1.00	16	0.312
384	A	4	3	1.00	16	0.188
385	A	4	3	1.00	16	0.188
386	A	4	3	1.00	16	0.188
387	A	8	5	1.00	17	0.294
388	A	8	5	1.00	19	0.263
389	A	8	5	1.00	15	0.333
390	A	8	5	1.00	17	0.294
391	A	16	10	1.00	23	0.435
392	A	17	10	1.00	21	0.476
393	F	0	0	N/A	0	N/A
394	A	15	11	1.00	17	0.647
395	A	13	9	1.00	17	0.529
396	A	13	9	1.00	15	0.600
397	A	9	6	1.00	9	0.667
398	A	17	12	1.00	17	0.706
399	A	17	12	1.00	17	0.706
400	A	17	12	1.00	17	0.706

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	16	12	1.00	17	0.706
402	A	14	10	1.00	17	0.588
403	A	14	10	1.00	15	0.667
404	A	10	7	1.00	9	0.778
405	A	31	14	1.00	17	0.824
406	A	31	14	1.00	17	0.824
407	A	31	14	1.00	17	0.824
408	A	15	11	1.00	17	0.647
409	A	15	10	1.00	17	0.588
410	A	15	10	1.00	15	0.667
411	A	11	7	1.00	9	0.778
412	A	46	15	1.00	17	0.882
413	A	46	15	1.00	17	0.882
414	A	46	15	1.00	17	0.882
415	A	4	4	1.00	14	0.286
416	A	5	5	1.00	13	0.385
417	A	4	4	1.00	16	0.250
418	A	5	5	1.00	18	0.278
419	A	3	2	1.00	14	0.143
420	A	4	3	1.00	16	0.188
421	A	2	2	1.00	11	0.182
422	A	1	0	1.00	17	0.000
423	A	1	1	1.00	11	0.091
424	B	1	0	6.23	73	0.000
425	A	11	7	1.00	13	0.538
426	A	13	9	1.00	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
427	A	3	2	1.00	15	0.133
428	A	3	2	1.00	11	0.182
429	A	1	0	1.00	11	0.000
430	A	1	0	1.00	11	0.000
431	A	5	4	1.00	16	0.250
432	A	2	1	1.00	16	0.062
433	A	4	3	1.00	15	0.200
434	A	1	1	1.00	15	0.067
435	A	1	1	1.00	20	0.050
436	A	3	2	1.00	15	0.133
437	A	6	5	1.00	13	0.385
438	A	1	1	1.00	22	0.045
439	A	3	2	1.00	18	0.111
440	A	3	3	1.00	20	0.150
441	A	3	2	1.00	16	0.125
442	A	6	6	1.00	17	0.353
443	A	1	1	1.00	17	0.059
444	A	4	3	1.00	25	0.120
445	A	2	2	1.00	20	0.100
446	A	3	2	1.00	18	0.111
447	A	6	5	1.00	15	0.333
448	A	2	1	1.00	11	0.091
449	A	5	5	1.00	13	0.385
450	A	3	2	1.00	20	0.100
451	A	2	1	1.00	16	0.062
452	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
453	A	2	1	1.00	16	0.062
454	A	1	1	1.00	9	0.111
455	A	2	2	1.00	7	0.286
456	A	2	2	1.00	11	0.182
457	A	1	1	1.00	7	0.143
458	A	1	1	1.00	9	0.111
459	A	1	1	1.00	9	0.111
460	A	2	2	1.00	10	0.200
461	A	2	1	1.00	13	0.077
462	A	2	1	1.00	11	0.091
463	A	2	1	1.00	14	0.071
464	A	4	4	1.00	16	0.250
465	A	2	1	1.00	7	0.143
466	A	2	1	1.00	11	0.091
467	A	5	5	1.00	13	0.385
468	A	5	5	1.00	13	0.385
469	A	5	4	1.00	14	0.286
470	A	2	1	0.78	13	0.077
471	A	3	2	1.00	20	0.100
472	A	4	4	1.00	18	0.222
473	A	2	1	1.00	12	0.083
474	A	2	1	1.00	10	0.100
475	A	3	2	1.00	16	0.125
476	A	2	1	1.00	11	0.091
477	A	1	1	1.00	7	0.143
478	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
479	A	2	1	1.00	12	0.083
480	A	3	3	1.00	16	0.188
481	A	2	1	1.00	11	0.091
482	A	2	1	1.00	17	0.059
483	A	2	1	1.00	29	0.034
484	A	2	1	1.00	18	0.056
485	A	3	2	1.00	14	0.143
486	A	2	1	1.00	24	0.042
487	A	2	1	1.00	11	0.091
488	A	3	2	1.00	18	0.111
489	A	3	3	1.00	15	0.200
490	A	3	3	1.00	16	0.188
491	A	10	7	1.00	15	0.467
492	A	5	2	1.00	50	0.040
493	F	0	0	N/A	0	N/A
494	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{2\sqrt{3}\,b^{3/2}-9bx+9x^3}\,dx$

Optimal. Leaf size=77

$$\frac{1}{3\sqrt{3}\,\sqrt{b}\,\left(\sqrt{3}\,\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}$$

```
[Out] -1/27*ln(-x*3^(1/2)+b^(1/2))/b+1/27*ln(x*3^(1/2)+2*b^(1/2))/b+1/9*3^(1/2)/b
^(1/2)/(-3*x+3^(1/2)*b^(1/2))
```

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2063, 44}

$$\frac{1}{3\sqrt{3}\,\sqrt{b}\,\left(\sqrt{3}\,\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqrt[3]*x]/(2
7*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rule 2063

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[1/(3^(3*p)*
a^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d},
 x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2\sqrt{3}\,b^{3/2}-9bx+9x^3}\,dx &= \left(324b^3\right)\int \frac{1}{\left(6\sqrt{3}\,b^{3/2}-18bx\right)^2\left(6\sqrt{3}\,b^{3/2}+9bx\right)}\,dx\\ &= \left(324b^3\right)\int\left(\frac{1}{324\sqrt{3}\,b^{7/2}\left(\sqrt{3}\sqrt{b}-3x\right)^2}+\frac{1}{2916b^4\left(\sqrt{3}\sqrt{b}-3x\right)}+\frac{1}{2916b^4\left(2\sqrt{3}\sqrt{b}\right.}\right.\\ &= \frac{1}{3\sqrt{3}\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}\end{aligned}$$

Mathematica [A] time = 0.07, size = 143, normalized size = 1.86

$$-\frac{\left(3x-\sqrt{3}\sqrt{b}\right)\left(2\sqrt{3}\sqrt{b}+3x\right)\left(\left(3x-\sqrt{3}\sqrt{b}\right)\log\left(3x-\sqrt{3}\sqrt{b}\right)+\left(\sqrt{3}\sqrt{b}-3x\right)\log\left(2\sqrt{3}\sqrt{b}+3x\right)+3\sqrt{3}\sqrt{b}\right.}{81b\left(2\sqrt{3}\,b^{3/2}-9bx+9x^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] -1/81*((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]*Sqrt
[b] + (-(Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (Sqrt[3]*S
qrt[b] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(b*(2*Sqrt[3]*b^(3/2) - 9*b*x
+ 9*x^3))
```

fricas [A] time = 0.59, size = 76, normalized size = 0.99

$$-\frac{3\sqrt{3}\sqrt{b}\,x-\left(3\,x^2-b\right)\log\left(2\sqrt{3}\sqrt{b}+3\,x\right)+\left(3\,x^2-b\right)\log\left(-\sqrt{3}\sqrt{b}+3\,x\right)+3\,b}{27\left(3\,bx^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/27*(3*sqrt(3)*sqrt(b)*x - (3*x^2 - b)*log(2*sqrt(3)*sqrt(b) + 3*x) + (3*
x^2 - b)*log(-sqrt(3)*sqrt(b) + 3*x) + 3*b)/(3*b*x^2 - b^2)
```

giac [A] time = 0.22, size = 55, normalized size = 0.71

$$\frac{\log\left(\left|9\sqrt{3}x+18\sqrt{b}\right|\right)}{27b}-\frac{\log\left(\left|-\sqrt{3}x+\sqrt{b}\right|\right)}{27b}-\frac{1}{9\left(\sqrt{3}x-\sqrt{b}\right)\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="giac")
```

```
[Out] 1/27*log(abs(9*sqrt(3)*x + 18*sqrt(b)))/b - 1/27*log(abs(-sqrt(3)*x + sqrt(
b)))/b - 1/9/((sqrt(3)*x - sqrt(b))*sqrt(b))
```

maple [C] time = 0.05, size = 43, normalized size = 0.56

$$\frac{\ln\left(-\mathrm{RootOf}\left(9\_Z^3-9\_Zb+2\sqrt{3}\,b^{\frac{3}{2}}\right)+x\right)}{27\,\mathrm{RootOf}\left(9\_Z^3-9\_Zb+2\sqrt{3}\,b^{\frac{3}{2}}\right)^2-9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x)
```

```
[Out] 1/9*sum(1/(3*_R^2-b)*ln(x-_R),_R=RootOf(-9*b*_Z+9*_Z^3+2*b^(3/2)*3^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{9x^3+2\sqrt{3}b^{\frac{3}{2}}-9bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)
```

mupad [B] time = 0.20, size = 51, normalized size = 0.66

$$\frac{2\sqrt{3}\sqrt{27}\,\mathrm{atanh}\left(\frac{\sqrt{3}\sqrt{27}}{27}+\frac{2\sqrt{27}\,x}{9\sqrt{b}}\right)}{243\,b}-\frac{\sqrt{3}}{27\sqrt{b}\left(x-\frac{\sqrt{3}\sqrt{b}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*3^(1/2)*b^(3/2) - 9*b*x + 9*x^3),x)
```

```
[Out] (2*3^(1/2)*27^(1/2)*atanh((3^(1/2)*27^(1/2))/27 + (2*27^(1/2)*x)/(9*b^(1/2)
)))/(243*b) - 3^(1/2)/(27*b^(1/2)*(x - (3^(1/2)*b^(1/2))/3))
```

sympy [A] time = 0.33, size = 60, normalized size = 0.78

$$-\frac{3\sqrt{3}}{81\sqrt{b}\,x-27\sqrt{3}\,b}+\frac{-\frac{\log\left(-\frac{\sqrt{3}\,\sqrt{b}}{3}+x\right)}{27}+\frac{\log\left(\frac{2\sqrt{3}\,\sqrt{b}}{3}+x\right)}{27}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)
```

```
[Out] -3*sqrt(3)/(81*sqrt(b)*x - 27*sqrt(3)*b) + (-log(-sqrt(3)*sqrt(b)/3 + x)/27
 + log(2*sqrt(3)*sqrt(b)/3 + x)/27)/b
```

3.2 $\int \left(a^3+3 a^2 b x+3 a b^2 x^2+b^3 x^3\right)^p \, dx$

Optimal. Leaf size=30

$$\frac{\left(\frac{a}{b}+x\right) \left(b^3 \left(\frac{a}{b}+x\right)^3\right)^p}{3 p+1}$$

```
[Out] (a/b+x)*(b^3*(a/b+x)^3)^p/(1+3*p)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2067, 15, 30}

$$\frac{\left(\frac{a}{b}+x\right) \left(b^3 \left(\frac{a}{b}+x\right)^3\right)^p}{3 p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

```
[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx = \text{Subst}\left(\int \left(b^3x^3\right)^p\,dx, x, \frac{a}{b}+x\right)$$
$$= \left(\left(\frac{a}{b}+x\right)^{-3p}\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p\right)\text{Subst}\left(\int x^{3p}\,dx, x, \frac{a}{b}+x\right)$$
$$= \frac{(a+bx)\left((a+bx)^3\right)^p}{b(1+3p)}$$

Mathematica [A] time = 0.11, size = 23, normalized size = 0.77

$$\frac{(a+bx)\left((a+bx)^3\right)^p}{3bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
[Out] ((a + b*x)*((a + b*x)^3)^p)/(b + 3*b*p)
```

fricas [A] time = 0.86, size = 43, normalized size = 1.43

$$\frac{(bx+a)\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="fricas")
[Out] (b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)
```

giac [B] time = 0.20, size = 73, normalized size = 2.43

$$\frac{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p bx+\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p a}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="giac")
[Out] ((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*b*x + (b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3)^p*a)/(3*b*p + b)
```

maple [A] time = 0.01, size = 46, normalized size = 1.53

$$\frac{(bx+a)\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p}{\left(3p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x)
[Out] (b*x+a)/b/(1+3*p)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p
```

maxima [A] time = 0.49, size = 25, normalized size = 0.83

$$\frac{(bx+a)(bx+a)^{3\,p}}{b\left(3\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="maxima")
[Out] (b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))
```

mupad [B] time = 2.13, size = 52, normalized size = 1.73

$$\left(\frac{x}{3\,p+1}+\frac{a}{b\,\left(3\,p+1\right)}\right)\,\left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p,x)
[Out] (x/(3*p + 1) + a/(b*(3*p + 1)))*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x}{\sqrt[3]{a^3}} & \text{for}\: b = 0 \wedge p = -\frac{1}{3} \\ x\left(a^3\right)^p & \text{for}\: b = 0 \\ \int \frac{1}{\sqrt[3]{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx & \text{for}\: p = -\frac{1}{3} \\ \frac{a\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p}{3bp+b} + \frac{bx\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p}{3bp+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)
```

```
[Out] Piecewise((x/(a**3)**(1/3), Eq(b, 0) & Eq(p, -1/3)), (x*(a**3)**p, Eq(b, 0)
), (Integral((a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**(-1/3), x), E
q(p, -1/3)), (a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p +
 b) + b*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b), T
rue))
```

3.3 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{10}}{10b}$$

```
[Out] 1/10*(b*x+a)^10/b
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2059, 32}

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3\,dx &= \int (a+bx)^9\,dx\\ &= \frac{(a+bx)^{10}}{10b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

fricas [B] time = 0.46, size = 97, normalized size = 6.93

$$\frac{1}{10}x^{10}b^9+x^9b^8a+\frac{9}{2}x^8b^7a^2+12x^7b^6a^3+21x^6b^5a^4+\frac{126}{5}x^5b^4a^5+21x^4b^3a^6+12x^3b^2a^7+\frac{9}{2}x^2ba^8+xa^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^9 + x^9*b^8*a + 9/2*x^8*b^7*a^2 + 12*x^7*b^6*a^3 + 21*x^6*b^5*a
^4 + 126/5*x^5*b^4*a^5 + 21*x^4*b^3*a^6 + 12*x^3*b^2*a^7 + 9/2*x^2*b*a^8 +
x*a^9
```

giac [B] time = 0.25, size = 97, normalized size = 6.93

$$\frac{1}{10}\,b^9x^{10}+ab^8x^9+\frac{9}{2}\,a^2b^7x^8+12\,a^3b^6x^7+21\,a^4b^5x^6+\frac{126}{5}\,a^5b^4x^5+21\,a^6b^3x^4+12\,a^7b^2x^3+\frac{9}{2}\,a^8bx^2+a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*x
^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2 +
a^9*x
```

maple [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{1}{10}b^9x^{10}+a\,b^8x^9+\frac{9}{2}a^2b^7x^8+12a^3b^6x^7+21a^4b^5x^6+\frac{126}{5}a^5b^4x^5+21a^6b^3x^4+12a^7b^2x^3+\frac{9}{2}a^8b\,x^2+a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

```
[Out] 1/10*b^9*x^10+a*b^8*x^9+9/2*a^2*b^7*x^8+12*a^3*b^6*x^7+21*a^4*b^5*x^6+126/5
*a^5*b^4*x^5+21*a^6*b^3*x^4+12*a^7*b^2*x^3+9/2*a^8*b*x^2+a^9*x
```

maxima [B] time = 0.69, size = 216, normalized size = 15.43

$$\frac{1}{10}\,b^9x^{10}+ab^8x^9+\frac{27}{8}\,a^2b^7x^8+\frac{27}{7}\,a^3b^6x^7+\frac{27}{4}\,a^6b^3x^4+a^9x+\frac{3}{4}\left(b^3x^4+4\,ab^2x^3+6\,a^2bx^2\right)a^6+\frac{9}{10}\left(5\,b^3x^6+18\,ab^2x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 27/8*a^2*b^7*x^8 + 27/7*a^3*b^6*x^7 + 27/4*a^6*
b^3*x^4 + a^9*x + 3/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^6 + 9/10*(5*b
^3*x^6 + 18*a*b^2*x^5)*a^4*b^2 + 3/70*(10*b^6*x^7 + 70*a*b^5*x^6 + 126*a^2*
b^4*x^5 + 210*a^4*b^2*x^3 + 21*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b)*a^3 + 9/56
*(7*b^6*x^8 + 48*a*b^5*x^7 + 84*a^2*b^4*x^6)*a^2*b
```

mupad [B] time = 2.07, size = 97, normalized size = 6.93

$$a^9\,x+\frac{9\,a^8\,b\,x^2}{2}+12\,a^7\,b^2\,x^3+21\,a^6\,b^3\,x^4+\frac{126\,a^5\,b^4\,x^5}{5}+21\,a^4\,b^5\,x^6+12\,a^3\,b^6\,x^7+\frac{9\,a^2\,b^7\,x^8}{2}+a\,b^8\,x^9+\frac{b^9\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

```
[Out] a^9*x + (b^9*x^10)/10 + (9*a^8*b*x^2)/2 + a*b^8*x^9 + 12*a^7*b^2*x^3 + 21*a
^6*b^3*x^4 + (126*a^5*b^4*x^5)/5 + 21*a^4*b^5*x^6 + 12*a^3*b^6*x^7 + (9*a^2
*b^7*x^8)/2
```

sympy [B] time = 0.09, size = 107, normalized size = 7.64

$$a^{9}x+\frac{9a^{8}bx^{2}}{2}+12a^{7}b^{2}x^{3}+21a^{6}b^{3}x^{4}+\frac{126a^{5}b^{4}x^{5}}{5}+21a^{4}b^{5}x^{6}+12a^{3}b^{6}x^{7}+\frac{9a^{2}b^{7}x^{8}}{2}+ab^{8}x^{9}+\frac{b^{9}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4 + 126*a**5
*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 + 9*a**2*b**7*x**8/2 +
 a*b**8*x**9 + b**9*x**10/10
```

3.4 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^7}{7b}$$

```
[Out] 1/7*(b*x+a)^7/b
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2059, 32}

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx &= \int (a+bx)^6\,dx\\ &= \frac{(a+bx)^7}{7b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

fricas [B] time = 0.43, size = 64, normalized size = 4.57

$$\frac{1}{7}x^7b^6 + x^6b^5a + 3x^5b^4a^2 + 5x^4b^3a^3 + 5x^3b^2a^4 + 3x^2ba^5 + xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^6 + x^6*b^5*a + 3*x^5*b^4*a^2 + 5*x^4*b^3*a^3 + 5*x^3*b^2*a^4 + 3
*x^2*b*a^5 + x*a^6
```

giac [B] time = 0.37, size = 64, normalized size = 4.57

$$\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3
*a^5*b*x^2 + a^6*x
```

maple [B] time = 0.00, size = 65, normalized size = 4.64

$$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5b\,x^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

```
[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^2
+a^6*x
```

maxima [B] time = 0.66, size = 99, normalized size = 7.07

$$\frac{1}{7}b^6x^7+ab^5x^6+\frac{9}{5}a^2b^4x^5+3a^4b^2x^3+a^6x+\frac{1}{2}\left(b^3x^4+4ab^2x^3+6a^2bx^2\right)a^3+\frac{3}{10}\left(4b^3x^5+15ab^2x^4\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x + 1/2*(b^
3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^5 + 15*a*b^2*x^4)*a^
2*b
```

mupad [B] time = 0.03, size = 64, normalized size = 4.57

$$a^6\,x+3\,a^5\,b\,x^2+5\,a^4\,b^2\,x^3+5\,a^3\,b^3\,x^4+3\,a^2\,b^4\,x^5+a\,b^5\,x^6+\frac{b^6\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

```
[Out] a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*x
^4 + 3*a^2*b^4*x^5
```

sympy [B] time = 0.08, size = 66, normalized size = 4.71

$$a^{6}x+3a^{5}bx^{2}+5a^{4}b^{2}x^{3}+5a^{3}b^{3}x^{4}+3a^{2}b^{4}x^{5}+ab^{5}x^{6}+\frac{b^{6}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*
x**5 + a*b**5*x**6 + b**6*x**7/7
```

3.5 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$

Optimal. Leaf size=35

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

```
[Out] a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{3}{2}a^2bx^2+a^3x+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Rubi steps

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

fricas [A] time = 0.52, size = 31, normalized size = 0.89

$$\frac{1}{4}x^4b^3+x^3b^2a+\frac{3}{2}x^2ba^2+xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3
```

giac [A] time = 0.28, size = 31, normalized size = 0.89

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="giac")
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{4}b^3x^4 + a\,b^2x^3 + \frac{3}{2}a^2b\,x^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x)
[Out] a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4
```

maxima [A] time = 0.74, size = 31, normalized size = 0.89

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$a^3\,x + \frac{3\,a^2\,b\,x^2}{2} + a\,b^2\,x^3 + \frac{b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x,x)
[Out] a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

sympy [A] time = 0.07, size = 32, normalized size = 0.91

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

3.6 $\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

```
[Out] -1/2/b/(b*x+a)^2
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] -1/(2*b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3} \, dx &= \int \frac{1}{(a+bx)^3} \, dx\\ &= -\frac{1}{2b(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
[Out] -1/2*1/(b*(a + b*x)^2)
```

fricas [A] time = 0.59, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^3x^2 + 2\,ab^2x + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="fricas")
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

giac [A] time = 0.26, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(bx + a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="giac")
[Out] -1/2/((b*x + a)^2*b)
```

maple [A] time = 0.02, size = 13, normalized size = 0.93

$$-\frac{1}{2\,(bx + a)^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)
[Out] -1/2/b/(b*x+a)^2
```

maxima [A] time = 0.65, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^3x^2 + 2\,ab^2x + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

mupad [B] time = 0.03, size = 26, normalized size = 1.86

$$-\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)
```

```
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

sympy [B] time = 0.18, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b+4ab^2x+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
```

```
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

3.7 $\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2} dx$

Optimal. Leaf size=14

$$-\frac{1}{5b(a+bx)^5}$$

```
[Out] -1/5/b/(b*x+a)^5
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
```

```
[Out] -1/(5*b*(a + b*x)^5)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2} dx &= \int \frac{1}{(a+bx)^6} dx \\ &= -\frac{1}{5b(a+bx)^5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
```

```
[Out] -1/5*1/(b*(a + b*x)^5)
```

fricas [B] time = 0.73, size = 57, normalized size = 4.07

$$-\frac{1}{5\left(b^{6}x^{5}+5\,ab^{5}x^{4}+10\,a^{2}b^{4}x^{3}+10\,a^{3}b^{3}x^{2}+5\,a^{4}b^{2}x+a^{5}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

giac [A] time = 0.29, size = 12, normalized size = 0.86

$$-\frac{1}{5\,(bx+a)^{5}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] -1/5/((b*x + a)^5*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{5\,(bx+a)^{5}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

```
[Out] -1/5/b/(b*x+a)^5
```

maxima [B] time = 0.71, size = 57, normalized size = 4.07

$$-\frac{1}{5\left(b^{6}x^{5}+5\,ab^{5}x^{4}+10\,a^{2}b^{4}x^{3}+10\,a^{3}b^{3}x^{2}+5\,a^{4}b^{2}x+a^{5}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

mupad [B] time = 2.05, size = 59, normalized size = 4.21

$$-\frac{1}{5\,a^5\,b+25\,a^4\,b^2\,x+50\,a^3\,b^3\,x^2+50\,a^2\,b^4\,x^3+25\,a\,b^5\,x^4+5\,b^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

```
[Out] -1/(5*a^5*b + 5*b^6*x^5 + 25*a^4*b^2*x + 25*a*b^5*x^4 + 50*a^3*b^3*x^2 + 50
*a^2*b^4*x^3)
```

sympy [B] time = 0.35, size = 61, normalized size = 4.36

$$-\frac{1}{5a^5b+25a^4b^2x+50a^3b^3x^2+50a^2b^4x^3+25ab^5x^4+5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] -1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25*
a*b**5*x**4 + 5*b**6*x**5)
```

3.8 $\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{8b(a+bx)^8}$$

```
[Out] -1/8/b/(b*x+a)^8
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/(8*b*(a + b*x)^8)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3}\,dx &= \int \frac{1}{(a+bx)^9}\,dx\\ &= -\frac{1}{8b(a+bx)^8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/8*1/(b*(a + b*x)^8)
```

fricas [B] time = 0.83, size = 90, normalized size = 6.43

$$-\frac{1}{8\left(b^{9}x^{8}+8\,ab^{8}x^{7}+28\,a^{2}b^{7}x^{6}+56\,a^{3}b^{6}x^{5}+70\,a^{4}b^{5}x^{4}+56\,a^{5}b^{4}x^{3}+28\,a^{6}b^{3}x^{2}+8\,a^{7}b^{2}x+a^{8}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

giac [A] time = 0.39, size = 12, normalized size = 0.86

$$-\frac{1}{8\,(bx+a)^{8}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] -1/8/((b*x + a)^8*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{8\,(bx+a)^{8}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

```
[Out] -1/8/b/(b*x+a)^8
```

maxima [B] time = 0.54, size = 90, normalized size = 6.43

$$-\frac{1}{8\left(b^{9}x^{8}+8\,ab^{8}x^{7}+28\,a^{2}b^{7}x^{6}+56\,a^{3}b^{6}x^{5}+70\,a^{4}b^{5}x^{4}+56\,a^{5}b^{4}x^{3}+28\,a^{6}b^{3}x^{2}+8\,a^{7}b^{2}x+a^{8}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

mupad [B] time = 2.07, size = 92, normalized size = 6.57

$$-\frac{1}{8\,a^8\,b+64\,a^7\,b^2\,x+224\,a^6\,b^3\,x^2+448\,a^5\,b^4\,x^3+560\,a^4\,b^5\,x^4+448\,a^3\,b^6\,x^5+224\,a^2\,b^7\,x^6+64\,a\,b^8\,x^7+8\,b^9\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

```
[Out] -1/(8*a^8*b + 8*b^9*x^8 + 64*a^7*b^2*x + 64*a*b^8*x^7 + 224*a^6*b^3*x^2 + 4
48*a^5*b^4*x^3 + 560*a^4*b^5*x^4 + 448*a^3*b^6*x^5 + 224*a^2*b^7*x^6)
```

sympy [B] time = 0.54, size = 97, normalized size = 6.93

$$-\frac{1}{8a^8b+64a^7b^2x+224a^6b^3x^2+448a^5b^4x^3+560a^4b^5x^4+448a^3b^6x^5+224a^2b^7x^6+64ab^8x^7+8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] -1/(8*a**8*b + 64*a**7*b**2*x + 224*a**6*b**3*x**2 + 448*a**5*b**4*x**3 + 5
60*a**4*b**5*x**4 + 448*a**3*b**6*x**5 + 224*a**2*b**7*x**6 + 64*a*b**8*x**
7 + 8*b**9*x**8)
```

3.9 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$

Optimal. Leaf size=84

$$-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{b^3x\left(b^2-3ac\right)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

```
[Out] -b^3*(-3*a*c+b^2)^3*x/c^3+3/4*b^2*(-3*a*c+b^2)^2*(c*x+b)^4/c^4-3/7*b*(-3*a*
c+b^2)*(c*x+b)^7/c^4+1/10*(c*x+b)^10/c^4
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2060, 194}

$$-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{b^3x\left(b^2-3ac\right)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)^4)/(4*c^4
) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)^10/(10*c^4)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rubi steps

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3 dx = \frac{1}{27}\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^3 dx, x, \frac{b}{c}+x\right)$$

$$= \frac{1}{27}\text{Subst}\left(\int \left(\frac{27\left(-b^3+3abc\right)^3}{c^3}+81\left(b^3-3abc\right)^2x^3-81bc^3\left(b^2-3ac\right)x^6\right.\right.$$

$$= -\frac{b^3\left(b^2-3ac\right)^3x}{c^3}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+$$

Mathematica [A] time = 0.02, size = 159, normalized size = 1.89

$$27a^3b^3x+\frac{81}{2}a^2b^4x^2+\frac{27}{4}b^2x^4\left(a^2c^2+6ab^2c+b^4\right)+\frac{9}{7}bc^3x^7\left(ac+9b^2\right)+9b^2c^2x^6\left(ac+2b^2\right)+\frac{27}{5}b^3cx^5\left(5ac+3b^2\right)+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] 27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (27*b^2*(b^4
 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a*c)*x^5)/5 + 9*b^2*c
^2*(2*b^2 + a*c)*x^6 + (9*b*c^3*(9*b^2 + a*c)*x^7)/7 + (9*b^2*c^4*x^8)/2 +
b*c^5*x^9 + (c^6*x^10)/10
```

fricas [B] time = 0.52, size = 166, normalized size = 1.98

$$\frac{1}{10}x^{10}c^6+x^9c^5b+\frac{9}{2}x^8c^4b^2+\frac{81}{7}x^7c^3b^3+\frac{9}{7}x^7c^4ba+18x^6c^2b^4+9x^6c^3b^2a+\frac{81}{5}x^5cb^5+27x^5c^2b^3a+\frac{27}{4}x^4b^6+\frac{81}{2}x^4cb^4a+\frac{27}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^6 + x^9*c^5*b + 9/2*x^8*c^4*b^2 + 81/7*x^7*c^3*b^3 + 9/7*x^7*c^
4*b*a + 18*x^6*c^2*b^4 + 9*x^6*c^3*b^2*a + 81/5*x^5*c*b^5 + 27*x^5*c^2*b^3*
a + 27/4*x^4*b^6 + 81/2*x^4*c*b^4*a + 27/4*x^4*c^2*b^2*a^2 + 27*x^3*b^5*a +
 27*x^3*c*b^3*a^2 + 81/2*x^2*b^4*a^2 + 27*x*b^3*a^3
```

giac [B] time = 0.27, size = 166, normalized size = 1.98

$$\frac{1}{10}c^6x^{10}+bc^5x^9+\frac{9}{2}b^2c^4x^8+\frac{81}{7}b^3c^3x^7+\frac{9}{7}abc^4x^7+18b^4c^2x^6+9ab^2c^3x^6+\frac{81}{5}b^5cx^5+27ab^3c^2x^5+\frac{27}{4}b^6x^4+\frac{81}{2}ab^4cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 + 9/7*a*b*c^
4*x^7 + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x^5 + 27*a*b^3*c^2*x^
5 + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^2*b^2*c^2*x^4 + 27*a*b^5*x^3 +
 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x
```

maple [B] time = 0.00, size = 295, normalized size = 3.51

$$\frac{c^6x^{10}}{10}+b\,c^5x^9+\frac{9b^2c^4x^8}{2}+\frac{81a^2b^4x^2}{2}+27a^3b^3x+\frac{\left(3ab\,c^4+63b^3c^3+\left(6ab\,c^2+18b^3c\right)c^2\right)x^7}{7}+\frac{\left(18a\,b^2c^3+45b^4c^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

```
[Out] 1/10*c^6*x^10+b*c^5*x^9+9/2*b^2*c^4*x^8+1/7*(3*a*b*c^4+63*b^3*c^3+c^2*(6*a*
b*c^2+18*b^3*c))*x^7+1/6*(18*a*b^2*c^3+45*b^4*c^2+3*b*c*(6*a*b*c^2+18*b^3*c
)+c^2*(18*a*b^2*c+9*b^4))*x^6+1/5*(63*a*b^3*c^2+3*b^2*(6*a*b*c^2+18*b^3*c)+
3*b*c*(18*a*b^2*c+9*b^4))*x^5+1/4*(3*a*b*(6*a*b*c^2+18*b^3*c)+3*b^2*(18*a*b
^2*c+9*b^4)+54*b^4*c*a+9*c^2*a^2*b^2)*x^4+1/3*(3*a*b*(18*a*b^2*c+9*b^4)+54*
b^5*a+27*b^3*c*a^2)*x^3+81/2*a^2*b^4*x^2+27*a^3*b^3*x
```

maxima [B] time = 0.68, size = 204, normalized size = 2.43

$$\frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{27}{8}\,b^2c^4x^8+\frac{27}{7}\,b^3c^3x^7+\frac{27}{4}\,b^6x^4+27\,a^3b^3x+\frac{27}{4}\left(c^2x^4+4\,bcx^3+6\,b^2x^2\right)a^2b^2+\frac{9}{10}\left(5\,c^2x^6+18\,bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 + 27/4*b^6*
x^4 + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a^2*b^2 + 9/10*
(5*c^2*x^6 + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x^7 + 70*b*c^3*x^6 + 126*b^2*c^
2*x^5 + 210*b^4*x^3 + 21*(4*c^2*x^5 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^4*x^
8 + 48*b*c^3*x^7 + 84*b^2*c^2*x^6)*b^2
```

mupad [B] time = 2.08, size = 149, normalized size = 1.77

$$x^4\left(\frac{27\,a^2\,b^2\,c^2}{4}+\frac{81\,a\,b^4\,c}{2}+\frac{27\,b^6}{4}\right)+\frac{c^6\,x^{10}}{10}+27\,a^3\,b^3\,x+b\,c^5\,x^9+\frac{81\,a^2\,b^4\,x^2}{2}+\frac{9\,b^2\,c^4\,x^8}{2}+9\,b^2\,c^2\,x^6\,\left(2\,b^2+a\,c\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

```
[Out] x^4*((27*b^6)/4 + (27*a^2*b^2*c^2)/4 + (81*a*b^4*c)/2) + (c^6*x^10)/10 + 27
*a^3*b^3*x + b*c^5*x^9 + (81*a^2*b^4*x^2)/2 + (9*b^2*c^4*x^8)/2 + 9*b^2*c^2
*x^6*(a*c + 2*b^2) + 27*a*b^3*x^3*(a*c + b^2) + (27*b^3*c*x^5*(5*a*c + 3*b^
2))/5 + (9*b*c^3*x^7*(a*c + 9*b^2))/7
```

sympy [B] time = 0.10, size = 175, normalized size = 2.08

$$27a^3b^3x+\frac{81a^2b^4x^2}{2}+\frac{9b^2c^4x^8}{2}+bc^5x^9+\frac{c^6x^{10}}{10}+x^7\left(\frac{9abc^4}{7}+\frac{81b^3c^3}{7}\right)+x^6\left(9ab^2c^3+18b^4c^2\right)+x^5\left(27ab^3c^2+\frac{81b^5c}{5}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] 27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**5*x**9 + c
**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x**6*(9*a*b**2*c**3 +
 18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b**5*c/5) + x**4*(27*a**2*b**2*c
**2/4 + 81*a*b**4*c/2 + 27*b**6/4) + x**3*(27*a**2*b**3*c + 27*a*b**5)
```

3.10 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx$

Optimal. Leaf size=56

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

```
[Out] b^2*(-3*a*c+b^2)^2*x/c^2-1/2*b*(-3*a*c+b^2)*(c*x+b)^4/c^3+1/7*(c*x+b)^7/c^3
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2060, 194}

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^3) + (b + 
c*x)^7/(7*c^3)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rubi steps

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 dx = \frac{1}{9}\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^2 dx, x, \frac{b}{c}+x\right)$$
$$= \frac{1}{9}\text{Subst}\left(\int \left(\frac{9\left(-b^3+3abc\right)^2}{c^2}-18bc\left(b^2-3ac\right)x^3+9c^4x^6\right) dx, x, \frac{b}{c}+x\right)$$
$$= \frac{b^2\left(b^2-3ac\right)^2 x}{c^2}-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{(b+cx)^7}{7c^3}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.46

$$9a^2b^2x+9ab^3x^2+\frac{3}{2}bcx^4\left(ac+3b^2\right)+3b^2x^3\left(2ac+b^2\right)+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] 9*a^2*b^2*x + 9*a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b^2 + a*c)*
x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7
```

fricas [A] time = 0.69, size = 83, normalized size = 1.48

$$\frac{1}{7}x^7c^4+x^6c^3b+3x^5c^2b^2+\frac{9}{2}x^4cb^3+\frac{3}{2}x^4c^2ba+3x^3b^4+6x^3cb^2a+9x^2b^3a+9xb^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^4 + x^6*c^3*b + 3*x^5*c^2*b^2 + 9/2*x^4*c*b^3 + 3/2*x^4*c^2*b*a +
 3*x^3*b^4 + 6*x^3*c*b^2*a + 9*x^2*b^3*a + 9*x*b^2*a^2
```

giac [A] time = 0.34, size = 83, normalized size = 1.48

$$\frac{1}{7}c^4x^7+bc^3x^6+3b^2c^2x^5+\frac{9}{2}b^3cx^4+\frac{3}{2}abc^2x^4+3b^4x^3+6ab^2cx^3+9ab^3x^2+9a^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b*c^2*x^4 +
 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x
```

maple [A] time = 0.00, size = 84, normalized size = 1.50

$$\frac{c^4x^7}{7}+b\,c^3x^6+3b^2c^2x^5+9a\,b^3x^2+9a^2b^2x+\frac{\left(6ab\,c^2+18b^3c\right)x^4}{4}+\frac{\left(18a\,b^2c+9b^4\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

```
[Out] 1/7*c^4*x^7+b*c^3*x^6+3*b^2*c^2*x^5+1/4*(6*a*b*c^2+18*b^3*c)*x^4+1/3*(18*a*
b^2*c+9*b^4)*x^3+9*a*b^3*x^2+9*a^2*b^2*x
```

maxima [A] time = 0.64, size = 93, normalized size = 1.66

$$\frac{1}{7}\,c^4x^7+bc^3x^6+\frac{9}{5}\,b^2c^2x^5+3\,b^4x^3+9\,a^2b^2x+\frac{3}{2}\left(c^2x^4+4\,bcx^3+6\,b^2x^2\right)ab+\frac{3}{10}\left(4\,c^2x^5+15\,bcx^4\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2*x + 3/2*(
c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5 + 15*b*c*x^4)*b^2
```

mupad [B] time = 0.04, size = 79, normalized size = 1.41

$$x^3\,\left(3\,b^4+6\,a\,c\,b^2\right)+\frac{c^4\,x^7}{7}+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+b\,c^3\,x^6+3\,b^2\,c^2\,x^5+\frac{3\,b\,c\,x^4\,\left(3\,b^2+a\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

```
[Out] x^3*(3*b^4 + 6*a*b^2*c) + (c^4*x^7)/7 + 9*a^2*b^2*x + 9*a*b^3*x^2 + b*c^3*x
^6 + 3*b^2*c^2*x^5 + (3*b*c*x^4*(a*c + 3*b^2))/2
```

sympy [A] time = 0.08, size = 87, normalized size = 1.55

$$9a^2b^2x+9ab^3x^2+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}+x^4\left(\frac{3abc^2}{2}+\frac{9b^3c}{2}\right)+x^3\left(6ab^2c+3b^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 + c**4*x**7/
7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c + 3*b**4)
```

3.11 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right) dx$

Optimal. Leaf size=32

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

```
[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

```
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Rubi steps

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right) dx = 3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

```
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

fricas [A] time = 0.60, size = 28, normalized size = 0.88

$$\frac{1}{4}x^4c^2+x^3cb+\frac{3}{2}x^2b^2+3xba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="fricas")
```

[Out] 1/4*x^4*c^2 + x^3*c*b + 3/2*x^2*b^2 + 3*x*b*a

giac [A] time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="giac")

[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{1}{4}c^2x^4 + bc\,x^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x)

[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4

maxima [A] time = 0.55, size = 28, normalized size = 0.88

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="maxima")

[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x

mupad [B] time = 0.04, size = 28, normalized size = 0.88

$$\frac{3\,b^2\,x^2}{2} + b\,c\,x^3 + 3\,a\,b\,x + \frac{c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2,x)

[Out] (3*b^2*x^2)/2 + (c^2*x^4)/4 + 3*a*b*x + b*c*x^3

sympy [A] time = 0.06, size = 31, normalized size = 0.97

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)
```

```
[Out] 3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4
```

3.12 $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\,\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}\,+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2(b+c}{\sqrt[3]{b^2-3}}}{\sqrt{3}}\right.}{\sqrt{3}\,b^{2/3}\left(b^2-\right.}$$

```
[Out] 1/3*ln(b-b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(2/3)/(-3*a*c+b^2)^(2/3)-1/6*ln(
b^(2/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)
^2)/b^(2/3)/(-3*a*c+b^2)^(2/3)-1/3*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^
2)^(1/3))/b^(1/3)*3^(1/2))/b^(2/3)/(-3*a*c+b^2)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2067, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\,\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}\,+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2(b+c}{\sqrt[3]{b^2-3}}}{\sqrt{3}}\right.}{\sqrt{3}\,b^{2/3}\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))]/(
Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3)
 + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) +
 b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(6*b^(2/3)*(b^2
 - 3*a*c)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}dx = \text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}dx, x, \frac{b}{c}+x\right)$$

$$= \frac{c^{2/3}\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}+c^{2/3}x}dx, x, \frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{c^{2/3}\text{Subst}\left(\int \frac{-\frac{2\sqrt[3]{b}\sqrt[3]{b^2-}}{\sqrt[3]{c}}}{\frac{b^{2/3}(b^2-3ac)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{}}\right.}{3b^{2/3}\left(b^2-\right.}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}+2c^{4/3}x}{\frac{b^{2/3}(b^2-3ac)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-3ac}x+c^{4/}}\right.}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}\left(b^2-3ac\right)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3ac}\right.}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}\left(b^2-\right.\right.}{}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.34

$$\frac{1}{3}\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&, \frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b
*c*#1 + c^2*#1^2) & ]/3
```

fricas [B] time = 0.86, size = 387, normalized size = 2.06

$$-\frac{2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{6}}\left(b^3-3abc\right)\arctan\left(\frac{2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}(cx+b)+\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)}{3\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{5}{6}}}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*(b^3 - 3*a*b*c)*arc
tan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt
(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4
*c + 9*a^2*b^2*c^2)^(5/6)) + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b
^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b
^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*
b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)
*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^
2*c^2)^(2/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)
```

giac [A] time = 0.36, size = 212, normalized size = 1.13

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\,cx+\sqrt{3}\,b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}}\right)}{3\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}\,cx+\sqrt{3}\,b-\sqrt{3}\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3\,abc\right)^{\frac{1}{3}}\right.\right.}{6\left(b^6-6\,ab^4c+9\,a^2b^2c^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3
))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1
/3) - 1/6*log(4*(sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^
2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^
2)^(1/3) + 1/3*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c
+ 9*a^2*b^2*c^2)^(1/3)
```

maple [C] time = 0.00, size = 57, normalized size = 0.30

$$\frac{\ln\left(-\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)+x\right)}{3\,\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)^2c^2+6\,\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)bc+3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x)
```

```
[Out] 1/3*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(-_R+x),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3
*_Z*b^2+3*a*b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c^2x^3 + 3\,bcx^2 + 3\,b^2x + 3\,ab}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="maxima")
```

```
[Out] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

mupad [B] time = 0.49, size = 174, normalized size = 0.93

$$\frac{\ln\left(b + b^{1/3}\left(3\,a\,c - b^2\right)^{1/3} + c\,x\right)}{3\,b^{2/3}\left(3\,a\,c - b^2\right)^{2/3}} + \frac{\ln\left(3\,b\,c^3 + 3\,c^4\,x + \frac{3\,b^{1/3}\,c^3\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(3\,a\,c-b^2\right)^{1/3}}{2}\right)\left(-1 + \sqrt{3}\,1\mathrm{i}\right)}{6\,b^{2/3}\left(3\,a\,c - b^2\right)^{2/3}} - \frac{\ln\left(3\,b\,c^3 + 3\,c^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2),x)
```

```
[Out] log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x)/(3*b^(2/3)*(3*a*c - b^2)^(2/3))
+ (log(3*b*c^3 + 3*c^4*x + (3*b^(1/3)*c^3*(3^(1/2)*1i - 1)*(3*a*c - b^2)^(1
/3))/2)*(3^(1/2)*1i - 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3)) - (log(3*b*c^3 +
3*c^4*x - (3*b^(1/3)*c^3*(3^(1/2)*1i + 1)*(3*a*c - b^2)^(1/3))/2)*(3^(1/2)*
1i + 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3))
```

sympy [A] time = 0.41, size = 53, normalized size = 0.28

$$\operatorname{RootSum}\left(t^3\left(243a^2b^2c^2 - 162ab^4c + 27b^6\right) - 1, \left(t \mapsto t\log\left(x + \frac{9tabc - 3tb^3 + b}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

```
[Out] RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1, Lambda(_t,
 _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))
```

3.13 $\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{c\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}-2$$

```
[Out] -1/3*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)-2/9*c*ln(b-
b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(5/3)/(-3*a*c+b^2)^(5/3)+1/9*c*ln(b^(2/3)
*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)^2)/b^(
5/3)/(-3*a*c+b^2)^(5/3)+2/9*c*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^2)^(1
/3))/b^(1/3)*3^(1/2))/b^(5/3)/(-3*a*c+b^2)^(5/3)*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{c\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}-2$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -(c*(b/c + x))/(3*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3))
+ (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3
))])/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3)) - (2*c*Log[b - b^(1/3)*(b^2 -
3*a*c)^(1/3) + c*x])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^2
- 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]
)/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^2}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\right.}{3b\left(b^2-3ac\right)}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{\left(2c^{5/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}}\right.}{9b^{5/3}\left(b^2-3a\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3a}\right.\right.}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3}\right.\right.}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{2c\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}\left(b^2-3ac\right)^{5/3}} - \frac{2}{-}$$

Mathematica [C] time = 0.07, size = 112, normalized size = 0.46

$$-\frac{2c\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]+\frac{3(b+cx)}{3ab+x\left(3b^2+3bcx+c^2x^2\right)}}{9\left(b^3-3abc\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -1/9*((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*RootSum[3
*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c
^2*#1^2) & ])/(b^3 - 3*a*b*c)
```

fricas [B] time = 0.82, size = 704, normalized size = 2.87

$$-\frac{3\,b^{7} - 18\,ab^{5}c + 27\,a^{2}b^{3}c^{2} - 2\,\sqrt{3}\left(b^{6} - 6\,ab^{4}c + 9\,a^{2}b^{2}c^{2}\right)^{\frac{1}{6}}\left(3\,ab^{4}c - 9\,a^{2}b^{2}c^{2} + \left(b^{3}c^{3} - 3\,abc^{4}\right)x^{3} + 3\left(b^{4}c^{2} - 3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b^7 - 18*a*b^5*c + 27*a^2*b^3*c^2 - 2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2)^(1/6)*(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3 +
 3*(b^4*c^2 - 3*a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x)*arctan(1/3*(2*s
qrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(3)*(b^6 - 6
*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*c + 9*a^2*b
^2*c^2)^(5/6)) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2
*x^2 + 3*b^2*c*x + 3*a*b*c)*log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^
2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*
x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) + 2*(b^6
- 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a
*b*c)*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a
^2*b^2*c^2)^(2/3)) + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3)*x)/(3*a*b^10 -
 27*a^2*b^8*c + 81*a^3*b^6*c^2 - 81*a^4*b^4*c^3 + (b^9*c^2 - 9*a*b^7*c^3 +
27*a^2*b^5*c^4 - 27*a^3*b^3*c^5)*x^3 + 3*(b^10*c - 9*a*b^8*c^2 + 27*a^2*b^6
*c^3 - 27*a^3*b^4*c^4)*x^2 + 3*(b^11 - 9*a*b^9*c + 27*a^2*b^7*c^2 - 27*a^3*
b^5*c^3)*x)
```

giac [A] time = 0.36, size = 289, normalized size = 1.18

$$-\frac{2\,\sqrt{3}\left(\frac{c^{3}}{b^{6}-6\,ab^{4}c+9\,a^{2}b^{2}c^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,cx+\sqrt{3}\,b-\sqrt{3}\left(-b^{3}+3\,abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^{3}+3\,abc\right)^{\frac{1}{3}}}\right)-\left(\frac{c^{3}}{b^{6}-6\,ab^{4}c+9\,a^{2}b^{2}c^{2}}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,cx+\sqrt{3}\,b-\sqrt{3}\left(-\right.\right.\right.}{9\left(b^{3}-3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(
3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a
*b*c)^(1/3))) - (c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3
```

```
)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3
+ 3*a*b*c)^(1/3))^2) + 2*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(
abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^3 - 3*a*b*c) - 1/3*(c*x + b)/((c
^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)*(b^3 - 3*a*b*c))
```

maple [C] time = 0.02, size = 136, normalized size = 0.56

$$\frac{2c\ln\left(-\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)+x\right)}{9\left(3ac-b^2\right)b\left(\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)^2c^2+2\,\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)bc+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

```
[Out] (1/3*c/b/(3*a*c-b^2)*x+1/3/(3*a*c-b^2))/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+2
/9*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(-_R+x),_R=RootOf(_Z^3*c
^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{3}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}}\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}+\frac{2\log\left(\left|cx+b+\left(-b^3+3abc\right)\right.\right.}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}\right.}{3\left(b^3-3abc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] -2/3*c*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^3 - 3*a*b
*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*
(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x)
```

mupad [B] time = 2.65, size = 247, normalized size = 1.01

$$\frac{\frac{1}{3\left(3ac-b^2\right)}+\frac{cx}{3b\left(3ac-b^2\right)}}{3b^2x+3bcx^2+3ab+c^2x^3}+\frac{2c\ln\left(b+b^{1/3}\left(3ac-b^2\right)^{1/3}+cx\right)}{9b^{5/3}\left(3ac-b^2\right)^{5/3}}-\frac{\ln\left(2b-b^{1/3}\left(3ac-b^2\right)^{1/3}+2cx-\sqrt{3}\,b^{1/3}\right.}{9b^{5/3}\left(3ac-b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

```
[Out] (1/(3*(3*a*c - b^2)) + (c*x)/(3*b*(3*a*c - b^2)))/(3*a*b + 3*b^2*x + c^2*x^
3 + 3*b*c*x^2) + (2*c*log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x))/(9*b^(5/3
)*(3*a*c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x - 3
^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c + 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*
c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x + 3^(1/2)*
b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c - 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*c - b^2
)^(5/3))
```

sympy [A] time = 1.22, size = 192, normalized size = 0.78

$$\frac{b+cx}{27a^2b^2c-9ab^4+x^3\left(9abc^3-3b^3c^2\right)+x^2\left(27ab^2c^2-9b^4c\right)+x\left(27ab^3c-9b^5\right)}+\text{RootSum}\left(t^3\left(177147a^5b^5c^5-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] (b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*c**2) + x*
*2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**5)) + RootSum(_t**3*
(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**4 + 196830*a**3*b**9*c**3 - 65
610*a**2*b**11*c**2 + 10935*a*b**13*c - 729*b**15) - 8*c**3, Lambda(_t, _t*
log(x + (81*_t*a**2*b**2*c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2*c**2
))))
```

3.14 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3} \, dx$

Optimal. Leaf size=305

$$\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-\right.}{27b^{8/3}\left(b^2-\right.}$$

```
[Out] -1/6*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/18*c^2*
(b/c+x)/b^2/(-3*a*c+b^2)^2/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+5/27*c^2*ln(b-
b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(8/3)/(-3*a*c+b^2)^(8/3)-5/54*c^2*ln(b^(2
/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)^2)/
b^(8/3)/(-3*a*c+b^2)^(8/3)-5/27*c^2*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b
^2)^(1/3))/b^(1/3)*3^(1/2))/b^(8/3)/(-3*a*c+b^2)^(8/3)*3^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-\right.}{27b^{8/3}\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] -(c*(b/c + x))/(6*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2
) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*x + 3*b*c*x^2
+ c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(
Sqrt[3]*b^(1/3))])/(9*Sqrt[3]*b^(8/3)*(b^2 - 3*a*c)^(8/3)) + (5*c^2*Log[b -
 b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (5*
c^2*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c +
x) + c^2*(b/c + x)^2])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
```

```
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^3}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} - \frac{(5c)\,\text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x\right.}\right.}{6b\left(b^2-3ac\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b+cx}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b\right.}$$

Mathematica [C] time = 0.09, size = 149, normalized size = 0.49

$$\frac{10c^2\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]-\frac{3(b+cx)\left(-3bc\left(8a+5cx^2\right)+3b^3-15b^2cx-5c^3x^3\right)}{\left(3ab+x\left(3b^2+3bcx+c^2x^2\right)\right)^2}}{54\left(b^3-3abc\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] ((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c*x^2)))/(3
*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSum[3*a*b + 3*b^2*#1 +
 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) & ])/(54
*(b^3 - 3*a*b*c)^2)
```

fricas [B] time = 0.78, size = 1268, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(9*b^10 - 126*a*b^8*c + 513*a^2*b^6*c^2 - 648*a^3*b^4*c^3 - 15*(b^6*c
^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^4 - 60*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b
^3*c^5)*x^3 - 90*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^2 + 10*sqrt(3)*(
9*a^2*b^5*c^2 - 27*a^3*b^3*c^3 + (b^3*c^6 - 3*a*b*c^7)*x^6 + 6*(b^4*c^5 - 3
*a*b^2*c^6)*x^5 + 15*(b^5*c^4 - 3*a*b^3*c^5)*x^4 + 6*(3*b^6*c^3 - 8*a*b^4*c
^4 - 3*a^2*b^2*c^5)*x^3 + 9*(b^7*c^2 - a*b^5*c^3 - 6*a^2*b^3*c^4)*x^2 + 18*
(a*b^6*c^2 - 3*a^2*b^4*c^3)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*arct
an(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(
3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(5/6)) + 5*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*
a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*
a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^5 + 3*a*b^3*
c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3
)*(b^3 - 3*a*b*c)) - 10*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*
c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*
c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^4 + 3*a*b^2*c - (b
^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)) - 36*(b^9*c
- 4*a*b^7*c^2 - 3*a^2*b^5*c^3 + 18*a^3*b^3*c^4)*x)/(9*a^2*b^14 - 108*a^3*b^
12*c + 486*a^4*b^10*c^2 - 972*a^5*b^8*c^3 + 729*a^6*b^6*c^4 + (b^12*c^4 - 1
2*a*b^10*c^5 + 54*a^2*b^8*c^6 - 108*a^3*b^6*c^7 + 81*a^4*b^4*c^8)*x^6 + 6*(
b^13*c^3 - 12*a*b^11*c^4 + 54*a^2*b^9*c^5 - 108*a^3*b^7*c^6 + 81*a^4*b^5*c^
```

```
7)*x^5 + 15*(b^14*c^2 - 12*a*b^12*c^3 + 54*a^2*b^10*c^4 - 108*a^3*b^8*c^5 +
 81*a^4*b^6*c^6)*x^4 + 6*(3*b^15*c - 35*a*b^13*c^2 + 150*a^2*b^11*c^3 - 270
*a^3*b^9*c^4 + 135*a^4*b^7*c^5 + 81*a^5*b^5*c^6)*x^3 + 9*(b^16 - 10*a*b^14*
c + 30*a^2*b^12*c^2 - 135*a^4*b^8*c^4 + 162*a^5*b^6*c^5)*x^2 + 18*(a*b^15 -
 12*a^2*b^13*c + 54*a^3*b^11*c^2 - 108*a^4*b^9*c^3 + 81*a^5*b^7*c^4)*x)
```

giac [A] time = 0.40, size = 366, normalized size = 1.20

$$\frac{5\left(2\sqrt{3}\left(\frac{c^6}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}}\right)-\left(\frac{c^6}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b\right.\right.\right.\right.}{54\left(b^6-6ab^4c+9\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 5/54*(2*sqrt(3)*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(
3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a
*b*c)^(1/3))) - (c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3
)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3
+ 3*a*b*c)^(1/3))^2) + 2*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(
abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2) +
 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 + 12*b^3*c*x + 24*a*b*c^2*
x - 3*b^4 + 24*a*b^2*c)/((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)*(c^2*x^3 + 3*b*c
*x^2 + 3*b^2*x + 3*a*b)^2)
```

maple [C] time = 0.02, size = 276, normalized size = 0.90

$$\frac{5c^2\ln\left(-\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)+x\right)}{27\left(9a^2c^2-6a\,b^2c+b^4\right)b^2\left(\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)^2c^2+2\,\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

```
[Out] (5/18*c^4/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^4+10/9/b*c^3/(9*a^2*c^2-6*a*b^2*c
+b^4)*x^3+5/3*c^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+2/3/b*(2*a*c+b^2)*c/(9*a^2*
c^2-6*a*b^2*c+b^4)*x+1/6*(8*a*c-b^2)/(9*a^2*c^2-6*a*b^2*c+b^4))/(c^2*x^3+3*
b*c*x^2+3*b^2*x+3*a*b)^2+5/27*c^2/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*sum(1/(_R^2
*c^2+2*_R*b*c+b^2)*ln(-_R+x),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{5}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}}\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}+\frac{2\log\left(\left|cx+b+\left(-b^3+3abc\right)\right.\right.}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}\right.}{9\left(b^6-6ab^4c+9a^2b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 5/9*c^2*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^6 - 6*a*
b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 -
3*b^4 + 24*a*b^2*c + 12*(b^3*c + 2*a*b*c^2)*x)/(9*a^2*b^8 - 54*a^3*b^6*c +
81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 -
 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c
^4)*x^4 + 6*(3*b^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 +
 9*(b^10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9 - 6*
a^2*b^7*c + 9*a^3*b^5*c^2)*x)
```

mupad [B] time = 2.98, size = 483, normalized size = 1.58

$$\frac{\frac{8\,a\,c-b^2}{6\,\left(9\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}+\frac{5\,c^2\,x^2}{3\,\left(9\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}+\frac{10\,c^3\,x^3}{9\,b\,\left(9\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}+\frac{5\,c^4\,x^4}{18\,b^2\,\left(9\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}+\frac{2\,c\,x\,\left(b^2+2\,a\,c\right)}{3\,b\,\left(9\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}}{x^2\,\left(9\,b^4+18\,a\,c\,b^2\right)+9\,a^2\,b^2+c^4\,x^6+x^3\,\left(18\,b^3\,c+6\,a\,b\,c^2\right)+6\,b\,c^3\,x^5+15\,b^2\,c^2\,x^4+18\,a\,b^3\,x}+\frac{5\,c^2\,\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

```
[Out] ((8*a*c - b^2)/(6*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (5*c^2*x^2)/(3*(b^4 + 9*
a^2*c^2 - 6*a*b^2*c)) + (10*c^3*x^3)/(9*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) +
(5*c^4*x^4)/(18*b^2*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/
(3*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)))/(x^2*(9*b^4 + 18*a*b^2*c) + 9*a^2*b^2
+ c^4*x^6 + x^3*(18*b^3*c + 6*a*b*c^2) + 6*b*c^3*x^5 + 15*b^2*c^2*x^4 + 18*
a*b^3*x) + (5*c^2*log(b*(3*a*c - b^2)^(8/3) - b^(19/3) + c*x*(3*a*c - b^2)^
(8/3) + 27*a^3*b^(1/3)*c^3 - 27*a^2*b^(7/3)*c^2 + 9*a*b^(13/3)*c))/(27*b^(8
/3)*(3*a*c - b^2)^(8/3)) - (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2
*c*x - 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2))/(27*
b^(8/3)*(3*a*c - b^2)^(8/3)) + (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3)
 + 2*c*x + 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2))/
(27*b^(8/3)*(3*a*c - b^2)^(8/3))
```

sympy [A] time = 2.54, size = 474, normalized size = 1.55

$$\overline{1458a^4b^4c^2 - 972a^3b^6c + 162a^2b^8 + x^6\left(162a^2b^2c^6 - 108ab^4c^5 + 18b^6c^4\right) + x^5\left(972a^2b^3c^5 - 648ab^5c^4 + 108b^7c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] (24*a*b**2*c - 3*b**4 + 30*b**2*c**2*x**2 + 20*b*c**3*x**3 + 5*c**4*x**4 +
x*(24*a*b*c**2 + 12*b**3*c))/(1458*a**4*b**4*c**2 - 972*a**3*b**6*c + 162*a
**2*b**8 + x**6*(162*a**2*b**2*c**6 - 108*a*b**4*c**5 + 18*b**6*c**4) + x**
5*(972*a**2*b**3*c**5 - 648*a*b**5*c**4 + 108*b**7*c**3) + x**4*(2430*a**2*
b**4*c**4 - 1620*a*b**6*c**3 + 270*b**8*c**2) + x**3*(972*a**3*b**3*c**4 +
2268*a**2*b**5*c**3 - 1836*a*b**7*c**2 + 324*b**9*c) + x**2*(2916*a**3*b**4
*c**3 - 486*a**2*b**6*c**2 - 648*a*b**8*c + 162*b**10) + x*(2916*a**3*b**5*
c**2 - 1944*a**2*b**7*c + 324*a*b**9)) + RootSum(_t**3*(129140163*a**8*b**8
*c**8 - 344373768*a**7*b**10*c**7 + 401769396*a**6*b**12*c**6 - 267846264*a
**5*b**14*c**5 + 111602610*a**4*b**16*c**4 - 29760696*a**3*b**18*c**3 + 496
0116*a**2*b**20*c**2 - 472392*a*b**22*c + 19683*b**24) - 125*c**6, Lambda(_
t, _t*log(x + (729*_t*a**3*b**3*c**3 - 729*_t*a**2*b**5*c**2 + 243*_t*a*b**
7*c - 27*_t*b**9 + 5*b*c**2)/(5*c**3))))
```

3.15 $\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + b\right.$

Optimal. Leaf size=361

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2\right.}{}$$

```
[Out] 1/4*(-a*d+b*c)^3*(-a*f+b*e)^3*(b*x+a)^4/b^7+3/5*(-a*d+b*c)^2*(-a*f+b*e)^2*(
-2*a*d*f+b*c*f+b*d*e)*(b*x+a)^5/b^7+1/2*(-a*d+b*c)*(-a*f+b*e)*(5*a^2*d^2*f^
2-5*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x+a)^6/b^7+1/7*(-
2*a*d*f+b*c*f+b*d*e)*(10*a^2*d^2*f^2-10*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+8*c*
d*e*f+d^2*e^2))*(b*x+a)^7/b^7+3/8*d*f*(5*a^2*d^2*f^2-5*a*b*d*f*(c*f+d*e)+b^
2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x+a)^8/b^7+1/3*d^2*f^2*(-2*a*d*f+b*c*f+b*
d*e)*(b*x+a)^9/b^7+1/10*d^3*f^3*(b*x+a)^10/b^7
```

Rubi [A] time = 0.66, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2059, 88}

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^3,x]
```

```
[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e -
 a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*e
- a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2
 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^
7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f - 2*a
*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3 \, dx = \int (a+bx)^3(c+dx)^3(e+fx)^3 \, dx$$
$$= \int \left(\frac{(bc-ad)^3(be-af)^3(a+bx)^3}{b^6} + \frac{3(bc-a\cdots}{\cdots}\right.$$
$$= \frac{(bc-ad)^3(be-af)^3(a+bx)^4}{4b^7} + \frac{3(bc-ad)^2\cdots}{\cdots}$$

Mathematica [A] time = 0.22, size = 653, normalized size = 1.81

$$a^3c^3e^3x+\frac{3}{8}bdfx^8\left(a^2d^2f^2+3abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)+acex^3\left(a^2\left(c^2f^2+3cdef+d^2e^2\right)+3abc\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^3,x]
```

```
[Out] a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a*c*e*(b^2*
c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^3
+ ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3*c*
d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))
*x^4)/4 + (3*(b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c*f) + b
^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c*d*e*f +
 c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^6)
/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + (3*b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f + a*d*f)*x^9)/3
 + (b^3*d^3*f^3*x^10)/10
```

fricas [B] time = 0.67, size = 987, normalized size = 2.73

$$\frac{1}{10}x^{10}f^3d^3b^3+\frac{1}{3}x^9f^2ed^3b^3+\frac{1}{3}x^9f^3d^2cb^3+\frac{1}{3}x^9f^3d^3b^2a+\frac{3}{8}x^8fe^2d^3b^3+\frac{9}{8}x^8f^2ed^2cb^3+\frac{3}{8}x^8f^3dc^2b^3+\frac{9}{8}x^8f^2ed^3b^2a+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*f^3*d^3*b^3 + 1/3*x^9*f^2*e*d^3*b^3 + 1/3*x^9*f^3*d^2*c*b^3 + 1/3
*x^9*f^3*d^3*b^2*a + 3/8*x^8*f*e^2*d^3*b^3 + 9/8*x^8*f^2*e*d^2*c*b^3 + 3/8*
x^8*f^3*d*c^2*b^3 + 9/8*x^8*f^2*e*d^3*b^2*a + 9/8*x^8*f^3*d^2*c*b^2*a + 3/8
*x^8*f^3*d^3*b*a^2 + 1/7*x^7*e^3*d^3*b^3 + 9/7*x^7*f*e^2*d^2*c*b^3 + 9/7*x^
7*f^2*e*d*c^2*b^3 + 1/7*x^7*f^3*c^3*b^3 + 9/7*x^7*f*e^2*d^3*b^2*a + 27/7*x^
7*f^2*e*d^2*c*b^2*a + 9/7*x^7*f^3*d*c^2*b^2*a + 9/7*x^7*f^2*e*d^3*b*a^2 + 9
/7*x^7*f^3*d^2*c*b*a^2 + 1/7*x^7*f^3*d^3*a^3 + 1/2*x^6*e^3*d^2*c*b^3 + 3/2*
x^6*f*e^2*d*c^2*b^3 + 1/2*x^6*f^2*e*c^3*b^3 + 1/2*x^6*e^3*d^3*b^2*a + 9/2*x
^6*f*e^2*d^2*c*b^2*a + 9/2*x^6*f^2*e*d*c^2*b^2*a + 1/2*x^6*f^3*c^3*b^2*a +
3/2*x^6*f*e^2*d^3*b*a^2 + 9/2*x^6*f^2*e*d^2*c*b*a^2 + 3/2*x^6*f^3*d*c^2*b*a
^2 + 1/2*x^6*f^2*e*d^3*a^3 + 1/2*x^6*f^3*d^2*c*a^3 + 3/5*x^5*e^3*d*c^2*b^3
+ 3/5*x^5*f*e^2*c^3*b^3 + 9/5*x^5*e^3*d^2*c*b^2*a + 27/5*x^5*f*e^2*d*c^2*b^
2*a + 9/5*x^5*f^2*e*c^3*b^2*a + 3/5*x^5*e^3*d^3*b*a^2 + 27/5*x^5*f*e^2*d^2*
c*b*a^2 + 27/5*x^5*f^2*e*d*c^2*b*a^2 + 3/5*x^5*f^3*c^3*b*a^2 + 3/5*x^5*f*e^
2*d^3*a^3 + 9/5*x^5*f^2*e*d^2*c*a^3 + 3/5*x^5*f^3*d*c^2*a^3 + 1/4*x^4*e^3*c
^3*b^3 + 9/4*x^4*e^3*d*c^2*b^2*a + 9/4*x^4*f*e^2*c^3*b^2*a + 9/4*x^4*e^3*d^
2*c*b*a^2 + 27/4*x^4*f*e^2*d*c^2*b*a^2 + 9/4*x^4*f^2*e*c^3*b*a^2 + 1/4*x^4*
e^3*d^3*a^3 + 9/4*x^4*f*e^2*d^2*c*a^3 + 9/4*x^4*f^2*e*d*c^2*a^3 + 1/4*x^4*f
^3*c^3*a^3 + x^3*e^3*c^3*b^2*a + 3*x^3*e^3*d*c^2*b*a^2 + 3*x^3*f*e^2*c^3*b*
a^2 + x^3*e^3*d^2*c*a^3 + 3*x^3*f*e^2*d*c^2*a^3 + x^3*f^2*e*c^3*a^3 + 3/2*x
^2*e^3*c^3*b*a^2 + 3/2*x^2*e^3*d*c^2*a^3 + 3/2*x^2*f*e^2*c^3*a^3 + x*e^3*c^
3*a^3
```

giac [B] time = 0.27, size = 971, normalized size = 2.69

$$\frac{1}{10}\,b^3d^3f^3x^{10}+\frac{1}{3}\,b^3cd^2f^3x^9+\frac{1}{3}\,ab^2d^3f^3x^9+\frac{1}{3}\,b^3d^3f^2x^9e+\frac{3}{8}\,b^3c^2df^3x^8+\frac{9}{8}\,ab^2cd^2f^3x^8+\frac{3}{8}\,a^2bd^3f^3x^8+\frac{9}{8}\,b^3cd^2f^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="giac")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*b^3*c*d^2*f^3*x^9 + 1/3*a*b^2*d^3*f^3*x^9 + 1/3
*b^3*d^3*f^2*x^9*e + 3/8*b^3*c^2*d*f^3*x^8 + 9/8*a*b^2*c*d^2*f^3*x^8 + 3/8*
a^2*b*d^3*f^3*x^8 + 9/8*b^3*c*d^2*f^2*x^8*e + 9/8*a*b^2*d^3*f^2*x^8*e + 1/7
*b^3*c^3*f^3*x^7 + 9/7*a*b^2*c^2*d*f^3*x^7 + 9/7*a^2*b*c*d^2*f^3*x^7 + 1/7*
a^3*d^3*f^3*x^7 + 3/8*b^3*d^3*f*x^8*e^2 + 9/7*b^3*c^2*d*f^2*x^7*e + 27/7*a*
b^2*c*d^2*f^2*x^7*e + 9/7*a^2*b*d^3*f^2*x^7*e + 1/2*a*b^2*c^3*f^3*x^6 + 3/2
*a^2*b*c^2*d*f^3*x^6 + 1/2*a^3*c*d^2*f^3*x^6 + 9/7*b^3*c*d^2*f*x^7*e^2 + 9/
7*a*b^2*d^3*f*x^7*e^2 + 1/2*b^3*c^3*f^2*x^6*e + 9/2*a*b^2*c^2*d*f^2*x^6*e +
 9/2*a^2*b*c*d^2*f^2*x^6*e + 1/2*a^3*d^3*f^2*x^6*e + 3/5*a^2*b*c^3*f^3*x^5
+ 3/5*a^3*c^2*d*f^3*x^5 + 1/7*b^3*d^3*x^7*e^3 + 3/2*b^3*c^2*d*f*x^6*e^2 + 9
```

```
/2*a*b^2*c*d^2*f*x^6*e^2 + 3/2*a^2*b*d^3*f*x^6*e^2 + 9/5*a*b^2*c^3*f^2*x^5*
e + 27/5*a^2*b*c^2*d*f^2*x^5*e + 9/5*a^3*c*d^2*f^2*x^5*e + 1/4*a^3*c^3*f^3*
x^4 + 1/2*b^3*c*d^2*x^6*e^3 + 1/2*a*b^2*d^3*x^6*e^3 + 3/5*b^3*c^3*f*x^5*e^2
 + 27/5*a*b^2*c^2*d*f*x^5*e^2 + 27/5*a^2*b*c*d^2*f*x^5*e^2 + 3/5*a^3*d^3*f*
x^5*e^2 + 9/4*a^2*b*c^3*f^2*x^4*e + 9/4*a^3*c^2*d*f^2*x^4*e + 3/5*b^3*c^2*d
*x^5*e^3 + 9/5*a*b^2*c*d^2*x^5*e^3 + 3/5*a^2*b*d^3*x^5*e^3 + 9/4*a*b^2*c^3*
f*x^4*e^2 + 27/4*a^2*b*c^2*d*f*x^4*e^2 + 9/4*a^3*c*d^2*f*x^4*e^2 + a^3*c^3*
f^2*x^3*e + 1/4*b^3*c^3*x^4*e^3 + 9/4*a*b^2*c^2*d*x^4*e^3 + 9/4*a^2*b*c*d^2
*x^4*e^3 + 1/4*a^3*d^3*x^4*e^3 + 3*a^2*b*c^3*f*x^3*e^2 + 3*a^3*c^2*d*f*x^3*
e^2 + a*b^2*c^3*x^3*e^3 + 3*a^2*b*c^2*d*x^3*e^3 + a^3*c*d^2*x^3*e^3 + 3/2*a
^3*c^3*f*x^2*e^2 + 3/2*a^2*b*c^3*x^2*e^3 + 3/2*a^3*c^2*d*x^2*e^3 + a^3*c^3*
x*e^3
```

maple [B] time = 0.00, size = 861, normalized size = 2.39

$$\frac{b^3d^3f^3x^{10}}{10}+\frac{\left(adf+bcf+bde\right)b^2d^2f^2x^9}{3}+a^3c^3e^3x+\frac{3\left(acf+ade+bce\right)a^2c^2e^2x^2}{2}+\frac{\left(\left(acf+ade+bce\right)b^2d^2f^2+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
```

```
[Out] 1/10*b^3*d^3*f^3*x^10+1/3*(a*d*f+b*c*f+b*d*e)*b^2*d^2*f^2*x^9+1/8*((a*c*f+a
*d*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^2*b*d*f+b*d*f*(2*(a*c*f+a*d*e
+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))*x^8+1/7*(a*c*e*b^2*d^2*f^2+2*(a*c*f+a
*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*d*f+b*c*f+b*d*e)*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+b*d*f*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*
e)*(a*d*f+b*c*f+b*d*e)))*x^7+1/6*(2*a*c*e*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*c*f+
a*d*e+b*c*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*d*f+b*c
*f+b*d*e)*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+b*d*f*(
2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2))*x^6+1/5*(a*c*e*(2*(a*c*
f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*b*
d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+(a*d*f+b*c*f+b*d*e)*(2*a*c*e
*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*b*d*f*a*c*e*(a*c*f+a*d*e+b*c*
e))*x^5+1/4*(a*c*e*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)
)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2
*(a*d*f+b*c*f+b*d*e)*a*c*e*(a*c*f+a*d*e+b*c*e)+b*d*f*a^2*c^2*e^2)*x^4+1/3*(
a*c*e*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*(a*c*f+a*d*e+b*
c*e)^2*a*c*e+(a*d*f+b*c*f+b*d*e)*a^2*c^2*e^2)*x^3+3/2*a^2*c^2*e^2*(a*c*f+a*
d*e+b*c*e)*x^2+a^3*c^3*e^3*x
```

maxima [A] time = 0.68, size = 461, normalized size = 1.28

$$\frac{1}{10}b^3d^3f^3x^{10}+\frac{1}{3}\left(bde+bcf+adf\right)b^2d^2f^2x^9+\frac{3}{8}\left(bde+bcf+adf\right)^2bdfx^8+a^3c^3e^3x+\frac{1}{7}\left(bde+bcf+adf\right)^3x^7+\frac{1}{4}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*(b*d*e + b*c*f + a*d*f)*b^2*d^2*f^2*x^9 + 3/8*(
b*d*e + b*c*f + a*d*f)^2*b*d*f*x^8 + a^3*c^3*e^3*x + 1/7*(b*d*e + b*c*f + a
*d*f)^3*x^7 + 1/4*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e +
 a*d*e + a*c*f)*x^2)*a^2*c^2*e^2 + 1/4*(b*c*e + a*d*e + a*c*f)^3*x^4 + 1/70
*(30*b^2*d^2*f^2*x^7 + 70*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + 42*(b*d*e + b
*c*f + a*d*f)^2*x^5 + 70*(b*c*e + a*d*e + a*c*f)^2*x^3 + 21*(4*b*d*f*x^5 +
5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f))*a*c*e + 1/10*(5*b*d
*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(b*c*e + a*d*e + a*c*f)^2 + 1/56*(2
1*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^7 + 28*(b^
2*d^2*e^2 + 2*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*f^2
)*x^6)*(b*c*e + a*d*e + a*c*f)
```

mupad [B]   time = 2.23, size = 787, normalized size = 2.18

$$x^7\left(\frac{a^3\,d^3\,f^3}{7}+\frac{9\,a^2\,b\,c\,d^2\,f^3}{7}+\frac{9\,a^2\,b\,d^3\,e\,f^2}{7}+\frac{9\,a\,b^2\,c^2\,d\,f^3}{7}+\frac{27\,a\,b^2\,c\,d^2\,e\,f^2}{7}+\frac{9\,a\,b^2\,d^3\,e^2\,f}{7}+\frac{b^3\,c^3\,f^3}{7}+\frac{9\,b^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^3,x)
```

```
[Out] x^7*((a^3*d^3*f^3)/7 + (b^3*c^3*f^3)/7 + (b^3*d^3*e^3)/7 + (9*a*b^2*c^2*d*f
^3)/7 + (9*a^2*b*c*d^2*f^3)/7 + (9*a*b^2*d^3*e^2*f)/7 + (9*a^2*b*d^3*e*f^2)
/7 + (9*b^3*c*d^2*e^2*f)/7 + (9*b^3*c^2*d*e*f^2)/7 + (27*a*b^2*c*d^2*e*f^2)
/7) + x^5*((3*a^2*b*c^3*f^3)/5 + (3*a^2*b*d^3*e^3)/5 + (3*a^3*c^2*d*f^3)/5
+ (3*b^3*c^2*d*e^3)/5 + (3*a^3*d^3*e^2*f)/5 + (3*b^3*c^3*e^2*f)/5 + (9*a*b^
2*c*d^2*e^3)/5 + (9*a*b^2*c^3*e*f^2)/5 + (9*a^3*c*d^2*e*f^2)/5 + (27*a*b^2*
c^2*d*e^2*f)/5 + (27*a^2*b*c*d^2*e^2*f)/5 + (27*a^2*b*c^2*d*e*f^2)/5) + x^6
*((a*b^2*c^3*f^3)/2 + (a*b^2*d^3*e^3)/2 + (a^3*c*d^2*f^3)/2 + (b^3*c*d^2*e^
3)/2 + (a^3*d^3*e*f^2)/2 + (b^3*c^3*e*f^2)/2 + (3*a^2*b*c^2*d*f^3)/2 + (3*a
^2*b*d^3*e^2*f)/2 + (3*b^3*c^2*d*e^2*f)/2 + (9*a*b^2*c*d^2*e^2*f)/2 + (9*a*
b^2*c^2*d*e*f^2)/2 + (9*a^2*b*c*d^2*e*f^2)/2) + x^4*((a^3*c^3*f^3)/4 + (a^3
*d^3*e^3)/4 + (b^3*c^3*e^3)/4 + (9*a*b^2*c^2*d*e^3)/4 + (9*a^2*b*c*d^2*e^3)
/4 + (9*a*b^2*c^3*e^2*f)/4 + (9*a^2*b*c^3*e*f^2)/4 + (9*a^3*c*d^2*e^2*f)/4
+ (9*a^3*c^2*d*e*f^2)/4 + (27*a^2*b*c^2*d*e^2*f)/4) + a^3*c^3*e^3*x + (b^3*
d^3*f^3*x^10)/10 + (3*a^2*c^2*e^2*x^2*(a*c*f + a*d*e + b*c*e))/2 + (b^2*d^2
*f^2*x^9*(a*d*f + b*c*f + b*d*e))/3 + a*c*e*x^3*(a^2*c^2*f^2 + a^2*d^2*e^2
+ b^2*c^2*e^2 + 3*a*b*c*d*e^2 + 3*a*b*c^2*e*f + 3*a^2*c*d*e*f) + (3*b*d*f*x
^8*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 3*a*b*c*d*f^2 + 3*a*b*d^2*e*f
 + 3*b^2*c*d*e*f))/8
```

sympy [B] time = 0.25, size = 1018, normalized size = 2.82

$$a^3c^3e^3x+\frac{b^3d^3f^3x^{10}}{10}+x^9\left(\frac{ab^2d^3f^3}{3}+\frac{b^3cd^2f^3}{3}+\frac{b^3d^3ef^2}{3}\right)+x^8\left(\frac{3a^2bd^3f^3}{8}+\frac{9ab^2cd^2f^3}{8}+\frac{9ab^2d^3ef^2}{8}+\frac{3b^3c^2df^3}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**3,x)
```

```
[Out] a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f**3/3 + b**
3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*d**3*f**3/8 + 9*a*b*
*2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3*b**3*c**2*d*f**3/8 + 9*b**3*c
*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f/8) + x**7*(a**3*d**3*f**3/7 + 9*a**2*b*
c*d**2*f**3/7 + 9*a**2*b*d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a*b**2
*c*d**2*e*f**2/7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3*c**2*
d*e*f**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3*c*d**2
*f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a**2*b*c*d**2*e*f
**2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/2 + 9*a*b**2*c**2*d*e*f**
2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*d**3*e**3/2 + b**3*c**3*e*f**2/2 +
3*b**3*c**2*d*e**2*f/2 + b**3*c*d**2*e**3/2) + x**5*(3*a**3*c**2*d*f**3/5 +
 9*a**3*c*d**2*e*f**2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*f**3/5 + 27*
a**2*b*c**2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*d**3*e**3/5 +
 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**2*c*d**2*e**3/
5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4*(a**3*c**3*f**3/4 +
 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*f/4 + a**3*d**3*e**3/4 + 9*a**
2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*d*e**2*f/4 + 9*a**2*b*c*d**2*e**3/4 + 9*
a*b**2*c**3*e**2*f/4 + 9*a*b**2*c**2*d*e**3/4 + b**3*c**3*e**3/4) + x**3*(a
**3*c**3*e*f**2 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**3 + 3*a**2*b*c**3*e
**2*f + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x**2*(3*a**3*c**3*e**2*f
/2 + 3*a**3*c**2*d*e**3/2 + 3*a**2*b*c**3*e**3/2)
```

3.16 $\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + b\right.$

Optimal. Leaf size=193

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4}{}$$

```
[Out] 1/3*(-a*d+b*c)^2*(-a*f+b*e)^2*(b*x+a)^3/b^5+1/2*(-a*d+b*c)*(-a*f+b*e)*(-2*a
*d*f+b*c*f+b*d*e)*(b*x+a)^4/b^5+1/5*(6*a^2*d^2*f^2-6*a*b*d*f*(c*f+d*e)+b^2*
(c^2*f^2+4*c*d*e*f+d^2*e^2))*(b*x+a)^5/b^5+1/3*d*f*(-2*a*d*f+b*c*f+b*d*e)*(
b*x+a)^6/b^5+1/7*d^2*f^2*(b*x+a)^7/b^5
```

Rubi [A] time = 0.23, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2059, 88}

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^2,x]
```

```
[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*f
)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*b*
d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b^5)
 + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)
^7)/(7*b^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^2 \, dx = \int (a+bx)^2(c+dx)^2(e+fx)^2 \, dx$$
$$= \int \left(\frac{(bc-ad)^2(be-af)^2(a+bx)^2}{b^4} + \frac{2(bc-a}{} \right.$$
$$= \frac{(bc-ad)^2(be-af)^2(a+bx)^3}{3b^5} + \frac{(bc-ad)(be}{}$$

Mathematica [A] time = 0.09, size = 241, normalized size = 1.25

$$\frac{1}{5}x^5\left(a^2d^2f^2+4abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{2}x^4\left(a^2df(cf+de)+ab\left(c^2f^2+4cdef+d^2e^2\right)+b^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^2,x]
```

```
[Out] a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2 + 4*a*b*c
*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^3)/3 + ((b^2*c*e*(d
*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^4)
/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2
*f^2))*x^5)/5 + (b*d*f*(b*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x^7)/7
```

fricas [A] time = 0.56, size = 346, normalized size = 1.79

$$\frac{1}{7}x^7f^2d^2b^2+\frac{1}{3}x^6fed^2b^2+\frac{1}{3}x^6f^2dcb^2+\frac{1}{3}x^6f^2d^2ba+\frac{1}{5}x^5e^2d^2b^2+\frac{4}{5}x^5fedcb^2+\frac{1}{5}x^5f^2c^2b^2+\frac{4}{5}x^5fed^2ba+\frac{4}{5}x^5f^2dcba+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*f^2*d^2*b^2 + 1/3*x^6*f*e*d^2*b^2 + 1/3*x^6*f^2*d*c*b^2 + 1/3*x^6*f
^2*d^2*b*a + 1/5*x^5*e^2*d^2*b^2 + 4/5*x^5*f*e*d*c*b^2 + 1/5*x^5*f^2*c^2*b^
2 + 4/5*x^5*f*e*d^2*b*a + 4/5*x^5*f^2*d*c*b*a + 1/5*x^5*f^2*d^2*a^2 + 1/2*x
^4*e^2*d*c*b^2 + 1/2*x^4*f*e*c^2*b^2 + 1/2*x^4*e^2*d^2*b*a + 2*x^4*f*e*d*c*
b*a + 1/2*x^4*f^2*c^2*b*a + 1/2*x^4*f*e*d^2*a^2 + 1/2*x^4*f^2*d*c*a^2 + 1/3
*x^3*e^2*c^2*b^2 + 4/3*x^3*e^2*d*c*b*a + 4/3*x^3*f*e*c^2*b*a + 1/3*x^3*e^2*
d^2*a^2 + 4/3*x^3*f*e*d*c*a^2 + 1/3*x^3*f^2*c^2*a^2 + x^2*e^2*c^2*b*a + x^2
*e^2*d*c*a^2 + x^2*f*e*c^2*a^2 + x*e^2*c^2*a^2
```

giac [A] time = 0.42, size = 346, normalized size = 1.79

$$\frac{1}{7}\,b^2d^2f^2x^7+\frac{1}{3}\,b^2cdf^2x^6+\frac{1}{3}\,abd^2f^2x^6+\frac{1}{3}\,b^2d^2fx^6e+\frac{1}{5}\,b^2c^2f^2x^5+\frac{4}{5}\,abcdf^2x^5+\frac{1}{5}\,a^2d^2f^2x^5+\frac{4}{5}\,b^2cdfx^5e+\frac{4}{5}\,abd^2f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*b^2*c*d*f^2*x^6 + 1/3*a*b*d^2*f^2*x^6 + 1/3*b^2*d
^2*f*x^6*e + 1/5*b^2*c^2*f^2*x^5 + 4/5*a*b*c*d*f^2*x^5 + 1/5*a^2*d^2*f^2*x^
5 + 4/5*b^2*c*d*f*x^5*e + 4/5*a*b*d^2*f*x^5*e + 1/2*a*b*c^2*f^2*x^4 + 1/2*a
^2*c*d*f^2*x^4 + 1/5*b^2*d^2*x^5*e^2 + 1/2*b^2*c^2*f*x^4*e + 2*a*b*c*d*f*x^
4*e + 1/2*a^2*d^2*f*x^4*e + 1/3*a^2*c^2*f^2*x^3 + 1/2*b^2*c*d*x^4*e^2 + 1/2
*a*b*d^2*x^4*e^2 + 4/3*a*b*c^2*f*x^3*e + 4/3*a^2*c*d*f*x^3*e + 1/3*b^2*c^2*
x^3*e^2 + 4/3*a*b*c*d*x^3*e^2 + 1/3*a^2*d^2*x^3*e^2 + a^2*c^2*f*x^2*e + a*b
*c^2*x^2*e^2 + a^2*c*d*x^2*e^2 + a^2*c^2*x*e^2
```

maple [A] time = 0.00, size = 188, normalized size = 0.97

$$\frac{b^2d^2f^2x^7}{7}+\frac{\left(adf+bcf+bde\right)bdf\,x^6}{3}+a^2c^2e^2x+\left(acf+ade+bce\right)ace\,x^2+\frac{\left(2\left(acf+ade+bce\right)bdf+\left(adf+bc\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
```

```
[Out] 1/7*b^2*d^2*f^2*x^7+1/3*(a*d*f+b*c*f+b*d*e)*b*d*f*x^6+1/5*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)*x^5+1/4*(2*a*b*c*d*e*f+2*(a*c*f+a*d*e+b*
c*e)*(a*d*f+b*c*f+b*d*e))*x^4+1/3*(2*(a*d*f+b*c*f+b*d*e)*a*c*e+(a*c*f+a*d*e
+b*c*e)^2)*x^3+a*c*e*(a*c*f+a*d*e+b*c*e)*x^2+a^2*c^2*e^2*x
```

maxima [A] time = 0.62, size = 180, normalized size = 0.93

$$\frac{1}{7}\,b^2d^2f^2x^7+\frac{1}{3}\left(bde+bcf+adf\right)bdfx^6+a^2c^2e^2x+\frac{1}{5}\left(bde+bcf+adf\right)^2x^5+\frac{1}{3}\left(bce+ade+acf\right)^2x^3+\frac{1}{6}\left(3\,bdfx^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + a^2*c^2*e^2*x
 + 1/5*(b*d*e + b*c*f + a*d*f)^2*x^5 + 1/3*(b*c*e + a*d*e + a*c*f)^2*x^3 +
1/6*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*c*f
```

```
)*x^2)*a*c*e + 1/10*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e +
a*d*e + a*c*f)
```

mupad [B] time = 0.08, size = 270, normalized size = 1.40

$$x^4\left(\frac{a^2\,c\,d\,f^2}{2}+\frac{a^2\,d^2\,e\,f}{2}+\frac{a\,b\,c^2\,f^2}{2}+2\,a\,b\,c\,d\,e\,f+\frac{a\,b\,d^2\,e^2}{2}+\frac{b^2\,c^2\,e\,f}{2}+\frac{b^2\,c\,d\,e^2}{2}\right)+x^3\left(\frac{a^2\,c^2\,f^2}{3}+\frac{4\,a^2\,c\,d\,e\,f}{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^2,x)
```

```
[Out] x^4*((a*b*c^2*f^2)/2 + (a*b*d^2*e^2)/2 + (a^2*c*d*f^2)/2 + (b^2*c*d*e^2)/2
+ (a^2*d^2*e*f)/2 + (b^2*c^2*e*f)/2 + 2*a*b*c*d*e*f) + x^3*((a^2*c^2*f^2)/3
 + (a^2*d^2*e^2)/3 + (b^2*c^2*e^2)/3 + (4*a*b*c*d*e^2)/3 + (4*a*b*c^2*e*f)/
3 + (4*a^2*c*d*e*f)/3) + x^5*((a^2*d^2*f^2)/5 + (b^2*c^2*f^2)/5 + (b^2*d^2*
e^2)/5 + (4*a*b*c*d*f^2)/5 + (4*a*b*d^2*e*f)/5 + (4*b^2*c*d*e*f)/5) + a^2*c
^2*e^2*x + (b^2*d^2*f^2*x^7)/7 + a*c*e*x^2*(a*c*f + a*d*e + b*c*e) + (b*d*f
*x^6*(a*d*f + b*c*f + b*d*e))/3
```

sympy [A] time = 0.14, size = 345, normalized size = 1.79

$$a^2c^2e^2x+\frac{b^2d^2f^2x^7}{7}+x^6\left(\frac{abd^2f^2}{3}+\frac{b^2cdf^2}{3}+\frac{b^2d^2ef}{3}\right)+x^5\left(\frac{a^2d^2f^2}{5}+\frac{4abcdf^2}{5}+\frac{4abd^2ef}{5}+\frac{b^2c^2f^2}{5}+\frac{4b^2cdef}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**2,x)
```

```
[Out] a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3 + b**2*c*d
*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 + 4*
a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5) +
x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e*f +
 a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**2*f**
2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e*
*2/3 + b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b*c**2*e
**2)
```

3.17 $\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + b$

Optimal. Leaf size=56

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

```
[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x
^4
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

```
[In] Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f
*x^3,x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/3
 + (b*d*f*x^4)/4
```

Rubi steps

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx = acex + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde + bcf$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.36

$$acex + \frac{1}{2}acfx^2 + \frac{1}{2}adex^2 + \frac{1}{3}adfx^3 + \frac{1}{2}bcex^2 + \frac{1}{3}bcfx^3 + \frac{1}{3}bdex^3 + \frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 +
 b*d*f*x^3,x]
```

```
[Out] a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*x^3)/3 + (
b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4
```

fricas [A] time = 0.70, size = 62, normalized size = 1.11

$$\frac{1}{4}x^4fdb + \frac{1}{3}x^3edb + \frac{1}{3}x^3fcb + \frac{1}{3}x^3fda + \frac{1}{2}x^2ecb + \frac{1}{2}x^2eda + \frac{1}{2}x^2fca + xeca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="fricas")
```

```
[Out] 1/4*x^4*f*d*b + 1/3*x^3*e*d*b + 1/3*x^3*f*c*b + 1/3*x^3*f*d*a + 1/2*x^2*e*c
*b + 1/2*x^2*e*d*a + 1/2*x^2*f*c*a + x*e*c*a
```

giac [A] time = 0.23, size = 54, normalized size = 0.96

$$\frac{1}{4}\,bdfx^4 + \frac{1}{3}\left(bcf + adf + bde\right)x^3 + acxe + \frac{1}{2}\left(acf + bce + ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="giac")
```

```
[Out] 1/4*b*d*f*x^4 + 1/3*(b*c*f + a*d*f + b*d*e)*x^3 + a*c*x*e + 1/2*(a*c*f + b*
c*e + a*d*e)*x^2
```

maple [A] time = 0.00, size = 51, normalized size = 0.91

$$\frac{bdf\,x^4}{4} + acex + \frac{\left(adf + bcf + bde\right)x^3}{3} + \frac{\left(acf + ade + bce\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x)
```

```
[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x
^4
```

maxima [A] time = 0.83, size = 50, normalized size = 0.89

$$\frac{1}{4}\,bdfx^4 + acex + \frac{1}{3}\left(bde + bcf + adf\right)x^3 + \frac{1}{2}\left(bce + ade + acf\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="maxima")
```

```
[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a*
d*e + a*c*f)*x^2
```

mupad [B] time = 0.04, size = 54, normalized size = 0.96

$$\frac{b\,d\,f\,x^4}{4} + \left(\frac{a\,d\,f}{3} + \frac{b\,c\,f}{3} + \frac{b\,d\,e}{3}\right)\,x^3 + \left(\frac{a\,c\,f}{2} + \frac{a\,d\,e}{2} + \frac{b\,c\,e}{2}\right)\,x^2 + a\,c\,e\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f
*x^3,x)
```

```
[Out] x^2*((a*c*f)/2 + (a*d*e)/2 + (b*c*e)/2) + x^3*((a*d*f)/3 + (b*c*f)/3 + (b*d
*e)/3) + a*c*e*x + (b*d*f*x^4)/4
```

sympy [A] time = 0.07, size = 63, normalized size = 1.12

$$acex + \frac{bdfx^4}{4} + x^3\left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right) + x^2\left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,x
)
```

```
[Out] a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**2*(a*c*f/2
 + a*d*e/2 + b*c*e/2)
```

3.18 $\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$

Optimal. Leaf size=86

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

```
[Out] b*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)-d*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)+f*ln(f*
x+e)/(-a*f+b*e)/(-c*f+d*e)
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2058}

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-1),x]
```

```
[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)*
(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx=\int \left(\frac{b^2}{(bc-ad)(be-af)(a+bx)}+\frac{d}{(bc-ad)(-de}\right.$$
$$=\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\,\mathrm{l}}{(be-}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.93

$$\frac{b\log(a+bx)(cf-de)+d(be-af)\log(c+dx)+f(ad-bc)\log(e+fx)}{(bc-ad)(be-af)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-1),x]
```

```
[Out] (b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(b*c) + a*d
)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f))
```

fricas [A] time = 10.24, size = 112, normalized size = 1.30

$$\frac{(bc-ad)f\log\left(fx+e\right)+\left(bde-bcf\right)\log\left(bx+a\right)-\left(bde-adf\right)\log\left(dx+c\right)}{\left(b^2cd-abd^2\right)e^2-\left(b^2c^2-a^2d^2\right)ef+\left(abc^2-a^2cd\right)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d*e - a*d*f
)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a^2*d^2)*e*f + (a*b*c
^2 - a^2*c*d)*f^2)
```

giac [A] time = 0.31, size = 137, normalized size = 1.59

$$-\frac{b^2\log\left(|bx+a|\right)}{ab^2cf-a^2bdf-b^3ce+ab^2de}+\frac{d^2\log\left(|dx+c|\right)}{bc^2df-acd^2f-bcd^2e+ad^3e}+\frac{f^2\log\left(\left|fx+e\right|\right)}{acf^3-bcf^2e-adf^2e+bdfe^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/(a*b^2*c*f - a^2*b*d*f - b^3*c*e + a*b^2*d*e) + d^2*
log(abs(d*x + c))/(b*c^2*d*f - a*c*d^2*f - b*c*d^2*e + a*d^3*e) + f^2*log(a
bs(f*x + e))/(a*c*f^3 - b*c*f^2*e - a*d*f^2*e + b*d*f*e^2)
```

maple [A] time = 0.01, size = 87, normalized size = 1.01

$$\frac{b\ln\left(bx+a\right)}{\left(af-be\right)\left(ad-bc\right)}-\frac{d\ln\left(dx+c\right)}{\left(cf-de\right)\left(ad-bc\right)}+\frac{f\ln\left(fx+e\right)}{\left(cf-de\right)\left(af-be\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x)
```

```
[Out] f/(c*f-d*e)/(a*f-b*e)*ln(f*x+e)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+b/(a*f-b*e)
/(a*d-b*c)*ln(b*x+a)
```

maxima [A] time = 0.65, size = 112, normalized size = 1.30

$$\frac{b\log\left(bx+a\right)}{\left(b^2c-abd\right)e-\left(abc-a^2d\right)f}-\frac{d\log\left(dx+c\right)}{\left(bcd-ad^2\right)e-\left(bc^2-acd\right)f}+\frac{f\log\left(fx+e\right)}{bde^2+acf^2-(bc+ad)ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f) - d*log(d*x + c)/((b
*c*d - a*d^2)*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(b*d*e^2 + a*c*f^2 -
(b*c + a*d)*e*f)
```

mupad [B] time = 2.33, size = 106, normalized size = 1.23

$$\frac{b\,\ln\left(a+b\,x\right)}{b^2\,c\,e+a^2\,d\,f-a\,b\,c\,f-a\,b\,d\,e}+\frac{d\,\ln\left(c+d\,x\right)}{a\,d^2\,e+b\,c^2\,f-a\,c\,d\,f-b\,c\,d\,e}+\frac{f\,\ln\left(e+f\,x\right)}{a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3),x)
```

```
[Out] (b*log(a + b*x))/(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e) + (d*log(c + d*x))
/(a*d^2*e + b*c^2*f - a*c*d*f - b*c*d*e) + (f*log(e + f*x))/(a*c*f^2 + b*d*
e^2 - a*d*e*f - b*c*e*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3),x)
```

```
[Out] Timed out
```

3.19 $\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+}{(bc-}$$

```
[Out] -b^3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)-d^3/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x+c
)-f^3/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x+e)-2*b^3*(-2*a*d*f+b*c*f+b*d*e)*ln(b*x
+a)/(-a*d+b*c)^3/(-a*f+b*e)^3+2*d^3*(a*d*f-2*b*c*f+b*d*e)*ln(d*x+c)/(-a*d+b
*c)^3/(-c*f+d*e)^3+2*f^3*(-a*d*f-b*c*f+2*b*d*e)*ln(f*x+e)/(-a*f+b*e)^3/(-c*
f+d*e)^3
```

Rubi [A] time = 0.41, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2058}

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+}{(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (
2*f^3*(2*b*d*e - b*c*f - a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2}\,dx=\int\left(\frac{b^4}{(bc-ad)^2(be-af)^2(a+bx)^2}-\frac{2b^4}{(bc-}\right.$$

$$=-\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)}-\frac{}{(bc-ad)^2}$$

Mathematica [A] time = 0.55, size = 232, normalized size = 0.99

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}-\frac{2d^3\log(c+d}{(bc-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) - (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(-(d*e) + c*f)^3)
- (2*f^3*(-2*b*d*e + b*c*f + a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f
)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.39, size = 1414, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="giac")
```

```
[Out] 2*(b^5*c*f - 2*a*b^4*d*f + b^5*d*e)*log(abs(b*x + a))/(a^3*b^4*c^3*f^3 - 3*
a^4*b^3*c^2*d*f^3 + 3*a^5*b^2*c*d^2*f^3 - a^6*b*d^3*f^3 - 3*a^2*b^5*c^3*f^2
*e + 9*a^3*b^4*c^2*d*f^2*e - 9*a^4*b^3*c*d^2*f^2*e + 3*a^5*b^2*d^3*f^2*e +
3*a*b^6*c^3*f*e^2 - 9*a^2*b^5*c^2*d*f*e^2 + 9*a^3*b^4*c*d^2*f*e^2 - 3*a^4*b
^3*d^3*f*e^2 - b^7*c^3*e^3 + 3*a*b^6*c^2*d*e^3 - 3*a^2*b^5*c*d^2*e^3 + a^3*
b^4*d^3*e^3) + 2*(2*b*c*d^4*f - a*d^5*f - b*d^5*e)*log(abs(d*x + c))/(b^3*c
^6*d*f^3 - 3*a*b^2*c^5*d^2*f^3 + 3*a^2*b*c^4*d^3*f^3 - a^3*c^3*d^4*f^3 - 3*
b^3*c^5*d^2*f^2*e + 9*a*b^2*c^4*d^3*f^2*e - 9*a^2*b*c^3*d^4*f^2*e + 3*a^3*c
^2*d^5*f^2*e + 3*b^3*c^4*d^3*f*e^2 - 9*a*b^2*c^3*d^4*f*e^2 + 9*a^2*b*c^2*d^
5*f*e^2 - 3*a^3*c*d^6*f*e^2 - b^3*c^3*d^4*e^3 + 3*a*b^2*c^2*d^5*e^3 - 3*a^2
```

```
*b*c*d^6*e^3 + a^3*d^7*e^3) - 2*(b*c*f^5 + a*d*f^5 - 2*b*d*f^4*e)*log(abs(f
*x + e))/(a^3*c^3*f^7 - 3*a^2*b*c^3*f^6*e - 3*a^3*c^2*d*f^6*e + 3*a*b^2*c^3
*f^5*e^2 + 9*a^2*b*c^2*d*f^5*e^2 + 3*a^3*c*d^2*f^5*e^2 - b^3*c^3*f^4*e^3 -
9*a*b^2*c^2*d*f^4*e^3 - 9*a^2*b*c*d^2*f^4*e^3 - a^3*d^3*f^4*e^3 + 3*b^3*c^2
*d*f^3*e^4 + 9*a*b^2*c*d^2*f^3*e^4 + 3*a^2*b*d^3*f^3*e^4 - 3*b^3*c*d^2*f^2*
e^5 - 3*a*b^2*d^3*f^2*e^5 + b^3*d^3*f*e^6) - (2*b^3*c^2*d*f^3*x^2 - 2*a*b^2
*c*d^2*f^3*x^2 + 2*a^2*b*d^3*f^3*x^2 - 2*b^3*c*d^2*f^2*x^2*e - 2*a*b^2*d^3*
f^2*x^2*e + 2*b^3*c^3*f^3*x - a*b^2*c^2*d*f^3*x - a^2*b*c*d^2*f^3*x + 2*a^3
*d^3*f^3*x + 2*b^3*d^3*f*x^2*e^2 - b^3*c^2*d*f^2*x*e - a^2*b*d^3*f^2*x*e +
a*b^2*c^3*f^3 - 2*a^2*b*c^2*d*f^3 + a^3*c*d^2*f^3 - b^3*c*d^2*f*x*e^2 - a*b
^2*d^3*f*x*e^2 + b^3*c^3*f^2*e + a^3*d^3*f^2*e + 2*b^3*d^3*x*e^3 - 2*b^3*c^
2*d*f*e^2 - 2*a^2*b*d^3*f*e^2 + b^3*c*d^2*e^3 + a*b^2*d^3*e^3)/((a^2*b^2*c^
4*f^4 - 2*a^3*b*c^3*d*f^4 + a^4*c^2*d^2*f^4 - 2*a*b^3*c^4*f^3*e + 2*a^2*b^2
*c^3*d*f^3*e + 2*a^3*b*c^2*d^2*f^3*e - 2*a^4*c*d^3*f^3*e + b^4*c^4*f^2*e^2
+ 2*a*b^3*c^3*d*f^2*e^2 - 6*a^2*b^2*c^2*d^2*f^2*e^2 + 2*a^3*b*c*d^3*f^2*e^2
 + a^4*d^4*f^2*e^2 - 2*b^4*c^3*d*f*e^3 + 2*a*b^3*c^2*d^2*f*e^3 + 2*a^2*b^2*
c*d^3*f*e^3 - 2*a^3*b*d^4*f*e^3 + b^4*c^2*d^2*e^4 - 2*a*b^3*c*d^3*e^4 + a^2
*b^2*d^4*e^4)*(b*d*f*x^3 + b*c*f*x^2 + a*d*f*x^2 + b*d*x^2*e + a*c*f*x + b*
c*x*e + a*d*x*e + a*c*e))
```

maple [A] time = 0.03, size = 398, normalized size = 1.70

$$\frac{4a\,b^3 df\ln(bx+a)}{(af-be)^3(ad-bc)^3}+\frac{2a\,d^4 f\ln(dx+c)}{(cf-de)^3(ad-bc)^3}-\frac{2ad\,f^4\ln(fx+e)}{(cf-de)^3(af-be)^3}-\frac{2b^4 cf\ln(bx+a)}{(af-be)^3(ad-bc)^3}-\frac{2b^4 de\ln(bx+a)}{(af-be)^3(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
```

```
[Out] -f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e
)*a*d-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)*b*c+4*f^3/(c*f-d*e)^3/(a*f-b*
e)^3*ln(f*x+e)*b*d*e-d^3/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+2*d^4/(c*f-d*e)^3/
(a*d-b*c)^3*ln(d*x+c)*a*f-4*d^3/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*c*f+2*d
^4/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*e-b^3/(a*f-b*e)^2/(a*d-b*c)^2/(b*x+a
)+4*b^3/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*a*d*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)
^3*ln(b*x+a)*c*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*d*e
```

maxima [B] time = 1.54, size = 2096, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="maxima")
```

```
[Out] -2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d
 + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*
a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a
^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*
c*d^2 - a^6*d^3)*f^3) + 2*(b*d^4*e - (2*b*c*d^3 - a*d^4)*f)*log(d*x + c)/((
b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*e^3 - 3*(b^3*c^4*d
^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*e^2*f + 3*(b^3*c^5*d -
3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*f^2 - (b^3*c^6 - 3*a*b^2
*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + 2*(2*b*d*e*f^3 - (b*c + a*d)
*f^4)*log(f*x + e)/(b^3*d^3*e^6 + a^3*c^3*f^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*e
^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3*c^3 + 9*a*b
^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d
+ a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5) - ((b^3*c*d^2 + a*b
^2*d^3)*e^3 - 2*(b^3*c^2*d + a^2*b*d^3)*e^2*f + (b^3*c^3 + a^3*d^3)*e*f^2 +
 (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3 + 2*(b^3*d^3*e^2*f - (b^3*c*d^
2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^2 + (2*
b^3*d^3*e^3 - (b^3*c*d^2 + a*b^2*d^3)*e^2*f - (b^3*c^2*d + a^2*b*d^3)*e*f^2
 + (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*f^3)*x)/((a*b^4*c^3*
d^2 - 2*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*e^5 - 2*(a*b^4*c^4*d - a^2*b^3*c^3
*d^2 - a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e^4*f + (a*b^4*c^5 + 2*a^2*b^3*c^4*d
- 6*a^3*b^2*c^3*d^2 + 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^3*f^2 - 2*(a^2*b^3*c^5
 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^2*f^3 + (a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2)*e*f^4 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*
d^5)*e^4*f - 2*(b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*
e^3*f^2 + (b^5*c^4*d + 2*a*b^4*c^3*d^2 - 6*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^
4 + a^4*b*d^5)*e^2*f^3 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3
 + a^4*b*c*d^4)*e*f^4 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)
*f^5)*x^3 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*e^5 - (b^5*c^3*d^2
 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^4*f - (b^5*c^4*d - 2*a*b^
4*c^3*d^2 + 2*a^2*b^3*c^2*d^3 - 2*a^3*b^2*c*d^4 + a^4*b*d^5)*e^3*f^2 + (b^5
*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 +
a^5*d^5)*e^2*f^3 - (2*a*b^4*c^5 - a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 - a^4*b
*c^2*d^3 + 2*a^5*c*d^4)*e*f^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^
2 + a^5*c^2*d^3)*f^5)*x^2 + ((b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 +
 a^3*b^2*d^5)*e^5 - (2*b^5*c^4*d - a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 - a^3*
b^2*c*d^4 + 2*a^4*b*d^5)*e^4*f + (b^5*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2
 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + a^5*d^5)*e^3*f^2 - (a*b^4*c^5 - 2*a^2*
b^3*c^4*d + 2*a^3*b^2*c^3*d^2 - 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^2*f^3 - (a^2
*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e*f^4 + (a^3*b^2*c^
5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*f^5)*x)
```

mupad [B] time = 8.18, size = 1940, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3)^2,x)
```

```
[Out] - ((a*b^2*c^3*f^3 + a*b^2*d^3*e^3 + a^3*c*d^2*f^3 + b^3*c*d^2*e^3 + a^3*d^3
*e*f^2 + b^3*c^3*e*f^2 - 2*a^2*b*c^2*d*f^3 - 2*a^2*b*d^3*e^2*f - 2*b^3*c^2*
d*e^2*f)/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2
*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*
d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c
^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^
3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e
*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) + (2*x^2*(a^2*b*d^3*f^3 + b^3*c^2*d*f^3 +
 b^3*d^3*e^2*f - a*b^2*c*d^2*f^3 - a*b^2*d^3*e*f^2 - b^3*c*d^2*e*f^2))/(a^2
*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^
4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b
^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f +
2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a
^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*
b^2*c^2*d^2*e^2*f^2) - (x*(a*b^2*c^2*d*f^3 - 2*b^3*c^3*f^3 - 2*b^3*d^3*e^3
- 2*a^3*d^3*f^3 + a^2*b*c*d^2*f^3 + a*b^2*d^3*e^2*f + a^2*b*d^3*e*f^2 + b^3
*c*d^2*e^2*f + b^3*c^2*d*e*f^2))/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c
^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*
c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a
^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*
e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2
*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2))/(x^2*(a*d*f + b*
c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3) - (log(a + b*
x)*(b^4*(2*c*f + 2*d*e) - 4*a*b^3*d*f))/(b^6*c^3*e^3 + a^6*d^3*f^3 - a^3*b^
3*c^3*f^3 - a^3*b^3*d^3*e^3 - 3*a*b^5*c^2*d*e^3 - 3*a^5*b*c*d^2*f^3 - 3*a*b
^5*c^3*e^2*f - 3*a^5*b*d^3*e*f^2 + 3*a^2*b^4*c*d^2*e^3 + 3*a^4*b^2*c^2*d*f^
3 + 3*a^2*b^4*c^3*e*f^2 + 3*a^4*b^2*d^3*e^2*f + 9*a^2*b^4*c^2*d*e^2*f - 9*a
^3*b^3*c*d^2*e^2*f - 9*a^3*b^3*c^2*d*e*f^2 + 9*a^4*b^2*c*d^2*e*f^2) - (log(
c + d*x)*(d^4*(2*a*f + 2*b*e) - 4*b*c*d^3*f))/(a^3*d^6*e^3 + b^3*c^6*f^3 -
a^3*c^3*d^3*f^3 - b^3*c^3*d^3*e^3 - 3*a^2*b*c*d^5*e^3 - 3*a*b^2*c^5*d*f^3 -
 3*a^3*c*d^5*e^2*f - 3*b^3*c^5*d*e*f^2 + 3*a*b^2*c^2*d^4*e^3 + 3*a^2*b*c^4*
d^2*f^3 + 3*a^3*c^2*d^4*e*f^2 + 3*b^3*c^4*d^2*e^2*f - 9*a*b^2*c^3*d^3*e^2*f
 + 9*a*b^2*c^4*d^2*e*f^2 + 9*a^2*b*c^2*d^4*e^2*f - 9*a^2*b*c^3*d^3*e*f^2) -
 (log(e + f*x)*(f^4*(2*a*d + 2*b*c) - 4*b*d*e*f^3))/(a^3*c^3*f^6 + b^3*d^3*
e^6 - a^3*d^3*e^3*f^3 - b^3*c^3*e^3*f^3 - 3*a^2*b*c^3*e*f^5 - 3*a*b^2*d^3*e
^5*f - 3*a^3*c^2*d*e*f^5 - 3*b^3*c*d^2*e^5*f + 3*a*b^2*c^3*e^2*f^4 + 3*a^2*
b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4 + 3*b^3*c^2*d*e^4*f^2 + 9*a*b^2*c*d^2*e
^4*f^2 - 9*a*b^2*c^2*d*e^3*f^3 - 9*a^2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*e^2*
f^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**2,x)
```

```
[Out] Timed out
```

3.20 $\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx$

Optimal. Leaf size=495

$$\frac{3f^5\log(e+fx)\left(2a^2d^2f^2-abdf(7de-3cf)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5}-\frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3\right.}{(bc-a}$$

```
[Out] -1/2*b^5/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2+3*b^5*(-2*a*d*f+b*c*f+b*d*e)/(
-a*d+b*c)^4/(-a*f+b*e)^4/(b*x+a)+1/2*d^5/(-a*d+b*c)^3/(-c*f+d*e)^3/(d*x+c)^
2+3*d^5*(a*d*f-2*b*c*f+b*d*e)/(-a*d+b*c)^4/(-c*f+d*e)^4/(d*x+c)-1/2*f^5/(-a
*f+b*e)^3/(-c*f+d*e)^3/(f*x+e)^2-3*f^5*(-a*d*f-b*c*f+2*b*d*e)/(-a*f+b*e)^4/
(-c*f+d*e)^4/(f*x+e)+3*b^5*(7*a^2*d^2*f^2-7*a*b*d*f*(c*f+d*e)+b^2*(2*c^2*f^
2+3*c*d*e*f+2*d^2*e^2))*ln(b*x+a)/(-a*d+b*c)^5/(-a*f+b*e)^5-3*d^5*(2*a^2*d^
2*f^2+a*b*d*f*(-7*c*f+3*d*e)+b^2*(7*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*ln(d*x+c)
/(-a*d+b*c)^5/(-c*f+d*e)^5+3*f^5*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+7*d*e)+b^2*
(2*c^2*f^2-7*c*d*e*f+7*d^2*e^2))*ln(f*x+e)/(-a*f+b*e)^5/(-c*f+d*e)^5
```

Rubi [A] time = 1.46, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2058}

$$\frac{3b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5}-\frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3\right.}{(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-3),x]
```

```
[Out] -b^5/(2*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f -
2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^3*(d
*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)^4
*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
 - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)
) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*
e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5*(2*a
^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f
^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2*d^2*f^2 -
a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e +
f*x])/((b*e - a*f)^5*(d*e - c*f)^5)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx=\int\left(\frac{b^6}{(bc-ad)^3(be-af)^3(a+bx)^3}-\frac{3b^6(}{(bc-ad}\right.$$

$$=-\frac{b^5}{2(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{3b^5(bc}{(bc-ad)}$$

Mathematica [A] time = 1.19, size = 490, normalized size = 0.99

$$\frac{1}{2}\left(\frac{6f^5\log(e+fx)\left(2a^2d^2f^2+abdf(3cf-7de)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5}+\frac{6d^5\log(c+dx)\left(2a^2d^2f^2+abc\right.}{(bc}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-3),x]
```

```
[Out] (-(b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e + b*c*f -
 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^5/((b*c - a*d)^3*(-(
d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)
^4*(d*e - c*f)^4*(c + d*x)) - f^5/((b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
 + (6*f^5*(-2*b*d*e + b*c*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x
)) + (6*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d
*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*d^5*(2*
a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*
f^2))*Log[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^5) + (6*f^5*(2*a^2*d^2*f^
2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log
[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5))/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.10, size = 6908, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="giac")
```

```
[Out] -3*(2*b^8*c^2*f^2 - 7*a*b^7*c*d*f^2 + 7*a^2*b^6*d^2*f^2 + 3*b^8*c*d*f*e - 7
*a*b^7*d^2*f*e + 2*b^8*d^2*e^2)*log(abs(b*x + a))/(a^5*b^6*c^5*f^5 - 5*a^6*
b^5*c^4*d*f^5 + 10*a^7*b^4*c^3*d^2*f^5 - 10*a^8*b^3*c^2*d^3*f^5 + 5*a^9*b^2
*c*d^4*f^5 - a^10*b*d^5*f^5 - 5*a^4*b^7*c^5*f^4*e + 25*a^5*b^6*c^4*d*f^4*e
- 50*a^6*b^5*c^3*d^2*f^4*e + 50*a^7*b^4*c^2*d^3*f^4*e - 25*a^8*b^3*c*d^4*f^
4*e + 5*a^9*b^2*d^5*f^4*e + 10*a^3*b^8*c^5*f^3*e^2 - 50*a^4*b^7*c^4*d*f^3*e
^2 + 100*a^5*b^6*c^3*d^2*f^3*e^2 - 100*a^6*b^5*c^2*d^3*f^3*e^2 + 50*a^7*b^4
*c*d^4*f^3*e^2 - 10*a^8*b^3*d^5*f^3*e^2 - 10*a^2*b^9*c^5*f^2*e^3 + 50*a^3*b
^8*c^4*d*f^2*e^3 - 100*a^4*b^7*c^3*d^2*f^2*e^3 + 100*a^5*b^6*c^2*d^3*f^2*e^
3 - 50*a^6*b^5*c*d^4*f^2*e^3 + 10*a^7*b^4*d^5*f^2*e^3 + 5*a*b^10*c^5*f*e^4
- 25*a^2*b^9*c^4*d*f*e^4 + 50*a^3*b^8*c^3*d^2*f*e^4 - 50*a^4*b^7*c^2*d^3*f*
e^4 + 25*a^5*b^6*c*d^4*f*e^4 - 5*a^6*b^5*d^5*f*e^4 - b^11*c^5*e^5 + 5*a*b^1
0*c^4*d*e^5 - 10*a^2*b^9*c^3*d^2*e^5 + 10*a^3*b^8*c^2*d^3*e^5 - 5*a^4*b^7*c
*d^4*e^5 + a^5*b^6*d^5*e^5) + 3*(7*b^2*c^2*d^6*f^2 - 7*a*b*c*d^7*f^2 + 2*a^
2*d^8*f^2 - 7*b^2*c*d^7*f*e + 3*a*b*d^8*f*e + 2*b^2*d^8*e^2)*log(abs(d*x +
c))/(b^5*c^10*d*f^5 - 5*a*b^4*c^9*d^2*f^5 + 10*a^2*b^3*c^8*d^3*f^5 - 10*a^3
*b^2*c^7*d^4*f^5 + 5*a^4*b*c^6*d^5*f^5 - a^5*c^5*d^6*f^5 - 5*b^5*c^9*d^2*f^
4*e + 25*a*b^4*c^8*d^3*f^4*e - 50*a^2*b^3*c^7*d^4*f^4*e + 50*a^3*b^2*c^6*d^
5*f^4*e - 25*a^4*b*c^5*d^6*f^4*e + 5*a^5*c^4*d^7*f^4*e + 10*b^5*c^8*d^3*f^3
*e^2 - 50*a*b^4*c^7*d^4*f^3*e^2 + 100*a^2*b^3*c^6*d^5*f^3*e^2 - 100*a^3*b^2
*c^5*d^6*f^3*e^2 + 50*a^4*b*c^4*d^7*f^3*e^2 - 10*a^5*c^3*d^8*f^3*e^2 - 10*b
^5*c^7*d^4*f^2*e^3 + 50*a*b^4*c^6*d^5*f^2*e^3 - 100*a^2*b^3*c^5*d^6*f^2*e^3
 + 100*a^3*b^2*c^4*d^7*f^2*e^3 - 50*a^4*b*c^3*d^8*f^2*e^3 + 10*a^5*c^2*d^9*
f^2*e^3 + 5*b^5*c^6*d^5*f*e^4 - 25*a*b^4*c^5*d^6*f*e^4 + 50*a^2*b^3*c^4*d^7
*f*e^4 - 50*a^3*b^2*c^3*d^8*f*e^4 + 25*a^4*b*c^2*d^9*f*e^4 - 5*a^5*c*d^10*f
*e^4 - b^5*c^5*d^6*e^5 + 5*a*b^4*c^4*d^7*e^5 - 10*a^2*b^3*c^3*d^8*e^5 + 10*
a^3*b^2*c^2*d^9*e^5 - 5*a^4*b*c*d^10*e^5 + a^5*d^11*e^5) + 3*(2*b^2*c^2*f^8
 + 3*a*b*c*d*f^8 + 2*a^2*d^2*f^8 - 7*b^2*c*d*f^7*e - 7*a*b*d^2*f^7*e + 7*b^
2*d^2*f^6*e^2)*log(abs(f*x + e))/(a^5*c^5*f^11 - 5*a^4*b*c^5*f^10*e - 5*a^5
*c^4*d*f^10*e + 10*a^3*b^2*c^5*f^9*e^2 + 25*a^4*b*c^4*d*f^9*e^2 + 10*a^5*c^
3*d^2*f^9*e^2 - 10*a^2*b^3*c^5*f^8*e^3 - 50*a^3*b^2*c^4*d*f^8*e^3 - 50*a^4*
b*c^3*d^2*f^8*e^3 - 10*a^5*c^2*d^3*f^8*e^3 + 5*a*b^4*c^5*f^7*e^4 + 50*a^2*b
^3*c^4*d*f^7*e^4 + 100*a^3*b^2*c^3*d^2*f^7*e^4 + 50*a^4*b*c^2*d^3*f^7*e^4 +
 5*a^5*c*d^4*f^7*e^4 - b^5*c^5*f^6*e^5 - 25*a*b^4*c^4*d*f^6*e^5 - 100*a^2*b
^3*c^3*d^2*f^6*e^5 - 100*a^3*b^2*c^2*d^3*f^6*e^5 - 25*a^4*b*c*d^4*f^6*e^5 -
```

a^5*d^5*f^6*e^5 + 5*b^5*c^4*d*f^5*e^6 + 50*a*b^4*c^3*d^2*f^5*e^6 + 100*a^2
*b^3*c^2*d^3*f^5*e^6 + 50*a^3*b^2*c*d^4*f^5*e^6 + 5*a^4*b*d^5*f^5*e^6 - 10*
b^5*c^3*d^2*f^4*e^7 - 50*a*b^4*c^2*d^3*f^4*e^7 - 50*a^2*b^3*c*d^4*f^4*e^7 -
10*a^3*b^2*d^5*f^4*e^7 + 10*b^5*c^2*d^3*f^3*e^8 + 25*a*b^4*c*d^4*f^3*e^8 +
10*a^2*b^3*d^5*f^3*e^8 - 5*b^5*c*d^4*f^2*e^9 - 5*a*b^4*d^5*f^2*e^9 + b^5*d
^5*f*e^10) + 1/2*(12*b^7*c^5*d^2*f^7*x^5 - 30*a*b^6*c^4*d^3*f^7*x^5 + 12*a^
2*b^5*c^3*d^4*f^7*x^5 + 12*a^3*b^4*c^2*d^5*f^7*x^5 - 30*a^4*b^3*c*d^6*f^7*x
^5 + 12*a^5*b^2*d^7*f^7*x^5 - 30*b^7*c^4*d^3*f^6*x^5*e + 96*a*b^6*c^3*d^4*f
^6*x^5*e - 72*a^2*b^5*c^2*d^5*f^6*x^5*e + 96*a^3*b^4*c*d^6*f^6*x^5*e - 30*a
^4*b^3*d^7*f^6*x^5*e + 24*b^7*c^6*d*f^7*x^4 - 42*a*b^6*c^5*d^2*f^7*x^4 - 21
*a^2*b^5*c^4*d^3*f^7*x^4 + 42*a^3*b^4*c^3*d^4*f^7*x^4 - 21*a^4*b^3*c^2*d^5*
f^7*x^4 - 42*a^5*b^2*c*d^6*f^7*x^4 + 24*a^6*b*d^7*f^7*x^4 + 12*b^7*c^3*d^4*
f^5*x^5*e^2 - 72*a*b^6*c^2*d^5*f^5*x^5*e^2 - 72*a^2*b^5*c*d^6*f^5*x^5*e^2 +
12*a^3*b^4*d^7*f^5*x^5*e^2 - 42*b^7*c^5*d^2*f^6*x^4*e + 102*a*b^6*c^4*d^3*
f^6*x^4*e + 18*a^2*b^5*c^3*d^4*f^6*x^4*e + 18*a^3*b^4*c^2*d^5*f^6*x^4*e + 1
02*a^4*b^3*c*d^6*f^6*x^4*e - 42*a^5*b^2*d^7*f^6*x^4*e + 12*b^7*c^7*f^7*x^3
+ 6*a*b^6*c^6*d*f^7*x^3 - 74*a^2*b^5*c^5*d^2*f^7*x^3 + 38*a^3*b^4*c^4*d^3*f
^7*x^3 + 38*a^4*b^3*c^3*d^4*f^7*x^3 - 74*a^5*b^2*c^2*d^5*f^7*x^3 + 6*a^6*b*
c*d^6*f^7*x^3 + 12*a^7*d^7*f^7*x^3 + 12*b^7*c^2*d^5*f^4*x^5*e^3 + 96*a*b^6*
c*d^6*f^4*x^5*e^3 + 12*a^2*b^5*d^7*f^4*x^5*e^3 - 21*b^7*c^4*d^3*f^5*x^4*e^2
+ 18*a*b^6*c^3*d^4*f^5*x^4*e^2 - 234*a^2*b^5*c^2*d^5*f^5*x^4*e^2 + 18*a^3*
b^4*c*d^6*f^5*x^4*e^2 - 21*a^4*b^3*d^7*f^5*x^4*e^2 + 6*b^7*c^6*d*f^6*x^3*e
- 56*a*b^6*c^5*d^2*f^6*x^3*e + 172*a^2*b^5*c^4*d^3*f^6*x^3*e - 136*a^3*b^4*
c^3*d^4*f^6*x^3*e + 172*a^4*b^3*c^2*d^5*f^6*x^3*e - 56*a^5*b^2*c*d^6*f^6*x^
3*e + 6*a^6*b*d^7*f^6*x^3*e + 18*a*b^6*c^7*f^7*x^2 - 37*a^2*b^5*c^6*d*f^7*x
^2 - 3*a^3*b^4*c^5*d^2*f^7*x^2 + 32*a^4*b^3*c^4*d^3*f^7*x^2 - 3*a^5*b^2*c^3
*d^4*f^7*x^2 - 37*a^6*b*c^2*d^5*f^7*x^2 + 18*a^7*c*d^6*f^7*x^2 - 30*b^7*c*d
^6*f^3*x^5*e^4 - 30*a*b^6*d^7*f^3*x^5*e^4 + 42*b^7*c^3*d^4*f^4*x^4*e^3 + 18
*a*b^6*c^2*d^5*f^4*x^4*e^3 + 18*a^2*b^5*c*d^6*f^4*x^4*e^3 + 42*a^3*b^4*d^7*
f^4*x^4*e^3 - 74*b^7*c^5*d^2*f^5*x^3*e^2 + 172*a*b^6*c^4*d^3*f^5*x^3*e^2 -
104*a^2*b^5*c^3*d^4*f^5*x^3*e^2 - 104*a^3*b^4*c^2*d^5*f^5*x^3*e^2 + 172*a^4
*b^3*c*d^6*f^5*x^3*e^2 - 74*a^5*b^2*d^7*f^5*x^3*e^2 + 18*b^7*c^7*f^6*x^2*e
- 34*a*b^6*c^6*d*f^6*x^2*e + 9*a^2*b^5*c^5*d^2*f^6*x^2*e + a^3*b^4*c^4*d^3*
f^6*x^2*e + a^4*b^3*c^3*d^4*f^6*x^2*e + 9*a^5*b^2*c^2*d^5*f^6*x^2*e - 34*a^
6*b*c*d^6*f^6*x^2*e + 18*a^7*d^7*f^6*x^2*e + 4*a^2*b^5*c^7*f^7*x - 12*a^3*b
^4*c^6*d*f^7*x + 8*a^4*b^3*c^5*d^2*f^7*x + 8*a^5*b^2*c^4*d^3*f^7*x - 12*a^6
*b*c^3*d^4*f^7*x + 4*a^7*c^2*d^5*f^7*x + 12*b^7*d^7*f^2*x^5*e^5 - 21*b^7*c^
2*d^5*f^3*x^4*e^4 + 102*a*b^6*c*d^6*f^3*x^4*e^4 - 21*a^2*b^5*d^7*f^3*x^4*e^
4 + 38*b^7*c^4*d^3*f^4*x^3*e^3 - 136*a*b^6*c^3*d^4*f^4*x^3*e^3 - 104*a^2*b^
5*c^2*d^5*f^4*x^3*e^3 - 136*a^3*b^4*c*d^6*f^4*x^3*e^3 + 38*a^4*b^3*d^7*f^4*
x^3*e^3 - 37*b^7*c^6*d*f^5*x^2*e^2 + 9*a*b^6*c^5*d^2*f^5*x^2*e^2 + 234*a^2*
b^5*c^4*d^3*f^5*x^2*e^2 - 208*a^3*b^4*c^3*d^4*f^5*x^2*e^2 + 234*a^4*b^3*c^2
*d^5*f^5*x^2*e^2 + 9*a^5*b^2*c*d^6*f^5*x^2*e^2 - 37*a^6*b*d^7*f^5*x^2*e^2 +
28*a*b^6*c^7*f^6*x*e - 66*a^2*b^5*c^6*d*f^6*x*e + 34*a^3*b^4*c^5*d^2*f^6*x
*e - 16*a^4*b^3*c^4*d^3*f^6*x*e + 34*a^5*b^2*c^3*d^4*f^6*x*e - 66*a^6*b*c^2

*d^5*f^6*x*e + 28*a^7*c*d^6*f^6*x*e - a^3*b^4*c^7*f^7 + 4*a^4*b^3*c^6*d*f^7
 - 6*a^5*b^2*c^5*d^2*f^7 + 4*a^6*b*c^4*d^3*f^7 - a^7*c^3*d^4*f^7 - 42*b^7*c
*d^6*f^2*x^4*e^5 - 42*a*b^6*d^7*f^2*x^4*e^5 + 38*b^7*c^3*d^4*f^3*x^3*e^4 +
172*a*b^6*c^2*d^5*f^3*x^3*e^4 + 172*a^2*b^5*c*d^6*f^3*x^3*e^4 + 38*a^3*b^4*
d^7*f^3*x^3*e^4 - 3*b^7*c^5*d^2*f^4*x^2*e^3 + a*b^6*c^4*d^3*f^4*x^2*e^3 - 2
08*a^2*b^5*c^3*d^4*f^4*x^2*e^3 - 208*a^3*b^4*c^2*d^5*f^4*x^2*e^3 + a^4*b^3*
c*d^6*f^4*x^2*e^3 - 3*a^5*b^2*d^7*f^4*x^2*e^3 + 4*b^7*c^7*f^5*x*e^2 - 66*a*
b^6*c^6*d*f^5*x*e^2 + 156*a^2*b^5*c^5*d^2*f^5*x*e^2 - 52*a^3*b^4*c^4*d^3*f^
5*x*e^2 - 52*a^4*b^3*c^3*d^4*f^5*x*e^2 + 156*a^5*b^2*c^2*d^5*f^5*x*e^2 - 66
*a^6*b*c*d^6*f^5*x*e^2 + 4*a^7*d^7*f^5*x*e^2 + 7*a^2*b^5*c^7*f^6*e - 21*a^3
*b^4*c^6*d*f^6*e + 14*a^4*b^3*c^5*d^2*f^6*e + 14*a^5*b^2*c^4*d^3*f^6*e - 21
*a^6*b*c^3*d^4*f^6*e + 7*a^7*c^2*d^5*f^6*e + 24*b^7*d^7*f*x^4*e^6 - 74*b^7*
c^2*d^5*f^2*x^3*e^5 - 56*a*b^6*c*d^6*f^2*x^3*e^5 - 74*a^2*b^5*d^7*f^2*x^3*e
^5 + 32*b^7*c^4*d^3*f^3*x^2*e^4 + a*b^6*c^3*d^4*f^3*x^2*e^4 + 234*a^2*b^5*c
^2*d^5*f^3*x^2*e^4 + a^3*b^4*c*d^6*f^3*x^2*e^4 + 32*a^4*b^3*d^7*f^3*x^2*e^4
 - 12*b^7*c^6*d*f^4*x*e^3 + 34*a*b^6*c^5*d^2*f^4*x*e^3 - 52*a^2*b^5*c^4*d^3
*f^4*x*e^3 - 52*a^4*b^3*c^2*d^5*f^4*x*e^3 + 34*a^5*b^2*c*d^6*f^4*x*e^3 - 12
*a^6*b*d^7*f^4*x*e^3 + 7*a*b^6*c^7*f^5*e^2 - 26*a^2*b^5*c^6*d*f^5*e^2 + 52*
a^3*b^4*c^5*d^2*f^5*e^2 - 78*a^4*b^3*c^4*d^3*f^5*e^2 + 52*a^5*b^2*c^3*d^4*f
^5*e^2 - 26*a^6*b*c^2*d^5*f^5*e^2 + 7*a^7*c*d^6*f^5*e^2 + 6*b^7*c*d^6*f*x^3
*e^6 + 6*a*b^6*d^7*f*x^3*e^6 - 3*b^7*c^3*d^4*f^2*x^2*e^5 + 9*a*b^6*c^2*d^5*
f^2*x^2*e^5 + 9*a^2*b^5*c*d^6*f^2*x^2*e^5 - 3*a^3*b^4*d^7*f^2*x^2*e^5 + 8*b
^7*c^5*d^2*f^3*x*e^4 - 16*a*b^6*c^4*d^3*f^3*x*e^4 - 52*a^2*b^5*c^3*d^4*f^3*
x*e^4 - 52*a^3*b^4*c^2*d^5*f^3*x*e^4 - 16*a^4*b^3*c*d^6*f^3*x*e^4 + 8*a^5*b
^2*d^7*f^3*x*e^4 - b^7*c^7*f^4*e^3 - 21*a*b^6*c^6*d*f^4*e^3 + 52*a^2*b^5*c^
5*d^2*f^4*e^3 + 52*a^5*b^2*c^2*d^5*f^4*e^3 - 21*a^6*b*c*d^6*f^4*e^3 - a^7*d
^7*f^4*e^3 + 12*b^7*d^7*x^3*e^7 - 37*b^7*c^2*d^5*f*x^2*e^6 - 34*a*b^6*c*d^6
*f*x^2*e^6 - 37*a^2*b^5*d^7*f*x^2*e^6 + 8*b^7*c^4*d^3*f^2*x*e^5 + 34*a*b^6*
c^3*d^4*f^2*x*e^5 + 156*a^2*b^5*c^2*d^5*f^2*x*e^5 + 34*a^3*b^4*c*d^6*f^2*x*
e^5 + 8*a^4*b^3*d^7*f^2*x*e^5 + 4*b^7*c^6*d*f^3*e^4 + 14*a*b^6*c^5*d^2*f^3*
e^4 - 78*a^2*b^5*c^4*d^3*f^3*e^4 - 78*a^4*b^3*c^2*d^5*f^3*e^4 + 14*a^5*b^2*
c*d^6*f^3*e^4 + 4*a^6*b*d^7*f^3*e^4 + 18*b^7*c*d^6*x^2*e^7 + 18*a*b^6*d^7*x
^2*e^7 - 12*b^7*c^3*d^4*f*x*e^6 - 66*a*b^6*c^2*d^5*f*x*e^6 - 66*a^2*b^5*c*d
^6*f*x*e^6 - 12*a^3*b^4*d^7*f*x*e^6 - 6*b^7*c^5*d^2*f^2*e^5 + 14*a*b^6*c^4*
d^3*f^2*e^5 + 52*a^2*b^5*c^3*d^4*f^2*e^5 + 52*a^3*b^4*c^2*d^5*f^2*e^5 + 14*
a^4*b^3*c*d^6*f^2*e^5 - 6*a^5*b^2*d^7*f^2*e^5 + 4*b^7*c^2*d^5*x*e^7 + 28*a*
b^6*c*d^6*x*e^7 + 4*a^2*b^5*d^7*x*e^7 + 4*b^7*c^4*d^3*f*e^6 - 21*a*b^6*c^3*
d^4*f*e^6 - 26*a^2*b^5*c^2*d^5*f*e^6 - 21*a^3*b^4*c*d^6*f*e^6 + 4*a^4*b^3*d
^7*f*e^6 - b^7*c^3*d^4*e^7 + 7*a*b^6*c^2*d^5*e^7 + 7*a^2*b^5*c*d^6*e^7 - a^
3*b^4*d^7*e^7)/((a^4*b^4*c^8*f^8 - 4*a^5*b^3*c^7*d*f^8 + 6*a^6*b^2*c^6*d^2*
f^8 - 4*a^7*b*c^5*d^3*f^8 + a^8*c^4*d^4*f^8 - 4*a^3*b^5*c^8*f^7*e + 12*a^4*
b^4*c^7*d*f^7*e - 8*a^5*b^3*c^6*d^2*f^7*e - 8*a^6*b^2*c^5*d^3*f^7*e + 12*a^
7*b*c^4*d^4*f^7*e - 4*a^8*c^3*d^5*f^7*e + 6*a^2*b^6*c^8*f^6*e^2 - 8*a^3*b^5
*c^7*d*f^6*e^2 - 22*a^4*b^4*c^6*d^2*f^6*e^2 + 48*a^5*b^3*c^5*d^3*f^6*e^2 -
22*a^6*b^2*c^4*d^4*f^6*e^2 - 8*a^7*b*c^3*d^5*f^6*e^2 + 6*a^8*c^2*d^6*f^6*e^

```
2 - 4*a*b^7*c^8*f^5*e^3 - 8*a^2*b^6*c^7*d*f^5*e^3 + 48*a^3*b^5*c^6*d^2*f^5*
e^3 - 36*a^4*b^4*c^5*d^3*f^5*e^3 - 36*a^5*b^3*c^4*d^4*f^5*e^3 + 48*a^6*b^2*
c^3*d^5*f^5*e^3 - 8*a^7*b*c^2*d^6*f^5*e^3 - 4*a^8*c*d^7*f^5*e^3 + b^8*c^8*f
^4*e^4 + 12*a*b^7*c^7*d*f^4*e^4 - 22*a^2*b^6*c^6*d^2*f^4*e^4 - 36*a^3*b^5*c
^5*d^3*f^4*e^4 + 90*a^4*b^4*c^4*d^4*f^4*e^4 - 36*a^5*b^3*c^3*d^5*f^4*e^4 -
22*a^6*b^2*c^2*d^6*f^4*e^4 + 12*a^7*b*c*d^7*f^4*e^4 + a^8*d^8*f^4*e^4 - 4*b
^8*c^7*d*f^3*e^5 - 8*a*b^7*c^6*d^2*f^3*e^5 + 48*a^2*b^6*c^5*d^3*f^3*e^5 - 3
6*a^3*b^5*c^4*d^4*f^3*e^5 - 36*a^4*b^4*c^3*d^5*f^3*e^5 + 48*a^5*b^3*c^2*d^6
*f^3*e^5 - 8*a^6*b^2*c*d^7*f^3*e^5 - 4*a^7*b*d^8*f^3*e^5 + 6*b^8*c^6*d^2*f^
2*e^6 - 8*a*b^7*c^5*d^3*f^2*e^6 - 22*a^2*b^6*c^4*d^4*f^2*e^6 + 48*a^3*b^5*c
^3*d^5*f^2*e^6 - 22*a^4*b^4*c^2*d^6*f^2*e^6 - 8*a^5*b^3*c*d^7*f^2*e^6 + 6*a
^6*b^2*d^8*f^2*e^6 - 4*b^8*c^5*d^3*f*e^7 + 12*a*b^7*c^4*d^4*f*e^7 - 8*a^2*b
^6*c^3*d^5*f*e^7 - 8*a^3*b^5*c^2*d^6*f*e^7 + 12*a^4*b^4*c*d^7*f*e^7 - 4*a^5
*b^3*d^8*f*e^7 + b^8*c^4*d^4*e^8 - 4*a*b^7*c^3*d^5*e^8 + 6*a^2*b^6*c^2*d^6*
e^8 - 4*a^3*b^5*c*d^7*e^8 + a^4*b^4*d^8*e^8)*(b*d*f*x^3 + b*c*f*x^2 + a*d*f
*x^2 + b*d*x^2*e + a*c*f*x + b*c*x*e + a*d*x*e + a*c*e)^2)
```

maple [B] time = 0.03, size = 1076, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
```

```
[Out] 21*d^6/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*b^2*c*e*f-21*b^6/(a*f-b*e)^5/(a*d-
b*c)^5*ln(b*x+a)*a*c*d*f^2-21*b^6/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a*d^2*e
*f+9*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*c*d*e*f+9*f^7/(c*f-d*e)^5/(a*f-b
*e)^5*ln(f*x+e)*a*b*c*d-21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a*b*d^2*e-
21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*b^2*c*d*e+21*d^6/(c*f-d*e)^5/(a*d-
b*c)^5*ln(d*x+c)*a*b*c*f^2-9*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a*b*e*f-
6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*e^2*b^2+3*b^6/(a*f-b*e)^4/(a*d-b*c)
^4/(b*x+a)*c*f+3*b^6/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*d*e+6*b^7/(a*f-b*e)^5/
(a*d-b*c)^5*ln(b*x+a)*c^2*f^2+6*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*d^2*e
^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*c+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*
ln(f*x+e)*a^2*d^2+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*b^2*c^2+3*d^6/(c*
f-d*e)^4/(a*d-b*c)^4/(d*x+c)*a*f+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*e-
6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a^2*f^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)
^4/(f*x+e)*a*d-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2+1/2*d^5/(c*f-d*e)^
3/(a*d-b*c)^3/(d*x+c)^2-1/2*f^5/(c*f-d*e)^3/(a*f-b*e)^3/(f*x+e)^2-6*f^5/(c*
f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*d*e+21*f^5/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)
*d^2*e^2*b^2-6*d^5/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*c*f-21*d^5/(c*f-d*e)^5
/(a*d-b*c)^5*ln(d*x+c)*b^2*c^2*f^2-6*b^5/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*a*
d*f+21*b^5/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a^2*d^2*f^2
```

maxima [B] time = 5.72, size = 11005, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="maxima")
```

```
[Out] 3*(2*b^7*d^2*e^2 + (3*b^7*c*d - 7*a*b^6*d^2)*e*f + (2*b^7*c^2 - 7*a*b^6*c*d
 + 7*a^2*b^5*d^2)*f^2)*log(b*x + a)/((b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8
*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*e^5 - 5*(a*b
^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*
b^5*c*d^4 - a^6*b^4*d^5)*e^4*f + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4
*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^3*f^2
- 10*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d
^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*e^2*f^3 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^
4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5
)*e*f^4 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*
c^2*d^3 + 5*a^9*b*c*d^4 - a^10*d^5)*f^5) - 3*(2*b^2*d^7*e^2 - (7*b^2*c*d^6
- 3*a*b*d^7)*e*f + (7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^2)*log(d*x +
 c)/((b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d
^8 + 5*a^4*b*c*d^9 - a^5*d^10)*e^5 - 5*(b^5*c^6*d^4 - 5*a*b^4*c^5*d^5 + 10*
a^2*b^3*c^4*d^6 - 10*a^3*b^2*c^3*d^7 + 5*a^4*b*c^2*d^8 - a^5*c*d^9)*e^4*f +
 10*(b^5*c^7*d^3 - 5*a*b^4*c^6*d^4 + 10*a^2*b^3*c^5*d^5 - 10*a^3*b^2*c^4*d^
6 + 5*a^4*b*c^3*d^7 - a^5*c^2*d^8)*e^3*f^2 - 10*(b^5*c^8*d^2 - 5*a*b^4*c^7*
d^3 + 10*a^2*b^3*c^6*d^4 - 10*a^3*b^2*c^5*d^5 + 5*a^4*b*c^4*d^6 - a^5*c^3*d
^7)*e^2*f^3 + 5*(b^5*c^9*d - 5*a*b^4*c^8*d^2 + 10*a^2*b^3*c^7*d^3 - 10*a^3*
b^2*c^6*d^4 + 5*a^4*b*c^5*d^5 - a^5*c^4*d^6)*e*f^4 - (b^5*c^10 - 5*a*b^4*c^
9*d + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 + 5*a^4*b*c^6*d^4 - a^5*c^5*d
^5)*f^5) + 3*(7*b^2*d^2*e^2*f^5 - 7*(b^2*c*d + a*b*d^2)*e*f^6 + (2*b^2*c^2
+ 3*a*b*c*d + 2*a^2*d^2)*f^7)*log(f*x + e)/(b^5*d^5*e^10 + a^5*c^5*f^10 - 5
*(b^5*c*d^4 + a*b^4*d^5)*e^9*f + 5*(2*b^5*c^2*d^3 + 5*a*b^4*c*d^4 + 2*a^2*b
^3*d^5)*e^8*f^2 - 10*(b^5*c^3*d^2 + 5*a*b^4*c^2*d^3 + 5*a^2*b^3*c*d^4 + a^3
*b^2*d^5)*e^7*f^3 + 5*(b^5*c^4*d + 10*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 +
10*a^3*b^2*c*d^4 + a^4*b*d^5)*e^6*f^4 - (b^5*c^5 + 25*a*b^4*c^4*d + 100*a^2
*b^3*c^3*d^2 + 100*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*e^5*f^5 + 5*
(a*b^4*c^5 + 10*a^2*b^3*c^4*d + 20*a^3*b^2*c^3*d^2 + 10*a^4*b*c^2*d^3 + a^5
*c*d^4)*e^4*f^6 - 10*(a^2*b^3*c^5 + 5*a^3*b^2*c^4*d + 5*a^4*b*c^3*d^2 + a^5
*c^2*d^3)*e^3*f^7 + 5*(2*a^3*b^2*c^5 + 5*a^4*b*c^4*d + 2*a^5*c^3*d^2)*e^2*f
^8 - 5*(a^4*b*c^5 + a^5*c^4*d)*e*f^9) - 1/2*((b^7*c^3*d^4 - 7*a*b^6*c^2*d^5
 - 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^7 - (4*b^7*c^4*d^3 - 21*a*b^6*c^3*d^4 -
 26*a^2*b^5*c^2*d^5 - 21*a^3*b^4*c*d^6 + 4*a^4*b^3*d^7)*e^6*f + 2*(3*b^7*c^
5*d^2 - 7*a*b^6*c^4*d^3 - 26*a^2*b^5*c^3*d^4 - 26*a^3*b^4*c^2*d^5 - 7*a^4*b
^3*c*d^6 + 3*a^5*b^2*d^7)*e^5*f^2 - 2*(2*b^7*c^6*d + 7*a*b^6*c^5*d^2 - 39*a
```

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^2*b^5*c^4*d^3 - 39*a^4*b^3*c^2*d^5 + 7*a^5*b^2*c*d^6 + 2*a^6*b*d^7)*e^4*f^
3 + (b^7*c^7 + 21*a*b^6*c^6*d - 52*a^2*b^5*c^5*d^2 - 52*a^5*b^2*c^2*d^5 + 2
1*a^6*b*c*d^6 + a^7*d^7)*e^3*f^4 - (7*a*b^6*c^7 - 26*a^2*b^5*c^6*d + 52*a^3
*b^4*c^5*d^2 - 78*a^4*b^3*c^4*d^3 + 52*a^5*b^2*c^3*d^4 - 26*a^6*b*c^2*d^5 +
 7*a^7*c*d^6)*e^2*f^5 - 7*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^
2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*e*f^6 + (a^3*b^4*c^7
 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*f^7
 - 6*(2*b^7*d^7*e^5*f^2 - 5*(b^7*c*d^6 + a*b^6*d^7)*e^4*f^3 + 2*(b^7*c^2*d^
5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*e^3*f^4 + 2*(b^7*c^3*d^4 - 6*a*b^6*c^2*d^5
 - 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^2*f^5 - (5*b^7*c^4*d^3 - 16*a*b^6*c^3*d
^4 + 12*a^2*b^5*c^2*d^5 - 16*a^3*b^4*c*d^6 + 5*a^4*b^3*d^7)*e*f^6 + (2*b^7*
c^5*d^2 - 5*a*b^6*c^4*d^3 + 2*a^2*b^5*c^3*d^4 + 2*a^3*b^4*c^2*d^5 - 5*a^4*b
^3*c*d^6 + 2*a^5*b^2*d^7)*f^7)*x^5 - 3*(8*b^7*d^7*e^6*f - 14*(b^7*c*d^6 + a
*b^6*d^7)*e^5*f^2 - (7*b^7*c^2*d^5 - 34*a*b^6*c*d^6 + 7*a^2*b^5*d^7)*e^4*f^
3 + 2*(7*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 + 7*a^3*b^4*d^7)*e
^3*f^4 - (7*b^7*c^4*d^3 - 6*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 - 6*a^3*b^4*
c*d^6 + 7*a^4*b^3*d^7)*e^2*f^5 - 2*(7*b^7*c^5*d^2 - 17*a*b^6*c^4*d^3 - 3*a^
2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 - 17*a^4*b^3*c*d^6 + 7*a^5*b^2*d^7)*e*f^6
 + (8*b^7*c^6*d - 14*a*b^6*c^5*d^2 - 7*a^2*b^5*c^4*d^3 + 14*a^3*b^4*c^3*d^4
 - 7*a^4*b^3*c^2*d^5 - 14*a^5*b^2*c*d^6 + 8*a^6*b*d^7)*f^7)*x^4 - 2*(6*b^7*
d^7*e^7 + 3*(b^7*c*d^6 + a*b^6*d^7)*e^6*f - (37*b^7*c^2*d^5 + 28*a*b^6*c*d^
6 + 37*a^2*b^5*d^7)*e^5*f^2 + (19*b^7*c^3*d^4 + 86*a*b^6*c^2*d^5 + 86*a^2*b
^5*c*d^6 + 19*a^3*b^4*d^7)*e^4*f^3 + (19*b^7*c^4*d^3 - 68*a*b^6*c^3*d^4 - 5
2*a^2*b^5*c^2*d^5 - 68*a^3*b^4*c*d^6 + 19*a^4*b^3*d^7)*e^3*f^4 - (37*b^7*c^
5*d^2 - 86*a*b^6*c^4*d^3 + 52*a^2*b^5*c^3*d^4 + 52*a^3*b^4*c^2*d^5 - 86*a^4
*b^3*c*d^6 + 37*a^5*b^2*d^7)*e^2*f^5 + (3*b^7*c^6*d - 28*a*b^6*c^5*d^2 + 86
*a^2*b^5*c^4*d^3 - 68*a^3*b^4*c^3*d^4 + 86*a^4*b^3*c^2*d^5 - 28*a^5*b^2*c*d
^6 + 3*a^6*b*d^7)*e*f^6 + (6*b^7*c^7 + 3*a*b^6*c^6*d - 37*a^2*b^5*c^5*d^2 +
 19*a^3*b^4*c^4*d^3 + 19*a^4*b^3*c^3*d^4 - 37*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + 6*a^7*d^7)*f^7)*x^3 - (18*(b^7*c*d^6 + a*b^6*d^7)*e^7 - (37*b^7*c^2*d^
5 + 34*a*b^6*c*d^6 + 37*a^2*b^5*d^7)*e^6*f - 3*(b^7*c^3*d^4 - 3*a*b^6*c^2*d
^5 - 3*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^5*f^2 + (32*b^7*c^4*d^3 + a*b^6*c^3*d
^4 + 234*a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + 32*a^4*b^3*d^7)*e^4*f^3 - (3*b^7
*c^5*d^2 - a*b^6*c^4*d^3 + 208*a^2*b^5*c^3*d^4 + 208*a^3*b^4*c^2*d^5 - a^4*
b^3*c*d^6 + 3*a^5*b^2*d^7)*e^3*f^4 - (37*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 234*
a^2*b^5*c^4*d^3 + 208*a^3*b^4*c^3*d^4 - 234*a^4*b^3*c^2*d^5 - 9*a^5*b^2*c*d
^6 + 37*a^6*b*d^7)*e^2*f^5 + (18*b^7*c^7 - 34*a*b^6*c^6*d + 9*a^2*b^5*c^5*d
^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 - 34*a^6*b*c*d^6
 + 18*a^7*d^7)*e*f^6 + (18*a*b^6*c^7 - 37*a^2*b^5*c^6*d - 3*a^3*b^4*c^5*d^2
 + 32*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 37*a^6*b*c^2*d^5 + 18*a^7*c*d^6
)*f^7)*x^2 - 2*(2*(b^7*c^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*e^7 - 3*(2*b^
7*c^3*d^4 + 11*a*b^6*c^2*d^5 + 11*a^2*b^5*c*d^6 + 2*a^3*b^4*d^7)*e^6*f + (4
*b^7*c^4*d^3 + 17*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 + 17*a^3*b^4*c*d^6 + 4
*a^4*b^3*d^7)*e^5*f^2 + 2*(2*b^7*c^5*d^2 - 4*a*b^6*c^4*d^3 - 13*a^2*b^5*c^3
*d^4 - 13*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + 2*a^5*b^2*d^7)*e^4*f^3 - (6*b
```

^7*c^6*d - 17*a*b^6*c^5*d^2 + 26*a^2*b^5*c^4*d^3 + 26*a^4*b^3*c^2*d^5 - 17*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*e^3*f^4 + (2*b^7*c^7 - 33*a*b^6*c^6*d + 78*a^2*b^5*c^5*d^2 - 26*a^3*b^4*c^4*d^3 - 26*a^4*b^3*c^3*d^4 + 78*a^5*b^2*c^2*d^5 - 33*a^6*b*c*d^6 + 2*a^7*d^7)*e^2*f^5 + (14*a*b^6*c^7 - 33*a^2*b^5*c^6*d + 17*a^3*b^4*c^5*d^2 - 8*a^4*b^3*c^4*d^3 + 17*a^5*b^2*c^3*d^4 - 33*a^6*b*c^2*d^5 + 14*a^7*c*d^6)*e*f^6 + 2*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*f^7)*x)/((a^2*b^8*c^6*d^4 - 4*a^3*b^7*c^5*d^5 + 6*a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8)*e^10 - 4*(a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 2*a^4*b^6*c^5*d^5 + 2*a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 + a^7*b^3*c^2*d^8)*e^9*f + 2*(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + 24*a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8)*e^8*f^2 - 4*(a^2*b^8*c^9*d + 2*a^3*b^7*c^8*d^2 - 12*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^6*d^4 + 9*a^6*b^4*c^5*d^5 - 12*a^7*b^3*c^4*d^6 + 2*a^8*b^2*c^3*d^7 + a^9*b*c^2*d^8)*e^7*f^3 + (a^2*b^8*c^10 + 12*a^3*b^7*c^9*d - 22*a^4*b^6*c^8*d^2 - 36*a^5*b^5*c^7*d^3 + 90*a^6*b^4*c^6*d^4 - 36*a^7*b^3*c^5*d^5 - 22*a^8*b^2*c^4*d^6 + 12*a^9*b*c^3*d^7 + a^10*c^2*d^8)*e^6*f^4 - 4*(a^3*b^7*c^10 + 2*a^4*b^6*c^9*d - 12*a^5*b^5*c^8*d^2 + 9*a^6*b^4*c^7*d^3 + 9*a^7*b^3*c^6*d^4 - 12*a^8*b^2*c^5*d^5 + 2*a^9*b*c^4*d^6 + a^10*c^3*d^7)*e^5*f^5 + 2*(3*a^4*b^6*c^10 - 4*a^5*b^5*c^9*d - 11*a^6*b^4*c^8*d^2 + 24*a^7*b^3*c^7*d^3 - 11*a^8*b^2*c^6*d^4 - 4*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^4*f^6 - 4*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*e^3*f^7 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*e^2*f^8 + ((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^8*f^2 - 4*(b^10*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b^5*d^10)*e^7*f^3 + 2*(3*b^10*c^6*d^4 - 4*a*b^9*c^5*d^5 - 11*a^2*b^8*c^4*d^6 + 24*a^3*b^7*c^3*d^7 - 11*a^4*b^6*c^2*d^8 - 4*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^6*f^4 - 4*(b^10*c^7*d^3 + 2*a*b^9*c^6*d^4 - 12*a^2*b^8*c^5*d^5 + 9*a^3*b^7*c^4*d^6 + 9*a^4*b^6*c^3*d^7 - 12*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^5*f^5 + (b^10*c^8*d^2 + 12*a*b^9*c^7*d^3 - 22*a^2*b^8*c^6*d^4 - 36*a^3*b^7*c^5*d^5 + 90*a^4*b^6*c^4*d^6 - 36*a^5*b^5*c^3*d^7 - 22*a^6*b^4*c^2*d^8 + 12*a^7*b^3*c*d^9 + a^8*b^2*d^10)*e^4*f^6 - 4*(a*b^9*c^8*d^2 + 2*a^2*b^8*c^7*d^3 - 12*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^5 + 9*a^5*b^5*c^4*d^6 - 12*a^6*b^4*c^3*d^7 + 2*a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9)*e^3*f^7 + 2*(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + 24*a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 2*a^5*b^5*c^6*d^4 + 2*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 + a^8*b^2*c^3*d^7)*e*f^9 + (a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6)*f^10)*x^6 + 2*((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^9*f - 3*(b^10*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b^5*d^10)*e^8*f^2 + 2*(b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7 - 9*a^4*b^6*c^2*d^8 + a^6*b^4*d^10)*e^7*f^3 + 2*(b^10*c^7*d^3 - 5*a*b^9*c^

6*d^4 + 9*a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 - 5*a^4*b^6*c^3*d^7 + 9*a^5*b
^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^6*f^4 - 3*(b^10*c^8*d^2 - 6*
a^2*b^8*c^6*d^4 + 8*a^3*b^7*c^5*d^5 - 6*a^4*b^6*c^4*d^6 + 8*a^5*b^5*c^3*d^7
 - 6*a^6*b^4*c^2*d^8 + a^8*b^2*d^10)*e^5*f^5 + (b^10*c^9*d + 9*a*b^9*c^8*d^
2 - 18*a^2*b^8*c^7*d^3 - 10*a^3*b^7*c^6*d^4 + 18*a^4*b^6*c^5*d^5 + 18*a^5*b
^5*c^4*d^6 - 10*a^6*b^4*c^3*d^7 - 18*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^
9*b*d^10)*e^4*f^6 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d
^3 + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^
3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^3*f^7 + 6*(a^2*b^8*c^9*d -
 a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 3*a^5*b^5*c^6*d^4 + 3*a^6*b^4*c^5*d^
5 - 3*a^7*b^3*c^4*d^6 - a^8*b^2*c^3*d^7 + a^9*b*c^2*d^8)*e^2*f^8 - (4*a^3*b
^7*c^9*d - 9*a^4*b^6*c^8*d^2 + 10*a^6*b^4*c^6*d^4 - 9*a^8*b^2*c^4*d^6 + 4*a
^9*b*c^3*d^7)*e*f^9 + (a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^
3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*f^10)*x^5 + ((b^
10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*e^10 - 3*(3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 5*
a^4*b^6*c^2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^8*f^2 + 4*(4*b^10*c^7
*d^3 - 5*a*b^9*c^6*d^4 - 9*a^2*b^8*c^5*d^5 + 10*a^3*b^7*c^4*d^6 + 10*a^4*b^
6*c^3*d^7 - 9*a^5*b^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + 4*a^7*b^3*d^10)*e^7*f^3 -
 (9*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 - 90*a^2*b^8*c^6*d^4 + 36*a^3*b^7*c^5*d
^5 + 50*a^4*b^6*c^4*d^6 + 36*a^5*b^5*c^3*d^7 - 90*a^6*b^4*c^2*d^8 + 20*a^7*
b^3*c*d^9 + 9*a^8*b^2*d^10)*e^6*f^4 + 12*(2*a*b^9*c^8*d^2 - 3*a^2*b^8*c^7*d
^3 - 3*a^3*b^7*c^6*d^4 + 4*a^4*b^6*c^5*d^5 + 4*a^5*b^5*c^4*d^6 - 3*a^6*b^4*
c^3*d^7 - 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9)*e^5*f^5 + (b^10*c^10 - 15*a^
2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48*a^5*b^5*c^5*d^
5 - 50*a^6*b^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^1
0)*e^4*f^6 - 4*(a*b^9*c^10 - 10*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^6*d^4 + 9*a^6
*b^4*c^5*d^5 - 10*a^7*b^3*c^4*d^6 + a^10*c*d^9)*e^3*f^7 + 3*(2*a^2*b^8*c^10
 - 5*a^4*b^6*c^8*d^2 - 12*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 12*a^7*b^3
*c^5*d^5 - 5*a^8*b^2*c^4*d^6 + 2*a^10*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^10 -
6*a^5*b^5*c^8*d^2 + 5*a^6*b^4*c^7*d^3 + 5*a^7*b^3*c^6*d^4 - 6*a^8*b^2*c^5*d
^5 + a^10*c^3*d^7)*e*f^9 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c
^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*f^10)*x^4 + 2*((b^10*c^5*d^5 - 3
*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 +
a^5*b^5*d^10)*e^10 - (3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
+ 5*a^4*b^6*c^2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^9*f + (2*b^10*c^7
*d^3 - 5*a*b^9*c^6*d^4 + 3*a^2*b^8*c^5*d^5 + 3*a^5*b^5*c^2*d^8 - 5*a^6*b^4*
c*d^9 + 2*a^7*b^3*d^10)*e^8*f^2 + 2*(b^10*c^8*d^2 - 16*a^3*b^7*c^5*d^5 + 30
*a^4*b^6*c^4*d^6 - 16*a^5*b^5*c^3*d^7 + a^8*b^2*d^10)*e^7*f^3 - (3*b^10*c^9
*d + 5*a*b^9*c^8*d^2 - 60*a^3*b^7*c^6*d^4 + 52*a^4*b^6*c^5*d^5 + 52*a^5*b^5
*c^4*d^6 - 60*a^6*b^4*c^3*d^7 + 5*a^8*b^2*c*d^9 + 3*a^9*b*d^10)*e^6*f^4 + (
b^10*c^10 + 8*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 32*a^3*b^7*c^7*d^3 - 52*a^4
*b^6*c^6*d^4 + 144*a^5*b^5*c^5*d^5 - 52*a^6*b^4*c^4*d^6 - 32*a^7*b^3*c^3*d^
7 + 3*a^8*b^2*c^2*d^8 + 8*a^9*b*c*d^9 + a^10*d^10)*e^5*f^5 - (3*a*b^9*c^10
+ 5*a^2*b^8*c^9*d - 60*a^4*b^6*c^7*d^3 + 52*a^5*b^5*c^6*d^4 + 52*a^6*b^4*c^

```
5*d^5 - 60*a^7*b^3*c^4*d^6 + 5*a^9*b*c^2*d^8 + 3*a^10*c*d^9)*e^4*f^6 + 2*(a
^2*b^8*c^10 - 16*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 16*a^7*b^3*c^5*d^5
+ a^10*c^2*d^8)*e^3*f^7 + (2*a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 3*a^5*b^5*c^8
*d^2 + 3*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*e^2*f^8 - (3*a
^4*b^6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8*a
^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e*f^9 + (a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*
a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*f^10)
*x^3 + ((b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7 - 9*a^4*b^6*
c^2*d^8 + a^6*b^4*d^10)*e^10 - 4*(b^10*c^7*d^3 - 6*a^2*b^8*c^5*d^5 + 5*a^3*
b^7*c^4*d^6 + 5*a^4*b^6*c^3*d^7 - 6*a^5*b^5*c^2*d^8 + a^7*b^3*d^10)*e^9*f +
 3*(2*b^10*c^8*d^2 - 5*a^2*b^8*c^6*d^4 - 12*a^3*b^7*c^5*d^5 + 30*a^4*b^6*c^
4*d^6 - 12*a^5*b^5*c^3*d^7 - 5*a^6*b^4*c^2*d^8 + 2*a^8*b^2*d^10)*e^8*f^2 -
4*(b^10*c^9*d - 10*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^5 + 9*a^5*b^5*c^4*d^6
- 10*a^6*b^4*c^3*d^7 + a^9*b*d^10)*e^7*f^3 + (b^10*c^10 - 15*a^2*b^8*c^8*d^
2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48*a^5*b^5*c^5*d^5 - 50*a^6*b
^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^10)*e^6*f^4 +
 12*(2*a^2*b^8*c^9*d - 3*a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 4*a^5*b^5*c^
6*d^4 + 4*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 - 3*a^8*b^2*c^3*d^7 + 2*a^9*b
*c^2*d^8)*e^5*f^5 - (9*a^2*b^8*c^10 + 20*a^3*b^7*c^9*d - 90*a^4*b^6*c^8*d^2
 + 36*a^5*b^5*c^7*d^3 + 50*a^6*b^4*c^6*d^4 + 36*a^7*b^3*c^5*d^5 - 90*a^8*b^
2*c^4*d^6 + 20*a^9*b*c^3*d^7 + 9*a^10*c^2*d^8)*e^4*f^6 + 4*(4*a^3*b^7*c^10
- 5*a^4*b^6*c^9*d - 9*a^5*b^5*c^8*d^2 + 10*a^6*b^4*c^7*d^3 + 10*a^7*b^3*c^6
*d^4 - 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 4*a^10*c^3*d^7)*e^3*f^7 - 3*(3
*a^4*b^6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8
*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d
+ 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*f^10)*x^2 + 2*((a*b^9
*c^6*d^4 - 3*a^2*b^8*c^5*d^5 + 2*a^3*b^7*c^4*d^6 + 2*a^4*b^6*c^3*d^7 - 3*a^
5*b^5*c^2*d^8 + a^6*b^4*c*d^9)*e^10 - (4*a*b^9*c^7*d^3 - 9*a^2*b^8*c^6*d^4
+ 10*a^4*b^6*c^4*d^6 - 9*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9)*e^9*f + 6*(a*b^
9*c^8*d^2 - a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 3*a^4*b^6*c^5*d^5 + 3*a^5
*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 - a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9)*e^8*f^2
 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3 + 5*a^4*b^6*c^
6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^3*c^3*d^7 + 3*a^8
*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^7*f^3 + (a*b^9*c^10 + 9*a^2*b^8*c^9*d - 18*
a^3*b^7*c^8*d^2 - 10*a^4*b^6*c^7*d^3 + 18*a^5*b^5*c^6*d^4 + 18*a^6*b^4*c^5*
d^5 - 10*a^7*b^3*c^4*d^6 - 18*a^8*b^2*c^3*d^7 + 9*a^9*b*c^2*d^8 + a^10*c*d^
9)*e^6*f^4 - 3*(a^2*b^8*c^10 - 6*a^4*b^6*c^8*d^2 + 8*a^5*b^5*c^7*d^3 - 6*a^
6*b^4*c^6*d^4 + 8*a^7*b^3*c^5*d^5 - 6*a^8*b^2*c^4*d^6 + a^10*c^2*d^8)*e^5*f
^5 + 2*(a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 9*a^5*b^5*c^8*d^2 - 5*a^6*b^4*c^7*
d^3 - 5*a^7*b^3*c^6*d^4 + 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + a^10*c^3*d^
7)*e^4*f^6 + 2*(a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a
^8*b^2*c^6*d^4 + a^10*c^4*d^6)*e^3*f^7 - 3*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d
+ 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*e
^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*
d^3 + a^10*c^6*d^4)*e*f^9)*x)
```

mupad [B] time = 20.46, size = 82532, normalized size = 166.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3)^3,x)
```

```
[Out] symsum(log(root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 573300*a^12*b^8*
c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 + 573300*a^11
*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*z^3 + 573300
*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11*f^9*z^3 - 3
43980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^11*e^10*f^10*
z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10*c^9*d^11*e^
11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^9*b^11*c^10*
d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 326340*a^13*b^
7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z^3 + 326340*
a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10*f^10*z^3 +
326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d^10*e^8*f^12
*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^10*c^12*d^8*e
^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*a^8*b^12*c^12
*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 + 245700*a^14*b
^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14*z^3 + 245700*a
^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^8*f^12*z^3 + 245
700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*d^8*e^12*f^8*z^3
- 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7*c^8*d^12*e^9*f^11
*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^12*b^8*c^7*d^13*e^1
1*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - 191100*a^11*b^9*c^7*d^1
3*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100*a^9*b^11*c^
8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 191100*a^8*b^
12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^3 - 191100*a
^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^13*z^3 - 123
900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e^6*f^14*z^3
- 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13*d^7*e^6*f^14
*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11*c^14*d^6*e^7
*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7*b^13*c^14*d^
6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900*a^6*b^14*c^
13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 101700*a^15*b
^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^3 + 101700*a
^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f^9*z^3 + 101
700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e^14*f^6*z^3
+ 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d^14*e^15*f^5
*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14*c^9*d^11*e^1
5*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5*b^15*c^11*d^
```

9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820*a^14*b^6*c^6
*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 65820*a^10*b^
10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^3 - 65820*a^
6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^13*z^3 - 5670
0*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8*f^12*z^3 + 5
6700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e^5*f^15*z^3
- 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^16*e^13*f^7*z
^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5*d^15*e^13*f^
7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c^7*d^13*e^15
*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 - 56700*a^7*b^13*c^15*d^5*e
^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3 + 56700*a^7*b^13*c^7*d^1
3*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*z^3 - 56700*a^5*b^15*c^12
*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7*z^3 - 48252*a^15*b^5*c
^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10*z^3 - 48252*a^10*
b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5*z^3 - 48252*
a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c^10*d^10*e^15*f^5*z^3 - 3
2400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b^4*c^6*d^14*e^8*f^12*z^3
- 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^14*b^6*c^4*d^16*e^12*f^8*z
^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*a^12*b^8*c^4*d^16*e^14*f^
6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 32400*a^8*b^12*c^6*d^14*e^16
*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 - 32400*a^6*b^14*c^8*d^12*e
^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3 - 32400*a^4*b^16*c^12*d^
8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16*z^3 + 20565*a^16*b^4*c^4
*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16*z^3 + 20565*a^10*b^
10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10*z^3 + 20565*a^
4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^15*z^3 + 1566
0*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*c^12*d^8*e^3*f^17*z^3 + 1
5660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b^8*c^15*d^5*e^3*f^17*z^3
+ 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8*b^12*c^17*d^3*e^5*f^15*z
^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*a^5*b^15*c^17*d^3*e^8*f^1
2*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 15660*a^3*b^17*c^15*d^5*e^12
*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 - 9750*a^17*b^3*c^9*d^11*e^
4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 - 9750*a^16*b^4*c^11*d^9*e
^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3 - 9750*a^11*b^9*c^16*d^4*
e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3 - 9750*a^9*b^11*c^17*d^3
*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^3 - 9750*a^4*b^16*c^17*d^
3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z^3 - 9750*a^3*b^17*c^16*d
^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*z^3 - 8100*a^17*b^3*c^7*d
^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13*z^3 - 8100*a^14*b^6*c^13
*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^7*z^3 - 8100*a^13*b^7*c^1
4*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f^6*z^3 - 8100*a^7*b^13*c^
17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*f^3*z^3 - 8100*a^6*b^14*c
^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17*f^3*z^3 - 8100*a^3*b^17*
c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^14*f^6*z^3 - 7980*a^16*b^4
*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^9*f^11*z^3 - 7980*a^15*b^

5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e^11*f^9*z^3 - 7980*a^11*b
^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*e^15*f^5*z^3 - 7980*a^9*b
^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15*e^16*f^4*z^3 - 7980*a^5*
b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^11*e^16*f^4*z^3 - 7980*a^4
*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d^9*e^15*f^5*z^3 + 6300*a^
18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*d^6*e^2*f^18*z^3 + 6300*a
^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18*d^2*e^6*f^14*z^3 + 6300*
a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^14*d^6*e^14*f^6*z^3 - 4260
*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^5*d^15*e^7*f^13*z^3 - 426
0*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c^2*d^18*e^13*f^7*z^3 - 42
60*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*c^2*d^18*e^15*f^5*z^3 - 4
260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13*c^5*d^15*e^18*f^2*z^3 -
4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^15*c^7*d^13*e^18*f^2*z^3 -
4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^18*c^13*d^7*e^15*f^5*z^3
+ 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17*b^3*c^3*d^17*e^10*f^10*z
^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a^10*b^10*c^3*d^17*e^17*f^
3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 1470*a^3*b^17*c^10*d^10*e^17
*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1350*a^18*b^2*c^3*d^17*e^9
*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 + 1350*a^17*b^3*c^2*d^18*e^
11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 + 1350*a^11*b^9*c^2*d^18*e
^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3 + 1350*a^9*b^11*c^3*d^17*
e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3 + 1350*a^3*b^17*c^9*d^11
*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^3 + 1350*a^2*b^18*c^11*d^
9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*z^3 - 1070*a^18*b^2*c^2*d
^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^18*z^3 - 1070*a^10*b^10*c
^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10*f^10*z^3 - 1070*a^2*b^18
*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^4*f^16*z^3 + 525*a^18*b^2
*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2*f^18*z^3 + 525*a^16*b^4*
c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*f^18*z^3 + 525*a^12*b^8*c
^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f^16*z^3 + 525*a^8*b^12*c^
4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^12*z^3 + 525*a^4*b^16*c^8
*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^8*z^3 + 525*a^2*b^18*c^12
*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z^3 + 900*a^19*b*c^4*d^16*
e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 + 900*a^16*b^4*c*d^19*e^13*
f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a^13*b^7*c*d^19*e^16*f^4*z
^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^13*c^4*d^16*e^19*f*z^3 +
900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^7*d^13*e^19*f*z^3 + 900*a
*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*e^16*f^4*z^3 - 750*a^19*b
*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f^12*z^3 - 750*a^17*b^3*c^
12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z^3 - 750*a^12*b^8*c^17*d^
3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 - 750*a^8*b^12*c^19*d*e^3*f
^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a^3*b^17*c^19*d*e^8*f^12*z
^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19*c^17*d^3*e^12*f^8*z^3 -
750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*d^14*e^5*f^15*z^3 - 420*a
^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^6*e*f^19*z^3 - 420*a^15*b

^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f^19*z^3 - 420*a^14*b^6*c*
d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z^3 - 420*a^6*b^14*c^5*d^1
5*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 - 420*a^5*b^15*c^6*d^14*e^1
9*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a*b^19*c^14*d^6*e^15*f^5*z
^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b*c^2*d^18*e^9*f^11*z^3 +
350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*d^19*e^11*f^9*z^3 + 350*a
^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*e^18*f^2*z^3 + 350*a^9*b^
11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^19*f*z^3 + 350*a^2*b^18*c^
19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z^3 + 350*a*b^19*c^18*d^2*
e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 - 90*a^19*b*c^10*d^10*e*f^1
9*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b^10*c^19*d*e*f^19*z^3 - 9
0*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e^10*f^10*z^3 - 90*a*b^19*
c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3 + 10*b^20*c^11*d^9*e^19*
f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^19*e^9*f^11*z^3 + 10*a^19
*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3 + 10*a^9*b^11*c^20*e*f^1
9*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^11*d^9*f^20*z^3 + 10*a^11
*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3 + 10*a*b^19*c^9*d^11*e^2
0*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*c^16*d^4*e^14*f^6*z^3 - 2
10*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*e^13*f^7*z^3 + 120*b^20*c
^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z^3 - 45*b^20*c^12*d^8*e^1
8*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a^20*c^6*d^14*e^4*f^16*z^3
 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d^13*e^3*f^17*z^3 + 120*a^
20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^18*z^3 - 45*a^20*c^2*d^18
*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 210*a^16*b^4*d^20*e^14*f^6
*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b^3*d^20*e^13*f^7*z^3 + 12
0*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^12*f^8*z^3 - 45*a^12*b^8*
d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3 - 210*a^6*b^14*c^20*e^4*
f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^7*b^13*c^20*e^3*f^17*z^3
+ 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^20*e^2*f^18*z^3 - 45*a^2*b
^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20*z^3 - 210*a^16*b^4*c^14*
d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 120*a^17*b^3*c^13*d^7*f^20*
z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2*c^12*d^8*f^20*z^3 - 45*a
^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*e^20*z^3 - 210*a^6*b^14*c
^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3 + 120*a^7*b^13*c^3*d^17*e
^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*b^12*c^2*d^18*e^20*z^3 -
45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^10*z^3 - a^20*d^20*e^10*f^
10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*f^20*z^3 - a^10*b^10*d^20
*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^2*c*d^13*e*f^13*z + 1890*
a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12*f^2*z + 92610*a^6*b^8*c^
4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^10*z + 92610*a^4*b^10*c^4*
d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z - 66150*a^7*b^7*c^4*d^10
*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z - 66150*a^4*b^10*c^7*d^7*e^
3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 66150*a^3*b^11*c^8*d^6*e^3*f
^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*a^3*b^11*c^4*d^10*e^7*f^7
*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5*b^9*c^5*d^9*e^4*f^10*z -

55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10*c^5*d^9*e^5*f^9*z - 3213
0*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*d^12*e^3*f^11*z - 32130*a
^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^12*e^9*f^5*z - 32130*a^2*
b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*e^9*f^5*z + 22680*a^8*b^6
*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*f^10*z + 22680*a^4*b^10*c
^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^6*z + 22680*a^2*b^12*c^8*
d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z + 19278*a^10*b^4*c^2*d^1
2*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z + 19278*a^2*b^12*c^2*d^12
*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 18522*a^6*b^8*c^3*d^11*e^5
*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d^11*e^6*f^8*
z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e^6*f^8*z - 1
3230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^8*z - 13230*
a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z + 3402*a^7*b^7
*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^5*b^9*c^2*d^
12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*c^5*d^9*e^7*
f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*e^3*f^11*z +
 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^4*z + 7938*a
*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5670*a^11*b^3
*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2*b^12*c^11*d
^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 5670*a*b^13*c^11*d^3*e^2*f^
12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*e*f^13*z - 3
780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z - 3780*a^4*b
^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*b^13*c^4*d^1
0*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^13*e^5*f^9*z
 - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*z - 2268*a*b
^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^7*b^7*c^6*d^
8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d^7*e*f^13*z
+ 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z + 1890*a*b^
13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^13*e^13*f*z
- 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*f*z - 252*a^13*b*c*d^13*f
^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^7*e^7*f^7*z - 882*b^14*c^
11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z + 693*b^14*c^12*d^2*e^2*f^
12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8*d^6*e^6*f^8*z + 567*b^14
*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z + 441*b^14*c^4*d^10*e^10*
f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*d^9*e^9*f^5*z - 918*a^7*b
^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z - 882*a^3*b^11*d^14*e^11*f
^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12*d^14*e^12*f^2*z + 567*a^
8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z + 441*a^10*b^4*d^14*e^4*f
^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5*d^14*e^5*f^9*z - 126*a^5
*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z - 882*a^11*b^3*c^3*d^11*f^
14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2*c^2*d^12*f^14*z + 693*a^
2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*z + 567*a^6*b^8*c^8*d^6*f
^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^10*c^10*d^4*f^14*z - 126*a
^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z + 36*b^14*d^14*e^14*z +
36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*a^2*b^9*c^2*d^9*e^2*f^9 +

9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10*e^2*f^9 + 9018*a^2*b^9*c
^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018*a*b^10*c^3*d^8*e^2*f^9 +
9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10*e*f^10 - 9018*a*b^10*c^4
*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^11*c^5*d^6*e*f^10 + 2268*
b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2268*a*b^10*d^11*e^5*f^6 +
2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^11 - 1458*b^11*c^3*d^8*e^
3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*d^9*e^4*f^7 - 1458*a^3*b^
8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*a^2*b^9*d^11*e^4*f^7 - 14
58*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^11 - 1161*a^2*b^9*c^4*d^7*
f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^11 - 756*a^6*b^5*d^11*f^1
1, z, k)*((39*a^5*b^11*c^14*d^2*f^16 - 102*a^6*b^10*c^13*d^3*f^16 + 132*a^7
*b^9*c^12*d^4*f^16 - 84*a^8*b^8*c^11*d^5*f^16 + 21*a^9*b^7*c^10*d^6*f^16 +
21*a^10*b^6*c^9*d^7*f^16 - 84*a^11*b^5*c^8*d^8*f^16 + 132*a^12*b^4*c^7*d^9*
f^16 - 102*a^13*b^3*c^6*d^10*f^16 + 39*a^14*b^2*c^5*d^11*f^16 + 39*a^5*b^11
*d^16*e^14*f^2 - 102*a^6*b^10*d^16*e^13*f^3 + 132*a^7*b^9*d^16*e^12*f^4 - 8
4*a^8*b^8*d^16*e^11*f^5 + 21*a^9*b^7*d^16*e^10*f^6 + 21*a^10*b^6*d^16*e^9*f
^7 - 84*a^11*b^5*d^16*e^8*f^8 + 132*a^12*b^4*d^16*e^7*f^9 - 102*a^13*b^3*d^
16*e^6*f^10 + 39*a^14*b^2*d^16*e^5*f^11 + 39*b^16*c^5*d^11*e^14*f^2 - 102*b
^16*c^6*d^10*e^13*f^3 + 132*b^16*c^7*d^9*e^12*f^4 - 84*b^16*c^8*d^8*e^11*f^
5 + 21*b^16*c^9*d^7*e^10*f^6 + 21*b^16*c^10*d^6*e^9*f^7 - 84*b^16*c^11*d^5*
e^8*f^8 + 132*b^16*c^12*d^4*e^7*f^9 - 102*b^16*c^13*d^3*e^6*f^10 + 39*b^16*
c^14*d^2*e^5*f^11 - 6*a^4*b^12*c^15*d*f^16 - 6*a^15*b*c^4*d^12*f^16 - 6*a^4
*b^12*d^16*e^15*f - 6*a^15*b*d^16*e^4*f^12 - 6*b^16*c^4*d^12*e^15*f - 6*b^1
6*c^15*d*e^4*f^12 + 24*a*b^15*c^3*d^13*e^15*f + 24*a*b^15*c^15*d*e^3*f^13 +
24*a^3*b^13*c*d^15*e^15*f + 24*a^3*b^13*c^15*d*e*f^15 + 24*a^15*b*c*d^15*e
^3*f^13 + 24*a^15*b*c^3*d^13*e*f^15 - 117*a*b^15*c^4*d^12*e^14*f^2 + 150*a*
b^15*c^5*d^11*e^13*f^3 + 159*a*b^15*c^6*d^10*e^12*f^4 - 546*a*b^15*c^7*d^9*
e^11*f^5 + 414*a*b^15*c^8*d^8*e^10*f^6 - 168*a*b^15*c^9*d^7*e^9*f^7 + 414*a
*b^15*c^10*d^6*e^8*f^8 - 546*a*b^15*c^11*d^5*e^7*f^9 + 159*a*b^15*c^12*d^4*
e^6*f^10 + 150*a*b^15*c^13*d^3*e^5*f^11 - 117*a*b^15*c^14*d^2*e^4*f^12 - 36
*a^2*b^14*c^2*d^14*e^15*f - 36*a^2*b^14*c^15*d*e^2*f^14 - 117*a^4*b^12*c*d^
15*e^14*f^2 - 117*a^4*b^12*c^14*d^2*e*f^15 + 150*a^5*b^11*c*d^15*e^13*f^3 +
150*a^5*b^11*c^13*d^3*e*f^15 + 159*a^6*b^10*c*d^15*e^12*f^4 + 159*a^6*b^10
*c^12*d^4*e*f^15 - 546*a^7*b^9*c*d^15*e^11*f^5 - 546*a^7*b^9*c^11*d^5*e*f^1
5 + 414*a^8*b^8*c*d^15*e^10*f^6 + 414*a^8*b^8*c^10*d^6*e*f^15 - 168*a^9*b^7
*c*d^15*e^9*f^7 - 168*a^9*b^7*c^9*d^7*e*f^15 + 414*a^10*b^6*c*d^15*e^8*f^8
+ 414*a^10*b^6*c^8*d^8*e*f^15 - 546*a^11*b^5*c*d^15*e^7*f^9 - 546*a^11*b^5*
c^7*d^9*e*f^15 + 159*a^12*b^4*c*d^15*e^6*f^10 + 159*a^12*b^4*c^6*d^10*e*f^1
5 + 150*a^13*b^3*c*d^15*e^5*f^11 + 150*a^13*b^3*c^5*d^11*e*f^15 - 117*a^14*
b^2*c*d^15*e^4*f^12 - 117*a^14*b^2*c^4*d^12*e*f^15 - 36*a^15*b*c^2*d^14*e^2
*f^14 + 78*a^2*b^14*c^3*d^13*e^14*f^2 + 318*a^2*b^14*c^4*d^12*e^13*f^3 - 12
69*a^2*b^14*c^5*d^11*e^12*f^4 + 1134*a^2*b^14*c^6*d^10*e^11*f^5 + 618*a^2*b
^14*c^7*d^9*e^10*f^6 - 843*a^2*b^14*c^8*d^8*e^9*f^7 - 843*a^2*b^14*c^9*d^7*
e^8*f^8 + 618*a^2*b^14*c^10*d^6*e^7*f^9 + 1134*a^2*b^14*c^11*d^5*e^6*f^10 -
1269*a^2*b^14*c^12*d^4*e^5*f^11 + 318*a^2*b^14*c^13*d^3*e^4*f^12 + 78*a^2*

b^14*c^14*d^2*e^3*f^13 + 78*a^3*b^13*c^2*d^14*e^14*f^2 - 732*a^3*b^13*c^3*d
^13*e^13*f^3 + 978*a^3*b^13*c^4*d^12*e^12*f^4 + 1722*a^3*b^13*c^5*d^11*e^11
*f^5 - 4548*a^3*b^13*c^6*d^10*e^10*f^6 + 1362*a^3*b^13*c^7*d^9*e^9*f^7 + 22
32*a^3*b^13*c^8*d^8*e^8*f^8 + 1362*a^3*b^13*c^9*d^7*e^7*f^9 - 4548*a^3*b^13
*c^10*d^6*e^6*f^10 + 1722*a^3*b^13*c^11*d^5*e^5*f^11 + 978*a^3*b^13*c^12*d^
4*e^4*f^12 - 732*a^3*b^13*c^13*d^3*e^3*f^13 + 78*a^3*b^13*c^14*d^2*e^2*f^14
 + 318*a^4*b^12*c^2*d^14*e^13*f^3 + 978*a^4*b^12*c^3*d^13*e^12*f^4 - 4452*a
^4*b^12*c^4*d^12*e^11*f^5 + 3495*a^4*b^12*c^5*d^11*e^10*f^6 + 4302*a^4*b^12
*c^6*d^10*e^9*f^7 - 4518*a^4*b^12*c^7*d^9*e^8*f^8 - 4518*a^4*b^12*c^8*d^8*e
^7*f^9 + 4302*a^4*b^12*c^9*d^7*e^6*f^10 + 3495*a^4*b^12*c^10*d^6*e^5*f^11 -
 4452*a^4*b^12*c^11*d^5*e^4*f^12 + 978*a^4*b^12*c^12*d^4*e^3*f^13 + 318*a^4
*b^12*c^13*d^3*e^2*f^14 - 1269*a^5*b^11*c^2*d^14*e^12*f^4 + 1722*a^5*b^11*c
^3*d^13*e^11*f^5 + 3495*a^5*b^11*c^4*d^12*e^10*f^6 - 9348*a^5*b^11*c^5*d^11
*e^9*f^7 + 2799*a^5*b^11*c^6*d^10*e^8*f^8 + 4824*a^5*b^11*c^7*d^9*e^7*f^9 +
 2799*a^5*b^11*c^8*d^8*e^6*f^10 - 9348*a^5*b^11*c^9*d^7*e^5*f^11 + 3495*a^5
*b^11*c^10*d^6*e^4*f^12 + 1722*a^5*b^11*c^11*d^5*e^3*f^13 - 1269*a^5*b^11*c
^12*d^4*e^2*f^14 + 1134*a^6*b^10*c^2*d^14*e^11*f^5 - 4548*a^6*b^10*c^3*d^13
*e^10*f^6 + 4302*a^6*b^10*c^4*d^12*e^9*f^7 + 2799*a^6*b^10*c^5*d^11*e^8*f^8
 - 3744*a^6*b^10*c^6*d^10*e^7*f^9 - 3744*a^6*b^10*c^7*d^9*e^6*f^10 + 2799*a
^6*b^10*c^8*d^8*e^5*f^11 + 4302*a^6*b^10*c^9*d^7*e^4*f^12 - 4548*a^6*b^10*c
^10*d^6*e^3*f^13 + 1134*a^6*b^10*c^11*d^5*e^2*f^14 + 618*a^7*b^9*c^2*d^14*e
^10*f^6 + 1362*a^7*b^9*c^3*d^13*e^9*f^7 - 4518*a^7*b^9*c^4*d^12*e^8*f^8 + 4
824*a^7*b^9*c^5*d^11*e^7*f^9 - 3744*a^7*b^9*c^6*d^10*e^6*f^10 + 4824*a^7*b^
9*c^7*d^9*e^5*f^11 - 4518*a^7*b^9*c^8*d^8*e^4*f^12 + 1362*a^7*b^9*c^9*d^7*e
^3*f^13 + 618*a^7*b^9*c^10*d^6*e^2*f^14 - 843*a^8*b^8*c^2*d^14*e^9*f^7 + 22
32*a^8*b^8*c^3*d^13*e^8*f^8 - 4518*a^8*b^8*c^4*d^12*e^7*f^9 + 2799*a^8*b^8*
c^5*d^11*e^6*f^10 + 2799*a^8*b^8*c^6*d^10*e^5*f^11 - 4518*a^8*b^8*c^7*d^9*e
^4*f^12 + 2232*a^8*b^8*c^8*d^8*e^3*f^13 - 843*a^8*b^8*c^9*d^7*e^2*f^14 - 84
3*a^9*b^7*c^2*d^14*e^8*f^8 + 1362*a^9*b^7*c^3*d^13*e^7*f^9 + 4302*a^9*b^7*c
^4*d^12*e^6*f^10 - 9348*a^9*b^7*c^5*d^11*e^5*f^11 + 4302*a^9*b^7*c^6*d^10*e
^4*f^12 + 1362*a^9*b^7*c^7*d^9*e^3*f^13 - 843*a^9*b^7*c^8*d^8*e^2*f^14 + 61
8*a^10*b^6*c^2*d^14*e^7*f^9 - 4548*a^10*b^6*c^3*d^13*e^6*f^10 + 3495*a^10*b
^6*c^4*d^12*e^5*f^11 + 3495*a^10*b^6*c^5*d^11*e^4*f^12 - 4548*a^10*b^6*c^6*
d^10*e^3*f^13 + 618*a^10*b^6*c^7*d^9*e^2*f^14 + 1134*a^11*b^5*c^2*d^14*e^6*
f^10 + 1722*a^11*b^5*c^3*d^13*e^5*f^11 - 4452*a^11*b^5*c^4*d^12*e^4*f^12 +
1722*a^11*b^5*c^5*d^11*e^3*f^13 + 1134*a^11*b^5*c^6*d^10*e^2*f^14 - 1269*a^
12*b^4*c^2*d^14*e^5*f^11 + 978*a^12*b^4*c^3*d^13*e^4*f^12 + 978*a^12*b^4*c^
4*d^12*e^3*f^13 - 1269*a^12*b^4*c^5*d^11*e^2*f^14 + 318*a^13*b^3*c^2*d^14*e
^4*f^12 - 732*a^13*b^3*c^3*d^13*e^3*f^13 + 318*a^13*b^3*c^4*d^12*e^2*f^14 +
 78*a^14*b^2*c^2*d^14*e^3*f^13 + 78*a^14*b^2*c^3*d^13*e^2*f^14)/(56*a^3*b^1
3*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16
 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*
b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*
a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f
^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c

^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a
^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^
14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^
16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^
16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^1
2 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^
6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^1
6*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8
*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*
b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*
b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9
*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*
c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a
^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f
^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^
15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e
^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^
3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^
11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 1
12*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c
^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14
+ 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*
c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f
^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12
*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6
*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a
^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12
*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 1
60*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c
^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*
e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 -
2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^
3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c
^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^
13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 -
8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4
*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14
*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^
13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 +
7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^
5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^
13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e
^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4
+ 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640
*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10

*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^
3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^
3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640
*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c
^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6
*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 3
00*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^
8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d
^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^
8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264
*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*
c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e
^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 -
24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9
*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d
^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^
13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 862
4*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*
b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*
d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*
f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1
344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^1
1*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^
6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^
5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 -
1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^
12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^
5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^
5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 +
840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13
*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6
*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3
*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 13
44*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b
^2*c^8*d^8*e^2*f^14) + root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 5733
00*a^12*b^8*c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 +
573300*a^11*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*
z^3 + 573300*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11
*f^9*z^3 - 343980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^1
1*e^10*f^10*z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10
*c^9*d^11*e^11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^
9*b^11*c^10*d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 32
6340*a^13*b^7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z
^3 + 326340*a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10
*f^10*z^3 + 326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d

^10*e^8*f^12*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^1
0*c^12*d^8*e^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*a
^8*b^12*c^12*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 + 2
45700*a^14*b^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14*z^
3 + 245700*a^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^8*f^
12*z^3 + 245700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*d^8*e
^12*f^8*z^3 - 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7*c^8*d
^12*e^9*f^11*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^12*b^8*
c^7*d^13*e^11*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - 191100*a^11
*b^9*c^7*d^13*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100
*a^9*b^11*c^8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 1
91100*a^8*b^12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^
3 - 191100*a^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^
13*z^3 - 123900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e
^6*f^14*z^3 - 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13*
d^7*e^6*f^14*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11*
c^14*d^6*e^7*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7*
b^13*c^14*d^6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900
*a^6*b^14*c^13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 1
01700*a^15*b^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^
3 + 101700*a^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f
^9*z^3 + 101700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e
^14*f^6*z^3 + 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d
^14*e^15*f^5*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14*
c^9*d^11*e^15*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5*
b^15*c^11*d^9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820*
a^14*b^6*c^6*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 6
5820*a^10*b^10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^
3 - 65820*a^6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^1
3*z^3 - 56700*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8*
f^12*z^3 + 56700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e
^5*f^15*z^3 - 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^1
6*e^13*f^7*z^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5*
d^15*e^13*f^7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c
^7*d^13*e^15*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 - 56700*a^7*b^1
3*c^15*d^5*e^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3 + 56700*a^7*
b^13*c^7*d^13*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*z^3 - 56700*a
^5*b^15*c^12*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7*z^3 - 4825
2*a^15*b^5*c^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10*z^3 -
 48252*a^10*b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5*
z^3 - 48252*a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c^10*d^10*e^15
*f^5*z^3 - 32400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b^4*c^6*d^14*e
^8*f^12*z^3 - 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^14*b^6*c^4*d^1
6*e^12*f^8*z^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*a^12*b^8*c^4*
d^16*e^14*f^6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 32400*a^8*b^12*c

^6*d^14*e^16*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 - 32400*a^6*b^1
4*c^8*d^12*e^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3 - 32400*a^4*
b^16*c^12*d^8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16*z^3 + 20565*
a^16*b^4*c^4*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16*z^3 + 2
0565*a^10*b^10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10*z^
3 + 20565*a^4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^1
5*z^3 + 15660*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*c^12*d^8*e^3*
f^17*z^3 + 15660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b^8*c^15*d^5*e
^3*f^17*z^3 + 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8*b^12*c^17*d^
3*e^5*f^15*z^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*a^5*b^15*c^17
*d^3*e^8*f^12*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 15660*a^3*b^17*c
^15*d^5*e^12*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 - 9750*a^17*b^3
*c^9*d^11*e^4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 - 9750*a^16*b^
4*c^11*d^9*e^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3 - 9750*a^11*b
^9*c^16*d^4*e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3 - 9750*a^9*b
^11*c^17*d^3*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^3 - 9750*a^4*
b^16*c^17*d^3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z^3 - 9750*a^3
*b^17*c^16*d^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*z^3 - 8100*a^
17*b^3*c^7*d^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13*z^3 - 8100*a
^14*b^6*c^13*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^7*z^3 - 8100*
a^13*b^7*c^14*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f^6*z^3 - 8100
*a^7*b^13*c^17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*f^3*z^3 - 810
0*a^6*b^14*c^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17*f^3*z^3 - 81
00*a^3*b^17*c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^14*f^6*z^3 - 7
980*a^16*b^4*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^9*f^11*z^3 -
7980*a^15*b^5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e^11*f^9*z^3 -
7980*a^11*b^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*e^15*f^5*z^3
- 7980*a^9*b^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15*e^16*f^4*z^3
- 7980*a^5*b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^11*e^16*f^4*z^
3 - 7980*a^4*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d^9*e^15*f^5*z
^3 + 6300*a^18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*d^6*e^2*f^18*
z^3 + 6300*a^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18*d^2*e^6*f^14
*z^3 + 6300*a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^14*d^6*e^14*f^
6*z^3 - 4260*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^5*d^15*e^7*f^
13*z^3 - 4260*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c^2*d^18*e^13*
f^7*z^3 - 4260*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*c^2*d^18*e^15
*f^5*z^3 - 4260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13*c^5*d^15*e^1
8*f^2*z^3 - 4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^15*c^7*d^13*e^
18*f^2*z^3 - 4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^18*c^13*d^7*e
^15*f^5*z^3 + 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17*b^3*c^3*d^17
*e^10*f^10*z^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a^10*b^10*c^3*
d^17*e^17*f^3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 1470*a^3*b^17*c^
10*d^10*e^17*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1350*a^18*b^2*
c^3*d^17*e^9*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 + 1350*a^17*b^3
*c^2*d^18*e^11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 + 1350*a^11*b^

9*c^2*d^18*e^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3 + 1350*a^9*b^
11*c^3*d^17*e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3 + 1350*a^3*b
^17*c^9*d^11*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^3 + 1350*a^2*
b^18*c^11*d^9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*z^3 - 1070*a^
18*b^2*c^2*d^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^18*z^3 - 1070
*a^10*b^10*c^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10*f^10*z^3 - 1
070*a^2*b^18*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^4*f^16*z^3 +
525*a^18*b^2*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2*f^18*z^3 + 5
25*a^16*b^4*c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*f^18*z^3 + 52
5*a^12*b^8*c^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f^16*z^3 + 525
*a^8*b^12*c^4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^12*z^3 + 525*
a^4*b^16*c^8*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^8*z^3 + 525*a
^2*b^18*c^12*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z^3 + 900*a^19
*b*c^4*d^16*e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 + 900*a^16*b^4*
c*d^19*e^13*f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a^13*b^7*c*d^1
9*e^16*f^4*z^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^13*c^4*d^16*e
^19*f*z^3 + 900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^7*d^13*e^19*f
*z^3 + 900*a*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*e^16*f^4*z^3
- 750*a^19*b*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f^12*z^3 - 750
*a^17*b^3*c^12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z^3 - 750*a^12
*b^8*c^17*d^3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 - 750*a^8*b^12*
c^19*d*e^3*f^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a^3*b^17*c^19*
d*e^8*f^12*z^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19*c^17*d^3*e^1
2*f^8*z^3 - 750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*d^14*e^5*f^15
*z^3 - 420*a^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^6*e*f^19*z^3
- 420*a^15*b^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f^19*z^3 - 420
*a^14*b^6*c*d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z^3 - 420*a^6*
b^14*c^5*d^15*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 - 420*a^5*b^15*
c^6*d^14*e^19*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a*b^19*c^14*d^
6*e^15*f^5*z^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b*c^2*d^18*e^9
*f^11*z^3 + 350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*d^19*e^11*f^9
*z^3 + 350*a^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*e^18*f^2*z^3
+ 350*a^9*b^11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^19*f*z^3 + 350
*a^2*b^18*c^19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z^3 + 350*a*b^
19*c^18*d^2*e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 - 90*a^19*b*c^1
0*d^10*e*f^19*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b^10*c^19*d*e*
f^19*z^3 - 90*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e^10*f^10*z^3
- 90*a*b^19*c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3 + 10*b^20*c^
11*d^9*e^19*f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^19*e^9*f^11*z
^3 + 10*a^19*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3 + 10*a^9*b^1
1*c^20*e*f^19*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^11*d^9*f^20*z
^3 + 10*a^11*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3 + 10*a*b^19*
c^9*d^11*e^20*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*c^16*d^4*e^14
*f^6*z^3 - 210*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*e^13*f^7*z^3
+ 120*b^20*c^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z^3 - 45*b^20*

c^12*d^8*e^18*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a^20*c^6*d^14*
e^4*f^16*z^3 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d^13*e^3*f^17*
z^3 + 120*a^20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^18*z^3 - 45*a
^20*c^2*d^18*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 210*a^16*b^4*d
^20*e^14*f^6*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b^3*d^20*e^13*
f^7*z^3 + 120*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^12*f^8*z^3 -
45*a^12*b^8*d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3 - 210*a^6*b^
14*c^20*e^4*f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^7*b^13*c^20*e
^3*f^17*z^3 + 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^20*e^2*f^18*z^
3 - 45*a^2*b^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20*z^3 - 210*a^
16*b^4*c^14*d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 120*a^17*b^3*c^
13*d^7*f^20*z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2*c^12*d^8*f^2
0*z^3 - 45*a^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*e^20*z^3 - 21
0*a^6*b^14*c^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3 + 120*a^7*b^1
3*c^3*d^17*e^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*b^12*c^2*d^18
*e^20*z^3 - 45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^10*z^3 - a^20*
d^20*e^10*f^10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*f^20*z^3 - a^
10*b^10*d^20*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^2*c*d^13*e*f^
13*z + 1890*a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12*f^2*z + 9261
0*a^6*b^8*c^4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^10*z + 92610*a
^4*b^10*c^4*d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z - 66150*a^7*
b^7*c^4*d^10*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z - 66150*a^4*b^1
0*c^7*d^7*e^3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 66150*a^3*b^11*c
^8*d^6*e^3*f^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*a^3*b^11*c^4*
d^10*e^7*f^7*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5*b^9*c^5*d^9*
e^4*f^10*z - 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10*c^5*d^9*e^5*
f^9*z - 32130*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*d^12*e^3*f^11
*z - 32130*a^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^12*e^9*f^5*z
- 32130*a^2*b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*e^9*f^5*z + 2
2680*a^8*b^6*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*f^10*z + 2268
0*a^4*b^10*c^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^6*z + 22680*a
^2*b^12*c^8*d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z + 19278*a^10
*b^4*c^2*d^12*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z + 19278*a^2*b
^12*c^2*d^12*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 18522*a^6*b^8*
c^3*d^11*e^5*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d
^11*e^6*f^8*z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e
^6*f^8*z - 13230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^
8*z - 13230*a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z +
3402*a^7*b^7*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^
5*b^9*c^2*d^12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*
c^5*d^9*e^7*f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*
e^3*f^11*z + 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^
4*z + 7938*a*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5
670*a^11*b^3*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2
*b^12*c^11*d^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 5670*a*b^13*c^1

1*d^3*e^2*f^12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*
e*f^13*z - 3780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z
- 3780*a^4*b^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*
b^13*c^4*d^10*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^
13*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*
z - 2268*a*b^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^
7*b^7*c^6*d^8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d
^7*e*f^13*z + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z
 + 1890*a*b^13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^
13*e^13*f*z - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*f*z - 252*a^1
3*b*c*d^13*f^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^7*e^7*f^7*z -
 882*b^14*c^11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z + 693*b^14*c^1
2*d^2*e^2*f^12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8*d^6*e^6*f^8*
z + 567*b^14*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z + 441*b^14*c^
4*d^10*e^10*f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*d^9*e^9*f^5*z
 - 918*a^7*b^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z - 882*a^3*b^11
*d^14*e^11*f^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12*d^14*e^12*f^
2*z + 567*a^8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z + 441*a^10*b^
4*d^14*e^4*f^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5*d^14*e^5*f^9
*z - 126*a^5*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z - 882*a^11*b^3
*c^3*d^11*f^14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2*c^2*d^12*f^1
4*z + 693*a^2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*z + 567*a^6*b
^8*c^8*d^6*f^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^10*c^10*d^4*f^
14*z - 126*a^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z + 36*b^14*d^
14*e^14*z + 36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*a^2*b^9*c^2*d
^9*e^2*f^9 + 9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10*e^2*f^9 + 90
18*a^2*b^9*c^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018*a*b^10*c^3*d
^8*e^2*f^9 + 9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10*e*f^10 - 901
8*a*b^10*c^4*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^11*c^5*d^6*e*
f^10 + 2268*b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2268*a*b^10*d^
11*e^5*f^6 + 2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^11 - 1458*b^1
1*c^3*d^8*e^3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*d^9*e^4*f^7 -
 1458*a^3*b^8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*a^2*b^9*d^11*
e^4*f^7 - 1458*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^11 - 1161*a^2*
b^9*c^4*d^7*f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^11 - 756*a^6*
b^5*d^11*f^11, z, k)*((20*a^11*b^8*c^16*d^3*f^19 - 7*a^10*b^9*c^17*d^2*f^19
 - 28*a^12*b^7*c^15*d^4*f^19 + 14*a^13*b^6*c^14*d^5*f^19 + 14*a^14*b^5*c^13
*d^6*f^19 - 28*a^15*b^4*c^12*d^7*f^19 + 20*a^16*b^3*c^11*d^8*f^19 - 7*a^17*
b^2*c^10*d^9*f^19 - 7*a^10*b^9*d^19*e^17*f^2 + 20*a^11*b^8*d^19*e^16*f^3 -
28*a^12*b^7*d^19*e^15*f^4 + 14*a^13*b^6*d^19*e^14*f^5 + 14*a^14*b^5*d^19*e^
13*f^6 - 28*a^15*b^4*d^19*e^12*f^7 + 20*a^16*b^3*d^19*e^11*f^8 - 7*a^17*b^2
*d^19*e^10*f^9 - 7*b^19*c^10*d^9*e^17*f^2 + 20*b^19*c^11*d^8*e^16*f^3 - 28*
b^19*c^12*d^7*e^15*f^4 + 14*b^19*c^13*d^6*e^14*f^5 + 14*b^19*c^14*d^5*e^13*
f^6 - 28*b^19*c^15*d^4*e^12*f^7 + 20*b^19*c^16*d^3*e^11*f^8 - 7*b^19*c^17*d
^2*e^10*f^9 + a^9*b^10*c^18*d*f^19 + a^18*b*c^9*d^10*f^19 + a^9*b^10*d^19*e

^18*f + a^18*b*d^19*e^9*f^10 + b^19*c^9*d^10*e^18*f + b^19*c^18*d*e^9*f^10
- 7*a*b^18*c^8*d^11*e^18*f - 7*a*b^18*c^18*d*e^8*f^11 - 7*a^8*b^11*c*d^18*e
^18*f - 7*a^8*b^11*c^18*d*e*f^18 - 7*a^18*b*c*d^18*e^8*f^11 - 7*a^18*b*c^8*
d^11*e*f^18 + 34*a*b^18*c^9*d^10*e^17*f^2 - 27*a*b^18*c^10*d^9*e^16*f^3 - 1
68*a*b^18*c^11*d^8*e^15*f^4 + 546*a*b^18*c^12*d^7*e^14*f^5 - 756*a*b^18*c^1
3*d^6*e^13*f^6 + 546*a*b^18*c^14*d^5*e^12*f^7 - 168*a*b^18*c^15*d^4*e^11*f^
8 - 27*a*b^18*c^16*d^3*e^10*f^9 + 34*a*b^18*c^17*d^2*e^9*f^10 + 20*a^2*b^17
*c^7*d^12*e^18*f + 20*a^2*b^17*c^18*d*e^7*f^12 - 28*a^3*b^16*c^6*d^13*e^18*
f - 28*a^3*b^16*c^18*d*e^6*f^13 + 14*a^4*b^15*c^5*d^14*e^18*f + 14*a^4*b^15
*c^18*d*e^5*f^14 + 14*a^5*b^14*c^4*d^15*e^18*f + 14*a^5*b^14*c^18*d*e^4*f^1
5 - 28*a^6*b^13*c^3*d^16*e^18*f - 28*a^6*b^13*c^18*d*e^3*f^16 + 20*a^7*b^12
*c^2*d^17*e^18*f + 20*a^7*b^12*c^18*d*e^2*f^17 + 34*a^9*b^10*c*d^18*e^17*f^
2 + 34*a^9*b^10*c^17*d^2*e*f^18 - 27*a^10*b^9*c*d^18*e^16*f^3 - 27*a^10*b^9
*c^16*d^3*e*f^18 - 168*a^11*b^8*c*d^18*e^15*f^4 - 168*a^11*b^8*c^15*d^4*e*f
^18 + 546*a^12*b^7*c*d^18*e^14*f^5 + 546*a^12*b^7*c^14*d^5*e*f^18 - 756*a^1
3*b^6*c*d^18*e^13*f^6 - 756*a^13*b^6*c^13*d^6*e*f^18 + 546*a^14*b^5*c*d^18*
e^12*f^7 + 546*a^14*b^5*c^12*d^7*e*f^18 - 168*a^15*b^4*c*d^18*e^11*f^8 - 16
8*a^15*b^4*c^11*d^8*e*f^18 - 27*a^16*b^3*c*d^18*e^10*f^9 - 27*a^16*b^3*c^10
*d^9*e*f^18 + 34*a^17*b^2*c*d^18*e^9*f^10 + 34*a^17*b^2*c^9*d^10*e*f^18 + 2
0*a^18*b*c^2*d^17*e^7*f^12 - 28*a^18*b*c^3*d^16*e^6*f^13 + 14*a^18*b*c^4*d^
15*e^5*f^14 + 14*a^18*b*c^5*d^14*e^4*f^15 - 28*a^18*b*c^6*d^13*e^3*f^16 + 2
0*a^18*b*c^7*d^12*e^2*f^17 - 27*a^2*b^17*c^8*d^11*e^17*f^2 - 371*a^2*b^17*c
^9*d^10*e^16*f^3 + 1560*a^2*b^17*c^10*d^9*e^15*f^4 - 2484*a^2*b^17*c^11*d^8
*e^14*f^5 + 1302*a^2*b^17*c^12*d^7*e^13*f^6 + 1302*a^2*b^17*c^13*d^6*e^12*f
^7 - 2484*a^2*b^17*c^14*d^5*e^11*f^8 + 1560*a^2*b^17*c^15*d^4*e^10*f^9 - 37
1*a^2*b^17*c^16*d^3*e^9*f^10 - 27*a^2*b^17*c^17*d^2*e^8*f^11 - 168*a^3*b^16
*c^7*d^12*e^17*f^2 + 1560*a^3*b^16*c^8*d^11*e^16*f^3 - 3464*a^3*b^16*c^9*d^
10*e^15*f^4 + 924*a^3*b^16*c^10*d^9*e^14*f^5 + 7728*a^3*b^16*c^11*d^8*e^13*
f^6 - 13104*a^3*b^16*c^12*d^7*e^12*f^7 + 7728*a^3*b^16*c^13*d^6*e^11*f^8 +
924*a^3*b^16*c^14*d^5*e^10*f^9 - 3464*a^3*b^16*c^15*d^4*e^9*f^10 + 1560*a^3
*b^16*c^16*d^3*e^8*f^11 - 168*a^3*b^16*c^17*d^2*e^7*f^12 + 546*a^4*b^15*c^6
*d^13*e^17*f^2 - 2484*a^4*b^15*c^7*d^12*e^16*f^3 + 924*a^4*b^15*c^8*d^11*e^
15*f^4 + 12550*a^4*b^15*c^9*d^10*e^14*f^5 - 26838*a^4*b^15*c^10*d^9*e^13*f^
6 + 15288*a^4*b^15*c^11*d^8*e^12*f^7 + 15288*a^4*b^15*c^12*d^7*e^11*f^8 - 2
6838*a^4*b^15*c^13*d^6*e^10*f^9 + 12550*a^4*b^15*c^14*d^5*e^9*f^10 + 924*a^
4*b^15*c^15*d^4*e^8*f^11 - 2484*a^4*b^15*c^16*d^3*e^7*f^12 + 546*a^4*b^15*c
^17*d^2*e^6*f^13 - 756*a^5*b^14*c^5*d^14*e^17*f^2 + 1302*a^5*b^14*c^6*d^13*
e^16*f^3 + 7728*a^5*b^14*c^7*d^12*e^15*f^4 - 26838*a^5*b^14*c^8*d^11*e^14*f
^5 + 18004*a^5*b^14*c^9*d^10*e^13*f^6 + 39858*a^5*b^14*c^10*d^9*e^12*f^7 -
78624*a^5*b^14*c^11*d^8*e^11*f^8 + 39858*a^5*b^14*c^12*d^7*e^10*f^9 + 18004
*a^5*b^14*c^13*d^6*e^9*f^10 - 26838*a^5*b^14*c^14*d^5*e^8*f^11 + 7728*a^5*b
^14*c^15*d^4*e^7*f^12 + 1302*a^5*b^14*c^16*d^3*e^6*f^13 - 756*a^5*b^14*c^17
*d^2*e^5*f^14 + 546*a^6*b^13*c^4*d^15*e^17*f^2 + 1302*a^6*b^13*c^5*d^14*e^1
6*f^3 - 13104*a^6*b^13*c^6*d^13*e^15*f^4 + 15288*a^6*b^13*c^7*d^12*e^14*f^5
 + 39858*a^6*b^13*c^8*d^11*e^13*f^6 - 110474*a^6*b^13*c^9*d^10*e^12*f^7 + 6

6612*a^6*b^13*c^10*d^9*e^11*f^8 + 66612*a^6*b^13*c^11*d^8*e^10*f^9 - 110474
*a^6*b^13*c^12*d^7*e^9*f^10 + 39858*a^6*b^13*c^13*d^6*e^8*f^11 + 15288*a^6*
b^13*c^14*d^5*e^7*f^12 - 13104*a^6*b^13*c^15*d^4*e^6*f^13 + 1302*a^6*b^13*c
^16*d^3*e^5*f^14 + 546*a^6*b^13*c^17*d^2*e^4*f^15 - 168*a^7*b^12*c^3*d^16*e
^17*f^2 - 2484*a^7*b^12*c^4*d^15*e^16*f^3 + 7728*a^7*b^12*c^5*d^14*e^15*f^4
+ 15288*a^7*b^12*c^6*d^13*e^14*f^5 - 78624*a^7*b^12*c^7*d^12*e^13*f^6 + 66
612*a^7*b^12*c^8*d^11*e^12*f^7 + 99736*a^7*b^12*c^9*d^10*e^11*f^8 - 216216*
a^7*b^12*c^10*d^9*e^10*f^9 + 99736*a^7*b^12*c^11*d^8*e^9*f^10 + 66612*a^7*b
^12*c^12*d^7*e^8*f^11 - 78624*a^7*b^12*c^13*d^6*e^7*f^12 + 15288*a^7*b^12*c
^14*d^5*e^6*f^13 + 7728*a^7*b^12*c^15*d^4*e^5*f^14 - 2484*a^7*b^12*c^16*d^3
*e^4*f^15 - 168*a^7*b^12*c^17*d^2*e^3*f^16 - 27*a^8*b^11*c^2*d^17*e^17*f^2
+ 1560*a^8*b^11*c^3*d^16*e^16*f^3 + 924*a^8*b^11*c^4*d^15*e^15*f^4 - 26838*
a^8*b^11*c^5*d^14*e^14*f^5 + 39858*a^8*b^11*c^6*d^13*e^13*f^6 + 66612*a^8*b
^11*c^7*d^12*e^12*f^7 - 216216*a^8*b^11*c^8*d^11*e^11*f^8 + 134134*a^8*b^11
*c^9*d^10*e^10*f^9 + 134134*a^8*b^11*c^10*d^9*e^9*f^10 - 216216*a^8*b^11*c^
11*d^8*e^8*f^11 + 66612*a^8*b^11*c^12*d^7*e^7*f^12 + 39858*a^8*b^11*c^13*d^
6*e^6*f^13 - 26838*a^8*b^11*c^14*d^5*e^5*f^14 + 924*a^8*b^11*c^15*d^4*e^4*f
^15 + 1560*a^8*b^11*c^16*d^3*e^3*f^16 - 27*a^8*b^11*c^17*d^2*e^2*f^17 - 371
*a^9*b^10*c^2*d^17*e^16*f^3 - 3464*a^9*b^10*c^3*d^16*e^15*f^4 + 12550*a^9*b
^10*c^4*d^15*e^14*f^5 + 18004*a^9*b^10*c^5*d^14*e^13*f^6 - 110474*a^9*b^10*
c^6*d^13*e^12*f^7 + 99736*a^9*b^10*c^7*d^12*e^11*f^8 + 134134*a^9*b^10*c^8*
d^11*e^10*f^9 - 300300*a^9*b^10*c^9*d^10*e^9*f^10 + 134134*a^9*b^10*c^10*d^
9*e^8*f^11 + 99736*a^9*b^10*c^11*d^8*e^7*f^12 - 110474*a^9*b^10*c^12*d^7*e^
6*f^13 + 18004*a^9*b^10*c^13*d^6*e^5*f^14 + 12550*a^9*b^10*c^14*d^5*e^4*f^1
5 - 3464*a^9*b^10*c^15*d^4*e^3*f^16 - 371*a^9*b^10*c^16*d^3*e^2*f^17 + 1560
*a^10*b^9*c^2*d^17*e^15*f^4 + 924*a^10*b^9*c^3*d^16*e^14*f^5 - 26838*a^10*b
^9*c^4*d^15*e^13*f^6 + 39858*a^10*b^9*c^5*d^14*e^12*f^7 + 66612*a^10*b^9*c^
6*d^13*e^11*f^8 - 216216*a^10*b^9*c^7*d^12*e^10*f^9 + 134134*a^10*b^9*c^8*d
^11*e^9*f^10 + 134134*a^10*b^9*c^9*d^10*e^8*f^11 - 216216*a^10*b^9*c^10*d^9
*e^7*f^12 + 66612*a^10*b^9*c^11*d^8*e^6*f^13 + 39858*a^10*b^9*c^12*d^7*e^5*
f^14 - 26838*a^10*b^9*c^13*d^6*e^4*f^15 + 924*a^10*b^9*c^14*d^5*e^3*f^16 +
1560*a^10*b^9*c^15*d^4*e^2*f^17 - 2484*a^11*b^8*c^2*d^17*e^14*f^5 + 7728*a^
11*b^8*c^3*d^16*e^13*f^6 + 15288*a^11*b^8*c^4*d^15*e^12*f^7 - 78624*a^11*b^
8*c^5*d^14*e^11*f^8 + 66612*a^11*b^8*c^6*d^13*e^10*f^9 + 99736*a^11*b^8*c^7
*d^12*e^9*f^10 - 216216*a^11*b^8*c^8*d^11*e^8*f^11 + 99736*a^11*b^8*c^9*d^1
0*e^7*f^12 + 66612*a^11*b^8*c^10*d^9*e^6*f^13 - 78624*a^11*b^8*c^11*d^8*e^5
*f^14 + 15288*a^11*b^8*c^12*d^7*e^4*f^15 + 7728*a^11*b^8*c^13*d^6*e^3*f^16
- 2484*a^11*b^8*c^14*d^5*e^2*f^17 + 1302*a^12*b^7*c^2*d^17*e^13*f^6 - 13104
*a^12*b^7*c^3*d^16*e^12*f^7 + 15288*a^12*b^7*c^4*d^15*e^11*f^8 + 39858*a^12
*b^7*c^5*d^14*e^10*f^9 - 110474*a^12*b^7*c^6*d^13*e^9*f^10 + 66612*a^12*b^7
*c^7*d^12*e^8*f^11 + 66612*a^12*b^7*c^8*d^11*e^7*f^12 - 110474*a^12*b^7*c^9
*d^10*e^6*f^13 + 39858*a^12*b^7*c^10*d^9*e^5*f^14 + 15288*a^12*b^7*c^11*d^8
*e^4*f^15 - 13104*a^12*b^7*c^12*d^7*e^3*f^16 + 1302*a^12*b^7*c^13*d^6*e^2*f
^17 + 1302*a^13*b^6*c^2*d^17*e^12*f^7 + 7728*a^13*b^6*c^3*d^16*e^11*f^8 - 2
6838*a^13*b^6*c^4*d^15*e^10*f^9 + 18004*a^13*b^6*c^5*d^14*e^9*f^10 + 39858*

a^13*b^6*c^6*d^13*e^8*f^11 - 78624*a^13*b^6*c^7*d^12*e^7*f^12 + 39858*a^13*
b^6*c^8*d^11*e^6*f^13 + 18004*a^13*b^6*c^9*d^10*e^5*f^14 - 26838*a^13*b^6*c
^10*d^9*e^4*f^15 + 7728*a^13*b^6*c^11*d^8*e^3*f^16 + 1302*a^13*b^6*c^12*d^7
*e^2*f^17 - 2484*a^14*b^5*c^2*d^17*e^11*f^8 + 924*a^14*b^5*c^3*d^16*e^10*f^
9 + 12550*a^14*b^5*c^4*d^15*e^9*f^10 - 26838*a^14*b^5*c^5*d^14*e^8*f^11 + 1
5288*a^14*b^5*c^6*d^13*e^7*f^12 + 15288*a^14*b^5*c^7*d^12*e^6*f^13 - 26838*
a^14*b^5*c^8*d^11*e^5*f^14 + 12550*a^14*b^5*c^9*d^10*e^4*f^15 + 924*a^14*b^
5*c^10*d^9*e^3*f^16 - 2484*a^14*b^5*c^11*d^8*e^2*f^17 + 1560*a^15*b^4*c^2*d
^17*e^10*f^9 - 3464*a^15*b^4*c^3*d^16*e^9*f^10 + 924*a^15*b^4*c^4*d^15*e^8*
f^11 + 7728*a^15*b^4*c^5*d^14*e^7*f^12 - 13104*a^15*b^4*c^6*d^13*e^6*f^13 +
 7728*a^15*b^4*c^7*d^12*e^5*f^14 + 924*a^15*b^4*c^8*d^11*e^4*f^15 - 3464*a^
15*b^4*c^9*d^10*e^3*f^16 + 1560*a^15*b^4*c^10*d^9*e^2*f^17 - 371*a^16*b^3*c
^2*d^17*e^9*f^10 + 1560*a^16*b^3*c^3*d^16*e^8*f^11 - 2484*a^16*b^3*c^4*d^15
*e^7*f^12 + 1302*a^16*b^3*c^5*d^14*e^6*f^13 + 1302*a^16*b^3*c^6*d^13*e^5*f^
14 - 2484*a^16*b^3*c^7*d^12*e^4*f^15 + 1560*a^16*b^3*c^8*d^11*e^3*f^16 - 37
1*a^16*b^3*c^9*d^10*e^2*f^17 - 27*a^17*b^2*c^2*d^17*e^8*f^11 - 168*a^17*b^2
*c^3*d^16*e^7*f^12 + 546*a^17*b^2*c^4*d^15*e^6*f^13 - 756*a^17*b^2*c^5*d^14
*e^5*f^14 + 546*a^17*b^2*c^6*d^13*e^4*f^15 - 168*a^17*b^2*c^7*d^12*e^3*f^16
 - 27*a^17*b^2*c^8*d^11*e^2*f^17)/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16
*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^
16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^
4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^
10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^1
6 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^1
6*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5
*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2
 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16
*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16
*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13
 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*
e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*
c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b
^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*
c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15
*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^
9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*
d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15
*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*
f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b
^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*
e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a
^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*
d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 -
224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c
^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2

+ 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6
*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f
^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^1
3*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*
e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a
^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^1
1*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 3
00*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^
14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d
^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^
4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 280
0*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12
*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8
*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^
6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*
a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*
c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9
*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^
6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480
*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^1
1*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^
11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*
f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11
396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b
^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14
*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12
*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15
400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9
*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*
e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 +
1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^
8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c
^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*
f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1
750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^
8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^1
2*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^
7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a
^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^1
1*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^1
2*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 +
7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a
^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c
^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*
e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5

+ 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264
*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5
*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7
*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^
14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750
*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4
*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8
*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14
+ 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a
^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3
*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*
e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11
+ 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f^14) - (x*(18*
a^9*b^10*c^17*d^2*f^19 - 74*a^10*b^9*c^16*d^3*f^19 + 184*a^11*b^8*c^15*d^4*
f^19 - 308*a^12*b^7*c^14*d^5*f^19 + 364*a^13*b^6*c^13*d^6*f^19 - 308*a^14*b
^5*c^12*d^7*f^19 + 184*a^15*b^4*c^11*d^8*f^19 - 74*a^16*b^3*c^10*d^9*f^19 +
18*a^17*b^2*c^9*d^10*f^19 + 18*a^9*b^10*d^19*e^17*f^2 - 74*a^10*b^9*d^19*e
^16*f^3 + 184*a^11*b^8*d^19*e^15*f^4 - 308*a^12*b^7*d^19*e^14*f^5 + 364*a^1
3*b^6*d^19*e^13*f^6 - 308*a^14*b^5*d^19*e^12*f^7 + 184*a^15*b^4*d^19*e^11*f
^8 - 74*a^16*b^3*d^19*e^10*f^9 + 18*a^17*b^2*d^19*e^9*f^10 + 18*b^19*c^9*d^
10*e^17*f^2 - 74*b^19*c^10*d^9*e^16*f^3 + 184*b^19*c^11*d^8*e^15*f^4 - 308*
b^19*c^12*d^7*e^14*f^5 + 364*b^19*c^13*d^6*e^13*f^6 - 308*b^19*c^14*d^5*e^1
2*f^7 + 184*b^19*c^15*d^4*e^11*f^8 - 74*b^19*c^16*d^3*e^10*f^9 + 18*b^19*c^
17*d^2*e^9*f^10 - 2*a^8*b^11*c^18*d*f^19 - 2*a^18*b*c^8*d^11*f^19 - 2*a^8*b
^11*d^19*e^18*f - 2*a^18*b*d^19*e^8*f^11 - 2*b^19*c^8*d^11*e^18*f - 2*b^19*
c^18*d*e^8*f^11 + 16*a*b^18*c^7*d^12*e^18*f + 16*a*b^18*c^18*d*e^7*f^12 + 1
6*a^7*b^12*c*d^18*e^18*f + 16*a^7*b^12*c^18*d*e*f^18 + 16*a^18*b*c*d^18*e^7
*f^12 + 16*a^18*b*c^7*d^12*e*f^18 - 126*a*b^18*c^8*d^11*e^17*f^2 + 434*a*b^
18*c^9*d^10*e^16*f^3 - 840*a*b^18*c^10*d^9*e^15*f^4 + 936*a*b^18*c^11*d^8*e
^14*f^5 - 420*a*b^18*c^12*d^7*e^13*f^6 - 420*a*b^18*c^13*d^6*e^12*f^7 + 936
*a*b^18*c^14*d^5*e^11*f^8 - 840*a*b^18*c^15*d^4*e^10*f^9 + 434*a*b^18*c^16*
d^3*e^9*f^10 - 126*a*b^18*c^17*d^2*e^8*f^11 - 56*a^2*b^17*c^6*d^13*e^18*f -
56*a^2*b^17*c^18*d*e^6*f^13 + 112*a^3*b^16*c^5*d^14*e^18*f + 112*a^3*b^16*
c^18*d*e^5*f^14 - 140*a^4*b^15*c^4*d^15*e^18*f - 140*a^4*b^15*c^18*d*e^4*f^
15 + 112*a^5*b^14*c^3*d^16*e^18*f + 112*a^5*b^14*c^18*d*e^3*f^16 - 56*a^6*b
^13*c^2*d^17*e^18*f - 56*a^6*b^13*c^18*d*e^2*f^17 - 126*a^8*b^11*c*d^18*e^1
7*f^2 - 126*a^8*b^11*c^17*d^2*e*f^18 + 434*a^9*b^10*c*d^18*e^16*f^3 + 434*a
^9*b^10*c^16*d^3*e*f^18 - 840*a^10*b^9*c*d^18*e^15*f^4 - 840*a^10*b^9*c^15*
d^4*e*f^18 + 936*a^11*b^8*c*d^18*e^14*f^5 + 936*a^11*b^8*c^14*d^5*e*f^18 -
420*a^12*b^7*c*d^18*e^13*f^6 - 420*a^12*b^7*c^13*d^6*e*f^18 - 420*a^13*b^6*
c*d^18*e^12*f^7 - 420*a^13*b^6*c^12*d^7*e*f^18 + 936*a^14*b^5*c*d^18*e^11*f
^8 + 936*a^14*b^5*c^11*d^8*e*f^18 - 840*a^15*b^4*c*d^18*e^10*f^9 - 840*a^15
*b^4*c^10*d^9*e*f^18 + 434*a^16*b^3*c*d^18*e^9*f^10 + 434*a^16*b^3*c^9*d^10
*e*f^18 - 126*a^17*b^2*c*d^18*e^8*f^11 - 126*a^17*b^2*c^8*d^11*e*f^18 - 56*
a^18*b*c^2*d^17*e^6*f^13 + 112*a^18*b*c^3*d^16*e^5*f^14 - 140*a^18*b*c^4*d^

```
15*e^4*f^15 + 112*a^18*b*c^5*d^14*e^3*f^16 - 56*a^18*b*c^6*d^13*e^2*f^17 +
360*a^2*b^17*c^7*d^12*e^17*f^2 - 882*a^2*b^17*c^8*d^11*e^16*f^3 + 728*a^2*b
^17*c^9*d^10*e^15*f^4 + 1152*a^2*b^17*c^10*d^9*e^14*f^5 - 4032*a^2*b^17*c^1
1*d^8*e^13*f^6 + 5460*a^2*b^17*c^12*d^7*e^12*f^7 - 4032*a^2*b^17*c^13*d^6*e
^11*f^8 + 1152*a^2*b^17*c^14*d^5*e^10*f^9 + 728*a^2*b^17*c^15*d^4*e^9*f^10
- 882*a^2*b^17*c^16*d^3*e^8*f^11 + 360*a^2*b^17*c^17*d^2*e^7*f^12 - 504*a^3
*b^16*c^6*d^13*e^17*f^2 + 312*a^3*b^16*c^7*d^12*e^16*f^3 + 2520*a^3*b^16*c^
8*d^11*e^15*f^4 - 7480*a^3*b^16*c^9*d^10*e^14*f^5 + 9408*a^3*b^16*c^10*d^9*
e^13*f^6 - 4368*a^3*b^16*c^11*d^8*e^12*f^7 - 4368*a^3*b^16*c^12*d^7*e^11*f^
8 + 9408*a^3*b^16*c^13*d^6*e^10*f^9 - 7480*a^3*b^16*c^14*d^5*e^9*f^10 + 252
0*a^3*b^16*c^15*d^4*e^8*f^11 + 312*a^3*b^16*c^16*d^3*e^7*f^12 - 504*a^3*b^1
6*c^17*d^2*e^6*f^13 + 252*a^4*b^15*c^5*d^14*e^17*f^2 + 1596*a^4*b^15*c^6*d^
13*e^16*f^3 - 6288*a^4*b^15*c^7*d^12*e^15*f^4 + 7380*a^4*b^15*c^8*d^11*e^14
*f^5 + 2660*a^4*b^15*c^9*d^10*e^13*f^6 - 18564*a^4*b^15*c^10*d^9*e^12*f^7 +
 26208*a^4*b^15*c^11*d^8*e^11*f^8 - 18564*a^4*b^15*c^12*d^7*e^10*f^9 + 2660
*a^4*b^15*c^13*d^6*e^9*f^10 + 7380*a^4*b^15*c^14*d^5*e^8*f^11 - 6288*a^4*b^
15*c^15*d^4*e^7*f^12 + 1596*a^4*b^15*c^16*d^3*e^6*f^13 + 252*a^4*b^15*c^17*
d^2*e^5*f^14 + 252*a^5*b^14*c^4*d^15*e^17*f^2 - 2772*a^5*b^14*c^5*d^14*e^16
*f^3 + 3696*a^5*b^14*c^6*d^13*e^15*f^4 + 7056*a^5*b^14*c^7*d^12*e^14*f^5 -
25452*a^5*b^14*c^8*d^11*e^13*f^6 + 30212*a^5*b^14*c^9*d^10*e^12*f^7 - 13104
*a^5*b^14*c^10*d^9*e^11*f^8 - 13104*a^5*b^14*c^11*d^8*e^10*f^9 + 30212*a^5*
b^14*c^12*d^7*e^9*f^10 - 25452*a^5*b^14*c^13*d^6*e^8*f^11 + 7056*a^5*b^14*c
^14*d^5*e^7*f^12 + 3696*a^5*b^14*c^15*d^4*e^6*f^13 - 2772*a^5*b^14*c^16*d^3
*e^5*f^14 + 252*a^5*b^14*c^17*d^2*e^4*f^15 - 504*a^6*b^13*c^3*d^16*e^17*f^2
 + 1596*a^6*b^13*c^4*d^15*e^16*f^3 + 3696*a^6*b^13*c^5*d^14*e^15*f^4 - 1747
2*a^6*b^13*c^6*d^13*e^14*f^5 + 17472*a^6*b^13*c^7*d^12*e^13*f^6 + 9828*a^6*
b^13*c^8*d^11*e^12*f^7 - 38584*a^6*b^13*c^9*d^10*e^11*f^8 + 48048*a^6*b^13*
c^10*d^9*e^10*f^9 - 38584*a^6*b^13*c^11*d^8*e^9*f^10 + 9828*a^6*b^13*c^12*d
^7*e^8*f^11 + 17472*a^6*b^13*c^13*d^6*e^7*f^12 - 17472*a^6*b^13*c^14*d^5*e^
6*f^13 + 3696*a^6*b^13*c^15*d^4*e^5*f^14 + 1596*a^6*b^13*c^16*d^3*e^4*f^15
- 504*a^6*b^13*c^17*d^2*e^3*f^16 + 360*a^7*b^12*c^2*d^17*e^17*f^2 + 312*a^7
*b^12*c^3*d^16*e^16*f^3 - 6288*a^7*b^12*c^4*d^15*e^15*f^4 + 7056*a^7*b^12*c
^5*d^14*e^14*f^5 + 17472*a^7*b^12*c^6*d^13*e^13*f^6 - 43680*a^7*b^12*c^7*d^
12*e^12*f^7 + 32760*a^7*b^12*c^8*d^11*e^11*f^8 - 8008*a^7*b^12*c^9*d^10*e^1
0*f^9 - 8008*a^7*b^12*c^10*d^9*e^9*f^10 + 32760*a^7*b^12*c^11*d^8*e^8*f^11
- 43680*a^7*b^12*c^12*d^7*e^7*f^12 + 17472*a^7*b^12*c^13*d^6*e^6*f^13 + 705
6*a^7*b^12*c^14*d^5*e^5*f^14 - 6288*a^7*b^12*c^15*d^4*e^4*f^15 + 312*a^7*b^
12*c^16*d^3*e^3*f^16 + 360*a^7*b^12*c^17*d^2*e^2*f^17 - 882*a^8*b^11*c^2*d^
17*e^16*f^3 + 2520*a^8*b^11*c^3*d^16*e^15*f^4 + 7380*a^8*b^11*c^4*d^15*e^14
*f^5 - 25452*a^8*b^11*c^5*d^14*e^13*f^6 + 9828*a^8*b^11*c^6*d^13*e^12*f^7 +
 32760*a^8*b^11*c^7*d^12*e^11*f^8 - 36036*a^8*b^11*c^8*d^11*e^10*f^9 + 2002
0*a^8*b^11*c^9*d^10*e^9*f^10 - 36036*a^8*b^11*c^10*d^9*e^8*f^11 + 32760*a^8
*b^11*c^11*d^8*e^7*f^12 + 9828*a^8*b^11*c^12*d^7*e^6*f^13 - 25452*a^8*b^11*
c^13*d^6*e^5*f^14 + 7380*a^8*b^11*c^14*d^5*e^4*f^15 + 2520*a^8*b^11*c^15*d^
4*e^3*f^16 - 882*a^8*b^11*c^16*d^3*e^2*f^17 + 728*a^9*b^10*c^2*d^17*e^15*f^
```

4 - 7480*a^9*b^10*c^3*d^16*e^14*f^5 + 2660*a^9*b^10*c^4*d^15*e^13*f^6 + 302
12*a^9*b^10*c^5*d^14*e^12*f^7 - 38584*a^9*b^10*c^6*d^13*e^11*f^8 - 8008*a^9
*b^10*c^7*d^12*e^10*f^9 + 20020*a^9*b^10*c^8*d^11*e^9*f^10 + 20020*a^9*b^10
*c^9*d^10*e^8*f^11 - 8008*a^9*b^10*c^10*d^9*e^7*f^12 - 38584*a^9*b^10*c^11*
d^8*e^6*f^13 + 30212*a^9*b^10*c^12*d^7*e^5*f^14 + 2660*a^9*b^10*c^13*d^6*e^
4*f^15 - 7480*a^9*b^10*c^14*d^5*e^3*f^16 + 728*a^9*b^10*c^15*d^4*e^2*f^17 +
1152*a^10*b^9*c^2*d^17*e^14*f^5 + 9408*a^10*b^9*c^3*d^16*e^13*f^6 - 18564*
a^10*b^9*c^4*d^15*e^12*f^7 - 13104*a^10*b^9*c^5*d^14*e^11*f^8 + 48048*a^10*
b^9*c^6*d^13*e^10*f^9 - 8008*a^10*b^9*c^7*d^12*e^9*f^10 - 36036*a^10*b^9*c^
8*d^11*e^8*f^11 - 8008*a^10*b^9*c^9*d^10*e^7*f^12 + 48048*a^10*b^9*c^10*d^9
*e^6*f^13 - 13104*a^10*b^9*c^11*d^8*e^5*f^14 - 18564*a^10*b^9*c^12*d^7*e^4*
f^15 + 9408*a^10*b^9*c^13*d^6*e^3*f^16 + 1152*a^10*b^9*c^14*d^5*e^2*f^17 -
4032*a^11*b^8*c^2*d^17*e^13*f^6 - 4368*a^11*b^8*c^3*d^16*e^12*f^7 + 26208*a
^11*b^8*c^4*d^15*e^11*f^8 - 13104*a^11*b^8*c^5*d^14*e^10*f^9 - 38584*a^11*b
^8*c^6*d^13*e^9*f^10 + 32760*a^11*b^8*c^7*d^12*e^8*f^11 + 32760*a^11*b^8*c^
8*d^11*e^7*f^12 - 38584*a^11*b^8*c^9*d^10*e^6*f^13 - 13104*a^11*b^8*c^10*d^
9*e^5*f^14 + 26208*a^11*b^8*c^11*d^8*e^4*f^15 - 4368*a^11*b^8*c^12*d^7*e^3*
f^16 - 4032*a^11*b^8*c^13*d^6*e^2*f^17 + 5460*a^12*b^7*c^2*d^17*e^12*f^7 -
4368*a^12*b^7*c^3*d^16*e^11*f^8 - 18564*a^12*b^7*c^4*d^15*e^10*f^9 + 30212*
a^12*b^7*c^5*d^14*e^9*f^10 + 9828*a^12*b^7*c^6*d^13*e^8*f^11 - 43680*a^12*b
^7*c^7*d^12*e^7*f^12 + 9828*a^12*b^7*c^8*d^11*e^6*f^13 + 30212*a^12*b^7*c^9
*d^10*e^5*f^14 - 18564*a^12*b^7*c^10*d^9*e^4*f^15 - 4368*a^12*b^7*c^11*d^8*
e^3*f^16 + 5460*a^12*b^7*c^12*d^7*e^2*f^17 - 4032*a^13*b^6*c^2*d^17*e^11*f^
8 + 9408*a^13*b^6*c^3*d^16*e^10*f^9 + 2660*a^13*b^6*c^4*d^15*e^9*f^10 - 254
52*a^13*b^6*c^5*d^14*e^8*f^11 + 17472*a^13*b^6*c^6*d^13*e^7*f^12 + 17472*a^
13*b^6*c^7*d^12*e^6*f^13 - 25452*a^13*b^6*c^8*d^11*e^5*f^14 + 2660*a^13*b^6
*c^9*d^10*e^4*f^15 + 9408*a^13*b^6*c^10*d^9*e^3*f^16 - 4032*a^13*b^6*c^11*d
^8*e^2*f^17 + 1152*a^14*b^5*c^2*d^17*e^10*f^9 - 7480*a^14*b^5*c^3*d^16*e^9*
f^10 + 7380*a^14*b^5*c^4*d^15*e^8*f^11 + 7056*a^14*b^5*c^5*d^14*e^7*f^12 -
17472*a^14*b^5*c^6*d^13*e^6*f^13 + 7056*a^14*b^5*c^7*d^12*e^5*f^14 + 7380*a
^14*b^5*c^8*d^11*e^4*f^15 - 7480*a^14*b^5*c^9*d^10*e^3*f^16 + 1152*a^14*b^5
*c^10*d^9*e^2*f^17 + 728*a^15*b^4*c^2*d^17*e^9*f^10 + 2520*a^15*b^4*c^3*d^1
6*e^8*f^11 - 6288*a^15*b^4*c^4*d^15*e^7*f^12 + 3696*a^15*b^4*c^5*d^14*e^6*f
^13 + 3696*a^15*b^4*c^6*d^13*e^5*f^14 - 6288*a^15*b^4*c^7*d^12*e^4*f^15 + 2
520*a^15*b^4*c^8*d^11*e^3*f^16 + 728*a^15*b^4*c^9*d^10*e^2*f^17 - 882*a^16*
b^3*c^2*d^17*e^8*f^11 + 312*a^16*b^3*c^3*d^16*e^7*f^12 + 1596*a^16*b^3*c^4*
d^15*e^6*f^13 - 2772*a^16*b^3*c^5*d^14*e^5*f^14 + 1596*a^16*b^3*c^6*d^13*e^
4*f^15 + 312*a^16*b^3*c^7*d^12*e^3*f^16 - 882*a^16*b^3*c^8*d^11*e^2*f^17 +
360*a^17*b^2*c^2*d^17*e^7*f^12 - 504*a^17*b^2*c^3*d^16*e^6*f^13 + 252*a^17*
b^2*c^4*d^15*e^5*f^14 + 252*a^17*b^2*c^5*d^14*e^4*f^15 - 504*a^17*b^2*c^6*d
^13*e^3*f^16 + 360*a^17*b^2*c^7*d^12*e^2*f^17))/(56*a^3*b^13*c^5*d^11*e^16
- a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8
*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 -
70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^1
4*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^

4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 2
8*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4
*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6
*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56
*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*
f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^12 + 56*a^16*c^5*
d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^6*e^14*f^2 + 56*
b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^16*c^13*d^3*e^11*
f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8*a^7*b^9*c*d^15*
e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*b^15*c^16*e^7*f^
9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*b*d^16*e^9*f^7 +
 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9*d^7*e^15*f + 8*
b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*c^15*d*e^8*f^8 -
 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a^15*b*c*d^15*e^8
*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f^2 - 224*a*b^15*
c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^15*c^12*d^4*e^11
*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e^9*f^7 + 160*a^2
*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^3*b^13*c^6*d^10*
e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^11*e^15*f + 112*
a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 112*a^5*b^11*c^15
*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c^15*d*e^3*f^13 +
 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14 + 160*a^9*b^7*c*
d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*c*d^15*e^13*f^3
- 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f^4 + 112*a^11*b^
5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12*b^4*c^11*d^5*e*
f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6*e*f^15 + 160*a^
14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a^15*b*c^2*d^14*e
^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12*e^5*f^11 + 112*
a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 160*a^15*b*c^7*d^
9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c^10*d^6*e^12*f^4
 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*e^10*f^6 - 300*a
^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 - 2800*a^3*b^13*c
^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^3*b^13*c^11*d^5*
e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c^13*d^3*e^8*f^8
+ 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^13*f^3 + 1750*a^
4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 - 8624*a^4*b^12*c^
10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4*b^12*c^12*d^4*e
^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14*d^2*e^6*f^10 -
1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^13*f^3 + 4480*a^
5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 + 7392*a^5*b^11*c
^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^5*b^11*c^11*d^5*
e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^13*d^3*e^6*f^10
- 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e^14*f^2 + 1568*a
^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4 + 7392*a^6*b^10
*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640*a^6*b^10*c^9*d^

7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10*c^11*d^5*e^7*f^
9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^3*e^5*f^11 + 840
*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^3 + 4480*a^7*b^9
*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640*a^7*b^9*c^7*d^9
*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 -
 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6*f^10 + 4480*a^7
*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 300*a^8*b^8*c^2*d
^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^8*c^4*d^12*e^12*
f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d^10*e^10*f^6 + 1
5400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^
8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264*a^8*b^8*c^11*d^
5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*c^13*d^3*e^3*f^1
3 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e^12*f^4 + 4480*a
^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 - 24640*a^9*b^7*c
^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7
*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d^6*e^5*f^11 + 44
80*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^13 + 840*a^10*b^
6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 8624*a^10*b^6*c^4*d
^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*b^6*c^6*d^10*e^8
*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*d^8*e^6*f^10 + 7
392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*f^12 + 1568*a^10
*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1344*a^11*b^5*c^2
*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^11*b^5*c^4*d^12*e
^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 +
 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^
11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c
^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*
e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 -
 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^
12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^1
0*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^
7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 -
 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14
*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*
d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f
^14)) + (x*(84*a^5*b^11*c^13*d^3*f^16 - 12*a^4*b^12*c^14*d^2*f^16 - 243*a^6
*b^10*c^12*d^4*f^16 + 366*a^7*b^9*c^11*d^5*f^16 - 321*a^8*b^8*c^10*d^6*f^16
 + 252*a^9*b^7*c^9*d^7*f^16 - 321*a^10*b^6*c^8*d^8*f^16 + 366*a^11*b^5*c^7*
d^9*f^16 - 243*a^12*b^4*c^6*d^10*f^16 + 84*a^13*b^3*c^5*d^11*f^16 - 12*a^14
*b^2*c^4*d^12*f^16 - 12*a^4*b^12*d^16*e^14*f^2 + 84*a^5*b^11*d^16*e^13*f^3
- 243*a^6*b^10*d^16*e^12*f^4 + 366*a^7*b^9*d^16*e^11*f^5 - 321*a^8*b^8*d^16
*e^10*f^6 + 252*a^9*b^7*d^16*e^9*f^7 - 321*a^10*b^6*d^16*e^8*f^8 + 366*a^11
*b^5*d^16*e^7*f^9 - 243*a^12*b^4*d^16*e^6*f^10 + 84*a^13*b^3*d^16*e^5*f^11
- 12*a^14*b^2*d^16*e^4*f^12 - 12*b^16*c^4*d^12*e^14*f^2 + 84*b^16*c^5*d^11*
e^13*f^3 - 243*b^16*c^6*d^10*e^12*f^4 + 366*b^16*c^7*d^9*e^11*f^5 - 321*b^1

6*c^8*d^8*e^10*f^6 + 252*b^16*c^9*d^7*e^9*f^7 - 321*b^16*c^10*d^6*e^8*f^8 +
366*b^16*c^11*d^5*e^7*f^9 - 243*b^16*c^12*d^4*e^6*f^10 + 84*b^16*c^13*d^3*
e^5*f^11 - 12*b^16*c^14*d^2*e^4*f^12 + 48*a*b^15*c^3*d^13*e^14*f^2 - 252*a*
b^15*c^4*d^12*e^13*f^3 + 366*a*b^15*c^5*d^11*e^12*f^4 + 354*a*b^15*c^6*d^10
*e^11*f^5 - 1458*a*b^15*c^7*d^9*e^10*f^6 + 942*a*b^15*c^8*d^8*e^9*f^7 + 942
*a*b^15*c^9*d^7*e^8*f^8 - 1458*a*b^15*c^10*d^6*e^7*f^9 + 354*a*b^15*c^11*d^
5*e^6*f^10 + 366*a*b^15*c^12*d^4*e^5*f^11 - 252*a*b^15*c^13*d^3*e^4*f^12 +
48*a*b^15*c^14*d^2*e^3*f^13 + 48*a^3*b^13*c*d^15*e^14*f^2 + 48*a^3*b^13*c^1
4*d^2*e*f^15 - 252*a^4*b^12*c*d^15*e^13*f^3 - 252*a^4*b^12*c^13*d^3*e*f^15
+ 366*a^5*b^11*c*d^15*e^12*f^4 + 366*a^5*b^11*c^12*d^4*e*f^15 + 354*a^6*b^1
0*c*d^15*e^11*f^5 + 354*a^6*b^10*c^11*d^5*e*f^15 - 1458*a^7*b^9*c*d^15*e^10
*f^6 - 1458*a^7*b^9*c^10*d^6*e*f^15 + 942*a^8*b^8*c*d^15*e^9*f^7 + 942*a^8*
b^8*c^9*d^7*e*f^15 + 942*a^9*b^7*c*d^15*e^8*f^8 + 942*a^9*b^7*c^8*d^8*e*f^1
5 - 1458*a^10*b^6*c*d^15*e^7*f^9 - 1458*a^10*b^6*c^7*d^9*e*f^15 + 354*a^11*
b^5*c*d^15*e^6*f^10 + 354*a^11*b^5*c^6*d^10*e*f^15 + 366*a^12*b^4*c*d^15*e^
5*f^11 + 366*a^12*b^4*c^5*d^11*e*f^15 - 252*a^13*b^3*c*d^15*e^4*f^12 - 252*
a^13*b^3*c^4*d^12*e*f^15 + 48*a^14*b^2*c*d^15*e^3*f^13 + 48*a^14*b^2*c^3*d^
13*e*f^15 - 72*a^2*b^14*c^2*d^14*e^14*f^2 + 168*a^2*b^14*c^3*d^13*e^13*f^3
+ 723*a^2*b^14*c^4*d^12*e^12*f^4 - 3258*a^2*b^14*c^5*d^11*e^11*f^5 + 3156*a
^2*b^14*c^6*d^10*e^10*f^6 + 3522*a^2*b^14*c^7*d^9*e^9*f^7 - 8478*a^2*b^14*c
^8*d^8*e^8*f^8 + 3522*a^2*b^14*c^9*d^7*e^7*f^9 + 3156*a^2*b^14*c^10*d^6*e^6
*f^10 - 3258*a^2*b^14*c^11*d^5*e^5*f^11 + 723*a^2*b^14*c^12*d^4*e^4*f^12 +
168*a^2*b^14*c^13*d^3*e^3*f^13 - 72*a^2*b^14*c^14*d^2*e^2*f^14 + 168*a^3*b^
13*c^2*d^14*e^13*f^3 - 1692*a^3*b^13*c^3*d^13*e^12*f^4 + 2538*a^3*b^13*c^4*
d^12*e^11*f^5 + 5634*a^3*b^13*c^5*d^11*e^10*f^6 - 18738*a^3*b^13*c^6*d^10*e
^9*f^7 + 12042*a^3*b^13*c^7*d^9*e^8*f^8 + 12042*a^3*b^13*c^8*d^8*e^7*f^9 -
18738*a^3*b^13*c^9*d^7*e^6*f^10 + 5634*a^3*b^13*c^10*d^6*e^5*f^11 + 2538*a^
3*b^13*c^11*d^5*e^4*f^12 - 1692*a^3*b^13*c^12*d^4*e^3*f^13 + 168*a^3*b^13*c
^13*d^3*e^2*f^14 + 723*a^4*b^12*c^2*d^14*e^12*f^4 + 2538*a^4*b^12*c^3*d^13*
e^11*f^5 - 14022*a^4*b^12*c^4*d^12*e^10*f^6 + 14022*a^4*b^12*c^5*d^11*e^9*f
^7 + 21087*a^4*b^12*c^6*d^10*e^8*f^8 - 48168*a^4*b^12*c^7*d^9*e^7*f^9 + 210
87*a^4*b^12*c^8*d^8*e^6*f^10 + 14022*a^4*b^12*c^9*d^7*e^5*f^11 - 14022*a^4*
b^12*c^10*d^6*e^4*f^12 + 2538*a^4*b^12*c^11*d^5*e^3*f^13 + 723*a^4*b^12*c^1
2*d^4*e^2*f^14 - 3258*a^5*b^11*c^2*d^14*e^11*f^5 + 5634*a^5*b^11*c^3*d^13*e
^10*f^6 + 14022*a^5*b^11*c^4*d^12*e^9*f^7 - 50544*a^5*b^11*c^5*d^11*e^8*f^8
+ 33696*a^5*b^11*c^6*d^10*e^7*f^9 + 33696*a^5*b^11*c^7*d^9*e^6*f^10 - 5054
4*a^5*b^11*c^8*d^8*e^5*f^11 + 14022*a^5*b^11*c^9*d^7*e^4*f^12 + 5634*a^5*b^
11*c^10*d^6*e^3*f^13 - 3258*a^5*b^11*c^11*d^5*e^2*f^14 + 3156*a^6*b^10*c^2*
d^14*e^10*f^6 - 18738*a^6*b^10*c^3*d^13*e^9*f^7 + 21087*a^6*b^10*c^4*d^12*e
^8*f^8 + 33696*a^6*b^10*c^5*d^11*e^7*f^9 - 78624*a^6*b^10*c^6*d^10*e^6*f^10
+ 33696*a^6*b^10*c^7*d^9*e^5*f^11 + 21087*a^6*b^10*c^8*d^8*e^4*f^12 - 1873
8*a^6*b^10*c^9*d^7*e^3*f^13 + 3156*a^6*b^10*c^10*d^6*e^2*f^14 + 3522*a^7*b^
9*c^2*d^14*e^9*f^7 + 12042*a^7*b^9*c^3*d^13*e^8*f^8 - 48168*a^7*b^9*c^4*d^1
2*e^7*f^9 + 33696*a^7*b^9*c^5*d^11*e^6*f^10 + 33696*a^7*b^9*c^6*d^10*e^5*f^
11 - 48168*a^7*b^9*c^7*d^9*e^4*f^12 + 12042*a^7*b^9*c^8*d^8*e^3*f^13 + 3522

*a^7*b^9*c^9*d^7*e^2*f^14 - 8478*a^8*b^8*c^2*d^14*e^8*f^8 + 12042*a^8*b^8*c^3*d^13*e^7*f^9 + 21087*a^8*b^8*c^4*d^12*e^6*f^10 - 50544*a^8*b^8*c^5*d^11*e^5*f^11 + 21087*a^8*b^8*c^6*d^10*e^4*f^12 + 12042*a^8*b^8*c^7*d^9*e^3*f^13 - 8478*a^8*b^8*c^8*d^8*e^2*f^14 + 3522*a^9*b^7*c^2*d^14*e^7*f^9 - 18738*a^9*b^7*c^3*d^13*e^6*f^10 + 14022*a^9*b^7*c^4*d^12*e^5*f^11 + 14022*a^9*b^7*c^5*d^11*e^4*f^12 - 18738*a^9*b^7*c^6*d^10*e^3*f^13 + 3522*a^9*b^7*c^7*d^9*e^2*f^14 + 3156*a^10*b^6*c^2*d^14*e^6*f^10 + 5634*a^10*b^6*c^3*d^13*e^5*f^11 - 14022*a^10*b^6*c^4*d^12*e^4*f^12 + 5634*a^10*b^6*c^5*d^11*e^3*f^13 + 3156*a^10*b^6*c^6*d^10*e^2*f^14 - 3258*a^11*b^5*c^2*d^14*e^5*f^11 + 2538*a^11*b^5*c^3*d^13*e^4*f^12 + 2538*a^11*b^5*c^4*d^12*e^3*f^13 - 3258*a^11*b^5*c^5*d^11*e^2*f^14 + 723*a^12*b^4*c^2*d^14*e^4*f^12 - 1692*a^12*b^4*c^3*d^13*e^3*f^13 + 723*a^12*b^4*c^4*d^12*e^2*f^14 + 168*a^13*b^3*c^2*d^14*e^3*f^13 + 168*a^13*b^3*c^3*d^13*e^2*f^14 - 72*a^14*b^2*c^2*d^14*e^2*f^14))/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^1

2*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c

^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f^14)) - (36*a^11*b^2*d^13*f^13 + 36*b^13*c^11*d^2*f^13 + 36*b^13*d^13*e^11*f^2 + 297*a^2*b^11*c^9*d^4*f^13 - 108*a^3*b^10*c^8*d^5*f^13 - 198*a^4*b^9*c^7*d^6*f^13 + 153*a^5*b^8*c^6*d^7*f^13 + 153*a^6*b^7*c^5*d^8*f^13 - 198*a^7*b^6*c^4*d^9*f^13 - 108*a^8*b^5*c^3*d^10*f^13 + 297*a^9*b^4*c^2*d^11*f^13 + 297*a^2*b^11*d^13*e^9*f^4 - 108*a^3*b^10*d^13*e^8*f^5 - 198*a^4*b^9*d^13*e^7*f^6 + 153*a^5*b^8*d^13*e^6*f^7 + 153*a^6*b^7*d^13*e^5*f^8 - 198*a^7*b^6*d^13*e^4*f^9 - 108*a^8*b^5*d^13*e^3*f^10 + 297*a^9*b^4*d^13*e^2*f^11 + 297*b^13*c^2*d^11*e^9*f^4 - 108*b^13*c^3*d^10*e^8*f^5 - 198*b^13*c^4*d^9*e^7*f^6 + 153*b^13*c^5*d^8*e^6*f^7 + 153*b^13*c^6*d^7*e^5*f^8 - 198*b^13*c^7*d^6*e^4*f^9 - 108*b^13*c^8*d^5*e^3*f^10 + 297*b^13*c^9*d^4*e^2*f^11 - 180*a*b^12*c^10*d^3*f^13 - 180*a^10*b^3*c*d^12*f^13 - 180*a*b^12*d^13*e^10*f^3 - 180*a^10*b^3*d^13*e*f^12 - 180*b^13*c*d^12*e^10*f^3 - 180*b^13*c^10*d^3*e*f^12 + 1026*a*b^12*c*d^12*e^9*f^4 + 1026*a*b^12*c^9*d^4*e*f^12 + 1026*a^9*b^4*c*d^12*e*f^12 - 2052*a*b^12*c^2*d^11*e^8*f^5 + 1548*a*b^12*c^3*d^10*e^7*f^6 + 297*a*b^12*c^4*d^9*e^6*f^7 - 1242*a*b^12*c^5*d^8*e^5*f^8 + 297*a*b^12*c^6*d^7*e^4*f^9 + 1548*a*b^12*c^7*d^6*e^3*f^10 - 2052*a*b^12*c^8*d^5*e^2*f^11 - 2052*a^2*b^11*c*d^12*e^8*f^5 - 2052*a^2*b^11*c^8*d^5*e*f^12 + 1548*a^3*b^10*c*d^12*e^7*f^6 + 1548*a^3*b^10*c^7*d^6*e*f^12 + 297*a^4*b^9*c*d^12*e^6*f^7 + 297*a^4*b^9*c^6*d^7*e*f^12 - 1242*a^5*b^8*c*d^12*e^5*f^8 - 1242*a^5*b^8*c^5*d^8*e*f^12 + 297*a^6*b^7*c*d^12*e^4*f^9 + 297*a^6*b^7*c^4*d^9*e*f^12 + 1548*a^7*b^6*c*d^12*e^3*f^10 + 1548*a^7*b^6*c^3*d^10*e*f^12 - 2052*a^8*b^5*c*d^12*e^2*f^11 - 2052*a^8*b^5*c^2*d^11*e*f^12 + 4860*a^2*b^11*c^2*d^11*e^7*f^6 - 4986*a^2*b^11*c^3*d^10*e^6*f^7 + 1701*a^2*b^11*c^4*d^9*e^5*f^8 + 1701*a^2*b^11*c^5*d^8*e^4*f^9 - 4986*a^2*b^11*c^6*d^7*e^3*f^10 + 4860*a^2*b^11*c^7*d^6*e^2*f^11 - 4986*a^3*b^10*c^2*d^11*e^6*f^7 + 6336*a^3*b^10*c^3*d^10*e^5*f^8 - 3960*a^3*b^10*c^4*d^9*e^4*f^9 + 6336*a^3*b^10*c^5*d^8*e^3*f^10 - 4986*a^3*b^10*c^6*d^7*e^2*f^11 + 1701*a^4*b^9*c^2*d^11*e^5*f^8 - 3960*a^4*b^9*c^3*d^10*e^4*f^9 - 3960*a^4*b^9*c^4*d^9*e^3*f^10 + 1701*a^4*b^9*c^5*d^8*e^2*f^11 + 1701*a^5*b^8*c^2*d^11*e^4*f^9 + 6336*a^5*b^8*c^3*d^10*e^3*f^10 + 1701*a^5*b^8*c^4*d^9*e^2*f^11 - 4986*a^6*b^7*c^2*d^11*e^3*f^10 - 4986*a^6*b^7*c^3*d^10*e^2*f^11 + 4860*a^7*b^6*c^2*d^11*e^2*f^11)/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10

*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70
*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10
*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*
d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*
b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*
f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*
d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7
*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15
*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15
+ 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f
- 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e
*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^
9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^
4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^1
5*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*
f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4
*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^1
2*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 22
4*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15
*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 2
24*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c
*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^
5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*
b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*
f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^1
5*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*
e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 84
0*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^
14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^
8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f
^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 140
0*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^1
2*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7
*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^
7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a
^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11
*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^
8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^
7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568
*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^1
0*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d
^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10
*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7
392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*
b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*

d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11
*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 154
00*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*
c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*
e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 +
 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a
^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^
8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*
f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1
400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^
7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^1
1*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8
 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^
9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^1
2*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^
11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 +
 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a
^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c
^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*
e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^
6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392
*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^
5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^
6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f
^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480
*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^
4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7
*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8
 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*
a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3
*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e
^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12
 - 300*a^14*b^2*c^8*d^8*e^2*f^14) + (x*(108*a^3*b^10*c^7*d^6*f^13 - 36*b^13*
c^10*d^3*f^13 - 36*b^13*d^13*e^10*f^3 - 297*a^2*b^11*c^8*d^5*f^13 - 36*a^10
*b^3*d^13*f^13 + 324*a^4*b^9*c^6*d^7*f^13 - 594*a^5*b^8*c^5*d^8*f^13 + 324*
a^6*b^7*c^4*d^9*f^13 + 108*a^7*b^6*c^3*d^10*f^13 - 297*a^8*b^5*c^2*d^11*f^1
3 - 297*a^2*b^11*d^13*e^8*f^5 + 108*a^3*b^10*d^13*e^7*f^6 + 324*a^4*b^9*d^1
3*e^6*f^7 - 594*a^5*b^8*d^13*e^5*f^8 + 324*a^6*b^7*d^13*e^4*f^9 + 108*a^7*b
^6*d^13*e^3*f^10 - 297*a^8*b^5*d^13*e^2*f^11 - 297*b^13*c^2*d^11*e^8*f^5 +
108*b^13*c^3*d^10*e^7*f^6 + 324*b^13*c^4*d^9*e^6*f^7 - 594*b^13*c^5*d^8*e^5
*f^8 + 324*b^13*c^6*d^7*e^4*f^9 + 108*b^13*c^7*d^6*e^3*f^10 - 297*b^13*c^8*
d^5*e^2*f^11 + 180*a*b^12*c^9*d^4*f^13 + 180*a^9*b^4*c*d^12*f^13 + 180*a*b^
12*d^13*e^9*f^4 + 180*a^9*b^4*d^13*e*f^12 + 180*b^13*c*d^12*e^9*f^4 + 180*b
^13*c^9*d^4*e*f^12 - 1026*a*b^12*c*d^12*e^8*f^5 - 1026*a*b^12*c^8*d^5*e*f^1
2 - 1026*a^8*b^5*c*d^12*e*f^12 + 2052*a*b^12*c^2*d^11*e^7*f^6 - 2052*a*b^12

*c^3*d^10*e^6*f^7 + 1026*a*b^12*c^4*d^9*e^5*f^8 + 1026*a*b^12*c^5*d^8*e^4*f^9 - 2052*a*b^12*c^6*d^7*e^3*f^10 + 2052*a*b^12*c^7*d^6*e^2*f^11 + 2052*a^2*b^11*c*d^12*e^7*f^6 + 2052*a^2*b^11*c^7*d^6*e*f^12 - 2052*a^3*b^10*c*d^12*e^6*f^7 - 2052*a^3*b^10*c^6*d^7*e*f^12 + 1026*a^4*b^9*c*d^12*e^5*f^8 + 1026*a^4*b^9*c^5*d^8*e*f^12 + 1026*a^5*b^8*c*d^12*e^4*f^9 + 1026*a^5*b^8*c^4*d^9*e*f^12 - 2052*a^6*b^7*c*d^12*e^3*f^10 - 2052*a^6*b^7*c^3*d^10*e*f^12 + 2052*a^7*b^6*c*d^12*e^2*f^11 + 2052*a^7*b^6*c^2*d^11*e*f^12 - 4104*a^2*b^11*c^2*d^11*e^6*f^7 + 4104*a^2*b^11*c^3*d^10*e^5*f^8 - 5130*a^2*b^11*c^4*d^9*e^4*f^9 + 4104*a^2*b^11*c^5*d^8*e^3*f^10 - 4104*a^2*b^11*c^6*d^7*e^2*f^11 + 4104*a^3*b^10*c^2*d^11*e^5*f^8 + 4104*a^3*b^10*c^5*d^8*e^2*f^11 - 5130*a^4*b^9*c^2*d^11*e^4*f^9 - 5130*a^4*b^9*c^4*d^9*e^2*f^11 + 4104*a^5*b^8*c^2*d^11*e^3*f^10 + 4104*a^5*b^8*c^3*d^10*e^2*f^11 - 4104*a^6*b^7*c^2*d^11*e^2*f^11))/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^1

0*e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 +
840*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*
b^14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*
d^8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11
*f^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1
400*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b
^12*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d
^7*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*
f^7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840
*a^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^
11*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*
d^8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*
f^7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 15
68*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b
^10*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6
*d^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^
10*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 +
 7392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^
6*b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^
4*d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^
11*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 1
5400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^
9*c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^
3*e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3
 + 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396
*a^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*
c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^
6*f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 +
 1400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*
b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d
^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f
^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*
a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c
^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*
e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7
 + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396
*a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6
*c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^
4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*
f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 73
92*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*
b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*
d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10
*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 44
80*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*

b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d
^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f
^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 156
8*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b
^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12
*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^1
2 - 300*a^14*b^2*c^8*d^8*e^2*f^14))*root(756756*a^10*b^10*c^10*d^10*e^10*f^
10*z^3 + 573300*a^12*b^8*c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e
^8*f^12*z^3 + 573300*a^11*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*
d^8*e^9*f^11*z^3 + 573300*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*
c^11*d^9*e^11*f^9*z^3 - 343980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^1
1*b^9*c^9*d^11*e^10*f^10*z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343
980*a^10*b^10*c^9*d^11*e^11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^
3 - 343980*a^9*b^11*c^10*d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*
f^13*z^3 + 326340*a^13*b^7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d
^7*e^7*f^13*z^3 + 326340*a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*
c^13*d^7*e^10*f^10*z^3 + 326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^
12*b^8*c^10*d^10*e^8*f^12*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 26
7540*a^10*b^10*c^12*d^8*e^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z
^3 - 267540*a^8*b^12*c^12*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^1
2*f^8*z^3 + 245700*a^14*b^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^
8*e^6*f^14*z^3 + 245700*a^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^
14*d^6*e^8*f^12*z^3 + 245700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^
14*c^12*d^8*e^12*f^8*z^3 - 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a
^13*b^7*c^8*d^12*e^9*f^11*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191
100*a^12*b^8*c^7*d^13*e^11*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3
- 191100*a^11*b^9*c^7*d^13*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12
*z^3 - 191100*a^9*b^11*c^8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9
*f^11*z^3 - 191100*a^8*b^12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^
8*e^11*f^9*z^3 - 191100*a^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^
9*d^11*e^7*f^13*z^3 - 123900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b
^7*c^11*d^9*e^6*f^14*z^3 - 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a
^11*b^9*c^13*d^7*e^6*f^14*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123
900*a^9*b^11*c^14*d^6*e^7*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3
- 123900*a^7*b^13*c^14*d^6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6
*z^3 - 123900*a^6*b^14*c^13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^1
3*f^7*z^3 + 101700*a^15*b^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^1
4*e^9*f^11*z^3 + 101700*a^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^
5*d^15*e^11*f^9*z^3 + 101700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b
^9*c^5*d^15*e^14*f^6*z^3 + 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a
^9*b^11*c^6*d^14*e^15*f^5*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101
700*a^6*b^14*c^9*d^11*e^15*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3
+ 101700*a^5*b^15*c^11*d^9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14
*z^3 - 65820*a^14*b^6*c^6*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6
*f^14*z^3 - 65820*a^10*b^10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6

*e^10*f^10*z^3 - 65820*a^6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7
*d^13*e^7*f^13*z^3 - 56700*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*
c^7*d^13*e^8*f^12*z^3 + 56700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b
^7*c^12*d^8*e^5*f^15*z^3 - 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^1
3*b^7*c^4*d^16*e^13*f^7*z^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*
a^12*b^8*c^5*d^15*e^13*f^7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 567
00*a^8*b^12*c^7*d^13*e^15*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 -
56700*a^7*b^13*c^15*d^5*e^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3
 + 56700*a^7*b^13*c^7*d^13*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*
z^3 - 56700*a^5*b^15*c^12*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f
^7*z^3 - 48252*a^15*b^5*c^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^
10*f^10*z^3 - 48252*a^10*b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d
^15*e^15*f^5*z^3 - 48252*a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c
^10*d^10*e^15*f^5*z^3 - 32400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b
^4*c^6*d^14*e^8*f^12*z^3 - 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^1
4*b^6*c^4*d^16*e^12*f^8*z^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*
a^12*b^8*c^4*d^16*e^14*f^6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 324
00*a^8*b^12*c^6*d^14*e^16*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 -
32400*a^6*b^14*c^8*d^12*e^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3
 - 32400*a^4*b^16*c^12*d^8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16
*z^3 + 20565*a^16*b^4*c^4*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4
*f^16*z^3 + 20565*a^10*b^10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4
*e^10*f^10*z^3 + 20565*a^4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8
*d^12*e^5*f^15*z^3 + 15660*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*
c^12*d^8*e^3*f^17*z^3 + 15660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b
^8*c^15*d^5*e^3*f^17*z^3 + 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8
*b^12*c^17*d^3*e^5*f^15*z^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*
a^5*b^15*c^17*d^3*e^8*f^12*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 156
60*a^3*b^17*c^15*d^5*e^12*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 -
9750*a^17*b^3*c^9*d^11*e^4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 -
 9750*a^16*b^4*c^11*d^9*e^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3
- 9750*a^11*b^9*c^16*d^4*e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3
 - 9750*a^9*b^11*c^17*d^3*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^
3 - 9750*a^4*b^16*c^17*d^3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z
^3 - 9750*a^3*b^17*c^16*d^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*
z^3 - 8100*a^17*b^3*c^7*d^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13
*z^3 - 8100*a^14*b^6*c^13*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^
7*z^3 - 8100*a^13*b^7*c^14*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f
^6*z^3 - 8100*a^7*b^13*c^17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*
f^3*z^3 - 8100*a^6*b^14*c^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17
*f^3*z^3 - 8100*a^3*b^17*c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^1
4*f^6*z^3 - 7980*a^16*b^4*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^
9*f^11*z^3 - 7980*a^15*b^5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e
^11*f^9*z^3 - 7980*a^11*b^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*
e^15*f^5*z^3 - 7980*a^9*b^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15

*e^16*f^4*z^3 - 7980*a^5*b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^1
1*e^16*f^4*z^3 - 7980*a^4*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d
^9*e^15*f^5*z^3 + 6300*a^18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*
d^6*e^2*f^18*z^3 + 6300*a^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18
*d^2*e^6*f^14*z^3 + 6300*a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^1
4*d^6*e^14*f^6*z^3 - 4260*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^
5*d^15*e^7*f^13*z^3 - 4260*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c
^2*d^18*e^13*f^7*z^3 - 4260*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*
c^2*d^18*e^15*f^5*z^3 - 4260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13
*c^5*d^15*e^18*f^2*z^3 - 4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^1
5*c^7*d^13*e^18*f^2*z^3 - 4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^
18*c^13*d^7*e^15*f^5*z^3 + 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17
*b^3*c^3*d^17*e^10*f^10*z^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a
^10*b^10*c^3*d^17*e^17*f^3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 147
0*a^3*b^17*c^10*d^10*e^17*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1
350*a^18*b^2*c^3*d^17*e^9*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 +
1350*a^17*b^3*c^2*d^18*e^11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 +
 1350*a^11*b^9*c^2*d^18*e^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3
+ 1350*a^9*b^11*c^3*d^17*e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3
 + 1350*a^3*b^17*c^9*d^11*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^
3 + 1350*a^2*b^18*c^11*d^9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*
z^3 - 1070*a^18*b^2*c^2*d^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^
18*z^3 - 1070*a^10*b^10*c^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10
*f^10*z^3 - 1070*a^2*b^18*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^
4*f^16*z^3 + 525*a^18*b^2*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2
*f^18*z^3 + 525*a^16*b^4*c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*
f^18*z^3 + 525*a^12*b^8*c^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f
^16*z^3 + 525*a^8*b^12*c^4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^
12*z^3 + 525*a^4*b^16*c^8*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^
8*z^3 + 525*a^2*b^18*c^12*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z
^3 + 900*a^19*b*c^4*d^16*e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 +
900*a^16*b^4*c*d^19*e^13*f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a
^13*b^7*c*d^19*e^16*f^4*z^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^
13*c^4*d^16*e^19*f*z^3 + 900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^
7*d^13*e^19*f*z^3 + 900*a*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*
e^16*f^4*z^3 - 750*a^19*b*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f
^12*z^3 - 750*a^17*b^3*c^12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z
^3 - 750*a^12*b^8*c^17*d^3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 -
750*a^8*b^12*c^19*d*e^3*f^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a
^3*b^17*c^19*d*e^8*f^12*z^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19
*c^17*d^3*e^12*f^8*z^3 - 750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*
d^14*e^5*f^15*z^3 - 420*a^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^
6*e*f^19*z^3 - 420*a^15*b^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f
^19*z^3 - 420*a^14*b^6*c*d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z
^3 - 420*a^6*b^14*c^5*d^15*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 -

420*a^5*b^15*c^6*d^14*e^19*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a
*b^19*c^14*d^6*e^15*f^5*z^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b
*c^2*d^18*e^9*f^11*z^3 + 350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*
d^19*e^11*f^9*z^3 + 350*a^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*
e^18*f^2*z^3 + 350*a^9*b^11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^1
9*f*z^3 + 350*a^2*b^18*c^19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z
^3 + 350*a*b^19*c^18*d^2*e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 -
90*a^19*b*c^10*d^10*e*f^19*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b
^10*c^19*d*e*f^19*z^3 - 90*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e
^10*f^10*z^3 - 90*a*b^19*c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3
 + 10*b^20*c^11*d^9*e^19*f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^
19*e^9*f^11*z^3 + 10*a^19*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3
 + 10*a^9*b^11*c^20*e*f^19*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^
11*d^9*f^20*z^3 + 10*a^11*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3
 + 10*a*b^19*c^9*d^11*e^20*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*
c^16*d^4*e^14*f^6*z^3 - 210*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*
e^13*f^7*z^3 + 120*b^20*c^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z
^3 - 45*b^20*c^12*d^8*e^18*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a
^20*c^6*d^14*e^4*f^16*z^3 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d
^13*e^3*f^17*z^3 + 120*a^20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^
18*z^3 - 45*a^20*c^2*d^18*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 2
10*a^16*b^4*d^20*e^14*f^6*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b
^3*d^20*e^13*f^7*z^3 + 120*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^
12*f^8*z^3 - 45*a^12*b^8*d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3
 - 210*a^6*b^14*c^20*e^4*f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^
7*b^13*c^20*e^3*f^17*z^3 + 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^2
0*e^2*f^18*z^3 - 45*a^2*b^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20
*z^3 - 210*a^16*b^4*c^14*d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 12
0*a^17*b^3*c^13*d^7*f^20*z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2
*c^12*d^8*f^20*z^3 - 45*a^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*
e^20*z^3 - 210*a^6*b^14*c^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3
+ 120*a^7*b^13*c^3*d^17*e^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*
b^12*c^2*d^18*e^20*z^3 - 45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^1
0*z^3 - a^20*d^20*e^10*f^10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*
f^20*z^3 - a^10*b^10*d^20*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^
2*c*d^13*e*f^13*z + 1890*a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12
*f^2*z + 92610*a^6*b^8*c^4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^1
0*z + 92610*a^4*b^10*c^4*d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z
 - 66150*a^7*b^7*c^4*d^10*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z -
66150*a^4*b^10*c^7*d^7*e^3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 661
50*a^3*b^11*c^8*d^6*e^3*f^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*
a^3*b^11*c^4*d^10*e^7*f^7*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5
*b^9*c^5*d^9*e^4*f^10*z - 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10
*c^5*d^9*e^5*f^9*z - 32130*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*
d^12*e^3*f^11*z - 32130*a^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^

12*e^9*f^5*z - 32130*a^2*b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*
e^9*f^5*z + 22680*a^8*b^6*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*
f^10*z + 22680*a^4*b^10*c^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^
6*z + 22680*a^2*b^12*c^8*d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z
 + 19278*a^10*b^4*c^2*d^12*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z
+ 19278*a^2*b^12*c^2*d^12*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 1
8522*a^6*b^8*c^3*d^11*e^5*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*
a^5*b^9*c^3*d^11*e^6*f^8*z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b
^11*c^5*d^9*e^6*f^8*z - 13230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^
2*d^12*e^6*f^8*z - 13230*a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*
e^2*f^12*z + 3402*a^7*b^7*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^1
2*z + 3402*a^5*b^9*c^2*d^12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3
402*a^2*b^12*c^5*d^9*e^7*f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^1
0*b^4*c*d^13*e^3*f^11*z + 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c
*d^13*e^10*f^4*z + 7938*a*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e
^10*f^4*z - 5670*a^11*b^3*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12
*z - 5670*a^2*b^12*c^11*d^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 56
70*a*b^13*c^11*d^3*e^2*f^12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*
b^5*c^4*d^10*e*f^13*z - 3780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*
d^5*e*f^13*z - 3780*a^4*b^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^1
0*z - 3780*a*b^13*c^4*d^10*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268
*a^8*b^6*c*d^13*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*
d^13*e^8*f^6*z - 2268*a*b^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^
6*z + 1890*a^7*b^7*c^6*d^8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*
a^6*b^8*c^7*d^7*e*f^13*z + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*
d^7*e^6*f^8*z + 1890*a*b^13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z -
252*b^14*c*d^13*e^13*f*z - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*
f*z - 252*a^13*b*c*d^13*f^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^
7*e^7*f^7*z - 882*b^14*c^11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z +
 693*b^14*c^12*d^2*e^2*f^12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8
*d^6*e^6*f^8*z + 567*b^14*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z
+ 441*b^14*c^4*d^10*e^10*f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*
d^9*e^9*f^5*z - 918*a^7*b^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z -
 882*a^3*b^11*d^14*e^11*f^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12
*d^14*e^12*f^2*z + 567*a^8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z
+ 441*a^10*b^4*d^14*e^4*f^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5
*d^14*e^5*f^9*z - 126*a^5*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z -
 882*a^11*b^3*c^3*d^11*f^14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2
*c^2*d^12*f^14*z + 693*a^2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*
z + 567*a^6*b^8*c^8*d^6*f^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^1
0*c^10*d^4*f^14*z - 126*a^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z
 + 36*b^14*d^14*e^14*z + 36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*
a^2*b^9*c^2*d^9*e^2*f^9 + 9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10
*e^2*f^9 + 9018*a^2*b^9*c^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018
*a*b^10*c^3*d^8*e^2*f^9 + 9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10

*e*f^10 - 9018*a*b^10*c^4*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^11*c^5*d^6*e*f^10 + 2268*b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2268*a*b^10*d^11*e^5*f^6 + 2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^11 - 1458*b^11*c^3*d^8*e^3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*d^9*e^4*f^7 - 1458*a^3*b^8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*a^2*b^9*d^11*e^4*f^7 - 1458*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^11 - 1161*a^2*b^9*c^4*d^7*f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^11 - 756*a^6*b^5*d^11*f^11, z, k), k, 1, 3) - ((7*a*b^6*c^2*d^5*e^7 - a^3*b^4*d^7*e^7 - a^7*c^3*d^4*f^7 - b^7*c^3*d^4*e^7 - a^7*d^7*e^3*f^4 - b^7*c^7*e^3*f^4 - 6*a^5*b^2*c^5*d^2*f^7 - 6*a^5*b^2*d^7*e^5*f^2 - 6*b^7*c^5*d^2*e^5*f^2 - a^3*b^4*c^7*f^7 + 7*a^2*b^5*c*d^6*e^7 + 4*a^4*b^3*c^6*d*f^7 + 4*a^6*b*c^4*d^3*f^7 + 7*a*b^6*c^7*e^2*f^5 + 7*a^2*b^5*c^7*e*f^6 + 4*a^4*b^3*d^7*e^6*f + 4*a^6*b*d^7*e^4*f^3 + 7*a^7*c*d^6*e^2*f^5 + 7*a^7*c^2*d^5*e*f^6 + 4*b^7*c^4*d^3*e^6*f + 4*b^7*c^6*d*e^4*f^3 - 21*a*b^6*c^3*d^4*e^6*f - 21*a*b^6*c^6*d*e^3*f^4 - 21*a^3*b^4*c*d^6*e^6*f - 21*a^3*b^4*c^6*d*e*f^6 - 21*a^6*b*c*d^6*e^3*f^4 - 21*a^6*b*c^3*d^4*e*f^6 + 14*a*b^6*c^4*d^3*e^5*f^2 + 14*a*b^6*c^5*d^2*e^4*f^3 - 26*a^2*b^5*c^2*d^5*e^6*f - 26*a^2*b^5*c^6*d*e^2*f^5 + 14*a^4*b^3*c*d^6*e^5*f^2 + 14*a^4*b^3*c^5*d^2*e*f^6 + 14*a^5*b^2*c*d^6*e^4*f^3 + 14*a^5*b^2*c^4*d^3*e*f^6 - 26*a^6*b*c^2*d^5*e^2*f^5 + 52*a^2*b^5*c^3*d^4*e^5*f^2 - 78*a^2*b^5*c^4*d^3*e^4*f^3 + 52*a^2*b^5*c^5*d^2*e^3*f^4 + 52*a^3*b^4*c^2*d^5*e^5*f^2 + 52*a^3*b^4*c^5*d^2*e^2*f^5 - 78*a^4*b^3*c^2*d^5*e^4*f^3 - 78*a^4*b^3*c^4*d^3*e^2*f^5 + 52*a^5*b^2*c^2*d^5*e^3*f^4 + 52*a^5*b^2*c^3*d^4*e^2*f^5)/(2*(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (3*x^5*(2*a^5*b^2*d^7*f^7 + 2*b^7*c^5*d^2*f^7 + 2*b^7*d^7*e^5*f^2 + 2*a^2*b^5*c^3*d^4*f^7 + 2*a^3*b^4*c^2*d^5*f^7 + 2*a^2*b^5*d^7*e^3*f^4 + 2*a^3*b^4*d^7*e^2*f^5 + 2*b^7*

c^2*d^5*e^3*f^4 + 2*b^7*c^3*d^4*e^2*f^5 - 5*a*b^6*c^4*d^3*f^7 - 5*a^4*b^3*c
*d^6*f^7 - 5*a*b^6*d^7*e^4*f^3 - 5*a^4*b^3*d^7*e*f^6 - 5*b^7*c*d^6*e^4*f^3
- 5*b^7*c^4*d^3*e*f^6 + 16*a*b^6*c*d^6*e^3*f^4 + 16*a*b^6*c^3*d^4*e*f^6 + 1
6*a^3*b^4*c*d^6*e*f^6 - 12*a*b^6*c^2*d^5*e^2*f^5 - 12*a^2*b^5*c*d^6*e^2*f^5
 - 12*a^2*b^5*c^2*d^5*e*f^6))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*
c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b
^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*
d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f
^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*
b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e
^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f +
4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*
a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*
a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8
*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f
 + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f
^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^
3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c
^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 +
36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^
2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^
4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2
*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3
*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4
- 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6) + (3*x^4*(8*a^6*
b*d^7*f^7 + 8*b^7*c^6*d*f^7 + 8*b^7*d^7*e^6*f - 7*a^2*b^5*c^4*d^3*f^7 + 14*
a^3*b^4*c^3*d^4*f^7 - 7*a^4*b^3*c^2*d^5*f^7 - 7*a^2*b^5*d^7*e^4*f^3 + 14*a^
3*b^4*d^7*e^3*f^4 - 7*a^4*b^3*d^7*e^2*f^5 - 7*b^7*c^2*d^5*e^4*f^3 + 14*b^7*
c^3*d^4*e^3*f^4 - 7*b^7*c^4*d^3*e^2*f^5 - 14*a*b^6*c^5*d^2*f^7 - 14*a^5*b^2
*c*d^6*f^7 - 14*a*b^6*d^7*e^5*f^2 - 14*a^5*b^2*d^7*e*f^6 - 14*b^7*c*d^6*e^5
*f^2 - 14*b^7*c^5*d^2*e*f^6 + 34*a*b^6*c*d^6*e^4*f^3 + 34*a*b^6*c^4*d^3*e*f
^6 + 34*a^4*b^3*c*d^6*e*f^6 + 6*a*b^6*c^2*d^5*e^3*f^4 + 6*a*b^6*c^3*d^4*e^2
*f^5 + 6*a^2*b^5*c*d^6*e^3*f^4 + 6*a^2*b^5*c^3*d^4*e*f^6 + 6*a^3*b^4*c*d^6*
e^2*f^5 + 6*a^3*b^4*c^2*d^5*e*f^6 - 78*a^2*b^5*c^2*d^5*e^2*f^5))/(2*(4*a*b^
7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d
^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^
8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 -
 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*
c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7
 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*
c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^
4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*
d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3
*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^
7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3

*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (x^2*(18*a*b^6*c^7*f^7 + 18*a*b^6*d^7*e^7 + 18*a^7*c*d^6*f^7 + 18*b^7*c*d^6*e^7 + 18*a^7*d^7*e*f^6 + 18*b^7*c^7*e*f^6 - 3*a^3*b^4*c^5*d^2*f^7 + 32*a^4*b^3*c^4*d^3*f^7 - 3*a^5*b^2*c^3*d^4*f^7 - 3*a^3*b^4*d^7*e^5*f^2 + 32*a^4*b^3*d^7*e^4*f^3 - 3*a^5*b^2*d^7*e^3*f^4 - 3*b^7*c^3*d^4*e^5*f^2 + 32*b^7*c^4*d^3*e^4*f^3 - 3*b^7*c^5*d^2*e^3*f^4 - 37*a^2*b^5*c^6*d*f^7 - 37*a^6*b*c^2*d^5*f^7 - 37*a^2*b^5*d^7*e^6*f - 37*a^6*b*d^7*e^2*f^5 - 37*b^7*c^2*d^5*e^6*f - 37*b^7*c^6*d*e^2*f^5 + 9*a*b^6*c^2*d^5*e^5*f^2 + a*b^6*c^3*d^4*e^4*f^3 + a*b^6*c^4*d^3*e^3*f^4 + 9*a*b^6*c^5*d^2*e^2*f^5 + 9*a^2*b^5*c*d^6*e^5*f^2 + 9*a^2*b^5*c^5*d^2*e*f^6 + a^3*b^4*c*d^6*e^4*f^3 + a^3*b^4*c^4*d^3*e*f^6 + a^4*b^3*c*d^6*e^3*f^4 + a^4*b^3*c^3*d^4*e*f^6 + 9*a^5*b^2*c*d^6*e^2*f^5 + 9*a^5*b^2*c^2*d^5*e*f^6 - 34*a*b^6*c*d^6*e^6*f - 34*a*b^6*c^6*d*e*f^6 - 34*a^6*b*c*d^6*e*f^6 + 234*a^2*b^5*c^2*d^5*e^4*f^3 - 208*a^2*b^5*c^3*d^4*e^3*f^4 + 234*a^2*b^5*c^4*d^3*e^2*f^5 - 208*a^3*b^4*c^2*d^5*e^3*f^4 - 208*a^3*b^4*c^3*d^4*e^2*f^5 + 234*a^4*b^3*c^2*d^5*e^2*f^5)) /(2*(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (x*(2*a^2*b^5*c^7*f^7 + 2*a^2*b^5*d^7*e^7 + 2*a^7*c^2*d^5*f^7 + 2*b^7*c^2*d^5*e^7 + 2*a^7*d^7*e^2*f^5 + 2*b^7*c^7* e^2*f^5 + 4*a^4*b^3*c^5*d^2*f^7 + 4*a^5*b^2*c^4*d^3*f^7 + 4*a^4*b^3*d^7*e^5*f^2 + 4*a^5*b^2*d^7*e^4*f^3 + 4*b^7*c^4*d^3*e^5*f^2 + 4*b^7*c^5*d^2*e^4*f^3 + 14*a*b^6*c*d^6*e^7 + 14*a*b^6*c^7*e*f^6 + 14*a^7*c*d^6*e*f^6 - 6*a^3*b^4*c^6*d*f^7 - 6*a^6*b*c^3*d^4*f^7 - 6*a^3*b^4*d^7*e^6*f - 6*a^6*b*d^7*e^3*f^4 - 6*b^7*c^3*d^4*e^6*f - 6*b^7*c^6*d*e^3*f^4 - 33*a*b^6*c^2*d^5*e^6*f - 33*a*b^6*c^6*d*e^2*f^5 - 33*a^2*b^5*c*d^6*e^6*f - 33*a^2*b^5*c^6*d*e*f^6 - 33*a^6*b*c*d^6*e^2*f^5 - 33*a^6*b*c^2*d^5*e*f^6 + 17*a*b^6*c^3*d^4*e^5*f^2 - 8*a*b^6*c^4*d^3*e^4*f^3 + 17*a*b^6*c^5*d^2*e^3*f^4 + 17*a^3*b^4*c*d^6*e^5*f^2 + 17*a^3*b^4*c^5*d^2*e*f^6 - 8*a^4*b^3*c*d^6*e^4*f^3 - 8*a^4*b^3*c^4*d^3*e*f^6 + 17*a^5*b^2*c*d^6*e^3*f^4 + 17*a^5*b^2*c^3*d^4*e*f^6 + 78*a^2*b^5*c^2*d^5*e^5*f^2 - 26*a^2*b^5*c^3*d^4*e^4*f^3 - 26*a^2*b^5*c^4*d^3*e^3*f^4 + 78*a^2*b^5*c^5*d^2*e^2*f^5 - 26*a^3*b^4*c^2*d^5*e^4*f^3 - 26*a^3*b^4*c^4*d^3*e^2*f^5 - 26*a^4*b^3*c^2*d^5*e^3*f^4 - 26*a^4*b^3*c^3*d^4*e^2*f^5 + 78*a^5*b^2*c^2*d^5*e^2*f^5))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6) + (x^3*(6*a^7*d^7*f^7 + 6*b^7*c^7*f^7 + 6*b^7*d^7*e^7 - 37*a^2*b^5*c^5*d^2*f^7 + 19*a^3*b^4*c^4*d^3*f^7 + 19*a^4*b^3*c^3*d^4*f^7 - 37*a^5*b^2*c^2*d^5*f^7 - 37*a^2*b^5*d^7*e^5*f^2 + 19*a^3*b^4*d^7*e^4*f^3 + 19*a^4*b^3*d^7*e^3*f^4 - 37*a^5*b^2*d^7*e^2*f^5 - 37*b^7*c^2*d^5*e^5*f^2 + 19*b^7*c^3*d^4*e^4*f^3 + 19*b^7*c^4*d^3*e^3*f^4 - 37*b^7*c^5*d^2*e^2*f^5 + 3*a*b^6*c^6*d*f^7 + 3*a^6*b*c*d^6*f^7 + 3*a*b^6*d^7*e^6*f + 3*a^6*b*d^7*e*f^6 + 3*b^7*c*d^6*e^6*f + 3*b^7*c^6*d*e*f^6 - 28*a*b^6*c*d^6*e^5*f^2 - 28*a*b^6*c^5*d^2*e*f^6 - 28*a^5*b^2*c*d^6*e*f^6 + 86*a*b^6*c^2*d^5*e^4*f^3 - 68*a*b^6*c^3*d^4*e^3*f^4 + 86*a*b^6*c^4*d^3*e^2*f^5 + 86*a^2*b^5*c*d^6*e^4*f^3 + 86*a^2*b^5*c^4*d^3*e*f^6 - 68*a^3*b^4*c*d^6*e^3*f^4 - 68*a^3*b^4*c^3*d^4*e*f^6 + 86*a^4*b^3*c*d^6*e^2*f^5 + 86*a^4*b^3*c^2*d^5*e*f^6 - 52*a^2*b^5*c^2*d^5*e^3*f^4 - 52*a^2*b^5*c^3*d^4*e^2*

```
f^5 - 52*a^3*b^4*c^2*d^5*e^2*f^5))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 -
 a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*
a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6
*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*
c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 +
 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*
d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7
*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4
- 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4
- 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^
3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e
^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^
2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d
^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*
b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f
^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c
^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 -
90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^
2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^
5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4
*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6))/(x*(2*a*b*
c^2*e^2 + 2*a^2*c*d*e^2 + 2*a^2*c^2*e*f) + x^3*(2*a*b*c^2*f^2 + 2*a*b*d^2*e
^2 + 2*a^2*c*d*f^2 + 2*b^2*c*d*e^2 + 2*a^2*d^2*e*f + 2*b^2*c^2*e*f + 8*a*b*
c*d*e*f) + x^2*(a^2*c^2*f^2 + a^2*d^2*e^2 + b^2*c^2*e^2 + 4*a*b*c*d*e^2 + 4
*a*b*c^2*e*f + 4*a^2*c*d*e*f) + x^5*(2*a*b*d^2*f^2 + 2*b^2*c*d*f^2 + 2*b^2*
d^2*e*f) + x^4*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 4*a*b*c*d*f^2 + 4
*a*b*d^2*e*f + 4*b^2*c*d*e*f) + a^2*c^2*e^2 + b^2*d^2*f^2*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**3,x)

[Out] Timed out
```

3.21 $\int \frac{1}{1+x+x^2+x^3}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{1+x+x^2+x^3}\,dx = \int \left(\frac{1}{2(1+x)} + \frac{1-x}{2\left(1+x^2\right)}\right) dx$$
$$= \frac{1}{2}\log(1+x) + \frac{1}{2}\int \frac{1-x}{1+x^2}\,dx$$
$$= \frac{1}{2}\log(1+x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{1}{2}\int \frac{x}{1+x^2}\,dx$$
$$= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1) + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^(-1),x]
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

fricas [A] time = 0.61, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

giac [A] time = 0.31, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} + \frac{\ln(x+1)}{2} - \frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+x^2+x+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

maxima [A] time = 1.45, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

mupad [B] time = 2.20, size = 25, normalized size = 1.00

$$\frac{\ln\left(x+1\right)}{2} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^2 + x^3 + 1),x)
```

```
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

sympy [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log\left(x+1\right)}{2} - \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.22 $\int \frac{1}{-1+4x-4x^2+16x^3}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

```
[Out] -1/10*arctan(2*x)+1/5*ln(1-4*x)-1/10*ln(4*x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

```
[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = \int \left(\frac{4}{5(-1+4x)} + \frac{-1-4x}{5\left(1+4x^2\right)}\right) dx$$
$$= \frac{1}{5}\log(1-4x) + \frac{1}{5}\int \frac{-1-4x}{1+4x^2}\,dx$$
$$= \frac{1}{5}\log(1-4x) - \frac{1}{5}\int \frac{1}{1+4x^2}\,dx - \frac{4}{5}\int \frac{x}{1+4x^2}\,dx$$
$$= -\frac{1}{10}\tan^{-1}(2x) + \frac{1}{5}\log(1-4x) - \frac{1}{10}\log\left(1+4x^2\right)$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{10}\log\left(4x^2+1\right) + \frac{1}{5}\log(1-4x) - \frac{1}{10}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
[Out] -1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

fricas [A] time = 0.75, size = 25, normalized size = 0.81

$$-\frac{1}{10}\arctan\left(2x\right) - \frac{1}{10}\log\left(4x^2+1\right) + \frac{1}{5}\log\left(4x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="fricas")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

giac [A] time = 0.35, size = 26, normalized size = 0.84

$$-\frac{1}{10}\arctan\left(2x\right) - \frac{1}{10}\log\left(4x^2+1\right) + \frac{1}{5}\log\left(|4x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="giac")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(abs(4*x - 1))
```

maple [A] time = 0.01, size = 26, normalized size = 0.84

$$-\frac{\arctan\left(2x\right)}{10} + \frac{\ln\left(4x-1\right)}{5} - \frac{\ln\left(4x^2+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(16*x^3-4*x^2+4*x-1),x)
```

```
[Out] -1/10*ln(4*x^2+1)-1/10*arctan(2*x)+1/5*ln(-1+4*x)
```

maxima [A] time = 1.28, size = 25, normalized size = 0.81

$$-\frac{1}{10}\arctan(2x) - \frac{1}{10}\log\left(4x^2+1\right) + \frac{1}{5}\log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="maxima")
```

```
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln\left(x-\frac{1}{4}\right)}{5} + \ln\left(x-\frac{1}{2}\mathrm{i}\right)\left(-\frac{1}{10}+\frac{1}{20}\mathrm{i}\right) + \ln\left(x+\frac{1}{2}\mathrm{i}\right)\left(-\frac{1}{10}-\frac{1}{20}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 4*x^2 + 16*x^3 - 1),x)
```

```
[Out] log(x - 1/4)/5 - log(x - 1i/2)*(1/10 - 1i/20) - log(x + 1i/2)*(1/10 + 1i/20
)
```

sympy [A] time = 0.15, size = 24, normalized size = 0.77

$$\frac{\log\left(x-\frac{1}{4}\right)}{5} - \frac{\log\left(x^2+\frac{1}{4}\right)}{10} - \frac{\operatorname{atan}(2x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x**3-4*x**2+4*x-1),x)
```

```
[Out] log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10
```

3.23 $\int \frac{1}{dx^3} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{2dx^2}$$

```
[Out] -1/2/d/x^2
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 30}

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(d*x^3),x]
```

```
[Out] -1/(2*d*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{dx^3} \, dx &= \frac{\int \frac{1}{x^3} \, dx}{d}\\ &= -\frac{1}{2dx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(d*x^3),x]
[Out] -1/2*1/(d*x^2)
```

fricas [A] time = 0.70, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="fricas")
[Out] -1/2/(d*x^2)
```

giac [A] time = 0.27, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="giac")
[Out] -1/2/(d*x^2)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{1}{2d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/d/x^3,x)
[Out] -1/2/d/x^2
```

maxima [A] time = 0.63, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="maxima")
[Out] -1/2/(d*x^2)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$-\frac{1}{2\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3),x)
```

```
[Out] -1/(2*d*x^2)
```

sympy [A] time = 0.06, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x**3,x)
```

```
[Out] -1/(2*d*x**2)
```

3.24 $\int \frac{1}{cx^2+dx^3}\,dx$

Optimal. Leaf size=28

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

```
[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{cx^2+dx^3}\,dx &= \int \frac{1}{x^2(c+dx)}\,dx\\ &= \int \left(\frac{1}{cx^2}-\frac{d}{c^2x}+\frac{d^2}{c^2(c+dx)}\right)dx\\ &= -\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)^(-1),x]
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

fricas [A] time = 0.89, size = 26, normalized size = 0.93

$$\frac{dx\log\left(dx+c\right)-dx\log(x)-c}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="fricas")
[Out] (d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)
```

giac [A] time = 0.36, size = 30, normalized size = 1.07

$$\frac{d\log\left(|dx+c|\right)}{c^2}-\frac{d\log\left(|x|\right)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="giac")
[Out] d*log(abs(d*x + c))/c^2 - d*log(abs(x))/c^2 - 1/(c*x)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\frac{d\ln(x)}{c^2}+\frac{d\ln\left(dx+c\right)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2),x)
[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

maxima [A] time = 0.59, size = 28, normalized size = 1.00

$$\frac{d\log\left(dx+c\right)}{c^2}-\frac{d\log(x)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="maxima")
```

```
[Out] d*log(d*x + c)/c^2 - d*log(x)/c^2 - 1/(c*x)
```

mupad [B] time = 0.06, size = 25, normalized size = 0.89

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{2\,d\,x}{c}+1\right)}{c^2}-\frac{1}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2 + d*x^3),x)
```

```
[Out] (2*d*atanh((2*d*x)/c + 1))/c^2 - 1/(c*x)
```

sympy [A] time = 0.18, size = 19, normalized size = 0.68

$$-\frac{1}{cx}+\frac{d\left(-\log\left(x\right)+\log\left(\frac{c}{d}+x\right)\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+c*x**2),x)
```

```
[Out] -1/(c*x) + d*(-log(x) + log(c/d + x))/c**2
```

3.25 $\int \frac{1}{bx+dx^3}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

```
[Out] ln(x)/b-1/2*ln(d*x^2+b)/b
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)^(-1),x]
```

```
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{bx+dx^3}\,dx &= \int \frac{1}{x\left(b+dx^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(b+dx)}\,dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2b}-\frac{d\operatorname{Subst}\left(\int \frac{1}{b+dx}\,dx,x,x^2\right)}{2b}\\
&= \frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)^(-1),x]
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

fricas [A] time = 0.78, size = 18, normalized size = 0.82

$$-\frac{\log\left(dx^2+b\right)-2\,\log(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="fricas")
[Out] -1/2*(log(d*x^2 + b) - 2*log(x))/b
```

giac [A] time = 0.30, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,b}-\frac{\log\left(\left|dx^2+b\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="giac")
[Out] 1/2*log(x^2)/b - 1/2*log(abs(d*x^2 + b))/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{b}-\frac{\ln\left(d\,x^{2}+b\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+b*x),x)
[Out] ln(x)/b-1/2*ln(d*x^2+b)/b
```

maxima [A] time = 0.79, size = 20, normalized size = 0.91

$$-\frac{\log\left(dx^{2}+b\right)}{2\,b}+\frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="maxima")
[Out] -1/2*log(d*x^2 + b)/b + log(x)/b
```

mupad [B] time = 2.13, size = 18, normalized size = 0.82

$$-\frac{\ln\left(d\,x^{2}+b\right)-2\,\ln(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + d*x^3),x)
[Out] -(log(b + d*x^2) - 2*log(x))/(2*b)
```

sympy [A] time = 0.20, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{b}{d}+x^{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+b*x),x)
[Out] log(x)/b - log(b/d + x**2)/(2*b)
```

3.26 $\int \frac{1}{bx+cx^2+dx^3}\,dx$

Optimal. Leaf size=62

$$\frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{\log(x)}{b}$$

```
[Out] ln(x)/b-1/2*ln(d*x^2+c*x+b)/b+c*arctanh((2*d*x+c)/(-4*b*d+c^2)^(1/2))/b/(-4
*b*d+c^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1594, 705, 29, 634, 618, 206, 628}

$$\frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d]) + Log[x]/b
 - Log[b + c*x + d*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{bx+cx^2+dx^3}\,dx &= \int \frac{1}{x\left(b+cx+dx^2\right)}\,dx \\
&= \frac{\int \frac{1}{x}\,dx}{b}+\frac{\int \frac{-c-dx}{b+cx+dx^2}\,dx}{b} \\
&= \frac{\log(x)}{b}-\frac{\int \frac{c+2dx}{b+cx+dx^2}\,dx}{2b}-\frac{c\int \frac{1}{b+cx+dx^2}\,dx}{2b} \\
&= \frac{\log(x)}{b}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{c\,\text{Subst}\left(\int \frac{1}{c^2-4bd-x^2}\,dx,x,c+2dx\right)}{b} \\
&= \frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}+\frac{\log(x)}{b}-\frac{\log\left(b+cx+dx^2\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 0.98

$$-\frac{\frac{2c\tan^{-1}\left(\frac{c+2dx}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}}+\log(b+x(c+dx))-2\log(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -1/2*((2*c*ArcTan[(c + 2*d*x)/Sqrt[-c^2 + 4*b*d]])/Sqrt[-c^2 + 4*b*d] - 2*L
og[x] + Log[b + x*(c + d*x)])/b
```

fricas [A] time = 0.86, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{c^2-4\,bd}\,c\log\left(\frac{2\,d^2x^2+2\,cdx+c^2-2\,bd+\sqrt{c^2-4\,bd}\,(2\,dx+c)}{dx^2+cx+b}\right)-\left(c^2-4\,bd\right)\log\left(dx^2+cx+b\right)+2\left(c^2-4\,bd\right)\log(x)}{2\left(bc^2-4\,b^2d\right)}, \frac{2\,\sqrt{-c}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c^2 - 4*b*d)*c*log((2*d^2*x^2 + 2*c*d*x + c^2 - 2*b*d + sqrt(c^2
 - 4*b*d)*(2*d*x + c))/(d*x^2 + c*x + b)) - (c^2 - 4*b*d)*log(d*x^2 + c*x +
 b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d), 1/2*(2*sqrt(-c^2 + 4*b*d)*
c*arctan(-sqrt(-c^2 + 4*b*d)*(2*d*x + c)/(c^2 - 4*b*d)) - (c^2 - 4*b*d)*log
(d*x^2 + c*x + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d)]
```

giac [A] time = 0.29, size = 62, normalized size = 1.00

$$-\frac{c\arctan\left(\frac{2\,dx+c}{\sqrt{-c^2+4\,bd}}\right)}{\sqrt{-c^2+4\,bd}\,b}-\frac{\log\left(dx^2+cx+b\right)}{2\,b}+\frac{\log\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -c*arctan((2*d*x + c)/sqrt(-c^2 + 4*b*d))/(sqrt(-c^2 + 4*b*d)*b) - 1/2*log(
d*x^2 + c*x + b)/b + log(abs(x))/b
```

maple [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{c\arctan\left(\frac{2dx+c}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}\;b}+\frac{\ln(x)}{b}-\frac{\ln\left(d\,x^2+cx+b\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2+b*x),x)
```

```
[Out] -1/2*ln(d*x^2+c*x+b)/b-1/b*c/(4*b*d-c^2)^(1/2)*arctan((2*d*x+c)/(4*b*d-c^2)
^(1/2))+1/b*ln(x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b*d-c^2>0)', see `assume?` for mo
re details)Is 4*b*d-c^2 positive or negative?
```

mupad [B] time = 0.47, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{b} - \ln\left(\left(x\left(6\,b\,d^2 - 2\,c^2\,d\right) - b\,c\,d\right)\left(\frac{1}{2\,b} - \frac{c\,\sqrt{c^2 - 4\,b\,d}}{2\,\left(b\,c^2 - 4\,b^2\,d\right)}\right) - c\,d - 3\,d^2\,x\right)\left(\frac{1}{2\,b} - \frac{c\,\sqrt{c^2 - 4\,b\,d}}{2\,\left(b\,c^2 - 4\,b^2\,d\right)}\right) - \ln\left(\left(x\,\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2 + d*x^3),x)
```

```
[Out] log(x)/b - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b) - (c*(c^2 - 4*b*d)^
(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b) - (c*(c^2 - 4*b*d)^
(1/2))/(2*(b*c^2 - 4*b^2*d))) - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b
) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b
) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d)))
```

sympy [B] time = 4.19, size = 564, normalized size = 9.10

$$\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)} - \frac{1}{2b}\right)\log\left(x + \frac{24b^4d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)} - \frac{1}{2b}\right)^2 - 14b^3c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)} - \frac{1}{2b}\right)^2 - 12b^3d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}\right.}{9b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+c*x**2+b*x),x)
```

```
[Out] (-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**
2*d*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b
*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c**3*d)) + (c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
 c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c*
*3*d)) + log(x)/b
```

3.27 $\int \frac{1}{a+dx^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}}$$

```
[Out] 1/3*ln(a^(1/3)+d^(1/3)*x)/a^(2/3)/d^(1/3)-1/6*ln(a^(2/3)-a^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/a^(2/3)/d^(1/3)-1/3*arctan(1/3*(a^(1/3)-2*d^(1/3)*x)/a^(1/3)*3
^(1/2))/a^(2/3)/d^(1/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d^(1/3
))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Log[a^(2/3) - a^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+dx^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{d}\,x}\,dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{d}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3a^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}+\frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{2\sqrt[3]{a}}-\frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{d}+2d^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{d}}\\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{d}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.77

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}\,x+d^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
 + d^(1/3)*x] + Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2]])/(a^(2/3)*d^
(1/3))
```

fricas [A] time = 0.83, size = 299, normalized size = 2.60

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ad\sqrt{-\frac{\left(a^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,adx^3-3\left(a^2d\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,adx^2+\left(a^2d\right)^{\frac{2}{3}}x-\left(a^2d\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+a}\right)-\left(a^2d\right)^{\frac{2}{3}}\log\left(adx^2-\left(a^2d\right)^{\frac{2}{3}}x+\right.}{6\,a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*d*sqrt(-(a^2*d)^(1/3)/d)*log((2*a*d*x^3 - 3*(a^2*d)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*d*x^2 + (a^2*d)^(2/3)*x - (a^2*d)^(1/3)*a)*
sqrt(-(a^2*d)^(1/3)/d))/(d*x^3 + a)) - (a^2*d)^(2/3)*log(a*d*x^2 - (a^2*d)^
(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x + (a^2*d)^(2/3)))/(a
^2*d), 1/6*(6*sqrt(1/3)*a*d*sqrt((a^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(a^2*
d)^(2/3)*x - (a^2*d)^(1/3)*a)*sqrt((a^2*d)^(1/3)/d)/a^2) - (a^2*d)^(2/3)*lo
g(a*d*x^2 - (a^2*d)^(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x
+ (a^2*d)^(2/3)))/(a^2*d)]
```

giac [A] time = 0.26, size = 112, normalized size = 0.97

$$-\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-ad^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,ad}+\frac{\left(-ad^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{d}\right)^{\frac{1}{3}}+\left(-\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(-a/d)^(1/3)*log(abs(x - (-a/d)^(1/3)))/a + 1/3*sqrt(3)*(-a*d^2)^(1/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/d)^(1/3))/(-a/d)^(1/3))/(a*d) + 1/6*(-a*d^2)
^(1/3)*log(x^2 + x*(-a/d)^(1/3) + (-a/d)^(2/3))/(a*d)
```

maple [A] time = 0.01, size = 91, normalized size = 0.79

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{d}\right)^{\frac{2}{3}} d}+\frac{\ln\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{2}{3}} d}-\frac{\ln\left(x^{2}-\left(\frac{a}{d}\right)^{\frac{1}{3}} x+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{d}\right)^{\frac{2}{3}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+a),x)
```

```
[Out] 1/3/d/(a/d)^(2/3)*ln(x+(a/d)^(1/3))-1/6/d/(a/d)^(2/3)*ln(x^2-(a/d)^(1/3)*x+
(a/d)^(2/3))+1/3/d/(a/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/d)^(1/3)*x-
1))
```

maxima [A] time = 1.28, size = 98, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{2}-x\left(\frac{a}{d}\right)^{\frac{1}{3}}+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/d)^(1/3))/(a/d)^(1/3))/(d*(a/d)^(2
/3)) - 1/6*log(x^2 - x*(a/d)^(1/3) + (a/d)^(2/3))/(d*(a/d)^(2/3)) + 1/3*log
(x + (a/d)^(1/3))/(d*(a/d)^(2/3))
```

mupad [B] time = 0.23, size = 99, normalized size = 0.86

$$\frac{\ln\left(d^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,d^{1/3}}+\frac{\ln\left(3\,d^2\,x+\frac{3\,a^{1/3}\,d^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}}-\frac{\ln\left(3\,d^2\,x-\frac{3\,a^{1/3}\,d^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + d*x^3),x)
```

```
[Out] log(d^(1/3)*x + a^(1/3))/(3*a^(2/3)*d^(1/3)) + (log(3*d^2*x + (3*a^(1/3)*d^
(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*d^(1/3)) - (log(3*d
^2*x - (3*a^(1/3)*d^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)
*d^(1/3))
```

sympy [A] time = 0.16, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3a^2d - 1, \left(t \mapsto t\log\left(3ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

3.28 $\int (dx^3)^n \, dx$

Optimal. Leaf size=16

$$\frac{x (dx^3)^n}{3n+1}$$

[Out] x*(d*x^3)^n/(1+3*n)

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x (dx^3)^n}{3n+1}$$

Antiderivative was successfully verified.

[In] Int[(d*x^3)^n,x]

[Out] (x*(d*x^3)^n)/(1 + 3*n)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (dx^3)^n \, dx &= \left(x^{-3n} (dx^3)^n\right) \int x^{3n} \, dx \\ &= \frac{x (dx^3)^n}{1+3n} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x (dx^3)^n}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^3)^n,x]
```

```
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

fricas [A] time = 0.85, size = 16, normalized size = 1.00

$$\frac{\left(dx^3\right)^n x}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3)^n*x/(3*n + 1)
```

giac [A] time = 0.31, size = 16, normalized size = 1.00

$$\frac{\left(dx^3\right)^n x}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="giac")
```

```
[Out] (d*x^3)^n*x/(3*n + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x\left(d\,x^3\right)^n}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)^n,x)
```

```
[Out] x*(d*x^3)^n/(1+3*n)
```

maxima [A] time = 0.57, size = 17, normalized size = 1.06

$$\frac{d^n x x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="maxima")
```

```
[Out] d^n*x*x^(3*n)/(3*n + 1)
```

mupad [B] time = 2.50, size = 16, normalized size = 1.00

$$\frac{x\left(d\,x^3\right)^n}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)^n,x)
[Out] (x*(d*x^3)^n)/(3*n + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{d^n x\left(x^3\right)^n}{3n+1} & \text{for } n \neq -\frac{1}{3} \\ \int \frac{1}{\sqrt[3]{dx^3}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3)**n,x)
[Out] Piecewise((d**n*x*(x**3)**n/(3*n + 1), Ne(n, -1/3)), (Integral((d*x**3)**(-
1/3), x), True))
```

3.29 $\int \left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=55

$$\frac{x\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n {}_2F_1\left(-n,2n+1;2(n+1);-\frac{dx}{c}\right)}{2n+1}$$

```
[Out] x*(d*x^3+c*x^2)^n*hypergeom([-n, 1+2*n],[2+2*n],-d*x/c)/(1+2*n)/((1+d*x/c)^
n)
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2011, 66, 64}

$$\frac{x\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n \text{Hypergeometric2F1}\left(-n,2n+1,2(n+1),-\frac{dx}{c}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(c*x^2 + d*x^3)^n*Hypergeometric2F1[-n, 1 + 2*n, 2*(1 + n), -((d*x)/c)])
/((1 + 2*n)*(1 + (d*x)/c)^n)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \left(cx^2+dx^3\right)^n\,dx &= \left(x^{-2n}(c+dx)^{-n}\left(cx^2+dx^3\right)^n\right)\int x^{2n}(c+dx)^n\,dx\\ &= \left(x^{-2n}\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n\right)\int x^{2n}\left(1+\frac{dx}{c}\right)^n\,dx\\ &= \frac{x\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n\,{}_2F_1\left(-n,1+2n;2(1+n);-\frac{dx}{c}\right)}{1+2n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.96

$$\frac{x\left(x^2(c+dx)\right)^n\left(\frac{dx}{c}+1\right)^{-n}\,{}_2F_1\left(-n,2n+1;2n+2;-\frac{dx}{c}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)^n,x]
[Out] (x*(x^2*(c + d*x))^n*Hypergeometric2F1[-n, 1 + 2*n, 2 + 2*n, -((d*x)/c)])/(
(1 + 2*n)*(1 + (d*x)/c)^n)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3+cx^2\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="fricas")
[Out] integral((d*x^3 + c*x^2)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3+cx^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="giac")
```

[Out] integrate((d*x^3 + c*x^2)^n, x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(d\,x^{3}+c\,x^{2}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2)^n,x)

[Out] int((d*x^3+c*x^2)^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^{3}+cx^{2}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2)^n,x, algorithm="maxima")

[Out] integrate((d*x^3 + c*x^2)^n, x)

mupad [B] time = 2.21, size = 56, normalized size = 1.02

$$\frac{x\left(d\,x^{3}+c\,x^{2}\right)^{n}\,{}_2\mathrm{F}_1\left(2\,n+1,-n;\;2\,n+2;\;-\frac{d\,x}{c}\right)}{(2\,n+1)\,\left(\frac{d\,x}{c}+1\right)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2 + d*x^3)^n,x)

[Out] (x*(c*x^2 + d*x^3)^n*hypergeom([2*n + 1, -n], 2*n + 2, -(d*x)/c))/((2*n + 1
)*((d*x)/c + 1)^n)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2}+dx^{3}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+c*x**2)**n,x)

[Out] Integral((c*x**2 + d*x**3)**n, x)

3.30 $\int \left(bx+dx^3\right)^n\,dx$

Optimal. Leaf size=53

$$\frac{x\left(b+dx^2\right)\left(bx+dx^3\right)^n\,{}_2F_1\left(1,\frac{3(n+1)}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)}{b(n+1)}$$

```
[Out] x*(d*x^2+b)*(d*x^3+b*x)^n*hypergeom([1, 3/2+3/2*n],[3/2+1/2*n],-d*x^2/b)/b/
(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2011, 365, 364}

$$\frac{x\left(\frac{dx^2}{b}+1\right)^{-n}\left(bx+dx^3\right)^n\text{Hypergeometric2F1}\left(-n,\frac{n+1}{2},\frac{n+3}{2},-\frac{dx^2}{b}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)^n,x]
```

```
[Out] (x*(b*x + d*x^3)^n*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b)
])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \left(bx+dx^3\right)^n\,dx &= \left(x^{-n}\left(b+dx^2\right)^{-n}\left(bx+dx^3\right)^n\right)\int x^n\left(b+dx^2\right)^n\,dx\\ &= \left(x^{-n}\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\right)\int x^n\left(1+\frac{dx^2}{b}\right)^n\,dx\\ &= \frac{x\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\,{}_2F_1\left(-n,\frac{1+n}{2};\frac{3+n}{2};-\frac{dx^2}{b}\right)}{1+n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.15

$$\frac{x\left(x\left(b+dx^2\right)\right)^n\left(\frac{dx^2}{b}+1\right)^{-n}\,{}_2F_1\left(-n,\frac{n+1}{2};\frac{n+1}{2}+1;-\frac{dx^2}{b}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)^n,x]
[Out] (x*(x*(b + d*x^2))^n*Hypergeometric2F1[-n, (1 + n)/2, 1 + (1 + n)/2, -((d*x
^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3+bx\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="fricas")
[Out] integral((d*x^3 + b*x)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(dx^3+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + b*x)^n, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(d\,x^3+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)^n,x)
```

```
[Out] int((d*x^3+b*x)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + b*x)^n, x)
```

mupad [B] time = 2.22, size = 56, normalized size = 1.06

$$\frac{x\left(d\,x^3+b\,x\right)^n\,{}_2\mathrm{F}_1\left(\frac{n}{2}+\frac{1}{2},-n;\ \frac{n}{2}+\frac{3}{2};\ -\frac{d\,x^2}{b}\right)}{\left(\frac{d\,x^2}{b}+1\right)^n\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^n,x)
```

```
[Out] (x*(b*x + d*x^3)^n*hypergeom([n/2 + 1/2, -n], n/2 + 3/2, -(d*x^2)/b))/(((d*
x^2)/b + 1)^n*(n + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx+dx^3\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)**n,x)
```

```
[Out] Integral((b*x + d*x**3)**n, x)
```

3.31 $\int \left(bx + cx^2 + dx^3\right)^n \, dx$

Optimal. Leaf size=132

$$\frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^n F_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{n+1}$$

```
[Out] x*(d*x^3+c*x^2+b*x)^n*AppellF1(1+n,-n,-n,2+n,-2*d*x/(c-(-4*b*d+c^2)^(1/2)),
-2*d*x/(c+(-4*b*d+c^2)^(1/2)))/(1+n)/((1+2*d*x/(c-(-4*b*d+c^2)^(1/2)))^n)/(
(1+2*d*x/(c+(-4*b*d+c^2)^(1/2)))^n)
```

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1908, 759, 133}

$$\frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}\left(bx+cx^2+dx^3\right)^n F_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c - Sqr
t[c^2 - 4*b*d]), (-2*d*x)/(c + Sqrt[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)/(
c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 1908

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n - q) + c*x
^(2*(n - q)))^p), Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !Int
egerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2+dx^3\right)^n\,dx &= \left(x^{-n}\left(b+cx+dx^2\right)^{-n}\left(bx+cx^2+dx^3\right)^n\right)\int x^n\left(b+cx+dx^2\right)^n\,dx\\ &= \left(x^{-n}\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}\left(bx+cx^2+dx^3\right)^n\right)\text{Subst}\left(\int x^n\left(\right.\right.\\ &= \frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}\left(bx+cx^2+dx^3\right)^n F_1\left(1+n;-n,-n;2+n;-\frac{}{c-\sqrt{}}\right.}{1+n}\end{aligned}$$

Mathematica [A] time = 0.36, size = 157, normalized size = 1.19

$$\frac{x\left(\frac{-\sqrt{c^2-4bd}+c+2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(\frac{\sqrt{c^2-4bd}+c+2dx}{\sqrt{c^2-4bd}+c}\right)^{-n}(x(b+x(c+dx)))^nF_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c+\sqrt{c^2-4bd}},\frac{2dx}{\sqrt{c^2-4bd}-c}\right)}{n+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + x*(c + d*x)))^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c + Sqr
t[c^2 - 4*b*d]), (2*d*x)/(-c + Sqrt[c^2 - 4*b*d])])/((1 + n)*((c - Sqrt[c^2
 - 4*b*d] + 2*d*x)/(c - Sqrt[c^2 - 4*b*d]))^n*((c + Sqrt[c^2 - 4*b*d] + 2*d
*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3+cx^2+bx\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c*x^2 + b*x)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3+cx^2+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(d\,x^3 + c\,x^2 + bx\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)^n,x)
```

```
[Out] int((d*x^3+c*x^2+b*x)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3 + cx^2 + bx\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,x^3 + c\,x^2 + b\,x\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + d*x^3)^n,x)
```

```
[Out] int((b*x + c*x^2 + d*x^3)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx + cx^2 + dx^3\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)**n,x)
```

```
[Out] Integral((b*x + c*x**2 + d*x**3)**n, x)
```

3.32 $\int \left(a+dx^3\right)^n\,dx$

Optimal. Leaf size=35

$$\frac{x\left(a+dx^3\right)^{n+1}\,{}_2F_1\left(1,n+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{a}\right)}{a}$$

```
[Out] x*(d*x^3+a)^(1+n)*hypergeom([1, 4/3+n],[4/3],-d*x^3/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+dx^3\right)^n\left(\frac{dx^3}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{1}{3},-n;\frac{4}{3};-\frac{dx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)^n,x]
```

```
[Out] (x*(a + d*x^3)^n*Hypergeometric2F1[1/3, -n, 4/3, -((d*x^3)/a)])/(1 + (d*x^3
)/a)^n
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+dx^3\right)^n\,dx &= \left(\left(a+dx^3\right)^n\left(1+\frac{dx^3}{a}\right)^{-n}\right)\int \left(1+\frac{dx^3}{a}\right)^n\,dx\\ &= x\left(a+dx^3\right)^n\left(1+\frac{dx^3}{a}\right)^{-n}\,{}_2F_1\left(\frac{1}{3},-n;\frac{4}{3};-\frac{dx^3}{a}\right)\end{aligned}$$

Mathematica [C] time = 0.20, size = 196, normalized size = 5.60

$$\frac{2^{-n}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{d}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{d}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-n}\left(\frac{i\left(\frac{\sqrt[3]{d}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-n}\left(a+dx^3\right)^n F_1\left(n+1;-n,-n;n+2;-\frac{i\left(\sqrt[3]{d}x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\sqrt[3]{a}},-\cdots\right.}{\sqrt[3]{d}(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + d*x^3)^n,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + d^(1/3)*x)*(a + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 +
 n, ((-I)*((-1)^(2/3)*a^(1/3) + d^(1/3)*x))/(Sqrt[3]*a^(1/3)), (I + Sqrt[3]
 - ((2*I)*d^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^n*d^(1/3)*(1 + n)*((a^(1
/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^n*((I*(1 + (d^(1/3)
*x)/a^(1/3)))/(3*I + Sqrt[3]))^n)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(dx^3+a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + a)^n, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\left(d\,x^3+a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)^n,x)
```

```
[Out] int((d*x^3+a)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3 + a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + a)^n, x)
```

mupad [B] time = 2.18, size = 41, normalized size = 1.17

$$\frac{x \left(d\,x^3 + a\right)^n {}_2\mathrm{F}_1\left(\frac{1}{3}, -n;\ \frac{4}{3};\ -\frac{d\,x^3}{a}\right)}{\left(\frac{d\,x^3}{a} + 1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + d*x^3)^n,x)
```

```
[Out] (x*(a + d*x^3)^n*hypergeom([1/3, -n], 4/3, -(d*x^3)/a))/((d*x^3)/a + 1)^n
```

sympy [C] time = 10.54, size = 34, normalized size = 0.97

$$\frac{a^n x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -n \\ \frac{4}{3} \end{matrix}\middle| \frac{dx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+a)**n,x)
```

```
[Out] a**n*x*gamma(1/3)*hyper((1/3, -n), (4/3,), d*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3))
```

3.33 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx$

Optimal. Leaf size=270

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}+\frac{4c^3}{}$$

```
[Out] c^4*(4*a*d^2+c^3)^4*x/d^8-8/3*c^5*(4*a*d^2+c^3)^3*(c/d+x)^3/d^6+4/5*c^3*(4*
a*d^2+c^3)^2*(4*a*d^2+7*c^3)*(c/d+x)^5/d^4-8/7*c^4*(4*a*d^2+c^3)*(12*a*d^2+
7*c^3)*(c/d+x)^7/d^2+2/9*c^2*(48*a^2*d^4+120*a*c^3*d^2+35*c^6)*(c/d+x)^9-8/
11*c^3*d^2*(12*a*d^2+7*c^3)*(c/d+x)^11+4/13*c*d^4*(4*a*d^2+7*c^3)*(c/d+x)^1
3-8/15*c^2*d^6*(c/d+x)^15+1/17*d^8*(c/d+x)^17
```

Rubi [A] time = 0.54, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1106, 1090}

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}-\frac{8c^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)^3)/(3*d^
6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^4) - (8*c
^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*c^2*(35*c^6
 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c^3 + 12*a*d^
2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)/13 - (8*c^2*
d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^4\,dx, x, \frac{c}{d}+x\right)$$

$$= \text{Subst}\left(\int \left(\frac{\left(c^4+4acd^2\right)^4}{d^8}-\frac{8c^5\left(c^3+4ad^2\right)^3x^2}{d^6}+\frac{24c^6\left(c^3+4ad^2\right)^2\left(\frac{7}{6}+\right.}{d^4}\right.\right.$$

$$= \frac{c^4\left(c^3+4ad^2\right)^4x}{d^8}-\frac{8c^5\left(c^3+4ad^2\right)^3\left(\frac{c}{d}+x\right)^3}{3d^6}+\frac{4c^3\left(c^3+4ad^2\right)^2\left(7c^3+4\right.}{5d^4}$$

Mathematica [A] time = 0.04, size = 285, normalized size = 1.06

$$256a^4c^4x+\frac{1024}{3}a^3c^5x^3+256a^3c^4dx^4+512a^2c^5dx^6+\frac{256}{5}a^2c^3x^5\left(ad^2+6c^3\right)+\frac{32}{9}c^2x^9\left(3a^2d^4+120ac^3d^2+8c^6\right)+\frac{64}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] 256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6*
c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^7)
/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3*a^
2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*c^3
+ 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*d^4*(
70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + c*d^7*
x^16 + (d^8*x^17)/17
```

fricas [A] time = 0.39, size = 277, normalized size = 1.03

$$\frac{1}{17}x^{17}d^8+x^{16}d^7c+\frac{112}{15}x^{15}d^6c^2+32x^{14}d^5c^3+\frac{1120}{13}x^{13}d^4c^4+\frac{16}{13}x^{13}d^6ca+\frac{448}{3}x^{12}d^3c^5+16x^{12}d^5c^2a+\frac{1792}{11}x^{11}d^2c^6+\frac{960}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^8 + x^16*d^7*c + 112/15*x^15*d^6*c^2 + 32*x^14*d^5*c^3 + 1120/1
3*x^13*d^4*c^4 + 16/13*x^13*d^6*c*a + 448/3*x^12*d^3*c^5 + 16*x^12*d^5*c^2*
a + 1792/11*x^11*d^2*c^6 + 960/11*x^11*d^4*c^3*a + 512/5*x^10*d*c^7 + 256*x
^10*d^3*c^4*a + 256/9*x^9*c^8 + 1280/3*x^9*d^2*c^5*a + 32/3*x^9*d^4*c^2*a^2
 + 384*x^8*d*c^6*a + 96*x^8*d^3*c^3*a^2 + 1024/7*x^7*c^7*a + 2304/7*x^7*d^2
```

```
*c^4*a^2 + 512*x^6*d*c^5*a^2 + 1536/5*x^5*c^6*a^2 + 256/5*x^5*d^2*c^3*a^3 +
 256*x^4*d*c^4*a^3 + 1024/3*x^3*c^5*a^3 + 256*x*c^4*a^4
```

giac [A] time = 0.24, size = 277, normalized size = 1.03

$$\frac{1}{17}d^8x^{17}+cd^7x^{16}+\frac{112}{15}c^2d^6x^{15}+32c^3d^5x^{14}+\frac{1120}{13}c^4d^4x^{13}+\frac{16}{13}acd^6x^{13}+\frac{448}{3}c^5d^3x^{12}+16ac^2d^5x^{12}+\frac{1792}{11}c^6d^2x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1120/1
3*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^12 + 16*a*c^2*d^5*x^1
2 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x^11 + 512/5*c^7*d*x^10 + 256*a
*c^4*d^3*x^10 + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4*x^9
 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^2*c^4
*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2*x^5 +
 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x
```

maple [A] time = 0.00, size = 392, normalized size = 1.45

$$\frac{d^8x^{17}}{17}+c\,d^7x^{16}+\frac{112c^2d^6x^{15}}{15}+32c^3d^5x^{14}+512a^2c^5d\,x^6+\frac{\left(1088c^4d^4+2\left(8ac\,d^2+16c^4\right)d^4\right)x^{13}}{13}+256a^3c^4d\,x^4+\frac{(64a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x)
```

```
[Out] 1/17*d^8*x^17+c*d^7*x^16+112/15*c^2*d^6*x^15+32*c^3*d^5*x^14+1/13*(2*(8*a*c
*d^2+16*c^4)*d^4+1088*c^4*d^4)*x^13+1/12*(64*a*c^2*d^5+16*(8*a*c*d^2+16*c^4
)*c*d^3+1536*c^5*d^3)*x^12+1/11*(576*a*c^3*d^4+48*(8*a*c*d^2+16*c^4)*c^2*d^
2+1024*c^6*d^2)*x^11+1/10*(2048*a*c^4*d^3+64*(8*a*c*d^2+16*c^4)*c^3*d)*x^10
+1/9*(32*a^2*c^2*d^4+3584*a*c^5*d^2+(8*a*c*d^2+16*c^4)^2)*x^9+1/8*(256*a^2*
c^3*d^3+2048*a*c^6*d+64*a*c^2*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(1792*a^2*c^4*d
^2+64*a*c^3*(8*a*c*d^2+16*c^4))*x^7+512*a^2*c^5*d*x^6+1/5*(32*a^2*c^2*(8*a*
c*d^2+16*c^4)+1024*a^2*c^6)*x^5+256*a^3*c^4*d*x^4+1024/3*a^3*c^5*x^3+256*a^
4*c^4*x
```

maxima [A] time = 0.63, size = 372, normalized size = 1.38

$$\frac{1}{17}d^8x^{17}+cd^7x^{16}+\frac{32}{5}c^2d^6x^{15}+\frac{128}{7}c^3d^5x^{14}+\frac{256}{13}c^4d^4x^{13}+\frac{256}{9}c^8x^9+256a^4c^4x+\frac{256}{15}\left(3d^2x^5+15cdx^4+20c^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/1
3*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c*d
*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/105
*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*
x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12 + 624
*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 65520*c^2
*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 + 9*c*d*
x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2)*a*c +
16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*c^3*d^3*x
^12)*c^2
```

mupad [B] time = 2.30, size = 261, normalized size = 0.97

$$x^{10}\left(\frac{512\,c^7\,d}{5}+256\,a\,c^4\,d^3\right)+x^{13}\left(\frac{1120\,c^4\,d^4}{13}+\frac{16\,a\,c\,d^6}{13}\right)+x^9\left(\frac{32\,a^2\,c^2\,d^4}{3}+\frac{1280\,a\,c^5\,d^2}{3}+\frac{256\,c^8}{9}\right)+x^{12}\left(\frac{448\,c^5}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^4,x)
```

```
[Out] x^10*((512*c^7*d)/5 + 256*a*c^4*d^3) + x^13*((1120*c^4*d^4)/13 + (16*a*c*d^
6)/13) + x^9*((256*c^8)/9 + (1280*a*c^5*d^2)/3 + (32*a^2*c^2*d^4)/3) + x^12
*((448*c^5*d^3)/3 + 16*a*c^2*d^5) + x^11*((1792*c^6*d^2)/11 + (960*a*c^3*d^
4)/11) + (d^8*x^17)/17 + 256*a^4*c^4*x + c*d^7*x^16 + (1024*a^3*c^5*x^3)/3
+ 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + 256*a^3*c^4*d*x^4 + 512*a^2*c^5
*d*x^6 + (256*a*c^4*x^7*(9*a*d^2 + 4*c^3))/7 + (256*a^2*c^3*x^5*(a*d^2 + 6*
c^3))/5 + 96*a*c^3*d*x^8*(a*d^2 + 4*c^3)
```

sympy [A] time = 0.13, size = 299, normalized size = 1.11

$$256a^4c^4x+\frac{1024a^3c^5x^3}{3}+256a^3c^4dx^4+512a^2c^5dx^6+32c^3d^5x^{14}+\frac{112c^2d^6x^{15}}{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}+x^{13}\left(\frac{16acd^6}{13}+\frac{1120c^4}{13}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

```
[Out] 256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*c
**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 + d
**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*c**
2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/11)
+ x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 + 1280
*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d) + x*
*7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**2/5 + 1
536*a**2*c**6/5)
```

3.34 $\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^3 \, dx$

Optimal. Leaf size=171

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{48}{5}$$

```
[Out] 64*a^3*c^3*x+64*a^2*c^4*x^3+48*a^2*c^3*d*x^4+48/5*a*c^2*(a*d^2+4*c^3)*x^5+6
4*a*c^4*d*x^6+32/7*c^3*(9*a*d^2+2*c^3)*x^7+12*c^2*d*(a*d^2+2*c^3)*x^8+4/3*c
*d^2*(a*d^2+20*c^3)*x^9+16*c^3*d^3*x^10+60/11*c^2*d^4*x^11+c*d^5*x^12+1/13*
d^6*x^13
```

Rubi [A] time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2061}

$$48a^2c^3dx^4+64a^2c^4x^3+64a^3c^3x+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 \, dx = \int \left(64a^3c^3+192a^2c^4x^2+192a^2c^3dx^3+48ac^2\left(4c^3+ad^2\right)x^4+384ac^4d\right.$$

$$= 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5+64ac^4dx^6+$$

Mathematica [A] time = 0.03, size = 171, normalized size = 1.00

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{48}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

fricas [A] time = 0.55, size = 166, normalized size = 0.97

$$\frac{1}{13}x^{13}d^6+x^{12}d^5c+\frac{60}{11}x^{11}d^4c^2+16x^{10}d^3c^3+\frac{80}{3}x^9d^2c^4+\frac{4}{3}x^9d^4ca+24x^8dc^5+12x^8d^3c^2a+\frac{64}{7}x^7c^6+\frac{288}{7}x^7d^2c^3a+64x^6d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^6 + x^12*d^5*c + 60/11*x^11*d^4*c^2 + 16*x^10*d^3*c^3 + 80/3*x^
9*d^2*c^4 + 4/3*x^9*d^4*c*a + 24*x^8*d*c^5 + 12*x^8*d^3*c^2*a + 64/7*x^7*c^
6 + 288/7*x^7*d^2*c^3*a + 64*x^6*d*c^4*a + 192/5*x^5*c^5*a + 48/5*x^5*d^2*c
^2*a^2 + 48*x^4*d*c^3*a^2 + 64*x^3*c^4*a^2 + 64*x*c^3*a^3
```

giac [A] time = 0.24, size = 166, normalized size = 0.97

$$\frac{1}{13}d^6x^{13}+cd^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{80}{3}c^4d^2x^9+\frac{4}{3}acd^4x^9+24c^5dx^8+12ac^2d^3x^8+\frac{64}{7}c^6x^7+\frac{288}{7}ac^3d^2x^7+64$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="giac")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 80/3*c^
4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*x^
7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^2*d
^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x
```

maple [A] time = 0.00, size = 231, normalized size = 1.35

$$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+64a\,c^4d\,x^6+48a^2c^3d\,x^4+64a^2c^4x^3+\frac{\left(4ac\,d^4+224c^4d^2+\left(8ac\,d^2+16c^4\right)d^2\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x)
```

```
[Out] 1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+1/9*(4*a*c*d^4+
224*c^4*d^2+d^2*(8*a*c*d^2+16*c^4))*x^9+1/8*(64*a*c^2*d^3+128*c^5*d+4*c*d*(
```

```
8*a*c*d^2+16*c^4))*x^8+1/7*(256*a*c^3*d^2+4*c^2*(8*a*c*d^2+16*c^4))*x^7+64*
a*c^4*d*x^6+1/5*(4*a*c*(8*a*c*d^2+16*c^4)+128*c^5*a+16*d^2*a^2*c^2)*x^5+48*
a^2*c^3*d*x^4+64*a^2*c^4*x^3+64*a^3*c^3*x
```

maxima [A] time = 0.80, size = 205, normalized size = 1.20

$$\frac{1}{13}d^6x^{13}+cd^5x^{12}+\frac{48}{11}c^2d^4x^{11}+\frac{32}{5}c^3d^3x^{10}+\frac{64}{7}c^6x^7+64a^3c^3x+\frac{16}{5}\left(3d^2x^5+15cdx^4+20c^2x^3\right)a^2c^2+\frac{8}{3}\left(2d^2x^9+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7*
c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c^2
 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 72
0*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c + 4/16
5*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
```

mupad [B] time = 2.16, size = 160, normalized size = 0.94

$$x^8\left(24c^5d+12ac^2d^3\right)+x^9\left(\frac{80c^4d^2}{3}+\frac{4acd^4}{3}\right)+\frac{d^6x^{13}}{13}+x^7\left(\frac{64c^6}{7}+\frac{288ac^3d^2}{7}\right)+64a^3c^3x+cd^5x^{12}+64a^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^3,x)
```

```
[Out] x^8*(24*c^5*d + 12*a*c^2*d^3) + x^9*((80*c^4*d^2)/3 + (4*a*c*d^4)/3) + (d^6
*x^13)/13 + x^7*((64*c^6)/7 + (288*a*c^3*d^2)/7) + 64*a^3*c^3*x + c*d^5*x^1
2 + 64*a^2*c^4*x^3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + 48*a^2*c^3*d*
x^4 + (48*a*c^2*x^5*(a*d^2 + 4*c^3))/5 + 64*a*c^4*d*x^6
```

sympy [A] time = 0.10, size = 180, normalized size = 1.05

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+16c^3d^3x^{10}+\frac{60c^2d^4x^{11}}{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}+x^9\left(\frac{4acd^4}{3}+\frac{80c^4d^2}{3}\right)+x^8\left(12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

```
[Out] 64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**6
 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13/1
3 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**5*d
) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 + 192*
a*c**5/5)
```

3.35 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx$

Optimal. Leaf size=92

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

```
[Out] 16*a^2*c^2*x+32/3*a*c^3*x^3+8*a*c^2*d*x^4+8/5*c*(a*d^2+2*c^3)*x^5+16/3*c^3*
d*x^6+24/7*c^2*d^2*x^7+c*d^3*x^8+1/9*d^4*x^9
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2061}

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+8ac^2dx^4+\frac{32}{3}ac^3x^3+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3dx^6+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx &= \int \left(16a^2c^2+32ac^3x^2+32ac^2dx^3+8c\left(2c^3+ad^2\right)x^4+32c^3dx^5+24c^2d^2\right.\\ &= 16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c\left(2c^3+ad^2\right)x^5+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7\end{aligned}$$

Mathematica [A] time = 0.02, size = 92, normalized size = 1.00

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

fricas [A] time = 0.40, size = 83, normalized size = 0.90

$$\frac{1}{9}x^9d^4 + x^8d^3c + \frac{24}{7}x^7d^2c^2 + \frac{16}{3}x^6dc^3 + \frac{16}{5}x^5c^4 + \frac{8}{5}x^5d^2ca + 8x^4dc^2a + \frac{32}{3}x^3c^3a + 16xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^4 + x^8*d^3*c + 24/7*x^7*d^2*c^2 + 16/3*x^6*d*c^3 + 16/5*x^5*c^4
+ 8/5*x^5*d^2*c*a + 8*x^4*d*c^2*a + 32/3*x^3*c^3*a + 16*x*c^2*a^2
```

giac [A] time = 0.25, size = 83, normalized size = 0.90

$$\frac{1}{9}\,d^4x^9 + cd^3x^8 + \frac{24}{7}\,c^2d^2x^7 + \frac{16}{3}\,c^3dx^6 + \frac{16}{5}\,c^4x^5 + \frac{8}{5}\,acd^2x^5 + 8\,ac^2dx^4 + \frac{32}{3}\,ac^3x^3 + 16\,a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5
+ 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x
```

maple [A] time = 0.00, size = 84, normalized size = 0.91

$$\frac{d^4x^9}{9} + c\,d^3x^8 + \frac{24c^2d^2x^7}{7} + \frac{16c^3d\,x^6}{3} + 8a\,c^2d\,x^4 + \frac{32a\,c^3x^3}{3} + 16a^2c^2x + \frac{\left(8ac\,d^2 + 16c^4\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

```
[Out] 1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+1/5*(8*a*c*d^2+16*c^4
)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x
```

maxima [A] time = 0.57, size = 94, normalized size = 1.02

$$\frac{1}{9}\,d^4x^9+cd^3x^8+\frac{16}{7}\,c^2d^2x^7+\frac{16}{5}\,c^4x^5+16\,a^2c^2x+\frac{8}{15}\left(3\,d^2x^5 + 15\,cdx^4 + 20\,c^2x^3\right)ac+\frac{8}{21}\left(3\,d^2x^7 + 14\,cdx^6\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x +
8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d*x
^6)*c^2
```

mupad [B] time = 0.04, size = 82, normalized size = 0.89

$$x^5\left(\frac{16\,c^4}{5}+\frac{8\,a\,c\,d^2}{5}\right)+\frac{d^4\,x^9}{9}+16\,a^2\,c^2\,x+\frac{32\,a\,c^3\,x^3}{3}+\frac{16\,c^3\,d\,x^6}{3}+c\,d^3\,x^8+\frac{24\,c^2\,d^2\,x^7}{7}+8\,a\,c^2\,d\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

```
[Out] x^5*((16*c^4)/5 + (8*a*c*d^2)/5) + (d^4*x^9)/9 + 16*a^2*c^2*x + (32*a*c^3*x
^3)/3 + (16*c^3*d*x^6)/3 + c*d^3*x^8 + (24*c^2*d^2*x^7)/7 + 8*a*c^2*d*x^4
```

sympy [A] time = 0.08, size = 95, normalized size = 1.03

$$16a^2c^2x+\frac{32ac^3x^3}{3}+8ac^2dx^4+\frac{16c^3dx^6}{3}+\frac{24c^2d^2x^7}{7}+cd^3x^8+\frac{d^4x^9}{9}+x^5\left(\frac{8acd^2}{5}+\frac{16c^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] 16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 24
*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c**
4/5)
```

3.36 $\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right) dx$

Optimal. Leaf size=32

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

```
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi steps

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right) dx = 4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

fricas [A] time = 0.39, size = 28, normalized size = 0.88

$$\frac{1}{5}x^5d^2 + x^4dc + \frac{4}{3}x^3c^2 + 4xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^2 + x^4*d*c + 4/3*x^3*c^2 + 4*x*c*a
```

giac [A] time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{5}\,d^2x^5 + cdx^4 + \frac{4}{3}\,c^2x^3 + 4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{1}{5}d^2x^5 + cd\,x^4 + \frac{4}{3}c^2x^3 + 4acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x)
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

maxima [A] time = 0.65, size = 28, normalized size = 0.88

$$\frac{1}{5}\,d^2x^5 + cdx^4 + \frac{4}{3}\,c^2x^3 + 4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

mupad [B] time = 0.04, size = 28, normalized size = 0.88

$$\frac{4\,c^2\,x^3}{3} + c\,d\,x^4 + 4\,a\,c\,x + \frac{d^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3,x)
[Out] (4*c^2*x^3)/3 + (d^2*x^5)/5 + 4*a*c*x + c*d*x^4
```

sympy [A] time = 0.07, size = 31, normalized size = 0.97

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)
```

```
[Out] 4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

3.37 $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal. Leaf size=529

$$-\frac{d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}\,d\right.}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2}}$$

```
[Out] -1/4*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))
^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d
^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/4*d*arctanh((-(d*x+c)*2
^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d
^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2
+c^3)^(1/2))^(1/2)-1/8*d*ln(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)-c^(1/
4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(
4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/8*d*ln(d^2*(c/d+x)
^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+
c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^
3)^(1/2))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1106, 1094, 634, 618, 206, 628}

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+\sqrt{c}\sqrt{4ad^2+c^3}+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+\right.}{4\sqrt{2}\,c^{3/4}\sqrt{4ad}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] -(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + Sqrt
[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)
*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*ArcTanh[(c^(
1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt
[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*S
qrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] -
Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d
+ x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4
*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(
```

```
3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(4*Sqrt[2]*c^(3/4
)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Subst}\left(\int \frac{1}{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{d\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}-x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,x}{d}+x^2}\,dx, x, \frac{c}{d}+x\right)}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{c}\sqrt{}}{}}\right.}{2\sqrt{2}\,c^{3/4}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,x}{d}+x^2}\,dx, x, \frac{c}{d}+x\right)}{4\sqrt{c}\sqrt{c^3+4ad^2}} + \frac{\text{Subst}\left(\int \frac{}{\frac{\sqrt{c}\sqrt{c^3+4a}}{d^2}}\right.}{}$$

$$= -\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,(c+dx)+(c+dx)^2\right)}{4\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$= -\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}\,c^{3/4}-\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}\,dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}} - \frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}\,c^{3/4}+\sqrt{c^{3/2}+\sqrt{c}}\right.}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3}}}\right.}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^4d^2+4\#1^3cd+4\#1^2c^2+4ac\&, \frac{\log(x-\#1)}{\#1^3d^2+3\#1^2cd+2\#1c^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*#
1 + 3*c*d*#1^2 + d^2*#1^3) & ]/4
```

fricas [B] time = 0.49, size = 905, normalized size = 1.71

$$\frac{1}{8}\sqrt{-\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}+1}{ac^3+4a^2d^2}}\log\left(d^2x+cd+\left(2acd^2+\left(ac^7+4a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*
c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 + (a*c^7 +
 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-
(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4))
+ 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*
c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x
+ c*d - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d
^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^
2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) + 1/8*sqrt((2*(a*c^
3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*
c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sq
rt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d
^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2
*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2
+ 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d - (2*a*c*d^2 -
 (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)
))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^
3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2)))
```

giac [A] time = 0.34, size = 603, normalized size = 1.14

$$-\frac{\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)\right)}+\frac{}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")
```

```
[Out] -1/4*log(x + sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*
d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)
*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d))
 + 1/4*log(x - sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^
2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*
c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d
)) - 1/4*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((
c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c
/d)) + 1/4*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt
((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt
```

```
(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) -
 c/d))
```

maple [C] time = 0.06, size = 64, normalized size = 0.12

$$\frac{\ln\left(-\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)+x\right)}{4\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^3d^2+12\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^2cd+8\,\mathrm{RootO}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x)
```

```
[Out] 1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(-_R+x),_R=RootOf(_Z^4*d^2+4*_Z^
3*c*d+4*_Z^2*c^2+4*a*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="maxima")
```

```
[Out] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

mupad [B] time = 4.54, size = 1551, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3),x)
```

```
[Out] atan(((-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2
)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*
(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a
*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i + (-(2*
d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*
c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 +
4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)/((-(2*d*(-a^3*c^3
)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 2
56*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d
^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4
*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - (-(2*d*(-a^3*c^3)^(1/2) + a*c^
3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x
```

```
)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) +
64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))
)^(1/2) + 4*c*d^5 + 4*d^6*x)))*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^
6 + 4*a^3*c^3*d^2)))^(1/2)*2i + atan((((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*
(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^
6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) +
4*c*d^5 + 4*d^6*x)*1i + ((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^
3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/
2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*((2*d*(-a^3
*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^
6*x)*1i)/(((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(
1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(6
4*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) -
a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - ((2*d*(
-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4
*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^
3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c
^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)))*((2*d*(-a^3*c^3)^(1/2) -
 a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*2i
```

sympy [A] time = 1.15, size = 88, normalized size = 0.17

$$\operatorname{RootSum}\left(t^4\left(16384a^3c^3d^2+4096a^2c^6\right)+128t^2ac^3+1,\left(t\mapsto t\log\left(x+\frac{-1024t^3a^2c^4d^2-256t^3ac^7+16tacd^2-}{d^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

```
[Out] RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 +
1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 + 1
6*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))
```

3.38 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$

Optimal. Leaf size=746

$$-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\cdots$$

```
[Out] -1/16*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c*
d*x^3+4*c^2*x^2+4*a*c)-1/64*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(3/
2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))
*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4)/(4*a*d^2+c^3)^(3/2)*2
^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/64*d*arctanh((-(d*x+c)*2^(1/2)
+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3
)^(1/2))^(1/2))*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4)/(4*a*d
^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)-1/128*d*ln(d^2*(c
/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)-c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a
*d^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4
)/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/128*d*l
n(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3
/2)+(4*a*d^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))/
a/c^(7/4)/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)
```

Rubi [A] time = 1.33, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1106, 1092, 1169, 634, 618, 206, 628}

$$-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(-\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+\cdots}\right.}{64\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(16*a*c*(c^3 + 4*a*d^2)*(4
*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqr
t[c^3 + 4*a*d^2])*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*
a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(32*
Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]
) + (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sqrt
```

```
[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2)
- Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[
c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3 + 4
*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) +
 Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(64*Sqrt[2]*a*c^(7/4)*(
c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*
d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2
]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2
])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*
a*d^2]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^2}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{\text{Subst}\left(\int \frac{4c^4-2c\left(4a+\frac{c}{d}\right.}{}\right.}{}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/}}}{}\right.}{}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{\left(d\left(c^3+12ad^2-c^{3/2}\right.\right.}{}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2-c^{3/2}\right.}{}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2+c^{3/2}\right.}{32\sqrt{2}}$$

Mathematica [C] time = 0.12, size = 182, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^4d^2+4\#1^3cd+4\#1^2c^2+4ac\&, \frac{\#1^2cd^2\log(x-\#1)+12ad^2\log(x-\#1)+2c^3\log(x-\#1)+2\#1c^2d\log(x-\#1)}{\#1^3d^2+3\#1^2cd+2\#1c^2}\&\right]+\frac{4(c+dx)(}{4ac+}}{64ac\left(4ad^2+c^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] ((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + Root
Sum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , (2*c^3*Log[x - #1] + 12*
a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 + c*d^2*Log[x - #1]*#1^2)/(2*c^2
*#1 + 3*c*d*#1^2 + d^2*#1^3) & ])/(64*a*c*(c^3 + 4*a*d^2))
```

fricas [B] time = 0.55, size = 3222, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3*c^2*d^2 +
 (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6
 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11
 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 36
0*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4
 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^
7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*lo
g(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 32
4*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^1
9 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8
)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^2
2*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8
*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 +
2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c
^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a
^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 +
4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c
^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4
*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6
+ 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4
 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*
c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^
13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d
^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a
^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 + 96
*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13
*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^
8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^
6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*
sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*
a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*
d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 +
3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 1
```

```
2*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + (4*a^2*c^5 + 16*a^3*c^
2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4
*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a
^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d
^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c
^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096
*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d
^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d
^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (
a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*
c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*
a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6
144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2
*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt
(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2
+ 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*
d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 6
4*a^6*c^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*
x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt
(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*
c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10
)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840
*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^
4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5
+ 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d
^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a
^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a
*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 +
1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d
^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^
2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 12
96*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16
*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c
^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + 8*(c^3 + 2*a*d^
2)*x)/(4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^
5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)
```

giac [A] time = 0.37, size = 1057, normalized size = 1.42

$$-\frac{\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^2-2\,c^2d\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)+2\,c^3+12\,ad^2\right)\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)}{d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)}-\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}-\frac{c}{d}\right)^2+2\,c^2d\left(\sqrt{\frac{c^2}{}}\right.\right.}{d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}-\frac{c}{d}\right)^3+3\,cd}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] -1/64*((c*d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(s
qrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x + sq
rt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a
*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/
d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt
((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 + 2*sq
rt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2 + 2*sqr
t(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d
)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt(
(c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) + (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(
-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4)
+ c/d) + 2*c^3 + 12*a*d^2)*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) +
 c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((
c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c
/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12
*a*d^2)*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c
^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a
*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/
d)))/(a*c^4 + 4*a^2*c*d^2) + 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 2*c^3*x + 4*a*
d^2*x + 4*a*c*d)/((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)*(a*c^4 + 4*a^2*
c*d^2))
```

maple [C] time = 0.02, size = 232, normalized size = 0.31

$$\frac{\left(\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^2 c\,d^2+2\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)c^2d+12a\,d^2+\right.}{64\left(4a\,d^2+c^3\right)ac\left(\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^3d^2+3\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

```
[Out] (1/16*d^2/a/(4*a*d^2+c^3)*x^3+3/16/a*c*d/(4*a*d^2+c^3)*x^2+1/8/c*(2*a*d^2+c
^3)/(4*a*d^2+c^3)/a*x+1/4*d/(4*a*d^2+c^3))/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a
*c)+1/64/(4*a*d^2+c^3)/a/c*sum((_R^2*c*d^2+2*_R*c^2*d+12*a*d^2+2*c^3)/(_R^3
*d^2+3*_R^2*c*d+2*_R*c^2)*ln(-_R+x),_R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2*c^
2+4*a*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5 +
 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^
3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*c^2
*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/(a*c
^4 + 4*a^2*c*d^2)
```

mupad [B] time = 4.30, size = 5844, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

```
[Out] (d/(4*(4*a*d^2 + c^3)) + (d^2*x^3)/(16*a*(4*a*d^2 + c^3)) + (x*(2*a*d^2 + c
^3))/(8*a*c*(4*a*d^2 + c^3)) + (3*c*d*x^2)/(16*a*(4*a*d^2 + c^3)))/(4*a*c +
 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3) - atan(((-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^
7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(40
96*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*
((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*
(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768
*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c
^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a
^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 +
 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*
c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*
d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*
c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48
*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 +
704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36
*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*
c^2*d^4)))*1i + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2
+ 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13
*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 +
2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 +
 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c
^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10
*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^
9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*
c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d
^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^
3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c
```

```
^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7
*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^
3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a
*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i)/((9*a*d^8 +
 c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-(a^3*c^11 +
10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-
a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^
9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*
a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a
^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3
*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15
*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16
+ 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c
^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^
5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^
4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 1
2*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5
+ 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5
*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^
3*c^5*d^2 + 16*a^4*c^2*d^4))) + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) +
 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^
16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144
*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 +
 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d
^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*
(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4
 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^
10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 +
196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(
a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 +
72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*
d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^
4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10
+ c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))
))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*
d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8
*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*2i - atan(((-(a^3*c^11 - 10*c^3*d^3*(-a
^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2)
)/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(
1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(
1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 +
32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*
a^4*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 +
 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*
d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536
```

*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i + (-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i)/((9*a*d^8 + c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))) + (-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d

```
^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4))
)*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d
^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*
c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a
^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*
d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d
^4)))))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5
*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 4
8*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*2i
```

sympy [A] time = 109.97, size = 427, normalized size = 0.57

$$\frac{4acd + 3c^2dx^2 + cd^2x^3 + x\left(4ad^2 + 2c^3\right)}{256a^3c^2d^2 + 64a^2c^5 + x^4\left(64a^2cd^4 + 16ac^4d^2\right) + x^3\left(256a^2c^2d^3 + 64ac^5d\right) + x^2\left(256a^2c^3d^2 + 64ac^6\right)} + \mathrm{RootSum}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] (4*a*c*d + 3*c**2*d*x**2 + c*d**2*x**3 + x*(4*a*d**2 + 2*c**3))/(256*a**3*c
**2*d**2 + 64*a**2*c**5 + x**4*(64*a**2*c*d**4 + 16*a*c**4*d**2) + x**3*(25
6*a**2*c**2*d**3 + 64*a*c**5*d) + x**2*(256*a**2*c**3*d**2 + 64*a*c**6)) +
RootSum(_t**4*(1073741824*a**9*c**7*d**6 + 805306368*a**8*c**10*d**4 + 2013
26592*a**7*c**13*d**2 + 16777216*a**6*c**16) + _t**2*(491520*a**5*c**5*d**4
 + 122880*a**4*c**8*d**2 + 8192*a**3*c**11) + 81*a**2*d**4 + 18*a*c**3*d**2
 + c**6, Lambda(_t, _t*log(x + (-67108864*_t**3*a**7*c**7*d**8 - 58720256*_
t**3*a**6*c**10*d**6 - 18874368*_t**3*a**5*c**13*d**4 - 2621440*_t**3*a**4*
c**16*d**2 - 131072*_t**3*a**3*c**19 + 27648*_t*a**4*c**2*d**8 - 9216*_t*a*
*3*c**5*d**6 - 5440*_t*a**2*c**8*d**4 - 736*_t*a*c**11*d**2 - 32*_t*c**14 +
 324*a**2*c*d**7 + 81*a*c**4*d**5 + 5*c**7*d**3)/(324*a**2*d**8 + 81*a*c**3
*d**6 + 5*c**6*d**4))))
```

3.39 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4 \, dx$

Optimal. Leaf size=295

$$\frac{1}{24}e^4\left(65536a^2e^6+20992ad^4e^3+601d^8\right)\left(\frac{d}{4e}+x\right)^9+\frac{\left(256ae^3+5d^4\right)^2\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^5}{5120}+\frac{64}{13}e^8\left(256ae^3\right.$$

```
[Out] 1/1048576*(256*a*e^3+5*d^4)^4*x/e^4-1/8192*d^2*(256*a*e^3+5*d^4)^3*(1/4*d/e
+x)^3/e^2+1/5120*(256*a*e^3+5*d^4)^2*(256*a*e^3+59*d^4)*(1/4*d/e+x)^5-9/224
*d^2*e^2*(256*a*e^3+5*d^4)*(256*a*e^3+17*d^4)*(1/4*d/e+x)^7+1/24*e^4*(65536
*a^2*e^6+20992*a*d^4*e^3+601*d^8)*(1/4*d/e+x)^9-72/11*d^2*e^6*(256*a*e^3+17
*d^4)*(1/4*d/e+x)^11+64/13*e^8*(256*a*e^3+59*d^4)*(1/4*d/e+x)^13-2048/5*d^2
*e^10*(1/4*d/e+x)^15+4096/17*e^12*(1/4*d/e+x)^17
```

Rubi [A] time = 0.53, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1106, 1090}

$$\frac{1}{24}e^4\left(65536a^2e^6+20992ad^4e^3+601d^8\right)\left(\frac{d}{4e}+x\right)^9+\frac{64}{13}e^8\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^{13}-\frac{72}{11}d^2e^6\left(256ae^3+17d^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3)^3*(d/(4*
e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*a*e^3)*(d/(4*e
) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 256*a*e^3)*(d/(4*
e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e)
+ x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^
8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^
15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
```

```
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4 dx = \text{Subst}\left(\int \left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^4 dx, x, \frac{d}{4e}+x\right)$$

$$= \text{Subst}\left(\int \left(\frac{\left(5d^4+256ae^3\right)^4}{1048576e^4}-\frac{3d^2\left(5d^4+256ae^3\right)^3x^2}{8192e^2}+\frac{27}{512}d^4\left(5d^4+2\right.\right.\right.$$

$$= \frac{\left(5d^4+256ae^3\right)^4 x}{1048576e^4}-\frac{d^2\left(5d^4+256ae^3\right)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2}+\frac{\left(5d^4+256ae^3\right)^2\left(5\right.}{}$$

Mathematica [A] time = 0.05, size = 345, normalized size = 1.17

$$4096a^4e^8x-1024a^3d^3e^6x^2+8ade^2x^4\left(512a^2e^6-d^8\right)+128a^2d^6e^4x^3-4de^3x^8\left(-1536a^2e^6+192ad^4e^3+d^8\right)+\frac{128}{3}e^4x^9\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] 4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d
^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 -
 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768
*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*
e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e
^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4
- 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 +
 (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17
```

fricas [A] time = 0.39, size = 353, normalized size = 1.20

$$\frac{4096}{17}x^{17}e^{12}+1024x^{16}e^{11}d+\frac{8192}{5}x^{15}e^{10}d^2+1024x^{14}e^9d^3-\frac{2048}{13}x^{13}e^8d^4+\frac{16384}{13}x^{13}e^{11}a-512x^{12}e^7d^5+4096x^{12}e^{10}da-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17*e^12 + 1024*x^16*e^11*d + 8192/5*x^15*e^10*d^2 + 1024*x^14*e^9
*d^3 - 2048/13*x^13*e^8*d^4 + 16384/13*x^13*e^11*a - 512*x^12*e^7*d^5 + 409
6*x^12*e^10*d*a - 1664/11*x^11*e^6*d^6 + 49152/11*x^11*e^9*d^2*a + 384/5*x^
10*e^5*d^7 + 1024*x^10*e^8*d^3*a + 128/3*x^9*e^4*d^8 - 4096/3*x^9*e^7*d^4*a
 + 8192/3*x^9*e^10*a^2 - 4*x^8*e^3*d^9 - 768*x^8*e^6*d^5*a + 6144*x^8*e^9*d
*a^2 - 32/7*x^7*e^2*d^10 + 768/7*x^7*e^5*d^6*a + 24576/7*x^7*e^8*d^2*a^2 +
128*x^6*e^4*d^7*a - 1024*x^6*e^7*d^3*a^2 + 1/5*x^5*d^12 - 6144/5*x^5*e^6*d^
4*a^2 + 16384/5*x^5*e^9*a^3 - 8*x^4*e^2*d^9*a + 4096*x^4*e^8*d*a^3 + 128*x^
3*e^4*d^6*a^2 - 1024*x^2*e^6*d^3*a^3 + 4096*x*e^8*a^4
```

giac [A] time = 0.31, size = 323, normalized size = 1.09

$$\frac{4096}{17}x^{17}e^{12}+1024\,dx^{16}e^{11}+\frac{8192}{5}\,d^2x^{15}e^{10}+1024\,d^3x^{14}e^9-\frac{2048}{13}\,d^4x^{13}e^8-512\,d^5x^{12}e^7-\frac{1664}{11}\,d^6x^{11}e^6+\frac{384}{5}\,d^7x^{10}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17*e^12 + 1024*d*x^16*e^11 + 8192/5*d^2*x^15*e^10 + 1024*d^3*x^14
*e^9 - 2048/13*d^4*x^13*e^8 - 512*d^5*x^12*e^7 - 1664/11*d^6*x^11*e^6 + 384
/5*d^7*x^10*e^5 + 128/3*d^8*x^9*e^4 - 4*d^9*x^8*e^3 - 32/7*d^10*x^7*e^2 + 1
/5*d^12*x^5 + 16384/13*a*x^13*e^11 + 4096*a*d*x^12*e^10 + 49152/11*a*d^2*x^
11*e^9 + 1024*a*d^3*x^10*e^8 - 4096/3*a*d^4*x^9*e^7 - 768*a*d^5*x^8*e^6 + 7
68/7*a*d^6*x^7*e^5 + 128*a*d^7*x^6*e^4 - 8*a*d^9*x^4*e^2 + 8192/3*a^2*x^9*e
^10 + 6144*a^2*d*x^8*e^9 + 24576/7*a^2*d^2*x^7*e^8 - 1024*a^2*d^3*x^6*e^7 -
 6144/5*a^2*d^4*x^5*e^6 + 128*a^2*d^6*x^3*e^4 + 16384/5*a^3*x^5*e^9 + 4096*
a^3*d*x^4*e^8 - 1024*a^3*d^3*x^2*e^6 + 4096*a^4*x*e^8
```

maple [A] time = 0.00, size = 500, normalized size = 1.69

$$\frac{4096e^{12}x^{17}}{17}+1024d\,e^{11}x^{16}+\frac{8192d^2e^{10}x^{15}}{5}+1024d^3e^9x^{14}+\frac{128\left(128a\,e^5-16d^4e^2\right)e^6x^{13}}{13}+128a^2d^6e^4x^3-1024a^3d^3e^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x)
```

```
[Out] 4096/17*e^12*x^17+1024*d*e^11*x^16+8192/5*d^2*e^10*x^15+1024*d^3*e^9*x^14+1
28/13*(128*a*e^5-16*d^4*e^2)*e^6*x^13+1/12*(16384*a*e^10*d+256*(128*a*e^5-1
6*d^4*e^2)*d*e^5-2048*d^5*e^7)*x^12+1/11*(384*d^6*e^6+32768*a*e^9*d^2+128*(
128*a*e^5-16*d^4*e^2)*d^2*e^4)*x^11+1/10*(14336*a*e^8*d^3+256*d^7*e^5-32*(1
28*a*e^5-16*d^4*e^2)*d^3*e^3)*x^10+1/9*(8192*a^2*e^10-8192*a*e^7*d^4+128*d^
8*e^4+(128*a*e^5-16*d^4*e^2)^2)*x^9+1/8*(16384*a^2*e^9*d-2048*a*e^6*d^5-32*
d^9*e^3+256*a*e^4*d*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(24576*a^2*e^8*d^2+512*
a*e^5*d^6+2*d^6*(128*a*e^5-16*d^4*e^2))*x^7+1/6*(-2048*a^2*e^7*d^3-32*a*e^2
```

```
*d^3*(128*a*e^5-16*d^4*e^2)+256*d^7*a*e^4)*x^6+1/5*(128*a^2*e^4*(128*a*e^5-
16*d^4*e^2)-4096*a^2*e^6*d^4+d^12)*x^5+1/4*(16384*a^3*d*e^8-32*a*d^9*e^2)*x
^4+128*a^2*e^4*d^6*x^3-1024*a^3*e^6*d^3*x^2+4096*a^4*e^8*x
```

maxima [A] time = 0.75, size = 383, normalized size = 1.30

$$\frac{4096}{17}e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}d^2e^{10}x^{15}+\frac{8192}{7}d^3e^9x^{14}+\frac{4096}{13}d^4e^8x^{13}+\frac{1}{5}d^{12}x^5+4096\,a^4e^8x-\frac{4}{7}\left(7\,e^3x^8+8\,de^2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 8192/7*d^3*e^
9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4*e^8*x - 4/7*(7*e^3*
x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^3
*e^6 + 128/165*(45*e^6*x^11 + 99*d*e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128/105
*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e
^3*x^6 + 6*d*e^2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e^8*x^1
3 + 1001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9*x^13 +
 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6*x^10 - 15015*d
^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 32032*(36*e^6*x^10 + 80*d*e^
5*x^9 + 45*d^2*e^4*x^8)*d^3)*a*e^2
```

mupad [B] time = 0.27, size = 331, normalized size = 1.12

$$x^5\left(\frac{16384\,a^3\,e^9}{5}-\frac{6144\,a^2\,d^4\,e^6}{5}+\frac{d^{12}}{5}\right)+x^{10}\left(\frac{384\,d^7\,e^5}{5}+1024\,a\,d^3\,e^8\right)-x^{11}\left(\frac{1664\,d^6\,e^6}{11}-\frac{49152\,a\,d^2\,e^9}{11}\right)+\frac{4096\,e}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^4,x)
```

```
[Out] x^5*(d^12/5 + (16384*a^3*e^9)/5 - (6144*a^2*d^4*e^6)/5) + x^10*((384*d^7*e^
5)/5 + 1024*a*d^3*e^8) - x^11*((1664*d^6*e^6)/11 - (49152*a*d^2*e^9)/11) +
(4096*e^12*x^17)/17 + (2048*e^8*x^13*(8*a*e^3 - d^4))/13 + (128*e^4*x^9*(d^
8 + 64*a^2*e^6 - 32*a*d^4*e^3))/3 + 4096*a^4*e^8*x + 1024*d*e^11*x^16 + 102
4*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 512*d*e^7*x^12*(8*a*e^3 - d^4) +
(32*d^2*e^2*x^7*(768*a^2*e^6 - d^8 + 24*a*d^4*e^3))/7 - 1024*a^3*d^3*e^6*x^
2 + 128*a^2*d^6*e^4*x^3 - 4*d*e^3*x^8*(d^8 - 1536*a^2*e^6 + 192*a*d^4*e^3)
- 128*a*d^3*e^4*x^6*(8*a*e^3 - d^4) - 8*a*d*e^2*x^4*(d^8 - 512*a^2*e^6)
```

sympy [A] time = 0.14, size = 366, normalized size = 1.24

$$4096a^4e^8x-1024a^3d^3e^6x^2+128a^2d^6e^4x^3+1024d^3e^9x^14+\frac{8192d^2e^{10}x^{15}}{5}+1024de^{11}x^{16}+\frac{4096e^{12}x^{17}}{17}+x^{13}\left(\frac{16384ae^{11}}{13}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

```
[Out] 4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 102
4*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e**
12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096*a*
d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6/11)
 + x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/3 - 4
096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a*d**5*
e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5/7 - 32
*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x**5*(1638
4*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3*d*e**8 -
 8*a*d**9*e**2)
```

3.40 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^3 \, dx$

Optimal. Leaf size=203

$$512a^3e^6x - \frac{1}{4}dx^4\left(d^8 - 1536a^2e^6\right) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{128}{3}e^5x^9\left(d^4 - 4ae^3\right) - 24de^4x^8\left(d^4 - 16ae^3\right) - \frac{384}{5}ae^4x^5\left(d\right.$$

```
[Out] 512*a^3*e^6*x-96*a^2*d^3*e^4*x^2+8*a*d^6*e^2*x^3-1/4*d*(-1536*a^2*e^6+d^8)*
x^4-384/5*a*e^4*(-4*a*e^3+d^4)*x^5+4*d^3*e^2*(-16*a*e^3+d^4)*x^6+24/7*d^2*e
^3*(64*a*e^3+d^4)*x^7-24*d*e^4*(-16*a*e^3+d^4)*x^8-128/3*e^5*(-4*a*e^3+d^4)
*x^9+32*d^3*e^6*x^10+1536/11*d^2*e^7*x^11+128*d*e^8*x^12+512/13*e^9*x^13
```

Rubi [A] time = 0.12, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {2061}

$$-\frac{1}{4}dx^4\left(d^8 - 1536a^2e^6\right) - 96a^2d^3e^4x^2 + 512a^3e^6x - \frac{128}{3}e^5x^9\left(d^4 - 4ae^3\right) - 24de^4x^8\left(d^4 - 16ae^3\right) + \frac{24}{7}d^2e^3x^7\left(64ae^3 + d\right.$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)
*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
- (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/1
1 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^3 \, dx = \int \left(512a^3e^6 - 192a^2d^3e^4x + 24ad^6e^2x^2 - d\left(d^8 - 1536a^2e^6\right)x^3 - 384ae^4\left(\right.\right.$$
$$= 512a^3e^6x - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{1}{4}d\left(d^8 - 1536a^2e^6\right)x^4 - \frac{384}{5}ae^4\left(d\right.$$

Mathematica [A] time = 0.03, size = 207, normalized size = 1.02

$$512a^3e^6x - \frac{1}{4}dx^4\left(d^8 - 1536a^2e^6\right) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + \frac{128}{3}e^5x^9\left(4ae^3 - d^4\right) - 24de^4x^8\left(d^4 - 16ae^3\right) + \frac{384}{5}ae^4x^5\left(4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3
)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)
/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

fricas [A] time = 0.38, size = 205, normalized size = 1.01

$$\frac{512}{13}x^{13}e^{9}+128x^{12}e^{8}d+\frac{1536}{11}x^{11}e^{7}d^{2}+32x^{10}e^{6}d^{3}-\frac{128}{3}x^{9}e^{5}d^{4}+\frac{512}{3}x^{9}e^{8}a-24x^{8}e^{4}d^{5}+384x^{8}e^{7}da+\frac{24}{7}x^{7}e^{3}d^{6}+\frac{1536}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13*e^9 + 128*x^12*e^8*d + 1536/11*x^11*e^7*d^2 + 32*x^10*e^6*d^3 -
 128/3*x^9*e^5*d^4 + 512/3*x^9*e^8*a - 24*x^8*e^4*d^5 + 384*x^8*e^7*d*a + 2
4/7*x^7*e^3*d^6 + 1536/7*x^7*e^6*d^2*a + 4*x^6*e^2*d^7 - 64*x^6*e^5*d^3*a -
 384/5*x^5*e^4*d^4*a + 1536/5*x^5*e^7*a^2 - 1/4*x^4*d^9 + 384*x^4*e^6*d*a^2
 + 8*x^3*e^2*d^6*a - 96*x^2*e^4*d^3*a^2 + 512*x*e^6*a^3
```

giac [A] time = 0.24, size = 187, normalized size = 0.92

$$\frac{512}{13}\,x^{13}e^{9}+128\,dx^{12}e^{8}+\frac{1536}{11}\,d^{2}x^{11}e^{7}+32\,d^{3}x^{10}e^{6}-\frac{128}{3}\,d^{4}x^{9}e^{5}-24\,d^{5}x^{8}e^{4}+\frac{24}{7}\,d^{6}x^{7}e^{3}+4\,d^{7}x^{6}e^{2}-\frac{1}{4}\,d^{9}x^{4}+\frac{512}{3}\,ax^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13*e^9 + 128*d*x^12*e^8 + 1536/11*d^2*x^11*e^7 + 32*d^3*x^10*e^6 -
 128/3*d^4*x^9*e^5 - 24*d^5*x^8*e^4 + 24/7*d^6*x^7*e^3 + 4*d^7*x^6*e^2 - 1/
4*d^9*x^4 + 512/3*a*x^9*e^8 + 384*a*d*x^8*e^7 + 1536/7*a*d^2*x^7*e^6 - 64*a
*d^3*x^6*e^5 - 384/5*a*d^4*x^5*e^4 + 8*a*d^6*x^3*e^2 + 1536/5*a^2*x^5*e^7 +
 384*a^2*d*x^4*e^6 - 96*a^2*d^3*x^2*e^4 + 512*a^3*x*e^6
```

maple [A] time = 0.00, size = 288, normalized size = 1.42

$$\frac{512e^{9}x^{13}}{13}+128d\,e^{8}x^{12}+\frac{1536d^{2}e^{7}x^{11}}{11}+32d^{3}e^{6}x^{10}+8a\,d^{6}e^{2}x^{3}-96a^{2}d^{3}e^{4}x^{2}+512a^{3}e^{6}x+\frac{\left(512a\,e^{8}-256d^{4}e^{5}+8\left(128\right.\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x)
```

```
[Out] 512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^10+1/9*(51
2*a*e^8-256*d^4*e^5+8*e^3*(128*a*e^5-16*d^4*e^2))*x^9+1/8*(2048*a*e^7*d-64*
d^5*e^4+8*d*e^2*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(1536*a*d^2*e^6+24*d^6*e^3)
*x^7+1/6*(-256*a*e^5*d^3-d^3*(128*a*e^5-16*d^4*e^2)+8*d^7*e^2)*x^6+1/5*(8*a
*e^2*(128*a*e^5-16*d^4*e^2)-256*d^4*a*e^4+512*e^7*a^2)*x^5+1/4*(1536*a^2*d*
e^6-d^9)*x^4+8*a*d^6*e^2*x^3-96*a^2*d^3*e^4*x^2+512*a^3*e^6*x
```

maxima [A] time = 0.60, size = 214, normalized size = 1.05

$$\frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+\frac{256}{5}\,d^3e^6x^{10}-\frac{1}{4}\,d^9x^4+512\,a^3e^6x+\frac{4}{7}\left(6\,e^3x^7+7\,de^2x^6\right)d^6+\frac{96}{5}\left(16\,e^3x^5+20\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="maxima")
```

```
[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 256/5*d^3*e^6*x^1
0 - 1/4*d^9*x^4 + 512*a^3*e^6*x + 4/7*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 + 96/5*
(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^2*e^4 - 8/15*(36*e^6*x^10 + 80*d*
e^5*x^9 + 45*d^2*e^4*x^8)*d^3 + 8/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880
*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a*e^2
```

mupad [B] time = 2.24, size = 201, normalized size = 0.99

$$\frac{512\,e^9\,x^{13}}{13}-x^4\,\left(\frac{d^9}{4}-384\,a^2\,d\,e^6\right)+\frac{128\,e^5\,x^9\,\left(4\,a\,e^3-d^4\right)}{3}+512\,a^3\,e^6\,x+128\,d\,e^8\,x^{12}+32\,d^3\,e^6\,x^{10}+\frac{1536\,d^2\,e^7\,x^{11}}{11}+8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^3,x)
```

```
[Out] (512*e^9*x^13)/13 - x^4*(d^9/4 - 384*a^2*d*e^6) + (128*e^5*x^9*(4*a*e^3 - d
^4))/3 + 512*a^3*e^6*x + 128*d*e^8*x^12 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x
^11)/11 + 8*a*d^6*e^2*x^3 + (384*a*e^4*x^5*(4*a*e^3 - d^4))/5 + 24*d*e^4*x^
8*(16*a*e^3 - d^4) + (24*d^2*e^3*x^7*(64*a*e^3 + d^4))/7 - 96*a^2*d^3*e^4*x
^2 - 4*d^3*e^2*x^6*(16*a*e^3 - d^4)
```

sympy [A] time = 0.11, size = 218, normalized size = 1.07

$$512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3+32d^3e^6x^{10}+\frac{1536d^2e^7x^{11}}{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}+x^9\left(\frac{512ae^8}{3}-\frac{128d^4e^5}{3}\right)+x^8\left(38\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

```
[Out] 512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e**
6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13 +
```

```
x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**4)
+ x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 + 4*d*
*7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**2*d*e
**6 - d**9/4)
```

3.41 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^2 \, dx$

Optimal. Leaf size=107

$$64a^2e^4x-\frac{16}{5}e^2x^5\left(d^4-8ae^3\right)-8ad^3e^2x^2+32ade^4x^4+\frac{d^6x^3}{3}-\frac{8}{3}d^3e^3x^6+\frac{64}{7}d^2e^4x^7+16de^5x^8+\frac{64e^6x^9}{9}$$

```
[Out] 64*a^2*e^4*x-8*a*d^3*e^2*x^2+1/3*d^6*x^3+32*a*d*e^4*x^4-16/5*e^2*(-8*a*e^3+
d^4)*x^5-8/3*d^3*e^3*x^6+64/7*d^2*e^4*x^7+16*d*e^5*x^8+64/9*e^6*x^9
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {2061}

$$64a^2e^4x-\frac{16}{5}e^2x^5\left(d^4-8ae^3\right)-8ad^3e^2x^2+32ade^4x^4+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{d^6x^3}{3}+16de^5x^8+\frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (16*e^2*(d^
4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8
 + (64*e^6*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 \, dx = \int \left(64a^2e^4-16ad^3e^2x+d^6x^2+128ade^4x^3-16e^2\left(d^4-8ae^3\right)x^4-16d^3e\right.$$
$$= 64a^2e^4x-8ad^3e^2x^2+\frac{d^6x^3}{3}+32ade^4x^4-\frac{16}{5}e^2\left(d^4-8ae^3\right)x^5-\frac{8}{3}d^3e^3x^6$$

Mathematica [A] time = 0.01, size = 109, normalized size = 1.02

$$64a^2e^4x+\frac{16}{5}e^2x^5\left(8ae^3-d^4\right)-8ad^3e^2x^2+32ade^4x^4+\frac{d^6x^3}{3}-\frac{8}{3}d^3e^3x^6+\frac{64}{7}d^2e^4x^7+16de^5x^8+\frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (16*e^2*(-d
^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^
8 + (64*e^6*x^9)/9
```

fricas [A] time = 0.39, size = 99, normalized size = 0.93

$$\frac{64}{9}x^9e^6+16x^8e^5d+\frac{64}{7}x^7e^4d^2-\frac{8}{3}x^6e^3d^3-\frac{16}{5}x^5e^2d^4+\frac{128}{5}x^5e^5a+32x^4e^4da+\frac{1}{3}x^3d^6-8x^2e^2d^3a+64xe^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9*e^6 + 16*x^8*e^5*d + 64/7*x^7*e^4*d^2 - 8/3*x^6*e^3*d^3 - 16/5*x^5
*e^2*d^4 + 128/5*x^5*e^5*a + 32*x^4*e^4*d*a + 1/3*x^3*d^6 - 8*x^2*e^2*d^3*a
 + 64*x*e^4*a^2
```

giac [A] time = 0.29, size = 90, normalized size = 0.84

$$\frac{64}{9}x^9e^6+16\,dx^8e^5+\frac{64}{7}\,d^2x^7e^4-\frac{8}{3}\,d^3x^6e^3-\frac{16}{5}\,d^4x^5e^2+\frac{1}{3}\,d^6x^3+\frac{128}{5}\,ax^5e^5+32\,adx^4e^4-8\,ad^3x^2e^2+64\,a^2xe^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9*e^6 + 16*d*x^8*e^5 + 64/7*d^2*x^7*e^4 - 8/3*d^3*x^6*e^3 - 16/5*d^4
*x^5*e^2 + 1/3*d^6*x^3 + 128/5*a*x^5*e^5 + 32*a*d*x^4*e^4 - 8*a*d^3*x^2*e^2
 + 64*a^2*x*e^4
```

maple [A] time = 0.00, size = 100, normalized size = 0.93

$$\frac{64e^6x^9}{9}+16d\,e^5x^8+\frac{64d^2e^4x^7}{7}-\frac{8d^3e^3x^6}{3}+32ad\,e^4x^4+\frac{d^6x^3}{3}-8a\,d^3e^2x^2+64a^2e^4x+\frac{\left(128a\,e^5-16d^4e^2\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] 64/9*e^6*x^9+16*d*e^5*x^8+64/7*d^2*e^4*x^7-8/3*d^3*e^3*x^6+1/5*(128*a*e^5-1
6*d^4*e^2)*x^5+32*a*d*e^4*x^4+1/3*d^6*x^3-8*a*d^3*e^2*x^2+64*a^2*e^4*x
```

maxima [A] time = 0.61, size = 101, normalized size = 0.94

$$\frac{64}{9}\,e^6x^9+16\,de^5x^8+\frac{64}{7}\,d^2e^4x^7+\frac{1}{3}\,d^6x^3+64\,a^2e^4x-\frac{8}{15}\left(5\,e^3x^6+6\,de^2x^5\right)d^3+\frac{8}{5}\left(16\,e^3x^5+20\,de^2x^4-5\,d^3x^2\right)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] 64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64*a^2*e^4*x
 - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*
d^3*x^2)*a*e^2
```

mupad [B] time = 0.04, size = 98, normalized size = 0.92

$$x^5\left(\frac{128\,a\,e^5}{5}-\frac{16\,d^4\,e^2}{5}\right)+\frac{d^6\,x^3}{3}+\frac{64\,e^6\,x^9}{9}+64\,a^2\,e^4\,x+16\,d\,e^5\,x^8-\frac{8\,d^3\,e^3\,x^6}{3}+\frac{64\,d^2\,e^4\,x^7}{7}-8\,a\,d^3\,e^2\,x^2+32\,a\,d\,e^4\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

```
[Out] x^5*((128*a*e^5)/5 - (16*d^4*e^2)/5) + (d^6*x^3)/3 + (64*e^6*x^9)/9 + 64*a^
2*e^4*x + 16*d*e^5*x^8 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 - 8*a*d^3*e
^2*x^2 + 32*a*d*e^4*x^4
```

sympy [A] time = 0.09, size = 112, normalized size = 1.05

$$64a^2e^4x-8ad^3e^2x^2+32ade^4x^4+\frac{d^6x^3}{3}-\frac{8d^3e^3x^6}{3}+\frac{64d^2e^4x^7}{7}+16de^5x^8+\frac{64e^6x^9}{9}+x^5\left(\frac{128ae^5}{5}-\frac{16d^4e^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] 64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d*
*3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 + x*
*5*(128*a*e**5/5 - 16*d**4*e**2/5)
```

3.42 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx$

Optimal. Leaf size=37

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

```
[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

```
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi steps

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

```
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

fricas [A] time = 0.38, size = 33, normalized size = 0.89

$$\frac{8}{5}x^5e^3 + 2x^4e^2d - \frac{1}{2}x^2d^3 + 8xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")
```

```
[Out] 8/5*x^5*e^3 + 2*x^4*e^2*d - 1/2*x^2*d^3 + 8*x*e^2*a
```

giac [A] time = 0.38, size = 30, normalized size = 0.81

$$\frac{8}{5}x^5e^3 + 2\,dx^4e^2 - \frac{1}{2}\,d^3x^2 + 8\,axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")
[Out] 8/5*x^5*e^3 + 2*d*x^4*e^2 - 1/2*d^3*x^2 + 8*a*x*e^2
```

maple [A] time = 0.00, size = 34, normalized size = 0.92

$$\frac{8}{5}e^3x^5 + 2d\,e^2x^4 - \frac{1}{2}d^3x^2 + 8a\,e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x)
[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

maxima [A] time = 0.67, size = 33, normalized size = 0.89

$$\frac{8}{5}\,e^3x^5 + 2\,de^2x^4 - \frac{1}{2}\,d^3x^2 + 8\,ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")
[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

mupad [B] time = 0.04, size = 33, normalized size = 0.89

$$-\frac{d^3\,x^2}{2} + 2\,d\,e^2\,x^4 + \frac{8\,e^3\,x^5}{5} + 8\,a\,e^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3,x)
[Out] (8*e^3*x^5)/5 - (d^3*x^2)/2 + 2*d*e^2*x^4 + 8*a*e^2*x
```

sympy [A] time = 0.07, size = 36, normalized size = 0.97

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)
```

```
[Out] 8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

3.43 $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal. Leaf size=153

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

```
[Out] 2*arctanh((4*e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^
(1/2)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2)-2*arctanh((4*e*x+d)/(3*d^2+2*(-
64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^(1/2)/(3*d^2+2*(-64*a*e^3+d^4)^
(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1106, 1093, 208}

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 6
4*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(d + 4*e*x)/Sqr
t[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sq
rt[d^4 - 64*a*e^3]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx = \text{Subst}\left(\int \frac{1}{\frac{1}{32}\left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4}\,dx, x, \frac{d}{4e} + x\right)$$

$$= \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2} - e\sqrt{d^4-64ae^3} + 8e^3x^2}\,dx, x, \frac{d}{4e} + x\right)}{\sqrt{d^4 - 64ae^3}} - \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{}{-\frac{3d^2e}{2}}\right.}{\sqrt{}}$$

$$= \frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4 - 64ae^3}\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} - \frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4 - 64ae^3}\sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.46

$$-\text{RootSum}\left[8\#1^4e^3 + 8\#1^3de^2 - \#1d^3 + 8ae^2\&, \frac{\log(x - \#1)}{-32\#1^3e^3 - 24\#1^2de^2 + d^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] -RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x - #1]/(d^3
- 24*d*e^2*#1^2 - 32*e^3*#1^3) & ]
```

fricas [B] time = 0.48, size = 1115, normalized size = 7.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")
```

```
[Out] -sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*
a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 1
6384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3
- 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419
4304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(
25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64
*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3
 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419430
4*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 -
128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 96
0*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8
- 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^
4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) -
sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a
*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16
384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 -
 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194
304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(2
5*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*
a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3
- 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304
*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 - 1
28*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960
*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 -
 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4
*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")
```

```
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

maple [C] time = 0.07, size = 67, normalized size = 0.44

$$\frac{\ln\left(-\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\,\_Z^3-d^3\_Z+8a\,e^2\right)+x\right)}{32\,\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\,\_Z^3-d^3\_Z+8a\,e^2\right)^3e^3+24\,\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\,\_Z^3-d^3\_Z+8a\,e^2\right)^2d\,e^2-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x)
```

```
[Out] sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(-_R+x),_R=RootOf(8*_Z^4*e^3+8*_Z^3
*d*e^2-_Z*d^3+8*a*e^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

mupad [B] time = 3.73, size = 1264, normalized size = 8.26

$$-\mathrm{atan}\left(\frac{d^3\,\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6-192\,a\,d^8\,e^3+d^{12}}\;3\mathrm{i}+d^9\,2\mathrm{i}-a\,d}{5\,d^{12}\,\sqrt{\frac{2\,\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6-192\,a\,d^8\,e^3+d^{12}}+3\,d^6-192\,a\,d^2\,e^3}{1048576\,a^3\,e^9-12288\,a^2\,d^4\,e^6-384\,a\,d^8\,e^3+5\,d^{12}}}+1048576\,a^3\,e^9\,\sqrt{\frac{2\,\sqrt{-262144\,a^3\,e^9+12288\,a^2\,d^4\,e^6}}{1048576\,a^3\,e^9-12288\,a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3),x)
```

```
[Out] atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)
*3i - d^9*2i + a*d^5*e^3*256i - a^2*d*e^6*8192i - a^2*e^7*x*32768i - d^8*e*
x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^
(1/2)*12i + a*d^4*e^4*x*1024i)/(5*d^12*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*
d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 10485
76*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) + 1048576*a^3*e^9*(-
(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^
6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^
4*e^6))^(1/2) - 384*a*d^8*e^3*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 +
 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^
9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 12288*a^2*d^4*e^6*(-(2*(d^1
2 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192
*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))
^(1/2)))*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(
1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 1
2288*a^2*d^4*e^6))^(1/2)*2i - atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*
e^3 + 12288*a^2*d^4*e^6)^(1/2)*3i + d^9*2i - a*d^5*e^3*256i + a^2*d*e^6*819
2i + a^2*e^7*x*32768i + d^8*e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a
*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i - a*d^4*e^4*x*1024i)/(5*d^12*((2*(d
^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 1
92*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6
```

```
))^(1/2) + 1048576*a^3*e^9*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 122
88*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 -
384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 384*a*d^8*e^3*((2*(d^12 - 26214
4*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^
3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) -
12288*a^2*d^4*e^6*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^
4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8
*e^3 - 12288*a^2*d^4*e^6))^(1/2)))*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e
^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^
3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)*2i
```

sympy [A] time = 1.83, size = 122, normalized size = 0.80

$$\operatorname{RootSum}\left(t^4\left(1048576a^3e^9-12288a^2d^4e^6-384ad^8e^3+5d^{12}\right)+t^2\left(384ad^2e^3-6d^6\right)+1,\left(t\mapsto t\log\left(x+\frac{-4915}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

```
[Out] RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3 +
 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (-4
9152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256*_t
*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))
```

3.44 $\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2} dx$

Optimal. Leaf size=342

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{\left(d^4-64ae^3\right)^{3/2}\left(256ae^3+5d^4\right)\sqrt{3d^2-2\sqrt{a}}}$$

```
[Out] 2*e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^6
-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)-24*e*arctanh((4*
e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))*(d^4+128*a*e^3-d^2*(-64*a*e^3
+d^4)^(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e^3+5*d^4)/(3*d^2-2*(-64*a*e^3+d^
4)^(1/2))^(1/2)+24*e*arctanh((4*e*x+d)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2
))*(d^4+128*a*e^3+d^2*(-64*a*e^3+d^4)^(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e
^3+5*d^4)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1106, 1092, 1166, 208}

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2+13d^4\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{\left(d^4-64ae^3\right)^{3/2}\left(256ae^3+5d^4\right)\sqrt{3d^2-2\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] (2*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*
x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 +
256*a*e^3)*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 +
 d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 + 2*Sqrt[
d^4 - 64*a*e^3]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^2}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} - \frac{4\,\text{Subst}}{}$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} + \frac{\left(48e^3\left(\right.\right.}{}$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} - \frac{24e\left(d^4\right.}{\left(d^4\right.}$$

Mathematica [C] time = 0.19, size = 234, normalized size = 0.68

$$\frac{48e^2\text{RootSum}\left[8\#1^4e^3+8\#1^3de^2-\#1d^3+8ae^2\&, \frac{2\#1^2d^2e\log(x-\#1)+32ae^2\log(x-\#1)+\#1d^3\log(x-\#1)}{32\#1^3e^3+24\#1^2de^2-d^3}\&\right]}{16384a^2e^6+64ad^4e^3-5d^8} + \frac{(d+4ex)}{\left(d^4-64ae^3\right)\left(2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] ((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64*
a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) + (
48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , (32*a*e^2*L
og[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*d*e^
2*#1^2 + 32*e^3*#1^3) & ])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)
```

fricas [B] time = 0.73, size = 4285, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] -(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt(2)*(40*a*
d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 -
16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (
5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^
5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6
+ 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^1
5 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)
/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^
3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27
44381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 51881467707
30811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a
*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d
^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*
d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 1382
4*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*
e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22
*e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a
^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737
86976294838206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^
2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d
^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804651110
4*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1
800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150
994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a
^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4
*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167
360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 515396
07552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 -
 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2
*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 -
64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960
*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 3119513
6*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 43980
46511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d
^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^
9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27443810229
28896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*
a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3
 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 -
 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 +
 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)
```

```
*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 26
8435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 56
6493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e
^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^
8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 -
115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e
^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931
694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737869762948
38206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (
125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 +
 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^1
8)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*
d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000
*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e
^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7
3786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d
^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5
*d^4*e^15 - 4398046511104*a^6*e^18))) + 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*
d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4
 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*
e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*
e^8 - (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^1
2*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*
a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 180
0000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 15099
4944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6
*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e
^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 116736
0*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607
552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 22649241
6*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 13824*sqrt(2)*(d^16*e^
2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a
^4*e^14 + (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a
^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 307
86325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 5
12*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000
*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780
82505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085
312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*
a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24
- 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 38252052
48*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((
d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3
- 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20
*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709
```

```
31694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629
4838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 +
 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15
 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 -
 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d
^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 - 163
84*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (12
5*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3
825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737
8697629483820646 4*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^1
6*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d
^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 226492416*a^2*d*e
^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)*(d^16*e^2 - 128*a
*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a^4*e^14 +
 (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a^3*d^18*e
^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 307863255777
28*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 512*a*d^4*
e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28
*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780825054412
80*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d
^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)
))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24 - 4800*a*
d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^
8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 +
 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 1152000
00*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 7
8082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709316941310
85312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629483820646
4*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136
*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804
6511104*a^6*e^18))) - 8*(d^4*e - 64*a*e^4)*x)/(40*a*d^8*e^2 - 512*a^2*d^4*e
^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*
(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 -
 16384*a^2*d^3*e^6)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.02, size = 288, normalized size = 0.84

$$\frac{384e^2\left(2\operatorname{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8a\,e^2\right)^2d^2e+\operatorname{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8a\,e^2\right)d^3+3}{\left(2048a\,e^3+40d^4\right)\left(64a\,e^3-d^4\right)\left(32\operatorname{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8a\,e^2\right)^3e^3+24\operatorname{RootOf}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] (12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^4
)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384*a
^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/(20
48*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*_R^3
*e^3+24*_R^2*d*e^2-d^3)*ln(-_R+x),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+
8*a*e^2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 7.11, size = 10351, normalized size = 30.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

```
[Out] ((8*e*x)/(256*a*e^3 + 5*d^4) - (5*d^5 - 128*a*d*e^3)/((64*a*e^3 - d^4)*(256
*a*e^3 + 5*d^4)) + (72*d^3*e^2*x^2)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4))
+ (96*d^2*e^3*x^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)))/(8*a*e^2 - d^3*x
 + 8*e^3*x^4 + 8*d*e^2*x^3) + atan((((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 -
 d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11
 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^
3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32
*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20
*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 79164837
```

1998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(6871
9476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 -
5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5
*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 1342
17728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^
19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^
5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 -
 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a
^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^
15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*
e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6
+ 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/
2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440
*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^
(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 12902
40*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 9663
6764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*
d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 616038
4*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a
e^5(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27
 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250
489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*
e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2
) + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^
2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2
*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(78643
2*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12
- 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*1i + ((288*(d
^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d
^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*
d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460
6846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*
d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062
494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8
*d^4*e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e
^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e
^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) -
 ((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2
*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17
179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d
^8*e^9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 +
 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 2
5769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 171798691
84*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9

+ 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 11878
4*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 1717986
9184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744
*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3
- d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^1
1 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e
^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^3
2*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^2
0*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7916483
71998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22
*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14
*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2
*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460684
6976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^2
4*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494
720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^
4*e^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))
/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097
152*a^3*d^4*e^9))*1i)/(((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2
) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*
a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(
1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 129024
0*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636
764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d
^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(68719476736*a^5*e
^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 5242880*a^3*d
^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15 - 2240*
a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4
e^12) - ((1536(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^14 + 199
22944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7*e^23 +
1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*
e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12)
+ (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 + 12320768
*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24))/(25*d^
16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3
*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^1
8*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14
 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*
d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e
^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d
^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40
532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3
 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^
11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*
e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^

32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) - ((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) + (113246208*a*d^2*e^14)/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 225

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28*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418
240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152
921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577
856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 +
44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 4053239664633
4464*a^8*d^4*e^24))^(1/2)*2i + atan(((-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^
(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^
11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*
e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^
32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^
20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648
371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(68
719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15
- 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a
^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 13
4217728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*
d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*
a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15
 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728
*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*
e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^
2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^
6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^2
2*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373
440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^
9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 12
90240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 9
6636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a
^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2*(-(64
*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 61
60384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 2
56*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*
e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 1
5250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d
^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^
(1/2) + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 19660
8*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784
*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(7
86432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e
^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*1i + (-(2
88*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*
a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240
*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921
504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856
*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 443
```

24062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 4053239664633446
4*a^8*d^4*e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^
3*d*e^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d
^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^
12) - ((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 77004
8*a^2*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20
 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*
a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e
^11 + 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^2
0 - 25769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 1717
9869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8
*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 -
118784*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17
179869184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 1
59744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4
)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^
10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(
64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800
*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a
^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7
91648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(28
8*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a
^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*
a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 11529215
04606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*
a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 4432
4062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464
*a^8*d^4*e^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4
*e^14))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6
 + 2097152*a^3*d^4*e^9))*1i)/((-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) -
 d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 46
1373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d
^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3
+ 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12
 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7916483719987
20*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(687194767
36*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 52428
80*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15
 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728
*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^
14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7
*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 2240
*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^
4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 +
12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24)

)/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 209
7152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 +
 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4
*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2
)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a
^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764
160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*
e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2*(-(64*a*e^3
- d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a
^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^
5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 -
28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489
344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^1
8 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) +
 (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d
^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^
12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a
^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 6
40*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) - (-(288*(d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e
^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e
^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 11529215046068469
76*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*
e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 4432406249472
0*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*
e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 +
 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 -
118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((15
36*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12
*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 1717986
9184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^
9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 2457
6*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 257698
03776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^
5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134
217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2
*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*
a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*
d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2)
 - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 -
461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 -
 d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^
3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^
12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 79164837199
8720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2

```
*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^
8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^
17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460684697
6*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e
^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720
*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e
^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(2
5*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152
*a^3*d^4*e^9)) + (113246208*a*d^2*e^14)/(25*d^20 - 17179869184*a^5*e^15 - 2
240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4
*d^4*e^12)))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*
d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e
^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(1
25*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^2
8*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^
5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 -
 40532396646334464*a^8*d^4*e^24))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
[Out] Timed out
```

3.45 $\int \left(8+8x-x^3+8x^4\right)^4 dx$

Optimal. Leaf size=96

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x$$

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+14
08*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^
15-128*x^16+4096/17*x^17
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+8x-x^3+8x^4\right)^4 dx = \int \left(4096+16384x+24576x^2+14336x^3+14336x^4+43008x^5+47488x^6+110\right.$$

$$= 4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+1$$

Mathematica [A] time = 0.00, size = 96, normalized size = 1.00

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

fricas [A] time = 0.35, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17}-128x^{16}+\frac{128}{5}x^{15}+1168x^{14}+\frac{10241}{13}x^{13}-448x^{12}+\frac{25312}{11}x^{11}+\frac{21488}{5}x^{10}+1408x^{9}+1376x^{8}+6784x^{7}+7168x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

giac [A] time = 0.31, size = 84, normalized size = 0.88

$$\frac{4096}{17}\,x^{17}-128\,x^{16}+\frac{128}{5}\,x^{15}+1168\,x^{14}+\frac{10241}{13}\,x^{13}-448\,x^{12}+\frac{25312}{11}\,x^{11}+\frac{21488}{5}\,x^{10}+1408\,x^{9}+1376\,x^{8}+6784\,x^{7}+71$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

maple [A] time = 0.00, size = 85, normalized size = 0.89

$$\frac{4096}{17}x^{17}-128x^{16}+\frac{128}{5}x^{15}+1168x^{14}+\frac{10241}{13}x^{13}-448x^{12}+\frac{25312}{11}x^{11}+\frac{21488}{5}x^{10}+1408x^{9}+1376x^{8}+6784x^{7}+7168x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^4,x)
```

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+14
08*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^
15-128*x^16+4096/17*x^17
```

maxima [A] time = 1.25, size = 84, normalized size = 0.88

$$\frac{4096}{17}\,x^{17}-128\,x^{16}+\frac{128}{5}\,x^{15}+1168\,x^{14}+\frac{10241}{13}\,x^{13}-448\,x^{12}+\frac{25312}{11}\,x^{11}+\frac{21488}{5}\,x^{10}+1408\,x^{9}+1376\,x^{8}+6784\,x^{7}+71$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

mupad [B] time = 0.19, size = 84, normalized size = 0.88

$$\frac{4096\,x^{17}}{17}-128\,x^{16}+\frac{128\,x^{15}}{5}+1168\,x^{14}+\frac{10241\,x^{13}}{13}-448\,x^{12}+\frac{25312\,x^{11}}{11}+\frac{21488\,x^{10}}{5}+1408\,x^9+1376\,x^8+6784\,x^7+7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^4,x)
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

sympy [A] time = 0.07, size = 94, normalized size = 0.98

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**13/13 - 448
*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1376*x**8 + 6784*x**7
 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 8192*x**3 + 8192*x**2 + 4096*x
```

3.46 $\int \left(8+8x-x^3+8x^4\right)^3\,dx$

Optimal. Leaf size=74

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

```
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+3
07/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+8x-x^3+8x^4\right)^3\,dx = \int \left(512+1536x+1536x^2+320x^3+1152x^4+2880x^5+1560x^6-360x^7+1152x^8\right.$$
$$= 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+\frac{307x}{2}$$

Mathematica [A] time = 0.00, size = 74, normalized size = 1.00

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

fricas [A] time = 0.35, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

giac [A] time = 0.33, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.88

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^3,x)
```

```
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+3
07/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

maxima [A] time = 0.94, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

mupad [B] time = 0.08, size = 64, normalized size = 0.86

$$\frac{512\,x^{13}}{13}-16\,x^{12}+\frac{24\,x^{11}}{11}+\frac{307\,x^{10}}{2}+128\,x^9-45\,x^8+\frac{1560\,x^7}{7}+480\,x^6+\frac{1152\,x^5}{5}+80\,x^4+512\,x^3+768\,x^2+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^3,x)
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

sympy [A] time = 0.07, size = 71, normalized size = 0.96

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 +
1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 512*
x
```

3.47 $\int \left(8+8x-x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(8+8x-x^3+8x^4\right)^2 \, dx &= \int \left(64+128x+64x^2-16x^3+112x^4+128x^5+x^6-16x^7+64x^8\right) dx\\ &= 64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

fricas [A] time = 0.38, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

giac [A] time = 0.33, size = 44, normalized size = 0.81

$$\frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^2,x)
```

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

maxima [A] time = 0.88, size = 44, normalized size = 0.81

$$\frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.81

$$\frac{64\,x^9}{9}-2\,x^8+\frac{x^7}{7}+\frac{64\,x^6}{3}+\frac{112\,x^5}{5}-4\,x^4+\frac{64\,x^3}{3}+64\,x^2+64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

sympy [A] time = 0.06, size = 49, normalized size = 0.91

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3 +
 64*x**2 + 64*x
```

3.48 $\int \left(8+8x-x^3+8x^4\right) dx$

Optimal. Leaf size=23

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

```
[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

```
[In] Int[8 + 8*x - x^3 + 8*x^4,x]
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Rubi steps

$$\int \left(8+8x-x^3+8x^4\right) dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

```
[In] Integrate[8 + 8*x - x^3 + 8*x^4,x]
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

fricas [A] time = 0.35, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5-\frac{1}{4}x^4+4x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

giac [A] time = 0.32, size = 19, normalized size = 0.83

$$\frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x^4-x^3+8*x+8,x)
```

```
[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5
```

maxima [A] time = 0.91, size = 19, normalized size = 0.83

$$\frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

mupad [B] time = 0.03, size = 19, normalized size = 0.83

$$\frac{8\,x^5}{5} - \frac{x^4}{4} + 4\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x - x^3 + 8*x^4 + 8,x)
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

sympy [A] time = 0.06, size = 19, normalized size = 0.83

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-x**3+8*x+8,x)
```

```
[Out] 8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```

3.49 $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

Optimal. Leaf size=268

$$-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{}\right.$$

```
[Out] -1/84*arctan(1/42*(3-(1+4/x)^2)*7^(1/2))*7^(1/2)-1/29232*ln((1+4/x)^2+3*29^
(1/2)-(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/2))^(1/2)+1/29232*
ln((1+4/x)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/
2))^(1/2)-1/14616*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2)
)*(132762+81606*29^(1/2))^(1/2)-1/14616*arctan((2+8/x+(6+6*29^(1/2))^(1/2))
/(-6+6*29^(1/2))^(1/2))*(132762+81606*29^(1/2))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] -ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/(12*Sqrt[7]) - (Sqrt[(109 + 67*Sqrt[
29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])
]])/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])]
 + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/12 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log
[3*Sqrt[29] - Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(
-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x)
 + (1 + 4/x)^2])/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(8-32x)^2}{8\left(1069056-393216x^2+1048576x^4\right)}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int \frac{(8-32x)^2}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int -\frac{512x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right) - 128\,\text{Subst}\Big(\int$$

$$= 65536\,\text{Subst}\left(\int \frac{x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right) - \frac{\text{Subst}\Big(\int \frac{16\sqrt{6(1}}{\frac{3\sqrt{29}}{16}}}{76}$$

$$= 32768\,\text{Subst}\left(\int \frac{1}{1069056-393216x+1048576x^2}\,dx, x, \left(\frac{1}{4}+\frac{1}{x}\right)^2\right) - \frac{\left(87+\sqrt{29}\right)S}{}$$

$$= -\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\,\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right) + \frac{1}{24}\sqrt{\frac{-10}{}}$$

$$= -\frac{\tan^{-1}\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}} - \frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\,\tan^{-1}\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right) - \frac{1}{12}\sqrt{\frac{109}{}}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.17

$$\text{RootSum}\left[8\#1^4-\#1^3+8\#1+8\&, \frac{\log(x-\#1)}{32\#1^3-3\#1^2+8}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) & ]
```

fricas [C] time = 1.73, size = 1015, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")
```

```
[Out] -1/168*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696))*log(287314195
392*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^3 - 120389
06880*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 + 1687
8104*x + 4897683*I*sqrt(7) - 411405372*sqrt(65/43848*I*sqrt(7) - 109/87696)
 + 6055613) - 1/168*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696))*
log(-35914274424*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696
))^3 + 16443*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/87696))
^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/87696) - 91520
) + 609*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
)^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)) + 2109763
*x - 1911147/8*I*sqrt(7) + 40134087/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
- 1461344) + 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/43
848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848
*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7
) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) - 7)
 + 261*sqrt(65/43848*I*sqrt(7) - 109/87696) + 261*sqrt(-65/43848*I*sqrt(7)
- 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7)
 - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/
87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt
(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/8
7696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/876
96))^2 + 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7)
- 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) -
 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) -
109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/
43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/438
48*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt
(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) -
7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt(7) - 10
9/87696) + 220336) - 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sq
rt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-
65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*
```

```
I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/8769
6)) - 7) - 261*sqrt(65/43848*I*sqrt(7) - 109/87696) - 261*sqrt(-65/43848*I*
sqrt(7) - 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I
*sqrt(7) - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7
) - 109/87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/4384
8*I*sqrt(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7)
 - 109/87696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) -
 109/87696))^2 - 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*
sqrt(7) - 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*s
qrt(7) - 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sq
rt(7) - 109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*
sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt
(-65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/4384
8*I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87
696)) - 7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt
(7) - 109/87696) + 220336)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="giac")
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

maple [C] time = 0.02, size = 41, normalized size = 0.15

$$\frac{\ln\left(-\mathrm{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)+x\right)}{32\,\mathrm{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)^3-3\,\mathrm{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8),x)
[Out] sum(1/(32*_R^3-3*_R^2+8)*ln(-_R+x),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

mupad [B] time = 2.45, size = 123, normalized size = 0.46

$$\sum_{k=1}^{4} \ln\left(-\frac{\mathrm{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)\left(8064\,\mathrm{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)+256\,x+\mathrm{root}\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8),x)
```

```
[Out] symsum(log(-(root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*(8
064*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k) + 256*x + 1
2285*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*x + 148176
*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2*x + 198072*r
oot(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2 - 8))/4096)*ro
ot(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k), k, 1, 4)
```

sympy [A] time = 0.96, size = 41, normalized size = 0.15

$$\mathrm{RootSum}\left(66298176t^4+74088t^2+4095t+64,\left(t\mapsto t\log\left(\frac{35914274424t^3}{2109763}-\frac{1504863360t^2}{2109763}+\frac{102851343t}{2109763}+x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8),x)
```

```
[Out] RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(3591
4274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 + x
 + 6055613/16878104)))
```

3.50 $\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2} \, dx$

Optimal. Leaf size=357

$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right.\right.}{17}$$

```
[Out] 1/336*(-207-29*(1+4/x)^2)/(261-6*(1+4/x)^2+(1+4/x)^4)+5/87696*(5157+199*(1+
4/x)^2)*(1+4/x)/(261-6*(1+4/x)^2+(1+4/x)^4)-17/7056*arctan(1/42*(3-(1+4/x)^
2)*7^(1/2))*7^(1/2)-1/213627456*ln((1+4/x)^2+3*29^(1/2)-(1+4/x)*(6+6*29^(1/
2))^(1/2))*(-220437694722+55934612286*29^(1/2))^(1/2)+1/213627456*ln((1+4/x
)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-220437694722+55934612286*29^
(1/2))^(1/2)-1/106813728*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2)
)^(1/2))*(220437694722+55934612286*29^(1/2))^(1/2)-1/106813728*arctan((2+8/
x+(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2))*(220437694722+55934612286*29
^(1/2))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right.\right.}{17}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] -(207 + 29*(1 + 4/x)^2)/(336*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)) + (5*(515
7 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4))
 - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])])/(1008*Sqrt[7]) - (Sqrt[(18098
3329 + 45923327*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/S
qrt[6*(-1 + Sqrt[29])]])/87696 - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218
]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/87696
 - (Sqrt[(-180983329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1
+ Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392 + (Sqrt[(-180983329 + 4592332
```

```
7*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 +
4/x)^2])/175392
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
```

```
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(8-32x)^6}{64\left(1069056-393216x^2+1048576x^4\right)^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{(8-32x)^6}{\left(1069056-393216x^2+1048576x^4\right)^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{x\left(-6291456-335544320x^2-1610612736x^4\right)}{\left(1069056-393216x^2+1048576x^4\right)^2}\,dx,x,\frac{1}{4}+\frac{1}{x}\right)\right)-16$$

$$= \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{278927740761415247462 4+7758008804499}{1069056-393216x^2+1048576x^4}\right.}{57853663025666457}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \text{Sub}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{1392}{21}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \sqrt{\frac{-1}{}}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{17\,\text{ta}}{}$$

Mathematica [C] time = 0.02, size = 113, normalized size = 0.32

$$\frac{\text{RootSum}\left[8\#1^4-\#1^3+8\#1+8\&,\frac{392\#1^2\log(x-\#1)-1097\#1\log(x-\#1)+2243\log(x-\#1)}{32\#1^3-3\#1^2+8}\&\right]}{21924}+\frac{784x^3-1146x^2+1539x+544}{43848\left(8x^4-x^3+8x+8\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4)) + RootS
um[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 1097*Log[x - #1]*#1 + 3
92*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) & ]/21924
```

fricas [C] time = 2.23, size = 1201, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 1/213627456*(3819648*x^3 - 15138*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*log(621
78505678730656543599738593328*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/3345405
96096*I*sqrt(7) - 180983329/4683568345344))^3 - 100287672431797174786327756
80*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 + 67481665655469287031416*x + 320944207138750561964778*
I*sqrt(7) - 1332107250335896450131455O4*sqrt(4550065/334540596096*I*sqrt(7)
 - 180983329/4683568345344) + 33397908111320253309O737) - 15138*(8*x^4 - x^
3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344))*log(-7772313209841332067949967324l6*(17/14112*I*sqr
t(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^
3 + 878169064752*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sq
rt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqrt(7) + 44252943
5492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) -
 1427510892508480) + 7569*(727651150781043057307 2*(17/14112*I*sqrt(7) - 1/2
*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 233594
23554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344)) + 8435208206933660878927*x - 14844919514132868277726
33/4*I*sqrt(7) + 154037870723119880241720 36*sqrt(4550065/334540596096*I*sqr
t(7) - 180983329/4683568345344) - 4739360660669659506761 6) - 5583312*x^2 +
(56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-125452723536*(17/14112*I*sqrt(7)
 - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 -
125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7
) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-45
50065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 66301
91) + 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/33454
0596096*I*sqrt(7) - 180983329/4683568345344)) + 7569*(8*x^4 - x^3 + 8*x + 8
)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468
3568345344)))*log(-439084532376*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/33
```

4540596096*I*sqrt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqr
t(7) + 442529435492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4
683568345344) - 1427510892508480) - 7569/2*(72765115078104305730 72*(17/1411
2*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356834
5344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/3345405960
96*I*sqrt(7) - 180983329/4683568345344)) + 626797952698732342414548480*(17/
14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835
68345344))^2 + 1/16*(261*(62716756730859*sqrt(87)*(-17*I*sqrt(7) + 7056*sqr
t(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 142751089250
8480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) -
180983329/4683568345344)) - 372580342944713280*sqrt(87)*(-17*I*sqrt(7) + 70
56*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 104650
21752358451264*sqrt(87))*sqrt(-125452723536*(17/14112*I*sqrt(7) - 1/2*sqrt(
4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 125452723536
*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540
596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7056*sqrt(455
0065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 6630191) + 843520
8206933660878927*x - 300572710701164955243 9/2*I*sqrt(7) + 62377677844335880
1235576*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) + 22
959102208397854107 04) - (56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-12545272
3536*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983
329/4683568345344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-45500
65/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I
*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345
344))*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329
/4683568345344)) - 6630191) - 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) +
7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 756
9*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I
*sqrt(7) - 180983329/4683568345344)))*log(-439084532376*(-17/14112*I*sqrt(7
) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2*
(-1066184864424603*I*sqrt(7) + 442529435492941104*sqrt(4550065/334540596096
*I*sqrt(7) - 180983329/4683568345344) - 1427510892508480) - 7569/2*(7276511
507810430573072*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(
7) - 180983329/4683568345344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*
sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 62679795
2698732342414548480*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*s
qrt(7) - 180983329/4683568345344))^2 - 1/16*(261*(62716756730859*sqrt(87)*(
-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356
8345344)) - 1427510892508480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/3
34540596096*I*sqrt(7) - 180983329/4683568345344)) - 372580342944713280*sqrt
(87)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/
4683568345344)) + 10465021752358451264*sqrt(87))*sqrt(-125452723536*(17/141
12*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835683
45344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596

```
096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7
056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I
*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835683453
44)) - 6630191) + 84352082069336608789 27*x - 300572710701164955243 9/2*I*sqr
t(7) + 623776778443358801235576*sqrt(4550065/334540596096*I*sqrt(7) - 18098
3329/4683568345344) + 229591022083978541070 4) + 7498008*x + 2650368)/(8*x^4
 - x^3 + 8*x + 8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^4 - x^3 + 8\,x + 8\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)
```

maple [C] time = 0.01, size = 83, normalized size = 0.23

$$\frac{\left(392\,\mathrm{RootOf}\left(8\_Z^4 - \_Z^3 + 8\_Z + 8\right)^2 - 1097\,\mathrm{RootOf}\left(8\_Z^4 - \_Z^3 + 8\_Z + 8\right) + 2243\right)\ln\left(-\mathrm{RootOf}\left(8\_Z^4 -\right.\right.}{701568\,\mathrm{RootOf}\left(8\_Z^4 - \_Z^3 + 8\_Z + 8\right)^3 - 65772\,\mathrm{RootOf}\left(8\_Z^4 - \_Z^3 + 8\_Z + 8\right)^2 + 1753}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8)^2,x)
```

```
[Out] (7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*su
m((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(-_R+x),_R=RootOf(8*_Z^4-_Z^
3+8*_Z+8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{784\,x^3 - 1146\,x^2 + 1539\,x + 544}{43848\left(8\,x^4 - x^3 + 8\,x + 8\right)} + \frac{1}{21924}\int \frac{392\,x^2 - 1097\,x + 2243}{8\,x^4 - x^3 + 8\,x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/219
24*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)
```

mupad [B] time = 0.21, size = 176, normalized size = 0.49

$$\left(\sum_{k=1}^{4} \ln\left(\frac{2615257\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)}{72918171648}+\frac{4225\,x}{40375589184}-\frac{\mathrm{root}\left(z^4+\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] symsum(log((2615257*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/67
4433841729536 + 1114096/13723971258377709, z, k))/72918171648 + (4225*x)/40
375589184 - (34885379*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/
674433841729536 + 1114096/13723971258377709, z, k)*x)/72918171648 - (191555
*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 111
4096/13723971258377709, z, k)^2*x)/475136 - (9261*root(z^4 + (6630191*z^2)/
167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z,
 k)^3*x)/256 - (11205*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/
674433841729536 + 1114096/13723971258377709, z, k)^2)/59392 - (24759*root(z
^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13
723971258377709, z, k)^3)/512 + 10901/107668237824)*root(z^4 + (6630191*z^2
)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709,
z, k), k, 1, 4) + ((57*x)/12992 - (191*x^2)/58464 + (7*x^3)/3132 + 17/10962
)/(x - x^3/8 + x^4 + 1)
```

sympy [B] time = 3.22, size = 3834, normalized size = 10.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] (784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x
+ 350784) - sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/1292018853888
)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329
 + 45923327*sqrt(29)) + 40699873480352667)/227008323264998681573683424 - 26
7658292345340*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-
180983329 + 45923327*sqrt(29)) + 40699873480352667)/8435208206933660878927
- 2157374520970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/11
3504161632499340786841712 + 3881045239007430*sqrt(29)/5326727264361229 + 43
5853770857118353330297/33740832827734643515708 + 20905585576953*sqrt(42)*sq
rt(-180983329 + 45923327*sqrt(29))/85227636229779664) - 2942814074101429415
084030510182204250067556953*sqrt(214095423017213*sqrt(29) + 47106822945*sqr
```

t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/888496186
751485201253966401139075287452416534006272 - 142576256328563148358311429727
651026090105395593510093/277655058359839125391864500355961027328880166876 96
- 751846315028188373888759000608813558 71*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-
180983329 + 45923327*sqrt(29)) + 40699873480352667)/30637799543154662112205
737970312940946635052896768 - 963314181796141259748858766106570487809406229
9*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/30637799543154662112205 73
79703129409466350528967 68 - 13988883340016523668552372 55*sqrt(42)*sqrt(-180
983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/359456428
29149701654794474681089537026 4 + 912459816900304989677782332140155916 79*sqr
t(42)*sqrt(-180983329 + 45923327*sqrt(29))/230052114106558090590684637958 97
303696896 + 10304175351841941260676745569701505519*sqrt(29)*sqrt(2140954230
17213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)
) + 40699873480352667)/1934738279636153541867657805234963240908953 6 + 63911
1088489748962499984017403917984374085485*sqrt(29)/48368456990903838546691 44
513087408102272384) + sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/129
2018853888)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 4592
3327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29)) + 214095423017213*sqrt(29) + 40699873480352667)/2270083232649986815 73
683424 - 20905585576953*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/85227
636229779664 + 3881045239007430*sqrt(29)/5326727264361229 + 267658292345340
*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095
423017213*sqrt(29) + 40699873480352667)/843520820693366087 8927 + 2157374520
970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/11350416163249
9340786841712 + 435853770857118353330297/337408328277346435157 08) - 1425762
563285631483583114297276510260901053955935109 3/277655058359839125391864500 3
559610273288801668769 6 - 10304175351841941260676745569701505519*sqrt(29)*sq
rt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423
017213*sqrt(29) + 40699873480352667)/193473827963615354186765780523496324 09
089536 - 91245981690030498967778233214015591679*sqrt(42)*sqrt(-180983329 +
45923327*sqrt(29))/23005211410655809059068463795897303696896 - 751846315028
188373888759000608813558 71*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))*
sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954
23017213*sqrt(29) + 40699873480352667)/306377995431546621122057379703129409
46635052896768 - 13988883340016523668552372 55*sqrt(42)*sqrt(-180983329 + 45
923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sq
rt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/359456428291497016 5
4794474681089537026 4 + 963314181796141259748858766106570487809406229 9*sqrt(
1218)*sqrt(-180983329 + 45923327*sqrt(29))/306377995431546621122057379703 12
940946635052896768 + 294281407410142941508403051018220425006755695 3*sqrt(-4
7106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 21409542301721
3*sqrt(29) + 40699873480352667)/888496186751485201253966401139075287452416 5
34006272 + 639111088489748962499984017403917984374085485*sqrt(29)/483684569

```
9090383854669144513087408102272384) - 2*sqrt(199631405/37468546762752 + sqr
t(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954230
17213*sqrt(29) + 40699873480352667)/9367136690688 + 1583563*sqrt(29)/430672
951296)*atan(454016646529997363147366848*x/(-4509673516272467429860*sqrt(12
18)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*s
qrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(19
9631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29
)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) +
20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*s
qrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 4069987348
0352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329
 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)) + 29
32424170326692281206238216/(-4509673516272467429860*sqrt(1218)*sqrt(1996314
05 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) +
214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 36016
09981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt
(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 21409542301
7213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 20905585576953*s
qrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 +
 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 13776
9981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29)) + 214095423017213*sqrt(29) + 40699873480352667)) + 431474904194070573
3646*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/(-45096735162724674298
60*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-1809833
29 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 1
37769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29
))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 459233
27*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sq
rt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sq
rt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) +
40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(
-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352
667)) + 7203219963597790080*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 +
45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/(-450967
3516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)
*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873
480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45
923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-1809
83329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)
+ 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-
47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954230172
13*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sq
rt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) +
40699873480352667)) + 165397912920614705160598080*sqrt(29)/(-45096735162724
```

```
67429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-1
80983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 4069987348035266
7) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*s
qrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 +
45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769
981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822
945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(
29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)
*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873
480352667)) - 55683134469459984392598*sqrt(42)*sqrt(-180983329 + 45923327*s
qrt(29))/(-4509673516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106
822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sq
rt(29) + 40699873480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqr
t(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt
(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40
699873480352667) + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199
631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)
) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqr
t(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954230
17213*sqrt(29) + 40699873480352667)) - 62716756730859*sqrt(1218)*sqrt(-1809
83329 + 45923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 4
5923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/(-4509673
516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*
sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 406998734
80352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 459
23327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-18098
3329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) +
 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-4
7106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 21409542301721
3*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 4
0699873480352667))) + 2*sqrt(-sqrt(214095423017213*sqrt(29) + 47106822945*s
qrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/9367136
690688 + 199631405/37468546762752 + 1583563*sqrt(29)/430672951296)*atan(454
016646529997363147366848*x/(3601609981798895040*sqrt(-180983329 + 45923327*
sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sq
rt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 13776
9981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(21409542301
7213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))
 + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sq
rt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-18098
3329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213
*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4
0699873480352667) + 199631405 + 137769981*sqrt(29))) - 62716756730859*sqrt(
1218)*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) +
```

47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4069987348035
2667)/(3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(-4*sqr
t(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 4509
673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106
822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)
 + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(2140954
23017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 471068229
45*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 1
99631405 + 137769981*sqrt(29))) - 7203219963597790080*sqrt(214095423017213*
sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40
699873480352667)/(3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*
sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983
329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(
29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(
29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4069987
3480352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*s
qrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 459
23327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29)
+ 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480
352667) + 199631405 + 137769981*sqrt(29))) - 431474904194070573364 6*sqrt(12
18)*sqrt(-180983329 + 45923327*sqrt(29))/(3601609981798895040*sqrt(-1809833
29 + 45923327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945
*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199
631405 + 137769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sq
rt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 4592
3327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 209
05585576953*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(121
8)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(2
14095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327
*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))) + 165397
912920614705160598080*sqrt(29)/(3601609981798895040*sqrt(-180983329 + 45923
327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218
)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 1
37769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(2140954
23017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 2090558557695
3*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-1
80983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(21409542301
7213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))
 + 40699873480352667) + 199631405 + 137769981*sqrt(29))) + 2932424170326692
281206238216/(3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt
(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329
+ 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))

```
 + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(29)
+ 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480
352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(
214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 4592332
7*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47
106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 406998734803526
67) + 199631405 + 137769981*sqrt(29))) + 5568313446945999843925 98*sqrt(42)*s
qrt(-180983329 + 45923327*sqrt(29))/(3601609981798895040*sqrt(-180983329 +
45923327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt
(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 19963140
5 + 137769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(21
4095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*
sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 20905585
576953*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sq
rt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095
423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))))
```

3.51 $\int \left(1+4x+4x^2+4x^4\right)^4 dx$

Optimal. Leaf size=97

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112$$

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^
9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/1
7*x^17
```

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right)^4 dx = \int \left(1+16x+112x^2+448x^3+1136x^4+1984x^5+2752x^6+3584x^7+4192x^8+3\right.$$

$$= x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.00

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

fricas [A] time = 0.34, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^{9}+448x^{8}+\frac{2752}{7}x^{7}+\frac{992}{3}x^{6}+\frac{1136}{5}x^{5}+11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

giac [A] time = 0.36, size = 77, normalized size = 0.79

$$\frac{256}{17}\,x^{17}+\frac{1024}{15}\,x^{15}+\frac{512}{7}\,x^{14}+\frac{1792}{13}\,x^{13}+256\,x^{12}+\frac{3328}{11}\,x^{11}+384\,x^{10}+\frac{4192}{9}\,x^{9}+448\,x^{8}+\frac{2752}{7}\,x^{7}+\frac{992}{3}\,x^{6}+\frac{1136}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

maple [A] time = 0.00, size = 78, normalized size = 0.80

$$\frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^{9}+448x^{8}+\frac{2752}{7}x^{7}+\frac{992}{3}x^{6}+\frac{1136}{5}x^{5}+11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^4,x)
```

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^
9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/1
7*x^17
```

maxima [A] time = 0.57, size = 77, normalized size = 0.79

$$\frac{256}{17}\,x^{17}+\frac{1024}{15}\,x^{15}+\frac{512}{7}\,x^{14}+\frac{1792}{13}\,x^{13}+256\,x^{12}+\frac{3328}{11}\,x^{11}+384\,x^{10}+\frac{4192}{9}\,x^{9}+448\,x^{8}+\frac{2752}{7}\,x^{7}+\frac{992}{3}\,x^{6}+\frac{1136}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

mupad [B] time = 0.15, size = 77, normalized size = 0.79

$$\frac{256\,x^{17}}{17}+\frac{1024\,x^{15}}{15}+\frac{512\,x^{14}}{7}+\frac{1792\,x^{13}}{13}+256\,x^{12}+\frac{3328\,x^{11}}{11}+384\,x^{10}+\frac{4192\,x^{9}}{9}+448\,x^{8}+\frac{2752\,x^{7}}{7}+\frac{992\,x^{6}}{3}+\frac{1136\,x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^4,x)
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

sympy [A] time = 0.07, size = 94, normalized size = 0.97

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^{9}}{9}+448x^{8}+\frac{2752x^{7}}{7}+\frac{992x^{6}}{3}+\frac{1136x^{5}}{5}+112$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

```
[Out] 256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*x**12 + 33
28*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752*x**7/7 + 992*x**6/3
 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x**2 + x
```

3.52 $\int \left(1+4x+4x^2+4x^4\right)^3 \, dx$

Optimal. Leaf size=69

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

```
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+
192/11*x^11+64/13*x^13
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \int \left(1+12x+60x^2+160x^3+252x^4+288x^5+352x^6+384x^7+240x^8+192x^9$$

$$= x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192}{11}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

fricas [A] time = 0.36, size = 57, normalized size = 0.83

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

giac [A] time = 0.36, size = 57, normalized size = 0.83

$$\frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^9 + 48\,x^8 + \frac{352}{7}\,x^7 + 48\,x^6 + \frac{252}{5}\,x^5 + 40\,x^4 + 20\,x^3 + 6\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^3,x)
```

```
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+
192/11*x^11+64/13*x^13
```

maxima [A] time = 0.46, size = 57, normalized size = 0.83

$$\frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^9 + 48\,x^8 + \frac{352}{7}\,x^7 + 48\,x^6 + \frac{252}{5}\,x^5 + 40\,x^4 + 20\,x^3 + 6\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

mupad [B] time = 0.06, size = 57, normalized size = 0.83

$$\frac{64\,x^{13}}{13}+\frac{192\,x^{11}}{11}+\frac{96\,x^{10}}{5}+\frac{80\,x^9}{3}+48\,x^8+\frac{352\,x^7}{7}+48\,x^6+\frac{252\,x^5}{5}+40\,x^4+20\,x^3+6\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^3,x)
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

sympy [A] time = 0.07, size = 66, normalized size = 0.96

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

```
[Out] 64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 352*x**7/7
+ 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

3.53 $\int \left(1+4x+4x^2+4x^4\right)^2 \, dx$

Optimal. Leaf size=45

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

```
[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1+4x+4x^2+4x^4\right)^2 \, dx &= \int \left(1+8x+24x^2+32x^3+24x^4+32x^5+32x^6+16x^8\right) dx\\ &= x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.00

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9

fricas [A] time = 0.34, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")

[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x

giac [A] time = 0.25, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")

[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x

maple [A] time = 0.00, size = 38, normalized size = 0.84

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^4+4*x^2+4*x+1)^2,x)

[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9

maxima [A] time = 0.57, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")

[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x

mupad [B] time = 0.03, size = 37, normalized size = 0.82

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

sympy **[A]** time = 0.06, size = 42, normalized size = 0.93

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 +
x
```

3.54 $\int \left(1+4x+4x^2+4x^4\right) dx$

Optimal. Leaf size=21

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

```
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[1 + 4*x + 4*x^2 + 4*x^4,x]
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right) dx = x+2x^2+\frac{4x^3}{3}+\frac{4x^5}{5}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

fricas [A] time = 0.34, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5+\frac{4}{3}x^3+2x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

giac [A] time = 0.36, size = 17, normalized size = 0.81

$$\frac{4}{5}\,x^{5}+\frac{4}{3}\,x^{3}+2\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{4}{5}x^{5}+\frac{4}{3}x^{3}+2x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^4+4*x^2+4*x+1,x)
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

maxima [A] time = 0.57, size = 17, normalized size = 0.81

$$\frac{4}{5}\,x^{5}+\frac{4}{3}\,x^{3}+2\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{4\,x^{5}}{5}+\frac{4\,x^{3}}{3}+2\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + 4*x^2 + 4*x^4 + 1,x)
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

sympy [A] time = 0.06, size = 19, normalized size = 0.90

$$\frac{4x^{5}}{5}+\frac{4x^{3}}{3}+2x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**4+4*x**2+4*x+1,x)
[Out] 4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

3.55 $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

Optimal. Leaf size=234

$$-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\right.$$

```
[Out] 1/2*arctan(-1/2+1/2*(1+1/x)^2)-1/20*ln((1+1/x)^2+5^(1/2)-(1+1/x)*(2+2*5^(1/
2))^(1/2))*(-10+5*5^(1/2))^(1/2)+1/20*ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(
1/2))^(1/2))*(-10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/
(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x+(2+2*5^(1/2))
^(1/2))/(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2069, 1673, 1169, 634, 618, 204, 628, 12, 1107}

$$-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 - Sqrt
[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(2 + Sqrt[5])/5
]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sq
rt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1
+ x^(-1))^2])/4 + (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5]
)]*(1 + x^(-1)) + (1 + x^(-1))^2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
```

```
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = -\left(16\,\text{Subst}\left(\int \frac{(4-4x)^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int -\frac{32x}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\right) - 16\,\text{Subst}\left(\int \frac{16+16\cdots}{1280-512x^2\cdots}\right.$$

$$= 512\,\text{Subst}\left(\int \frac{x}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{16\sqrt{2\left(1+\sqrt{5}\right)}-\left(16-16\sqrt{5}\right)x\cdots}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\,x+x^2}\cdots\right.}{32\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$= 256\,\text{Subst}\left(\int \frac{1}{1280-512x+256x^2}\,dx, x, \left(1+\frac{1}{x}\right)^2\right) + \frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{-\sqrt{2(1+\cdots}}{\sqrt{5}-\sqrt{2(1+\cdots}}\right.}{4\sqrt{10\left(1+\cdots\right.}}$$

$$= -\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\,\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)+\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\,\log\left(\cdots\right.$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) - \frac{\left(1+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)}{4\sqrt{10}} - \frac{\left(1+\sqrt{5}\right)^{3/2}\tan\cdots}{4\cdots}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.20

$$\frac{1}{4}\text{RootSum}\left[4\#1^4+4\#1^2+4\#1+1\&, \frac{\log(x-\#1)}{4\#1^3+2\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) & ]/
4
```

fricas [C] time = 1.33, size = 499, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] -1/20*(sqrt(10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*
I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I
)^2 - 9) - 5*sqrt(1/10*I - 1/5) - 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/
10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I -
 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^
2 + ((6*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1
/5) - I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I
- 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)
- 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) + 1/20*(sqrt(10)*sqrt(
-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt
(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 5*sqrt(1/1
0*I - 1/5) + 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/10*I - 1/5) - I)^2*(1
2*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1
/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^2 - ((6*sqrt(10)*(2*s
qrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1/5) - I) - sqrt(10)*(
2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(
2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*
I - 1/5) + I)^2 - 9) + 8*x + 3) - 1/4*(2*sqrt(1/10*I - 1/5) - I)*log(-5*(2*
sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) - 30*(2*sqrt(1
/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 30*(2*sqrt(-1/10*I - 1/5)
 + I)^3 + 8*x - 216*sqrt(-1/10*I - 1/5) - 108*I + 21) - 1/4*(2*sqrt(-1/10*I
 - 1/5) + I)*log(30*(2*sqrt(-1/10*I - 1/5) + I)^3 + 5*(2*sqrt(-1/10*I - 1/5
) + I)^2 + 8*x + 216*sqrt(-1/10*I - 1/5) + 108*I - 27)
```

giac [C] time = 0.52, size = 265, normalized size = 1.13

$$-\frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(\frac{i}{\sqrt{5}-2}+1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)x+29\sqrt{5}\sqrt{29\sqrt{5}+62}+(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] -1/20*((I + 2)*sqrt(sqrt(5) - 2)*(I/(sqrt(5) - 2) + 1) + 5*I)*log((406*I +
174)*sqrt(5)*x + (868*I + 372)*x + 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I
 - 203)*sqrt(5) + (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*(
(I + 2)*sqrt(sqrt(5) - 2)*(-I/(sqrt(5) - 2) - 1) + 5*I)*log((406*I + 174)*s
qrt(5)*x + (868*I + 372)*x - 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I - 203
)*sqrt(5) - (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*((2*I +
 1)*sqrt(sqrt(5) + 2)*(-I/(sqrt(5) + 2) - 1) - 5*I)*log((26*I + 130)*sqrt(5
)*x - (44*I + 220)*x + 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(
5) + (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22) - 1/20*((2*I + 1)*sqrt
```

```
(sqrt(5) + 2)*(I/(sqrt(5) + 2) + 1) - 5*I)*log((26*I + 130)*sqrt(5)*x - (44
*I + 220)*x - 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(5) - (19*
I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22)
```

maple [C] time = 0.01, size = 41, normalized size = 0.18

$$\frac{\ln\left(-\operatorname{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+x\right)}{16\operatorname{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)^3+8\operatorname{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1),x)
```

```
[Out] 1/4*sum(1/(4*_R^3+2*_R+1)*ln(-_R+x),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

mupad [B] time = 2.36, size = 87, normalized size = 0.37

$$\sum_{k=1}^{4}\ln\left(-\operatorname{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(\frac{x}{4}+\operatorname{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(6\,x+\operatorname{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1),x)
```

```
[Out] symsum(log(-root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(x/4 + root(z^4 +
(9*z^2)/40 + z/40 + 1/1280, z, k)*(6*x + root(z^4 + (9*z^2)/40 + z/40 + 1/1
280, z, k)*(36*x + 16))))*root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k), k,
1, 4)
```

sympy [B] time = 2.57, size = 3432, normalized size = 14.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

```
[Out] sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(-8 - 21*sqrt(5)*sqrt(-2 + sqrt(5))/1
0 - sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 - sqrt(5)/2 + 12*s
qrt(-2 + sqrt(5)) + 9*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 +
sqrt(5)) + sqrt(5) + 19)/5) - 841*sqrt(5)*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)
) + sqrt(5) + 19)/20 - 14351/40 - 441*sqrt(-2 + sqrt(5))/4 - 75*sqrt(5)*sqr
t(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8 - 3*sq
rt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 301*s
qrt(5)*sqrt(-2 + sqrt(5))/10 + 7407*sqrt(5)/40 + 3913*sqrt(-2*sqrt(5)*sqrt(
-2 + sqrt(5)) + sqrt(5) + 19)/40) - sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(
-8 - 12*sqrt(-2 + sqrt(5)) - sqrt(5)/2 + 21*sqrt(5)*sqrt(-2 + sqrt(5))/10 +
 sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 + 9*sqrt(5)*sqrt(-2 +
sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/5) - 3913*sqrt(2
*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40 - 14351/40 - 75*sqrt(5)*sqrt
(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8 - 301*sq
rt(5)*sqrt(-2 + sqrt(5))/10 - 3*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 +
 sqrt(5)) + sqrt(5) + 19) + 441*sqrt(-2 + sqrt(5))/4 + 7407*sqrt(5)/40 + 84
1*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/20) - 2*sqrt(3/
80 + 3*sqrt(5)/80 + sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40)*
atan(-20*x/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(
-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(
5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*
sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) - 18*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2
*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5)
 + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt
(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) - 120*
sqrt(-2 + sqrt(5))/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt
(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) +
 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 +
 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*
sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(5)/(-27*sqrt(5)*sqrt(3
 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sq
rt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2
 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
 19)) + 5*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(-27*sqrt(5)*s
qrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) +
 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt
(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sq
rt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)) + 21*sqrt(5)*sqrt(-2 + sqrt(5))/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5)
+ 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(
5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
```

```
9)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) +
 sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 80/(-
27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sq
rt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-
2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqr
t(5)) + sqrt(5) + 19))) - 2*sqrt(-sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(
5) + 19)/40 + 3/80 + 3*sqrt(5)/80)*atan(20*x/(5*sqrt(-2 + sqrt(5))*sqrt(-2*
sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sqr
t(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt
(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*
sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))) - 80/(5
*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(
5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9) + 3 + 3*sqrt(5))) - 120*sqrt(-2 + sqrt(5))/(5*sqrt(-2 + sqrt(5))*sqrt(-2
*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sq
rt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqr
t(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2
*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))) - 5*sq
rt(5)/(5*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqr
t(5) + 19) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + s
qrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2
 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqr
t(5) + 19) + 3 + 3*sqrt(5))) + 21*sqrt(5)*sqrt(-2 + sqrt(5))/(5*sqrt(-2 + s
qrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*s
qrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5
) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*s
qrt(5))) + 5*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(5*sqrt(-2 +
 sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3
*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5)
+ 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3
*sqrt(5))) + 18*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)
) + sqrt(5) + 19)/(5*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)
*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqr
t(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))))
```

3.56 $\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^2} \, dx$

Optimal. Leaf size=317

$$-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{\right.$$

```
[Out] 1/2*(-17+(1+1/x)^2)/(5-2*(1+1/x)^2+(1+1/x)^4)+1/10*(59-17*(1+1/x)^2)*(1+1/x
)/(5-2*(1+1/x)^2+(1+1/x)^4)+7/4*arctan(-1/2+1/2*(1+1/x)^2)+1/400*ln((1+1/x)
^2+5^(1/2)-(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1/2)-1/400*
ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1
/2)-1/200*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(59590+2
6650*5^(1/2))^(1/2)-1/200*arctan((2+2/x+(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))
^(1/2))*(59590+26650*5^(1/2))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {2069, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660, 12}

$$-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] -(17 - (1 + x^(-1))^2)/(2*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)) + ((59 -
 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^
4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 - (Sqrt[(5959 + 2665*Sqrt[5])/1
0]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (
Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqr
t[2*(-1 + Sqrt[5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - S
qrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959 + 26
65*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(
-1))^2])/40
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^2}\,dx = -\left(16\,\text{Subst}\left(\int \frac{(4-4x)^6}{\left(1280-512x^2+256x^4\right)^2}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{x\left(-24576-81920x^2-24576x^4\right)}{\left(1280-512x^2+256x^4\right)^2}\,dx, x, 1+\frac{1}{x}\right)\right) - 16\,\text{Subst}\left(\int \frac{4}{}\right.$$

$$= \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)}{167772160}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int -\frac{}{128}\right.}{}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + 896\,\text{Subst}\left(\int\right.$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{1}{40}\sqrt{-\frac{5959}{10}+}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{7}{4}\tan^{-1}\left(\frac{1}{2}\left(-\right.\right.$$

Mathematica [C] time = 0.02, size = 108, normalized size = 0.34

$$\frac{1}{40}\left(\text{RootSum}\left[4\#1^4+4\#1^2+4\#1+1\&, \frac{18\#1^2\log(x-\#1)-16\#1\log(x-\#1)+27\log(x-\#1)}{4\#1^3+2\#1+1}\&\right] + \frac{72x^3-32x}{4x^4+4x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] ((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#1
 + 4*#1^2 + 4*#1^4 & , (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #1]
*#1^2)/(1 + 2*#1 + 4*#1^3) & ])/40
```

fricas [C] time = 1.34, size = 704, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(720*x^3 - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000
) + 7*I)*log(33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 11755375/4*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 541735337*x + 25784243612*sqrt(19
/1000*I - 5959/2000) + 45122426321*I - 71080995) - 50*(4*x^4 + 4*x^2 + 4*x
+ 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)*log(-33368250*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^3 - 125/4*(4271136*sqrt(19/1000*I - 5959/2000) + 74744
88*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 25*(1334730*(4*sqr
t(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000)
 - 7*I) + 541735337*x - 25806203712*sqrt(19/1000*I - 5959/2000) - 451608564
96*I - 355111539) - 320*x^2 - (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-3
75/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I -
5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-
19/1000*I - 5959/2000) - 7*I)^2 - 3021) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-1
9/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000
) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/
8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 1
/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19
/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32
*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 59
59/2000) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 8
78404*sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 3843
0175/2*I + 213096267) + (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2
000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/100
0*I - 5959/2000) - 7*I)^2 - 3021) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000
*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000) + 74
74488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/8*(4*s
qrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I - 595
9/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 1/2*sqr
t(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*
```

```
I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sq
rt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 5959/200
0) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 878404*
sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 38430175/2
*I + 213096267) + 840*x + 380)/(4*x^4 + 4*x^2 + 4*x + 1)
```

giac [C] time = 0.72, size = 315, normalized size = 0.99

$$-\frac{1}{400}\left(-(i+3)\sqrt{2665\sqrt{5}-4790}\left(\frac{709i}{533\sqrt{5}-958}+1\right)-350i\right)\log\left((2534636224790i+16853816172010)\sqrt{5}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

```
[Out] -1/400*(-(I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1)
- 350*I)*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*
I + 26022625108780)*x + 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118)
- (8426908086005*I - 1267318112395)*sqrt(5) + (8166407345*I - 7795873310)*s
qrt(1424281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*
((I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1) - 350*I)
*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*I + 2602
2625108780)*x - 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118) - (84269
08086005*I - 1267318112395)*sqrt(5) - (8166407345*I - 7795873310)*sqrt(1424
281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*((3*I +
1)*sqrt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((
16722951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 381438558
5140)*x + 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (12404110944
55*I - 8361475596225)*sqrt(5) + (8153361745*I + 7721428310)*sqrt(1416617*sq
rt(5) + 2178118) + 1907192792570*I - 12856178136150) - 1/400*(-(3*I + 1)*sq
rt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((16722
951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 3814385585140)
*x - 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (1240411094455*I
- 8361475596225)*sqrt(5) - (8153361745*I + 7721428310)*sqrt(1416617*sqrt(5)
 + 2178118) + 1907192792570*I - 12856178136150) + 1/20*(36*x^3 - 16*x^2 + 4
2*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1)
```

maple [C] time = 0.01, size = 79, normalized size = 0.25

$$\frac{\left(18\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)^2-16\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+27\right)\ln\left(-\mathrm{RootOf}\left(4\_Z^4+4\_Z\right.\right.}{160\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)^3+80\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^2,x)
```

```
[Out] (9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^2-16*_R+27
)/(4*_R^3+2*_R+1)*ln(-_R+x),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{36\,x^3-16\,x^2+42\,x+19}{20\left(4\,x^4+4\,x^2+4\,x+1\right)}+\frac{1}{10}\int\frac{18\,x^2-16\,x+27}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*integra
te((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

mupad [B] time = 2.21, size = 174, normalized size = 0.55

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{169\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)}{100}+\frac{11\,x}{1600}+\frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\,x\,131}{100}-\frac{\mathrm{roo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

```
[Out] symsum(log((11*x)/1600 - (169*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 2
9/64000, z, k))/100 + (131*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/6
4000, z, k)*x)/100 - (72*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/640
00, z, k)^2*x)/5 - 36*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000,
 z, k)^3*x + (59*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k
)^2)/20 - 16*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^3
+ 27/1600)*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k), k,
1, 4) + ((21*x)/40 - x^2/5 + (9*x^3)/20 + 19/80)/(x + x^2 + x^4 + 1/4)
```

sympy [B] time = 3.66, size = 3834, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] (36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) - sqrt(-595
9/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-1601676*sqrt(10)*sqrt(-5959 + 26
```

65*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)
+ 36004639)/13543383425 - 1067784*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/101638
9 + 3131659367*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/
1083470674 + 470215*sqrt(5)/2032778 + 94043*sqrt(-665*sqrt(10)*sqrt(-5959 +
 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337) - 40634464149111451*
sqrt(5)*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36
004639)/27530691871904650 - 2885835544225227917282997/146738587677251784500
 - 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806926 - 5020
8805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093 - 53848575489193
3*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*
sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450 - 92532195509690
1411*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/29347717535450356900 + 48430461193
8766076267*sqrt(5)/55061383743809300 + 22013036087014785403*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/666993580351144
4750) + sqrt(-5959/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-94043*sqrt(665*
sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337
- 1601676*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sqrt(-5959
+ 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/13543383425 - 3131659367*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/1083470674 + 1067784
*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/1016389 + 470215*sqrt(5)/2032778) - 220
13036087014785403*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639)/6669935803511444750 - 2885835544225227917282997/1467385876
77251784500 - 50208805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093
 - 538485754891933*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450
+ 925321955096901411*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/293477175354503569
00 + 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806926 + 48
4304611938766076267*sqrt(5)/55061383743809300 + 40634464149111451*sqrt(5)*s
qrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/27
530691871904650) + 2*sqrt(6291/16000 + 1599*sqrt(5)/3200 + sqrt(-665*sqrt(1
0)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/4000)*atan(54173
533700*x/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt
(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt
(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqr
t(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 2
21195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
221195*sqrt(5) + 36004639)) - 3203352*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*s
qrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/(
-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2
665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2
665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7

995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639)) - 28456443600*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/(-644057
0878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqr
t(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqr
t(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) +
36004639)) + 6263318734*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/(-6440570878*sq
rt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sq
rt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqrt(5))*s
qrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqrt(5) +
4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3600463
9)) + 7292234875/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-6
65*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351
075*sqrt(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt
(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt
(5)) + 221195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)) + 6265614875*sqrt(5)/(-6440570878*sqrt(
10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(
5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt
(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22
1195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*s
qrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639))*
sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639))
+ 4702150*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) +
36004639)/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqr
t(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sq
rt(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*s
qrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
221195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))
+ 221195*sqrt(5) + 36004639))) - 2*sqrt(-sqrt(665*sqrt(10)*sqrt(-5959 + 266
5*sqrt(5)) + 221195*sqrt(5) + 36004639)/4000 + 6291/16000 + 1599*sqrt(5)/32
00)*atan(54173533700*x/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt(665
*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 +
7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 1067784
*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639) + 6291 + 7995*sqrt(5))) - 4702150*sqrt(665*sqrt(10)*sqrt(-5

```
959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/(2351075*sqrt(-5959 + 2665
*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(
665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291
 + 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2
665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) - 3203352
*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sq
rt(5)) + 221195*sqrt(5) + 36004639)/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqr
t(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 360046
39) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*
sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(
5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22119
5*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))
 + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) - 6263318734*sqrt(10)*
sqrt(-5959 + 2665*sqrt(5))/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt
(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 629
1 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-595
9 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 106
7784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)
 + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*
sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) + 7292234875/(2351075*sqrt(-595
9 + 2665*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22
1195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(
-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
) + 6291 + 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) +
 28456443600*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/(2351075*sqrt(-5959 + 2665*
sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(6
65*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291
+ 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt
(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 26
65*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) + 62656148
75*sqrt(5)/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)
) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)
) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 1067784*sqrt(10)*sq
rt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)*sqr
t(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 360046
39) + 6291 + 7995*sqrt(5))))
```

3.57 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx$

Optimal. Leaf size=104

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8$$

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^
7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x
^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

Rubi [A] time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx = \int \left(4096+49152x+237568x^2+559104x^3+538624x^4-184320x^5-\right.$$

$$= 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-$$

Mathematica [A] time = 0.00, size = 104, normalized size = 1.00

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

fricas [A] time = 0.37, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^{9}+36384x^{8}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

giac [A] time = 0.36, size = 84, normalized size = 0.81

$$\frac{4096}{17}\,x^{17}-1920\,x^{16}+\frac{102784}{15}\,x^{15}-\frac{75504}{7}\,x^{14}-\frac{12095}{13}\,x^{13}+31128\,x^{12}-\frac{331040}{11}\,x^{11}-\frac{169584}{5}\,x^{10}+\frac{641152}{9}\,x^{9}+36384$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

maple [A] time = 0.00, size = 85, normalized size = 0.82

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^{9}+36384x^{8}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x)
```

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^
7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x
^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

maxima [A] time = 0.66, size = 84, normalized size = 0.81

$$\frac{4096}{17}\,x^{17}-1920\,x^{16}+\frac{102784}{15}\,x^{15}-\frac{75504}{7}\,x^{14}-\frac{12095}{13}\,x^{13}+31128\,x^{12}-\frac{331040}{11}\,x^{11}-\frac{169584}{5}\,x^{10}+\frac{641152}{9}\,x^{9}+3638$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

mupad [B] time = 2.23, size = 84, normalized size = 0.81

$$\frac{4096\,x^{17}}{17}-1920\,x^{16}+\frac{102784\,x^{15}}{15}-\frac{75504\,x^{14}}{7}-\frac{12095\,x^{13}}{13}+31128\,x^{12}-\frac{331040\,x^{11}}{11}-\frac{169584\,x^{10}}{5}+\frac{641152\,x^{9}}{9}+3638$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^4,x)
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

sympy [A] time = 0.08, size = 100, normalized size = 0.96

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^{9}}{9}+36384x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13/
13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 36384
*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 237568*x
**3/3 + 24576*x**2 + 4096*x
```

3.58 $\int \left(8 + 24x + 8x^2 - 15x^3 + 8x^4\right)^3 dx$

Optimal. Leaf size=76

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-293
6/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(8 + 24x + 8x^2 - 15x^3 + 8x^4\right)^3 dx &= \int \left(512 + 4608x + 15360x^2 + 20160x^3 - 384x^4 - 17856x^5 + 5528x^6 + 16 \right. \\ &= 512x + 2304x^2 + 5120x^3 + 5040x^4 - \frac{384x^5}{5} - 2976x^6 + \frac{5528x^7}{7} + 2097x \end{aligned}$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.00

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

fricas [A] time = 0.34, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13}-240x^{12}+\frac{6936}{11}x^{11}-\frac{4527}{10}x^{10}-\frac{2936}{3}x^{9}+2097x^{8}+\frac{5528}{7}x^{7}-2976x^{6}-\frac{384}{5}x^{5}+5040x^{4}+5120x^{3}+2304x^{2}+51$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

giac [A] time = 0.38, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13}-240x^{12}+\frac{6936}{11}x^{11}-\frac{4527}{10}x^{10}-\frac{2936}{3}x^{9}+2097x^{8}+\frac{5528}{7}x^{7}-2976x^{6}-\frac{384}{5}x^{5}+5040x^{4}+5120x^{3}+2304x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.86

$$\frac{512}{13}x^{13}-240x^{12}+\frac{6936}{11}x^{11}-\frac{4527}{10}x^{10}-\frac{2936}{3}x^{9}+2097x^{8}+\frac{5528}{7}x^{7}-2976x^{6}-\frac{384}{5}x^{5}+5040x^{4}+5120x^{3}+2304x^{2}+51$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x)
```

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-293
6/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

maxima [A] time = 0.65, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13}-240x^{12}+\frac{6936}{11}x^{11}-\frac{4527}{10}x^{10}-\frac{2936}{3}x^{9}+2097x^{8}+\frac{5528}{7}x^{7}-2976x^{6}-\frac{384}{5}x^{5}+5040x^{4}+5120x^{3}+2304x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

mupad [B] time = 0.08, size = 64, normalized size = 0.84

$$\frac{512\,x^{13}}{13}-240\,x^{12}+\frac{6936\,x^{11}}{11}-\frac{4527\,x^{10}}{10}-\frac{2936\,x^9}{3}+2097\,x^8+\frac{5528\,x^7}{7}-2976\,x^6-\frac{384\,x^5}{5}+5040\,x^4+5120\,x^3+2304\,x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^3,x)
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

sympy [A] time = 0.08, size = 73, normalized size = 0.96

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x**9/3 + 20
97*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x**4 + 5120*x**3 + 23
04*x**2 + 512*x
```

3.59 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=52

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

```
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx &= \int \left(64+384x+704x^2+144x^3-528x^4+144x^5+353x^6-240x^7+64\right.\\ &= 64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 52, normalized size = 1.00

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

fricas [A] time = 0.34, size = 44, normalized size = 0.85

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

giac [A] time = 0.36, size = 44, normalized size = 0.85

$$\frac{64}{9}\,x^9 - 30\,x^8 + \frac{353}{7}\,x^7 + 24\,x^6 - \frac{528}{5}\,x^5 + 36\,x^4 + \frac{704}{3}\,x^3 + 192\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.87

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

```
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

maxima [A] time = 0.61, size = 44, normalized size = 0.85

$$\frac{64}{9}\,x^9 - 30\,x^8 + \frac{353}{7}\,x^7 + 24\,x^6 - \frac{528}{5}\,x^5 + 36\,x^4 + \frac{704}{3}\,x^3 + 192\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.85

$$\frac{64\,x^9}{9}-30\,x^8+\frac{353\,x^7}{7}+24\,x^6-\frac{528\,x^5}{5}+36\,x^4+\frac{704\,x^3}{3}+192\,x^2+64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

sympy [A] time = 0.07, size = 49, normalized size = 0.94

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x**
3/3 + 192*x**2 + 64*x
```

3.60 $\int \left(8+24x+8x^2-15x^3+8x^4\right) dx$

Optimal. Leaf size=30

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

```
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Antiderivative was successfully verified.

```
[In] Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
```

```
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Antiderivative was successfully verified.

```
[In] Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
```

```
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

fricas [A] time = 0.34, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5-\frac{15}{4}x^4+\frac{8}{3}x^3+12x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

giac [A] time = 0.30, size = 24, normalized size = 0.80

$$\frac{8}{5}\,x^{5}-\frac{15}{4}\,x^{4}+\frac{8}{3}\,x^{3}+12\,x^{2}+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{8}{5}x^{5}-\frac{15}{4}x^{4}+\frac{8}{3}x^{3}+12x^{2}+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x^4-15*x^3+8*x^2+24*x+8,x)
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

maxima [A] time = 0.46, size = 24, normalized size = 0.80

$$\frac{8}{5}\,x^{5}-\frac{15}{4}\,x^{4}+\frac{8}{3}\,x^{3}+12\,x^{2}+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

mupad [B] time = 0.02, size = 24, normalized size = 0.80

$$\frac{8\,x^5}{5}-\frac{15\,x^4}{4}+\frac{8\,x^3}{3}+12\,x^2+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8,x)
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

sympy [A] time = 0.06, size = 27, normalized size = 0.90

$$\frac{8x^{5}}{5}-\frac{15x^{4}}{4}+\frac{8x^{3}}{3}+12x^{2}+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)
```

```
[Out] 8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x
```

3.61 $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

Optimal. Leaf size=263

$$\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+\frac{1}{8}$$

```
[Out] 1/52*arctan(1/39*(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/322608*ln((3+4/x)
^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))*(-208364442+9476610*517^(1/2))
^(1/2)+1/322608*ln((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-20
8364442+9476610*517^(1/2))^(1/2)-1/161304*arctan((6+8/x-(38+2*517^(1/2))^(1
/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9476610*517^(1/2))^(1/2)-1/161304*
arctan((6+8/x+(38+2*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9
476610*517^(1/2))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+\frac{1}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] -(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])] +
 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 - (Sqrt[(5167 + 235*Sqrt[517])/40326]*A
rcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 +
(Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/4 - (Sqrt[(-5167 + 2
35*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) + (
3 + 4/x)^2])/8 + (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[
2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(24-32x)^2}{8\left(2117632-2490368x^2+1048576x^4\right)}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int \frac{(24-32x)^2}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int -\frac{1536x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right) - 12$$

Sub

$$= 196608\,\text{Subst}\left(\int \frac{x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right) -$$

(5

$$= 98304\,\text{Subst}\left(\int \frac{1}{2117632-2490368x+1048576x^2}\,dx, x, \left(\frac{3}{4}+\frac{1}{x}\right)^2\right) -$$

$$= -\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\,\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right.\right.$$

$$= -\frac{1}{4}\sqrt{\frac{3}{13}}\,\tan^{-1}\left(\frac{19-\left(3+\frac{4}{x}\right)^2}{2\sqrt{39}}\right) - \frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\,\tan^{-1}\left(\frac{6+\sqrt{2(1}}{\sqrt{2(-}}\right.$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.21

$$\text{RootSum}\left[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&, \frac{\log(x-\#1)}{32\#1^3-45\#1^2+16\#1+24}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1 -
 45*#1^2 + 32*#1^3) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8 x^4 - 15 x^3 + 8 x^2 + 24 x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

maple [C] time = 0.01, size = 49, normalized size = 0.19

$$\frac{\ln \left(-\operatorname{RootOf}\left(8\_Z^4 - 15\_Z^3 + 8\_Z^2 + 24\_Z + 8\right) + x\right)}{32 \operatorname{RootOf}\left(8\_Z^4 - 15\_Z^3 + 8\_Z^2 + 24\_Z + 8\right)^3 - 45 \operatorname{RootOf}\left(8\_Z^4 - 15\_Z^3 + 8\_Z^2 + 24\_Z + 8\right)^2 + 16 \operatorname{RootOf}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x)
```

```
[Out] sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(-_R+x),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+
24*_Z+8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8 x^4 - 15 x^3 + 8 x^2 + 24 x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

mupad [B] time = 0.41, size = 123, normalized size = 0.47

$$\sum_{k=1}^{4} \ln\left(-\frac{\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)\left(2184\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)+256\,x+\mathrm{root}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8),x)
```

```
[Out] symsum(log(-(root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k
)*(2184*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) + 2
56*x + 38259*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k
)*x + 1531920*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z,
k)^2*x + 805896*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z
, k)^2 - 120))/4096)*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/7863
57, z, k), k, 1, 4)
```

sympy [A] time = 2.37, size = 41, normalized size = 0.16

$$\mathrm{RootSum}\left(50326848t^4+765960t^2+12753t+64,\left(t\mapsto t\log\left(\frac{100785893208t^3}{4758335}-\frac{1430512512t^2}{4758335}+\frac{729823525}{2236417}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
```

```
[Out] RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(10
0785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/223641
745 + x + 2270349121/1789133960)))
```

3.62 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx$

Optimal. Leaf size=366

$$\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}$$

```
[Out] -3/208*(3359-107*(3+4/x)^2)/(517-38*(3+4/x)^2+(3+4/x)^4)+1/322608*(3327931-
129631*(3+4/x)^2)*(3+4/x)/(517-38*(3+4/x)^2+(3+4/x)^4)+73/2704*arctan(1/39*
(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/26018980416*arctan((6+8/x-(38+2*51
7^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+
40326*517^(1/2))^(1/2)-1/26018980416*arctan((6+8/x+(38+2*517^(1/2))^(1/2))/
(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+40326*517^(1/2))
^(1/2)-1/26018980416*ln((3+4/x)^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))
*(-2405208568240933026+105781971094684170*517^(1/2))^(1/2)+1/26018980416*ln
((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-2405208568240933026+
105781971094684170*517^(1/2))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) +
((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517 - 38*(3 + 4/x)^2 +
(3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*A
rcTan[(6 - Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/6452
16 - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 +
Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sq
rt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-596441146
71451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[5
```

```
17])]*(3 + 4/x) + (3 + 4/x)^2])/645216 + (Sqrt[(-59644114671451 + 262317043
8295*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) +
 (3 + 4/x)^2])/645216
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(24-32x)^6}{64\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\right.\right.$$

$$= -\left(16\,\text{Subst}\left(\int \frac{(24-32x)^6}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{x\left(-1528823808-9059696640x^2-4831838208x^4\right)}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx\right.\right.$$

$$= \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{1209256852201639415 64}{2117632-2}\right.}{70942}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(\right.\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(\right.\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(\right.\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(\right.\right.}$$

Mathematica [C] time = 0.03, size = 128, normalized size = 0.35

$$\frac{\text{RootSum}\left[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&, \frac{19640\#1^2\log(x-\#1)-57489\#1\log(x-\#1)+74897\log(x-\#1)}{32\#1^3-45\#1^2+16\#1+24}\&\right]}{80652}+\frac{39280x^3-943}{161304\left(8x^4-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Log[
x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 45*#
1^2 + 32*#1^3) & ]/80652
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^{4} - 15\,x^{3} + 8\,x^{2} + 24\,x + 8\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)
```

maple [C] time = 0.01, size = 96, normalized size = 0.26

$$\frac{\left(19640\,\mathrm{RootOf}\left(8\_Z^{4}-15\_Z^{3}+8\_Z^{2}+24\_Z+8\right)^{2}-57489\,\mathrm{RootOf}\left(8\_Z^{4}-15\_Z^{3}+8\_Z^{2}+24\_Z+8\right)+\right.}{2580864\,\mathrm{RootOf}\left(8\_Z^{4}-15\_Z^{3}+8\_Z^{2}+24\_Z+8\right)^{3}-3629340\,\mathrm{RootOf}\left(8\_Z^{4}-15\_Z^{3}+8\_Z^{2}+24\_Z+8\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

```
[Out] (2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x^
2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+24)
*ln(-_R+x),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{39280\,x^{3} - 94314\,x^{2} + 89033\,x + 72888}{161304\,\left(8\,x^{4} - 15\,x^{3} + 8\,x^{2} + 24\,x + 8\right)} + \frac{1}{80652}\int \frac{19640\,x^{2} - 57489\,x + 74897}{8\,x^{4} - 15\,x^{3} + 8\,x^{2} + 24\,x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2
+ 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15*x
^3 + 8*x^2 + 24*x + 8), x)
```

mupad [B] time = 0.21, size = 181, normalized size = 0.49

$$\frac{\frac{2455\,x^3}{80652}-\frac{1429\,x^2}{19552}+\frac{89033\,x}{1290432}+\frac{3037}{53768}}{x^4-\frac{15\,x^3}{8}+x^2+3\,x+1}+\left(\sum_{k=1}^{4}\ln\left(\frac{2146659825\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023}{4420451055}\right.}{2960381771776}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] ((89033*x)/1290432 - (1429*x^2)/19552 + (2455*x^3)/80652 + 3037/53768)/(3*x
 + x^2 - (15*x^3)/8 + x^4 + 1) + symsum(log((2146659825*root(z^4 + (1491162
5619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/
44204510553294663, z, k))/2960381771776 + (2222183*x)/338246745408 + (92412
4364159*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3
730636104019968 + 43023440/44204510553294663, z, k)*x)/26643435945984 - (72
451101*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37
30636104019968 + 43023440/44204510553294663, z, k)^2*x)/8470528 - (95745*ro
ot(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37306361040
19968 + 43023440/44204510553294663, z, k)^3*x)/256 + (389551*root(z^4 + (14
911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 4302
3440/44204510553294663, z, k)^2)/264704 - (100737*root(z^4 + (1491162561931
1*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/442045
10553294663, z, k)^3)/512 + 271033/624455529984)*root(z^4 + (14911625619311
*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/4420451
0553294663, z, k), k, 1, 4)
```

sympy [B] time = 3.95, size = 3839, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] (39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 +
1290432*x**2 + 3871296*x + 1290432) + sqrt(-59644114671451/1678786246808985
6 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-1123969950204685033
```

```
06932567484755463/603722125611976319526135612861060 - 296438698298128332309
07750777733957*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/
1936419398792394461637855141912238396080 - 181533261043120360732*sqrt(-7120
427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) +
626362156858715004293 5*sqrt(517) + 3557579971691991294769382675)/1509305314
02994079881533903215265 - 46926347979646613249222*sqrt(517)/297468603626329
12338339 + 994065243322493861977*sqrt(78)*sqrt(-59644114671451 + 2623170438
295*sqrt(517))/1427849297406379792240272 + 994065243322493861977*sqrt(40326
)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sq
rt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715
0042935*sqrt(517) + 3557579971691991294769382675)/12909462658615963077585 70
094608158930720) - 459714970677306696892185479122356023880918931359173517 60
29*sqrt(517)*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623
170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912
94769382675)/184327671869986986264506040483747638901487480538062757284195 58
40 - 102213276372026717588278042506361313108860193595830387808115871094971 5
459967411486447/302201812380681690634631534385892067350866441656553353409 62
4708723614680800 - 106380947173342801261761115266827766437283828355653383 69
93*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/689619370306997
24367447235196266078627061899493829808665 60 - 890360389298500646731845595 59
303467032604459587082416931 3*sqrt(40326)*sqrt(-59644114671451 + 26231704382
95*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262
3170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991
294769382675)/935247388412167707960174961348988989867284925822989101461120 9
554309158400 - 45113976327488809325094501633826014671791*sqrt(78)*sqrt(-596
44114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*s
qrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqr
t(517) + 3557579971691991294769382675)/107753026610468319324136304994165747
854784217959109076040 + 4269800963651546871890094273427400521225528229955 28
02371283821308121207*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(
517))/93524738841216770796017496134898898986728492582298910146112095543091 5
8400 + 685487767096696740818928514074132093732180070609343531371525732094 05
073*sqrt(517)/460819179674967465661265101209369097253718701345156893210488 9
60 + 274196431933554153007434570764680602132735098624644758583157528631167 0
33*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*
sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129476938267
5)/483522899809090705015410455017427307761386306650485365455399533957783489
2800) - sqrt(-59644114671451/16787862468089856 + 5073830635*sqrt(517)/32471
687559168)*log(x**2 + x*(-11239699502046850330693256748475546 3/603722125611
976319526135612861060 - 994065243322493861977*sqrt(78)*sqrt(-59644114671451
 + 2623170438295*sqrt(517))/1427849297406379792240272 - 46926347979646613 24
9222*sqrt(517)/29746860362632912338339 + 181533261043120360732*sqrt(7120427
417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626
3621568587150042935*sqrt(517) + 3557579971691991294769382675)/1509305314029
94079881533903215265 + 29643869829812833230907750777733957*sqrt(40326)*sqrt
```

(-59644114671451 + 2623170438295*sqrt(517))/19364193987923944616378551419122
38396080 + 994065243322493861977*sqrt(40326)*sqrt(-59644114671451 + 262317
0438295*sqrt(517))*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 +
 2623170438295*sqrt(517)) + 626362156858715004293 5*sqrt(517) + 355757997169
1991294769382675)/12909462658615963077585700946081589307 20) - 2741964319335
5415300743457076468060213273509862464475858315752863116 7033*sqrt(7120427417
275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362
1568587150042935*sqrt(517) + 3557579971691991294769382675)/4835228998090907
0501541045501742730776138630665048536545539953395778348 92800 - 102213276372
0267175882780425063613131088601935958303878081158710949715459967411486447/3
0220181238068169063463153438589206735086644165655335340962470872361468 0800
- 890360389298500646731845595593034670326044595870824169313*sqrt(40326)*sqr
t(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042 93
5*sqrt(517) + 3557579971691991294769382675)/9352473884121677079601749613489
8898986728492582298910146112095543091 58400 - 4269800963651546871890094273 42
74005212255282299552802371283821308121207*sqrt(40326)*sqrt(-59644114671451
+ 2623170438295*sqrt(517))/935247388412167707960174961348988989867284925822
989101461120955430915 8400 - 4511397632748880932509450163382601467 1791*sqrt(
78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*s
qrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685871
50042935*sqrt(517) + 3557579971691991294769382675)/10775302661046831932413 6
30499416574785478421795910907604 0 + 1063809471733428012617611152668277664 37
2838283556533836993*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517)
)/689619370306997243674472351962660786270618994938298086656 0 + 685487767096
69674081892851407413209373218007060934353137152573209405073*sqrt(517)/46081
917967496746566126510120936909725371870134515689321048896 0 + 45971497067730
66968921854791223560238809189313591735176029*sqrt(517)*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675)/18432767186998698626 4
50604048374763890148748053806275728419558 40) - 2*sqrt(59653665894623/167878
62468089856 + 5073830635*sqrt(517)/10823895853056 + sqrt(-7120427417275887*
sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587
150042935*sqrt(517) + 3557579971691991294769382675)/4196965617022464)*atan(
-774567759516957784655142056764895358432 0*x/(-59292486929118917272637172801
533436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-
7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)
) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 232
9048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*s
qrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqr
t(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150
042935*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*s
qrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204274
17275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263
621568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-7120427

417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626
362156858715004293 5*sqrt(517) + 35575799716919912947693826 75)) - 2982195729
967481585931*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sq
rt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(
517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/(-
5929248692911891727263717280153343 6*sqrt(40326)*sqrt(59653665894623 + 78695
11314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671
451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579
971691991294769382675)) + 2329048502925820708785386304*sqrt(-59644114671451
 + 2623170438295*sqrt(517))*sqrt(59653665894623 + 7869511314885*sqrt(517) +
 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*
sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129476938267
5)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885
*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26
23170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199
1294769382675))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675)) - 269626104706077517551711257226632831 0*sqrt(78)*sqrt(-596
44114671451 + 2623170438295*sqrt(517))/(-592924869291189172726371728015334 3
6*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204
27417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6
263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 23290485
02925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(5
9653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004293
5*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqrt(4
0326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275
887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156
8587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-712042741727
5887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215
68587150042935*sqrt(517) + 3557579971691991294769382675)) + 610949118223023
869853714934815457011168 0*sqrt(517)/(-592924869291189172726371728015343 36*s
qrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204274
17275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263
621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 23290485029
25820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(5965
3665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)
*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*s
qrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqrt(4032
6)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858
7150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675)) + 465809700585164141
7570772608*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262317

0438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294
769382675)/(-5929248692911891727263717280153436*sqrt(40326)*sqrt(596536658
94623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt
(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(5
17) + 3557579971691991294769382675)) + 23290485029258207087853863O4*sqrt(-5
9644114671451 + 2623170438295*sqrt(517))*sqrt(59653665894623 + 786951131488
5*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 +
7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-596441
14671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35
57579971691991294769382675))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644
114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3
557579971691991294769382675)) + 5928773965962566646181550155546791 4*sqrt(40
326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/(-59292486929118917272
637172801533436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) +
 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*
sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129476938267
5)) + 2329048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqr
t(517))*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204274172
75887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621
568587150042935*sqrt(517) + 3557579971691991294769382675)) + 99406524332249
3861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(
-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517
)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt
(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(51
7)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 7
210195296486241387297607767303872707953368/(-5929248692911891727263717280153
3436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71
20427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))
+ 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 23290
48502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqr
t(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(
40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004
2935*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqr
t(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417
275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362
1568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-712042741
7275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636
21568587150042935*sqrt(517) + 3557579971691991294769382675))) - 2*sqrt(-sqr
t(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(51
7)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/4196
965617022464 + 59653665894623/16787862468089856 + 5073830635*sqrt(517)/1082
3895853056)*atan(774567759516957784655142056764895358432O*x/(23290485029258
20708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt

(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517
)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675) + 596
53665894623 + 7869511314885*sqrt(517)) + 5929248692911891727263717280153343
6*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-5964411467145
1 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997
1691991294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 99406524
3322493861977*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441146
71451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575
79971691991294769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59
644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517)
+ 3557579971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517))
) - 721019529648624138729760776730387270795368/(232904850292582070878538630
4*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042741727
5887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215
68587150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 +
 7869511314885*sqrt(517)) + 59292486929118917272637172801533436*sqrt(40326)
*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262317043
8295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769
382675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493861977
*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26231
70438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129
4769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451
+ 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716
91991294769382675) + 59653665894623 + 7869511314885*sqrt(517))) - 269626104
7060775175517112572266328310*sqrt(78)*sqrt(-59644114671451 + 2623170438295*
sqrt(517))/(2329048502925820708785386304*sqrt(-59644114671451 + 26231704382
95*sqrt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-5964411467145
1 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997
1691991294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 59292486
929118917272637172801533436*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(
40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004
2935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 786951131
4885*sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(7120427417275887*s
qrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685871
50042935*sqrt(517) + 3557579971691991294769382675)*sqrt(-4*sqrt(71204274172
75887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621
568587150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623
+ 7869511314885*sqrt(517))) - 6109491182230238698537149348154570111680*sqrt
(517)/(2329048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sq
rt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 5929248692911
8917272637172801533436*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326
)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*
sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 7869511314885*

sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(7120427417275887*sqrt(4
0326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042
935*sqrt(517) + 3557579971691991294769382675)*sqrt(-4*sqrt(7120427417275887
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858
7150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 786
9511314885*sqrt(517))) + 4658097005851641417570772608*sqrt(7120427417275887
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858
7150042935*sqrt(517) + 3557579971691991294769382675)/(232904850292582070878
5386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042
7417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62
63621568587150042935*sqrt(517) + 3557579971691991294769382675) + 5965366589
4623 + 7869511314885*sqrt(517)) + 592924869291189172726371728015334 36*sqrt(
40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262
3170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991
294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493
861977*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 +
 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169
1991294769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441146
71451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575
79971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517))) + 592
8773965962566646181550155546791 4*sqrt(40326)*sqrt(-59644114671451 + 2623170
438295*sqrt(517))/(2329048502925820708785386304*sqrt(-59644114671451 + 2623
170438295*sqrt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441
14671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35
57579971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 5
92924869291189172726371728015334 36*sqrt(40326)*sqrt(-4*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 78
69511314885*sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(71204274172
75887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621
568587150042935*sqrt(517) + 3557579971691991294769382675)*sqrt(-4*sqrt(7120
427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) +
6263621568587150042935*sqrt(517) + 3557579971691991294769382675) + 59653665
894623 + 7869511314885*sqrt(517))) + 298219572996748158593 1*sqrt(40326)*sqr
t(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004293
5*sqrt(517) + 3557579971691991294769382675)/(2329048502925820708785386304*s
qrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 78
69511314885*sqrt(517)) + 592924869291189172726371728015334 36*sqrt(40326)*sq
rt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262317043829
5*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382
675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493861977*sq
rt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26231704

```
38295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517))))
```

3.63 $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3$

Optimal. Leaf size=14

$$\frac{(a+bx)^{16}}{16b}$$

```
[Out] 1/16*(b*x+a)^16/b
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2059, 32}

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx &= \int (a+bx)^{15}\,dx\\ &= \frac{(a+bx)^{16}}{16b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

fricas [B] time = 0.71, size = 163, normalized size = 11.64

$$\frac{1}{16}x^{16}b^{15}+x^{15}b^{14}a+\frac{15}{2}x^{14}b^{13}a^{2}+35x^{13}b^{12}a^{3}+\frac{455}{4}x^{12}b^{11}a^{4}+273x^{11}b^{10}a^{5}+\frac{1001}{2}x^{10}b^{9}a^{6}+715x^{9}b^{8}a^{7}+\frac{6435}{8}x^{8}b^{7}a^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^15 + x^15*b^14*a + 15/2*x^14*b^13*a^2 + 35*x^13*b^12*a^3 + 455/
4*x^12*b^11*a^4 + 273*x^11*b^10*a^5 + 1001/2*x^10*b^9*a^6 + 715*x^9*b^8*a^7
 + 6435/8*x^8*b^7*a^8 + 715*x^7*b^6*a^9 + 1001/2*x^6*b^5*a^10 + 273*x^5*b^4
*a^11 + 455/4*x^4*b^3*a^12 + 35*x^3*b^2*a^13 + 15/2*x^2*b*a^14 + x*a^15
```

giac [B] time = 0.28, size = 163, normalized size = 11.64

$$\frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{15}{2}\,a^{2}b^{13}x^{14}+35\,a^{3}b^{12}x^{13}+\frac{455}{4}\,a^{4}b^{11}x^{12}+273\,a^{5}b^{10}x^{11}+\frac{1001}{2}\,a^{6}b^{9}x^{10}+715\,a^{7}b^{8}x^{9}+\frac{6435}{8}\,a^{8}b^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="giac")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455/
4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x^9
 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11*b^
4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

maple [B] time = 0.00, size = 164, normalized size = 11.71

$$\frac{1}{16}b^{15}x^{16}+a\,b^{14}x^{15}+\frac{15}{2}a^{2}b^{13}x^{14}+35a^{3}b^{12}x^{13}+\frac{455}{4}a^{4}b^{11}x^{12}+273a^{5}b^{10}x^{11}+\frac{1001}{2}a^{6}b^{9}x^{10}+715a^{7}b^{8}x^{9}+\frac{6435}{8}a^{8}b^{7}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)
```

```
[Out] 1/16*b^15*x^16+a*b^14*x^15+15/2*a^2*b^13*x^14+35*a^3*b^12*x^13+455/4*a^4*b^
11*x^12+273*a^5*b^10*x^11+1001/2*a^6*b^9*x^10+715*a^7*b^8*x^9+6435/8*a^8*b^
```

```
7*x^8+715*a^9*b^6*x^7+1001/2*a^10*b^5*x^6+273*a^11*b^4*x^5+455/4*a^12*b^3*x
^4+35*a^13*b^2*x^3+15/2*a^14*b*x^2+a^15*x
```

maxima [B] time = 0.68, size = 592, normalized size = 42.29

$$\frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{75}{14}\,a^2b^{13}x^{14}+\frac{125}{13}\,a^3b^{12}x^{13}+100\,a^6b^9x^{10}+\frac{1000}{7}\,a^9b^6x^7+\frac{125}{4}\,a^{12}b^3x^4+a^{15}x+\frac{1}{2}\left(b^5x^6+6\,ab^4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 75/14*a^2*b^13*x^14 + 125/13*a^3*b^12*x^13 +
 100*a^6*b^9*x^10 + 1000/7*a^9*b^6*x^7 + 125/4*a^12*b^3*x^4 + a^15*x + 1/2*
(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^
10 + 25/56*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*x^5)
*a^8*b^2 + 5/3*(18*b^5*x^10 + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4 + 25
/11*(11*b^5*x^12 + 60*a*b^4*x^11)*a^4*b^6 + 1/462*(126*b^10*x^11 + 1386*a*b
^9*x^10 + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 + 11550*
a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*
x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 3
85*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a^5 + 5/308*(77*b^10*x^12 + 840*a*b^
9*x^11 + 4158*a^2*b^8*x^10 + 12320*a^3*b^7*x^9 + 23100*a^4*b^6*x^8 + 26400*
a^5*b^5*x^7 + 15400*a^6*b^4*x^6)*a^4*b + 5/429*(198*b^10*x^13 + 2145*a*b^9*
x^12 + 10530*a^2*b^8*x^11 + 25740*a^3*b^7*x^10 + 28600*a^4*b^6*x^9)*a^3*b^2
 + 5/182*(78*b^10*x^14 + 840*a*b^9*x^13 + 2275*a^2*b^8*x^12)*a^2*b^3
```

mupad [B] time = 0.17, size = 163, normalized size = 11.64

$$a^{15}\,x+\frac{15\,a^{14}\,b\,x^2}{2}+35\,a^{13}\,b^2\,x^3+\frac{455\,a^{12}\,b^3\,x^4}{4}+273\,a^{11}\,b^4\,x^5+\frac{1001\,a^{10}\,b^5\,x^6}{2}+715\,a^9\,b^6\,x^7+\frac{6435\,a^8\,b^7\,x^8}{8}+715\,a^7\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x)^3,x)
```

```
[Out] a^15*x + (b^15*x^16)/16 + (15*a^14*b*x^2)/2 + a*b^14*x^15 + 35*a^13*b^2*x^3
 + (455*a^12*b^3*x^4)/4 + 273*a^11*b^4*x^5 + (1001*a^10*b^5*x^6)/2 + 715*a^
9*b^6*x^7 + (6435*a^8*b^7*x^8)/8 + 715*a^7*b^8*x^9 + (1001*a^6*b^9*x^10)/2
+ 273*a^5*b^10*x^11 + (455*a^4*b^11*x^12)/4 + 35*a^3*b^12*x^13 + (15*a^2*b^
13*x^14)/2
```

sympy [B] time = 0.11, size = 185, normalized size = 13.21

$$a^{15}x+\frac{15a^{14}bx^2}{2}+35a^{13}b^2x^3+\frac{455a^{12}b^3x^4}{4}+273a^{11}b^4x^5+\frac{1001a^{10}b^5x^6}{2}+715a^9b^6x^7+\frac{6435a^8b^7x^8}{8}+715a^7b^8x^9+\frac{1001}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**3,x)
```

```
[Out] a**15*x + 15*a**14*b*x**2/2 + 35*a**13*b**2*x**3 + 455*a**12*b**3*x**4/4 +
273*a**11*b**4*x**5 + 1001*a**10*b**5*x**6/2 + 715*a**9*b**6*x**7 + 6435*a*
*8*b**7*x**8/8 + 715*a**7*b**8*x**9 + 1001*a**6*b**9*x**10/2 + 273*a**5*b**
10*x**11 + 455*a**4*b**11*x**12/4 + 35*a**3*b**12*x**13 + 15*a**2*b**13*x**
14/2 + a*b**14*x**15 + b**15*x**16/16
```

3.64 $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

```
[Out] 1/11*(b*x+a)^11/b
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2059, 32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx &= \int (a+bx)^{10}\,dx\\ &= \frac{(a+bx)^{11}}{11b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

fricas [B] time = 0.68, size = 108, normalized size = 7.71

$$\frac{1}{11}x^{11}b^{10}+x^{10}b^9a+5x^9b^8a^2+15x^8b^7a^3+30x^7b^6a^4+42x^6b^5a^5+42x^5b^4a^6+30x^4b^3a^7+15x^3b^2a^8+5x^2ba^9+xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^10 + x^10*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30*x^7*b^6*a
^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 + 15*x^3*b^2*a^8 + 5*
x^2*b*a^9 + x*a^10
```

giac [B] time = 0.24, size = 108, normalized size = 7.71

$$\frac{1}{11}\,b^{10}x^{11}+ab^9x^{10}+5\,a^2b^8x^9+15\,a^3b^7x^8+30\,a^4b^6x^7+42\,a^5b^5x^6+42\,a^6b^4x^5+30\,a^7b^3x^4+15\,a^8b^2x^3+5\,a^9bx^2+a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="giac")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

maple [B] time = 0.00, size = 109, normalized size = 7.79

$$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a^8b^2x^3+5a^9b\,x^2+a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)
```

```
[Out] 1/11*b^10*x^11+a*b^9*x^10+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*a^4*b^6*x^7+42*a^
5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+5*a^9*b*x^2+a^10*x
```

maxima [B] time = 0.49, size = 228, normalized size = 16.29

$$\frac{1}{11}\,b^{10}x^{11}+ab^9x^{10}+\frac{25}{9}\,a^2b^8x^9+\frac{100}{7}\,a^4b^6x^7+20\,a^6b^4x^5+\frac{25}{3}\,a^8b^2x^3+a^{10}x+\frac{1}{3}\left(b^5x^6+6\,ab^4x^5+15\,a^2b^3x^4+20\,a^3b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^7 + 20*a^6
*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^10*x + 1/3*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*
b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5 + 5/21*(6*b^5*x^7 + 35*a*b^4*x
^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a*b^4
*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3
```

mupad [B] time = 0.06, size = 108, normalized size = 7.71

$$a^{10}\,x+5\,a^9\,b\,x^2+15\,a^8\,b^2\,x^3+30\,a^7\,b^3\,x^4+42\,a^6\,b^4\,x^5+42\,a^5\,b^5\,x^6+30\,a^4\,b^6\,x^7+15\,a^3\,b^7\,x^8+5\,a^2\,b^8\,x^9+a\,b^9\,x^{10}+\frac{b^{10}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x)^2,x)
```

```
[Out] a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a^
7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x
^8 + 5*a^2*b^8*x^9
```

sympy [B] time = 0.10, size = 114, normalized size = 8.14

$$a^{10}x+5a^9bx^2+15a^8b^2x^3+30a^7b^3x^4+42a^6b^4x^5+42a^5b^5x^6+30a^4b^6x^7+15a^3b^7x^8+5a^2b^8x^9+ab^9x^{10}+\frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**2,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

3.65 $\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

```
[Out] 1/6*(b*x+a)^6/b
```

Rubi [B] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 4.36, number of steps used = 1, number of rules used = 0, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4bx^2+a^5x+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x
^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

Rubi steps

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx=a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5}{6}$$

Mathematica [B] time = 0.00, size = 61, normalized size = 4.36

$$a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 +
 b^5*x^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

fricas [B] time = 0.56, size = 53, normalized size = 3.79

$$\frac{1}{6}x^6b^5+x^5b^4a+\frac{5}{2}x^4b^3a^2+\frac{10}{3}x^3b^2a^3+\frac{5}{2}x^2ba^4+xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/2*x^2*b*a^
4 + x*a^5
```

giac [B] time = 0.29, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

maple [B] time = 0.00, size = 54, normalized size = 3.86

$$\frac{1}{6}b^5x^6 + a\,b^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4b\,x^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x)
```

```
[Out] a^5*x+5/2*a^4*b*x^2+10/3*a^3*b^2*x^3+5/2*a^2*b^3*x^4+a*b^4*x^5+1/6*b^5*x^6
```

maxima [B] time = 0.46, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

mupad [B] time = 0.02, size = 53, normalized size = 3.79

$$a^5\,x + \frac{5\,a^4\,b\,x^2}{2} + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{5\,a^2\,b^3\,x^4}{2} + a\,b^4\,x^5 + \frac{b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b
*x,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5
*a^2*b^3*x^4)/2
```

sympy [B] time = 0.07, size = 60, normalized size = 4.29

$$a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**
4*b*x+a**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

3.66 $\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5} dx$

Optimal. Leaf size=14

$$-\frac{1}{4b(a+bx)^4}$$

```
[Out] -1/4/b/(b*x+a)^4
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2058, 32}

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-1),x]
```

```
[Out] -1/(4*b*(a + b*x)^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx &= \int \frac{1}{(a+bx)^5}\,dx\\ &= -\frac{1}{4b(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-1),x]
```

```
[Out] -1/4*1/(b*(a + b*x)^4)
```

fricas [B] time = 0.83, size = 46, normalized size = 3.29

$$-\frac{1}{4\left(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="fricas")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

giac [A] time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{4\,(bx+a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="giac")
```

```
[Out] -1/4/((b*x + a)^4*b)
```

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$-\frac{1}{4\,(bx+a)^4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x)
```

```
[Out] -1/4/b/(b*x+a)^4
```

maxima [B] time = 0.69, size = 46, normalized size = 3.29

$$-\frac{1}{4\left(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="maxima")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

mupad [B] time = 0.05, size = 48, normalized size = 3.43

$$-\frac{1}{4\,a^4\,b+16\,a^3\,b^2\,x+24\,a^2\,b^3\,x^2+16\,a\,b^4\,x^3+4\,b^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x),x)
```

```
[Out] -1/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2)
```

sympy [B] time = 0.29, size = 49, normalized size = 3.50

$$-\frac{1}{4a^4b+16a^3b^2x+24a^2b^3x^2+16ab^4x^3+4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5),x)
```

```
[Out] -1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**5
*x**4)
```

3.67 $\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2} dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

```
[Out] -1/9/b/(b*x+a)^9
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2058, 32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-2),x]
```

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2} dx &= \int \frac{1}{(a+bx)^{10}} dx \\ &= -\frac{1}{9b(a+bx)^9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-2),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

fricas [B] time = 0.62, size = 101, normalized size = 7.21

$$-\frac{1}{9\left(b^{10}x^{9}+9\,ab^{9}x^{8}+36\,a^{2}b^{8}x^{7}+84\,a^{3}b^{7}x^{6}+126\,a^{4}b^{6}x^{5}+126\,a^{5}b^{5}x^{4}+84\,a^{6}b^{4}x^{3}+36\,a^{7}b^{3}x^{2}+9\,a^{8}b^{2}x+a^{9}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

giac [A] time = 0.28, size = 12, normalized size = 0.86

$$-\frac{1}{9\,{\left(bx+a\right)}^{9}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{9\left(bx+a\right)^{9}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x
)
```

```
[Out] -1/9/b/(b*x+a)^9
```

maxima [B] time = 0.60, size = 101, normalized size = 7.21

$$-\frac{1}{9\left(b^{10}x^{9}+9\,ab^{9}x^{8}+36\,a^{2}b^{8}x^{7}+84\,a^{3}b^{7}x^{6}+126\,a^{4}b^{6}x^{5}+126\,a^{5}b^{5}x^{4}+84\,a^{6}b^{4}x^{3}+36\,a^{7}b^{3}x^{2}+9\,a^{8}b^{2}x+a^{9}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="maxima")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

mupad [B] time = 2.10, size = 103, normalized size = 7.36

$$-\frac{1}{9\,a^9\,b+81\,a^8\,b^2\,x+324\,a^7\,b^3\,x^2+756\,a^6\,b^4\,x^3+1134\,a^5\,b^5\,x^4+1134\,a^4\,b^6\,x^5+756\,a^3\,b^7\,x^6+324\,a^2\,b^8\,x^7+81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)^2,x)
```

```
[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 3
24*a^2*b^8*x^7)
```

sympy [B] time = 0.59, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b+81a^8b^2x+324a^7b^3x^2+756a^6b^4x^3+1134a^5b^5x^4+1134a^4b^6x^5+756a^3b^7x^6+324a^2b^8x^7+81ab^9x^8+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**2,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

3.68 $\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{14b(a+bx)^{14}}$$

```
[Out] -1/14/b/(b*x+a)^14
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2058, 32}

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-3),x]
```

```
[Out] -1/(14*b*(a + b*x)^14)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3}\,dx &= \int \frac{1}{(a+bx)^{15}}\,dx\\ &= -\frac{1}{14b(a+bx)^{14}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-3),x]
```

```
[Out] -1/14*1/(b*(a + b*x)^14)
```

fricas [B] time = 0.79, size = 156, normalized size = 11.14

$$-\frac{1}{14\left(b^{15}x^{14} + 14\,ab^{14}x^{13} + 91\,a^{2}b^{13}x^{12} + 364\,a^{3}b^{12}x^{11} + 1001\,a^{4}b^{11}x^{10} + 2002\,a^{5}b^{10}x^{9} + 3003\,a^{6}b^{9}x^{8} + 3432\,a^{7}b^{8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="fricas")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

giac [A] time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{14\,(bx + a)^{14}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="giac")
```

```
[Out] -1/14/((b*x + a)^14*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{14\,(bx + a)^{14}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x
)
```

```
[Out] -1/14/b/(b*x+a)^14
```

maxima [B] time = 0.54, size = 156, normalized size = 11.14

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^{2}b^{13}x^{12}+364\,a^{3}b^{12}x^{11}+1001\,a^{4}b^{11}x^{10}+2002\,a^{5}b^{10}x^{9}+3003\,a^{6}b^{9}x^{8}+3432\,a^{7}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="maxima")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

mupad [B] time = 3.03, size = 158, normalized size = 11.29

$$-\frac{}{14\,a^{14}\,b+196\,a^{13}\,b^{2}\,x+1274\,a^{12}\,b^{3}\,x^{2}+5096\,a^{11}\,b^{4}\,x^{3}+14014\,a^{10}\,b^{5}\,x^{4}+28028\,a^{9}\,b^{6}\,x^{5}+42042\,a^{8}\,b^{7}\,x^{6}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)^3,x)
```

```
[Out] -1/(14*a^14*b + 14*b^15*x^14 + 196*a^13*b^2*x + 196*a*b^14*x^13 + 1274*a^12
*b^3*x^2 + 5096*a^11*b^4*x^3 + 14014*a^10*b^5*x^4 + 28028*a^9*b^6*x^5 + 420
42*a^8*b^7*x^6 + 48048*a^7*b^8*x^7 + 42042*a^6*b^9*x^8 + 28028*a^5*b^10*x^9
 + 14014*a^4*b^11*x^10 + 5096*a^3*b^12*x^11 + 1274*a^2*b^13*x^12)
```

sympy [B] time = 0.94, size = 168, normalized size = 12.00

$$-\frac{}{14a^{14}b+196a^{13}b^{2}x+1274a^{12}b^{3}x^{2}+5096a^{11}b^{4}x^{3}+14014a^{10}b^{5}x^{4}+28028a^{9}b^{6}x^{5}+42042a^{8}b^{7}x^{6}+48048a^{7}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**3,x)
```

```
[Out] -1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a**11*b**4*
x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 42042*a**8*b**7*x**6
+ 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8 + 28028*a**5*b**10*x**9 + 140
14*a**4*b**11*x**10 + 5096*a**3*b**12*x**11 + 1274*a**2*b**13*x**12 + 196*a
*b**14*x**13 + 14*b**15*x**14)
```

3.69 $\int \frac{1}{1+x^2+x^3+x^5} \, dx$

Optimal. Leaf size=38

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/6*ln(1+x)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2058, 635, 203, 260, 628}

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3 + x^5)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+x^2+x^3+x^5}\,dx &= \int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx\\
&= \frac{1}{6}\log(1+x)+\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx\\
&= \frac{1}{6}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{6}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)-\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3 + x^5)^(-1),x]
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

fricas [A] time = 0.79, size = 30, normalized size = 0.79

$$\frac{1}{2}\arctan(x)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

giac [A] time = 0.29, size = 31, normalized size = 0.82

$$\frac{1}{2}\arctan(x)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(abs(x + 1
))
```

maple [A] time = 0.01, size = 31, normalized size = 0.82

$$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{6}+\frac{\ln\left(x^2+1\right)}{4}-\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5+x^3+x^2+1),x)
```

```
[Out] 1/2*arctan(x)+1/6*ln(x+1)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

maxima [A] time = 1.11, size = 30, normalized size = 0.79

$$\frac{1}{2}\arctan(x)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

mupad [B] time = 2.16, size = 36, normalized size = 0.95

$$\frac{\ln(x+1)}{6}-\frac{\ln\left(x^2-x+1\right)}{3}+\ln(x-\mathrm{i})\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^3 + x^5 + 1),x)
```

```
[Out] log(x + 1)/6 + log(x - 1i)*(1/4 - 1i/4) + log(x + 1i)*(1/4 + 1i/4) - log(x^
2 - x + 1)/3
```

sympy [A] time = 0.15, size = 29, normalized size = 0.76

$$\frac{\log(x+1)}{6}+\frac{\log\left(x^2+1\right)}{4}-\frac{\log\left(x^2-x+1\right)}{3}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**5+x**3+x**2+1),x)
```

```
[Out] log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2
```

3.70 $\int \left(3-19x^2+32x^4-16x^6\right)^4 \, dx$

Optimal. Leaf size=84

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634\ldots}{}$$

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*
x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/
23*x^23+65536/25*x^25
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx = \int (-1+x)^4(1+x)^4(-1+2x)^4(1+2x)^4\left(-3+4x^2\right)^4\,dx$$
$$= \int (-1+2x)^4(1+2x)^4\left(-1+x^2\right)^4\left(-3+4x^2\right)^4\,dx$$
$$= \int \left(-1+x^2\right)^4\left(-3+4x^2\right)^4\left(-1+4x^2\right)^4\,dx$$
$$= \int \left(81-2052x^2+22950x^4-149700x^6+634321x^8-1841600x^{10}+3764416$$
$$= 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}$$

Mathematica [A] time = 0.00, size = 84, normalized size = 1.00

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{6343}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

fricas [A] time = 0.49, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{6343}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="fricas")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

giac [A] time = 0.38, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="giac")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.77

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^4,x)
```

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*
x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/
23*x^23+65536/25*x^25
```

maxima [A] time = 0.49, size = 64, normalized size = 0.76

$$\frac{65536}{25}\,x^{25}-\frac{524288}{23}\,x^{23}+\frac{1884160}{21}\,x^{21}-\frac{4014080}{19}\,x^{19}+\frac{5633536}{17}\,x^{17}-\frac{1094656}{3}\,x^{15}+\frac{3764416}{13}\,x^{13}-\frac{1841600}{11}\,x^{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="maxima")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

mupad [B] time = 2.17, size = 64, normalized size = 0.76

$$\frac{65536\,x^{25}}{25}-\frac{524288\,x^{23}}{23}+\frac{1884160\,x^{21}}{21}-\frac{4014080\,x^{19}}{19}+\frac{5633536\,x^{17}}{17}-\frac{1094656\,x^{15}}{3}+\frac{3764416\,x^{13}}{13}-\frac{1841600\,x^{11}}{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^4,x)
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

sympy [A] time = 0.08, size = 80, normalized size = 0.95

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)
```

```
[Out] 65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**19/19 + 56
33536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1841600*x**11/11 + 63
4321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684*x**3 + 81*x
```

3.71 $\int \left(3-19x^2+32x^4-16x^6\right)^3 \, dx$

Optimal. Leaf size=63

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

```
[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21
248/5*x^15+24576/17*x^17-4096/19*x^19
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(3-19x^2+32x^4-16x^6\right)^3 dx = -\int (-1+x)^3(1+x)^3(-1+2x)^3(1+2x)^3\left(-3+4x^2\right)^3 dx$$
$$= -\int (-1+2x)^3(1+2x)^3\left(-1+x^2\right)^3\left(-3+4x^2\right)^3 dx$$
$$= -\int \left(-1+x^2\right)^3\left(-3+4x^2\right)^3\left(-1+4x^2\right)^3 dx$$
$$= -\int \left(-27+513x^2-4113x^4+18235x^6-49344x^8+84912x^{10}-93440x^{12}+\ldots\right.$$
$$= 27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}+\frac{93440x^{13}}{13}-\frac{21248\ldots}{5}$$

Mathematica [A] time = 0.00, size = 63, normalized size = 1.00

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

fricas [A] time = 0.78, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^9-2605x^7+\frac{4113}{5}x^5-171x^3+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

giac [A] time = 0.36, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^9-2605x^7+\frac{4113}{5}x^5-171x^3+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

maple [A] time = 0.00, size = 50, normalized size = 0.79

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^{9}-2605x^{7}+\frac{4113}{5}x^{5}-171x^{3}+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^3,x)
```

```
[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21
248/5*x^15+24576/17*x^17-4096/19*x^19
```

maxima [A] time = 0.73, size = 49, normalized size = 0.78

$$-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

mupad [B] time = 0.05, size = 49, normalized size = 0.78

$$-\frac{4096\,x^{19}}{19}+\frac{24576\,x^{17}}{17}-\frac{21248\,x^{15}}{5}+\frac{93440\,x^{13}}{13}-\frac{84912\,x^{11}}{11}+\frac{16448\,x^{9}}{3}-2605\,x^{7}+\frac{4113\,x^{5}}{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

sympy [A] time = 0.07, size = 60, normalized size = 0.95

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^{9}}{3}-2605x^{7}+\frac{4113x^{5}}{5}-171x^{3}+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -4096*x**19/19 + 24576*x**17/17 - 21248*x**15/5 + 93440*x**13/13 - 84912*x*
*11/11 + 16448*x**9/3 - 2605*x**7 + 4113*x**5/5 - 171*x**3 + 27*x
```

3.72 $\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx$

Optimal. Leaf size=44

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

```
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx &= \int (-1+x)^2(1+x)^2(-1+2x)^2(1+2x)^2\left(-3+4x^2\right)^2\,dx\\ &= \int (-1+2x)^2(1+2x)^2\left(-1+x^2\right)^2\left(-3+4x^2\right)^2\,dx\\ &= \int \left(-1+x^2\right)^2\left(-3+4x^2\right)^2\left(-1+4x^2\right)^2\,dx\\ &= \int \left(9-114x^2+553x^4-1312x^6+1632x^8-1024x^{10}+256x^{12}\right)dx\\ &= 9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

fricas [A] time = 0.65, size = 34, normalized size = 0.77

$$\frac{256}{13}x^{13}-\frac{1024}{11}x^{11}+\frac{544}{3}x^9-\frac{1312}{7}x^7+\frac{553}{5}x^5-38x^3+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

giac [A] time = 0.36, size = 34, normalized size = 0.77

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

maple [A] time = 0.00, size = 35, normalized size = 0.80

$$\frac{256}{13}x^{13}-\frac{1024}{11}x^{11}+\frac{544}{3}x^{9}-\frac{1312}{7}x^{7}+\frac{553}{5}x^{5}-38x^{3}+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^2,x)
```

```
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

maxima [A] time = 0.75, size = 34, normalized size = 0.77

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^{9}-\frac{1312}{7}\,x^{7}+\frac{553}{5}\,x^{5}-38\,x^{3}+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

mupad [B] time = 0.02, size = 34, normalized size = 0.77

$$\frac{256\,x^{13}}{13}-\frac{1024\,x^{11}}{11}+\frac{544\,x^{9}}{3}-\frac{1312\,x^{7}}{7}+\frac{553\,x^{5}}{5}-38\,x^{3}+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

sympy [A] time = 0.06, size = 41, normalized size = 0.93

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^{9}}{3}-\frac{1312x^{7}}{7}+\frac{553x^{5}}{5}-38x^{3}+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] 256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**5/5 - 38*x
**3 + 9*x
```

3.73 $\int \left(3-19x^2+32x^4-16x^6\right) dx$

Optimal. Leaf size=25

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

```
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In] Int[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Rubi steps

$$\int \left(3-19x^2+32x^4-16x^6\right) dx = 3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In] Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

fricas [A] time = 0.72, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7+\frac{32}{5}x^5-\frac{19}{3}x^3+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="fricas")
```

```
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

giac [A] time = 0.28, size = 19, normalized size = 0.76

$$-\frac{16}{7}\,x^7+\frac{32}{5}\,x^5-\frac{19}{3}\,x^3+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="giac")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{16}{7}x^7+\frac{32}{5}x^5-\frac{19}{3}x^3+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-16*x^6+32*x^4-19*x^2+3,x)
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

maxima [A] time = 0.64, size = 19, normalized size = 0.76

$$-\frac{16}{7}\,x^7+\frac{32}{5}\,x^5-\frac{19}{3}\,x^3+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="maxima")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$-\frac{16\,x^7}{7}+\frac{32\,x^5}{5}-\frac{19\,x^3}{3}+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(32*x^4 - 19*x^2 - 16*x^6 + 3,x)
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x**6+32*x**4-19*x**2+3,x)
```

```
[Out] -16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x
```

3.74 $\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(x)+1/3*arctanh(2*x)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2057, 207}

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx = \int \left(-\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right)dx$$
$$= -\left(\frac{1}{3}\int \frac{1}{-1+x^2}\,dx\right)-\frac{2}{3}\int \frac{1}{-1+4x^2}\,dx+2\int \frac{1}{-3+4x^2}\,dx$$
$$= \frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 2.00

$$\frac{1}{6}\left(-\log\left(2x^2-3x+1\right)+\log\left(2x^2+3x+1\right)+\sqrt{3}\log\left(\sqrt{3}-2x\right)-\sqrt{3}\log\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] (Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1 - 3*x + 2*
x^2] + Log[1 + 3*x + 2*x^2])/6
```

fricas [B] time = 0.88, size = 56, normalized size = 1.81

$$\frac{1}{6}\sqrt{3}\log\left(\frac{4x^2-4\sqrt{3}x+3}{4x^2-3}\right)+\frac{1}{6}\log\left(2x^2+3x+1\right)-\frac{1}{6}\log\left(2x^2-3x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 1/6*log(2*x^2 + 3*
x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

giac [B] time = 0.28, size = 62, normalized size = 2.00

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|8x-4\sqrt{3}\right|}{\left|8x+4\sqrt{3}\right|}\right)+\frac{1}{6}\log\left(|2x+1|\right)-\frac{1}{6}\log\left(|2x-1|\right)+\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{6}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*log(abs(2*
x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 42, normalized size = 1.35

$$-\frac{\sqrt{3}\ \operatorname{arctanh}\left(\frac{2\sqrt{3}\,x}{3}\right)}{3}-\frac{\ln\left(x-1\right)}{6}-\frac{\ln\left(2x-1\right)}{6}+\frac{\ln\left(x+1\right)}{6}+\frac{\ln\left(2x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3),x)
```

```
[Out] -1/6*ln(-1+x)-1/6*ln(-1+2*x)+1/6*ln(1+2*x)+1/6*ln(x+1)-1/3*arctanh(2/3*x*3^
(1/2))*3^(1/2)
```

maxima [B] time = 1.05, size = 54, normalized size = 1.74

$$\frac{1}{6}\,\sqrt{3}\,\log\left(\frac{2\,x-\sqrt{3}}{2\,x+\sqrt{3}}\right)+\frac{1}{6}\,\log\left(2\,x+1\right)-\frac{1}{6}\,\log\left(2\,x-1\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{6}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x + 1) - 1/6*l
og(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

mupad [B] time = 0.07, size = 27, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{x}{4608\left(\frac{x^2}{6912}+\frac{1}{13824}\right)}\right)}{3}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\,\sqrt{3}\,x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3),x)
```

```
[Out] atanh(x/(4608*(x^2/6912 + 1/13824)))/3 - (3^(1/2)*atanh((2*3^(1/2)*x)/3))/3
```

sympy [B] time = 0.15, size = 63, normalized size = 2.03

$$\frac{\sqrt{3}\,\log\left(x-\frac{\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\,\log\left(x+\frac{\sqrt{3}}{2}\right)}{6}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)
```

```
[Out] sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log(x**2 - 3*
x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

3.75 $\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{2x}{3\left(3-4x^2\right)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/18/(1-2*x)+1/36/(1-x)-1/36/(1+x)-1/18/(1+2*x)+2/3*x/(-4*x^2+3)+67/54*arct
anh(x)-7/27*arctanh(2*x)-5/9*arctanh(2/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2057, 207, 199}

$$\frac{2x}{3\left(3-4x^2\right)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2*
x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTanh[
(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
```

```
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^2}\,dx = \int \left(\frac{1}{36(-1+x)^2}+\frac{1}{36(1+x)^2}+\frac{1}{9(-1+2x)^2}+\frac{1}{9(1+2x)^2}-\frac{67}{54\left(-1+x^2\right)}+\right.$$

$$= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{14}{27}\int \frac{1}{-1+4x^2}\,dx-\frac{67}{54}$$

$$= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\left(3-4x^2\right)}+\frac{67}{54}\tanh^{-}$$

$$= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\left(3-4x^2\right)}+\frac{67}{54}\tanh^{-}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.16

$$\frac{1}{108}\left(-\frac{6x\left(80x^4-104x^2+27\right)}{16x^6-32x^4+19x^2-3}+14\log(1-2x)+30\sqrt{3}\,\log\left(\sqrt{3}\,-2x\right)-67\log(1-x)+67\log(x+1)-14\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] ((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) + 14*Log[1
- 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x] + 67*Log[1 + x] - 14
*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x])/108
```

fricas [B] time = 0.82, size = 177, normalized size = 1.99

$$-\frac{480\,x^5-624\,x^3-30\,\sqrt{3}\left(16\,x^6-32\,x^4+19\,x^2-3\right)\log\left(\frac{4\,x^2-4\,\sqrt{3}\,x+3}{4\,x^2-3}\right)+14\left(16\,x^6-32\,x^4+19\,x^2-3\right)\log\left(2\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(480*x^5 - 624*x^3 - 30*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log((
4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*l
```

```
og(2*x + 1) - 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x - 1) - 67*(16*x^6 -
 32*x^4 + 19*x^2 - 3)*log(x + 1) + 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x
- 1) + 162*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)
```

giac [A] time = 0.28, size = 97, normalized size = 1.09

$$\frac{5}{18}\sqrt{3}\log\left(\frac{\left|8x-4\sqrt{3}\right|}{\left|8x+4\sqrt{3}\right|}\right)-\frac{80x^5-104x^3+27x}{18\left(16x^6-32x^4+19x^2-3\right)}-\frac{7}{54}\log\left(\left|2x+1\right|\right)+\frac{7}{54}\log\left(\left|2x-1\right|\right)+\frac{67}{108}\log\left(\left|x+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/18*(80*x^5
- 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(abs(2*x + 1)) +
 7/54*log(abs(2*x - 1)) + 67/108*log(abs(x + 1)) - 67/108*log(abs(x - 1))
```

maple [A] time = 0.02, size = 84, normalized size = 0.94

$$-\frac{x}{6\left(x^2-\frac{3}{4}\right)}-\frac{5\sqrt{3}\ \mathrm{arctanh}\left(\frac{2\sqrt{3}\,x}{3}\right)}{9}-\frac{67\ln\left(x-1\right)}{108}+\frac{7\ln\left(2x-1\right)}{54}+\frac{67\ln\left(x+1\right)}{108}-\frac{7\ln\left(2x+1\right)}{54}-\frac{1}{36\left(x-1\right)}-\frac{1}{18\left(2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x)
```

```
[Out] -1/36/(x-1)-67/108*ln(x-1)-1/18/(2*x-1)+7/54*ln(2*x-1)-1/18/(2*x+1)-7/54*ln
(2*x+1)-1/36/(x+1)+67/108*ln(x+1)-1/6*x/(x^2-3/4)-5/9*arctanh(2/3*3^(1/2)*x
)*3^(1/2)
```

maxima [A] time = 1.29, size = 89, normalized size = 1.00

$$\frac{5}{18}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)-\frac{80x^5-104x^3+27x}{18\left(16x^6-32x^4+19x^2-3\right)}-\frac{7}{54}\log\left(2x+1\right)+\frac{7}{54}\log\left(2x-1\right)+\frac{67}{108}\log\left(x+1\right)-\frac{67}{108}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/18*(80*x^5 - 104*x^3
+ 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(2*x + 1) + 7/54*log(2*x -
 1) + 67/108*log(x + 1) - 67/108*log(x - 1)
```

mupad [B] time = 0.08, size = 64, normalized size = 0.72

$$-\frac{\operatorname{atan}(x\,1\mathrm{i})\ 67\mathrm{i}}{54}+\frac{\operatorname{atan}(x\,2\mathrm{i})\ 7\mathrm{i}}{27}-\frac{\frac{5\,x^5}{18}-\frac{13\,x^3}{36}+\frac{3\,x}{32}}{x^6-2\,x^4+\frac{19\,x^2}{16}-\frac{3}{16}}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)\ 5\mathrm{i}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

```
[Out] (atan(x*2i)*7i)/27 - (atan(x*1i)*67i)/54 - ((3*x)/32 - (13*x^3)/36 + (5*x^5
)/18)/((19*x^2)/16 - 2*x^4 + x^6 - 3/16) + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*
5i)/9
```

sympy [A] time = 1.36, size = 104, normalized size = 1.17

$$\frac{-80x^5+104x^3-27x}{288x^6-576x^4+342x^2-54}-\frac{67\log(x-1)}{108}+\frac{7\log\left(x-\frac{1}{2}\right)}{54}-\frac{7\log\left(x+\frac{1}{2}\right)}{54}+\frac{67\log(x+1)}{108}+\frac{5\sqrt{3}\ \log\left(x-\frac{\sqrt{3}}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] (-80*x**5 + 104*x**3 - 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54) - 67*log
(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 67*log(x + 1)/108 + 5
*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*log(x + sqrt(3)/2)/18
```

3.76 $\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^3}\,dx$

Optimal. Leaf size=161

$$\frac{5x}{3\left(3-4x^2\right)}-\frac{2x}{3\left(3-4x^2\right)^2}-\frac{7}{108(1-2x)}+\frac{67}{432(1-x)}-\frac{67}{432(x+1)}+\frac{7}{108(2x+1)}+\frac{1}{108(1-2x)^2}+\frac{1}{432(1-x)^2}-\frac{1}{432(}$$

```
[Out] 1/108/(1-2*x)^2-7/108/(1-2*x)+1/432/(1-x)^2+67/432/(1-x)-1/432/(1+x)^2-67/4
32/(1+x)-1/108/(1+2*x)^2+7/108/(1+2*x)-2/3*x/(-4*x^2+3)^2+5/3*x/(-4*x^2+3)+
3913/648*arctanh(x)+67/162*arctanh(2*x)-67/18*arctanh(2/3*x*3^(1/2))*3^(1/2
)
```

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2057, 207, 199}

$$\frac{5x}{3\left(3-4x^2\right)}-\frac{2x}{3\left(3-4x^2\right)^2}-\frac{7}{108(1-2x)}+\frac{67}{432(1-x)}-\frac{67}{432(x+1)}+\frac{7}{108(2x+1)}+\frac{1}{108(1-2x)^2}+\frac{1}{432(1-x)^2}-\frac{1}{432(}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 -
x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108*(
1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcTanh
[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) -
 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^3}\,dx = \int \left(-\frac{1}{216(-1+x)^3}+\frac{67}{432(-1+x)^2}+\frac{1}{216(1+x)^3}+\frac{67}{432(1+x)^2}-\frac{1}{27(-1+\right.$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{}{43}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{}{43}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{}{43}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{}{43}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 0.85

$$\frac{\frac{36x\left(80x^4-104x^2+27\right)}{\left(-16x^6+32x^4-19x^2+3\right)^2}-\frac{6x\left(2288x^4-2384x^2+345\right)}{16x^6-32x^4+19x^2-3}-268\log(1-2x)+2412\sqrt{3}\log\left(\sqrt{3}-2x\right)-3913\log(1-x)+3913}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] ((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2 - (6*x*(34
5 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) - 268*Log[1 - 2*x
] + 2412*Sqrt[3]*Log[Sqrt[3] - 2*x] - 3913*Log[1 - x] + 3913*Log[1 + x] + 2
68*Log[1 + 2*x] - 2412*Sqrt[3]*Log[Sqrt[3] + 2*x])/1296
```

fricas [B] time = 0.88, size = 282, normalized size = 1.75

$$-\frac{219648\,x^{11}-668160\,x^{9}+751680\,x^{7}-382080\,x^{5}+85986\,x^{3}-2412\,\sqrt{3}\left(256\,x^{12}-1024\,x^{10}+1632\,x^{8}-1312\,x^{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/1296*(219648*x^11 - 668160*x^9 + 751680*x^7 - 382080*x^5 + 85986*x^3 - 2
412*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2
 + 9)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) - 268*(256*x^12 - 1024*x^1
0 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) + 268*(256*x^
12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x - 1)
- 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)
*log(x + 1) + 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 -
114*x^2 + 9)*log(x - 1) - 7182*x)/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x
^6 + 553*x^4 - 114*x^2 + 9)
```

giac [A] time = 0.31, size = 112, normalized size = 0.70

$$\frac{67}{36}\,\sqrt{3}\,\log\left(\frac{\left|8\,x-4\,\sqrt{3}\right|}{\left|8\,x+4\,\sqrt{3}\right|}\right)-\frac{36608\,x^{11}-111360\,x^{9}+125280\,x^{7}-63680\,x^{5}+14331\,x^{3}-1197\,x}{216\,\left(16\,x^{6}-32\,x^{4}+19\,x^{2}-3\right)^{2}}+\frac{67}{324}\,\log\left(\left|2\,x+1\right|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 67/36*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/216*(36608
*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(16*x^6 -
 32*x^4 + 19*x^2 - 3)^2 + 67/324*log(abs(2*x + 1)) - 67/324*log(abs(2*x - 1
)) + 3913/1296*log(abs(x + 1)) - 3913/1296*log(abs(x - 1))
```

maple [A] time = 0.02, size = 126, normalized size = 0.78

$$-\frac{67\sqrt{3}\,\operatorname{arctanh}\left(\frac{2\sqrt{3}\,x}{3}\right)}{18}-\frac{3913\ln\left(x-1\right)}{1296}-\frac{67\ln\left(2x-1\right)}{324}+\frac{3913\ln\left(x+1\right)}{1296}+\frac{67\ln\left(2x+1\right)}{324}+\frac{1}{432\left(x-1\right)^{2}}-\frac{67}{432\left(x-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x)
```

```
[Out] 1/432/(x-1)^2-67/432/(x-1)-3913/1296*ln(x-1)+1/108/(2*x-1)^2+7/108/(2*x-1)-
67/324*ln(2*x-1)-1/108/(2*x+1)^2+7/108/(2*x+1)+67/324*ln(2*x+1)-1/432/(x+1)
```

```
^2-67/432/(x+1)+3913/1296*ln(x+1)+64*(-5/48*x^3+13/192*x)/(4*x^2-3)^2-67/18
*arctanh(2/3*3^(1/2)*x)*3^(1/2)
```

maxima [A] time = 1.32, size = 119, normalized size = 0.74

$$\frac{67}{36}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{216\left(256x^{12}-1024x^{10}+1632x^{8}-1312x^{6}+553x^{4}-114x^{2}+9\right)}+\frac{67}{324}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/216*(36608*x^11 - 11
1360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(256*x^12 - 1024*x^
10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9) + 67/324*log(2*x + 1) - 6
7/324*log(2*x - 1) + 3913/1296*log(x + 1) - 3913/1296*log(x - 1)
```

mupad [B] time = 0.09, size = 93, normalized size = 0.58

$$\frac{-\frac{143x^{11}}{216}+\frac{145x^{9}}{72}-\frac{145x^{7}}{64}+\frac{995x^{5}}{864}-\frac{4777x^{3}}{18432}+\frac{133x}{6144}}{x^{12}-4x^{10}+\frac{51x^{8}}{8}-\frac{41x^{6}}{8}+\frac{553x^{4}}{256}-\frac{57x^{2}}{128}+\frac{9}{256}}-\frac{\operatorname{atan}(x\,2\mathrm{i})\,67\mathrm{i}}{162}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,3913\mathrm{i}}{648}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)\,67\mathrm{i}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

```
[Out] ((133*x)/6144 - (4777*x^3)/18432 + (995*x^5)/864 - (145*x^7)/64 + (145*x^9)
/72 - (143*x^11)/216)/((553*x^4)/256 - (57*x^2)/128 - (41*x^6)/8 + (51*x^8)
/8 - 4*x^10 + x^12 + 9/256) - (atan(x*2i)*67i)/162 - (atan(x*1i)*3913i)/648
 + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*67i)/18
```

sympy [A] time = 1.45, size = 134, normalized size = 0.83

$$-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{55296x^{12}-221184x^{10}+352512x^{8}-283392x^{6}+119448x^{4}-24624x^{2}+1944}-\frac{3913\log(x-1)}{1296}-\frac{67\log\left(x-\right.}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -(36608*x**11 - 111360*x**9 + 125280*x**7 - 63680*x**5 + 14331*x**3 - 1197*
x)/(55296*x**12 - 221184*x**10 + 352512*x**8 - 283392*x**6 + 119448*x**4 -
24624*x**2 + 1944) - 3913*log(x - 1)/1296 - 67*log(x - 1/2)/324 + 67*log(x
+ 1/2)/324 + 3913*log(x + 1)/1296 + 67*sqrt(3)*log(x - sqrt(3)/2)/36 - 67*s
qrt(3)*log(x + sqrt(3)/2)/36
```

3.77 $\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{x}{32\left(1-x^2\right)}+\frac{\left(99-17x^2\right)x}{128\left(x^4-6x^2+1\right)}+\frac{5}{32}\tanh^{-1}(x)+\frac{1}{512}\left(3\sqrt{2}-4\right)\tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right)+\frac{1}{512}\left(4+3\sqrt{2}\right)\tanh^{-1}\left(\left(1\right.\right.$$

```
[Out] 1/32*x/(-x^2+1)+1/128*x*(-17*x^2+99)/(x^4-6*x^2+1)+5/32*arctanh(x)+1/512*ar
ctanh(x*(2^(1/2)-1))*(-4+3*2^(1/2))+1/512*arctanh(x*(1+2^(1/2)))*(4+3*2^(1/
2))
```

Rubi [B] time = 0.13, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2057, 207, 638, 618, 206, 632, 31}

$$-\frac{41-17x}{256\left(-x^2+2x+1\right)}+\frac{17x+41}{256\left(-x^2-2x+1\right)}+\frac{1}{64(1-x)}-\frac{1}{64(x+1)}+\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)+\frac{1}{512}\left(2+7\right.$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] 1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2)) - (41 -
 17*x)/(256*(1 + 2*x - x^2)) - (17*ArcTanh[(1 - x)/Sqrt[2]])/(256*Sqrt[2])
+ (5*ArcTanh[x])/32 + (17*ArcTanh[(1 + x)/Sqrt[2]])/(256*Sqrt[2]) + ((2 - 7
*Sqrt[2])*Log[1 - Sqrt[2] - x])/512 + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x]
)/512 - ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] + x])/512 - ((2 + 7*Sqrt[2])*Log[1
 + Sqrt[2] + x])/512
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx = \int\left(\frac{1}{64(-1+x)^2}+\frac{1}{64(1+x)^2}-\frac{5}{32\left(-1+x^2\right)}+\frac{29-12x}{64\left(-1-2x+x^2\right)^2}+\frac{6+}{128\left(-1-\right.}\right.$$

$$=\frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{1}{128}\int\frac{6+x}{-1-2x+x^2}\,dx+\frac{1}{128}\int\frac{6-x}{-1+2x+x^2}\,dx+\frac{1}{64}$$

$$=\frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}+\frac{5}{32}\tanh^{-1}(x)$$

$$=\frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}+\frac{5}{32}\tanh^{-1}(x)$$

$$=\frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}-\frac{17\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right.}{256\sqrt{2}}$$

Mathematica [A] time = 0.10, size = 132, normalized size = 1.45

$$\frac{-\frac{8x\left(21x^4-140x^2+103\right)}{x^6-7x^4+7x^2-1}-80\log(1-x)-\left(4+3\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)+\left(4-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)+80\log(x+}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] ((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x]
 - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2]
- x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*Sqrt
[2])*Log[1 + Sqrt[2] + x])/1024
```

fricas [B] time = 0.87, size = 223, normalized size = 2.45

$$-\frac{168\,x^5-1120\,x^3-3\,\sqrt{2}\left(x^6-7\,x^4+7\,x^2-1\right)\log\left(\frac{x^2+2\,\sqrt{2}\,(x+1)+2\,x+3}{x^2+2\,x-1}\right)-3\,\sqrt{2}\left(x^6-7\,x^4+7\,x^2-1\right)\log\left(\frac{x^2+2\,\sqrt{2}\,(}{x^2-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -1/1024*(168*x^5 - 1120*x^3 - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2
+ 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*sqrt(2)*(x^6 - 7*x^4 +
7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*(x^
```

```
6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)*log
(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 80*(x^6 - 7*x^4
 + 7*x^2 - 1)*log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

giac [A] time = 0.44, size = 134, normalized size = 1.47

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)-\frac{3}{1024}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}-2\right|}{\left|2x+2\sqrt{2}-2\right|}\right)-\frac{21x^5-140x^3+103x}{128\left(x^6-7x^4+7x^2-1\right)}-\frac{1}{256}\log\left(\left|x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")
```

```
[Out] -3/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) - 3/
1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/128
*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(abs(x^2 +
 2*x - 1)) + 1/256*log(abs(x^2 - 2*x - 1)) + 5/64*log(abs(x + 1)) - 5/64*lo
g(abs(x - 1))
```

maple [A] time = 0.02, size = 116, normalized size = 1.27

$$\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{512}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{512}-\frac{5\ln(x-1)}{64}+\frac{5\ln(x+1)}{64}+\frac{\ln\left(x^2-2x-1\right)}{256}-\frac{\ln\left(x^2+2x-\right.}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-7*x^4+7*x^2-1)^2,x)
```

```
[Out] -1/64/(x-1)-5/64*ln(x-1)-1/128*(17/2*x+41/2)/(x^2+2*x-1)-1/256*ln(x^2+2*x-1
)+3/512*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))-1/64/(x+1)+5/64*ln(x+1)+1/128*
(-17/2*x+41/2)/(x^2-2*x-1)+1/256*ln(x^2-2*x-1)+3/512*2^(1/2)*arctanh(1/4*(2
*x-2)*2^(1/2))
```

maxima [A] time = 1.27, size = 114, normalized size = 1.25

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right)-\frac{21x^5-140x^3+103x}{128\left(x^6-7x^4+7x^2-1\right)}-\frac{1}{256}\log\left(x^2+2x-1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -3/1024*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) - 3/1024*sqrt(2)*l
og((x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/
```

```
(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(x^2 + 2*x - 1) + 1/256*log(x^2 - 2*x
- 1) + 5/64*log(x + 1) - 5/64*log(x - 1)
```

mupad [B] time = 2.18, size = 126, normalized size = 1.38

$$-\frac{\operatorname{atan}(x\,1\mathrm{i})\,5\mathrm{i}}{32}-\frac{\frac{21\,x^5}{128}-\frac{35\,x^3}{32}+\frac{103\,x}{128}}{x^6-7\,x^4+7\,x^2-1}-\operatorname{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}-\frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] - (atan(x*1i)*5i)/32 - ((103*x)/128 - (35*x^3)/32 + (21*x^5)/128)/(7*x^2 -
7*x^4 + x^6 - 1) - atan((x*940311i)/(134217728*((275445*2^(1/2))/134217728
- 389421/134217728)) - (2^(1/2)*x*332433i)/(67108864*((275445*2^(1/2))/1342
17728 - 389421/134217728)))*((2^(1/2)*3i)/512 - 1i/128) - atan((x*940311i)/
(134217728*((275445*2^(1/2))/134217728 + 389421/134217728)) + (2^(1/2)*x*33
2433i)/(67108864*((275445*2^(1/2))/134217728 + 389421/134217728)))*((2^(1/2
)*3i)/512 + 1i/128)
```

sympy [B] time = 1.43, size = 296, normalized size = 3.25

$$\frac{-21x^5+140x^3-103x}{128x^6-896x^4+896x^2-128}-\frac{5\log(x-1)}{64}+\frac{5\log(x+1)}{64}+\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001}{202624020}-\frac{471550901\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

```
[Out] (-21*x**5 + 140*x**3 - 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*lo
g(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/1024)*log(x - 807126400
1/202624020 - 471550901878784*(-1/256 + 3*sqrt(2)/1024)**3/2979765 + 129955
2375287054336*(-1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001*sqrt(2)/27
0165360) + (-3*sqrt(2)/1024 - 1/256)*log(x - 8071264001*sqrt(2)/270165360 -
 8071264001/202624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)**5/50
656005 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (1/256 - 3
*sqrt(2)/1024)*log(x - 8071264001*sqrt(2)/270165360 + 1299552375287054336*(
1/256 - 3*sqrt(2)/1024)**5/50656005 - 471550901878784*(1/256 - 3*sqrt(2)/10
24)**3/2979765 + 8071264001/202624020) + (1/256 + 3*sqrt(2)/1024)*log(x - 4
71550901878784*(1/256 + 3*sqrt(2)/1024)**3/2979765 + 1299552375287054336*(1
/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001/202624020 + 8071264001*sqrt
(2)/270165360)
```

3.78 $\int \frac{x^3}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=78

$$\frac{(3a^2-c)\log\left((a+bx)^2+c\right)}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

```
[Out] -3*a*x/b^3+1/2*(b*x+a)^2/b^4+1/2*(3*a^2-c)*ln(c+(b*x+a)^2)/b^4-a*(a^2-3*c)*
arctan((b*x+a)/c^(1/2))/b^4/c^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 702, 635, 203, 260}

$$\frac{(3a^2-c)\log\left((a+bx)^2+c\right)}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c
]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*b^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^3}{c+x^2}\,dx, x, a+bx\right)}{b^4} \\
&= \frac{\text{Subst}\left(\int \left(-3a+x-\frac{a^3-3ac-\left(3a^2-c\right)x}{c+x^2}\right)dx, x, a+bx\right)}{b^4} \\
&= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{\text{Subst}\left(\int \frac{a^3-3ac-\left(3a^2-c\right)x}{c+x^2}\,dx, x, a+bx\right)}{b^4} \\
&= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{\left(a\left(a^2-3c\right)\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^4}+\frac{\left(3a^2-c\right)\text{Subst}\left(\int \frac{x}{c+x^2}\,dx\right.}{b^4} \\
&= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{\left(3a^2-c\right)\log\left(c+(a+bx)^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.94

$$
\frac{-\frac{2\left(a^3-3ac\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\left(3a^2-c\right)\log\left(a^2+2abx+b^2x^2+c\right)+bx(bx-4a)}{2b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + (
3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)
```

fricas [A] time = 0.87, size = 198, normalized size = 2.54

$$\left[\frac{b^2cx^2 - 4\,abcx + \left(a^3 - 3\,ac\right)\sqrt{-c}\,\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right) + \left(3\,a^2c - c^2\right)\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{2\,b^4c},\ b^2c\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x
 + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^2*
c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a*b*
c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^2)*l
og(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]
```

giac [A] time = 0.34, size = 77, normalized size = 0.99

$$\frac{\left(3\,a^2 - c\right)\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{2\,b^4} - \frac{\left(a^3 - 3\,ac\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{b^2x^2 - 4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan
((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4
```

maple [A] time = 0.01, size = 127, normalized size = 1.63

$$\frac{x^2}{2b^2} - \frac{a^3\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{3a^2\ln\left(b^2x^2 + 2abx + a^2 + c\right)}{2b^4} - \frac{2ax}{b^3} + \frac{3a\sqrt{c}\,\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^4} - \frac{c\ln\left(b^2x^2 + 2abx + a\cdots\right.}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c+(b*x+a)^2),x)
```

```
[Out] 1/2/b^2*x^2-2*a*x/b^3+3/2/b^4*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2-1/2/b^4*ln(b^2*
x^2+2*a*b*x+a^2+c)*c-1/b^4/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^
3+3/b^4*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a
```

maxima [A] time = 1.46, size = 81, normalized size = 1.04

$$\frac{bx^2 - 4\,ax}{2\,b^3} + \frac{\left(3\,a^2 - c\right)\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{2\,b^4} - \frac{\left(a^3 - 3\,ac\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 - 4*a*x)/b^3 + 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/
b^4 - (a^3 - 3*a*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^4*sqrt(c))
```

mupad [B] time = 2.27, size = 87, normalized size = 1.12

$$\frac{x^2}{2\,b^2}-\frac{2\,a\,x}{b^3}-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)\,\left(4\,b^4\,c^2-12\,a^2\,b^4\,c\right)}{8\,b^8\,c}+\frac{a\,\mathrm{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)\,\left(3\,c-a^2\right)}{b^4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c + (a + b*x)^2),x)
```

```
[Out] x^2/(2*b^2) - (2*a*x)/b^3 - (log(c + a^2 + b^2*x^2 + 2*a*b*x)*(4*b^4*c^2 -
12*a^2*b^4*c))/(8*b^8*c) + (a*atan((a + b*x)/c^(1/2))*(3*c - a^2))/(b^4*c^(
1/2))
```

sympy [B] time = 0.69, size = 209, normalized size = 2.68

$$-\frac{2ax}{b^3}+\left(-\frac{a\sqrt{-c}\,\left(a^2-3c\right)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)\log\left(x+\frac{a^4-2b^4c\left(-\frac{a\sqrt{-c}\,\left(a^2-3c\right)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)-c^2}{a^3b-3abc}\right)+\left(\frac{a\sqrt{-c}\,\left(a^2-3c\right)}{2b^4c}+\frac{3a^2-}{2b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4))
*log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 -
c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a*sqrt(-c)*(a**2 - 3*c)/(2*b**4
*c) + (3*a**2 - c)/(2*b**4))*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2 - 3
*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + x**2/
(2*b**2)
```

3.79 $\int \frac{x^2}{c+(a+bx)^2} dx$

Optimal. Leaf size=50

$$\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left((a+bx)^2+c\right)}{b^3}+\frac{x}{b^2}$$

```
[Out] x/b^2-a*ln(c+(b*x+a)^2)/b^3+(a^2-c)*arctan((b*x+a)/c^(1/2))/b^3/c^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 702, 635, 203, 260}

$$\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left((a+bx)^2+c\right)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + (a + b*x)^2),x]
```

```
[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*Log[c + (a
 + b*x)^2])/b^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a^2-c-2ax}{c+x^2}\right)dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}+\frac{\text{Subst}\left(\int \frac{a^2-c-2ax}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}-\frac{(2a)\,\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{b^3}+\frac{\left(a^2-c\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}+\frac{\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left(c+(a+bx)^2\right)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.08

$$\frac{-a\log\left(a^2+2abx+b^2x^2+c\right)+\frac{\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + (a + b*x)^2),x]
```

```
[Out] (b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a*
b*x + b^2*x^2])/b^3
```

fricas [A] time = 0.80, size = 157, normalized size = 3.14

$$\left[\frac{2\,bcx - 2\,ac\log\left(b^2x^2 + 2\,abx + a^2 + c\right) + \left(a^2 - c\right)\sqrt{-c}\,\log\left(\frac{b^2x^2 + 2\,abx + a^2 + 2\,(bx + a)\sqrt{-c} - c}{b^2x^2 + 2\,abx + a^2 + c}\right)}{2\,b^3c}, \frac{bcx - ac\log\left(b^2x^2 + 2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c)
*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*
x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^
2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]
```

giac [A] time = 0.31, size = 54, normalized size = 1.08

$$\frac{x}{b^2} - \frac{a\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{b^3} + \frac{\left(a^2 - c\right)\arctan\left(\frac{bx + a}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a)
/sqrt(c))/(b^3*sqrt(c))
```

maple [A] time = 0.01, size = 89, normalized size = 1.78

$$\frac{a^2\arctan\left(\frac{2b^2x + 2ba}{2b\sqrt{c}}\right)}{b^3\sqrt{c}} - \frac{a\ln\left(b^2x^2 + 2abx + a^2 + c\right)}{b^3} + \frac{x}{b^2} - \frac{\sqrt{c}\,\arctan\left(\frac{2b^2x + 2ba}{2b\sqrt{c}}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c+(b*x+a)^2),x)
```

```
[Out] x/b^2-1/b^3*a*ln(b^2*x^2+2*a*b*x+a^2+c)+1/b^3/c^(1/2)*arctan(1/2*(2*b^2*x+2
*a*b)/b/c^(1/2))*a^2-1/b^3*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

maxima [A] time = 1.65, size = 61, normalized size = 1.22

$$\frac{x}{b^2} - \frac{a\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{b^3} + \frac{\left(a^2 - c\right)\arctan\left(\frac{b^2x + ab}{b\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b^2*x +
a*b)/(b*sqrt(c)))/(b^3*sqrt(c))
```

mupad [B] time = 0.09, size = 206, normalized size = 4.12

$$\frac{x}{b^2}-\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{b^3}+\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,\left(c-a^2\right)}-\frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}}-\frac{a\,\sqrt{c}}{c-a^2}+\frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3}-\frac{a^2\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,\left(c-a^2\right)}-\frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}}-\frac{a}{c-}\right.}{b^3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c + (a + b*x)^2),x)
```

```
[Out] x/b^2 - (a*log(c + a^2 + b^2*x^2 + 2*a*b*x))/b^3 + (c^(1/2)*atan(a^3/(c^(1/
2)*(c - a^2)) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)
/(c^(1/2)*(c/b - a^2/b))))/b^3 - (a^2*atan(a^3/(c^(1/2)*(c - a^2)) - (c^(1/
2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a^2/b
))))/(b^3*c^(1/2))
```

sympy [B] time = 0.46, size = 153, normalized size = 3.06

$$\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\,\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)}{a^2b-bc}\right)+\left(-\frac{a}{b^3}+\frac{\sqrt{-c}\,\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c+(b*x+a)**2),x)
```

```
[Out] (-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(
-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + sqr
t(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + sqrt
(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2
```

3.80 $\int \frac{x}{c+(a+bx)^2} dx$

Optimal. Leaf size=41

$$\frac{\log\left((a+bx)^2+c\right)}{2b^2}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

```
[Out] 1/2*ln(c+(b*x+a)^2)/b^2-a*arctan((b*x+a)/c^(1/2))/b^2/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {371, 635, 203, 260}

$$\frac{\log\left((a+bx)^2+c\right)}{2b^2}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + (a + b*x)^2),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x)^2]/(2*b^
2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[-(a*c)]
```

Rubi steps

$$\int \frac{x}{c+(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-a+x}{c+x^2}\,dx, x, a+bx\right)}{b^2}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^2}$$
$$= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(c+(a+bx)^2\right)}{2b^2}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.93

$$\frac{\log\left((a+bx)^2+c\right) - \frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + (a + b*x)^2),x]
```

```
[Out] ((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)
```

fricas [A] time = 0.82, size = 136, normalized size = 3.32

$$\left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right) - c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}, -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right) - c\log\left(b^2x^2+2\,a\right.}{2\,b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c
), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) - c*log(b^2*x^2 + 2*a*b*x +
a^2 + c))/(b^2*c)]
```

giac [A] time = 0.36, size = 43, normalized size = 1.05

$$-\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*b*x + a^
2 + c)/b^2
```

maple [A] time = 0.00, size = 54, normalized size = 1.32

$$-\frac{a\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\ln\left(b^2x^2+2abx+a^2+c\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+(b*x+a)^2),x)
```

```
[Out] 1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/
b/c^(1/2))
```

maxima [A] time = 1.36, size = 50, normalized size = 1.22

$$-\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*
b*x + a^2 + c)/b^2
```

mupad [B] time = 2.08, size = 46, normalized size = 1.12

$$\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{2\,b^2}-\frac{a\,\mathrm{atan}\left(\frac{a}{\sqrt{c}}+\frac{b\,x}{\sqrt{c}}\right)}{b^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + (a + b*x)^2),x)
```

```
[Out] log(c + a^2 + b^2*x^2 + 2*a*b*x)/(2*b^2) - (a*atan(a/c^(1/2) + (b*x)/c^(1/2
)))/(b^2*c^(1/2))
```

sympy [B] time = 0.25, size = 124, normalized size = 3.02

$$\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)+\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(-a*sqrt(-c)
/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b)) + (a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)
)*log(x + (a**2 - 2*b**2*c*(a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b))
```

3.81 $\int \frac{1}{c+(a+bx)^2} \, dx$

Optimal. Leaf size=21

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[Out] arctan((b*x+a)/c^(1/2))/b/c^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {247, 203}

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + (a + b*x)^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{c+(a+bx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{c+x^2} \, dx, x, a+bx\right)}{b} \\ &= \frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a + b*x)^2)^(-1),x]
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

fricas [A] time = 0.66, size = 83, normalized size = 3.95

$$\left[-\frac{\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,bc}, \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^
2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]
```

giac [A] time = 0.38, size = 17, normalized size = 0.81

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="giac")
[Out] arctan((b*x + a)/sqrt(c))/(b*sqrt(c))
```

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+(b*x+a)^2),x)
```

```
[Out] 1/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

maxima [A] time = 1.52, size = 24, normalized size = 1.14

$$\frac{\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] arctan((b^2*x + a*b)/(b*sqrt(c)))/(b*sqrt(c))
```

mupad [B] time = 0.04, size = 17, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)}{b\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + (a + b*x)^2),x)
```

```
[Out] atan((a + b*x)/c^(1/2))/(b*c^(1/2))
```

sympy [B] time = 0.19, size = 54, normalized size = 2.57

$$\frac{-\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)**2),x)
```

```
[Out] (-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*sq
rt(-1/c))/b)/2)/b
```

3.82 $\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=59

$$-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

```
[Out] ln(x)/(a^2+c)-1/2*ln(c+(b*x+a)^2)/(a^2+c)-a*arctan((b*x+a)/c^(1/2))/(a^2+c)
/c^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {371, 706, 31, 635, 203, 260}

$$-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^2 + c) - L
og[c + (a + b*x)^2]/(2*(a^2 + c))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx &= \text{Subst}\left(\int \frac{1}{(-a+x)\left(c+x^2\right)}\,dx,x,a+bx\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-a+x}\,dx,x,a+bx\right)}{a^2+c}+\frac{\text{Subst}\left(\int \frac{-a-x}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}\\ &= \frac{\log(x)}{a^2+c}-\frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}-\frac{a\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}\\ &= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}-\frac{\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.81

$$-\frac{\log\left((a+bx)^2+c\right)+\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-2\log(bx)}{2\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c + (a + b
*x)^2])/(a^2 + c)
```

fricas [A] time = 0.84, size = 154, normalized size = 2.61

$$\left[-\frac{a\sqrt{-c}\,\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log(x)}{2\left(a^2c+c^2\right)},-\frac{2\,a\sqrt{c}\,\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+c\log\ldots}{2\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*l
og(x))/(a^2*c + c^2), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) + c*log(b
^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2)]
```

giac [A] time = 0.38, size = 62, normalized size = 1.05

$$-\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^2+c\right)\sqrt{c}}-\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^2+c\right)}+\frac{\log\left(|x|\right)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2 + 2*a*b*
x + a^2 + c)/(a^2 + c) + log(abs(x))/(a^2 + c)
```

maple [A] time = 0.01, size = 72, normalized size = 1.22

$$-\frac{a\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{\left(a^2+c\right)\sqrt{c}}+\frac{\ln(x)}{a^2+c}-\frac{\ln\left(b^2x^2+2abx+a^2+c\right)}{2\left(a^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c+(b*x+a)^2),x)
```

```
[Out] ln(x)/(a^2+c)-1/2/(a^2+c)*ln(b^2*x^2+2*a*b*x+a^2+c)-1/(a^2+c)*a/c^(1/2)*arc
tan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

maxima [A] time = 1.52, size = 68, normalized size = 1.15

$$-\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{\left(a^2+c\right)\sqrt{c}}-\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2
+ 2*a*b*x + a^2 + c)/(a^2 + c) + log(x)/(a^2 + c)
```

mupad [B] time = 2.59, size = 173, normalized size = 2.93

$$\frac{\ln(x)}{a^2+c}-\frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c+a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,\left(a^2+c\right)}\right)\,\left(c+a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)}-\frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c-a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2\right.}{c\,\left(a^2+c\right)}\right.}{2\,\left(a^2\,c+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + (a + b*x)^2)),x)
```

```
[Out] log(x)/(c + a^2) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c + a*(-c)^(1/2))*(a*c +
a^3 - 3*b*c*x + a^2*b*x))/(c*(c + a^2)))*(c + a*(-c)^(1/2)))/(2*(a^2*c + c^
2)) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c - a*(-c)^(1/2))*(a*c + a^3 - 3*b*c*x
 + a^2*b*x))/(c*(c + a^2)))*(c - a*(-c)^(1/2)))/(2*(a^2*c + c^2))
```

sympy [B] time = 3.40, size = 738, normalized size = 12.51

$$\left(-\frac{a\sqrt{-c}}{2c\left(a^2+c\right)}-\frac{1}{2\left(a^2+c\right)}\right)\log\left(x+\frac{-4a^6c\left(-\frac{a\sqrt{-c}}{2c\left(a^2+c\right)}-\frac{1}{2\left(a^2+c\right)}\right)^2+4a^4c^2\left(-\frac{a\sqrt{-c}}{2c\left(a^2+c\right)}-\frac{1}{2\left(a^2+c\right)}\right)^2-6a^4c\left(-\frac{a\sqrt{-c}}{2c\left(a^2+c\right)}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*log(x + (-4*a**6*c*(-a*sq
rt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a**4*c**2*(-a*sqrt(-c)/(
2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(-a*sqrt(-c)/(2*c*(a**2 +
 c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(
2*(a**2 + c)))**2 - 12*a**2*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) + 10*a**2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 +
c)))**2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) - 6*c**2
)/(a**3*b + 9*a*b*c)) + (a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*lo
g(x + (-4*a**6*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a*
*4*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(a*sqrt(-c)/(2*
```

```
c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(a*sqrt(-c)/(2*c*(a**2
+ c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*sqrt(-c)/(2*c*(a**2 + c)
) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + log(x + (-4*a**6*c/(a**2 + c)**2 +
4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 + c) + 20*a**2*c**3/(a**2 + c)**
2 - 12*a**2*c**2/(a**2 + c) + 10*a**2*c + 12*c**4/(a**2 + c)**2 - 6*c**3/(a
**2 + c) - 6*c**2)/(a**3*b + 9*a*b*c))/(a**2 + c)
```

3.83 $\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=79

$$-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{\left(a^2+c\right)^2}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{1}{x\left(a^2+c\right)}$$

```
[Out] -1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+a*b*ln(c+(b*x+a)^2)/(a^2+c)^2+b*(a^2-c)*
arctan((b*x+a)/c^(1/2))/(a^2+c)^2/c^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {371, 710, 801, 635, 203, 260}

$$-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{\left(a^2+c\right)^2}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{1}{x\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2
+ c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a + b*x)^2])/(a^2 + c)
^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx &= b\,\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(c+x^2\right)}\,dx, x, a+bx\right)\\
&= -\frac{1}{\left(a^2+c\right)x} + \frac{b\,\text{Subst}\left(\int \frac{-a-x}{(-a+x)\left(c+x^2\right)}\,dx, x, a+bx\right)}{a^2+c}\\
&= -\frac{1}{\left(a^2+c\right)x} + \frac{b\,\text{Subst}\left(\int \left(\frac{2a}{\left(a^2+c\right)(a-x)} + \frac{a^2-c+2ax}{\left(a^2+c\right)\left(c+x^2\right)}\right)dx, x, a+bx\right)}{a^2+c}\\
&= -\frac{1}{\left(a^2+c\right)x} - \frac{2ab\log(x)}{\left(a^2+c\right)^2} + \frac{b\,\text{Subst}\left(\int \frac{a^2-c+2ax}{c+x^2}\,dx, x, a+bx\right)}{\left(a^2+c\right)^2}\\
&= -\frac{1}{\left(a^2+c\right)x} - \frac{2ab\log(x)}{\left(a^2+c\right)^2} + \frac{(2ab)\,\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{\left(a^2+c\right)^2} + \frac{\left(b\left(a^2-c\right)\right)\text{Subst}\left(\int \right.}{\left(a^2+\right.}\\
&= -\frac{1}{\left(a^2+c\right)x} + \frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2} - \frac{2ab\log(x)}{\left(a^2+c\right)^2} + \frac{ab\log\left(c+(a+bx)^2\right)}{\left(a^2+c\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.03

$$\frac{bx\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)-\sqrt{c}\left(-abx\log\left(a^2+2abx+b^2x^2+c\right)+a^2+2abx\log(x)+c\right)}{\sqrt{c}\,x\left(a^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] (b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[x
] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)
```

fricas [A] time = 0.92, size = 229, normalized size = 2.90

$$\left[\frac{2\,abcx\log\left(b^2x^2+2\,abx+a^2+c\right)-4\,abcx\log(x)+\left(a^2b-bc\right)\sqrt{-c}\,x\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-2\,a^2c-2\right.}{2\left(a^4c+2\,a^2c^2+c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2*
b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c
)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2 + c
^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) + (a^2
*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c + 2*a^
2*c^2 + c^3)*x)]
```

giac [A] time = 0.37, size = 117, normalized size = 1.48

$$\frac{ab\log\left(b^2x^2+2\,abx+a^2+c\right)}{a^4+2\,a^2c+c^2}-\frac{2\,ab\log\left(|x|\right)}{a^4+2\,a^2c+c^2}+\frac{\left(a^2b^2-b^2c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^4+2\,a^2c+c^2\right)b\sqrt{c}}-\frac{1}{\left(a^2+c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(abs(
x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x + a)/sqrt(c))/((a
^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

maple [A] time = 0.01, size = 123, normalized size = 1.56

$$\frac{a^2b\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{\left(a^2+c\right)^2\sqrt{c}}-\frac{2ab\ln(x)}{\left(a^2+c\right)^2}+\frac{ab\ln\left(b^2x^2+2abx+a^2+c\right)}{\left(a^2+c\right)^2}-\frac{b\sqrt{c}\,\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{\left(a^2+c\right)^2}-\frac{1}{\left(a^2+c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c+(b*x+a)^2),x)
```

```
[Out] -1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+b/(a^2+c)^2*a*ln(b^2*x^2+2*a*b*x+a^2+c)+
b/(a^2+c)^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^2-b/(a^2+c)^2*c
^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

maxima [A] time = 1.59, size = 123, normalized size = 1.56

$$\frac{ab\log\left(b^2x^2+2\,abx+a^2+c\right)}{a^4+2\,a^2c+c^2}-\frac{2\,ab\log(x)}{a^4+2\,a^2c+c^2}+\frac{\left(a^2b^2-b^2c\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{\left(a^4+2\,a^2c+c^2\right)b\sqrt{c}}-\frac{1}{\left(a^2+c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(x)/(
a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/
((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

mupad [B] time = 2.58, size = 425, normalized size = 5.38

$$\frac{\ln\left((-c)^{13/2}-35\,a^2\,(-c)^{11/2}+34\,a^4\,(-c)^{9/2}+34\,a^6\,(-c)^{7/2}-35\,a^8\,(-c)^{5/2}+a^{10}\,(-c)^{3/2}+a\,c^6-a^{11}\,c+35\,a^3\,c^5+3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + (a + b*x)^2)),x)
```

```
[Out] (log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/
2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) + a*c^6 - a^11*c + 35*a^3*c^5 + 34
*a^5*c^4 - 34*a^7*c^3 - 35*a^9*c^2 + b*c^6*x - a^10*b*c*x + 35*a^2*b*c^5*x
+ 34*a^4*b*c^4*x - 34*a^6*b*c^3*x - 35*a^8*b*c^2*x)*(b*(-c)^(3/2) + 2*a*b*c
 + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - 1/(x*(c + a^2)) - (lo
g((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/2)
- 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) - a*c^6 + a^11*c - 35*a^3*c^5 - 34*a^
5*c^4 + 34*a^7*c^3 + 35*a^9*c^2 - b*c^6*x + a^10*b*c*x - 35*a^2*b*c^5*x - 3
4*a^4*b*c^4*x + 34*a^6*b*c^3*x + 35*a^8*b*c^2*x)*(b*(-c)^(3/2) - 2*a*b*c +
a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - (2*a*b*log(x))/(c + a^2)
^2
```

sympy [B] time = 11.12, size = 1620, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*b*log(x + (-16*a**13*b**2*c/(a**2 + c)**4 + 48*a**11*b**2*c**2/(a**2 +
 c)**4 + 352*a**9*b**2*c**3/(a**2 + c)**4 - 20*a**9*b**2*c/(a**2 + c)**2 +
608*a**7*b**2*c**4/(a**2 + c)**4 - 64*a**7*b**2*c**2/(a**2 + c)**2 + 432*a*
*5*b**2*c**5/(a**2 + c)**4 - 72*a**5*b**2*c**3/(a**2 + c)**2 + 36*a**5*b**2
*c + 112*a**3*b**2*c**6/(a**2 + c)**4 - 32*a**3*b**2*c**4/(a**2 + c)**2 - 8
8*a**3*b**2*c**2 - 4*a*b**2*c**5/(a**2 + c)**2 + 4*a*b**2*c**3)/(a**6*b**3
+ 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3))/(a**2 + c)**2 + (a*b/(a*
*2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*log(x +
(-4*a**11*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*
c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2
*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 - b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a**2 +
c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32*a**6*
b*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c
**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a*
*2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**2 + c)
**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3*b**2*
c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a
**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*(a*b/(
a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 2
*b*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c +
c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)) + (a
*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*lo
g(x + (-4*a**11*c*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2
*a**2*c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 -
 c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 + b
*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a
**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32
*a**6*b*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2
*c + c**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 + b*sqrt(-
c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**
2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3
*b**2*c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*
(a**4 + 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 + b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*
(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
*2 + 2*b*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**
2*c + c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)
) - 1/(x*(a**2 + c))
```

3.84 $\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=121

$$\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

```
[Out] -1/2/(a^2+c)/x^2+2*a*b/(a^2+c)^2/x+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3-1/2*b^2*(3
*a^2-c)*ln(c+(b*x+a)^2)/(a^2+c)^3-a*b^2*(a^2-3*c)*arctan((b*x+a)/c^(1/2))/(
a^2+c)^3/c^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {371, 710, 801, 635, 203, 260}

$$\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/(2*(a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*c)*ArcTan[
(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2 - c)*Log[x])/(a^2 +
 c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx = b^2\,\text{Subst}\left(\int \frac{1}{(-a+x)^3\left(c+x^2\right)}\,dx, x, a+bx\right)$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{b^2\,\text{Subst}\left(\int \frac{-a-x}{(-a+x)^2\left(c+x^2\right)}\,dx, x, a+bx\right)}{a^2+c}$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{b^2\,\text{Subst}\left(\int\left(-\frac{2a}{\left(a^2+c\right)(a-x)^2} + \frac{-3a^2+c}{\left(a^2+c\right)^2(a-x)} + \frac{-a\left(a^2-3c\right)-\left(3a^2-c\right)x}{\left(a^2+c\right)^2\left(c+x^2\right)}\right)dx, x, a +\right.}{a^2+c}$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{2ab}{\left(a^2+c\right)^2 x} + \frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3} + \frac{b^2\,\text{Subst}\left(\int \frac{-a\left(a^2-3c\right)-\left(3a^2-c\right)x}{c+x^2}\,dx, \right.}{\left(a^2+c\right)^3}$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{2ab}{\left(a^2+c\right)^2 x} + \frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3} - \frac{\left(ab^2\left(a^2-3c\right)\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, \right.}{\left(a^2+c\right)^3}$$

$$= -\frac{1}{2\left(a^2+c\right)x^2} + \frac{2ab}{\left(a^2+c\right)^2 x} - \frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3} + \frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3} - \frac{b^2\left(3\right.}{}$$

Mathematica [A] time = 0.17, size = 106, normalized size = 0.88

$$-\frac{b^2\left(3a^2-c\right)\log\left(a^2+2abx+b^2x^2+c\right)+2b^2\left(c-3a^2\right)\log(x)+\frac{2ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\left(a^2+c\right)\left(a^2-4abx+c\right)}{x^2}}{2\left(a^2+c\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*(((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcTan[(a
+ b*x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*(3*a^2 - c)*Log[
a^2 + c + 2*a*b*x + b^2*x^2])/(a^2 + c)^3
```

fricas [A] time = 0.85, size = 371, normalized size = 3.07

$$\left[-\frac{a^4c-\left(a^3b^2-3\,ab^2c\right)\sqrt{-c}\,x^2\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+2\,a^2c^2+\left(3\,a^2b^2c-b^2c^2\right)x^2\log\left(b^2x^2+2\,abx+a\right.}{2\left(a^6c+3\,a^4c^2+3\,a^2c^3+c^4\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x +
a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^2
+ (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b
^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4
*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sqrt(c)
*x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*x^2*lo
g(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3
 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2)]
```

giac [A] time = 0.42, size = 195, normalized size = 1.61

$$-\frac{\left(3\,a^2b^2-b^2c\right)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)}+\frac{\left(3\,a^2b^2-b^2c\right)\log\left(|x|\right)}{a^6+3\,a^4c+3\,a^2c^2+c^3}-\frac{\left(a^3b^3-3\,ab^3c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)b\sqrt{c}}-\frac{a^4+2\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(abs(x))/(a^6 + 3*a^4*c + 3*a^2*c
^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x + a)/sqrt(c))/((a^6 + 3*a^4*c
 + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b + a*b*
c)*x)/((a^2 + c)^3*x^2)
```

maple [A] time = 0.01, size = 198, normalized size = 1.64

$$-\frac{a^3b^2\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{\left(a^2+c\right)^3\sqrt{c}}+\frac{3a^2b^2\ln(x)}{\left(a^2+c\right)^3}-\frac{3a^2b^2\ln\left(b^2x^2+2abx+a^2+c\right)}{2\left(a^2+c\right)^3}+\frac{3a\,b^2\sqrt{c}\,\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{\left(a^2+c\right)^3}-\frac{b^2c\ln(x)}{\left(a^2+c\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c+(b*x+a)^2),x)
```

```
[Out] -1/2/(a^2+c)/x^2+3*b^2/(a^2+c)^3*ln(x)*a^2-b^2/(a^2+c)^3*ln(x)*c+2*a*b/(a^2
+c)^2/x-3/2*b^2/(a^2+c)^3*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2+1/2*b^2/(a^2+c)^3*l
n(b^2*x^2+2*a*b*x+a^2+c)*c-b^2/(a^2+c)^3/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)
/b/c^(1/2))*a^3+3*b^2/(a^2+c)^3*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2
))*a
```

maxima [A] time = 1.61, size = 197, normalized size = 1.63

$$-\frac{\left(3\,a^2b^2-b^2c\right)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)}+\frac{\left(3\,a^2b^2-b^2c\right)\log(x)}{a^6+3\,a^4c+3\,a^2c^2+c^3}-\frac{\left(a^3b^3-3\,ab^3c\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{\left(a^6+3\,a^4c+3\,a^2c^2+c^3\right)b\sqrt{c}}+\frac{4\,abx}{2\left(a^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(x)/(a^6 + 3*a^4*c + 3*a^2*c^2 +
c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^6 + 3*a^
4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) + 1/2*(4*a*b*x - a^2 - c)/((a^4 + 2*a^2*c
 + c^2)*x^2)
```

mupad [B] time = 2.77, size = 573, normalized size = 4.74

$$\ln(x)\left(\frac{3\,b^2}{\left(a^2+c\right)^2}-\frac{4\,b^2\,c}{\left(a^2+c\right)^3}\right)-\frac{\frac{1}{2\left(a^2+c\right)}-\frac{2\,a\,b\,x}{\left(a^2+c\right)^2}}{x^2}-\frac{\ln\left(27\,(-c)^{15/2}+90\,a^2\,(-c)^{13/2}+9\,a^4\,(-c)^{11/2}-324\,a^6\,(-c)^{9/2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + (a + b*x)^2)),x)
```

```
[Out] log(x)*((3*b^2)/(c + a^2)^2 - (4*b^2*c)/(c + a^2)^3) - (1/(2*(c + a^2)) - (
2*a*b*x)/(c + a^2)^2)/x^2 - (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) + 9*a^
4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^(5/2
) - a^12*(-c)^(3/2) - 27*a*c^7 + a^13*c + 90*a^3*c^6 - 9*a^5*c^5 - 324*a^7*
c^4 - 125*a^9*c^3 + 74*a^11*c^2 - 27*b*c^7*x + a^12*b*c*x + 90*a^2*b*c^6*x
- 9*a^4*b*c^5*x - 324*a^6*b*c^4*x - 125*a^8*b*c^3*x + 74*a^10*b*c^2*x)*(a^3
*b^2*(-c)^(1/2) - b^2*c^2 + 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*c +
c^4 + 3*a^2*c^3 + 3*a^4*c^2)) + (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) +
9*a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^
(5/2) - a^12*(-c)^(3/2) + 27*a*c^7 - a^13*c - 90*a^3*c^6 + 9*a^5*c^5 + 324*
a^7*c^4 + 125*a^9*c^3 - 74*a^11*c^2 + 27*b*c^7*x - a^12*b*c*x - 90*a^2*b*c^
6*x + 9*a^4*b*c^5*x + 324*a^6*b*c^4*x + 125*a^8*b*c^3*x - 74*a^10*b*c^2*x)*
(b^2*c^2 + a^3*b^2*(-c)^(1/2) - 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*
c + c^4 + 3*a^2*c^3 + 3*a^4*c^2))
```

sympy [B] time = 38.26, size = 3284, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c+(b*x+a)**2),x)

[Out] b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2 + c)**6 +
24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*a**12*b**4*c**3*(3*a
**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(3*a**2 - c)/(a**2 + c)**3 + 56
8*a**10*b**4*c**4*(3*a**2 - c)**2/(a**2 + c)**6 - 44*a**10*b**4*c**2*(3*a**
2 - c)/(a**2 + c)**3 + 720*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**6 - 4
2*a**8*b**4*c**3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a**6*b**
4*c**6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)/(a**2
+ c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 - c)**2/(a**2 + c
)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3 + 380*a**4*b**4*c**3 - 2
4*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 + c)**6 - 12*a**2*b**4*c**6*(3*a**2
- c)/(a**2 + c)**3 - 96*a**2*b**4*c**4 - 12*b**4*c**9*(3*a**2 - c)**2/(a**2
 + c)**6 - 6*b**4*c**7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5)/(a**9*b**5
 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**
4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**
16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c
**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14*c**2*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) + 216*a**12*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 +
 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2
- 44*a**10*b**2*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 568*a**10*c*
*4*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**
3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 78*a**8*b**4*c - 42*a**8*b*
*2*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 720*a**8*c**5*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*b**4*c**2 - 8*a**6*b**2*c**4*(-a
*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) -
b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a**6*c**6*(-a*b**2*sqrt(-c)*(a**
2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(
2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2*a**4*b**2*c**5*(-a*b**2*sqrt(
-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**
2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2
*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c
)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c**6*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) - 24*a**2*c**8*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3
*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 +
6*b**4*c**5 - 6*b**2*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**
4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 12*c**9
```

```
*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)
) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*b**5 + 72*a**7*b**5*c -
270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**4)) + (a*b**2*sqrt(-
c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2
 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**16*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/
(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 +
 c)**3))**2 + 24*a**14*c**2*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a*
*4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 14*
a**12*b**2*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c
**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 216*a**12*c**3*(a*b**
2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2
*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 44*a**10*b**2*c**2*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3)) + 568*a**10*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a
**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)
)**2 + 78*a**8*b**4*c - 42*a**8*b**2*c**3*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*
c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)
**3)) + 720*a**8*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*
b**4*c**2 - 8*a**6*b**2*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a
**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a
**6*c**6*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2
*a**4*b**2*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**
2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(a*b
**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b*
*2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c*
*6*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3
)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 24*a**2*c**8*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3))**2 + 6*b**4*c**5 - 6*b**2*c**7*(a*b**2*sqrt(-c)*(a**2
- 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*
(a**2 + c)**3)) - 12*c**9*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4
*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*
b**5 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5
*c**4)) + (-a**2 + 4*a*b*x - c)/(x**2*(2*a**4 + 4*a**2*c + 2*c**2))
```

3.85 $\int \frac{1}{a+b(c+dx)^2} \, dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

```
[Out] arctan((d*x+c)*b^(1/2)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} \, dx, x, c+dx\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

fricas [A] time = 0.72, size = 109, normalized size = 3.52

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}\,(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{2\,abd}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,(dx+c)}{a}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c
) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a*b*d), sqrt(a*b)*arctan(sqrt(
a*b)*(d*x + c)/a)/(a*b*d)]
```

giac [A] time = 0.45, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*d)
```

maple [A] time = 0.01, size = 34, normalized size = 1.10

$$\frac{\arctan\left(\frac{2b\,d^2x+2bdc}{2\sqrt{ab}\,d}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2),x)
```

```
[Out] 1/d/(b*a)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(b*a)^(1/2))
```

maxima [A] time = 1.56, size = 30, normalized size = 0.97

$$\frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}\,d}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*d)
```

mupad [B] time = 0.06, size = 27, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2),x)
```

```
[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

sympy [B] time = 0.21, size = 61, normalized size = 1.97

$$\frac{-\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{-a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2),x)
```

```
[Out] (-sqrt(-1/(a*b))*log(x + (-a*sqrt(-1/(a*b)) + c)/d)/2 + sqrt(-1/(a*b))*log(
x + (a*sqrt(-1/(a*b)) + c)/d)/2)/d
```

3.86 $\int \frac{1}{\left(a+b(c+dx)^2\right)^2} dx$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}$$

```
[Out] 1/2*(d*x+c)/a/d/(a+b*(d*x+c)^2)+1/2*arctan((d*x+c)*b^(1/2)/a^(1/2))/a^(3/2)
/d/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]
/(2*a^(3/2)*Sqrt[b]*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{2ad\left(a+b(c+dx)^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{2ad}$$

$$= \frac{c+dx}{2ad\left(a+b(c+dx)^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.95

$$\frac{\frac{\sqrt{a}\,(c+dx)}{a+b(c+dx)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] ((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[
a]]/Sqrt[b])/(2*a^(3/2)*d)
```

fricas [A] time = 0.56, size = 253, normalized size = 4.02

$$\left[\frac{2\,abdx + 2\,abc - \left(bd^2x^2 + 2\,bcdx + bc^2 + a\right)\sqrt{-ab}\,\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}\,(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{4\left(a^2b^2d^3x^2 + 2\,a^2b^2cd^2x + \left(a^2b^2c^2 + a^3b\right)d\right)}, \frac{abdx + abc + \left(bd^2x^2 + 2\right.}{2\left(a^2b^2d^3x^2 + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*d*x + 2*a*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(-a*b)*
log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2
 + 2*b*c*d*x + b*c^2 + a)))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2
*c^2 + a^3*b)*d), 1/2*(a*b*d*x + a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d
^2*x + (a^2*b^2*c^2 + a^3*b)*d)]
```

giac [A] time = 0.36, size = 65, normalized size = 1.03

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ad}+\frac{dx+c}{2\left(bd^2x^2+2\,bcdx+bc^2+a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x + c)/((b*d^2
*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)
```

maple [A] time = 0.00, size = 86, normalized size = 1.37

$$\frac{\arctan\left(\frac{2b\,d^2x+2bdc}{2\sqrt{ab}\,d}\right)}{2\sqrt{ab}\,ad}+\frac{2b\,d^2x+2bdc}{4\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)ab\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2)^2,x)
```

```
[Out] 1/4*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)
^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/(a*b)^(1/2)/d)
```

maxima [A] time = 1.56, size = 75, normalized size = 1.19

$$\frac{dx+c}{2\left(abd^3x^2+2\,abcd^2x+\left(abc^2+a^2\right)d\right)}+\frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}\,d}\right)}{2\sqrt{ab}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d) + 1/2*arcta
n((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a*d)
```

mupad [B] time = 0.10, size = 76, normalized size = 1.21

$$\frac{\frac{x}{2\,a}+\frac{c}{2\,a\,d}}{b\,c^2+2\,b\,c\,d\,x+b\,d^2\,x^2+a}+\frac{\mathrm{atan}\left(2\,a\,\left(\frac{\sqrt{b}\,c}{2\,a^{3/2}}+\frac{\sqrt{b}\,d\,x}{2\,a^{3/2}}\right)\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2)^2,x)
```

```
[Out] (x/(2*a) + c/(2*a*d))/(a + b*c^2 + b*d^2*x^2 + 2*b*c*d*x) + atan(2*a*((b^(1
/2)*c)/(2*a^(3/2)) + (b^(1/2)*d*x)/(2*a^(3/2))))/(2*a^(3/2)*b^(1/2)*d)
```

sympy [B] time = 0.58, size = 117, normalized size = 1.86

$$\frac{c+dx}{2a^2d+2abc^2d+4abcd^2x+2abd^3x^2}+\frac{-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{-a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*x**2) + (-
sqrt(-1/(a**3*b))*log(x + (-a**2*sqrt(-1/(a**3*b)) + c)/d)/4 + sqrt(-1/(a**
3*b))*log(x + (a**2*sqrt(-1/(a**3*b)) + c)/d)/4)/d
```

3.87 $\int \frac{1}{\left(a+b(c+dx)^2\right)^3}\,dx$

Optimal. Leaf size=91

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

```
[Out] 1/4*(d*x+c)/a/d/(a+b*(d*x+c)^2)^2+3/8*(d*x+c)/a^2/d/(a+b*(d*x+c)^2)+3/8*arc
tan((d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/d/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 205}

$$\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*(a + b*(c
+ d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
```

```
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, c+dx\right)}{4ad}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{8a^2d}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.82

$$\frac{\frac{\sqrt{a}\,(c+dx)\left(5a+3b(c+dx)^2\right)}{\left(a+b(c+dx)^2\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] ((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^2 + (3*Arc
Tan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/Sqrt[b])/(8*a^(5/2)*d)
```

fricas [B] time = 1.01, size = 595, normalized size = 6.54

$$\left[\frac{6\,ab^2d^3x^3 + 18\,ab^2cd^2x^2 + 6\,ab^2c^3 + 10\,a^2bc + 2\left(9\,ab^2c^2 + 5\,a^2b\right)dx - 3\left(b^2d^4x^4 + 4\,b^2cd^3x^3 + b^2c^4 + 2\left(3\,b^2c^2\right.\right.}{16\left(a^3b^3d^5x^4 + 4\,a^3b^3cd^4x^3 + 2\left(3\,a^3b^3c^2 + a^4b^2\right)d^3x^2 + 4\left(a^3b^3c^3 +\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*d^3*x^3 + 18*a*b^2*c*d^2*x^2 + 6*a*b^2*c^3 + 10*a^2*b*c + 2*
(9*a*b^2*c^2 + 5*a^2*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 +
2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sq
rt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(
b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3
 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x
+ (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d), 1/8*(3*a*b^2*d^3*x^3 + 9*a*b^2*
c*d^2*x^2 + 3*a*b^2*c^3 + 5*a^2*b*c + (9*a*b^2*c^2 + 5*a^2*b)*d*x + 3*(b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c
^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a)
)/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*
x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^
5*b)*d)]
```

giac [A] time = 0.41, size = 103, normalized size = 1.13

$$\frac{3 \arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8 \sqrt{ab}\, a^2 d} + \frac{3 \, bd^3x^3 + 9 \, bcd^2x^2 + 9 \, bc^2dx + 3 \, bc^3 + 5 \, adx + 5 \, ac}{8 \left(bd^2x^2 + 2 \, bcdx + bc^2 + a\right)^2 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/8*(3*b*d^3*x^3 +
9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c)/((b*d^2*x^2 + 2*b*
c*d*x + b*c^2 + a)^2*a^2*d)
```

maple [A] time = 0.00, size = 147, normalized size = 1.62

$$\frac{3x}{8 \left(b\, d^2 x^2 + 2bcdx + b\, c^2 + a\right) a^2} + \frac{3c}{8 \left(b\, d^2 x^2 + 2bcdx + b\, c^2 + a\right) a^2 d} + \frac{3 \arctan\left(\frac{2b\, d^2 x + 2bdc}{2\sqrt{ab}\, d}\right)}{8\sqrt{ab}\, a^2 d} + \frac{2b\, d^2 x + 2}{8 \left(b\, d^2 x^2 + 2bcdx + b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2)^3,x)
```

```
[Out] 1/8*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^2+3/8/a^2/(b*
d^2*x^2+2*b*c*d*x+b*c^2+a)*x+3/8/a^2/d/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)*c+3/8/
a^2/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/(a*b)^(1/2)/d)
```

maxima [B] time = 1.59, size = 184, normalized size = 2.02

$$\frac{3 \, bd^3x^3 + 9 \, bcd^2x^2 + 3 \, bc^3 + \left(9 \, bc^2 + 5 \, a\right)dx + 5 \, ac}{8 \left(a^2b^2d^5x^4 + 4 \, a^2b^2cd^4x^3 + 2 \left(3 \, a^2b^2c^2 + a^3b\right)d^3x^2 + 4 \left(a^2b^2c^3 + a^3bc\right)d^2x + \left(a^2b^2c^4 + 2 \, a^3bc^2 + a^4\right)d\right)} + \frac{3 \arctan}{8 \sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*d^3*x^3 + 9*b*c*d^2*x^2 + 3*b*c^3 + (9*b*c^2 + 5*a)*d*x + 5*a*c)/(
a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + 2*(3*a^2*b^2*c^2 + a^3*b)*d^3*x^2 +
 4*(a^2*b^2*c^3 + a^3*b*c)*d^2*x + (a^2*b^2*c^4 + 2*a^3*b*c^2 + a^4)*d) + 3
/8*arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a^2*d)
```

mupad [B] time = 2.22, size = 181, normalized size = 1.99

$$\frac{\frac{x\left(9\,b\,c^2+5\,a\right)}{8\,a^2}+\frac{3\,b\,c^3+5\,a\,c}{8\,a^2\,d}+\frac{3\,b\,d^2\,x^3}{8\,a^2}+\frac{9\,b\,c\,d\,x^2}{8\,a^2}}{x^2\,\left(6\,b^2\,c^2\,d^2+2\,a\,b\,d^2\right)+x\,\left(4\,d\,b^2\,c^3+4\,a\,d\,b\,c\right)+a^2+b^2\,c^4+b^2\,d^4\,x^4+2\,a\,b\,c^2+4\,b^2\,c\,d^3\,x^3}+\frac{3\,\mathrm{atan}\left(\frac{8\,a^2\,\left(\frac{3\,\ldots}{8\,a\ldots}\right.}{\ldots}\right.}{8\,a^{5/2}\,\sqrt{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2)^3,x)
```

```
[Out] ((x*(5*a + 9*b*c^2))/(8*a^2) + (5*a*c + 3*b*c^3)/(8*a^2*d) + (3*b*d^2*x^3)/
(8*a^2) + (9*b*c*d*x^2)/(8*a^2))/(x^2*(6*b^2*c^2*d^2 + 2*a*b*d^2) + x*(4*b^
2*c^3*d + 4*a*b*c*d) + a^2 + b^2*c^4 + b^2*d^4*x^4 + 2*a*b*c^2 + 4*b^2*c*d^
3*x^3) + (3*atan((8*a^2*((3*b^(1/2)*c)/(8*a^(5/2)) + (3*b^(1/2)*d*x)/(8*a^(
5/2))))/3))/(8*a^(5/2)*b^(1/2)*d)
```

sympy [B] time = 1.25, size = 257, normalized size = 2.82

$$\frac{5ac+3bc^3+9bcd^2x^2+3bd^3x^3+x\left(5ad+9bc^2d\right)}{8a^4d+16a^3bc^2d+8a^2b^2c^4d+32a^2b^2cd^4x^3+8a^2b^2d^5x^4+x^2\left(16a^3bd^3+48a^2b^2c^2d^3\right)+x\left(32a^3bcd^2+32a^2b^2c^{\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2)**3,x)
```

```
[Out] (5*a*c + 3*b*c**3 + 9*b*c*d**2*x**2 + 3*b*d**3*x**3 + x*(5*a*d + 9*b*c**2*d
))/(8*a**4*d + 16*a**3*b*c**2*d + 8*a**2*b**2*c**4*d + 32*a**2*b**2*c*d**4*
x**3 + 8*a**2*b**2*d**5*x**4 + x**2*(16*a**3*b*d**3 + 48*a**2*b**2*c**2*d**
3) + x*(32*a**3*b*c*d**2 + 32*a**2*b**2*c**3*d**2)) + (-3*sqrt(-1/(a**5*b))
*log(x + (-3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16 + 3*sqrt(-1/(a**5*b))*
log(x + (3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16)/d
```

3.88 $\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\,\sqrt{b}\,d}$$

```
[Out] arctan((d*x+c)*b^(1/2)/(-a)^(1/4))/(-a)^(1/4)/d/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+bx^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\,\sqrt{b}\,d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

fricas [A] time = 0.87, size = 279, normalized size = 7.97

$$\left[\frac{\sqrt{\frac{\sqrt{-a}}{ab}}\log\left(\frac{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4-2\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{-a}+2\left(abdx+abc+\left(b^2d^3x^3+3\,b^2cd^2x^2+3\,b^2c^2dx+b^2c^3\right)\sqrt{-a}\right.}{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4+a}\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(sqrt(-a)/(a*b))*log((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^
2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-a
) + 2*(a*b*d*x + a*b*c + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*sqrt(-a))*sqrt(sqrt(-a)/(a*b)) - a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
+ 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + a))/d, sqrt(-sqrt(-a)/(a*b)
)*arctan((b*d*x + b*c)*sqrt(-sqrt(-a)/(a*b)))/d]
```

giac [A] time = 0.45, size = 30, normalized size = 0.86

$$\frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)
```

maple [A] time = 0.01, size = 42, normalized size = 1.20

$$\frac{\arctan\left(\frac{2b\,d^{2}x+2bdc}{2\sqrt{\sqrt{-a}\;b\;d}}\right)}{\sqrt{\sqrt{-a}\;b}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(d*x+c)^2+(-a)^(1/2)),x)
```

```
[Out] 1/d/((-a)^(1/2)*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/((-a)^(1/2)*b)^(1
/2))
```

maxima [B] time = 1.40, size = 66, normalized size = 1.89

$$\frac{\log\left(\frac{bd^{2}x+bcd-\sqrt{-\sqrt{-a}\,b}\,d}{bd^{2}x+bcd+\sqrt{-\sqrt{-a}\,b}\,d}\right)}{2\sqrt{-\sqrt{-a}\,b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d + sqrt(-sq
rt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)
```

mupad [B] time = 0.10, size = 31, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\;c+\sqrt{b}\;d\,x}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,\sqrt{b}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c + d*x)^2 + (-a)^(1/2)),x)
```

```
[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/(-a)^(1/4))/((-a)^(1/4)*b^(1/2)*d)
```

sympy [B] time = 0.22, size = 92, normalized size = 2.63

$$\frac{-\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c-\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c+\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)
```

```
[Out] (-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2 +
 sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2)/d
```

3.89 $\int \frac{1}{1+(c+dx)^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}(c+dx)}{d}$$

```
[Out] arctan(d*x+c)/d
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {247, 203}

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[c + d*x]/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+(c+dx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{\tan^{-1}(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-1),x]
[Out] ArcTan[c + d*x]/d
```

fricas [A] time = 0.85, size = 10, normalized size = 1.00

$$\frac{\arctan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="fricas")
[Out] arctan(d*x + c)/d
```

giac [A] time = 0.32, size = 10, normalized size = 1.00

$$\frac{\arctan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="giac")
[Out] arctan(d*x + c)/d
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2),x)
[Out] arctan(d*x+c)/d
```

maxima [A] time = 1.57, size = 18, normalized size = 1.80

$$\frac{\arctan\left(\frac{d^{2}x+cd}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="maxima")
[Out] arctan((d^2*x + c*d)/d)/d
```

mupad [B] time = 0.04, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1),x)
```

```
[Out] atan(c + d*x)/d
```

sympy [C] time = 0.17, size = 24, normalized size = 2.40

$$\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{2}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2),x)
```

```
[Out] (-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d
```

3.90 $\int \frac{1}{\left(1+(c+dx)^2\right)^2} dx$

Optimal. Leaf size=37

$$\frac{c + dx}{2d\left((c + dx)^2 + 1\right)} + \frac{\tan^{-1}(c + dx)}{2d}$$

```
[Out] 1/2*(d*x+c)/d/(1+(d*x+c)^2)+1/2*arctan(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 203}

$$\frac{c + dx}{2d\left((c + dx)^2 + 1\right)} + \frac{\tan^{-1}(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{2d\left(1+(c+dx)^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{2d}$$

$$= \frac{c+dx}{2d\left(1+(c+dx)^2\right)} + \frac{\tan^{-1}(c+dx)}{2d}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.84

$$\frac{\frac{c+dx}{(c+dx)^2+1} + \tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] ((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)
```

fricas [A] time = 0.81, size = 55, normalized size = 1.49

$$\frac{dx + \left(d^2x^2 + 2\,cdx + c^2 + 1\right)\arctan\left(dx + c\right) + c}{2\left(d^3x^2 + 2\,cd^2x + \left(c^2 + 1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c) + c)/(d^3*x^2 + 2*
c*d^2*x + (c^2 + 1)*d)
```

giac [A] time = 0.38, size = 41, normalized size = 1.11

$$\frac{\arctan\left(dx + c\right)}{2\,d} + \frac{dx + c}{2\left(d^2x^2 + 2\,cdx + c^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(d*x + c)/d + 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)*d)
```

maple [A] time = 0.01, size = 59, normalized size = 1.59

$$\frac{\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{2d}+\frac{2d^2x+2cd}{4\left(d^2x^2+2cdx+c^2+1\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^2,x)
```

```
[Out] 1/4*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)+1/2/d*arctan(1/2*(2*d^2*x+2
*c*d)/d)
```

maxima [A] time = 1.32, size = 51, normalized size = 1.38

$$\frac{dx+c}{2\left(d^3x^2+2\,cd^2x+\left(c^2+1\right)d\right)}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + 1/2*arctan((d^2*x + c*d
)/d)/d
```

mupad [B] time = 2.07, size = 42, normalized size = 1.14

$$\frac{\frac{x}{2}+\frac{c}{2\,d}}{c^2+2\,c\,d\,x+d^2\,x^2+1}+\frac{\mathrm{atan}\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1)^2,x)
```

```
[Out] (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x + 1) + atan(c + d*x)/(2*d)
```

sympy [C] time = 0.44, size = 56, normalized size = 1.51

$$\frac{c+dx}{2c^2d+4cd^2x+2d^3x^2+2d}+\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{4}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*log(x + (c - I)
/d)/4 + I*log(x + (c + I)/d)/4)/d
```

3.91 $\int \frac{1}{\left(1+(c+dx)^2\right)^3} dx$

Optimal. Leaf size=60

$$\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}+\frac{3\tan^{-1}(c+dx)}{8d}$$

```
[Out] 1/4*(d*x+c)/d/(1+(d*x+c)^2)^2+3/8*(d*x+c)/d/(1+(d*x+c)^2)+3/8*arctan(d*x+c)
/d
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 203}

$$\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}+\frac{3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c + d*x)^2))
 + (3*ArcTan[c + d*x])/(8*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx, x, c+dx\right)}{4d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{8d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)} + \frac{3\tan^{-1}(c+dx)}{8d}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.87

$$\frac{\frac{3(c+dx)}{(c+dx)^2+1} + \frac{2(c+dx)}{\left((c+dx)^2+1\right)^2} + 3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] ((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)^2) + 3*Ar
cTan[c + d*x])/(8*d)
```

fricas [B] time = 0.84, size = 153, normalized size = 2.55

$$\frac{3\,d^3x^3 + 9\,cd^2x^2 + 3\,c^3 + \left(9\,c^2+5\right)dx + 3\left(d^4x^4 + 4\,cd^3x^3 + 2\left(3\,c^2+1\right)d^2x^2 + c^4 + 4\left(c^3+c\right)dx + 2\,c^2+1\right)\text{arct}}{8\left(d^5x^4 + 4\,cd^4x^3 + 2\left(3\,c^2+1\right)d^3x^2 + 4\left(c^3+c\right)d^2x + \left(c^4+2\,c^2+1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x^4 + 4*c*d
^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x + 2*c^2 + 1)*arctan(
d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c
)*d^2*x + (c^4 + 2*c^2 + 1)*d)
```

giac [A] time = 0.30, size = 73, normalized size = 1.22

$$\frac{3\arctan(dx+c)}{8d}+\frac{3d^3x^3+9cd^2x^2+9c^2dx+3c^3+5dx+5c}{8(d^2x^2+2cdx+c^2+1)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 +
5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)
```

maple [A] time = 0.01, size = 94, normalized size = 1.57

$$\frac{3\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{8d}+\frac{2d^2x+2cd}{8(d^2x^2+2cdx+c^2+1)^2d^2}+\frac{\frac{3}{8}d^2x+\frac{3}{8}cd}{(d^2x^2+2cdx+c^2+1)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^3,x)
```

```
[Out] 1/8*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)^2+3/16*(2*d^2*x+2*c*d)/(d^2
*x^2+2*c*d*x+c^2+1)/d^2+3/8/d*arctan(1/2*(2*d^2*x+2*c*d)/d)
```

maxima [B] time = 1.51, size = 115, normalized size = 1.92

$$\frac{3d^3x^3+9cd^2x^2+3c^3+(9c^2+5)dx+5c}{8(d^5x^4+4cd^4x^3+2(3c^2+1)d^3x^2+4(c^3+c)d^2x+(c^4+2c^2+1)d)}+\frac{3\arctan\left(\frac{d^2x+cd}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 5*c)/(d^5*x^4 + 4*
c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d
) + 3/8*arctan((d^2*x + c*d)/d)/d
```

mupad [B] time = 0.12, size = 111, normalized size = 1.85

$$\frac{3\,\mathrm{atan}(c+d\,x)}{8\,d}+\frac{x\,\left(\frac{9\,c^2}{8}+\frac{5}{8}\right)+\frac{3\,c^3+5\,c}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{x^2\,\left(6\,c^2\,d^2+2\,d^2\right)+2\,c^2+c^4+x\,\left(4\,d\,c^3+4\,d\,c\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1)^3,x)
```

```
[Out] (3*atan(c + d*x))/(8*d) + (x*((9*c^2)/8 + 5/8) + (5*c + 3*c^3)/(8*d) + (3*d
^2*x^3)/8 + (9*c*d*x^2)/8)/(x^2*(2*d^2 + 6*c^2*d^2) + 2*c^2 + c^4 + x*(4*c*
d + 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

sympy [C] time = 0.93, size = 146, normalized size = 2.43

$$\frac{3c^3 + 9cd^2x^2 + 5c + 3d^3x^3 + x\left(9c^2d + 5d\right)}{8c^4d + 16c^2d + 32cd^4x^3 + 8d^5x^4 + 8d + x^2\left(48c^2d^3 + 16d^3\right) + x\left(32c^3d^2 + 32cd^2\right)} + \frac{-\frac{3i\log\left(x + \frac{3c-3i}{3d}\right)}{16} + \frac{3i\log\left(x + \frac{3c+3}{3d}\right.}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2)**3,x)
```

```
[Out] (3*c**3 + 9*c*d**2*x**2 + 5*c + 3*d**3*x**3 + x*(9*c**2*d + 5*d))/(8*c**4*d
 + 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 + 16
*d**3) + x*(32*c**3*d**2 + 32*c*d**2)) + (-3*I*log(x + (3*c - 3*I)/(3*d))/1
6 + 3*I*log(x + (3*c + 3*I)/(3*d))/16)/d
```

3.92 $\int \frac{1}{1-(c+dx)^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}(c+dx)}{d}$$

[Out] arctanh(d*x+c)/d

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {247, 206}

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(1 - (c + d*x)^2)^(-1),x]

[Out] ArcTanh[c + d*x]/d

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-(c+dx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{\tanh^{-1}(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.01, size = 32, normalized size = 3.20

$$\frac{\log(c+dx+1)}{2d}-\frac{\log(-c-dx+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-1),x]
[Out] -1/2*Log[1 - c - d*x]/d + Log[1 + c + d*x]/(2*d)
```

fricas [B] time = 0.79, size = 22, normalized size = 2.20

$$\frac{\log\left(dx+c+1\right)-\log\left(dx+c-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d
```

giac [B] time = 0.38, size = 27, normalized size = 2.70

$$\frac{\log\left(|dx+c+1|\right)}{2\,d}-\frac{\log\left(|dx+c-1|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*log(abs(d*x + c + 1))/d - 1/2*log(abs(d*x + c - 1))/d
```

maple [B] time = 0.01, size = 26, normalized size = 2.60

$$-\frac{\ln\left(dx+c-1\right)}{2d}+\frac{\ln\left(dx+c+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2),x)
[Out] -1/2/d*ln(d*x+c-1)+1/2/d*ln(d*x+c+1)
```

maxima [B] time = 0.49, size = 25, normalized size = 2.50

$$\frac{\log\left(dx+c+1\right)}{2\,d}-\frac{\log\left(dx+c-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d
```

mupad [B] time = 2.05, size = 10, normalized size = 1.00

$$\frac{\operatorname{atanh}(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((c + d*x)^2 - 1),x)
```

```
[Out] atanh(c + d*x)/d
```

sympy [B] time = 0.18, size = 22, normalized size = 2.20

$$-\frac{\frac{\log\left(x+\frac{c-1}{d}\right)}{2}-\frac{\log\left(x+\frac{c+1}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2),x)
```

```
[Out] -(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d
```

3.93 $\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx$

Optimal. Leaf size=39

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

```
[Out] 1/2*(d*x+c)/d/(1-(d*x+c)^2)+1/2*arctanh(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 206}

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1-(c+dx)^2\right)^2} dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2} dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{2d\left(1-(c+dx)^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c+dx\right)}{2d}$$

$$= \frac{c+dx}{2d\left(1-(c+dx)^2\right)} + \frac{\tanh^{-1}(c+dx)}{2d}$$

Mathematica [**A**] time = 0.03, size = 45, normalized size = 1.15

$$\frac{-\frac{2(c+dx)}{(c+dx)^2-1} - \log(-c-dx+1) + \log(c+dx+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] ((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c + d*x])/(
4*d)
```

fricas [**B**] time = 0.83, size = 85, normalized size = 2.18

$$-\frac{2\,dx - \left(d^2x^2 + 2\,cdx + c^2 - 1\right)\log\left(dx + c + 1\right) + \left(d^2x^2 + 2\,cdx + c^2 - 1\right)\log\left(dx + c - 1\right) + 2\,c}{4\left(d^3x^2 + 2\,cd^2x + \left(c^2 - 1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + (d^2*x^2 + 2
*c*d*x + c^2 - 1)*log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*
d)
```

giac [**A**] time = 0.35, size = 56, normalized size = 1.44

$$\frac{\log\left(|dx + c + 1|\right)}{4\,d} - \frac{\log\left(|dx + c - 1|\right)}{4\,d} - \frac{dx + c}{2\left(d^2x^2 + 2\,cdx + c^2 - 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(d*x + c + 1))/d - 1/4*log(abs(d*x + c - 1))/d - 1/2*(d*x + c)/(
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
```

maple [A] time = 0.01, size = 52, normalized size = 1.33

$$-\frac{\ln\left(dx+c-1\right)}{4d}+\frac{\ln\left(dx+c+1\right)}{4d}-\frac{1}{4\left(dx+c-1\right)d}-\frac{1}{4\left(dx+c+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2)^2,x)
```

```
[Out] -1/4/d/(d*x+c-1)-1/4/d*ln(d*x+c-1)-1/4/d/(d*x+c+1)+1/4/d*ln(d*x+c+1)
```

maxima [A] time = 0.62, size = 56, normalized size = 1.44

$$-\frac{dx+c}{2\left(d^{3}x^{2}+2\,cd^{2}x+\left(c^{2}-1\right)d\right)}+\frac{\log\left(dx+c+1\right)}{4\,d}-\frac{\log\left(dx+c-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*log(d*x + c + 1)/d
 - 1/4*log(d*x + c - 1)/d
```

mupad [B] time = 2.06, size = 43, normalized size = 1.10

$$\frac{\mathrm{atanh}\left(c+d\,x\right)}{2\,d}-\frac{\frac{x}{2}+\frac{c}{2\,d}}{c^2+2\,c\,d\,x+d^2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 - 1)^2,x)
```

```
[Out] atanh(c + d*x)/(2*d) - (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x - 1)
```

sympy [A] time = 0.47, size = 54, normalized size = 1.38

$$\frac{-c-dx}{2c^{2}d+4cd^{2}x+2d^{3}x^{2}-2d}+\frac{-\frac{\log\left(x+\frac{c-1}{d}\right)}{4}+\frac{\log\left(x+\frac{c+1}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2)**2,x)
```

```
[Out] (-c - d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-log(x + (c - 1)/
d)/4 + log(x + (c + 1)/d)/4)/d
```

3.94 $\int \frac{1}{\left(1-(c+dx)^2\right)^3} dx$

Optimal. Leaf size=64

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

```
[Out] 1/4*(d*x+c)/d/(1-(d*x+c)^2)^2+3/8*(d*x+c)/d/(1-(d*x+c)^2)+3/8*arctanh(d*x+c
)/d
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 206}

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c + d*x)^2))
 + (3*ArcTanh[c + d*x])/(8*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, c+dx\right)}{4d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{8d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)} + \frac{3\tanh^{-1}(c+dx)}{8d}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.02

$$\frac{-\frac{6(c+dx)}{(c+dx)^2-1} + \frac{4(c+dx)}{\left((c+dx)^2-1\right)^2} - 3\log(-c-dx+1) + 3\log(c+dx+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] ((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*x)^2) - 3*
Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)
```

fricas [B] time = 0.84, size = 220, normalized size = 3.44

$$-\frac{6\,d^3x^3 + 18\,cd^2x^2 + 6\,c^3 + 2\left(9\,c^2-5\right)dx - 3\left(d^4x^4 + 4\,cd^3x^3 + 2\left(3\,c^2-1\right)d^2x^2 + c^4 + 4\left(c^3-c\right)dx - 2\,c^2+1\right)\cdots}{16\left(d^5x^4 + 4\,cd^4x^3 + 2\left(3\,c^2-1\right)d^3x^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(d^4*x^4 +
4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*lo
g(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4
```

```
*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4*x^3
 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)
```

giac [A] time = 0.36, size = 88, normalized size = 1.38

$$\frac{3\log\left(|dx+c+1|\right)}{16\,d}-\frac{3\log\left(|dx+c-1|\right)}{16\,d}-\frac{3\,d^3x^3+9\,cd^2x^2+9\,c^2dx+3\,c^3-5\,dx-5\,c}{8\left(d^2x^2+2\,cdx+c^2-1\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(d*x + c + 1))/d - 3/16*log(abs(d*x + c - 1))/d - 1/8*(3*d^3*x^
3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*x^2 + 2*c*d*x + c^
2 - 1)^2*d)
```

maple [A] time = 0.01, size = 78, normalized size = 1.22

$$-\frac{3\ln\left(dx+c-1\right)}{16d}+\frac{3\ln\left(dx+c+1\right)}{16d}+\frac{1}{16\left(dx+c-1\right)^2d}-\frac{3}{16\left(dx+c-1\right)d}-\frac{1}{16\left(dx+c+1\right)^2d}-\frac{3}{16\left(dx+c+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2)^3,x)
```

```
[Out] 1/16/d/(d*x+c-1)^2-3/16/(d*x+c-1)/d-3/16/d*ln(d*x+c-1)-1/16/d/(d*x+c+1)^2-3
/16/(d*x+c+1)/d+3/16/d*ln(d*x+c+1)
```

maxima [B] time = 0.64, size = 122, normalized size = 1.91

$$-\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+\left(9\,c^2-5\right)dx-5\,c}{8\left(d^5x^4+4\,cd^4x^3+2\left(3\,c^2-1\right)d^3x^2+4\left(c^3-c\right)d^2x+\left(c^4-2\,c^2+1\right)d\right)}+\frac{3\log\left(dx+c+1\right)}{16\,d}-\frac{3\log\left(dx+c-1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d^5*x^4 + 4
*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*
d) + 3/16*log(d*x + c + 1)/d - 3/16*log(d*x + c - 1)/d
```

mupad [B] time = 2.12, size = 114, normalized size = 1.78

$$\frac{3\,\mathrm{atanh}\left(c+d\,x\right)}{8\,d}-\frac{x\,\left(\frac{9\,c^2}{8}-\frac{5}{8}\right)-\frac{5\,c-3\,c^3}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{c^4-2\,c^2-x^2\,\left(2\,d^2-6\,c^2\,d^2\right)-x\,\left(4\,c\,d-4\,c^3\,d\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((c + d*x)^2 - 1)^3,x)
```

```
[Out] (3*atanh(c + d*x))/(8*d) - (x*((9*c^2)/8 - 5/8) - (5*c - 3*c^3)/(8*d) + (3*
d^2*x^3)/8 + (9*c*d*x^2)/8)/(c^4 - 2*c^2 - x^2*(2*d^2 - 6*c^2*d^2) - x*(4*c
*d - 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

sympy [B] time = 1.03, size = 141, normalized size = 2.20

$$-\frac{3c^3+9cd^2x^2-5c+3d^3x^3+x\left(9c^2d-5d\right)}{8c^4d-16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\left(48c^2d^3-16d^3\right)+x\left(32c^3d^2-32cd^2\right)}-\frac{\frac{3\log\left(x+\frac{3c-3}{3d}\right)}{16}-\frac{3\log\left(x+\frac{3c+3}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2)**3,x)
```

```
[Out] -(3*c**3 + 9*c*d**2*x**2 - 5*c + 3*d**3*x**3 + x*(9*c**2*d - 5*d))/(8*c**4*
d - 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 - 1
6*d**3) + x*(32*c**3*d**2 - 32*c*d**2)) - (3*log(x + (3*c - 3)/(3*d))/16 -
3*log(x + (3*c + 3)/(3*d))/16)/d
```

3.95 $\int \frac{1}{1-(1+x)^2} dx$

Optimal. Leaf size=4

$$\tanh^{-1}(x+1)$$

```
[Out] arctanh(1+x)
```

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {247, 206}

$$\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] ArcTanh[1 + x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1-(1+x)^2} dx &= \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, 1+x\right) \\ &= \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 15, normalized size = 3.75

$$\frac{1}{2}\log(x+2)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-1),x]
[Out] -1/2*Log[x] + Log[2 + x]/2
```

fricas [B] time = 0.89, size = 11, normalized size = 2.75

$$\frac{1}{2}\,\log\left(x+2\right)-\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="fricas")
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

giac [B] time = 0.35, size = 13, normalized size = 3.25

$$\frac{1}{2}\,\log\left(|x+2|\right)-\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="giac")
[Out] 1/2*log(abs(x + 2)) - 1/2*log(abs(x))
```

maple [B] time = 0.00, size = 12, normalized size = 3.00

$$-\frac{\ln(x)}{2}+\frac{\ln\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(x+1)^2),x)
[Out] 1/2*ln(2+x)-1/2*ln(x)
```

maxima [B] time = 0.57, size = 11, normalized size = 2.75

$$\frac{1}{2}\,\log\left(x+2\right)-\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="maxima")
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

mupad [B] time = 0.15, size = 4, normalized size = 1.00

$$\mathrm{atanh}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^2 - 1),x)
```

```
[Out] atanh(x + 1)
```

sympy [B] time = 0.10, size = 10, normalized size = 2.50

$$-\frac{\log(x)}{2}+\frac{\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2),x)
```

```
[Out] -log(x)/2 + log(x + 2)/2
```

3.96 $\int \frac{1}{\left(1-(1+x)^2\right)^2} \, dx$

Optimal. Leaf size=27

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

```
[Out] 1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 206}

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1-(1+x)^2\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{2\left(1-(1+x)^2\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{2\left(1-(1+x)^2\right)} + \frac{1}{2}\tanh^{-1}(1+x)$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(x+1)}{x(x+2)} - \log(x) + \log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-2),x]
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

fricas [A] time = 0.79, size = 39, normalized size = 1.44

$$\frac{\left(x^2+2\,x\right)\log\left(x+2\right) - \left(x^2+2\,x\right)\log(x) - 2\,x - 2}{4\left(x^2+2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

giac [A] time = 0.33, size = 27, normalized size = 1.00

$$-\frac{x+1}{2\left(x^2+2\,x\right)} + \frac{1}{4}\,\log\left(|x+2|\right) - \frac{1}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="giac")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}-\frac{1}{4x}-\frac{1}{4(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(x+1)^2)^2,x)
```

```
[Out] -1/4/(x+2)+1/4*ln(x+2)-1/4/x-1/4*ln(x)
```

maxima [A] time = 0.65, size = 25, normalized size = 0.93

$$-\frac{x+1}{2\left(x^2+2x\right)}+\frac{1}{4}\log(x+2)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

mupad [B] time = 0.07, size = 23, normalized size = 0.85

$$\frac{\operatorname{atanh}(x+1)}{2}-\frac{x+1}{2\left((x+1)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^2 - 1)^2,x)
```

```
[Out] atanh(x + 1)/2 - (x + 1)/(2*((x + 1)^2 - 1))
```

sympy [A] time = 0.12, size = 24, normalized size = 0.89

$$\frac{-x-1}{2x^2+4x}-\frac{\log(x)}{4}+\frac{\log(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**2,x)
```

```
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

3.97 $\int \frac{1}{\left(1-(1+x)^2\right)^3} \, dx$

Optimal. Leaf size=45

$$\frac{3(x+1)}{8\left(1-(x+1)^2\right)}+\frac{x+1}{4\left(1-(x+1)^2\right)^2}+\frac{3}{8}\tanh^{-1}(x+1)$$

```
[Out] 1/4*(1+x)/(1-(1+x)^2)^2+3/8*(1+x)/(1-(1+x)^2)+3/8*arctanh(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 206}

$$\frac{3(x+1)}{8\left(1-(x+1)^2\right)}+\frac{x+1}{4\left(1-(x+1)^2\right)^2}+\frac{3}{8}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-3),x]
```

```
[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) + (3*ArcTan
h[1 + x])/8
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{4\left(1-(1+x)^2\right)^2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{4\left(1-(1+x)^2\right)^2} + \frac{3(1+x)}{8\left(1-(1+x)^2\right)} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{4\left(1-(1+x)^2\right)^2} + \frac{3(1+x)}{8\left(1-(1+x)^2\right)} + \frac{3}{8}\tanh^{-1}(1+x)$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.82

$$\frac{1}{16}\left(\frac{1}{x^2} - \frac{3}{x} - \frac{3}{x+2} - \frac{1}{(x+2)^2} - 3\log(x) + 3\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-3),x]
[Out] (x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x])/16
```

fricas [B] time = 0.83, size = 71, normalized size = 1.58

$$-\frac{6\,x^3 + 18\,x^2 - 3\left(x^4 + 4\,x^3 + 4\,x^2\right)\log\left(x+2\right) + 3\left(x^4 + 4\,x^3 + 4\,x^2\right)\log(x) + 8\,x - 4}{16\left(x^4 + 4\,x^3 + 4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="fricas")
[Out] -1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*log(x + 2) + 3*(x^4 + 4*x^3
 + 4*x^2)*log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)
```

giac [A] time = 0.34, size = 39, normalized size = 0.87

$$-\frac{3\,x^3 + 9\,x^2 + 4\,x - 2}{8\left(x^2 + 2\,x\right)^2} + \frac{3}{16}\log\left(|x+2|\right) - \frac{3}{16}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*log(abs(x + 2)) - 3/16*
log(abs(x))
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{3\ln(x)}{16}+\frac{3\ln(x+2)}{16}-\frac{3}{16x}+\frac{1}{16x^2}-\frac{1}{16(x+2)^2}-\frac{3}{16(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(x+1)^2)^3,x)
```

```
[Out] -1/16/(x+2)^2-3/16/(x+2)+3/16*ln(x+2)+1/16/x^2-3/16/x-3/16*ln(x)
```

maxima [A] time = 0.64, size = 44, normalized size = 0.98

$$-\frac{3x^3+9x^2+4x-2}{8\left(x^4+4x^3+4x^2\right)}+\frac{3}{16}\log(x+2)-\frac{3}{16}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*log(x + 2) - 3/
16*log(x)
```

mupad [B] time = 2.09, size = 36, normalized size = 0.80

$$\frac{3\,\mathrm{atanh}(x+1)}{8}+\frac{\frac{5x}{8}-\frac{3(x+1)^3}{8}+\frac{5}{8}}{(x+1)^4-2(x+1)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^2 - 1)^3,x)
```

```
[Out] (3*atanh(x + 1))/8 + ((5*x)/8 - (3*(x + 1)^3)/8 + 5/8)/((x + 1)^4 - 2*(x +
1)^2 + 1)
```

sympy [A] time = 0.14, size = 44, normalized size = 0.98

$$-\frac{3\log(x)}{16}+\frac{3\log(x+2)}{16}-\frac{3x^3+9x^2+4x-2}{8x^4+32x^3+32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**3,x)
```

```
[Out] -3*log(x)/16 + 3*log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x**4 + 32*x
**3 + 32*x**2)
```

3.98 $\int \frac{\left(1+(a+bx)^2\right)^2}{x} \, dx$

Optimal. Leaf size=59

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

```
[Out] a*(a^2+2)*b*x+1/2*(a^2+2)*(b*x+a)^2+1/3*a*(b*x+a)^3+1/4*(b*x+a)^4+(a^2+1)^2
*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {371, 697}

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)
^4/4 + (1 + a^2)^2*Log[x]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{-a+x}\,dx, x, a+bx\right)$$
$$= \text{Subst}\left(\int \left(a\left(2+a^2\right) - \frac{\left(1+a^2\right)^2}{a-x} + \left(2+a^2\right)x + ax^2 + x^3\right)dx, x, a+bx\right)$$
$$= a\left(2+a^2\right)bx + \frac{1}{2}\left(2+a^2\right)(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + \left(1+a^2\right)^2\log(x)$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.08

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2 + a\left(a^2+2\right)(a+bx) + \left(a^2+1\right)^2\log(bx) + \frac{1}{4}(a+bx)^4 + \frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
+ b*x)^4/4 + (1 + a^2)^2*Log[b*x]
```

fricas [A] time = 0.84, size = 54, normalized size = 0.92

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + \left(3a^2+1\right)b^2x^2 + 4\left(a^3+a\right)bx + \left(a^4+2a^2+1\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

giac [A] time = 0.31, size = 62, normalized size = 1.05

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + 3a^2b^2x^2 + 4a^3bx + b^2x^2 + 4abx + \left(a^4+2a^2+1\right)\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*x
 + (a^4 + 2*a^2 + 1)*log(abs(x))
```

maple [A] time = 0.00, size = 64, normalized size = 1.08

$$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+a^4\ln(x)+4a^3bx+b^2x^2+2a^2\ln(x)+4abx+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b*x+a)^2)^2/x,x)
```

```
[Out] 1/4*b^4*x^4+4/3*a*b^3*x^3+3*x^2*a^2*b^2+4*a^3*b*x+b^2*x^2+4*a*b*x+ln(x)*a^4
+2*ln(x)*a^2+ln(x)
```

maxima [A] time = 0.54, size = 54, normalized size = 0.92

$$\frac{1}{4}b^4x^4+\frac{4}{3}ab^3x^3+\left(3a^2+1\right)b^2x^2+4\left(a^3+a\right)bx+\left(a^4+2a^2+1\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

mupad [B] time = 0.05, size = 55, normalized size = 0.93

$$\ln(x)\left(a^4+2a^2+1\right)+\frac{b^4x^4}{4}+\frac{4ab^3x^3}{3}+b^2x^2\left(3a^2+1\right)+4abx\left(a^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^2/x,x)
```

```
[Out] log(x)*(2*a^2 + a^4 + 1) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + b^2*x^2*(3*a^2 +
 1) + 4*a*b*x*(a^2 + 1)
```

sympy [A] time = 0.17, size = 58, normalized size = 0.98

$$\frac{4ab^3x^3}{3}+\frac{b^4x^4}{4}+x^2\left(3a^2b^2+b^2\right)+x\left(4a^3b+4ab\right)+\left(a^2+1\right)^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)**2)**2/x,x)
```

```
[Out] 4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b + 4
*a*b) + (a**2 + 1)**2*log(x)
```

3.99 $\int \frac{x^2}{1+(-1+x)^2} \, dx$

Optimal. Leaf size=10

$$x + \log\left((x-1)^2 + 1\right)$$

```
[Out] x+ln(1+(-1+x)^2)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {371, 702, 260}

$$x + \log\left((x-1)^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + (-1 + x)^2),x]
```

```
[Out] x + Log[1 + (-1 + x)^2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{1+(-1+x)^2}\,dx &= \text{Subst}\left(\int \frac{(1+x)^2}{1+x^2}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{2x}{1+x^2}\right)dx, x, -1+x\right)\\
&= x + 2\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, -1+x\right)\\
&= x + \log\left(1+(-1+x)^2\right)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 11, normalized size = 1.10

$$\log\left(x^2-2x+2\right)+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x)^2),x]
[Out] x + Log[2 - 2*x + x^2]
```

fricas [**A**] time = 0.68, size = 11, normalized size = 1.10

$$x+\log\left(x^2-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="fricas")
[Out] x + log(x^2 - 2*x + 2)
```

giac [**A**] time = 0.35, size = 11, normalized size = 1.10

$$x+\log\left(x^2-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="giac")
[Out] x + log(x^2 - 2*x + 2)
```

maple [**A**] time = 0.00, size = 12, normalized size = 1.20

$$x+\ln\left(x^2-2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(x-1)^2),x)
```

```
[Out] x+ln(x^2-2*x+2)
```

maxima [A] time = 0.68, size = 11, normalized size = 1.10

$$x + \log\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="maxima")
```

```
[Out] x + log(x^2 - 2*x + 2)
```

mupad [B] time = 0.03, size = 11, normalized size = 1.10

$$x + \ln\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)^2 + 1),x)
```

```
[Out] x + log(x^2 - 2*x + 2)
```

sympy [A] time = 0.09, size = 10, normalized size = 1.00

$$x + \log\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(-1+x)**2),x)
```

```
[Out] x + log(x**2 - 2*x + 2)
```

3.100 $\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{1-(x+1)^2}\,x+\frac{3}{2}\sqrt{1-(x+1)^2}\,+\frac{3}{2}\sin^{-1}(x+1)$$

```
[Out] 3/2*arcsin(1+x)+3/2*(1-(1+x)^2)^(1/2)-1/2*x*(1-(1+x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {371, 671, 641, 216}

$$-\frac{1}{2}\sqrt{1-(x+1)^2}\,x+\frac{3}{2}\sqrt{1-(x+1)^2}\,+\frac{3}{2}\sin^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - (1 + x)^2],x]
```

```
[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin[1 + x])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
```

```
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx &= \text{Subst}\left(\int \frac{(-1+x)^2}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= -\frac{1}{2}x\sqrt{1-(1+x)^2} - \frac{3}{2}\text{Subst}\left(\int \frac{-1+x}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2}\sin^{-1}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 1.16

$$\frac{x\left(x^2-x-6\right)+6\sqrt{x}\sqrt{x+2}\,\sinh^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}}\right)}{2\sqrt{-x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - (1 + x)^2],x]
```

```
[Out] (x*(-6 - x + x^2) + 6*Sqrt[x]*Sqrt[2 + x]*ArcSinh[Sqrt[x]/Sqrt[2]])/(2*Sqrt
[-(x*(2 + x))])
```

fricas [A] time = 0.87, size = 35, normalized size = 0.80

$$-\frac{1}{2}\sqrt{-x^2-2\,x}\,(x-3)-3\,\arctan\left(\frac{\sqrt{-x^2-2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)
```

giac [A] time = 0.39, size = 23, normalized size = 0.52

$$-\frac{1}{2}\sqrt{-(x+1)^2+1}\,(x-3)+\frac{3}{2}\,\arcsin\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-(x + 1)^2 + 1)*(x - 3) + 3/2*arcsin(x + 1)
```

maple [A] time = 0.01, size = 35, normalized size = 0.80

$$-\frac{\sqrt{-x^2-2x}\,x}{2}+\frac{3\arcsin\left(x+1\right)}{2}+\frac{3\sqrt{-x^2-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(x+1)^2)^(1/2),x)
[Out] -1/2*x*(-x^2-2*x)^(1/2)+3/2*(-x^2-2*x)^(1/2)+3/2*arcsin(x+1)
```

maxima [A] time = 1.29, size = 36, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2-2\,x}\,x+\frac{3}{2}\sqrt{-x^2-2\,x}-\frac{3}{2}\arcsin\left(-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^2}{\sqrt{1-{\left(x+1\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - (x + 1)^2)^(1/2),x)
[Out] int(x^2/(1 - (x + 1)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{-x\left(x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(1+x)**2)**(1/2),x)
[Out] Integral(x**2/sqrt(-x*(x + 2)), x)
```

3.101 $\int \frac{x^2}{\sqrt{1-(a+bx)^2}} dx$

Optimal. Leaf size=67

$$\frac{(2a^2+1)\sin^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

```
[Out] 1/2*(2*a^2+1)*arcsin(b*x+a)/b^3+3/2*a*(1-(b*x+a)^2)^(1/2)/b^3-1/2*x*(1-(b*x
+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {371, 743, 641, 216}

$$\frac{(2a^2+1)\sin^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(2*b^2) + (
(1 + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b^3}\\
&= -\frac{x\sqrt{1-(a+bx)^2}}{2b^2} - \frac{\text{Subst}\left(\int \frac{-1-2a^2+3ax}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3}\\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{\left(1+2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3}\\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{\left(1+2a^2\right)\sin^{-1}(a+bx)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 55, normalized size = 0.82

$$\frac{\sqrt{-a^2-2abx-b^2x^2+1}\,(3a-bx)+\left(2a^2+1\right)\sin^{-1}(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - (a + b*x)^2],x]
[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + (1 + 2*a^2)*ArcSin[a + b*x
])/(2*b^3)
```

fricas [A] time = 0.93, size = 92, normalized size = 1.37

$$-\frac{\left(2\,a^2+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx-3\,a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*
x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a))
/b^3
```

giac [A] time = 0.49, size = 55, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{\left(2\,a^2+1\right)\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1)*arcsin(-b*x
 - a)*sgn(b)/(b^2*abs(b))
```

maple [B] time = 0.02, size = 152, normalized size = 2.27

$$\frac{a^2\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{\sqrt{b^2}\;b^2}-\frac{\sqrt{-b^2x^2-2abx-a^2+1}\;x}{2b^2}+\frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2\sqrt{b^2}\;b^2}+\frac{3\sqrt{-b^2x^2-2abx-a^2+}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)+a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))+1/2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))
```

maxima [B] time = 1.46, size = 139, normalized size = 2.07

$$-\frac{3\,a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-\left(a^2-1\right)b^2}}\right)}{2\,b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,x}{2\,b^2}+\frac{\left(a^2-1\right)\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-\left(a^2-1\right)b^2}}\right)}{2\,b^3}+\frac{3\,\sqrt{-b^2x^2-2\,ab}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 1/2*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 + 1/2*(a^2 - 1)*arcsin(-(b^2*x + a*b)
/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{1-(a+b\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - (a + b*x)^2)^(1/2),x)
[Out] int(x^2/(1 - (a + b*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
[Out] Integral(x**2/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

3.102 $\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

```
[Out] -1/2*(-2*a^2+1)*arcsinh(b*x+a)/b^3-3/2*a*(1+(b*x+a)^2)^(1/2)/b^3+1/2*x*(1+(
b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {371, 743, 641, 215}

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/(2*b^2) -
((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x\sqrt{1+(a+bx)^2}}{2b^2} + \frac{\text{Subst}\left(\int \frac{-1+2a^2-3ax}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^3}\\
&= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3} + \frac{x\sqrt{1+(a+bx)^2}}{2b^2} - \frac{\left(1-2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^3}\\
&= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3} + \frac{x\sqrt{1+(a+bx)^2}}{2b^2} - \frac{\left(1-2a^2\right)\sinh^{-1}(a+bx)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.81

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\,(bx-3a)+\left(2a^2-1\right)\sinh^{-1}(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 + 2*a^2)*ArcSinh[a +
b*x])/(2*b^3)
```

fricas [A] time = 0.85, size = 70, normalized size = 1.11

$$-\frac{\left(2\,a^2-1\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx-3\,a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3
```

giac [A] time = 0.42, size = 70, normalized size = 1.11

$$\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{\left(2\,a^2-1\right)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt((b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*log(-a*b - (x
*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

maple [B] time = 0.01, size = 146, normalized size = 2.32

$$\frac{a^2\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}\;b^2}+\frac{\sqrt{b^2x^2+2abx+a^2+1}\;x}{2b^2}-\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2\sqrt{b^2}\;b^2}-\frac{3\sqrt{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)+a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2
)^(1/2)-1/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(
b^2)^(1/2)
```

maxima [B] time = 0.59, size = 135, normalized size = 2.14

$$\frac{3\,a^2\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^3}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,x}{2\,b^2}-\frac{\left(a^2+1\right)\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^3}-\frac{3\,\sqrt{b^2x^2+2\,a}}{2\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 1
/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 1/2*(a^2 + 1)*arcsinh(2*(b^2*x
 + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{(a+b\,x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^2 + 1)^(1/2),x)
[Out] int(x^2/((a + b*x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
[Out] Integral(x**2/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

3.103 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+d\right.}{6a^{2/3}b^{4/3}d^4}$$

```
[Out] x/b/d^3-1/3*(a+3*a^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(
2/3)/b^(4/3)/d^4+1/6*(a+3*a^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(4/3)/d^4-c*ln(a+b*(d*x+c)^3)/b/d
^4+1/3*(a-3*a^(1/3)*b^(2/3)*c^2+b*c^3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c
))/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)/d^4*3^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+d\right.}{6a^{2/3}b^{4/3}d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^(1/3
)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)*d^4) - ((a + 3*a^
(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(
4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a +
b*(c + d*x)^3])/(b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a+bc^3-3bc^2x+3bcx^2}{b\left(a+bx^3\right)}\right)dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x+3bcx^2}{a+bx^3}\,dx, x, c+dx\right)}{bd^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x}{a+bx^3}\,dx, x, c+dx\right)}{bd^4} - \frac{(3c)\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-3\sqrt[3]{a}\,bc^2+2\sqrt[3]{b}\left(a+bc^3\right)\right)+\sqrt[3]{b}\left(-3\sqrt[3]{a}\,bc^2-\sqrt[3]{b}\left(a+bc^3\right)\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\right.}{3a^{2/3}b^{4/3}d^4}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\left(a-3\right.}{}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} + \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(a^{2/3}\right.}{6a^{2/3}}$$

$$= \frac{x}{bd^3} + \frac{\left(a-3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}d^4} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3a^{2/3}b^{4/3}d^4}$$

Mathematica [C] time = 0.06, size = 132, normalized size = 0.56

$$-\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&, \frac{3\#1^2bcd^2\log(x-\#1)+a\log(x-\#1)+bc^3\log(x-\#1)+3\#1bc^2d\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] -1/3*(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*
#1^3 & , (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 + 3*
b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(b^2*d^4)
```

fricas [C] time = 30.34, size = 6315, normalized size = 26.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)
)/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 +
 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2
*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(
b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*
a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*b*d^4*log(-3/4*((-I*s
qrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^
12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 +
3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2
*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c
^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3
)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + b^3*c^10 - 9*a*b
^2*c^7 - 12*a^2*b*c^4 + 1/2*(a*b^3*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*sqrt(
3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12)
- 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^
2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*
c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a
^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*d^4 - 2*a^3*c + (b^3*c^9 - 24*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)*d*x) - 12*d*x - (((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) +
(b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(
a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^1
2) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3
) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^1
2) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54
*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(
b*d^4))*b*d^4 - 3*sqrt(1/3)*b*d^4*sqrt(-(((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8)
```

```
 + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*
c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*
d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(
1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*
d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1
/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*
c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2
*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12
) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*
(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*s
qrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9
- 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a*
b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*d^8)) - 18*c)*log(3/4*((-I*sqrt(3) +
1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2
*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c
^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3
)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 -
 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^
4*d^12))^(1/3) + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + 2*b^3*c^10 - 63*a*b^2*c^7
 + 21*a^2*b*c^4 - 1/2*(a*b^3*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*sqrt(3) + 1
)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*
(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^
3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 -
2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/
(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4
*d^12))^(1/3) + 6*c/(b*d^4))*d^4 + 5*a^3*c + 2*(b^3*c^9 - 24*a*b^2*c^6 + 3*
a^2*b*c^3 + a^3)*d*x + 3/4*sqrt(1/3)*(3*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8)
+ (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c
/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d
^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1
/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d
^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/
54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c
/(b*d^4))*a^2*b^3*c^2*d^8 + 2*(a*b^3*c^6 - 7*a^2*b^2*c^3 + a^3*b)*d^4)*sqrt
(-(((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^
3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 +
 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^
12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 +
3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(
3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12)
```

```
- 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^
2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*
c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a
^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a*b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*
d^8))) - (((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8
))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^
2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/
(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2
*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*b*d^4 + 3*sqrt(1/3)*
b*d^4*sqrt(-(((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*
d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c
^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a
*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c
^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*
b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*
((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(
b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*
a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12)
 - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3
 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a*b*c*d^4 - 48*b*c^5 - 12*a*c^
2)/(a*b^2*d^8)) - 18*c)*log(3/4*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5
 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*
d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1
/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*
(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1
/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*
c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4)
)^2*a^2*b^3*c^2*d^8 + 2*b^3*c^10 - 63*a*b^2*c^7 + 21*a^2*b*c^4 - 1/2*(a*b^3
*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5
- 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d
^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/
54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(
I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/
54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c
^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))
*d^4 + 5*a^3*c + 2*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*d*x - 3/4*s
qrt(1/3)*(3*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d
^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^
```

```
9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*
b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^
3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 +
 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a^2*b^3*c^2*d^8 +
2*(a*b^3*c^6 - 7*a^2*b^2*c^3 + a^3*b)*d^4)*sqrt(-(((-I*sqrt(3) + 1)*(3*c^2/
(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 -
2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/
(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4
*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*
c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*
d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(
1/3) + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (
b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a
*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12
) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3)
 + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12
) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*
(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b
*d^4))*a*b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*d^8))))/(b*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^3*b + a), x)
```

maple [C] time = 0.01, size = 108, normalized size = 0.46

$$\frac{x}{b\,d^3}+\frac{\left(-3\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2 bc\,d^2-3\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\right.}{3b^2d^4\left(d^2\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2+2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^3),x)
```

```
[Out] x/b/d^3+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_R^2*d^2+2*
_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^
3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{bd^3} - \frac{\int \frac{3\,bcd^2x^2+3\,bc^2dx+bc^3+a}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] x/(b*d^3) - integrate((3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*d^3)
```

mupad [B] time = 2.46, size = 374, normalized size = 1.60

$$\left(\sum_{k=1}^{3}\ln\left(\frac{3\left(b\,c^5+a\,c^2\right)}{d^2}-\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log((3*(a*c^2 + b*c^5))/d^2 - root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*
c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + b^3*c^9 + a^3, z, k)*((3*(b^2*c^4*d^4 - 5*a*b*c*d^4))/d^2 + (3*x*(
b^2*c^3*d^4 + a*b*d^4))/d - 9*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d^8*z
^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ b^3*c^9 + a^3, z, k)*a*b^2*d^6) - (3*x*(a*c - 2*b*c^4))/d)*root(27*a^2*b^
4*d^12*z^3 + 81*a^2*b^3*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4
*z + 3*a*b^2*c^6 + 3*a^2*b*c^3 + b^3*c^9 + a^3, z, k), k, 1, 3) + x/(b*d^3)
```

sympy [A] time = 2.90, size = 238, normalized size = 1.02

$$\mathrm{RootSum}\left(27t^3a^2b^4d^{12}+81t^2a^2b^3cd^8+t\left(54a^2b^2c^2d^4-27ab^3c^5d^4\right)+a^3+3a^2bc^3+3ab^2c^6+b^3c^9,\left(t\mapsto t\log\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2*
b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c**
6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*_t*
a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**3*c
- 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3*d -
24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)
```

3.104 $\int \frac{x^2}{a+b(c+dx)^3} \, dx$

Optimal. Leaf size=210

$$\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}-\right.}{}$$

```
[Out] 1/3*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^3
-1/6*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*
x+c)^2)/a^(2/3)/b^(2/3)/d^3+1/3*ln(a+b*(d*x+c)^3)/b/d^3+1/3*c*(2*a^(1/3)-b^
(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^
(2/3)/d^3*3^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {371, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3) + b^(1/3)*c)*Log[a
^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^3) - (c*(2*a^(1/3) + b^(1
/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a
^(2/3)*b^(2/3)*d^3) + Log[a + b*(c + d*x)^3]/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(-c+x)^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3} + \frac{\operatorname{Subst}\left(\int \frac{c^2-2cx}{a+bx^3}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{a}\,c+2\sqrt[3]{b}\,c^2\right)+\sqrt[3]{b}\left(-2\sqrt[3]{a}\,c-\sqrt[3]{b}\,c^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}\,d^3} + \frac{(-\cdots}{\cdots}$$

$$= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} - \frac{\left(c\left(\frac{2}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\right)\operatorname{Subst}\cdots}{\cdots}$$

$$= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+\cdots\right.}{6a^{2/3}b^{2/3}d^3}$$

$$= \frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^3} + \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.03, size = 81, normalized size = 0.39

$$\frac{\operatorname{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&, \frac{\#1^2\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

fricas [C] time = 2.97, size = 4759, normalized size = 22.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^
2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/
(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d
^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b
*d^3))*b*d^3*log(-1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2
*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*
b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) +
 (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) - 2/(b*d^3))^2*a^2*b^2*d^6 + b^2*c^6 - a*b*c^3 - 1/2*(a*b^2*c^3 + 4*
a^2*b)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*
d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b
^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9
) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d
^3))*d^3 + (b^2*c^5 - 8*a*b*c^2)*d*x - 2*a^2) - ((2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2
*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/
(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*
b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 - 3*sqrt(1/3)*b*d^3*sq
rt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^
6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3
*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9)
+ 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3
))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^
6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3
*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a
)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1
/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) + 6)*log(1/2*(2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3
 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^
2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))^2*a^2*b^
2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*d^3 + 2*(b^2*c^5 -
 8*a*b*c^2)*d*x + 2*a^2 + 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((
```

```
2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) +
3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^
2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2
*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b^2*c^3 - 2*a^2*b)*
d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a
*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)
/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*
c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d
^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6))) - ((2*(1/2)
^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9)
 + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 + 3*
sqrt(1/3)*b*d^3*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^
2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a
*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3)
+ (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9)
)^(1/3) - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*
c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2
*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^
3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9
) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)
/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) +
 6)*log(1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a^2*b^2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b
)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))
/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2
/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*
d^3 + 2*(b^2*c^5 - 8*a*b*c^2)*d*x + 2*a^2 - 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
```

```
3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b
^2*c^3 - 2*a^2*b)*d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a
)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(
2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b
^3*d^9))^(1/3) - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9)
 + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/
(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*
b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3
 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*
d^6))))/(b*d^3)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^3 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

maple [C]   time = 0.00, size = 74, normalized size = 0.35

$$\frac{\mathrm{RootOf}\left(b d^3\_Z^3+3b d^2c\_Z^2+3bd c^2\_Z+b c^3+a\right)^2 \ln\left(-\mathrm{RootOf}\left(b d^3\_Z^3+3b d^2c\_Z^2+3bd c^2\_Z+b c\right.\right.}{3bd\left(d^2\,\mathrm{RootOf}\left(b d^3\_Z^3+3b d^2c\_Z^2+3bd c^2\_Z+b c^3+a\right)^2+2cd\,\mathrm{RootOf}\left(b d^3\_Z^3+3b d^2c\_Z^2+3bd c^2\_Z+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_
Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^3 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

mupad [B] time = 2.30, size = 437, normalized size = 2.08

$$\sum_{k=1}^{3} \ln\left(a + b\,c^3 - \mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\right)\,a\,b\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log(a + b*c^3 - 6*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*
a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*a*b*d^3
+ 3*b*c^2*d*x + 9*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c
^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)^2*a*b^2*d^6 + 3
*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*
b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^3*d^3 + 3*root(27*a^2*b^3*
d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c
^3 - b^2*c^6 - a^2, z, k)*b^2*c^2*d^4*x)*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b
^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2
, z, k), k, 1, 3)
```

sympy [A] time = 0.99, size = 158, normalized size = 0.75

$$\mathrm{RootSum}\left(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t\left(9a^2bd^3 - 18ab^2c^3d^3\right) - a^2 - 2abc^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{18t^2a^2b^2d^6 -}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d*
*3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*lo
g(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d**3
+ 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))
```

3.105 $\int \frac{x}{a+b(c+dx)^3} \, dx$

Optimal. Leaf size=180

$$-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\mathrm{t}}{\sqrt{3}}$$

```
[Out] -1/3*(a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^2+1/
6*(a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)
/a^(2/3)/b^(2/3)/d^2-1/3*(a^(1/3)-b^(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)/d^2*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {371, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\mathrm{t}}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c + d*x)^3),x]
```

```
[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b^(1/3)*c)*Log[a^(1/3
) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^2) + ((a^(1/3) + b^(1/3)*c)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^
(2/3)*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^3}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}\,c\right)+\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}\,d^2} - \frac{\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c\right.}{3a^{2/3}d^2}$$

$$= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\frac{1}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+\right.}{2d^2}$$

$$= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c\right.}{6a^{2/3}b^{2/3}d^2}$$

$$= -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^2} - \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)}{}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.44

$$\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{\#1\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

fricas [C] time = 2.57, size = 1950, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)
/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3
 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3)
```

```
 + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2
*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*
(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*
log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3
- a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/
(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(
9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*
b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d
^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*
c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a
)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1
)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6
))^(1/3)))*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(
3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b
^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4))) + 1/36*(9*(I*sqrt(3) + 1)*(
-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + 3
*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/5
4*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*
c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4
- 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b
^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*log(1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)
 + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*
(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*s
qrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c - 1/
12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b
*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3
+ a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a^2*b*d^4 + 6*
a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1
/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(
b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^
4 - 144*c)/(a*b*d^4))) - 1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^
4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))
)*log(-1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 +
a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 1/
6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + b*c^4 + (b*c
^3 - a)*d*x + a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

maple [C] time = 0.00, size = 72, normalized size = 0.40

$$\frac{\mathrm{RootOf}\left(b\,d^3\_Z^3 + 3b\,d^2c\,\_Z^2 + 3bd\,c^2\_Z + b\,c^3 + a\right)\ln\left(-\mathrm{RootOf}\left(b\,d^3\_Z^3 + 3b\,d^2c\,\_Z^2 + 3bd\,c^2\_Z + b\,c^3\right.\right.}{3bd\left(d^2\,\mathrm{RootOf}\left(b\,d^3\_Z^3 + 3b\,d^2c\,\_Z^2 + 3bd\,c^2\_Z + b\,c^3 + a\right)^2 + 2cd\,\mathrm{RootOf}\left(b\,d^3\_Z^3 + 3b\,d^2c\,\_Z^2 + 3bd\,c^2\_Z +\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^
2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

mupad [B] time = 0.26, size = 145, normalized size = 0.81

$$\sum_{k=1}^{3} \ln\left(-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3 - 9\,a\,b\,c\,d^2\,z + b\,c^3 + a, z, k\right)\left(3\,b^2\,c^2\,d^4 - \mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3 - 9\,a\,b\,c\,d^2\,z + b\,c^3 + a, z\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log(b*d^3*x - root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z
, k)*(3*b^2*c^2*d^4 - 9*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a
, z, k)*a*b^2*d^6 + 3*b^2*c*d^5*x))*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z
 + b*c^3 + a, z, k), k, 1, 3)
```

sympy [A] time = 0.70, size = 83, normalized size = 0.46

$$\text{RootSum}\left(27t^3a^2b^2d^6 - 9tabcd^2 + a + bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd^4 + 3tabc^2d^2 - ac - bc^4}{ad - bc^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t,
_t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d -
 b*c**3*d))))
```

3.106 $\int \frac{1}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)*b^
(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/3*(a^(1/3)-
2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {247, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(
1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}d}$$

$$= \frac{\log\left(\sqrt[3]{a}\,+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}\,d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}\,-}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}}\right.}{6a^{2/}}$$

$$= \frac{\log\left(\sqrt[3]{a}\,+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{a^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d} + \frac{\log\left(\sqrt[3]{a}\,+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx\right.}{6a^{2/3}\sqrt[3]{b}\,d}$$

Mathematica [A] time = 0.03, size = 116, normalized size = 0.83

$$\frac{-\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)+2\log\left(\sqrt[3]{a}\,+\sqrt[3]{b}\,(c+dx)\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*L
og[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

fricas [A] time = 0.92, size = 442, normalized size = 3.16

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abd^3x^3+6abcd^2x^2+6abc^2dx+2abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2abd^2x^2+4abcdx+2abc^2+\left(a^2b\right)^{\frac{2}{3}}(dx+c)-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d
^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a
*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
 3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(
a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2
*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
 a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

giac [A] time = 0.45, size = 160, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)-\frac{1}{6}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(sq
rt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))^(
1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d
*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*
c + (a*b^2)^(1/3)))
```

maple [C] time = 0.00, size = 71, normalized size = 0.51

$$\frac{\ln\left(-\operatorname{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)+x\right)}{3bd\left(d^2\operatorname{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^3*b + a), x)
```

mupad [B] time = 2.31, size = 144, normalized size = 1.03

$$\frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{2/3}\,b^{1/3}\,d}+\frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x+\frac{3\,a^{1/3}\,b^{5/3}\,d^5\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}-\frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^3),x)
```

```
[Out] log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*c
*d^5 + 3*b^2*d^6*x + (3*a^(1/3)*b^(5/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1
i - 1))/(6*a^(2/3)*b^(1/3)*d) - (log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^(1/3)
*b^(5/3)*d^5*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*d)
```

sympy [A] time = 0.26, size = 26, normalized size = 0.19

$$\frac{\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

3.107 $\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=224

$$-\frac{\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}+\frac{\sqrt[3]{b}c\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c\right.}{3a^{2/3}\left(\sqrt[3]{a}+\right.}$$

```
[Out] ln(x)/(b*c^3+a)-1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(a^(1/3)+b^(1/3)*c)
-1/6*(2*a^(1/3)-b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+
c)^2)/a^(2/3)/(a^(2/3)-a^(1/3)*b^(1/3)*c+b^(2/3)*c^2)+1/3*b^(1/3)*c*arctan(
1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^(2/3)-a^(1/3)*b
^(1/3)*c+b^(2/3)*c^2)*3^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] (b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[x]/(a + b*c^3) + (b^(1
/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a
 + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)) - Log[a + b*(c +
 d*x)^3]/(3*(a + b*c^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \text{Subst}\left(\int \frac{1}{(-c+x)\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= \text{Subst}\left(\int \left(-\frac{1}{\left(a+bc^3\right)(c-x)} - \frac{b\left(c^2+cx+x^2\right)}{\left(a+bc^3\right)\left(a+bx^3\right)}\right) dx, x, c+dx\right)$$

$$= \frac{\log(x)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{c^2+cx+x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}$$

$$= \frac{\log(x)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{c^2+cx}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}$$

$$= \frac{\log(x)}{a+bc^3} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}\,c+2\sqrt[3]{b}\,c^2\right)+\sqrt[3]{b}\left(\sqrt[3]{a}\,c-\sqrt[3]{b}\,c^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c\right.}{3a^{2/3}\left(a+bc^3\right)}$$

$$= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{\left(\sqrt[3]{b}\,c\right.}{}$$

$$= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\right.}{6a^{2/3}\,(}$$

$$= \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)} + \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(\right.}{3a^{2/3}\left(a+bc^3\right)}$$

Mathematica [C] time = 0.06, size = 119, normalized size = 0.53

$$-\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{\#1^2d^2\log(x-\#1)+3c^2\log(x-\#1)+3\#1cd\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]-3\log(x)}{3\left(a+bc^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/3*(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3
*#1^3 & , (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*Log[x - #1]*#1^2)
/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(a + b*c^3)
```

fricas [C] time = 2.82, size = 4370, normalized size = 19.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(b*c^3 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))*log(b*c^2*d*x + b*c^3
+ 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2)
- 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a
*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3
) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2
)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 - 1/2*(a*b*c^3 -
 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^
2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + a) - ((b*c^3 + a)*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)
^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3
+ a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - 2/(b*c^3 + a)) + 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 +
 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
```

```
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*
(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^
3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a
)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 +
a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^
3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) +
6)*log(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2)
 - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^
(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c
^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b
*c^3 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*
b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 +
 a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1
/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((
a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 3/4
*sqrt(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(
1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*
c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/
2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) +
 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))
 + 2*a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2
*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 +
a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a
^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3)
 - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(
a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3
 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^
(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/
((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4
*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) - ((b*c^3 + a)*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a
^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)
^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2
*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) -
2/(b*c^3 + a)) - 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a
^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c
^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 +
 a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(
b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3
 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2
```

```
*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 +
a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) + 6)*l
og(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 +
 a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3
 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^
3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3
) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*
(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*
c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) - 3/4*sqr
t(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a
*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3
+ a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(
1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/(
(a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 2*
a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2
) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3
)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b
*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/
(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*
c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a
^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3
)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*
b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/
(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) + 12*log(x))/(b*c^3 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^3 b + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^3*b + a)*x), x)

maple [C] time = 0.01, size = 105, normalized size = 0.47

$$\frac{\ln(x)}{b\,c^3+a}-\frac{\left(d^2\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2+3cd\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c\right.}{3\left(b\,c^3+a\right)\left(d^2\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2+2cd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/3/(b*c^3+a)*sum((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R
+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))+ln(x)/(b*c^3
+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd\int \frac{d^2x^2+3\,cdx+3\,c^2}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{bc^3+a}+\frac{\log(x)}{bc^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*d*integrate((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + log(x)/(b*c^3 + a)
```

mupad [B] time = 0.12, size = 553, normalized size = 2.47

$$\frac{\ln(x)}{b\,c^3+a}+\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2 b^4\,c^4\,d^8\,3-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^3)),x)
```

```
[Out] log(x)/(a + b*c^3) + symsum(log(3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a
^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^4*d^8 - 3*root(27*a^2*b*c^3*z^3 + 27*a^3*
z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*c*d^8 - 4*root(27*a^2*b*c^3*z^3 + 2
7*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*d^9*x - 6*root(27*a^2*b*c^3*z
^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*c*d^8 - 24*root(27*
a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*d^9*x +
9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*
b^3*c*d^8 + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1,
z, k)^3*a*b^4*c^4*d^8 - 36*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2
+ 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x + 3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 +
 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^3*d^9*x + 18*root(27*a^2*b*c^3*z^3 +
 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^3*d^9*x)*root(27*a^2*
b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.108 $\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=314

$$-\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}\,c\left(2a-bc^3\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{\vphantom{b}}\right.}{3}$$

```
[Out] -1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)*(-2*b*c^3+a
)-b^(1/3)*c*(-b*c^3+2*a))*d*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^2
-1/6*b^(1/3)*(a^(1/3)*(-2*b*c^3+a)-b^(1/3)*c*(-b*c^3+2*a))*d*ln(a^(2/3)-a^(
1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^2+b*c^2*d*ln(a+b*
(d*x+c)^3)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)-b^(1/3)*c)*(a^(1/3)+b^(1/3)*c)^
3*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+
a)^2*3^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}d\left(-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}+2ac-bc^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}\,c\left(2a-b\right.\right.}{3a^{2/3}\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1/3)*c)
^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a + b*c^3)^2 + (b^(1/3)*(a^(1/3)
*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*a^(2/3)*(a + b*c^3)^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*(a - 2*b*
c^3))/b^(1/3))*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x
)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a + b*c
^3)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = d\,\text{Subst}\left(\int \frac{1}{(-c+x)^2\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= d\,\text{Subst}\left(\int\left(\frac{1}{\left(a+bc^3\right)(c-x)^2}+\frac{3bc^2}{\left(a+bc^3\right)^2(c-x)}+\frac{b\left(-c\left(2a-bc^3\right)-\left(a-2bc^3\right)x\right.}{\left(a+bc^3\right)^2\left(a+bx^3\right)}\right.\right.$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{-c\left(2a-bc^3\right)-\left(a-2bc^3\right)x+3bc^2x^2}{a+bx^3}\,dx, x, c+dx\right)}{\left(a+bc^3\right)^2}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{-c\left(2a-bc^3\right)+\left(-a+2bc^3\right)x}{a+bx^3}\,dx, x, c+dx\right)}{\left(a+bc^3\right)^2}+\frac{(3b}{}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2}+\frac{\left(b^{2/3}d\right)\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{b}\right.}{}\right.}{}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-\frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}\right)d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-\frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}\right)d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}$$

$$= -\frac{1}{\left(a+bc^3\right)x}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3 d\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^2}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-$$

Mathematica [C] time = 0.10, size = 173, normalized size = 0.55

$$\frac{dx\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{3\#1^2bc^2d^2\log(x-\#1)-3ac\log(x-\#1)-\#1ad\log(x-\#1)+6bc^4\log(x-\#1)+8\#1}{\#1^2d^2+2\#1cd+c^2}\right.}{3x\left(a+bc^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#1
 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x - #1
] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x - #1]
*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3)^2*x)
```

fricas [C] time = 4.14, size = 8919, normalized size = 28.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(36*b*c^2*d*x*log(x) + 12*b*c^3 - 2*(b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*
c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +
 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3
) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b
^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c
^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1
/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((
a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^
2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(
I*sqrt(3) + 1))*x*log((b^3*c^6 - a^2*b)*d^3*x + 1/4*(2*a^2*b^3*c^9 + 3*a^3*
b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^
2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^
3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)
) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a
)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3
 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a
^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
+ a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 + 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 1
0*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*
c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3
+ a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18
*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2))
+ b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^
3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2
)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 +
a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (b^3*c^7 + 5*a*b^2*c^4 - 5*a^2*b*c)*d
^2) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a
*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2
- 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
```

```
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^
6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*
c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
+ a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*x -
3*sqrt(1/3)*(b^2*c^6 + 2*a*b*c^3 + a^2)*x*sqrt(-((a*b^4*c^12 + 4*a^2*b^3*c^
9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^
2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2
/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6
 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^
2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^
3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*
c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*
(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b
*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 12*(a*b^3*c
^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*
(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2
*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*
b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 +
 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*
b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2
*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^
6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 -
a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + 4*(8*b^3*c^7 - 11*
a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4
*a^4*b*c^3 + a^5)))*log(2*(b^3*c^6 - a^2*b)*d^3*x - 1/4*(2*a^2*b^3*c^9 + 3*
a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(
9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*
b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^
3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^
2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3
 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*
c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5
 + 10*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*
b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*
c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3
- 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^
2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 +
 a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)
^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
 a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^
3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (2*b^3*c^7 - 5*a*b^2*c^4 + 2*a^2*
b*c)*d^2 + 3/4*sqrt(1/3)*((2*a^2*b^3*c^9 + 3*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/
(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b
```

```
*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)
/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6
 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2
*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1
/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*
c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6
+ 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt
(3) + 1)) - 2*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*d)*sqrt(-((a*b^4*c^12
 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 +
 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2
)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c
^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b
*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^
3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^
2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*
c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^
2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*
c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2
*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/
(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 +
a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)
 + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + 4*(
8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*
a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5))) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 +
 a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2
/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))
*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^
3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3
/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))
^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2
*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*
d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^
2))^(1/3)*(I*sqrt(3) + 1))*x + 3*sqrt(1/3)*(b^2*c^6 + 2*a*b*c^3 + a^2)*x*sq
rt(-((a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*
c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +
 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3
) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b
^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c
^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1
/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((
a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^
```

```
2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(
I*sqrt(3) + 1))^2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(
b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*
c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/
(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6
+ 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*
a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/
3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c
^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 +
 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(
3) + 1))*d + 4*(8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*
a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)))*log(2*(b^3*c^6 - a^2*b)*
d^3*x - 1/4*(2*a^2*b^3*c^9 + 3*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a
*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2
- 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^
6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*
c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
+ a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 -
1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 10*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b
*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 -
2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3
/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 +
 a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4
) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*
d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^
3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 +
a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (2
*b^3*c^7 - 5*a*b^2*c^4 + 2*a^2*b*c)*d^2 - 3/4*sqrt(1/3)*((2*a^2*b^3*c^9 + 3
*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*
(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2
*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)
^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
 a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^
3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a
^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^
3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b
*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1)) - 2*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a
^3*b*c^2)*d)*sqrt(-((a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c
^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4
*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a
^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^
2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b
```

```
*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a
^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18
*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2))
+ b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^
3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^
2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b
^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-
I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d
^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a
^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1
/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^
3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3
/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))
^(1/3)*(I*sqrt(3) + 1))*d + 4*(8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(
a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5))) + 12*a)/(
(b^2*c^6 + 2*a*b*c^3 + a^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^3 b + a\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^2), x)
```

maple [C] time = 0.01, size = 144, normalized size = 0.46

$$-\frac{3b\,c^2 d\ln(x)}{\left(b\,c^3+a\right)^2}+\frac{d\left(3\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2 b\,c^2d^2+8\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\right.\right.}{3\left(b\,c^3+a\right)^2\left(d^2\,\mathrm{RootOf}\left(b\,d^3\_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3*d/(b*c^3+a)^2*sum((3*_R^2*b*c^2*d^2+8*_R*b*c^3*d+6*b*c^4-_R*a*d-3*a*c)/
(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*
b*c^2*d+b*c^3+a))-1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\,bc^2d\log(x)}{b^2c^6+2\,abc^3+a^2}+\frac{bd^2\int\frac{3\,bc^2d^2x^2+6\,bc^4+\left(8\,bc^3-a\right)dx-3\,ac}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{b^2c^6+2\,abc^3+a^2}-\frac{1}{\left(bc^3+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -3*b*c^2*d*log(x)/(b^2*c^6 + 2*a*b*c^3 + a^2) + b*d^2*integrate((3*b*c^2*d^
2*x^2 + 6*b*c^4 + (8*b*c^3 - a)*d*x - 3*a*c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3 + a), x)/(b^2*c^6 + 2*a*b*c^3 + a^2) - 1/((b*c^3 + a)*x)
```

mupad [B] time = 2.33, size = 1588, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((b^4*d^12*x - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*
a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^3*b^3*d^9
- 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*
d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^9*d^9 - 9*root(27*a^2*b^2*c^6
*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z
- b*d^3, z, k)*b^5*c^5*d^10 + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3
 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^
4*c^3*d^9 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81
*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^4*d^8 + 27*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^2*b^5*c^7*d^8 - 9*root(27*a^2*b^2*c^6*z^
3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b
*d^3, z, k)*a*b^4*c^2*d^10 - 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 +
 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*b^5*c^4*d^
11*x + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b
*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*c*d^8 + 18*root(27*a^2
*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*
c*d^2*z - b*d^3, z, k)^2*a*b^5*c^6*d^9 + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3
*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k
)^3*a*b^6*c^10*d^8 - 36*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*d^9*x -
 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^8*d^10*x + 48*root(27*a^2*b^2*
c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2
*z - b*d^3, z, k)^2*a*b^5*c^5*d^10*x + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*
b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)
^3*a*b^6*c^9*d^9*x - 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*a*b^4*c*d^11*x +
51*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^4*c^2*d^10*x - 54*root(27*a^2*
```

```
b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c
*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^3*d^9*x)/(a^2 + b^2*c^6 + 2*a*b*c^3))*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
 18*a*b*c*d^2*z - b*d^3, z, k), k, 1, 3) - 1/(a*x + b*c^3*x) - (3*b*c^2*d*l
og(x))/(a^2 + b^2*c^6 + 2*a*b*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.109 $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$

Optimal. Leaf size=393

$$\frac{b^{2/3}d^2\left(-3a^{2/3}\sqrt[3]{b}\,c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^3}-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}\,b^{5/3}c^5-7abc^3+\right.}{3a^{2/3}\left(a+bc^3\right)^3}$$

```
[Out] -1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*ln(x)/(b*
c^3+a)^3-1/3*b^(2/3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)
*c^5+b^2*c^6)*d^2*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^3+1/6*b^(2/
3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)*c^5+b^2*c^6)*d^2*
ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^3+b
*c*(-2*b*c^3+a)*d^2*ln(a+b*(d*x+c)^3)/(b*c^3+a)^3+1/3*b^(2/3)*(a^(1/3)+b^(1
/3)*c)^3*(a-3*a^(2/3)*b^(1/3)*c+b*c^3)*d^2*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d
*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+a)^3*3^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}\,b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}+\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}\,b^{5/\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/(2*(a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)*(a^(1/3)
+ b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*ArcTan[(a^(1/3) - 2*b^
(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)^3) - (3*b
*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2
/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(1/3) + b^
(1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b^(
2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(2/3) - a
^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^3)
+ (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
```

```
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = d^2\,\mathrm{Subst}\left(\int \frac{1}{(-c+x)^3\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= d^2\,\mathrm{Subst}\left(\int\left(-\frac{1}{\left(a+bc^3\right)(c-x)^3} - \frac{3bc^2}{\left(a+bc^3\right)^2(c-x)^2} - \frac{3bc\left(-a+2bc^3\right)}{\left(a+bc^3\right)^3(c-x)} + \frac{b\left(-a\right.}{}\right.\right.$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{\left(bd^2\right)\mathrm{Subst}\left(\int \frac{-a^2+7abc}{}\right.}{}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{\left(bd^2\right)\mathrm{Subst}\left(\int \frac{-a^2+7abc}{}\right.}{(}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{bc\left(a-2bc^3\right)d^2\log\left(a\right.}{\left(a+bc^3\right)^3}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-\right.}{}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-\right.}{}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} + \frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}\,c+bc^3\right)d^2\tan^{-1}\left(\right.}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^3}$$

Mathematica [C] time = 0.17, size = 244, normalized size = 0.62

$$-\frac{2d^2x^2\mathrm{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&, \frac{-3\#1^2abcd^2\log(x-\#1)+6\#1^2b^2c^4d^2\log(x-\#1)+a^2\log(x-\#1)-16ab}{\#1^2}\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/6*(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^2*x^2*
Log[x] + 2*d^2*x^2*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^
3*#1^3 & , (a^2*Log[x - #1] - 16*a*b*c^3*Log[x - #1] + 10*b^2*c^6*Log[x - #
1] - 12*a*b*c^2*d*Log[x - #1]*#1 + 15*b^2*c^5*d*Log[x - #1]*#1 - 3*a*b*c*d^
2*Log[x - #1]*#1^2 + 6*b^2*c^4*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*
#1^2) & ])/((a + b*c^3)^3*x^2)
```

fricas [C] time = 11.50, size = 14765, normalized size = 37.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*c^6 - 36*(2*b^2*c^4 - a*b*c)*d^2*x^2*log(x) + 12*a*b*c^3 - 2*(
b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b
^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)
^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2
*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2*
log((b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (b^4*c^10 + 15
*a*b^3*c^7 - 63*a^2*b^2*c^4 + 4*a^3*b*c)*d^4 - 1/2*(a*b^4*c^12 - 50*a^2*b^3
*c^9 + 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a
*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^
2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2
*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b
*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3
*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2
*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^
3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2
*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b
*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3
*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2
*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^
3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*
(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^
```

```
2 + 3/4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*
c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 +
 a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^
3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((
a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5
) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2
) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((
a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5
) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2
) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3
*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2) - 36*(b^2*c^5 + a*b*c^2)*d*x + 6*a^2 +
(18*(2*b^2*c^4 - a*b*c)*d^2*x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^
3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a
^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5)
+ (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2)
- 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5)
+ (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2)
- 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a
*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2 + 3*sqrt(1/3)*(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)*x^2*sqrt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5
 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 -
 a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*
c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*
d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6
+ 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6
/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*
d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6
+ 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6
/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*
```

```
c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a
^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c
^3 + a^7)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d
^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3
+ a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^
6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d
^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3
+ a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^
6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^
9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*
a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)))*log(2
*(b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (2*b^4*c^10 - 6*a
*b^3*c^7 - 9*a^2*b^2*c^4 - a^3*b*c)*d^4 + 1/2*(a*b^4*c^12 - 50*a^2*b^3*c^9
+ 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*
c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*
c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3
+ a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9
+ 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^
3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^
3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*
c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3
+ a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9
+ 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^
3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^
3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^
2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 - 3
/4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*
(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
 - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
```

```
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^
2*c^6 + 3*a^2*b*c^3 + a^3))^2 + 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^
9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11
 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2
)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*
b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*
c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*
b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3
/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*
b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*
c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*
b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3
/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(
2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)))*sq
rt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 +
12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*
(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
 - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^
2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4
*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)
*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^
4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqr
t(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b
^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) -
b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*
b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d
^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1
/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b
^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) -
b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*
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b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d
^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*s
qrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*
b*c^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^
3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7))) + (18*(2*b^2*c^4 - a*b*c)*d^2
*x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*
d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*
b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/
(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3
*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a
^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 -
24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c
*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*
(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3
*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a
^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 -
24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c
*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1
) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^
3))*x^2 - 3*sqrt(1/3)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*x^2*sqrt(
-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*
(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*(6*
(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) -
3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^
9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3
 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3
*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(
2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)
^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^
9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3
 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3
*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(
2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)
^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c
^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^
12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)*(b
^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d
^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3
) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*
c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2
*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2
*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2
- a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)
^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*
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c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2
*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2
*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2
- a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt
(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c
^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c
^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)))*log(2*(b^4*c^9 + 3*a*b^3*c^6 - 2
4*a^2*b^2*c^3 + a^3*b)*d^5*x + (2*b^4*c^10 - 6*a*b^3*c^7 - 9*a^2*b^2*c^4 -
a^3*b*c)*d^4 + 1/2*(a*b^4*c^12 - 50*a^2*b^3*c^9 + 141*a^3*b^2*c^6 - 50*a^4*
b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3
*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*
a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^
2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c
^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c
^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 +
a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^
2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c
^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c
^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 +
a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3
*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 - 3/4*(a^2*b^4*c^14 + a^3*b^3*
c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4
/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c
*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) -
 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))
^2 - 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b
*c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*
c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3
*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^
2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4
*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^
```

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3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^
2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4
*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^
3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)))*sqrt(-(12*(4*b^5*c^11 - 24*a*
b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^
4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^(2/3)*(b^2*c^2*d^4
/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c
*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) -
 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))
*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15
*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 +
3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c
^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^
2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^
3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^
3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^
2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^
3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^
3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*
d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2)/(a*b^6*c^1
8 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*
a^6*b*c^3 + a^7))))/((b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^3 b+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^3), x)
```

maple [C] time = 0.01, size = 217, normalized size = 0.55

$$\frac{6b^2c^4d^2\ln(x)}{\left(bc^3+a\right)^3}-\frac{3abcd^2\ln(x)}{\left(bc^3+a\right)^3}+\frac{3bc^2d}{\left(bc^3+a\right)^2x}+\frac{d^2\left(-6\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\,\_Z^2+3bdc^2\_Z+bc^3+a\right)^2b^2c^4d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3*d^2/(b*c^3+a)^3*sum((-6*_R^2*b^2*c^4*d^2-15*_R*b^2*c^5*d-10*b^2*c^6+3*_
R^2*a*b*c*d^2+12*_R*a*b*c^2*d+16*a*b*c^3-a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_
R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))-1/2/(b*c^3+
a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x+6*b^2*d^2*c^4/(b*c^3+a)^3*ln(x)-3*b*d^2*c/(b
*c^3+a)^3*ln(x)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd^3\int\frac{10\,b^2c^6-16\,abc^3+3\left(2\,b^2c^4-abc\right)d^2x^2+3\left(5\,b^2c^5-4\,abc^2\right)dx+a^2}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{b^3c^9+3\,ab^2c^6+3\,a^2bc^3+a^3}+\frac{3\left(2\,b^2c^4-abc\right)d^2\log(x)}{b^3c^9+3\,ab^2c^6+3\,a^2bc^3+a^3}+\frac{6\,bc^2dx-bc^3-a}{2\left(b^2c^6+2\,abc^3+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*d^3*integrate((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)*d^2*x^2 +
 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 3*(2*b^2
*c^4 - a*b*c)*d^2*log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 1/2*
(6*b*c^2*d*x - b*c^3 - a)/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)
```

mupad [B] time = 2.49, size = 1328, normalized size = 3.38

$$\left(\sum_{k=1}^{3}\ln\left(\frac{6\,b^6\,c^4\,d^{14}-3\,a\,b^5\,c\,d^{14}}{a^4+4\,a^3\,b\,c^3+6\,a^2\,b^2\,c^6+4\,a\,b^3\,c^9+b^4\,c^{12}}-\mathrm{root}\left(81\,a^3\,b^2\,c^6\,z^3+27\,a^2\,b^3\,c^9\,z^3+81\,a^4\,b\,c^3\,z^3+27\,a^5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((6*b^6*c^4*d^14 - 3*a*b^5*c*d^14)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
+ 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z
^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d
^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((a^3*b^4*d^12 + 19*b^7*c^9*d^
12 + 12*a*b^6*c^6*d^12 - 6*a^2*b^5*c^3*d^12)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
+ 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z
^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d
^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*(root(81*a^3*b^2*c^6*z^3 + 27*
a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*
a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((9*a^6*b^3*c*d^8
 + 9*a*b^8*c^16*d^8 + 45*a^5*b^4*c^4*d^8 + 90*a^4*b^5*c^7*d^8 + 90*a^3*b^6*
c^10*d^8 + 45*a^2*b^7*c^13*d^8)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9
 + 6*a^2*b^2*c^6) - (x*(36*a^6*b^3*d^9 - 18*a*b^8*c^15*d^9 + 126*a^5*b^4*c^
3*d^9 + 144*a^4*b^5*c^6*d^9 + 36*a^3*b^6*c^9*d^9 - 36*a^2*b^7*c^12*d^9))/(a
^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)) + (3*b^8*c^14*d
^10 - 42*a*b^7*c^11*d^10 + 30*a^4*b^4*c^2*d^10 + 12*a^3*b^5*c^5*d^10 - 63*a
^2*b^6*c^8*d^10)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^
6) + (x*(3*b^8*c^13*d^11 + 66*a^4*b^4*c*d^11 - 87*a*b^7*c^10*d^11 + 39*a^3*
b^5*c^4*d^11 - 117*a^2*b^6*c^7*d^11))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b
^3*c^9 + 6*a^2*b^2*c^6)) + (x*(18*b^7*c^8*d^13 + 90*a*b^6*c^5*d^13 - 9*a^2*
b^5*c^2*d^13))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)
) - (x*(a*b^5*d^15 + b^6*c^3*d^15))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3
*c^9 + 6*a^2*b^2*c^6))*root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^
4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 2
7*a*b^2*c^2*d^4*z + b^2*d^6, z, k), k, 1, 3) - 1/(2*(a*x^2 + b*c^3*x^2)) +
(3*b*c^2*d)/(a^2*x + b^2*c^6*x + 2*a*b*c^3*x) + (6*b^2*c^4*d^2*log(x))/(a^3
 + b^3*c^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6) - (3*a*b*c*d^2*log(x))/(a^3 + b^3*c
^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.110 $\int \frac{x^3}{a+b(c+dx)^4} \, dx$

Optimal. Leaf size=356

$$-\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\cdots\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}$$

```
[Out] 1/4*ln(a+b*(d*x+c)^4)/b/d^4+3/2*c^2*arctan((d*x+c)^2*b^(1/2)/a^(1/2))/d^4/a
^(1/2)/b^(1/2)-1/8*c*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*
b^(1/2))*(3*a^(1/2)-b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)+1/8*c*ln(a^(1/
4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(3*a^(1/2)-b^(1/2)*c^
2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)-1/4*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1
/4))*(3*a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)-1/4*c*arctan(1+b^(
1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(3*a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2
^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {371, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\cdots\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*d^4) + (c*
(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(
3*Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) +
Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)^4]/(4*b*d^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^4}\,dx, x, c+dx\right)}{d^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{x(3c^2+x^2)}{a+bx^4} + \frac{-c^3-3cx^2}{a+bx^4}\right) dx, x, c+dx\right)}{d^4}$$
$$= \frac{\text{Subst}\left(\int \frac{x(3c^2+x^2)}{a+bx^4}\,dx, x, c+dx\right)}{d^4} + \frac{\text{Subst}\left(\int \frac{-c^3-3cx^2}{a+bx^4}\,dx, x, c+dx\right)}{d^4}$$
$$= \frac{\text{Subst}\left(\int \frac{3c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{\left(c\left(3-\frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2bd^4} - \frac{\left(c\left(3\right.\right.}{}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} - \frac{\left(c\left(3\sqrt{a}-\sqrt{b}c\right.\right.}{}$$
$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} - \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} +$$
$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}}{2\sqrt{2}a^{3/4}b}$$

Mathematica [C] time = 0.06, size = 106, normalized size = 0.30

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&, \frac{\#1^3\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

maple [C] time = 0.02, size = 97, normalized size = 0.27

$$\frac{\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}{4bd\left(d^3\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)^3+3\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf
(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

mupad [B] time = 2.69, size = 1003, normalized size = 2.82

$$\sum_{k=1}^{4} \ln\left(b\,c^2\,d\,\left(2\,a\,c+2\,b\,c^5-3\,a\,d\,x+5\,b\,c^4\,d\,x-\mathrm{root}\left(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+9\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(2*b*c^2*d*(2*a*c + 2*b*c^5 - 3*a*d*x + 5*b*c^4*d*x - 2*root(256*
a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*
b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z +
 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^5*d^4 + 32*root(25
6*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^
3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z
 + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*c*d^8 + 24*roo
t(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 9
6*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d
^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*d^9*x - 2*
root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2
+ 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*
b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^4*d^5*x +
 38*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*
z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*
a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*c*d^4 +
 6*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z
^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a
^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*d^5*x))*
root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2
+ 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*
b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k), k, 1, 4)
```

sympy [A] time = 3.74, size = 374, normalized size = 1.05

$$\operatorname{RootSum}\left(256t^4a^3b^4d^{16} - 256t^3a^3b^3d^{12} + t^2\left(96a^3b^2d^8 + 480a^2b^3c^4d^8\right) + t\left(-16a^3bd^4 + 192a^2b^2c^4d^4 - 48ab^3c^8d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*a
**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2*b
**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c**8
 + b**3*c**12, Lambda(_t, _t*log(x + (-1728*_t**3*a**4*b**3*d**12 - 960*_t*
*3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*b**3*
c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*_t*a**
3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d**4 + 2
7*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c**12 + b**4*c**1
6)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 117*a*b**3*c**11*d + b**4*c
**15*d))))
```

3.111 $\int \frac{x^2}{a+b(c+dx)^4} \, dx$

Optimal. Leaf size=318

$$\frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3}$$

```
[Out] -c*arctan((d*x+c)^2*b^(1/2)/a^(1/2))/d^3/a^(1/2)/b^(1/2)+1/8*ln(-a^(1/4)*b^
(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3
/4)/b^(3/4)/d^3*2^(1/2)-1/8*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x
+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^3*2^(1/2)+1/4*arctan
(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/
d^3*2^(1/2)+1/4*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*
c^2)/a^(3/4)/b^(3/4)/d^3*2^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)) - ((Sqrt
[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a
^(3/4)*b^(3/4)*d^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^4}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{2cx}{a+bx^4}+\frac{c^2+x^2}{a+bx^4}\right)dx, x, c+dx\right)}{d^3}$$

$$= \frac{\text{Subst}\left(\int \frac{c^2+x^2}{a+bx^4}\,dx, x, c+dx\right)}{d^3} - \frac{(2c)\,\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx, x, c+dx\right)}{d^3}$$

$$= -\frac{c\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{d^3} - \frac{\left(1-\frac{\sqrt{b}\,c^2}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2bd^3} + \frac{\left(1 -\right.}{}$$

$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} + \frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}-\sqrt{\right.}{}$$

$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} + \frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3} - \frac{\left(\right.}{}$$

$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} - \frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1+\right.}{2\sqrt{2}\,a^{3/4}b^{3/4}}$$

Mathematica [C] time = 0.04, size = 106, normalized size = 0.33

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&,\frac{\#1^2\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

maple [C] time = 0.00, size = 97, normalized size = 0.31

$$\frac{\text{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\,\_Z+b\,c^4+a\right)^2}{4bd\left(d^3\,\text{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\,\_Z+b\,c^4+a\right)^3+3\,\text{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf
(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

mupad [B] time = 2.59, size = 625, normalized size = 1.97

$$\sum_{k=1}^{4} \ln\Big(-b\, d^4\, \Big(a + b\, c^4 + 4\, b\, c^3\, d\, x + \mathrm{root}\Big(256\, a^3\, b^3\, d^{12}\, z^4 + 192\, a^2\, b^2\, c^2\, d^6\, z^2 + 32\, a\, b^2\, c^5\, d^3\, z - 32\, a^2\, b\, c\, d^3\, z + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(-b*d^4*(a + b*c^4 + 4*b*c^3*d*x + 4*root(256*a^3*b^3*d^12*z^4 +
192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4
 + b^2*c^8 + a^2, z, k)*b^2*c^5*d^3 + 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2
*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*
c^8 + a^2, z, k)*b^2*c^4*d^4*x - 20*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2
*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8
+ a^2, z, k)*a*b*c*d^3 - 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*
z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z,
 k)*a*b*d^4*x + 48*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32
*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*
b^2*c^2*d^6 + 32*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a
*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*b^
2*c*d^7*x))*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*
c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k), k, 1, 4)
```

sympy [A] time = 2.67, size = 274, normalized size = 0.86

$$\mathrm{RootSum}\left(256t^4a^3b^3d^{12} + 192t^2a^2b^2c^2d^6 + t\left(-32a^2bcd^3 + 32ab^2c^5d^3\right) + a^2 + 2abc^4 + b^2c^8, \left(t \mapsto t\log\left(x + \frac{64}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-32
*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, Lamb
```

```
da(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d**9
+ 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*a**3
*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5*a**3
*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*c**4*d
 - 33*a*b**2*c**8*d + b**3*c**12*d))))
```

3.112 $\int \frac{x}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=261

$$\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{c\tan^{-1}\left(1-\right.}{2\sqrt{2}a}$$

```
[Out] -1/4*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d^2*2^(1/
2)-1/4*c*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d^2*2^(1
/2)+1/8*c*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))/a^
(3/4)/b^(1/4)/d^2*2^(1/2)-1/8*c*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+
(d*x+c)^2*b^(1/2))/a^(3/4)/b^(1/4)/d^2*2^(1/2)+1/2*arctan((d*x+c)^2*b^(1/2)
/a^(1/2))/d^2/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {371, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{c\tan^{-1}\left(1-\right.}{2\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c + d*x)^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) -
(c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1
/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
+ d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^4}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{a+bx^4}+\frac{x}{a+bx^4}\right)dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^2} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d^2} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{\ldots}}{a+bx\ldots}\right.}{2\sqrt{\ldots}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\,\sqrt{b}\,d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,d\ldots\right.}{4\sqrt{a}\,\sqrt{b}\,d^2}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2} - \frac{c\log\left(\sqrt{a}+\sqrt{2}\ldots\right.}{\ldots}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2} + \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2} - \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2} + \frac{c\log\left(\sqrt{a}-\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.03, size = 104, normalized size = 0.40

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&, \frac{\#1\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &
]/(4*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

maple [C] time = 0.01, size = 95, normalized size = 0.36

$$\frac{\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\,\_Z+b\,c^4+a\right)\mathrm{l}}{4bd\left(d^3\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\,\_Z+b\,c^4+a\right)^3+3\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf(_
Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

mupad [B] time = 2.36, size = 205, normalized size = 0.79

$$\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\,\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(b^2*d^8*x - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b
*c^2*d^2*z + b*c^4 + a, z, k)*(4*b^3*c^3*d^9 - root(256*a^3*b^2*d^8*z^4 + 3
2*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k)*(32*a*b^3*c*d^11 + 16
*a*b^3*d^12*x) + 4*b^3*c^2*d^10*x))*root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4
*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k), k, 1, 4)
```

sympy [A] time = 0.88, size = 131, normalized size = 0.50

$$\mathrm{RootSum}\left(256t^4a^3b^2d^8+32t^2a^2bd^4-16tabc^2d^2+a+bc^4,\left(t\mapsto t\log\left(x+\frac{128t^3a^3bd^6+16t^2a^2bc^2d^4+8ta^2d^2}{4acd-bc^5d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d*
*2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*a*
*2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/(4*
a*c*d - b*c**5*d))))
```

3.113 $\int \frac{1}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=221

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{2\sqrt{2}a^{3/4}}$$

```
[Out] 1/4*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d*2^(1/2)+1/
4*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d*2^(1/2)-1/8*l
n(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))/a^(3/4)/b^(1/
4)/d*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/
2))/a^(3/4)/b^(1/4)/d*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {247, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{2\sqrt{2}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4)
*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^
(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d
*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}\,d} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}\,d} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}\,d}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\,d} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\,d} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}\,d}$$

Mathematica [A] time = 0.10, size = 161, normalized size = 0.73

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)-2\tan^{-1}\left(1-\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d
*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x
) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

fricas [A] time = 0.49, size = 189, normalized size = 0.86

$$\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \arctan\left(a^2bd^4\sqrt{\frac{a^2d^2\sqrt{-\frac{1}{a^3bd^4}}+d^2x^2+2\,cdx+c^2}{d^2}}\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}} - \left(a^2bd^4x+a^2bcd^3\right)\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*b*d^4))^(1/4)*arctan(a^2*b*d^4*sqrt((a^2*d^2*sqrt(-1/(a^3*b*d^4))
+ d^2*x^2 + 2*c*d*x + c^2)/d^2)*(-1/(a^3*b*d^4))^(3/4) - (a^2*b*d^4*x + a^2
*b*c*d^3)*(-1/(a^3*b*d^4))^(3/4)) + 1/4*(-1/(a^3*b*d^4))^(1/4)*log(a*d*(-1/
(a^3*b*d^4))^(1/4) + d*x + c) - 1/4*(-1/(a^3*b*d^4))^(1/4)*log(-a*d*(-1/(a^
3*b*d^4))^(1/4) + d*x + c)
```

giac [A] time = 0.50, size = 103, normalized size = 0.47

$$-\frac{1}{2}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{bdx+bc}{\left(-ab^3\right)^{\frac{1}{4}}}\right)+\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|bdx+bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right)-\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|-bdx-bc+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(-1/(a^3*b*d^4))^(1/4)*arctan(-(b*d*x + b*c)/(-a*b^3)^(1/4)) + 1/4*(-1
/(a^3*b*d^4))^(1/4)*log(abs(b*d*x + b*c + (-a*b^3)^(1/4))) - 1/4*(-1/(a^3*b
*d^4))^(1/4)*log(abs(-b*d*x - b*c + (-a*b^3)^(1/4)))
```

maple [C] time = 0.00, size = 94, normalized size = 0.43

$$\frac{\ln\left(-\operatorname{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_\right.\right.}{4bd\left(d^3\operatorname{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)^3+3\operatorname{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf(_Z
^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^4*b + a), x)
```

mupad [B] time = 0.12, size = 60, normalized size = 0.27

$$-\frac{\operatorname{atan}\left(\frac{b^{1/4}\,c}{{\left(-a\right)}^{1/4}}+\frac{b^{1/4}\,d\,x}{{\left(-a\right)}^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\,c}{{\left(-a\right)}^{1/4}}+\frac{b^{1/4}\,d\,x}{{\left(-a\right)}^{1/4}}\right)}{2\,{\left(-a\right)}^{3/4}\,b^{1/4}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^4),x)
```

```
[Out] -(atan((b^(1/4)*c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)) + atanh((b^(1/4)*
c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)))/(2*(-a)^(3/4)*b^(1/4)*d)
```

sympy [A] time = 0.30, size = 26, normalized size = 0.12

$$\frac{\operatorname{RootSum}\left(256t^4a^3b+1,\left(t\mapsto t\log\left(x+\frac{4ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/d)))/d
```

3.114 $\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx$

Optimal. Leaf size=393

$$-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx\right.}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right.}$$

```
[Out] ln(x)/(b*c^4+a)-1/4*ln(a+b*(d*x+c)^4)/(b*c^4+a)-1/2*c^2*arctan((d*x+c)^2*b^
(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)/a^(1/2)-1/8*b^(1/4)*c*ln(-a^(1/4)*b^(1/4)*
(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b
*c^4+a)*2^(1/2)+1/8*b^(1/4)*c*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d
*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4
)*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4
)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4)*c*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))
*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx\right.}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*(a + b*c^4)
) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x
))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b
]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(a + b*c^4)) + Log[x]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt
[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/
4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \text{Subst}\left(\int \frac{1}{(-c+x)\left(a+bx^4\right)}\,dx, x, c+dx\right)$$

$$= \text{Subst}\left(\int \left(-\frac{1}{\left(a+bc^4\right)(c-x)} - \frac{b\left(c^3+c^2x+cx^2+x^3\right)}{\left(a+bc^4\right)\left(a+bx^4\right)}\right)dx, x, c+dx\right)$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^3+c^2x+cx^2+x^3}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \left(\frac{x\left(c^2+x^2\right)}{a+bx^4} + \frac{c^3+cx^2}{a+bx^4}\right)dx, x, c+dx\right)}{a+bc^4}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{x\left(c^2+x^2\right)}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^3+cx^2}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} + \frac{\left(c\left(1-\frac{\sqrt{b}\,c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx, x,\right.}{2\left(a+bc^4\right)}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} - \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} -$$

$$= -\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\log(x)}{a+bc^4} - \frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx\right.}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}$$

$$= -\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(a+bc^4\right)} - \frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\right.}{2}$$

Mathematica [C] time = 0.07, size = 163, normalized size = 0.41

$$-\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&, \frac{\#1^3d^3\log(x-\#1)+4\#1^2cd^2\log(x-\#1)+4c^3\log(x-\#1)+6\#1c}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\right.}{4\left(a+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -1/4*(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b
*c*d^3*#1^3 + b*d^4*#1^4 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1 +
4*c*d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^
2*#1^2 + d^3*#1^3) & ])/(a + b*c^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^4 b + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x), x)
```

maple [C] time = 0.01, size = 139, normalized size = 0.35

$$\frac{\ln(x)}{b\,c^4+a}-\frac{\left(d^3 \operatorname{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)^3+4\operatorname{RootOf}\left(b\,d^4\_Z^4+4b\,d^3\right.\right.}{4\left(b\,c^4+a\right)\left(d^3 \operatorname{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^4),x)
```

```
[Out] -1/4/(b*c^4+a)*sum((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2*d+4*c^3)/(_R^3*d^3+3*_R^
2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^
2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))+ln(x)/(b*c^4+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd\int \frac{d^3x^3+4\,cd^2x^2+6\,c^2dx+4\,c^3}{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}\,dx}{bc^4+a}+\frac{\log(x)}{bc^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -b*d*integrate((d^3*x^3 + 4*c*d^2*x^2 + 6*c^2*d*x + 4*c^3)/(b*d^4*x^4 + 4*b
*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b*c^4 + a) + l
og(x)/(b*c^4 + a)
```

mupad [B] time = 2.18, size = 882, normalized size = 2.24

$$\frac{\ln(x)}{b\,c^4+a}+\left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(256\,a^3\,b\,c^4\,z^4+256\,a^4\,z^4+256\,a^3\,z^3+96\,a^2\,z^2+16\,a\,z+1,z,k\right)^2\,b^5\,c^5\,d^{15}\,4+\mathrm{root}\left(256\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^4)),x)
```

```
[Out] log(x)/(a + b*c^4) + symsum(log(4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 25
6*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)*b^4*c*d^15 - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
5*d^15 + 5*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2
+ 16*a*z + 1, z, k)*b^4*d^16*x - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 +
256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^5*d^15 + 28*root(2
56*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z,
k)^2*a*b^4*c*d^15 + 60*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 +
 96*a^2*z^2 + 16*a*z + 1, z, k)^2*a*b^4*d^16*x + 32*root(256*a^3*b*c^4*z^4
+ 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^2*b^4*c*d^
15 - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 1
6*a*z + 1, z, k)^4*a^3*b^4*c*d^15 - 32*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4
 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^5*d^15 + 240*root
(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z
, k)^3*a^2*b^4*d^16*x + 320*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*
z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*d^16*x - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
4*d^16*x - 48*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z
^2 + 16*a*z + 1, z, k)^3*a*b^5*c^4*d^16*x - 192*root(256*a^3*b*c^4*z^4 + 25
6*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^4*d^16
*x)*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*
z + 1, z, k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.115 $\int \frac{1}{x^2(a+b(c+dx)^4)}\,dx$

Optimal. Leaf size=496

$$-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\,\left(a-3bc^4\right)-\sqrt{b}\,c^2\left(3a-bc^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\,\left(a-3bc^4\right.\right.}{}$$

```
[Out] -1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2+b*c^3*d*ln(a+b*(d*x+c)^4)/(b*c^4
+a)^2-c*(-b*c^4+a)*d*arctan((d*x+c)^2*b^(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)^2/
a^(1/2)-1/8*b^(1/4)*d*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2
*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)
^2*2^(1/2)+1/8*b^(1/4)*d*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)
^2*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+
a)^2*2^(1/2)-1/4*b^(1/4)*d*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*
b*c^4+a)*a^(1/2)+c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)-1/4*
b^(1/4)*d*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*b*c^4+a)*a^(1/2)+c
^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\,\left(a-3bc^4\right)-\sqrt{b}\,c^2\left(3a-bc^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\,\left(a-3bc^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(c + d*x)^2
)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqr
t[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[
b]*c^2*(3*a - b*c^4))*d*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/
4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a
 + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))
*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/
```

```
(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b
*c^4)^2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx = d\,\text{Subst}\left(\int \frac{1}{(-c+x)^2\left(a+bx^4\right)}\,dx, x, c+dx\right)$$

$$= d\,\text{Subst}\left(\int\left(\frac{1}{\left(a+bc^4\right)(c-x)^2}+\frac{4bc^3}{\left(a+bc^4\right)^2(c-x)}+\frac{b\left(-c^2\left(3a-bc^4\right)-2c\left(a-bc^4\right)\right.}{\left(a+bc^4\right)^2\left(\right.}\right.\right.$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{-c^2\left(3a-bc^4\right)-2c\left(a-bc^4\right)x-\left(a-3bc^4\right)x^2+4bc^3x^3}{a+bx^4}\,dx,\right.}{\left(a+bc^4\right)^2}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\text{Subst}\left(\int\left(\frac{x\left(-2c\left(a-bc^4\right)+4bc^3x^2\right)}{a+bx^4}+\frac{-c^2\left(3a-bc^4\right)+\left(-a+3bc\right.}{a+bx^4}\right.\right.}{\left(a+bc^4\right)^2}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{x\left(-2c\left(a-bc^4\right)+4bc^3x^2\right)}{a+bx^4}\,dx, x, c+dx\right)}{\left(a+bc^4\right)^2}+\frac{(bd)}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{-2c\left(a-bc^4\right)+4bc^3x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)^2}+\frac{\left(\left(a-\right.\right.}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{\left(2b^2c^3d\right)\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{\left(a+bc^4\right)^2}-\frac{\left(bc\left(a-bc\right.\right.}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{\sqrt{b}\,c\left(a-bc^4\right)d\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}-\frac{\sqrt[4]{b}\left(a-3bc^4-\right.}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x}-\frac{\sqrt{b}\,c\left(a-bc^4\right)d\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}\left(a-3bc^4+\frac{\sqrt{b}\,c^2\left(3a-bc^4\right)}{\sqrt{a}}\right)d\,\text{ta}}{2\sqrt{2}\,\sqrt[4]{a}\left(a+bc\right.}$$

Mathematica [C] time = 0.13, size = 238, normalized size = 0.48

$$\frac{dx\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&, \frac{4\#1^3bc^3d^3\log(x-\#1)-\#1^2ad^2\log(x-\#1)+15\#1^2bc^4d^2\log}{\#1}\right.}{4x\left(a+bc^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] (-4*(a + b*c^4 + 4*b*c^3*d*x*Log[x]) + d*x*RootSum[a + b*c^4 + 4*b*c^3*d*#1
 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (-6*a*c^2*Log[x - #1]
 + 10*b*c^6*Log[x - #1] - 4*a*c*d*Log[x - #1]*#1 + 20*b*c^5*d*Log[x - #1]*#
1 - a*d^2*Log[x - #1]*#1^2 + 15*b*c^4*d^2*Log[x - #1]*#1^2 + 4*b*c^3*d^3*Lo
g[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(a + b
*c^4)^2*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^4 b + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x^2), x)
```

maple [C] time = 0.01, size = 188, normalized size = 0.38

$$-\frac{4b\,c^3 d\ln(x)}{\left(b\,c^4+a\right)^2}+\frac{d\left(4\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)^3 b\,c^3d^3+10b\,c^6+\left(15b\,c\right.\right.}{4\left(b\,c^4+a\right)^2\left(d^3\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\,\_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4*d/(b*c^4+a)^2*sum((4*b*d^3*c^3*_R^3+d^2*(15*b*c^4-a)*_R^2+4*c*d*(5*b*c^
4-a)*_R+10*b*c^6-6*a*c^2)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),
_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
-1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\,bc^{3}d\log\left(x\right)}{b^{2}c^{8}+2\,abc^{4}+a^{2}}+\frac{bd^{2}\int\frac{4\,bc^{3}d^{3}x^{3}+10\,bc^{6}+\left(15\,bc^{4}-a\right)d^{2}x^{2}-6\,ac^{2}+4\,\left(5\,bc^{5}-ac\right)dx}{bd^{4}x^{4}+4\,bcd^{3}x^{3}+6\,bc^{2}d^{2}x^{2}+4\,bc^{3}dx+bc^{4}+a}\,dx}{b^{2}c^{8}+2\,abc^{4}+a^{2}}-\frac{1}{\left(bc^{4}+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -4*b*c^3*d*log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*integrate((4*b*c^3*d^
3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4*(5*b*c^5 - a*c)*d*x
)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a),
x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/((b*c^4 + a)*x)
```

mupad [B] time = 2.48, size = 2440, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^4)),x)
```

```
[Out] symsum(log(-(4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z,
k)^2*b^7*c^11*d^17 - 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*
a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b
*d^4, z, k)^3*a^4*b^4*d^16 - b^5*d^20*x + 16*root(256*a^3*b^2*c^8*z^4 + 512
*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2
 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^6*d^18 - 60*root(256*a^3*b^2*c^8*z^4
 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d
^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^3*d^17 + 176*root(256*a^
3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 32
0*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^4*d^16 + 19
2*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c
^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*
c^5*d^15 + 144*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z,
k)^3*a^2*b^6*c^8*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 +
256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z
 + b*d^4, z, k)^4*a^3*b^6*c^9*d^15 + 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*
b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32
*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^13*d^15 + 16*root(256*a^3*b^2*c^8*z
^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2
*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^5*d^19*x + 64*root(256*a^3*b
^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a
```

```
^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*c*d^15 - 184*roo
t(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*
z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^7*d^1
7 - 48*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^
3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b
^7*c^12*d^16 - 320*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z
^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4,
 z, k)^4*a^5*b^4*d^16*x + 4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 +
256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z
 + b*d^4, z, k)^2*b^7*c^10*d^18*x - 248*root(256*a^3*b^2*c^8*z^4 + 512*a^4*
b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32
*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^6*d^18*x - 64*root(256*a^3*b^2*c^8*z^
4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*
d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^11*d^17*x + 32*root(256*a
^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 3
20*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*a*b^5*c*d^19*x - 316*r
oot(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*
d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^2
*d^18*x + 704*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k
)^3*a^3*b^5*c^3*d^17*x - 448*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 +
 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*
z + b*d^4, z, k)^4*a^4*b^5*c^4*d^16*x + 640*root(256*a^3*b^2*c^8*z^4 + 512*
a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2
- 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^7*d^17*x + 64*root(256*a^3*b^2*
c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*
b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^8*d^16*x + 192*ro
ot(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d
*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^12
*d^16*x)/(a^2 + b^2*c^8 + 2*a*b*c^4))*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*
c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a
*b*c*d^3*z + b*d^4, z, k), k, 1, 4) - 1/(a*x + b*c^4*x) - (4*b*c^3*d*log(x)
)/(a^2 + b^2*c^8 + 2*a*b*c^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.116 $\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=123

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2(x-1)^5-\frac{8}{3}$$

```
[Out] -8/3*(3+a)^3*(-1+x)^3+4/5*(3-a)*(3+a)^2*(-1+x)^5+8/7*(3+a)*(5+3*a)*(-1+x)^7
-2/9*(-3*a^2+6*a+37)*(-1+x)^9-8/11*(5+3*a)*(-1+x)^11+4/13*(3-a)*(-1+x)^13+8
/15*(-1+x)^15+1/17*(-1+x)^17+(3+a)^4*x
```

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1106, 1090}

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2(x-1)^5-\frac{8}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5 + (8*(3 +
a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1 + x)^9)/9 - (8*(5 +
3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13)/13 + (8*(-1 + x)^15)/15 + (-
1 + x)^17/17 + (3 + a)^4*x
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4 dx = \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^4 dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \left(81\left(1+\frac{1}{81}a\left(108+54a+12a^2+a^3\right)\right)-216\left(1+a\left(1+\frac{1}{27}a(9+\right.\right.\right.\right.$$
$$= -\frac{8}{3}(3+a)^3(-1+x)^3+\frac{4}{5}(3-a)(3+a)^2(-1+x)^5+\frac{8}{7}(3+a)(5+3a)(-1+$$

Mathematica [A] time = 0.04, size = 195, normalized size = 1.59

$$a^4x+16a^3x^2+\frac{2}{9}\left(3a^2-1536a+20480\right)x^9-6\left(a^2-128a+896\right)x^8+\frac{64}{7}\left(3a^2-140a+512\right)x^7-\frac{16}{3}\left(15a^2-288a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a + a^2)*x^4
- (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512 - 288*a + 15*a^2)*x
^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*(896 - 128*a + a^2)*x^8 + (2*(
20480 - 1536*a + 3*a^2)*x^9)/9 + (16*(-928 + 35*a)*x^10)/5 - (32*(-524 + 9*
a)*x^11)/11 + (4*(-464 + 3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^14 +
(128*x^15)/15 - x^16 + x^17/17
```

fricas [B] time = 0.37, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17}-x^{16}+\frac{128}{15}x^{15}-48x^{14}-\frac{4}{13}x^{13}a+\frac{2560}{13}x^{13}+4x^{12}a-\frac{1856}{3}x^{12}-\frac{288}{11}x^{11}a+\frac{16768}{11}x^{11}+112x^{10}a+\frac{2}{3}x^9a^2-\frac{14848}{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 - 4/13*x^13*a + 2560/13*x^13 + 4*x
^12*a - 1856/3*x^12 - 288/11*x^11*a + 16768/11*x^11 + 112*x^10*a + 2/3*x^9*
a^2 - 14848/5*x^10 - 1024/3*x^9*a - 6*x^8*a^2 + 40960/9*x^9 + 768*x^8*a + 1
92/7*x^7*a^2 - 5376*x^8 - 1280*x^7*a - 80*x^6*a^2 - 4/5*x^5*a^3 + 32768/7*x
^7 + 1536*x^6*a + 768/5*x^5*a^2 + 4*x^4*a^3 - 8192/3*x^6 - 6144/5*x^5*a - 1
92*x^4*a^2 - 32/3*x^3*a^3 + 4096/5*x^5 + 512*x^4*a + 128*x^3*a^2 + 16*x^2*a
^3 + x*a^4
```

giac [B] time = 0.33, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17}-x^{16}+\frac{128}{15}x^{15}-\frac{4}{13}ax^{13}-48x^{14}+4ax^{12}+\frac{2560}{13}x^{13}-\frac{288}{11}ax^{11}-\frac{1856}{3}x^{12}+\frac{2}{3}a^2x^9+112ax^{10}+\frac{16768}{11}x^{11}-6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 4/13*a*x^13 - 48*x^14 + 4*a*x^12 + 2560/13
*x^13 - 288/11*a*x^11 - 1856/3*x^12 + 2/3*a^2*x^9 + 112*a*x^10 + 16768/11*x
^11 - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^10 + 192/7*a^2*x^7 + 768*a*x^8 +
 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3*x^4
 + 768/5*a^2*x^5 + 1536*a*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x^4 -
6144/5*a*x^5 - 8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*x^4 +
4096/5*x^5
```

maple [B] time = 0.00, size = 264, normalized size = 2.15

$$\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+\frac{(-4a+2560)\,x^{13}}{13}+\frac{(48a-7424)\,x^{12}}{12}+\frac{(-288a+16768)\,x^{11}}{11}+\frac{(1120a-29696)\,x^{10}}{10}+\frac{(2a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/17*x^17-x^16+128/15*x^15-48*x^14+1/13*(-4*a+2560)*x^13+1/12*(48*a-7424)*x
^12+1/11*(-288*a+16768)*x^11+1/10*(1120*a-29696)*x^10+1/9*(2*a^2-2560*a+245
76+(-2*a+128)^2)*x^9+1/8*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128))*x^8+
1/7*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^7+1/6*(-160*a^2+3
2*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^6+1/5*(2*a^2*(-2*a+128)+32*a*(8*a-
128)+(-16*a+64)^2)*x^5+1/4*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^4+1/3*(2*a^2
*(-16*a+64)+256*a^2)*x^3+16*a^3*x^2+a^4*x
```

maxima [A] time = 0.69, size = 192, normalized size = 1.56

$$\frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-48\,x^{14}+\frac{2560}{13}\,x^{13}-\frac{1856}{3}\,x^{12}+\frac{16768}{11}\,x^{11}-\frac{14848}{5}\,x^{10}+\frac{40960}{9}\,x^{9}-5376\,x^{8}+\frac{32768}{7}\,x^{7}-\frac{8192}{3}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*x^12 + 167
68/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32768/7*x^7 - 8192/3*x
^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^3 + 2/1
05*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^
3)*a^2 - 4/2145*(165*x^13 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 183040*x^
9 - 411840*x^8 + 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a
```

mupad [B] time = 0.21, size = 175, normalized size = 1.42

$$x^{12}\,\left(4\,a-\frac{1856}{3}\right)-x^{13}\,\left(\frac{4\,a}{13}-\frac{2560}{13}\right)+x^{10}\,\left(112\,a-\frac{14848}{5}\right)-x^{11}\,\left(\frac{288\,a}{11}-\frac{16768}{11}\right)-x^{8}\,\left(6\,a^2-768\,a+5376\right)-x^{6}\,\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^12*(4*a - 1856/3) - x^13*((4*a)/13 - 2560/13) + x^10*(112*a - 14848/5) -
x^11*((288*a)/11 - 16768/11) - x^8*(6*a^2 - 768*a + 5376) - x^6*(80*a^2 - 1
536*a + 8192/3) + x^7*((192*a^2)/7 - 1280*a + 32768/7) + x^9*((2*a^2)/3 - (
1024*a)/3 + 40960/9) - x^5*((6144*a)/5 - (768*a^2)/5 + (4*a^3)/5 - 4096/5)
+ a^4*x - 48*x^14 + (128*x^15)/15 - x^16 + x^17/17 + 16*a^3*x^2 + 4*a*x^4*(
a^2 - 48*a + 128) - (32*a^2*x^3*(a - 12))/3
```

sympy [A] time = 0.12, size = 199, normalized size = 1.62

$$a^4x+16a^3x^2+\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+x^{13}\left(\frac{2560}{13}-\frac{4a}{13}\right)+x^{12}\left(4a-\frac{1856}{3}\right)+x^{11}\left(\frac{16768}{11}-\frac{288a}{11}\right)+x^{10}\left(112a-\frac{1}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**14 + x**13*
(2560/13 - 4*a/13) + x**12*(4*a - 1856/3) + x**11*(16768/11 - 288*a/11) + x
**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1024*a/3 + 40960/9) + x**8*(-6*a*
*2 + 768*a - 5376) + x**7*(192*a**2/7 - 1280*a + 32768/7) + x**6*(-80*a**2
+ 1536*a - 8192/3) + x**5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5) + x*
*4*(4*a**3 - 192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)
```

3.117 $\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx$

Optimal. Leaf size=120

$$a^3x-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48-5a)x^6$$

```
[Out] a^3*x+12*a^2*x^2+8*(8-a)*a*x^3+(3*a^2-96*a+128)*x^4-3/5*(a^2-128*a+512)*x^5
+8*(48-5*a)*x^6-32/7*(70-3*a)*x^7+3*(64-a)*x^8-1/3*(256-a)*x^9+28*x^10-72/1
1*x^11+x^12-1/13*x^13
```

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2+a^3x-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48-5a)x$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512 -
 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a)*x^7)/7 + 3*(64 - a
)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx = \int \left(a^3+24a^2x+24(8-a)ax^2+4\left(128-96a+3a^2\right)x^3-3\left(512-128a+a^2\right.\right.$$

$$= a^3x+12a^2x^2+8(8-a)ax^3+\left(128-96a+3a^2\right)x^4-\frac{3}{5}\left(512-128a+a^2\right)x^5$$

Mathematica [A] time = 0.02, size = 114, normalized size = 0.95

$$a^3x-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2+\frac{1}{3}(a-256)x^9-3(a-64)x^8+\frac{32}{7}(3a-70)x^7-8(5a-48)x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512
- 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 + 3*a)*x^7)/7 - 3*(-64
 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

fricas [A] time = 0.36, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^9a-\frac{256}{3}x^9-3x^8a+192x^8+\frac{96}{7}x^7a-320x^7-40x^6a-\frac{3}{5}x^5a^2+384x^6+\frac{384}{5}x^5a+3x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 + 1/3*x^9*a - 256/3*x^9 - 3*x^8*a
+ 192*x^8 + 96/7*x^7*a - 320*x^7 - 40*x^6*a - 3/5*x^5*a^2 + 384*x^6 + 384/5
*x^5*a + 3*x^4*a^2 - 1536/5*x^5 - 96*x^4*a - 8*x^3*a^2 + 128*x^4 + 64*x^3*a
 + 12*x^2*a^2 + x*a^3
```

giac [A] time = 0.30, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+\frac{1}{3}ax^9+28x^{10}-3ax^8-\frac{256}{3}x^9+\frac{96}{7}ax^7+192x^8-\frac{3}{5}a^2x^5-40ax^6-320x^7+3a^2x^4+\frac{384}{5}ax^5+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 - 256/3*x^9
+ 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 384
/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*x^2
 + 64*a*x^3 + 128*x^4
```

maple [A] time = 0.00, size = 138, normalized size = 1.15

$$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\frac{(3a-768)x^9}{9}+\frac{(-24a+1536)x^8}{8}+\frac{(96a-2240)x^7}{7}+\frac{(-240a+2304)x^6}{6}+\frac{\left(-a^2+(-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/13*x^13+x^12-72/11*x^11+28*x^10+1/9*(3*a-768)*x^9+1/8*(-24*a+1536)*x^8+1
/7*(96*a-2240)*x^7+1/6*(-240*a+2304)*x^6+1/5*((-2*a+128)*a+256*a-1536-a^2)*
```

```
x^5+1/4*((8*a-128)*a-256*a+512+4*a^2)*x^4+1/3*((-16*a+64)*a+128*a-8*a^2)*x^
3+12*a^2*x^2+a^3*x
```

maxima [A] time = 0.67, size = 119, normalized size = 0.99

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}-\frac{256}{3}x^9+192x^8-320x^7+384x^6-\frac{1536}{5}x^5+a^3x+128x^4-\frac{1}{5}\left(3x^5-15x^4+40x^3-60\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 - 256/3*x^9 + 192*x^8 - 320*x^7 +
384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 60*
x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080
*x^4 + 6720*x^3)*a
```

mupad [B] time = 0.10, size = 108, normalized size = 0.90

$$x^9\left(\frac{a}{3}-\frac{256}{3}\right)-x^8\left(3a-192\right)-x^6\left(40a-384\right)+x^7\left(\frac{96a}{7}-320\right)+x^4\left(3a^2-96a+128\right)-x^5\left(\frac{3a^2}{5}-\frac{384a}{5}+\frac{153}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^9*(a/3 - 256/3) - x^8*(3*a - 192) - x^6*(40*a - 384) + x^7*((96*a)/7 - 32
0) + x^4*(3*a^2 - 96*a + 128) - x^5*((3*a^2)/5 - (384*a)/5 + 1536/5) + a^3*
x + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13 + 12*a^2*x^2 - 8*a*x^3*(a - 8)
```

sympy [A] time = 0.09, size = 114, normalized size = 0.95

$$a^3x+12a^2x^2-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+x^9\left(\frac{a}{3}-\frac{256}{3}\right)+x^8\left(192-3a\right)+x^7\left(\frac{96a}{7}-320\right)+x^6\left(384-40a\right)+x^5\left(-\frac{3a^2}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(a
/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a) +
x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*(-8
*a**2 + 64*a)
```

3.118 $\int \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx$

Optimal. Leaf size=72

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

```
[Out] a^2*x+8*a*x^2+16/3*(4-a)*x^3-2*(16-a)*x^4+2/5*(64-a)*x^5-40/3*x^6+32/7*x^7-
x^8+1/9*x^9
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 - a)*x^5)/5
- (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx = \int \left(a^2+16ax+16(4-a)x^2-8(16-a)x^3+2(64-a)x^4-80x^5+32x^6-8x\right.$$

$$= a^2x+8ax^2+\frac{16}{3}(4-a)x^3-2(16-a)x^4+\frac{2}{5}(64-a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8$$

Mathematica [A] time = 0.01, size = 66, normalized size = 0.92

$$a^2x-\frac{2}{5}(a-64)x^5+2(a-16)x^4-\frac{16}{3}(a-4)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5)
/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

fricas [A] time = 0.37, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{40}{3}x^6 - \frac{2}{5}x^5a + \frac{128}{5}x^5 + 2x^4a - 32x^4 - \frac{16}{3}x^3a + \frac{64}{3}x^3 + 8x^2a + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 - 2/5*x^5*a + 128/5*x^5 + 2*x^4*a - 32*
x^4 - 16/3*x^3*a + 64/3*x^3 + 8*x^2*a + x*a^2
```

giac [A] time = 0.31, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{2}{5}ax^5 - \frac{40}{3}x^6 + 2ax^4 + \frac{128}{5}x^5 - \frac{16}{3}ax^3 - 32x^4 + a^2x + 8ax^2 + \frac{64}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16/
3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3
```

maple [A] time = 0.00, size = 63, normalized size = 0.88

$$\frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + \frac{(-2a+128)x^5}{5} + \frac{(8a-128)x^4}{4} + a^2x + 8ax^2 + \frac{(-16a+64)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/9*x^9-x^8+32/7*x^7-40/3*x^6+1/5*(-2*a+128)*x^5+1/4*(8*a-128)*x^4+1/3*(-16
*a+64)*x^3+8*a*x^2+a^2*x
```

maxima [A] time = 0.71, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{40}{3}x^6 + \frac{128}{5}x^5 - 32x^4 + a^2x + \frac{64}{3}x^3 - \frac{2}{15}\left(3x^5 - 15x^4 + 40x^3 - 60x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^3
 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a
```

mupad [B] time = 0.04, size = 61, normalized size = 0.85

$$x^4\,\left(2\,a-32\right)-x^3\,\left(\frac{16\,a}{3}-\frac{64}{3}\right)-x^5\,\left(\frac{2\,a}{5}-\frac{128}{5}\right)+8\,a\,x^2+a^2\,x-\frac{40\,x^6}{3}+\frac{32\,x^7}{7}-x^8+\frac{x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^4*(2*a - 32) - x^3*((16*a)/3 - 64/3) - x^5*((2*a)/5 - 128/5) + 8*a*x^2 +
a^2*x - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

sympy [A] time = 0.08, size = 65, normalized size = 0.90

$$a^2x+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+x^5\left(\frac{128}{5}-\frac{2a}{5}\right)+x^4\left(2a-32\right)+x^3\left(\frac{64}{3}-\frac{16a}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 - 2
*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)
```

3.119 $\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right) dx$

Optimal. Leaf size=26

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

```
[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Antiderivative was successfully verified.

```
[In] Int[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Rubi steps

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right) dx = ax + 4x^2 - \frac{8x^3}{3} + x^4 - \frac{x^5}{5}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

fricas [A] time = 0.35, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + 4x^2 + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + 4*x^2 + x*a
```

giac [A] time = 0.30, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4+4*x^3-8*x^2+a+8*x,x)
[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

maxima [A] time = 0.63, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

mupad [B] time = 0.02, size = 22, normalized size = 0.85

$$-\frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + 8*x - 8*x^2 + 4*x^3 - x^4,x)
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

sympy [A] time = 0.06, size = 22, normalized size = 0.85

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**4+4*x**3-8*x**2+a+8*x,x)
```

```
[Out] a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2
```

3.120 $\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=89

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

```
[Out] -1/2*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)
+1/2*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1106, 1093, 204}

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]
]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4
+ a]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}}+\frac{\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}}\\ &= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}}-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.64

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 + 4*#1 -
 3*#1^2 + #1^3) & ]
```

fricas [B] time = 0.45, size = 457, normalized size = 5.13

$$\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)-\frac{1}{4}\sqrt{\frac{\frac{a^2+7}{\sqrt{a^3+10a^2}}}{a^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a +
 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((
```

```
a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x -
 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 +
 7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*
sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)
) + x - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1
)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36
) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7
*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a +
 36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 +
33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)
/(a^2 + 7*a + 12)) + x - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a]=[86]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a]=[12]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a]=[48]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a]=[34]-sqrt(1/16)*sqrt(1/256*(256*sqrt(a+4)*(a+4)-256*a-1024)/(-a^3
-11*a^2-40*a-48))*ln(abs(-a^5*sqrt(a+4)+a^5+a^4*sqrt(sqrt(a+4)*(-a^2-7*a-12
)+a^2+7*a+12)*x-a^4*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-17*a^4*sqrt(a+
4)+17*a^4-a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x+a^3*sqrt
(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)+14*a^3*sqrt(sqrt(a+4)*(-a^2-
7*a-12)+a^2+7*a+12)*x-14*a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-111*a
^3*sqrt(a+4)+111*a^3-10*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a
+4)*x+10*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)+69*a^2*sqrt
(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x-69*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+
a^2+7*a+12)-351*a^2*sqrt(a+4)+351*a^2-29*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2
+7*a+12)*sqrt(a+4)*x+29*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4
)+144*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x-144*a*sqrt(sqrt(a+4)*(-a
^2-7*a-12)+a^2+7*a+12)-544*a*sqrt(a+4)+544*a-28*sqrt(sqrt(a+4)*(-a^2-7*a-12
)+a^2+7*a+12)*sqrt(a+4)*x+28*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(
a+4)+112*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x-112*sqrt(sqrt(a+4)*(-a^
2-7*a-12)+a^2+7*a+12)-336*sqrt(a+4)+336))+sqrt(1/16)*sqrt(1/256*(256*sqrt(a
+4)*(a+4)-256*a-1024)/(-a^3-11*a^2-40*a-48))*ln(abs(-a^5*sqrt(a+4)+a^5-a^4*
sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x+a^4*sqrt(sqrt(a+4)*(-a^2-7*a-12)
```

```
+a^2+7*a+12)-17*a^4*sqrt(a+4)+17*a^4+a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7
*a+12)*sqrt(a+4)*x-a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)-1
4*a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x+14*a^3*sqrt(sqrt(a+4)*(-a^
2-7*a-12)+a^2+7*a+12)-111*a^3*sqrt(a+4)+111*a^3+10*a^2*sqrt(sqrt(a+4)*(-a^2
-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x-10*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*
a+12)*sqrt(a+4)-69*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x+69*a^2*sq
rt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-351*a^2*sqrt(a+4)+351*a^2+29*a*sqrt(
sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x-29*a*sqrt(sqrt(a+4)*(-a^2-7
*a-12)+a^2+7*a+12)*sqrt(a+4)-144*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)
*x+144*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-544*a*sqrt(a+4)+544*a+28*
sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x-28*sqrt(sqrt(a+4)*(-a^
2-7*a-12)+a^2+7*a+12)*sqrt(a+4)-112*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12
)*x+112*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-336*sqrt(a+4)+336))-sqrt(1
/16)*sqrt(1/256*(-256*sqrt(a+4)*(a+4)-256*a-1024)/(-a^3-11*a^2-40*a-48))*ln
(abs(a^5*sqrt(a+4)+a^5+a^4*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*x-a^4*sq
rt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+17*a^4*sqrt(a+4)+17*a^4+a^3*sqrt(sqrt
(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x-a^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+
a^2+7*a+12)*sqrt(a+4)+14*a^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*x-14*a
^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+111*a^3*sqrt(a+4)+111*a^3+10*a^2
*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x-10*a^2*sqrt(sqrt(a+4)*
(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)+69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7
*a+12)*x-69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+351*a^2*sqrt(a+4)+3
51*a^2+29*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x-29*a*sqrt(s
qrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)+144*a*sqrt(sqrt(a+4)*(a^2+7*a+1
2)+a^2+7*a+12)*x-144*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+544*a*sqrt(a
+4)+544*a+28*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x-28*sqrt(sq
rt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)+112*sqrt(sqrt(a+4)*(a^2+7*a+12)+
a^2+7*a+12)*x-112*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+336*sqrt(a+4)+336
))+sqrt(1/16)*sqrt(1/256*(-256*sqrt(a+4)*(a+4)-256*a-1024)/(-a^3-11*a^2-40*
a-48))*ln(abs(a^5*sqrt(a+4)+a^5-a^4*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)
*x+a^4*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+17*a^4*sqrt(a+4)+17*a^4-a^3*
sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+a^3*sqrt(sqrt(a+4)*(a^2
+7*a+12)+a^2+7*a+12)*sqrt(a+4)-14*a^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+1
2)*x+14*a^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+111*a^3*sqrt(a+4)+111*a
^3-10*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+10*a^2*sqrt(s
qrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)-69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+
12)+a^2+7*a+12)*x+69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+351*a^2*sq
rt(a+4)+351*a^2-29*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+29
*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)-144*a*sqrt(sqrt(a+4)*(
a^2+7*a+12)+a^2+7*a+12)*x+144*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+544
*a*sqrt(a+4)+544*a-28*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+2
8*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)-112*sqrt(sqrt(a+4)*(a^2
+7*a+12)+a^2+7*a+12)*x+112*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+336*sqrt
(a+4)+336))
```

maple [C] time = 0.02, size = 49, normalized size = 0.55

$$-\frac{\ln\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+x\right)}{4\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\,\mathrm{RootOf}\left(\_Z^4-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(1/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

mupad [B] time = 2.58, size = 571, normalized size = 6.42

$$-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}+x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}-\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2\,x\,1\mathrm{i}+a}{44\,a^2\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+160\,a\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192\,\sqrt{\frac{a-}{1}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] - atan(-(a*8i - x*16i + x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i - (4
8*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(44*a^2*((a -
(48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/
2) + 4*a^3*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 1
6*a^3 + 768))^(1/2) + 160*a*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(64
0*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a - (48*a + 12*a^2 + a^3 + 64)
^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a - (48*a + 12*a^2
+ a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i - atan(-(
a*8i - x*16i - x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i + (48*a + 12*
a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(160*a*((a + (48*a + 12
*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*(
```

```
(a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768)
)^(1/2) + 44*a^2*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a
^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) +
4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a + (48*a + 12*a^2 + a^3 + 6
4)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i
```

sympy [A] time = 0.93, size = 66, normalized size = 0.74

$$-\operatorname{RootSum}\left(t^{4}\left(256a^{3}+2816a^{2}+10240a+12288\right)+t^{2}\left(-32a-128\right)-1,\left(t\mapsto t\log\left(64t^{3}a^{2}+448t^{3}a+768t^{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 12
8) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*a
- 20*_t + x - 1)))
```

3.121 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal. Leaf size=169

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan}{8(a+3)(a+4)^{3/2}\sqrt{}}$$

```
[Out] 1/4*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)-1/8*arctan
((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1-(4
+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(10+3*a-(4+a)^(1/
2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1106, 1092, 1166, 204}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan}{8(a+3)(a+4)^{3/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 +
 a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[
4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sq
rt[1 + Sqrt[4 + a]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
```

```
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-2(4+4(3\cdots}{3+a-2x^2-x\cdots}\right.}{8\left(12+7a\cdots\right.}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} - \frac{\left(10+3a-\sqrt{4+a}\right)\text{Sub}\cdots}{8(3+\cdots}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{\left(10+3a+\sqrt{4+a}\right)\tan\cdots}{8(3+a)(4+a)^{3/2}\sqrt{1\cdots}}$$

Mathematica [C] time = 0.07, size = 150, normalized size = 0.89

$$\frac{(x-1)\left(a+x^2-2x+6\right)}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)} - \frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1^2\log(x-\#1)+3a\log(x-\#1)-2\cdots}{\#1^3-3\#1^2\cdots}\right.}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
+ x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (12*Log[x - #1] +
3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2
+ #1^3) & ]/(16*(12 + 7*a + a^2))
```

fricas [B] time = 0.44, size = 1948, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^
2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^
6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 5
58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 1
83*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a
 + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 29
75*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 96
1)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3
+ 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21
*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a +
 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a
^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4
 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7*a + 12)*
x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^
2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 +
2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 302
4*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 408*a^3 + 2
309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 +
24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088
*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))
 + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a
^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3
088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 4665
6)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a +
1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^
3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*
```

```
a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((8
1*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598
*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 2
1*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^
2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513
*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 5
58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 -
(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2
+ 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 +
 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5
+ 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 8
47*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^
8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 +
128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*
a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 40
8*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 214
20*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 5800*a + 5456)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3
 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 39
9*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304
*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 +
3024*a + 1728)) - 567*a - 992) + 4*(a + 8)*x - 12*x^2 - 4*a - 24)/((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a]=[86]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a]=[12]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a]=[48]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a]=[34](x^3-3*x^2+x*a+8*x-a-6)/(-4*a^2-28*a-48)/(x^4-4*x^3+8*x^2-8*x
```

-a)+(sqrt(1/16)*sqrt(1/256*(256*sqrt(a+4)*(9*a^3+103*a^2+392*a+496)+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*ln(abs(243*a^10*sqrt(a+4)+324*a^10+8640*a^9*sqrt(a+4)+11466*a^9+81*a^8*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-81*a^8*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+138027*a^8*sqrt(a+4)+182314*a^8+81*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-81*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+2340*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2340*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+1304648*a^7*sqrt(a+4)+1715172*a^7+2016*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-2016*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+29518*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-29518*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+8079749*a^6*sqrt(a+4)+10572392*a^6+21454*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-21454*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+212356*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-212356*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+34255200*a^5*sqrt(a+4)+44613658*a^5+126540*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-126540*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+952845*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-952845*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+100679657*a^4*sqrt(a+4)+130513730*a^4+446685*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-446685*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+2730184*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2730184*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+202540404*a^3*sqrt(a+4)+261341928*a^3+943444*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-943444*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+4877364*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-4877364*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+266882676*a^2*sqrt(a+4)+342778384*a^2+1103588*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-1103588*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+4965684*a*sqrt(sqrt(

a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+220
8)*x-4965684*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+21
0*a^3+1099*a^2+2548*a+2208)+207974132*a*sqrt(a+4)+265897256*a+551332*sqrt(s
qrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a
+2208)*sqrt(a+4)*x-551332*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488
)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+2205328*sqrt(sqrt(a+4)*(9*
a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-220
5328*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099
*a^2+2548*a+2208)+72775824*sqrt(a+4)+92623776))-sqrt(1/16)*sqrt(1/256*(256*
sqrt(a+4)*(9*a^3+103*a^2+392*a+496)+3840*a^3+42240*a^2+154624*a+188416)/(a^
3+11*a^2+40*a+48))*ln(abs(243*a^10*sqrt(a+4)+324*a^10+8640*a^9*sqrt(a+4)+11
466*a^9-81*a^8*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+21
0*a^3+1099*a^2+2548*a+2208)*x+81*a^8*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+
1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+138027*a^8*sqrt(a+4)+1823
14*a^8-81*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210
*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+81*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3
+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2340*a
^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a
^2+2548*a+2208)*x+2340*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+148
8)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+1304648*a^7*sqrt(a+4)+1715172*a^7-2
016*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1
099*a^2+2548*a+2208)*sqrt(a+4)*x+2016*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701
*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-29518*a^6*
sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+
2548*a+2208)*x+29518*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)
+15*a^4+210*a^3+1099*a^2+2548*a+2208)+8079749*a^6*sqrt(a+4)+10572392*a^6-21
454*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1
099*a^2+2548*a+2208)*sqrt(a+4)*x+21454*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+70
1*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-212356*a^
5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*x+212356*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+14
88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+34255200*a^5*sqrt(a+4)+44613658*a^
5-126540*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*
a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+126540*a^4*sqrt(sqrt(a+4)*(9*a^4+130*
a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-952
845*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1
099*a^2+2548*a+2208)*x+952845*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+167
2*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+100679657*a^4*sqrt(a+4)+1305
13730*a^4-446685*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*
a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+446685*a^3*sqrt(sqrt(a+4)*(9*
a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(
a+4)-2730184*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+
210*a^3+1099*a^2+2548*a+2208)*x+2730184*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+7
01*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+202540404*a^3*sqrt
(a+4)+261341928*a^3-943444*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a

+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+943444*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-4877364*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4877364*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+266882676*a^2*sqrt(a+4)+342778384*a^2-1103588*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+1103588*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-4965684*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4965684*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+207974132*a*sqrt(a+4)+265897256*a-551332*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+551332*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2205328*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2205328*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+72775824*sqrt(a+4)+92623776))+sqrt(1/16)*sqrt(1/256*(-256*sqrt(a+4)*(9*a^3+103*a^2+392*a+496)+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*ln(abs(-243*a^10*sqrt(a+4)+324*a^10-8640*a^9*sqrt(a+4)+11466*a^9+81*a^8*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-81*a^8*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-138027*a^8*sqrt(a+4)+182314*a^8-81*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+81*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+2340*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2340*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-1304648*a^7*sqrt(a+4)+1715172*a^7-2016*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+2016*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+29518*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-29518*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-8079749*a^6*sqrt(a+4)+10572392*a^6-21454*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+21454*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+212356*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-212356*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-34255200*a^5*sqrt(a+4)+44613658*a^5-126540*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+126540*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+952845*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*

a+2208)*x-952845*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15
*a^4+210*a^3+1099*a^2+2548*a+2208)-100679657*a^4*sqrt(a+4)+130513730*a^4-44
6685*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3
+1099*a^2+2548*a+2208)*sqrt(a+4)*x+446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^
3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+27301
84*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1
099*a^2+2548*a+2208)*x-2730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1
672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-202540404*a^3*sqrt(a+4)+26
1341928*a^3-943444*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+
15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+943444*a^2*sqrt(sqrt(a+4)*
(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*s
qrt(a+4)+4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15
*a^4+210*a^3+1099*a^2+2548*a+2208)*x-4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4-130
*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-266882676*a^
2*sqrt(a+4)+342778384*a^2-1103588*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-
1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+1103588*a*sqr
t(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+25
48*a+2208)*sqrt(a+4)+4965684*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*
a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-4965684*a*sqrt(sqrt(a+4)*(-9
*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-2079
74132*a*sqrt(a+4)+265897256*a-551332*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2
-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+551332*sqrt(
sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548
*a+2208)*sqrt(a+4)+2205328*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-14
88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2205328*sqrt(sqrt(a+4)*(-9*a^4-1
30*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-72775824*s
qrt(a+4)+92623776))-sqrt(1/16)*sqrt(1/256*(-256*sqrt(a+4)*(9*a^3+103*a^2+39
2*a+496)+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*ln(abs(-
243*a^10*sqrt(a+4)+324*a^10-8640*a^9*sqrt(a+4)+11466*a^9-81*a^8*sqrt(sqrt(a
+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+220
8)*x+81*a^8*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*
a^3+1099*a^2+2548*a+2208)-138027*a^8*sqrt(a+4)+182314*a^8+81*a^7*sqrt(sqrt(
a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+22
08)*sqrt(a+4)*x-81*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+
15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2340*a^7*sqrt(sqrt(a+4)*(-9*
a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+234
0*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+10
99*a^2+2548*a+2208)-1304648*a^7*sqrt(a+4)+1715172*a^7+2016*a^6*sqrt(sqrt(a+
4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208
)*sqrt(a+4)*x-2016*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+
15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-29518*a^6*sqrt(sqrt(a+4)*(-9
*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+29
518*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+
1099*a^2+2548*a+2208)-8079749*a^6*sqrt(a+4)+10572392*a^6+21454*a^5*sqrt(sqr
t(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+

```
2208)*sqrt(a+4)*x-21454*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1
488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-212356*a^5*sqrt(sqrt(a+
4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208
)*x+212356*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+2
10*a^3+1099*a^2+2548*a+2208)-34255200*a^5*sqrt(a+4)+44613658*a^5+126540*a^4
*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*sqrt(a+4)*x-126540*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^
2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-952845*a^4*sq
rt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2
548*a+2208)*x+952845*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488
)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-100679657*a^4*sqrt(a+4)+130513730*a^
4+446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210
*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-13
0*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2
730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a
^3+1099*a^2+2548*a+2208)*x+2730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a
^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-202540404*a^3*sqrt(a+4
)+261341928*a^3+943444*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-14
88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-943444*a^2*sqrt(sqrt(a
+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+220
8)*sqrt(a+4)-4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488
)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4
-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-26688267
6*a^2*sqrt(a+4)+342778384*a^2+1103588*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*
a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-1103588*a
*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*sqrt(a+4)-4965684*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1
672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4965684*a*sqrt(sqrt(a+4)
*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-
207974132*a*sqrt(a+4)+265897256*a+551332*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701
*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-551332*s
qrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+
2548*a+2208)*sqrt(a+4)-2205328*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*
a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2205328*sqrt(sqrt(a+4)*(-9*a
^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-727758
24*sqrt(a+4)+92623776)))/(4*a^2+28*a+48)
```

maple [C] time = 0.01, size = 158, normalized size = 0.93

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+2\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-3a-12\right)\ln\left(-\mathrm{Ro}\right.}{16\,(a+3)\,(a+4)\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\,\mathrm{I}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a^2+7*a+12)*x^3+3/4/(a^2+7*a+12)*x^2-1/4*(8+a)/(a^2+7*a+12)*x+1/4*(6
+a)/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(3+a)/(4+a)*sum((-_R^2+2*_R-
3*a-12)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^3+(a+8)x-3\,x^2-a-6}{4\left(\left(a^2+7\,a+12\right)x^4-4\left(a^2+7\,a+12\right)x^3-a^3+8\left(a^2+7\,a+12\right)x^2-7\,a^2-8\left(a^2+7\,a+12\right)x-12\,a\right)}-\frac{\int\frac{x^2+}{x^4-4}}{4\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a
 + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x -
12*a) - 1/4*integrate((x^2 + 3*a - 2*x + 12)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x
), x)/(a^2 + 7*a + 12)
```

mupad [B] time = 5.35, size = 4591, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] atan(-(((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2
+ 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632
*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592))
)^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^
5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576
)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/
(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2
) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/
(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6
 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 1067
52*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2)
 + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*
a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^
9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460
*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a
+ 73*a^2 + 14*a^3 + a^4 + 144)))*1i + ((15552*a - 9*a*((a + 4)^9)^(1/2) - 3
```

```
1*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*
(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 48
3*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997
696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2
 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 +
 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*(
(15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a
^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 8
1744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)
+ (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(
816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4
115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 1
44*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(
61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i)/((9*a +
 32)/(32*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + ((15552*a - 9*
a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4
+ 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22
488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*
a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 943718
4)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 1
17760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 +
14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1
/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 3064
32*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 +
 a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 +
768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((
15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^
3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81
744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) +
 (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a
^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a
^4 + 144))) - ((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 82
08*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 +
 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 1
10592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 9
0112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^
5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 14
7456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4
115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2
 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^
3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9
```

```
)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a +
306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a
^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*
a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*
(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))))*((15552*a - 9*a*((a + 4)^9)^(1/2)
 - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(
256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6
+ 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*2i + atan(-(((15552*a + 9*a*((a
+ 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a
^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^
5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10
878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64
*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*
a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3
 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) +
8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2
 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 +
 110592)))^(1/2) - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^
5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*
a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 28
5*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^
4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568
*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a
^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 1
44)))*1i + ((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*
a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 19
7632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 1105
92)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 9011
2*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 +
 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 14745
6))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^
(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 117
76)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115
*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2 +
106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 +
 18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(
1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306
432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8
+ a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a +
 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(16
8*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i)/((9*a + 32)/(32*(816*a + 460*a^2 +
 129*a^3 + 18*a^4 + a^5 + 576)) + ((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((
a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276
480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^
7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*
```

```
a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 1
29*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 460
8*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((155
52*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 +
 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744
*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (7
33184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*
a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^(
1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 1177
6)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*
a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a
^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a
 + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))) - ((15552*a + 9
*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4
 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 2
2488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640
*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 94371
84)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a +
117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 +
 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(
1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306
432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8
+ a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 +
 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*(
(15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a
^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 8
1744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)
+ (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 +
a^4 + 144)))))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8
208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2
+ 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 +
110592)))^(1/2)*2i + (x^3/(4*(7*a + a^2 + 12)) - (a + 6)/(4*(a + 3)*(a + 4)
) - (3*x^2)/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(4*(a + 3)*(a + 4)))/(a + 8*x
 - 8*x^2 + 4*x^3 - x^4)
```

sympy [B] time = 6.32, size = 294, normalized size = 1.74

$$\frac{a - x^3 + 3x^2 + x(-a-8) + 6}{-4a^3 - 28a^2 - 48a + x^4\left(4a^2 + 28a + 48\right) + x^3\left(-16a^2 - 112a - 192\right) + x^2\left(32a^2 + 224a + 384\right) + x\left(-32a^2 - 22\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (a - x**3 + 3*x**2 + x*(-a - 8) + 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a*
*2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 3
84) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**
8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12
952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-7
680*a**5 - 145920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 6029312)
 - 81*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7 - 40140
8*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 84549632*_t**3*a*
*3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 99090432*_t**3 + 432*_t*a**
4 + 7488*_t*a**3 + 47024*_t*a**2 + 128096*_t*a + 128512*_t - 81*a**2 - 567*
a - 992)/(81*a**2 + 567*a + 992))))
```

3.122 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$

Optimal. Leaf size=252

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

```
[Out] 1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+1/32*((6
+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+
x)^4)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)^(1/2)+a*
(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/64*arctan((
-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2))/(3+a)^2/
(4+a)^2/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1106, 1092, 1178, 1166, 204}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a
^2 + 14*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt
[4 + a]]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*
a - (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]
)/(64*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^3}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{8\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-4(4+}{(3+a-2x}\right.}{16\left(12\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-\right.}{8\left(12+7a+a^2\right)\left(3+a\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-\right.}{8\left(12+7a+a^2\right)\left(3+a\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-\right.}{8\left(12+7a+a^2\right)\left(3+a\right.}$$

Mathematica [C] time = 0.15, size = 254, normalized size = 1.01

$$\frac{1}{128}\left(-\frac{3\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{4\#1^2 a\log(x-\#1)+14\#1^2\log(x-\#1)+7a^2\log(x-\#1)+55a\log(x-\#1)-8\#1a\log(}{\#1^3-3\#1^2+4\#1-2}\right.}{\left(a^2+7a+12\right)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^
2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x +
12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootS
um[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (108*Log[x - #1] + 55*a*Log[x - #1
] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Log[x -
 #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*
a + a^2)^2)/128
```

fricas [B] time = 0.52, size = 3971, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a + 1116)*x^5
 - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1968)*x^3 + 44*a^3 -
8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^
8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2
 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 +
14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^
4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16
*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 +
14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 +
 (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4
+ 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 +
 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 2
41870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^
2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 +
5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 +
725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^
4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6
+ 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11
 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410
692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 338
41152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 +
 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^
9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940
*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a
 + 3543424)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8
+ 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2
+ 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 3
46921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^1
0 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a
^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176
)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129
367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 1122
8868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 +
 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 1
44)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(
a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 -
 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a
^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*
a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*
a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3
```

+ 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2
+ 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11
+ 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6
+ 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 27713664
0*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 3
1085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 2
48832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^
3 + 177061*a^2 + 415884*a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5
+ 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10
+ 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 11
7844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt(
(2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1
165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320
*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 78207
1200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*
sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 +
31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a +
248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15
+ 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*
a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 7143179
40*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a
+ 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 37
3020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 992
3472) - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^
6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4
- 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151
*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3
- 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^
2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^
8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^
2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a +
346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a
^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313
*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 604661
76)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 1
29367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(
-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a
^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^
4 + 2354874*a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^
9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4
+ 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 +
33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 1
6780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320
045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 5

92064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^
4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 +
 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqr
t((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 +
 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 285703
20*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782
071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a
^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 7
35840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 3*((
a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a
 + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4
+ 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 -
 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*
a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 -
528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x
)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7
 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a
 + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^
15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 672213
0*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 71431
7940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228
*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 +
373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 -
 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*
a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*
a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 810205*a
^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400*
a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 + 33124*a^3 +
171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 24
1870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2
 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^4 + 1470*a^3
 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5
+ 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4
+ 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 +
 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 943
20045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 +
 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9
 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 +
950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 524*a^2 - 4*(11*
a^3 + 107*a^2 - 84*a - 1152)*x + 1632*a + 1152)/((a^4 + 14*a^3 + 73*a^2 + 1
68*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 1
4*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a
 + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 921
6)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3

```
+ 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^
2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a]=[86]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a]=[12]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a]=[48]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a]=[34]-(-12*x^7*a-42*x^7+84*x^6*a+294*x^6-7*x^5*a^2-343*x^5*a-1116*
x^5+35*x^4*a^2+875*x^4*a+2640*x^4-68*x^3*a^2-1358*x^3*a-3936*x^3+64*x^2*a^2
+1246*x^2*a+3600*x^2+11*x*a^3+107*x*a^2-84*x*a-1152*x-11*a^3-131*a^2-408*a-
288)/(-32*a^4-448*a^3-2336*a^2-5376*a-4608)/(x^4-4*x^3+8*x^2-8*x-a)^2-(sqrt
(9/16)*sqrt(1/256*(256*sqrt(a+4)*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a
+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-22798336*a-16531456)/
(-a^3-11*a^2-40*a-48))*ln(abs(-16807*a^15*sqrt(a+4)+26411*a^15-908950*a^14*
sqrt(a+4)+1420804*a^14-22929088*a^13*sqrt(a+4)+35650176*a^13+2401*a^12*sqrt
(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)
+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2401*a^
12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-
113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-35
7887692*a^12*sqrt(a+4)+553458148*a^12-2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-107
3*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2401*a^11*sqrt(sqrt(
a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a
^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+10515
4*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18740
8*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)*x-105154*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*
a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744
*a+193728)-3865394166*a^11*sqrt(a+4)+5945365998*a^11-95550*a^10*sqrt(sqrt(a
+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+9555
0*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18740
8*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)*sqrt(a+4)+2109279*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^
```

3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a
^2+331744*a+193728)*x-2109279*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^
4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^
3+236728*a^2+331744*a+193728)-30600511272*a^10*sqrt(a+4)+46810709868*a^10-1
727079*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-1
87408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+19
3728)*sqrt(a+4)*x+1727079*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-474
19*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*sqrt(a+4)+25624212*a^9*sqrt(sqrt(a+4)*(-49*a^6-107
3*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-25624212*a^9*sqrt(sqrt(a+4)*(-
49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205
*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-183431725500*a^9*sqrt(
a+4)+279067335420*a^9-18715896*a^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^
4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^
3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+18715896*a^8*sqrt(sqrt(a+4)*(-49*
a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^
5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+209972223*a^8*s
qrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-1130
88)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2099
72223*a^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18
7408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193
728)-847810669320*a^8*sqrt(a+4)+1282741275000*a^8-135108639*a^7*sqrt(sqrt(a
+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+1351
08639*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18
7408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193
728)*sqrt(a+4)+1222644882*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-474
19*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*x-1222644882*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-
9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+9
0111*a^3+236728*a^2+331744*a+193728)-3046208716923*a^7*sqrt(a+4)+4583471076
759*a^7-682210326*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-1
29188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+
331744*a+193728)*sqrt(a+4)*x+682210326*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5
-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+
90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+5187446733*a^6*sqrt(sqrt(a+
4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6
+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-5187446733*a^6*
sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113
088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-85085
27261290*a^6*sqrt(a+4)+12731345334296*a^6-2458605429*a^5*sqrt(sqrt(a+4)*(-4
9*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*
a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2458605429*
a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a

-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*s
qrt(a+4)+16158435972*a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^
3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a
^2+331744*a+193728)*x-16158435972*a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775
*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111
*a^3+236728*a^2+331744*a+193728)-18324012543204*a^5*sqrt(a+4)+2726574704738
0*a^5-6324014256*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-12
9188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3
31744*a+193728)*sqrt(a+4)*x+6324014256*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5
-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+
90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+36673732452*a^4*sqrt(sqrt(a
+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-36673732452*a^
4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-1
13088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-298
80339194272*a^4*sqrt(a+4)+44213263379848*a^4-11377675428*a^3*sqrt(sqrt(a+4)
*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2
205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+1137767
5428*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187
408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)+59146408708*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-474
19*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*x-59146408708*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5
-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+
90111*a^3+236728*a^2+331744*a+193728)-35712575419864*a^3*sqrt(a+4)+52547642
661032*a^3-13635706996*a^2*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*
a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728
*a^2+331744*a+193728)*sqrt(a+4)*x+13635706996*a^2*sqrt(sqrt(a+4)*(-49*a^6-1
073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+192
99*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+64340036640*a^2*sqrt
(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)
+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-6434003
6640*a^2*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187
408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)-29533943648028*a^2*sqrt(a+4)+43213040637212*a^2-9797208656*a*sqrt(sqrt(
a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a
^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+979
7208656*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18
7408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193
728)*sqrt(a+4)+42385864836*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-4741
9*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2367
28*a^2+331744*a+193728)*x-42385864836*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-97
75*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+901
11*a^3+236728*a^2+331744*a+193728)-15111479733208*a*sqrt(a+4)+2198667320430
4*a-3197030212*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a

^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+3197030212*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+12788120848*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-12788120848*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3606250079136*sqrt(a+4)+5217553305984))-sqrt(9/16)*sqrt(1/256*(256*sqrt(a+4)*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*ln(abs(-16807*a^15*sqrt(a+4)+26411*a^15-908950*a^14*sqrt(a+4)+1420804*a^14-22929088*a^13*sqrt(a+4)+35650176*a^13-2401*a^12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+2401*a^12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-357887692*a^12*sqrt(a+4)+553458148*a^12+2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-105154*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+105154*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3865394166*a^11*sqrt(a+4)+5945365998*a^11+95550*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-95550*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-2109279*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+2109279*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-30600511272*a^10*sqrt(a+4)+46810709868*a^10+1727079*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-1727079*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-25624212*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+25624212*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-183431725500*a^9*sqrt(a+4)+279067335420*a^9+18715896*a^8*sqrt(sqrt(a+4)*(-

49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205
*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-18715896*a
^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-
113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sq
rt(a+4)-209972223*a^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-1
29188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+
331744*a+193728)*x+209972223*a^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-
47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+
236728*a^2+331744*a+193728)-847810669320*a^8*sqrt(a+4)+1282741275000*a^8+13
5108639*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-
187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*sqrt(a+4)*x-135108639*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-
47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+
236728*a^2+331744*a+193728)*sqrt(a+4)-1222644882*a^7*sqrt(sqrt(a+4)*(-49*a^
6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+
19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+1222644882*a^7*sqrt(sqrt(
a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a
^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3046208716923*a
^7*sqrt(a+4)+4583471076759*a^7+682210326*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a
^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^
4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-682210326*a^6*sqrt(sqrt
(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-5187
446733*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-1
87408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+19
3728)*x+5187446733*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-
129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2
+331744*a+193728)-8508527261290*a^6*sqrt(a+4)+12731345334296*a^6+2458605429
*a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*
a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*
sqrt(a+4)*x-2458605429*a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*
a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728
*a^2+331744*a+193728)*sqrt(a+4)-16158435972*a^5*sqrt(sqrt(a+4)*(-49*a^6-107
3*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+16158435972*a^5*sqrt(sqrt(a+4)
*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2
205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-18324012543204*a^5*
sqrt(a+4)+27265747047380*a^5+6324014256*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^
5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4
+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-6324014256*a^4*sqrt(sqrt
(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-3667
3732452*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-
187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*x+36673732452*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^

3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a
^2+331744*a+193728)-29880339194272*a^4*sqrt(a+4)+44213263379848*a^4+1137767
5428*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187
408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)*x-11377675428*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-4
7419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2
36728*a^2+331744*a+193728)*sqrt(a+4)-59146408708*a^3*sqrt(sqrt(a+4)*(-49*a^
6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+
19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+59146408708*a^3*sqrt(sqrt
(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-35712575419864
*a^3*sqrt(a+4)+52547642661032*a^3+13635706996*a^2*sqrt(sqrt(a+4)*(-49*a^6-1
073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+192
99*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-13635706996*a^2*sq
rt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-11308
8)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+
4)-64340036640*a^2*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-1291
88*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331
744*a+193728)*x+64340036640*a^2*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-4
7419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2
36728*a^2+331744*a+193728)-29533943648028*a^2*sqrt(a+4)+43213040637212*a^2+
9797208656*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2
-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+
193728)*sqrt(a+4)*x-9797208656*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-
47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+
236728*a^2+331744*a+193728)*sqrt(a+4)-42385864836*a*sqrt(sqrt(a+4)*(-49*a^6
-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+1
9299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+42385864836*a*sqrt(sqrt(a+
4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6
+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-15111479733208*a*
sqrt(a+4)+21986673204304*a+3197030212*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775
*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111
*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-3197030212*sqrt(sqrt(a+4)*(-49
*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a
^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-12788120848*sq
rt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-11308
8)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+12788
120848*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18740
8*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)-3606250079136*sqrt(a+4)+5217553305984))+sqrt(9/16)*sqrt(1/256*(-256*sqrt(
a+4)*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4
-3489024*a^3-12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*ln(ab
s(16807*a^15*sqrt(a+4)+26411*a^15+908950*a^14*sqrt(a+4)+1420804*a^14+229290
88*a^13*sqrt(a+4)+35650176*a^13+2401*a^12*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9
775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90

111*a^3+236728*a^2+331744*a+193728)*x-2401*a^12*sqrt(sqrt(a+4)*(49*a^6+1073
*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*
a^4+90111*a^3+236728*a^2+331744*a+193728)+357887692*a^12*sqrt(a+4)+55345814
8*a^12+2401*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*
a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744
*a+193728)*sqrt(a+4)*x-2401*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+4
7419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2
36728*a^2+331744*a+193728)*sqrt(a+4)+105154*a^11*sqrt(sqrt(a+4)*(49*a^6+107
3*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-105154*a^11*sqrt(sqrt(a+4)*(49
*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a
^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3865394166*a^11*sqrt(a+4
)+5945365998*a^11+95550*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)*sqrt(a+4)*x-95550*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a
^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^
4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+2109279*a^10*sqrt(sqrt(a+
4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+
2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2109279*a^10*sqr
t(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)
+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+306005112
72*a^10*sqrt(a+4)+46810709868*a^10+1727079*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*
a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a
^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-1727079*a^9*sqrt(sqrt(
a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+256242
12*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408
*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)
*x-25624212*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a
^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*
a+193728)+183431725500*a^9*sqrt(a+4)+279067335420*a^9+18715896*a^8*sqrt(sqr
t(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-18
715896*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+18
7408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193
728)*sqrt(a+4)+209972223*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)*x-209972223*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775
*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111
*a^3+236728*a^2+331744*a+193728)+847810669320*a^8*sqrt(a+4)+1282741275000*a
^8+135108639*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*
a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744
*a+193728)*sqrt(a+4)*x-135108639*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a
^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a
^3+236728*a^2+331744*a+193728)*sqrt(a+4)+1222644882*a^7*sqrt(sqrt(a+4)*(49*

a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-1222644882*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3046208716923*a^7*sqrt(a+4)+4583471076759*a^7+682210326*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-682210326*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+5187446733*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-5187446733*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+8508527261290*a^6*sqrt(a+4)+12731345334296*a^6+2458605429*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-2458605429*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+18324012543204*a^5*sqrt(a+4)+27265747047380*a^5+6324014256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-6324014256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+36673732452*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-36673732452*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29880339194272*a^4*sqrt(a+4)+44213263379848*a^4+11377675428*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-11377675428*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+59146408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-59146408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+35712575419864*a^3*sqrt(a+4)+52547642661032*a^3+13635706996*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-13635706996*a^2*sqrt(sqrt(a+4)*(49 *a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+64340036640*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-64340036640*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29533943648028*a^2*sqrt(a+4)+43213040637212*a^2+9797208656*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-9797208656*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+42385864836*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-42385864836*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+15111479733208*a*sqrt(a+4)+21986673204304*a+3197030212*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-3197030212*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+12788120848*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-12788120848*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3606250079136*sqrt(a+4)+5217553305984))-sqrt(9/16)*sqrt(1/256*(-256*sqrt(a+4)*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*ln(abs(16807*a^15*sqrt(a+4)+26411*a^15+908950*a^14*sqrt(a+4)+1420804*a^14+22929088*a^13*sqrt(a+4)+35650176*a^13-2401*a^12*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+2401*a^12*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+357887692*a^12*sqrt(a+4)+553458148*a^12-2401*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2401*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-105154*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+105154*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3865394166*a^11*sqrt(a+4)+5945365998*a^11-95550*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt

(a+4)*x+95550*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+12918
8*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3317
44*a+193728)*sqrt(a+4)-2109279*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^
4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^
3+236728*a^2+331744*a+193728)*x+2109279*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^
5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4
+90111*a^3+236728*a^2+331744*a+193728)+30600511272*a^10*sqrt(a+4)+468107098
68*a^10-1727079*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+1291
88*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331
744*a+193728)*sqrt(a+4)*x+1727079*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*
a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*
a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-25624212*a^9*sqrt(sqrt(a+4)*(49*a
^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5
+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+25624212*a^9*sqrt(sqrt(a
+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6
+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+183431725500*a^9*
sqrt(a+4)+279067335420*a^9-18715896*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+977
5*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9011
1*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+18715896*a^8*sqrt(sqrt(a+4)*(
49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205
*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-209972223*a^
8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+11
3088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+20
9972223*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+1
87408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+19
3728)+847810669320*a^8*sqrt(a+4)+1282741275000*a^8-135108639*a^7*sqrt(sqrt(
a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+1351
08639*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187
408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)-1222644882*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)*x+1222644882*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+977
5*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9011
1*a^3+236728*a^2+331744*a+193728)+3046208716923*a^7*sqrt(a+4)+4583471076759
*a^7-682210326*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+12918
8*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3317
44*a+193728)*sqrt(a+4)*x+682210326*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775
*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111
*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-5187446733*a^6*sqrt(sqrt(a+4)*(4
9*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*
a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+5187446733*a^6*sqrt(s
qrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+10
5*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+850852726129
0*a^6*sqrt(a+4)+12731345334296*a^6-2458605429*a^5*sqrt(sqrt(a+4)*(49*a^6+10

```
73*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+1929
9*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2458605429*a^5*sqrt
(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+
105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-
16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a
^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*
a+193728)*x+16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*
a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728
*a^2+331744*a+193728)+18324012543204*a^5*sqrt(a+4)+27265747047380*a^5-63240
14256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187
408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)*x+6324014256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+474
19*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*sqrt(a+4)-36673732452*a^4*sqrt(sqrt(a+4)*(49*a^6+1
073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+192
99*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+36673732452*a^4*sqrt(sqrt(a+
4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+
2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29880339194272*a^4
*sqrt(a+4)+44213263379848*a^4-11377675428*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a
^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^
4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+11377675428*a^3*sqrt(sq
rt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105
*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-591
46408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+
187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*x+59146408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3
+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^
2+331744*a+193728)+35712575419864*a^3*sqrt(a+4)+52547642661032*a^3-13635706
996*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+18740
8*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)*sqrt(a+4)*x+13635706996*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+4741
9*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2367
28*a^2+331744*a+193728)*sqrt(a+4)-64340036640*a^2*sqrt(sqrt(a+4)*(49*a^6+10
73*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+1929
9*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+64340036640*a^2*sqrt(sqrt(a+4
)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2
205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29533943648028*a^2*
sqrt(a+4)+43213040637212*a^2-9797208656*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9
775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90
111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+9797208656*a*sqrt(sqrt(a+4)
*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+22
05*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-4238586483
6*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+
113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+
42385864836*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2
```

```
+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+
193728)+15111479733208*a*sqrt(a+4)+21986673204304*a-3197030212*sqrt(sqrt(a+
4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+
2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+319703
0212*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a
+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*s
qrt(a+4)-12788120848*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129
188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+33
1744*a+193728)*x+12788120848*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)+3606250079136*sqrt(a+4)+5217553305984)))/(32*a^4+448
*a^3+2336*a^2+5376*a+4608)
```

maple [C] time = 0.02, size = 398, normalized size = 1.58

$$-\frac{3\left(2\left(2a+7\right)\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+7a^2+4\left(-2a-7\right)\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z\right.\right.}{128\left(a^3+10a^2+33a+36\right)\left(a+4\right)\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16)
/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^5-
5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+679*a
+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+14*a
^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+73*a^2
+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(a^4+14*a^3+73*a^2+168*a+144)
)/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(a+4)*sum((108+2*(7+
2*a)*_R^2+4*(-2*a-7)*_R+7*a^2+55*a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootO
f(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{6\left(2\,a+7\right)}{32\left(\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^8-8\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^7+32\left(a^4+14\,a^3+73\,a^2+16\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116)*x^5 - 5*
(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x^3 + 11*a^3 - 2*(32*
a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 + 107*a^2 - 84*a - 1152)*x + 40
```

```
8*a + 288)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 7
3*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a
^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^
4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 15
1*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3
 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a
^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 7*a^2 - 4*(2*a + 7)*x +
55*a + 108)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73*a^2 + 16
8*a + 144)
```

mupad [B] time = 6.41, size = 8242, normalized size = 32.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] atan(((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^
4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^
9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 +
 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (
(4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^
4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8
 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(94
0032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5
564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 402024038
4*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 74
38336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2
 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9
*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*(
(a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^
5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168
3200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5
+ 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^1
0 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1
/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) -
49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 26
95744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384
*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703
040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 196
6491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548039
68)))^(1/2) + (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 61
4016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) - (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*
```

(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)*1i - (((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - ((4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) - (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)*1i)/((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(9400

32*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 556
4*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + ((4290672328704*a + 6001143054
336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 30255
6119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a
^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328
*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 +
a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144
324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 1638
4*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 23
80*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)
^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*
a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 +
3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247
4311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a
^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12
+ 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*
((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 4
1598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 5394
9*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a
^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203
166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 223
50*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (108343296*a +
74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 662077
44)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 +
34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(73476*a +
31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*
a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3932979
2*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^1
5)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 47560
8*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a +
2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5736217
60*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 24496
5*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (((
52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 40271
70816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290
287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a
^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - ((42906723
28704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 10902
00272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266
176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2
678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7
+ 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*
a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3932979

2*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^1
5)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 47560
8*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a +
2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5736217
60*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 24496
5*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*
(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((
a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5
 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683
200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 +
 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10
 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/
2) - (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5
+ 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 44286
4*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 33177
6)) + (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a
+ 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 2
0736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2)
 - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 +
 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16
384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247
703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 +
1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548
03968)))^(1/2) - (99468*a + 28053*a^2 + 2646*a^3 + 117936)/(8192*(940032*a
+ 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7
 + 582*a^8 + 36*a^9 + a^10 + 331776))))*((9*(39329792*a - 338*a*((a + 4)^15
)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)*2i + atan(((((52357496832*a
+ 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 72
8506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384
*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6
 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + ((4290672328704*a + 6001
143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 +
 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 4194
3040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 +
899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*
a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3
+ 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8
+ 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^
4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((
a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 415

98976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*
a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2
 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 20316
6720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350
*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a +
338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/
2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6
+ 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 206143
4880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6
 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11
 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (10834329
6*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 +
66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208
*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(7347
6*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 +
28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3
9329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a
+ 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 +
 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168320
0*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5
73621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 +
 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)
*1i - (((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^
4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^
9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 +
 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (
(4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^
4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8
 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(94
0032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5
564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 402024038
4*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 74
38336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2
 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9
*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*(
(a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^
5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168
3200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5
+ 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^1
0 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1
/2))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) +
49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 26
95744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384
*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703
040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 196

6491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548039
68)))^(1/2) - (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 61
4016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) + (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*
(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7
+ a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^
15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187
840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531
456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a
^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442
00*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^
15 + 254803968)))^(1/2)*1i)/((((52357496832*a + 57139003392*a^2 + 363221483
52*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 +
5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 89
9328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^
9 + a^10 + 331776)) + ((4290672328704*a + 6001143054336*a^2 + 5025917042688
*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 598621552
64*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 13
91569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149
208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3
510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*
a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(2
56*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a
^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a +
4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10
187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16
531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 205320192
0*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119
44200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 +
a^15 + 254803968)))^(1/2))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 58
9*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*
a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*
a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2
053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a
^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 5
5*a^14 + a^15 + 254803968)))^(1/2) + (108343296*a + 74059776*a^2 + 27065088
*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(73476*a + 31545*a^2 + 6066*a^3 +
441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*
a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15
)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
+ 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431

1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - ((4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) - (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) - (99468*a + 28053*a^2 + 2646*a^3 + 117936)/(8192*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442

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00*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^
15 + 254803968)))^(1/2)*2i - ((408*a + 131*a^2 + 11*a^3 + 288)/(32*(a + 4)*
(33*a + 10*a^2 + a^3 + 36)) - (21*x^6*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a
^3 + a^4 + 144)) + (3*x^7*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 1
44)) + (x*(84*a - 107*a^2 - 11*a^3 + 1152))/(32*(a + 4)*(33*a + 10*a^2 + a^
3 + 36)) - (5*x^4*(175*a + 7*a^2 + 528))/(32*(a + 4)*(33*a + 10*a^2 + a^3 +
 36)) + (x^5*(343*a + 7*a^2 + 1116))/(32*(a + 4)*(33*a + 10*a^2 + a^3 + 36)
) - (x^2*(623*a + 32*a^2 + 1800))/(16*(a + 4)*(33*a + 10*a^2 + a^3 + 36)) +
 (x^3*(679*a + 34*a^2 + 1968))/(16*(a + 4)*(33*a + 10*a^2 + a^3 + 36)))/(16
*a*x - x^2*(16*a - 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 +
 32*x^6 - 8*x^7 + x^8)
```

sympy [B] time = 15.62, size = 697, normalized size = 2.77

$$-\frac{}{32a^6 + 448a^5 + 2336a^4 + 5376a^3 + 4608a^2 + x^8\left(32a^4 + 448a^3 + 2336a^2 + 5376a + 4608\right) + x^7\left(-256a^4 - 358\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] -(11*a**3 + 131*a**2 + 408*a + x**7*(12*a + 42) + x**6*(-84*a - 294) + x**5
*(7*a**2 + 343*a + 1116) + x**4*(-35*a**2 - 875*a - 2640) + x**3*(68*a**2 +
 1358*a + 3936) + x**2*(-64*a**2 - 1246*a - 3600) + x*(-11*a**3 - 107*a**2
+ 84*a + 1152) + 288)/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 + 4608*a*
*2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-256*a**
4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14336*a**
3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3 - 18688
0*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**3 + 288
256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a**3 - 25
6000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*a**3 +
63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a**3 + 86
016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*a**14 +
 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 52787590
8304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 5453715112
7224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 55115219
3655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2 + 28499
3413919539200*a + 68398419340689408) + _t**2*(-30965760*a**9 - 1052835840*a
**8 - 15910207488*a**7 - 140262506496*a**6 - 795007254528*a**5 - 3004516270
080*a**4 - 7571263979520*a**3 - 12268037210112*a**2 - 11598827618304*a - 48
75324751872) - 194481*a**4 - 2762424*a**3 - 14762736*a**2 - 35178624*a - 31
539456, Lambda(_t, _t*log(x + (23068672*_t**3*a**12 + 968884224*_t**3*a**11
 + 18624806912*_t**3*a**10 + 216677744640*_t**3*a**9 + 1699123036160*_t**3*
a**8 + 9461389328384*_t**3*a**7 + 38361186172928*_t**3*a**6 + 1141074915491
84*_t**3*a**5 + 247138458009600*_t**3*a**4 + 380084473036800*_t**3*a**3 + 3
```

```
94002582994944*_t**3*a**2 + 247177515368448*_t**3*a + 70970039599104*_t**3
- 395136*_t*a**7 - 11676672*_t*a**6 - 144076032*_t*a**5 - 969518592*_t*a**4
 - 3861475200*_t*a**3 - 9133300224*_t*a**2 - 11906574336*_t*a - 6611337216*
_t - 64827*a**4 - 907578*a**3 - 4780647*a**2 - 11228868*a - 9923472)/(64827
*a**4 + 907578*a**3 + 4780647*a**2 + 11228868*a + 9923472))))
```

3.123 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx$

Optimal. Leaf size=210

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48\right.$$

```
[Out] 1/2*a^4*x^2+32/3*a^3*x^3+8*(12-a)*a^2*x^4+16/5*a*(a^2-48*a+128)*x^5+2/3*(-a
^3+192*a^2-1536*a+1024)*x^6-32/7*(15*a^2-288*a+512)*x^7+8*(128-3*a)*(4-a)*x
^8-16/3*(a^2-128*a+896)*x^9+1/5*(3*a^2-1536*a+20480)*x^10-32/11*(928-35*a)*
x^11+8/3*(524-9*a)*x^12-16/13*(464-3*a)*x^13+2/7*(640-a)*x^14-224/5*x^15+8*
x^16-16/17*x^17+1/18*x^18
```

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{2}{3}\left(-a^3+192a^2-1536a+10\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^2
)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a + 1
5*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)/3
+ ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(524 -
 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15
)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\int\left(a^4x+32a^3x^2-32(-12+a)a^2x^3+16a\left(128-48a+a^2\right)x^4-4\left(-1024\right.\right.$$

$$=\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5+\frac{2}{3}\left(1024-153\right.$$

Mathematica [A] time = 0.04, size = 204, normalized size = 0.97

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9+8\left(3a^2-140a+512\right)x^8-\frac{32}{7}\left(15a^2-288\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 - 8*(-12 + a)*a^2*x^4 + (16*a*(128 - 48*a + a^
2)*x^5)/5 - (2*(-1024 + 1536*a - 192*a^2 + a^3)*x^6)/3 - (32*(512 - 288*a +
 15*a^2)*x^7)/7 + 8*(512 - 140*a + 3*a^2)*x^8 - (16*(896 - 128*a + a^2)*x^9
)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 + (32*(-928 + 35*a)*x^11)/11 - (8*(
-524 + 9*a)*x^12)/3 + (16*(-464 + 3*a)*x^13)/13 - (2*(-640 + a)*x^14)/7 - (
224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

fricas [A] time = 0.36, size = 222, normalized size = 1.06

$$\frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{224}{5}x^{15}-\frac{2}{7}x^{14}a+\frac{1280}{7}x^{14}+\frac{48}{13}x^{13}a-\frac{7424}{13}x^{13}-24x^{12}a+\frac{4192}{3}x^{12}+\frac{1120}{11}x^{11}a+\frac{3}{5}x^{10}a^2-\frac{29696}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 224/5*x^15 - 2/7*x^14*a + 1280/7*x^14 + 4
8/13*x^13*a - 7424/13*x^13 - 24*x^12*a + 4192/3*x^12 + 1120/11*x^11*a + 3/5
*x^10*a^2 - 29696/11*x^11 - 1536/5*x^10*a - 16/3*x^9*a^2 + 4096*x^10 + 2048
/3*x^9*a + 24*x^8*a^2 - 14336/3*x^9 - 1120*x^8*a - 480/7*x^7*a^2 - 2/3*x^6*
a^3 + 4096*x^8 + 9216/7*x^7*a + 128*x^6*a^2 + 16/5*x^5*a^3 - 16384/7*x^7 -
1024*x^6*a - 768/5*x^5*a^2 - 8*x^4*a^3 + 2048/3*x^6 + 2048/5*x^5*a + 96*x^4
*a^2 + 32/3*x^3*a^3 + 1/2*x^2*a^4
```

giac [A] time = 0.39, size = 222, normalized size = 1.06

$$\frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}ax^{14}-\frac{224}{5}x^{15}+\frac{48}{13}ax^{13}+\frac{1280}{7}x^{14}-24ax^{12}-\frac{7424}{13}x^{13}+\frac{3}{5}a^2x^{10}+\frac{1120}{11}ax^{11}+\frac{4192}{3}x^{12}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*a*x^14 - 224/5*x^15 + 48/13*a*x^13 +
1280/7*x^14 - 24*a*x^12 - 7424/13*x^13 + 3/5*a^2*x^10 + 1120/11*a*x^11 + 41
92/3*x^12 - 16/3*a^2*x^9 - 1536/5*a*x^10 - 29696/11*x^11 + 24*a^2*x^8 + 204
8/3*a*x^9 + 4096*x^10 - 2/3*a^3*x^6 - 480/7*a^2*x^7 - 1120*a*x^8 - 14336/3*
x^9 + 16/5*a^3*x^5 + 128*a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4 - 76
```

```
8/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 96*a^
2*x^4 + 2048/5*a*x^5 + 2048/3*x^6
```

maple [A] time = 0.00, size = 267, normalized size = 1.27

$$\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+\frac{(-4a+2560)\,x^{14}}{14}+\frac{(48a-7424)\,x^{13}}{13}+\frac{(-288a+16768)\,x^{12}}{12}+\frac{(1120a-29696)\,x^{11}}{11}+\frac{(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/18*x^18-16/17*x^17+8*x^16-224/5*x^15+1/14*(-4*a+2560)*x^14+1/13*(48*a-742
4)*x^13+1/12*(-288*a+16768)*x^12+1/11*(1120*a-29696)*x^11+1/10*(2*a^2-2560*
a+24576+(-2*a+128)^2)*x^10+1/9*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128)
)*x^9+1/8*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^8+1/7*(-160
*a^2+32*(-2*a+128)*a+2*(-16*a+64)*(8*a-128))*x^7+1/6*(2*(-2*a+128)*a^2+32*(
8*a-128)*a+(-16*a+64)^2)*x^6+1/5*(2*(8*a-128)*a^2+32*(-16*a+64)*a)*x^5+1/4*
(2*(-16*a+64)*a^2+256*a^2)*x^4+32/3*a^3*x^3+1/2*a^4*x^2
```

maxima [A] time = 0.62, size = 182, normalized size = 0.87

$$\frac{1}{18}\,x^{18}-\frac{16}{17}\,x^{17}+8\,x^{16}-\frac{2}{7}\,(a-640)x^{14}-\frac{224}{5}\,x^{15}+\frac{16}{13}\,(3\,a-464)x^{13}-\frac{8}{3}\,(9\,a-524)x^{12}+\frac{32}{11}\,(35\,a-928)x^{11}+\frac{1}{5}\,\Big(3\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*(
3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3*a
^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a
 + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a
 - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5
 - 8*(a^3 - 12*a^2)*x^4
```

mupad [B] time = 0.22, size = 178, normalized size = 0.85

$$x^{13}\,\left(\frac{48\,a}{13}-\frac{7424}{13}\right)-x^{12}\,\left(24\,a-\frac{4192}{3}\right)-x^{14}\,\left(\frac{2\,a}{7}-\frac{1280}{7}\right)+x^{11}\,\left(\frac{1120\,a}{11}-\frac{29696}{11}\right)+x^8\,\left(24\,a^2-1120\,a+4096\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^13*((48*a)/13 - 7424/13) - x^12*(24*a - 4192/3) - x^14*((2*a)/7 - 1280/7)
 + x^11*((1120*a)/11 - 29696/11) + x^8*(24*a^2 - 1120*a + 4096) + x^10*((3*
```

```
a^2)/5 - (1536*a)/5 + 4096) - x^9*((16*a^2)/3 - (2048*a)/3 + 14336/3) - x^7
*((480*a^2)/7 - (9216*a)/7 + 16384/7) - x^6*(1024*a - 128*a^2 + (2*a^3)/3 -
 2048/3) - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18 + (32*a^3*x^3)/3
+ (a^4*x^2)/2 + (16*a*x^5*(a^2 - 48*a + 128))/5 - 8*a^2*x^4*(a - 12)
```

sympy [A] time = 0.12, size = 212, normalized size = 1.01

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+x^{14}\left(\frac{1280}{7}-\frac{2a}{7}\right)+x^{13}\left(\frac{48a}{13}-\frac{7424}{13}\right)+x^{12}\left(\frac{4192}{3}-24a\right)+x^{11}\left(\frac{1120a}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**15
/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 - 2
4*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096) +
x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096) + x
**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 - 1024*
a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**3 + 96
*a**2)
```

3.124 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx$

Optimal. Leaf size=134

$$\frac{a^3x^2}{2}-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48-5$$

```
[Out] 1/2*a^3*x^2+8*a^2*x^3+6*(8-a)*a*x^4+4/5*(3*a^2-96*a+128)*x^5-1/2*(a^2-128*a
+512)*x^6+48/7*(48-5*a)*x^7-4*(70-3*a)*x^8+8/3*(64-a)*x^9-3/10*(256-a)*x^10
+280/11*x^11-6*x^12+12/13*x^13-1/14*x^14
```

Rubi [A] time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3+\frac{a^3x^2}{2}-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48-$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
- ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8 +
(8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (12*x
^13)/13 - x^14/14
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\int\left(a^3x+24a^2x^2-24(-8+a)ax^3+4\left(128-96a+3a^2\right)x^4-3\left(512-128a\right.\right.$$
$$=\frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5-\frac{1}{2}\left(512-128a+a^2\right.$$

Mathematica [A] time = 0.02, size = 130, normalized size = 0.97

$$\frac{a^3x^2}{2}+\frac{1}{2}\left(-a^2+128a-512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3+\frac{3}{10}(a-256)x^{10}-\frac{8}{3}(a-64)x^9+4(3a-70)x^8-\frac{48}{7}(5a-$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 - 6*(-8 + a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
 + ((-512 + 128*a - a^2)*x^6)/2 - (48*(-48 + 5*a)*x^7)/7 + 4*(-70 + 3*a)*x^
8 - (8*(-64 + a)*x^9)/3 + (3*(-256 + a)*x^10)/10 + (280*x^11)/11 - 6*x^12 +
 (12*x^13)/13 - x^14/14
```

fricas [A] time = 0.35, size = 133, normalized size = 0.99

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{280}{11}x^{11}+\frac{3}{10}x^{10}a-\frac{384}{5}x^{10}-\frac{8}{3}x^{9}a+\frac{512}{3}x^{9}+12x^{8}a-280x^{8}-\frac{240}{7}x^{7}a-\frac{1}{2}x^{6}a^{2}+\frac{2304}{7}x^{7}+64x^{6}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 280/11*x^11 + 3/10*x^10*a - 384/5*x^10 -
 8/3*x^9*a + 512/3*x^9 + 12*x^8*a - 280*x^8 - 240/7*x^7*a - 1/2*x^6*a^2 + 2
304/7*x^7 + 64*x^6*a + 12/5*x^5*a^2 - 256*x^6 - 384/5*x^5*a - 6*x^4*a^2 + 5
12/5*x^5 + 48*x^4*a + 8*x^3*a^2 + 1/2*x^2*a^3
```

giac [A] time = 0.35, size = 133, normalized size = 0.99

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{3}{10}ax^{10}+\frac{280}{11}x^{11}-\frac{8}{3}ax^{9}-\frac{384}{5}x^{10}+12ax^{8}+\frac{512}{3}x^{9}-\frac{1}{2}a^{2}x^{6}-\frac{240}{7}ax^{7}-280x^{8}+\frac{12}{5}a^{2}x^{5}+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*a*x^10 + 280/11*x^11 - 8/3*a*x^9 -
384/5*x^10 + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 + 1
2/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6 + 1
/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5
```

maple [A] time = 0.00, size = 143, normalized size = 1.07

$$-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+\frac{(3a-768)\,x^{10}}{10}+\frac{(-24a+1536)\,x^{9}}{9}+\frac{(96a-2240)\,x^{8}}{8}+\frac{(-240a+2304)\,x^{7}}{7}+\frac{(-a^{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/14*x^14+12/13*x^13-6*x^12+280/11*x^11+1/10*(3*a-768)*x^10+1/9*(-24*a+153
6)*x^9+1/8*(96*a-2240)*x^8+1/7*(-240*a+2304)*x^7+1/6*(-a^2+(-2*a+128)*a+256
```

```
*a-1536)*x^6+1/5*(4*a^2+(8*a-128)*a-256*a+512)*x^5+1/4*(-8*a^2+(-16*a+64)*a
+128*a)*x^4+8*a^2*x^3+1/2*a^3*x^2
```

maxima [A] time = 0.62, size = 113, normalized size = 0.84

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{3}{10}(a-256)x^{10}+\frac{280}{11}x^{11}-\frac{8}{3}(a-64)x^9+4(3a-70)x^8-\frac{48}{7}(5a-48)x^7-\frac{1}{2}\left(a^2-128a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3*
(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a +
512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2
- 8*a)*x^4
```

mupad [B] time = 2.12, size = 113, normalized size = 0.84

$$x^8(12a-280)+x^{10}\left(\frac{3a}{10}-\frac{384}{5}\right)-x^9\left(\frac{8a}{3}-\frac{512}{3}\right)-x^7\left(\frac{240a}{7}-\frac{2304}{7}\right)-x^6\left(\frac{a^2}{2}-64a+256\right)+x^5\left(\frac{12a^2}{5}-\frac{384a}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^8*(12*a - 280) + x^10*((3*a)/10 - 384/5) - x^9*((8*a)/3 - 512/3) - x^7*((
240*a)/7 - 2304/7) - x^6*(a^2/2 - 64*a + 256) + x^5*((12*a^2)/5 - (384*a)/5
 + 512/5) + (280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14 + 8*a^2*x^3 + (
a^3*x^2)/2 - 6*a*x^4*(a - 8)
```

sympy [A] time = 0.10, size = 128, normalized size = 0.96

$$\frac{a^3x^2}{2}+8a^2x^3-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+x^{10}\left(\frac{3a}{10}-\frac{384}{5}\right)+x^9\left(\frac{512}{3}-\frac{8a}{3}\right)+x^8(12a-280)+x^7\left(\frac{2304}{7}-\frac{240a}{7}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/11
 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x**7
*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 384*a
/5 + 512/5) + x**4*(-6*a**2 + 48*a)
```

3.125 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx$

Optimal. Leaf size=79

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

```
[Out] 1/2*a^2*x^2+16/3*a*x^3+4*(4-a)*x^4-8/5*(16-a)*x^5+1/3*(64-a)*x^6-80/7*x^7+4
*x^8-8/9*x^9+1/10*x^10
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a)
*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx &= \int \left(a^2x+16ax^2-16(-4+a)x^3+8(-16+a)x^4-2(-64+a)x^5-80x^6+\cdots\right.\\ &= \frac{a^2x^2}{2}+\frac{16ax^3}{3}+4(4-a)x^4-\frac{8}{5}(16-a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\cdots\end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 0.95

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6+\frac{8}{5}(a-16)x^5-4(a-4)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 -
a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

fricas [A] time = 0.39, size = 68, normalized size = 0.86

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{80}{7}x^7 - \frac{1}{3}x^6a + \frac{64}{3}x^6 + \frac{8}{5}x^5a - \frac{128}{5}x^5 - 4x^4a + 16x^4 + \frac{16}{3}x^3a + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 80/7*x^7 - 1/3*x^6*a + 64/3*x^6 + 8/5*x^5*a -
 128/5*x^5 - 4*x^4*a + 16*x^4 + 16/3*x^3*a + 1/2*x^2*a^2
```

giac [A] time = 0.35, size = 68, normalized size = 0.86

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{1}{3}ax^6 - \frac{80}{7}x^7 + \frac{8}{5}ax^5 + \frac{64}{3}x^6 - 4ax^4 - \frac{128}{5}x^5 + \frac{1}{2}a^2x^2 + \frac{16}{3}ax^3 + 16x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6 -
 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4
```

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7} + \frac{(-2a+128)x^6}{6} + \frac{(8a-128)x^5}{5} + \frac{a^2x^2}{2} + \frac{16a\,x^3}{3} + \frac{(-16a+64)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+1/6*(-2*a+128)*x^6+1/5*(8*a-128)*x^5+1/4*(
-16*a+64)*x^4+16/3*a*x^3+1/2*a^2*x^2
```

maxima [A] time = 0.62, size = 59, normalized size = 0.75

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{1}{3}(a-64)x^6 - \frac{80}{7}x^7 + \frac{8}{5}(a-16)x^5 - 4(a-4)x^4 + \frac{1}{2}a^2x^2 + \frac{16}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^
5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

mupad [B] time = 0.04, size = 64, normalized size = 0.81

$$x^5\left(\frac{8\,a}{5}-\frac{128}{5}\right)-x^6\left(\frac{a}{3}-\frac{64}{3}\right)-x^4\,(4\,a-16)+\frac{16\,a\,x^3}{3}-\frac{80\,x^7}{7}+4\,x^8-\frac{8\,x^9}{9}+\frac{x^{10}}{10}+\frac{a^2\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^5*((8*a)/5 - 128/5) - x^6*(a/3 - 64/3) - x^4*(4*a - 16) + (16*a*x^3)/3 -
(80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10 + (a^2*x^2)/2
```

sympy [A] time = 0.08, size = 70, normalized size = 0.89

$$\frac{a^2x^2}{2}+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+x^6\left(\frac{64}{3}-\frac{a}{3}\right)+x^5\left(\frac{8a}{5}-\frac{128}{5}\right)+x^4\,(16-4a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**6
*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)
```

3.126 $\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx$

Optimal. Leaf size=35

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

```
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx &= \int \left(ax+8x^2-8x^3+4x^4-x^5\right)\,dx\\ &= \frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

fricas [A] time = 0.40, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{8}{3}x^3 + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 8/3*x^3 + 1/2*x^2*a
```

giac [A] time = 0.36, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}a\,x^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x),x)
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

maxima [A] time = 0.61, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

mupad [B] time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^6}{6} + \frac{4\,x^5}{5} - 2\,x^4 + \frac{8\,x^3}{3} + \frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

sympy [A] time = 0.06, size = 29, normalized size = 0.83

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3
```

3.127 $\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=116

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

```
[Out] 1/2*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a
)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)+1/2*arctan((-1+x)/(1+(4+a
)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1680, 1673, 1093, 204, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]
]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4
+ a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*Sqrt[4 + a])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \frac{1+x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{3+a-2x^2-x^4}\,dx, x, -\right.$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right) - \frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1\right.}{2\sqrt{4+a}}$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} - \text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.51

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(-2 + 4
*#1 - 3*#1^2 + #1^3) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{x^4-4x^3+8x^2-a-8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

maple [C] time = 0.00, size = 50, normalized size = 0.43

$$-\frac{\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)\right.}{4\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\,\mathrm{RootOf}\left(\_Z^4-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x),x)
[Out] -1/4*sum(_R/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_
Z-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

mupad [B] time = 2.58, size = 275, normalized size = 2.37

$$\sum_{k=1}^{4}\ln\left(-x-\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] symsum(log(- x - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k)*(root(2816*a^
2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 51
2*z^2 + 16*a*z + 64*z + a, z, k)*(32*a - root(2816*a^2*z^4 + 256*a^3*z^4 +
10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z
+ a, z, k)*(64*a - x*(64*a + 256) + 256) - x*(16*a + 64) + 128) - 8))*root(
2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z
^2 - 512*z^2 + 16*a*z + 64*z + a, z, k), k, 1, 4)
```

sympy [A] time = 4.43, size = 155, normalized size = 1.34

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a^2-256a-512\right)+t\left(-16a-64\right)+a,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2 -
 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a**4
 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*_t*
*2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_t*a*
*2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

3.128 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2} dx$

Optimal. Leaf size=231

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-}}{8(a+3)(a+4)^{3/2}\sqrt{1-}}$$

```
[Out] 1/4*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(5+a+(-1+x)^2)*(-1+x)/
(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*arctanh((1+(-1+x)^2)/(4+a)^(1/2)
)/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))
/(3+a)/(4+a)^(3/2)/(1-(4+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^
(1/2))*(10+3*a-(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1680, 1673, 1092, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-}}{8(a+3)(a+4)^{3/2}\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
+ (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x
)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(
8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*
ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + S
qrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1+x}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\left(3+a-2x^2-x^4\right)^2}\right.$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(3+a-2x\right.}\right.$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2\right.}$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2\right.}$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-1+}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2\right.}$$

Mathematica [C] time = 0.07, size = 166, normalized size = 0.72

$$\frac{ax^2-ax+a+x^3+2x}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)} - \frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1^2\log(x-\#1)+2\#1a\log(x-\#1)}{\#1^3}\right.}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (6*Log[x - #1] + a*
Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2
 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

maple [C] time = 0.01, size = 162, normalized size = 0.70

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+2\left(-a-2\right)\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-a-6\right)\ln\left(\right.}{16\left(a^2+7a+12\right)\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a^2+7*a+12)*x^3-1/4*a/(a^2+7*a+12)*x^2+1/4*(a-2)/(a^2+7*a+12)*x-1/4*
a/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a^2+7*a+12)*sum((-6-_R^2+2*(-
a-2)*_R-a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ax^2+x^3-(a-2)x+a}{4\left(\left(a^2+7\,a+12\right)x^4-4\left(a^2+7\,a+12\right)x^3-a^3+8\left(a^2+7\,a+12\right)x^2-7\,a^2-8\left(a^2+7\,a+12\right)x-12\,a\right)}-\frac{\int\frac{2(}{x^4-}}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 1
2)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a
) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)/(x^4 - 4*x^3 + 8*x^2 - a - 8*
x), x)/(a^2 + 7*a + 12)
```

mupad [B] time = 2.82, size = 1167, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] symsum(log((35*a + 4*a^2 + 68)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^
5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 +
 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z
^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*
z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 141
55776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 26419
2*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((12800*a + 3600*a
^2 + 336*a^3 + 15104)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))
 + root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^
9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357
174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 284
2624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z
^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 2
53952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*(root(12952010752*a^3*z^4 +
 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 2
0082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^
6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2
 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2
*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16
*a^3 - 2064, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 +
 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
a^5 + 576)) - (x*(3932160*a + 2719744*a^2 + 999424*a^3 + 205824*a^4 + 22528
*a^5 + 1024*a^6 + 2359296))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 +
 576))) - (1150976*a + 631808*a^2 + 172800*a^3 + 23552*a^4 + 1280*a^5 + 835
584)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a +
 58880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(16*(816*a + 460*a^2
+ 129*a^3 + 18*a^4 + a^5 + 576))) - (x*(864*a + 228*a^2 + 20*a^3 + 1088))/(
16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) + (x*(9*a + 2*a^2 + 8
))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*root(12952010752*
```

```
a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*
a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269
680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 52070
4*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 1
02912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57
*a^2 + 16*a^3 - 2064, z, k), k, 1, 4) + (x^3/(4*(7*a + a^2 + 12)) + a/(4*(a
 + 3)*(a + 4)) - (x*(a - 2))/(4*(a + 3)*(a + 4)) + (a*x^2)/(4*(a + 3)*(a +
4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)
```

sympy [B] time = 31.42, size = 539, normalized size = 2.33

$$\frac{-ax^2 - a - x^3 + x(a-2)}{-4a^3 - 28a^2 - 48a + x^4\left(4a^2 + 28a + 48\right) + x^3\left(-16a^2 - 112a - 192\right) + x^2\left(32a^2 + 224a + 384\right) + x\left(-32a^2 - 22\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (-a*x**2 - a - x**3 + x*(a - 2))/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 +
 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384)
+ x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 +
31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 129520
10752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-2048*
a**6 - 50688*a**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155776*a
- 9568256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 253952)
+ 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (98304*_t**3*a**1
2 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 + 793837568*_t**3*a**9 + 583
9372288*_t**3*a**8 + 30226464768*_t**3*a**7 + 112668450816*_t**3*a**6 + 303
864643584*_t**3*a**5 + 586157391872*_t**3*a**4 + 784017129472*_t**3*a**3 +
683648483328*_t**3*a**2 + 343136010240*_t**3*a + 72477573120*_t**3 + 30208*
_t**2*a**10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156928*_t**2*a**
7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312*_t**2*a**4 +
12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820569600*_t**2*a + 2944
401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8 - 757040*_t*a**7 - 6200656*_t*a
**6 - 31380496*_t*a**5 - 100736416*_t*a**4 - 200813696*_t*a**3 - 228144640*
_t*a**2 - 114632704*_t*a - 2490368*_t + 248*a**7 + 6797*a**6 + 71132*a**5 +
 369745*a**4 + 987758*a**3 + 1128896*a**2 - 129568*a - 956416)/(576*a**7 +
10985*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3 + 1826304*a**2 + 18677
76*a + 917504))))
```

3.129 $\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$

Optimal. Leaf size=349

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

```
[Out] 1/8*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+3/16*(1+(-1+x)^2)/(4+a)^
2/(3+a-2*(-1+x)^2-(-1+x)^4)+1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(
-1+x)^2-(-1+x)^4)^2+1/32*((6+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*
a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)+3/16*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4
+a)^(5/2)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)^(1/2
)+a*(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/64*arct
an((-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2))/(3+a
)^2/(4+a)^2/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1680, 1673, 1092, 1178, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (3*(1
+ (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
 + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 +
x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(32*(3
+ a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14
*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]
]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*a - (80
 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(64*(
3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]]) + (3*ArcTanh[(1 + (-1 + x)^2)/Sqr
t[4 + a]])/(16*(4 + a)^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right) + \text{Subst}\Big(\int \frac{x}{(3+a-2x^2-x^4)}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{1}{2}\text{Subst}\Big(\int \frac{1}{(3+a-}$$

$$= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} - \frac{((6+a)(25+7a)+6(7+2a)(1}{32(12+7a+a^2)^2(3+a-2(1-}$$

$$= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1}$$

$$= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1}$$

$$= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1}$$

Mathematica [C] time = 0.16, size = 284, normalized size = 0.81

$$\frac{1}{128}\Bigg(-\frac{3\text{RootSum}\Big[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{4\#1^2a\log(x-\#1)+14\#1^2\log(x-\#1)+3a^2\log(x-\#1)+4\#1a^2\log(x-\#1)+31a\log}{\#1^3-3\#1^2+4\#1-2}}{(a^2+7a+12)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] ((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x
^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^3)
 + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 +
```

```
 x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (72*Log[x - #1]
+ 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x - #1
]*#1 + 4*a^2*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(
-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a^2)^2)/128
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)
```

maple [C] time = 0.02, size = 405, normalized size = 1.16

$$-\frac{3\left(2\left(2a+7\right)\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+3a^2+4\left(a^2+4a+2\right)\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-\right.\right.}{128\left(a^4+14a^3+73a^2+168a+144\right)\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -(3/16*(2*a+7)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14*
a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*a+
144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16*(6
2*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+140*
a+1008)/(a^4+14*a^3+73*a^2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)/(a^4
+14*a^3+73*a^2+168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+168*a+
144))/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^4+14*a^3+73*a^2+168*a+144)*sum((72
+2*(2*a+7)*_R^2+4*(a^2+4*a+2)*_R+3*a^2+31*a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x)
,_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{6\,(2\,a+7}{32\,\left(\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^8-8\,\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^7+32\,\left(a^4+14\,a^3+73\,a^2+1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 + 6*(a^2 - 8*a - 40)*x^6 - (29*a^2 - 127*a - 792)*x^
5 + (73*a^2 - 227*a - 1668)*x^4 - 2*(62*a^2 - 103*a - 1104)*x^3 - 9*a^3 - 2
*(5*a^3 - 26*a^2 + 140*a + 1008)*x^2 - 21*a^2 + 3*(3*a^3 - 17*a^2 - 40*a +
192)*x + 36*a)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3
 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6
 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 5
0*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4
- 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17
*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 1
68*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 3*a^2 + 4*(a^2 + 4*a
 + 2)*x + 31*a + 72)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)
```

mupad [B] time = 3.39, size = 2200, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] symsum(log(root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 153
980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a
^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 59995324
41600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 3206
246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 378
493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a
^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2
- 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 11
043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 60494614
36416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 728801
28*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 3335100825
6*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 68
345856, z, k)*((242823168*a + 170044416*a^2 + 63509760*a^3 + 13340736*a^4 +
 1494144*a^5 + 69696*a^6 + 144506880)/(16384*(940032*a + 1195776*a^2 + 8993
28*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9
```

+ a^10 + 331776)) + root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 68345856, z, k)*(root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 68345856, z, k)*((4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(536334041088*a + 750142881792*a^2 + 628239630336*a^3 + 350065000448*a^4 + 136275034112*a^5 + 37819514880*a^6 + 7482769408*a^7 + 1034420224*a^8 + 95158272*a^9 + 5242880*a^10 + 131072*a^11 + 173946175488))/(2048*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))) - (73421291520*a + 81260445696*a^2 + 52393672704*a^3 + 21688418304*a^4 + 5977620480*a^5 + 1096949760*a^6 + 129245184*a^7 + 8871936*a^8 + 270336*a^9 + 29444014080)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(2632974336*a + 3015180288*a^2 + 2008940544*a^3 + 858243072*a^4 + 243806208*a^5 + 46055424*a^6 + 5578752*a^7 + 393216*a^8 + 12288*a^9 + 1019215872))/(2048*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))) - (x*(10805760*a + 7173504*a^2 + 2539872*a^3 + 505800*a^4 + 53712*a^5 + 2376*a^6 + 6782976))/(2048*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))) - (133812*a + 56187*a^2 + 10098*a^3 + 648*a^4 + 115776)/(16384

```
*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6
 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(1971*a^2 - 1539*a +
918*a^3 + 108*a^4 - 6372))/(2048*(940032*a + 1195776*a^2 + 899328*a^3 + 442
864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331
776)))*root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 1539804
18717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z
^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 599953244160
0*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 32062467
73555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 3784939
92960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a^10*
z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2 - 16
752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 110433
92716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 604946143641
6*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 72880128*a
^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 33351008256*z
- 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 683458
56, z, k), k, 1, 4) + ((3*(7*a^2 - 12*a + 3*a^3))/(32*(6*a + a^2 + 9)*(8*a
+ a^2 + 16)) - (3*x^7*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))
 + (x^2*(140*a - 26*a^2 + 5*a^3 + 1008))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 1
6)) + (3*x*(40*a + 17*a^2 - 3*a^3 - 192))/(32*(6*a + a^2 + 9)*(8*a + a^2 +
16)) + (3*x^6*(8*a - a^2 + 40))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 16)) - (x^
5*(127*a - 29*a^2 + 792))/(32*(6*a + a^2 + 9)*(8*a + a^2 + 16)) - (x^3*(103
*a - 62*a^2 + 1104))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 16)) + (x^4*(227*a -
73*a^2 + 1668))/(32*(6*a + a^2 + 9)*(8*a + a^2 + 16)))/(16*a*x - x^2*(16*a
- 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 + 32*x^6 - 8*x^7 +
 x^8)
```

sympy [B] time = 88.09, size = 1102, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] -(-9*a**3 - 21*a**2 + 36*a + x**7*(12*a + 42) + x**6*(6*a**2 - 48*a - 240)
+ x**5*(-29*a**2 + 127*a + 792) + x**4*(73*a**2 - 227*a - 1668) + x**3*(-12
4*a**2 + 206*a + 2208) + x**2*(-10*a**3 + 52*a**2 - 280*a - 2016) + x*(9*a*
*3 - 51*a**2 - 120*a + 576))/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 +
4608*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-
256*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14
336*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3
- 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**
3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a*
*3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*
a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a*
```

```
*3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*
a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 5
27875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 545
37151127224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 5
51152193655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2
+ 284993413919539200*a + 68398419340689408) + _t**2*(-4718592*a**10 - 19611
6480*a**9 - 3648061440*a**8 - 40022212608*a**7 - 286939938816*a**6 - 140543
7345792*a**5 - 4764645457920*a**4 - 11043392716800*a**3 - 16752587046912*a*
*2 - 15023392948224*a - 6049461436416) + _t*(-2709504*a**7 - 72880128*a**6
- 839890944*a**5 - 5375877120*a**4 - 20640890880*a**3 - 47542173696*a**2 -
60827369472*a - 33351008256) + 20736*a**5 - 155601*a**4 - 4706424*a**3 - 29
249424*a**2 - 74027520*a - 68345856, Lambda(_t, _t*log(x + (-469762048*_t**
3*a**20 - 31417434112*_t**3*a**19 - 992305217536*_t**3*a**18 - 196635766292
48*_t**3*a**17 - 273880031690752*_t**3*a**16 - 2846116194287616*_t**3*a**15
 - 22853982892326912*_t**3*a**14 - 144840417605582848*_t**3*a**13 - 7331931
54773123072*_t**3*a**12 - 2977941469704224768*_t**3*a**11 - 967719737311730
0736*_t**3*a**10 - 24850421452415959040*_t**3*a**9 - 48984708931769073664*_
t**3*a**8 - 69124682329943441408*_t**3*a**7 - 54921507243737219072*_t**3*a*
*6 + 18833423088924753920*_t**3*a**5 + 128767022044444360704*_t**3*a**4 + 1
97893824476545548288*_t**3*a**3 + 170576989286005997568*_t**3*a**2 + 837098
68624400351232*_t**3*a + 18392762450832261120*_t**3 + 136642560*_t**2*a**17
 + 7616593920*_t**2*a**16 + 198980665344*_t**2*a**15 + 3234300690432*_t**2*
a**14 + 36614363283456*_t**2*a**13 + 306155605721088*_t**2*a**12 + 19563396
56687616*_t**2*a**11 + 9747894775578624*_t**2*a**10 + 38291841445330944*_t*
*2*a**9 + 119050488573591552*_t**2*a**8 + 292236772188880896*_t**2*a**7 + 5
61261720373297152*_t**2*a**6 + 828898581078343680*_t**2*a**5 + 914439454498
750464*_t**2*a**4 + 718255692208668672*_t**2*a**3 + 369227414724673536*_t**
2*a**2 + 104815442748506112*_t**2*a + 10263520138493952*_t**2 + 4128768*_t*
a**15 + 235608192*_t*a**14 + 6050117376*_t*a**13 + 92875570560*_t*a**12 + 9
50838962688*_t*a**11 + 6825858397056*_t*a**10 + 34932826734336*_t*a**9 + 12
5262778564224*_t*a**8 + 287989861404672*_t*a**7 + 257684685023232*_t*a**6 -
 836263788945408*_t*a**5 - 4002432415137792*_t*a**4 - 8409454278082560*_t*a
**3 - 10371340262965248*_t*a**2 - 7285247072796672*_t*a - 2270140431335424*
_t + 1000512*a**12 + 42546357*a**11 + 777344580*a**10 + 7998006582*a**9 + 5
0045408388*a**8 + 182866499613*a**7 + 247394170512*a**6 - 1063305068832*a**
5 - 6960658344192*a**4 - 19132655580288*a**3 - 30001872614400*a**2 - 261928
92672000*a - 9953981595648)/(1354752*a**12 + 44550027*a**11 + 663517980*a**
10 + 5951170602*a**9 + 36270700668*a**8 + 162289912419*a**7 + 567868212432*
a**6 + 1626099007104*a**5 + 3825839091456*a**4 + 7035734732544*a**3 + 92167
60449024*a**2 + 7467334520832*a + 2773884911616)))
```

3.130 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=210

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a\right.$$

```
[Out] 1/3*a^4*x^3+8*a^3*x^4+32/5*(12-a)*a^2*x^5+8/3*a*(a^2-48*a+128)*x^6+4/7*(-a^
3+192*a^2-1536*a+1024)*x^7-4*(15*a^2-288*a+512)*x^8+64/9*(128-3*a)*(4-a)*x^
9-24/5*(a^2-128*a+896)*x^10+2/11*(3*a^2-1536*a+20480)*x^11-8/3*(928-35*a)*x
^12+32/13*(524-9*a)*x^13-8/7*(464-3*a)*x^14+4/15*(640-a)*x^15-42*x^16+128/1
7*x^17-8/9*x^18+1/19*x^19
```

Rubi [A] time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6742}

$$\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{4}{7}\left(-a^3+192a^2-1536a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2)
*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15*a
^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)/5
+ (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524
 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16
 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\int\left(a^4x^2+32a^3x^3-32(-12+a)a^2x^4+16a\left(128-48a+a^2\right)x^5-4\left(-10\right.\right.$$
$$=\frac{a^4x^3}{3}+8a^3x^4+\frac{32}{5}(12-a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6+\frac{4}{7}\left(1024-15\right.$$

Mathematica [A] time = 0.04, size = 204, normalized size = 0.97

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}+\frac{64}{9}\left(3a^2-140a+512\right)x^9-4\left(15a^2-288a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 - (32*(-12 + a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2
)*x^6)/3 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^7)/7 - 4*(512 - 288*a + 15
*a^2)*x^8 + (64*(512 - 140*a + 3*a^2)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10
)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 + (8*(-928 + 35*a)*x^12)/3 - (32
*(-524 + 9*a)*x^13)/13 + (8*(-464 + 3*a)*x^14)/7 - (4*(-640 + a)*x^15)/15 -
 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

fricas [A] time = 0.36, size = 222, normalized size = 1.06

$$\frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-42x^{16}-\frac{4}{15}x^{15}a+\frac{512}{3}x^{15}+\frac{24}{7}x^{14}a-\frac{3712}{7}x^{14}-\frac{288}{13}x^{13}a+\frac{16768}{13}x^{13}+\frac{280}{3}x^{12}a+\frac{6}{11}x^{11}a^2-\frac{7424}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 42*x^16 - 4/15*x^15*a + 512/3*x^15 + 2
4/7*x^14*a - 3712/7*x^14 - 288/13*x^13*a + 16768/13*x^13 + 280/3*x^12*a + 6
/11*x^11*a^2 - 7424/3*x^12 - 3072/11*x^11*a - 24/5*x^10*a^2 + 40960/11*x^11
 + 3072/5*x^10*a + 64/3*x^9*a^2 - 21504/5*x^10 - 8960/9*x^9*a - 60*x^8*a^2
- 4/7*x^7*a^3 + 32768/9*x^9 + 1152*x^8*a + 768/7*x^7*a^2 + 8/3*x^6*a^3 - 20
48*x^8 - 6144/7*x^7*a - 128*x^6*a^2 - 32/5*x^5*a^3 + 4096/7*x^7 + 1024/3*x^
6*a + 384/5*x^5*a^2 + 8*x^4*a^3 + 1/3*x^3*a^4
```

giac [A] time = 0.24, size = 222, normalized size = 1.06

$$\frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-\frac{4}{15}ax^{15}-42x^{16}+\frac{24}{7}ax^{14}+\frac{512}{3}x^{15}-\frac{288}{13}ax^{13}-\frac{3712}{7}x^{14}+\frac{6}{11}a^2x^{11}+\frac{280}{3}ax^{12}+\frac{16768}{13}x^{13}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*a*x^15 - 42*x^16 + 24/7*a*x^14 +
512/3*x^15 - 288/13*a*x^13 - 3712/7*x^14 + 6/11*a^2*x^11 + 280/3*a*x^12 + 1
6768/13*x^13 - 24/5*a^2*x^10 - 3072/11*a*x^11 - 7424/3*x^12 + 64/3*a^2*x^9
+ 3072/5*a*x^10 + 40960/11*x^11 - 4/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x^9 -
 21504/5*x^10 + 8/3*a^3*x^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9 - 32
```

```
/5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*a^3*x^
4 + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7
```

maple [A] time = 0.00, size = 267, normalized size = 1.27

$$\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+\frac{(-4a+2560)\,x^{15}}{15}+\frac{(48a-7424)\,x^{14}}{14}+\frac{(-288a+16768)\,x^{13}}{13}+\frac{(1120a-29696)\,x^{12}}{12}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/19*x^19-8/9*x^18+128/17*x^17-42*x^16+1/15*(-4*a+2560)*x^15+1/14*(48*a-742
4)*x^14+1/13*(-288*a+16768)*x^13+1/12*(1120*a-29696)*x^12+1/11*(2*a^2-2560*
a+24576+(-2*a+128)^2)*x^11+1/10*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128
))*x^10+1/9*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^9+1/8*(-1
60*a^2+32*(-2*a+128)*a+2*(-16*a+64)*(8*a-128))*x^8+1/7*(2*(-2*a+128)*a^2+32
*(8*a-128)*a+(-16*a+64)^2)*x^7+1/6*(2*(8*a-128)*a^2+32*(-16*a+64)*a)*x^6+1/
5*(2*(-16*a+64)*a^2+256*a^2)*x^5+8*a^3*x^4+1/3*a^4*x^3
```

maxima [A] time = 0.64, size = 182, normalized size = 0.87

$$\frac{1}{19}\,x^{19}-\frac{8}{9}\,x^{18}+\frac{128}{17}\,x^{17}-\frac{4}{15}\,(a-640)x^{15}-42\,x^{16}+\frac{8}{7}\,(3\,a-464)x^{14}-\frac{32}{13}\,(9\,a-524)x^{13}+\frac{8}{3}\,(35\,a-928)x^{12}+\frac{2}{11}\,\big(3\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(3
*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3*a
^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2 - 1
40*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*
a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 -
32/5*(a^3 - 12*a^2)*x^5
```

mupad [B] time = 2.29, size = 178, normalized size = 0.85

$$x^{14}\,\left(\frac{24\,a}{7}-\frac{3712}{7}\right)-x^{15}\,\left(\frac{4\,a}{15}-\frac{512}{3}\right)+x^{12}\,\left(\frac{280\,a}{3}-\frac{7424}{3}\right)-x^{13}\,\left(\frac{288\,a}{13}-\frac{16768}{13}\right)-x^{8}\,\left(60\,a^2-1152\,a+2048\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^14*((24*a)/7 - 3712/7) - x^15*((4*a)/15 - 512/3) + x^12*((280*a)/3 - 7424
/3) - x^13*((288*a)/13 - 16768/13) - x^8*(60*a^2 - 1152*a + 2048) - x^10*((
```

```
24*a^2)/5 - (3072*a)/5 + 21504/5) + x^9*((64*a^2)/3 - (8960*a)/9 + 32768/9)
 + x^11*((6*a^2)/11 - (3072*a)/11 + 40960/11) - x^7*((6144*a)/7 - (768*a^2)
/7 + (4*a^3)/7 - 4096/7) - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19 +
 8*a^3*x^4 + (a^4*x^3)/3 + (8*a*x^6*(a^2 - 48*a + 128))/3 - (32*a^2*x^5*(a
- 12))/5
```

sympy [A] time = 0.15, size = 219, normalized size = 1.04

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+x^{15}\left(\frac{512}{3}-\frac{4a}{15}\right)+x^{14}\left(\frac{24a}{7}-\frac{3712}{7}\right)+x^{13}\left(\frac{16768}{13}-\frac{288a}{13}\right)+x^{12}\left(\frac{280a}{3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16
+ x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 288*
a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/11)
 + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9 + 3
2768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2/7 -
6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-32*a**3
/5 + 384*a**2/5)
```

3.131 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx$

Optimal. Leaf size=138

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6(4$$

```
[Out] 1/3*a^3*x^3+6*a^2*x^4+24/5*(8-a)*a*x^5+2/3*(3*a^2-96*a+128)*x^6-3/7*(a^2-12
8*a+512)*x^7+6*(48-5*a)*x^8-32/9*(70-3*a)*x^9+12/5*(64-a)*x^10-3/11*(256-a)
*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15
```

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6742}

$$-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4+\frac{a^3x^3}{3}-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6($$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^
6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*x^
9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*x^1
3)/13 + (6*x^14)/7 - x^15/15
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx &= \int \left(a^3x^2+24a^2x^3-24(-8+a)ax^4+4\left(128-96a+3a^2\right)x^5-3\left(512-1\right.\right.\\ &= \frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 132, normalized size = 0.96

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4+\frac{3}{11}(a-256)x^{11}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x
^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x^8 + (32*(-70 + 3*a)
*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 + a)*x^11)/11 + (70*x^12)/3 - (7
2*x^13)/13 + (6*x^14)/7 - x^15/15
```

fricas [A] time = 0.34, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{70}{3}x^{12}+\frac{3}{11}x^{11}a-\frac{768}{11}x^{11}-\frac{12}{5}x^{10}a+\frac{768}{5}x^{10}+\frac{32}{3}x^{9}a-\frac{2240}{9}x^{9}-30x^{8}a-\frac{3}{7}x^{7}a^{2}+288x^{8}+\frac{384}{7}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 70/3*x^12 + 3/11*x^11*a - 768/11*x^11
- 12/5*x^10*a + 768/5*x^10 + 32/3*x^9*a - 2240/9*x^9 - 30*x^8*a - 3/7*x^7*a
^2 + 288*x^8 + 384/7*x^7*a + 2*x^6*a^2 - 1536/7*x^7 - 64*x^6*a - 24/5*x^5*a
^2 + 256/3*x^6 + 192/5*x^5*a + 6*x^4*a^2 + 1/3*x^3*a^3
```

giac [A] time = 0.29, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}ax^{11}+\frac{70}{3}x^{12}-\frac{12}{5}ax^{10}-\frac{768}{11}x^{11}+\frac{32}{3}ax^{9}+\frac{768}{5}x^{10}-\frac{3}{7}a^{2}x^{7}-30ax^{8}-\frac{2240}{9}x^{9}+2a^{2}x^{6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*a*x^11 + 70/3*x^12 - 12/5*a*x^10
- 768/11*x^11 + 32/3*a*x^9 + 768/5*x^10 - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9*x
^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/7*x
^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6
```

maple [A] time = 0.00, size = 143, normalized size = 1.04

$$-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+\frac{(3a-768)\,x^{11}}{11}+\frac{(-24a+1536)\,x^{10}}{10}+\frac{(96a-2240)\,x^{9}}{9}+\frac{(-240a+2304)\,x^{8}}{8}+\frac{\left(-a^{2}+(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/15*x^15+6/7*x^14-72/13*x^13+70/3*x^12+1/11*(3*a-768)*x^11+1/10*(-24*a+15
36)*x^10+1/9*(96*a-2240)*x^9+1/8*(-240*a+2304)*x^8+1/7*(-a^2+(-2*a+128)*a+2
```

```
56*a-1536)*x^7+1/6*(4*a^2+(8*a-128)*a-256*a+512)*x^6+1/5*(-8*a^2+(-16*a+64)
*a+128*a)*x^5+6*a^2*x^4+1/3*a^3*x^3
```

maxima [A] time = 0.56, size = 113, normalized size = 0.82

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}(a-256)x^{11}+\frac{70}{3}x^{12}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5a-48)x^8-\frac{3}{7}\left(a^2-128a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/5
*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*a
+ 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*
(a^2 - 8*a)*x^5
```

mupad [B] time = 0.09, size = 113, normalized size = 0.82

$$x^{11}\left(\frac{3a}{11}-\frac{768}{11}\right)-x^{10}\left(\frac{12a}{5}-\frac{768}{5}\right)-x^8(30a-288)+x^9\left(\frac{32a}{3}-\frac{2240}{9}\right)+x^6\left(2a^2-64a+\frac{256}{3}\right)-x^7\left(\frac{3a^2}{7}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^11*((3*a)/11 - 768/11) - x^10*((12*a)/5 - 768/5) - x^8*(30*a - 288) + x^9
*((32*a)/3 - 2240/9) + x^6*(2*a^2 - 64*a + 256/3) - x^7*((3*a^2)/7 - (384*a
)/7 + 1536/7) + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15 + 6*a^2*x
^4 + (a^3*x^3)/3 - (24*a*x^5*(a - 8))/5
```

sympy [A] time = 0.10, size = 134, normalized size = 0.97

$$\frac{a^3x^3}{3}+6a^2x^4-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+x^{11}\left(\frac{3a}{11}-\frac{768}{11}\right)+x^{10}\left(\frac{768}{5}-\frac{12a}{5}\right)+x^9\left(\frac{32a}{3}-\frac{2240}{9}\right)+x^8(288-30a)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/3
 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240/9
) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a**2
- 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)
```

3.132 $\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx$

Optimal. Leaf size=79

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

```
[Out] 1/3*a^2*x^3+4*a*x^4+16/5*(4-a)*x^5-4/3*(16-a)*x^6+2/7*(64-a)*x^7-10*x^8+32/
9*x^9-4/5*x^10+1/11*x^11
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6742}

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 -
a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx &= \int \left(a^2x^2+16ax^3-16(-4+a)x^4+8(-16+a)x^5-2(-64+a)x^6-80x^7+\right.\\ &= \frac{a^2x^3}{3}+4ax^4+\frac{16}{5}(4-a)x^5-\frac{4}{3}(16-a)x^6+\frac{2}{7}(64-a)x^7-10x^8+\frac{32x^9}{9}-\end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3}-\frac{2}{7}(a-64)x^7+\frac{4}{3}(a-16)x^6-\frac{16}{5}(a-4)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-64
 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

fricas [A] time = 0.36, size = 68, normalized size = 0.86

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - 10x^8 - \frac{2}{7}x^7a + \frac{128}{7}x^7 + \frac{4}{3}x^6a - \frac{64}{3}x^6 - \frac{16}{5}x^5a + \frac{64}{5}x^5 + 4x^4a + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 10*x^8 - 2/7*x^7*a + 128/7*x^7 + 4/3*x^6*
a - 64/3*x^6 - 16/5*x^5*a + 64/5*x^5 + 4*x^4*a + 1/3*x^3*a^2
```

giac [A] time = 0.29, size = 68, normalized size = 0.86

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}ax^7 - 10x^8 + \frac{4}{3}ax^6 + \frac{128}{7}x^7 - \frac{16}{5}ax^5 - \frac{64}{3}x^6 + \frac{1}{3}a^2x^3 + 4ax^4 + \frac{64}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x^
7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5
```

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + \frac{(-2a + 128)\,x^7}{7} + \frac{(8a - 128)\,x^6}{6} + \frac{a^2x^3}{3} + 4a\,x^4 + \frac{(-16a + 64)\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+1/7*(-2*a+128)*x^7+1/6*(8*a-128)*x^6+1/5
*(-16*a+64)*x^5+4*a*x^4+1/3*a^2*x^3
```

maxima [A] time = 0.55, size = 59, normalized size = 0.75

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}(a - 64)x^7 - 10x^8 + \frac{4}{3}(a - 16)x^6 - \frac{16}{5}(a - 4)x^5 + \frac{1}{3}a^2x^3 + 4ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*
x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

mupad [B] time = 0.04, size = 64, normalized size = 0.81

$$x^6\left(\frac{4\,a}{3}-\frac{64}{3}\right)-x^5\left(\frac{16\,a}{5}-\frac{64}{5}\right)-x^7\left(\frac{2\,a}{7}-\frac{128}{7}\right)+4\,a\,x^4-10\,x^8+\frac{32\,x^9}{9}-\frac{4\,x^{10}}{5}+\frac{x^{11}}{11}+\frac{a^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^6*((4*a)/3 - 64/3) - x^5*((16*a)/5 - 64/5) - x^7*((2*a)/7 - 128/7) + 4*a*
x^4 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11 + (a^2*x^3)/3
```

sympy [A] time = 0.08, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3}+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8+x^7\left(\frac{128}{7}-\frac{2a}{7}\right)+x^6\left(\frac{4a}{3}-\frac{64}{3}\right)+x^5\left(\frac{64}{5}-\frac{16a}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7*
(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)
```

3.133 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right) dx$

Optimal. Leaf size=35

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

```
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {14}

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)dx &= \int \left(ax^2+8x^3-8x^4+4x^5-x^6\right)dx\\ &= \frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

fricas [A] time = 0.35, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + 2x^4 + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 2*x^4 + 1/3*x^3*a
```

giac [A] time = 0.31, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}a\,x^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x)
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

maxima [A] time = 0.61, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

mupad [B] time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^7}{7} + \frac{2\,x^6}{3} - \frac{8\,x^5}{5} + 2\,x^4 + \frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

sympy [A] time = 0.06, size = 29, normalized size = 0.83

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)
[Out] a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4
```

3.134 $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=99

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

```
[Out] arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a)^(1
/2))^(1/2))/(1-(4+a)^(1/2))^(1/2)-1/2*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))/
(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1680, 1673, 1166, 204, 12, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[1 - Sqrt[4 + a]]) - ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-
1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{3+a-2x^2-x^4}\,dx, x, -1$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,d$$

$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} + \text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+$$

$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} - 2\,\text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2\left(1-\right.\right.$$

$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.62

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1^2\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1^2)/(-2 +
 4*#1 - 3*#1^2 + #1^3) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

maple [C] time = 0.00, size = 52, normalized size = 0.53

$$-\frac{\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - \right.\right.}{4\left(\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)^3 - 3\,\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)^2 + 4\,\mathrm{RootOf}\left(\_Z^4 - \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(_R^2/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8
*_Z-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

mupad [B] time = 2.78, size = 878, normalized size = 8.87

$$\sum_{k=1}^{4} \ln\left(64\,\mathrm{root}\left(2816\,a^2\,z^4 + 256\,a^3\,z^4 + 10240\,a\,z^4 + 12288\,z^4 - 160\,a^2\,z^2 - 1152\,a\,z^2 - 2048\,z^2 + 32\,a^2\,z + 256\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] symsum(log(64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 1
60*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k
) - a - 8*x + 20*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
```

```
- 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)*a - 48*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160
*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^
2*a + 64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^
2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a
 + 128*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*
z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*x -
 256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^
2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*x - 1
92*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2
- 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2 + 256*r
oot(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 11
52*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3 - 4*root(28
16*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z
^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a*x + 32*root(2816*
a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2
- 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*a*x - 64*root(2816*a
^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 -
 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a*x)*root(2816*a^2*z^
4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048
*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k), k, 1, 4)
```

sympy [B] time = 7.61, size = 172, normalized size = 1.74

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-160a^2-1152a-2048\right)+t\left(-32a^2-256a-512\right)-a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2
- 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, Lambda(_t, _t*log(x
+ (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 6144*_
t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 56*_t
*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a**3 +
60*a**2 + 320*a + 448)))
```

3.135 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$

Optimal. Leaf size=225

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(a+\sqrt{a+4}+4\right)\tan^{-1}}{8(a+3)(a+4)\sqrt{1-}}$$

```
[Out] 1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(4+a)*(2+(-1+x)^2)*(-1
+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/2*arctanh((1+(-1+x)^2)/(4+a)^(
1/2))/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(4+a+(4+a)^(1/2)
)/(3+a)/(4+a)/(1-(4+a)^(1/2))^(1/2)-1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2)
)*(4+a-(4+a)^(1/2))/(3+a)/(4+a)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1680, 1673, 1178, 1166, 204, 12, 1107, 614, 618, 206}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(a+\sqrt{a+4}+4\right)\tan^{-1}}{8(a+3)(a+4)\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((4 + a)
*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/
(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(
-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) +
 ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\left(3+a-2x^2-x^4\right)^2}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + 2\,\text{Subst}\left(\int \frac{x}{\left(3+a-2x^2\right.}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{\left(4+a-\sqrt{4+a}\right)\text{Subst}}{8\left(12\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)(}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)(}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)(}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

Mathematica [C] time = 0.08, size = 182, normalized size = 0.81

$$\frac{a\left(x^3-x^2+x+1\right)+2x\left(2x^2-3x+4\right)}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)} - \frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1^2 a\log(x-\#1)+4\#1^2\log(x-\#1}{\#}\right.}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*(
-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (
-(a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]*#1
```

```
^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^
2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

maple [C] time = 0.01, size = 160, normalized size = 0.71

$$\frac{\left((-a-4)\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+2\,(-a-2)\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+a\right)\mathrm{l}}{16\,(a+3)\,(a+4)\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a+3)*x^3+1/4*(a+6)/(a+3)/(a+4)*x^2-1/4*(a+8)/(a+3)/(a+4)*x-1/4*a/(a+
3)/(a+4))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a+3)/(a+4)*sum(((-a-4)*_R^2+2*(-a-2
)*_R+a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(a+4)x^3-(a+6)x^2+(a+8)x+a}{4\left(\left(a^2+7\,a+12\right)x^4-4\left(a^2+7\,a+12\right)x^3-a^3+8\left(a^2+7\,a+12\right)x^2-7\,a^2-8\left(a^2+7\,a+12\right)x-12\,a\right)}-\frac{\int \frac{(a+}{x^4-}}{4\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((a + 4)*x^3 - (a + 6)*x^2 + (a + 8)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*
(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a
+ 12)*x - 12*a) - 1/4*integrate(((a + 4)*x^2 + 2*(a + 2)*x - a)/(x^4 - 4*x^
3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)
```

mupad [B] time = 2.85, size = 1218, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] symsum(log((x*(40*a + 7*a^2 + 56))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
 a^5 + 576)) - (48*a + 12*a^2 - a^3)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^
4 + a^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8
*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568
*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 242155
52*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2 - 9728*a^6*
z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*z + 5888*a^4
*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 144*a^3 - a^4,
 z, k)*((28160*a + 11328*a^2 + 2064*a^3 + 144*a^4 + 26624)/(64*(816*a + 460
*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + root(12952010752*a^3*z^4 + 31653888
*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 2008232755
2*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7
247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 2094
08*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z +
53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 102
4*a^2 + 144*a^3 - a^4, z, k)*(root(12952010752*a^3*z^4 + 31653888*a^7*z^4 +
 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4
+ 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*
z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2
 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*
z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 14
4*a^3 - a^4, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 +
 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
a^5 + 576)) - (x*(1966080*a + 1359872*a^2 + 499712*a^3 + 102912*a^4 + 11264
*a^5 + 512*a^6 + 1179648))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 5
76))) - (1359872*a + 749568*a^2 + 205824*a^3 + 28160*a^4 + 1536*a^5 + 98304
0)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a + 5
8880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(8*(816*a + 460*a^2 + 1
29*a^3 + 18*a^4 + a^5 + 576))) + (x*(448*a + 104*a^2 - 2*a^3 - 2*a^4 + 512)
)/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))))*root(12952010752*a
^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a
```

```
*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 2696
80640*a^6*z^4 + 7247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 18780
16*a^4*z^2 - 209408*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2
+ 237568*a^2*z + 53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752
*z + 1792*a + 1024*a^2 + 144*a^3 - a^4, z, k), k, 1, 4) + (x^3/(4*(a + 3))
+ a/(4*(a + 3)*(a + 4)) - (x^2*(a + 6))/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(
4*(a + 3)*(a + 4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)
```

sympy [B] time = 43.73, size = 561, normalized size = 2.49

$$\frac{-a + x^3(-a-4) + x^2(a+6) + x(-a-8)}{-4a^3 - 28a^2 - 48a + x^4\left(4a^2+28a+48\right) + x^3\left(-16a^2-112a-192\right) + x^2\left(32a^2+224a+384\right) + x\left(-32a^2-22\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (-a + x**3*(-a - 4) + x**2*(a + 6) + x*(-a - 8))/(-4*a**3 - 28*a**2 - 48*a
+ x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2
 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 +
 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 53571747
84*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312)
 + _t**2*(-9728*a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24215552
*a**2 - 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a**3 + 23
7568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2 + 1792*a, Lamb
da(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a**11 - 5480448*_t**3*a**
10 - 111403008*_t**3*a**9 - 1227173888*_t**3*a**8 - 8682876928*_t**3*a**7 -
 42187440128*_t**3*a**6 - 144630284288*_t**3*a**5 - 350972280832*_t**3*a**4
 - 591750234112*_t**3*a**3 - 660716126208*_t**3*a**2 - 439848271872*_t**3*a
 - 132271570944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9 - 15400960*_t
**2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 3414427648*_t**2*a
**5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 22938255360*_t**2*a*
*2 - 16873684992*_t**2*a - 5549064192*_t**2 - 848*_t*a**9 - 6096*_t*a**8 +
174608*_t*a**7 + 3323792*_t*a**6 + 26276224*_t*a**5 + 119009280*_t*a**4 + 3
32017664*_t*a**3 + 566497280*_t*a**2 + 544112640*_t*a + 225837056*_t + 11*a
**8 + 958*a**7 + 17419*a**6 + 142964*a**5 + 632632*a**4 + 1567552*a**3 + 20
49792*a**2 + 1100800*a)/(a**8 + 870*a**7 + 18289*a**6 + 165176*a**5 + 82456
0*a**4 + 2452288*a**3 + 4340224*a**2 + 4229120*a + 1748992)))
```

3.136 $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=545

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{(-1)^{2/3}\,(\ldots}{3\sqrt{3}\,(1-\ldots}$$

```
[Out] -1/18*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(1/3)+1/6*ln(3*a-3*(-
1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(2/3)/b^2/c^(1/3)+1/18
*(-1)^(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(1/3
)-1/27*(2*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)
/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4*b-3*
a^(1/3)*c^(2/3))^(1/2)-1/9*(-1)^(2/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ar
ctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(
1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(5/6)/b^2/c^
(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/9*(-1)^(1/3)*(2*(-
1)^(1/3)*b+3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*
x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))
^2/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] time = 1.48, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{(-1)^{2/3}\,(\ldots}{3\sqrt{3}\,(1-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2
/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)])])/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b - 3*(-1)^(2/3)*a^
(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1
/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*a
^(5/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sq
rt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(3*Sqrt[3]*(1 - (
```

```
-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c
^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c
^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^(1
/3))^2*a^(2/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^
(1/3)*x + b*x^2])/(18*a^(2/3)*b^2*c^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}\ -\sqrt[3]{c}\,x}{59049\left(1+\sqrt[3]{-1}\right)^2 a^{20/3}bc^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2/3}\right.}\right.$$

$$= \frac{\int \frac{-\sqrt[3]{a}-\sqrt[3]{c}\,x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{9a^{2/3}bc^{2/3}} - \frac{(-1)^{2/3}\int \frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{c}\,x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{9a^{2/3}bc^{2/3}} + \frac{\int}{3}$$

$$= \frac{\left(3-\frac{2b}{\sqrt[3]{a}\,c^{2/3}}\right)\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{18b^2} + \frac{\left(3-\frac{2(-1)^{2/3}b}{\sqrt[3]{a}\,c^{2/3}}\right)\int \frac{}{3a+3(-1)^{2/}}}{18b^2}$$

$$= -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}} + \frac{\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+b\right.}{6\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}b^2\sqrt[3]{c}}$$

$$= -\frac{\left(3ib+\sqrt{3}\left(b+3\sqrt[3]{a}\,c^{2/3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.18

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^3\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

maple [C] time = 0.02, size = 93, normalized size = 0.17

$$\frac{\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z\right.}{6\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3 + 27b\,a^2\_Z^2 + 27a^3\right)^5 b^3 + 36\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

mupad [B] time = 3.18, size = 1563, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-19683*a^8*b^3*(c*x - b + 6561*root(918330048*a^5*b^9*c^4*z^6 -
387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z
^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*
a^3*c^4 + 2*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 15
94323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 3280
5*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*b^4*x - 198*root(918330048*a^5*b
^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*
a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z
+ 1, z, k)*a*b^2*c - 8991*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^
6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*
c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^3*c^2 - 19
683*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^
4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^
2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^4*c^3 + 104976*root(918330048*a^
5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 10235
16*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c
*z + 1, z, k)^4*a^3*b^8*c^2 - 8503056*root(918330048*a^5*b^9*c^4*z^6 - 3874
20489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 -
 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*
b^9*c^3 + 4782969*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^
6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3
+ 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^5*b^6*c^5 + 108*root(9
18330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*
z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2
+ 324*a*b*c*z + 1, z, k)^2*a*b^5*c*x + 108*root(918330048*a^5*b^9*c^4*z^6 -
 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*
z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*a
*b*c^2*x + 1458*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6
+ 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 +
32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^2*c^3*x - 2916*root(
918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4
*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2
 + 324*a*b*c*z + 1, z, k)^3*a^2*b^6*c^2*x + 78732*root(918330048*a^5*b^9*c^
4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b
^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1,
z, k)^4*a^3*b^7*c^3*x + 1062882*root(918330048*a^5*b^9*c^4*z^6 - 387420489*
a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 53144
1*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*b^8*c^
4*x))*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*
a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*
b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

3.137 $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=487

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}$$

```
[Out] 1/54*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c^(2/3)-1/18*(-1)^(2/3)*ln
(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(4/3)/b/c^(2/
3)+1/54*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c
^(2/3)-1/27*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(
1/3)*c^(2/3))^(1/2))/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2
)+1/9*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^
(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3
))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/9
*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1
)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/
(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqr
t[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3)
+ 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7/6
)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/
3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3
)*c^(2/3)])])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqrt
[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)
*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(
```

```
2/3)*c^(1/3)*x + b*x^2])/(18*(1 + (-1)^(1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1)^
(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(54*a^(4/3)*b*c^(2
/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{(-1)^{2/3}x}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2/3}\right.}\right.$$

$$= \frac{\int \frac{x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-}{}$$

$$= \frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{54a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{54a^{4/3}bc^{2/3}} - \frac{(-}{}$$

$$= \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\right.}{18\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}bc^{2/}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\sqrt[3]{c}} - \frac{\tan^{-1}}{9\sqrt{3}\,a^{7/}}$$

Mathematica [C] time = 0.05, size = 99, normalized size = 0.20

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^2\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

maple [C] time = 0.00, size = 93, normalized size = 0.19

$$\frac{\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z\right.}{6\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3 + 27b\,a^2\_Z^2 + 27a^3\right)^5 b^3 + 36\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

mupad [B] time = 3.10, size = 1354, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(4782969*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c
^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4
*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^9*b^6*c^3 - 729*a^5*b^7*x + 12914
0163*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348
907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a
^3*b*c^2*z^2 - 1, z, k)^3*a^10*b^8*c^3 + 1549681956*root(10460353203*a^9*b^
3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928
*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^
11*b^10*c^3 + 167365651248*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a
^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441
*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^12*c^3 - 94143178827
*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*
a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b
*c^2*z^2 - 1, z, k)^5*a^13*b^9*c^5 + 98415*root(10460353203*a^9*b^3*c^6*z^6
 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*
c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^7*c + 4
374*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 143489
07*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^
3*b*c^2*z^2 - 1, z, k)*a^6*b^9*x - 2125764*root(10460353203*a^9*b^3*c^6*z^6
 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*
c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b^9*c -
 59049*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 143
48907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683
*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^6*c^2*x - 531441*root(10460353203*a^9*b^3*c
^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^
4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b
^8*c^2*x - 688747536*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6
*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c
^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^10*b^12*c^2*x + 1162261467*root
(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b
^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*
z^2 - 1, z, k)^4*a^11*b^9*c^4*x - 20920706406*root(10460353203*a^9*b^3*c^6*
z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b
^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^1
1*c^4*x)*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 1
4348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 196
83*a^3*b*c^2*z^2 - 1, z, k), k, 1, 6)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

3.138 $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=334

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3(\cdots}{\sqrt{3}\sqrt{\cdots}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a\cdots}$$

```
[Out] 2/81*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^
(2/3))^(1/2))/a^(11/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+2/27*(
-1)^(2/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(
4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^
(11/6)/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+2/27*(-1)^(
2/3)*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3
*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(11/6)/c^(2/3)*3^(1/
2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2097, 618, 204}

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3(\cdots}{\sqrt{3}\sqrt{\cdots}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a
]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 + (-1)^(1/3))^2
*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*ArcTan[(3*
a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/
(27*Sqrt[3]*a^(11/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)
*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b +
3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3
))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{(-1)^{2/3}}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2}\right.}\right.$$

$$= \frac{\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + $$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}\,c^{2/3}\right)-x^2}\,dx, x, 3a^{2/3}\sqrt[3]{c}\,+2bx\right)}{27a^{4/3}c^{2/3}} - \frac{\left(2(\right.}{}$$

$$= \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}} + \frac{2\tan^{}}{27\sqrt{3}\,a}$$

Mathematica [C] time = 0.06, size = 97, normalized size = 0.29

$$\frac{1}{3}\mathrm{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &
]/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

maple [C] time = 0.00, size = 93, normalized size = 0.28

$$\frac{\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3\right.}{6\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3 + 27b\,a^2\_Z^2 + 27a^3\right)^5 b^3 + 36\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

mupad [B] time = 3.34, size = 825, normalized size = 2.47

$$\sum_{k=1}^{6} \ln\left(-a^3\,b^9\,\left(-\mathrm{root}\left(669462604992\,a^{11}\,b^3\,c^4\,z^6-282429536481\,a^{12}\,c^6\,z^6+129140163\,a^8\,c^4\,z^4-19683\,a^4\,c^2\,z\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-27*a^3*b^9*(43046721*root(669462604992*a^11*b^3*c^4*z^6 - 28242
9536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)
^4*a^8*c^4 - 1062882*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12
*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^6*c^3 -
 13122*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129
140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^4*c^2 + 3486784401*ro
ot(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^
8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^10*c^5 + 81*root(669462604992*
a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 1968
3*a^4*c^2*z^2 + 1, z, k)*a^2*c + 18*root(669462604992*a^11*b^3*c^4*z^6 - 28
2429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z,
 k)*a*b^2*x - 25509168*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^
12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^3
*c^2 - 6198727824*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^
6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^3*c^3
+ 5832*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129
140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^3*b^2*c*x + 708588*ro
ot(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^
8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^5*b^2*c^2*x + 38263752*root(66
9462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4
*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^2*c^3*x + 774840978*root(669462
604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4
 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^2*c^4*x + 1))*root(669462604992*a^1
1*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a
^4*c^2*z^2 + 1, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

3.139 $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=469

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx\right.}{162a^{7/3}c^{2/3}}$$

```
[Out] -1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c^(2/3)+1/54*(-1)^(2/3)*ln
(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(7/3)/c^(2/3)
-1/162*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c^(2
/3)-1/81*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3
)*c^(2/3))^(1/2))/a^(13/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/
27*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/
2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^
2/a^(13/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/27*ar
ctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(
2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(13/6)/c^(1/3)*3^(1/2)/(4*b
-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx\right.}{162a^{7/3}c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt
[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3) +
 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(13/
6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3
)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)
*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt[4
*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x
 + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2/3
```

```
)*c^(1/3)*x + b*x^2])/(54*(1 + (-1)^(1/3))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/3)
*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*a^(7/3)*c^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-3a^{2/3}\sqrt[3]{c}-(-1)^{2/3}bx}{531441\left(1+\sqrt[3]{-1}\right)^2 a^{25/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2}\right.}\right.$$

$$= \frac{\int \frac{-3a^{2/3}\sqrt[3]{c}-bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{81a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{81a^{7/3}c^{2/3}} + \cdots$$

$$= -\frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{162a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{162a^{7/3}c^{2/3}} + \cdots$$

$$= -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\cdots\right.}{54\left(1+\sqrt[3]{-1}\right)^2 a^{7/3}\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}} - \frac{\tan\cdots}{27\sqrt{3}\cdots}$$

Mathematica [C] time = 0.05, size = 95, normalized size = 0.20

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),
x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ]/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

maple [C] time = 0.00, size = 91, normalized size = 0.19

$$\frac{\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z}{6\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3 + 27b\,a^2\_Z^2 + 27a^3\right)^5 b^3 + 36\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_
R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

mupad [B] time = 2.90, size = 1057, normalized size = 2.25

$$\sum_{k=1}^{6} \ln\Big(b^{12}\,x + \mathrm{root}\Big(18075490334784\,a^{14}\,b^3\,c^4\,z^6 - 7625597484987\,a^{15}\,c^6\,z^6 + 1162261467\,a^{10}\,b\,c^4\,z^4 + 8503056$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(b^12*x + 1033121304*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4*a^10*b^11*c
^3 + 167365651248*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15
*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a
^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^12*c^3 - 94143178
827*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 116
2261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 1
77147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^13*b^9*c^5 + 54*root(18075490334784*
a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 +
 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 +
b^3, z, k)*a^2*b^13*x + 177147*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^2*a^5*b^11*c^
2*x + 17006112*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^
6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*
c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^7*b^12*c^2*x - 14348907*r
oot(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 11622614
67*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147
*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^8*b^9*c^4*x + 229582512*root(180754903347
84*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^
4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2
 + b^3, z, k)^4*a^9*b^13*c^2*x + 387420489*root(18075490334784*a^14*b^3*c^4
*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7
*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4
*a^10*b^10*c^4*x - 20920706406*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^11*c
^4*x)*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1
162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 +
 177147*a^5*b^2*c^2*z^2 + b^3, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

3.140 $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=522

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\,a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2(-1\cdots\right.}{27\sqrt{3}\left(1\cdots\right.}$$

```
[Out] 1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/54*ln(3*a-3*(-1)^
(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(8/3)/c^(1/3)-1/162*(-1)^
(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/243*(2
*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/
(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(17/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2
/3))^(1/2)-1/81*(2*(-1)^(2/3)*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)
*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^
(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(17/6)/c^(2/3)*3^(1/2)/(4*b+3*(-1)
^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/81*(-1)^(1/3)*(2*(-1)^(1/3)*b+3*a^(1/3)*c^(
2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-
3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(17/6)/c^(2/3)*3^(1
/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2070, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\,a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2(-1\cdots\right.}{27\sqrt{3}\left(1\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1),x]
```

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2
/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)]])])/(27*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1
/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*a^
(17/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1/3
)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*S
qrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 - (-1)^(1/3))*(1
+ (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
```

```
+ Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(8/3)*c^(1/3)) - Log[3*a -
3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^(1
/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*
a^(8/3)*c^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2070

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}\,b-3\sqrt[3]{-1}\,a^{2/3}c^{2/3}+b\sqrt[3]{c}}{531441\left(1+\sqrt[3]{-1}\right)^2a^{26/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2/3}\right.}\right.$$

$$=\frac{\int\frac{-\sqrt[3]{a}\,b+3a^{2/3}c^{2/3}+b\sqrt[3]{c}\,x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{81a^{8/3}c^{2/3}}-\frac{\int\frac{(-1)^{2/3}\sqrt[3]{a}\,b-3a^{2/3}c^{2/3}+\sqrt[3]{-1}\,b\sqrt[3]{c}\,x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{81a^{8/3}c^{2/3}}-$$

$$=-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\int\frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{162a^{7/3}c^{2/3}}-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}\,c^{2/}\right.}{16}$$

$$=\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+bx^2\right.}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}$$

$$=-\frac{\left(2(-1)^{2/3}b+3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.19

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\log(x-\#1)}{2\#1^5b^3+12\#1^3ab^2+27\#1^2a^2c+18\#1a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-
1),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &
]/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

maple [C] time = 0.00, size = 90, normalized size = 0.17

$$\frac{\ln\left(-\mathrm{R}\right.}{6\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^5 b^3+36\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R
=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

mupad [B] time = 0.71, size = 1394, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(6561*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^1
8*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 3874
20489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k
)^2*a^4*b^12*c^2 - 6*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209464
9*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 +
 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6,
 z, k)*b^15*x - 4782969*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209
4649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^
3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b
^6, z, k)^3*a^7*b^11*c^3 - 229582512*root(488038239039168*a^17*b^3*c^4*z^6
- 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a
^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^
3*b^5*c*z + b^6, z, k)^4*a^9*b^13*c^2 - 387420489*root(488038239039168*a^17
*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4
- 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*
z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^10*c^4 + 167365651248*root(488
038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*
a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 265
7205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^12*c^3 - 9414
3178827*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^
10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^13
*b^9*c^5 + 14580*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^
18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387
420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z,
k)^2*a^3*b^14*c*x - 10628820*root(488038239039168*a^17*b^3*c^4*z^6 - 205891
132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c
^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*
z + b^6, z, k)^3*a^6*b^13*c^2*x + 2238429492*root(488038239039168*a^17*b^3*
c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746
143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 -
 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^12*c^3*x - 1162261467*root(488038239
039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b
^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a
^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^9*c^5*x - 209207064
06*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 1
0460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^
5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^11
*c^4*x)*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^
10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k), k, 1,
 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

3.141 $\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx$

Optimal. Leaf size=563

$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}\,c^{2/3}+i\sqrt{3}\,b+b\right)\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1}{c}\right.}{}$$

```
[Out] 1/27*ln(x)/a^3-1/486*(3*a^(1/3)-b/c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2
)/a^(10/3)-1/486*(3*a^(1/3)-(-1)^(2/3)*b/c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/
3)*c^(1/3)*x+b*x^2)/a^(10/3)-1/972*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*
x^2)*(b+6*a^(1/3)*c^(2/3)+I*b*3^(1/2))/a^(10/3)/c^(2/3)+1/81*(b-a^(1/3)*c^(
2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c
^(2/3))^(1/2))/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/27*
(-1)^(2/3)*((-1)^(2/3)*b-a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*
c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1
-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^
(1/3)*c^(2/3))^(1/2)+1/27*(b-(-1)^(2/3)*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)
^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(
2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^
(1/3)*c^(2/3))^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}\,c^{2/3}+i\sqrt{3}\,b+b\right)\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1}{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]
```

```
[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*
b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3
]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1
/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*S
qrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]])])/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a
^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*Arc
Tan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-
```

```
1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2
*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(27*a^
3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*
a^(10/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/
3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(1
0/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x}+\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\sqrt[3]{c}+b}{4782969a^{28/3}c^{2/3}\left(3a+3a\right.}\right.$$

$$=\frac{\log(x)}{27a^3}+\frac{\int\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\sqrt[3]{c}+b\left(b-3\sqrt[3]{a}\,c^{2/3}\right)x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{243a^{10/3}c^{2/3}}+\frac{(-1)^{2/3}}{}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{486a^{10/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-}{c}\right.}{}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{486a^{10/3}}-\frac{\left(b+\right.}{}$$

$$=\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\sqrt[3]{c}}+\frac{\left(b\right.}{}$$

Mathematica [C] time = 0.12, size = 157, normalized size = 0.28

$$-\frac{\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^4b^3\log(x-\#1)+9\#1^2ab^2\log(x-\#1)+27a^2b\log(x-\#1)+27\#1a^2c}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\right.}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6)),x]
```

```
[Out] -1/81*(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2
*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*
b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 12*a
*b^2*#1^2 + 2*b^3*#1^4) & ])/a^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x), x)
```

maple [C] time = 0.01, size = 134, normalized size = 0.24

$$\frac{\ln(x)}{27a^3} - \frac{\left(\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3 + 27b\,a^2\_Z^2 + 27a^3\right)^5 b^3 + 9\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a\right.\right.}{81a^3\left(2\,\mathrm{RootOf}\left(b^3\_Z^6 + 9b^2a\,\_Z^4 + 27a^2c\,\_Z^3 + 27b\,a^2\_Z^2 + 27a^3\right)^5 b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] -1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2*b)/(2*_R^5*b^3
+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z
^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))+1/27*ln(x)/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.55, size = 4002, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),x)

[Out] log(x)/(27*a^3) + symsum(log(7*root(13177032454057536*a^20*b^3*c^4*z^6 - 55
59060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911320
94649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^
6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046035
3203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6
561*a^4*b^6*c^2*z + b^9, z, k)*b^18*x - 162*root(13177032454057536*a^20*b^3
*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5
 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 25418658
28329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*
z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^
6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^3*b^18*x + 86093442*root(13
177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 48803823
9039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^1
4*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 -
 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c
^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^8*b
^13*c^3 + 34867844010*root(13177032454057536*a^20*b^3*c^4*z^6 - 55590605665
55523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^1
8*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 2
7506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12
*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b
^6*c^2*z + b^9, z, k)^4*a^11*b^13*c^3 - 10460353203*root(13177032454057536*
a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3
*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 -
2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*
b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009737
9*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^12*b^10*c^5 + 15062
90861232*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^
6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6
119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^
11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14
348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^
9, z, k)^5*a^14*b^13*c^3 - 564859072962*root(13177032454057536*a^20*b^3*c^4
*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2
05891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 254186582832
9*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3
- 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^
2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^15*b^10*c^5 - 67783088755440*ro
ot(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488
038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 611930662375
5*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*
z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*
b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*
```

a^17*b^13*c^3 + 22876792454961*root(13177032454057536*a^20*b^3*c^4*z^6 - 55
59060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911320
94649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^
6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046035
3203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6
561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^10*c^5 + 17496*root(131770324540575
36*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*
b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4
 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^
10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009
7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^4*b^16*c - 47239
2*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 +
 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 61193066
23755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*
c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*
a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k
)^3*a^7*b^16*c - 39366*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566
555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^
18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 +
27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^1
2*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*
b^6*c^2*z + b^9, z, k)^2*a^4*b^15*c^2*x + 51372630*root(13177032454057536*a
^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*
c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2
541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b
^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379
*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^7*b^15*c^2*x + 71744
535*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6
 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 611930
6623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^
3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 1434890
7*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z,
 k)^3*a^8*b^12*c^4*x - 2008846980*root(13177032454057536*a^20*b^3*c^4*z^6 -
 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911
32094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15
*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046
0353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2
 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^10*b^15*c^2*x + 108477736920*root(131
77032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239
039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14
*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 -
229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^
4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^11*b
^12*c^4*x - 41841412812*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056
6555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a

```
^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 +
 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^
12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4
*b^6*c^2*z + b^9, z, k)^4*a^12*b^9*c^6*x + 18596183472*root(131770324540575
36*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*
b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4
 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^
10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009
7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^13*b^15*c^2*x +
16129864639026*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a
^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z
^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854
719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^
3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*
z + b^9, z, k)^5*a^14*b^12*c^4*x - 6778308875544*root(13177032454057536*a^2
0*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^
4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 254
1865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6
*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a
^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^15*b^9*c^6*x + 6456339
20395566*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^
6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6
119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^
11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14
348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^
9, z, k)^6*a^17*b^12*c^4*x - 274521509459532*root(13177032454057536*a^20*b^
3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^
5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865
828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2
*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b
^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^9*c^6*x)*root(1317703
2454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 4880382390391
68*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3
*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 2295
82512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^
2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
3),x)
```

```
[Out] Timed out
```

3.142 $\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal. Leaf size=645

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}\,bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}\,bc^{2/3}+2b^2\right)\tan^{-1}\left(\right.}{243\sqrt{3}\,a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{}}}$$

```
[Out] -1/27/a^3/x-1/486*(2*b-3*a^(1/3)*c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)
/a^(11/3)/c^(1/3)+1/162*(2*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))*ln(3*a-3*(-1)^(1
/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(11/3)/c^(1/3)+1/486*(-1)^(
1/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)
*x+b*x^2)/a^(11/3)/c^(1/3)+1/729*(2*b^2-12*a^(1/3)*b*c^(2/3)+9*a^(2/3)*c^(4
/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^
(2/3))^(1/2))/a^(23/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/243*
(-1)^(2/3)*(2*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*(-1)^(2/3)*a^(2/3)*c^(4
/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3
*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(23/6
)/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+1/243*(2*(-1)^(2
/3)*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*a^(2/3)*c^(4/3))*arctan(1/3*(3*(-
1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c
^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(23/6)/c^(2/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*
a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A] time = 1.38, antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}\,bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}\,bc^{2/3}+2b^2\right)\tan^{-1}\left(\right.}{243\sqrt{3}\,a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),
x]
```

```
[Out] -1/(27*a^3*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3)*b*c^(2/3) + 9*a^
(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[
a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]])/(81*Sqrt[3]*(1 + (-1)^(1/3))
^2*a^(23/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*b^2 - 1
```

```
2*a^(1/3)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)
/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(243*Sqrt[3]*a^(23/6)*Sq
rt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*(-1)^(2/3)*b^2 - 12*a^(1/3)*b*c^
(2/3) - 9*(-1)^(1/3)*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3)
+ 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(81*S
qrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(23/6)*Sqrt[4*b + 3*(-1)^(1/3)
*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a^(2/3)
*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/3)*a^(1/3)*
c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*(1 + (-1)^
(1/3))^2*a^(11/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3
))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
```

```
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x^2}+\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}\,bc^{2/3}+\right.}{1594323\left(1-\sqrt[3]{-1}\right)\left(1+\right.}\right.$$

$$=-\frac{1}{27a^3x}+\frac{\int\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}\,bc^{2/3}+9a^{2/3}c^{4/3}\right)-b\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\sqrt[3]{c}\,x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{243a^{11/3}c^{2/3}}$$

$$=-\frac{1}{27a^3x}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{486a^{11/3}\sqrt[3]{c}}+\frac{\left(\sqrt[3]{-1}\right.}{}$$

$$=-\frac{1}{27a^3x}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}+$$

$$=-\frac{1}{27a^3x}+\frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\,\sqrt[3]{a}\,bc^{2/3}+9a^{2/3}c^{4/3}\right)\tan}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b-3(}}$$

Mathematica [C] time = 0.14, size = 163, normalized size = 0.25

$$-\frac{x\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^4b^3\log(x-\#1)+9\#1^2ab^2\log(x-\#1)+27a^2b\log(x-\#1)+27\#1a^2}{2\#1^5b^3+12\#1^3ab^2+27\#1^2a^2c+18\#1a^2b}\right.}{81a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*
x^6)),x]
```

```
[Out] -1/81*(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4
+ b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Lo
g[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*
b^2*#1^3 + 2*b^3*#1^5) & ])/(a^3*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x^2), x)
```

maple [C] time = 0.01, size = 133, normalized size = 0.21

$$\frac{\left(-\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^4 b^3-9\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z\right.\right.}{81a^3\left(2\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^5 b^3+12\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] -1/27/a^3/x+1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_
R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b
^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.72, size = 2663, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),
x)

[Out] symsum(log(-282429536481*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946
35296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*
a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^
3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a
^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^23*b^9*(2*b^10*x + 2541
865828329*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^2
4*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^
4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 2824295
36481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2
+ 17496*a^4*b^10*c*z + b^12, z, k)^4*a^17*c^5 - 45*a*b^8*c + 387420489*root
(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45
753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 7531454306
16*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7
*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^
10*c*z + b^12, z, k)^2*a^10*c^6*x - 401769396*root(355779876259553472*a^23*
b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z
^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207
657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*
c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^
9*b^4*c^3 - 2066242608*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635
296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^
16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*
z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8
*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^12*b^5*c^2 + 6973568802
*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6
 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 75314
5430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^1
4*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a
^4*b^10*c*z + b^12, z, k)^3*a^13*b^2*c^4 - 4518872583696*root(3557798762595
53472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a
^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^
5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 2582803
26*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12,
 z, k)^4*a^16*b^3*c^3 - 328050*root(355779876259553472*a^23*b^3*c^4*z^6 - 1
50094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 1093002306
18147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*
b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 10044
2349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^5*b^6*c^2 - 17714
7*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^
6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 7531
```

```
45430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^
14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*
a^4*b^10*c*z + b^12, z, k)*a^6*b^3*c^4 + 387420489*root(355779876259553472*
a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*
c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3
+ 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9
*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)
^2*a^10*b*c^5 + 23328*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946352
96999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^1
6*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z
^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*
b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^4*b^8*c*x + 196830*root(35
5779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753
584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*
a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^
3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*
c*z + b^12, z, k)*a^5*b^5*c^3*x - 20920706406*root(355779876259553472*a^23*
b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z
^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207
657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*
c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^
13*b*c^5*x + 74401740*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946352
96999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^1
6*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z
^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*
b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^8*b^6*c^2*x - 746143164*
root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6
- 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145
430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14
*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^
4*b^10*c*z + b^12, z, k)^2*a^9*b^3*c^4*x + 55788550416*root(355779876259553
472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^1
7*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*
z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326
*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z
, k)^3*a^12*b^4*c^3*x + 564859072962*root(355779876259553472*a^23*b^3*c^4*z
^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 1093
00230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104
*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 +
 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^16*b^2*c^
4*x))*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^
6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 -
753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 28242953648
1*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17
496*a^4*b^10*c*z + b^12, z, k), k, 1, 6) - 1/(27*a^3*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
a**3),x)
```

```
[Out] Timed out
```

3.143 $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6} \, dx$

Optimal. Leaf size=395

$$\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)+\frac{}{1\ldots}$$

```
[Out] 1/108*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/108*(18-2^
(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/216*ln(6-3*(-3)^(1/3)*2^(2/3
)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))+1/2*3^(1/6)*2^(5/6)*(1-(-3)^(2/
3)*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3
))^(1/2))/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/6*(1-2^(1/3)*3^
(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))
*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/3*(-2)^(1/3)*(1+(-2)^(1/3)*
3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2)
)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 1.44, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)+\frac{}{1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -(((-2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/
Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6
) + 3*2^(1/3)*3^(2/3)])) + ((3/2)^(1/6)*(1 - (-3)^(2/3)*2^(1/3))*ArcTan[(2^
(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1
+ (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/
(2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3)*(1
+ I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + ((18 - (-2)^(2/3
)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/108 + ((18 - 2^(2/3)*3^(1
/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/108
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}\,2^{2/3}+\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x\right)}{3779136\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+ -$$

$$= \frac{1}{54}\int \frac{6\sqrt[3]{2}\,3^{2/3}+\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx + \frac{(-1)^{2/3}\int \frac{3(-2)^{2/3}\sqrt[3]{3}-\left(1+3\sqrt[3]{-2}\,3\right.}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}}{9\sqrt[3]{2}\,3^{2/3}}$$

$$= \frac{\left((-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\right)\int \frac{-3\sqrt[3]{-3}\,2^{2/3}+2x}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}\,dx}{6\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\left((-1)^{2/3}\sqrt[3]{\frac{3}{2}}\left(6+\right.\right.}{2}$$

$$= \frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}+i2^{2/3}3^{5/6}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)+\frac{1}{108}\left(18-\right.$$

$$= \frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{6^{5/6}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}} - \frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\right.}{\sqrt[6]{6}\left(1+\sqrt[3]{-}\right.}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.15

$$\frac{1}{6}\mathrm{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5 \ln\left(-\right.}{6\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x)
[Out] 1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 0.65, size = 427, normalized size = 1.08

$$\sum_{k=1}^{6} \ln\left(\frac{362797056\,\left(19236852\,x\,\mathrm{root}\left(z^6+4374\,z^5+6626610\,z^4+2646786132\,z^3-24163559388\,z^2+7266286\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log((362797056*(19236852*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 26467
86132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 19131876
```

```
*x - 6482268*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 2416355
9388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 742851*x*root(z^6 + 4374*
z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 7266
2865048, z, k)^3 - 4130*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^
3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + x*root(z^6 + 4
374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z -
72662865048, z, k)^5 - 154944576*root(z^6 + 4374*z^5 + 6626610*z^4 + 264678
6132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 1704742
2*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72
662865048*z - 72662865048, z, k)^3 + 27054*root(z^6 + 4374*z^5 + 6626610*z^
4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4
 + 9*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 +
 72662865048*z - 72662865048, z, k)^5 + 465542316*root(z^6 + 4374*z^5 + 662
6610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048,
z, k) - 465542316))/root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24
163559388*z^2 + 72662865048*z - 72662865048, z, k)^5)*root(z^6 - z^5 + (421
*z^4)/1266 - (100853*z^3)/2768742 - (505*z^2)/5537484 - z/16612452 - 1/7266
2865048, z, k), k, 1, 6)
```

sympy [A] time = 0.26, size = 70, normalized size = 0.18

$$\text{RootSum}\left(72662865048t^6 - 72662865048t^5 + 24163559388t^4 - 2646786132t^3 - 6626610t^2 - 4374t - 1, \left(t \mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 2646786
132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-892364171310473
76*_t**5/833243797 + 89301949532998128*_t**4/833243797 - 29740560281805852*
_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/8332437
97 + x + 5365044886/2499731391)))
```

3.144 $\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{18\,2^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{18\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2}\right.}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^{2}}$$

```
[Out] 1/36*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2+1/10
8*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)-1/108*ln(6+3*
2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/27*(-1)^(2/3)*(3*(-3)^(2/3)-2^(2/3
))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(5
/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/27*(9-2^(2/3)*3^(1/3)
)*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(-24
+18*2^(1/3)*3^(2/3))^(1/2)+1/27*(9-(-2)^(2/3)*3^(1/3))*arctan((3*(-2)^(2/3)
*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^(1/3)*3^(1/6)+9*
2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{18\,2^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{18\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2}\right.}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4
- 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*
3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(27*Sqrt[3*(8 + (9*I)*2
^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(
1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*Sqrt[6
*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(6*2^(2
/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3
)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(18*2^(2/3)*3
^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(-2+\sqrt[3]{-3}\,2^{2/3}x\right)}{7558272\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)}+ \cdots\right.$$

$$= -\frac{(-1)^{2/3}\int\frac{2+(-2)^{2/3}\sqrt[3]{3}\,x}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{18\sqrt[3]{2}\,3^{2/3}} - \frac{\int\frac{\sqrt[3]{2}+\sqrt[3]{3}\,x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{9\,6^{2/3}} + \frac{(-1)^{2/3}\int\frac{-2+\cdots}{-6+3\sqrt[3]{\cdots}}}{6\sqrt[3]{2}\,3^{2/3}\left(1 \cdots\right.}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{18\,2^{2/3}} - \frac{\int\frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{18\,2^{2/3}\sqrt[3]{3}} + \frac{\int\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2}}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-\cdots}\right.}$$

$$= \frac{\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)}{18\,2^{2/3}} - \frac{\log \cdots}{\cdots}$$

$$= -\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \frac{\left((-2)^{2/3}-3\,3^{2/3}\right)\tan \cdots}{27\sqrt[6]{3}\sqrt{2\left(4 \cdots\right.}}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^3\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.15

$$\frac{\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4 \ln}{6\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x)
[Out] 1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.70, size = 390, normalized size = 1.03

$$\sum_{k=1}^{6} \ln\left(-\frac{5038848\,\left(1377495072\,x+17006112\,x\,\mathrm{root}\left(z^6+1944\,z^5+1180980\,z^4-1845163152\,z^3+20662426\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log(-(5038848*(1377495072*x + 17006112*x*root(z^6 + 1944*z^5 + 11809
80*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) - 104976*x
```

```
*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 1569
5178850368, z, k)^2 + 158112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 18451631
52*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 + 1946*x*root(z^6 + 1944*
z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)
^4 + 3*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^
2 - 15695178850368, z, k)^5 - 4251528*root(z^6 + 1944*z^5 + 1180980*z^4 - 1
845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 3927852*root(z^6
 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 1569517885036
8, z, k)^3 - 1188*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066
242608*z^2 - 15695178850368, z, k)^4 - root(z^6 + 1944*z^5 + 1180980*z^4 -
1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 + 7558272*root(z^
6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 156951788503
68, z, k) + 33519046752))/root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^
3 + 2066242608*z^2 - 15695178850368, z, k)^5)*root(z^6 - z^4/7596 + (217*z^
3)/1845828 - (5*z^2)/66449808 - z/8073651672 - 1/15695178850368, z, k), k,
1, 6)
```

sympy [A] time = 0.28, size = 65, normalized size = 0.17

$$\operatorname{RootSum}\left(15695178850368t^6 - 2066242608t^4 + 1845163152t^3 - 1180980t^2 - 1944t - 1, \left(t \mapsto t\log\left(\frac{614714526}{571}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 118098
0*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(61471452617855174620 8*_t**5/571212
95165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/571212
95165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/57121295165
 + x - 44532180783/57121295165)))
```

3.145 $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{36\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}-\frac{\text{ta}}{6\sqrt[6]{2}\,3^{5/6}\,(}$$

```
[Out] -1/216*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^
(1/3))^2+1/648*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)+
1/648*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/36*arctan((3*(-3)^(1/
3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1+(-1)^(
1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/108*arctanh(2^(1/6)*(3*3^(1/3)+2^(
1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3
))^(1/2)+1/54*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3
)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))
^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{36\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}-\frac{\text{ta}}{6\sqrt[6]{2}\,3^{5/6}\,(}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(6
*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)
^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3)
)]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]
) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3
))]]/(18*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[6
- 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) +
((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1/3)*3^(2/3)) +
Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)}-\right.$$

$$=\frac{\int\frac{x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{}{-6+3\sqrt[3]{}}}{18\sqrt[3]{2}\,3^{2/3}\left(1\right.}$$

$$=\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\,2^{2/3}}+\frac{\int\frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{3(-2)^{2/}}{6+3(-2)^{2}}}{108\sqrt[3]{2}\,3}$$

$$=-\frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{36\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x\right.}{108\sqrt[3]{2}\,3^{2/3}}$$

$$=-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3\right.}}\right.}{18\sqrt[6]{2}\,3^{5/6}\sqrt{4+3\sqrt[3]{-2}\,3}}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.17

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^2\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.16

$$\frac{\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^3 \ln\left(-\right.}{6\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^5 + 72\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^3 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x)
[Out] 1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 0.52, size = 276, normalized size = 0.76

$$\sum_{k=1}^{6} \ln\left(-\frac{23328\left(297538935552\,x - 7992872640\,x\,\mathrm{root}\left(z^6 + 1417176\,z^4 + 1332145440\,z^3 + 74384733888\,z^2 - 3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log(-(23328*(297538935552*x - 7992872640*x*root(z^6 + 1417176*z^4 +
1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 52488*x*root(z
```

```
^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z,
k)^3 + 2904*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3
390158631679488, z, k)^4 + x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 7438
4733888*z^2 - 3390158631679488, z, k)^5 - 153055008*root(z^6 + 1417176*z^4
+ 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^2 - 2764368*ro
ot(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488,
 z, k)^3 - 1620*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 -
 3390158631679488, z, k)^4 - 3673320192*root(z^6 + 1417176*z^4 + 1332145440
*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 7240114098432))/root(z^6
 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)
^5)*root(z^6 - z^4/45576 - (235*z^3)/598048272 - z^2/2392193088 - 1/3390158
631679488, z, k), k, 1, 6)
```

sympy [A] time = 0.25, size = 61, normalized size = 0.17

$$\mathrm{RootSum}\left(3390158631679488t^6 - 74384733888t^4 - 1332145440t^3 - 1417176t^2 - 1, \left(t \mapsto t\log\left(-\frac{8482372214}{4158}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 141
7176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 22160559
10930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/4158
17 - 416583756*_t/415817 + x - 89938/415817)))
```

3.146 $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=248

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

```
[Out] 1/162*(-1)^(2/3)*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3
))^(1/2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1
/486*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2
^(1/6)*3^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/486*(-1)^(2/3)*arctan((3*(-2)
^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(5/6)/(8+9*
I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2097, 618, 204, 206}

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^
(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1
/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^
(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^
(1/3)*3^(2/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}}{22674816\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)}-\right.$$

$$=\frac{\int\frac{1}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{}{-6+3\sqrt[3]{}}}{18\sqrt[3]{2}\,3^{2/3}\left(1\right.}$$

$$=-\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}\,3^{2/3}\right)-x^2}\,dx,x,3\,2^{2/3}\sqrt[3]{3}+2x\right)}{27\sqrt[3]{2}\,3^{2/3}}-\frac{(-1)^{2/3}\,\text{Subst}\left(\int\right.}{}$$

$$=\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{}}{\sqrt{6\left(4+3\sqrt[3]{-}\right.}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}}}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.24

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(36
 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

fricas [B] time = 2.00, size = 1277, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] 1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)*
(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867
)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt(1/6
33)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(6*18^(
2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2*x + 72
9/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/136728*sqrt(1266)*sqrt(-2
/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 4
8*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(
1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36
*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)
 - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273
*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*
18^(2/3) - 3888*18^(1/3) - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3)
+ sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) +
 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 18*sqrt
(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*
(6*18^(2/3) + 8*18^(1/3) + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/
3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) +
48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(
1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))
*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(
```

```
2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3
888*18^(1/3) - 16578) - 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27
*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*
18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18
^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3)
+ 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 -
 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3)
+ 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 124
7*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*1
8^(2/3) - sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18
^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3)
 - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27*(6*18^(2/3)
 + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 1
8)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18^(2/3) + 8*1
8^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3)
 + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 9*sqrt(-1/2
7*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sq
rt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)
*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) - sq
rt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371
) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.23

$$\frac{\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^2 \ln}{6\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^5 + 72\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.68, size = 247, normalized size = 1.00

$$\sum_{k=1}^{6} \ln\left(-\frac{216\,\left(32134205039616\,x - 1836660096\,\mathrm{root}\left(z^6 - 2834352\,z^4 + 2677850419968\,z^2 - 73227426444276\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(216*(32134205039616*x - 1836660096*root(z^6 - 2834352*z^4 + 26
77850419968*z^2 - 732274264442769408, z, k)^2 - 1889568*root(z^6 - 2834352*
z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 972*root(z^6 - 2834
352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + root(z^6 - 2834
352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5 + 132239526912*x*
root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 20
4073344*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408,
z, k)^2 + 139968*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 73227426444
2769408, z, k)^3 + 36*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274
264442769408, z, k)^4 + 863230245120*root(z^6 - 2834352*z^4 + 2677850419968
*z^2 - 732274264442769408, z, k) + 781932322630656))/root(z^6 - 2834352*z^4
 + 2677850419968*z^2 - 732274264442769408, z, k)^5)*root(z^6 - z^4/273456 +
 z^2/258356853504 - 1/732274264442769408, z, k), k, 1, 6)
```

sympy [A] time = 0.21, size = 48, normalized size = 0.19

$$\mathrm{RootSum}\left(732274264442769408t^6 - 2677850419968t^4 + 2834352t^2 - 1, \left(t \mapsto t\log\left(10170475895038464t^5 - 5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1,
 Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 31809932
496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))
```

3.147 $\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\tan}{36\sqrt[6]{2}\,3^{5/6}\left(\right.}$$

```
[Out] 1/1296*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^
(1/3))^2-1/3888*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)
-1/3888*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/216*arctan((3*(-3)^
(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1+(-1
)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/648*arctanh(2^(1/6)*(3*3^(1/3)+
2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(
2/3))^(1/2)+1/324*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^
(1/3)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2
/3))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\tan}{36\sqrt[6]{2}\,3^{5/6}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(3
6*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1
)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3
))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3
)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]]/(108*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2
) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(1/3)*3^(2/3)
) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}\,2^{2/3}-x\right)}{136048896\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)}\cdots\right.$$

$$= -\frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{324\sqrt[3]{2}\,3^{2/3}} - \frac{\int\frac{6\sqrt[3]{3}+\sqrt[3]{2}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{324\,6^{2/3}} + \frac{(-1)^{2/3}\int\frac{3\cdots}{-6+3\cdots}}{108\sqrt[3]{2}\,3^{2/3}\cdots}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\,2^{2/3}} - \frac{\int\frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\sqrt[3]{2}\,3^{2/3}} - \frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\cdots}{6+3(-2)^{2/3}\cdots}}{648\sqrt[3]{2}\,3\cdots}$$

$$= \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{216\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+\cdots\right)}{648\sqrt[3]{2}\,3^{2/3}}\cdots$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\,\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}\cdots}{\sqrt{6\left(4+3\sqrt[3]{-2}\cdots\right)}}\right)}{108\sqrt[6]{2}\,3^{5/6}\sqrt{4+3\sqrt[3]{-2}\cdots}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 16
2*#1 + 12*#1^2 + #1^4) & ]/6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 54, normalized size = 0.15

$$\frac{\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right) \ln\left(-\right.}{6\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^5 + 72\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^3 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+3
24*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.42, size = 176, normalized size = 0.49

$$\sum_{k=1}^{6} \ln\left(x + \mathrm{root}\left(z^6 - \frac{z^4}{1640736} + \frac{235\,z^3}{129178426752} - \frac{z^2}{3100282242048} - \frac{1}{15817124111963819212 8}, z, k\right) \left(216\,x + \mathrm{r}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/310028
2242048 - 1/158171241119638192128, z, k)*(216*x + root(z^6 - z^4/1640736 +
(235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)
*(51018336*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/310028
```

```
2242048 - 1/158171241119638192128, z, k)*(277947894528*x - root(z^6 - z^4/1
640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/1581712411196381921
28, z, k)*(33192121254912*x - root(z^6 - z^4/1640736 + (235*z^3)/1291784267
52 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(6940988288557056*x
 + 168897381688221696) + 28563737812992)))))*root(z^6 - z^4/1640736 + (235*
z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k), k,
1, 6)
```

sympy [A] time = 0.26, size = 61, normalized size = 0.17

$$\operatorname{RootSum}\left(158171241119638192128t^6 - 96402615118848t^4 + 287743415040t^3 - 51018336t^2 - 1, \left(t \mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 287743415040*_
t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(65418399445721140961280*_t**5/
415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/415817
+ 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415817)
))
```

3.148 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$-\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\,2^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2\right.}{324\sqrt[6]{3}\left(1+\sqrt[3]{-\right.}}$$

```
[Out] -1/1296*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2-1
/3888*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)+1/3888*ln
(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/972*(-1)^(2/3)*(3*(-3)^(2/3)-
2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2)
)*3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/972*(9-2^(2/3)*
3^(1/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))/(-24+18*2^(1/3)*3^(2/3))^(1/2)+1/972*(9-(-2)^(2/3)*3^(1/3))*arctan((3*(-
2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^(1/3)*3^
(1/6)+9*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2070, 634, 618, 204, 628, 206}

$$-\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\,2^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2\right.}{324\sqrt[6]{3}\left(1+\sqrt[3]{-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(324*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(
4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(972*Sqrt[3*(8 + (9*I
)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(972*Sq
rt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(21
6*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2])/(648*2^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2
^(2/3)*3^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2070

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(-2+6(-3)^{2/3}\sqrt[3]{2}-\sqrt[3]{-3}\,2^{2/3}x\right)}{272097792\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)}+\right.$$

$$=\frac{\int\frac{18-2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}\,3^{2/3}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}dx}{1944}-\frac{\int\frac{2(-1)^{2/3}-6\sqrt[3]{2}\,3^{2/3}+\sqrt[3]{-3}\,2^{2/3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{648\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{-2+}{}}{216\sqrt[3]{2}\,}$$

$$=-\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}dx}{648\,2^{2/3}}+\frac{\int\frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}dx}{648\,2^{2/3}\sqrt[3]{3}}-\frac{\int\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{-6+3\sqrt[3]{-3}\,2^{2/3}x-}}{216\,2^{2/3}\sqrt[3]{3}\left(1+\right.}$$

$$=-\frac{\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\,2^{2/3}}+\frac{\log}{}$$

$$=-\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}-\frac{\left((-2)^{2/3}-3\,3^{2/3}\right)\tan}{972\sqrt[6]{3}\sqrt{2\left(4\right.}}$$

Mathematica [C] time = 0.01, size = 62, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1 +
 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 53, normalized size = 0.14

$$\frac{\ln\left(-\operatorname{RootOf}\left(\_Z^6+18\_Z^4\right.\right.}{6\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+32
4*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.67, size = 306, normalized size = 0.81

$$\sum_{k=1}^{6}\ln\left(-\operatorname{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/
111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2
*x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071
```

```
3728 + z/488182842961846272 - 1/34164988081841849499648, z, k)*x - 61222003
20*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728
 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3*x - 2582637960
59136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713
728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4*x - 6940988
288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016
0713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5*x + 944
784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071372
8 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 - 16529940864
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
 z/488182842961846272 - 1/34164988081841849499648, z, k)^3 - 33192121254912
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
 z/488182842961846272 - 1/34164988081841849499648, z, k)^4 - 16889738168822
1696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607137
28 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5)*root(z^6 -
z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842
961846272 - 1/34164988081841849499648, z, k), k, 1, 6)
```

sympy [A] time = 0.27, size = 65, normalized size = 0.17

$$\mathrm{RootSum}\left(34164988081841849499648t^6 - 3470494144278528t^4 - 86087932019712t^3 - 1530550080t^2 + 69984\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(34164988081841849499648*_t**6 - 3470494144278528*_t**4 - 8608793201
9712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(18590444669
9109611410573787136*_t**5/57121295165 + 6377301253267917382766592*_t**4/571
21295165 - 18904636002388564311552*_t**3/57121295165 - 469080552915181723966
8*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895956/5
7121295165)))
```

3.149 $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)} dx$

Optimal. Leaf size=415

$$-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{23328}-\frac{\left(18-2^{2\ldots\right.}{}$$

```
[Out] 1/216*ln(x)-1/23328*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2
)-1/23328*(18-2^(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/46656*ln(6-3
*(-3)^(1/3)*2^(2/3)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))-1/1296*(-1)^(
2/3)*((-3)^(1/3)+3*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(
-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))
^(1/2)-1/1296*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-1
2+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/
1296*(-1)^(2/3)*((-2)^(2/3)-2*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(2
4+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^
(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{23328}-\frac{\left(18-2^{2\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sq
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(216*2^(1/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*Ar
cTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))
]])/(216*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 -
 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + Lo
g[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(
2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1
```

```
/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x +
 x^2])/23328
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = 1259712\int\left(\frac{1}{272097792x}+\frac{(-1)^{2/3}\left(6\left(9+\sqrt[3]{-3}\,2^{2/3}\right)-\left(1-3\right.\right.}{816293376\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-\right.}\right.$$

$$=\frac{\log(x)}{216}+\frac{\int\frac{-6\sqrt[3]{6}\left(9\sqrt[3]{2}-2\sqrt[3]{3}\right)-\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{11664}+\frac{(-1)^{2/3}\int\frac{-6\left(9-(-2)^{2/3}\right.}{6+3}}{1944}$$

$$=\frac{\log(x)}{216}+\frac{\left(\left(-\frac{1}{6}\right)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\right)\int\frac{1}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}\,dx}{72\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left((-1)^{2/3}\right.}{}$$

$$=\frac{\log(x)}{216}-\frac{(-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(}{}$$

$$=\frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{216\,6^{5/6}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}\right.}{216\sqrt[6]{6}\left(1\right.}$$

Mathematica [C] time = 0.02, size = 103, normalized size = 0.25

$$\frac{\log(x)}{216}-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)+18\#1^2\log(x-\#1)+324\#1\log(x-\#1)+108\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\right.}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6 + 162*#1 + 12*#1^2 + #1^4) & ]/1296
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)
```

maple [C] time = 0.01, size = 75, normalized size = 0.18

$$\frac{\ln(x)}{216}-\frac{\left(\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+18\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+2\right.\right.}{1296\left(\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+12\,\mathrm{RootOf}\left(\_Z^6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
[Out] 1/216*ln(x)-1/1296*sum((_R^5+18*_R^3+324*_R^2+108*_R)/(_R^5+12*_R^3+162*_R^
2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{216}\int\frac{x^5+18\,x^3+324\,x^2+108\,x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx+\frac{1}{216}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] -1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3 +
 108*x^2 + 216), x) + 1/216*log(x)
```

mupad [B] time = 2.33, size = 432, normalized size = 1.04

$$\frac{\ln(x)}{216}+\left(\sum_{k=1}^{6}\ln\left(\mathrm{root}\left(z^6+\frac{z^5}{216}+\frac{421\,z^4}{59066496}+\frac{100853\,z^3}{27902540178432}-\frac{505\,z^2}{12053897357082624}+\frac{z}{78109254873895403}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

```
[Out] log(x)/216 + symsum(log(7*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853
*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352
- 1/7379637425677839491923968, z, k)*x - 5670000*root(z^6 + z^5/216 + (421*
z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 +
 z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2*x + 154687594
7520*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^3*x - 106961147905609728*root(z^6 + z^5/216 + (421*z^4)/590
66496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/78109
25487389540352 - 1/7379637425677839491923968, z, k)^4*x - 14051199585413401
8048*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^5*x - 4560729056738761900320*root(z^6 + z^5/216 + (421*z^4
)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/
7810925487389540352 - 1/7379637425677839491923968, z, k)^6*x + 839808*root(
z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2
)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968,
z, k)^2 + 594896472576*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^
3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1
/7379637425677839491923968, z, k)^3 - 8483430130458624*root(z^6 + z^5/216 +
 (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1205389735708
2624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4 - 38314
25535283494912*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902
540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637
425677839491923968, z, k)^5 + 1217393817906599165952*root(z^6 + z^5/216 + (
421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/120538973570826
24 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6)*root(z^6
 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1
2053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z,
k), k, 1, 6)
```

sympy [A] time = 0.42, size = 82, normalized size = 0.20

$$\frac{\log(x)}{216}+\mathrm{RootSum}\left(7379637425677839491923968t^6+3416498808184184949 9648t^5+5259880925068537 03\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 34164988081841849499
648*_t**5 + 52598809250685370368*_t**4 + 26673506015311872*_t**3 - 30917111
6160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(814557009966881793678336211511
9297360560128*_t**6/143425799309052440063 + 977068766770806381087358257564
745728*_t**5/143425799309052440063 - 116529526608851264288400971539061538 81
6*_t**4/143425799309052440063 - 2393597949852422025425014407107665 92*_t**3/
```

```
143425799309052440063 - 13667831263813709443988734141824O*_t**2/14342579930
9052440063 + 15631155690676637957354136 96*_t/143425799309052440063 + x - 31
6444631507523619O044/143425799309052440063)))
```

3.150 $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal. Leaf size=448

$$-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\right.}{}$$

```
[Out] -1/216/x-1/7776*(-1)^(2/3)*(9+(-3)^(1/3)*2^(2/3))*ln(6-3*(-3)^(1/3)*2^(2/3)
*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^2+1/23328*(3*(-6)^(2/3)+2*(-2)^(1/3)
)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*3^(2/3)-1/23328*(2^(2/3)-3*3^(2/3))*ln(6
+3*2^(2/3)*3^(1/3)*x+x^2)*6^(2/3)-1/11664*(-1)^(2/3)*(6*(-6)^(2/3)+27*(-3)^
(1/3)-2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(
1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/3499
2*(2^(1/3)+27*3^(1/3)-6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12
+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/17496*(27
*(-6)^(1/3)-(-2)^(2/3)+12*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18
*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/(216*x) - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(5832*3^(1/6)*Sqrt[8
+ (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(6*(-6)^(2/3) +
 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[
3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3
*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6
)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(5832*6^(1/6)
*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6
```

```
 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2)
 + ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7
776*3^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(
3888*6^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = 1259712\int\left(\frac{1}{272097792x^2}+\frac{(-1)^{2/3}\left(-1+9(-3)^{2/3}\sqrt[3]{2}+27\sqrt[3]{-\right.}{816293376\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}\right.$$

$$= -\frac{1}{216x}+\frac{\int\frac{-54+2^{2/3}\sqrt[3]{3}+54\sqrt[3]{2}\,3^{2/3}-6^{2/3}\left(2^{2/3}-3\ 3^{2/3}\right)x}{6+3\ 2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{11664}+\frac{(-1)^{2/3}\int\frac{1+}{}}{}$$

$$= -\frac{1}{216x}-\frac{\left((-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\right)\int\frac{-3\sqrt[3]{-3}\,2^{2/3}+2x}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}\,dx}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\left((-1)^{2/3}\right.}{}$$

$$= -\frac{1}{216x}-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-}{}$$

$$= -\frac{1}{216x}+\frac{(-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2}}{\sqrt{6\left(4+3\right.}}\right.}{5832\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.24

$$-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)+18\#1^2\log(x-\#1)+324\#1\log(x-\#1)+108\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{1296}-\frac{}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/216*1/x - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/1296
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
```

maple [C] time = 0.01, size = 74, normalized size = 0.17

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-18\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2}{1296\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+15552\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
[Out] -1/216/x+1/1296*sum((-_R^4-18*_R^2-324*_R-108)/(_R^5+12*_R^3+162*_R^2+36*_R
)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{216\,x}-\frac{1}{216}\int \frac{x^4+18\,x^2+324\,x+108}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] -1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 324
*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 0.29, size = 340, normalized size = 0.76

$$\left(\sum_{k=1}^{6}\ln\left(\frac{5\,\mathrm{root}\left(z^6+\frac{281\,z^4}{118132992}-\frac{50435\,z^3}{9300846726144}-\frac{331\,z^2}{48215589428330496}-\frac{z}{1898054893435658305536}-\frac{1}{1594001683946413330255577}\right.}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

```
[Out] symsum(log((5*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 -
(331*z^2)/48215589428330496 - z/18980548934356583055 36 - 1/1594001683946413
330255577088, z, k))/8 - (root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300
846726144 - (331*z^2)/48215589428330496 - z/18980548934356583055 36 - 1/1594
0016839464133302555770 88, z, k)*x)/216 - 396252*root(z^6 + (281*z^4)/118132
992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189805489
3435658305536 - 1/159400168394641333025557 7088, z, k)^2*x - 598229670528*ro
ot(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482155
89428330496 - z/18980548934356583055 36 - 1/159400168394641333025557 7088, z,
 k)^3*x + 82120746212352*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/93008
46726144 - (331*z^2)/48215589428330496 - z/18980548934356583055 36 - 1/15940
016839464133302555770 88, z, k)^4*x - 6940988288557056*root(z^6 + (281*z^4)/
118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189
8054893435658305536 - 1/159400168394641333025557 7088, z, k)^5*x + 2344464*r
oot(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215
589428330496 - z/18980548934356583055 36 - 1/159400168394641333025557 7088, z
, k)^2 - 210297580992*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/93008467
26144 - (331*z^2)/48215589428330496 - z/18980548934356583055 36 - 1/15940016
8394641333025557 7088, z, k)^3 - 10535082310656*root(z^6 + (281*z^4)/1181329
92 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893
435658305536 - 1/159400168394641333025557 7088, z, k)^4 - 1688973816882216 96
*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482
15589428330496 - z/18980548934356583055 36 - 1/159400168394641333025557 7088,
 z, k)^5)*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331
*z^2)/48215589428330496 - z/18980548934356583055 36 - 1/15940016839464133302
55577088, z, k), k, 1, 6) - 1/(216*x)
```

sympy [A] time = 0.32, size = 70, normalized size = 0.16

$$\mathrm{RootSum}\Big(1594001683946413330255577088t^6 + 3791612026460331638784t^4 - 8643672699589509120t^3 - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(1594001683946413330255577088*_t**6 + 3791612026460331638784*_t**4 -
 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda(_
t, _t*log(-4987553276190249600329356123691446802 8416*_t**5/1235044978470399
1795 + 1262548987243162038800597520049766 4*_t**4/12350449784703991795 - 118
6376926075737712385507988526448 64*_t**3/12350449784703991795 + 270486324927
832147818193778754816*_t**2/12350449784703991795 + 2739141948974794029611 99
352*_t/12350449784703991795 + x - 12798926329353908292/12350449784703991795
))) - 1/(216*x)
```

3.151 $\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=1064

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{729\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-1}\left(\right.}{162\sqrt[6]{2}\,3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\right.}$$

```
[Out] -1/972*(-1)^(1/3)*3^(2/3)*(54+9*(-3)^(1/3)*2^(2/3)+(2-2^(2/3)*(6*(-6)^(2/3)
+27*(-3)^(1/3)))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*
(-3)^(1/3)*2^(2/3)*x+x^2)-1/4374*(-1)^(1/3)*3^(2/3)*(54-9*(-2)^(2/3)*3^(1/3
)+(2+27*(-2)^(2/3)*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)
*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/8748*(54-9*2^(
2/3)*3^(1/3)+(2+2^(2/3)*(27*3^(1/3)-6*6^(2/3)))*x)*2^(1/3)*3^(2/3)/(4-3*2^(
1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/972*(-1)^(1/3)*(2+27*(-2)^(2/3)
*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-
2)^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/
(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/5832*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2
/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/5832*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2
/3)*3^(5/6)/(1+(-1)^(1/3))^5-1/52488*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*
3^(1/3)-1/486*(-1)^(1/3)*(6*(-6)^(2/3)+27*(-3)^(1/3)-2^(1/3))*arctan(2^(1/6
)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/6)/(1+(-1)
^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)+1/486*(2^(1/3)+27*3^(1/3)-
6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1
/2))*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)
*2^(1/2)+1/972*(I*2^(2/3)-9*3^(1/6)-3*I*3^(2/3))*arctan(2^(1/6)*(3*(-3)^(1/
3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)*3^(2/3)/(1+(-1)^(1/3
))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/972*I*((-2)^(2/3)+6*3^(2/3))*arctan((
3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(2/3)
/(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/8748*(1+3*2^(1/3)*3^(2/3
))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(
5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 2.50, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26,

$\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{729\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-1}}{162\sqrt[6]{2}\,3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4(4+}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)) + (2 - 3*2^(2/3)*(2*(-6)^(2/3) +
 9*(-3)^(1/3)))*x))/(162*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/
3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^(1/3)*(9*(6 - (-2)^(2/3)*
3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))*x))/(729*2^(
2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) + (2 + 2^(2/3)*(27*3^(1/3) - 6*6^(
2/3)))*x)/(1458*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1
/3)*x + x^2)) - ((I/162)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(1/3)*(1 + (-1)^
(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)*3
^(1/3) + 12*(-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*
(4 + 3*(-2)^(1/3)*3^(2/3))]])/(162*2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1 +
(-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*(6*(-6)^(2/3)
 + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqr
t[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(81*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(4
 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((I*2^(2/3) - 9*3^(1/6) - (3*I)*3^(2/3))*
ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3
))]])/(162*2^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)
]) - ((1 + 3*2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqr
t[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(1458*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3
^(2/3)]) + ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3)
+ 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(81*Sqrt[2]*3^(5/6)*(1 - (
-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - Log[6 - 3
*(-3)^(1/3)*2^(2/3)*x + x^2]/(972*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I
/972)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/
3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(8748*2^(1/3)*3^(2/3))
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

<u>Rule 206</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(\frac{\sqrt[3]{-\frac{1}{3}}\left(-1+3(-3)^{2/3}\sqrt[3]{2}+\left(9+\sqrt[3]{-3}\,2\right.\right.}{42845606719488\, 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(6-3\sqrt[3]{-3}\right.}\right.$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{-1-3\sqrt[3]{-2}\,3^{2/3}+\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{243\, 2^{2/3}} - \frac{\int \frac{-27+x}{6+3\, 2^{2/3}\sqrt[3]{3}x+x^2}dx}{4374\sqrt[3]{2}\,3^{2/3}} + \frac{\int \frac{1-}{}}{}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\, 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\, 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} -$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\, 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\, 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} -$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\, 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\, 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} -$$

Mathematica [C] time = 0.04, size = 167, normalized size = 0.16

$$\frac{-9x^5-203x^4-11610x^3-3990x^2+324x-7884}{34182\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&, \frac{9\#1^4\log}{}\right.}{20}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(216 + 108*
x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^
4 + #1^6 & , (324*Log[x - #1] - 96*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 +
406*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1
^5) & ]/205092
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.02, size = 122, normalized size = 0.11

$$\frac{\left(-9\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-406\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}{205092\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+2461104\,\mathrm{RootOf}\left(\_Z^6+18\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+1
8*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R^3-324*_R^2+96*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+
108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{9\,x^5+203\,x^4+11610\,x^3+3990\,x^2-324\,x+7884}{34182\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}-\frac{1}{34182}\int\frac{9\,x^4+406\,x^3+324\,x^2-96\,x+324}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18*
x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x^4 + 406*x^3 + 324*x
^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.34, size = 388, normalized size = 0.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((239491904*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/141
49992416343982992 + (5171*z^2)/5093997269883838387712 + (505*z)/133686864350
83133627113728 + 4513/852560170529641874151233606645 76, z, k)*x)/876306843
- (275536*x)/638827688547 - (3848128*root(z^6 + (326*z^4)/554702231619 + (8
113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883838387712 + (505*
z)/1336868643508313362711 3728 + 4513/85256017052964187415123360664576, z, k
))/3606201 - (152363520*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/1
4149992416343982992 + (5171*z^2)/5093997269883838387712 + (505*z)/1336868643
5083133627113728 + 4513/85256017052964187415123360664576, z, k)^2*x)/44521
- (698075283456*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/141499924
16343982992 + (5171*z^2)/5093997269883838387712 + (505*z)/133686864350831336
27113728 + 4513/85256017052964187415123360664576, z, k)^3*x)/44521 + (13078
9789876224*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343
982992 + (5171*z^2)/5093997269883838387712 + (505*z)/1336868643508313362711 3
728 + 4513/85256017052964187415123360664576, z, k)^4*x)/211 - 6940988288557
056*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992
+ (5171*z^2)/5093997269883838387712 + (505*z)/13368686435083133627113728 + 4
513/85256017052964187415123360664576, z, k)^5*x - (4264220928*root(z^6 + (3
26*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093
997269883838387712 + (505*z)/13368686435083133627113728 + 4513/8525601705296
4187415123360664576, z, k)^2)/44521 - (5086414725120*root(z^6 + (326*z^4)/5
54702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883
83387712 + (505*z)/13368686435083133627113728 + 4513/852560170529641874151 2
3360664576, z, k)^3)/44521 + (243585208571904*root(z^6 + (326*z^4)/55470223
1619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387 71
2 + (505*z)/13368686435083133627113728 + 4513/8525601705296418741512336066 4
576, z, k)^4)/211 - 16889738168822169 6*root(z^6 + (326*z^4)/554702231619 +
(8113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883838387712 + (50
5*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z,
 k)^5 - 48160/23660284761)*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3
)/14149992416343982992 + (5171*z^2)/5093997269883838387712 + (505*z)/1336868
6435083133627113728 + 4513/85256017052964187415123360664576, z, k), k, 1, 6
) - ((665*x^2)/5697 - (2*x)/211 + (215*x^3)/633 + (203*x^4)/34182 + x^5/379
8 + 146/633)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.40, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(85256017052964187415123360664576t^6 + 5010519153385434568704t^4 + 4888574805127748601\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(852560170529641874151233606645 76*_t**6 + 501051915333854345687 04*_t
**4 + 48885748051277486016*_t**3 + 865447782603408*_t**2 + 3220532460*_t +
4513, Lambda(_t, _t*log(354920362040841744041191931354834874665907640 32*_t*
*5/356900697070792948475845 - 1947416006721883708682680963101702230822 4*_t*
*4/713801394141585896951 69 + 20779963076545132233894582764903396544*_t**3/3
56900697070792948475845 + 20265219154367004972162198012037344*_t**2/3569006
97070792948475845 + 2751924689492105320490751453 72*_t/356900697070792948475
845 + x + 129028519129217728962 2012/10707020912123788454275 35))) + (-9*x**5
 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 + 615276*x
**4 + 11074968*x**3 + 3691656*x**2 + 7383312)
```

3.152 $\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=1005

$$\frac{2\left(2-3\sqrt[3]{2}\,3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}+\frac{\left(9i+\sqrt[3]{3}\left(2i2^{2/3}-9\sqrt[6]{3}+2\,2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right.}}\right.}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

```
[Out] 1/1944*(-4*(-1)^(1/3)*3^(2/3)-18*6^(1/3)+9*((-2)^(2/3)+2*(-1)^(1/3)*3^(2/3)
)*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2
/3)*x+x^2)+1/4374*(-(-6)^(1/3)*(9*(-2)^(1/3)+2*3^(1/3))+9*(1+(-2)^(1/3)*3^(
2/3))*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*
x+x^2)+1/17496*(4-6*2^(1/3)*3^(2/3)-3*(6-2^(2/3)*3^(1/3))*x)*2^(1/3)*3^(2/3
)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/3888*I*ln(6-3*(-3)^(1
/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5-1/104976*ln(6+3*2^(2/3)
*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)-1/324*(-1)^(1/3)*((-3)^(1/3)+3*2^(1/3))*arc
tan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/
6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)-1/7776*ln(6+3*(-
2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/324*
(1+(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*
3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^
(3/2)*6^(1/2)+1/324*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*
x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2
^(1/3)*3^(2/3))^(3/2)*6^(1/2)+1/5832*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-
18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I+3^(1/3)*(2*I*2^(2/3)-9*3^(1/6)+2*2^(2/3)
*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)+1/5832*(9*3^(1/6
)+I*(4*2^(2/3)-3*3^(2/3)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1
/3)*3^(2/3))^(1/2))*3^(1/3)/(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3))^(1/2)
+1/78732*(2*2^(2/3)+3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9
*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 2.40, antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 634, 618, 204, 628, 638, 206}

$$\frac{2\left(2-3\sqrt[3]{2}\,3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}+\frac{\left(9i+\sqrt[3]{3}\left(2i2^{2/3}-9\sqrt[6]{3}+2\,2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right.}}\right.}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/
3))*x)/(972*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-
3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) - 9*(1
+ (-2)^(1/3)*3^(2/3))*x)/(4374*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/
3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*(6
 - 2^(2/3)*3^(1/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3
*2^(2/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1/6) + 2
*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(
2/3)*2^(1/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))
]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(
4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3)
)^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) + ((9*3^(1/6) + I*(4*2^(2/3) - 3*3^(2
/3)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]])/(1944*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3))]) -
((-1)^(1/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3
)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(54*Sqrt[2]*3^(5/6)*(1 + (-1)^(1
/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(54*Sqr
t[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2))
+ ((2*2^(2/3) + 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3
*(-4 + 3*2^(1/3)*3^(2/3))]])/(26244*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))
]) + ((I/648)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 +
(-1)^(1/3))^5) - ((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(129
6*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/
(17496*2^(2/3)*3^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^7}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(\frac{-27+3\,2^{2/3}\sqrt[3]{3}+9i\sqrt{3}+i2^{2/3}3^{5/6}-3i\sqrt[3]{2}}{9254651051409408\left(1+\sqrt[3]{-1}\right)^5\left(-6+3\sqrt[3]{-3}\,2\right.}\right.$$

$$= \frac{\int \frac{-18-2\,2^{2/3}\sqrt[3]{3}-\sqrt[3]{2}\,3^{2/3}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{52488} + \frac{\int \frac{9\,2^{2/3}+\sqrt[3]{-1}\,3^{2/3}\left(1+3\sqrt[3]{-2}\,3^{2/3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{4374\,2^{2/3}} + \frac{\int \frac{3\,2}{(}}{}$$

$$= -\frac{2\left(2\sqrt[3]{-1}\,3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)x}{972\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{8}$$

$$= -\frac{2\left(2\sqrt[3]{-1}\,3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)x}{972\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{8}$$

$$= -\frac{2\left(2\sqrt[3]{-1}\,3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)x}{972\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{8}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.17

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{73\#1^4\log(x-\#1)-36\#1^3\log(x-\#1)+96\#1^2\log(x-\#1)-216\#1\log(x-\#1)+96\log(}{\#1^5+12\#1^3+162\#1^2+36\#1}\right.}{410184}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 +
324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1
^6 & , (96*Log[x - #1] - 216*Log[x - #1]*#1 + 96*Log[x - #1]*#1^2 - 36*Log[
x - #1]*#1^3 + 73*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]
/410184
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.02, size = 122, normalized size = 0.12

$$\frac{\left(73\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-36\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}{410184\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+4922208\,\mathrm{RootOf}\left(\_Z^6+18\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x^
4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(_R
^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z
^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{73\,x^5-18\,x^4+908\,x^3+432\,x^2-96\,x+648}{68364\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}+\frac{1}{68364}\int\frac{73\,x^4-36\,x^3+96\,x^2-216\,x+96}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 +
324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 - 21
6*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.30, size = 387, normalized size = 0.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((8336932*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3
)/75466626220501242624 - (2479189*z^2)/26407281847077794818990008 - (1989787
*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913
549312, z, k))/97367427 - (480227*x)/851770251396 - (759164282*root(z^6 - (
292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (247918
9*z^2)/26407281847077794818990008 - (1989787*z)/311864717157619341253309046 7
84 - 7197829/589289589870088463413332668913549312, z, k)*x)/7886761587 - (2
07565888*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262
20501242624 - (2479189*z^2)/26407281847077794818990008 - (1989787*z)/3118647
171576193412533090467 84 - 7197829/589289589870088463413332668913549312, z,
k)^2*x)/400689 - (108430970112*root(z^6 - (292589*z^4)/319508485412544 + (1
1805253*z^3)/75466626220501242624 - (2479189*z^2)/26407281847077794818990008
 - (1989787*z)/311864717157619341253309046784 - 7197829/5892895898700884634
13332668913549312, z, k)^3*x)/44521 - (147138513610752*root(z^6 - (292589*z
^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2
6407281847077794818990008 - (1989787*z)/311864717157619341253309046784 - 719
7829/589289589870088463413332668913549312, z, k)^4*x)/211 - 694098828855705
6*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546662622050124
2624 - (2479189*z^2)/26407281847077794818990008 - (1989787*z)/31186471715761
9341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5*x
- (1156135728*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546
6626220501242624 - (2479189*z^2)/26407281847077794818990008 - (1989787*z)/31
1864717157619341253309046784 - 7197829/589289589870088463413332668913549312
, z, k)^2)/44521 + (6458021903232*root(z^6 - (292589*z^4)/319508485412544 +
 (11805253*z^3)/75466626220501242624 - (2479189*z^2)/26407281847077794818990
008 - (1989787*z)/311864717157619341253309046784 - 7197829/5892895898700884
63413332668913549312, z, k)^3)/44521 - (102226052063232*root(z^6 - (292589*
z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/
26407281847077794818990008 - (1989787*z)/311864717157619341253309046784 - 71
97829/589289589870088463413332668913549312, z, k)^4)/211 - 1688973816882216
96*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012
42624 - (2479189*z^2)/26407281847077794818990008 - (1989787*z)/3118647171576
19341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5 +
 2207561/7665932262564)*root(z^6 - (292589*z^4)/319508485412544 + (11805253
*z^3)/75466626220501242624 - (2479189*z^2)/26407281847077794818990008 - (198
9787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332 66
8913549312, z, k), k, 1, 6) + ((4*x^2)/633 - (8*x)/5697 + (227*x^3)/17091 -
 x^4/3798 + (73*x^5)/68364 + 2/211)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216
)
```

sympy [A] time = 0.40, size = 112, normalized size = 0.11

$$\text{RootSum}\Big(5892895898700884634133326689135493 12t^6 - 539640290266075248405737472t^4 + 92182638168$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(589289589870088463413332668913549312*_t**6 - 5396402902660752484057
37472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 - 37
59837842016*_t - 7197829, Lambda(_t, _t*log(429960276397274477140037433051б
0746111018438501025999323136*_t**5/154206009791052044490694380303237521 - 4
2584766259508194684689715474422251405157209835847680*_t**4/1542060097910520
44490694380303237521 - 37512446128849588150108369449323754078317341082112*_
t**3/154206009791052044490694380303237521 + 7152037594021675267638890715531
672481920222144*_t**2/154206009791052044490694380303237521 - 44227546998835
297723830291794974310524032*_t/154206009791052044490694380303237521 + x - 1
74573349036676047734132569583024855/154206009791052044490694380303237521)))
 + (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364*x**6 + 123
0552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

3.153 $\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=677

$$\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x+9(-2)^{2/3}}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\,3^{2/3}\left(2+3\sqrt[3]{-2}\,3^{2/3}\right)x+9\,2^{2/3}}{13122\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2\right.}$$

```
[Out] 1/5832*(9*(-2)^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3))*x)*2^(1/3)/(1+(-1)^(1/3
))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/26244*(9*2^(
2/3)+(-1)^(1/3)*3^(2/3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*
3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(3*2^(2/3
)*3^(1/3)-(2-3*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6
+3*2^(2/3)*3^(1/3)*x+x^2)+1/2916*(-1)^(1/3)*(3*(-3)^(2/3)-2^(2/3))*arctan((
3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*6^(1/6)/(1+(-1)
^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)+1/2916*(3*(-3)^(2/3)+(-1)^(1/3)*2^
(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*
6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/
2916*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/
3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*
3^(2/3))^(3/2)+1/34992*(-1)^(1/6)*3^(5/6)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*
2^(2/3)/(1+(-1)^(1/3))^5-1/34992*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)
*3^(5/6)/(1+(-1)^(1/3))^5+1/314928*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^
(1/3)
```

Rubi [A] time = 1.55, antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 638, 618, 204, 628, 206}

$$\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x+9(-2)^{2/3}}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\,3^{2/3}\left(2+3\sqrt[3]{-2}\,3^{2/3}\right)x+9\,2^{2/3}}{13122\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)*(1 + (-1)
^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) +
```

```
(9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13122*2^(2/3
)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)
*x + x^2)) + (3*2^(2/3)*3^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*
3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1
/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4
- 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/
3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-
1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((2^(2/3
) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3
*2^(1/3)*3^(2/3))^(3/2)) + ((-1/3)^(1/6)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x
^2])/(5832*2^(1/3)*(1 + (-1)^(1/3))^5) - ((I/5832)*Log[6 + 3*(-2)^(2/3)*3^(
1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(
1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{-2\sqrt[3]{-1}\,3^{2/3}+3(-2)^{2/3}x}{1542441841901568\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\right.}\right.$$

$$= -\frac{\int\frac{-2\sqrt[3]{-1}\,3^{2/3}+3\,2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{8748\,2^{2/3}} - \frac{\int\frac{2+2^{2/3}\sqrt[3]{3}\,x}{\left(6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{2916\,2^{2/3}\sqrt[3]{3}} + \frac{\int\frac{3\sqrt[3]{3}+\sqrt[3]{2}\,x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x}}{26244\,6^{2/3}}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} + \frac{}{2}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} + \frac{}{2}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} + \frac{}{2}$$

Mathematica [C] time = 0.05, size = 167, normalized size = 0.25

$$\frac{-3x^5+73x^4-72x^3-64x^2+108x-96}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{3\#1^4\log(x-\#1)-146}{}\right.}{410184}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 32
4*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6
 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*Log[
x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/
410184
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.02, size = 122, normalized size = 0.18

$$\frac{\left(-3\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^4 + 146\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right.\right.}{410184\,\mathrm{RootOf}\left(\_Z^6 + 18\_Z^4 + 324\_Z^3 + 108\_Z^2 + 216\right)^5 + 4922208\,\mathrm{RootOf}\left(\_Z^6 + 1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/410184*sum((-3*_R^4+146*_R^3-108*_R^2+32*_R-108)
/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+10
8*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\,x^5-73\,x^4+72\,x^3+64\,x^2-108\,x+96}{68364\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}-\frac{1}{68364}\int\frac{3\,x^4-146\,x^3+108\,x^2-32\,x+108}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216) - 1/68364*integrate((3*x^4 - 146*x^3 + 108*x^2 - 32*
x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.33, size = 388, normalized size = 0.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((7028852*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)
/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)
/311864717157619341253309046784 - 880007/3977704731623097128039995515166457
856, z, k))/2628920529 - (1980083*x)/310470256633842 - (235710556*root(z^6
- (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (16
8169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090467
84 - 880007/3977704731623097128039995515166457856, z, k)*x)/70980854283 - (
6628544*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619
30300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31186471715
7619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^2
*x)/44521 - (141776759808*root(z^6 - (60865*z^4)/239631364059408 - (1549690
9*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3
971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515
166457856, z, k)^3*x)/44521 + (183701926508544*root(z^6 - (60865*z^4)/23963
1364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/594163841
5592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704
731623097128039995515166457856, z, k)^4*x)/211 - 6940988288557056*root(z^6
- (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (16
8169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090467
84 - 880007/3977704731623097128039995515166457856, z, k)^5*x + (100886752*r
oot(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619303003262
72 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31186471715761934125
3309046784 - 880007/3977704731623097128039995515166457856, z, k)^2)/133563
+ (1715052538368*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30
56398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31
```

```
1864717157619341253309046784 - 880007/3977704731623097128039995515166457856
, z, k)^3)/44521 + (115004308571136*root(z^6 - (60865*z^4)/239631364059408
- (15496909*z^3)/305639836193030326272 - (168169*z^2)/59416384155925038342
72768 - (3971*z)/311864717157619341253309046784 - 880007/397770473162309712
8039995515166457856, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^
4)/239631364059408 - (15496909*z^3)/305639836193030326272 - (168169*z^2)/5
941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007
/3977704731623097128039995515166457856, z, k)^5 - 265/5749449196923)*root(z
^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/305639836193030326272 -
(168169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090
46784 - 880007/3977704731623097128039995515166457856, z, k), k, 1, 6) - ((1
6*x^2)/17091 - x/633 + (2*x^3)/1899 - (73*x^4)/68364 + x^5/22788 + 8/5697)/
(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.38, size = 112, normalized size = 0.17

$$\text{RootSum}\Big(3977704731623097128039995515166457856t^6 - 1010314319415295961050951680t^4 - 20168224477$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(3977704731623097128039995515166457856*_t**6 - 101031431941529596105
0951680*_t**4 - 20168224477093957151232*_t**3 - 112582856818899648*_t**2 -
50648453064*_t - 880007, Lambda(_t, _t*log(-2736555670900189915706499414143
95560986199688040644608*_t**5/49797855396139900267573395695 + 1183700847019
6046085308646230764354292805044570112*_t**4/49797855396139900267573395695 -
 10570581900446717266374077482873315047787008*_t**3/49797855396139900267573
395695 - 1552547411569469872387563218792789323392*_t**2/4979785539613990026
7573395695 - 12542923791159140826909003250295928*_t/49797855396139900267573
395695 + x - 23066533870320322410834348296/49797855396139900267573395695)))
 + (-3*x**5 + 73*x**4 - 72*x**3 - 64*x**2 + 108*x - 96)/(68364*x**6 + 12305
52*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

3.154 $\int \frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=682

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}\,x+4\right)}{8748\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)\right.}$$

```
[Out] 1/11664*(-1)^(1/3)*3^(2/3)*(4-(-3)^(1/3)*2^(2/3)*x)*2^(1/3)/(1+(-1)^(1/3))^
4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/52488*(-1)^(1/3
)*3^(2/3)*(4+(-2)^(2/3)*3^(1/3)*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)
*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-4-2^(2/3)*3^(1/3)*x)*2^
(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/13122*arc
tan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))/(8-9*I*2^(
1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)-1/13122*arctan((3*(-2)^(2/3)*
3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(8+9*I*2^(1/3)*3^(1/6)+3*2^(
1/3)*3^(2/3))^(3/2)*3^(1/2)-1/52488*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(
-12+9*2^(1/3)*3^(2/3))^(1/2))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)-1/26244*
arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)
*3^(5/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/8748*I*arctan((3
*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(1/3)/
(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/236196*arctanh(2^(1/6)*(3
*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(5/6)/(-4+3*2^
(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2097, 638, 618, 204, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}\,x+4\right)}{8748\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)^(1/3))^4*
(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + ((-1/3)^(1
```

```
/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/
3)*x)/(17496*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)
*x + x^2)) - ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2
^(1/3))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^
(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1
/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
 - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3
^(2/3))]])/(2^(5/6)*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3
)]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]]/(4374*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/
(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3)
 + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(39366*2^(5/6)*3^(1/6)*Sqr
t[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
```

```
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{\sqrt[3]{-\frac{1}{3}}\,x}{1542441841901568\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\right.}\right.$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{8748\,2^{2/3}}+\frac{\int\frac{1}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{26244\sqrt[3]{2}\,3^{2/3}}+\frac{\int\frac{x}{\left(6+3\,2^{2/3}\sqrt[3]{\right.}}}{8748\,2}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\frac{}{17}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\frac{}{17}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\frac{}{17}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{4\#1^4\log(x-\#1)-54\#1^3\log(x-\#1)+2043\#1^2\log(x-\#1)-324\#1\log(x-\#1)+144\,\text{lo}}{\#1^5+12\#1^3+162\#1^2+36\#1}\right.}{3691656}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2 +
 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #
1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 - 54*
Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/3691656
```

fricas [B] time = 2.97, size = 1445, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] 1/28041818976*(182304*x^5 - 1230552*x^4 + 33224904*x^3 + 422*sqrt(1/633)*(x
^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)*log(2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3)
 + 9367856*18^(1/3) + 44687457)^2 - 447902374840040617697673*18^(2/3) - 83
34306522507661258645112*18^(1/3) - 2686255981142288534712047 7)*sqrt(5034474
*18^(2/3) + 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 513225545496
0803463351330/9393931*18^(2/3) + 9549802036377046040753520/9393931*18^(1/3)
 + 27278928233033940032425830/9393931) - 422*sqrt(1/633)*(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
*log(-2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)^2 - 447902374840040617697673*18^(2/3) - 83343065225076612
58645112*18^(1/3) - 26862559811422885347120477)*sqrt(5034474*18^(2/3) + 936
7856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2/3) + 9367856*
18^(1/3) + 44687457)^2 + 247458158879850620*x + 5132255454960803463351330/9
393931*18^(2/3) + 9549802036377046040753520/9393931*18^(1/3) + 272789282330
33940032425830/9393931) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 2
16)*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/
3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397
4962699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 3064230/211*sqrt(-1/19683*(5034474*
18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787
01088/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*
18^(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 243*sqrt(-1/19683*(5034474*18^(2/3)
 + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81
*18^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 936785
6*18^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 177934129698570542
9*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805
69051992480475*sqrt(422))*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387
```

65959317812080*x - 102645109099216069267026660/211*18^(2/3) - 19099604072754
092081507040/211*18^(1/3) - 545578564660678800648516660/211) + 9*sqrt(422)*(
x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) + sqrt(-1/1968
3*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3)
 + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 36779
8)*log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
 3064230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895
278433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*
(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
 243*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228
60116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*
sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061
5865*sqrt(422)) - 177934129698570542 9*sqrt(422)*(5034474*18^(2/3) + 9367856
*18^(1/3) + 44687457) + 26505855880569051992480475*sqrt(422))*sqrt(-20718*1
8^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 936785
6/243*18^(1/3) + 367798) + 44068338765959317812080*x - 10264510909921606926
702660/211*18^(2/3) - 19099604072754092081507040/211*18^(1/3) - 54557856466
067880064851660/211) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)
*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3)
+ 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397496
2699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 3064230/211*sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*18^
(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 243*sqrt(-1/19683*(5034474*18^(2/3) +
9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18
^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*1
8^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 1779341296985705429*s
qrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805690
51992480475*sqrt(422))*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/
3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/
81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387659
59317812080*x - 10264510909921606926702660/211*18^(2/3) - 19099604072754092
081507040/211*18^(1/3) - 54557856466067880064851660/211) + 9*sqrt(422)*(x^6
 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(
5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) +
3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798)*
log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 30
64230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895278
433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*(14
476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 24

```
3*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228601
16892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*sqr
t(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061586
5*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856*18
^(1/3) + 44687457) + 265058558805690519924804 75*sqrt(422))*sqrt(-20718*18^(
2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 2
2860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/2
43*18^(1/3) + 367798) + 44068338765959317812080*x - 10264510909921606926702
660/211*18^(2/3) - 19099604072754092081507040/211*18^(1/3) - 54557856466067
880064851660/211) + 29533248*x^2 - 6562944*x + 44299872)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.01, size = 122, normalized size = 0.18

$$\frac{\left(4\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-54\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3}{3691656\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+44299872\,\mathrm{RootOf}\left(\_Z^6+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^4
+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)/(
_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*
_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,x^5-27\,x^4+729\,x^3+648\,x^2-144\,x+972}{615276\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}+\frac{1}{615276}\int \frac{4\,x^4-54\,x^3+2043\,x^2-324\,x+144}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4 +
 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2 -
 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 0.31, size = 299, normalized size = 0.44

$$\left(\sum_{k=1}^{6}\ln\left(-\frac{4477969\,\mathrm{root}\left(z^6-\frac{183899\,z^4}{3834101824950528}+\frac{6209\,z^2}{14083883651774823903461376}-\frac{39753025}{27493895104978847349012449000830556700672},\right.}{189282278088}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((6305*x)/4967524106141472 - (4477969*root(z^6 - (183899*z^4)/383
4101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/2749389510
4978847349012449000830556700672, z, k))/189282278088 - (16340881*root(z^6 -
 (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39
753025/27493895104978847349012449000830556700672, z, k)*x)/5110621508376 -
(43348696*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651
774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)
^2*x)/10818603 - (65333687616*root(z^6 - (183899*z^4)/3834101824950528 + (6
209*z^2)/14083883651774823903461376 - 39753025/2749389510497884734901244900
0830556700672, z, k)^3*x)/44521 - (40024496812032*root(z^6 - (183899*z^4)/3
834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895
104978847349012449000830556700672, z, k)^4*x)/211 - 6940988288557056*root(z
^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376
- 39753025/27493895104978847349012449000830556700672, z, k)^5*x + (5943884*
root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/140838836517748239034
61376 - 39753025/27493895104978847349012449000830556700672, z, k)^2)/400689
 + (224442467136*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/1408
3883651774823903461376 - 39753025/27493895104978847349012449000830556700672
, z, k)^3)/44521 - (137087493272064*root(z^6 - (183899*z^4)/383410182495052
8 + (6209*z^2)/14083883651774823903461376 - 39753025/2749389510497884734901
2449000830556700672, z, k)^4)/211 - 168897381688221696*root(z^6 - (183899*z
^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/274
93895104978847349012449000830556700672, z, k)^5 - 13082875/1788308678210929
92)*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823
903461376 - 39753025/27493895104978847349012449000830556700672, z, k), k, 1
, 6) + ((2*x^2)/1899 - (4*x)/17091 + x^3/844 - x^4/22788 + x^5/153819 + 1/6
33)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.31, size = 104, normalized size = 0.15

$$\mathrm{RootSum}\left(27493895104978847349012449000830556700672t^6-1318718189226950088862983192576t^4+121\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(274938951049788473490124490008305567006720*_t**6 - 13187181892269500
88862983192576*_t**4 + 121209177047767764480*_t**2 - 39753025, Lambda(_t, _t
*log(9478422590012887239098320545502099502420459520*_t**5/61864539719962655
- 24345864681777560763965488948081459200*_t**4/9811980923071 - 41682556475067
500431787310779667456*_t**3/61864539719962655 + 12026877442664328616462272*
_t**2/9811980923071 + 21614261848885979366842800*_t/61864539719962655 + x - 3
08574300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 648*x**2
- 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3 + 66449808*x**
2 + 132899616)
```

3.155 $\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=850

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{729\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2}\right.}$$

```
[Out] 1/34992*(-1)^(1/3)*3^(2/3)*(3*(-3)^(1/3)*2^(2/3)-2*x)*2^(1/3)/(1+(-1)^(1/3)
)^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/157464*(-1)^(
1/3)*3^(2/3)*(3*(-2)^(2/3)*3^(1/3)+2*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^
(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(-3*3^(1/3)-2^(1/3)*x
)/(9*2^(1/3)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/4374*(-1)^(1/3)*arcta
n((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/3)*3^(1
/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)-1/1749
6*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(
1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(
2/3))^(3/2)+1/157464*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3
^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/209952*ln(6-3
*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/209952*I*ln(6
+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5-1/1889568*ln(
6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/34992*I*arctan(2^(1/6)*(3*(-3)
^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^
(1/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/69984*arctan((3*(-2)^(2/3)*3^(1/3
)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/6)*3^(2/3)/(1+(-1
)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)+1/314928*arctanh(2^(1/6)*(3*3^(1/
3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*
3^(2/3))^(1/2)
```

Rubi [A] time = 1.47, antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 614, 618, 204, 634, 628, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{729\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(3*(-3)^(1/3)*2^(2/3) - 2*x))/(5832*2^(2/3)*(1 + (-1)^(1/3))^
4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^
(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(26244*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/
6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (3*3^(1/3) +
2^(1/3)*x)/(52488*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2))
+ ((-1)^(1/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*
2^(1/3))]])/(729*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1
/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3)
+ 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2916*2^(1/6)*3^(5/6)*(1 - (-1)
^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((I + Sqrt
[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]
])/(11664*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]
) - ((I/5832)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]]/(26244*2^(1/6)*3^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(
52488*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2]/(34992*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/34992)*Lo
g[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) -
 Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(314928*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{\sqrt[3]{-\frac{1}{3}}}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\right.}\right.$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{8748\ 2^{2/3}} - \frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+x}{6+3\ 2^{2/3}\sqrt[3]{3}x+x^2}dx}{157464\sqrt[3]{2}\ 3^{2/3}} + \frac{\int\frac{}{\left(6+3\ 2^{2/3}\right.}}{8748}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} -$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} -$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} -$$

Mathematica [C] time = 0.04, size = 167, normalized size = 0.20

$$\frac{-9x^5+8x^4-216x^3-1458x^2+324x-288}{1230552\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{9\#1^4\log(x-\#1)}{}\right.}{7383312}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^2
 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
 #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 1
6*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5
) & ]/7383312
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.01, size = 122, normalized size = 0.14

$$\frac{\left(-9\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4+16\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3}{7383312\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+88599744\,\mathrm{RootOf}\left(\_Z^6+18\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+
108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{9\,x^5 - 8\,x^4 + 216\,x^3 + 1458\,x^2 - 324\,x + 288}{1230552\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)} - \frac{1}{1230552}\int \frac{9\,x^4 - 16\,x^3 + 324\,x^2 - 2628\,x + 324}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^4
 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x^2
 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.42, size = 388, normalized size = 0.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((24389*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/
660182046176944870474752 - (168169*z^2)/77003633866078849692175073 28 + (397
1*z)/24250600406176479975857311477 92384 - 880007/1855837919586072196058340 3
0755606257729536, z, k))/851770251396 + (288041*x)/804738905194918464 - (10
90723*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176
944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060
0406176479975857311477 92384 - 880007/18558379195860721960583403075560625772
9536, z, k)*x)/22997796787692 + (5850124*root(z^6 - (60865*z^4)/86267291061
38688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770036338660
7884969217507328 + (3971*z)/24250600406176479975857311477 92384 - 880007/185
58379195860721960583403075560625772 9536, z, k)^2*x)/3606201 - (64554687936*
root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870
474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617
6479975857311477 92384 - 880007/18558379195860721960583403075560625772 9536,
z, k)^3*x)/44521 + (31535589897216*root(z^6 - (60865*z^4)/8626729106138688
+ (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770036338660788496
9217507328 + (3971*z)/24250600406176479975857311477 92384 - 880007/185583791
95860721960583403075560625772 9536, z, k)^4*x)/211 - 6940988288557056*root(z
^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752
 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/24250600406176479997
585731147792384 - 880007/18558379195860721960583403075560625772 9536, z, k)^
5*x - (1697552*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660
182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z
)/24250600406176479975857311477 92384 - 880007/18558379195860721960583403075
5606257729536, z, k)^2)/10818603 + (12229983936*root(z^6 - (60865*z^4)/8626
729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/77003
63386607884969217507328 + (3971*z)/24250600406176479975857311477 92384 - 880
007/18558379195860721960583403075560625772 9536, z, k)^3)/44521 + (253679492
45952*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176
944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060
0406176479975857311477 92384 - 880007/18558379195860721960583403075560625772
9536, z, k)^4)/211 - 1688973816882216 96*root(z^6 - (60865*z^4)/862672910613
8688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607
884969217507328 + (3971*z)/24250600406176479975857311477 92384 - 880007/1855
8379195860721960583403075560625772 9536, z, k)^5 - 971/22353858477636624)*ro
ot(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870047
4752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/242506004061764
7997585731147792384 - 880007/18558379195860721960583403075560625772 9536, z,
 k), k, 1, 6) - (x^2/844 - x/3798 + x^3/5697 - x^4/153819 + x^5/136728 + 4/
17091)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.39, size = 112, normalized size = 0.13

$$\text{RootSum}\Big(1855837919586072196058340307556062577295 36t^6 - 1309367357962223565522033377280t^4 + 43$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(185583791958607219605834030755606257729536*_t**6 - 1309367357962223
565522033377280*_t**4 + 435633648705229474466611 2*_t**3 - 40529828454803873
28*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(39083462657955593476
841044707333565976412952759280634691584*_t**5/4979785539613990026757339569 5
 + 88369793462237855389128176014147111023968044625756 16*_t**4/4979785539613
9900267573395695 - 26493058134830853258884424959713469570680506 7776*_t**3/4
97978553961399002675733956 95 + 8861353335473631852015151098261583762506 24*_
t**2/497978553961399002675733956 95 - 6823214795749099065113946358556 01936*_
t/4979785539613990026757339569 5 + x - 21375560770846486224291519568/4979785
5396139900267573395695))) + (-9*x**5 + 8*x**4 - 216*x**3 - 1458*x**2 + 324*
x - 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899616*x**2 +
265799232)
```

3.156 $\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=873

$$\frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}-\frac{\left(9i-\sqrt[3]{3}\left(2i2^{2/3}+9\sqrt[6]{3}+2\,2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\cdots}{\sqrt{6\left(4-3\cdots\right.}}\right.}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

```
[Out] 1/157464*((-6)^(1/3)*(2*(-3)^(1/3)+9*2^(1/3))-3*x)/(8-9*I*2^(1/3)*3^(1/6)+3
*2^(1/3)*3^(2/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/157464*(-(-6)^(1/3)*(9*(
-2)^(1/3)+2*3^(1/3))-3*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-
2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-2*2^(1/3)+3*6^(2/3)+3^(1/3)*x)/(9*2^(1/3
)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/139968*I*ln(6-3*(-3)^(1/3)*2^(2/
3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/3779136*ln(6+3*2^(2/3)*3^(1/3)
*x+x^2)*2^(1/3)*3^(2/3)+1/78732*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-
3)^(2/3)*2^(1/3))^(1/2))/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^
(1/2)-1/78732*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^
(1/2))/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)+1/279936*ln(
6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5-
1/314928*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)-1/209952*arctan((3*(-3)^(1/3)*2^(2/
3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I-3^(1/3)*(2*I*2^(2/3)+9*3^(1/
6)+2*2^(2/3)*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)+1/20
9952*(9*I+3^(1/3)*(4*I*2^(2/3)-9*3^(1/6)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x
)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3
))^(1/2)+1/2834352*(2*2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)
*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 1.92, antiderivative size = 873, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$\frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}-\frac{\left(9i-\sqrt[3]{3}\left(2i2^{2/3}+9\sqrt[6]{3}+2\,2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\cdots}{\sqrt{6\left(4-3\cdots\right.}}\right.}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-6)^(1/3)*(2*(-3)^(1/3) + 9*2^(1/3)) - 3*x)/(157464*(8 - (9*I)*2^(1/3)*3^
(1/6) + 3*2^(1/3)*3^(2/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3
)*(9*(-2)^(1/3) + 2*3^(1/3)) + 3*x)/(157464*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*
2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (2*2^(1/3) - 3*6^(2/
3) - 3^(1/3)*x)/(104976*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x +
x^2)) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3
))]]/(26244*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
- ((9*I - 3^(1/3)*((2*I)*2^(2/3) + 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(
3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(209952*(1
 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ArcTan[(3*(-2)^(2/3)
*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(26244*Sqrt[3]*(8 + (9*
I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((4*I)*2^(
2/3) - 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(209952*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3
))]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) - 3*3^
(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]])/(944784*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) +
((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656*2^(2/3)*3^(5/6
)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(629856*2^(2/3)*
3^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{9(-2)^{2/3}-\sqrt[3]{-1}\,3^{2/3}x}{27763953154228224\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+\right.}\right.$$

$$= \frac{\int \frac{18-2\,2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}\,3^{2/3}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{1889568} - \frac{\int \frac{9\,2^{2/3}-\sqrt[3]{-1}\,3^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{157464\,2^{2/3}} - \frac{\int \frac{3\,2^{2/3}\sqrt[3]{3}+}{\left(6+3\,2^{2/3}\sqrt[3]{3}x-\right.}}{52488\,2^{2/3}}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{314928}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{314928}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{314928}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.19

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{2\#1^4\log(x-\#1)-27\#1^3\log(x-\#1)+72\#1^2\log(x-\#1)-162\#1\log(x-\#1)+1971\log}{\#1^5+12\#1^3+162\#1^2+36\#1}\right.}{11074968}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2
+ 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
#1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 - 27*
Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/11074968
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.01, size = 122, normalized size = 0.14

$$\frac{\left(2\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-27\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3}{11074968\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+132899616\,\mathrm{RootOf}\left(\_Z^6+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6+
18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R+1
971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^
3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,x^5-27\,x^4+96\,x^3+648\,x^2-3942\,x+972}{3691656\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}+\frac{1}{1845828}\int \frac{2\,x^4-27\,x^3+72\,x^2-162\,x+1971}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4
+ 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2 -
 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.42, size = 387, normalized size = 0.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((11*x)/603554178896188848 - (14059*root(z^6 - (292589*z^4)/41408
2997094657024 - (11805253*z^3)/352097091294370597586534 4 - (2479189*z^2)/44
35409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233
3577984 - 7197829/12827551700178931019155248205827504534265528 32, z, k))/30
663729050256 - (5658601*root(z^6 - (292589*z^4)/414082997094657024 - (11805
253*z^3)/352097091294370597586534 4 - (2479189*z^2)/44354093106861417422692 8
4220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282
7551700178931019155248205827504534265528 32, z, k)*x)/6623365474855296 + (66
03523*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/352097091
2943705975865344 - (2479189*z^2)/44354093106861417422692842209 28 + (1989787
*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915
5248205827504534265528 32, z, k)^2*x)/584204562 - (1762321104*root(z^6 - (29
2589*z^4)/414082997094657024 - (11805253*z^3)/352097091294370597586534 4 - (
2479189*z^2)/44354093106861417422692842209 28 + (1989787*z)/1885726687584283
0829226645405233577984 - 7197829/12827551700178931019155248205827504534265 5
2832, z, k)^3*x)/44521 - (59633904436992*root(z^6 - (292589*z^4)/4140829970
94657024 - (11805253*z^3)/352097091294370597586534 4 - (2479189*z^2)/4435409
3106861417422692842209 28 + (1989787*z)/18857266875842830829226645405233577 9
84 - 7197829/12827551700178931019155248205827504534265528 32, z, k)^4*x)/211
 - 6940988288557056*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*
z^3)/352097091294370597586534 4 - (2479189*z^2)/4435409310686141742269284220
928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/12827551
7001789310191552482058275045342655 2832, z, k)^5*x + (166697*root(z^6 - (292
589*z^4)/414082997094657024 - (11805253*z^3)/352097091294370597586534 4 - (2
479189*z^2)/44354093106861417422692842209 28 + (1989787*z)/18857266875842830
8292266454052335779 84 - 7197829/12827551700178931019155248205827504534265 52
832, z, k)^2)/43274412 + (639193032*root(z^6 - (292589*z^4)/414082997094657
024 - (11805253*z^3)/352097091294370597586534 4 - (2479189*z^2)/443540931068
6141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 -
7197829/12827551700178931019155248205827504534265528 32, z, k)^3)/44521 - (9
815247601920*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/35
2097091294370597586534 4 - (2479189*z^2)/44354093106861417422692842209 28 + (
1989787*z)/18857266875842830829226645405233577984 - 7197829/128275517001789
3101915524820582750453426552832, z, k)^4)/211 - 1688973816882216 96*root(z^6
 - (292589*z^4)/414082997094657024 - (11805253*z^3)/352097091294370597586 53
44 - (2479189*z^2)/44354093106861417422692842209 28 + (1989787*z)/1885726687
5842830829226645405233577984 - 7197829/1282755170017893101915524820582750 45
3426552832, z, k)^5 + 661/28970600587017064704)*root(z^6 - (292589*z^4)/414
082997094657024 - (11805253*z^3)/352097091294370597586534 4 - (2479189*z^2)/
44354093106861417422692842209 28 + (1989787*z)/188572668758428308292266454 05
233577984 - 7197829/12827551700178931019155248205827504534265528 32, z, k),
```

```
k, 1, 6) + (x^2/5697 - (73*x)/68364 + (4*x^3)/153819 - x^4/136728 + x^5/922
914 + 1/3798)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.40, size = 112, normalized size = 0.13

$$\text{RootSum}\Big(1282755170017893101915524820582750453426552832t^6 - 9063884657755442444262511497707$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(1282755170017893101915524820582750453426552832*_t**6 - 906388465775
5442444262511497707 52*_t**4 - 43008731663899877416841379 84*_t**3 - 71700090
8921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17257
93559281044990140955659789188299560400108333936804136148 0613888*_t**5/15420
6009791052044490694380303237521 + 2389607400620985524376358853572652207181 9
5632456058768405299 2*_t**4/154206009791052044490694380303237521 - 122860721
608832839307117159488782600789969921934883880 96*_t**3/154206009791052044490
694380303237521 - 594905535739591731611254960135279097541565584107 52*_t**2/
154206009791052044490694380303237521 - 175201496798366911123670641977137530
04827200*_t/154206009791052044490694380303237521 + x + 766422988707229615 05
5855287040887332/154206009791052044490694380303237521))) + (4*x**5 - 27*x**
4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808*x**4 + 1196
096544*x**3 + 398698848*x**2 + 797397696)
```

3.157 $\int \frac{x^2}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=986

$$-\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}-\frac{\left(1+i\sqrt{3}+3\sqrt[3]{2}\,3^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-\cdots}{\sqrt{6\left(4-3(-3)^{2/3}\cdots\right.}}\right.}{8748\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\cdots\right.}$$

```
[Out] 1/209952*(-27*(-2)^(2/3)-54*(-1)^(1/3)*3^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3
))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(
2/3)*x+x^2)+1/944784*(-27*2^(2/3)-54*(-1)^(1/3)*3^(2/3)+(-1)^(1/3)*3^(2/3)*
(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/1889568*(54-9*2^(2/3)*3^(1/3)-(2-3*2^(1
/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*
x+x^2)+1/104976*(3*(-3)^(2/3)+(-1)^(1/3)*2^(2/3))*arctan((3*(-2)^(2/3)*3^(1
/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)
^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/104976*(2^(2/3)-3*3^(2/3))*arcta
nh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-
(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/1259712*I*ln(
6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/11337408*l
n(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/52488*arctan((3*(-3)^(1/3)*2
^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(1+3*2^(1/3)*3^(2/3)+I*3^(1/2
))*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))^(3/2)+1/2519424*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*(3^(1/2)+I)*2^(2/3)*3^(
5/6)/(1+(-1)^(1/3))^5+1/104976*I*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(1
2-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^(
2/3)*2^(1/3))^(1/2)+1/209952*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^
(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-2
)^(1/3)*3^(2/3))^(1/2)-1/944784*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+
9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A] time = 1.93, antiderivative size = 986, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$-\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}-\frac{\left(1+i\sqrt{3}+3\sqrt[3]{2}\,3^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-\cdots}{\sqrt{6\left(4-3(-3)^{2/3}\cdots\right.}}\right.}{8748\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))
*x)/(104976*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-
3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 + (-2)^(1/3)*3^(2/3)) - (-1)^(1
/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1/3
)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6
- 2^(2/3)*3^(1/3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(4
- 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3] + 3
*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2
/3)*2^(1/3))]])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)
*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))
*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(
17496*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/
3))^(3/2)) + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 +
 3*(-2)^(1/3)*3^(2/3))]])/(34992*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4
+ 3*(-2)^(1/3)*3^(2/3)]) + ((I/17496)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/
3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3
))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1
/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(17496*6^(5
/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) -
 ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]
]/(157464*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((I + Sqrt[3])*Lo
g[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(419904*2^(1/3)*3^(1/6)*(1 + (-1)^(1/3
))^5) - ((I/209952)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)
*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(1889568*2^(1/3)*
3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(\frac{2\sqrt[3]{-1}\,3^{2/3}+18\sqrt[3]{6}+3(-2)^{2/3}}{55527906308456448\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+\right.}\right.$$

$$= \frac{\int \frac{2\sqrt[3]{-1}\,3^{2/3}+18(-1)^{2/3}\sqrt[3]{6}+3\,2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{314928\,2^{2/3}} + \frac{\int \frac{-2+6\sqrt[3]{2}\,3^{2/3}+2^{2/3}\sqrt[3]{3}x}{\left(6+3\,2^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{104976\,2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{}{6+}}{9}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right.}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right.}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right.}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.17

$$\frac{-9x^5+8x^4-216x^3-2724x^2+324x-7884}{7383312\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{9\#1^4\log(x-\#1}{}\right.}{4429987}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x^
2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4
+ #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 -
16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^
5) & ]/44299872
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.01, size = 122, normalized size = 0.12

$$\frac{\left(-9\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4+16\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3}{44299872\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+531598464\,\mathrm{RootOf}\left(\_Z^6+18\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-73/68364)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*sum((-9*_R^4+16*_R^3-324*_R^2-2436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{9\,x^5-8\,x^4+216\,x^3+2724\,x^2-324\,x+7884}{7383312\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}-\frac{1}{7383312}\int\frac{9\,x^4-16\,x^3+324\,x^2+2436\,x+324}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] -1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x^4 - 16*x^3 + 324*x^2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.48, size = 388, normalized size = 0.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((4897*x)/18772949180387057928192 - (8147*root(z^6 + (163*z^4)/12
940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/11
08852327671535435567321055232 - (505*z)/628575562528094360974221513507785593
28 + 4513/8658597397620778437929792538933565560629231616, z, k))/1103894245
809216 - (1197643*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/14
2599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - (5
05*z)/6285755625280943609742215135077859328 + 4513/865859739762077843792979
2538933565560629231616, z, k)*x)/29805144636848832 + (452809*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517
1*z^2)/1108852327671535435567321055232 - (505*z)/62857556252809436097422151
35077859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^2*
x)/194734854 - (1241776944*root(z^6 + (163*z^4)/12940093659208032 - (811359
7*z^3)/142599321974220092022546432 + (5171*z^2)/110885232767153543556732105
5232 - (505*z)/6285755625280943609742215135077859328 + 4513/865859739762077
8437929792538933565560629231616, z, k)^3*x)/44521 + (452407928832*root(z^6
+ (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 +
 (5171*z^2)/1108852327671535435567321055232 - (505*z)/628575562528094360974
2215135077859328 + 4513/8658597397620778437929792538933565560629231616, z,
k)^4*x)/211 - 6940988288557056*root(z^6 + (163*z^4)/12940093659208032 - (81
13597*z^3)/142599321974220092022546432 + (5171*z^2)/11088523276715354355673
21055232 - (505*z)/6285755625280943609742215135077859328 + 4513/86585973976
20778437929792538933565560629231616, z, k)^5*x + (114155*root(z^6 + (163*z^
4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^
2)/1108852327671535435567321055232 - (505*z)/628575562528094360974221513507
7859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^2)/292
102281 - (163984176*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/
142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 -
(505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929
792538933565560629231616, z, k)^3)/44521 + (94281884928*root(z^6 + (163*z^4
)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2
)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077
859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^4)/211
- 168897381688221696*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)
/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 -
 (505*z)/6285755625280943609742215135077859328 + 4513/865859739762077843792
9792538933565560629231616, z, k)^5 + 1/19313733724678043136)*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517
1*z^2)/1108852327671535435567321055232 - (505*z)/62857556252809436097422151
35077859328 + 4513/8658597397620778437929792538933565560629231616, z, k), k
, 1, 6) - ((227*x^2)/615276 - x/22788 + x^3/34182 - x^4/922914 + x^5/820368
 + 73/68364)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.39, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(8658597397620778437929792538933565560629231616t^6 + 10906809587177016824883864561254\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(8658597397620778437929792538933565560629231616*_t**6 + 109068095871
770168248838645612544*_t**4 - 492655707593366915713499136*_t**3 + 403783317
45144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(1014425315618
04181113161287039859349851881619653631712165888*_t**5/356900697070792948475
845 - 149796550082359335112709434971975088967050210050048*_t**4/35690069707
0792948475845 + 122240975445827281850589877768670783617236992*_t**3/356900
697070792948475845 - 5775055524251595723022901938558261453824*_t**2/3569006
97070792948475845 + 96165242200260265765603930470432*_t/7138013941415858969
5169 + x - 17059152341129698120545584/1070702091212378845427535))) + (-9*x*
*5 + 8*x**4 - 216*x**3 - 2724*x**2 + 324*x - 7884)/(7383312*x**6 + 13289961
6*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)
```

3.158 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx &= \int \left(a^2+2abx^2+b^2x^4\right)dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

fricas [A] time = 0.71, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

giac [A] time = 0.37, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5 + \frac{2}{3}ab\,x^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

maxima [A] time = 0.44, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B] time = 0.04, size = 21, normalized size = 0.84

$$a^2\,x+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
c + d*x),x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.09, size = 22, normalized size = 0.88

$$a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c),x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.159 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2} dx$

Optimal. Leaf size=94

$$\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}-\frac{bcx\left(2ad^2+bc^2\right)}{d^4}+\frac{bx^2\left(2ad^2+bc^2\right)}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

```
[Out] -b*c*(2*a*d^2+b*c^2)*x/d^4+1/2*b*(2*a*d^2+b*c^2)*x^2/d^3-1/3*b^2*c*x^3/d^2+
1/4*b^2*x^4/d+(a*d^2+b*c^2)^2*ln(d*x+c)/d^5
```

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {1586, 28, 697}

$$\frac{bx^2\left(2ad^2+bc^2\right)}{2d^3}-\frac{bcx\left(2ad^2+bc^2\right)}{d^4}+\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x)^2,x]
```

```
[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^2
*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = \int \frac{a^2+2abx^2+b^2x^4}{c+dx}\,dx$$
$$= \frac{\int \frac{(ab+b^2x^2)^2}{c+dx}\,dx}{b^2}$$
$$= \frac{\int \left(-\frac{b^3c(bc^2+2ad^2)}{d^4}+\frac{b^3(bc^2+2ad^2)x}{d^3}-\frac{b^4cx^2}{d^2}+\frac{b^4x^3}{d}+\frac{b^2(bc^2+ad^2)}{d^4(c+dx)}\right.}{b^2}$$
$$= -\frac{bc(bc^2+2ad^2)x}{d^4}+\frac{b(bc^2+2ad^2)x^2}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.84

$$\frac{12\left(ad^2+bc^2\right)^2\log(c+dx)+bdx\left(12ad^2(dx-2c)+b\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)\right)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x)^2,x]
```

```
[Out] (b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^
3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)
```

fricas [A] time = 0.96, size = 105, normalized size = 1.12

$$\frac{3\,b^2d^4x^4-4\,b^2cd^3x^3+6\left(b^2c^2d^2+2\,abd^4\right)x^2-12\left(b^2c^3d+2\,abcd^3\right)x+12\left(b^2c^4+2\,abc^2d^2+a^2d^4\right)\log\left(dx+c\right)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 - 1
2*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(
d*x + c))/d^5
```

giac [B] time = 0.26, size = 365, normalized size = 3.88

$$-\frac{1}{12}\,b^2d\left(\frac{(dx+c)^4\left(\frac{20\,c}{dx+c}-\frac{60\,c^2}{(dx+c)^2}+\frac{120\,c^3}{(dx+c)^3}-3\right)}{d^6}+\frac{60\,c^4\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6}-\frac{12\,c^5}{(dx+c)d^6}\right)-\frac{1}{3}\,b^2c\left(\frac{(dx+c)^3\left(\frac{6\,c}{dx+c}-\frac{1}{(d}\right.}{d^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="giac")
```

```
[Out] -1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d*
x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 12*
c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d*x
+ c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3*c^4
/((d*x + c)*d^5)) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*log(
abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a*b*c*(
2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x
+ c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d
)) - a^2*c/((d*x + c)*d)
```

maple [A] time = 0.00, size = 114, normalized size = 1.21

$$\frac{b^2x^4}{4d}-\frac{b^2c\,x^3}{3d^2}+\frac{ab\,x^2}{d}+\frac{b^2c^2x^2}{2d^3}+\frac{a^2\ln(dx+c)}{d}+\frac{2ab\,c^2\ln(dx+c)}{d^3}-\frac{2abcx}{d^2}+\frac{b^2c^4\ln(dx+c)}{d^5}-\frac{b^2c^3x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x
)
```

```
[Out] 1/4*b^2*x^4/d-1/3*b^2*c*x^3/d^2+b/d*x^2*a+1/2*b^2/d^3*x^2*c^2-2*b/d^2*a*c*x
-b^2/d^4*c^3*x+1/d*ln(d*x+c)*a^2+2/d^3*ln(d*x+c)*a*b*c^2+1/d^5*ln(d*x+c)*b^
2*c^4
```

maxima [A] time = 0.44, size = 105, normalized size = 1.12

$$\frac{3\,b^2d^3x^4-4\,b^2cd^2x^3+6\left(b^2c^2d+2\,abd^3\right)x^2-12\left(b^2c^3+2\,abcd^2\right)x}{12\,d^4}+\frac{\left(b^2c^4+2\,abc^2d^2+a^2d^4\right)\log(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12*
(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*
x + c)/d^5
```

mupad [B] time = 0.06, size = 106, normalized size = 1.13

$$x^2\left(\frac{b^2\,c^2}{2\,d^3}+\frac{a\,b}{d}\right)+\frac{\ln(c+d\,x)\,\left(a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4\right)}{d^5}+\frac{b^2\,x^4}{4\,d}-\frac{b^2\,c\,x^3}{3\,d^2}-\frac{c\,x\,\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
c + d*x)^2,x)
```

```
[Out] x^2*((b^2*c^2)/(2*d^3) + (a*b)/d) + (log(c + d*x)*(a^2*d^4 + b^2*c^4 + 2*a*
b*c^2*d^2))/d^5 + (b^2*x^4)/(4*d) - (b^2*c*x^3)/(3*d^2) - (c*x*((b^2*c^2)/d
^3 + (2*a*b)/d))/d
```

sympy [A] time = 0.31, size = 88, normalized size = 0.94

$$-\frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d} + x^2\left(\frac{ab}{d} + \frac{b^2c^2}{2d^3}\right) + x\left(-\frac{2abc}{d^2} - \frac{b^2c^3}{d^4}\right) + \frac{\left(ad^2 + bc^2\right)^2 \log\left(c + dx\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(a*b/d + b**2*c**2/(2*d**3))
 + x*(-2*a*b*c/d**2 - b**2*c**3/d**4) + (a*d**2 + b*c**2)**2*log(c + d*x)/d
**5
```

3.160 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=15

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[Out] 1/14*(c*x^2+b*x)^14
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

fricas [B] time = 0.66, size = 154, normalized size = 10.27

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

maple [B] time = 0.00, size = 155, normalized size = 10.33

$$\frac{1}{14}c^{14}x^{28}+b\,c^{13}x^{27}+\frac{13}{2}b^2c^{12}x^{26}+26b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

mupad [B] time = 2.22, size = 154, normalized size = 10.27

$$\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

sympy [B] time = 0.13, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

3.161 $\int x^{14} \left(b+2cx^2\right) \left(bx+cx^3\right)^{13} dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx=\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$$
$$=\frac{1}{2}\operatorname{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right)$$
$$=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{14}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

fricas [B] time = 0.82, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14}+\frac{1}{2}x^{54}c^{13}b+\frac{13}{4}x^{52}c^{12}b^2+13x^{50}c^{11}b^3+\frac{143}{4}x^{48}c^{10}b^4+\frac{143}{2}x^{46}c^9b^5+\frac{429}{4}x^{44}c^8b^6+\frac{858}{7}x^{42}c^7b^7+\frac{429}{4}x^{40}c^6b^8+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

giac [B] time = 0.31, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}bc^{13}x^{54}+\frac{13}{4}b^2c^{12}x^{52}+13b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}b\,c^{13}x^{54}+\frac{13}{4}b^{2}c^{12}x^{52}+13b^{3}c^{11}x^{50}+\frac{143}{4}b^{4}c^{10}x^{48}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{858}{7}b^{7}c^{7}x^{42}+\frac{429}{4}b^{8}c^{6}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

maxima [B] time = 0.61, size = 156, normalized size = 9.75

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^{2}c^{12}x^{52}+13\,b^{3}c^{11}x^{50}+\frac{143}{4}\,b^{4}c^{10}x^{48}+\frac{143}{2}\,b^{5}c^{9}x^{46}+\frac{429}{4}\,b^{6}c^{8}x^{44}+\frac{858}{7}\,b^{7}c^{7}x^{42}+\frac{429}{4}\,b^{8}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

mupad [B] time = 0.14, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^{2}\,x^{32}}{4}+13\,b^{11}\,c^{3}\,x^{34}+\frac{143\,b^{10}\,c^{4}\,x^{36}}{4}+\frac{143\,b^{9}\,c^{5}\,x^{38}}{2}+\frac{429\,b^{8}\,c^{6}\,x^{40}}{4}+\frac{858\,b^{7}\,c^{7}\,x^{42}}{7}+\frac{429\,b^{6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(b*x + c*x^3)^13*(b + 2*c*x^2),x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

sympy [B] time = 0.14, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^{2}x^{32}}{4}+13b^{11}c^{3}x^{34}+\frac{143b^{10}c^{4}x^{36}}{4}+\frac{143b^{9}c^{5}x^{38}}{2}+\frac{429b^{8}c^{6}x^{40}}{4}+\frac{858b^{7}c^{7}x^{42}}{7}+\frac{429b^{6}c^{8}x^{44}}{4}+\frac{143}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

3.162 $\int x^{28} \left(b+2cx^3\right) \left(bx+cx^4\right)^{13} \, dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx=\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$$
$$=\frac{1}{3}\operatorname{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)$$
$$=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Mathematica [B] time = 0.01, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

fricas [B] time = 0.68, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14}+\frac{1}{3}x^{81}c^{13}b+\frac{13}{6}x^{78}c^{12}b^2+\frac{26}{3}x^{75}c^{11}b^3+\frac{143}{6}x^{72}c^{10}b^4+\frac{143}{3}x^{69}c^9b^5+\frac{143}{2}x^{66}c^8b^6+\frac{572}{7}x^{63}c^7b^7+\frac{143}{2}x^{60}c^6b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

giac [B] time = 0.33, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}b\,c^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

maxima [B] time = 0.55, size = 156, normalized size = 9.75

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^{2}c^{12}x^{78}+\frac{26}{3}\,b^{3}c^{11}x^{75}+\frac{143}{6}\,b^{4}c^{10}x^{72}+\frac{143}{3}\,b^{5}c^{9}x^{69}+\frac{143}{2}\,b^{6}c^{8}x^{66}+\frac{572}{7}\,b^{7}c^{7}x^{63}+\frac{143}{2}\,b^{8}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

mupad [B] time = 2.17, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^{2}\,x^{48}}{6}+\frac{26\,b^{11}\,c^{3}\,x^{51}}{3}+\frac{143\,b^{10}\,c^{4}\,x^{54}}{6}+\frac{143\,b^{9}\,c^{5}\,x^{57}}{3}+\frac{143\,b^{8}\,c^{6}\,x^{60}}{2}+\frac{572\,b^{7}\,c^{7}\,x^{63}}{7}+\frac{143\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^28*(b*x + c*x^4)^13*(b + 2*c*x^3),x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

sympy [B] time = 0.14, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^{2}x^{48}}{6}+\frac{26b^{11}c^{3}x^{51}}{3}+\frac{143b^{10}c^{4}x^{54}}{6}+\frac{143b^{9}c^{5}x^{57}}{3}+\frac{143b^{8}c^{6}x^{60}}{2}+\frac{572b^{7}c^{7}x^{63}}{7}+\frac{143b^{6}c^{8}x^{66}}{2}+14$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

3.163 $\int x^{14(-1+n)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^{13}\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

```
[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^{14(-1+n)} (b+2cx^n) \left(bx+cx^{1+n}\right)^{13} dx = \int x^{13+14(-1+n)} (b+cx^n)^{13} (b+2cx^n)\, dx$$
$$= \frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^n\right)}{n}$$
$$= \frac{x^{14n} (b+cx^n)^{14}}{14n}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

fricas [B] time = 1.04, size = 262, normalized size = 12.48

$$b^{14}x^{14}x^{14n+14} + 14\,b^{13}cx^{13}x^{15n+15} + 91\,b^{12}c^2x^{12}x^{16n+16} + 364\,b^{11}c^3x^{11}x^{17n+17} + 1001\,b^{10}c^4x^{10}x^{18n+18} + 2002\,b^9c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2*
x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10*x
^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n +
20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) + 200
2*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^3*c^1
1*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(27*n +
 27) + c^14*x^(28*n + 28))/(n*x^28)
```

giac [B] time = 1.86, size = 189, normalized size = 9.00

$$c^{14}x^{28n} + 14\,bc^{13}x^{27n} + 91\,b^2c^{12}x^{26n} + 364\,b^3c^{11}x^{25n} + 1001\,b^4c^{10}x^{24n} + 2002\,b^5c^9x^{23n} + 3003\,b^6c^8x^{22n} + 3432$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

maple [B] time = 0.05, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n}+\frac{b^{13}c\,x^{15n}}{n}+\frac{13b^{12}c^{2}x^{16n}}{2n}+\frac{26b^{11}c^{3}x^{17n}}{n}+\frac{143b^{10}c^{4}x^{18n}}{2n}+\frac{143b^{9}c^{5}x^{19n}}{n}+\frac{429b^{8}c^{6}x^{20n}}{2n}+\frac{1716b^{7}c^{7}x^{21n}}{7n}+429b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

maxima [B] time = 0.65, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^{2}c^{12}x^{26\,n}}{2\,n}+\frac{26\,b^{3}c^{11}x^{25\,n}}{n}+\frac{143\,b^{4}c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^{5}c^{9}x^{23\,n}}{n}+\frac{429\,b^{6}c^{8}x^{22\,n}}{2\,n}+\frac{1716\,b^{7}c^{7}x^{21\,n}}{7\,n}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima"
)
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

mupad [B] time = 4.02, size = 229, normalized size = 10.90

$$\frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^{2}\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^{3}\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^{4}\,x^{18\,n}}{2\,n}+\frac{143\,b^{9}\,c^{5}\,x^{19\,n}}{n}+\frac{429\,b^{8}\,c^{6}\,x^{20\,n}}{2\,n}+\frac{1716\,b^{7}\,c^{7}\,x^{2}}{7\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(14*n - 14)*(b*x + c*x^(n + 1))^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

```
[Out] Timed out
```

3.164 $\int \frac{b+2cx}{bx+cx^2} \, dx$

Optimal. Leaf size=10

$$\log \left(bx + cx^2\right)$$

[Out] ln(c*x^2+b*x)

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {628}

$$\log \left(bx + cx^2\right)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]

[Out] Log[b*x + c*x^2]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b + 2cx}{bx + cx^2} \, dx = \log \left(bx + cx^2\right)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.90

$$\log(b + cx) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]

[Out] Log[x] + Log[b + c*x]

fricas [A] time = 0.78, size = 10, normalized size = 1.00

$$\log \left(cx^2 + bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
[Out] log(c*x^2 + b*x)
```

giac [A] time = 0.23, size = 11, normalized size = 1.10

$$\log\left(\left|cx^2+bx\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
[Out] log(abs(c*x^2 + b*x))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\ln\left(\left(cx+b\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x)
[Out] ln(x*(c*x+b))
```

maxima [A] time = 0.55, size = 10, normalized size = 1.00

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
[Out] log(c*x^2 + b*x)
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\ln\left(x\,\left(b+c\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
[Out] log(x*(b + c*x))
```

sympy [A] time = 0.13, size = 8, normalized size = 0.80

$$\log\left(bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
[Out] log(b*x + c*x**2)
```

3.165 $\int \frac{b+2cx^2}{bx+cx^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

[Out] ln(x)+1/2*ln(c*x^2+b)

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1593, 446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(b*x + c*x^3),x]

[Out] Log[x] + Log[b + c*x^2]/2

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^2}{bx+cx^3}\,dx &= \int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^2\right)\\ &= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

fricas [**A**] time = 0.85, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

giac [**A**] time = 0.33, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|cx^2+b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

maple [**A**] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x)+\frac{\ln\left(c\,x^2+b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/(c*x^3+b*x),x)
```

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

maxima [A] time = 0.64, size = 13, normalized size = 0.87

$$\frac{1}{2}\,\log\left(cx^2+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

mupad [B] time = 2.08, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2+b\right)}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(b*x + c*x^3),x)
```

```
[Out] log(b + c*x^2)/2 + log(x)
```

sympy [A] time = 0.18, size = 12, normalized size = 0.80

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/(c*x**3+b*x),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

3.166 $\int \frac{b+2cx^3}{bx+cx^4}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1593, 446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+2cx^3}{bx+cx^4}\,dx &= \int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^3\right) \\
&= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$
\frac{1}{3}\log\left(b+cx^3\right)+\log(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

fricas [A] time = 1.00, size = 13, normalized size = 0.87

$$
\frac{1}{3}\log\left(cx^3+b\right)+\log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

giac [A] time = 0.31, size = 15, normalized size = 1.00

$$
\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(|x|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$
\ln(x)+\frac{\ln\left(c\,x^3+b\right)}{3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/(c*x^4+b*x),x)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

maxima [A] time = 0.52, size = 13, normalized size = 0.87

$$\frac{1}{3}\,\log\left(cx^3+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^3+b\right)}{3}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(b*x + c*x^4),x)
```

```
[Out] log(b + c*x^3)/3 + log(x)
```

sympy [A] time = 0.20, size = 12, normalized size = 0.80

$$\log(x)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/(c*x**4+b*x),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

3.167 $\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

[Out] ln(x)+ln(b+c*x^n)/n

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1593, 446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]

[Out] Log[x] + Log[b + c*x^n]/n

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = \int \frac{b+2cx^n}{x(b+cx^n)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^n\right)}{n}$$
$$= \log(x) + \frac{\log(b+cx^n)}{n}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

fricas [A] time = 0.80, size = 23, normalized size = 1.53

$$\frac{(n-1)\log(x) + \log\left(bx + cx^{n+1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="fricas")
```

```
[Out] ((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,cx^n + b}{bx + cx^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)
```

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\ln(x) + \frac{\ln\left(c\,\mathrm{e}^{n\ln(x)} + b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(b*x+c*x^(1+n)),x)
```

```
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

maxima [B] time = 0.58, size = 47, normalized size = 3.13

$$b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\,\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")
```

```
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{b + 2\,c\,x^n}{b\,x + c\,x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)),x)
```

```
[Out] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)), x)
```

sympy [A] time = 1.48, size = 29, normalized size = 1.93

$$\begin{cases} \log{\left(x \right)} & \text{for}\: c = 0 \wedge n = 0 \\ \frac{\left(b + 2 c\right) \log{\left(x \right)}}{b + c} & \text{for}\: n = 0 \\ \log{\left(x \right)} & \text{for}\: c = 0 \\ \log{\left(x \right)} + \frac{\log{\left(\frac{b}{c} + x^{n} \right)}}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))
```

3.168 $\int \frac{b+2cx}{(bx+cx^2)^8} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

```
[Out] -1/7/(c*x^2+b*x)^7
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} \, dx = -\frac{1}{7\left(bx+cx^2\right)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

fricas [B] time = 0.74, size = 81, normalized size = 5.40

$$-\frac{1}{7\left(c^{7}x^{14}+7\,bc^{6}x^{13}+21\,b^{2}c^{5}x^{12}+35\,b^{3}c^{4}x^{11}+35\,b^{4}c^{3}x^{10}+21\,b^{5}c^{2}x^{9}+7\,b^{6}cx^{8}+b^{7}x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

giac [A] time = 0.27, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^{2}+bx\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

maple [B] time = 0.02, size = 177, normalized size = 11.80

$$\frac{c^{7}}{7\left(cx+b\right)^{7}b^{7}}+\frac{c^{7}}{\left(cx+b\right)^{6}b^{8}}+\frac{4c^{7}}{\left(cx+b\right)^{5}b^{9}}+\frac{12c^{7}}{\left(cx+b\right)^{4}b^{10}}+\frac{30c^{7}}{\left(cx+b\right)^{3}b^{11}}+\frac{66c^{7}}{\left(cx+b\right)^{2}b^{12}}+\frac{132c^{7}}{\left(cx+b\right)b^{13}}-\frac{132c^{6}}{b^{13}x}+\frac{66c^{5}}{b^{12}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

```
[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^
7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+1/b^8*c^7/(c*x+b)^6+1/7/b^7*c^7/(c*x+b)^7-1
/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4
/b^9*c^2/x^5+1/b^8*c/x^6
```

maxima [A] time = 0.53, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^{2}+bx\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

mupad [B] time = 4.38, size = 12, normalized size = 0.80

$$-\frac{1}{7\,x^7\,{\left(b+c\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

sympy [B] time = 0.93, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

3.169 $\int \frac{b+2cx^2}{x^7(bx+cx^3)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/14/x^14/(c*x^2+b)^7
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = \int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right)$$
$$= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

fricas [B] time = 0.82, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

giac [A] time = 0.41, size = 15, normalized size = 0.94

$$-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^{6}}{7\left(c\,x^{2}+b\right)^{7} c}-\frac{b^{5}}{\left(c\,x^{2}+b\right)^{6} c}-\frac{4b^{4}}{\left(c\,x^{2}+b\right)^{5} c}-\frac{12b^{3}}{\left(c\,x^{2}+b\right)^{4} c}-\frac{30b^{2}}{\left(c\,x^{2}+b\right)^{3} c}-\frac{66b}{\left(c\,x^{2}+b\right)^{2} c}-\frac{132}{\left(c\,x^{2}+b\right) c}\right) c^{8}}{2b^{13}}-\frac{66c^{6}}{b^{13}x^{2}}+\frac{33c^{5}}{b^{12}x^{4}}-\frac{15c^{4}}{b^{11}x^{6}}+\frac{6c^{3}}{b^{10}x^{8}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x)
```

```
[Out] -1/2/b^13*c^8*(-1/7*b^6/c/(c*x^2+b)^7-b^5/c/(c*x^2+b)^6-132/c/(c*x^2+b)-66*
b/c/(c*x^2+b)^2-4*b^4/c/(c*x^2+b)^5-30*b^2/c/(c*x^2+b)^3-12*b^3/c/(c*x^2+b)
^4)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^
3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

maxima [B] time = 0.67, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

mupad [B] time = 2.22, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\left(c\,x^{2}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

sympy [B] time = 1.43, size = 87, normalized size = 5.44

$$-\frac{1}{14b^{7}x^{14}+98b^{6}cx^{16}+294b^{5}c^{2}x^{18}+490b^{4}c^{3}x^{20}+490b^{3}c^{4}x^{22}+294b^{2}c^{5}x^{24}+98bc^{6}x^{26}+14c^{7}x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

3.170 $\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx &= \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx \\
&= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right) \\
&= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}
\end{aligned}
$$

Mathematica [**A**] time = 0.04, size = 16, normalized size = 1.00

$$
-\frac{1}{21x^{21}\left(b+cx^3\right)^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

fricas [**B**] time = 0.81, size = 81, normalized size = 5.06

$$
-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

giac [**A**] time = 0.30, size = 15, normalized size = 0.94

$$
-\frac{1}{21\left(cx^6+bx^3\right)^7}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^{6}}{7\left(c\,x^{3}+b\right)^{7} c}-\frac{b^{5}}{\left(c\,x^{3}+b\right)^{6} c}-\frac{4 b^{4}}{\left(c\,x^{3}+b\right)^{5} c}-\frac{12 b^{3}}{\left(c\,x^{3}+b\right)^{4} c}-\frac{30 b^{2}}{\left(c\,x^{3}+b\right)^{3} c}-\frac{66 b}{\left(c\,x^{3}+b\right)^{2} c}-\frac{132}{\left(c\,x^{3}+b\right) c}\right) c^{8}}{3 b^{13}}-\frac{44 c^{6}}{b^{13} x^{3}}+\frac{22 c^{5}}{b^{12} x^{6}}-\frac{10 c^{4}}{b^{11} x^{9}}+\frac{4 c^{3}}{b^{10} x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-1/7*b^6/c/(c*x^3+b)^7-b^5/c/(c*x^3+b)^6-132/c/(c*x^3+b)-66*
b/c/(c*x^3+b)^2-4*b^4/c/(c*x^3+b)^5-30*b^2/c/(c*x^3+b)^3-12*b^3/c/(c*x^3+b)
^4)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^
3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

maxima [B] time = 0.80, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^{7} x^{42}+7\,b c^{6} x^{39}+21\,b^{2} c^{5} x^{36}+35\,b^{3} c^{4} x^{33}+35\,b^{4} c^{3} x^{30}+21\,b^{5} c^{2} x^{27}+7\,b^{6} c x^{24}+b^{7} x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

mupad [B] time = 7.22, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\left(c\,x^{3}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x)
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

sympy [B] time = 2.06, size = 87, normalized size = 5.44

$$-\frac{1}{21 b^{7} x^{21}+147 b^{6} c x^{24}+441 b^{5} c^{2} x^{27}+735 b^{4} c^{3} x^{30}+735 b^{3} c^{4} x^{33}+441 b^{2} c^{5} x^{36}+147 b c^{6} x^{39}+21 c^{7} x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

3.171 $\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{\left(bx+cx^{1+n}\right)^8}\,dx = \int \frac{x^{-8-7(-1+n)}(b+2cx^n)}{(b+cx^n)^8}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

fricas [B] time = 1.01, size = 143, normalized size = 6.81

$$-\frac{x^{14}}{7\left(b^7nx^7x^{7\,n+7}+7\,b^6cnx^6x^{8\,n+8}+21\,b^5c^2nx^5x^{9\,n+9}+35\,b^4c^3nx^4x^{10\,n+10}+35\,b^3c^4nx^3x^{11\,n+11}+21\,b^2c^5nx^2x^{12\,n}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas")
```

```
[Out] -1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*n
*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(11*
n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c^7*
n*x^(14*n + 14))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,cx^n+b}{\left(bx+cx^{n+1}\right)^8x^{7\,n-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)
```

maple [B] time = 0.06, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^7n}+\frac{c\,x^{-6n}}{b^8n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{66c^5x^{-2n}}{b^{12}n}+\frac{\left(9009b^5c\,x^n+20020b^4c^2x^{2n}+24024b^3c^3x^{3n}+16380\right.}{7\left(c\,x^n+b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

maxima [B] time = 1.03, size = 612, normalized size = 29.14

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}+401}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5nx^{12\,n}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{7-7\,n}\,\left(b+2\,c\,x^n\right)}{{\left(b\,x+c\,x^{n+1}\right)}^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8,x)
[Out] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
[Out] Timed out
```

3.172 $\int (b+2cx)\left(bx+cx^2\right)^p \, dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (c*x^2+b*x)^(1+p)/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^p \, dx = \frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

fricas [A] time = 0.78, size = 26, normalized size = 1.37

$$\frac{\left(cx^2+bx\right)\left(cx^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

giac [A] time = 0.37, size = 19, normalized size = 1.00

$$\frac{\left(cx^2+bx\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$\frac{(cx+b)\,x\left(c\,x^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)
[Out] (c*x+b)*x/(1+p)*(c*x^2+b*x)^p
```

maxima [A] time = 0.56, size = 19, normalized size = 1.00

$$\frac{\left(cx^2+bx\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

mupad [B] time = 2.10, size = 23, normalized size = 1.21

$$\frac{x\,{\left(c\,x^2+b\,x\right)}^p\,\left(b+c\,x\right)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

```
[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

sympy [A] time = 0.71, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx\left(bx+cx^2\right)^p}{p+1}+\frac{cx^2\left(bx+cx^2\right)^p}{p+1} & \text{for } p\neq -1 \\ \log\left(x\right)+\log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

3.173 $\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

```
[Out] 1/2*x^(1+p)*(c*x^3+b*x)^(1+p)/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1590}

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx=\frac{x^{1+p}\left(bx+cx^3\right)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 3.59

$$\frac{x^{p+2}\left(x\left(b+cx^2\right)\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

fricas [A] time = 1.21, size = 32, normalized size = 1.19

$$\frac{\left(cx^3+bx\right)\left(cx^3+bx\right)^p x^{p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)
```

giac [B] time = 0.33, size = 54, normalized size = 2.00

$$\frac{cx^3e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)+\log(x)\right)}+bxe^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)+\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
+ b) + 2*p*log(x) + log(x)))/(p + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 1.15

$$\frac{\left(c\,x^2+b\right)x^{p+2}\left(c\,x^3+bx\right)^p}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x)
```

```
[Out] 1/2*x^(2+p)*(c*x^2+b)/(p+1)*(c*x^3+b*x)^p
```

maxima [A] time = 0.99, size = 35, normalized size = 1.30

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

mupad [B] time = 2.21, size = 45, normalized size = 1.67

$$\left(c\,x^3+b\,x\right)^p\,\left(\frac{b\,x\,x^{p+1}}{2\,p+2}+\frac{c\,x^{p+1}\,x^3}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(p + 1)*(b*x + c*x^3)^p*(b + 2*c*x^2),x)
```

```
[Out] (b*x + c*x^3)^p*((b*x*x^(p + 1))/(2*p + 2) + (c*x^(p + 1)*x^3)/(2*p + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)
```

```
[Out] Timed out
```

3.174 $\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)dx$

Optimal. Leaf size=27

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

```
[Out] 1/2*x^(1+p)*(c*x^3+b*x)^(1+p)/(1+p)
```

Rubi [C] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 4.30, number of steps used = 7, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2032, 365, 364}

$$\frac{bx^{p+2}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)}{2(p+1)}+\frac{cx^{p+4}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)}{p+2}$$

Antiderivative was successfully verified.

```
[In] Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)/
b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hypergeom
etric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)dx = b\int x^{1+p}\left(bx+cx^3\right)^p\,dx+(2c)\int x^{3+p}\left(bx+cx^3\right)^p\,dx$$
$$=\left(bx^{-p}\left(b+cx^2\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(b+cx^2\right)^p\,dx+\left(2cx^{-}\right.$$
$$=\left(bx^{-p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(1+\frac{cx^2}{b}\right)^p\,dx+\left(2cx\right.$$
$$=\frac{bx^{2+p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\,{}_2F_1\left(-p,1+p;2+p;-\frac{cx^2}{b}\right)}{2(1+p)}+$$

Mathematica [C] time = 0.03, size = 97, normalized size = 3.59

$$\frac{x^{p+2}\left(x\left(b+cx^2\right)\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

fricas [A] time = 0.75, size = 33, normalized size = 1.22

$$\frac{\left(cx^2+b\right)\left(cx^3+bx\right)^p x^{p+3}}{2\left(p+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="f
ricas")
```

```
[Out] 1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)
```

giac [B] time = 0.49, size = 54, normalized size = 2.00

$$\frac{cx^3e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)+\log(x)\right)}+bxe^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)+\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="g
iac")
```

```
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
+ b) + 2*p*log(x) + log(x)))/(p + 1)
```

maple [C] time = 0.26, size = 142, normalized size = 5.26

$$\frac{\left(c\,x^2+b\right)x\,x^{p+1}\mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}\left(i\left(c\,x^2+b\right)\right)\mathrm{csgn}\left(i\left(c\,x^2+b\right)x\right)+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}\left(i\left(c\,x^2+b\right)x\right)^2+i\pi\,\mathrm{csgn}\left(i\left(c\,x^2+b\right)\right)\mathrm{csgn}\left(i\left(c\,x^2+b\right)x\right)^2-i\pi\mathrm{csgn}\left(i\left(c\,x^2+b\right)x\right)^3+2\ln(x)+2\ln(}{2}}}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x)
```

```
[Out] 1/2*(c*x^2+b)*x*x^(p+1)/(p+1)*exp(1/2*p*(-I*csgn(I*x*(c*x^2+b))^3*Pi+I*csgn
(I*x*(c*x^2+b))^2*csgn(I*x)*Pi+I*csgn(I*x*(c*x^2+b))^2*csgn(I*(c*x^2+b))*Pi
-I*csgn(I*x*(c*x^2+b))*csgn(I*x)*csgn(I*(c*x^2+b))*Pi+2*ln(x)+2*ln(c*x^2+b)
))
```

maxima [A] time = 1.07, size = 35, normalized size = 1.30

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="m
axima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int b\,x^{p+1}\left(c\,x^3+b\,x\right)^p+2\,c\,x^{p+3}\left(c\,x^3+b\,x\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p,x)
```

```
[Out] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
```

```
[Out] Timed out
```

3.175 $\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$

Optimal. Leaf size=29

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

```
[Out] 1/3*x^(2*p+2)*(c*x^4+b*x)^(1+p)/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1590}

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.08, size = 99, normalized size = 3.41

$$\frac{x^{2p+3}\left(x\left(b+cx^3\right)\right)^p\left(\frac{cx^3}{b}+1\right)^{-p}\left(2c(p+1)x^3\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] (x^(3 + 2*p)*(x*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 +
p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c
*x^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

fricas [A] time = 1.01, size = 34, normalized size = 1.17

$$\frac{\left(cx^4+bx\right)\left(cx^4+bx\right)^p x^{2p+2}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)
```

giac [B] time = 0.35, size = 58, normalized size = 2.00

$$\frac{cx^4e^{\left(p\log\left(cx^3+b\right)+3p\log(x)+2\log(x)\right)}+bxe^{\left(p\log\left(cx^3+b\right)+3p\log(x)+2\log(x)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x^
3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 1.14

$$\frac{\left(c\,x^3+b\right)x^{2p+3}\left(c\,x^4+bx\right)^p}{3+3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x)
```

```
[Out] 1/3*x^(3+2*p)*(c*x^3+b)/(p+1)*(c*x^4+b*x)^p
```

maxima [A] time = 1.01, size = 35, normalized size = 1.21

$$\frac{\left(cx^6+bx^3\right)e^{\left(p\log\left(cx^3+b\right)+3p\log(x)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

mupad [B] time = 2.21, size = 49, normalized size = 1.69

$$\left(c\,x^4+b\,x\right)^p\,\left(\frac{c\,x^{2\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{2\,p+2}}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*p + 2)*(b*x + c*x^4)^p*(b + 2*c*x^3),x)
```

```
[Out] (b*x + c*x^4)^p*((c*x^(2*p + 2)*x^4)/(3*p + 3) + (b*x*x^(2*p + 2))/(3*p + 3
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)
```

```
[Out] Timed out
```

3.176 $\int x^{(-1+n)(1+p)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx$

Optimal. Leaf size=36

$$\frac{x^{-((1-n)(p+1))}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

```
[Out] (b*x+c*x^(1+n))^(1+p)/n/(1+p)/(x^((1-n)*(1+p)))
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2036}

$$\frac{x^{-(1-n)(p+1)}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

```
[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Rule 2036

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Rubi steps

$$\int x^{(-1+n)(1+p)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{-(1-n)(1+p)}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.17, size = 108, normalized size = 3.00

$$\frac{x^{-p}\left(x\left(b+cx^n\right)\right)^p\left(\frac{cx^n}{b}+1\right)^{-p}\left(b(p+2)x^{n(p+1)}\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^n}{b}\right)+2c(p+1)x^{n(p+2)}\,{}_2F_1\left(-p,p+2;p+3\right.\right.}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

```
[Out] ((x*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2
+ p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p,
 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^p*(1 + (c*x^n)/b)^p)
```

fricas [A] time = 0.87, size = 42, normalized size = 1.17

$$\frac{\left(bx+cx^{n+1}\right)\left(bx+cx^{n+1}\right)^{p}x^{(n-1)p+n-1}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fric
as")
```

```
[Out] (b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2\,cx^{n}+b\right)\left(bx+cx^{n+1}\right)^{p}x^{(n-1)\left(p+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="giac
")
```

```
[Out] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(2c\,x^{n}+b\right)x^{(n-1)\left(p+1\right)}\left(bx+c\,x^{n+1}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

```
[Out] int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

maxima [A] time = 1.07, size = 39, normalized size = 1.08

$$\frac{\left(cx^{2\,n}+bx^{n}\right)e^{\left(np\log(x)+p\log(cx^{n}+b)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="maxi
ma")
```

```
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int x^{(n-1)\,(p+1)}\,{\left(b\,x+c\,x^{n+1}\right)}^p\,\left(b+2\,c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n),x)
```

```
[Out] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)
```

```
[Out] Timed out
```

3.177 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx$

Optimal. Leaf size=32

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx &= \int \left(ac + adx + bcx^2 + bdx^3\right) dx\\ &= acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4\end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

fricas [A] time = 0.84, size = 26, normalized size = 0.81

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="fricas")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

giac [A] time = 0.31, size = 26, normalized size = 0.81

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{1}{4}bd\,x^4 + \frac{1}{3}bc\,x^3 + \frac{1}{2}ad\,x^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x
)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

maxima [A] time = 0.78, size = 26, normalized size = 0.81

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

mupad [B] time = 2.14, size = 26, normalized size = 0.81

$$\frac{b\,d\,x^4}{4}+\frac{b\,c\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2),x)
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

sympy [A] time = 0.10, size = 29, normalized size = 0.91

$$acx+\frac{adx^2}{2}+\frac{bcx^3}{3}+\frac{bdx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4
```

3.178 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2
```

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^2}\,dx &= \int \frac{ac+adx+bcx^2+bdx^3}{a+bx^2}\,dx\\
&= \int (c+dx)\,dx\\
&= cx+\frac{dx^2}{2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

fricas [A] time = 0.82, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + c*x
```

giac [A] time = 0.40, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}d\,x^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2
,x)
```

```
[Out] c*x+1/2*d*x^2
```

maxima [A] time = 0.47, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2}+c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2)^2,x)
```

```
[Out] c*x + (d*x^2)/2
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$cx+\frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**2,x)
```

```
[Out] c*x + d*x**2/2
```

3.179 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=42

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*d*ln(b*x^2+a)/b+c*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1586, 635, 205, 260}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3} dx = \int \frac{ac + adx + bcx^2 + bdx^3}{(a + bx^2)^2} dx$$
$$= \int \frac{c + dx}{a + bx^2} dx$$
$$= c\int \frac{1}{a + bx^2} dx + d\int \frac{x}{a + bx^2} dx$$
$$= \frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{d\log\left(a + bx^2\right)}{2b}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{d\log\left(a + bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

fricas [A] time = 1.01, size = 98, normalized size = 2.33

$$\left[\frac{ad\log\left(bx^2 + a\right) - \sqrt{-ab}\,c\log\left(\frac{bx^2 - 2\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{2\,ab}, \frac{ad\log\left(bx^2 + a\right) + 2\sqrt{ab}\,c\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*b)
*x/a))/(a*b)]
```

giac [A] time = 0.40, size = 31, normalized size = 0.74

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{d\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.76

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{d\ln\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3
,x)
```

```
[Out] 1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

maxima [A] time = 1.49, size = 31, normalized size = 0.74

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{d\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="maxima")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

mupad [B] time = 2.13, size = 32, normalized size = 0.76

$$\frac{d\,\ln\left(b\,x^2+a\right)}{2\,b}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2)^3,x)
```

```
[Out] (d*log(a + b*x^2))/(2*b) + (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.30, size = 124, normalized size = 2.95

$$\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x + \frac{2ab\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right) + \left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x + \frac{2ab\left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**3,x)
```

```
[Out] (d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) - c*sqrt(-a*
b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2))*lo
g(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c))
```

3.180 $\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

[Out] (d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1588}

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{\left(a+bx+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{(a+x(b+x(c+dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

fricas [A] time = 0.84, size = 38, normalized size = 1.52

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)
```

giac [A] time = 0.29, size = 25, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + bx + a\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 26, normalized size = 1.04

$$\frac{\left(d\,x^3 + c\,x^2 + bx + a\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x)
[Out] (d*x^3+c*x^2+b*x+a)^(n+1)/(n+1)
```

maxima [A] time = 0.67, size = 25, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + bx + a\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

mupad [B] time = 2.19, size = 54, normalized size = 2.16

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^n\,\left(\frac{a}{n+1}+\frac{b\,x}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^n*(a/(n + 1) + (b*x)/(n + 1) + (c*x^2)/(n + 1) +
(d*x^3)/(n + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)
```

```
[Out] Timed out
```

3.181 $\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=24

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (d*x^3+c*x^2+b*x)^(1+n)/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1588}

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{\left(bx+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.05, size = 21, normalized size = 0.88

$$\frac{(x(b+x(c+dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

fricas [A] time = 0.87, size = 36, normalized size = 1.50

$$\frac{\left(dx^3+cx^2+bx\right)\left(dx^3+cx^2+bx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)
```

giac [A] time = 0.31, size = 24, normalized size = 1.00

$$\frac{\left(dx^3+cx^2+bx\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 34, normalized size = 1.42

$$\frac{\left(d\,x^2+cx+b\right)x\left(d\,x^3+c\,x^2+bx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x)
[Out] x*(d*x^2+c*x+b)/(n+1)*(d*x^3+c*x^2+b*x)^n
```

maxima [A] time = 0.63, size = 24, normalized size = 1.00

$$\frac{\left(dx^3+cx^2+bx\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

mupad [B] time = 2.12, size = 46, normalized size = 1.92

$$\left(\frac{b\,x}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)\,{\left(d\,x^3+c\,x^2+b\,x\right)}^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x)
[Out] ((b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(b*x + c*x^2 + d*x^3)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)
[Out] Timed out
```

3.182 $\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right) dx$

Optimal. Leaf size=25

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

```
[Out] x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1590}

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right) dx = \frac{x^{1+n}\left(b+cx+dx^2\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.96

$$\frac{x^{n+1}(b+x(c+dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + x*(c + d*x))^(1 + n))/(1 + n)
```

fricas [A] time = 1.02, size = 35, normalized size = 1.40

$$\frac{\left(dx^3+cx^2+bx\right)\left(dx^2+cx+b\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)
```

giac [B] time = 0.40, size = 65, normalized size = 2.60

$$\frac{\left(dx^2+cx+b\right)^n dx^3x^n+\left(dx^2+cx+b\right)^n cx^2x^n+\left(dx^2+cx+b\right)^n bxx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] ((d*x^2 + c*x + b)^n*d*x^3*x^n + (d*x^2 + c*x + b)^n*c*x^2*x^n + (d*x^2 + c
*x + b)^n*b*x*x^n)/(n + 1)
```

maple [A] time = 0.01, size = 26, normalized size = 1.04

$$\frac{x^{n+1}\left(d\,x^2+cx+b\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x)
```

```
[Out] x^(n+1)*(d*x^2+c*x+b)^(n+1)/(n+1)
```

maxima [A] time = 0.86, size = 39, normalized size = 1.56

$$\frac{\left(dx^3+cx^2+bx\right)e^{\left(n\log\left(dx^2+cx+b\right)+n\log\left(x\right)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n + 1)
```

mupad [B] time = 2.15, size = 51, normalized size = 2.04

$$\left(\frac{c\,x^n\,x^2}{n+1}+\frac{d\,x^n\,x^3}{n+1}+\frac{b\,x\,x^n}{n+1}\right)\,{\left(d\,x^2+c\,x+b\right)}^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x)
```

```
[Out] ((c*x^n*x^2)/(n + 1) + (d*x^n*x^3)/(n + 1) + (b*x*x^n)/(n + 1))*(b + c*x +
d*x^2)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)
```

```
[Out] Timed out
```

3.183 $\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (d*x^3+b*x+a)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\frac{\left(a+bx+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

fricas [A] time = 0.83, size = 28, normalized size = 1.40

$$\frac{\left(dx^3+bx+a\right)\left(dx^3+bx+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)
```

giac [A] time = 0.28, size = 20, normalized size = 1.00

$$\frac{\left(dx^3+bx+a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="giac")
[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{\left(d\,x^3+bx+a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x)
[Out] (d*x^3+b*x+a)^(n+1)/(n+1)
```

maxima [A] time = 0.65, size = 20, normalized size = 1.00

$$\frac{\left(dx^3+bx+a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

mupad [B] time = 2.14, size = 39, normalized size = 1.95

$$\left(\frac{a}{n+1}+\frac{b\,x}{n+1}+\frac{d\,x^3}{n+1}\right)\,{\left(d\,x^3+b\,x+a\right)}^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^n,x)
[Out] (a/(n + 1) + (b*x)/(n + 1) + (d*x^3)/(n + 1))*(a + b*x + d*x^3)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)
[Out] Timed out
```

3.184 $\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (d*x^3+b*x)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1588}

$$\frac{\left(bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] (b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx=\frac{\left(bx+dx^3\right)^{1+n}}{1+n}$$

Mathematica [C] time = 0.07, size = 106, normalized size = 5.58

$$\frac{x\left(x\left(b+dx^2\right)\right)^n\left(\frac{dx^2}{b}+1\right)^{-n}\left(3d(n+1)x^2\,{}_2F_1\left(-n,\frac{n+3}{2};\frac{n+5}{2};-\frac{dx^2}{b}\right)+b(n+3)\,{}_2F_1\left(-n,\frac{n+1}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + d*x^2))^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2,
 -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 + n)/2
, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

fricas [A] time = 0.91, size = 26, normalized size = 1.37

$$\frac{(dx^3 + bx)(dx^3 + bx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)
```

giac [A] time = 0.34, size = 19, normalized size = 1.00

$$\frac{(dx^3 + bx)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 26, normalized size = 1.37

$$\frac{(d\,x^2 + b)\,x\,(d\,x^3 + bx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^n,x)
```

```
[Out] x*(d*x^2+b)/(n+1)*(d*x^3+b*x)^n
```

maxima [A] time = 0.53, size = 19, normalized size = 1.00

$$\frac{(dx^3 + bx)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)
```

mupad [B] time = 2.13, size = 25, normalized size = 1.32

$$\frac{x\left(d\,x^3+b\,x\right)^n\,\left(d\,x^2+b\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^n*(b + 3*d*x^2),x)
```

```
[Out] (x*(b*x + d*x^3)^n*(b + d*x^2))/(n + 1)
```

sympy [B] time = 11.34, size = 73, normalized size = 3.84

$$\begin{cases} \frac{bx\left(bx+dx^3\right)^n}{n+1}+\frac{dx^3\left(bx+dx^3\right)^n}{n+1} & \text{for } n\neq -1 \\ \log\left(x\right)+\log\left(-i\sqrt{b}\sqrt{\frac{1}{d}}+x\right)+\log\left(i\sqrt{b}\sqrt{\frac{1}{d}}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)
```

```
[Out] Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)**n/(n + 1)
, Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)*sqrt(
1/d) + x), True))
```

3.185 $\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right) dx$

Optimal. Leaf size=22

$$\frac{x^{n+1}\left(b+dx^2\right)^{n+1}}{n+1}$$

```
[Out] x^(1+n)*(d*x^2+b)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {449}

$$\frac{x^{n+1}\left(b+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right) dx = \frac{x^{1+n}\left(b+dx^2\right)^{1+n}}{1+n}$$

Mathematica [C] time = 0.04, size = 108, normalized size = 4.91

$$\frac{x^{n+1}\left(b+dx^2\right)^n\left(\frac{dx^2}{b}+1\right)^{-n}\left(3d(n+1)x^2\,{}_2F_1\left(-n,\frac{n+3}{2};\frac{n+5}{2};-\frac{dx^2}{b}\right)+b(n+3)\,{}_2F_1\left(-n,\frac{n+1}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + d*x^2)^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n
)/2, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 +
n)/2, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

fricas [A] time = 1.07, size = 27, normalized size = 1.23

$$\frac{\left(dx^3+bx\right)\left(dx^2+b\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)
```

giac [A] time = 0.34, size = 39, normalized size = 1.77

$$\frac{\left(dx^2+b\right)^n dx^3 x^n+\left(dx^2+b\right)^n bxx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="giac")
```

```
[Out] ((d*x^2 + b)^n*d*x^3*x^n + (d*x^2 + b)^n*b*x*x^n)/(n + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^{n+1}\left(d\,x^2+b\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+b)^n*(3*d*x^2+b),x)
```

```
[Out] x^(n+1)*(d*x^2+b)^(n+1)/(n+1)
```

maxima [A] time = 1.21, size = 31, normalized size = 1.41

$$\frac{\left(dx^3+bx\right)e^{\left(n\log\left(dx^2+b\right)+n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="maxima")
```

```
[Out] (d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)
```

mupad [B] time = 2.16, size = 26, normalized size = 1.18

$$\frac{x\,x^n\,{\left(d\,x^2+b\right)}^n\,\left(d\,x^2+b\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(b + d*x^2)^n*(b + 3*d*x^2),x)
```

```
[Out] (x*x^n*(b + d*x^2)^n*(b + d*x^2))/(n + 1)
```

sympy [B] time = 52.84, size = 76, normalized size = 3.45

$$\begin{cases} \frac{bxx^n\left(b+dx^2\right)^n}{n+1} + \frac{dx^3x^n\left(b+dx^2\right)^n}{n+1} & \text{for } n \neq -1 \\ \log\left(x\right) + \log\left(-i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) + \log\left(i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)
```

```
[Out] Piecewise((b*x*x**n*(b + d*x**2)**n/(n + 1) + d*x**3*x**n*(b + d*x**2)**n/(
n + 1), Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)
*sqrt(1/d) + x), True))
```

3.186 $\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1588}

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

fricas [A] time = 0.77, size = 32, normalized size = 1.45

$$\frac{\left(dx^3 + cx^2 + a\right)\left(dx^3 + cx^2 + a\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

giac [A] time = 0.34, size = 22, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + a\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{\left(d\,x^3 + c\,x^2 + a\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x)
[Out] (d*x^3+c*x^2+a)^(n+1)/(n+1)
```

maxima [A] time = 0.66, size = 22, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + a\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

mupad [B] time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)\,{\left(d\,x^3+c\,x^2+a\right)}^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x)
[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)
[Out] Timed out
```

3.187 $\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

[Out] (d*x^3+c*x^2)^(1+n)/(1+n)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1588}

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$\frac{\left(x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

```
[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

fricas [A] time = 0.80, size = 30, normalized size = 1.43

$$\frac{\left(dx^3 + cx^2\right)\left(dx^3 + cx^2\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

giac [A] time = 0.31, size = 21, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{(dx + c)\, x^2 \left(d\, x^3 + c\, x^2\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x)
[Out] (d*x^3+c*x^2)^n*x^2*(d*x+c)/(n+1)
```

maxima [A] time = 0.59, size = 21, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

mupad [B] time = 2.16, size = 27, normalized size = 1.29

$$\frac{x^2\,{\left(d\,x^3+c\,x^2\right)}^n\,\left(c+d\,x\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x)
```

```
[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

sympy [A] time = 1.16, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2\left(cx^2+dx^3\right)^n}{n+1}+\frac{dx^3\left(cx^2+dx^3\right)^n}{n+1} & \text{for } n\neq -1 \\ 2\log\left(x\right)+\log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.188 $\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right)\,dx$

Optimal. Leaf size=24

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

```
[Out] x^(1+n)*(d*x^2+c*x)^(1+n)/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 763}

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 763

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right)\,dx &= \int x^{1+n}(2c+3dx)\left(cx+dx^2\right)^n\,dx\\ &= \frac{x^{1+n}\left(cx+dx^2\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{x^{n+1}(x(c+dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
[Out] (x^(1 + n)*(x*(c + d*x))^(1 + n))/(1 + n)
```

fricas [A] time = 0.91, size = 31, normalized size = 1.29

$$\frac{\left(dx^3+cx^2\right)\left(dx^2+cx\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)
```

giac [B] time = 0.46, size = 51, normalized size = 2.12

$$\frac{dx^3x^ne^{\left(n\log(dx+c)+n\log(x)\right)}+cx^2x^ne^{\left(n\log(dx+c)+n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
[Out] (d*x^3*x^n*e^(n*log(d*x + c) + n*log(x)) + c*x^2*x^n*e^(n*log(d*x + c) + n*
log(x)))/(n + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 1.17

$$\frac{(dx+c)\,x^{n+2}\left(d\,x^2+cx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x)
[Out] (d*x^2+c*x)^n*x^(2+n)*(d*x+c)/(n+1)
```

maxima [A] time = 0.83, size = 32, normalized size = 1.33

$$\frac{\left(dx^3+cx^2\right)e^{\left(n\log(dx+c)+2\,n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

mupad [B] time = 2.20, size = 28, normalized size = 1.17

$$\frac{x^n\,x^2\,{\left(d\,x^2+c\,x\right)}^n\,\left(c+d\,x\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x)
```

```
[Out] (x^n*x^2*(c*x + d*x^2)^n*(c + d*x))/(n + 1)
```

sympy [A] time = 6.17, size = 56, normalized size = 2.33

$$\begin{cases} \frac{cx^{2}x^{n}\left(cx+dx^{2}\right)^{n}}{n+1} + \frac{dx^{3}x^{n}\left(cx+dx^{2}\right)^{n}}{n+1} & \text{for}\: n \neq -1 \\ 2\log{\left(x \right)} + \log{\left(\frac{c}{d} + x \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c*x + d*x**
2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.189 $\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx$

Optimal. Leaf size=22

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

```
[Out] x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {845}

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\int x^{2n}(c+dx)^n\left(2cx+3dx^2\right)\,dx=\frac{x^{2(1+n)}(c+dx)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x^{2n+2}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2 + 2*n)*(c + d*x)^(1 + n))/(1 + n)
```

fricas [A] time = 0.98, size = 29, normalized size = 1.32

$$\frac{\left(dx^3 + cx^2\right)(dx + c)^n x^{2n}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x + c)^n*x^(2*n)/(n + 1)
```

giac [A] time = 0.31, size = 41, normalized size = 1.86

$$\frac{(dx + c)^n dx^3 x^{2n} + (dx + c)^n cx^2 x^{2n}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*d*x^3*x^(2*n) + (d*x + c)^n*c*x^2*x^(2*n))/(n + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^{2n+2}\,(dx + c)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x)
```

```
[Out] x^(2+2*n)*(d*x+c)^(n+1)/(n+1)
```

maxima [A] time = 0.90, size = 32, normalized size = 1.45

$$\frac{\left(dx^3 + cx^2\right)e^{\left(n\log(dx+c)+2\,n\log(x)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

mupad [B] time = 2.19, size = 26, normalized size = 1.18

$$\frac{x^{2n}\,x^2\,(c + d\,x)^n\,(c + d\,x)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n)*(2*c*x + 3*d*x^2)*(c + d*x)^n,x)
```

```
[Out] (x^(2*n)*x^2*(c + d*x)^n*(c + d*x))/(n + 1)
```

sympy [A] time = 6.15, size = 53, normalized size = 2.41

$$\begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)*(c + d*x)
**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.190 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1588}

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

fricas [A] time = 1.23, size = 32, normalized size = 1.45

$$\frac{\left(dx^3+cx^2+a\right)\left(dx^3+cx^2+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

giac [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{\left(dx^3+cx^2+a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{\left(d\,x^3+c\,x^2+a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x)
[Out] 1/(n+1)*(d*x^3+c*x^2+a)^(n+1)
```

maxima [A] time = 0.75, size = 32, normalized size = 1.45

$$\frac{\left(dx^3+cx^2+a\right)\left(dx^3+cx^2+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

mupad [B] time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)\,{\left(d\,x^3+c\,x^2+a\right)}^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x)
[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)
[Out] Timed out
```

3.191 $\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

```
[Out] (d*x^3+c*x^2)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1588}

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.90

$$\frac{\left(x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)

fricas [A] time = 0.76, size = 30, normalized size = 1.43

$$\frac{\left(dx^3 + cx^2\right)\left(dx^3 + cx^2\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)

giac [A] time = 0.40, size = 21, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2\right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{(dx + c)\, x^2 \left(d\, x^3 + c\, x^2\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x)

[Out] (d*x+c)/(n+1)*x^2*(d*x^3+c*x^2)^n

maxima [A] time = 0.93, size = 32, normalized size = 1.52

$$\frac{\left(dx^3 + cx^2\right)e^{\left(n\log(dx+c)+2\,n\log(x)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)

mupad [B] time = 2.15, size = 27, normalized size = 1.29

$$\frac{x^2\,{\left(d\,x^3+c\,x^2\right)}^n\,\left(c+d\,x\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x)
```

```
[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

sympy [A] time = 1.14, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2\left(cx^2+dx^3\right)^n}{n+1}+\frac{dx^3\left(cx^2+dx^3\right)^n}{n+1} & \text{for } n\neq -1 \\ 2\log\left(x\right)+\log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.192 $\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=21

$$\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

[Out] 1/8*(d*x^3+c*x^2+b*x+a)^8

Rubi [A] time = 0.13, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1588}

$$\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]

[Out] (a + b*x + c*x^2 + d*x^3)^8/8

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.15, size = 143, normalized size = 6.81

$$\frac{1}{8}x(b+x(c+dx))\left(8a^7+28a^6x(b+x(c+dx))+56a^5x^2(b+x(c+dx))^2+70a^4x^3(b+x(c+dx))^3+56a^3x^4(b+x(c\right.$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]

[Out] (x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5*x^2*(b + x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^3*x^4*(b + x*(c + d*

```
x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*x^6*(b + x*(c + d*x))^6 + x^7*
(b + x*(c + d*x))^7))/8
```

fricas [B] time = 0.56, size = 1956, normalized size = 93.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + x^21*d^7*a + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2
*x^20*d^6*b^2 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^
19*d^5*c*b^2 + 21*x^19*d^5*c^2*a + 7*x^19*d^6*b*a + 7/2*x^18*d^2*c^6 + 35*x
^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + 35*x^18*d^4*c^3*a
 + 42*x^18*d^5*c*b*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 21*x^17*d^2*c^5*b +
70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 35*x^17*d^3*c^4*a + 105*x^17*d^4*
c^2*b*a + 21*x^17*d^5*b^2*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 7*x^16*d*c
^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*x^16*d^4*b^4 + 2
1*x^16*d^2*c^5*a + 140*x^16*d^3*c^3*b*a + 105*x^16*d^4*c*b^2*a + 105/2*x^16
*d^4*c^2*a^2 + 21*x^16*d^5*b*a^2 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15
*d^2*c^3*b^3 + 35*x^15*d^3*c*b^4 + 7*x^15*d*c^6*a + 105*x^15*d^2*c^4*b*a +
210*x^15*d^3*c^2*b^2*a + 35*x^15*d^4*b^3*a + 70*x^15*d^3*c^3*a^2 + 105*x^15
*d^4*c*b*a^2 + 7*x^15*d^5*a^3 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/
2*x^14*d^2*c^2*b^4 + 7*x^14*d^3*b^5 + x^14*c^7*a + 42*x^14*d*c^5*b*a + 210*
x^14*d^2*c^3*b^2*a + 140*x^14*d^3*c*b^3*a + 105/2*x^14*d^2*c^4*a^2 + 210*x^
14*d^3*c^2*b*a^2 + 105/2*x^14*d^4*b^2*a^2 + 35*x^14*d^4*c*a^3 + 7*x^13*c^5*
b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5 + 7*x^13*c^6*b*a + 105*x^13*d*c
^4*b^2*a + 210*x^13*d^2*c^2*b^3*a + 35*x^13*d^3*b^4*a + 21*x^13*d*c^5*a^2 +
 210*x^13*d^2*c^3*b*a^2 + 210*x^13*d^3*c*b^2*a^2 + 70*x^13*d^3*c^2*a^3 + 35
*x^13*d^4*b*a^3 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6
+ 21*x^12*c^5*b^2*a + 140*x^12*d*c^3*b^3*a + 105*x^12*d^2*c*b^4*a + 7/2*x^1
2*c^6*a^2 + 105*x^12*d*c^4*b*a^2 + 315*x^12*d^2*c^2*b^2*a^2 + 70*x^12*d^3*b
^3*a^2 + 70*x^12*d^2*c^3*a^3 + 140*x^12*d^3*c*b*a^3 + 35/4*x^12*d^4*a^4 + 7
*x^11*c^3*b^5 + 7*x^11*d*c*b^6 + 35*x^11*c^4*b^3*a + 105*x^11*d*c^2*b^4*a +
 21*x^11*d^2*b^5*a + 21*x^11*c^5*b*a^2 + 210*x^11*d*c^3*b^2*a^2 + 210*x^11*
d^2*c*b^3*a^2 + 35*x^11*d*c^4*a^3 + 210*x^11*d^2*c^2*b*a^3 + 70*x^11*d^3*b^
2*a^3 + 35*x^11*d^3*c*a^4 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + 35*x^10*c^3*b^4
*a + 42*x^10*d*c*b^5*a + 105/2*x^10*c^4*b^2*a^2 + 210*x^10*d*c^2*b^3*a^2 +
105/2*x^10*d^2*b^4*a^2 + 7*x^10*c^5*a^3 + 140*x^10*d*c^3*b*a^3 + 210*x^10*d
^2*c*b^2*a^3 + 105/2*x^10*d^2*c^2*a^4 + 35*x^10*d^3*b*a^4 + x^9*c*b^7 + 21*
x^9*c^2*b^5*a + 7*x^9*d*b^6*a + 70*x^9*c^3*b^3*a^2 + 105*x^9*d*c*b^4*a^2 +
35*x^9*c^4*b*a^3 + 210*x^9*d*c^2*b^2*a^3 + 70*x^9*d^2*b^3*a^3 + 35*x^9*d*c^
3*a^4 + 105*x^9*d^2*c*b*a^4 + 7*x^9*d^3*a^5 + 1/8*x^8*b^8 + 7*x^8*c*b^6*a +
 105/2*x^8*c^2*b^4*a^2 + 21*x^8*d*b^5*a^2 + 70*x^8*c^3*b^2*a^3 + 140*x^8*d*
```

```
c*b^3*a^3 + 35/4*x^8*c^4*a^4 + 105*x^8*d*c^2*b*a^4 + 105/2*x^8*d^2*b^2*a^4
+ 21*x^8*d^2*c*a^5 + x^7*b^7*a + 21*x^7*c*b^5*a^2 + 70*x^7*c^2*b^3*a^3 + 35
*x^7*d*b^4*a^3 + 35*x^7*c^3*b*a^4 + 105*x^7*d*c*b^2*a^4 + 21*x^7*d*c^2*a^5
+ 21*x^7*d^2*b*a^5 + 7/2*x^6*b^6*a^2 + 35*x^6*c*b^4*a^3 + 105/2*x^6*c^2*b^2
*a^4 + 35*x^6*d*b^3*a^4 + 7*x^6*c^3*a^5 + 42*x^6*d*c*b*a^5 + 7/2*x^6*d^2*a^
6 + 7*x^5*b^5*a^3 + 35*x^5*c*b^3*a^4 + 21*x^5*c^2*b*a^5 + 21*x^5*d*b^2*a^5
+ 7*x^5*d*c*a^6 + 35/4*x^4*b^4*a^4 + 21*x^4*c*b^2*a^5 + 7/2*x^4*c^2*a^6 + 7
*x^4*d*b*a^6 + 7*x^3*b^3*a^5 + 7*x^3*c*b*a^6 + x^3*d*a^7 + 7/2*x^2*b^2*a^6
+ x^2*c*a^7 + x*b*a^7
```

giac [B] time = 0.30, size = 160, normalized size = 7.62

$$\frac{1}{8}\left(dx^3+cx^2+bx\right)^8+\left(dx^3+cx^2+bx\right)^7a+\frac{7}{2}\left(dx^3+cx^2+bx\right)^6a^2+7\left(dx^3+cx^2+bx\right)^5a^3+\frac{35}{4}\left(dx^3+cx^2+bx\right)^4a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8 + (d*x^3 + c*x^2 + b*x)^7*a + 7/2*(d*x^3 + c*x^
2 + b*x)^6*a^2 + 7*(d*x^3 + c*x^2 + b*x)^5*a^3 + 35/4*(d*x^3 + c*x^2 + b*x)
^4*a^4 + 7*(d*x^3 + c*x^2 + b*x)^3*a^5 + 7/2*(d*x^3 + c*x^2 + b*x)^2*a^6 +
(d*x^3 + c*x^2 + b*x)*a^7
```

maple [B] time = 0.00, size = 25686, normalized size = 1223.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.74, size = 19, normalized size = 0.90

$$\frac{1}{8}\left(dx^3+cx^2+bx+a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x + a)^8
```

mupad [B] time = 2.98, size = 1576, normalized size = 75.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + (35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*
a*b^2*c^5 + 21*b^5*c^2*d + 70*a^2*b^3*d^3 + 70*a^3*c^3*d^2 + 315*a^2*b^2*c^
2*d^2 + 140*a*b^3*c^3*d + 105*a*b^4*c*d^2 + 105*a^2*b*c^4*d + 140*a^3*b*c*d
^3) + x^11*(7*b^5*c^3 + 35*a*b^3*c^4 + 21*a^2*b*c^5 + 21*a*b^5*d^2 + 35*a^3
*c^4*d + 35*a^4*c*d^3 + 70*a^3*b^2*d^3 + 7*b^6*c*d + 210*a^2*b^2*c^3*d + 21
0*a^2*b^3*c*d^2 + 210*a^3*b*c^2*d^2 + 105*a*b^4*c^2*d) + x^13*(7*b^3*c^5 +
35*a*b^4*d^3 + 35*a^3*b*d^4 + 21*a^2*c^5*d + 35*b^4*c^3*d + 21*b^5*c*d^2 +
70*a^3*c^2*d^3 + 7*a*b*c^6 + 210*a*b^3*c^2*d^2 + 210*a^2*b*c^3*d^2 + 210*a^
2*b^2*c*d^3 + 105*a*b^2*c^4*d) + x^5*(7*a^3*b^5 + 35*a^4*b^3*c + 21*a^5*b*c
^2 + 21*a^5*b^2*d + 7*a^6*c*d) + x^19*(7*c^5*d^3 + 21*a*c^2*d^5 + 35*b*c^3*
d^4 + 21*b^2*c*d^5 + 7*a*b*d^6) + x^8*(b^8/8 + (35*a^4*c^4)/4 + 21*a^2*b^5*
d + 21*a^5*c*d^2 + (105*a^2*b^4*c^2)/2 + 70*a^3*b^2*c^3 + (105*a^4*b^2*d^2)
/2 + 7*a*b^6*c + 140*a^3*b^3*c*d + 105*a^4*b*c^2*d) + x^9*(b^7*c + 7*a^5*d^
3 + 21*a*b^5*c^2 + 35*a^3*b*c^4 + 35*a^4*c^3*d + 70*a^2*b^3*c^3 + 70*a^3*b^
3*d^2 + 7*a*b^6*d + 210*a^3*b^2*c^2*d + 105*a^2*b^4*c*d + 105*a^4*b*c*d^2)
+ x^16*(c^8/8 + (35*b^4*d^4)/4 + 21*a^2*b*d^5 + 21*a*c^5*d^2 + (105*a^2*c^2
*d^4)/2 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d + 140*a*b*c^3*d^
3 + 105*a*b^2*c*d^4) + x^10*(b^7*d + 7*a^3*c^5 + (7*b^6*c^2)/2 + 35*a*b^4*c
^3 + 35*a^4*b*d^3 + (105*a^2*b^2*c^4)/2 + (105*a^2*b^4*d^2)/2 + (105*a^4*c^
2*d^2)/2 + 210*a^2*b^3*c^2*d + 210*a^3*b^2*c*d^2 + 42*a*b^5*c*d + 140*a^3*b
*c^3*d) + x^15*(b*c^7 + 7*a^3*d^5 + 35*a*b^3*d^4 + 21*b^2*c^5*d + 35*b^4*c*
d^3 + 70*a^2*c^3*d^3 + 70*b^3*c^3*d^2 + 7*a*c^6*d + 210*a*b^2*c^2*d^3 + 105
*a*b*c^4*d^2 + 105*a^2*b*c*d^4) + x^14*(a*c^7 + (7*b^2*c^6)/2 + 7*b^5*d^3 +
 35*a^3*c*d^4 + 35*b^3*c^4*d + (105*a^2*b^2*d^4)/2 + (105*a^2*c^4*d^2)/2 +
(105*b^4*c^2*d^2)/2 + 210*a*b^2*c^3*d^2 + 210*a^2*b*c^2*d^3 + 42*a*b*c^5*d
+ 140*a*b^3*c*d^3) + x^4*((35*a^4*b^4)/4 + (7*a^6*c^2)/2 + 21*a^5*b^2*c + 7
*a^6*b*d) + x^20*((7*b^2*d^6)/2 + (35*c^4*d^4)/4 + 21*b*c^2*d^5 + 7*a*c*d^6
) + x^6*((7*a^2*b^6)/2 + 7*a^5*c^3 + (7*a^6*d^2)/2 + 35*a^3*b^4*c + 35*a^4*
b^3*d + (105*a^4*b^2*c^2)/2 + 42*a^5*b*c*d) + x^7*(a*b^7 + 21*a^2*b^5*c + 3
5*a^4*b*c^3 + 35*a^3*b^4*d + 21*a^5*b*d^2 + 21*a^5*c^2*d + 70*a^3*b^3*c^2 +
 105*a^4*b^2*c*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5 + (7*c^6*d^2)/2 + 35*a*
c^3*d^4 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2 + 42*a*b*c*d^5) + x^17*(c^7*d
+ 21*a*b^2*d^5 + 35*a*c^4*d^3 + 21*a^2*c*d^5 + 21*b*c^5*d^2 + 35*b^3*c*d^4
+ 70*b^2*c^3*d^3 + 105*a*b*c^2*d^4) + x^3*(a^7*d + 7*a^5*b^3 + 7*a^6*b*c) +
 (d^8*x^24)/8 + x^2*(a^7*c + (7*a^6*b^2)/2) + c*d^7*x^23 + d^5*x^21*(a*d^2
+ 7*c^3 + 7*b*c*d) + (d^6*x^22*(2*b*d + 7*c^2))/2 + a^7*b*x
```

sympy [B] time = 0.42, size = 1771, normalized size = 84.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)
```

```
[Out] a**7*b*x + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) + x
**21*(a*d**7 + 7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 7*b**2*d**6/
2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(7*a*b*d**6 + 21*a*c**2*d**5 +
 21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 42
*a*b*c*d**5 + 35*a*c**3*d**4 + 7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c*
*4*d**3 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 21*a*b**2*d**5 + 105*a*b
*c**2*d**4 + 35*a*c**4*d**3 + 35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**
5*d**2 + c**7*d) + x**16*(21*a**2*b*d**5 + 105*a**2*c**2*d**4/2 + 105*a*b**
2*c*d**4 + 140*a*b*c**3*d**3 + 21*a*c**5*d**2 + 35*b**4*d**4/4 + 70*b**3*c*
*2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(7*a**3*d**5
+ 105*a**2*b*c*d**4 + 70*a**2*c**3*d**3 + 35*a*b**3*d**4 + 210*a*b**2*c**2*
d**3 + 105*a*b*c**4*d**2 + 7*a*c**6*d + 35*b**4*c*d**3 + 70*b**3*c**3*d**2
+ 21*b**2*c**5*d + b*c**7) + x**14*(35*a**3*c*d**4 + 105*a**2*b**2*d**4/2 +
 210*a**2*b*c**2*d**3 + 105*a**2*c**4*d**2/2 + 140*a*b**3*c*d**3 + 210*a*b*
*2*c**3*d**2 + 42*a*b*c**5*d + a*c**7 + 7*b**5*d**3 + 105*b**4*c**2*d**2/2
+ 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(35*a**3*b*d**4 + 70*a**3*c**2*d*
*3 + 210*a**2*b**2*c*d**3 + 210*a**2*b*c**3*d**2 + 21*a**2*c**5*d + 35*a*b*
*4*d**3 + 210*a*b**3*c**2*d**2 + 105*a*b**2*c**4*d + 7*a*b*c**6 + 21*b**5*c
*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(35*a**4*d**4/4 + 140*a**3*b*
c*d**3 + 70*a**3*c**3*d**2 + 70*a**2*b**3*d**3 + 315*a**2*b**2*c**2*d**2 +
105*a**2*b*c**4*d + 7*a**2*c**6/2 + 105*a*b**4*c*d**2 + 140*a*b**3*c**3*d +
 21*a*b**2*c**5 + 7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*
(35*a**4*c*d**3 + 70*a**3*b**2*d**3 + 210*a**3*b*c**2*d**2 + 35*a**3*c**4*d
 + 210*a**2*b**3*c*d**2 + 210*a**2*b**2*c**3*d + 21*a**2*b*c**5 + 21*a*b**5
*d**2 + 105*a*b**4*c**2*d + 35*a*b**3*c**4 + 7*b**6*c*d + 7*b**5*c**3) + x*
*10*(35*a**4*b*d**3 + 105*a**4*c**2*d**2/2 + 210*a**3*b**2*c*d**2 + 140*a**
3*b*c**3*d + 7*a**3*c**5 + 105*a**2*b**4*d**2/2 + 210*a**2*b**3*c**2*d + 10
5*a**2*b**2*c**4/2 + 42*a*b**5*c*d + 35*a*b**4*c**3 + b**7*d + 7*b**6*c**2/
2) + x**9*(7*a**5*d**3 + 105*a**4*b*c*d**2 + 35*a**4*c**3*d + 70*a**3*b**3*
d**2 + 210*a**3*b**2*c**2*d + 35*a**3*b*c**4 + 105*a**2*b**4*c*d + 70*a**2*
b**3*c**3 + 7*a*b**6*d + 21*a*b**5*c**2 + b**7*c) + x**8*(21*a**5*c*d**2 +
105*a**4*b**2*d**2/2 + 105*a**4*b*c**2*d + 35*a**4*c**4/4 + 140*a**3*b**3*c
*d + 70*a**3*b**2*c**3 + 21*a**2*b**5*d + 105*a**2*b**4*c**2/2 + 7*a*b**6*c
 + b**8/8) + x**7*(21*a**5*b*d**2 + 21*a**5*c**2*d + 105*a**4*b**2*c*d + 35
*a**4*b*c**3 + 35*a**3*b**4*d + 70*a**3*b**3*c**2 + 21*a**2*b**5*c + a*b**7
) + x**6*(7*a**6*d**2/2 + 42*a**5*b*c*d + 7*a**5*c**3 + 35*a**4*b**3*d + 10
5*a**4*b**2*c**2/2 + 35*a**3*b**4*c + 7*a**2*b**6/2) + x**5*(7*a**6*c*d + 2
1*a**5*b**2*d + 21*a**5*b*c**2 + 35*a**4*b**3*c + 7*a**3*b**5) + x**4*(7*a*
*6*b*d + 7*a**6*c**2/2 + 21*a**5*b**2*c + 35*a**4*b**4/4) + x**3*(a**7*d +
7*a**6*b*c + 7*a**5*b**3) + x**2*(a**7*c + 7*a**6*b**2/2)
```

3.193 $\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=20

$$\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+b*x)^8
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1588}

$$\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(bx+cx^2+dx^3\right)^8$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.90

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

fricas [B] time = 0.50, size = 496, normalized size = 24.80

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+x^{22}d^7b+7x^{21}d^5c^3+7x^{21}d^6cb+\frac{35}{4}x^{20}d^4c^4+21x^{20}d^5c^2b+\frac{7}{2}x^{20}d^6b^2+7x^{19}d^3c^5+35x^{19}d^4c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2
 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c*b^2 + 7/2*x^18*d^2*c^
6 + 35*x^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d*c^
7 + 21*x^17*d^2*c^5*b + 70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*
c^8 + 7*x^16*d*c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*
x^16*d^4*b^4 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 + 35*x^
15*d^3*c*b^4 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/2*x^14*d^2*c^2*b^
4 + 7*x^14*d^3*b^5 + 7*x^13*c^5*b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5
 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6 + 7*x^11*c^3*b^
5 + 7*x^11*d*c*b^6 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + x^9*c*b^7 + 1/8*x^8*b^
8
```

giac [A] time = 0.31, size = 18, normalized size = 0.90

$$\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

maple [B] time = 0.00, size = 5596, normalized size = 279.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.63, size = 18, normalized size = 0.90

$$\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

mupad [B] time = 2.32, size = 418, normalized size = 20.90

$$x^{14}\left(7\,b^5\,d^3+\frac{105\,b^4\,c^2\,d^2}{2}+35\,b^3\,c^4\,d+\frac{7\,b^2\,c^6}{2}\right)+x^{18}\left(7\,b^3\,d^5+\frac{105\,b^2\,c^2\,d^4}{2}+35\,b\,c^4\,d^3+\frac{7\,c^6\,d^2}{2}\right)+x^{12}\left(\frac{7\,b^6\,d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^1
8*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*(
(35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*
c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^
2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7
*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2
 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d
^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d)
 + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11
*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

sympy [B] time = 0.18, size = 469, normalized size = 23.45

$$\frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\left(7bcd^6+7c^3d^5\right)+x^{20}\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{19}\left(21\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

3.194 $\int x^7 \left(b+cx+dx^2\right)^7 \left(b+2cx+3dx^2\right) dx$

Optimal. Leaf size=19

$$\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

```
[Out] 1/8*x^8*(d*x^2+c*x+b)^8
```

Rubi [A] time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1588}

$$\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(b + c*x + d*x^2)^8)/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^7\left(b+cx+dx^2\right)^7\left(b+2cx+3dx^2\right)\,dx=\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

fricas [B] time = 0.52, size = 496, normalized size = 26.11

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+x^{22}d^7b+7x^{21}d^5c^3+7x^{21}d^6cb+\frac{35}{4}x^{20}d^4c^4+21x^{20}d^5c^2b+\frac{7}{2}x^{20}d^6b^2+7x^{19}d^3c^5+35x^{19}d^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2
 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c*b^2 + 7/2*x^18*d^2*c^
6 + 35*x^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d*c^
7 + 21*x^17*d^2*c^5*b + 70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*
c^8 + 7*x^16*d*c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*
x^16*d^4*b^4 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 + 35*x^
15*d^3*c*b^4 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/2*x^14*d^2*c^2*b^
4 + 7*x^14*d^3*b^5 + 7*x^13*c^5*b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5
 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6 + 7*x^11*c^3*b^
5 + 7*x^11*d*c*b^6 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + x^9*c*b^7 + 1/8*x^8*b^
8
```

giac [A] time = 0.30, size = 18, normalized size = 0.95

$$\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

maple [B] time = 0.00, size = 5596, normalized size = 294.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.60, size = 441, normalized size = 23.21

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{1}{2}\left(7c^2d^6+2bd^7\right)x^{22}+7\left(c^3d^5+bcd^6\right)x^{21}+\frac{7}{4}\left(5c^4d^4+12bc^2d^5+2b^2d^6\right)x^{20}+7\left(c^5d^3+5bc^3d^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

mupad [B] time = 2.27, size = 418, normalized size = 22.00

$$x^{14}\left(7\,b^5\,d^3+\frac{105\,b^4\,c^2\,d^2}{2}+35\,b^3\,c^4\,d+\frac{7\,b^2\,c^6}{2}\right)+x^{18}\left(7\,b^3\,d^5+\frac{105\,b^2\,c^2\,d^4}{2}+35\,b\,c^4\,d^3+\frac{7\,c^6\,d^2}{2}\right)+x^{12}\left(\frac{7\,b^6\,d^2}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x)
```

```
[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^1
8*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*(
(35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*
c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^
2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7
*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2
 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d
^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d)
 + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11
*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

sympy [B] time = 0.17, size = 469, normalized size = 24.68

$$\frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\left(7bcd^6+7c^3d^5\right)+x^{20}\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{19}\left(21b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
```

```
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

3.195 $\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^7\,dx$

Optimal. Leaf size=16

$$\frac{1}{8}\left(a+bx+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+b*x+a)^8
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{1}{8}\left(a+bx+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (a + b*x + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^7\,dx=\frac{1}{8}\left(a+bx+dx^3\right)^8$$

Mathematica [B] time = 0.06, size = 127, normalized size = 7.94

$$\frac{1}{8}x\left(b+dx^2\right)\left(8a^7+28a^6x\left(b+dx^2\right)+56a^5x^2\left(b+dx^2\right)^2+70a^4x^3\left(b+dx^2\right)^3+56a^3x^4\left(b+dx^2\right)^4+28a^2x^5\left(b+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*x^2)^2 + 7
0*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28*a^2*x^5*(b + d*x^2)
^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^7))/8
```

fricas [B] time = 0.53, size = 486, normalized size = 30.38

$$\frac{1}{8}x^{24}d^8+x^{22}d^7b+x^{21}d^7a+\frac{7}{2}x^{20}d^6b^2+7x^{19}d^6ba+7x^{18}d^5b^3+\frac{7}{2}x^{18}d^6a^2+21x^{17}d^5b^2a+\frac{35}{4}x^{16}d^4b^4+21x^{16}d^5ba^2+35x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + x^21*d^7*a + 7/2*x^20*d^6*b^2 + 7*x^19*d^6*b*a
+ 7*x^18*d^5*b^3 + 7/2*x^18*d^6*a^2 + 21*x^17*d^5*b^2*a + 35/4*x^16*d^4*b^4
 + 21*x^16*d^5*b*a^2 + 35*x^15*d^4*b^3*a + 7*x^15*d^5*a^3 + 7*x^14*d^3*b^5
+ 105/2*x^14*d^4*b^2*a^2 + 35*x^13*d^3*b^4*a + 35*x^13*d^4*b*a^3 + 7/2*x^12
*d^2*b^6 + 70*x^12*d^3*b^3*a^2 + 35/4*x^12*d^4*a^4 + 21*x^11*d^2*b^5*a + 70
*x^11*d^3*b^2*a^3 + x^10*d*b^7 + 105/2*x^10*d^2*b^4*a^2 + 35*x^10*d^3*b*a^4
 + 7*x^9*d*b^6*a + 70*x^9*d^2*b^3*a^3 + 7*x^9*d^3*a^5 + 1/8*x^8*b^8 + 21*x^
8*d*b^5*a^2 + 105/2*x^8*d^2*b^2*a^4 + x^7*b^7*a + 35*x^7*d*b^4*a^3 + 21*x^7
*d^2*b*a^5 + 7/2*x^6*b^6*a^2 + 35*x^6*d*b^3*a^4 + 7/2*x^6*d^2*a^6 + 7*x^5*b
^5*a^3 + 21*x^5*d*b^2*a^5 + 35/4*x^4*b^4*a^4 + 7*x^4*d*b*a^6 + 7*x^3*b^3*a^
5 + x^3*d*a^7 + 7/2*x^2*b^2*a^6 + x*b*a^7
```

giac [B] time = 0.41, size = 120, normalized size = 7.50

$$\frac{1}{8}\left(dx^3+bx\right)^8+\left(dx^3+bx\right)^7a+\frac{7}{2}\left(dx^3+bx\right)^6a^2+7\left(dx^3+bx\right)^5a^3+\frac{35}{4}\left(dx^3+bx\right)^4a^4+7\left(dx^3+bx\right)^3a^5+\frac{7}{2}\left(dx^3+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8 + (d*x^3 + b*x)^7*a + 7/2*(d*x^3 + b*x)^6*a^2 + 7*(d*x^
3 + b*x)^5*a^3 + 35/4*(d*x^3 + b*x)^4*a^4 + 7*(d*x^3 + b*x)^3*a^5 + 7/2*(d*
x^3 + b*x)^2*a^6 + (d*x^3 + b*x)*a^7
```

maple [B] time = 0.00, size = 2185, normalized size = 136.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+d^7*a*x^21+7/2*b^2*d^6*x^20+7*b*a*d^6*x^19+1/18*(21
*b^3*d^5+3*d*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*
d^4)))*x^18+21*b^2*a*d^5*x^17+1/16*(b*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b
^3)*d^3+18*b^3*d^3+9*a^2*d^4))+3*d*(30*a^2*d^4*b+b*(2*(3*a^2*d+b^3)*d^3+18*
b^3*d^3+9*a^2*d^4)+d*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4)))*x^16+
```

```
1/15*(105*b^3*a*d^4+3*d*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^
3*a*d^3+d*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)))*x^15+1/14*(b*(30
*a^2*d^4*b+b*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+d*(42*b*a^2*d^3+6*(
3*a^2*d+b^3)*b*d^2+9*d^2*b^4))+3*d*(60*a^2*b^2*d^3+b*(42*b*a^2*d^3+6*(3*a^2
*d+b^3)*b*d^2+9*d^2*b^4)+d*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)))*x^14+1/
13*(b*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^3*a*d^3+d*(2*a^3*d
^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2))+3*d*(a*(42*b*a^2*d^3+6*(3*a^2*d+b^3
)*b*d^2+9*d^2*b^4)+b*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+d*(24*a
^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)))*x^13+1/12*(b*(60*a^2*b^2*d^3+
b*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4)+d*(72*b^2*a^2*d^2+6*(3*a^2
*d+b^3)*d*b^2))+3*d*(a*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+b*(72
*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)
^2)))*x^12+1/11*(b*(a*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4)+b*(2*a
^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+d*(24*a^3*b*d^2+18*b^4*a*d+12*(3
*a^2*d+b^3)*d*a*b))+3*d*(a*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+b*(24*a^3
*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)+d*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+
b^3))))*x^11+1/10*(b*(a*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+b*(7
2*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3
)^2))+3*d*(a*(24*a^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)+b*(6*a^4*d^2+
54*b^3*a^2*d+(3*a^2*d+b^3)^2)+d*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2
)))*x^10+1/9*(b*(a*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+b*(24*a^3*b*d^2+1
8*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)+d*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3)))+3
*d*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2)+b*(42*a^3*d*b^2+6*b^2*a*(3*a
^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)))*x^9+1/8*(b*(a*(24*a^3*b*d^2
+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)+b*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)
^2)+d*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2))+3*d*(a*(42*a^3*d*b^2+6*
b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^
2*a^4))*x^8+1/7*(b*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2)+b*(42*a^3*d*
b^2+6*b^2*a*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3))+3*d*(a*(12*a
^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+b*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+
6*d*a^5*b))*x^7+1/6*(b*(a*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*
b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^2*a^4)+3*d*(a*(2*a^3*(3*a^2*d+b^3
)+18*b^3*a^3)+15*b^3*a^4+d*a^6))*x^6+1/5*(b*(a*(12*a^4*d*b+6*b*a^2*(3*a^2*d
+b^3)+9*b^4*a^2)+b*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+6*d*a^5*b)+63*d*b^2*a^5
)*x^5+1/4*(b*(a*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+15*b^3*a^4+d*a^6)+21*d*a^6
*b)*x^4+1/3*(3*a^7*d+21*a^5*b^3)*x^3+7/2*b^2*a^6*x^2+b*a^7*x
```

maxima [A] time = 0.55, size = 14, normalized size = 0.88

$$\frac{1}{8}\left(dx^3+bx+a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x + a)^8
```

mupad [B] time = 2.63, size = 438, normalized size = 27.38

$$x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^2\,b^3\,d^3+\frac{7\,b^6\,d^2}{2}\right)+x^4\left(7\,d\,a^6\,b+\frac{35\,a^4\,b^4}{4}\right)+x^{18}\left(\frac{7\,a^2\,d^6}{2}+7\,b^3\,d^5\right)+x^6\left(\frac{7\,a^6\,d^2}{2}+35\,a^4\,b^3\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^7,x)
```

```
[Out] x^12*((35*a^4*d^4)/4 + (7*b^6*d^2)/2 + 70*a^2*b^3*d^3) + x^4*((35*a^4*b^4)/
4 + 7*a^6*b*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5) + x^6*((7*a^2*b^6)/2 + (7
*a^6*d^2)/2 + 35*a^4*b^3*d) + x^8*(b^8/8 + 21*a^2*b^5*d + (105*a^4*b^2*d^2)
/2) + (d^8*x^24)/8 + x^3*(a^7*d + 7*a^5*b^3) + a*d^7*x^21 + b*d^7*x^22 + (7
*a^6*b^2*x^2)/2 + (7*b^2*d^6*x^20)/2 + a^7*b*x + 21*a*b^2*d^5*x^17 + a*b*x^
7*(b^6 + 21*a^4*d^2 + 35*a^2*b^3*d) + 7*a*d*x^9*(b^6 + a^4*d^2 + 10*a^2*b^3
*d) + 7*a^3*b^2*x^5*(3*a^2*d + b^3) + 7*a*d^4*x^15*(a^2*d + 5*b^3) + (7*b*d
^4*x^16*(12*a^2*d + 5*b^3))/4 + (b*d*x^10*(2*b^6 + 70*a^4*d^2 + 105*a^2*b^3
*d))/2 + 7*a*b*d^6*x^19 + (7*b^2*d^3*x^14*(15*a^2*d + 2*b^3))/2 + 7*a*b^2*d
^2*x^11*(10*a^2*d + 3*b^3) + 35*a*b*d^3*x^13*(a^2*d + b^3)
```

sympy [B] time = 0.17, size = 483, normalized size = 30.19

$$a^7bx+\frac{7a^6b^2x^2}{2}+21ab^2d^5x^{17}+7abd^6x^{19}+ad^7x^{21}+\frac{7b^2d^6x^{20}}{2}+bd^7x^{22}+\frac{d^8x^{24}}{8}+x^{18}\left(\frac{7a^2d^6}{2}+7b^3d^5\right)+x^{16}\left(21a^2bd^5+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)
```

```
[Out] a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6*x**19 + a
*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8 + x**18*(7*
a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b*d**5 + 35*b**4*d**4/4) + x**1
5*(7*a**3*d**5 + 35*a*b**3*d**4) + x**14*(105*a**2*b**2*d**4/2 + 7*b**5*d**
3) + x**13*(35*a**3*b*d**4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 + 70*a
**2*b**3*d**3 + 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5*d**2)
 + x**10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*a**5*d*
*3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d**2/2 + 21*a**2
*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3*b**4*d + a*b**7) + x**6*
(7*a**6*d**2/2 + 35*a**4*b**3*d + 7*a**2*b**6/2) + x**5*(21*a**5*b**2*d + 7
*a**3*b**5) + x**4*(7*a**6*b*d + 35*a**4*b**4/4) + x**3*(a**7*d + 7*a**5*b*
*3)
```

3.196 $\int \left(b+3dx^2\right)\left(bx+dx^3\right)^7\,dx$

Optimal. Leaf size=15

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+b*x)^8
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1588}

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] (b*x + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^7\,dx=\frac{1}{8}\left(bx+dx^3\right)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 6.53

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

fricas [B] time = 0.65, size = 88, normalized size = 5.87

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d
^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8
```

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{8}\left(dx^3 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

maple [B] time = 0.00, size = 89, normalized size = 5.93

$$\frac{1}{8}d^8x^{24} + b\,d^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7d\,x^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*d^4*x^16+7
*b^5*d^3*x^14+7/2*b^6*d^2*x^12+d*b^7*x^10+1/8*b^8*x^8
```

maxima [A] time = 0.51, size = 13, normalized size = 0.87

$$\frac{1}{8}\left(dx^3 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

mupad [B] time = 0.05, size = 88, normalized size = 5.87

$$\frac{b^8\,x^8}{8}+b^7\,d\,x^{10}+\frac{7\,b^6\,d^2\,x^{12}}{2}+7\,b^5\,d^3\,x^{14}+\frac{35\,b^4\,d^4\,x^{16}}{4}+7\,b^3\,d^5\,x^{18}+\frac{7\,b^2\,d^6\,x^{20}}{2}+b\,d^7\,x^{22}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^7*(b + 3*d*x^2),x)
```

```
[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 +
 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2
```

sympy [B] time = 0.10, size = 97, normalized size = 6.47

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7b^6d^2x^{12}}{2}+7b^5d^3x^{14}+\frac{35b^4d^4x^{16}}{4}+7b^3d^5x^{18}+\frac{7b^2d^6x^{20}}{2}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

3.197 $\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx$

Optimal. Leaf size=16

$$\frac{1}{8}x^8 \left(b+dx^2\right)^8$$

```
[Out] 1/8*x^8*(d*x^2+b)^8
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{8}x^8 \left(b+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (x^8*(b + d*x^2)^8)/8
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(b+dx)^7(b+3dx) \, dx, x, x^2\right) \\ &= \frac{1}{8}x^8 \left(b+dx^2\right)^8 \end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 6.12

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

fricas [B] time = 0.77, size = 88, normalized size = 5.50

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d
^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8
```

giac [A] time = 0.27, size = 13, normalized size = 0.81

$$\frac{1}{8}\left(dx^3 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

maple [B] time = 0.00, size = 89, normalized size = 5.56

$$\frac{1}{8}d^8x^{24} + b\,d^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7d\,x^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+b)^7*(3*d*x^2+b),x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*d^4*x^16+7
*b^5*d^3*x^14+7/2*b^6*d^2*x^12+b^7*d*x^10+1/8*b^8*x^8
```

maxima [B] time = 0.64, size = 88, normalized size = 5.50

$$\frac{1}{8}\,d^8x^{24} + bd^7x^{22} + \frac{7}{2}\,b^2d^6x^{20} + 7\,b^3d^5x^{18} + \frac{35}{4}\,b^4d^4x^{16} + 7\,b^5d^3x^{14} + \frac{7}{2}\,b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}\,b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

mupad [B] time = 0.04, size = 88, normalized size = 5.50

$$\frac{b^8\,x^8}{8}+b^7\,d\,x^{10}+\frac{7\,b^6\,d^2\,x^{12}}{2}+7\,b^5\,d^3\,x^{14}+\frac{35\,b^4\,d^4\,x^{16}}{4}+7\,b^3\,d^5\,x^{18}+\frac{7\,b^2\,d^6\,x^{20}}{2}+b\,d^7\,x^{22}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b + d*x^2)^7*(b + 3*d*x^2),x)
```

```
[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 +
 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2
```

sympy [B] time = 0.09, size = 97, normalized size = 6.06

$$\frac{b^{8}x^{8}}{8} + b^{7}dx^{10} + \frac{7b^{6}d^{2}x^{12}}{2} + 7b^{5}d^{3}x^{14} + \frac{35b^{4}d^{4}x^{16}}{4} + 7b^{3}d^{5}x^{18} + \frac{7b^{2}d^{6}x^{20}}{2} + bd^{7}x^{22} + \frac{d^{8}x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

3.198 $\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+a)^8
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1588}

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.05, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*
a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*
a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

fricas [B] time = 0.73, size = 488, normalized size = 27.11

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+x^{21}d^7a+\frac{35}{4}x^{20}d^4c^4+7x^{20}d^6ca+7x^{19}d^3c^5+21x^{19}d^5c^2a+\frac{7}{2}x^{18}d^2c^6+35x^{18}d^4c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x^21*d^7*a
+ 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 21*x^19*d^5*c^2*a +
 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 35*
x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105
/2*x^16*d^4*c^2*a^2 + 7*x^15*d*c^6*a + 70*x^15*d^3*c^3*a^2 + 7*x^15*d^5*a^3
 + x^14*c^7*a + 105/2*x^14*d^2*c^4*a^2 + 35*x^14*d^4*c*a^3 + 21*x^13*d*c^5*
a^2 + 70*x^13*d^3*c^2*a^3 + 7/2*x^12*c^6*a^2 + 70*x^12*d^2*c^3*a^3 + 35/4*x
^12*d^4*a^4 + 35*x^11*d*c^4*a^3 + 35*x^11*d^3*c*a^4 + 7*x^10*c^5*a^3 + 105/
2*x^10*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*a^5 + 35/4*x^8*c^4*a^4 +
21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x
^5*d*c*a^6 + 7/2*x^4*c^2*a^6 + x^3*d*a^7 + x^2*c*a^7
```

giac [B] time = 0.43, size = 136, normalized size = 7.56

$$\frac{1}{8}\left(dx^3+cx^2\right)^8+\left(dx^3+cx^2\right)^7a+\frac{7}{2}\left(dx^3+cx^2\right)^6a^2+7\left(dx^3+cx^2\right)^5a^3+\frac{35}{4}\left(dx^3+cx^2\right)^4a^4+7\left(dx^3+cx^2\right)^3a^5+\frac{7}{2}\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8 + (d*x^3 + c*x^2)^7*a + 7/2*(d*x^3 + c*x^2)^6*a^2 + 7
*(d*x^3 + c*x^2)^5*a^3 + 35/4*(d*x^3 + c*x^2)^4*a^4 + 7*(d*x^3 + c*x^2)^3*a
^5 + 7/2*(d*x^3 + c*x^2)^2*a^6 + (d*x^3 + c*x^2)*a^7
```

maple [B] time = 0.00, size = 2205, normalized size = 122.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*d*(a*d^6+15*c^3
*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)))*x^21+1/20*(2*c*(a*d^6+15*c^3*d^4+
d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+3*d*(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+1
8*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)))*x^20+1/19*(2*c*
(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)
```

```
*c*d^2+9*c^4*d^2))+3*d*(15*a*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*
c^4*d^2)+d*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)))*x^19+1/18*(2*c*(15*a*c^2*
d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+d*(42*a*c^2*d^3+6*(3*a*d
^2+c^3)*c^2*d))+3*d*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(
3*a*d^2+c^3)*c^2*d)+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)))*x^18+1/17*
(2*c*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^
2*d)+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2))+3*d*(a*(12*a*c*d^4+6*(3*a*
d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+d*(24*
c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))))*x^17+1/16*(2*c*(a*(12*a*c*d^
4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^
2)+d*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3)))+3*d*(a*(42*a*c^2*d^3
+6*(3*a*d^2+c^3)*c^2*d)+c*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+
d*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))))*x^16+1/15*(2*c*(a*(42*a*c^2*d^3+
6*(3*a*d^2+c^3)*c^2*d)+c*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+d
*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+3*d*(a*(6*a^2*d^4+54*a*c^3*d^2+(3*
a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*c^3*
a^2*d+6*d*a^2*(3*a*d^2+c^3))))*x^15+1/14*(2*c*(a*(6*a^2*d^4+54*a*c^3*d^2+(3
*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*c^3
*a^2*d+6*d*a^2*(3*a*d^2+c^3)))+3*d*(a*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*
a*d^2+c^3))+c*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3))+d*(42*a^3*c*d^
2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4)))*x^14+1/13*(2*c*(a*(24*c*a^2*d^3+18*a*c
^4*d+12*a*c*d*(3*a*d^2+c^3))+c*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3
))+d*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4))+3*d*(a*(72*a^2*c^2*d^2
+6*a*c^2*(3*a*d^2+c^3))+c*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4)+60
*d^2*a^3*c^2))*x^13+1/12*(2*c*(a*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+c*(
42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4)+60*d^2*a^3*c^2)+3*d*(a*(2*a^3
*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3))+60*c^3*a^3*d+d*(2*a^3*(3*a*d^2+c^3
)+18*c^3*a^3+9*a^4*d^2)))*x^12+1/11*(2*c*(a*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2
*(3*a*d^2+c^3))+60*c^3*a^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*c^3*a^3+9*a^4*d^2))+
3*d*(a*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4)+c*(2*a^3*(3*a*d^2+c^3
)+18*c^3*a^3+9*a^4*d^2)+30*d^2*a^4*c))*x^11+1/10*(2*c*(a*(42*a^3*c*d^2+6*c*
a^2*(3*a*d^2+c^3)+9*a^2*c^4)+c*(2*a^3*(3*a*d^2+c^3)+18*c^3*a^3+9*a^4*d^2)+3
0*d^2*a^4*c)+315*d^2*a^4*c^2)*x^10+1/9*(210*c^3*a^4*d+3*d*(a*(2*a^3*(3*a*d^
2+c^3)+18*c^3*a^3+9*a^4*d^2)+15*c^3*a^4+6*d^2*a^5))*x^9+1/8*(2*c*(a*(2*a^3*
(3*a*d^2+c^3)+18*c^3*a^3+9*a^4*d^2)+15*c^3*a^4+6*d^2*a^5)+126*d^2*a^5*c)*x^
8+21*c^2*a^5*d*x^7+1/6*(21*a^6*d^2+42*a^5*c^3)*x^6+7*c*a^6*d*x^5+7/2*c^2*a^
6*x^4+d*a^7*x^3+c*a^7*x^2
```

maxima [A] time = 0.62, size = 16, normalized size = 0.89

$$\frac{1}{8}\left(dx^3 + cx^2 + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + a)^8
```

mupad [B] time = 2.63, size = 440, normalized size = 24.44

$$x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^3\,c^3\,d^2+\frac{7\,a^2\,c^6}{2}\right)+x^6\left(\frac{7\,a^6\,d^2}{2}+7\,a^5\,c^3\right)+x^{20}\left(\frac{35\,c^4\,d^4}{4}+7\,a\,c\,d^6\right)+x^{16}\left(\frac{105\,a^2\,c^2\,d^4}{2}+21\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (
7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*
d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3
*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c
*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a
*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*
c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3)
 + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c
^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*
c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

sympy [B] time = 0.17, size = 484, normalized size = 26.89

$$a^7cx^2+a^7dx^3+\frac{7a^6c^2x^4}{2}+7a^6cdx^5+21a^5c^2dx^7+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{21}\left(ad^7+7c^3d^5\right)+x^{20}\left(7acd^6+\frac{35c^4d^4}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

3.199 $\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^7 \, dx$

Optimal. Leaf size=17

$$\frac{1}{8}\left(cx^2+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2)^8
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1588}

$$\frac{1}{8}\left(cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^7 \, dx=\frac{1}{8}\left(cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 5.76

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

fricas [B] time = 0.57, size = 88, normalized size = 5.18

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [A] time = 0.37, size = 15, normalized size = 0.88

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

maple [B] time = 0.00, size = 89, normalized size = 5.24

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [A] time = 0.66, size = 15, normalized size = 0.88

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

mupad [B] time = 2.07, size = 88, normalized size = 5.18

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.10, size = 97, normalized size = 5.71

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.200 $\int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right) dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A] time = 0.23, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 845}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right) dx &= \int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) dx\\ &= \frac{1}{8}x^{16}(c+dx)^8\end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

fricas [B] time = 0.73, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [A] time = 0.23, size = 15, normalized size = 1.07

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.52, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.05, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.10, size = 97, normalized size = 6.93

$$\frac{c^{8}x^{16}}{8} + c^{7}dx^{17} + \frac{7c^{6}d^{2}x^{18}}{2} + 7c^{5}d^{3}x^{19} + \frac{35c^{4}d^{4}x^{20}}{4} + 7c^{3}d^{5}x^{21} + \frac{7c^{2}d^{6}x^{22}}{2} + cd^{7}x^{23} + \frac{d^{8}x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.201 $\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {845}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

fricas [B] time = 0.62, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [A] time = 0.26, size = 15, normalized size = 1.07

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.66, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(2*c*x + 3*d*x^2)*(c + d*x)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.202 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+a)^8
```

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1588}

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.01, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*
a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*
a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

fricas [B] time = 1.06, size = 488, normalized size = 27.11

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+x^{21}d^7a+\frac{35}{4}x^{20}d^4c^4+7x^{20}d^6ca+7x^{19}d^3c^5+21x^{19}d^5c^2a+\frac{7}{2}x^{18}d^2c^6+35x^{18}d^4c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x^21*d^7*a
+ 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 21*x^19*d^5*c^2*a +
 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 35*
x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105
/2*x^16*d^4*c^2*a^2 + 7*x^15*d*c^6*a + 70*x^15*d^3*c^3*a^2 + 7*x^15*d^5*a^3
 + x^14*c^7*a + 105/2*x^14*d^2*c^4*a^2 + 35*x^14*d^4*c*a^3 + 21*x^13*d*c^5*
a^2 + 70*x^13*d^3*c^2*a^3 + 7/2*x^12*c^6*a^2 + 70*x^12*d^2*c^3*a^3 + 35/4*x
^12*d^4*a^4 + 35*x^11*d*c^4*a^3 + 35*x^11*d^3*c*a^4 + 7*x^10*c^5*a^3 + 105/
2*x^10*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*a^5 + 35/4*x^8*c^4*a^4 +
21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x
^5*d*c*a^6 + 7/2*x^4*c^2*a^6 + x^3*d*a^7 + x^2*c*a^7
```

giac [B] time = 0.32, size = 488, normalized size = 27.11

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+ad^7x^{21}+\frac{35}{4}c^4d^4x^{20}+7acd^6x^{20}+7c^5d^3x^{19}+21ac^2d^5x^{19}+\frac{7}{2}c^6d^2x^{18}+35ac^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a*d^7*x^21
+ 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 21*a*c^2*d^5*x^19 +
 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 + 35*
a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105
/2*a^2*c^2*d^4*x^16 + 7*a*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15
 + a*c^7*x^14 + 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x
^13 + 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 35/4*a
^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*c^5*x^10 + 105/
2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c^4*x^8 +
21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a
^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2
```

maple [B] time = 0.00, size = 2205, normalized size = 122.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*(a*d^6+15*c^3*d
^4+(18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*d)*d)*x^21+1/20*(2*(a*d^6+15*c^3*d^4+(1
8*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*d)*c+3*(6*a*c*d^5+(18*c^3*d^3+2*(3*a*d^2+c^3
)*d^3)*c+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*d)*d)*x^20+1/19*(2*(6
*a*c*d^5+(18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*c+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^
2+c^3)*c*d^2)*d)*c+3*(15*a*c^2*d^4+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*
d^2)*c+(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*d)*d)*x^19+1/18*(2*(15*a*c^2*d^
4+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*c+(42*a*c^2*d^3+6*(3*a*d^2+c
^3)*c^2*d)*d)*c+3*((18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*a+(42*a*c^2*d^3+6*(3*a*
d^2+c^3)*c^2*d)*c+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*d)*d)*x^18+1/17*
(2*((18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*a+(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)
*c+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*d)*c+3*((12*a*c*d^4+9*c^4*d^2+6
*(3*a*d^2+c^3)*c*d^2)*a+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*c+(24*a^2*
c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*d)*d)*x^17+1/16*(2*((12*a*c*d^4+9*
c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*a+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*c
+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*d)*c+3*((42*a*c^2*d^3+6*(
3*a*d^2+c^3)*c^2*d)*a+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*c+(7
2*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*d)*d)*x^16+21*a^5*c^2*d*x^7+1/15*(2*((
42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*a+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+
c^3)*a*c*d)*c+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*d)*c+3*((6*a^2*d^4+54*
a*c^3*d^2+(3*a*d^2+c^3)^2)*a+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*c+(2*a^
3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*d)*d)*x^15+1/14*(2*((6*a^2*d^4+54
*a*c^3*d^2+(3*a*d^2+c^3)^2)*a+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*c+(2*a
^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*d)*c+3*((24*a^2*c*d^3+18*a*c^4*d
+12*(3*a*d^2+c^3)*a*c*d)*a+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*c
+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*d)*d)*x^14+7*a^6*c*d*x^5+1/
13*(2*((24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*a+(2*a^3*d^3+54*a^2
*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*c+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2
*c)*d)*c+3*(60*a^3*c^2*d^2+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*a+(42*a^3
*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*c)*d)*x^13+7/2*a^6*c^2*x^4+1/12*(2*
(60*a^3*c^2*d^2+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*a+(42*a^3*c*d^2+9*a^
2*c^4+6*(3*a*d^2+c^3)*a^2*c)*c)*c+3*(60*a^3*c^3*d+(2*a^3*d^3+54*a^2*c^3*d+6
*(3*a*d^2+c^3)*a^2*d)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*d)*d)*x^
12+a^7*d*x^3+1/11*(2*(60*a^3*c^3*d+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*
a^2*d)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*d)*c+3*(30*a^4*c*d^2+(4
2*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a
*d^2+c^3)*a^3)*c)*d)*x^11+a^7*c*x^2+1/10*(315*a^4*c^2*d^2+2*(30*a^4*c*d^2+(
42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*
a*d^2+c^3)*a^3)*c)*c)*x^10+1/9*(210*a^4*c^3*d+3*(6*a^5*d^2+15*a^4*c^3+(9*a^
4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*a)*d)*x^9+1/8*(126*a^5*c*d^2+2*(6*a^5
*d^2+15*a^4*c^3+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*a)*c)*x^8+1/6*(2
1*a^6*d^2+42*a^5*c^3)*x^6
```

maxima [B] time = 0.59, size = 458, normalized size = 25.44

$$\frac{1}{8}\,d^{8}x^{24}+cd^{7}x^{23}+\frac{7}{2}\,c^{2}d^{6}x^{22}+\left(7\,c^{3}d^{5}+ad^{7}\right)x^{21}+\frac{7}{4}\left(5\,c^{4}d^{4}+4\,acd^{6}\right)x^{20}+7\left(c^{5}d^{3}+3\,ac^{2}d^{5}\right)x^{19}+\frac{7}{2}\left(c^{6}d^{2}+10\,ac^{3}d^{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7
/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6
*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)
*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*
a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4
*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c
*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^
4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c
^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 +
 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

mupad [B] time = 0.57, size = 440, normalized size = 24.44

$$x^{12}\left(\frac{35\,a^{4}\,d^{4}}{4}+70\,a^{3}\,c^{3}\,d^{2}+\frac{7\,a^{2}\,c^{6}}{2}\right)+x^{6}\left(\frac{7\,a^{6}\,d^{2}}{2}+7\,a^{5}\,c^{3}\right)+x^{20}\left(\frac{35\,c^{4}\,d^{4}}{4}+7\,a\,c\,d^{6}\right)+x^{16}\left(\frac{105\,a^{2}\,c^{2}\,d^{4}}{2}+21\,a\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (
7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*
d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3
*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c
*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a
*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*
c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3)
 + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c
^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*
c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

sympy [B] time = 0.17, size = 484, normalized size = 26.89

$$a^{7}cx^{2}+a^{7}dx^{3}+\frac{7a^{6}c^{2}x^{4}}{2}+7a^{6}cdx^{5}+21a^{5}c^{2}dx^{7}+\frac{7c^{2}d^{6}x^{22}}{2}+cd^{7}x^{23}+\frac{d^{8}x^{24}}{8}+x^{21}\left(ad^{7}+7c^{3}d^{5}\right)+x^{20}\left(7acd^{6}+\frac{35c^{4}d^{4}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

3.203 $\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

[Out] 1/8*x^16*(d*x+c)^8

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 74}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]

[Out] (x^16*(c + d*x)^8)/8

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx &= \int x^{15}(c+dx)^7(2c+3dx)\,dx\\ &= \frac{1}{8}x^{16}(c+dx)^8\end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

fricas [B] time = 0.71, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [B] time = 0.31, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.60, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^{8}x^{16}}{8} + c^{7}dx^{17} + \frac{7c^{6}d^{2}x^{18}}{2} + 7c^{5}d^{3}x^{19} + \frac{35c^{4}d^{4}x^{20}}{4} + 7c^{3}d^{5}x^{21} + \frac{7c^{2}d^{6}x^{22}}{2} + cd^{7}x^{23} + \frac{d^{8}x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.204 $\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

```
[Out] 1/8*x^8*(d*x^2+c*x)^8
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {763}

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rule 763

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 5.44

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

fricas [B] time = 0.56, size = 88, normalized size = 4.89

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [B] time = 0.29, size = 88, normalized size = 4.89

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

maple [B] time = 0.00, size = 89, normalized size = 4.94

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.57, size = 88, normalized size = 4.89

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.04, size = 88, normalized size = 4.89

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x + d*x^2)^7*(2*c + 3*d*x),x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.10, size = 97, normalized size = 5.39

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.205 $\int x^{15}(c+dx)^7(2c+3dx)\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {74}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

fricas [B] time = 0.79, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^{8} + x^{23}d^{7}c + \frac{7}{2}x^{22}d^{6}c^{2} + 7x^{21}d^{5}c^{3} + \frac{35}{4}x^{20}d^{4}c^{4} + 7x^{19}d^{3}c^{5} + \frac{7}{2}x^{18}d^{2}c^{6} + x^{17}dc^{7} + \frac{1}{8}x^{16}c^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [B] time = 0.24, size = 88, normalized size = 6.29

$$\frac{1}{8}d^{8}x^{24} + cd^{7}x^{23} + \frac{7}{2}c^{2}d^{6}x^{22} + 7c^{3}d^{5}x^{21} + \frac{35}{4}c^{4}d^{4}x^{20} + 7c^{5}d^{3}x^{19} + \frac{7}{2}c^{6}d^{2}x^{18} + c^{7}dx^{17} + \frac{1}{8}c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^{8}x^{24} + c\,d^{7}x^{23} + \frac{7}{2}c^{2}d^{6}x^{22} + 7c^{3}d^{5}x^{21} + \frac{35}{4}c^{4}d^{4}x^{20} + 7c^{5}d^{3}x^{19} + \frac{7}{2}c^{6}d^{2}x^{18} + c^{7}d\,x^{17} + \frac{1}{8}c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(d*x+c)^7*(3*d*x+2*c),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.67, size = 88, normalized size = 6.29

$$\frac{1}{8}d^{8}x^{24} + cd^{7}x^{23} + \frac{7}{2}c^{2}d^{6}x^{22} + 7c^{3}d^{5}x^{21} + \frac{35}{4}c^{4}d^{4}x^{20} + 7c^{5}d^{3}x^{19} + \frac{7}{2}c^{6}d^{2}x^{18} + c^{7}dx^{17} + \frac{1}{8}c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(2*c + 3*d*x)*(c + d*x)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.206 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx$

Optimal. Leaf size=28

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2+1/160*x^5*(b*x+2*a)^5
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1591}

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx &= \text{Subst}\left(\int \left(1+x^4\right)dx, x, ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{1}{160}x^5(2a+bx)^5\end{aligned}$$

Mathematica [B] time = 0.01, size = 80, normalized size = 2.86

$$\frac{a^5x^5}{5}+\frac{1}{2}a^4bx^6+\frac{1}{2}a^3b^2x^7+\frac{1}{4}a^2b^3x^8+\frac{1}{16}ab^4x^9+ax+\frac{b^5x^{10}}{160}+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3*
x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160
```

fricas [B] time = 0.71, size = 66, normalized size = 2.36

$$\frac{1}{160}x^{10}b^5 + \frac{1}{16}x^9b^4a + \frac{1}{4}x^8b^3a^2 + \frac{1}{2}x^7b^2a^3 + \frac{1}{2}x^6ba^4 + \frac{1}{5}x^5a^5 + \frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/4*x^8*b^3*a^2 + 1/2*x^7*b^2*a^3 + 1/2*x
^6*b*a^4 + 1/5*x^5*a^5 + 1/2*x^2*b + x*a
```

giac [A] time = 0.37, size = 24, normalized size = 0.86

$$\frac{1}{160}\left(bx^2 + 2\,ax\right)^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/2*b*x^2 + a*x
```

maple [B] time = 0.00, size = 67, normalized size = 2.39

$$\frac{1}{160}b^5x^{10} + \frac{1}{16}a\,b^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4b\,x^6 + \frac{1}{5}a^5x^5 + \frac{1}{2}b\,x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x)
```

```
[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/4*a^2*b^3*x^8+1/2*a^3*b^2*x^7+1/2*a^4*b*x^6
+1/5*a^5*x^5+1/2*b*x^2+a*x
```

maxima [B] time = 0.61, size = 66, normalized size = 2.36

$$\frac{1}{160}\,b^5x^{10} + \frac{1}{16}\,ab^4x^9 + \frac{1}{4}\,a^2b^3x^8 + \frac{1}{2}\,a^3b^2x^7 + \frac{1}{2}\,a^4bx^6 + \frac{1}{5}\,a^5x^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a
^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

mupad [B] time = 0.05, size = 66, normalized size = 2.36

$$\frac{a^5\,x^5}{5}+\frac{a^4\,b\,x^6}{2}+\frac{a^3\,b^2\,x^7}{2}+\frac{a^2\,b^3\,x^8}{4}+\frac{a\,b^4\,x^9}{16}+a\,x+\frac{b^5\,x^{10}}{160}+\frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (b^5*x^10)/160 + (a^4*b*x^6)/2 + (a*b^4*x^9
)/16 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4
```

sympy [B] time = 0.09, size = 70, normalized size = 2.50

$$\frac{a^{5}x^{5}}{5}+\frac{a^{4}bx^{6}}{2}+\frac{a^{3}b^{2}x^{7}}{2}+\frac{a^{2}b^{3}x^{8}}{4}+\frac{ab^{4}x^{9}}{16}+ax+\frac{b^{5}x^{10}}{160}+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)
```

```
[Out] a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4*
x**9/16 + a*x + b**5*x**10/160 + b*x**2/2
```

3.207 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx$

Optimal. Leaf size=31

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2+1/5*(c+a*x+1/2*b*x^2)^5
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1591}

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx &= \text{Subst}\left(\int \left(1+x^4\right)dx, x, c+ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{1}{5}\left(c+ax+\frac{bx^2}{2}\right)^5\end{aligned}$$

Mathematica [B] time = 0.04, size = 108, normalized size = 3.48

$$\frac{1}{160}x(2a+bx)\left(16a^4x^4+32a^3bx^5+24a^2b^2x^6+8ab^3x^7+80c^3x(2a+bx)+40c^2x^2(2a+bx)^2+10cx^3(2a+bx)^3+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] (x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 +
8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 + 1
0*c*x^3*(2*a + b*x)^3))/160
```

fricas [B] time = 0.55, size = 208, normalized size = 6.71

$$\frac{1}{160}x^{10}b^5+\frac{1}{16}x^9b^4a+\frac{1}{16}x^8cb^4+\frac{1}{4}x^8b^3a^2+\frac{1}{2}x^7cb^3a+\frac{1}{2}x^7b^2a^3+\frac{1}{4}x^6c^2b^3+\frac{3}{2}x^6cb^2a^2+\frac{1}{2}x^6ba^4+\frac{3}{2}x^5c^2b^2a+2x^5cba^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/16*x^8*c*b^4 + 1/4*x^8*b^3*a^2 + 1/2*x^
7*c*b^3*a + 1/2*x^7*b^2*a^3 + 1/4*x^6*c^2*b^3 + 3/2*x^6*c*b^2*a^2 + 1/2*x^6
*b*a^4 + 3/2*x^5*c^2*b^2*a + 2*x^5*c*b*a^3 + 1/5*x^5*a^5 + 1/2*x^4*c^3*b^2
+ 3*x^4*c^2*b*a^2 + x^4*c*a^4 + 2*x^3*c^3*b*a + 2*x^3*c^2*a^3 + 1/2*x^2*c^4
*b + 2*x^2*c^3*a^2 + x*c^4*a + 1/2*x^2*b + x*a
```

giac [B] time = 0.37, size = 88, normalized size = 2.84

$$\frac{1}{160}\left(bx^2+2\,ax\right)^5+\frac{1}{16}\left(bx^2+2\,ax\right)^4c+\frac{1}{4}\left(bx^2+2\,ax\right)^3c^2+\frac{1}{2}\left(bx^2+2\,ax\right)^2c^3+\frac{1}{2}\left(bx^2+2\,ax\right)c^4+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/16*(b*x^2 + 2*a*x)^4*c + 1/4*(b*x^2 + 2*a*x)^3*
c^2 + 1/2*(b*x^2 + 2*a*x)^2*c^3 + 1/2*(b*x^2 + 2*a*x)*c^4 + 1/2*b*x^2 + a*x
```

maple [B] time = 0.00, size = 325, normalized size = 10.48

$$\frac{b^5x^{10}}{160}+\frac{a\,b^4x^9}{16}+\frac{\left(\frac{a^2b^3}{2}+\left(a^2b^2+\frac{\left(a^2+bc\right)b^2}{2}\right)b\right)x^8}{8}+\frac{\left(\left(a^2b^2+\frac{\left(a^2+bc\right)b^2}{2}\right)a+\left(a\,b^2c+2\left(a^2+bc\right)ab\right)b\right)x^7}{7}+\frac{\left(\left(a\,b^2c+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x)
```

```
[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/8*(1/2*a^2*b^3+b*(1/2*(a^2+b*c)*b^2+a^2*b^2
))*x^8+1/7*(a*(1/2*(a^2+b*c)*b^2+a^2*b^2)+b*(a*c*b^2+2*(a^2+b*c)*a*b))*x^7+
1/6*(a*(a*c*b^2+2*(a^2+b*c)*a*b)+b*(1/2*c^2*b^2+4*a^2*c*b+(a^2+b*c)^2))*x^6
```

```
+1/5*(a*(1/2*c^2*b^2+4*a^2*c*b+(a^2+b*c)^2)+b*(2*c^2*a*b+4*a*c*(a^2+b*c)))*
x^5+1/4*(a*(2*c^2*a*b+4*a*c*(a^2+b*c))+b*(2*c^2*(a^2+b*c)+4*a^2*c^2))*x^4+1
/3*(a*(2*c^2*(a^2+b*c)+4*a^2*c^2)+4*a*b*c^3)*x^3+1/2*(4*a^2*c^3+b*(c^4+1))*
x^2+a*(c^4+1)*x
```

maxima [B] time = 0.44, size = 187, normalized size = 6.03

$$\frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{16}\left(4\,a^2b^3+b^4c\right)x^8+\frac{1}{2}\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\left(2\,a^4b+6\,a^2b^2c+b^3c^2\right)x^6+\frac{1}{10}\left(2\,a^5+20\,a^3bc+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b
^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5
 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3)*x
^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 +
 a)*x
```

mupad [B] time = 0.10, size = 180, normalized size = 5.81

$$x^6\left(\frac{a^4\,b}{2}+\frac{3\,a^2\,b^2\,c}{2}+\frac{b^3\,c^2}{4}\right)+x^4\left(a^4\,c+3\,a^2\,b\,c^2+\frac{b^2\,c^3}{2}\right)+x^2\left(2\,a^2\,c^3+\frac{b\,c^4}{2}+\frac{b}{2}\right)+x^5\left(\frac{a^5}{5}+2\,a^3\,b\,c+\frac{3\,a\,b^2\,c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

```
[Out] x^6*((a^4*b)/2 + (b^3*c^2)/4 + (3*a^2*b^2*c)/2) + x^4*(a^4*c + (b^2*c^3)/2
+ 3*a^2*b*c^2) + x^2*(b/2 + (b*c^4)/2 + 2*a^2*c^3) + x^5*(a^5/5 + (3*a*b^2*
c^2)/2 + 2*a^3*b*c) + (b^5*x^10)/160 + x^8*((b^4*c)/16 + (a^2*b^3)/4) + (a*
b^4*x^9)/16 + a*x*(c^4 + 1) + (a*b^2*x^7*(b*c + a^2))/2 + 2*a*c^2*x^3*(b*c
+ a^2)
```

sympy [B] time = 0.11, size = 194, normalized size = 6.26

$$\frac{ab^4x^9}{16}+\frac{b^5x^{10}}{160}+x^8\left(\frac{a^2b^3}{4}+\frac{b^4c}{16}\right)+x^7\left(\frac{a^3b^2}{2}+\frac{ab^3c}{2}\right)+x^6\left(\frac{a^4b}{2}+\frac{3a^2b^2c}{2}+\frac{b^3c^2}{4}\right)+x^5\left(\frac{a^5}{5}+2a^3bc+\frac{3ab^2c^2}{2}\right)+x^4\left(a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)
```

```
[Out] a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16) + x**7*(a*
*3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*c/2 + b**3*c**2/4) +
 x**5*(a**5/5 + 2*a**3*b*c + 3*a*b**2*c**2/2) + x**4*(a**4*c + 3*a**2*b*c**
2 + b**2*c**3/2) + x**3*(2*a**3*c**2 + 2*a*b*c**3) + x**2*(2*a**2*c**3 + b*
c**4/2 + b/2) + x*(a*c**4 + a)
```

3.208 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right) dx$

Optimal. Leaf size=34

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right) dx &= \text{Subst}\left(\int (1+x^n)\, dx, x, ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.00

$$\frac{x(2a+bx)\left(\left(ax+\frac{bx^2}{2}\right)^n+n+1\right)}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] (x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

fricas [A] time = 0.79, size = 48, normalized size = 1.41

$$\frac{(bn+b)x^2 + \left(bx^2 + 2\,ax\right)\left(\frac{1}{2}\,bx^2 + ax\right)^n + 2\,(an+a)x}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

```
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n + a)*x)/(
n + 1)
```

giac [A] time = 0.40, size = 30, normalized size = 0.88

$$\frac{1}{2}\,bx^2 + ax + \frac{\left(\frac{1}{2}\,bx^2 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{b\,x^2}{2} + ax + \frac{\left(\frac{1}{2}b\,x^2 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x)
```

```
[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(n+1)/(n+1)
```

maxima [A] time = 1.13, size = 52, normalized size = 1.53

$$\frac{1}{2}\,bx^2 + ax + \frac{\left(bx^2 + 2\,ax\right)e^{\left(n\log(bx+2\,a)+n\log(x)\right)}}{2^{n+1}n + 2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^(n*log(b*x + 2*a) + n*log(x))/(2^(n + 1
)*n + 2^(n + 1))
```

mupad [B] time = 2.12, size = 31, normalized size = 0.91

$$\frac{x\,\left(2\,a+b\,x\right)\,\left(n+{\left(\frac{b\,x^2}{2}+a\,x\right)}^n+1\right)}{2\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

```
[Out] (x*(2*a + b*x)*(n + (a*x + (b*x^2)/2)^n + 1))/(2*(n + 1))
```

sympy [A] time = 50.75, size = 230, normalized size = 6.76

$$\begin{cases} a \left(x + \frac{\log{\left(x \right)}}{a}\right) & \text{for}\: b = 0 \wedge n = -1 \\ a \left(\frac{a^{n} x x^{n}}{n + 1} + \frac{n x}{n + 1} + \frac{x}{n + 1}\right) & \text{for}\: b = 0 \\ a x + \frac{b x^{2}}{2} + \log{\left(x \right)} + \log{\left(\frac{2 a}{b} + x \right)} & \text{for}\: n = -1 \\ \frac{2 \cdot 2^{n} a b n x}{2 \cdot 2^{n} b n + 2 \cdot 2^{n} b} + \frac{2 \cdot 2^{n} a b x}{2 \cdot 2^{n} b n + 2 \cdot 2^{n} b} + \frac{2^{n} b^{2} n x^{2}}{2 \cdot 2^{n} b n + 2 \cdot 2^{n} b} + \frac{2^{n} b^{2} x^{2}}{2 \cdot 2^{n} b n + 2 \cdot 2^{n} b} + \frac{2 a b x \left(2 a x + b x^{2}\right)^{n}}{2 \cdot 2^{n} b n + 2 \cdot 2^{n} b} + \frac{b^{2} x^{2} \left(2 a x + b x^{2}\right)^{n}}{2 \cdot 2^{n} b n + 2 \cdot 2^{n} b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)
```

```
[Out] Piecewise((a*(x + log(x)/a), Eq(b, 0) & Eq(n, -1)), (a*(a**n*x*x**n/(n + 1)
 + n*x/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a/
b + x), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*2**n*b) + 2*2**n*a*b*x/
(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b*n + 2*2**n*b) + 2**n*b
**2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2*2**n*b*n
+ 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b), True))
```

3.209 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right) dx$

Optimal. Leaf size=35

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right) dx &= \text{Subst}\left(\int (1+x^n)\ dx, x, c+ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 1.00

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

fricas [A] time = 0.95, size = 52, normalized size = 1.49

$$\frac{(bn+b)x^2+\left(bx^2+2\,ax+2\,c\right)\left(\frac{1}{2}\,bx^2+ax+c\right)^n+2\,(an+a)x}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

```
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*n
 + a)*x)/(n + 1)
```

giac [A] time = 0.25, size = 32, normalized size = 0.91

$$\frac{1}{2}\,bx^2+ax+c+\frac{\left(\frac{1}{2}\,bx^2+ax+c\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + c + (1/2*b*x^2 + a*x + c)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.94

$$\frac{b\,x^2}{2}+ax+c+\frac{\left(\frac{1}{2}b\,x^2+ax+c\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x)
```

```
[Out] c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(n+1)/(n+1)
```

maxima [A] time = 1.16, size = 54, normalized size = 1.54

$$\frac{1}{2}\,bx^2+ax+\frac{\left(bx^2+2\,ax+2\,c\right)\left(bx^2+2\,ax+2\,c\right)^n}{2^{n+1}n+2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^(n + 1)*
n + 2^(n + 1))
```

mupad [B] time = 2.11, size = 58, normalized size = 1.66

$$a\,x+{\left(\frac{b\,x^2}{2}+a\,x+c\right)}^n\,\left(\frac{2\,c}{2\,n+2}+\frac{b\,x^2}{2\,n+2}+\frac{2\,a\,x}{2\,n+2}\right)+\frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

```
[Out] a*x + (c + a*x + (b*x^2)/2)^n*((2*c)/(2*n + 2) + (b*x^2)/(2*n + 2) + (2*a*x
)/(2*n + 2)) + (b*x^2)/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
```

```
[Out] Timed out
```

3.210 $\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=30

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

```
[Out] a*x+1/3*c*x^3+1/6*(a*x+1/3*c*x^3)^6
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int \left(1+x^5\right)\,dx,x,ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.01, size = 93, normalized size = 3.10

$$\frac{a^6x^6}{6}+\frac{1}{3}a^5cx^8+\frac{5}{18}a^4c^2x^{10}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{162}a^2c^4x^{14}+\frac{1}{243}ac^5x^{16}+ax+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*a
^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/4374
```

fricas [B] time = 0.76, size = 77, normalized size = 2.57

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{243}x^{16}c^5a + \frac{5}{162}x^{14}c^4a^2 + \frac{10}{81}x^{12}c^3a^3 + \frac{5}{18}x^{10}c^2a^4 + \frac{1}{3}x^8ca^5 + \frac{1}{6}x^6a^6 + \frac{1}{3}x^3c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 5/162*x^14*c^4*a^2 + 10/81*x^12*c^3*a^
3 + 5/18*x^10*c^2*a^4 + 1/3*x^8*c*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + x*a
```

giac [A] time = 0.22, size = 24, normalized size = 0.80

$$\frac{1}{4374}\left(cx^3 + 3\,ax\right)^6 + \frac{1}{3}\,cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/3*c*x^3 + a*x
```

maple [B] time = 0.00, size = 78, normalized size = 2.60

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}a\,c^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5c\,x^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}c\,x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/243*a*c^5*x^16+5/162*a^2*c^4*x^14+10/81*a^3*c^3*x^12+5/18
*a^4*c^2*x^10+1/3*a^5*c*x^8+1/6*a^6*x^6+1/3*c*x^3+a*x
```

maxima [B] time = 0.44, size = 77, normalized size = 2.57

$$\frac{1}{4374}\,c^6x^{18} + \frac{1}{243}\,ac^5x^{16} + \frac{5}{162}\,a^2c^4x^{14} + \frac{10}{81}\,a^3c^3x^{12} + \frac{5}{18}\,a^4c^2x^{10} + \frac{1}{3}\,a^5cx^8 + \frac{1}{6}\,a^6x^6 + \frac{1}{3}\,cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^1
2 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

mupad [B] time = 0.05, size = 77, normalized size = 2.57

$$\frac{a^6\,x^6}{6}+\frac{a^5\,c\,x^8}{3}+\frac{5\,a^4\,c^2\,x^{10}}{18}+\frac{10\,a^3\,c^3\,x^{12}}{81}+\frac{5\,a^2\,c^4\,x^{14}}{162}+\frac{a\,c^5\,x^{16}}{243}+a\,x+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^5 + 1),x)
```

```
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (a^5*c*x^8)/3 + (a*c^5*x^
16)/243 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162
```

sympy [B] time = 0.09, size = 87, normalized size = 2.90

$$\frac{a^{6}x^{6}}{6}+\frac{a^{5}cx^{8}}{3}+\frac{5a^{4}c^{2}x^{10}}{18}+\frac{10a^{3}c^{3}x^{12}}{81}+\frac{5a^{2}c^{4}x^{14}}{162}+\frac{ac^{5}x^{16}}{243}+ax+\frac{c^{6}x^{18}}{4374}+\frac{cx^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81
+ 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x**3
/3
```

3.211 $\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}$$

```
[Out] a*x+1/3*c*x^3+1/6*(d+a*x+1/3*c*x^3)^6
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, d+ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.05, size = 140, normalized size = 4.52

$$\frac{x\left(3a+cx^2\right)\left(243a^5x^5+405a^4cx^7+270a^3c^2x^9+90a^2c^3x^{11}+15ac^4x^{13}+1215d^4\left(3ax+cx^3\right)+540d^3\left(3ax+cx^3\right.\right.}{4374}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] (x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*c
^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c*x
^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*x +
 c*x^3)^4))/4374
```

fricas [B] time = 0.99, size = 291, normalized size = 9.39

$$\frac{1}{4374}x^{18}c^6+\frac{1}{243}x^{16}c^5a+\frac{1}{243}x^{15}dc^5+\frac{5}{162}x^{14}c^4a^2+\frac{5}{81}x^{13}dc^4a+\frac{5}{162}x^{12}d^2c^4+\frac{10}{81}x^{12}c^3a^3+\frac{10}{27}x^{11}dc^3a^2+\frac{10}{27}x^{10}d^2c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 1/243*x^15*d*c^5 + 5/162*x^14*c^4*a^2
+ 5/81*x^13*d*c^4*a + 5/162*x^12*d^2*c^4 + 10/81*x^12*c^3*a^3 + 10/27*x^11*
d*c^3*a^2 + 10/27*x^10*d^2*c^3*a + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 +
10/9*x^9*d*c^2*a^3 + 5/3*x^8*d^2*c^2*a^2 + 1/3*x^8*c*a^5 + 10/9*x^7*d^3*c^2
*a + 5/3*x^7*d*c*a^4 + 5/18*x^6*d^4*c^2 + 10/3*x^6*d^2*c*a^3 + 1/6*x^6*a^6
+ 10/3*x^5*d^3*c*a^2 + x^5*d*a^5 + 5/3*x^4*d^4*c*a + 5/2*x^4*d^2*a^4 + 1/3*
x^3*d^5*c + 10/3*x^3*d^3*a^3 + 5/2*x^2*d^4*a^2 + x*d^5*a + 1/3*x^3*c + x*a
```

giac [B] time = 0.34, size = 105, normalized size = 3.39

$$\frac{1}{4374}\left(cx^3+3\,ax\right)^6+\frac{1}{243}\left(cx^3+3\,ax\right)^5d+\frac{5}{162}\left(cx^3+3\,ax\right)^4d^2+\frac{10}{81}\left(cx^3+3\,ax\right)^3d^3+\frac{5}{18}\left(cx^3+3\,ax\right)^2d^4+\frac{1}{3}\left(cx^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/243*(c*x^3 + 3*a*x)^5*d + 5/162*(c*x^3 + 3*a*x
)^4*d^2 + 10/81*(c*x^3 + 3*a*x)^3*d^3 + 5/18*(c*x^3 + 3*a*x)^2*d^4 + 1/3*(c
*x^3 + 3*a*x)*d^5 + 1/3*c*x^3 + a*x
```

maple [B] time = 0.00, size = 618, normalized size = 19.94

$$\frac{c^6x^{18}}{4374}+\frac{a\,c^5x^{16}}{243}+\frac{c^5d\,x^{15}}{243}+\frac{5a^2c^4x^{14}}{162}+\frac{5a\,c^4d\,x^{13}}{81}+\frac{10a^2c^3d\,x^{11}}{27}+\frac{\left(\frac{10a^3c^3}{27}+\left(\frac{2a^3c^2}{3}+\frac{4c^3d^2}{27}+\frac{\left(\frac{4}{3}a^3c+\frac{2}{3}c^2d^2\right)c}{3}\right)c\right)x^{12}}{12}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/243*a*c^5*x^16+1/243*c^5*d*x^15+5/162*a^2*c^4*x^14+5/81*a
*c^4*d*x^13+1/12*(10/27*a^3*c^3+c*(4/27*d^2*c^3+2/3*a^3*c^2+1/3*c*(2/3*d^2*
c^2+4/3*a^3*c)))*x^12+10/27*a^2*c^3*d*x^11+1/10*(a*(4/27*d^2*c^3+2/3*a^3*c^
2+1/3*c*(2/3*d^2*c^2+4/3*a^3*c))+c*(4/3*d^2*a*c^2+a*(2/3*d^2*c^2+4/3*a^3*c)
+1/3*c*(a^4+4*a*c*d^2)))*x^10+1/9*(10/3*a^3*d*c^2+c*(d*(2/3*d^2*c^2+4/3*a^3
*c)+4*a^3*d*c+1/3*c*(4/3*d^3*c+4*a^3*d)))*x^9+1/8*(a*(4/3*d^2*a*c^2+a*(2/3*
d^2*c^2+4/3*a^3*c)+1/3*c*(a^4+4*a*c*d^2))+c*(6*d^2*a^2*c+a*(a^4+4*a*c*d^2))
)*x^8+1/7*(a*(d*(2/3*d^2*c^2+4/3*a^3*c)+4*a^3*d*c+1/3*c*(4/3*d^3*c+4*a^3*d)
)+c*(d*(a^4+4*a*c*d^2)+a*(4/3*d^3*c+4*a^3*d)+4/3*c*d^3*a))*x^7+1/6*(a*(6*d^
2*a^2*c+a*(a^4+4*a*c*d^2))+c*(d*(4/3*d^3*c+4*a^3*d)+6*a^3*d^2+1/3*c*d^4))*x
^6+1/5*(a*(d*(a^4+4*a*c*d^2)+a*(4/3*d^3*c+4*a^3*d)+4/3*c*d^3*a)+10*a^2*c*d^
3)*x^5+1/4*(a*(d*(4/3*d^3*c+4*a^3*d)+6*a^3*d^2+1/3*c*d^4)+5*a*c*d^4)*x^4+1/
3*(10*a^3*d^3+c*(d^5+1))*x^3+5/2*a^2*d^4*x^2+a*(d^5+1)*x
```

maxima [B] time = 0.46, size = 280, normalized size = 9.03

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{243}\,ac^5x^{16}+\frac{1}{243}\,c^5dx^{15}+\frac{5}{162}\,a^2c^4x^{14}+\frac{5}{81}\,ac^4dx^{13}+\frac{10}{27}\,a^2c^3dx^{11}+\frac{5}{162}\left(4\,a^3c^3+c^4d^2\right)x^{12}+\frac{5}{54}\left(3\,a^4c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
+ 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*x^
12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)*x^
9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*
a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d
 + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 +
c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

mupad [B] time = 2.27, size = 266, normalized size = 8.58

$$x^5\left(a^5\,d+\frac{10\,c\,a^2\,d^3}{3}\right)+x^4\left(\frac{5\,a^4\,d^2}{2}+\frac{5\,c\,a\,d^4}{3}\right)+x^3\left(\frac{10\,a^3\,d^3}{3}+\frac{c\,d^5}{3}+\frac{c}{3}\right)+x^6\left(\frac{a^6}{6}+\frac{10\,a^3\,c\,d^2}{3}+\frac{5\,c^2\,d^4}{18}\right)+\frac{c^6\,x^{18}}{4374}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + a*x + (c*x^3)/3)^5 + 1)*(a + c*x^2),x)
```

```
[Out] x^5*(a^5*d + (10*a^2*c*d^3)/3) + x^4*((5*a^4*d^2)/2 + (5*a*c*d^4)/3) + x^3*
(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3) + x^6*(a^6/6 + (5*c^2*d^4)/18 + (10*a^3*
c*d^2)/3) + (c^6*x^18)/4374 + (a*c^5*x^16)/243 + a*x*(d^5 + 1) + (c^5*d*x^1
5)/243 + (5*a^2*c^4*x^14)/162 + (5*a^2*d^4*x^2)/2 + (5*c^3*x^12*(c*d^2 + 4*
a^3))/162 + (a^2*c*x^8*(5*c*d^2 + a^3))/3 + (10*a^2*c^3*d*x^11)/27 + (5*a*c
^2*x^10*(4*c*d^2 + 3*a^3))/54 + (10*c^2*d*x^9*(c*d^2 + 9*a^3))/81 + (5*a*c^
4*d*x^13)/81 + (5*a*c*d*x^7*(2*c*d^2 + 3*a^3))/9
```

sympy [B] time = 0.14, size = 314, normalized size = 10.13

$$\frac{5a^2c^4x^{14}}{162}+\frac{10a^2c^3dx^{11}}{27}+\frac{5a^2d^4x^2}{2}+\frac{ac^5x^{16}}{243}+\frac{5ac^4dx^{13}}{81}+\frac{c^6x^{18}}{4374}+\frac{c^5dx^{15}}{243}+x^{12}\left(\frac{10a^3c^3}{81}+\frac{5c^4d^2}{162}\right)+x^{10}\left(\frac{5a^4c^2}{18}+\frac{10ac}{27}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)
```

```
[Out] 5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c*
*5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 + x
**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*c**
3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5
*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a**6/6
+ 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/3) + x
**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3 + c/3)
 + x*(a*d**5 + a)
```

3.212 $\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=34

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+1/279936*x^12*(2*c*x+3*b)^6
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1591}

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{x^{12}(3b+2cx)^6}{279936}\end{aligned}$$

Mathematica [B] time = 0.01, size = 98, normalized size = 2.88

$$\frac{b^6x^{12}}{384}+\frac{1}{96}b^5cx^{13}+\frac{5}{288}b^4c^2x^{14}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{486}bc^5x^{17}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14)
/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (c^
6*x^18)/4374
```

fricas [B] time = 0.68, size = 80, normalized size = 2.35

$$\frac{1}{4374}x^{18}c^6+\frac{1}{486}x^{17}c^5b+\frac{5}{648}x^{16}c^4b^2+\frac{5}{324}x^{15}c^3b^3+\frac{5}{288}x^{14}c^2b^4+\frac{1}{96}x^{13}cb^5+\frac{1}{384}x^{12}b^6+\frac{1}{3}x^3c+\frac{1}{2}x^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 5/324*x^15*c^3*b^
3 + 5/288*x^14*c^2*b^4 + 1/96*x^13*c*b^5 + 1/384*x^12*b^6 + 1/3*x^3*c + 1/2
*x^2*b
```

giac [A] time = 0.29, size = 30, normalized size = 0.88

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2\right)^6+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/3*c*x^3 + 1/2*b*x^2
```

maple [B] time = 0.00, size = 81, normalized size = 2.38

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}b\,c^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{288}b^4c^2x^{14}+\frac{1}{96}b^5c\,x^{13}+\frac{1}{384}b^6x^{12}+\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+5/324*b^3*c^3*x^15+5/28
8*b^4*c^2*x^14+1/96*b^5*c*x^13+1/384*b^6*x^12+1/3*c*x^3+1/2*b*x^2
```

maxima [B] time = 0.44, size = 80, normalized size = 2.35

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{5}{648}\,b^2c^4x^{16}+\frac{5}{324}\,b^3c^3x^{15}+\frac{5}{288}\,b^4c^2x^{14}+\frac{1}{96}\,b^5cx^{13}+\frac{1}{384}\,b^6x^{12}+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1/2
*b*x^2
```

mupad [B] time = 0.07, size = 80, normalized size = 2.35

$$\frac{b^6\,x^{12}}{384}+\frac{b^5\,c\,x^{13}}{96}+\frac{5\,b^4\,c^2\,x^{14}}{288}+\frac{5\,b^3\,c^3\,x^{15}}{324}+\frac{5\,b^2\,c^4\,x^{16}}{648}+\frac{b\,c^5\,x^{17}}{486}+\frac{b\,x^2}{2}+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (c^6*x^18)/4374 + (b^5*c*x^13)/96
+ (b*c^5*x^17)/486 + (5*b^4*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c
^4*x^16)/648
```

sympy [B] time = 0.10, size = 90, normalized size = 2.65

$$\frac{b^{6}x^{12}}{384}+\frac{b^{5}cx^{13}}{96}+\frac{5b^{4}c^{2}x^{14}}{288}+\frac{5b^{3}c^{3}x^{15}}{324}+\frac{5b^{2}c^{4}x^{16}}{648}+\frac{bc^{5}x^{17}}{486}+\frac{bx^{2}}{2}+\frac{c^{6}x^{18}}{4374}+\frac{cx^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**1
5/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/43
74 + c*x**3/3
```

3.213 $\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$

Optimal. Leaf size=41

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+1/6*(d+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1591}

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.05, size = 146, normalized size = 3.56

$$\frac{x^2(3b+2cx)\left(243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+19440d^4x^2(3b+2cx)+4320d^3\right.}{279936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 108
0*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*d^
4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b + 2*c
*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936
```

fricas [B] time = 0.93, size = 298, normalized size = 7.27

$$\frac{1}{4374}x^{18}c^6+\frac{1}{486}x^{17}c^5b+\frac{5}{648}x^{16}c^4b^2+\frac{1}{243}x^{15}dc^5+\frac{5}{324}x^{15}c^3b^3+\frac{5}{162}x^{14}dc^4b+\frac{5}{288}x^{14}c^2b^4+\frac{5}{54}x^{13}dc^3b^2+\frac{1}{96}x^{13}cb^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^15*d*c^5
+ 5/324*x^15*c^3*b^3 + 5/162*x^14*d*c^4*b + 5/288*x^14*c^2*b^4 + 5/54*x^13*
d*c^3*b^2 + 1/96*x^13*c*b^5 + 5/162*x^12*d^2*c^4 + 5/36*x^12*d*c^2*b^3 + 1/
384*x^12*b^6 + 5/27*x^11*d^2*c^3*b + 5/48*x^11*d*c*b^4 + 5/12*x^10*d^2*c^2*
b^2 + 1/32*x^10*d*b^5 + 10/81*x^9*d^3*c^3 + 5/12*x^9*d^2*c*b^3 + 5/9*x^8*d^
3*c^2*b + 5/32*x^8*d^2*b^4 + 5/6*x^7*d^3*c*b^2 + 5/18*x^6*d^4*c^2 + 5/12*x^
6*d^3*b^3 + 5/6*x^5*d^4*c*b + 5/8*x^4*d^4*b^2 + 1/3*x^3*d^5*c + 1/2*x^2*d^5
*b + 1/3*x^3*c + 1/2*x^2*b
```

giac [B] time = 0.31, size = 126, normalized size = 3.07

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2\right)^6+\frac{1}{7776}\left(2\,cx^3+3\,bx^2\right)^5d+\frac{5}{2592}\left(2\,cx^3+3\,bx^2\right)^4d^2+\frac{5}{324}\left(2\,cx^3+3\,bx^2\right)^3d^3+\frac{5}{72}\left(2\,cx^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/7776*(2*c*x^3 + 3*b*x^2)^5*d + 5/2592*(2
*c*x^3 + 3*b*x^2)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2)^3*d^3 + 5/72*(2*c*x^3 +
 3*b*x^2)^2*d^4 + 1/6*(2*c*x^3 + 3*b*x^2)*d^5 + 1/3*c*x^3 + 1/2*b*x^2
```

maple [B] time = 0.00, size = 646, normalized size = 15.76

$$\frac{c^6x^{18}}{4374}+\frac{b\,c^5x^{17}}{486}+\frac{5b^2c^4x^{16}}{648}+\frac{\left(\frac{5b^3c^3}{54}+\left(\frac{b^3c^2}{12}+\frac{c^4d}{81}+\frac{\left(\frac{1}{6}b^3c+\frac{4}{27}c^3d\right)c}{3}\right)c\right)x^{15}}{15}+\frac{\left(\left(\left(\frac{b^3c^2}{12}+\frac{c^4d}{81}+\frac{\left(\frac{1}{6}b^3c+\frac{4}{27}c^3d\right)c}{3}\right)b+\left(\frac{2b\,c^3d}{27}+\cdots\right.\right.\right.}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+1/15*(5/54*b^3*c^3+c*(1
/81*d*c^4+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c)))*x^15+1/14*(b*(1/81*d*
c^4+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c))+c*(2/27*d*b*c^3+1/2*b*(4/27*
c^3*d+1/6*b^3*c)+1/3*c*(2/3*d*b*c^2+1/16*b^4)))*x^14+1/13*(b*(2/27*d*b*c^3+
1/2*b*(4/27*c^3*d+1/6*b^3*c)+1/3*c*(2/3*d*b*c^2+1/16*b^4))+c*(1/2*d*b^2*c^2
+1/2*b*(2/3*d*b*c^2+1/16*b^4)))*x^13+1/12*(b*(1/2*d*b^2*c^2+1/2*b*(2/3*d*b*
c^2+1/16*b^4))+c*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*d*c+1/3*c*(2/3*c^2*d^2+1
/2*d*b^3)))*x^12+1/11*(b*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*d*c+1/3*c*(2/3*c
^2*d^2+1/2*d*b^3))+c*(d*(2/3*d*b*c^2+1/16*b^4)+1/2*b*(2/3*c^2*d^2+1/2*d*b^3
)+2/3*c^2*d^2*b))*x^11+1/10*(b*(d*(2/3*d*b*c^2+1/16*b^4)+1/2*b*(2/3*c^2*d^2
+1/2*d*b^3)+2/3*c^2*d^2*b)+5/2*b^2*c^2*d^2)*x^10+1/9*(5/2*b^3*c*d^2+c*(d*(2
/3*c^2*d^2+1/2*d*b^3)+3/4*b^3*d^2+4/9*c^2*d^3))*x^9+1/8*(b*(d*(2/3*c^2*d^2+
1/2*d*b^3)+3/4*b^3*d^2+4/9*c^2*d^3)+10/3*c^2*d^3*b)*x^8+5/6*b^2*c*d^3*x^7+1
/6*(5/2*b^3*d^3+5/3*c^2*d^4)*x^6+5/6*b*c*d^4*x^5+5/8*b^2*d^4*x^4+1/3*(d^5+1
)*c*x^3+1/2*b*(d^5+1)*x^2
```

maxima [B] time = 0.45, size = 289, normalized size = 7.05

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{5}{648}\,b^2c^4x^{16}+\frac{1}{972}\left(15\,b^3c^3+4\,c^5d\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+16\,bc^4d\right)x^{14}+\frac{1}{864}\left(9\,b^5c+80\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^3
 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c +
80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*d +
 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^
5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*
c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3
*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

mupad [B] time = 2.28, size = 273, normalized size = 6.66

$$x^{13}\left(\frac{b^5\,c}{96}+\frac{5\,d\,b^2\,c^3}{54}\right)+x^{14}\left(\frac{5\,b^4\,c^2}{288}+\frac{5\,d\,b\,c^4}{162}\right)+x^{12}\left(\frac{b^6}{384}+\frac{5\,b^3\,c^2\,d}{36}+\frac{5\,c^4\,d^2}{162}\right)+\frac{c^6\,x^{18}}{4374}+x^{15}\left(\frac{5\,b^3\,c^3}{324}+\frac{d\,c^5}{243}\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*((d + (b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

```
[Out] x^13*((b^5*c)/96 + (5*b^2*c^3*d)/54) + x^14*((5*b^4*c^2)/288 + (5*b*c^4*d)/
162) + x^12*(b^6/384 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36) + (c^6*x^18)/437
4 + x^15*((c^5*d)/243 + (5*b^3*c^3)/324) + (5*d^3*x^6*(2*c^2*d + 3*b^3))/36
```

```
 + (b*c^5*x^17)/486 + (5*b^2*c^4*x^16)/648 + (b*x^2*(d^5 + 1))/2 + (5*b^2*d
^4*x^4)/8 + (c*x^3*(d^5 + 1))/3 + (5*b^2*c*d^3*x^7)/6 + (5*b*d^2*x^8*(32*c^
2*d + 9*b^3))/288 + (b^2*d*x^10*(40*c^2*d + 3*b^3))/96 + (5*c*d^2*x^9*(8*c^
2*d + 27*b^3))/324 + (5*b*c*d^4*x^5)/6 + (5*b*c*d*x^11*(16*c^2*d + 9*b^3))/
432
```

sympy [B] time = 0.15, size = 321, normalized size = 7.83

$$\frac{5b^2c^4x^{16}}{648}+\frac{5b^2cd^3x^7}{6}+\frac{5b^2d^4x^4}{8}+\frac{bc^5x^{17}}{486}+\frac{5bcd^4x^5}{6}+\frac{c^6x^{18}}{4374}+x^{15}\left(\frac{5b^3c^3}{324}+\frac{c^5d}{243}\right)+x^{14}\left(\frac{5b^4c^2}{288}+\frac{5bc^4d}{162}\right)+x^{13}\left(\frac{b^5c}{96}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] 5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8 + b*c**5*
x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15*(5*b**3*c**3/324 +
c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(b**5*c/96 +
 5*b**2*c**3*d/54) + x**12*(b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162)
+ x**11*(5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2*c**2
*d**2/12) + x**9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*d**2/3
2 + 5*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3*(c*d**
5/3 + c/3) + x**2*(b*d**5/2 + b/2)
```

3.214 $\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(a*x+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.06, size = 244, normalized size = 5.30

$$\frac{a^6x^6}{6}+\frac{1}{6}a^5x^7(3b+2cx)+\frac{5}{72}a^4x^8(3b+2cx)^2+\frac{5}{324}a^3x^9(3b+2cx)^3+\frac{5a^2x^{10}(3b+2cx)^4}{2592}+a\left(\frac{b^5x^{11}}{32}+\frac{5}{48}b^4cx^{12}+\frac{5}{36}b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 +
(5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(x
+ (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x^14
)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916*b^5
*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 + 576*b
*(243 + c^5*x^15) + 64*c*x*(1458 + c^5*x^15)))/279936
```

fricas [B] time = 0.68, size = 309, normalized size = 6.72

$$\frac{1}{4374}x^{18}c^6+\frac{1}{486}x^{17}c^5b+\frac{5}{648}x^{16}c^4b^2+\frac{1}{243}x^{16}c^5a+\frac{5}{324}x^{15}c^3b^3+\frac{5}{162}x^{15}c^4ba+\frac{5}{288}x^{14}c^2b^4+\frac{5}{54}x^{14}c^3b^2a+\frac{5}{162}x^{14}c^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="frica
s")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^16*c^5*a
+ 5/324*x^15*c^3*b^3 + 5/162*x^15*c^4*b*a + 5/288*x^14*c^2*b^4 + 5/54*x^14*
c^3*b^2*a + 5/162*x^14*c^4*a^2 + 1/96*x^13*c*b^5 + 5/36*x^13*c^2*b^3*a + 5/
27*x^13*c^3*b*a^2 + 1/384*x^12*b^6 + 5/48*x^12*c*b^4*a + 5/12*x^12*c^2*b^2*
a^2 + 10/81*x^12*c^3*a^3 + 1/32*x^11*b^5*a + 5/12*x^11*c*b^3*a^2 + 5/9*x^11
*c^2*b*a^3 + 5/32*x^10*b^4*a^2 + 5/6*x^10*c*b^2*a^3 + 5/18*x^10*c^2*a^4 + 5
/12*x^9*b^3*a^3 + 5/6*x^9*c*b*a^4 + 5/8*x^8*b^2*a^4 + 1/3*x^8*c*a^5 + 1/2*x
^7*b*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + 1/2*x^2*b + x*a
```

giac [A] time = 0.31, size = 37, normalized size = 0.80

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac"
)
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

maple [B] time = 0.00, size = 1523, normalized size = 33.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*a*c^5+5/162*b^2*c^4+c*(1/81*a*
c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)))*x^16+1/15*
(5/162*a*b*c^4+b*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+
1/9*b^2*c^2))+c*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)
+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)))*x^15+1/14*(a*(1/81*a*c^4+1
/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2))+b*(2/27*a*b*c^3+
1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a
*c+1/4*b^2)*b*c))+c*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a
*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1
/4*b^2)^2)))*x^14+1/13*(a*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/
9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+b*(a*(2/9*(2/3*a*
c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1
/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+c*(a*(2/9*a*b*c^2+2/3*(
2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1
/3*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))))*x^13+1/12*(a*(a*(2/9*(2/3*a*c+
1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3
*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+b*(a*(2/9*a*b*c^2+2/3*(2/
3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3
*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+c*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/
3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*
(2/3*a*c+1/4*b^2)+a^2*b^2)))*x^12+1/11*(a*(a*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*
b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(2/3*a^
2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+b*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*
b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*(2/3*a*c+1
/4*b^2)+a^2*b^2))+c*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(
2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b))*x^11+1/10*(a*(a*(2/9*a^2*c^2+2/3*a*
b^2*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*
c*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2))+b*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b
^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b)+c*(a*(2*a^2*(2/3*
a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4))*x^10+1/9*(a*(a*(2/3*a^2*b*c+2*a*b
*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b)+b*
(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4)+5/2*c*a^4*b)*x^9+1/
8*(a*(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4)+5/2*b^2*a^4+c*
a^5)*x^8+1/2*a^5*b*x^7+1/6*a^6*x^6+1/3*c*x^3+1/2*b*x^2+a*x
```

maxima [B] time = 0.45, size = 289, normalized size = 6.28

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}\left(15\,b^2c^4+8\,ac^5\right)x^{16}+\frac{5}{324}\left(b^3c^3+2\,abc^4\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+48\,ab^2c^3+16\,a^2c^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxim
a")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5
/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2
*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*a^5
*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*
x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 +
 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4
*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

mupad [B] time = 2.24, size = 270, normalized size = 5.87

$$x^{12}\left(\frac{10\,a^3\,c^3}{81}+\frac{5\,a^2\,b^2\,c^2}{12}+\frac{5\,a\,b^4\,c}{48}+\frac{b^6}{384}\right)+a\,x+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{a^6\,x^6}{6}+\frac{c^6\,x^{18}}{4374}+\frac{5\,a^2\,x^{10}\,\left(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4\right)}{288}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

```
[Out] x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48) + a*
x + (b*x^2)/2 + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (5*a^2*x^10*(9*
b^4 + 16*a^2*c^2 + 48*a*b^2*c))/288 + (5*c^2*x^14*(9*b^4 + 16*a^2*c^2 + 48*
a*b^2*c))/2592 + (a^5*b*x^7)/2 + (b*c^5*x^17)/486 + (a^4*x^8*(8*a*c + 15*b^
2))/24 + (c^4*x^16*(8*a*c + 15*b^2))/1944 + (a*b*x^11*(9*b^4 + 160*a^2*c^2
+ 120*a*b^2*c))/288 + (b*c*x^13*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/864 +
(5*a^3*b*x^9*(2*a*c + b^2))/12 + (5*b*c^3*x^15*(2*a*c + b^2))/324
```

sympy [B] time = 0.16, size = 323, normalized size = 7.02

$$\frac{a^6x^6}{6}+\frac{a^5bx^7}{2}+ax+\frac{bc^5x^{17}}{486}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}\right)+x^{14}\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**1
8/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**
4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b*
*4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96) + x*
*12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384) + x*
*11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*c**2/1
8 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12
) + x**8*(a**5*c/3 + 5*a**4*b**2/8)
```

3.215 $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=47

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(d+a*x+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.12, size = 248, normalized size = 5.28

$$x(6a+x(3b+2cx))\left(7776a^5x^5+6480a^4x^6(3b+2cx)+2160a^3x^7(3b+2cx)^2+360a^2x^8(3b+2cx)^3+19440d^4x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^10
 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 +
 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2 +
360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6*a +
 x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x^3*(6
*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936
```

fricas [B] time = 0.63, size = 928, normalized size = 19.74

$$\frac{1}{4374}x^{18}c^6+\frac{1}{486}x^{17}c^5b+\frac{5}{648}x^{16}c^4b^2+\frac{1}{243}x^{16}c^5a+\frac{1}{243}x^{15}dc^5+\frac{5}{324}x^{15}c^3b^3+\frac{5}{162}x^{15}c^4ba+\frac{5}{162}x^{14}dc^4b+\frac{5}{288}x^{14}c^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fri
cas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^16*c^5*a
+ 1/243*x^15*d*c^5 + 5/324*x^15*c^3*b^3 + 5/162*x^15*c^4*b*a + 5/162*x^14*d
*c^4*b + 5/288*x^14*c^2*b^4 + 5/54*x^14*c^3*b^2*a + 5/162*x^14*c^4*a^2 + 5/
54*x^13*d*c^3*b^2 + 1/96*x^13*c*b^5 + 5/81*x^13*d*c^4*a + 5/36*x^13*c^2*b^3
*a + 5/27*x^13*c^3*b*a^2 + 5/162*x^12*d^2*c^4 + 5/36*x^12*d*c^2*b^3 + 1/384
*x^12*b^6 + 10/27*x^12*d*c^3*b*a + 5/48*x^12*c*b^4*a + 5/12*x^12*c^2*b^2*a^
2 + 10/81*x^12*c^3*a^3 + 5/27*x^11*d^2*c^3*b + 5/48*x^11*d*c*b^4 + 5/6*x^11
*d*c^2*b^2*a + 1/32*x^11*b^5*a + 10/27*x^11*d*c^3*a^2 + 5/12*x^11*c*b^3*a^2
 + 5/9*x^11*c^2*b*a^3 + 5/12*x^10*d^2*c^2*b^2 + 1/32*x^10*d*b^5 + 10/27*x^1
0*d^2*c^3*a + 5/6*x^10*d*c*b^3*a + 5/3*x^10*d*c^2*b*a^2 + 5/32*x^10*b^4*a^2
 + 5/6*x^10*c*b^2*a^3 + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 + 5/12*x^9*d^
2*c*b^3 + 5/3*x^9*d^2*c^2*b*a + 5/16*x^9*d*b^4*a + 5/2*x^9*d*c*b^2*a^2 + 10
/9*x^9*d*c^2*a^3 + 5/12*x^9*b^3*a^3 + 5/6*x^9*c*b*a^4 + 5/9*x^8*d^3*c^2*b +
 5/32*x^8*d^2*b^4 + 5/2*x^8*d^2*c*b^2*a + 5/3*x^8*d^2*c^2*a^2 + 5/4*x^8*d*b
^3*a^2 + 10/3*x^8*d*c*b*a^3 + 5/8*x^8*b^2*a^4 + 1/3*x^8*c*a^5 + 5/6*x^7*d^3
*c*b^2 + 10/9*x^7*d^3*c^2*a + 5/4*x^7*d^2*b^3*a + 5*x^7*d^2*c*b*a^2 + 5/2*x
^7*d*b^2*a^3 + 5/3*x^7*d*c*a^4 + 1/2*x^7*b*a^5 + 5/18*x^6*d^4*c^2 + 5/12*x^
6*d^3*b^3 + 10/3*x^6*d^3*c*b*a + 15/4*x^6*d^2*b^2*a^2 + 10/3*x^6*d^2*c*a^3
+ 5/2*x^6*d*b*a^4 + 1/6*x^6*a^6 + 5/6*x^5*d^4*c*b + 5/2*x^5*d^3*b^2*a + 10/
3*x^5*d^3*c*a^2 + 5*x^5*d^2*b*a^3 + x^5*d*a^5 + 5/8*x^4*d^4*b^2 + 5/3*x^4*d
^4*c*a + 5*x^4*d^3*b*a^2 + 5/2*x^4*d^2*a^4 + 1/3*x^3*d^5*c + 5/2*x^3*d^4*b*
a + 10/3*x^3*d^3*a^3 + 1/2*x^2*d^5*b + 5/2*x^2*d^4*a^2 + x*d^5*a + 1/3*x^3*
c + 1/2*x^2*b + x*a
```

giac [B] time = 0.44, size = 153, normalized size = 3.26

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{7776}\left(2\,cx^3+3\,bx^2+6\,ax\right)^5d+\frac{5}{2592}\left(2\,cx^3+3\,bx^2+6\,ax\right)^4d^2+\frac{5}{324}\left(2\,cx^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="gia
c")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/7776*(2*c*x^3 + 3*b*x^2 + 6*a*x)
^5*d + 5/2592*(2*c*x^3 + 3*b*x^2 + 6*a*x)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2
+ 6*a*x)^3*d^3 + 5/72*(2*c*x^3 + 3*b*x^2 + 6*a*x)^2*d^4 + 1/6*(2*c*x^3 + 3*
b*x^2 + 6*a*x)*d^5 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

maple [B] time = 0.00, size = 4284, normalized size = 91.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*a*c^5+5/162*b^2*c^4+(1/81*a*c^
4+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*c)*c)*x^16+1/15*
(5/162*a*b*c^4+(1/81*a*c^4+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b
^2)*c^2)*c)*b+c*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*
b^2)*c^2)*b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)))*x^15+
1/14*((1/81*a*c^4+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*
c)*a+b*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)
*b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+c*(2/27*b*c^3*d
+(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(2/9*(2/3*c*d+a*b)*c^2+2/3
*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3
*a*c+1/4*b^2)^2)))*x^14+1/13*(a*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2
/9*(2/3*a*c+1/4*b^2)*c^2)*b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b
^2)*b*c))+b*(2/27*b*c^3*d+(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(
2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2
/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2))+c*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1
/4*b^2)*c^2)+a*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9
*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^
2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))))*x^13+1/12*(a*(2/27
*b*c^3*d+(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(2/9*(2/3*c*d+a*b)
*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*
b*c+(2/3*a*c+1/4*b^2)^2))+b*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)+a*(2
/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/
3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^2+2/3*(a^2+b*d)*b
```

```
*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2)))+c*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3
*a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*
b^2)^2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
^2))+1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+
a*b)^2)))*x^12+1/11*(a*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)+a*(2/9*(2
/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/3*(2/
3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*
(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2)))+b*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+
1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^
2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+
1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^
2))+c*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4
/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9*
c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2
/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))))*x^11+1/10*(
a*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2
+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d
)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(
a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2))+b*(d*(2/9*(a^2+b*d)*c^2+2/3*(2
/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/
3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2
/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)
+2*(a^2+b*d)*(2/3*c*d+a*b)))+c*(d*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d
+a*b)*(2/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/
4*b^2)+(2/3*c*d+a*b)^2)+1/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b
*d)*(2/3*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+
b*d)^2)))*x^10+1/9*(a*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+
1/4*b^2)^2)+a*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
^2))+1/2*b*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+
a*b)^2)+1/3*c*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b
)))+b*(d*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+
a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1
/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/3*c*
(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))+c*(d*(2/9*c^2*d^
2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c
+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1
/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(
a^2+b*d))))*x^9+1/8*(a*(d*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2
/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(
2/3*c*d+a*b)^2)+1/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3
*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))
+b*(d*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^
2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/2*b*
(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3
*c*d+a*b)+4*a*d*(a^2+b*d)))+c*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^
```

```
2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b
*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^2*(a^2+b*d)+4
*a^2*d^2)))*x^8+1/7*(a*(d*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4
*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(
2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^
2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d)))+b*(d*(2/3*d^2*b*c+4*a*d*(2/
3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*
(2/3*c*d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*
c*(2*d^2*(a^2+b*d)+4*a^2*d^2))+c*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d
+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^
2+b*d)+4*a^2*d^2)+4/3*a*c*d^3))*x^7+1/6*(a*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1
/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*
d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^
2*(a^2+b*d)+4*a^2*d^2))+b*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(
a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^2+b*d)+
4*a^2*d^2)+4/3*a*c*d^3)+c*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2
*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4))*x^6+1/5*(a*(d*(2*d^2*(2/3*a*c+1
/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+
b*d))+1/2*b*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4/3*a*c*d^3)+b*(d*(2*d^2*(2/3*c*d+a
*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4)+c*(
d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4))*x^5+1/4*(a*(d*(2*d^2*(2
/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*
d^4)+b*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*a*c*d^4)*x^4+1
/3*(a*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*b*d^4*a+(d^5+1)
*c)*x^3+1/2*(5*a^2*d^4+b*(d^5+1))*x^2+(d^5+1)*a*x
```

maxima [B] time = 0.48, size = 773, normalized size = 16.45

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{1}{1944}\left(15\,b^2c^4+8\,ac^5\right)x^{16}+\frac{1}{972}\left(15\,b^3c^3+30\,abc^4+4\,c^5d\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+48\,ab^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="max
ima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1
/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^
2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2 +
480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a
*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8
*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b
*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(135*a^
2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2 + 9*(3*b
^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 216*a^4*b*
c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c +
```

```
32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*
b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1
/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^2*d^4 + 15
*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 3
0*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2
 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 +
 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

mupad [B] time = 2.45, size = 753, normalized size = 16.02

$$x^{10}\left(\frac{5\,a^4\,c^2}{18}+\frac{5\,a^3\,b^2\,c}{6}+\frac{5\,a^2\,b^4}{32}+\frac{5\,a^2\,b\,c^2\,d}{3}+\frac{5\,a\,b^3\,c\,d}{6}+\frac{10\,a\,c^3\,d^2}{27}+\frac{b^5\,d}{32}+\frac{5\,b^2\,c^2\,d^2}{12}\right)+x^8\left(\frac{a^5\,c}{3}+\frac{5\,a^4\,b^2}{8}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

```
[Out] x^10*((b^5*d)/32 + (5*a^2*b^4)/32 + (5*a^4*c^2)/18 + (5*a^3*b^2*c)/6 + (10*
a*c^3*d^2)/27 + (5*b^2*c^2*d^2)/12 + (5*a*b^3*c*d)/6 + (5*a^2*b*c^2*d)/3) +
 x^8*((a^5*c)/3 + (5*a^4*b^2)/8 + (5*b^4*d^2)/32 + (5*a^2*b^3*d)/4 + (5*b*c
^2*d^3)/9 + (5*a^2*c^2*d^2)/3 + (10*a^3*b*c*d)/3 + (5*a*b^2*c*d^2)/2) + x^9
*((5*a^3*b^3)/12 + (10*c^3*d^3)/81 + (10*a^3*c^2*d)/9 + (5*b^3*c*d^2)/12 +
(5*a^4*b*c)/6 + (5*a*b^4*d)/16 + (5*a*b*c^2*d^2)/3 + (5*a^2*b^2*c*d)/2) + x
^14*((5*a^2*c^4)/162 + (5*b^4*c^2)/288 + (5*a*b^2*c^3)/54 + (5*b*c^4*d)/162
) + x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36 +
(5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48 + (10*a*b*c^3*d)/27) + x^6*(a^6/6 + (5*
b^3*d^3)/12 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3 + (15*a^2*b^2*d^2)/4 + (5*a
^4*b*d)/2 + (10*a*b*c*d^3)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3 + (5*
a*b*d^4)/2) + x^11*((a*b^5)/32 + (5*a^2*b^3*c)/12 + (5*a^3*b*c^2)/9 + (10*a
^2*c^3*d)/27 + (5*b*c^3*d^2)/27 + (5*b^4*c*d)/48 + (5*a*b^2*c^2*d)/6) + x^7
*((a^5*b)/2 + (5*a*b^3*d^2)/4 + (5*a^3*b^2*d)/2 + (10*a*c^2*d^3)/9 + (5*b^2
*c*d^3)/6 + (5*a^4*c*d)/3 + 5*a^2*b*c*d^2) + x^2*(b/2 + (b*d^5)/2 + (5*a^2*
d^4)/2) + x^13*((b^5*c)/96 + (5*a*b^3*c^2)/36 + (5*a^2*b*c^3)/27 + (5*b^2*c
^3*d)/54 + (5*a*c^4*d)/81) + x^5*(a^5*d + (5*a*b^2*d^3)/2 + 5*a^3*b*d^2 + (
10*a^2*c*d^3)/3 + (5*b*c*d^4)/6) + (c^6*x^18)/4374 + (5*d^2*x^4*(12*a^4 + 3
*b^2*d^2 + 24*a^2*b*d + 8*a*c*d^2))/24 + a*x*(d^5 + 1) + (b*c^5*x^17)/486 +
 (c^3*x^15*(4*c^2*d + 15*b^3 + 30*a*b*c))/972 + (c^4*x^16*(8*a*c + 15*b^2))
/1944
```

sympy [B] time = 0.27, size = 930, normalized size = 19.79

$$\frac{bc^5x^{17}}{486}+\frac{c^6x^{18}}{4374}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}+\frac{c^5d}{243}\right)+x^{14}\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\frac{5b^4c^2}{288}+\frac{5bc^4d}{162}\right)+x^{13}\left(\frac{5}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648) +
 x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c**4
/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a**2
*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3*d/5
4) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10*a*b*
c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*a**3*
b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b**2*c**2
*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 + 5*a**3*b
**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10*a*c**3*d
**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 + 5*a**3*b**
3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*a*b*c**2*d*
*2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**4*b**2/8
 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*a*b**2*c*d**2
/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a**4*c*d/3 + 5*
a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**2*d**3/9 + 5*b*
*2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2/3 + 15*a**2*b**
2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/18) + x**5*(a**5*
d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 + 5*b*c*d**4/6) + x*
*4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**2*d**4/8) + x**3*(1
0*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**2*(5*a**2*d**4/2 + b*d*
*5/2 + b/2) + x*(a*d**5 + a)
```

3.216 $\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=34

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

```
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1591}

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int \left(1+x^n\right)\,dx,x,ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 1.06

$$\frac{x\left(3a+cx^2\right)\left(\left(ax+\frac{cx^3}{3}\right)^n+n+1\right)}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

```
[Out] (x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))
```

fricas [A] time = 1.02, size = 48, normalized size = 1.41

$$\frac{(cn+c)x^3 + \left(cx^3 + 3\,ax\right)\left(\frac{1}{3}\,cx^3 + ax\right)^n + 3\,(an+a)x}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="fricas")
```

```
[Out] 1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/(
n + 1)
```

giac [A] time = 0.42, size = 30, normalized size = 0.88

$$\frac{1}{3}\,cx^3 + ax + \frac{\left(\frac{1}{3}\,cx^3 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{c\,x^3}{3} + ax + \frac{\left(\frac{1}{3}c\,x^3 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x)
```

```
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(n+1)/(n+1)
```

maxima [A] time = 1.14, size = 54, normalized size = 1.59

$$\frac{1}{3}\,cx^3 + ax + \frac{\left(cx^3 + 3\,ax\right)e^{\left(n\log\left(cx^2+3\,a\right)+n\log(x)\right)}}{3^{n+1}n + 3^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x))/(3^(n +
 1)*n + 3^(n + 1))
```

mupad [B] time = 2.11, size = 33, normalized size = 0.97

$$\frac{x\,\left(c\,x^2+3\,a\right)\,\left(n+{\left(\frac{c\,x^3}{3}+a\,x\right)}^n+1\right)}{3\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^n + 1),x)
```

```
[Out] (x*(3*a + c*x^2)*(n + (a*x + (c*x^3)/3)^n + 1))/(3*(n + 1))
```

sympy [B] time = 112.00, size = 201, normalized size = 5.91

$$\begin{cases} \frac{3\cdot 3^{n} a n x}{3\cdot 3^{n} n+3\cdot 3^{n}} + \frac{3\cdot 3^{n} a x}{3\cdot 3^{n} n+3\cdot 3^{n}} + \frac{3^{n} c n x^{3}}{3\cdot 3^{n} n+3\cdot 3^{n}} + \frac{3^{n} c x^{3}}{3\cdot 3^{n} n+3\cdot 3^{n}} + \frac{3 a x \left(3 a x+c x^{3}\right)^{n}}{3\cdot 3^{n} n+3\cdot 3^{n}} + \frac{c x^{3} \left(3 a x+c x^{3}\right)^{n}}{3\cdot 3^{n} n+3\cdot 3^{n}} & \text{for}\: n \neq -1 \\ a x + \frac{c x^{3}}{3} + \log{\left (x \right)} + \log{\left (- \sqrt{3} i \sqrt{a} \sqrt{\frac{1}{c}} + x \right)} + \log{\left (\sqrt{3} i \sqrt{a} \sqrt{\frac{1}{c}} + x \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)
```

```
[Out] Piecewise((3*3**n*a*n*x/(3*3**n*n + 3*3**n) + 3*3**n*a*x/(3*3**n*n + 3*3**n
) + 3**n*c*n*x**3/(3*3**n*n + 3*3**n) + 3**n*c*x**3/(3*3**n*n + 3*3**n) + 3
*a*x*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n) + c*x**3*(3*a*x + c*x**3)**n/(
3*3**n*n + 3*3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(-sqrt(3)*I*s
qrt(a)*sqrt(1/c) + x) + log(sqrt(3)*I*sqrt(a)*sqrt(1/c) + x), True))
```

3.217 $\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=44

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1591}

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int \left(1+x^n\right)\,dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 0.95

$$\frac{x^2(3b+2cx)\left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+n+1\right)}{6(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n))
```

fricas [A] time = 0.64, size = 57, normalized size = 1.30

$$\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^n}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 + 1
/2*b*x^2)^n)/(n + 1)
```

giac [A] time = 0.39, size = 36, normalized size = 0.82

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 37, normalized size = 0.84

$$\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+\frac{\left(\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x)
```

```
[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(n+1)/(n+1)
```

maxima [A] time = 1.17, size = 71, normalized size = 1.61

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{\left(2\,cx^3+3\,bx^2\right)e^{\left(n\log(2\,cx+3\,b)+2\,n\log(x)\right)}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^(n*log(2*c*x + 3*b) + 2*n*log
(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

mupad [**B**] time = 2.21, size = 37, normalized size = 0.84

$$\frac{x^2\,\left(3\,b+2\,c\,x\right)\,\left(n+{\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}\right)}^n+1\right)}{6\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^n + 1),x)
```

```
[Out] (x^2*(3*b + 2*c*x)*(n + ((b*x^2)/2 + (c*x^3)/3)^n + 1))/(6*(n + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

3.218 $\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=50

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int (1+x^n)\,dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.19, size = 49, normalized size = 0.98

$$\frac{x(6a+x(3b+2cx))\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+n+1\right)}{6(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n))/(6*(1
 + n))
```

fricas [A] time = 0.83, size = 72, normalized size = 1.44

$$\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2+6\,ax\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^n+6\,(an+a)x}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="frica
s")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*c
*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)
```

giac [A] time = 0.39, size = 42, normalized size = 0.84

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac"
)
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 43, normalized size = 0.86

$$\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+ax+\frac{\left(\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(n+1)/(n+1)
```

maxima [A] time = 1.22, size = 83, normalized size = 1.66

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(2\,cx^3+3\,bx^2+6\,ax\right)e^{\left(n\log\left(2\,cx^2+3\,bx+6\,a\right)+n\log\left(x\right)\right)}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxim
a")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2
+ 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

mupad [B] time = 2.17, size = 73, normalized size = 1.46

$$a\,x+\left(\frac{3\,b\,x^2}{6\,n+6}+\frac{2\,c\,x^3}{6\,n+6}+\frac{6\,a\,x}{6\,n+6}\right)\,{\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x\right)}^n+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^n + 1)*(a + b*x + c*x^2),x)
```

```
[Out] a*x + ((3*b*x^2)/(6*n + 6) + (2*c*x^3)/(6*n + 6) + (6*a*x)/(6*n + 6))*(a*x
+ (b*x^2)/2 + (c*x^3)/3)^n + (b*x^2)/2 + (c*x^3)/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

3.219 $\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx$

Optimal. Leaf size=19

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

```
[Out] 1/6*(x^3+6*x^2-12*x+5)^2
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1588}

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] (5 - 12*x + 6*x^2 + x^3)^2/6
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=\frac{1}{6}\left(5-12x+6x^2+x^3\right)^2$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.74

$$\frac{x^6}{6}+2x^5+2x^4-\frac{67x^3}{3}+34x^2-20x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] -20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

fricas [A] time = 0.77, size = 29, normalized size = 1.53

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="fricas")
[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

giac [A] time = 0.35, size = 30, normalized size = 1.58

$$\frac{5}{3}x^3 + \frac{1}{6}\left(x^3 + 6x^2 - 12x\right)^2 + 10x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="giac")
[Out] 5/3*x^3 + 1/6*(x^3 + 6*x^2 - 12*x)^2 + 10*x^2 - 20*x
```

maple [A] time = 0.00, size = 30, normalized size = 1.58

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x)
[Out] 1/6*x^6+2*x^5+2*x^4-67/3*x^3+34*x^2-20*x
```

maxima [A] time = 0.43, size = 17, normalized size = 0.89

$$\frac{1}{6}\left(x^3 + 6x^2 - 12x + 5\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")
[Out] 1/6*(x^3 + 6*x^2 - 12*x + 5)^2
```

mupad [B] time = 0.03, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + x^2 - 4)*(6*x^2 - 12*x + x^3 + 5),x)
[Out] 34*x^2 - 20*x - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

sympy [A] time = 0.06, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
[Out] x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x
```

3.220 $\int \left(2x+x^3\right)\left(1+4x^2+x^4\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

[Out] 1/8*(x^4+4*x^2+1)^2

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1588}

$$\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] (1 + 4*x^2 + x^4)^2/8

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2x+x^3\right)\left(1+4x^2+x^4\right)\,dx=\frac{1}{8}\left(1+4x^2+x^4\right)^2$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.31

$$\frac{x^8}{8}+x^6+\frac{9x^4}{4}+x^2$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8

fricas [A] time = 0.82, size = 17, normalized size = 1.06

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

giac [A] time = 0.23, size = 22, normalized size = 1.38

$$\frac{1}{4}x^4 + \frac{1}{8}\left(x^4 + 4x^2\right)^2 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/4*x^4 + 1/8*(x^4 + 4*x^2)^2 + x^2
```

maple [A] time = 0.00, size = 18, normalized size = 1.12

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2*x)*(x^4+4*x^2+1),x)
[Out] 1/8*x^8+x^6+9/4*x^4+x^2
```

maxima [A] time = 0.44, size = 14, normalized size = 0.88

$$\frac{1}{8}\left(x^4 + 4x^2 + 1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")
[Out] 1/8*(x^4 + 4*x^2 + 1)^2
```

mupad [B] time = 0.03, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^3)*(4*x^2 + x^4 + 1),x)
```

```
[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8
```

sympy [A] time = 0.06, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2*x)*(x**4+4*x**2+1),x)
```

```
[Out] x**8/8 + x**6 + 9*x**4/4 + x**2
```

3.221 $\int(1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10}-36x^7(x+1)^7+81x^4(x+1)^4$$

```
[Out] 81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

Rubi [B] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1593, 1612}

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \int x^3(1+x)^3(1+2x)\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$
$$= \int \left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804x^8-125\right.$$
$$= 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-12$$

Mathematica [**B**] time = 0.01, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

fricas [**B**] time = 0.72, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

giac [**A**] time = 0.29, size = 28, normalized size = 0.85

$$\frac{49}{10}\left(x^2+x\right)^{10}-36\left(x^2+x\right)^7+81\left(x^2+x\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

maple [B] time = 0.00, size = 87, normalized size = 2.64

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-171x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x)
```

```
[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*x^14+336*x
^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8+288*x^7+486*x^6+324
*x^5+81*x^4
```

maxima [B] time = 0.44, size = 86, normalized size = 2.61

$$\frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13}-\frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^{9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

mupad [B] time = 0.22, size = 86, normalized size = 2.61

$$\frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13}-\frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756\,x^{9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(x + x^2)^3*(7*(x + x^2)^3 - 18)^2,x)
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

sympy [B] time = 0.08, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^{9}-171x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

3.222 $\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10}-36x^7(x+1)^7+81x^4(x+1)^4$$

```
[Out] 81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

Rubi [B] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1612}

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx=\int\left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804x^8-12\right.$$

$$=81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1$$

Mathematica [B] time = 0.00, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

fricas [B] time = 0.72, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-171x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

giac [A] time = 0.36, size = 28, normalized size = 0.85

$$\frac{49}{10}\left(x^{2}+x\right)^{10}-36\left(x^{2}+x\right)^{7}+81\left(x^{2}+x\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

maple [B] time = 0.00, size = 87, normalized size = 2.64

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-171x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x+1)^3*(2*x+1)*(-18+7*x^3*(x+1)^3)^2,x)
```

```
[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*x^14+336*x
^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8+288*x^7+486*x^6+324
*x^5+81*x^4
```

maxima [B] time = 0.45, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

mupad [B] time = 0.19, size = 86, normalized size = 2.61

$$\frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13}-\frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756\,x^9-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x + 1)*(7*x^3*(x + 1)^3 - 18)^2*(x + 1)^3,x)
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

sympy [B] time = 0.09, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

3.223 $\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} dx$

Optimal. Leaf size=14

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

```
[Out] 1/12/(x^3-6*x+1)^4
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1588}

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} dx = \frac{1}{12\left(1-6x+x^3\right)^4}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

fricas [**B**] time = 0.83, size = 57, normalized size = 4.07

$$\frac{1}{12\left(x^{12}-24\,x^{10}+4\,x^{9}+216\,x^{8}-72\,x^{7}-858\,x^{6}+432\,x^{5}+1224\,x^{4}-860\,x^{3}+216\,x^{2}-24\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224*
x^4 - 860*x^3 + 216*x^2 - 24*x + 1)
```

giac [**A**] time = 0.32, size = 12, normalized size = 0.86

$$\frac{1}{12\left(x^{3}-6\,x+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="giac")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{12\left(x^{3}-6x+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2)/(x^3-6*x+1)^5,x)
```

```
[Out] 1/12/(x^3-6*x+1)^4
```

maxima [**A**] time = 0.44, size = 12, normalized size = 0.86

$$\frac{1}{12\left(x^{3}-6\,x+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

mupad [B] time = 2.10, size = 12, normalized size = 0.86

$$\frac{1}{12\left(x^3-6\,x+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x)
```

```
[Out] 1/(12*(x^3 - 6*x + 1)^4)
```

sympy [B] time = 0.19, size = 56, normalized size = 4.00

$$\frac{1}{12x^{12}-288x^{10}+48x^9+2592x^8-864x^7-10296x^6+5184x^5+14688x^4-10320x^3+2592x^2-288x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2)/(x**3-6*x+1)**5,x)
```

```
[Out] 1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 518
4*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)
```

3.224 $\int \frac{2x+x^2}{4+3x^2+x^3} \, dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1587}

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{2x+x^2}{4+3x^2+x^3} \, dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

fricas [A] time = 0.79, size = 13, normalized size = 0.87

$$\frac{1}{3}\,\log\left(x^{3}+3\,x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

giac [A] time = 0.37, size = 14, normalized size = 0.93

$$\frac{1}{3}\,\log\left(\left|x^{3}+3\,x^{2}+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(x^{3}+3x^{2}+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x^3+3*x^2+4),x)
[Out] 1/3*ln(x^3+3*x^2+4)
```

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{3}\,\log\left(x^{3}+3\,x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^{3}+3\,x^{2}+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2)/(3*x^2 + x^3 + 4),x)
```

```
[Out] log(3*x^2 + x^3 + 4)/3
```

sympy [A] time = 0.09, size = 12, normalized size = 0.80

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

```
[Out] log(x**3 + 3*x**2 + 4)/3
```

3.225 $\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

```
[Out] 1/4*ln(x^4+2*x^2+4*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1587}

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^3+2x+4\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

fricas [A]   time = 0.80, size = 15, normalized size = 0.88

$$\frac{1}{4}\,\log\left(x^4+2\,x^2+4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

giac [A]   time = 0.27, size = 18, normalized size = 1.06

$$\frac{1}{4}\,\log\left(4\left|\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
[Out] 1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))
```

maple [A]   time = 0.00, size = 14, normalized size = 0.82

$$\frac{\ln\left(\left(x^3+2x+4\right)x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x)
[Out] 1/4*ln(x*(x^3+2*x+4))
```

maxima [A]   time = 0.45, size = 15, normalized size = 0.88

$$\frac{1}{4}\,\log\left(x^4+2\,x^2+4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

mupad [B]   time = 0.07, size = 13, normalized size = 0.76

$$\frac{\ln\left(x\,\left(x^3+2\,x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)
```

```
[Out] log(x*(2*x + x^3 + 4))/4
```

sympy [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4+2x^2+4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

```
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

3.226 $\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{\left(c+dx+ex^2+fx^3\right)^2} dx$

Optimal. Leaf size=40

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

```
[Out] a/(f*x^3+e*x^2+d*x+c)+b*x/(f*x^3+e*x^2+d*x+c)
```

Rubi [A] time = 0.10, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {6, 2102, 1588}

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^
2 + f*x^3)^2,x]
```

```
[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2102

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn
, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{bc - ad - 2aex - bex^2 - 3afx^2 - 2bfx^3}{\left(c + dx + ex^2 + fx^3\right)^2}\,dx = \int \frac{bc - ad - 2aex + (-be - 3af)x^2 - 2bfx^3}{\left(c + dx + ex^2 + fx^3\right)^2}\,dx$$

$$= \frac{bx}{c + dx + ex^2 + fx^3} - \frac{\int \frac{2adf + 4aefx + 6af^2x^2}{\left(c+dx+ex^2+fx^3\right)^2}\,dx}{2f}$$

$$= \frac{a}{c + dx + ex^2 + fx^3} + \frac{bx}{c + dx + ex^2 + fx^3}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 0.58

$$\frac{a + bx}{c + dx + ex^2 + fx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x
+ e*x^2 + f*x^3)^2,x]
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

fricas [A] time = 0.86, size = 23, normalized size = 0.58

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="fricas")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

giac [A] time = 0.54, size = 24, normalized size = 0.60

$$\frac{bx + a}{fx^3 + x^2e + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/(f*x^3 + x^2*e + d*x + c)
```

maple [A] time = 0.01, size = 28, normalized size = 0.70

$$-\frac{-bx-a}{f\,x^3+e\,x^2+dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x)
```

```
[Out] -(-b*x-a)/(f*x^3+e*x^2+d*x+c)
```

maxima [A] time = 0.50, size = 23, normalized size = 0.58

$$\frac{bx+a}{fx^3+ex^2+dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

mupad [B] time = 0.11, size = 23, normalized size = 0.58

$$\frac{a+b\,x}{f\,x^3+e\,x^2+d\,x+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*d - b*c + 2*a*e*x + 3*a*f*x^2 + b*e*x^2 + 2*b*f*x^3)/(c + d*x + e*x
^2 + f*x^3)^2,x)
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

sympy [A] time = 32.56, size = 22, normalized size = 0.55

$$-\frac{-a-bx}{c+dx+ex^2+fx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2+
d*x+c)**2,x)
```

```
[Out] -(-a - b*x)/(c + d*x + e*x**2 + f*x**3)
```

3.227 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$

Optimal. Leaf size=605

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b\right.\right.}{\sqrt{2}\,a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

```
[Out] -1/4*ln(2*a+2*a*x^2+x*(b-(8*a^2-4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b-(8*a^2-4
*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c+b^2)^(1/2)+1/4*ln(2*a+2*a*x^2+x*(b+(8*a^2-
4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b+(8*a^2-4*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c
+b^2)^(1/2)+1/2*arctan(1/2*(b+4*a*x-(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4*a^2
+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*(4*a^2*B+b*D*(b-(8*a^2-4*a*c+b
^2)^(1/2))-a*(b*C+2*c*D+A*(b-(8*a^2-4*a*c+b^2)^(1/2))-C*(8*a^2-4*a*c+b^2)^(
1/2)))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2
)^(1/2)))^(1/2)-1/2*arctan(1/2*(b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4
*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*(4*a^2*B+b*D*(b+(8*a^2-4*a
*c+b^2)^(1/2))-a*(b*C+2*c*D+C*(8*a^2-4*a*c+b^2)^(1/2)+A*(b+(8*a^2-4*a*c+b^2
)^(1/2))))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c
+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 4.54, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2086, 634, 618, 204, 628}

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b\right.\right.}{\sqrt{2}\,a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 + b^
2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sqrt[
8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a
^2 + b^2 - 4*a*c])])])/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*
a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 +
 b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2
+ b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(
```

```
Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]
*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 -
 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (
b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]
) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[
8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = -\frac{\int \frac{Ab-2aB-A\sqrt{8a^2+b^2-4ac}+2aD+\left(2aA-2aC+bD-\sqrt{8a^2+b^2-4ac}\,D\right)x}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{\sqrt{8a^2+b^2-4ac}} + \frac{\int \frac{Ab-2aB+A\sqrt{8a}}{}}{}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\int \frac{b-\sqrt{8a^2+b^2-4ac}+4ax}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{4a\sqrt{8a^2+b^2-4ac}} + \frac{(2a}{}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x\right.}{4a\sqrt{8a^2+b^2-4ac}}$$

$$= \frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8}\right.\right.}{\sqrt{2}\,a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(\right.}}$$

Mathematica [C] time = 0.07, size = 98, normalized size = 0.16

$$\text{RootSum}\left[\#1^4a+\#1^3b+\#1^2c+\#1b+a\&,\frac{\#1^3D\log(x-\#1)+\#1^2C\log(x-\#1)+A\log(x-\#1)+\#1B\log(x}{4\#1^3a+3\#1^2b+2\#1c+b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] + B*Log[x -
#1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2 +
4*a*#1^3) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.16Not invertible Error: B
ad Argument Value
```

maple [B] time = 0.09, size = 2105, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x)
```

```
[Out] -1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)
*A+1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*
a)*C+1/4/a*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D-1/4/a/(8*a^2-4*
a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D*b-1/(8*a^2+
4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+
b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*A+1/(8
*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)
*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*
a*c+b^2)^(1/2))^(1/2))*b*A-1/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2)
)^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8
*a^2-4*a*c+b^2)^(1/2))^(1/2))*C+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^
2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)
-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*C+2/(8*a^2-4*a
*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan(
(-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)
^(1/2))^(1/2))*D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^
2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+
4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b^2+1/a/(8*a^2+4*a*c-2*b^
2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)
-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b-4*a/(8*a^2-4
*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arcta
n((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^
2)^(1/2))^(1/2))*B+1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2)
^(1/2)*x+b*x+2*a)*A-1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2
)^(1/2)*x+b*x+2*a)*C+1/4/a*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+b*x+2*a)*D+
```

```
1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+b*x+2*a)
*D*b+1/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*
x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^
(1/2))*A+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)
^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2
*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*A+1/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*
c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2
*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*C+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2
+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*
c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*C+2/
(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/
2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4
*a*c+b^2)^(1/2))^(1/2))*D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-
2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))
/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b^2-1/a/(8*a^2+4*
a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b
^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b-4*a/(
8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2
)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*
a*c+b^2)^(1/2))^(1/2))*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Dx^3 + Cx^2 + Bx + A}{ax^4 + bx^3 + cx^2 + bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxim
a")
```

```
[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x + C\,x^2 + x^3\,D}{a\,x^4 + b\,x^3 + c\,x^2 + b\,x + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.228 $\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx$

Optimal. Leaf size=63

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

```
[Out] -2*ln(2+2*x^2-x*(-5^(1/2)+1))/(-5^(1/2)+1)-2*ln(2+2*x^2-x*(5^(1/2)+1))/(5^(
1/2)+1)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2086, 628}

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt[
5])*x + 2*x^2])/(1 + Sqrt[5])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{\int \frac{-2\sqrt{5}+\left(10-2\sqrt{5}\right)x}{2+\left(-1-\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}} + \frac{\int \frac{2\sqrt{5}+\left(10+2\sqrt{5}\right)x}{2+\left(-1+\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}}$$
$$= -\frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}} - \frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.87

$$\frac{1}{2}\left(\left(1+\sqrt{5}\right)\log\left(2x^2+\left(\sqrt{5}-1\right)x+2\right)-\left(\sqrt{5}-1\right)\log\left(-2x^2+\sqrt{5}x+x-2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5])*Log[2 +
(-1 + Sqrt[5])*x + 2*x^2])/2
```

fricas [A] time = 0.76, size = 83, normalized size = 1.32

$$\frac{1}{2}\sqrt{5}\log\left(\frac{2x^4-2x^3+7x^2+\sqrt{5}\left(2x^3-x^2+2x\right)-2x+2}{x^4-x^3+x^2-x+1}\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2*x) - 2*x
+ 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

giac [A] time = 0.27, size = 58, normalized size = 0.92

$$-\frac{1}{2}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{2}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/2*sqrt(5)*log(x^2 + 1/2
*x*(sqrt(5) - 1) + 1) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

maple [A] time = 0.05, size = 82, normalized size = 1.30

$$\frac{\ln\left(2x^2-\sqrt{5}\,x-x+2\right)}{2}-\frac{\sqrt{5}\,\ln\left(2x^2-\sqrt{5}\,x-x+2\right)}{2}+\frac{\ln\left(2x^2+\sqrt{5}\,x-x+2\right)}{2}+\frac{\sqrt{5}\,\ln\left(2x^2+\sqrt{5}\,x-x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x)
```

```
[Out] 1/2*ln(x*5^(1/2)+2*x^2-x+2)+1/2*ln(x*5^(1/2)+2*x^2-x+2)*5^(1/2)+1/2*ln(-x*5
^(1/2)+2*x^2-x+2)-1/2*ln(-x*5^(1/2)+2*x^2-x+2)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3-4\,x^2+x+2}{x^4-x^3+x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)
```

mupad [B] time = 0.18, size = 75, normalized size = 1.19

$$\frac{\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}-\frac{\sqrt{5}\,\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\sqrt{5}\,\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 4*x^2 + 2*x^3 + 2)/(x^2 - x - x^3 + x^4 + 1),x)
```

```
[Out] log(x^2 - (5^(1/2)*x)/2 - x/2 + 1)/2 + log((5^(1/2)*x)/2 - x/2 + x^2 + 1)/2
 - (5^(1/2)*log(x^2 - (5^(1/2)*x)/2 - x/2 + 1))/2 + (5^(1/2)*log((5^(1/2)*x
)/2 - x/2 + x^2 + 1))/2
```

sympy [A] time = 0.13, size = 58, normalized size = 0.92

$$\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+1\right)+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
```

```
[Out] (1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)*
log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```

3.229 $\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

```
[Out] 1/3/(1+x)^3+ln(1+x)
```

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1594, 1680, 14}

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \int \frac{x\left(3+3x+x^2\right)}{1+4x+6x^2+4x^3+x^4}\,dx$$
$$= \mathrm{Subst}\left(\int \frac{-1+x^3}{x^4}\,dx, x, 1+x\right)$$
$$= \mathrm{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x}\right)dx, x, 1+x\right)$$
$$= \frac{1}{3(1+x)^3}+\log(1+x)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

fricas [B] time = 0.89, size = 38, normalized size = 2.71

$$\frac{3\left(x^3+3\,x^2+3\,x+1\right)\log\left(x+1\right)+1}{3\left(x^3+3\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
[Out] 1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)
```

giac [A] time = 0.29, size = 13, normalized size = 0.93

$$\frac{1}{3\left(x+1\right)^3}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
[Out] 1/3/(x + 1)^3 + log(abs(x + 1))
```

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$\ln(x+1)+\frac{1}{3(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x)
[Out] 1/3/(x+1)^3+ln(x+1)
```

maxima [A] time = 0.44, size = 22, normalized size = 1.57

$$\frac{1}{3\left(x^3+3x^2+3x+1\right)}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
[Out] 1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.86

$$\ln(x+1)+\frac{1}{3(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 3*x^2 + x^3)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
[Out] log(x + 1) + 1/(3*(x + 1)^3)
```

sympy [A] time = 0.10, size = 20, normalized size = 1.43

$$\log(x+1)+\frac{1}{3x^3+9x^2+9x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
[Out] log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

3.230 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=28

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

```
[Out] 8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1680, 43}

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \text{Subst}\left(\int \frac{(-2+x)^3}{x^4}\,dx, x, 1+x\right)$$
$$= \text{Subst}\left(\int \left(-\frac{8}{x^4}+\frac{12}{x^3}-\frac{6}{x^2}+\frac{1}{x}\right)dx, x, 1+x\right)$$
$$= \frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\log(1+x)$$

Mathematica [**A**] time = 0.01, size = 24, normalized size = 0.86

$$\frac{2\left(9x^2+9x+4\right)}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

fricas [**A**] time = 1.00, size = 46, normalized size = 1.64

$$\frac{18\,x^2+3\left(x^3+3\,x^2+3\,x+1\right)\log\left(x+1\right)+18\,x+8}{3\left(x^3+3\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^2
 + 3*x + 1)
```

giac [**A**] time = 0.39, size = 23, normalized size = 0.82

$$\frac{2\left(9\,x^2+9\,x+4\right)}{3\,(x+1)^3}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

maple [A] time = 0.01, size = 27, normalized size = 0.96

$$\ln(x+1)+\frac{8}{3(x+1)^3}-\frac{6}{(x+1)^2}+\frac{6}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)
[Out] 8/3/(x+1)^3-6/(x+1)^2+6/(x+1)+ln(x+1)
```

maxima [A] time = 0.45, size = 32, normalized size = 1.14

$$\frac{2\left(9x^2+9x+4\right)}{3\left(x^3+3x^2+3x+1\right)}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.75

$$\ln(x+1)+\frac{6x^2+6x+\frac{8}{3}}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
[Out] log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3
```

sympy [A] time = 0.11, size = 29, normalized size = 1.04

$$\frac{18x^2+18x+8}{3x^3+9x^2+9x+3}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

3.231 $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=59

$$\frac{13x}{x^4+2x^2+3}-\frac{2\left(13x^2+18\right)x}{\left(x^4+2x^2+3\right)^2}+\frac{2\left(1-2x^2\right)}{\left(x^4+2x^2+3\right)^2}$$

```
[Out] 2*(-2*x^2+1)/(x^4+2*x^2+3)^2-2*x*(13*x^2+18)/(x^4+2*x^2+3)^2+13*x/(x^4+2*x^
2+3)
```

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {1673, 1678, 1588, 1663, 1660, 8}

$$\frac{13x}{x^4+2x^2+3}-\frac{2\left(13x^2+18\right)x}{\left(x^4+2x^2+3\right)^2}+\frac{2\left(1-2x^2\right)}{\left(x^4+2x^2+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x
^4)^3,x]
```

```
[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4)
^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
```

```
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{\left(3+2x^2+x^4\right)^3}\,dx = \int \frac{x\left(-40+24x^4\right)}{\left(3+2x^2+x^4\right)^3}\,dx + \int \frac{9-18x^2+174x^4+26x^6-39}{\left(3+2x^2+x^4\right)^3}$$

$$= -\frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{3744-2496x^2-3744x^4}{\left(3+2x^2+x^4\right)^2}\,dx +$$

$$= \frac{2\left(1-2x^2\right)}{\left(3+2x^2+x^4\right)^2} - \frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{13x}{3+2x^2+x^4} + \frac{1}{32}$$

$$= \frac{2\left(1-2x^2\right)}{\left(3+2x^2+x^4\right)^2} - \frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{13x}{3+2x^2+x^4}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{13x^5-4x^2+3x+2}{\left(x^4+2x^2+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x
^2 + x^4)^3,x]
```

```
[Out] (2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

fricas [A] time = 0.84, size = 38, normalized size = 0.64

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="fricas")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [A] time = 1.44, size = 28, normalized size = 0.47

$$\frac{13\,x^5-4\,x^2+3\,x+2}{\left(x^4+2\,x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="giac")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2
```

maple [A] time = 0.01, size = 30, normalized size = 0.51

$$-\frac{-13x^5+4x^2-3x-2}{\left(x^4+2x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x)
```

```
[Out] -(-13*x^5+4*x^2-3*x-2)/(x^4+2*x^2+3)^2
```

maxima [A] time = 0.46, size = 38, normalized size = 0.64

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="maxima")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

mupad [B] time = 0.05, size = 28, normalized size = 0.47

$$\frac{13\,x^5-4\,x^2+3\,x+2}{\left(x^4+2\,x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((174*x^4 - 18*x^2 - 40*x + 24*x^5 + 26*x^6 - 39*x^8 + 9)/(2*x^2 + x^4 +
 3)^3,x)
```

```
[Out] (3*x - 4*x^2 + 13*x^5 + 2)/(2*x^2 + x^4 + 3)^2
```

sympy [A] time = 0.18, size = 36, normalized size = 0.61

$$-\frac{-13x^5+4x^2-3x-2}{x^8+4x^6+10x^4+12x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+3
)**3,x)
```

```
[Out] -(-13*x**5 + 4*x**2 - 3*x - 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

3.232 $\int \frac{-1+4x^5}{\left(1+x+x^5\right)^2} \, dx$

Optimal. Leaf size=11

$$-\frac{x}{x^5+x+1}$$

[Out] -x/(x^5+x+1)

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1588}

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+4x^5}{\left(1+x+x^5\right)^2} \, dx = -\frac{x}{1+x+x^5}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

fricas [A] time = 0.73, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
[Out] -x/(x^5 + x + 1)
```

giac [A] time = 0.33, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
[Out] -x/(x^5 + x + 1)
```

maple [B] time = 0.01, size = 41, normalized size = 3.73

$$-\frac{-3x^2+5x-1}{7\left(x^3-x^2+1\right)}+\frac{-3x-1}{7x^2+7x+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x)
[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)
```

maxima [A] time = 0.44, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
[Out] -x/(x^5 + x + 1)
```

mupad [B] time = 2.30, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
```

```
[Out] -x/(x + x^5 + 1)
```

sympy [A] time = 0.12, size = 8, normalized size = 0.73

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

3.233 $\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{x}{16\left(1-x^2\right)}+\frac{\left(29-5x^2\right)x}{32\left(x^4-6x^2+1\right)}+\frac{1}{4}\tanh^{-1}(x)+\frac{1}{64}\left(\left(3-2\sqrt{2}\right)\tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right)-\left(3+2\sqrt{2}\right)\tanh^{-1}\left(\left(1+\sqrt{2}\right.\right.\right.$$

```
[Out] 1/16*x/(-x^2+1)+1/32*x*(-5*x^2+29)/(x^4-6*x^2+1)+1/4*arctanh(x)+1/64*arctan
h(x*(2^(1/2)-1))*(3-2*2^(1/2))-1/64*arctanh(x*(1+2^(1/2)))*(3+2*2^(1/2))
```

Rubi [B] time = 0.15, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2073, 207, 638, 618, 206, 632, 31}

$$-\frac{12-5x}{64\left(-x^2+2x+1\right)}+\frac{5x+12}{64\left(-x^2-2x+1\right)}+\frac{1}{32(1-x)}-\frac{1}{32(x+1)}-\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)-\frac{3}{256}\left(2-3\sqrt{\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] 1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 - 5
*x)/(64*(1 + 2*x - x^2)) - (5*ArcTanh[(1 - x)/Sqrt[2]])/(64*Sqrt[2]) + ArcT
anh[x]/4 + (5*ArcTanh[(1 + x)/Sqrt[2]])/(64*Sqrt[2]) - (3*(2 + 3*Sqrt[2])*L
og[1 - Sqrt[2] - x])/256 - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 + (
3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] + x])/256 + (3*(2 - 3*Sqrt[2])*Log[1 + Sq
rt[2] + x])/256
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx = \int \left(\frac{1}{32(-1+x)^2} + \frac{1}{32(1+x)^2} - \frac{1}{4\left(-1+x^2\right)} + \frac{17-7x}{32\left(-1-2x+x^2\right)^2} - \frac{3(-4+x}{64\left(-1-2x\right.}\right.$$

$$= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{1}{32}\int \frac{17-7x}{\left(-1-2x+x^2\right)^2}\,dx + \frac{1}{32}\int \frac{17+7x}{\left(-1+2x+x^2\right)^2}\,dx -$$

$$= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64\left(1-2x-x^2\right)} - \frac{12-5x}{64\left(1+2x-x^2\right)} + \frac{1}{4}\tanh^{-1}(x) - \frac{5}{64}$$

$$= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64\left(1-2x-x^2\right)} - \frac{12-5x}{64\left(1+2x-x^2\right)} + \frac{1}{4}\tanh^{-1}(x) - \frac{3}{256}$$

$$= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64\left(1-2x-x^2\right)} - \frac{12-5x}{64\left(1+2x-x^2\right)} - \frac{5\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}} +$$

Mathematica [A] time = 0.09, size = 132, normalized size = 1.45

$$\frac{1}{128}\left(-\frac{4x\left(7x^4-46x^2+31\right)}{x^6-7x^4+7x^2-1} - 16\log(1-x) + \left(3+2\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right) + \left(2\sqrt{2}-3\right)\log\left(-x+\sqrt{2}+1\right) + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] ((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] +
(3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2] -
x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt[2]
)*Log[1 + Sqrt[2] + x])/128
```

fricas [B] time = 0.94, size = 223, normalized size = 2.45

$$-\frac{28x^5 - 184x^3 - 2\sqrt{2}\left(x^6-7x^4+7x^2-1\right)\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right) - 2\sqrt{2}\left(x^6-7x^4+7x^2-1\right)\log\left(\frac{x^2-2\sqrt{2}(x-1}{x^2-2x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2
*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x
^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^6 -
```

```
 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^
2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x^4 +
7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

giac [A] time = 0.37, size = 134, normalized size = 1.47

$$\frac{1}{64}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}-2\right|}{\left|2x+2\sqrt{2}-2\right|}\right)-\frac{7x^5-46x^3+31x}{32\left(x^6-7x^4+7x^2-1\right)}+\frac{3}{128}\log\left(\left|x^2+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/64*
sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/32*(7*x^
5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(abs(x^2 + 2*x - 1)
) - 3/128*log(abs(x^2 - 2*x - 1)) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1
))
```

maple [A] time = 0.02, size = 116, normalized size = 1.27

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{32}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{32}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{3\ln\left(x^2-2x-1\right)}{128}+\frac{3\ln\left(x^2+2x-1\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x)
```

```
[Out] -1/32/(x-1)-1/8*ln(x-1)+1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x^2+2*x-1)-1/32
*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/32/(x+1)+1/8*ln(x+1)-1/64*(-12+5*x)
/(x^2-2*x-1)-3/128*ln(x^2-2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*x-2)*2^(1/2))
```

maxima [A] time = 0.99, size = 114, normalized size = 1.25

$$\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right)-\frac{7x^5-46x^3+31x}{32\left(x^6-7x^4+7x^2-1\right)}+\frac{3}{128}\log\left(x^2+2x-1\right)-\frac{3}{128}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/64*sqrt(2)*log((x
 - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*
x^4 + 7*x^2 - 1) + 3/128*log(x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) + 1/
8*log(x + 1) - 1/8*log(x - 1)
```

mupad [B] time = 2.37, size = 124, normalized size = 1.36

$$-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}-\frac{\frac{7\,x^5}{32}-\frac{23\,x^3}{16}+\frac{31\,x}{32}}{x^6-7\,x^4+7\,x^2-1}+\operatorname{atan}\left(\frac{x\,23313\mathrm{i}}{8192\,\left(\frac{27309\,\sqrt{2}}{32768}-\frac{19317}{16384}\right)}-\frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\,\left(\frac{27309\,\sqrt{2}}{32768}-\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}-\frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] atan((x*23313i)/(8192*((27309*2^(1/2))/32768 - 19317/16384)) - (2^(1/2)*x*6
5943i)/(32768*((27309*2^(1/2))/32768 - 19317/16384)))*((2^(1/2)*1i)/32 - 3i
/64) - ((31*x)/32 - (23*x^3)/16 + (7*x^5)/32)/(7*x^2 - 7*x^4 + x^6 - 1) - (
atan(x*1i)*1i)/4 + atan((x*23313i)/(8192*((27309*2^(1/2))/32768 + 19317/163
84)) + (2^(1/2)*x*65943i)/(32768*((27309*2^(1/2))/32768 + 19317/16384)))*((
2^(1/2)*1i)/32 + 3i/64)
```

sympy [B] time = 1.36, size = 272, normalized size = 2.99

$$\frac{-7x^5+46x^3-31x}{32x^6-224x^4+224x^2-32}-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}+\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555}{909328}-\frac{38423555\sqrt{2}}{1363992}+\frac{9549}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

```
[Out] (-7*x**5 + 46*x**3 - 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) - log(x - 1
)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 38423555/909328 - 384235
55*sqrt(2)/1363992 + 9549859782656*(-3/128 - sqrt(2)/64)**5/170499 - 562673
74592*(-3/128 - sqrt(2)/64)**3/56833) + (-3/128 + sqrt(2)/64)*log(x - 38423
555/909328 + 9549859782656*(-3/128 + sqrt(2)/64)**5/170499 - 56267374592*(-
3/128 + sqrt(2)/64)**3/56833 + 38423555*sqrt(2)/1363992) + (3/128 - sqrt(2)
/64)*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(3/128 - sqrt(2)/64)**3
/56833 + 9549859782656*(3/128 - sqrt(2)/64)**5/170499 + 38423555/909328) +
(sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(2)/64 + 3/128)**3/56833 + 95
49859782656*(sqrt(2)/64 + 3/128)**5/170499 + 38423555*sqrt(2)/1363992 + 384
23555/909328)
```

3.234 $\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$

Optimal. Leaf size=25

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {1590}

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3
 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = x^{1+m}\left(a+\right.$$

Mathematica [A] time = 0.37, size = 23, normalized size = 0.92

$$x^{m+1}(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x
*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

fricas [A] time = 1.25, size = 40, normalized size = 1.60

$$\left(dx^4 + cx^3 + bx^2 + ax\right)\left(dx^3 + cx^2 + bx + a\right)^p x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

giac [B] time = 2.09, size = 99, normalized size = 3.96

$$\left(dx^3 + cx^2 + bx + a\right)^p dx^4x^m + \left(dx^3 + cx^2 + bx + a\right)^p cx^3x^m + \left(dx^3 + cx^2 + bx + a\right)^p bx^2x^m + \left(dx^3 + cx^2 + bx + a\right)^p ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*x
^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*a*
x*x^m
```

maple [A] time = 0.01, size = 26, normalized size = 1.04

$$x^{m+1} \left(d\,x^3 + c\,x^2 + bx + a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3
*p)*x))),x)
```

```
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.70, size = 44, normalized size = 1.76

$$\left(dx^4 + cx^3 + bx^2 + ax\right)e^{\left(p\log\left(dx^3+cx^2+bx+a\right)+m\log\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x))
```

mupad [B] time = 2.66, size = 49, normalized size = 1.96

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(a\,x\,x^m+b\,x^m\,x^2+c\,x^m\,x^3+d\,x^m\,x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*(m + 1) + x*(x*(c*(m + 2*p + 3) + d*x*(m + 3*p + 4)) + b*(m + p
+ 2)))*(a + b*x + c*x^2 + d*x^3)^p,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x*x^m + b*x^m*x^2 + c*x^m*x^3 + d*x^m*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p
)+d*(4+m+3*p)*x))),x)
```

```
[Out] Timed out
```

3.235 $\int x^2 \left(a+bx+cx^2+dx^3\right)^p \left(3a+b(4+p)x+c(5+2p)x\right.$

Optimal. Leaf size=23

$$x^3\left(a+bx+cx^2+dx^3\right)^{p+1}$$

[Out] x^3*(d*x^3+c*x^2+b*x+a)^(1+p)

Rubi [A] time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1588}

$$x^3\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3),x]

[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx=x^3\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.23, size = 21, normalized size = 0.91

$$x^3(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3),x]

[Out] x^3*(a + x*(b + x*(c + d*x)))^(1 + p)

fricas [A] time = 0.71, size = 39, normalized size = 1.70

$$\left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="fricas")
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 1.89, size = 89, normalized size = 3.87

$$\left(dx^3+cx^2+bx+a\right)^p dx^6+\left(dx^3+cx^2+bx+a\right)^p cx^5+\left(dx^3+cx^2+bx+a\right)^p bx^4+\left(dx^3+cx^2+bx+a\right)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$x^3\left(d\,x^3+c\,x^2+bx+a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x
)
```

```
[Out] x^3*(d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.66, size = 39, normalized size = 1.70

$$\left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="maxima")
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.55, size = 39, normalized size = 1.70

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(d\,x^6+c\,x^5+b\,x^4+a\,x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) +
d*x^3*(3*p + 6)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6+
3*p)*x**3),x)
```

```
[Out] Timed out
```

3.236 $\int x\left(a+bx+cx^2+dx^3\right)^p \left(2a+b(3+p)x+c(4+2p)x\right.$

Optimal. Leaf size=23

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1588}

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*
(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx=x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 0.91

$$x^2(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^
2 + d*(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

fricas [A] time = 0.89, size = 39, normalized size = 1.70

$$\left(dx^5 + cx^4 + bx^3 + ax^2\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="fricas")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 0.73, size = 89, normalized size = 3.87

$$\left(dx^3 + cx^2 + bx + a\right)^p dx^5 + \left(dx^3 + cx^2 + bx + a\right)^p cx^4 + \left(dx^3 + cx^2 + bx + a\right)^p bx^3 + \left(dx^3 + cx^2 + bx + a\right)^p ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$x^2 \left(d\, x^3 + c\, x^2 + bx + a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x)
```

```
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.63, size = 39, normalized size = 1.70

$$\left(dx^5 + cx^4 + bx^3 + ax^2\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="maxima")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.32, size = 39, normalized size = 1.70

$$\left(d\,x^3 + c\,x^2 + b\,x + a\right)^p\,\left(d\,x^5 + c\,x^4 + b\,x^3 + a\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d*
x^3*(3*p + 5)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*p
)*x**3),x)
```

```
[Out] Timed out
```

3.237 $\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+\right.$

Optimal. Leaf size=21

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] x*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1588}

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 +
 3*p)*x^3),x]
```

```
[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = x\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.12, size = 19, normalized size = 0.90

$$x(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 +
d*(4 + 3*p)*x^3),x]
```

```
[Out] x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

fricas [A] time = 0.87, size = 37, normalized size = 1.76

$$\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 1.08, size = 87, normalized size = 4.14

$$\left(dx^3+cx^2+bx+a\right)^p dx^4+\left(dx^3+cx^2+bx+a\right)^p cx^3+\left(dx^3+cx^2+bx+a\right)^p bx^2+\left(dx^3+cx^2+bx+a\right)^p ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x
```

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$x\left(d\,x^3+c\,x^2+bx+a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(a+b*(p+2)*x+c*(2*p+3)*x^2+d*(4+3*p)*x^3),x)
```

```
[Out] x*(d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.61, size = 37, normalized size = 1.76

$$\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.27, size = 37, normalized size = 1.76

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(d\,x^4+c\,x^3+b\,x^2+a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3*
(3*p + 4)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x*
*3),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{(a+bx+cx^2+dx^3)^p (b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x} dx$

Optimal. Leaf size=19

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1585, 1588}

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*
p)*x^3))/x,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx+cx^2+dx^3\right)^p \left(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3\right)}{x}\,dx &= \int \left(b(1+p)+c(2+2p)x+d(3+3p)x^2\right. \\
&= \left(a+bx+cx^2+dx^3\right)^{1+p}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(
3 + 3*p)*x^3))/x,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)
```

fricas [A] time = 0.90, size = 33, normalized size = 1.74

$$\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 0.30, size = 52, normalized size = 2.74

$$\frac{\left(dx^3+cx^2+bx+a\right)^{p+1}p}{p+1}+\frac{\left(dx^3+cx^2+bx+a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p
+ 1)/(p + 1)
```

maple [A] time = 0.01, size = 20, normalized size = 1.05

$$\left(d\,x^3+c\,x^2+bx+a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.62, size = 33, normalized size = 1.74

$$\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.19, size = 19, normalized size = 1.00

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 + d
*x^3)^p)/x,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**3
)/x,x)
```

```
[Out] Timed out
```

3.239 $\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(-a+bpx+c(1+2p)x^2+d(2+3p)x^3\right)}{x^2}\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx+cx^2+dx^3\right)^{p+1}}{x}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1590}

$$\frac{\left(a+bx+cx^2+dx^3\right)^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p
)*x^3))/x^2,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(-a+bpx+c(1+2p)x^2+d(2+3p)x^3\right)}{x^2}\,dx=\frac{\left(a+bx+cx^2+dx^3\right)^{1+p}}{x}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2
 + 3*p)*x^3))/x^2,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x
```

fricas [A] time = 0.81, size = 36, normalized size = 1.57

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(3\,p + 2\right)x^3 + c\left(2\,p + 1\right)x^2 + bpx - a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 +
b*x + a)^p/x^2, x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{\left(d\,x^3 + c\,x^2 + bx + a\right)^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x
```

maxima [A] time = 0.62, size = 36, normalized size = 1.57

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

mupad [B] time = 3.20, size = 23, normalized size = 1.00

$$\frac{\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p
+ 2)))/x^2,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3)
/x**2,x)
```

```
[Out] Timed out
```

3.240 $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^2
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 +
3*p)*x^3))/x^3,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

Mathematica [A] time = 0.20, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d
*(1 + 3*p)*x^3))/x^3,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

fricas [A] time = 1.04, size = 36, normalized size = 1.57

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(3\,p+1\right)x^3+2\,cpx^2+b\left(p-1\right)x-2\,a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^3, x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{\left(d\,x^3+c\,x^2+bx+a\right)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(3*p+1)*x^3)/x^3,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x^2
```

maxima [A] time = 0.64, size = 36, normalized size = 1.57

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

mupad [B] time = 3.32, size = 23, normalized size = 1.00

$$\frac{\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3*
p + 1)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x*
*3)/x**3,x)
```

```
[Out] Timed out
```

3.241 $\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4} dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^3
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p\left(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3\right)}{x^4}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

Mathematica [A] time = 0.19, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*
x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

fricas [A] time = 1.04, size = 36, normalized size = 1.57

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,dpx^3 + c\left(2\,p - 1\right)x^2 + b\left(p - 2\right)x - 3\,a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="giac")
```

```
[Out] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^4, x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{\left(d\,x^3 + c\,x^2 + bx + a\right)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x^3
```

maxima [A] time = 0.64, size = 36, normalized size = 1.57

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

mupad [B] time = 3.35, size = 23, normalized size = 1.00

$$\frac{\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2*
p - 1)))/x^4,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*x
**3)/x**4,x)
```

```
[Out] Timed out
```

3.242 $\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 5/4*x-3/4*x^2+1/3*x^3+1/4*x^4+1/3*ln(x^2+x+1)-13/48*ln(2*x^2-x+2)+1/72*arct
an(1/15*(1-4*x)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]
)/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 - (1
3*Log[2 - x + 2*x^2])/48
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{x^4\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(\frac{5}{4}-\frac{3x}{2}+x^2+x^3+\frac{2(-2+x)}{3\left(1+x+x^2\right)}+\frac{2-13x}{12\left(2-x+2x^2\right)}\right)dx$$

$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{12}\int \frac{2-13x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx$$

$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx-\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1-}{1+}$$

$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{3}\log\left(1+x+x^2\right)-\frac{13}{48}\log\left(2-x+2x^2\right)+\frac{5}{24}\,\mathrm{Subst}\Big(\int$$

$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x\right.$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.86

$$\frac{1}{144}\left(36x^4+48x^3-108x^2+48\log\left(x^2+x+1\right)-39\log\left(2x^2-x+2\right)+180x-160\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
- 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2 -
 x + 2*x^2])/144
```

fricas [A] time = 0.69, size = 79, normalized size = 0.81

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{5}\,\sqrt{3}\,\arctan\left(\frac{1}{15}\,\sqrt{5}\,\sqrt{3}\,(4\,x-1)\right)-\frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{5}{4}\,x-\frac{13}{48}\,\log\left(2\,x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt
(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48
*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

giac [A] time = 0.31, size = 73, normalized size = 0.75

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{15}\,\arctan\left(\frac{1}{15}\,\sqrt{15}\,(4\,x-1)\right)-\frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{5}{4}\,x-\frac{13}{48}\,\log\left(2\,x^2-x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.76

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}-\frac{10\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^2+x+1\right)}{3}-\frac{13\ln\left(2x^2-x+2\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/4*x^4+1/3*x^3-3/4*x^2+5/4*x-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15
*(4*x-1)*15^(1/2))+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)
```

maxima [A] time = 1.52, size = 73, normalized size = 0.75

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{15}\,\arctan\left(\frac{1}{15}\,\sqrt{15}\,(4\,x-1)\right)-\frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{5}{4}\,x-\frac{13}{48}\,\log\left(2\,x^2-x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

mupad [B] time = 0.19, size = 97, normalized size = 1.00

$$\frac{5x}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5i}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5i}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{13}{48}+\frac{\sqrt{15}\,1i}{144}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] (5*x)/4 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3
^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 - 1/4)
*((15^(1/2)*1i)/144 - 13/48) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i
)/144 + 13/48) - (3*x^2)/4 + x^3/3 + x^4/4
```

sympy [A] time = 0.25, size = 97, normalized size = 1.00

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{72}-\frac{10\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*log(x**2 - x/2 + 1)/48 + log(x**2 +
 x + 1)/3 - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 - 10*sqrt(3)*at
an(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.243 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=90

$$\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -3/2*x+1/2*x^2+1/3*x^3+2/3*ln(x^2+x+1)-1/24*ln(2*x^2-x+2)+5/36*arctan(1/15*
(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (8
*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 -
x + 2*x^2]/24
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(-\frac{3}{2}+x+x^2+\frac{2(3+2x)}{3\left(1+x+x^2\right)}+\frac{-6-x}{6\left(2-x+2x^2\right)}\right)dx$$

$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{1}{6}\int \frac{-6-x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx$$

$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{1}{24}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{24}\int \frac{1}{2-x+2x^2}$$

$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right)+\frac{25}{12}\text{Subst}\left(\int \frac{}{-15}\right.$$

$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.87

$$\frac{1}{72}\left(24x^3+36x^2+48\log\left(x^2+x+1\right)-3\log\left(2x^2-x+2\right)-108x+64\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-10\sqrt{15}\tan^{-1}\left(\frac{4x}{\sqrt{1}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 10*Sqrt[
15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2
])/72
```

fricas [A] time = 0.80, size = 74, normalized size = 0.82

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x -
 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 -
 x + 2) + 2/3*log(x^2 + x + 1)
```

giac [A] time = 0.38, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{2}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

maple [A] time = 0.01, size = 69, normalized size = 0.77

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{5\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{2\ln\left(x^2+x+1\right)}{3}-\frac{\ln\left(2x^2-x+2\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/3*x^3+1/2*x^2-3/2*x-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*
15^(1/2))+2/3*ln(x^2+x+1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.88, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{2}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

mupad [B] time = 0.18, size = 92, normalized size = 1.02

$$\frac{x^2}{2}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - (3*x)/2 + log(x - (15^(1/2)*1i)/4 - 1/4)
*((15^(1/2)*5i)/72 - 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*5i)/
72 + 1/24) + x^2/2 + x^3/3
```

sympy [A] time = 0.25, size = 92, normalized size = 1.02

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 3*x/2 - log(x**2 - x/2 + 1)/24 + 2*log(x**2 + x + 1)/3 -
5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 + 8*sqrt(3)*atan(2*sqrt(3
)*x/3 + sqrt(3)/3)/9
```

3.244 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=77

$$\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x+1/2*x^2-ln(x^2+x+1)+1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))*
15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/
Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(1+x-\frac{2(1+3x)}{3\left(1+x+x^2\right)}+\frac{-2+3x}{3\left(2-x+2x^2\right)}\right)dx$$

$$= x+\frac{x^2}{2}+\frac{1}{3}\int \frac{-2+3x}{2-x+2x^2}\,dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx$$

$$= x+\frac{x^2}{2}+\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx-\frac{5}{12}\int \frac{1}{2-x+2x^2}\,dx-\int$$

$$= x+\frac{x^2}{2}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\right.$$

$$= x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-\right.$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.94

$$\frac{1}{36}\left(9\left(-4\log\left(x^2+x+1\right)+\log\left(2x^2-x+2\right)+2x(x+2)\right)+8\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36
```

fricas [A] time = 0.55, size = 67, normalized size = 0.87

$$\frac{1}{2}x^2-\frac{1}{18}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9
*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x
^2 + x + 1)
```

giac [A] time = 0.39, size = 61, normalized size = 0.79

$$\frac{1}{2}x^2 - \frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{4}\log\left(2x^2 - x + 2\right) - \log\left(x^2 + x +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 0.81

$$\frac{x^2}{2} + x - \frac{\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} - \ln\left(x^2 + x + 1\right) + \frac{\ln\left(2x^2 - x + 2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/2*x^2+x+1/4*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-ln(
x^2+x+1)+2/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.44, size = 61, normalized size = 0.79

$$\frac{1}{2}x^2 - \frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{4}\log\left(2x^2 - x + 2\right) - \log\left(x^2 + x +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

mupad [B] time = 2.29, size = 85, normalized size = 1.10

$$x-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right)-\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x + (3^(1/2)*1
i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2
)*1i)/36 + 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/36 - 1/4) +
 x^2/2
```

sympy [A] time = 0.23, size = 78, normalized size = 1.01

$$\frac{x^2}{2}+x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right)-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.245 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx$

Optimal. Leaf size=72

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x+1/3*ln(x^2+x+1)+1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1
/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2075, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3]
])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx &= \int\left(1+\frac{2(-2+x)}{3\left(1+x+x^2\right)}+\frac{2(1+x)}{3\left(2-x+2x^2\right)}\right)dx\\ &= x+\frac{2}{3}\int\frac{-2+x}{1+x+x^2}\,dx+\frac{2}{3}\int\frac{1+x}{2-x+2x^2}\,dx\\ &= x+\frac{1}{6}\int\frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int\frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int\frac{1}{2-x+2x^2}\,dx-\frac{5}{3}\int\frac{1}{1+x}\\ &= x+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)-\frac{5}{3}\mathrm{Subst}\left(\int\frac{1}{-15-x^2}\,dx,x,-1+\right.\\ &= x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.96

$$\frac{1}{18}\left(3\left(2\log\left(x^2+x+1\right)+\log\left(2x^2-x+2\right)+6x\right)-20\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18
```

fricas [A] time = 1.04, size = 62, normalized size = 0.86

$$\frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x
 + 1)
```

giac [A] time = 0.31, size = 56, normalized size = 0.78

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 57, normalized size = 0.79

$$x+\frac{\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^2+x+1\right)}{3}+\frac{\ln\left(2x^2-x+2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] x+1/6*ln(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+1/3*ln(x^2+x
+1)-10/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.59, size = 56, normalized size = 0.78

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

mupad [B] time = 2.29, size = 80, normalized size = 1.11

$$x+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3^(1/2)
*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^
(1/2)*1i)/18 - 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 + 1/
6)
```

sympy [A] time = 0.23, size = 75, normalized size = 1.04

$$x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*
x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.246 $\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=71

$$\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 2/3*ln(x^2+x+1)-1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2
)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2074, 634, 618, 204, 628}

$$\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx &= \int \left(\frac{2(3+2x)}{3\left(1+x+x^2\right)}+\frac{3-2x}{3\left(2-x+2x^2\right)}\right)dx\\
&= \frac{1}{3}\int \frac{3-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx\\
&= -\left(\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx\right)+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx+\frac{4}{3}\int \frac{}{1+x}\\
&= \frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)-\frac{5}{3}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+4x\right)\\
&= -\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2\right.
\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.92

$$\frac{1}{18}\left(12\log\left(x^2+x+1\right)-3\log\left(2x^2-x+2\right)+16\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18
```

fricas [A] time = 0.80, size = 61, normalized size = 0.86

$$\frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

giac [A] time = 0.29, size = 55, normalized size = 0.77

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

maple [A] time = 0.01, size = 56, normalized size = 0.79

$$\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{2\ln\left(x^2+x+1\right)}{3}-\frac{\ln\left(2x^2-x+2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] -1/6*ln(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+2/3*ln(x^2+x+
1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.32, size = 55, normalized size = 0.77

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

mupad [B] time = 0.15, size = 79, normalized size = 1.11

$$-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)+\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2
)*1i)/18 + 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 - 1/6)
```

sympy [A] time = 0.23, size = 75, normalized size = 1.06

$$-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{2\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*x
/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.247 $\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx$

Optimal. Leaf size=75

$$-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 5/2*ln(x)-ln(x^2+x+1)-1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))*
15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx &= -\left(\frac{1}{3}\int \frac{-6+4x}{x\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x\left(4+4x+4x^2\right)}\,dx\\
&= \frac{1}{3}\int \left(\frac{6}{x}-\frac{2(1+3x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x}+\frac{1+6x}{2\left(2-x+2x^2\right)}\right)dx\\
&= \frac{5\log(x)}{2}-\frac{1}{6}\int \frac{1+6x}{2-x+2x^2}\,dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx\\
&= \frac{5\log(x)}{2}-\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx-\frac{5}{12}\int \frac{1}{2-x+2x^2}\,dx\\
&= \frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-x+2x^2\right)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.\\
&= \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log
\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.92

$$\frac{1}{36}\left(-36\log\left(x^2+x+1\right)-9\log\left(2x^2-x+2\right)+90\log(x)+8\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36
```

fricas [A] time = 0.96, size = 65, normalized size = 0.87

$$-\frac{1}{18}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) +
 5/2*log(x)
```

giac [A] time = 0.31, size = 60, normalized size = 0.80

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\,1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(abs(x
))
```

maple [A] time = 0.01, size = 60, normalized size = 0.80

$$-\frac{\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{5\ln(x)}{2}-\ln\left(x^2+x+1\right)-\frac{\ln\left(2x^2-x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] -1/4*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+5/2*ln(x)-ln
(x^2+x+1)+2/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.57, size = 59, normalized size = 0.79

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\,(4\,x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{1}{4}\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

mupad [B] time = 0.15, size = 83, normalized size = 1.11

$$\frac{5\,\ln(x)}{2}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}}{36}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] (5*log(x))/2 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x +
 (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4
)*((15^(1/2)*1i)/36 - 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/
36 + 1/4)
```

sympy [A] time = 0.29, size = 78, normalized size = 1.04

$$\frac{5\log(x)}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right)-\frac{\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 - log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.248 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -5/2/x-3/4*ln(x)+1/3*ln(x^2+x+1)+1/24*ln(2*x^2-x+2)+5/36*arctan(1/15*(1-4*x
)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcTan[(1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x +
2*x^2]/24
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\left(\frac{1}{3}\int \frac{-6+4x}{x^2\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^2\left(4+4x+4x^2\right)}\,dx$$

$$= \frac{1}{3}\int \left(\frac{6}{x^2}-\frac{2}{x}+\frac{2(-2+x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x^2}+\frac{1}{4x}+\frac{13-2x}{4\left(2-x+2x^2\right)}\right)dx$$

$$= -\frac{5}{2x}-\frac{3\log(x)}{4}-\frac{1}{12}\int \frac{13-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx$$

$$= -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{24}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{24}\int \frac{1}{2-x}$$

$$= -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{24}\log\left(2-x+2x^2\right)+\frac{25}{12}\,\text{Subst}\Bigg(\int$$

$$= -\frac{5}{2x}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{3\log(x)}{4}+\frac{1}{3}\log\Big(1+x$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.93

$$-\frac{-24x\log\left(x^2+x+1\right)-3x\log\left(2x^2-x+2\right)+54x\log(x)+80\sqrt{3}\,x\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+10\sqrt{15}\,x\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)+18}{72x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/72*(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan[
(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 - x
+ 2*x^2])/x
```

fricas [A] time = 0.89, size = 76, normalized size = 0.90

$$-\frac{10\sqrt{5}\sqrt{3}\,x\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}\,(4x-1)\right)+80\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-3x\log\left(2x^2-x+2\right)-24x\log\left(x\right.}{72x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/72*(10*sqrt(5)*sqrt(3)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 80*sqr
t(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(x^
2 + x + 1) + 54*x*log(x) + 180)/x
```

giac [A] time = 0.26, size = 65, normalized size = 0.77

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\,(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(abs(x))
```

maple [A] time = 0.01, size = 65, normalized size = 0.77

$$-\frac{5\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}-\frac{10\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}-\frac{3\ln(x)}{4}+\frac{\ln\left(x^2+x+1\right)}{3}+\frac{\ln\left(2x^2-x+2\right)}{24}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/2/x-3/4*ln
(x)+1/3*ln(x^2+x+1)-10/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.44, size = 64, normalized size = 0.76

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(x)
```

mupad [B] time = 2.28, size = 88, normalized size = 1.05

$$-\frac{3\ln(x)}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5i}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5i}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - (3*log(x))/4 - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 -
 1/4)*((15^(1/2)*5i)/72 + 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)
*5i)/72 - 1/24) - 5/(2*x)
```

sympy [A] time = 0.31, size = 87, normalized size = 1.04

$$-\frac{3\log(x)}{4}+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}-\frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*ata
n(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(
3)/3)/9 - 5/(2*x)
```

3.249 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} \, dx$

Optimal. Leaf size=91

$$-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -5/4/x^2+3/4/x-15/8*ln(x)+2/3*ln(x^2+x+1)+13/48*ln(2*x^2-x+2)+1/72*arctan(1
/15*(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/3
+ (13*Log[2 - x + 2*x^2])/48
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx &= -\left(\frac{1}{3}\int \frac{-6+4x}{x^3\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^3\left(4+4x+4x^2\right)}\,dx \\
&= \frac{1}{3}\int \left(\frac{6}{x^3}-\frac{2}{x^2}-\frac{4}{x}+\frac{2(3+2x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x^3}+\frac{1}{4x^2}+\frac{13}{8x}+\frac{9-26}{8\left(2-x+\right.}\right. \\
&= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{1}{24}\int \frac{9-26x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx \\
&= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx+\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{2}{3} \\
&= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right)+\frac{5}{24}\mathrm{Su} \\
&= -\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{15\log(x)}{8}+\frac{2}{3}\log
\end{aligned}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.90

$$\frac{1}{144}\left(3\left(-\frac{60}{x^2}+32\log\left(x^2+x+1\right)+13\log\left(2x^2-x+2\right)+\frac{36}{x}-90\log(x)\right)+128\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\ \mathrm{t}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x +
 2*x^2]))/144
```

fricas [A] time = 0.97, size = 89, normalized size = 0.98

$$-\frac{2\sqrt{5}\sqrt{3}x^2\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-128\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-39x^2\log\left(2x^2-x+2\right)-96x^2\ \mathrm{l}}{144x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/144*(2*sqrt(5)*sqrt(3)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 128*
sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 96*
x^2*log(x^2 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2
```

giac [A] time = 0.29, size = 70, normalized size = 0.77

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{3x-5}{4x^2}+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(abs(x))
```

maple [A] time = 0.01, size = 70, normalized size = 0.77

$$-\frac{\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}+\frac{8\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}-\frac{15\ln(x)}{8}+\frac{2\ln\left(x^2+x+1\right)}{3}+\frac{13\ln\left(2x^2-x+2\right)}{48}+\frac{3}{4x}-\frac{5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/4/x^2+3/4
/x-15/8*ln(x)+2/3*ln(x^2+x+1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.26, size = 69, normalized size = 0.76

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{3x-5}{4x^2}+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(x)
```

mupad [B] time = 0.15, size = 92, normalized size = 1.01

$$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}-\frac{15\ln(x)}{8}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] ((3*x)/4 - 5/4)/x^2 - (15*log(x))/8 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/
2)*4i)/9 - 2/3) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) + lo
g(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 + 13/48) - log(x + (15^(1/2
)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 - 13/48)
```

sympy [A] time = 0.32, size = 94, normalized size = 1.03

$$-\frac{15\log(x)}{8}+\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}+\frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3 - sqrt(15)
*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sq
rt(3)/3)/9 + (3*x - 5)/(4*x**2)
```

3.250 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=307

$$\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\right.\right.$$

```
[Out] 1/28*x^2*(7-5*I*7^(1/2))+1/42*x^3*(7-5*I*7^(1/2))+1/28*x^2*(7+5*I*7^(1/2))+
1/42*x^3*(7+5*I*7^(1/2))-1/28*x*(35-9*I*7^(1/2))-1/28*x*(35+9*I*7^(1/2))+3/
112*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7-11*I*7^(1/2))+3/112*ln(4+4*x^2+x*(1+I*7^
(1/2)))*(7+11*I*7^(1/2))-11/4*arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/
2))*(9*I-5*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+11/4*arctan((1+8*x-I*7^(1/2))/
(70+2*I*7^(1/2))^(1/2))*(9*I+5*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -((35 - (9*I)*Sqrt[7])*x)/28 - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7 - (5*I)*Sq
rt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (5*I)*Sqrt[7])*x^3)/4
2 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I + 5*Sqrt[7])*ArcTan[(1 - I*Sqrt
[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11*(
9*I - 5*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4
*Sqrt[14*(35 - I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*
x^2])/112
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x^3\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^3\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{4}\left(-9+5i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{1}{2}\left(5-i\sqrt{7}\right)x^2+\frac{2\left(9-5i\sqrt{7}\right)-3\left(11+i\sqrt{7}\right)x}{2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\right) d}{\sqrt{7}}$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x - \frac{1}{28}\left(35+9i\sqrt{7}\right)x + \frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x - \frac{1}{28}\left(35+9i\sqrt{7}\right)x + \frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x - \frac{1}{28}\left(35+9i\sqrt{7}\right)x + \frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x - \frac{1}{28}\left(35+9i\sqrt{7}\right)x + \frac{1}{28}\left(7-5i\sqrt{7}\right)x^2 + \frac{1}{28}\left(7+5i\sqrt{7}\right)x$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.36

$$\frac{1}{6}\left(3\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&, \frac{3\#1^3\log(x-\#1)+19\#1^2\log(x-\#1)+\#1\log(x-\#1)+10\log(}{8\#1^3+3\#1^2+10\#1+1}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (10*
Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/(1
 + 10*#1 + 3*#1^2 + 8*#1^3) & ])/6
```

fricas [B] time = 2.94, size = 1202, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 1/112*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55
/32) - 21)*log(23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/
32) + 3/16)^3 - 23765*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^2 + 7744*x + 19470*I*sqrt(7) - 33040*sqrt(2101/1568*I*sqrt(7)
- 55/32) + 38950) - 1/112*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*log(-23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^3 + 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) -
55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) -
869) + 1/256*(53312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) -
36681)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 17493*
(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 7744*
x - 15708*I*sqrt(7) + 26656*sqrt(2101/1568*I*sqrt(7) - 55/32) - 29132) + 1/
112*(2*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) -
 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7)
- 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99
/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 28*sqrt(210
1/1568*I*sqrt(7) - 55/32) + 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 21)*log
(-49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(5
3312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 -
11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sq
rt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7
) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((17*sqrt(7)*(-
33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(3
3*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-3
3*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*s
qrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)
^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 8
4*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) +
6384*sqrt(2101/1568*I*sqrt(7) - 55/32) - 5946) - 1/112*(2*sqrt(7)*sqrt(-336
*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*
(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/5
6*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7
) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 1859/2) - 28*sqrt(2101/1568*I*sqrt(7) - 55/32
) - 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log(-49/4*(-33/112*I*sqrt(7
) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(53312*(33/112*I*sqrt(7) -
 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 11781*I*sqrt(7) + 19992*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I
```

```
*sqrt(7) - 55/32) + 3/16)^2 - 1/256*((17*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(21
01/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*sqrt(-336*(33/112*I*sqrt(
7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(
7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7)
+ 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101
/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(
7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) + 6384*sqrt(2101/1568*I*sq
rt(7) - 55/32) - 5946) - 5/2*x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+3\,x^2+x+5\right)x^3}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 74, normalized size = 0.24

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{\left(3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+19\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+\mathrm{Root}\right.}{16\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+6\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+1
)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{2}\,x+\frac{1}{2}\int \frac{3\,x^3+19\,x^2+x+10}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4
+ x^3 + 5*x^2 + x + 2), x)
```

mupad [B] time = 2.17, size = 128, normalized size = 0.42

$$\left(\sum_{k=1}^{4}\ln\left(-29\,x+\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\left(-\frac{289\,x}{4}+\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] symsum(log(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*(
root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((581*x)/16
- root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((147*x)/4
 - 49/16) + 1141/64) - (289*x)/4 + 47/4) - 29*x + 7)*root(z^4 - (3*z^3)/4 +
 (16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (5*x)/2 + x^2/2 + x^3/
3
```

sympy [A] time = 0.99, size = 61, normalized size = 0.20

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\mathrm{RootSum}\left(1372t^4-1029t^3+3136t^2+2688t+512,\left(t\mapsto t\log\left(\frac{5831t^3}{1936}-\frac{23765t^2}{7744}+\frac{2065t}{242}+x+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 26
88*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*_t
/242 + x + 415/121)))
```

3.251 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$

Optimal. Leaf size=269

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\right.$$

```
[Out] 1/14*x*(7-5*I*7^(1/2))+1/28*x^2*(7-5*I*7^(1/2))+1/14*x*(7+5*I*7^(1/2))+1/28
*x^2*(7+5*I*7^(1/2))-1/56*ln(4+4*x^2+x*(1+I*7^(1/2)))*(35-9*I*7^(1/2))-1/56
*ln(4+4*x^2+x*(1-I*7^(1/2)))*(35+9*I*7^(1/2))+1/2*arctan((1+8*x+I*7^(1/2))/
(70-2*I*7^(1/2))^(1/2))*(53*I-7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/2*arctan(
(1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(53*I+7^(1/2))/(490+14*I*7^(1/2))
^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (5*I)*Sqrt[
7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + Sqrt[7])*ArcTan[(1 -
I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) +
 ((53*I - Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/
(2*Sqrt[14*(35 - I*Sqrt[7])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2
])/56
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x^2 \left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x^2\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^2\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{2}\left(5-i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{i\left(2\left(5i+\sqrt{7}\right)+\left(9i+5\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\right)dx}{\sqrt{7}} - \frac{i\int \left(\frac{1}{2}\left(5+i\sqrt{7}\right)\right.}{}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-$$

Mathematica [C] time = 0.02, size = 101, normalized size = 0.38

$$-\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{5\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+3\#1\log(x-\#1)+2\log(x-\#}{8\#1^3+3\#1^2+10\#1+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x - #1] + 3*
Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2
 + 8*#1^3) & ]
```

fricas [B] time = 3.03, size = 1145, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(
49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*s
qrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 10290*(-9/56*I*sqrt(
7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 25725*(-9/56*I*sqrt(7)
- 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 3/64*(3920*(-9/56*I*sqrt(7
) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sq
rt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt
(7) + 79/56) + 35) + 8384*x + 6615/2*I*sqrt(7) + 10290*sqrt(-37/392*I*sqrt(
7) + 79/56) + 13373/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*
I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt
(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79
/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14
*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*s
qrt(7) + 79/56) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(135*I*s
qrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*
sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt
(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392
*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/5
6) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5
/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*
I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*
sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*
I*sqrt(7) + 79/56) - 1459)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56
) + 35) - 17856*I*sqrt(7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776)
+ 16768*x - 4941*I*sqrt(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391)
- 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/
8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 -
 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(
7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-3
7/392*I*sqrt(7) + 79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqr
t(-37/392*I*sqrt(7) + 79/56) + 5)*log(-49/4*(135*I*sqrt(7) + 420*sqrt(-37/3
92*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) +
 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5
587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-1
2*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56
*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt
(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/
392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7)
+ 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 14
59)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 17856*I*sqrt(
7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776) + 16768*x - 4941*I*sqrt
```

```
(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391) - 1/56*(9*I*sqrt(7) + 2
8*sqrt(-37/392*I*sqrt(7) + 79/56) + 35)*log(10290*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 + 1421*(-9/56*I*sqrt(7) - 1/2*sqrt(-
37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 8384*x + 3267/2*I*sqrt(7) + 5082*sqrt(
-37/392*I*sqrt(7) + 79/56) + 13793/2) + x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^3+3x^2+x+5\right)x^2}{2x^4+x^3+5x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 67, normalized size = 0.25

$$\frac{x^2}{2}+x+\frac{\left(-5\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2-3\,\mathrm{RootOf}\left(2\right.\right.}{8\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/2*x^2+x+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=Ro
otOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2+x-\int \frac{5x^3+x^2+3x+2}{2x^4+x^3+5x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x +
2), x)
```

mupad [B] time = 0.13, size = 188, normalized size = 0.70

$$x+\frac{x^2}{2}+\left(\sum_{k=1}^{4}\ln\left(-\frac{179\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)}{8}-7x-\frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)x\,45}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + x^2/2 + symsum(log((49*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^3)/16 - 7*x - (459*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128
/343, z, k)*x)/8 - (665*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k)^2*x)/8 - (147*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z,
 k)^3*x)/4 - (35*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^
2)/32 - (179*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k))/8 -
 15)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1, 4)
```

sympy [B] time = 2.73, size = 3662, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**2/2 + x + (-5/8 + sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-1459*sqrt(1
4)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 15*s
qrt(77)*sqrt(553 + 64*sqrt(77))/2096 - 10391*sqrt(553 + 64*sqrt(77))/268288
 + 1459*sqrt(77)/8384 + 522933/268288 + 45*sqrt(14)*sqrt(553 + 64*sqrt(77))
*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576) - 51089
5297*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71
978450944 - 6009493*sqrt(22)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 76
48*sqrt(77))/1124663296 - 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/2249326
592 - 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 153195*sqrt(22)*sqrt
(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7))/2249326592 + 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 + 290832444193/359892
25472 + 2303470247*sqrt(77)/2249326592) + (-5/8 - sqrt(79/448 + sqrt(77)/49
))*log(x**2 + x*(-45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 1459*sqrt(14)*sqrt(333*sqrt(5
53 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 + 10391*sqrt(553 + 64*sqr
t(77))/268288 + 1459*sqrt(77)/8384 + 522933/268288 + 15*sqrt(77)*sqrt(553 +
 64*sqrt(77))/2096) - 510895297*sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) +
 21975 + 7648*sqrt(77))/71978450944 - 6009493*sqrt(22)*sqrt(333*sqrt(553 +
64*sqrt(77)) + 21975 + 7648*sqrt(77))/1124663296 - 8313499*sqrt(14)*sqrt(55
3 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/
71978450944 - 153195*sqrt(22)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77))/2249326592 + 4417610843*sqrt(553 + 64*
sqrt(77))/35989225472 + 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/224932659
2 + 290832444193/35989225472 + 2303470247*sqrt(77)/2249326592) + 2*sqrt(-sq
rt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1568 + 5/1
```

```
4 + 3*sqrt(77)/49)*atan(1073152*x/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqr
t(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqr
t(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77
)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqr
t(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 45*sqrt
(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) +
 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sq
rt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)
)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*
sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 2
1975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 1459*sqrt(14)*sqrt(333*sqrt(5
53 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqr
t(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
 + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(
333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)))
+ 20782*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(
7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))
 + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(
77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 93376*sqr
t(77)/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333
*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt
(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(5
53 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 1045866/(4313*sqrt(2)*sqrt(-sqrt(14
)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt
(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21
975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*
sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(7
7))) + 3840*sqrt(77)*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*s
qrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77
)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7
648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqr
t(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
)) + 2*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt
(77))/1568 + 5/14 + 3*sqrt(77)/49)*atan(1073152*x/(-1459*sqrt(2)*sqrt(553 +
 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 76
```

48*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*s
qrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*s
qrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt
(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648
*sqrt(77))) + 45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*s
qrt(77)) + 21975 + 7648*sqrt(77))/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sq
rt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 5
60 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqr
t(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqr
t(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96
*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))) + 10
45866/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqr
t(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))) + 93376*sqrt(77)/(-1459*sqrt(2)*
sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) +
21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*s
qrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(7
7)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21
975 + 7648*sqrt(77))) - 20782*sqrt(553 + 64*sqrt(77))/(-1459*sqrt(2)*sqrt(5
53 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-3
33*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) +
30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*
sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
7648*sqrt(77))) - 3840*sqrt(77)*sqrt(553 + 64*sqrt(77))/(-1459*sqrt(2)*sqrt
(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 2197
5 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(
-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
+ 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77))) - 1459*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77))/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqr
t(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)
) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*
sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(
-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))))

3.252 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=230

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}$$

```
[Out] 1/14*x*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/1
4*x*(7+5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7+5*I*7^(1/2))+arctan
((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19*I-7*7^(1/2))/(490-14*I*7^(1/
2))^(1/2)-arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(19*I+7*7^(1/2))
/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2087, 773, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I + 7*Sqrt[7
])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*
Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I
*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 -
I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x
+ 4*x^2])/28
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{i\int \frac{-4\left(10-2i\sqrt{7}\right)+\left(-\left(1-i\sqrt{7}\right)\left(10-2i\sqrt{7}\right)+4\left(9-5i\sqrt{\right.}\right.}{4+\left(1-i\sqrt{7}\right)x+4x^2}}{4\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{\left(19i\right.}{}$$

Mathematica [C] time = 0.01, size = 94, normalized size = 0.41

$$2\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{\#1^3\log(x-\#1)-2\#1^2\log(x-\#1)+2\#1\log(x-\#1)-\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] + 2*Log[x -
 #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1
^3) & ]
```

fricas [B] time = 3.07, size = 1190, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(55*I*
sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqr
t(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/9
```

8*I*sqrt(7) - 1/14) + 1/4)^3 + 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqr
t(7) - 1/14) + 1/4)^2 + 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt
(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)
*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x + 1155/2*I*s
qrt(7) + 1617*sqrt(53/98*I*sqrt(7) - 1/14) + 1903/2) + 1/28*(2*sqrt(7)*sqrt
(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/
28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/
2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*
log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*s
qrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(-5/28*I*sqrt(7)
- 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*
I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) -
7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/
4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/
56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7)
 - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14
) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)
 + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 3456*s
qrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqrt(7) - 1/14) + 636) -
 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1
/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
2744*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(
196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqr
t(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98
*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I
sqrt(7) - 1/14) + 1/4)^2 - 21(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7)
- 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21
)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7
*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(5
3/98*I*sqrt(7) - 1/14) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I
*sqrt(7) - 1/14) - 7) + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7)
- 1/14) - 7) + 3456*sqrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqr
t(7) - 1/14) + 636) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) -
 7)*log(3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 -
 1029*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 304*x
- 715/2*I*sqrt(7) - 1001*sqrt(53/98*I*sqrt(7) - 1/14) - 2871/2) + x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{3}+3\,x^{2}+x+5\right)x}{2\,x^{4}+x^{3}+5\,x^{2}+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 62, normalized size = 0.27

$$x+\frac{2\left(\mathrm{RootOf}\left(2\_Z^{4}+\_Z^{3}+5\_Z^{2}+\_Z+2\right)^{3}-2\,\mathrm{RootOf}\left(2\_Z^{4}+\_Z^{3}+5\_Z^{2}+\_Z+2\right)^{2}+2\,\mathrm{RootOf}\left(2\_Z^{4}\right.\right.}{8\,\mathrm{RootOf}\left(2\_Z^{4}+\_Z^{3}+5\_Z^{2}+\_Z+2\right)^{3}+3\,\mathrm{RootOf}\left(2\_Z^{4}+\_Z^{3}+5\_Z^{2}+\_Z\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
[Out] x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*
_Z^4+_Z^3+5*_Z^2+_Z+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x+2\int \frac{x^{3}-2\,x^{2}+2\,x-1}{2\,x^{4}+x^{3}+5\,x^{2}+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
[Out] x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

mupad [B] time = 0.19, size = 183, normalized size = 0.80

$$x+\left(\sum_{k=1}^{4}\ln\left(\frac{115\,\mathrm{root}\left(z^{4}-z^{3}+\frac{6\,z^{2}}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8}+15\,x-\frac{\mathrm{root}\left(z^{4}-z^{3}+\frac{6\,z^{2}}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8}+\frac{\mathrm{root}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
[Out] x + symsum(log((115*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)
)/8 + 15*x - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x
```

```
)/8 + (133*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/8 -
 (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 - (189
*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z
^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 - 4)*root(z^4 - z^3
 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

sympy [A] time = 0.95, size = 48, normalized size = 0.21

$$x+\text{RootSum}\left(343t^4-343t^3+294t^2-336t+128,\left(t\mapsto t\log\left(\frac{3773t^3}{304}-\frac{1029t^2}{304}+\frac{1001t}{152}+x-\frac{121}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t
*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19)))
```

3.253 $\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=198

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\right.}{\sqrt{14\left(35+i\right.}}$$

```
[Out] 1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(
1/2)))*(7+5*I*7^(1/2))-arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19
*I-7*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(
1/2))^(1/2))*(19*I+7*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2086, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\right.}{\sqrt{14\left(35+i\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])
/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*
x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[
7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (
1 + I*Sqrt[7])*x + 4*x^2])/28
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= -\left(\frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx\right) + \frac{1}{28}\left(7+5i\sqrt{7}\right)\int \frac{1-i\sqrt{7}\ldots}{4+\left(1-i\ldots\right.}$$

$$= \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)\ldots\right.$$

$$= \frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{\left(19i-7\sqrt{7}\right)\tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\left(7+5\ldots\right.$$

Mathematica [C] time = 0.01, size = 90, normalized size = 0.45

$$\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{2\#1^3\log(x-\#1)+3\#1^2\log(x-\#1)+\#1\log(x-\#1)+5\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[x - #1]*#1
 + 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &
 ]
```

fricas [B] time = 3.23, size = 1189, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1
/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 1/16*(4
116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sq
rt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53
/98*I*sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/9
8*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(
7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) +
 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7)
- 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqr
t(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/9
8*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(
7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*
sqrt(7) - 1/14) - 1247) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sq
rt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98
*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) -
 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*
sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) -
 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*log(49/4*(105*I*sqrt(7) + 294
*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sq
```

```
rt(7) - 1/14) + 1/4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
 1/14) + 1/4)^2 + 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-
5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I
*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(
53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/
14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt
(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*
I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sq
rt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) - 1/28*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(-49/4*(105*I*sqrt(7) + 294*sqrt(53/9
8*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/
14) + 1/4)^2 + 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^3 - 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x - 2205/2*I*sqrt(7) - 3087*sqrt(5
3/98*I*sqrt(7) - 1/14) - 3025/2) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt
(7) - 1/14) - 7)*log(-7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/
14) + 1/4)^3 + 2303*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^2 + 304*x + 1555/2*I*sqrt(7) + 2177*sqrt(53/98*I*sqrt(7) - 1/14) + 5823
/2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 58, normalized size = 0.29

$$\frac{\left(2\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+\mathrm{RootOf}\left(2\_Z^4+\_Z^3\right.\right.}{8\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4
+_Z^3+5*_Z^2+_Z+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{2x^4 + x^3 + 5x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

mupad [B] time = 2.34, size = 181, normalized size = 0.91

$$\sum_{k=1}^{4} \ln\left(-\frac{193\,\mathrm{root}\left(z^4 - z^3 + \frac{6\,z^2}{7} - \frac{48\,z}{49} + \frac{128}{343}, z, k\right)}{8} + 4\,x - \frac{\mathrm{root}\left(z^4 - z^3 + \frac{6\,z^2}{7} - \frac{48\,z}{49} + \frac{128}{343}, z, k\right)\,x\,137}{8} + \frac{\mathrm{root}\left(z^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] symsum(log(4*x - (193*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z,
k))/8 - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x)/8 +
 (651*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/16 - (14
7*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 + (273*roo
t(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z^4 -
 z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 + 7)*root(z^4 - z^3 + (
6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

sympy [A] time = 0.91, size = 46, normalized size = 0.23

$$\mathrm{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t\log\left(-\frac{7203t^3}{304} + \frac{2303t^2}{304} - \frac{2177t}{152} + x + \frac{250}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*log
(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))
```

3.254 $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=245

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{7}\right)$$

```
[Out] 1/28*ln(x)*(35-9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(35-9*I*7^(1
/2))+1/28*ln(x)*(35+9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I
*7^(1/2))-1/2*arctanh((I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(53+I*7^(1/
2))/(490-14*I*7^(1/2))^(1/2)+1/2*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))
^(1/2))*(53-I*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{7}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]
])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7] +
 (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((35 -
 (9*I)*Sqrt[7])*Log[x])/28 + ((35 + (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I
)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7
])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/56
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x}+\frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{2\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x}+\frac{3\left(11i-\sqrt{7}\right)-2\left(9i+5\sqrt{7}\right)x}{2\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)}\right.}{\sqrt{7}}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{i\int \frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx}{2\sqrt{7}}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\int \frac{i}{4i+\left(\right.}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\right.$$

$$= -\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{1}{28}\left(\right.$$

Mathematica [C] time = 0.02, size = 101, normalized size = 0.41

$$\frac{5\log(x)}{2} - \frac{1}{2}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&, \frac{10\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+19\#1\log(x-\#1)+}{8\#1^3+3\#1^2+10\#1+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log[x - #1] +
 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 + 3
*#1^2 + 8*#1^3) & ]/2
```

fricas [B] time = 3.24, size = 1143, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(49/4*(27*
I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/
2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 2058*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 5145*(-9/56*I*sqrt(7) - 1/2*sqrt(-3
7/392*I*sqrt(7) + 79/56) - 5/8)^2 + 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(
-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*s
qrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56)
+ 35) + 8384*x + 1323/2*I*sqrt(7) + 2058*sqrt(-37/392*I*sqrt(7) + 79/56) +
16089/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) -
 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-
9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 1
0*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*sqrt(7) + 79/56
) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(27*I*sqrt(7) + 84*sqr
t(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sq
rt(7) + 79/56) - 5/8)^2 - 15680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(
7) + 79/56) - 5/8)^2 - 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqr
t(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/5
6) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 7/64*
sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12
*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9
*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*s
qrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*s
qrt(7) + 79/56) + 11/2)*((27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56)
 + 1385)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*
sqrt(7) + 35840*sqrt(-37/392*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I
*sqrt(7) + 10864*sqrt(-37/392*I*sqrt(7) + 79/56) + 5484) - 1/8*(2*sqrt(-12*
(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I
*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7
) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/39
2*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) +
79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqrt(-37/392*I*sqrt(7
) + 79/56) + 5)*log(-49/4*(27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56
) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1
5680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/64
*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 94
5*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) +
 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) -
1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((2
7*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(-9*I*sqrt(7) + 28
*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*sqrt(7) + 35840*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I*sqrt(7) + 10864*sqrt(-37/3
```

```
92*I*sqrt(7) + 79/56) + 5484) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(
7) + 79/56) + 35)*log(2058*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^3 + 20825*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/
56) - 5/8)^2 + 8384*x - 8307/2*I*sqrt(7) - 12922*sqrt(-37/392*I*sqrt(7) + 7
9/56) - 18673/2) + 5/2*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+3\,x^2+x+5}{\left(2\,x^4+x^3+5\,x^2+x+2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)
```

maple [C] time = 0.01, size = 67, normalized size = 0.27

$$\frac{5\ln(x)}{2}+\frac{\left(-10\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2-19\,\mathrm{RootOf}\left(\right.\right.}{16\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+6\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/2*sum((-10*_R^3-_R^2-19*_R-3)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf
(2*_Z^4+_Z^3+5*_Z^2+_Z+2))+5/2*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\int \frac{10\,x^3+x^2+19\,x+3}{2\,x^4+x^3+5\,x^2+x+2}\,dx+\frac{5}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
+ 5/2*log(x)
```

mupad [B] time = 2.34, size = 237, normalized size = 0.97

$$\frac{5\,\ln(x)}{2}+\left(\sum_{k=1}^{4}\ln\left(\frac{223\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8}-\frac{31\,x}{2}+\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,7}{16}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] (5*log(x))/2 + symsum(log((223*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k))/8 - (31*x)/2 + (71*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)*x)/16 - (4463*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k)^2*x)/64 + (1449*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 12
8/343, z, k)^3*x)/16 + (3675*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128
/343, z, k)^4*x)/32 + (257*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^2)/32 + (1673*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k)^3)/64 - (441*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z,
k)^4)/32 + 10)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k
, 1, 4)
```

sympy [A] time = 12.66, size = 60, normalized size = 0.24

$$\frac{5\log(x)}{2} + \mathrm{RootSum}\left(686t^4 + 1715t^3 + 1372t^2 + 448t + 256, \left(t \mapsto t\log\left(-\frac{160344611t^4}{532759184} - \frac{16880402t^3}{33297449} + \frac{40105}{2131}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, La
mbda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_t**3/33297449 + 4010
520787*_t**2/2131036736 + 1537535671*_t/532759184 + x + 46660495/66594898))
)
```

3.255 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=281

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-\frac{35}{}$$

```
[Out] 1/28*(-35+9*I*7^(1/2))/x+1/28*(-35-9*I*7^(1/2))/x-3/56*ln(x)*(7-11*I*7^(1/2
))+3/112*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(7-11*I*7^(1/2))-3/56*ln(x)*(7+11*I*
7^(1/2))+3/112*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(7+11*I*7^(1/2))+11/4*arctanh(
(I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(9+5*I*7^(1/2))/(490-14*I*7^(1/2)
)^(1/2)-11/4*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(9-5*I*7^(1/
2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-\frac{35}{}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -(35 - (9*I)*Sqrt[7])/(28*x) - (35 + (9*I)*Sqrt[7])/(28*x) + (11*(9 + (5*I)
*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqr
t[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8
*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (
11*I)*Sqrt[7])*Log[x])/56 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (1
1*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*
Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^2\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int\left(\frac{9+5i\sqrt{7}}{4x^2}+\frac{3\left(11-i\sqrt{7}\right)}{8x}+\frac{-7\left(9i-5\sqrt{7}\right)-6\left(11i+\sqrt{7}\right)x}{4\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int\left(\frac{9-5i\sqrt{7}}{4x^2}+\frac{3\left(11+i\right.}{8x}\right.}{}$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} + \frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}} - \frac{11\left(9-5\right.}{}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.39

$$\frac{1}{4}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{6\#1^3\log(x-\#1)-17\#1^2\log(x-\#1)+13\#1\log(x-\#1)-35\log(x\right.}{8\#1^3+3\#1^2+10\#1+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-35*
Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^3)
/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/4
```

fricas [B] time = 3.27, size = 1245, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/224*(2*x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log
(91924*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3
- 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839) - 1/256
*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33
*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 68943*(33/112*I*
sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x + 61908
*I*sqrt(7) - 105056*sqrt(2101/1568*I*sqrt(7) - 55/32) + 123428) + 2*x*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21)*log(-91924*(33/112*I
*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3 + 98735*(33/112*
I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x - 146
487/2*I*sqrt(7) + 124292*sqrt(2101/1568*I*sqrt(7) - 55/32) - 285347/2) + (4
*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32
) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) -
 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*s
qrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2)*x - x*(33*I*sqrt(7)
 + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - x*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) - 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1
/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqr
t(2101/1568*I*sqrt(7) - 55/32) - 3839) + 1/256*(210112*(33/112*I*sqrt(7) -
1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 46431*I*sqrt(7) + 78792*s
qrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) - 29792*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*
I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(
-2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) + 147456*sqrt(7))*sqrt(-336*(33/112*I*s
qrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*s
qrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt
(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*s
qrt(7) - 55/32) - 1859/2) + 30976*x + 22671/2*I*sqrt(7) - 19236*sqrt(2101/1
568*I*sqrt(7) - 55/32) + 53979/2) - (4*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7)
- 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7)
- 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 5
6*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/15
68*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7)
- 55/32) - 1859/2)*x + x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/
32) - 21) + x*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839
) + 1/256*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 11
```

```
7483)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 29792*(
33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/256*
((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) -
2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) -
 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 147456*sqrt(7))*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7)
 - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7
) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) -
55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) +
99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 30976*x +
 22671/2*I*sqrt(7) - 19236*sqrt(2101/1568*I*sqrt(7) - 55/32) + 53979/2) + 1
68*x*log(x) + 560)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3+3x^2+x+5}{(2x^4+x^3+5x^2+x+2)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2), x)
```

maple [C] time = 0.01, size = 72, normalized size = 0.26

$$-\frac{3\ln(x)}{4}+\frac{\left(6\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3-17\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+13\,\mathrm{RootC}}{32\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+12\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x)
[Out] 1/4*sum((6*_R^3-17*_R^2+13*_R-35)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=Root
Of(2*_Z^4+_Z^3+5*_Z^2+_Z+2))-5/2/x-3/4*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{5}{2x}+\frac{1}{4}\int\frac{6x^3-17x^2+13x-35}{2x^4+x^3+5x^2+x+2}\,dx-\frac{3}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

[Out] -5/2/x + 1/4*integrate((6*x^3 - 17*x^2 + 13*x - 35)/(2*x^4 + x^3 + 5*x^2 + x + 2), x) - 3/4*log(x)

mupad [B] time = 2.30, size = 242, normalized size = 0.86

$$\left(\sum_{k=1}^{4}\ln\left(\frac{1199\,\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)}{32}+25\,x+\frac{\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\,x\,4169}{32}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)

[Out] symsum(log((1199*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z , k))/32 + 25*x + (4169*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128 /343, z, k)*x)/32 + (43993*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)^2*x)/256 + 28*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)^3*x + (3675*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)^4*x)/32 + (11647*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)^2)/128 + (7273*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (9 6*z)/49 + 128/343, z, k)^3)/128 - (441*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)^4)/32 + 21/4)*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (3*log(x))/4 - 5/(2*x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)

[Out] Timed out

3.256 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=317

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right.\right.$$

```
[Out] 1/56*(-35+9*I*7^(1/2))/x^2-1/16*ln(x)*(35-9*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+
x*(I-7^(1/2)))*(35-9*I*7^(1/2))+1/56*(-35-9*I*7^(1/2))/x^2-1/16*ln(x)*(35+9
*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I*7^(1/2))+3/56*(7-11*
I*7^(1/2))/x+3/56*(7+11*I*7^(1/2))/x+1/8*arctanh((I+8*I*x-7^(1/2))/(70-2*I*
7^(1/2))^(1/2))*(355-73*I*7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/8*arctanh((I+
8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(355+73*I*7^(1/2))/(490+14*I*7^(1/2)
)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -(35 - (9*I)*Sqrt[7])/(56*x^2) - (35 + (9*I)*Sqrt[7])/(56*x^2) + (3*(7 - (1
1*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x) + ((355 - (73*I)*Sq
rt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(8*Sqrt[1
4*(35 - I*Sqrt[7])]) - ((355 + (73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)
*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)
*Sqrt[7])*Log[x])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt
[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7])*Log
[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^3\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^3\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^3}+\frac{3\left(11-i\sqrt{7}\right)}{8x^2}-\frac{7i\left(-9i+5\sqrt{7}\right)}{16x}+\frac{-223i-61\sqrt{7}+14\left(9i-5\sqrt{7}\right)x}{8\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \Big(\frac{9}{}}{}$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\Big(35-9i$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\Big(35-9i$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\Big(35-9i$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} + \frac{\Big(355-73i}{8}$$

Mathematica [C] time = 0.02, size = 116, normalized size = 0.37

$$\frac{1}{8}\text{RootSum}\Big[2\#1^4+\#1^3+5\#1^2+\#1+2\&, \frac{70\#1^3\log(x-\#1)+47\#1^2\log(x-\#1)+141\#1\log(x-\#1)+61\log}{8\#1^3+3\#1^2+10\#1+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#
1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*Log
[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/8
```

fricas [B] time = 3.43, size = 1274, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/448*(14*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
)*log(-49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896
) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35
/32)^2 + 9046968*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 35/32)^3 - 39580485*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) +
 2815/896) + 35/32)^2 - 21/1024*(13785856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803
/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 30156560*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/
6272*I*sqrt(7) + 2815/896) - 35) + 9662336*x - 68336919/4*I*sqrt(7) - 30371
964*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 257023549/4) + 14*x^2*(9*I*sqrt
(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)*log(-9046968*(-9/32*I*
sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^3 + 41411909*(
-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 96
62336*x + 70198191/4*I*sqrt(7) + 31199196*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 240366533/4) + 1960*x^2*log(x) + (4*sqrt(7)*sqrt(-1344*(9/32*I*sqrt(
7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqr
t(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*s
qrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9
803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 - 7*x^2*(9*I*sqrt(7) + 16*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 7*x^2*(-9*I*sqrt(7) + 16*sqrt(980
3/6272*I*sqrt(7) + 2815/896) - 35) - 980*x^2)*log(49/4*(207711*I*sqrt(7) +
369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7) - 1/
2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sqrt(7
) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(1378585
6*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 +
 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 6848
8563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 1/102
4*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 3
5/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
 + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) +
 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/
2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(2307
9*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
49504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
 - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sqrt(-
9803/6272*I*sqrt(7) + 2815/896) + 666914) - (4*sqrt(7)*sqrt(-1344*(9/32*I*s
qrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*
I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I
*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) +
16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqr
t(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 + 7*x^2*(9*I*sqrt(7) + 16*
sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) + 7*x^2*(-9*I*sqrt(7) + 16*sqrt
(9803/6272*I*sqrt(7) + 2815/896) - 35) + 980*x^2)*log(49/4*(207711*I*sqrt(7
```

```
) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7)
- 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(137
85856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)
^2 + 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) -
68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
/1024*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896)
 + 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2
205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(
23079*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)
 - 149504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896)
- 35) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sq
rt(-9803/6272*I*sqrt(7) + 2815/896) + 666914) - 336*x + 560)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+3\,x^2+x+5}{\left(2\,x^4+x^3+5\,x^2+x+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)
```

maple [C] time = 0.01, size = 77, normalized size = 0.24

$$-\frac{35\ln(x)}{8}+\frac{\left(70\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+47\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+141\,\mathrm{Ro}\right.}{64\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+24\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/8*sum((70*_R^3+47*_R^2+141*_R+61)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=Ro
otOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))-5/4/x^2+3/4/x-35/8*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,x-5}{4\,x^2}+\frac{1}{8}\int\frac{70\,x^3+47\,x^2+141\,x+61}{2\,x^4+x^3+5\,x^2+x+2}\,dx-\frac{35}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 + x
^3 + 5*x^2 + x + 2), x) - 35/8*log(x)
```

mupad [B] time = 2.25, size = 246, normalized size = 0.78

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{8939\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)}{128}-\frac{69\,x}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)\,x\,14945}{128}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] symsum(log((14945*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z
, k)*x)/128 - (69*x)/8 - (8939*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7
 + 128/343, z, k))/128 - (269991*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)
/7 + 128/343, z, k)^2*x)/1024 - (1393*root(z^4 - (35*z^3)/8 + (47*z^2)/7 -
(8*z)/7 + 128/343, z, k)^3*x)/8 + (3675*root(z^4 - (35*z^3)/8 + (47*z^2)/7
- (8*z)/7 + 128/343, z, k)^4*x)/32 - (35697*root(z^4 - (35*z^3)/8 + (47*z^2
)/7 - (8*z)/7 + 128/343, z, k)^2)/512 - (18487*root(z^4 - (35*z^3)/8 + (47*
z^2)/7 - (8*z)/7 + 128/343, z, k)^3)/256 - (441*root(z^4 - (35*z^3)/8 + (47
*z^2)/7 - (8*z)/7 + 128/343, z, k)^4)/32 + 245/8)*root(z^4 - (35*z^3)/8 + (
47*z^2)/7 - (8*z)/7 + 128/343, z, k), k, 1, 4) - (35*log(x))/8 + ((3*x)/4 -
 5/4)/x^2
```

sympy [A] time = 2.70, size = 70, normalized size = 0.22

$$-\frac{35\log\left(x\right)}{8}+\mathrm{RootSum}\left(2744t^4-12005t^3+18424t^2-3136t+1024,\left(t\mapsto t\log\left(-\frac{20101387287723t^4}{91907904361586}+\frac{9445}{4595}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t + 1
024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 944515214496*
_t**3/45953952180793 + 16572327093911939*_t**2/5882105879141504 - 456447174
9800865*_t/735263234892688 + x + 70084064010625/91907904361586))) + (3*x -
5)/(4*x**2)
```

3.257 $\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$

Optimal. Leaf size=19

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

```
[Out] arctan(c*x^3/(b*x^2+a))/c
```

Rubi [A] time = 0.10, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(3a+bx^2\right)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx &= \left(3a^2\right)\text{Subst}\left(\int \frac{1}{a^2+9a^2c^2x^2}\,dx, x, \frac{x^3}{3a+3bx^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}\end{aligned}$$

Mathematica [C] time = 0.05, size = 87, normalized size = 4.58

$$\frac{1}{2}\text{RootSum}\left[\#1^6c^2+\#1^4b^2+2\#1^2ab+a^2\&,\frac{\#1^3b\log(x-\#1)+3\#1a\log(x-\#1)}{3\#1^4c^2+2\#1^2b^2+2ab}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 & , (3*a*Log[x - #1]*#1 + b*
Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) & ]/2
```

fricas [B] time = 0.76, size = 83, normalized size = 4.37

$$\frac{\arctan\left(\frac{cx}{b}\right)-\arctan\left(\frac{bc^2x^5+ab^2x+\left(b^3-ac^2\right)x^3}{a^2c}\right)+\arctan\left(\frac{bc^2x^3+\left(b^3-ac^2\right)x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fri
cas")
```

```
[Out] (arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c))
+ arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c
```

giac [B] time = 4.08, size = 87, normalized size = 4.58

$$\frac{\arctan\left(\frac{cx}{b}\right)+\arctan\left(-\frac{bc^2x^5+b^3x^3-ac^2x^3+ab^2x}{a^2c}\right)-\arctan\left(-\frac{bc^2x^3+b^3x-ac^2x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="gia
c")
```

```
[Out] (arctan(c*x/b) + arctan(-(b*c^2*x^5 + b^3*x^3 - a*c^2*x^3 + a*b^2*x)/(a^2*c
)) - arctan(-(b*c^2*x^3 + b^3*x - a*c^2*x)/(a*b*c)))/c
```

maple [C] time = 0.10, size = 75, normalized size = 3.95

$$\frac{\left(\text{RootOf}\left(c^2\_Z^6+b^2\_Z^4+2ab\_Z^2+a^2\right)^4b+3\,\text{RootOf}\left(c^2\_Z^6+b^2\_Z^4+2ab\_Z^2+a^2\right)^2a\right)\ln\left(-\text{RootOf}\left(c^2\right.\right.}{6\,\text{RootOf}\left(c^2\_Z^6+b^2\_Z^4+2ab\_Z^2+a^2\right)^5c^2+4\,\text{RootOf}\left(c^2\_Z^6+b^2\_Z^4+2ab\_Z^2+a^2\right)^3b^2+4\,\text{RootOf}\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*sum((_R^4*b+3*_R^2*a)/(3*_R^5*c^2+2*_R^3*b^2+2*_R*a*b)*ln(-_R+x),_R=Roo
tOf(_Z^6*c^2+_Z^4*b^2+2*_Z^2*a*b+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + 3a)x^2}{c^2x^6 + b^2x^4 + 2abx^2 + a^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

mupad [B] time = 2.27, size = 252, normalized size = 13.26

$$\frac{\operatorname{atan}\left(\frac{27\,a\,c^5\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{27\,b\,c^5\,x^5}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{31\,b^3\,c^3\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^6\,c\,x^3}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}+\frac{4\,b^5\,c\,x}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^4\,c^3\,x^5}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}-\frac{27\,a\,b^2\,c^3}{27\,a^2\,c^4-4\,a}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*a + b*x^2))/(a^2 + b^2*x^4 + c^2*x^6 + 2*a*b*x^2),x)
```

```
[Out] (atan((27*a*c^5*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) - (27*b*c^5*x^5)/(27*a^2*c^
4 - 4*a*b^3*c^2) - (31*b^3*c^3*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^6*c*x
^3)/(27*a^3*c^4 - 4*a^2*b^3*c^2) + (4*b^5*c*x)/(27*a^2*c^4 - 4*a*b^3*c^2) +
 (4*b^4*c^3*x^5)/(27*a^3*c^4 - 4*a^2*b^3*c^2) - (27*a*b^2*c^3*x)/(27*a^2*c^
4 - 4*a*b^3*c^2)) + atan((c*x^3)/a - (c*x)/b + (b^2*x)/(a*c)) + atan((c*x)/
b))/c
```

sympy [C] time = 1.05, size = 44, normalized size = 2.32

$$\frac{-\frac{i \log\left(-\frac{ia}{c}-\frac{ibx^2}{c}+x^3\right)}{2}+\frac{i \log\left(\frac{ia}{c}+\frac{ibx^2}{c}+x^3\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2)
/c
```

3.258 $\int \frac{1-3x^4}{(-2+x)(1+x^2)^2} dx$

Optimal. Leaf size=43

$$-\frac{1-2x}{5(x^2+1)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)-\frac{46}{25}\tan^{-1}(x)$$

```
[Out] 1/5*(-1+2*x)/(x^2+1)-46/25*arctan(x)-47/25*ln(2-x)-14/25*ln(x^2+1)
```

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1647, 1629, 635, 203, 260}

$$-\frac{1-2x}{5(x^2+1)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)-\frac{46}{25}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] -(1 - 2*x)/(5*(1 + x^2)) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25 - (14*Log
[1 + x^2])/25
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx &= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{1}{2}\int \frac{-\frac{18}{5}-\frac{4x}{5}+6x^2}{(-2+x)\left(1+x^2\right)}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{1}{2}\int \left(\frac{94}{25(-2+x)}+\frac{4(23+14x)}{25\left(1+x^2\right)}\right)dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{47}{25}\log(2-x)-\frac{2}{25}\int \frac{23+14x}{1+x^2}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{47}{25}\log(2-x)-\frac{28}{25}\int \frac{x}{1+x^2}\,dx-\frac{46}{25}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1-2x}{5\left(1+x^2\right)}-\frac{46}{25}\tan^{-1}(x)-\frac{47}{25}\log(2-x)-\frac{14}{25}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.33

$$
\frac{2(x-2)+3}{5\left((x-2)^2+4(x-2)+5\right)}-\frac{14}{25}\log\left((x-2)^2+4(x-2)+5\right)-\frac{47}{25}\log(x-2)-\frac{46}{25}\tan^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] (3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x])/25 - (1
4*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x])/25
```

fricas [A] time = 0.81, size = 47, normalized size = 1.09

$$-\frac{46\left(x^{2}+1\right)\arctan(x)+14\left(x^{2}+1\right)\log\left(x^{2}+1\right)+47\left(x^{2}+1\right)\log(x-2)-10\,x+5}{25\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x^2 + 1)*lo
g(x - 2) - 10*x + 5)/(x^2 + 1)
```

giac [A] time = 0.25, size = 34, normalized size = 0.79

$$\frac{2\,x-1}{5\left(x^{2}+1\right)}-\frac{46}{25}\arctan(x)-\frac{14}{25}\log\left(x^{2}+1\right)-\frac{47}{25}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
abs(x - 2))
```

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$-\frac{46\arctan(x)}{25}-\frac{47\ln(x-2)}{25}-\frac{14\ln\left(x^{2}+1\right)}{25}-\frac{2\left(-5x+\frac{5}{2}\right)}{25\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^4+1)/(-2+x)/(x^2+1)^2,x)
```

```
[Out] -2/25*(-5*x+5/2)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)-47/25*ln(-2+x)
```

maxima [A] time = 2.10, size = 33, normalized size = 0.77

$$\frac{2\,x-1}{5\left(x^{2}+1\right)}-\frac{46}{25}\arctan(x)-\frac{14}{25}\log\left(x^{2}+1\right)-\frac{47}{25}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
x - 2)
```

mupad [B] time = 0.05, size = 38, normalized size = 0.88

$$\frac{\frac{2x}{5}-\frac{1}{5}}{x^2+1}-\frac{47\ \ln(x-2)}{25}+\ln(x-\mathrm{i})\left(-\frac{14}{25}+\frac{23}{25}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{14}{25}-\frac{23}{25}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x)
```

```
[Out] ((2*x)/5 - 1/5)/(x^2 + 1) - log(x - 1i)*(14/25 - 23i/25) - log(x + 1i)*(14/
25 + 23i/25) - (47*log(x - 2))/25
```

sympy [A] time = 0.17, size = 37, normalized size = 0.86

$$-\frac{1-2x}{5x^2+5}-\frac{47\log(x-2)}{25}-\frac{14\log\left(x^2+1\right)}{25}-\frac{46\,\mathrm{atan}(x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)
```

```
[Out] -(1 - 2*x)/(5*x**2 + 5) - 47*log(x - 2)/25 - 14*log(x**2 + 1)/25 - 46*atan(
x)/25
```

3.259 $\int \frac{-9-9x+2x^2}{-9x+x^3} \, dx$

Optimal. Leaf size=17

$$-\log(3-x)+\log(x)+2\log(x+3)$$

```
[Out] -ln(3-x)+ln(x)+2*ln(3+x)
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1593, 1802}

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-9-9x+2x^2}{-9x+x^3} \, dx &= \int \frac{-9-9x+2x^2}{x\left(-9+x^2\right)} \, dx\\ &= \int \left(\frac{1}{3-x}+\frac{1}{x}+\frac{2}{3+x}\right) dx\\ &= -\log(3-x)+\log(x)+2\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

fricas [A] time = 0.94, size = 15, normalized size = 0.88

$$2 \log(x+3) - \log(x-3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="fricas")
```

```
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

giac [A] time = 0.31, size = 18, normalized size = 1.06

$$2 \log(|x+3|) - \log(|x-3|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="giac")
```

```
[Out] 2*log(abs(x + 3)) - log(abs(x - 3)) + log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) - \ln(x-3) + 2\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-9*x-9)/(x^3-9*x),x)
```

```
[Out] 2*ln(3+x)-ln(-3+x)+ln(x)
```

maxima [A] time = 1.12, size = 15, normalized size = 0.88

$$2 \log(x+3) - \log(x-3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="maxima")
```

```
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

mupad [B] time = 2.20, size = 21, normalized size = 1.24

$$2 \ln(x+3) - 2 \operatorname{atanh}\left(\frac{1296}{18x+162} - 7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x - 2*x^2 + 9)/(9*x - x^3),x)
```

```
[Out] 2*log(x + 3) - 2*atanh(1296/(18*x + 162) - 7)
```

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$\log(x) - \log(x - 3) + 2\log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-9*x-9)/(x**3-9*x),x)
```

```
[Out] log(x) - log(x - 3) + 2*log(x + 3)
```

3.260 $\int \frac{1+2x^2+x^5}{-x+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

```
[Out] x+1/3*x^3+2*ln(1-x)-ln(x)+ln(1+x)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1593, 1802}

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^2+x^5}{-x+x^3}\,dx &= \int \frac{1+2x^2+x^5}{x\left(-1+x^2\right)}\,dx\\
&= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}+x^2+\frac{1}{1+x}\right)dx\\
&= x+\frac{x^3}{3}+2\log(1-x)-\log(x)+\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^3}{3} + x + 2\log(1-x) - \log(x) + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^5)/(-x + x^3),x]
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

fricas [A] time = 0.76, size = 21, normalized size = 0.84

$$\frac{1}{3}x^3 + x + \log(x+1) + 2\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="fricas")
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

giac [A] time = 0.25, size = 24, normalized size = 0.96

$$\frac{1}{3}x^3 + x + \log(|x+1|) + 2\log(|x-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="giac")
[Out] 1/3*x^3 + x + log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{x^3}{3} + x - \ln(x) + 2\ln(x-1) + \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+2*x^2+1)/(x^3-x),x)
[Out] 1/3*x^3+x+2*ln(x-1)+ln(x+1)-ln(x)
```

maxima [A] time = 1.15, size = 21, normalized size = 0.84

$$\frac{1}{3}x^3 + x + \log(x+1) + 2\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

mupad [B] time = 0.05, size = 30, normalized size = 1.20

$$x + 2 \ln(x-1) + \frac{x^3}{3} + \operatorname{atan}\left(\frac{48i}{11\ (22x-2)} + \frac{13}{11}i\right) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + x^5 + 1)/(x - x^3),x)
[Out] x + 2*log(x - 1) + atan(48i/(11*(22*x - 2)) + 13i/11)*2i + x^3/3
```

sympy [A] time = 0.13, size = 20, normalized size = 0.80

$$\frac{x^3}{3} + x - \log(x) + 2\log(x-1) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+2*x**2+1)/(x**3-x),x)
[Out] x**3/3 + x - log(x) + 2*log(x - 1) + log(x + 1)
```

3.261 $\int \frac{3+2x^2}{(-1+x)^2 x} \, dx$

Optimal. Leaf size=22

$$\frac{5}{1-x} - \log(1-x) + 3\log(x)$$

```
[Out] 5/(1-x)-ln(1-x)+3*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {894}

$$\frac{5}{1-x} - \log(1-x) + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

```
[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{3+2x^2}{(-1+x)^2 x} \, dx &= \int \left(\frac{1}{1-x} + \frac{5}{(-1+x)^2} + \frac{3}{x}\right) dx \\ &= \frac{5}{1-x} - \log(1-x) + 3\log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.91

$$-\frac{5}{x-1} - \log(1-x) + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

[Out] -5/(-1 + x) - Log[1 - x] + 3*Log[x]

fricas [A] time = 0.57, size = 24, normalized size = 1.09

$$-\frac{(x-1)\log(x-1) - 3(x-1)\log(x) + 5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="fricas")

[Out] -((x - 1)*log(x - 1) - 3*(x - 1)*log(x) + 5)/(x - 1)

giac [A] time = 0.37, size = 28, normalized size = 1.27

$$-\frac{5}{x-1} + 2\log(|x-1|) + 3\log\left(\left|-\frac{1}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="giac")

[Out] -5/(x - 1) + 2*log(abs(x - 1)) + 3*log(abs(-1/(x - 1) - 1))

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$3\ln(x) - \ln(x-1) - \frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+3)/(x-1)^2/x,x)

[Out] -5/(x-1)-ln(x-1)+3*ln(x)

maxima [A] time = 1.09, size = 18, normalized size = 0.82

$$-\frac{5}{x-1} - \log(x-1) + 3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="maxima")

[Out] -5/(x - 1) - log(x - 1) + 3*log(x)

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$3\ln(x) - \ln(x-1) - \frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 3)/(x*(x - 1)^2),x)
```

```
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

sympy [A] time = 0.11, size = 14, normalized size = 0.64

$$3\log(x) - \log(x - 1) - \frac{5}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(-1+x)**2/x,x)
```

```
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

3.262 $\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx$

Optimal. Leaf size=27

$$\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)+\frac{3}{17}\tan^{-1}(x)$$

```
[Out] 3/17*arctan(x)-7/34*ln(1-4*x)+6/17*ln(x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1629, 635, 203, 260}

$$\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)+\frac{3}{17}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

```
[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x^2}{(-1+4x)\left(1+x^2\right)}\,dx &= \int \left(-\frac{14}{17(-1+4x)}+\frac{3(1+4x)}{17\left(1+x^2\right)}\right)dx\\ &= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1+4x}{1+x^2}\,dx\\ &= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1}{1+x^2}\,dx+\frac{12}{17}\int \frac{x}{1+x^2}\,dx\\ &= \frac{3}{17}\tan^{-1}(x)-\frac{7}{34}\log(1-4x)+\frac{6}{17}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.41

$$-\frac{7}{34}\log(4x-1)+\frac{6}{17}\log\left((4x-1)^2+2(4x-1)+17\right)+\frac{3}{17}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
[Out] (3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x) + (-1 +
4*x)^2])/17
```

fricas [A] time = 0.95, size = 21, normalized size = 0.78

$$\frac{3}{17}\arctan(x)+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="fricas")
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

giac [A] time = 0.29, size = 22, normalized size = 0.81

$$\frac{3}{17}\arctan(x)+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(|4x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="giac")
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(abs(4*x - 1))
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{3\arctan(x)}{17}-\frac{7\ln(4x-1)}{34}+\frac{6\ln\left(x^2+1\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(4*x-1)/(x^2+1),x)
```

```
[Out] 6/17*ln(x^2+1)+3/17*arctan(x)-7/34*ln(4*x-1)
```

maxima [A] time = 2.31, size = 21, normalized size = 0.78

$$\frac{3}{17}\arctan(x)+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

mupad [B] time = 2.26, size = 25, normalized size = 0.93

$$-\frac{7\ln\left(x-\frac{1}{4}\right)}{34}+\ln(x-\mathrm{i})\left(\frac{6}{17}-\frac{3}{34}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{6}{17}+\frac{3}{34}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 1)/((4*x - 1)*(x^2 + 1)),x)
```

```
[Out] log(x - 1i)*(6/17 - 3i/34) - (7*log(x - 1/4))/34 + log(x + 1i)*(6/17 + 3i/3
4)
```

sympy [A] time = 0.14, size = 26, normalized size = 0.96

$$-\frac{7\log\left(x-\frac{1}{4}\right)}{34}+\frac{6\log\left(x^2+1\right)}{17}+\frac{3\operatorname{atan}(x)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)
```

```
[Out] -7*log(x - 1/4)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17
```

3.263 $\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

```
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1810, 260}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx &= \int \left(-3+x+\frac{x}{1+x^2}\right)dx\\ &= -3x+\frac{x^2}{2}+\int \frac{x}{1+x^2}\,dx\\ &= -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

fricas [A] time = 0.82, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

giac [A] time = 0.27, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="giac")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{x^2}{2} - 3x + \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+2*x-3)/(x^2+1),x)
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

maxima [A] time = 2.28, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

mupad [**B**] time = 0.03, size = 17, normalized size = 0.81

$$\frac{\ln\left(x^2+1\right)}{2}-3\,x+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3*x^2 + x^3 - 3)/(x^2 + 1),x)
```

```
[Out] log(x^2 + 1)/2 - 3*x + x^2/2
```

sympy [**A**] time = 0.08, size = 15, normalized size = 0.71

$$\frac{x^2}{2}-3x+\frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)
```

```
[Out] x**2/2 - 3*x + log(x**2 + 1)/2
```

3.264 $\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} dx$

Optimal. Leaf size=27

$$\frac{x^3}{3} + \frac{1}{2}\log\left(x^2+6x+10\right) - 3\tan^{-1}(x+3)$$

```
[Out] 1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{3} + \frac{1}{2}\log\left(x^2+6x+10\right) - 3\tan^{-1}(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
```

```
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx &= \int \left(x^2+\frac{x}{10+6x+x^2}\right)dx\\ &= \frac{x^3}{3}+\int \frac{x}{10+6x+x^2}\,dx\\ &= \frac{x^3}{3}+\frac{1}{2}\int \frac{6+2x}{10+6x+x^2}\,dx-3\int \frac{1}{10+6x+x^2}\,dx\\ &= \frac{x^3}{3}+\frac{1}{2}\log\left(10+6x+x^2\right)+6\,\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,6+2x\right)\\ &= \frac{x^3}{3}-3\tan^{-1}(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

fricas [A] time = 1.07, size = 23, normalized size = 0.85

$$\frac{1}{3}\,x^3-3\,\arctan\left(x+3\right)+\frac{1}{2}\,\log\left(x^2+6\,x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="fricas")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

giac [A] time = 0.28, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="giac")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^3}{3} - 3\arctan(x+3) + \frac{\ln\left(x^2+6x+10\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x)
[Out] 1/3*x^3-3*arctan(x+3)+1/2*ln(x^2+6*x+10)
```

maxima [A] time = 2.08, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="maxima")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

mupad [B] time = 2.14, size = 23, normalized size = 0.85

$$\frac{\ln\left(x^2+6x+10\right)}{2} - 3\,\mathrm{atan}(x+3) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 10*x^2 + 6*x^3 + x^4)/(6*x + x^2 + 10),x)
[Out] log(6*x + x^2 + 10)/2 - 3*atan(x + 3) + x^3/3
```

sympy [A] time = 0.11, size = 22, normalized size = 0.81

$$\frac{x^3}{3} + \frac{\log\left(x^2+6x+10\right)}{2} - 3\,\mathrm{atan}(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)
```

```
[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)
```

3.265 $\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

```
[Out] 1/8*ln(1-x)-1/5*ln(2-x)+1/12*ln(3-x)-1/120*ln(3+x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2058}

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx &= \int \left(\frac{1}{12(-3+x)}-\frac{1}{5(-2+x)}+\frac{1}{8(-1+x)}-\frac{1}{120(3+x)}\right)dx\\ &= \frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

fricas [A] time = 0.78, size = 25, normalized size = 0.64

$$-\frac{1}{120}\log\left(x+3\right)+\frac{1}{8}\log\left(x-1\right)-\frac{1}{5}\log\left(x-2\right)+\frac{1}{12}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="fricas")
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

giac [A] time = 0.32, size = 29, normalized size = 0.74

$$-\frac{1}{120}\log\left(|x+3|\right)+\frac{1}{8}\log\left(|x-1|\right)-\frac{1}{5}\log\left(|x-2|\right)+\frac{1}{12}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="giac")
[Out] -1/120*log(abs(x + 3)) + 1/8*log(abs(x - 1)) - 1/5*log(abs(x - 2)) + 1/12*l
og(abs(x - 3))
```

maple [A] time = 0.01, size = 26, normalized size = 0.67

$$\frac{\ln\left(x-3\right)}{12}-\frac{\ln\left(x-2\right)}{5}+\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x+3\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^3-7*x^2+27*x-18),x)
[Out] 1/8*ln(x-1)-1/5*ln(x-2)-1/120*ln(x+3)+1/12*ln(x-3)
```

maxima [A] time = 1.27, size = 25, normalized size = 0.64

$$-\frac{1}{120}\log\left(x+3\right)+\frac{1}{8}\log\left(x-1\right)-\frac{1}{5}\log\left(x-2\right)+\frac{1}{12}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="maxima")
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

mupad [B] time = 2.14, size = 25, normalized size = 0.64

$$\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x-2\right)}{5}+\frac{\ln\left(x-3\right)}{12}-\frac{\ln\left(x+3\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(7*x^2 - 27*x + 3*x^3 - x^4 + 18),x)
```

```
[Out] log(x - 1)/8 - log(x - 2)/5 + log(x - 3)/12 - log(x + 3)/120
```

sympy [A] time = 0.24, size = 26, normalized size = 0.67

$$\frac{\log\left(x-3\right)}{12}-\frac{\log\left(x-2\right)}{5}+\frac{\log\left(x-1\right)}{8}-\frac{\log\left(x+3\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)
```

```
[Out] log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120
```

3.266 $\int \frac{1+x^3}{-2+x} \, dx$

Optimal. Leaf size=22

$$\frac{x^3}{3} + x^2 + 4x + 9\log(2 - x)$$

```
[Out] 4*x+x^2+1/3*x^3+9*ln(2-x)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1850}

$$\frac{x^3}{3} + x^2 + 4x + 9\log(2 - x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-2 + x),x]
```

```
[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{1 + x^3}{-2 + x} \, dx &= \int \left(4 + \frac{9}{-2 + x} + 2x + x^2\right) dx \\ &= 4x + x^2 + \frac{x^3}{3} + 9\log(2 - x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^3}{3} + x^2 + 4x + 9\log(x - 2) - \frac{44}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-2 + x),x]
```

```
[Out] -44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]
```

fricas [A] time = 0.92, size = 18, normalized size = 0.82

$$\frac{1}{3}\,x^{3} + x^{2} + 4\,x + 9\,\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="fricas")
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

giac [A] time = 0.28, size = 19, normalized size = 0.86

$$\frac{1}{3}\,x^{3} + x^{2} + 4\,x + 9\,\log\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="giac")
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(abs(x - 2))
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^{3}}{3} + x^{2} + 4x + 9\ln\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x-2),x)
[Out] 1/3*x^3+x^2+4*x+9*ln(x-2)
```

maxima [A] time = 1.13, size = 18, normalized size = 0.82

$$\frac{1}{3}\,x^{3} + x^{2} + 4\,x + 9\,\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="maxima")
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$4\,x + 9\,\ln\left(x - 2\right) + x^{2} + \frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x - 2),x)
```

```
[Out] 4*x + 9*log(x - 2) + x^2 + x^3/3
```

sympy [A] time = 0.07, size = 17, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + 4x + 9\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(-2+x),x)
```

```
[Out] x**3/3 + x**2 + 4*x + 9*log(x - 2)
```

3.267 $\int \frac{3x-4x^2+3x^3}{1+x^2} dx$

Optimal. Leaf size=15

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

```
[Out] -4*x+3/2*x^2+4*arctan(x)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1594, 1802, 203}

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx &= \int \frac{x\left(3-4x+3x^2\right)}{1+x^2}\,dx\\ &= \int \left(-4+3x+\frac{4}{1+x^2}\right)dx\\ &= -4x+\frac{3x^2}{2}+4\int \frac{1}{1+x^2}\,dx\\ &= -4x+\frac{3x^2}{2}+4\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{3x^2}{2}-4x+4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

fricas [A] time = 0.82, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2-4\,x+4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="fricas")
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

giac [A] time = 0.34, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2-4\,x+4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="giac")
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{3x^2}{2}-4x+4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-4*x^2+3*x)/(x^2+1),x)
```

```
[Out] -4*x+3/2*x^2+4*arctan(x)
```

maxima [A] time = 2.30, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2 - 4\,x + 4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

mupad [B] time = 2.13, size = 13, normalized size = 0.87

$$4\,\mathrm{atan}(x) - 4\,x + \frac{3\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2 + 3*x^3)/(x^2 + 1),x)
```

```
[Out] 4*atan(x) - 4*x + (3*x^2)/2
```

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)
```

```
[Out] 3*x**2/2 - 4*x + 4*atan(x)
```

3.268 $\int \frac{5+3x}{1-x-x^2+x^3}\,dx$

Optimal. Leaf size=12

$$\frac{4}{1-x}+\tanh^{-1}(x)$$

```
[Out] 4/(1-x)+arctanh(x)
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2074, 206}

$$\frac{4}{1-x}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] 4/(1 - x) + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+3x}{1-x-x^2+x^3}\,dx &= \int \left(\frac{4}{(-1+x)^2}+\frac{1}{1-x^2}\right)dx\\
&= \frac{4}{1-x}+\int \frac{1}{1-x^2}\,dx\\
&= \frac{4}{1-x}+\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 2.00

$$-\frac{4}{x-1}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] -4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2
```

fricas [B] time = 0.73, size = 26, normalized size = 2.17

$$\frac{(x-1)\log(x+1)-(x-1)\log(x-1)-8}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)
```

giac [B] time = 0.30, size = 22, normalized size = 1.83

$$-\frac{4}{x-1}+\frac{1}{2}\log(|x+1|)-\frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="giac")
```

```
[Out] -4/(x - 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 21, normalized size = 1.75

$$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}-\frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+3*x)/(x^3-x^2-x+1),x)
```

```
[Out] -4/(x-1)-1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [A] time = 1.08, size = 20, normalized size = 1.67

$$-\frac{4}{x-1}+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="maxima")
```

```
[Out] -4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.07, size = 10, normalized size = 0.83

$$\operatorname{atanh}(x) - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + 5)/(x + x^2 - x^3 - 1),x)
```

```
[Out] atanh(x) - 4/(x - 1)
```

sympy [B] time = 0.10, size = 17, normalized size = 1.42

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x**3-x**2-x+1),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)
```

3.269 $\int \frac{-1-x-x^3+x^4}{-x^2+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

[Out] -1/x+1/2*x^2-2*ln(1-x)+2*ln(x)

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1593, 1620}

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]

[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-x-x^3+x^4}{-x^2+x^3} \, dx &= \int \frac{-1-x-x^3+x^4}{(-1+x)x^2} \, dx \\
&= \int \left(-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}+x\right) dx \\
&= -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

fricas [A] time = 0.87, size = 22, normalized size = 0.88

$$\frac{x^3-4\,x\log\left(x-1\right)+4\,x\log(x)-2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="fricas")
[Out] 1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x
```

giac [A] time = 0.27, size = 23, normalized size = 0.92

$$\frac{1}{2}\,x^2-\frac{1}{x}-2\,\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="giac")
[Out] 1/2*x^2 - 1/x - 2*log(abs(x - 1)) + 2*log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{x^2}{2}+2\ln(x)-2\ln\left(x-1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3-x-1)/(x^3-x^2),x)
[Out] 1/2*x^2-2*ln(x-1)-1/x+2*ln(x)
```

maxima [A] time = 1.13, size = 21, normalized size = 0.84

$$\frac{1}{2}\,x^2-\frac{1}{x}-2\,\log\left(x-1\right)+2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)
```

mupad [B] time = 2.14, size = 19, normalized size = 0.76

$$4\,\mathrm{atanh}\left(2\,x-1\right)-\frac{1}{x}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 - x^4 + 1)/(x^2 - x^3),x)
[Out] 4*atanh(2*x - 1) - 1/x + x^2/2
```

sympy [A] time = 0.10, size = 19, normalized size = 0.76

$$\frac{x^2}{2}+2\log\left(x\right)-2\log\left(x-1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3-x-1)/(x**3-x**2),x)
[Out] x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x
```

3.270 $\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

```
[Out] arctan(x)+1/2*ln(x^2+2)
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1673, 1149, 203, 1247, 626, 31}

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx &= \int \frac{x\left(1+x^2\right)}{2+3x^2+x^4}\,dx+\int \frac{2+x^2}{2+3x^2+x^4}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x}{2+3x+x^2}\,dx,x,x^2\right)+\int \frac{1}{1+x^2}\,dx\\
&= \tan^{-1}(x)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^2\right)\\
&= \tan^{-1}(x)+\frac{1}{2}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

fricas [A] time = 0.83, size = 11, normalized size = 0.85

$$\arctan(x)+\frac{1}{2}\,\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="fricas")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

giac [A] time = 0.31, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log\left(x^2 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="giac")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\arctan(x) + \frac{\ln\left(x^2 + 2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+2)/(x^4+3*x^2+2),x)
```

```
[Out] arctan(x)+1/2*ln(x^2+2)
```

maxima [A] time = 2.16, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log\left(x^2 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="maxima")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln\left(x^2 + 2\right)}{2} + \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 2)/(3*x^2 + x^4 + 2),x)
```

```
[Out] log(x^2 + 2)/2 + atan(x)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$\frac{\log\left(x^2+2\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)
```

```
[Out] log(x**2 + 2)/2 + atan(x)
```

3.271 $\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx$

Optimal. Leaf size=35

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1814, 1586, 635, 203, 260}

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
```

```
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{\left(2+x^2\right)^3}\,dx &= -\frac{1}{\left(2+x^2\right)^2}-\frac{1}{8}\int \frac{16-16x+8x^2-8x^3}{\left(2+x^2\right)^2}\,dx\\
&= -\frac{1}{\left(2+x^2\right)^2}-\frac{1}{8}\int \frac{8-8x}{2+x^2}\,dx\\
&= -\frac{1}{\left(2+x^2\right)^2}-\int \frac{1}{2+x^2}\,dx+\int \frac{x}{2+x^2}\,dx\\
&= -\frac{1}{\left(2+x^2\right)^2}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$
-\frac{1}{\left(x^2+2\right)^2}+\frac{1}{2}\log\left(x^2+2\right)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

fricas [A] time = 0.78, size = 55, normalized size = 1.57

$$-\frac{\sqrt{2}\left(x^4+4\,x^2+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-\left(x^4+4\,x^2+4\right)\log\left(x^2+2\right)+2}{2\left(x^4+4\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x^4 + 4*x^2 + 4)*arctan(1/2*sqrt(2)*x) - (x^4 + 4*x^2 + 4)*l
og(x^2 + 2) + 2)/(x^4 + 4*x^2 + 4)
```

giac [A] time = 0.33, size = 30, normalized size = 0.86

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-\frac{1}{\left(x^2+2\right)^2}+\frac{1}{2}\,\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*log(x^2 + 2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.89

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\ln\left(x^2+2\right)}{2}-\frac{1}{\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x)
```

```
[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

maxima [A] time = 2.22, size = 35, normalized size = 1.00

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-\frac{1}{x^4+4\,x^2+4}+\frac{1}{2}\,\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*log(x^2 + 2)
```

mupad [B] time = 2.12, size = 35, normalized size = 1.00

$$\frac{\ln\left(x^2+2\right)}{2}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}-\frac{1}{x^4+4\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - 4*x^2 + 4*x^3 - x^4 + x^5 - 4)/(x^2 + 2)^3,x)
[Out] log(x^2 + 2)/2 - (2^(1/2)*atan((2^(1/2)*x)/2))/2 - 1/(4*x^2 + x^4 + 4)
```

sympy [A] time = 0.15, size = 36, normalized size = 1.03

$$\frac{\log\left(x^2+2\right)}{2}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}-\frac{1}{x^4+4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)
[Out] log(x**2 + 2)/2 - sqrt(2)*atan(sqrt(2)*x/2)/2 - 1/(x**4 + 4*x**2 + 4)
```

3.272 $\int \frac{-1-3x+x^2}{-2x+x^2+x^3} dx$

Optimal. Leaf size=23

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

```
[Out] -ln(1-x)+1/2*ln(x)+3/2*ln(2+x)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1594, 1628}

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1-3x+x^2}{-2x+x^2+x^3} dx &= \int \frac{-1-3x+x^2}{x\left(-2+x+x^2\right)} dx\\ &= \int \left(\frac{1}{1-x}+\frac{1}{2x}+\frac{3}{2(2+x)}\right) dx\\ &= -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

fricas [A] time = 0.69, size = 17, normalized size = 0.74

$$\frac{3}{2}\log\left(x+2\right)-\log\left(x-1\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

giac [A] time = 0.24, size = 20, normalized size = 0.87

$$\frac{3}{2}\log\left(|x+2|\right)-\log\left(|x-1|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
[Out] 3/2*log(abs(x + 2)) - log(abs(x - 1)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{\ln(x)}{2}-\ln\left(x-1\right)+\frac{3\ln\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x-1)/(x^3+x^2-2*x),x)
[Out] -ln(x-1)+3/2*ln(x+2)+1/2*ln(x)
```

maxima [A] time = 1.40, size = 17, normalized size = 0.74

$$\frac{3}{2}\log\left(x+2\right)-\log\left(x-1\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

mupad [B] time = 2.19, size = 17, normalized size = 0.74

$$\frac{3\ \ln (x+2)}{2}-\ln (x-1)+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - x^2 + 1)/(x^2 - 2*x + x^3),x)
[Out] (3*log(x + 2))/2 - log(x - 1) + log(x)/2
```

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\frac{\log (x)}{2}-\log (x-1)+\frac{3\log (x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)
[Out] log(x)/2 - log(x - 1) + 3*log(x + 2)/2
```

3.273 $\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx$

Optimal. Leaf size=23

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2-2x+3\right) + \log(x)$$

```
[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1594, 1628, 628}

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2-2x+3\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
```

```
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx &= \int \frac{3-x+3x^2-2x^3+x^4}{x\left(3-2x+x^2\right)}\,dx \\
&= \int \left(\frac{1}{x}+x+\frac{1-x}{3-2x+x^2}\right)dx \\
&= \frac{x^2}{2}+\log(x)+\int \frac{1-x}{3-2x+x^2}\,dx \\
&= \frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

fricas [A] time = 0.92, size = 19, normalized size = 0.83

$$\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-2\,x+3\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

giac [A] time = 0.29, size = 20, normalized size = 0.87

$$\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-2\,x+3\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(abs(x))
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{x^2}{2}+\ln(x)-\frac{\ln\left(x^2-2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x)
```

```
[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

maxima [A] time = 1.12, size = 19, normalized size = 0.83

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2 - 2\,x + 3\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

mupad [B] time = 0.06, size = 19, normalized size = 0.83

$$\ln(x) - \frac{\ln\left(x^2 - 2\,x + 3\right)}{2} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - x - 2*x^3 + x^4 + 3)/(3*x - 2*x^2 + x^3),x)
```

```
[Out] log(x) - log(x^2 - 2*x + 3)/2 + x^2/2
```

sympy [A] time = 0.11, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + \log\left(x\right) - \frac{\log\left(x^2 - 2x + 3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)
```

```
[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2
```

3.274 $\int \frac{-1+x+x^3}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=29

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1814, 635, 203, 260}

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^3)/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) - ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
```

```
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x+x^3}{\left(1+x^2\right)^2}\,dx &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{1-2x}{1+x^2}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{1}{2}\left(-\frac{x}{x^2+1}+\log\left(x^2+1\right)-\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^3)/(1 + x^2)^2,x]
[Out] (-(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2
```

fricas [A] time = 0.89, size = 32, normalized size = 1.10

$$-\frac{\left(x^2+1\right)\arctan(x)-\left(x^2+1\right)\log\left(x^2+1\right)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + x)/(x^2 + 1)
```

giac [A] time = 0.37, size = 23, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.01, size = 24, normalized size = 0.83

$$-\frac{x}{2\left(x^{2}+1\right)}-\frac{\arctan(x)}{2}+\frac{\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x-1)/(x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

maxima [A] time = 2.33, size = 23, normalized size = 0.79

$$-\frac{x}{2\left(x^{2}+1\right)}-\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 2.13, size = 25, normalized size = 0.86

$$\frac{\ln\left(x^{2}+1\right)}{2}-\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 - 1)/(x^2 + 1)^2,x)
```

```
[Out] log(x^2 + 1)/2 - atan(x)/2 - x/(2*(x^2 + 1))
```

sympy [A] time = 0.12, size = 20, normalized size = 0.69

$$-\frac{x}{2x^{2}+2}+\frac{\log\left(x^{2}+1\right)}{2}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x-1)/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2
```

3.275 $\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} \, dx$

Optimal. Leaf size=44

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -3/(1+x)+ln(x)-2*ln(1+x)+ln(x^2-x+1)-2/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2
)
```

Rubi [A] time = 0.24, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 6725, 634, 618, 204, 628}

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
+ Log[1 - x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x-x^2+8x^3+x^4}{\left(x+x^2\right)\left(1+x^3\right)}\,dx &= \int \frac{1+2x-x^2+8x^3+x^4}{x(1+x)\left(1+x^3\right)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{3}{(1+x)^2}-\frac{2}{1+x}+\frac{2x}{1-x+x^2}\right)dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+2\int \frac{x}{1-x+x^2}\,dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\int \frac{1}{1-x+x^2}\,dx+\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)-2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-\right. \\
&= -\frac{3}{1+x}-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)+\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) + (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
 + Log[1 - x + x^2]
```

fricas [A] time = 0.88, size = 58, normalized size = 1.32

$$\frac{2\sqrt{3}\,(x+1)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+3\,(x+1)\log\left(x^2-x+1\right)-6\,(x+1)\log\left(x+1\right)+3\,(x+1)\log(x)-9}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*(x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x + 1)*log(x^2 -
x + 1) - 6*(x + 1)*log(x + 1) + 3*(x + 1)*log(x) - 9)/(x + 1)
```

giac [A] time = 0.36, size = 43, normalized size = 0.98

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{3}{x+1}+\log\left(x^2-x+1\right)-2\log\left(|x+1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.01, size = 42, normalized size = 0.95

$$\frac{2\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\ln(x)-2\ln\left(x+1\right)+\ln\left(x^2-x+1\right)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x)
```

```
[Out] ln(x^2-x+1)+2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-3/(x+1)-2*ln(x+1)+ln(x)
```

maxima [A] time = 2.25, size = 41, normalized size = 0.93

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{3}{x+1}+\log\left(x^2-x+1\right)-2\log\left(x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(x + 1) + log(x)
```

mupad [B] time = 0.13, size = 55, normalized size = 1.25

$$\ln(x)-2\ln(x+1)-\frac{3}{x+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-1+\frac{\sqrt{3}\,1i}{3}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(1+\frac{\sqrt{3}\,1i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 8*x^3 + x^4 + 1)/((x^3 + 1)*(x + x^2)),x)
```

```
[Out] log(x) - 2*log(x + 1) - 3/(x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/3 - 1) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/3 + 1)
```

sympy [A] time = 0.21, size = 49, normalized size = 1.11

$$\log(x)-2\log(x+1)+\log\left(x^2-x+1\right)+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/3 - 3/(x + 1)
```

3.276 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2
))*5^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{15-5x+x^2+x^3}{\left(5+x^2\right)\left(3+2x+x^2\right)}\,dx &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right) dx\\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$
\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

fricas [A] time = 0.88, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

giac [A] time = 0.30, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$-\sqrt{5}\arctan\left(\frac{\sqrt{5}\,x}{5}\right)+\frac{5\sqrt{2}\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}+\frac{\ln\left(x^2+2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

```
[Out] -arctan(1/5*5^(1/2)*x)*5^(1/2)+1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2*
x+2)*2^(1/2))
```

maxima [A] time = 2.29, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

mupad [B] time = 0.16, size = 88, normalized size = 1.91

$$\frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2}+\sqrt{5}\,\mathrm{atan}\left(\frac{2000\,\sqrt{5}}{2000\,x+1120}-\frac{224\,\sqrt{5}\,x}{2000\,x+1120}\right)-\frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

```
[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*
5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(
x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

sympy [A] time = 0.21, size = 51, normalized size = 1.11

$$\frac{\log\left(x^{2}+2x+3\right)}{2}-\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

3.277 $\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

Rubi [A] time = 0.17, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6742, 261, 260, 629, 628}

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 +
x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
```

```
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = \int \left(\frac{6x}{(1+x^2)^2}+\frac{2x}{1+x^2}+\frac{-1-2x}{(2+x+x^2)^2}+\frac{-1-2x}{2+x+x^2}\right)dx$$
$$= 2\int \frac{x}{1+x^2}\,dx + 6\int \frac{x}{(1+x^2)^2}\,dx + \int \frac{-1-2x}{(2+x+x^2)^2}\,dx +$$
$$= -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log(1+x^2)-\log(2+x+x^2)$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log(x^2+1)-\log(x^2+x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2
*(2 + x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

fricas [B] time = 0.85, size = 72, normalized size = 2.18

$$-\frac{2\,x^2+(x^4+x^3+3\,x^2+x+2)\log(x^2+x+2)-(x^4+x^3+3\,x^2+x+2)\log(x^2+1)+3\,x+5}{x^4+x^3+3\,x^2+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="fricas")
```

```
[Out] -(2*x^2 + (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + x + 2) - (x^4 + x^3 + 3*x^2
 + x + 2)*log(x^2 + 1) + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2)
```

giac [A] time = 0.30, size = 44, normalized size = 1.33

$$-\frac{2x^2+3x+5}{x^4+x^3+3x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="giac")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

maple [A] time = 0.01, size = 34, normalized size = 1.03

$$\ln\left(x^2+1\right)-\ln\left(x^2+x+2\right)-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x)
```

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

maxima [A] time = 1.16, size = 44, normalized size = 1.33

$$-\frac{2x^2+3x+5}{x^4+x^3+3x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="maxima")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

mupad [B] time = 2.16, size = 56, normalized size = 1.70

$$-\frac{2x^2+3x+5}{x^4+x^3+3x^2+x+2}+\operatorname{atan}\left(\frac{\frac{x\,224\mathrm{i}}{11}+\frac{224}{11}\mathrm{i}}{44x^2+16x+60}-\frac{3}{11}\mathrm{i}\right)2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6 - 3)/((x^2 + 1)^2*(x + x
^2 + 2)^2),x)
```

```
[Out] atan(((x*224i)/11 + 224i/11)/(16*x + 44*x^2 + 60) - 3i/11)*2i - (3*x + 2*x^
2 + 5)/(x + 3*x^2 + x^3 + x^4 + 2)
```

sympy [A] time = 0.19, size = 41, normalized size = 1.24

$$\frac{-2x^2-3x-5}{x^4+x^3+3x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x+
2)**2,x)
```

```
[Out] (-2*x**2 - 3*x - 5)/(x**4 + x**3 + 3*x**2 + x + 2) + log(x**2 + 1) - log(x*
*2 + x + 2)
```

3.278 $\int \frac{1}{(1+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {391, 203}

$$\frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/6 + ArcTan[x]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x^2)(4+x^2)} \, dx &= \frac{1}{3}\int \frac{1}{1+x^2} \, dx - \frac{1}{3}\int \frac{1}{4+x^2} \, dx \\ &= -\frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(4 + x^2)),x]
[Out] ArcTan[2/x]/6 + ArcTan[x]/3
```

fricas [A] time = 0.64, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

giac [A] time = 0.29, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="giac")
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

maple [A] time = 0.01, size = 12, normalized size = 0.71

$$\frac{\arctan(x)}{3}-\frac{\arctan\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^2+4),x)
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

maxima [A] time = 2.20, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.65

$$\frac{\operatorname{atan}(x)}{3}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x^2 + 4)),x)
```

```
[Out] atan(x)/3 - atan(x/2)/6
```

sympy [A] time = 0.14, size = 10, normalized size = 0.59

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}+\frac{\operatorname{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**2+4),x)
```

```
[Out] -atan(x/2)/6 + atan(x)/3
```

3.279 $\int \frac{a+bx^3}{1+x^2} dx$

Optimal. Leaf size=24

$$a\tan^{-1}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

```
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1810, 635, 203, 260}

$$a\tan^{-1}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{1+x^2}\,dx &= \int \left(bx+\frac{a-bx}{1+x^2}\right)dx\\ &= \frac{bx^2}{2}+\int \frac{a-bx}{1+x^2}\,dx\\ &= \frac{bx^2}{2}+a\int \frac{1}{1+x^2}\,dx-b\int \frac{x}{1+x^2}\,dx\\ &= \frac{bx^2}{2}+a\tan^{-1}(x)-\frac{1}{2}b\log\left(1+x^2\right)\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 22, normalized size = 0.92

$$a\tan^{-1}(x)+\frac{1}{2}b\left(x^2-\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(1 + x^2),x]
[Out] a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2
```

fricas [**A**] time = 0.76, size = 20, normalized size = 0.83

$$\frac{1}{2}\,bx^2+a\arctan(x)-\frac{1}{2}\,b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

giac [**A**] time = 0.29, size = 20, normalized size = 0.83

$$\frac{1}{2}\,bx^2+a\arctan(x)-\frac{1}{2}\,b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="giac")
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

maple [**A**] time = 0.00, size = 21, normalized size = 0.88

$$\frac{b\,x^2}{2}+a\arctan(x)-\frac{b\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(x^2+1),x)
```

```
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

maxima [A] time = 2.25, size = 20, normalized size = 0.83

$$\frac{1}{2}bx^2 + a\arctan(x) - \frac{1}{2}b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

mupad [B] time = 2.13, size = 20, normalized size = 0.83

$$\frac{b\,x^2}{2} - \frac{b\,\ln\left(x^2+1\right)}{2} + a\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(x^2 + 1),x)
```

```
[Out] (b*x^2)/2 - (b*log(x^2 + 1))/2 + a*atan(x)
```

sympy [C] time = 0.17, size = 34, normalized size = 1.42

$$\frac{bx^2}{2} + \left(-\frac{ia}{2} - \frac{b}{2}\right)\log\left(x - i\right) + \left(\frac{ia}{2} - \frac{b}{2}\right)\log\left(x + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(x**2+1),x)
```

```
[Out] b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)
```

3.280 $\int \frac{x+x^2}{(4+x)(-4+x^2)} dx$

Optimal. Leaf size=15

$$\log(x+4) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -1/2*arctanh(1/2*x)+ln(4+x)
```

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1593, 1629, 207}

$$\log(x+4) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

```
[Out] -ArcTanh[x/2]/2 + Log[4 + x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx = \int \frac{x(1+x)}{(4+x)(-4+x^2)}\,dx$$
$$= \int \left(\frac{1}{4+x}+\frac{1}{-4+x^2}\right)dx$$
$$= \log(4+x)+\int \frac{1}{-4+x^2}\,dx$$
$$= -\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)+\log(4+x)$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.53

$$\frac{1}{4}\log(2-x)-\frac{1}{4}\log(x+2)+\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/((4 + x)*(-4 + x^2)),x]
[Out] Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]
```

fricas [A] time = 0.96, size = 17, normalized size = 1.13

$$\log(x+4)-\frac{1}{4}\log(x+2)+\frac{1}{4}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="fricas")
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

giac [A] time = 0.32, size = 20, normalized size = 1.33

$$\log(|x+4|)-\frac{1}{4}\log(|x+2|)+\frac{1}{4}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="giac")
[Out] log(abs(x + 4)) - 1/4*log(abs(x + 2)) + 1/4*log(abs(x - 2))
```

maple [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{\ln(x-2)}{4}-\frac{\ln(x+2)}{4}+\ln(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(4+x)/(x^2-4),x)
```

```
[Out] ln(4+x)+1/4*ln(x-2)-1/4*ln(x+2)
```

maxima [A] time = 2.31, size = 17, normalized size = 1.13

$$\log(x+4) - \frac{1}{4}\log(x+2) + \frac{1}{4}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="maxima")
```

```
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

mupad [B] time = 0.06, size = 19, normalized size = 1.27

$$\ln(x+4) + \frac{\operatorname{atanh}\left(\frac{90}{7\,(21\,x+48)} - \frac{8}{7}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)/((x^2 - 4)*(x + 4)),x)
```

```
[Out] log(x + 4) + atanh(90/(7*(21*x + 48)) - 8/7)/2
```

sympy [A] time = 0.14, size = 17, normalized size = 1.13

$$\frac{\log(x-2)}{4} - \frac{\log(x+2)}{4} + \log(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(4+x)/(x**2-4),x)
```

```
[Out] log(x - 2)/4 - log(x + 2)/4 + log(x + 4)
```

3.281 $\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=20

$$3\tan^{-1}(x)-\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 3*arctan(x)-arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {522, 203}

$$3\tan^{-1}(x)-\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= -\left(2\int \frac{1}{2+x^2}\,dx\right)+3\int \frac{1}{1+x^2}\,dx\\ &= 3\tan^{-1}(x)-\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

fricas [A] time = 0.87, size = 17, normalized size = 0.85

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

giac [A] time = 0.38, size = 17, normalized size = 0.85

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.90

$$3\arctan(x)-\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)/(x^2+1)/(x^2+2),x)
```

```
[Out] 3*arctan(x)-2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.40, size = 17, normalized size = 0.85

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

mupad [B] time = 0.05, size = 17, normalized size = 0.85

$$3\,\mathrm{atan}(x)-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 4)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] 3*atan(x) - 2^(1/2)*atan((2^(1/2)*x)/2)
```

sympy [A] time = 0.15, size = 19, normalized size = 0.95

$$3\,\mathrm{atan}\,(x)-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)
```

3.282 $\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=37

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)+2\tan^{-1}(x)$$

```
[Out] 5/2/(1-x)+x+2*arctan(x)+1/2*ln(1-x)+3/4*ln(x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1629, 635, 203, 260}

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{5-4x+3x^2+x^4}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \left(1+\frac{5}{2(-1+x)^2}+\frac{1}{2(-1+x)}+\frac{4+3x}{2\left(1+x^2\right)}\right)dx\\ &= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{4+3x}{1+x^2}\,dx\\ &= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{3}{2}\int \frac{x}{1+x^2}\,dx+2\int \frac{1}{1+x^2}\,dx\\ &= \frac{5}{2(1-x)}+x+2\tan^{-1}(x)+\frac{1}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.89

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2-2x}+\frac{1}{2}\log(x-1)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] 5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/4
```

fricas [A] time = 0.51, size = 44, normalized size = 1.19

$$\frac{4\,x^2+8\,(x-1)\arctan(x)+3\,(x-1)\log\left(x^2+1\right)+2\,(x-1)\log\left(x-1\right)-4\,x-10}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x
 - 1) - 4*x - 10)/(x - 1)
```

giac [B] time = 0.29, size = 60, normalized size = 1.62

$$\frac{1}{2}\,\pi-2\,\pi\left\lfloor\frac{\pi+4\,\arctan(x)}{4\,\pi}+\frac{1}{2}\right\rfloor+x-\frac{5}{2\,(x-1)}+2\,\arctan(x)+\frac{3}{4}\,\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)+2\,\log\left(|x-1|\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x - 1) + 2*
arctan(x) + 3/4*log(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*log(abs(x - 1)) - 1
```

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$x+2\arctan(x)+\frac{\ln(x-1)}{2}+\frac{3\ln\left(x^2+1\right)}{4}-\frac{5}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2-4*x+5)/(x-1)^2/(x^2+1),x)
[Out] x-5/2/(x-1)+1/2*ln(x-1)+3/4*ln(x^2+1)+2*arctan(x)
```

maxima [A] time = 2.22, size = 27, normalized size = 0.73

$$x-\frac{5}{2(x-1)}+2\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)
```

mupad [B] time = 2.14, size = 35, normalized size = 0.95

$$x+\frac{\ln(x-1)}{2}-\frac{5}{2\ (x-1)}+\ln(x-\mathrm{i})\left(\frac{3}{4}-\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{3}{4}+1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 4*x + x^4 + 5)/((x^2 + 1)*(x - 1)^2),x)
[Out] x + log(x - 1)/2 + log(x - 1i)*(3/4 - 1i) + log(x + 1i)*(3/4 + 1i) - 5/(2*(
x - 1))
```

sympy [A] time = 0.16, size = 29, normalized size = 0.78

$$x+\frac{\log(x-1)}{2}+\frac{3\log\left(x^2+1\right)}{4}+2\,\mathrm{atan}\,(x)-\frac{5}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1),x)
[Out] x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)
```

3.283 $\int \frac{1+x^4}{2+x^2} \, dx$

Optimal. Leaf size=26

$$\frac{x^3}{3} - 2x + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -2*x+1/3*x^3+5/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1154, 203}

$$\frac{x^3}{3} - 2x + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(2 + x^2),x]
```

```
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{1+x^4}{2+x^2}\,dx = \int \left(-2+x^2+\frac{5}{2+x^2}\right)dx$$
$$= -2x+\frac{x^3}{3}+5\int \frac{1}{2+x^2}\,dx$$
$$= -2x+\frac{x^3}{3}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [**A**] time = 0.01, size = 26, normalized size = 1.00

$$\frac{x^3}{3}-2x+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(2 + x^2),x]
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

fricas [**A**] time = 0.60, size = 21, normalized size = 0.81

$$\frac{1}{3}\,x^3+\frac{5}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="fricas")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

giac [**A**] time = 0.31, size = 21, normalized size = 0.81

$$\frac{1}{3}\,x^3+\frac{5}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="giac")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.85

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^2+2),x)
[Out] -2*x+1/3*x^3+5/2*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.22, size = 21, normalized size = 0.81

$$\frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="maxima")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.81

$$\frac{5\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - 2\,x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^2 + 2),x)
[Out] (5*2^(1/2)*atan((2^(1/2)*x)/2))/2 - 2*x + x^3/3
```

sympy [A] time = 0.09, size = 26, normalized size = 1.00

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**2+2),x)
[Out] x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2
```

3.284 $\int \frac{2+2x+x^4}{x^4+x^5} \, dx$

Optimal. Leaf size=12

$$\log(x+1) - \frac{2}{3x^3}$$

```
[Out] -2/3/x^3+ln(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1593, 1620}

$$\log(x+1) - \frac{2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

```
[Out] -2/(3*x^3) + Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+2x+x^4}{x^4+x^5} \, dx &= \int \frac{2+2x+x^4}{x^4(1+x)} \, dx \\
&= \int \left(\frac{2}{x^4} + \frac{1}{1+x}\right) dx \\
&= -\frac{2}{3x^3} + \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(x+1)-\frac{2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x + x^4)/(x^4 + x^5),x]
[Out] -2/(3*x^3) + Log[1 + x]
```

fricas [A] time = 0.70, size = 16, normalized size = 1.33

$$\frac{3\,x^3\log\left(x+1\right)-2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="fricas")
[Out] 1/3*(3*x^3*log(x + 1) - 2)/x^3
```

giac [A] time = 0.29, size = 11, normalized size = 0.92

$$-\frac{2}{3\,x^3}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="giac")
[Out] -2/3/x^3 + log(abs(x + 1))
```

maple [A] time = 0.01, size = 11, normalized size = 0.92

$$\ln\left(x+1\right)-\frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x+2)/(x^5+x^4),x)
[Out] -2/3/x^3+ln(x+1)
```

maxima [A] time = 1.11, size = 10, normalized size = 0.83

$$-\frac{2}{3\,x^3}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="maxima")
```

```
[Out] -2/3/x^3 + log(x + 1)
```

mupad [B] time = 2.12, size = 10, normalized size = 0.83

$$\ln(x+1) - \frac{2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^4 + 2)/(x^4 + x^5),x)
```

```
[Out] log(x + 1) - 2/(3*x^3)
```

sympy [A] time = 0.09, size = 10, normalized size = 0.83

$$\log(x+1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x+2)/(x**5+x**4),x)
```

```
[Out] log(x + 1) - 2/(3*x**3)
```

3.285 $\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} \, dx$

Optimal. Leaf size=21

$$2\log(1-x) - \log(2-x) + \log(x+1)$$

```
[Out] 2*ln(1-x)-ln(2-x)+ln(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2074}

$$2\log(1-x) - \log(2-x) + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} \, dx &= \int \left(\frac{1}{2-x} + \frac{2}{-1+x} + \frac{1}{1+x}\right) dx \\
&= 2\log(1-x) - \log(2-x) + \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$2\log(1-x) - \log(2-x) + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

fricas [A] time = 0.79, size = 17, normalized size = 0.81

$$\log(x+1) + 2\log(x-1) - \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="fricas")
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

giac [A] time = 0.28, size = 20, normalized size = 0.95

$$\log(|x+1|) + 2\log(|x-1|) - \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="giac")
[Out] log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x - 2))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\ln(x-2) + 2\ln(x-1) + \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x)
[Out] 2*ln(x-1)-ln(x-2)+ln(x+1)
```

maxima [A] time = 1.09, size = 17, normalized size = 0.81

$$\log(x+1) + 2\log(x-1) - \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="maxima")
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

mupad [B] time = 2.14, size = 21, normalized size = 1.00

$$2\ln(x-1) - 2\,\mathrm{atanh}\left(\frac{144}{11\,(22\,x-50)} + \frac{13}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 2*x^2 + 1)/(x + 2*x^2 - x^3 - 2),x)
```

```
[Out] 2*log(x - 1) - 2*atanh(144/(11*(22*x - 50)) + 13/11)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.71

$$-\log(x-2)+2\log(x-1)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2),x)
[Out] -log(x - 2) + 2*log(x - 1) + log(x + 1)
```

3.286 $\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=22

$$\frac{x}{x^2+1} + \frac{1}{2}\log\left(x^2+1\right) + \tan^{-1}(x)$$

```
[Out] x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {28, 1814, 635, 203, 260}

$$\frac{x}{x^2+1} + \frac{1}{2}\log\left(x^2+1\right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx &= \int \frac{2+x+x^3}{\left(1+x^2\right)^2}\,dx\\ &= \frac{x}{1+x^2}-\frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx\\ &= \frac{x}{1+x^2}+\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= \frac{x}{1+x^2}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

fricas [A] time = 0.72, size = 34, normalized size = 1.55

$$\frac{2\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)+2\,x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 + 1)
```

giac [A] time = 0.28, size = 20, normalized size = 0.91

$$\frac{x}{x^2+1}+\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{x}{x^2+1}+\arctan(x)+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+2)/(x^4+2*x^2+1),x)
[Out] 1/(x^2+1)*x+arctan(x)+1/2*ln(x^2+1)
```

maxima [A] time = 2.23, size = 20, normalized size = 0.91

$$\frac{x}{x^2+1}+\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 2.12, size = 20, normalized size = 0.91

$$\frac{\ln\left(x^2+1\right)}{2}+\mathrm{atan}(x)+\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 2)/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + atan(x) + x/(x^2 + 1)
```

sympy [A] time = 0.11, size = 17, normalized size = 0.77

$$\frac{x}{x^2+1}+\frac{\log\left(x^2+1\right)}{2}+\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+2)/(x**4+2*x**2+1),x)
[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

3.287 $\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=24

$$-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

```
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {28, 1814, 635, 203, 260}

$$-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx &= \int \frac{1+2x+x^2+x^3}{\left(1+x^2\right)^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}+\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{1}{2\left(1+x^2\right)}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

fricas [A] time = 0.79, size = 32, normalized size = 1.33

$$\frac{2\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)-1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1)
```

giac [A] time = 0.40, size = 20, normalized size = 0.83

$$-\frac{1}{2\left(x^2+1\right)}+\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x)+\frac{\ln\left(x^2+1\right)}{2}-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x)
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

maxima [A] time = 2.18, size = 20, normalized size = 0.83

$$-\frac{1}{2\left(x^2+1\right)}+\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.92

$$\frac{\ln\left(x^2+1\right)}{2}+\mathrm{atan}(x)-\frac{1}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + x^3 + 1)/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + atan(x) - 1/(2*(x^2 + 1))
```

sympy [A] time = 0.12, size = 19, normalized size = 0.79

$$\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}\left(x\right)-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)
```

```
[Out] log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)
```

3.288 $\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1010, 391, 203, 444, 36, 31}

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
```

```
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3+4x}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= 3\int \frac{1}{\left(1+x^2\right)\left(2+x^2\right)}\,dx+4\int \frac{x}{\left(1+x^2\right)\left(2+x^2\right)}\,dx\\
&= 2\,\text{Subst}\left(\int \frac{1}{(1+x)(2+x)}\,dx,x,x^2\right)+3\int \frac{1}{1+x^2}\,dx-3\int \frac{1}{2+x^2}\,dx\\
&= 3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+2\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-2\,\text{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^2\right)\\
&= 3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+2\log\left(1+x^2\right)-2\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

fricas [A] time = 0.80, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)-2\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

giac [A] time = 0.25, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)-2\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

maple [A] time = 0.00, size = 34, normalized size = 0.94

$$3\arctan(x)-\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+2\ln\left(x^2+1\right)-2\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*x)/(x^2+1)/(x^2+2),x)
```

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.06, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)-2\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

mupad [B] time = 0.10, size = 56, normalized size = 1.56

$$\ln(x-\mathrm{i})\left(2-\frac{3}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(2+\frac{3}{2}\mathrm{i}\right)+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 1i)*(2 - 3i/2) + log(x + 1i)*(2 + 3i/2) + log(x - 2^(1/2)*1i)*((2^(
1/2)*3i)/4 - 2) - log(x + 2^(1/2)*1i)*((2^(1/2)*3i)/4 + 2)
```

sympy [A] time = 0.19, size = 39, normalized size = 1.08

$$2\log\left(x^{2}+1\right)-2\log\left(x^{2}+2\right)+3\,\mathrm{atan}\,(x)-\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x**2+1)/(x**2+2),x)
```

```
[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sqrt(2)*x/2)
/2
```

3.289 $\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

```
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1010, 391, 203, 444, 36, 31}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx &= 2\int \frac{1}{\left(1+x^2\right)\left(4+x^2\right)}\,dx+\int \frac{x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx,x,x^2\right)+\frac{2}{3}\int \frac{1}{1+x^2}\,dx-\frac{2}{3}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\ &= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((1 + x^2)*(4 + x^2)),x]
[Out] -1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

fricas [A] time = 0.75, size = 27, normalized size = 0.73

$$-\frac{1}{3}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{2}{3}\,\arctan(x)-\frac{1}{6}\,\log\left(x^2+4\right)+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

giac [A] time = 0.29, size = 27, normalized size = 0.73

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan(x)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="giac")
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 0.76

$$\frac{2\arctan(x)}{3}-\frac{\arctan\left(\frac{x}{2}\right)}{3}+\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+1)/(x^2+4),x)
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

maxima [A] time = 2.00, size = 27, normalized size = 0.73

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan(x)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="maxima")
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

mupad [B] time = 2.14, size = 37, normalized size = 1.00

$$\ln(x-\mathrm{i})\left(\frac{1}{6}-\frac{1}{3}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{6}+\frac{1}{3}\mathrm{i}\right)+\ln(x-2\mathrm{i})\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)+\ln(x+2\mathrm{i})\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x^2 + 1)*(x^2 + 4)),x)
[Out] log(x - 1i)*(1/6 - 1i/3) + log(x + 1i)*(1/6 + 1i/3) - log(x - 2i)*(1/6 - 1i
/6) - log(x + 2i)*(1/6 + 1i/6)
```

sympy [A] time = 0.17, size = 29, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3}+\frac{2\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+1)/(x**2+4),x)
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3
```

3.290 $\int \frac{2-x+x^3}{-7-6x+x^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

```
[Out] 6*x+1/2*x^2+169/4*ln(7-x)-1/4*ln(1+x)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx = \int \left(6+x+\frac{2(22+21x)}{-7-6x+x^2}\right)dx$$
$$= 6x+\frac{x^2}{2}+2\int \frac{22+21x}{-7-6x+x^2}\,dx$$
$$= 6x+\frac{x^2}{2}-\frac{1}{4}\int \frac{1}{1+x}\,dx+\frac{169}{4}\int \frac{1}{-7+x}\,dx$$
$$= 6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^3)/(-7 - 6*x + x^2),x]
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

fricas [A] time = 0.61, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2+6x-\frac{1}{4}\log(x+1)+\frac{169}{4}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="fricas")
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

giac [A] time = 0.37, size = 23, normalized size = 0.79

$$\frac{1}{2}x^2+6x-\frac{1}{4}\log(|x+1|)+\frac{169}{4}\log(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="giac")
[Out] 1/2*x^2 + 6*x - 1/4*log(abs(x + 1)) + 169/4*log(abs(x - 7))
```

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$\frac{x^2}{2}+6x-\frac{\ln(x+1)}{4}+\frac{169\ln(x-7)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x+2)/(x^2-6*x-7),x)
```

```
[Out] 1/2*x^2+6*x-1/4*ln(x+1)+169/4*ln(x-7)
```

maxima [A] time = 1.08, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(x+1) + \frac{169}{4}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

mupad [B] time = 2.21, size = 21, normalized size = 0.72

$$6x - \frac{\ln(x+1)}{4} + \frac{169\ \ln(x-7)}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - x + 2)/(6*x - x^2 + 7),x)
```

```
[Out] 6*x - log(x + 1)/4 + (169*log(x - 7))/4 + x^2/2
```

sympy [A] time = 0.11, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{169\log(x-7)}{4} - \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x+2)/(x**2-6*x-7),x)
```

```
[Out] x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4
```

3.291 $\int \frac{-1+x^5}{-1+x^2} \, dx$

Optimal. Leaf size=19

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

```
[Out] 1/2*x^2+1/4*x^4+ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1810, 627, 31}

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^5)/(-1 + x^2),x]
```

```
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \int \left(x+x^3-\frac{1-x}{-1+x^2}\right)dx$$
$$= \frac{x^2}{2}+\frac{x^4}{4}-\int \frac{1-x}{-1+x^2}\,dx$$
$$= \frac{x^2}{2}+\frac{x^4}{4}-\int \frac{1}{-1-x}\,dx$$
$$= \frac{x^2}{2}+\frac{x^4}{4}+\log(1+x)$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)/(-1 + x^2),x]
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

fricas [A] time = 0.80, size = 15, normalized size = 0.79

$$\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="fricas")
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

giac [A] time = 0.27, size = 16, normalized size = 0.84

$$\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="giac")
[Out] 1/4*x^4 + 1/2*x^2 + log(abs(x + 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^4}{4}+\frac{x^2}{2}+\ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)/(x^2-1),x)
```

```
[Out] 1/2*x^2+1/4*x^4+ln(x+1)
```

maxima [A] time = 1.07, size = 15, normalized size = 0.79

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$\ln(x+1) + \frac{x^2}{2} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 - 1)/(x^2 - 1),x)
```

```
[Out] log(x + 1) + x^2/2 + x^4/4
```

sympy [A] time = 0.08, size = 14, normalized size = 0.74

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)/(x**2-1),x)
```

```
[Out] x**4/4 + x**2/2 + log(x + 1)
```

3.292 $\int \frac{5+2x-x^2+x^3}{1+x+x^2} \, dx$

Optimal. Leaf size=41

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx &= \int \left(-2+x+\frac{7+3x}{1+x+x^2}\right) dx\\
&= -2x+\frac{x^2}{2}+\int \frac{7+3x}{1+x+x^2}\,dx\\
&= -2x+\frac{x^2}{2}+\frac{3}{2}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{11}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= -2x+\frac{x^2}{2}+\frac{3}{2}\log\left(1+x+x^2\right)-11\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -2x+\frac{x^2}{2}+\frac{11\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

fricas [A] time = 0.92, size = 34, normalized size = 0.83

$$\frac{1}{2}\,x^2+\frac{11}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-2\,x+\frac{3}{2}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

giac [A] time = 0.29, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

maple [A] time = 0.00, size = 35, normalized size = 0.85

$$\frac{x^2}{2} - 2x + \frac{11\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{3\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2+2*x+5)/(x^2+x+1),x)
```

```
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.18, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.88

$$\frac{3\ln\left(x^2+x+1\right)}{2} - 2x + \frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + x^3 + 5)/(x + x^2 + 1),x)
```

```
[Out] (3*log(x + x^2 + 1))/2 - 2*x + (11*3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3
))/3 + x^2/2
```

sympy [A] time = 0.12, size = 46, normalized size = 1.12

$$\frac{x^2}{2} - 2x + \frac{3\log\left(x^2 + x + 1\right)}{2} + \frac{11\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)
```

```
[Out] x**2/2 - 2*x + 3*log(x**2 + x + 1)/2 + 11*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

3.293 $\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} \, dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}+6\tan^{-1}(2-x)$$

```
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(-2+x)+3/4*ln(x^2-4*x+5)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}+6\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

```
[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx &= \int \left(\frac{3}{2}+x+\frac{x^2}{2}-\frac{3(6-x)}{10-8x+2x^2}\right)dx \\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-3\int \frac{6-x}{10-8x+2x^2}\,dx \\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\int \frac{-8+4x}{10-8x+2x^2}\,dx-12\int \frac{1}{10-8x+2x^2}\,dx \\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\log\left(5-4x+x^2\right)+24\,\mathrm{Subst}\left(\int \frac{1}{-16-x^2}\,dx,x,-8+4x\right) \\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+6\tan^{-1}(2-x)+\frac{3}{4}\log\left(5-4x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.95

$$\frac{1}{2}\left(\frac{x^3}{3}+x^2+\frac{3}{2}\log\left(x^2-4x+5\right)+3x+12\tan^{-1}(2-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
[Out] (3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/2)/2
```

fricas [A] time = 1.11, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3+\frac{1}{2}x^2+\frac{3}{2}x-6\arctan\left(x-2\right)+\frac{3}{4}\log\left(x^2-4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="fricas")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

giac [A] time = 0.28, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log\left(x^2 - 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="giac")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

maple [A] time = 0.00, size = 32, normalized size = 0.78

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} - 6\arctan(x-2) + \frac{3\ln\left(x^2 - 4x + 5\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x)
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(x-2)+3/4*ln(x^2-4*x+5)
```

maxima [A] time = 2.07, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log\left(x^2 - 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="maxima")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

mupad [B] time = 2.12, size = 31, normalized size = 0.76

$$\frac{3x}{2} - 6\,\mathrm{atan}(x-2) + \frac{3\ln\left(x^2 - 4x + 5\right)}{4} + \frac{x^2}{2} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^3 + x^4 - 3)/(2*x^2 - 8*x + 10),x)
[Out] (3*x)/2 - 6*atan(x - 2) + (3*log(x^2 - 4*x + 5))/4 + x^2/2 + x^3/6
```

sympy [A] time = 0.12, size = 34, normalized size = 0.83

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} + \frac{3\log\left(x^2 - 4x + 5\right)}{4} - 6\,\mathrm{atan}(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)
```

```
[Out] x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)
```

3.294 $\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=30

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

```
[Out] x+7/2*ln(1-x)-25*ln(2-x)+61/2*ln(3-x)
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1612}

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(1+\frac{61}{2(-3+x)}-\frac{25}{-2+x}+\frac{7}{2(-1+x)}\right)dx\\ &= x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.80

$$x+\frac{61}{2}\log(x-3)-25\log(x-2)+\frac{7}{2}\log(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
[Out] x + (61*Log[-3 + x])/2 - 25*Log[-2 + x] + (7*Log[-1 + x])/2
```

fricas [A] time = 0.96, size = 20, normalized size = 0.67

$$x + \frac{7}{2} \log(x-1) - 25 \log(x-2) + \frac{61}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

giac [A] time = 0.25, size = 23, normalized size = 0.77

$$x + \frac{7}{2} \log(|x-1|) - 25 \log(|x-2|) + \frac{61}{2} \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] x + 7/2*log(abs(x - 1)) - 25*log(abs(x - 2)) + 61/2*log(abs(x - 3))
```

maple [A] time = 0.01, size = 21, normalized size = 0.70

$$x + \frac{61 \ln(x-3)}{2} - 25 \ln(x-2) + \frac{7 \ln(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+2*x+1)/(x-3)/(x-2)/(x-1),x)
[Out] x+7/2*ln(x-1)-25*ln(x-2)+61/2*ln(x-3)
```

maxima [A] time = 1.04, size = 20, normalized size = 0.67

$$x + \frac{7}{2} \log(x-1) - 25 \log(x-2) + \frac{61}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

mupad [B] time = 2.13, size = 20, normalized size = 0.67

$$x + \frac{7 \ln(x-1)}{2} - 25 \ln(x-2) + \frac{61 \ln(x-3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2 + x^3 + 1)/((x - 1)*(x - 2)*(x - 3)),x)
```

```
[Out] x + (7*log(x - 1))/2 - 25*log(x - 2) + (61*log(x - 3))/2
```

sympy **[A]** time = 0.15, size = 24, normalized size = 0.80

$$x + \frac{61\log(x-3)}{2} - 25\log(x-2) + \frac{7\log(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2
```

3.295 $\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx$

Optimal. Leaf size=35

$$\frac{x^2}{2} - 2x + \frac{13}{3}\log(4-x) - \frac{22}{3}\log(x+2) + 20\log(x+3)$$

```
[Out] -2*x+1/2*x^2+13/3*ln(4-x)-22/3*ln(2+x)+20*ln(3+x)
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2074}

$$\frac{x^2}{2} - 2x + \frac{13}{3}\log(4-x) - \frac{22}{3}\log(x+2) + 20\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} \, dx &= \int \left(-2+\frac{13}{3(-4+x)}+x-\frac{22}{3(2+x)}+\frac{20}{3+x}\right) dx \\
&= -2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{x^2}{2} - 2x + \frac{13}{3}\log(4-x) - \frac{22}{3}\log(x+2) + 20\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

fricas [A] time = 0.75, size = 27, normalized size = 0.77

$$\frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(x+3\right) - \frac{22}{3}\,\log\left(x+2\right) + \frac{13}{3}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

giac [A] time = 0.32, size = 30, normalized size = 0.86

$$\frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(|x+3|\right) - \frac{22}{3}\,\log\left(|x+2|\right) + \frac{13}{3}\,\log\left(|x-4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="giac")
[Out] 1/2*x^2 - 2*x + 20*log(abs(x + 3)) - 22/3*log(abs(x + 2)) + 13/3*log(abs(x
- 4))
```

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{x^2}{2} - 2x - \frac{22\ln\left(x+2\right)}{3} + 20\ln\left(x+3\right) + \frac{13\ln\left(x-4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x)
[Out] -2*x+1/2*x^2-22/3*ln(x+2)+13/3*ln(x-4)+20*ln(x+3)
```

maxima [A] time = 1.12, size = 27, normalized size = 0.77

$$\frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(x+3\right) - \frac{22}{3}\,\log\left(x+2\right) + \frac{13}{3}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

mupad [B] time = 0.04, size = 27, normalized size = 0.77

$$20\,\ln\left(x+3\right) - \frac{22\,\ln\left(x+2\right)}{3} - 2\,x + \frac{13\,\ln\left(x-4\right)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 7*x - x^3 + x^4 + 2)/(14*x - x^2 - x^3 + 24),x)
[Out] 20*log(x + 3) - (22*log(x + 2))/3 - 2*x + (13*log(x - 4))/3 + x^2/2
```

sympy [A] time = 0.15, size = 31, normalized size = 0.89

$$\frac{x^2}{2} - 2x + \frac{13\log(x-4)}{3} - \frac{22\log(x+2)}{3} + 20\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)
[Out] x**2/2 - 2*x + 13*log(x - 4)/3 - 22*log(x + 2)/3 + 20*log(x + 3)
```

3.296 $\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx$

Optimal. Leaf size=34

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

```
[Out] 3/2/(1-x)-5/4*ln(1-x)+2*ln(x)-3/4*ln(1+x)
```

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1612}

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
```

```
[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx &= \int \left(\frac{3}{2(-1+x)^2}-\frac{5}{4(-1+x)}+\frac{2}{x}-\frac{3}{4(1+x)}\right)dx\\ &= \frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.94

$$-\frac{3}{2(x-1)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
```

```
[Out] -3/(2*(-1 + x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

fricas [A] time = 0.60, size = 34, normalized size = 1.00

$$-\frac{3\,(x-1)\log\left(x+1\right)+5\,(x-1)\log\left(x-1\right)-8\,(x-1)\log(x)+6}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="fricas")
```

```
[Out] -1/4*(3*(x - 1)*log(x + 1) + 5*(x - 1)*log(x - 1) - 8*(x - 1)*log(x) + 6)/(
x - 1)
```

giac [A] time = 0.39, size = 34, normalized size = 1.00

$$-\frac{3}{2\,(x-1)}+2\,\log\left(\left|-\frac{1}{x-1}-1\right|\right)-\frac{3}{4}\,\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="giac")
```

```
[Out] -3/2/(x - 1) + 2*log(abs(-1/(x - 1) - 1)) - 3/4*log(abs(-2/(x - 1) - 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.74

$$2\ln(x)-\frac{5\ln\left(x-1\right)}{4}-\frac{3\ln\left(x+1\right)}{4}-\frac{3}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x-1)^2/x/(x+1),x)
```

```
[Out] -3/2/(x-1)-5/4*ln(x-1)-3/4*ln(x+1)+2*ln(x)
```

maxima [A] time = 1.13, size = 24, normalized size = 0.71

$$-\frac{3}{2\,(x-1)}-\frac{3}{4}\,\log\left(x+1\right)-\frac{5}{4}\,\log\left(x-1\right)+2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="maxima")
```

```
[Out] -3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)
```

mupad [B] time = 2.11, size = 26, normalized size = 0.76

$$2\ \ln(x) - \frac{3\ \ln(x+1)}{4} - \frac{5\ \ln(x-1)}{4} - \frac{3}{2\ (x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/(x*(x - 1)^2*(x + 1)),x)
[Out] 2*log(x) - (3*log(x + 1))/4 - (5*log(x - 1))/4 - 3/(2*(x - 1))
```

sympy [A] time = 0.14, size = 27, normalized size = 0.79

$$2\log(x) - \frac{5\log(x-1)}{4} - \frac{3\log(x+1)}{4} - \frac{3}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(-1+x)**2/x/(1+x),x)
[Out] 2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)
```

3.297 $\int \frac{3+x^2+x^3}{(2+x^2)^2} \, dx$

Optimal. Leaf size=42

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] 1/4*(4+x)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1814, 635, 203, 260}

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{3+x^2+x^3}{\left(2+x^2\right)^2}\,dx &= \frac{4+x}{4\left(2+x^2\right)}-\frac{1}{4}\int \frac{-5-4x}{2+x^2}\,dx\\ &= \frac{4+x}{4\left(2+x^2\right)}+\frac{5}{4}\int \frac{1}{2+x^2}\,dx+\int \frac{x}{2+x^2}\,dx\\ &= \frac{4+x}{4\left(2+x^2\right)}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{x+4}{4\left(x^2+2\right)}+\frac{1}{2}\log\left(x^2+2\right)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2 + x^3)/(2 + x^2)^2,x]
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

fricas [A] time = 0.79, size = 44, normalized size = 1.05

$$\frac{5\sqrt{2}\left(x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+4\left(x^2+2\right)\log\left(x^2+2\right)+2x+8}{8\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="fricas")
[Out] 1/8*(5*sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 4*(x^2 + 2)*log(x^2 + 2) +
 2*x + 8)/(x^2 + 2)
```

giac [A] time = 0.30, size = 33, normalized size = 0.79

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\left(x^2+2\right)}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="giac")
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

maple [A] time = 0.01, size = 35, normalized size = 0.83

$$\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{8}+\frac{\ln\left(x^2+2\right)}{2}+\frac{\frac{x}{4}+1}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+3)/(x^2+2)^2,x)
[Out] (1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.36, size = 33, normalized size = 0.79

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\left(x^2+2\right)}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="maxima")
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

mupad [B] time = 2.19, size = 39, normalized size = 0.93

$$\frac{\ln\left(x^2+2\right)}{2}+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}+\frac{x}{4\left(x^2+2\right)}+\frac{1}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 3)/(x^2 + 2)^2,x)
```

```
[Out] log(x^2 + 2)/2 + (5*2^(1/2)*atan((2^(1/2)*x)/2))/8 + x/(4*(x^2 + 2)) + 1/(x
^2 + 2)
```

sympy [A] time = 0.13, size = 36, normalized size = 0.86

$$\frac{x+4}{4x^2+8}+\frac{\log\left(x^2+2\right)}{2}+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+3)/(x**2+2)**2,x)
```

```
[Out] (x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x/2)/8
```

3.298 $\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx$

Optimal. Leaf size=49

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+
26)+22/1025*ln(x^2-2*x+17)
```

Rubi [A] time = 0.16, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {6728, 634, 618, 204, 628}

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

```
[In] Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \int \left(\frac{5003+2006x}{1025(26-10x+x^2)}+\frac{-4651+44x}{1025(17-2x+x^2)}\right)dx$$
$$= \frac{\int \frac{5003+2006x}{26-10x+x^2}\,dx}{1025}+\frac{\int \frac{-4651+44x}{17-2x+x^2}\,dx}{1025}$$
$$= \frac{22\int \frac{-2+2x}{17-2x+x^2}\,dx}{1025}+\frac{1003\int \frac{-10+2x}{26-10x+x^2}\,dx}{1025}-\frac{4607\int \frac{1}{17-2x+x^2}\,dx}{1025}+\frac{15033}{}$$
$$= \frac{1003\log(26-10x+x^2)}{1025}+\frac{22\log(17-2x+x^2)}{1025}+\frac{9214\,\mathrm{Subst}\left(\int \frac{1}{-64}\right.}{10}$$
$$= -\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{1}{4}(-1+x)\right)}{4100}+\frac{1003\log(26-10x+}{1025}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{1003\log(x^2-10x+26)}{1025}+\frac{22\log(x^2-2x+17)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2))
,x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

fricas [A] time = 0.62, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="fr
icas")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

giac [A] time = 0.32, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="gi
ac")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{15033\arctan\left(x-5\right)}{1025}-\frac{4607\arctan\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{1003\ln\left(x^2-10x+26\right)}{1025}+\frac{22\ln\left(x^2-2x+17\right)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x)
```

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+
26)+22/1025*ln(x^2-2*x+17)
```

maxima [A] time = 2.80, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="ma
xima")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

mupad [B] time = 0.07, size = 41, normalized size = 0.84

$$\ln (x-1-4i) \left(\frac{22}{1025}+\frac{4607}{8200}i\right)+\ln (x-1+4i) \left(\frac{22}{1025}-\frac{4607}{8200}i\right)+\ln (x-5-i) \left(\frac{1003}{1025}-\frac{15033}{2050}i\right)+\ln (x-5+1i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((70*x - 4*x^2 + 2*x^3 - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x)
```

```
[Out] log(x - (1 + 4i))*(22/1025 + 4607i/8200) + log(x - (1 - 4i))*(22/1025 - 460
7i/8200) + log(x - (5 + 1i))*(1003/1025 - 15033i/2050) + log(x - (5 - 1i))*
(1003/1025 + 15033i/2050)
```

sympy [A] time = 0.23, size = 46, normalized size = 0.94

$$\frac{1003 \log \left(x^{2}-10 x+26\right)}{1025}+\frac{22 \log \left(x^{2}-2 x+17\right)}{1025}-\frac{4607 \operatorname{atan}\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{15033 \operatorname{atan}(x-5)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)
```

```
[Out] 1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 - 4607*atan(
x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025
```

3.299 $\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx$

Optimal. Leaf size=29

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

```
[Out] -11/14*ln(3-x)+3/2*ln(5-x)+2/7*ln(4+x)
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1612}

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
```

```
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx &= \int \left(\frac{3}{2(-5+x)}-\frac{11}{14(-3+x)}+\frac{2}{7(4+x)}\right)dx\\ &= -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

fricas [A] time = 0.58, size = 19, normalized size = 0.66

$$\frac{2}{7}\,\log\left(x+4\right)-\frac{11}{14}\,\log\left(x-3\right)+\frac{3}{2}\,\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="fricas")
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

giac [A] time = 0.27, size = 22, normalized size = 0.76

$$\frac{2}{7}\,\log\left(|x+4|\right)-\frac{11}{14}\,\log\left(|x-3|\right)+\frac{3}{2}\,\log\left(|x-5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="giac")
[Out] 2/7*log(abs(x + 4)) - 11/14*log(abs(x - 3)) + 3/2*log(abs(x - 5))
```

maple [A] time = 0.01, size = 20, normalized size = 0.69

$$\frac{3\ln\left(x-5\right)}{2}-\frac{11\ln\left(x-3\right)}{14}+\frac{2\ln\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x-5)/(x-3)/(x+4),x)
[Out] 2/7*ln(x+4)-11/14*ln(x-3)+3/2*ln(x-5)
```

maxima [A] time = 1.17, size = 19, normalized size = 0.66

$$\frac{2}{7}\,\log\left(x+4\right)-\frac{11}{14}\,\log\left(x-3\right)+\frac{3}{2}\,\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="maxima")
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

mupad [B] time = 2.18, size = 19, normalized size = 0.66

$$\frac{2\,\ln\left(x+4\right)}{7}-\frac{11\,\ln\left(x-3\right)}{14}+\frac{3\,\ln\left(x-5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/((x - 3)*(x + 4)*(x - 5)),x)
```

```
[Out] (2*log(x + 4))/7 - (11*log(x - 3))/14 + (3*log(x - 5))/2
```

sympy [A] time = 0.14, size = 24, normalized size = 0.83

$$\frac{3\log\left(x-5\right)}{2}-\frac{11\log\left(x-3\right)}{14}+\frac{2\log\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)
```

```
[Out] 3*log(x - 5)/2 - 11*log(x - 3)/14 + 2*log(x + 4)/7
```

3.300 $\int \frac{x^4}{(-1+x)\left(2+x^2\right)} \, dx$

Optimal. Leaf size=46

$$\frac{x^2}{2}-\frac{2}{3}\log\left(x^2+2\right)+x+\frac{1}{3}\log(1-x)-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] x+1/2*x^2+1/3*ln(1-x)-2/3*ln(x^2+2)-2/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1629, 635, 203, 260}

$$\frac{x^2}{2}-\frac{2}{3}\log\left(x^2+2\right)+x+\frac{1}{3}\log(1-x)-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*Log[2 + x^2
])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(-1+x)\left(2+x^2\right)}\,dx &= \int \left(1+\frac{1}{3(-1+x)}+x-\frac{4(1+x)}{3\left(2+x^2\right)}\right)dx\\ &= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1+x}{2+x^2}\,dx\\ &= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1}{2+x^2}\,dx-\frac{4}{3}\int \frac{x}{2+x^2}\,dx\\ &= x+\frac{x^2}{2}-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{3}\log(1-x)-\frac{2}{3}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.93

$$\frac{1}{6}\left(3x^2-4\log\left(x^2+2\right)+6x+2\log(x-1)-4\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] (-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] - 4*Log[2 +
 x^2])/6
```

fricas [A] time = 0.99, size = 33, normalized size = 0.72

$$\frac{1}{2}x^2-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+x-\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

giac [A] time = 0.32, size = 34, normalized size = 0.74

$$\frac{1}{2}x^2-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+x-\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(abs(x - 1))
```

maple [A] time = 0.01, size = 34, normalized size = 0.74

$$\frac{x^2}{2}+x-\frac{2\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{3}+\frac{\ln\left(x-1\right)}{3}-\frac{2\ln\left(x^2+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x-1)/(x^2+2),x)
```

```
[Out] 1/2*x^2+x+1/3*ln(x-1)-2/3*ln(x^2+2)-2/3*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.16, size = 33, normalized size = 0.72

$$\frac{1}{2}\,x^2-\frac{2}{3}\,\sqrt{2}\ \arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+x-\frac{2}{3}\,\log\left(x^2+2\right)+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

mupad [B] time = 0.09, size = 50, normalized size = 1.09

$$x+\frac{\ln\left(x-1\right)}{3}+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^2 + 2)*(x - 1)),x)
```

```
[Out] x + log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/3 - 2/3) - log(x + 2^(
1/2)*1i)*((2^(1/2)*1i)/3 + 2/3) + x^2/2
```

sympy [A] time = 0.14, size = 41, normalized size = 0.89

$$\frac{x^2}{2}+x+\frac{\log\left(x-1\right)}{3}-\frac{2\log\left(x^2+2\right)}{3}-\frac{2\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-1+x)/(x**2+2),x)
```

```
[Out] x**2/2 + x + log(x - 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/2)
/3
```

3.301 $\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} \, dx$

Optimal. Leaf size=16

$$2\log(1-x) - \frac{3}{x+1}$$

```
[Out] -3/(1+x)+2*ln(1-x)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2074}

$$2\log(1-x) - \frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
[Out] -3/(1 + x) + 2*Log[1 - x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} \, dx &= \int \left(\frac{2}{-1+x} + \frac{3}{(1+x)^2}\right) dx \\ &= -\frac{3}{1+x} + 2\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.88

$$2\log(x-1) - \frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
[Out] -3/(1 + x) + 2*Log[-1 + x]
```

fricas [A] time = 0.80, size = 17, normalized size = 1.06

$$\frac{2\,(x+1)\log\left(x-1\right)-3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="fricas")
[Out] (2*(x + 1)*log(x - 1) - 3)/(x + 1)
```

giac [A] time = 0.40, size = 15, normalized size = 0.94

$$-\frac{3}{x+1}+2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="giac")
[Out] -3/(x + 1) + 2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$2\ln\left(x-1\right)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+7*x-1)/(x^3+x^2-x-1),x)
[Out] 2*ln(x-1)-3/(x+1)
```

maxima [A] time = 1.09, size = 14, normalized size = 0.88

$$-\frac{3}{x+1}+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="maxima")
[Out] -3/(x + 1) + 2*log(x - 1)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$2\,\ln\left(x-1\right)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(7*x + 2*x^2 - 1)/(x - x^2 - x^3 + 1),x)
```

```
[Out] 2*log(x - 1) - 3/(x + 1)
```

sympy [A] time = 0.09, size = 10, normalized size = 0.62

$$2\log(x-1) - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)
```

```
[Out] 2*log(x - 1) - 3/(x + 1)
```

3.302 $\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$\frac{2}{1-x}-\frac{3}{2(1-x)^2}$$

```
[Out] -3/2/(1-x)^2+2/(1-x)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2074}

$$\frac{2}{1-x}-\frac{3}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x)
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx &= \int \left(\frac{3}{(-1+x)^3}+\frac{2}{(-1+x)^2}\right)dx\\ &= -\frac{3}{2(1-x)^2}+\frac{2}{1-x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{1-4x}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] (1 - 4*x)/(2*(-1 + x)^2)
```

fricas [A] time = 0.88, size = 17, normalized size = 0.81

$$-\frac{4\,x-1}{2\,\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="fricas")
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

giac [A] time = 0.30, size = 12, normalized size = 0.57

$$-\frac{4\,x-1}{2\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="giac")
[Out] -1/2*(4*x - 1)/(x - 1)^2
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{3}{2\,(x-1)^2}-\frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^3-3*x^2+3*x-1),x)
[Out] -3/2/(x-1)^2-2/(x-1)
```

maxima [A] time = 1.09, size = 17, normalized size = 0.81

$$-\frac{4\,x-1}{2\,\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

mupad [B] time = 2.09, size = 12, normalized size = 0.57

$$-\frac{4\,x-1}{2\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(3*x - 3*x^2 + x^3 - 1),x)
```

```
[Out] -(4*x - 1)/(2*(x - 1)^2)
```

sympy [A] time = 0.09, size = 14, normalized size = 0.67

$$\frac{1-4x}{2x^2-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)
```

```
[Out] (1 - 4*x)/(2*x**2 - 4*x + 2)
```

3.303 $\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} dx$

Optimal. Leaf size=15

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

```
[Out] 1/(1-x)-2/(1+x)^2
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1620}

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] (1 - x)^(-1) - 2/(1 + x)^2
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx &= \int \left(\frac{1}{(-1+x)^2}+\frac{4}{(1+x)^3}\right) dx\\ &= \frac{1}{1-x}-\frac{2}{(1+x)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2}{(x+1)^2}-\frac{1}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] -(-1 + x)^(-1) - 2/(1 + x)^2
```

fricas [A] time = 0.88, size = 23, normalized size = 1.53

$$-\frac{x^2 + 4x - 1}{x^3 + x^2 - x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

giac [A] time = 0.26, size = 30, normalized size = 2.00

$$-\frac{1}{x-1} + \frac{\frac{4}{x-1} + 1}{2\left(\frac{2}{x-1} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="giac")
[Out] -1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$-\frac{2}{(x+1)^2} - \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+7*x^2-5*x+5)/(x-1)^2/(x+1)^3,x)
[Out] -2/(x+1)^2-1/(x-1)
```

maxima [A] time = 1.27, size = 23, normalized size = 1.53

$$-\frac{x^2 + 4x - 1}{x^3 + x^2 - x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

mupad [B]   time = 2.09, size = 15, normalized size = 1.00

$$-\frac{1}{x-1}-\frac{2}{(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^2 - 5*x + x^3 + 5)/((x - 1)^2*(x + 1)^3),x)
[Out] - 1/(x - 1) - 2/(x + 1)^2
```

sympy [A]   time = 0.11, size = 17, normalized size = 1.13

$$\frac{-x^2-4x+1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)
[Out] (-x**2 - 4*x + 1)/(x**3 + x**2 - x - 1)
```

3.304 $\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} \, dx$

Optimal. Leaf size=31

$$\log \left(x^2+x+1\right)+\log (x+1)-\frac{2 \tan ^{-1}\left(\frac{2 x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ln(1+x)+ln(x^2+x+1)-2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2074, 634, 618, 204, 628}

$$\log \left(x^2+x+1\right)+\log (x+1)-\frac{2 \tan ^{-1}\left(\frac{2 x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx &= \int \left(\frac{1}{1+x}+\frac{2x}{1+x+x^2}\right)dx\\
&= \log(1+x)+2\int \frac{x}{1+x+x^2}\,dx\\
&= \log(1+x)-\int \frac{1}{1+x+x^2}\,dx+\int \frac{1+2x}{1+x+x^2}\,dx\\
&= \log(1+x)+\log\left(1+x+x^2\right)+2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

fricas [A] time = 0.89, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

giac [A] time = 0.28, size = 29, normalized size = 0.94

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="giac")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x +
 1))
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$-\frac{2\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\ln(x+1)+\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x)
[Out] ln(x+1)+ln(x^2+x+1)-2/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.36, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="maxima")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

mupad [B] time = 0.11, size = 57, normalized size = 1.84

$$\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln(x+1)+\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3}-\frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 3*x^2 + 1)/(2*x + 2*x^2 + x^3 + 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x + 1)
+ (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - (3^(1/2)*log(x + (3^(1/2)*
1i)/2 + 1/2)*1i)/3
```

sympy [A] time = 0.13, size = 3, normalized size = 0.10

$$\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)
```

```
[Out] log(x + 1)
```

3.305 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$

Optimal. Leaf size=25

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

```
[Out] 1/10*ln(1-2*x)+1/2*ln(x)-1/10*ln(2+x)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1594, 1628}

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx &= \int \frac{-1+2x+x^2}{x\left(-2+3x+2x^2\right)} dx \\
&= \int \left(\frac{1}{2x}-\frac{1}{10(2+x)}+\frac{1}{5(-1+2x)}\right) dx \\
&= \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

fricas [A] time = 0.80, size = 19, normalized size = 0.76

$$\frac{1}{10}\,\log\left(2\,x-1\right)-\frac{1}{10}\,\log\left(x+2\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

giac [A] time = 0.29, size = 22, normalized size = 0.88

$$\frac{1}{10}\,\log\left(|2\,x-1|\right)-\frac{1}{10}\,\log\left(|x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\ln(x)}{2}+\frac{\ln\left(2x-1\right)}{10}-\frac{\ln\left(x+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)
[Out] 1/10*ln(2*x-1)-1/10*ln(x+2)+1/2*ln(x)
```

maxima [A] time = 1.09, size = 19, normalized size = 0.76

$$\frac{1}{10}\,\log\left(2\,x-1\right)-\frac{1}{10}\,\log\left(x+2\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

mupad [B] time = 0.06, size = 19, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{24}{145\left(\frac{29x}{100}-\frac{11}{50}\right)}+\frac{35}{29}\right)}{5}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)
```

```
[Out] atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2
```

sympy [A] time = 0.14, size = 19, normalized size = 0.76

$$\frac{\log\left(x\right)}{2}+\frac{\log\left(x-\frac{1}{2}\right)}{10}-\frac{\log\left(x+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
```

```
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

3.306 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

```
[Out] 2/(1-x)+x+1/2*x^2+ln(1-x)-ln(1+x)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2074}

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx &= \int \left(1+\frac{1}{-1-x}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}+x\right)dx\\ &= \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2-\frac{2}{x-1}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]

fricas [A] time = 0.87, size = 36, normalized size = 1.20

$$\frac{x^3 + x^2 - 2\,(x-1)\log\left(x+1\right) + 2\,(x-1)\log\left(x-1\right) - 2\,x - 4}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")

[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x - 1)

giac [A] time = 0.28, size = 26, normalized size = 0.87

$$\frac{1}{2}\,x^2 + x - \frac{2}{x-1} - \log\left(|x+1|\right) + \log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")

[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{x^2}{2} + x + \ln\left(x-1\right) - \ln\left(x+1\right) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x)

[Out] 1/2*x^2+x+ln(x-1)-2/(x-1)-ln(x+1)

maxima [A] time = 1.21, size = 24, normalized size = 0.80

$$\frac{1}{2}\,x^2 + x - \frac{2}{x-1} - \log\left(x+1\right) + \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")

[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)

mupad [B] time = 2.11, size = 22, normalized size = 0.73

$$x - \frac{2}{x-1} + \frac{x^2}{2} + \mathrm{atan}\left(x\,1\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)
```

```
[Out] x + atan(x*1i)*2i - 2/(x - 1) + x^2/2
```

sympy [A] time = 0.10, size = 20, normalized size = 0.67

$$\frac{x^2}{2} + x + \log(x-1) - \log(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

3.307 $\int \frac{4-x+2x^2}{4x+x^3} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4-x+2x^2}{4x+x^3}\,dx &= \int \frac{4-x+2x^2}{x\left(4+x^2\right)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{-1+x}{4+x^2}\right)dx \\
&= \log(x)+\int \frac{-1+x}{4+x^2}\,dx \\
&= \log(x)-\int \frac{1}{4+x^2}\,dx+\int \frac{x}{4+x^2}\,dx \\
&= -\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

fricas [A] time = 0.73, size = 17, normalized size = 0.74

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

giac [A] time = 0.33, size = 18, normalized size = 0.78

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln(x)+\frac{\ln\left(x^{2}+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+4)/(x^3+4*x),x)
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

maxima [A] time = 2.08, size = 17, normalized size = 0.74

$$-\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\log\left(x^{2}+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

mupad [B] time = 2.12, size = 21, normalized size = 0.91

$$\ln(x)+\ln\left(x-2\mathrm{i}\right)\left(\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\left(\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 4)/(4*x + x^3),x)
[Out] log(x - 2i)*(1/2 + 1i/4) + log(x + 2i)*(1/2 - 1i/4) + log(x)
```

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\log\left(x\right)+\frac{\log\left(x^{2}+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+4)/(x**3+4*x),x)
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

3.308 $\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-$$

```
[Out] 1/8*(1+x)/(x^2+1)^2-3/8*(1-x)/(x^2+1)+3/16*x/(x^2+1)+7/16*arctan(x)+1/8*ln(
1-x)-ln(x)+15/16*ln(x^2+1)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6728, 639, 199, 203, 635, 260, 634, 618, 204, 628}

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \int \left(\frac{1}{8(-1+x)}-\frac{1}{x}+\frac{1-x}{2\left(1+x^2\right)^3}+\frac{3(1+x)}{4\left(1+x^2\right)^2}+\frac{-1+15x}{8\left(1+x^2\right)}+\frac{-1-x}{1+x+\cdots}\right.$$

$$=\frac{1}{8}\log(1-x)-\log(x)+\frac{1}{8}\int\frac{-1+15x}{1+x^2}\,dx+\frac{1}{2}\int\frac{1-x}{\left(1+x^2\right)^3}\,dx+\frac{3}{4}\int\cdots$$

$$=\frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{1}{8}\log(1-x)-\log(x)-\frac{1}{8}\int\frac{1}{1+x^2}\,dx+\frac{3}{8}\cdots$$

$$=\frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{1}{4}\tan^{-1}(x)+\frac{1}{8}\log(1-x)-\cdots$$

$$=\frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{7}{16}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\cdots$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.90

$$\frac{1}{48}\left(-14\log\left(1-x^3\right)+\frac{6(x+1)}{\left(x^2+1\right)^2}+\frac{9(3x-2)}{x^2+1}+45\log\left(x^2+1\right)-10\log\left(x^2+x+1\right)+20\log(1-x)-48\log(x\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2
] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

fricas [A] time = 0.70, size = 136, normalized size = 1.32

$$\frac{27\,x^3-16\,\sqrt{3}\left(x^4+2\,x^2+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-18\,x^2+21\left(x^4+2\,x^2+1\right)\arctan(x)-24\left(x^4+2\,x^2+1\right)\cdots}{48\left(x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) -
 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x
 + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1)
- 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

giac [**A**] time = 0.34, size = 74, normalized size = 0.72

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^3-6x^2+11x-4}{16\left(x^2+1\right)^2}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{15}{16}\log\left(x^2+1\right)+\frac{1}{8}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1)
+ 1/8*log(abs(x - 1)) - log(abs(x))
```

maple [**A**] time = 0.01, size = 73, normalized size = 0.71

$$\frac{7\arctan(x)}{16}-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\ln(x)+\frac{\ln(x-1)}{8}+\frac{15\ln\left(x^2+1\right)}{16}-\frac{\ln\left(x^2+x+1\right)}{2}+\frac{\frac{9}{2}x^3-3x^2+\frac{11}{2}x-2}{8\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x-1)/x/(x^2+1)^3/(x^2+x+1),x)
```

```
[Out] 1/8*ln(x-1)+1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arc
tan(x)-ln(x)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [**A**] time = 2.25, size = 77, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^3-6x^2+11x-4}{16\left(x^4+2x^2+1\right)}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{15}{16}\log\left(x^2+1\right)+\frac{1}{8}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2
 + 1) + 1/8*log(x - 1) - log(x)
```

mupad [**B**] time = 2.20, size = 96, normalized size = 0.93

$$\frac{\ln(x-1)}{8}-\ln(x)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{\frac{9x^3}{16}-\frac{3x^2}{8}+\frac{11x}{16}-\frac{1}{4}}{x^4+2x^2+1}+\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)
```

```
[Out] log(x - 1)/8 + log(x - 1i)*(15/16 - 7i/32) + log(x + 1i)*(15/16 + 7i/32) -
log(x) + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x + (3^
(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*x^3
)/16 - 1/4)/(2*x^2 + x^4 + 1)
```

sympy [A] time = 0.51, size = 88, normalized size = 0.85

$$-\log(x)+\frac{\log(x-1)}{8}+\frac{15\log(x^2+1)}{16}-\frac{\log(x^2+x+1)}{2}+\frac{7\,\mathrm{atan}(x)}{16}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{9x^3-6x^2+11x}{16x^4+32x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*atan
(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 11*
x - 4)/(16*x**4 + 32*x**2 + 16)
```

3.309 $\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{2-x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3}{2}\tan^{-1}(x)$$

```
[Out] 1/2*(2-x)/(x^2+1)+3/2*arctan(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1814, 635, 203, 260}

$$\frac{2-x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
```

```
[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
```

```
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx &= \frac{2-x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-3+2x}{1+x^2}\,dx\\ &= \frac{2-x}{2\left(1+x^2\right)}+\frac{3}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \frac{2-x}{2\left(1+x^2\right)}+\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{1}{2}\left(\frac{2-x}{x^2+1}-\log\left(x^2+1\right)+3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
[Out] ((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2
```

fricas [A] time = 0.92, size = 36, normalized size = 1.09

$$\frac{3\left(x^2+1\right)\arctan(x)-\left(x^2+1\right)\log\left(x^2+1\right)-x+2}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) - x + 2)/(x^2 + 1)
```

giac [A] time = 0.32, size = 25, normalized size = 0.76

$$-\frac{x-2}{2\left(x^2+1\right)}+\frac{3}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 0.85

$$\frac{3\arctan(x)}{2}-\frac{\ln\left(x^2+1\right)}{2}-\frac{\frac{x}{2}-1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x)
[Out] -(1/2*x-1)/(x^2+1)-1/2*ln(x^2+1)+3/2*arctan(x)
```

maxima [A] time = 2.15, size = 25, normalized size = 0.76

$$-\frac{x-2}{2\left(x^2+1\right)}+\frac{3}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.97

$$\frac{3\,\mathrm{atan}(x)}{2}-\frac{\ln\left(x^2+1\right)}{2}-\frac{x}{2\,\left(x^2+1\right)}+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x^2 + 1)^2,x)
[Out] (3*atan(x))/2 - log(x^2 + 1)/2 - x/(2*(x^2 + 1)) + 1/(x^2 + 1)
```

sympy [A] time = 0.13, size = 24, normalized size = 0.73

$$-\frac{x-2}{2x^2+2}-\frac{\log\left(x^2+1\right)}{2}+\frac{3\,\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)
[Out] -(x - 2)/(2*x**2 + 2) - log(x**2 + 1)/2 + 3*atan(x)/2
```

3.310 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx$

Optimal. Leaf size=33

$$-\frac{2x+1}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

```
[Out] 1/2*(-1-2*x)/(x^2+1)-2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1805, 801, 635, 203, 260}

$$-\frac{2x+1}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2+4x}{x\left(1+x^2\right)}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{2(2+x)}{1+x^2}\right)dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-\int \frac{2+x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-2\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-2\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{-2x-1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

fricas [A] time = 0.78, size = 44, normalized size = 1.33

$$-\frac{4\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log(x)+2\,x+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) +
 2*x + 1)/(x^2 + 1)
```

giac [A] time = 0.38, size = 30, normalized size = 0.91

$$-\frac{2x+1}{2\left(x^2+1\right)} - 2\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$-2\arctan(x) + \ln(x) - \frac{\ln\left(x^2+1\right)}{2} - \frac{x+\frac{1}{2}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)
```

```
[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)
```

maxima [A] time = 2.25, size = 29, normalized size = 0.88

$$-\frac{2x+1}{2\left(x^2+1\right)} - 2\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

mupad [B] time = 2.11, size = 33, normalized size = 1.00

$$\ln(x) - \frac{x+\frac{1}{2}}{x^2+1} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}+1\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 + 1i) - log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2 +
 1)
```

sympy [A] time = 0.15, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2}+\log(x)-\frac{\log\left(x^2+1\right)}{2}-2\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

3.311 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

```
[Out] x+1/2*x^2-ln(x)+1/2*ln(-x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1593, 1802, 260}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx = \int \frac{1-x-x^2+x^3+x^4}{x(-1+x^2)} dx$$
$$= \int \left(1 - \frac{1}{x} + x + \frac{x}{-1+x^2}\right) dx$$
$$= x + \frac{x^2}{2} - \log(x) + \int \frac{x}{-1+x^2} dx$$
$$= x + \frac{x^2}{2} - \log(x) + \frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2} + \frac{1}{2}\log\left(1-x^2\right) + x - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

fricas [A] time = 0.56, size = 19, normalized size = 0.76

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log\left(x^2-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

giac [A] time = 0.29, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{x^2}{2} + x - \ln(x) + \frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x)
[Out] 1/2*x^2+x+1/2*ln(x-1)+1/2*ln(x+1)-ln(x)
```

maxima [A] time = 1.05, size = 23, normalized size = 0.92

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.76

$$x + \frac{\ln\left(x^2-1\right)}{2} - \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)
[Out] x + log(x^2 - 1)/2 - log(x) + x^2/2
```

sympy [A] time = 0.10, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

3.312 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6725, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx &= \int \left(\frac{6-x}{1+x^2}+\frac{2(-5+x)}{2+x^2}\right)dx\\ &= 2\int \frac{-5+x}{2+x^2}\,dx+\int \frac{6-x}{1+x^2}\,dx\\ &= 2\int \frac{x}{2+x^2}\,dx+6\int \frac{1}{1+x^2}\,dx-10\int \frac{1}{2+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= 6\tan^{-1}(x)-5\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log(1+x^2)+\log(2+x^2)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

fricas [A] time = 0.84, size = 31, normalized size = 0.86

$$-5\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+6\,\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

giac [A] time = 0.34, size = 31, normalized size = 0.86

$$-5\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+6\,\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

maple [A] time = 0.00, size = 32, normalized size = 0.89

$$6\arctan(x)-5\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)-\frac{\ln\left(x^{2}+1\right)}{2}+\ln\left(x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x)
```

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.10, size = 31, normalized size = 0.86

$$-5\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+6\ \arctan(x)+\log\left(x^{2}+2\right)-\frac{1}{2}\ \log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

mupad [B] time = 0.11, size = 56, normalized size = 1.56

$$\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}-3\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}+3\mathrm{i}\right)+\ln\left(x-\sqrt{2}\ 1\mathrm{i}\right)\left(1+\frac{\sqrt{2}\ 5\mathrm{i}}{2}\right)-\ln\left(x+\sqrt{2}\ 1\mathrm{i}\right)\left(-1+\frac{\sqrt{2}\ 5\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x -
 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)
```

sympy [A] time = 0.20, size = 36, normalized size = 1.00

$$-\frac{\log\left(x^{2}+1\right)}{2}+\log\left(x^{2}+2\right)+6\operatorname{atan}\left(x\right)-5\sqrt{2}\ \operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

3.313 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} \, dx$

Optimal. Leaf size=29

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Rubi [A] time = 0.11, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6725, 203, 199}

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx = \int \left(\frac{1}{9(1+x^2)} - \frac{13}{3(4+x^2)^2} + \frac{8}{9(4+x^2)}\right) dx$$
$$= \frac{1}{9}\int \frac{1}{1+x^2}\,dx + \frac{8}{9}\int \frac{1}{4+x^2}\,dx - \frac{13}{3}\int \frac{1}{(4+x^2)^2}\,dx$$
$$= -\frac{13x}{24(4+x^2)} + \frac{4}{9}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x) - \frac{13}{24}\int \frac{1}{4+x^2}\,dx$$
$$= -\frac{13x}{24(4+x^2)} + \frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x)$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

fricas [A] time = 0.99, size = 33, normalized size = 1.14

$$\frac{25(x^2+4)\arctan\left(\frac{1}{2}x\right) + 16(x^2+4)\arctan(x) - 78x}{144(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4
)
```

giac [A] time = 0.23, size = 21, normalized size = 0.72

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144}\arctan\left(\frac{1}{2}x\right) + \frac{1}{9}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$-\frac{13x}{24\left(x^{2}+4\right)}+\frac{\arctan\left(x\right)}{9}+\frac{25\arctan\left(\frac{x}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

maxima [A] time = 2.09, size = 21, normalized size = 0.72

$$-\frac{13\,x}{24\left(x^{2}+4\right)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

mupad [B] time = 0.04, size = 23, normalized size = 0.79

$$\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\left(x\right)}{9}-\frac{13\,x}{24\,\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2),x)
[Out] (25*atan(x/2))/144 + atan(x)/9 - (13*x)/(24*(x^2 + 4))
```

sympy [A] time = 0.17, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^{2}+96}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

3.314 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx$

Optimal. Leaf size=46

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1594, 1628, 634, 618, 204, 628}

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx &= \int \frac{1+x^2+x^3}{x^2\left(2+x+x^2\right)}\,dx\\
&= \int \left(\frac{1}{2x^2}-\frac{1}{4x}+\frac{3+5x}{4\left(2+x+x^2\right)}\right)dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{4}\int \frac{3+5x}{2+x+x^2}\,dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{8}\int \frac{1}{2+x+x^2}\,dx+\frac{5}{8}\int \frac{1+2x}{2+x+x^2}\,dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)-\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right)\\
&= -\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

fricas [A] time = 0.81, size = 39, normalized size = 0.85

$$\frac{2\sqrt{7}\,x\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x+1)\right)+35\,x\log\left(x^2+x+2\right)-14\,x\log(x)-28}{56\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
```

```
[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 1
4*x*log(x) - 28)/x
```

giac [A] time = 0.30, size = 36, normalized size = 0.78

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x+1)\right)-\frac{1}{2\,x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(abs(x))
```

maple [A] time = 0.01, size = 36, normalized size = 0.78

$$\frac{\sqrt{7}\arctan\left(\frac{(2x+1)\sqrt{7}}{7}\right)}{28}-\frac{\ln(x)}{4}+\frac{5\ln\left(x^2+x+2\right)}{8}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x)
```

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(2*x+1)*7^(1/2))*7^(1/2)
```

maxima [A] time = 2.09, size = 35, normalized size = 0.76

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x+1)\right)-\frac{1}{2\,x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(x)
```

mupad [B] time = 2.17, size = 49, normalized size = 1.07

$$-\frac{\ln(x)}{4}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right)-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)
```

```
[Out] log(x + (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 + 5/8) - log(x - (7^(1/2)*1i
)/2 + 1/2)*((7^(1/2)*1i)/56 - 5/8) - log(x)/4 - 1/(2*x)
```

sympy [A] time = 0.16, size = 46, normalized size = 1.00

$$-\frac{\log(x)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)
/28 - 1/(2*x)
```

3.315 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx$

Optimal. Leaf size=22

$$\frac{x^2}{2} - \frac{2}{7}\tanh^{-1}\left(\frac{1}{7}(2x+1)\right)$$

```
[Out] 1/2*x^2-2/7*arctanh(1/7+2/7*x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1657, 616, 31}

$$\frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx &= \int \left(x+\frac{1}{-12+x+x^2}\right)dx \\
&= \frac{x^2}{2}+\int \frac{1}{-12+x+x^2}\,dx \\
&= \frac{x^2}{2}+\frac{1}{7}\int \frac{1}{-3+x}\,dx-\frac{1}{7}\int \frac{1}{4+x}\,dx \\
&= \frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.18

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

fricas [A] time = 0.85, size = 18, normalized size = 0.82

$$\frac{1}{2}\,x^2-\frac{1}{7}\,\log\left(x+4\right)+\frac{1}{7}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

giac [A] time = 0.28, size = 20, normalized size = 0.91

$$\frac{1}{2}\,x^2-\frac{1}{7}\,\log\left(|x+4|\right)+\frac{1}{7}\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^2}{2}+\frac{\ln\left(x-3\right)}{7}-\frac{\ln\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x)
```

```
[Out] 1/2*x^2-1/7*ln(x+4)+1/7*ln(x-3)
```

maxima [A] time = 0.97, size = 18, normalized size = 0.82

$$\frac{1}{2}\,x^{2}-\frac{1}{7}\,\log\left(x+4\right)+\frac{1}{7}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.64

$$\frac{x^2}{2}-\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{7}+\frac{1}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12),x)
```

```
[Out] x^2/2 - (2*atanh((2*x)/7 + 1/7))/7
```

sympy [A] time = 0.10, size = 17, normalized size = 0.77

$$\frac{x^{2}}{2}+\frac{\log\left(x-3\right)}{7}-\frac{\log\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
```

```
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

3.316 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$

Optimal. Leaf size=17

$$2\log(1-x)+\log(x)+3\log(x+3)$$

```
[Out] 2*ln(1-x)+ln(x)+3*ln(3+x)
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1594, 1628}

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx &= \int \frac{-3+5x+6x^2}{x\left(-3+2x+x^2\right)}\,dx\\ &= \int \left(\frac{2}{-1+x}+\frac{1}{x}+\frac{3}{3+x}\right)dx\\ &= 2\log(1-x)+\log(x)+3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

fricas [A] time = 1.13, size = 15, normalized size = 0.88

$$3 \log(x+3) + 2 \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

giac [A] time = 0.38, size = 18, normalized size = 1.06

$$3 \log(|x+3|) + 2 \log(|x-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) + 2\ln(x-1) + 3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x)
[Out] 2*ln(x-1)+3*ln(x+3)+ln(x)
```

maxima [A] time = 0.85, size = 15, normalized size = 0.88

$$3 \log(x+3) + 2 \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

mupad [B] time = 0.07, size = 15, normalized size = 0.88

$$2 \ln(x-1) + 3 \ln(x+3) + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)
```

```
[Out] 2*log(x - 1) + 3*log(x + 3) + log(x)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\log(x) + 2\log(x-1) + 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

```
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

3.317 $\int \frac{-2+3x+5x^2}{2x^2+x^3} dx$

Optimal. Leaf size=14

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

```
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 893}

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \int \frac{-2+3x+5x^2}{x^2(2+x)}\,dx$$
$$= \int \left(-\frac{1}{x^2}+\frac{2}{x}+\frac{3}{2+x}\right)dx$$
$$= \frac{1}{x}+2\log(x)+3\log(2+x)$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

fricas [A] time = 0.77, size = 18, normalized size = 1.29

$$\frac{3\,x\log\left(x+2\right)+2\,x\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

giac [A] time = 0.37, size = 16, normalized size = 1.14

$$\frac{1}{x}+3\,\log\left(|x+2|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

maple [A] time = 0.01, size = 15, normalized size = 1.07

$$2\ln(x)+3\ln\left(x+2\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x)
```

```
[Out] 1/x+2*ln(x)+3*ln(x+2)
```

maxima [A] time = 1.00, size = 14, normalized size = 1.00

$$\frac{1}{x} + 3\log(x+2) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

mupad [B] time = 2.12, size = 14, normalized size = 1.00

$$3\ln(x+2) + 2\ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)
```

```
[Out] 3*log(x + 2) + 2*log(x) + 1/x
```

sympy [A] time = 0.12, size = 14, normalized size = 1.00

$$2\log(x) + 3\log(x+2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

```
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

3.318 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx$

Optimal. Leaf size=19

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

```
[Out] ln(1-x)-2*ln(2+x)-3*ln(3+x)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2074}

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx &= \int \left(\frac{1}{-1+x}-\frac{2}{2+x}-\frac{3}{3+x}\right) dx \\ &= \log(1-x)-2\log(2+x)-3\log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.32

$$-2\left(-\frac{1}{2}\log(1-x)+\log(x+2)+\frac{3}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] -2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)
```

fricas [A] time = 1.14, size = 17, normalized size = 0.89

$$-3 \log(x+3) - 2 \log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

giac [A] time = 0.29, size = 20, normalized size = 1.05

$$-3 \log(|x+3|) - 2 \log(|x+2|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

maple [A] time = 0.01, size = 18, normalized size = 0.95

$$\ln(x-1) - 2\ln(x+2) - 3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)
[Out] ln(x-1)-2*ln(x+2)-3*ln(x+3)
```

maxima [A] time = 1.03, size = 17, normalized size = 0.89

$$-3 \log(x+3) - 2 \log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

mupad [B] time = 2.12, size = 17, normalized size = 0.89

$$\ln(x-1) - 2 \ln(x+2) - 3 \ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

sympy [A] time = 0.13, size = 17, normalized size = 0.89

$$\log(x-1) - 2\log(x+2) - 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

3.319 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1673, 1166, 203, 1247, 626, 31}

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx &= \int \frac{1-2x^2}{4+5x^2+x^4}\,dx+\int \frac{x\left(1+x^2\right)}{4+5x^2+x^4}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x}{4+5x+x^2}\,dx,x,x^2\right)-3\int \frac{1}{4+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\ &= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

fricas [A] time = 1.00, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

giac [A] time = 0.36, size = 17, normalized size = 0.74

$$-\frac{3}{2}\,\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\arctan\left(x\right)-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\frac{\ln\left(x^{2}+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x)
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

maxima [A] time = 2.12, size = 17, normalized size = 0.74

$$-\frac{3}{2}\,\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

mupad [B] time = 0.05, size = 33, normalized size = 1.43

$$-\mathrm{atan}\left(\frac{1305}{4\,\left(144\,x-162\right)}+\frac{9}{8}\right)+\ln\left(x-2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4),x)
```

```
[Out] log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x -
 162)) + 9/8)
```

sympy [A] time = 0.18, size = 19, normalized size = 0.83

$$\frac{\log\left(x^2+4\right)}{2}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2}+\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

```
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

3.320 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$

Optimal. Leaf size=63

$$\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}+\frac{3988\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

```
[Out] -3146/80155*ln(7-3*x)-334/323*ln(1+2*x)+4822/4879*ln(2+5*x)+11049/260015*ln
(x^2+x+5)+3988/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2074, 634, 618, 204, 628}

$$\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}+\frac{3988\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 +
 30*x^5),x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \int \left(-\frac{668}{323(1+2x)}-\frac{9438}{80155(-7+3x)}+\frac{24110}{4879(2+5x)}+\frac{48}{260}\right.$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+$$

$$= \frac{3988\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.90

$$\frac{453009\log\left(x^2+x+5\right)-418418\log(7-3x)-11023670\log(2x+1)+10536070\log(5x+2)+163508\sqrt{19}\ \mathrm{ta}}{10660615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5),x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

fricas [A] time = 0.98, size = 50, normalized size = 0.79

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5x+2)-\frac{3146}{80155}\log(3x-7)-\frac{334}{323}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="fricas")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

giac [A] time = 0.31, size = 53, normalized size = 0.84

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(|5x+2|)-\frac{3146}{80155}\log(|3x-7|)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="giac")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 33
4/323*log(abs(2*x + 1))
```

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{3988\sqrt{19}\arctan\left(\frac{(2x+1)\sqrt{19}}{19}\right)}{260015}-\frac{334\ln(2x+1)}{323}+\frac{4822\ln(5x+2)}{4879}-\frac{3146\ln(3x-7)}{80155}+\frac{11049\ln\left(x^2+x+5\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)
```

```
[Out] 4822/4879*ln(2+5*x)-3146/80155*ln(3*x-7)-334/323*ln(2*x+1)+11049/260015*ln(
x^2+x+5)+3988/260015*arctan(1/19*(2*x+1)*19^(1/2))*19^(1/2)
```

maxima [A] time = 2.11, size = 50, normalized size = 0.79

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5x+2)-\frac{3146}{80155}\log(3x-7)-\frac{334}{323}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="maxima")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

mupad [B] time = 2.21, size = 58, normalized size = 0.92

$$\frac{4822\ \ln\left(x+\frac{2}{5}\right)}{4879}-\frac{334\ \ln\left(x+\frac{1}{2}\right)}{323}-\frac{3146\ \ln\left(x-\frac{7}{3}\right)}{80155}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{19}\ 1\mathrm{i}}{2}\right)\left(-\frac{11049}{260015}+\frac{\sqrt{19}\ 1994\mathrm{i}}{260015}\right)+\ln\left(x+\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x^
5 + 70),x)
```

```
[Out] (4822*log(x + 2/5))/4879 - (334*log(x + 1/2))/323 - (3146*log(x - 7/3))/801
55 - log(x - (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 - 11049/260015
) + log(x + (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 + 11049/260015)
```

sympy [A] time = 0.35, size = 68, normalized size = 1.08

$$-\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\ \operatorname{atan}\left(\frac{2\sqrt{19}\,x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-7
0),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(
19)/19)/260015
```

3.321 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$

Optimal. Leaf size=69

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{503\tan^{-1}\left(\sqrt{2}\,x\right)}{7986\sqrt{2}}$$

```
[Out] 5828/9075/(2-5*x)+1/1452*(-313-502*x)/(2*x^2+1)-59096/99825*ln(2-5*x)+2843/
7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2074, 639, 203, 635, 260}

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}\,x\right)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 8
0*x^5 + 100*x^6),x]
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \int \left(\frac{5828}{1815(-2+5x)^2}-\frac{59096}{19965(-2+5x)}+\frac{-251+313x}{363\left(1+2x^2\right)}\right.$$

$$= \frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{2\int \frac{816+2843x}{1+2x^2}\,dx}{3993}-$$

$$= \frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}-\frac{59096\log(2-5x)}{99825}-$$

$$= \frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}-\frac{251\tan^{-1}\left(\sqrt{2}\,x\right)}{726\sqrt{2}}+$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.97

$$\frac{142150\log\left(2x^2+1\right)-\frac{33\left(36458x^2+4675x+2554\right)}{10x^3-4x^2+5x-2}-236384\log(2-5x)+12575\sqrt{2}\,\tan^{-1}\left(\sqrt{2}\,x\right)}{399300}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x
^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt
[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300
```

fricas [A] time = 0.79, size = 103, normalized size = 1.49

$$\frac{12575\sqrt{2}\left(10x^3-4x^2+5x-2\right)\arctan\left(\sqrt{2}\,x\right)-1203114x^2+142150\left(10x^3-4x^2+5x-2\right)\log\left(2x^2+1\right)-}{399300\left(10x^3-4x^2+5x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="fricas")
```

```
[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203
114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3
 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x
- 2)
```

giac [A] time = 0.24, size = 59, normalized size = 0.86

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}\,x\right)-\frac{36458\,x^2+4675\,x+2554}{12100\left(2\,x^2+1\right)(5\,x-2)}+\frac{2843}{7986}\log\left(2\,x^2+1\right)-\frac{59096}{99825}\log\left(|5\,x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="giac")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
(2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x
 - 2))
```

maple [A] time = 0.01, size = 54, normalized size = 0.78

$$\frac{503\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)}{15972}-\frac{59096\ln\left(5x-2\right)}{99825}+\frac{2843\ln\left(2x^2+1\right)}{7986}-\frac{5828}{9075\left(5x-2\right)}+\frac{-\frac{2761x}{4}-\frac{3443}{8}}{3993x^2+\frac{3993}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x
)
```

```
[Out] -5828/9075/(5*x-2)-59096/99825*ln(5*x-2)+1/3993*(-2761/4*x-3443/8)/(x^2+1/2
)+2843/7986*ln(2*x^2+1)+503/15972*arctan(2^(1/2)*x)*2^(1/2)
```

maxima [A] time = 2.07, size = 59, normalized size = 0.86

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}\,x\right)-\frac{36458\,x^2+4675\,x+2554}{12100\left(10\,x^3-4\,x^2+5\,x-2\right)}+\frac{2843}{7986}\log\left(2\,x^2+1\right)-\frac{59096}{99825}\log\left(5\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="maxima")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*x
- 2)
```

mupad [B] time = 2.18, size = 71, normalized size = 1.03

$$-\frac{59096\,\ln\left(x-\frac{2}{5}\right)}{99825}-\frac{\frac{18229\,x^2}{60500}+\frac{17\,x}{440}+\frac{1277}{60500}}{x^3-\frac{2\,x^2}{5}+\frac{x}{2}-\frac{1}{5}}-\ln\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(-\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)+\ln\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(\frac{2843}{7986}+\frac{\sqrt{2}}{319}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*x
^5 + 100*x^6 + 4),x)
```

```
[Out] log(x + (2^(1/2)*1i)/2)*((2^(1/2)*503i)/31944 + 2843/7986) - ((17*x)/440 +
(18229*x^2)/60500 + 1277/60500)/(x/2 - (2*x^2)/5 + x^3 - 1/5) - log(x - (2^
(1/2)*1i)/2)*((2^(1/2)*503i)/31944 - 2843/7986) - (59096*log(x - 2/5))/9982
5
```

sympy [A] time = 0.21, size = 65, normalized size = 0.94

$$\frac{-36458x^2-4675x-2554}{121000x^3-48400x^2+60500x-24200}-\frac{59096\log\left(x-\frac{2}{5}\right)}{99825}+\frac{2843\log\left(x^2+\frac{1}{2}\right)}{7986}+\frac{503\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*
x**2-20*x+4),x)
```

```
[Out] (-36458*x**2 - 4675*x - 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
- 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan(s
qrt(2)*x)/15972
```

3.322 $\int \frac{9+x^4}{x^2(9+x^2)}\,dx$

Optimal. Leaf size=17

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -1/x+x-10/3*arctan(1/3*x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1262, 203}

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx &= \int \left(1+\frac{1}{x^2}-\frac{10}{9+x^2}\right)dx\\ &= -\frac{1}{x}+x-10\int \frac{1}{9+x^2}\,dx\\ &= -\frac{1}{x}+x-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x^4)/(x^2*(9 + x^2)),x]
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

fricas [A] time = 0.89, size = 19, normalized size = 1.12

$$\frac{3\,x^2-10\,x\arctan\left(\frac{1}{3}\,x\right)-3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="fricas")
[Out] 1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x
```

giac [A] time = 0.30, size = 13, normalized size = 0.76

$$x-\frac{1}{x}-\frac{10}{3}\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="giac")
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+9)/x^2/(x^2+9),x)
[Out] -1/x+x-10/3*arctan(1/3*x)
```

maxima [A] time = 2.04, size = 13, normalized size = 0.76

$$x-\frac{1}{x}-\frac{10}{3}\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="maxima")
```

```
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$x-\frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 9)/(x^2*(x^2 + 9)),x)
```

```
[Out] x - (10*atan(x/3))/3 - 1/x
```

sympy [A] time = 0.11, size = 12, normalized size = 0.71

$$x-\frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+9)/x**2/(x**2+9),x)
```

```
[Out] x - 10*atan(x/3)/3 - 1/x
```

3.323 $\int \frac{2x+x^4}{1+x^2}\,dx$

Optimal. Leaf size=19

$$\frac{x^3}{3} + \log\left(x^2+1\right) - x + \tan^{-1}(x)$$

```
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{x^3}{3} + \log\left(x^2+1\right) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2x+x^4}{1+x^2}\,dx &= \int \frac{x\left(2+x^3\right)}{1+x^2}\,dx\\ &= \int \left(-1+x^2+\frac{1+2x}{1+x^2}\right)dx\\ &= -x+\frac{x^3}{3}+\int \frac{1+2x}{1+x^2}\,dx\\ &= -x+\frac{x^3}{3}+2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -x+\frac{x^3}{3}+\tan^{-1}(x)+\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^3}{3}+\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^4)/(1 + x^2),x]
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

fricas [A] time = 0.91, size = 17, normalized size = 0.89

$$\frac{1}{3}\,x^3-x+\arctan(x)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

giac [A] time = 0.27, size = 17, normalized size = 0.89

$$\frac{1}{3}\,x^3-x+\arctan(x)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x^3}{3} - x + \arctan(x) + \ln\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x)/(x^2+1),x)
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

maxima [A] time = 2.03, size = 17, normalized size = 0.89

$$\frac{1}{3}\,x^3 - x + \arctan(x) + \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

mupad [B] time = 2.10, size = 17, normalized size = 0.89

$$\ln\left(x^2 + 1\right) - x + \mathrm{atan}(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^4)/(x^2 + 1),x)
[Out] log(x^2 + 1) - x + atan(x) + x^3/3
```

sympy [A] time = 0.10, size = 15, normalized size = 0.79

$$\frac{x^3}{3} - x + \log\left(x^2 + 1\right) + \mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x)/(x**2+1),x)
[Out] x**3/3 - x + log(x**2 + 1) + atan(x)
```

3.324 $\int \frac{-x+x^3}{(-1+x)^2\left(1+x^2\right)}\,dx$

Optimal. Leaf size=9

$$\log(1-x)+\tan^{-1}(x)$$

```
[Out] arctan(x)+ln(1-x)
```

Rubi [A] time = 0.08, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1586, 1593, 1629, 203}

$$\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[1 - x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx = \int \frac{x+x^2}{(-1+x)(1+x^2)}\,dx$$
$$= \int \frac{x(1+x)}{(-1+x)(1+x^2)}\,dx$$
$$= \int \left(\frac{1}{-1+x} + \frac{1}{1+x^2}\right) dx$$
$$= \log(1-x) + \int \frac{1}{1+x^2}\,dx$$
$$= \tan^{-1}(x) + \log(1-x)$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\log(1-x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
[Out] ArcTan[x] + Log[1 - x]
```

fricas [A] time = 0.89, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] arctan(x) + log(x - 1)
```

giac [B] time = 0.29, size = 28, normalized size = 3.11

$$\frac{1}{4}\pi - \pi\left\lfloor \frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2} \right\rfloor + \arctan(x) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="giac")
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) + log(abs(x
- 1))
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$\arctan(x) + \ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(x-1)^2/(x^2+1),x)
[Out] ln(x-1)+arctan(x)
```

maxima [A] time = 2.17, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] arctan(x) + log(x - 1)
```

mupad [B] time = 2.10, size = 19, normalized size = 2.11

$$\ln(x-1) - \mathrm{atan}\left(\frac{5}{4\,x+2} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/((x^2 + 1)*(x - 1)^2),x)
[Out] log(x - 1) - atan(5/(4*x + 2) - 1/2)
```

sympy [A] time = 0.14, size = 7, normalized size = 0.78

$$\log(x-1) + \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(-1+x)**2/(x**2+1),x)
[Out] log(x - 1) + atan(x)
```

3.325 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx$

Optimal. Leaf size=12

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

```
[Out] x+x^2+ln(x^2+x+1)
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1657, 628}

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+5x+3x^2+2x^3}{1+x+x^2}\,dx &= \int \left(1+2x+\frac{1+2x}{1+x+x^2}\right)dx\\ &= x+x^2+\int \frac{1+2x}{1+x+x^2}\,dx\\ &= x+x^2+\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

fricas [A] time = 0.63, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

giac [A] time = 0.29, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$x^2 + x + \ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x)
```

```
[Out] x+x^2+ln(x^2+x+1)
```

maxima [A] time = 1.37, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$x + \ln\left(x^2 + x + 1\right) + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3*x^2 + 2*x^3 + 2)/(x + x^2 + 1),x)
```

```
[Out] x + log(x + x^2 + 1) + x^2
```

sympy [A] time = 0.09, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)
```

```
[Out] x**2 + x + log(x**2 + x + 1)
```

3.326 $\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx$

Optimal. Leaf size=65

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] 3/2/x^2-1/x+3*ln(x)-1/10*ln(1+2*x-5^(1/2))*(15-5^(1/2))-1/10*ln(1+2*x+5^(1/
2))*(15+5^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1628, 632, 31}

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10
- ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx = \int \left(-\frac{3}{x^3}+\frac{1}{x^2}+\frac{3}{x}+\frac{-1-3x}{-1+x+x^2}\right)dx$$
$$= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\int \frac{-1-3x}{-1+x+x^2}\,dx$$
$$= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\frac{1}{10}\left(-15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx-\frac{1}{10}\left(15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}}$$
$$= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}\right.$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.89

$$\frac{1}{10}\left(\frac{15}{x^2}-\frac{10}{x}+\left(\sqrt{5}-15\right)\log\left(-2x+\sqrt{5}-1\right)+30\log(x)-\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] (15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log[x] - (15
+ Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

fricas [A] time = 0.60, size = 66, normalized size = 1.02

$$\frac{\sqrt{5}\,x^2\log\left(\frac{2\,x^2-\sqrt{5}\,(2\,x+1)+2\,x+3}{x^2+x-1}\right)-15\,x^2\log\left(x^2+x-1\right)+30\,x^2\log(x)-10\,x+15}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^2*log((2*x^2 - sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1))
- 15*x^2*log(x^2 + x - 1) + 30*x^2*log(x) - 10*x + 15)/x^2
```

giac [A] time = 0.30, size = 55, normalized size = 0.85

$$\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2\,x-\sqrt{5}+1\right|}{\left|2\,x+\sqrt{5}+1\right|}\right)-\frac{2\,x-3}{2\,x^2}-\frac{3}{2}\log\left(\left|x^2+x-1\right|\right)+3\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="giac")
```

[Out] 1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/x^2 - 3/2*log(abs(x^2 + x - 1)) + 3*log(abs(x))

maple [A] time = 0.01, size = 41, normalized size = 0.63

$$-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5}+3\ln(x)-\frac{3\ln\left(x^2+x-1\right)}{2}-\frac{1}{x}+\frac{3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x)

[Out] -3/2*ln(x^2+x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))-1/x+3/2/x^2+3*ln(x)

maxima [A] time = 1.77, size = 51, normalized size = 0.78

$$\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)-\frac{2x-3}{2x^2}-\frac{3}{2}\log\left(x^2+x-1\right)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="maxima")

[Out] 1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/x^2 - 3/2*log(x^2 + x - 1) + 3*log(x)

mupad [B] time = 0.10, size = 48, normalized size = 0.74

$$3\ln(x)-\frac{x-\frac{3}{2}}{x^2}+\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}-\frac{3}{2}\right)-\ln\left(x+\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(4*x + 5*x^2 - 3*x^3 - 3)/(x^3*(x + x^2 - 1)),x)

[Out] 3*log(x) - (x - 3/2)/x^2 + log(x - 5^(1/2)/2 + 1/2)*(5^(1/2)/10 - 3/2) - log(x + 5^(1/2)/2 + 1/2)*(5^(1/2)/10 + 3/2)

sympy [A] time = 0.45, size = 99, normalized size = 1.52

$$3\log(x)+\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}-\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)^2}{101}\right)+\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}+\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}\right.}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)
```

```
[Out] 3*log(x) + (-3/2 + sqrt(5)/10)*log(x - 405/202 - 35*sqrt(5)/202 + 110*(-3/2
 + sqrt(5)/10)**2/101) + (-3/2 - sqrt(5)/10)*log(x - 405/202 + 35*sqrt(5)/2
02 + 110*(-3/2 - sqrt(5)/10)**2/101) + (3 - 2*x)/(2*x**2)
```

3.327 $\int \frac{4+8x+5x^2+2x^3}{\left(2+2x+x^2\right)^2} dx$

Optimal. Leaf size=28

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

```
[Out] -1/(x^2+2*x+2)-arctan(1+x)+ln(x^2+2*x+2)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1660, 634, 617, 204, 628}

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+8x+5x^2+2x^3}{\left(2+2x+x^2\right)^2}\,dx &= -\frac{1}{2+2x+x^2}+\frac{1}{4}\int \frac{4+8x}{2+2x+x^2}\,dx\\
&= -\frac{1}{2+2x+x^2}-\int \frac{1}{2+2x+x^2}\,dx+\int \frac{2+2x}{2+2x+x^2}\,dx\\
&= -\frac{1}{2+2x+x^2}+\log\left(2+2x+x^2\right)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right)\\
&= -\frac{1}{2+2x+x^2}-\tan^{-1}(1+x)+\log\left(2+2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

fricas [A] time = 0.62, size = 46, normalized size = 1.64

$$-\frac{\left(x^2+2\,x+2\right)\arctan\left(x+1\right)-\left(x^2+2\,x+2\right)\log\left(x^2+2\,x+2\right)+1}{x^2+2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="fricas")
```

```
[Out] -((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x + 2) + 1)/(
x^2 + 2*x + 2)
```

giac [A] time = 0.28, size = 28, normalized size = 1.00

$$-\frac{1}{x^2 + 2x + 2} - \arctan\left(x + 1\right) + \log\left(x^2 + 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="giac")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\arctan\left(x + 1\right) + \ln\left(x^2 + 2x + 2\right) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x)
```

```
[Out] -1/(x^2+2*x+2)-arctan(x+1)+ln(x^2+2*x+2)
```

maxima [A] time = 1.66, size = 28, normalized size = 1.00

$$-\frac{1}{x^2 + 2x + 2} - \arctan\left(x + 1\right) + \log\left(x^2 + 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="maxima")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

mupad [B] time = 0.04, size = 28, normalized size = 1.00

$$\ln\left(x^2 + 2x + 2\right) - \mathrm{atan}\left(x + 1\right) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x + 5*x^2 + 2*x^3 + 4)/(2*x + x^2 + 2)^2,x)
```

```
[Out] log(2*x + x^2 + 2) - atan(x + 1) - 1/(2*x + x^2 + 2)
```

sympy [**A**] time = 0.13, size = 24, normalized size = 0.86

$$\log\left(x^2+2x+2\right)-\operatorname{atan}(x+1)-\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)
```

```
[Out] log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)
```

3.328 $\int \frac{(-1+x)^4x^4}{1+x^2}\,dx$

Optimal. Leaf size=32

$$\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x-4\tan^{-1}(x)$$

```
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1629, 203}

$$\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x-4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)^4x^4}{1+x^2}\,dx &= \int \left(4-4x^2+5x^4-4x^5+x^6-\frac{4}{1+x^2}\right)dx\\ &= 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\int \frac{1}{1+x^2}\,dx\\ &= 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x-4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

fricas [A] time = 0.81, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7-\frac{2}{3}x^6+x^5-\frac{4}{3}x^3+4x-4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

giac [A] time = 0.37, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7-\frac{2}{3}x^6+x^5-\frac{4}{3}x^3+4x-4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x-4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^4*x^4/(x^2+1),x)
```

```
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

maxima [A] time = 1.51, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7-\frac{2}{3}x^6+x^5-\frac{4}{3}x^3+4x-4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

mupad [B] time = 0.02, size = 26, normalized size = 0.81

$$4\,x-4\,\mathrm{atan}(x)-\frac{4\,x^3}{3}+x^5-\frac{2\,x^6}{3}+\frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x - 1)^4)/(x^2 + 1),x)
```

```
[Out] 4*x - 4*atan(x) - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7
```

sympy [A] time = 0.10, size = 29, normalized size = 0.91

$$\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x-4\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**4*x**4/(x**2+1),x)
```

```
[Out] x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)
```

3.329 $\int \frac{-20x+4x^2}{9-10x^2+x^4} \, dx$

Optimal. Leaf size=31

$$\log(1-x) - \frac{1}{2}\log(3-x) + \frac{3}{2}\log(x+1) - 2\log(x+3)$$

```
[Out] ln(1-x)-1/2*ln(3-x)+3/2*ln(1+x)-2*ln(3+x)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1593, 1662, 12, 1107, 616, 31, 1130, 207}

$$\frac{5}{4}\log\left(1-x^2\right) - \frac{5}{4}\log\left(9-x^2\right) - \frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
```

```
[Out] (-3*ArcTanh[x/3])/2 + ArcTanh[x]/2 + (5*Log[1 - x^2])/4 - (5*Log[9 - x^2])/
4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx &= \int \frac{x(-20+4x)}{9-10x^2+x^4}\,dx\\
&= \int -\frac{20x}{9-10x^2+x^4}\,dx + \int \frac{4x^2}{9-10x^2+x^4}\,dx\\
&= 4\int \frac{x^2}{9-10x^2+x^4}\,dx - 20\int \frac{x}{9-10x^2+x^4}\,dx\\
&= -\left(\frac{1}{2}\int \frac{1}{-1+x^2}\,dx\right) + \frac{9}{2}\int \frac{1}{-9+x^2}\,dx - 10\,\text{Subst}\left(\int \frac{1}{9-10x+x^2}\,dx, x, x^2\right)\\
&= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) - \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{-9+x}\,dx, x, x^2\right) + \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx,\right.\\
&= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) + \frac{5}{4}\log\left(1-x^2\right) - \frac{5}{4}\log\left(9-x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.26

$$4\left(\frac{1}{4}\log(1-x) - \frac{1}{8}\log(3-x) + \frac{3}{8}\log(x+1) - \frac{1}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
[Out] 4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)
```

fricas [A] time = 0.84, size = 23, normalized size = 0.74

$$-2\,\log\left(x+3\right) + \frac{3}{2}\,\log\left(x+1\right) + \log\left(x-1\right) - \frac{1}{2}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="fricas")
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

giac [A] time = 0.31, size = 27, normalized size = 0.87

$$-2\,\log\left(|x+3|\right) + \frac{3}{2}\,\log\left(|x+1|\right) + \log\left(|x-1|\right) - \frac{1}{2}\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="giac")
```

```
[Out] -2*log(abs(x + 3)) + 3/2*log(abs(x + 1)) + log(abs(x - 1)) - 1/2*log(abs(x
- 3))
```

maple [A] time = 0.01, size = 24, normalized size = 0.77

$$-\frac{\ln (x-3)}{2}+\ln (x-1)+\frac{3 \ln (x+1)}{2}-2 \ln (x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-20*x)/(x^4-10*x^2+9),x)
```

```
[Out] ln(x-1)+3/2*ln(x+1)-2*ln(x+3)-1/2*ln(x-3)
```

maxima [A] time = 0.80, size = 23, normalized size = 0.74

$$-2 \log (x+3)+\frac{3}{2} \log (x+1)+\log (x-1)-\frac{1}{2} \log (x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="maxima")
```

```
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

mupad [B] time = 0.05, size = 23, normalized size = 0.74

$$\ln (x-1)+\frac{3 \ln (x+1)}{2}-\frac{\ln (x-3)}{2}-2 \ln (x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(20*x - 4*x^2)/(x^4 - 10*x^2 + 9),x)
```

```
[Out] log(x - 1) + (3*log(x + 1))/2 - log(x - 3)/2 - 2*log(x + 3)
```

sympy [A] time = 0.19, size = 26, normalized size = 0.84

$$-\frac{\log (x-3)}{2}+\log (x-1)+\frac{3 \log (x+1)}{2}-2 \log (x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)
```

```
[Out] -log(x - 3)/2 + log(x - 1) + 3*log(x + 1)/2 - 2*log(x + 3)
```

3.330 $\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)} \, dx$

Optimal. Leaf size=24

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

```
[Out] -1/x+arctan(x)+2*ln(1-x)-ln(x^2+1)
```

Rubi [A] time = 0.17, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6725, 635, 203, 260}

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
```

```
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{-1+x}+\frac{1}{x^2}+\frac{1-2x}{1+x^2}\right)dx\\
&= -\frac{1}{x}+2\log(1-x)+\int \frac{1-2x}{1+x^2}\,dx\\
&= -\frac{1}{x}+2\log(1-x)-2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1}{x}+\tan^{-1}(x)+2\log(1-x)-\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

fricas [A] time = 0.76, size = 26, normalized size = 1.08

$$\frac{x\arctan(x)-x\log\left(x^2+1\right)+2\,x\log\left(x-1\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] (x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x
```

giac [A] time = 0.29, size = 23, normalized size = 0.96

$$-\frac{1}{x}+\arctan(x)-\log\left(x^2+1\right)+2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="giac")
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$\arctan(x) + 2\ln(x-1) - \ln\left(x^2+1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3+x-1)/(x-1)/x^2/(x^2+1),x)
[Out] 2*ln(x-1)-1/x-ln(x^2+1)+arctan(x)
```

maxima [A] time = 1.22, size = 22, normalized size = 0.92

$$-\frac{1}{x} + \arctan(x) - \log\left(x^2+1\right) + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)
```

mupad [B] time = 2.13, size = 30, normalized size = 1.25

$$2\ln(x-1) - \frac{1}{x} + \ln(x-\mathrm{i})\left(-1 - \frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-1 + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^3 - 1)/(x^2*(x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) - log(x - 1i)*(1 + 1i/2) - log(x + 1i)*(1 - 1i/2) - 1/x
```

sympy [A] time = 0.15, size = 19, normalized size = 0.79

$$2\log(x-1) - \log\left(x^2+1\right) + \operatorname{atan}(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
[Out] 2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

3.331 $\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} \, dx$

Optimal. Leaf size=23

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

```
[Out] -1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1814, 12, 203}

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-4x^3+x^4}{\left(1+x^2\right)^3}\,dx &= -\frac{1}{4\left(1+x^2\right)^2}-\frac{1}{4}\int \frac{-4+16x-4x^2}{\left(1+x^2\right)^2}\,dx\\ &= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\frac{1}{8}\int \frac{8}{1+x^2}\,dx\\ &= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

fricas [A] time = 0.84, size = 35, normalized size = 1.52

$$\frac{8\,x^2+4\left(x^4+2\,x^2+1\right)\arctan(x)+7}{4\left(x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="fricas")
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

giac [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{8\,x^2+7}{4\left(x^2+1\right)^2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.83

$$\arctan(x) + \frac{2x^2 + \frac{7}{4}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x)
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

maxima [A] time = 1.43, size = 24, normalized size = 1.04

$$\frac{8\,x^2+7}{4\left(x^4+2\,x^2+1\right)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="maxima")
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{2\,x^2 + \frac{7}{4}}{x^4+2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(x^2 + 1)^3,x)
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

sympy [A] time = 0.13, size = 20, normalized size = 0.87

$$\frac{8x^2+7}{4x^4+8x^2+4} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

3.332 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx$

Optimal. Leaf size=23

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

```
[Out] -1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2073, 261, 203}

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = \int \left(\frac{x}{\left(1+x^2\right)^3} - \frac{4x}{\left(1+x^2\right)^2} + \frac{1}{1+x^2}\right) dx$$
$$= -\left(4\int \frac{x}{\left(1+x^2\right)^2}\,dx\right) + \int \frac{x}{\left(1+x^2\right)^3}\,dx + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{4\left(1+x^2\right)^2} + \frac{2}{1+x^2} + \tan^{-1}(x)$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2}{x^2+1} - \frac{1}{4\left(x^2+1\right)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

fricas [A] time = 1.24, size = 35, normalized size = 1.52

$$\frac{8\,x^2 + 4\left(x^4 + 2\,x^2 + 1\right)\arctan(x) + 7}{4\left(x^4 + 2\,x^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

giac [A] time = 0.27, size = 19, normalized size = 0.83

$$\frac{8\,x^2+7}{4\left(x^2+1\right)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\arctan(x)+\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x)
```

```
[Out] arctan(x)+(2*x^2+7/4)/(x^2+1)^2
```

maxima [A] time = 1.42, size = 24, normalized size = 1.04

$$\frac{8\,x^2+7}{4\,(x^4+2\,x^2+1)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x)+\frac{2\,x^2+\frac{7}{4}}{x^4+2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(3*x^2 + 3*x^4 + x^6 + 1),x)
```

```
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

sympy [A] time = 0.13, size = 20, normalized size = 0.87

$$\frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)
```

```
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

3.333 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx$

Optimal. Leaf size=13

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

```
[Out] -1/x+ln(x^2+x+1)
```

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1594, 1628, 628}

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = \int \frac{1+x+2x^2+2x^3}{x^2\left(1+x+x^2\right)}\,dx$$
$$= \int \left(\frac{1}{x^2}+\frac{1+2x}{1+x+x^2}\right)dx$$
$$= -\frac{1}{x}+\int \frac{1+2x}{1+x+x^2}\,dx$$
$$= -\frac{1}{x}+\log\left(1+x+x^2\right)$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
[Out] -x^(-1) + Log[1 + x + x^2]
```

fricas [A] time = 0.81, size = 15, normalized size = 1.15

$$\frac{x\log\left(x^2+x+1\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="fricas")
[Out] (x*log(x^2 + x + 1) - 1)/x
```

giac [A] time = 0.28, size = 13, normalized size = 1.00

$$-\frac{1}{x}+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="giac")
[Out] -1/x + log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$\ln\left(x^2+x+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x)
```

```
[Out] -1/x+ln(x^2+x+1)
```

maxima [A] time = 0.71, size = 13, normalized size = 1.00

$$-\frac{1}{x} + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="maxima")
```

```
[Out] -1/x + log(x^2 + x + 1)
```

mupad [B] time = 2.14, size = 13, normalized size = 1.00

$$\ln\left(x^2 + x + 1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*x^2 + 2*x^3 + 1)/(x^2 + x^3 + x^4),x)
```

```
[Out] log(x + x^2 + 1) - 1/x
```

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$\log\left(x^2 + x + 1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
```

```
[Out] log(x**2 + x + 1) - 1/x
```

3.334 $\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$

Optimal. Leaf size=206

$$\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right.}{\sqrt{3}}$$

```
[Out] 2*c*d*x/b+1/2*d^2*x^2/b-1/3*a^(1/3)*d*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^
(1/3)*x)/b^(5/3)+1/6*a^(1/3)*d*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/3*c^2*ln(b*x^3+a)/b+1/3*a^(1/3)*d*(2*b^(1/3
)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^
(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right.}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*d
*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)
*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)) + (c^2*Log[a + b*x^3])/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \int \left(\frac{2cd}{b} + \frac{d^2x}{b} - \frac{2acd + ad^2x - bc^2x^2}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\int \frac{2acd+ad^2x-bc^2x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\int \frac{2acd+ad^2x}{a+bx^3}\,dx}{b} + c^2\int \frac{x^2}{a+bx^3}\,dx$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{c^2\log\left(a+bx^3\right)}{3b} - \frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}\,cd+a^{4/3}d^2\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}\,cd+a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}} - \frac{\left(\sqrt[3]{a}\,d\left(2c - \right.\right.}{}$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c - \sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}{3b^{5/3}} + \frac{c^2\log\left(a+bx^3\right)}{3b} + \frac{\left(\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c - \right.\right.}{}$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c - \sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}{3b^{5/3}} + \frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c - \sqrt[3]{a}\,d\right)\log\left(a^{2/3} - \right.}{6b^{5/3}}$$

$$= \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c + \sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}} - \frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c - \sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a} + \right.}{3b^{5/3}}$$

Mathematica [A] time = 0.11, size = 193, normalized size = 0.94

$$\frac{-\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d - 2\sqrt[3]{b}\,c\right)\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,x + b^{2/3}x^2\right) + 2b^{2/3}c^2\log\left(a+bx^3\right) + 2\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d - 2\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a} + \right.}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*Sqrt[3]*a^(1/3)*d*(2*b^(1/3)*c +
a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(-2*b^
(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*d*(-2*b^(1/3)*c + a
^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Lo
g[a + b*x^3])/(6*b^(5/3))
```

fricas [C] time = 2.99, size = 4545, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*d^2*x^2 + 24*c*d*x - 2*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b*log(1/4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*s
qrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2
*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b
^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b
^3 + 3*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (
8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a
*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 + 5*b*c
^4 + 4*a*c*d^3 + (8*b*c^3*d + a*d^4)*x) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^
5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5
)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqr
t(3) + 1) - 2*c^2/b)*b + 6*c^2 + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b
^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^
3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a
^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
+ 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)
^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt
(3) + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)
^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8
*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*
b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3
)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^
2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/
b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d
^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 +
a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
- 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^
(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3
```

```
*d + a*d^4)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d
^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c
^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) +
(1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1)
- 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I
*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d
^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b
^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 +
2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*
a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3
) + 1) - 2*c^2/b)*b + 6*c^2 - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2
- (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*
a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*
d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*
(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3
)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2
*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1
/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a
*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3)
 + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)^(2
/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*
c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c
^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a
*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d
^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/b^2
 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)
*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2
*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3
*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/
3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3*d
+ a*d^4)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b
^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)
^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2
```

```
/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c
^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)
/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
+ 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/
2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c
^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2
*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sq
rt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)
*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*
c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^
2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*
c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)))/b
```

giac [A] time = 0.44, size = 208, normalized size = 1.01

$$\frac{c^2\log\left(\left|bx^3+a\right|\right)}{3\,b}-\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd-\left(-ab^2\right)^{\frac{2}{3}}d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3}+\frac{bd^2x^2+4\,bcdx}{2\,b^2}-\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c^2*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c*d - (-a*b
^2)^(2/3)*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 +
1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 - 1/6*(2*(-a*b^2)^(1/3)*b*c*d + (-a*b^2)^(2
/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*d^2*(-a/
b)^(1/3) + 2*a*b^4*c*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

maple [A] time = 0.01, size = 236, normalized size = 1.15

$$\frac{d^2x^2}{2b}-\frac{2\sqrt{3}\,acd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{2acd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{acd\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{\sqrt{3}\,a\,d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x+c)^2/(b*x^3+a),x)
```

```
[Out] 1/2*d^2*x^2/b+2*c*d*x/b-2/3/b^2*a*c*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/3/b^2
*a*c*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-2/3/b^2*a*c*d/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b^2*a*d^2/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))-1/6/b^2*a*d^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))-1/3/b^2*a*d^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))+1/3*c^2*ln(b*x^3+a)/b
```

maxima [A] time = 1.65, size = 199, normalized size = 0.97

$$-\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{d^2x^2+4\,cdx}{2\,b}+\frac{\left(2\,bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,acd\right)\log\left(x^2-x\left(\right.\right.}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) + 2*a*c*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(d^2*x^2 + 4*c*d*x)/b + 1/6*(2*
b*c^2*(a/b)^(2/3) - a*d^2*(a/b)^(1/3) + 2*a*c*d)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)^(1/3)
 - 2*a*c*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 0.23, size = 357, normalized size = 1.73

$$\left(\sum_{k=1}^{3}\ln\left(\frac{a\left(b\,c^4+\mathrm{root}\left(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k\right)^2\,b^3\,9-\mathrm{ro}\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x)^2)/(a + b*x^3),x)
```

```
[Out] symsum(log((a*(b*c^4 + 9*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*
z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)^2*b^3 - 6*root(2
7*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3
 - b^2*c^6 - a^2*d^6, z, k)*b^2*c^2 + 2*a*c*d^3 + a*d^4*x + 2*b*c^3*d*x - 6
*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*
c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)*b^2*c*d*x))/b)*root(27*b^5*z^3 - 27*b^4*
c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^
6, z, k), k, 1, 3) + (d^2*x^2)/(2*b) + (2*c*d*x)/b
```

sympy [A] time = 1.24, size = 138, normalized size = 0.67

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c^2 + t\left(18ab^2cd^3 + 9b^3c^4\right) - a^2d^6 + 2abc^3d^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{9t^2b^3 - 18tb^2c^2 + 4}{ad^4 + 8bc}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + _t*(18*a*b**2*c*d**3 + 9*b**3*
c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda(_t, _t*log(x + (9*_
t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*b*c**4)/(a*d**4 + 8*b*c**3*d))
)) + 2*c*d*x/b + d**2*x**2/(2*b)
```

3.335 $\int \frac{-x+2x^3+4x^5}{\left(3+2x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=45

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5-7x^2}{8\left(x^4+2x^2+3\right)}$$

```
[Out] 1/8*(-7*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1594, 1663, 1660, 12, 618, 204}

$$\frac{5-7x^2}{8\left(x^4+2x^2+3\right)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-x+2x^3+4x^5}{\left(3+2x^2+x^4\right)^2}\,dx &= \int \frac{x\left(-1+2x^2+4x^4\right)}{\left(3+2x^2+x^4\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+2x+4x^2}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)} + \frac{1}{16}\operatorname{Subst}\left(\int \frac{18}{3+2x+x^2}\,dx, x, x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)} + \frac{9}{8}\operatorname{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)} - \frac{9}{4}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5-7x^2}{8\left(x^4+2x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

fricas [A] time = 0.93, size = 47, normalized size = 1.04

$$\frac{9\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-14x^2+10}{16\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 1.11, size = 38, normalized size = 0.84

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{7x^2-5}{8\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2 +
 3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.91

$$\frac{9\sqrt{2}\ \arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}+\frac{-\frac{7x^2}{4}+\frac{5}{4}}{2x^4+4x^2+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/2*(-7/4*x^2+5/4)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7\,x^2-5}{8\left(x^4+2\,x^2+3\right)}+\frac{9}{4}\int\frac{x}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] -1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 2.19, size = 42, normalized size = 0.93

$$\frac{9\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}-\frac{\frac{7\,x^2}{8}-\frac{5}{8}}{x^4+2\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 - x + 4*x^5)/(2*x^2 + x^4 + 3)^2,x)
[Out] (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((7*x^2)/8 - 5/8)/(2*x^2
 + x^4 + 3)
```

sympy [A] time = 0.15, size = 44, normalized size = 0.98

$$\frac{5-7x^2}{8x^4+16x^2+24}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
[Out] (5 - 7*x**2)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

3.336 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx$

Optimal. Leaf size=59

$$\tan^{-1}\left(2x^2+1\right)+\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}+\frac{4x^2+3}{16\left(2x^4+2x^2+1\right)^2}$$

```
[Out] 1/16*(4*x^2+3)/(2*x^4+2*x^2+1)^2+1/2*(2*x^2+1)/(2*x^4+2*x^2+1)+arctan(2*x^2
+1)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1593, 1663, 1660, 12, 614, 617, 204}

$$\frac{2x^2+1}{2\left(2x^4+2x^2+1\right)}+\frac{4x^2+3}{16\left(2x^4+2x^2+1\right)^2}+\tan^{-1}\left(2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^2 + 2*x^4)
) + ArcTan[1 + 2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+x^5}{\left(1+2x^2+2x^4\right)^3}\,dx &= \int \frac{x\left(1+x^4\right)}{\left(1+2x^2+2x^4\right)^3}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{\left(1+2x+2x^2\right)^3}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1}{16}\operatorname{Subst}\left(\int \frac{16}{\left(1+2x+2x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \operatorname{Subst}\left(\int \frac{1}{\left(1+2x+2x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} + \operatorname{Subst}\left(\int \frac{1}{1+2x+2x^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} - \operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} + \tan^{-1}\left(1+2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.75

$$\tan^{-1}\left(2x^2+1\right) + \frac{32x^6+48x^4+36x^2+11}{16\left(2x^4+2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*x
^2]
```

fricas [A] time = 0.78, size = 75, normalized size = 1.27

$$\frac{32\,x^6+48\,x^4+36\,x^2+16\left(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1\right)\arctan\left(2\,x^2+1\right)+11}{16\left(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*arc
tan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)
```

giac [A] time = 1.80, size = 42, normalized size = 0.71

$$\frac{32\,x^6+48\,x^4+36\,x^2+11}{16\left(2\,x^4+2\,x^2+1\right)^2}+\arctan\left(2\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2 +
 1)
```

maple [A] time = 0.01, size = 41, normalized size = 0.69

$$\arctan\left(2x^2+1\right)+\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{\left(2x^4+2x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+x)/(2*x^4+2*x^2+1)^3,x)
```

```
[Out] 2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{32\,x^6+48\,x^4+36\,x^2+11}{16\left(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1\right)}+2\int\frac{x}{2\,x^4+2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) +
2*integrate(x/(2*x^4 + 2*x^2 + 1), x)
```

mupad [B] time = 0.05, size = 47, normalized size = 0.80

$$\mathrm{atan}\left(2\,x^2+1\right)+\frac{\frac{x^6}{2}+\frac{3\,x^4}{4}+\frac{9\,x^2}{16}+\frac{11}{64}}{x^8+2\,x^6+2\,x^4+x^2+\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^5)/(2*x^2 + 2*x^4 + 1)^3,x)
```

```
[Out] atan(2*x^2 + 1) + ((9*x^2)/16 + (3*x^4)/4 + x^6/2 + 11/64)/(x^2 + 2*x^4 + 2
*x^6 + x^8 + 1/4)
```

sympy [A] time = 0.18, size = 46, normalized size = 0.78

$$\frac{32x^6 + 48x^4 + 36x^2 + 11}{64x^8 + 128x^6 + 128x^4 + 64x^2 + 16} + \operatorname{atan}\left(2x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

```
[Out] (32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**2
 + 16) + atan(2*x**2 + 1)
```

3.337 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$

Optimal. Leaf size=209

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

```
[Out] -b*arctanh((2*f*x^2+e)/(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)+1/2*arctan(x*
2^(1/2)*f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2))*(c+(2*a*f-c*e)/(-4*d*f+e^2)^(
1/2))*2^(1/2)/f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*f^(
1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2))*(c+(-2*a*f+c*e)/(-4*d*f+e^2)^(1/2))*2^(1
/2)/f^(1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e -
Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c +
(c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e
^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(
e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \int \frac{bx}{d+ex^2+fx^4}\,dx + \int \frac{a+cx^2}{d+ex^2+fx^4}\,dx$$

$$= b\int \frac{x}{d+ex^2+fx^4}\,dx + \frac{1}{2}\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int \frac{1}{\frac{e}{2}-\frac{1}{2}\sqrt{e^2-4df}+fx^2}\,dx + \frac{1}{2}\Big(c+\frac{ce-2}{\sqrt{e^2-}}$$

$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} + \frac{1}{2}b\,\mathrm{Subst}\Big(\int \frac{}{d}$$

$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} - b\,\mathrm{Subst}\Big(\int \frac{}{e^2-}$$

$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} - \frac{b\tanh^{-1}\Big(\frac{e+2f}{\sqrt{e^2-4}}}{\sqrt{e^2-4df}}$$

Mathematica [A] time = 0.25, size = 234, normalized size = 1.12

$$\frac{\frac{\sqrt{2}\left(2af+c\left(\sqrt{e^2-4df}-e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\sqrt{2}\left(c\left(\sqrt{e^2-4df}+e\right)-2af\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}} + b\log\left(\sqrt{e^2-4df}-e-2fx^2\right) - b\,\mathrm{l}}{2\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/S
qrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqrt[
2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e +
 Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e + Sq
rt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(2*Sqr
t[e^2 - 4*d*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 4.70, size = 1587, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*sqrt(-4*d*f + e^2)*b*log(x^2 + 1/2*
(sqrt(-4*d*f + e^2) + e)/f)/((16*d^2*f^2 - 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2
+ f^2*e^2 - 2*f*e^3 + e^4)*f^2) - 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*s
qrt(-4*d*f + e^2)*b*log(x^2 - 1/2*(sqrt(-4*d*f + e^2) - e)/f)/((16*d^2*f^2
- 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2 + f^2*e^2 - 2*f*e^3 + e^4)*f^2) + 1/4*((1
6*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e + s
qrt(-4*d*f + e^2)*f)*d*f^3 - 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*f
+ e^2)*f)*d*f^2*e - 8*d*f^3*e + 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqr
t(-4*d*f + e^2)*f)*d*f*e - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*
f + e^2)*f)*f^2*e + 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*
f + e^2)*f)*d*f*e^2 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*f^2*e^2 + 16
*d*f^2*e^2 + 2*(4*d*f - e^2)*f^2*e + 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e
+ sqrt(-4*d*f + e^2)*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*
f*e^3 + 2*f^2*e^3 - 2*(4*d*f - e^2)*f*e^2 - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt
(f*e + sqrt(-4*d*f + e^2)*f)*e^3 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)
*e^4 - 2*f*e^4)*a + 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e +
sqrt(-4*d*f + e^2)*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4
*d*f + e^2)*f)*d*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*f
+ e^2)*f)*d*f*e - 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*f
 + e^2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sqr
t((sqrt(-4*d*f + e^2) + e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d
^2*f*e^2 + d*f^2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f)) + 1/4*((16*sqrt(2)*sqrt(f
*e - sqrt(-4*d*f + e^2)*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2
)*f)*d*f^3 + 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f^2*
e + 8*d*f^3*e - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*
f)*d*f*e + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f^2*
e - 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f*
e^2 + sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f^2*e^2 - 16*d*f^2*e^2 - 2*(
4*d*f - e^2)*f^2*e - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f +
e^2)*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f*e^3 - 2*f^2*e^
3 + 2*(4*d*f - e^2)*f*e^2 + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d
*f + e^2)*f)*e^3 + sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*e^4 + 2*f*e^4)*
```

```
a - 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^
2)*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d
*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f*e
- 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e
 - sqrt(-4*d*f + e^2)*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sqrt(-(sqrt(-4*d*f
+ e^2) - e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d^2*f*e^2 + d*f^
2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f))
```

maple [B] time = 0.06, size = 616, normalized size = 2.95

$$-\frac{2\sqrt{2}\, cdf \operatorname{arctanh}\left(\frac{\sqrt{2}\, fx}{\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}\right)}{\left(4df-e^2\right)\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}+\frac{2\sqrt{2}\, cdf \arctan\left(\frac{\sqrt{2}\, fx}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)}{\left(4df-e^2\right)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}+\frac{\sqrt{2}\, c e^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\, fx}{\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}\right)}{2\left(4df-e^2\right)\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x)
```

```
[Out] -1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*b*ln(-2*f*x^2+(-4*d*f+e^2)^(1/2)-e)-2*f
/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(1/2)/(
((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*c*d+1/2/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)
^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*c*
e^2+f*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/
2)*arctanh(f*x*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*a-1/2*(-4*d*f+e^2)
^(1/2)/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(
1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*c*e+1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^
2)*b*ln(2*f*x^2+(-4*d*f+e^2)^(1/2)+e)+2*f/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e
^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*
d-1/2/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/
2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*e^2+f*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)
*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+(-4*d*f+e^
2)^(1/2))*f)^(1/2))*a-1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*
f+e^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))
*c*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2+bx+a}{fx^4+ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d), x)
```

mupad [B] time = 3.44, size = 3942, normalized size = 18.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x)
```

```
[Out] symsum(log(a*b^2*f^2 - a^2*c*f^2 + b^3*f^2*x - c^3*d*f - 8*root(16*d*e^4*f*
z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d
^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 +
32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z +
 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a
^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*
f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*e^3*f^2*x + a*c^2*e*f - 16*roo
t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z
^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*
c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*d*f^3 - 4*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a^2*f^3*x + 4*root(16
*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 -
 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f
^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*
d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*
d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f
 + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*e^2*f^2 + 16*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*d*f^3*x + 2*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b^2*e*f^2*x + 4*root(1
```

6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*d*f^2*x - 2*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*e^2*f*x + 32*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*d*e*f^3*x - 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*e^2*f^2*x + 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a*b*e*f^2 - 8*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b*c*d*f^2 - 2*a*b*c*f^2*x + b*c^2*e*f*x + 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a*c*e*f^2*x)*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 +

```
4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16
*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e
+ b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z,
 k), k, 1, 4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
```

```
[Out] Timed out
```

3.338 $\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=224

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -2*d*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arcta
n(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e^2+(-b*e^2+2*c*d^2)/(-4
*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e^2+(b*e^2-2*c*d^2)/(-4*a*c+
b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
(2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \int \frac{2dex}{a+bx^2+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+bx^2+cx^4}\,dx$$
$$= (2de)\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(e^2 + \frac{2c}{\sqrt{}}\right.$$
$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + (de)\,\mathrm{Subst}\left(\int\right.$$
$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - (2de)\,\mathrm{Subst}\left(\int\right.$$
$$= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de\tanh^{-1}\left(\frac{b}{\sqrt{}}\right.}{\sqrt{b^2-4a}}$$

Mathematica [A] time = 0.25, size = 245, normalized size = 1.09

$$\frac{\frac{\sqrt{2}\left(e^2\left(\sqrt{b^2-4ac}-b\right)+2cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(e^2\left(\sqrt{b^2-4ac}+b\right)-2cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} + 2de\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2*d*e
*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*c] +
2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 4.56, size = 1625, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*d*e*log(x^2 + 1/2*(b
 + sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c
^2 + b^2*c^2 - 4*a*c^3)*c^2) + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2
 - 4*a*c)*d*e*log(x^2 + 1/2*(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c -
2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c +
16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*
(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 +
2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(
b^2 - 4*a*c)*a*c^2)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/
c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^
2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c
)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*
```

```
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e^2)*arctan(2*sq
rt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [B] time = 0.06, size = 633, normalized size = 2.83

$$-\frac{2\sqrt{2}\,ac\,e^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{2\sqrt{2}\,ac\,e^2\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,b^2e^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*d*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c/(
4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^2*a+1/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^
2*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e^2+c*(-4*a*
c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c
*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d^2+(-4*a*c+b^2)^(1/2)/(4*a*c-
b^2)*d*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*e^2*a-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*b^2-1/2*(-4*a*c+b^2)^(1/2)/(
4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 3.22, size = 3046, normalized size = 13.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + b*x^2 + c*x^4),x)

[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^3*b^3*c^2*x + 4*c^2*d^3*e^3*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*b^2*c^2*d^2 + b*c*d^2*e^4 - 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 32*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^3*a*b*c^3*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*b^2*c*e^4*x + 2*b*c*d*e^5*x - 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*d*e^3 + 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4

```
 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c
^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 +
8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2
*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*
e^8, z, k)*b*c^2*d^3*e + 32*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256
*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4
*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*
c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d
^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z
, k)^2*a*c^3*d*e*x + 12*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3
*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2
 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^
5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e
^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)
*b*c^2*d^2*e^2*x - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^
3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 +
192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e
*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2
+ 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*
b^2*c^2*d*e*x)*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4
- 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^
2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 3
2*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*
c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k), k, 1, 4
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.339 $\int \frac{x^2}{(a+bx)(c+dx)} dx$

Optimal. Leaf size=56

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

```
[Out] x/b/d+a^2*ln(b*x+a)/b^2/(-a*d+b*c)-c^2*ln(d*x+c)/d^2/(-a*d+b*c)
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {72}

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)(c+dx)} dx &= \int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right) dx \\ &= \frac{x}{bd} + \frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

fricas [A] time = 0.71, size = 65, normalized size = 1.16

$$\frac{a^2d^2 \log(bx+a) - b^2c^2 \log(dx+c) + \left(b^2cd - abd^2\right)x}{b^3cd^2 - ab^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x + a) - b^2*c^2*log(d*x + c) + (b^2*c*d - a*b*d^2)*x)/(b^3*
c*d^2 - a*b^2*d^3)
```

giac [A] time = 0.31, size = 62, normalized size = 1.11

$$\frac{a^2 \log(|bx+a|)}{b^3c - ab^2d} - \frac{c^2 \log(|dx+c|)}{bcd^2 - ad^3} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/(b^3*c - a*b^2*d) - c^2*log(abs(d*x + c))/(b*c*d^2 -
a*d^3) + x/(b*d)
```

maple [A] time = 0.01, size = 57, normalized size = 1.02

$$-\frac{a^2 \ln(bx+a)}{(ad-bc)\,b^2} + \frac{c^2 \ln(dx+c)}{(ad-bc)\,d^2} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(d*x+c),x)
```

```
[Out] x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*ln(b*x+a)
```

maxima [A] time = 0.68, size = 60, normalized size = 1.07

$$\frac{a^2 \log(bx+a)}{b^3c - ab^2d} - \frac{c^2 \log(dx+c)}{bcd^2 - ad^3} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 - a*d^3) + x
/(b*d)
```

mupad [B] time = 0.21, size = 61, normalized size = 1.09

$$-\frac{a^2\,d^2\,\ln\left(a+b\,x\right)-b^2\,c^2\,\ln\left(c+d\,x\right)-a\,b\,d^2\,x+b^2\,c\,d\,x}{b^2\,d^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)*(c + d*x)),x)
```

```
[Out] -(a^2*d^2*log(a + b*x) - b^2*c^2*log(c + d*x) - a*b*d^2*x + b^2*c*d*x)/(b^2
*d^2*(a*d - b*c))
```

sympy [B] time = 1.06, size = 190, normalized size = 3.39

$$-\frac{a^2\log\left(x+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{b^2(ad-bc)}+\frac{c^2\log\left(x+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{d^2(ad-bc)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] -a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a**2
*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(b**2
*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + 2*a*b
**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2 + b
**2*c**2))/(d**2*(a*d - b*c)) + x/(b*d)
```

3.340 $\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx$

Optimal. Leaf size=96

$$\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log(c+dx)}{d\left(ad^2+bc^2\right)}-\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}$$

```
[Out] c^2*ln(d*x+c)/d/(a*d^2+b*c^2)+1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-c*arctan(
x*b^(1/2)/a^(1/2))*a^(1/2)/(a*d^2+b*c^2)/b^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1629, 635, 205, 260}

$$\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log(c+dx)}{d\left(ad^2+bc^2\right)}-\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^2
*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a*d
^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(c+dx)\left(a+bx^2\right)}\,dx &= \int \left(\frac{c^2}{\left(bc^2+ad^2\right)(c+dx)}-\frac{a(c-dx)}{\left(bc^2+ad^2\right)\left(a+bx^2\right)}\right)dx\\ &= \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}-\frac{a\int \frac{c-dx}{a+bx^2}\,dx}{bc^2+ad^2}\\ &= \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}-\frac{(ac)\int \frac{1}{a+bx^2}\,dx}{bc^2+ad^2}+\frac{(ad)\int \frac{x}{a+bx^2}\,dx}{bc^2+ad^2}\\ &= -\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\left(bc^2+ad^2\right)}+\frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}+\frac{ad\log\left(a+bx^2\right)}{2b\left(bc^2+ad^2\right)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.76

$$\frac{-2\sqrt{a}\sqrt{b}\,cd\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+ad^2\log\left(a+bx^2\right)+2bc^2\log(c+dx)}{2abd^3+2b^2c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c + d*x]
+ a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)
```

fricas [A] time = 0.73, size = 162, normalized size = 1.69

$$\left[\frac{bcd\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+ad^2\log\left(bx^2+a\right)+2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)},-\frac{2\,bcd\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-ad^2\log\left(bx^2\right.}{2\left(b^2c^2d+abd^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a*
d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2*b
*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*log
(d*x + c))/(b^2*c^2*d + a*b*d^3)]
```

giac [A] time = 0.26, size = 85, normalized size = 0.89

$$\frac{ad\log\left(bx^2+a\right)}{2\left(b^2c^2+abd^2\right)}+\frac{c^2\log\left(|dx+c|\right)}{bc^2d+ad^3}-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bc^2+ad^2\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*d
 + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

maple [A] time = 0.01, size = 87, normalized size = 0.91

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a\,d^2+b\,c^2\right)\sqrt{ab}}+\frac{ad\ln\left(b\,x^2+a\right)}{2\left(a\,d^2+b\,c^2\right)b}+\frac{c^2\ln\left(dx+c\right)}{\left(a\,d^2+b\,c^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-a/(a*d^2+b*c^2)*c/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)+c^2*ln(d*x+c)/d/(a*d^2+b*c^2)
```

maxima [A] time = 1.22, size = 84, normalized size = 0.88

$$\frac{ad\log\left(bx^2+a\right)}{2\left(b^2c^2+abd^2\right)}+\frac{c^2\log\left(dx+c\right)}{bc^2d+ad^3}-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bc^2+ad^2\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(d*x + c)/(b*c^2*d + a*
d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

mupad [B] time = 1.13, size = 347, normalized size = 3.61

$$\frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\left(x\left(2\,b^2\,c^2-5\,a\,b\,d^2\right)-5\,a\,b\,c\,d+\frac{2\,b^2\,d\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}-\frac{\ln\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x)),x)
```

```
[Out] (log(a*c + a*d*x + ((c*(-a*b^3)^(1/2) + a*b*d)*(x*(2*b^2*c^2 - 5*a*b*d^2) -
 5*a*b*c*d + (2*b^2*d*(c*(-a*b^3)^(1/2) + a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c
^2*x))/(2*b^3*c^2 + 2*a*b^2*d^2)))/(2*b^3*c^2 + 2*a*b^2*d^2))*(c*(-a*b^3)^(
1/2) + a*b*d))/(2*b^3*c^2 + 2*a*b^2*d^2) - (log(a*c + a*d*x + ((c*(-a*b^3)^
(1/2) - a*b*d)*(b*x*(5*a*d^2 - 2*b*c^2) + 5*a*b*c*d + (d*(c*(-a*b^3)^(1/2)
- a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x))/(a*d^2 + b*c^2)))/(2*b^2*(a*d^2 +
 b*c^2)))*(c*(-a*b^3)^(1/2) - a*b*d))/(2*(b^3*c^2 + a*b^2*d^2)) + (c^2*log(
c + d*x))/(a*d^3 + b*c^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.341 $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}-\frac{\sqrt[3]{a}\,d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\,\sqrt[3]{b}\,cd+b^{2/3}c^2\right)}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log}{3b^{2/3}\left(bc^3-\right.}$$

```
[Out] 1/3*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d^3+b
*c^3)-c^2*ln(d*x+c)/(-a*d^3+b*c^3)-1/6*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/(-a*d^3+b*c^3)+1/3*c^2*ln(b*x
^3+a)/(-a*d^3+b*c^3)-1/3*a^(1/3)*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/b^(2/3)/(b^(2/3)*c^2+a^(1/3)*b^(1/3)*c*d+a^(2/3)*d^2)*3^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}-\frac{\sqrt[3]{a}\,d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\,\sqrt[3]{b}\,cd+b^{2/3}c^2\right)}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log}{3b^{2/3}\left(bc^3-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^
(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2))) + (a^(1/3)*d*(b^(
1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c^3 - a*d^3)) -
 (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^3)) +
 (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \frac{x^2}{(c+dx)\left(a+bx^3\right)}\,dx = \int\left(-\frac{c^2 d}{\left(bc^3-ad^3\right)(c+dx)} + \frac{acd-ad^2x+bc^2x^2}{\left(bc^3-ad^3\right)\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{\int \frac{acd-ad^2x+bc^2x^2}{a+bx^3}\,dx}{bc^3-ad^3}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{\int \frac{acd-ad^2x}{a+bx^3}\,dx}{bc^3-ad^3} + \frac{\left(bc^2\right)\int \frac{x^2}{a+bx^3}\,dx}{bc^3-ad^3}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)} + \frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}cd-a^{4/3}d^2\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}cd-a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}\left(bc^3-ad^3\right)} + \frac{\left(\sqrt[3]{a}\right.}{}$$

$$= \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)} + \frac{\left(a^{2/3}d\right)}{2\sqrt[3]{b}\left(b^{2/3}\right.}$$

$$= \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} - \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(a^{2/3}\right.}{6b^{2/3}\left(bc^3-\right.}$$

$$= -\frac{\sqrt[3]{a}\,d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)} + \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log}{bc^3}$$

Mathematica [A] time = 0.09, size = 228, normalized size = 0.86

$$\frac{-\sqrt[3]{a}\sqrt[3]{b}cd\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - a^{2/3}d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) + 2b^{2/3}c^2\log\left(a+bx^3\right) + 2\sqrt[3]{a}\,d\left(\right.}{6b^{2/3}\left(bc^3-ad^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3
```

```
)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b^(1/3)*c*d*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(2/3)*(b*c^3 - a*d^3))
```

fricas [C] time = 2.99, size = 5975, normalized size = 22.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(b*c^3 - a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3
 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 -
a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*
b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 -
a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*
b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*log(-3/2*(2*(1/2)^
(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c
^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/
((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c
^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/
((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) -
2*c^2/(b*c^3 - a*d^3))*b*c^2 - 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/
(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 -
 a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a
*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 -
 a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a
*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^
3 - a*d^3))^2 + d*x - 2*c) + 12*c^2*log(d*x + c) - ((b*c^3 - a*d^3)*(2*(1/2
)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2
*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^
3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2
*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^
3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1)
- 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 - 3*sqrt(1/3)*(b*c^3 - a*d^3)*sqrt(-(4*b*c
^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2/3)*(c
^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
- a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
- a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b
*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a
*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3
- 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^
2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3
```

- 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)))*log(3/2*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)) + 2*d*x + 2*c) - ((b*c^3 - a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 + 3*sqrt(1/3)*(b*c^3 - a*d^3)*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3)

```
 + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)
*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b
*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c
^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b
*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c
^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2
/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)))*log(3/2*(2*(1/
2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(
2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d
^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(
2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d
^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1)
 - 2*c^2/(b*c^3 - a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c
^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b
*c^3 - a*d^3))^2 - 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b
*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3
 - a*d^3))*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*
d^6)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqr
t(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*
d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1
/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*
d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*s
qrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)
^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*
c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3
/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*
c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3
/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) -
 2*c^2/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)) + 2*d*x +
 2*c))/(b*c^3 - a*d^3)
```

giac [A] time = 0.30, size = 320, normalized size = 1.21

$$-\frac{c^2 d\log\left(|dx+c|\right)}{bc^3 d - ad^4}+\frac{c^2\log\left(\left|bx^3+a\right|\right)}{3\left(bc^3-ad^3\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}} d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2-\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}} bcd+\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}} d^2}+\frac{\left(ab^2c^3d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bd^5\left(-\right.\right.}{3\left(ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -c^2*d*log(abs(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*log(abs(b*x^3 + a))/(b
*c^3 - a*d^3) + (-a*b^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(sqrt(3)*b^2*c^2 - sqrt(3)*(-a*b^2)^(1/3)*b*c*d + sqrt(3)*(-a*b
^2)^(2/3)*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^(1/3) - a^2*b*d^5*(-a/b)^(1/3) -
 a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3*
c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^(1/3)*b*c*d - (-a*b^2)
^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c^3 - a*b^2*d^3)
```

maple [A] time = 0.01, size = 336, normalized size = 1.27

$$-\frac{\sqrt{3}\,acd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(a\,d^3-b\,c^3\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{acd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a\,d^3-b\,c^3\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{acd\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a\,d^3-b\,c^3\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\sqrt{3}\,a\,d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(a\,d^3-b\,c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d^3-b*c^3)*
a*c*d/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d^3-b*c^3)*a*c
*d/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d^3
-b*c^3)*a*d^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/(a*d^3-b*c^3)*a*d^2/b/(a/
b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/(a*d^3-b*c^3)*a*d^2*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d^3-b*c^3)*c^2
*ln(b*x^3+a)+c^2/(a*d^3-b*c^3)*ln(d*x+c)
```

maxima [A] time = 1.71, size = 279, normalized size = 1.06

$$-\frac{c^2\log(dx+c)}{bc^3-ad^3}-\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(2bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-acd\right)\log\left(x^2-x\left(\frac{a}{b}\right.\right.}{6\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -c^2*log(d*x + c)/(b*c^3 - a*d^3) - 1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) - a*c*d*
(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^3*
(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3))*(a/b)^(1/3)) + 1/6*(2*b*c^2*(a/b)^(2/3)
- a*d^2*(a/b)^(1/3) - a*c*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^
3*(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)
^(1/3) + a*c*d)*log(x + (a/b)^(1/3))/(b^2*c^3*(a/b)^(2/3) - a*b*d^3*(a/b)^(
2/3))
```

mupad [B] time = 2.50, size = 570, normalized size = 2.16

$$\left(\sum_{k=1}^{3}\ln\left(-a\,b\,d\,\left(c+d\,x+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^2\,b^2\,c^3\,3+\mathrm{root}\left(27\,a\,b^2\,d\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x)),x)
```

```
[Out] symsum(log(-a*b*d*(c + d*x + 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^3 + 9*root(27*a*b^2*d^3*z^3 - 27*b
^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^4 - 5*root(27*a*b^
2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c^2 - 3*
root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k
)^2*a*b*d^3 - 8*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9
*b*c*z + 1, z, k)*b*c*d*x + 45*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*c*d^3 + 36*root(27*a*b^2*d^3*z^3 -
 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*d^4*x + 9*roo
t(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2
*b^2*c^2*d*x + 18*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 -
 9*b*c*z + 1, z, k)^3*b^3*c^3*d*x))*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3
+ 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k), k, 1, 3) + (c^2*log(c + d*x))/(a*d^3
 - b*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.342 $\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=417

$$\frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,\left(ad^4+bc^4\right)}-\frac{c^2d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}+\frac{c\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\left(ad^4+bc^4\right)}-$$

```
[Out] c^2*d*ln(d*x+c)/(a*d^4+b*c^4)-1/4*c^2*d*ln(b*x^4+a)/(a*d^4+b*c^4)+1/2*d^3*a
rctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/(a*d^4+b*c^4)/b^(1/2)+1/4*c*arctan(-1+b^
(1/4)*x*2^(1/2)/a^(1/4))*(-d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+
b*c^4)*2^(1/2)+1/4*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d^2*a^(1/2)+b^(1
/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)*2^(1/2)+1/8*c*ln(-a^(1/4)*b^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*b^(1/2))*(d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d
^4+b*c^4)*2^(1/2)-1/8*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(
d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)*2^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6725, 1461, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{c^2d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\right.}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\left(ad^4+bc^4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) - (
c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 +
a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*
d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c
^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1461

```
Int[((A_) + (B_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*
(x_)^(n2_))^(p_.), x_Symbol] :> Dist[A, Int[(d + e*x^n)^q*(a + c*x^(2*n))^p
, x], x] + Dist[B, Int[x^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ
[{a, c, d, e, A, B, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[m - n + 1, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx = \int \left(\frac{c^2d^2}{\left(bc^4+ad^4\right)(c+dx)} + \frac{(c-dx)\left(-ad^2+bc^2x^2\right)}{\left(bc^4+ad^4\right)\left(a+bx^4\right)}\right)dx$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\int \frac{(c-dx)\left(-ad^2+bc^2x^2\right)}{a+bx^4}\,dx}{bc^4+ad^4}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{c\int \frac{-ad^2+bc^2x^2}{a+bx^4}\,dx}{bc^4+ad^4} - \frac{d\int \frac{x\left(-ad^2+bc^2x^2\right)}{a+bx^4}\,dx}{bc^4+ad^4}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{d\,\mathrm{Subst}\left(\int \frac{-ad^2+bc^2x}{a+bx^2}\,dx, x, x^2\right)}{2\left(bc^4+ad^4\right)} + \frac{\left(c\left(c^2-\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\right)\int \frac{\sqrt{a}\,\sqrt{b}+bx^2}{a+bx^4}\,dx}{2\left(bc^4+ad^4\right)} - $$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{\left(bc^2d\right)\mathrm{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right)}{2\left(bc^4+ad^4\right)} + \frac{\left(ad^3\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2\left(bc^4+ad^4\right)} + $$

$$= \frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,\left(bc^4+ad^4\right)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\sqrt[4]{b}\,c\left(c^2+\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{\right.}{4\sqrt{2}\,\sqrt[4]{a}\,\left(bc^4+ad^4\right)}$$

$$= \frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,\left(bc^4+ad^4\right)} - \frac{\sqrt[4]{b}\,c\left(c^2-\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\left(bc^4+ad^4\right)} + \frac{\sqrt[4]{b}\,c\left(c^2-\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\tan^{-1}\left(\right.}{2\sqrt{2}\,\sqrt[4]{a}\,\left(bc^4+\right.}$$

Mathematica [A] time = 0.23, size = 370, normalized size = 0.89

$$-2\left(2a^{3/4}d^3-\sqrt{2}\,\sqrt{a}\,\sqrt[4]{b}\,cd^2+\sqrt{2}\,b^{3/4}c^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(-2a^{3/4}d^3-\sqrt{2}\,\sqrt{a}\,\sqrt[4]{b}\,cd^2+\sqrt{2}\,b^{3/4}c^3\right)\tan^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*S
qrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c^2 +
 Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt
[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sq
```

```
rt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4]))/(8*a^(1/4)*Sqrt[b]*(b*c^4 + a*d^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.57, size = 401, normalized size = 0.96

$$\frac{c^2d^2\log\left(|dx+c|\right)}{bc^4d+ad^5}-\frac{c^2d\log\left(\left|bx^4+a\right|\right)}{4\left(bc^4+ad^4\right)}+\frac{\left(\sqrt{2}\sqrt{ab}\,bd+\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^2d^2+\sqrt{2}\sqrt{ab}\,b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}cd\right)}+\frac{\left(\sqrt{2}\sqrt{ab}\,bd+\left(ab^3\right)\right.}{2\left(\sqrt{2}\,ab^2d^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] c^2*d^2*log(abs(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*log(abs(b*x^4 + a))
/(b*c^4 + a*d^4) + 1/2*(sqrt(2)*sqrt(a*b)*b*d + (a*b^3)^(1/4)*b*c)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^2*d^2 + sq
rt(2)*sqrt(a*b)*b^2*c^2 - 2*(a*b^3)^(3/4)*c*d) + 1/2*(sqrt(2)*sqrt(a*b)*b*d
 + (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^
(1/4))/(sqrt(2)*a*b^2*d^2 + sqrt(2)*sqrt(a*b)*b^2*c^2 + 2*(a*b^3)^(3/4)*c*d
) - 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4) + 1/4*((a
*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
+ sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4)
```

maple [A] time = 0.01, size = 422, normalized size = 1.01

$$\frac{a\,d^3\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\left(a\,d^4+b\,c^4\right)\sqrt{ab}}+\frac{\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,d^4+b\,c^4\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,d^4+b\,c^4\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,c^3\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(a\,d^4+b\,c^4\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{c^2d\ln\left(\right.}{4\left(a\,d^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^4+a),x)
```

```
[Out] c^2*d*ln(d*x+c)/(a*d^4+b*c^4)-1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/8/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*
ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2)))-1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)+1/2/(a*d^4+b*c^4)*a*d^3/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8/(
a*d^4+b*c^4)*c^3/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4/(a*d^4+b*c^4)*c^3/(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a*d^4+b*c^4)*c^3/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/4*c^2*d*ln(b*x^4+a)/(a*d^4+b*
c^4)
```

maxima [A] time = 1.48, size = 349, normalized size = 0.84

$$\frac{c^2 d \log(dx+c)}{bc^4+ad^4} - \frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}c^2 d+\sqrt{a}\,b^{\frac{3}{2}}c^3+abcd^2\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}c^2 d-\sqrt{a}\,b^{\frac{3}{2}}c^3-abcd^2\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{a^{\frac{3}{4}}b^{\frac{5}{4}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] c^2*d*log(d*x + c)/(b*c^4 + a*d^4) - 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*
c^2*d + sqrt(a)*b^(3/2)*c^3 + a*b*c*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*
b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c
^2*d - sqrt(a)*b^(3/2)*c^3 - a*b*c*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b
^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sq
rt(2)*a^(5/4)*b^(5/4)*c*d^2 - 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)
*sqrt(b))*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sqrt(2)*a^(5/4)*b^(5/
4)*c*d^2 + 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))
/(b*c^4 + a*d^4)
```

mupad [B] time = 2.45, size = 823, normalized size = 1.97

$$\left(\sum_{k=1}^{4} \ln\left(a\,b^2\,d\,\left(c\,d+d^2\,x-\mathrm{root}\left(256\,a^2\,b^2\,d^4\,z^4+256\,a\,b^3\,c^4\,z^4+256\,a\,b^2\,c^2\,d\,z^3+32\,a\,b\,d^2\,z^2+1,z,k\right)\,b\,c^3+\mathrm{ro}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)*(c + d*x)),x)
```

```
[Out] symsum(log(a*b^2*d*(c*d + d^2*x - root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*
z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^3 + 4*root(256*a^
2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 +
1, z, k)^2*b^2*c^4*x + 36*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 25
6*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*d^4*x - 128*root(256*a^
2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 +
1, z, k)^4*a*b^3*c^5*d - 5*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 2
56*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^2*d*x + 96*root(256*a^2*
b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1,
 z, k)^3*a*b^2*c^3*d^2 + 384*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 +
 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*c*d^5 + 320*root
(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2
*z^2 + 1, z, k)^4*a^2*b^2*d^6*x + 32*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c
^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*c*d^3 + 160*
root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b
*d^2*z^2 + 1, z, k)^3*a*b^2*c^2*d^3*x - 192*root(256*a^2*b^2*d^4*z^4 + 256*
a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a*b^3*c^4
*d^2*x))*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3
 + 32*a*b*d^2*z^2 + 1, z, k), k, 1, 4) + (c^2*d*log(c + d*x))/(a*d^4 + b*c^
4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.343 $\int \frac{x}{(1-x)(1+x)^2} \, dx$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)} + \frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2/(1+x)+1/2*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {77, 207}

$$\frac{1}{2(x+1)} + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTanh[x]/2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1-x)(1+x)^2}\,dx &= \int \left(-\frac{1}{2(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}\right)dx\\ &= \frac{1}{2(1+x)}-\frac{1}{2}\int \frac{1}{-1+x^2}\,dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.50

$$\frac{1}{4}\left(\frac{2}{x+1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x)*(1 + x)^2),x]
[Out] (2/(1 + x) - Log[1 - x] + Log[1 + x])/4
```

fricas [B] time = 0.67, size = 26, normalized size = 1.62

$$\frac{(x+1)\log(x+1)-(x+1)\log(x-1)+2}{4\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")
[Out] 1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)
```

giac [A] time = 0.24, size = 21, normalized size = 1.31

$$\frac{1}{2\,(x+1)}-\frac{1}{4}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")
[Out] 1/2/(x + 1) - 1/4*log(abs(-2/(x + 1) + 1))
```

maple [A] time = 0.01, size = 21, normalized size = 1.31

$$-\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x)/(x+1)^2,x)
```

```
[Out] -1/4*ln(x-1)+1/2/(x+1)+1/4*ln(x+1)
```

maxima [A] time = 0.75, size = 20, normalized size = 1.25

$$\frac{1}{2\,(x+1)}+\frac{1}{4}\,\log\left(x+1\right)-\frac{1}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.75

$$\frac{\operatorname{atanh}(x)}{2}+\frac{1}{2\,\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x - 1)*(x + 1)^2),x)
```

```
[Out] atanh(x)/2 + 1/(2*(x + 1))
```

sympy [A] time = 0.10, size = 19, normalized size = 1.19

$$-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)**2,x)
```

```
[Out] -log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)
```

3.344 $\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx$

Optimal. Leaf size=19

$$\frac{1}{4}\tanh^{-1}(x)-\frac{x}{4(x^2+1)}$$

```
[Out] -1/4*x/(x^2+1)+1/4*arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {471, 206}

$$\frac{1}{4}\tanh^{-1}(x)-\frac{x}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

```
[Out] -x/(4*(1 + x^2)) + ArcTanh[x]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{x}{4(1+x^2)} + \frac{1}{4}\int \frac{1}{1-x^2}\,dx$$
$$= -\frac{x}{4(1+x^2)} + \frac{1}{4}\tanh^{-1}(x)$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.42

$$\frac{1}{8}\left(-\frac{2x}{x^2+1} - \log(1-x) + \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*(1 + x^2)^2),x]
[Out] ((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8
```

fricas [B] time = 0.48, size = 34, normalized size = 1.79

$$\frac{(x^2+1)\log(x+1) - (x^2+1)\log(x-1) - 2x}{8(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)
```

giac [A] time = 0.30, size = 30, normalized size = 1.58

$$-\frac{1}{4\left(x+\frac{1}{x}\right)} + \frac{1}{16}\log\left(\left|x+\frac{1}{x}+2\right|\right) - \frac{1}{16}\log\left(\left|x+\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))
```

maple [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{x}{4(x^2+1)} - \frac{\ln(x-1)}{8} + \frac{\ln(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^2+1)^2,x)
```

```
[Out] -1/8*ln(x-1)-1/4/(x^2+1)*x+1/8*ln(x+1)
```

maxima [A] time = 0.65, size = 23, normalized size = 1.21

$$-\frac{x}{4\left(x^2+1\right)}+\frac{1}{8}\log\left(x+1\right)-\frac{1}{8}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

mupad [B] time = 2.14, size = 17, normalized size = 0.89

$$\frac{\operatorname{atanh}(x)}{4}-\frac{x}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(x^2 + 1)^2),x)
```

```
[Out] atanh(x)/4 - x/(4*(x^2 + 1))
```

sympy [A] time = 0.12, size = 20, normalized size = 1.05

$$-\frac{x}{4x^2+4}-\frac{\log\left(x-1\right)}{8}+\frac{\log\left(x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**2+1)**2,x)
```

```
[Out] -x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8
```

3.345 $\int \frac{x^3}{(1-x^3)(1+x^3)^2} \, dx$

Optimal. Leaf size=97

$$-\frac{x}{6(x^3+1)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -1/6*x/(x^3+1)-1/12*ln(1-x)-1/36*ln(1+x)+1/72*ln(x^2-x+1)+1/24*ln(x^2+x+1)+
1/36*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {471, 522, 200, 31, 634, 618, 204, 628}

$$-\frac{x}{6(x^3+1)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] -x/(6*(1 + x^3)) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTan[(1 + 2*x
)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36 + Log[1 - x + x^2]/7
2 + Log[1 + x + x^2]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{x}{6(1+x^3)} + \frac{1}{6}\int \frac{1+2x^3}{(1-x^3)(1+x^3)}\,dx$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\int \frac{1}{1+x^3}\,dx + \frac{1}{4}\int \frac{1}{1-x^3}\,dx$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{36}\int \frac{1}{1+x}\,dx - \frac{1}{36}\int \frac{2-x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1}{1-x}\,dx + \frac{1}{12}\int \frac{2+x}{1+x+}$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{24}\int \frac{1}{1-x+x^2}$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+x^2\right) + \frac{1}{24}\log\left(1+x+\right.$$
$$= -\frac{x}{6(1+x^3)} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.88

$$\frac{1}{72}\left(-\frac{12x}{x^3+1} + \log\left(x^2-x+1\right) + 3\log\left(x^2+x+1\right) - 6\log(1-x) - 2\log(x+1) - 2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 6\sqrt{3}\ \text{ta}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] ((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcTa
n[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + 3*L
og[1 + x + x^2])/72
```

fricas [A] time = 0.70, size = 106, normalized size = 1.09

$$\frac{6\sqrt{3}\left(x^3+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2\sqrt{3}\left(x^3+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 3\left(x^3+1\right)\log\left(x^2+x+1\right) + \left(x^3\right.}{72\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(x^3 +
1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x^3 + 1)*log(x^2 + x + 1) + (x^3 + 1)
```

```
*log(x^2 - x + 1) - 2*(x^3 + 1)*log(x + 1) - 6*(x^3 + 1)*log(x - 1) - 12*x)
/(x^3 + 1)
```

giac [A] time = 0.31, size = 77, normalized size = 0.79

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x}{6\left(x^3+1\right)}+\frac{1}{24}\log\left(x^2+x+1\right)+\frac{1}{72}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(abs(x + 1)) - 1/12*log(abs(x - 1))
```

maple [A] time = 0.01, size = 90, normalized size = 0.93

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x-1)}{12}-\frac{\ln(x+1)}{36}+\frac{\ln\left(x^2-x+1\right)}{72}+\frac{\ln\left(x^2+x+1\right)}{24}+\frac{}{36x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)/(x^3+1)^2,x)
```

```
[Out] -1/12*ln(x-1)+1/36*(-2*x-2)/(x^2-x+1)+1/72*ln(x^2-x+1)-1/36*3^(1/2)*arctan(
1/3*(2*x-1)*3^(1/2))+1/18/(x+1)-1/36*ln(x+1)+1/24*ln(x^2+x+1)+1/12*3^(1/2)*
arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.93, size = 75, normalized size = 0.77

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x}{6\left(x^3+1\right)}+\frac{1}{24}\log\left(x^2+x+1\right)+\frac{1}{72}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(x + 1) - 1/12*log(x - 1)
```

mupad [B] time = 0.18, size = 103, normalized size = 1.06

$$-\frac{\ln(x-1)}{12}-\frac{\ln(x+1)}{36}-\frac{x}{6\left(x^3+1\right)}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)+\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/((x^3 - 1)*(x^3 + 1)^2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x + 1)/36 - x/
(6*(x^3 + 1)) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 - 1/24) - lo
g(x - 1)/12 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 + 1/72) - log(
x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 - 1/72)
```

sympy [A] time = 0.36, size = 92, normalized size = 0.95

$$-\frac{x}{6x^3+6}-\frac{\log(x-1)}{12}-\frac{\log(x+1)}{36}+\frac{\log\left(x^2-x+1\right)}{72}+\frac{\log\left(x^2+x+1\right)}{24}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)/(x**3+1)**2,x)
```

```
[Out] -x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x + 1)/72 + lo
g(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/36 + sqrt(3)*a
tan(2*sqrt(3)*x/3 + sqrt(3)/3)/12
```

3.346 $\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

```
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

Rubi [A] time = 0.11, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6725, 203, 260}

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{9+x+3x^2+x^3}{\left(1+x^2\right)\left(3+x^2\right)}\,dx &= \int \left(\frac{3}{1+x^2}+\frac{x}{3+x^2}\right)dx\\ &= 3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{3+x^2}\,dx\\ &= 3\tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

fricas [A] time = 0.63, size = 13, normalized size = 0.87

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="fricas")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

giac [A] time = 0.29, size = 13, normalized size = 0.87

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="giac")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$3\arctan(x)+\frac{\ln\left(x^2+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x)
```

```
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

maxima [A] time = 1.40, size = 13, normalized size = 0.87

$$3 \arctan(x) + \frac{1}{2} \log\left(x^2 + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="maxima")
```

```
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

mupad [B] time = 2.12, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^2+3\right)}{2} + 3\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + x^3 + 9)/((x^2 + 1)*(x^2 + 3)),x)
```

```
[Out] log(x^2 + 3)/2 + 3*atan(x)
```

sympy [A] time = 0.12, size = 12, normalized size = 0.80

$$\frac{\log\left(x^2+3\right)}{2} + 3\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + 3*atan(x)
```

3.347 $\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

```
[Out] arctan(x)+1/2*ln(x^2+3)
```

Rubi [A] time = 0.09, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6725, 203, 260}

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)}\,dx = \int \left(\frac{1}{1+x^2}+\frac{x}{3+x^2}\right)dx$$
$$= \int \frac{1}{1+x^2}\,dx + \int \frac{x}{3+x^2}\,dx$$
$$= \tan^{-1}(x) + \frac{1}{2}\log\left(3+x^2\right)$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
[Out] ArcTan[x] + Log[3 + x^2]/2
```

fricas [A] time = 0.55, size = 11, normalized size = 0.85

$$\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="fricas")
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

giac [A] time = 0.30, size = 11, normalized size = 0.85

$$\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="giac")
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\arctan(x)+\frac{\ln\left(x^2+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x)
```

```
[Out] arctan(x)+1/2*ln(x^2+3)
```

maxima [A] time = 1.33, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log\left(x^2 + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="maxima")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln\left(x^2 + 3\right)}{2} + \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 3)/((x^2 + 1)*(x^2 + 3)),x)
```

```
[Out] log(x^2 + 3)/2 + atan(x)
```

sympy [A] time = 0.12, size = 10, normalized size = 0.77

$$\frac{\log\left(x^2 + 3\right)}{2} + \mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + atan(x)
```

3.348 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=29

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6725, 635, 203, 260}

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-4+6x-x^2+3x^3}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= \int \left(\frac{3(-1+x)}{1+x^2}+\frac{2}{2+x^2}\right)dx\\
&= 2\int \frac{1}{2+x^2}\,dx+3\int \frac{-1+x}{1+x^2}\,dx\\
&= \sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-3\int \frac{1}{1+x^2}\,dx+3\int \frac{x}{1+x^2}\,dx\\
&= -3\tan^{-1}(x)+\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

fricas [A] time = 0.57, size = 24, normalized size = 0.83

$$\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-3\,\arctan(x)+\frac{3}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

giac [A] time = 0.26, size = 24, normalized size = 0.83

$$\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-3\,\arctan(x)+\frac{3}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 25, normalized size = 0.86

$$-3\arctan(x)+\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)+\frac{3\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x)
```

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.48, size = 24, normalized size = 0.83

$$\sqrt{2}\ \arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-3\ \arctan(x)+\frac{3}{2}\ \log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

mupad [B] time = 2.15, size = 51, normalized size = 1.76

$$-\sqrt{2}\ \mathrm{atan}\left(\frac{24\,\sqrt{2}}{24\,x-64}+\frac{32\,\sqrt{2}\ x}{24\,x-64}\right)+\ln\left(x-\mathrm{i}\right)\,\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(1
/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))
```

sympy [A] time = 0.19, size = 29, normalized size = 1.00

$$\frac{3\log\left(x^{2}+1\right)}{2}-3\,\mathrm{atan}\,(x)+\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

3.349 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

```
[Out] 1/(2-x)-arctan(-2+x)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {27, 693, 618, 204}

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
```

```
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx &= \int \frac{1}{(-2+x)^2(5-4x+x^2)}\,dx\\ &= \frac{1}{2-x}-\int \frac{1}{5-4x+x^2}\,dx\\ &= \frac{1}{2-x}+2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,-4+2x\right)\\ &= \frac{1}{2-x}+\tan^{-1}(2-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\tan^{-1}(2-x)-\frac{1}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

fricas [A] time = 0.57, size = 17, normalized size = 1.21

$$-\frac{(x-2)\arctan(x-2)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$-\frac{1}{x-2}-\arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
[Out] -1/(x - 2) - arctan(x - 2)
```

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$-\arctan(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x)
[Out] -arctan(x-2)-1/(x-2)
```

maxima [A] time = 2.07, size = 14, normalized size = 1.00

$$-\frac{1}{x-2}-\arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
[Out] -1/(x - 2) - arctan(x - 2)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.00

$$-\mathrm{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
[Out] - atan(x - 2) - 1/(x - 2)
```

sympy [A] time = 0.14, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
[Out] -atan(x - 2) - 1/(x - 2)
```

3.350 $\int \frac{-3+x+x^2}{(-3+x)x^2} \, dx$

Optimal. Leaf size=12

$$\log(3-x) - \frac{1}{x}$$

```
[Out] -1/x+ln(3-x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {893}

$$\log(3-x) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned} \int \frac{-3+x+x^2}{(-3+x)x^2} \, dx &= \int \left(\frac{1}{-3+x} + \frac{1}{x^2}\right) dx \\ &= -\frac{1}{x} + \log(3-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(3-x) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x + x^2)/((-3 + x)*x^2),x]
[Out] -x^(-1) + Log[3 - x]
```

fricas [A] time = 0.60, size = 12, normalized size = 1.00

$$\frac{x\log(x-3)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")
[Out] (x*log(x - 3) - 1)/x
```

giac [A] time = 0.42, size = 11, normalized size = 0.92

$$-\frac{1}{x}+\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")
[Out] -1/x + log(abs(x - 3))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\ln(x-3)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-3)/(x-3)/x^2,x)
[Out] -1/x+ln(x-3)
```

maxima [A] time = 0.75, size = 10, normalized size = 0.83

$$-\frac{1}{x}+\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")
[Out] -1/x + log(x - 3)
```

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$\ln(x-3)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 3)/(x^2*(x - 3)),x)
[Out] log(x - 3) - 1/x
```

sympy [A] time = 0.09, size = 7, normalized size = 0.58

$$\log(x-3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-3)/(-3+x)/x**2,x)
[Out] log(x - 3) - 1/x
```

3.351 $\int \frac{1+x+4x^2}{x+4x^3} \, dx$

Optimal. Leaf size=11

$$\log(x) + \frac{1}{2}\tan^{-1}(2x)$$

```
[Out] 1/2*arctan(2*x)+ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1593, 1802, 203}

$$\log(x) + \frac{1}{2}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

```
[Out] ArcTan[2*x]/2 + Log[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \int \frac{1+x+4x^2}{x\left(1+4x^2\right)}\,dx$$
$$= \int \left(\frac{1}{x}+\frac{1}{1+4x^2}\right)dx$$
$$= \log(x) + \int \frac{1}{1+4x^2}\,dx$$
$$= \frac{1}{2}\tan^{-1}(2x) + \log(x)$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x) + \frac{1}{2}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]
[Out] ArcTan[2*x]/2 + Log[x]
```

fricas [A] time = 0.65, size = 9, normalized size = 0.82

$$\frac{1}{2}\arctan(2x) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="fricas")
[Out] 1/2*arctan(2*x) + log(x)
```

giac [A] time = 0.29, size = 10, normalized size = 0.91

$$\frac{1}{2}\arctan(2x) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")
[Out] 1/2*arctan(2*x) + log(abs(x))
```

maple [A] time = 0.01, size = 10, normalized size = 0.91

$$\frac{\arctan(2x)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(4*x^3+x),x)
```

```
[Out] 1/2*arctan(2*x)+ln(x)
```

maxima [A] time = 1.61, size = 9, normalized size = 0.82

$$\frac{1}{2}\arctan\left(2\,x\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x) + log(x)
```

mupad [B] time = 2.17, size = 17, normalized size = 1.55

$$\ln(x)-\frac{\mathrm{atan}\left(\frac{17}{32\left(\frac{x}{16}-\frac{1}{8}\right)}+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^2 + 1)/(x + 4*x^3),x)
```

```
[Out] log(x) - atan(17/(32*(x/16 - 1/8)) + 4)/2
```

sympy [A] time = 0.13, size = 8, normalized size = 0.73

$$\log\left(x\right)+\frac{\mathrm{atan}\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(4*x**3+x),x)
```

```
[Out] log(x) + atan(2*x)/2
```

3.352 $\int \frac{1-x+3x^2}{-x^2+x^3}\,dx$

Optimal. Leaf size=12

$$\frac{1}{x}+3\log(1-x)$$

[Out] 1/x+3*ln(1-x)

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 893}

$$\frac{1}{x}+3\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x + 3*x^2)/(-x^2 + x^3),x]

[Out] x^(-1) + 3*Log[1 - x]

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x+3x^2}{-x^2+x^3}\,dx &= \int \frac{1-x+3x^2}{(-1+x)x^2}\,dx\\ &= \int \left(\frac{3}{-1+x}-\frac{1}{x^2}\right)dx\\ &= \frac{1}{x}+3\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{x}+3\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + 3*Log[1 - x]
```

fricas [A] time = 0.52, size = 13, normalized size = 1.08

$$\frac{3\,x\log\left(x-1\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x - 1) + 1)/x
```

giac [A] time = 0.38, size = 11, normalized size = 0.92

$$\frac{1}{x}+3\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="giac")
```

```
[Out] 1/x + 3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$3\ln\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^3-x^2),x)
```

```
[Out] 3*ln(x-1)+1/x
```

maxima [A] time = 0.71, size = 10, normalized size = 0.83

$$\frac{1}{x}+3\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")
[Out] 1/x + 3*log(x - 1)
```

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$3\ \ln(x-1)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 - x + 1)/(x^2 - x^3),x)
[Out] 3*log(x - 1) + 1/x
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$3\log(x-1)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**3-x**2),x)
[Out] 3*log(x - 1) + 1/x
```

3.353 $\int \frac{4+3x+x^2}{x+x^2} dx$

Optimal. Leaf size=12

$$x + 4\log(x) - 2\log(x+1)$$

[Out] x+4*ln(x)-2*ln(1+x)

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1593, 893}

$$x + 4\log(x) - 2\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*x + x^2)/(x + x^2),x]

[Out] x + 4*Log[x] - 2*Log[1 + x]

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+3x+x^2}{x+x^2}\,dx &= \int \frac{4+3x+x^2}{x(1+x)}\,dx \\
&= \int \left(1+\frac{4}{x}-\frac{2}{1+x}\right) dx \\
&= x + 4\log(x) - 2\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + 4\log(x) - 2\log(x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x + x^2)/(x + x^2),x]
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

fricas [A] time = 0.56, size = 12, normalized size = 1.00

$$x - 2\log(x + 1) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")
[Out] x - 2*log(x + 1) + 4*log(x)
```

giac [A] time = 0.23, size = 14, normalized size = 1.17

$$x - 2\log(|x + 1|) + 4\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")
[Out] x - 2*log(abs(x + 1)) + 4*log(abs(x))
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$x + 4\ln(x) - 2\ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x+4)/(x^2+x),x)
[Out] x+4*ln(x)-2*ln(x+1)
```

maxima [A] time = 0.60, size = 12, normalized size = 1.00

$$x - 2\log(x + 1) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")
[Out] x - 2*log(x + 1) + 4*log(x)
```

mupad [B] time = 0.05, size = 12, normalized size = 1.00

$$x - 2\ \ln(x+1) + 4\ \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 + 4)/(x + x^2),x)
[Out] x - 2*log(x + 1) + 4*log(x)
```

sympy [A] time = 0.10, size = 12, normalized size = 1.00

$$x + 4\log(x) - 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(x**2+x),x)
[Out] x + 4*log(x) - 2*log(x + 1)
```

3.354 $\int \frac{4+x+3x^2}{x+x^3} \, dx$

Optimal. Leaf size=17

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

```
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1593, 1802, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + 3*x^2)/(x + x^3),x]
```

```
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x+3x^2}{x+x^3}\,dx &= \int \frac{4+x+3x^2}{x\left(1+x^2\right)}\,dx\\ &= \int \left(\frac{4}{x}+\frac{1-x}{1+x^2}\right)dx\\ &= 4\log(x)+\int \frac{1-x}{1+x^2}\,dx\\ &= 4\log(x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \tan^{-1}(x)+4\log(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + 3*x^2)/(x + x^3),x]
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

fricas [A] time = 0.57, size = 15, normalized size = 0.88

$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="fricas")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

giac [A] time = 0.24, size = 16, normalized size = 0.94

$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+4\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="giac")
```

```
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\arctan(x) + 4\ln(x) - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x+4)/(x^3+x),x)
```

```
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

maxima [A] time = 1.53, size = 15, normalized size = 0.88

$$\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

mupad [B] time = 2.28, size = 23, normalized size = 1.35

$$4\ln(x) + \ln(x - \mathrm{i})\left(-\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) + \ln(x + 1\mathrm{i})\left(-\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 4)/(x + x^3),x)
```

```
[Out] 4*log(x) - log(x + 1i)*(1/2 - 1i/2) - log(x - 1i)*(1/2 + 1i/2)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$4\log(x) - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x+4)/(x**3+x),x)
```

```
[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)
```

3.355 $\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx$

Optimal. Leaf size=13

$$2\log(4x+1)-\tan^{-1}(x)$$

```
[Out] -arctan(x)+2*ln(1+4*x)
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1629, 204}

$$2\log(4x+1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)}\,dx &= \int \left(\frac{8}{1+4x}+\frac{1}{-1-x^2}\right)dx\\ &= 2\log(1+4x)+\int \frac{1}{-1-x^2}\,dx\\ &= -\tan^{-1}(x)+2\log(1+4x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$2\log(4x+1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

fricas [A] time = 0.63, size = 13, normalized size = 1.00

$$-\arctan(x) + 2\log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -arctan(x) + 2*log(4*x + 1)
```

giac [A] time = 0.34, size = 14, normalized size = 1.08

$$-\arctan(x) + 2\log(|4x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] -arctan(x) + 2*log(abs(4*x + 1))
```

maple [A] time = 0.01, size = 14, normalized size = 1.08

$$-\arctan(x) + 2\ln(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x)
```

```
[Out] -arctan(x)+2*ln(1+4*x)
```

maxima [A] time = 1.41, size = 13, normalized size = 1.00

$$-\arctan(x) + 2\log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -arctan(x) + 2*log(4*x + 1)
```

mupad [B] time = 0.06, size = 19, normalized size = 1.46

$$\operatorname{atan}\left(\frac{4x + 1}{x - 4}\right) + 2\ln\left(x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^2 - 4*x + 7)/((4*x + 1)*(x^2 + 1)),x)
```

```
[Out] atan((4*x + 1)/(x - 4)) + 2*log(x + 1/4)
```

sympy [A] time = 0.13, size = 10, normalized size = 0.77

$$2\log\left(x+\frac{1}{4}\right)-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)
```

```
[Out] 2*log(x + 1/4) - atan(x)
```

3.356 $\int \frac{x^2}{(-1+x)(1+2x+x^2)}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

```
[Out] 1/2/(1+x)+1/4*ln(1-x)+3/4*ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {27, 88}

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)}\,dx &= \int \frac{x^2}{(-1+x)(1+x)^2}\,dx\\ &= \int \left(\frac{1}{4(-1+x)}-\frac{1}{2(1+x)^2}+\frac{3}{4(1+x)}\right)dx\\ &= \frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{4}\left(\frac{2}{x+1}+\log(x-1)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
[Out] (2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4
```

fricas [A] time = 0.59, size = 26, normalized size = 0.93

$$\frac{3\,(x+1)\log\,(x+1)+(x+1)\log\,(x-1)+2}{4\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="fricas")
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)
```

giac [A] time = 0.30, size = 22, normalized size = 0.79

$$\frac{1}{2\,(x+1)}+\frac{3}{4}\,\log\,(|x+1|)+\frac{1}{4}\,\log\,(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="giac")
[Out] 1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 21, normalized size = 0.75

$$\frac{\ln\,(x-1)}{4}+\frac{3\ln\,(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x-1)/(x^2+2*x+1),x)
[Out] 1/4*ln(x-1)+1/2/(x+1)+3/4*ln(x+1)
```

maxima [A] time = 0.66, size = 20, normalized size = 0.71

$$\frac{1}{2\,(x+1)}+\frac{3}{4}\,\log\,(x+1)+\frac{1}{4}\,\log\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="maxima")
[Out] 1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)
```

mupad [B] time = 0.07, size = 20, normalized size = 0.71

$$\frac{\ln(x-1)}{4}+\frac{3\ \ln(x+1)}{4}+\frac{1}{2\ (x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)*(2*x + x^2 + 1)),x)
[Out] log(x - 1)/4 + (3*log(x + 1))/4 + 1/(2*(x + 1))
```

sympy [A] time = 0.11, size = 20, normalized size = 0.71

$$\frac{\log(x-1)}{4}+\frac{3\log(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x)/(x**2+2*x+1),x)
[Out] log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)
```

3.357 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

```
[Out] -9/32/(1-2*x)+41/128*ln(1-2*x)-25/128*ln(3+2*x)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {893}

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx &= \int \left(-\frac{9}{16(-1+2x)^2}+\frac{41}{64(-1+2x)}-\frac{25}{64(3+2x)}\right)dx\\ &= -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{9}{32(2x-1)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

fricas [A] time = 0.53, size = 37, normalized size = 1.16

$$-\frac{25 (2 x-1) \log (2 x+3)-41 (2 x-1) \log (2 x-1)-36}{128 (2 x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x -
1)
```

giac [A] time = 0.30, size = 43, normalized size = 1.34

$$\frac{9}{32 (2 x-1)}-\frac{1}{8} \log \left(\frac{|2 x-1|}{2 (2 x-1)^2}\right)-\frac{25}{128} \log \left(\left|-\frac{4}{2 x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
```

```
[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/
(2*x - 1) - 1))
```

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{41 \ln (2x-1)}{128}-\frac{25 \ln (2x+3)}{128}+\frac{9}{32 (2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x)
```

```
[Out] 9/32/(2*x-1)+41/128*ln(2*x-1)-25/128*ln(3+2*x)
```

maxima [A] time = 0.75, size = 26, normalized size = 0.81

$$\frac{9}{32 (2 x-1)}-\frac{25}{128} \log (2 x+3)+\frac{41}{128} \log (2 x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
```

```
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

mupad [B] time = 2.23, size = 22, normalized size = 0.69

$$\frac{41\,\ln\left(x-\frac{1}{2}\right)}{128}-\frac{25\,\ln\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64\,\left(x-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)
```

```
[Out] (41*log(x - 1/2))/128 - (25*log(x + 3/2))/128 + 9/(64*(x - 1/2))
```

sympy [A] time = 0.13, size = 26, normalized size = 0.81

$$\frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
```

```
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

3.358 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

```
[Out] -3*arctan(x)+2*ln(1-x)+1/2*ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1629, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{-1+x}+\frac{-3+x}{1+x^2}\right)dx \\
&= 2\log(1-x)+\int \frac{-3+x}{1+x^2}\,dx \\
&= 2\log(1-x)-3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx \\
&= -3\tan^{-1}(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.22

$$\frac{1}{2}\log\left((x-1)^2+2(x-1)+2\right)+2\log(x-1)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

fricas [A] time = 0.71, size = 19, normalized size = 0.83

$$-3\ \arctan(x)+\frac{1}{2}\ \log\left(x^2+1\right)+2\ \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

giac [A] time = 0.31, size = 20, normalized size = 0.87

$$-3\ \arctan(x)+\frac{1}{2}\ \log\left(x^2+1\right)+2\ \log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-3\arctan(x)+2\ln\left(x-1\right)+\frac{\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-4*x+5)/(x-1)/(x^2+1),x)
[Out] 2*ln(x-1)+1/2*ln(x^2+1)-3*arctan(x)
```

maxima [A] time = 1.56, size = 19, normalized size = 0.83

$$-3\ \arctan(x)+\frac{1}{2}\ \log\left(x^{2}+1\right)+2\ \log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

mupad [B] time = 0.05, size = 25, normalized size = 1.09

$$2\ \ln\left(x-1\right)+\ln\left(x-\mathrm{i}\right)\left(\frac{1}{2}+\frac{3}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(\frac{1}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)
```

sympy [A] time = 0.14, size = 19, normalized size = 0.83

$$2\log\left(x-1\right)+\frac{\log\left(x^{2}+1\right)}{2}-3\,\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

3.359 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)} \, dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

```
[Out] 1/(-1+x)+arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1629, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\frac{1-x}{1+x^2}\right)dx\\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1-x}{1+x^2}\,dx\\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{-1+x}+\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$
-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(x-1)+\tan^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

fricas [A] time = 0.57, size = 36, normalized size = 1.50

$$
\frac{2\,(x-1)\arctan(x)-(x-1)\log\left(x^2+1\right)+2\,(x-1)\log\left(x-1\right)+2}{2\,(x-1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)
/(x - 1)
```

giac [B] time = 0.30, size = 47, normalized size = 1.96

$$
\frac{1}{4}\,\pi-\pi\left\lfloor\frac{\pi+4\,\arctan(x)}{4\,\pi}+\frac{1}{2}\right\rfloor+\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\,\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
- 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x) + \ln(x-1) - \frac{\ln(x^2+1)}{2} + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-1)/(x-1)^2/(x^2+1),x)
```

```
[Out] ln(x-1)+1/(x-1)-1/2*ln(x^2+1)+arctan(x)
```

maxima [A] time = 1.60, size = 20, normalized size = 0.83

$$\frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log(x^2+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

mupad [B] time = 2.11, size = 28, normalized size = 1.17

$$\ln(x-1) + \frac{1}{x-1} + \ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)
```

```
[Out] log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x - 1
)
```

sympy [A] time = 0.14, size = 20, normalized size = 0.83

$$\log(x-1) - \frac{\log(x^2+1)}{2} + \operatorname{atan}(x) + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

3.360 $\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx$

Optimal. Leaf size=49

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

```
[Out] 261/221*arctan(-1+2*x)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442*
ln(2*x^2-2*x+1)
```

Rubi [A] time = 0.14, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6728, 634, 618, 204, 628, 617}

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{5+x^3}{\left(10-6x+x^2\right)\left(\frac{1}{2}-x+x^2\right)}\,dx = \int \left(\frac{2(345+56x)}{221\left(10-6x+x^2\right)}+\frac{2(76+109x)}{221\left(1-2x+2x^2\right)}\right)dx$$

$$= \frac{2}{221}\int \frac{345+56x}{10-6x+x^2}\,dx+\frac{2}{221}\int \frac{76+109x}{1-2x+2x^2}\,dx$$

$$= \frac{109}{442}\int \frac{-2+4x}{1-2x+2x^2}\,dx+\frac{56}{221}\int \frac{-6+2x}{10-6x+x^2}\,dx+\frac{261}{221}\int \frac{1}{1-2x+2x^2}\,dx$$

$$= \frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)+\frac{261}{221}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,d$$

$$= -\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

fricas [A] time = 0.66, size = 37, normalized size = 0.76

$$\frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(x^2-x+\frac{1}{2}\right)+\frac{56}{221}\log\left(x^2-6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="fricas")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(x^2 - x + 1/
2) + 56/221*log(x^2 - 6*x + 10)
```

giac [A] time = 0.28, size = 39, normalized size = 0.80

$$\frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)+\frac{56}{221}\log\left(x^2-6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="giac")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

maple [A] time = 0.01, size = 40, normalized size = 0.82

$$\frac{1026\arctan\left(x-3\right)}{221}+\frac{261\arctan\left(2x-1\right)}{221}+\frac{56\ln\left(x^2-6x+10\right)}{221}+\frac{109\ln\left(2x^2-2x+1\right)}{442}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x)
```

```
[Out] 261/221*arctan(2*x-1)+1026/221*arctan(x-3)+56/221*ln(x^2-6*x+10)+109/442*ln
(2*x^2-2*x+1)
```

maxima [A] time = 1.71, size = 39, normalized size = 0.80

$$\frac{261}{221}\arctan\left(2x-1\right)+\frac{1026}{221}\arctan\left(x-3\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)+\frac{56}{221}\log\left(x^2-6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

mupad [B] time = 2.16, size = 41, normalized size = 0.84

$$\ln(x-3-\mathrm{i})\left(\frac{56}{221}-\frac{513}{221}\mathrm{i}\right)+\ln(x-3+1\mathrm{i})\left(\frac{56}{221}+\frac{513}{221}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}-\frac{261}{442}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 5)/((x^2 - x + 1/2)*(x^2 - 6*x + 10)),x)
```

```
[Out] log(x - (3 + 1i))*(56/221 - 513i/221) + log(x - (3 - 1i))*(56/221 + 513i/22
1) + log(x - (1/2 + 1i/2))*(109/442 - 261i/442) + log(x - (1/2 - 1i/2))*(10
9/442 + 261i/442)
```

sympy [A] time = 0.22, size = 44, normalized size = 0.90

$$\frac{56\log\left(x^{2}-6x+10\right)}{221}+\frac{109\log\left(x^{2}-x+\frac{1}{2}\right)}{442}+\frac{1026\,\mathrm{atan}\,(x-3)}{221}+\frac{261\,\mathrm{atan}\,(2x-1)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

```
[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x - 3
)/221 + 261*atan(2*x - 1)/221
```

3.361 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} \, dx$

Optimal. Leaf size=25

$$4\log(1-x) - 14\log(2-x) + 11\log(3-x)$$

```
[Out] 4*ln(1-x)-14*ln(2-x)+11*ln(3-x)
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1612}

$$4\log(1-x) - 14\log(2-x) + 11\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned} \int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} \, dx &= \int \left(\frac{11}{-3+x} - \frac{14}{-2+x} + \frac{4}{-1+x}\right) dx \\ &= 4\log(1-x) - 14\log(2-x) + 11\log(3-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.76

$$11\log(x-3) - 14\log(x-2) + 4\log(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] 11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]
```

fricas [A] time = 0.52, size = 19, normalized size = 0.76

$$4 \log(x-1) - 14 \log(x-2) + 11 \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

giac [A] time = 0.39, size = 22, normalized size = 0.88

$$4 \log(|x-1|) - 14 \log(|x-2|) + 11 \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] 4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$11 \ln(x-3) - 14 \ln(x-2) + 4 \ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x+4)/(x-3)/(x-2)/(x-1),x)
[Out] 4*ln(x-1)-14*ln(x-2)+11*ln(x-3)
```

maxima [A] time = 1.07, size = 19, normalized size = 0.76

$$4 \log(x-1) - 14 \log(x-2) + 11 \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

mupad [B] time = 2.14, size = 19, normalized size = 0.76

$$4 \ln(x-1) - 14 \ln(x-2) + 11 \ln(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 + 4)/((x - 1)*(x - 2)*(x - 3)),x)
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

sympy [A] time = 0.14, size = 19, normalized size = 0.76

$$11\log(x-3)-14\log(x-2)+4\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
[Out] 11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

3.362 $\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx$

Optimal. Leaf size=60

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

```
[Out] -79/273/(5+x)+200/3211*ln(3-2*x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+45
1/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6728, 634, 618, 204, 628}

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx &= \int \left(\frac{79}{273(5+x)^2}+\frac{2731}{24843(5+x)}+\frac{400}{3211(-3+2x)}+\frac{-15-481x}{2793\left(1+x+x^2\right)}\right.\\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{\int \frac{-15-481x}{1+x+x^2}\,dx}{2793}\\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{451\int \frac{1}{1+x+x^2}\,dx}{5586}-\\
&= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right.}{5586}\\
&= -\frac{79}{273(5+x)}+\frac{451\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.90

$$\frac{-243867\log\left(x^2+x+1\right)-\frac{819546}{x+5}+176400\log(3-2x)+311334\log(x+5)+152438\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
- 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

fricas [A] time = 0.73, size = 60, normalized size = 1.00

$$\frac{152438\,\sqrt{3}\,(x+5)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right) - 243867\,(x+5)\log\left(x^2+x+1\right) + 176400\,(x+5)\log\left(2\,x-3\right) + 311\ldots}{2832102\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x
 + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x
 + 5) - 819546)/(x + 5)
```

giac [A] time = 0.34, size = 60, normalized size = 1.00

$$\frac{451}{8379}\,\sqrt{3}\,\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\,\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right)+\frac{200}{3211}\,\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5
586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))
```

maple [A] time = 0.01, size = 48, normalized size = 0.80

$$\frac{451\sqrt{3}\;\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{8379}+\frac{200\ln(2x-3)}{3211}+\frac{2731\ln(x+5)}{24843}-\frac{481\ln\left(x^2+x+1\right)}{5586}-\frac{79}{273\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x)
```

```
[Out] -79/273/(5+x)+2731/24843*ln(5+x)+200/3211*ln(-3+2*x)-481/5586*ln(x^2+x+1)+4
51/8379*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.80, size = 47, normalized size = 0.78

$$\frac{451}{8379}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\,\log\left(x^2+x+1\right)+\frac{200}{3211}\,\log\left(2\,x-3\right)+\frac{2731}{24843}\,\log\left(x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*
log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

mupad [B] time = 0.13, size = 61, normalized size = 1.02

$$\frac{200\ \ln\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\ \ln\left(x+5\right)}{24843}-\frac{79}{273\ (x+5)}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{481}{5586}+\frac{\sqrt{3}\ 451\mathrm{i}}{16758}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)
```

```
[Out] (200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log(
x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^(1
/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)
```

sympy [A] time = 0.25, size = 63, normalized size = 1.05

$$\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log\left(x+5\right)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
+ 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

3.363 $\int \frac{-1+x^3}{1+x+x^2} \, dx$

Optimal. Leaf size=11

$$\frac{x^2}{2}-x$$

```
[Out] -x+1/2*x^2
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1586}

$$\frac{x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(1 + x + x^2),x]
```

```
[Out] -x + x^2/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{1+x+x^2} \, dx &= \int (-1+x) \, dx \\ &= -x+\frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(1 + x + x^2),x]
```

```
[Out] -x + x^2/2
```

fricas [**A**] time = 0.73, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="fricas")
[Out] 1/2*x^2 - x
```

giac [**A**] time = 0.23, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="giac")
[Out] 1/2*x^2 - x
```

maple [**A**] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^2+x+1),x)
[Out] -x+1/2*x^2
```

maxima [**A**] time = 0.84, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")
[Out] 1/2*x^2 - x
```

mupad [**B**] time = 0.02, size = 6, normalized size = 0.55

$$\frac{x\,(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x + x^2 + 1),x)
```

```
[Out] (x*(x - 2))/2
```

sympy [A] time = 0.06, size = 5, normalized size = 0.45

$$\frac{x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**2+x+1),x)
```

```
[Out] x**2/2 - x
```

3.364 $\int \frac{-3+x^3}{-7-6x+x^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

```
[Out] 6*x+1/2*x^2+85/2*ln(7-x)+1/2*ln(1+x)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1657, 632, 31}

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-3+x^3}{-7-6x+x^2}\,dx = \int \left(6+x+\frac{39+43x}{-7-6x+x^2}\right) dx$$
$$= 6x+\frac{x^2}{2}+\int \frac{39+43x}{-7-6x+x^2}\,dx$$
$$= 6x+\frac{x^2}{2}+\frac{1}{2}\int \frac{1}{1+x}\,dx+\frac{85}{2}\int \frac{1}{-7+x}\,dx$$
$$= 6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^3)/(-7 - 6*x + x^2),x]
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

fricas [A] time = 0.62, size = 21, normalized size = 0.72

$$\frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log\left(x+1\right)+\frac{85}{2}\,\log\left(x-7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="fricas")
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

giac [A] time = 0.35, size = 23, normalized size = 0.79

$$\frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log\left(|x+1|\right)+\frac{85}{2}\,\log\left(|x-7|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="giac")
[Out] 1/2*x^2 + 6*x + 1/2*log(abs(x + 1)) + 85/2*log(abs(x - 7))
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{x^2}{2}+6x+\frac{\ln\left(x+1\right)}{2}+\frac{85\ln\left(x-7\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3)/(x^2-6*x-7),x)
```

```
[Out] 1/2*x^2+6*x+1/2*ln(x+1)+85/2*ln(x-7)
```

maxima [A] time = 1.10, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2+6x+\frac{1}{2}\log(x+1)+\frac{85}{2}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

mupad [B] time = 2.12, size = 21, normalized size = 0.72

$$6x+\frac{\ln(x+1)}{2}+\frac{85\ln(x-7)}{2}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 3)/(6*x - x^2 + 7),x)
```

```
[Out] 6*x + log(x + 1)/2 + (85*log(x - 7))/2 + x^2/2
```

sympy [A] time = 0.11, size = 22, normalized size = 0.76

$$\frac{x^2}{2}+6x+\frac{85\log(x-7)}{2}+\frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3)/(x**2-6*x-7),x)
```

```
[Out] x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2
```

3.365 $\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

```
[Out] 1/18*(67+47*x)/(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)+1/2*ln(x^2+4*x+13)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1660, 634, 618, 204, 628}

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx &= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{36}\int \frac{-50+36x}{13+4x+x^2}\,dx\\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\int \frac{4+2x}{13+4x+x^2}\,dx-\frac{61}{18}\int \frac{1}{13+4x+x^2}\,dx\\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\log\left(13+4x+x^2\right)+\frac{61}{9}\operatorname{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,4+2x\right)\\
&= \frac{67+47x}{18\left(13+4x+x^2\right)}-\frac{61}{54}\tan^{-1}\left(\frac{2+x}{3}\right)+\frac{1}{2}\log\left(13+4x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.00

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

fricas [A] time = 0.71, size = 52, normalized size = 1.16

$$-\frac{61\left(x^2+4x+13\right)\arctan\left(\frac{1}{3}x+\frac{2}{3}\right)-27\left(x^2+4x+13\right)\log\left(x^2+4x+13\right)-141x-201}{54\left(x^2+4x+13\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="fricas")
```

```
[Out] -1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x + 13)*log(x^
2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)
```

giac [A] time = 0.30, size = 37, normalized size = 0.82

$$\frac{47x+67}{18\left(x^2+4x+13\right)}-\frac{61}{54}\arctan\left(\frac{1}{3}x+\frac{2}{3}\right)+\frac{1}{2}\log\left(x^2+4x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="giac")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

maple [A] time = 0.01, size = 37, normalized size = 0.82

$$-\frac{61\arctan\left(\frac{x}{3}+\frac{2}{3}\right)}{54}+\frac{\ln\left(x^2+4x+13\right)}{2}+\frac{\frac{47x}{18}+\frac{67}{18}}{x^2+4x+13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^2+4*x+13)^2,x)
```

```
[Out] (47/18*x+67/18)/(x^2+4*x+13)+1/2*ln(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)
```

maxima [A] time = 1.84, size = 37, normalized size = 0.82

$$\frac{47x+67}{18\left(x^2+4x+13\right)}-\frac{61}{54}\arctan\left(\frac{1}{3}x+\frac{2}{3}\right)+\frac{1}{2}\log\left(x^2+4x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

mupad [B] time = 0.04, size = 49, normalized size = 1.09

$$\frac{\ln\left(x^2+4\,x+13\right)}{2}-\frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}+\frac{47\,x}{18\,\left(x^2+4\,x+13\right)}+\frac{67}{18\,\left(x^2+4\,x+13\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(4*x + x^2 + 13)^2,x)
```

```
[Out] log(4*x + x^2 + 13)/2 - (61*atan(x/3 + 2/3))/54 + (47*x)/(18*(4*x + x^2 + 1
3)) + 67/(18*(4*x + x^2 + 13))
```

sympy [A] time = 0.14, size = 37, normalized size = 0.82

$$\frac{47x+67}{18x^2+72x+234}+\frac{\log\left(x^2+4x+13\right)}{2}-\frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**2+4*x+13)**2,x)
```

```
[Out] (47*x + 67)/(18*x**2 + 72*x + 234) + log(x**2 + 4*x + 13)/2 - 61*atan(x/3 +
 2/3)/54
```

3.366 $\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} \, dx$

Optimal. Leaf size=32

$$\frac{1}{x^2+4} + \log\left(x^2+4\right) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

Rubi [A] time = 0.25, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {6725, 203, 261, 635, 260}

$$\frac{1}{x^2+4} + \log\left(x^2+4\right) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^
2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx &= \int \left(-\frac{2}{x}+\frac{2}{1+x^2}-\frac{2x}{(4+x^2)^2}+\frac{1+2x}{4+x^2}\right)dx\\ &= -2\log(x)+2\int \frac{1}{1+x^2}\,dx-2\int \frac{x}{(4+x^2)^2}\,dx+\int \frac{1+2x}{4+x^2}\,dx\\ &= \frac{1}{4+x^2}+2\tan^{-1}(x)-2\log(x)+2\int \frac{x}{4+x^2}\,dx+\int \frac{1}{4+x^2}\,dx\\ &= \frac{1}{4+x^2}+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-2\log(x)+\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 +
 x^2)^2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

fricas [A] time = 0.63, size = 52, normalized size = 1.62

$$\frac{(x^2+4)\arctan\left(\frac{1}{2}x\right)+4(x^2+4)\arctan(x)+2(x^2+4)\log\left(x^2+4\right)-4(x^2+4)\log(x)+2}{2(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="fricas")
```

```
[Out] 1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4)*log(x^2
+ 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)
```

giac [A] time = 0.32, size = 29, normalized size = 0.91

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="giac")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(abs(x)
)
```

maple [A] time = 0.01, size = 29, normalized size = 0.91

$$2\arctan(x)+\frac{\arctan\left(\frac{x}{2}\right)}{2}-2\ln(x)+\ln\left(x^2+4\right)+\frac{1}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

maxima [A] time = 2.02, size = 28, normalized size = 0.88

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="maxima")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(x)
```

mupad [B] time = 0.07, size = 44, normalized size = 1.38

$$\frac{1}{x^2+4}-2\ln(x)-2\,\mathrm{atan}\left(\frac{328000}{7\,(36288\,x-19584)}+\frac{34}{63}\right)+\ln\left(x-2\mathrm{i}\right)\,\left(1-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(1+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5 - 32)/(x*(x^2 + 1)*(x^2 + 4)^2
),x)
```

```
[Out] log(x - 2i)*(1 - 1i/4) + log(x + 2i)*(1 + 1i/4) - 2*atan(328000/(7*(36288*x
 - 19584)) + 34/63) - 2*log(x) + 1/(x^2 + 4)
```

sympy [A] time = 0.26, size = 29, normalized size = 0.91

$$-2\log(x) + \log\left(x^2 + 4\right) + \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2} + 2\operatorname{atan}(x) + \frac{1}{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2,x
)
```

```
[Out] -2*log(x) + log(x**2 + 4) + atan(x/2)/2 + 2*atan(x) + 1/(x**2 + 4)
```

3.367 $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$

Optimal. Leaf size=148

$$\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}\,7^{3/4}}$$

```
[Out] 1/2*x^2-1/2*arctanh(x^2)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1
/2)+1/28*arctan(1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1/2)-1/56*ln(x^2-7^(1/4
)*x*2^(1/2)+7^(1/2))*7^(1/4)*2^(1/2)+1/56*ln(x^2+7^(1/4)*x*2^(1/2)+7^(1/2))
*7^(1/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1790, 1403, 211, 1165, 628, 1162, 617, 204, 1584, 1478, 275, 321, 207}

$$\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}\,7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (S
qrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - Sqr
t[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4)*x
 + x^2]/(4*Sqrt[2]*7^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1403

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /;
FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1790

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \int \left(\frac{-1+x^4}{-7+6x^4+x^8}+\frac{x\left(7x^4+x^8\right)}{-7+6x^4+x^8}\right)dx$$
$$= \int \frac{-1+x^4}{-7+6x^4+x^8}\,dx + \int \frac{x\left(7x^4+x^8\right)}{-7+6x^4+x^8}\,dx$$
$$= \int \frac{1}{7+x^4}\,dx + \int \frac{x^5\left(7+x^4\right)}{-7+6x^4+x^8}\,dx$$
$$= \frac{\int \frac{\sqrt{7}-x^2}{7+x^4}\,dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{7}+x^2}{7+x^4}\,dx}{2\sqrt{7}} + \int \frac{x^5}{-1+x^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, x^2\right) - \frac{\int \frac{\sqrt{2}\sqrt[4]{7}+2x}{-\sqrt{7}-\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}\,7^{3/4}} - \frac{\int \frac{\sqrt{2}\sqrt[4]{7}-2x}{-\sqrt{7}+\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}\,7^{3/4}} + \frac{\int -}{\sqrt{}}$$
$$= \frac{x^2}{2} - \frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}\,7^{3/4}} + \frac{\log\left(\sqrt{7}+\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}\,7^{3/4}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\right.$$
$$= \frac{x^2}{2} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}} - \frac{1}{2}\tanh^{-1}\left(x^2\right) - \frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+\right.}{4\sqrt{2}\,7^{3/4}}$$

Mathematica [A] time = 0.07, size = 159, normalized size = 1.07

$$\frac{1}{56}\left(28x^2-14\log\left(x^2+1\right)-\sqrt{2}\sqrt[4]{7}\log\left(\sqrt{7}x^2-\sqrt{2}\,7^{3/4}x+7\right)+\sqrt{2}\sqrt[4]{7}\log\left(\sqrt{7}x^2+\sqrt{2}\,7^{3/4}x+7\right)+14\log(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] (28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*Sqrt[2]*7^(
1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] + 14*Log[1 + x] - 14*L
og[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2] + Sq
rt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2])/56
```

fricas [A] time = 0.76, size = 178, normalized size = 1.20

$$-\frac{1}{686}\cdot 343^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{7}\cdot 343^{\frac{1}{4}}\sqrt{2}x+\frac{1}{49}\cdot 343^{\frac{1}{4}}\sqrt{2}\sqrt{343^{\frac{3}{4}}\sqrt{2}x+49x^2+49\sqrt{7}}-1\right)-\frac{1}{686}\cdot 343^{\frac{3}{4}}\sqrt{2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="fricas")
```

```
[Out] -1/686*343^(3/4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*s
qrt(2)*sqrt(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1) - 1/686*343^(3/
4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*sqrt(2)*sqrt(-3
43^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) + 1) + 1/2744*343^(3/4)*sqrt(2)*l
og(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1/2744*343^(3/4)*sqrt(2)*lo
g(-343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) + 1/2*x^2 - 1/4*log(x^2 + 1)
+ 1/4*log(x^2 - 1)
```

giac [A] time = 0.39, size = 122, normalized size = 0.82

$$\frac{1}{2}x^2+\frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x+7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x-7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2+7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2))
) + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/
56*28^(1/4)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*28^(1/4)*log(x^2
- 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(abs(x + 1)) + 1
/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 110, normalized size = 0.74

$$\frac{x^2}{2}+\frac{7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,7^{\frac{3}{4}}x}{7}-1\right)}{28}+\frac{7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,7^{\frac{3}{4}}x}{7}+1\right)}{28}+\frac{7^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{x^2+7^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{7}}{x^2-7^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{7}}\right)}{56}+\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x)
```

```
[Out] 1/2*x^2+1/4*ln(x-1)-1/4*ln(x^2+1)+1/56*7^(1/4)*2^(1/2)*ln((x^2+7^(1/4)*x*2^
(1/2)+7^(1/2))/(x^2-7^(1/4)*x*2^(1/2)+7^(1/2)))+1/28*arctan(1+1/7*x*2^(1/2)
*7^(3/4))*7^(1/4)*2^(1/2)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(
1/2)+1/4*ln(x+1)
```

maxima [A] time = 2.01, size = 132, normalized size = 0.89

$$\frac{1}{2}x^2+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x+7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x-7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*s
qrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*
sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1
/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 +
1) + 1/4*log(x + 1) + 1/4*log(x - 1)
```

mupad [B] time = 2.19, size = 124, normalized size = 0.84

$$\frac{\operatorname{atan}\left(x^2\,1\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{x^2}{2}+\sqrt{2}\,7^{1/4}\,\operatorname{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\,\left(\frac{89653248}{2401}+\frac{89653248}{2401}\mathrm{i}\right)}{-\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}+\frac{\sqrt{2}\,7^{3/4}\,x\,\left(-\frac{524288}{343}+\frac{524288}{343}\mathrm{i}\right)}{-\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}\right)\left(\frac{1}{28}+\frac{1}{28}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 7*x^5 + x^9 - 1)/(6*x^4 + x^8 - 7),x)
```

```
[Out] (atan(x^2*1i)*1i)/2 + x^2/2 + 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(8965
3248/2401 + 89653248i/2401))/((7^(1/2)*179306496i)/2401 - 1048576/49) - (2^
(1/2)*7^(3/4)*x*(524288/343 - 524288i/343))/((7^(1/2)*179306496i)/2401 - 10
48576/49))*(1/28 + 1i/28) - 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(896532
48/2401 - 89653248i/2401))/((7^(1/2)*179306496i)/2401 + 1048576/49) - (2^(1
/2)*7^(3/4)*x*(524288/343 + 524288i/343))/((7^(1/2)*179306496i)/2401 + 1048
576/49))*(1/28 - 1i/28)
```

sympy [A] time = 0.43, size = 146, normalized size = 0.99

$$\frac{x^2}{2}+\frac{\log\left(x^2-1\right)}{4}-\frac{\log\left(x^2+1\right)}{4}-\frac{\sqrt{2}\,\sqrt[4]{7}\,\log\left(x^2-\sqrt{2}\,\sqrt[4]{7}\,x+\sqrt{7}\right)}{56}+\frac{\sqrt{2}\,\sqrt[4]{7}\,\log\left(x^2+\sqrt{2}\,\sqrt[4]{7}\,x+\sqrt{7}\right)}{56}+\frac{\sqrt{2}\,\sqrt[4]{7}\ \ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)
```

```
[Out] x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log(x**2 - sq
rt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(1/
4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28 + s
qrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28
```

3.368 $\int \frac{1+x^3+x^6}{x+x^5} \, dx$

Optimal. Leaf size=112

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(x^2\right)+\log(x)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\right.}{2}$$

```
[Out] 1/2*x^2-1/2*arctan(x^2)+ln(x)-1/4*ln(x^4+1)+1/4*arctan(-1+x*2^(1/2))*2^(1/2
)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+
x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {1593, 1833, 297, 1162, 617, 204, 1165, 628, 1834, 1248, 635, 203, 260}

$$\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{2}\tan^{-1}\left(x^2\right)+\log(x)-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{\right.}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt
[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1
+ Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \int \frac{1+x^3+x^6}{x\left(1+x^4\right)}\,dx$$
$$= \int \left(\frac{x^2}{1+x^4} + \frac{1+x^6}{x\left(1+x^4\right)}\right)dx$$
$$= \int \frac{x^2}{1+x^4}\,dx + \int \frac{1+x^6}{x\left(1+x^4\right)}\,dx$$
$$= -\left(\frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx\right) + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx + \int \left(\frac{1}{x} + x + \frac{x\left(-1-x^2\right)}{1+x^4}\right)dx$$
$$= \frac{x^2}{2} + \log(x) + \frac{1}{4}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{\sqrt{2}-}{-1+\sqrt{2}}}{4\sqrt{}}$$
$$= \frac{x^2}{2} + \log(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{-1-x}{1+x^2}\,dx, x, x^2\right) +$$
$$= \frac{x^2}{2} - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}} + \log(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x\right.}{4\sqrt{2}}$$
$$= \frac{x^2}{2} - \frac{1}{2}\tan^{-1}\left(x^2\right) - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}} + \log(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} -$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.90

$$\frac{1}{8}\left(-2\log\left(x^4+1\right) + 4x^2 + \sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right) - \sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) + 8\log(x) - 2\left(\sqrt{2}-2\right)\tan^{-1}\left(1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] (4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1
+ Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1
+ Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8
```

fricas [C] time = 2.41, size = 515, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/4*(2*sqrt(1/4*I) + I + 1)*log((2*sqrt(1/4*I) + I + 1)^3 - 5*(2*
sqrt(1/4*I) + I + 1)^2 + 3*x + 20*sqrt(1/4*I) + 10*I + 5) - 1/4*(2*sqrt(-1/
4*I) - I + 1)*log(-(2*sqrt(1/4*I) + I + 1)^3 - (2*sqrt(1/4*I) + I + 2)*(2*s
qrt(-1/4*I) - I + 1)^2 + 4*(2*sqrt(1/4*I) + I + 1)^2 - ((2*sqrt(1/4*I) + I
+ 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) + 3*x - 16*sqrt(
1/4*I) - 8*I - 9) + 1/4*(sqrt(1/4*I) + sqrt(-1/4*I) - 2*sqrt(-3/16*(2*sqrt(
1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) -
3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(
2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I
+ 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(
-1/4*I) - I + 1) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4
*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + s
qrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)
 + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*sqrt(1/4*I) - I + 2) + 1/4*(sqrt(1/4*I)
 + sqrt(-1/4*I) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*
I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sq
rt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*
I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I + 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1
)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) - 2*sqrt(-3/16*(2*s
qrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1
) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1
/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1) + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*
sqrt(1/4*I) - I + 2) + log(x)
```

giac [A] time = 0.39, size = 92, normalized size = 0.82

$$\frac{1}{2}x^2+\frac{1}{4}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqrt
(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2
)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*log(x^4 + 1) + log(ab
s(x))
```

maple [A] time = 0.01, size = 79, normalized size = 0.71

$$\frac{x^2}{2}-\frac{\arctan\left(x^2\right)}{2}+\frac{\sqrt{2}\arctan\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\arctan\left(\sqrt{2}x+1\right)}{4}+\ln(x)+\frac{\sqrt{2}\ln\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right)}{8}-\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^3+1)/(x^5+x),x)
```

```
[Out] 1/2*x^2-1/2*arctan(x^2)+1/4*arctan(-1+2^(1/2)*x)*2^(1/2)+1/8*2^(1/2)*ln((1+
x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+1/4*arctan(1+2^(1/2)*x)*2^(1/2)-1/4*ln(x^
4+1)+ln(x)
```

maxima [A] time = 2.06, size = 99, normalized size = 0.88

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{4}\sqrt{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\left(\sqrt{2}+1\right)\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*sqrt(2)
*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*(sqrt(2) +
 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(sqrt(2) - 1)*log(x^2 - sqrt(2)*
x + 1) + 1/2*x^2 + log(x)
```

mupad [B] time = 2.23, size = 170, normalized size = 1.52

$$\ln(x)+\left(\sum_{k=1}^{4}\ln\left(\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)\left(8\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)+x+\mathrm{root}\left(z^4+z^3\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^6 + 1)/(x + x^5),x)
```

```
[Out] log(x) + symsum(log(root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*(8*root(z^
4 + z^3 + z^2/2 + z/16 + 1/256, z, k) + x + 96*root(z^4 + z^3 + z^2/2 + z/1
6 + 1/256, z, k)*x + 240*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2*x +
 320*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^3*x - 16*root(z^4 + z^3 +
 z^2/2 + z/16 + 1/256, z, k)^2 + 8))*root(z^4 + z^3 + z^2/2 + z/16 + 1/256,
 z, k), k, 1, 4) + x^2/2
```

sympy [A] time = 0.96, size = 61, normalized size = 0.54

$$\frac{x^2}{2}+\log(x)+\mathrm{RootSum}\left(256t^4+256t^3+128t^2+16t+1,\left(t\mapsto t\log\left(\frac{1792t^4}{73}+\frac{704t^3}{219}-\frac{3152t^2}{219}-\frac{2584t}{219}+x-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, La
mbda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/21
9 + x - 344/219)))
```

3.369 $\int \frac{1+x^2}{-x+x^2} \, dx$

Optimal. Leaf size=14

$$x + 2\log(1-x) - \log(x)$$

```
[Out] x+2*ln(1-x)-ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 894}

$$x + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{-x+x^2} \, dx &= \int \frac{1+x^2}{(-1+x)x} \, dx \\
&= \int \left(1 + \frac{2}{-1+x} - \frac{1}{x}\right) dx \\
&= x + 2\log(1-x) - \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$x + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-x + x^2),x]
[Out] x + 2*Log[1 - x] - Log[x]
```

fricas [A] time = 0.75, size = 12, normalized size = 0.86

$$x + 2\ \log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
[Out] x + 2*log(x - 1) - log(x)
```

giac [A] time = 0.26, size = 14, normalized size = 1.00

$$x + 2\ \log(|x-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="giac")
[Out] x + 2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$x - \ln(x) + 2\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-x),x)
[Out] x+2*ln(x-1)-ln(x)
```

maxima [A] time = 0.91, size = 12, normalized size = 0.86

$$x + 2\ \log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
[Out] x + 2*log(x - 1) - log(x)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.86

$$x + 2\ln(x-1) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/(x - x^2),x)
[Out] x + 2*log(x - 1) - log(x)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-x),x)
[Out] x - log(x) + 2*log(x - 1)
```

3.370 $\int \frac{1+x^3}{-x+x^3} \, dx$

Optimal. Leaf size=12

$$x + \log(1 - x) - \log(x)$$

```
[Out] x+ln(1-x)-ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.133, Rules used = {1593, 1802}

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x + x^3),x]
```

```
[Out] x + Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-x+x^3} \, dx &= \int \frac{1+x^3}{x\left(-1+x^2\right)} \, dx \\ &= \int \left(1+\frac{1}{-1+x}-\frac{1}{x}\right) dx \\ &= x + \log(1-x) - \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x + x^3),x]
[Out] x + Log[1 - x] - Log[x]
```

fricas [A] time = 0.63, size = 10, normalized size = 0.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="fricas")
[Out] x + log(x - 1) - log(x)
```

giac [A] time = 0.37, size = 12, normalized size = 1.00

$$x + \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="giac")
[Out] x + log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$x - \ln(x) + \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x),x)
[Out] x+ln(x-1)-ln(x)
```

maxima [A] time = 1.01, size = 10, normalized size = 0.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="maxima")
[Out] x + log(x - 1) - log(x)
```

mupad [B] time = 2.14, size = 10, normalized size = 0.83

$$x - 2\,\mathrm{atanh}\left(2\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/(x - x^3),x)
```

```
[Out] x - 2*atanh(2*x - 1)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$x - \log(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x),x)
```

```
[Out] x - log(x) + log(x - 1)
```

3.371 $\int \frac{1+x^3}{-x^2+x^3} \, dx$

Optimal. Leaf size=17

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

```
[Out] 1/x+x+2*ln(1-x)-ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 1620}

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{-x^2+x^3} \, dx &= \int \frac{1+x^3}{(-1+x)x^2} \, dx \\
&= \int \left(1+\frac{2}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right) dx \\
&= \frac{1}{x}+x+2\log(1-x)-\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x^2 + x^3),x]
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

fricas [A] time = 0.70, size = 21, normalized size = 1.24

$$\frac{x^2+2\,x\log\left(x-1\right)-x\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

giac [A] time = 0.32, size = 17, normalized size = 1.00

$$x+\frac{1}{x}+2\,\log\left(|x-1|\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$x-\ln(x)+2\ln\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x^2),x)
[Out] x+2*ln(x-1)+1/x-ln(x)
```

maxima [A] time = 0.89, size = 15, normalized size = 0.88

$$x+\frac{1}{x}+2\,\log\left(x-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$x + 2\ \ln\left(x - 1\right) - \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/(x^2 - x^3),x)
```

```
[Out] x + 2*log(x - 1) - log(x) + 1/x
```

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$x - \log\left(x\right) + 2\log\left(x - 1\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x**2),x)
```

```
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

3.372 $\int \frac{-1+x^5}{-x+x^3} \, dx$

Optimal. Leaf size=17

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

[Out] x+1/3*x^3+ln(x)-ln(1+x)

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 1802}

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^5)/(-x + x^3),x]

[Out] x + x^3/3 + Log[x] - Log[1 + x]

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^5}{-x+x^3} \, dx &= \int \frac{-1+x^5}{x\left(-1+x^2\right)} \, dx \\
&= \int \left(1+\frac{1}{-1-x}+\frac{1}{x}+x^2\right) dx \\
&= x+\frac{x^3}{3}+\log(x)-\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)/(-x + x^3),x]
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

fricas [A] time = 0.62, size = 15, normalized size = 0.88

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="fricas")
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

giac [A] time = 0.26, size = 17, normalized size = 1.00

$$\frac{1}{3}x^3 + x - \log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="giac")
[Out] 1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^3}{3} + x + \ln(x) - \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)/(x^3-x),x)
[Out] x+1/3*x^3+ln(x)-ln(x+1)
```

maxima [A] time = 0.90, size = 15, normalized size = 0.88

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="maxima")
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$x - 2\,\mathrm{atanh}\,(2\,x + 1) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5 - 1)/(x - x^3),x)
[Out] x - 2*atanh(2*x + 1) + x^3/3
```

sympy [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{x^3}{3} + x + \log\,(x) - \log\,(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)/(x**3-x),x)
[Out] x**3/3 + x + log(x) - log(x + 1)
```

3.373 $\int \frac{1+x^4}{x^3+x^5}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{2x^2}+\log\left(x^2+1\right)-\log(x)$$

```
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 1252, 894}

$$-\frac{1}{2x^2}+\log\left(x^2+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \int \frac{1+x^4}{x^3\left(1+x^2\right)}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{x^2(1+x)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{1+x}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2x^2}-\log(x)+\log\left(1+x^2\right)$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{2x^2}+\log\left(x^2+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/2*1/x^2 - Log[x] + Log[1 + x^2]
```

fricas [A] time = 0.67, size = 25, normalized size = 1.39

$$\frac{2\,x^2\log\left(x^2+1\right)-2\,x^2\log(x)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2
```

giac [A] time = 0.27, size = 23, normalized size = 1.28

$$\frac{x^2-1}{2\,x^2}+\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)
```

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$-\ln(x)+\ln\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^5+x^3),x)
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

maxima [A] time = 1.92, size = 16, normalized size = 0.89

$$-\frac{1}{2\,x^2}+\log\left(x^2+1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="maxima")
[Out] -1/2/x^2 + log(x^2 + 1) - log(x)
```

mupad [B] time = 0.05, size = 16, normalized size = 0.89

$$\ln\left(x^2+1\right)-\ln(x)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^3 + x^5),x)
[Out] log(x^2 + 1) - log(x) - 1/(2*x^2)
```

sympy [A] time = 0.11, size = 15, normalized size = 0.83

$$-\log(x)+\log\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**5+x**3),x)
[Out] -log(x) + log(x**2 + 1) - 1/(2*x**2)
```

3.374 $\int \frac{1+x^2}{x+2x^2+x^3} \, dx$

Optimal. Leaf size=10

$$\frac{2}{x+1} + \log(x)$$

[Out] 2/(1+x)+ln(x)

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1594, 27, 894}

$$\frac{2}{x+1} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(x + 2*x^2 + x^3),x]

[Out] 2/(1 + x) + Log[x]

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{x+2x^2+x^3}\,dx &= \int \frac{1+x^2}{x\left(1+2x+x^2\right)}\,dx\\ &= \int \frac{1+x^2}{x(1+x)^2}\,dx\\ &= \int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right)dx\\ &= \frac{2}{1+x}+\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(x + 2*x^2 + x^3),x]
[Out] 2/(1 + x) + Log[x]
```

fricas [A] time = 0.78, size = 14, normalized size = 1.40

$$\frac{(x+1)\log(x)+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="fricas")
[Out] ((x + 1)*log(x) + 2)/(x + 1)
```

giac [A] time = 0.36, size = 11, normalized size = 1.10

$$\frac{2}{x+1}+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="giac")
[Out] 2/(x + 1) + log(abs(x))
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\ln(x)+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^3+2*x^2+x),x)
```

```
[Out] 2/(x+1)+ln(x)
```

maxima [A] time = 0.89, size = 10, normalized size = 1.00

$$\frac{2}{x+1}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="maxima")
```

```
[Out] 2/(x + 1) + log(x)
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\ln(x)+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + 2*x^2 + x^3),x)
```

```
[Out] log(x) + 2/(x + 1)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.70

$$\log{(x)}+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+2*x**2+x),x)
```

```
[Out] log(x) + 2/(x + 1)
```

3.375 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal. Leaf size=42

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

```
[Out] 19*x+3/2*x^2+1/3*x^3+3126/35*ln(5-x)-1/10*ln(x)-31/14*ln(2+x)
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1594, 1628}

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx &= \int \frac{1+x^5}{x\left(-10-3x+x^2\right)}\,dx\\ &= \int \left(19+\frac{3126}{35(-5+x)}-\frac{1}{10x}+3x+x^2-\frac{31}{14(2+x)}\right)dx\\ &= 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

fricas [A] time = 0.59, size = 30, normalized size = 0.71

$$\frac{1}{3}\,x^3+\frac{3}{2}\,x^2+19\,x-\frac{31}{14}\,\log\left(x+2\right)+\frac{3126}{35}\,\log\left(x-5\right)-\frac{1}{10}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

giac [A] time = 0.29, size = 33, normalized size = 0.79

$$\frac{1}{3}\,x^3+\frac{3}{2}\,x^2+19\,x-\frac{31}{14}\,\log\left(|x+2|\right)+\frac{3126}{35}\,\log\left(|x-5|\right)-\frac{1}{10}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
- 1/10*log(abs(x))
```

maple [A] time = 0.01, size = 31, normalized size = 0.74

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln\left(x-5\right)}{35}-\frac{31\ln\left(x+2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)/(x^3-3*x^2-10*x),x)
```

```
[Out] 1/3*x^3+3/2*x^2+19*x-31/14*ln(x+2)-1/10*ln(x)+3126/35*ln(x-5)
```

maxima [A] time = 0.88, size = 30, normalized size = 0.71

$$\frac{1}{3}\,x^3+\frac{3}{2}\,x^2+19\,x-\frac{31}{14}\,\log\left(x+2\right)+\frac{3126}{35}\,\log\left(x-5\right)-\frac{1}{10}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

mupad [B] time = 0.05, size = 30, normalized size = 0.71

$$19\,x-\frac{31\,\ln\left(x+2\right)}{14}+\frac{3126\,\ln\left(x-5\right)}{35}-\frac{\ln(x)}{10}+\frac{3\,x^2}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)
```

```
[Out] 19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 +
x^3/3
```

sympy [A] time = 0.15, size = 36, normalized size = 0.86

$$\frac{x^{3}}{3}+\frac{3x^{2}}{2}+19x-\frac{\log\left(x\right)}{10}+\frac{3126\log\left(x-5\right)}{35}-\frac{31\log\left(x+2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/1
4
```

3.376 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$

Optimal. Leaf size=46

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2
))*5^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{15-5x+x^2+x^3}{\left(5+x^2\right)\left(3+2x+x^2\right)}\,dx &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right)dx\\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$
\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

fricas [A] time = 0.77, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

giac [A] time = 0.36, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$-\sqrt{5}\,\arctan\left(\frac{\sqrt{5}\,x}{5}\right)+\frac{5\sqrt{2}\,\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}+\frac{\ln\left(x^2+2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

```
[Out] -5^(1/2)*arctan(1/5*5^(1/2)*x)+5/2*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))+1/2*
ln(x^2+2*x+3)
```

maxima [A] time = 1.97, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

mupad [B] time = 0.00, size = 88, normalized size = 1.91

$$\frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2}+\sqrt{5}\,\operatorname{atan}\left(\frac{2000\,\sqrt{5}}{2000\,x+1120}-\frac{224\,\sqrt{5}\,x}{2000\,x+1120}\right)-\frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

```
[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*
5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(
x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

sympy [A] time = 0.21, size = 51, normalized size = 1.11

$$\frac{\log\left(x^2+2x+3\right)}{2}-\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,x}{5}\right)+\frac{5\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

3.377 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx$

Optimal. Leaf size=19

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

```
[Out] -1/8*ln(3+x)+1/8*ln(1+3*x)
```

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6688, 616, 31}

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

```
[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx = \int \frac{1}{3+10x+3x^2}\,dx$$
$$= \frac{3}{8}\int \frac{1}{1+3x}\,dx - \frac{3}{8}\int \frac{1}{9+3x}\,dx$$
$$= -\frac{1}{8}\log(3+x) + \frac{1}{8}\log(1+3x)$$

Mathematica [**A**] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
[Out] -1/8*Log[3 + x] + Log[1 + 3*x]/8
```

fricas [**A**] time = 0.52, size = 15, normalized size = 0.79

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

giac [**A**] time = 0.28, size = 17, normalized size = 0.89

$$\frac{1}{8}\log(|3x+1|) - \frac{1}{8}\log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")
[Out] 1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))
```

maple [**A**] time = 0.01, size = 16, normalized size = 0.84

$$\frac{\ln(3x+1)}{8} - \frac{\ln(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(3+10/(x^2+1)*x),x)
```

```
[Out] -1/8*ln(x+3)+1/8*ln(1+3*x)
```

maxima [A] time = 1.03, size = 15, normalized size = 0.79

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

mupad [B] time = 0.08, size = 8, normalized size = 0.42

$$-\frac{\operatorname{atanh}\left(\frac{3x}{4}+\frac{5}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*((10*x)/(x^2 + 1) + 3)),x)
```

```
[Out] -atanh((3*x)/4 + 5/4)/4
```

sympy [A] time = 0.11, size = 14, normalized size = 0.74

$$\frac{\log\left(x+\frac{1}{3}\right)}{8}-\frac{\log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)
```

```
[Out] log(x + 1/3)/8 - log(x + 3)/8
```

3.378 $\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1386, 701, 632, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^4}{2+13x+15x^2}\,dx \\
&= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\left(2+13x+15x^2\right)}\right)dx \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375} \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx \\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

fricas [A] time = 0.63, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

giac [A] time = 0.28, size = 32, normalized size = 0.80

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(|5\,x+1|\right)-\frac{16}{567}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

maple [A] time = 0.01, size = 31, normalized size = 0.78

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\ln\left(5x+1\right)}{4375}-\frac{16\ln\left(3x+2\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(13+2/x+15*x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

maxima [A] time = 0.99, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

mupad [B] time = 2.09, size = 26, normalized size = 0.65

$$\frac{139\,x}{3375}-\frac{16\,\ln\left(x+\frac{2}{3}\right)}{567}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{13\,x^2}{450}+\frac{x^3}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(15*x + 2/x + 13),x)
```

```
[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x
^3/45
```

sympy [A] time = 0.12, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x + \frac{1}{5}\right)}{4375} - \frac{16\log\left(x + \frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(13+2/x+15*x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

3.379 $\int \frac{x^2}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=33

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1386, 701, 632, 31}

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(13 + 2/x + 15*x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^3}{2+13x+15x^2}\,dx \\
&= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\left(2+13x+15x^2\right)}\right)dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(13 + 2/x + 15*x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

fricas [A] time = 0.65, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

giac [A] time = 0.34, size = 27, normalized size = 0.82

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(|5\,x+1|\right)+\frac{8}{189}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 26, normalized size = 0.79

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\ln(5x+1)}{875}+\frac{8\ln(3x+2)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(13+2/x+15*x),x)
```

```
[Out] -13/225*x+1/30*x^2+8/189*ln(3*x+2)-1/875*ln(5*x+1)
```

maxima [A] time = 1.02, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.64

$$\frac{8\,\ln\left(x+\frac{2}{3}\right)}{189}-\frac{13\,x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15*x + 2/x + 13),x)
```

```
[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

sympy [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(13+2/x+15*x),x)
```

```
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

3.380 $\int \frac{x}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1386, 703, 632, 31}

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(13 + 2/x + 15*x),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^2}{2+13x+15x^2}\,dx\\ &= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx\\ &= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx\\ &= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(13 + 2/x + 15*x),x]
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

fricas [A] time = 0.83, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

giac [A] time = 0.27, size = 22, normalized size = 0.85

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(|5\,x+1|\right)-\frac{4}{63}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{15}+\frac{\ln\left(5x+1\right)}{175}-\frac{4\ln\left(3x+2\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(13+2/x+15*x),x)
[Out] 1/15*x-4/63*ln(3*x+2)+1/175*ln(5*x+1)
```

maxima [A] time = 1.03, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

mupad [B] time = 2.12, size = 16, normalized size = 0.62

$$\frac{x}{15}-\frac{4\,\ln\left(x+\frac{2}{3}\right)}{63}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15*x + 2/x + 13),x)
[Out] x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

sympy [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x)
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

3.381 $\int \frac{1}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1350, 632, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(13 + 2/x + 15*x)^(-1),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1350

```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x] /; FreeQ[{a, b, c, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx = \int \frac{x}{2+13x+15x^2}\,dx$$
$$= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right) + \frac{10}{7}\int \frac{1}{10+15x}\,dx$$
$$= \frac{2}{21}\log(2+3x) - \frac{1}{35}\log(1+5x)$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21}\log(3x+2) - \frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(13 + 2/x + 15*x)^(-1),x]
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

fricas [A] time = 0.60, size = 17, normalized size = 0.81

$$-\frac{1}{35}\log(5x+1) + \frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="fricas")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

giac [A] time = 0.29, size = 19, normalized size = 0.90

$$-\frac{1}{35}\log(|5x+1|) + \frac{2}{21}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="giac")
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\ln(5x+1)}{35} + \frac{2\ln(3x+2)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(13+2/x+15*x),x)
[Out] 2/21*ln(3*x+2)-1/35*ln(5*x+1)
```

maxima [A] time = 1.17, size = 17, normalized size = 0.81

$$-\frac{1}{35}\log(5x+1)+\frac{2}{21}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="maxima")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

mupad [B] time = 2.11, size = 13, normalized size = 0.62

$$\frac{2\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x + 2/x + 13),x)
[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35
```

sympy [A] time = 0.11, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x)
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

3.382 $\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)} dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

```
[Out] -1/7*ln(2+3*x)+1/7*ln(1+5*x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 616, 31}

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(13 + 2/x + 15*x)),x]
```

```
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \int \frac{1}{2+13x+15x^2}\,dx$$
$$= \frac{15}{7}\int \frac{1}{3+15x}\,dx - \frac{15}{7}\int \frac{1}{10+15x}\,dx$$
$$= -\frac{1}{7}\log(2+3x) + \frac{1}{7}\log(1+5x)$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{7}\log(5x+1) - \frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(13 + 2/x + 15*x)),x]
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

fricas [A] time = 0.73, size = 17, normalized size = 0.81

$$\frac{1}{7}\log\left(5\,x+1\right) - \frac{1}{7}\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

giac [A] time = 0.27, size = 19, normalized size = 0.90

$$\frac{1}{7}\log\left(|5\,x+1|\right) - \frac{1}{7}\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln\left(5x+1\right)}{7} - \frac{\ln\left(3x+2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(13+2/x+15*x),x)
```

```
[Out] -1/7*ln(3*x+2)+1/7*ln(5*x+1)
```

maxima [A] time = 0.93, size = 17, normalized size = 0.81

$$\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

mupad [B] time = 0.08, size = 8, normalized size = 0.38

$$-\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(15*x + 2/x + 13)),x)
```

```
[Out] -(2*atanh((30*x)/7 + 13/7))/7
```

sympy [A] time = 0.11, size = 15, normalized size = 0.71

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x)
```

```
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

3.383 $\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1386, 705, 29, 632, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(13 + 2/x + 15*x)),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x\left(2+13x+15x^2\right)}\,dx\\ &= \frac{1}{2}\int \frac{1}{x}\,dx+\frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx\\ &= \frac{\log(x)}{2}+\frac{45}{14}\int \frac{1}{10+15x}\,dx-\frac{75}{7}\int \frac{1}{3+15x}\,dx\\ &= \frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(13 + 2/x + 15*x)),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

fricas [A] time = 0.73, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="fricas")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

giac [A] time = 0.37, size = 24, normalized size = 0.89

$$-\frac{5}{7}\log\left(|5x+1|\right)+\frac{3}{14}\log\left(|3x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="giac")
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\ln(x)}{2}-\frac{5\ln\left(5x+1\right)}{7}+\frac{3\ln\left(3x+2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(13+2/x+15*x),x)
[Out] 1/2*ln(x)+3/14*ln(3*x+2)-5/7*ln(5*x+1)
```

maxima [A] time = 0.92, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5\,x+1\right)+\frac{3}{14}\log\left(3\,x+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="maxima")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

mupad [B] time = 0.09, size = 17, normalized size = 0.63

$$\frac{3\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(15*x + 2/x + 13)),x)
[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{\log\left(x\right)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(13+2/x+15*x),x)
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

3.384 $\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(13 + 2/x + 15*x)),x]

[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx \\
&= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [**A**] time = 0.00, size = 34, normalized size = 1.00

$$
-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(13 + 2/x + 15*x)),x]
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

fricas [**A**] time = 0.54, size = 30, normalized size = 0.88

$$
\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log(x)-14}{28\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

giac [**A**] time = 0.27, size = 29, normalized size = 0.85

$$
-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="giac")
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{13\ln(x)}{4}+\frac{25\ln(5x+1)}{7}-\frac{9\ln(3x+2)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(13+2/x+15*x),x)
[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(5*x+1)
```

maxima [A] time = 1.06, size = 26, normalized size = 0.76

$$-\frac{1}{2x}+\frac{25}{7}\log(5x+1)-\frac{9}{28}\log(3x+2)-\frac{13}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="maxima")
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.65

$$\frac{25\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\ln(x)}{4}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(15*x + 2/x + 13)),x)
[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

sympy [A] time = 0.16, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(13+2/x+15*x),x)
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

3.385 $\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx\\
&= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

fricas [A] time = 0.59, size = 39, normalized size = 0.95

$$-\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log(x)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

giac [A] time = 0.28, size = 34, normalized size = 0.83

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(|5\,x+1|\right)+\frac{27}{56}\,\log\left(|3\,x+2|\right)+\frac{139}{8}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{139\ln(x)}{8}-\frac{125\ln(5x+1)}{7}+\frac{27\ln(3x+2)}{56}+\frac{13}{4x}-\frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(13+2/x+15*x),x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(5*x+1)
```

maxima [A] time = 1.02, size = 31, normalized size = 0.76

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log(5\,x+1)+\frac{27}{56}\,\log(3\,x+2)+\frac{139}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.63

$$\frac{27\,\ln\left(x+\frac{2}{3}\right)}{56}-\frac{125\,\ln\left(x+\frac{1}{5}\right)}{7}+\frac{139\,\ln(x)}{8}+\frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(15*x + 2/x + 13)),x)
```

```
[Out] (27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

sympy [A] time = 0.17, size = 36, normalized size = 0.88

$$\frac{139\log(x)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(13+2/x+15*x),x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

3.386 $\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx=\int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx$$
$$=-\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx$$
$$=-\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx$$
$$=-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

fricas [A] time = 0.88, size = 44, normalized size = 0.92

$$\frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log(x)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

giac [A] time = 0.24, size = 39, normalized size = 0.81

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log\left(|5\,x+1|\right)-\frac{81}{112}\log\left(|3\,x+2|\right)-\frac{1417}{16}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{1417 \ln(x)}{16}+\frac{625 \ln(5x+1)}{7}-\frac{81 \ln(3x+2)}{112}-\frac{139}{8x}+\frac{13}{8x^2}-\frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(13+2/x+15*x),x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(5*x+1)
```

maxima [A] time = 1.07, size = 36, normalized size = 0.75

$$-\frac{417 x^2-39 x+4}{24 x^3}+\frac{625}{7} \log(5 x+1)-\frac{81}{112} \log(3 x+2)-\frac{1417}{16} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.67

$$\frac{625 \ln\left(x+\frac{1}{5}\right)}{7}-\frac{81 \ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417 \ln(x)}{16}-\frac{\frac{139 x^2}{8}-\frac{13 x}{8}+\frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(15*x + 2/x + 13)),x)
```

```
[Out] (625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^2
)/8 - (13*x)/8 + 1/6)/x^3
```

sympy [A] time = 0.18, size = 41, normalized size = 0.85

$$-\frac{1417 \log(x)}{16}+\frac{625 \log\left(x+\frac{1}{5}\right)}{7}-\frac{81 \log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(13+2/x+15*x),x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 + 3
9*x - 4)/(24*x**3)
```

3.387 $\int \frac{x^2}{2-\left(1+x^2\right)^4} \, dx$

Optimal. Leaf size=157

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\ \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\ \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\ \tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\ \tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-}}\right.}{4\ 2^{3/4}}$$

```
[Out] 1/8*I*arctan(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)-1/8*I*arcta
n(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(-1+2^(1
/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(1+2^(1/4))^(1/2))*(1+2
^(1/4))^(1/2)*2^(1/4)
```

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6740, 206, 203, 1972, 205}

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\ \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\ \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\ \tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\ \tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-}}\right.}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) - ((I/4)*
Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) - (Sqrt[1 + 2^(1
/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcTanh
[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2-\left(1+x^2\right)^4}\,dx = \int \left(\frac{-\sqrt[4]{2}+\sqrt{2}}{8\left(-1+\sqrt[4]{2}-x^2\right)}+\frac{-\sqrt[4]{2}-\sqrt{2}}{8\left(1+\sqrt[4]{2}+x^2\right)}+\frac{-\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1+x^2\right)\right)}+\frac{-\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1+x^2\right)\right)}\right) d$$

$$= -\frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}-x^2}\,dx}{4\,2^{3/4}}-\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i\left(1+x^2\right)}\,dx}{4\,2^{3/4}}-\frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i\left(1+x^2\right)}\,dx}{4\,2^{3/4}}-\frac{(1}{}$$

$$= -\frac{\sqrt{1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt{-1+\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}-\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{i+\sqrt[4]{2}+ix^2}\,dx}{4\,2^{3/4}}$$

$$= \frac{i\sqrt{1-i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[\#1^8+4\#1^6+6\#1^4+4\#1^2-1\&,\frac{\#1\log(x-\#1)}{\#1^6+3\#1^4+3\#1^2+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] -1/8*RootSum[-1 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 +
 3*#1^2 + 3*#1^4 + #1^6) & ]
```

fricas [B] time = 2.68, size = 1506, normalized size = 9.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^
(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log
(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(
2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(
3/4) - sqrt(2)) + 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(
2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4
) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) -
 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) +
 sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(1/2*sqrt
(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
 sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 -
 (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) + 4*sqrt(-
3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) -
3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2
^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2
^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4)
+ sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4)
- sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4
) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2
))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2
+ 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2
+ 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)
*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(
2))*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sq
rt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16
*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4)
 + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4
*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(
2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4)
 - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*
(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*s
qrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) - sqr
```

```
t(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2))
 - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) - sqrt(2))*l
og(1/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))
^2 - ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2
) - 6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) - sqrt(2))*log(-1
/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))^2 -
 ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2) -
6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*lo
g(4*((2^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4
) - 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*log(-4*((2
^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4) - 1/2
56*sqrt(2)) + 3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(x^2+1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 + 1)^4 - 2), x)
```

maple [C] time = 0.01, size = 54, normalized size = 0.34

$$-\frac{\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)^2\ln\left(-\mathrm{RootOf}\left(\_Z^8+\right.\right.}{8\left(\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)^7+3\,\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)^5+3\,\mathrm{RootOf}\left(\_Z\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2-(x^2+1)^4),x)
```

```
[Out] -1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(-_R+x),_R=RootOf(_Z^8+4*_Z^6+6*_Z^
4+4*_Z^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(x^2+1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 + 1)^4 - 2), x)
```

mupad [B] time = 2.75, size = 144, normalized size = 0.92

$$\sum_{k=1}^{8}\ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56\,x-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 + 1)^4 - 2),x)
```

```
[Out] symsum(log(- root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(56*x
 - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4
/16384 + z^2/1048576 - 1/1073741824, z, k)*(4096*x - root(z^8 - z^4/16384 +
 z^2/1048576 - 1/1073741824, z, k)^2*(262144*x + 67108864*root(z^8 - z^4/16
384 + z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/1
6384 + z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.22, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8-65536t^4+1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}-\frac{262144t^5}{3}-\frac{40t}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2-(x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x)))
```

3.388 $\int \frac{x^2}{2-(1-x^2)^4}\,dx$

Optimal. Leaf size=157

$$-\frac{\sqrt{\sqrt[4]{2}-1}\,\tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\,2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\,\tanh^{-1}\left(\right.}{4\,2^{3/4}}$$

```
[Out] -1/8*I*arctanh(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)+1/8*I*arc
tanh(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(-1+2^
(1/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(1+2^(1/4))^(1/2))*(
1+2^(1/4))^(1/2)*2^(1/4)
```

Rubi [A] time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6740, 206, 203, 1972, 208}

$$-\frac{\sqrt{\sqrt[4]{2}-1}\,\tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\,2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\,\tanh^{-1}\left(\right.}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4)) - ((I/4)*Sqr
t[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) + ((I/4)*Sqrt[1 +
I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) + (Sqrt[1 + 2^(1/4)]*Arc
Tanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2-\left(1-x^2\right)^4}\,dx = \int \left(\frac{\sqrt[4]{2}+\sqrt{2}}{8\left(1+\sqrt[4]{2}-x^2\right)}+\frac{\sqrt[4]{2}-\sqrt{2}}{8\left(-1+\sqrt[4]{2}+x^2\right)}+\frac{\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1-x^2\right)\right)}\right) dx$$

$$= \frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}+x^2}\,dx}{4\ 2^{3/4}}+\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i\left(1-x^2\right)}\,dx}{4\ 2^{3/4}}+\frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i\left(1-x^2\right)}\,dx}{4\ 2^{3/4}}+\frac{(1+$$

$$= -\frac{\sqrt{-1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{i+\sqrt[4]{2}-ix^2}\,dx}{4\ 2^{3/4}}+$$

$$= -\frac{\sqrt{-1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+}}\right.}{4\ 2^{3/4}}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^6+6\#1^4-4\#1^2-1\&,\frac{\#1\log(x-\#1)}{\#1^6-3\#1^4+3\#1^2-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - (1 - x^2)^4),x]

[Out] -1/8*RootSum[-1 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1
+ 3*#1^2 - 3*#1^4 + #1^6) & ]
```

fricas [B] time = 2.52, size = 1546, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2
^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*lo
g(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt
(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^
(3/4) + sqrt(2)) + 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sq
rt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2)
 + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sq
rt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(-1/2*s
qrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4
) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4)
 - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^
2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) + 4*((s
qrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4
) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) +
 sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)
*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) +
 sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) -
 sqrt(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(
3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(
3/4) - sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2
^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - s
qrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)
) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))
*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*
sqrt(2))*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4
) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x)
+ 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(
2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*l
og(-1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sq
rt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(
2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) +
sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + s
qrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - s
qrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(
```

```
2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) + sq
rt(2))*log(1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sq
rt(2) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) +
4*sqrt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) + sqrt(2)
)*log(-1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sqrt(2
) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) + 4*sq
rt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) + 1/256*sqr
t(2))*log(4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/25
6*2^(3/4) + 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) + 1/256*sqrt(2))*lo
g(-4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/256*2^(3/
4) + 1/256*sqrt(2)) + 3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(x^2-1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 - 1)^4 - 2), x)
```

maple [C] time = 0.02, size = 56, normalized size = 0.36

$$-\frac{\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)^2\ln\left(-\mathrm{RootOf}\left(\_Z^8-4\_\right.\right.}{8\left(\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)^7-3\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)^5+3\,\mathrm{RootOf}\left(\_Z^8-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2-(-x^2+1)^4),x)
```

```
[Out] -1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8-4*_Z^6+6*_Z^
4-4*_Z^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(x^2-1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 - 1)^4 - 2), x)
```

mupad [B] time = 2.80, size = 142, normalized size = 0.90

$$\sum_{k=1}^{8} \ln\left(-\mathrm{root}\left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k\right)\left(56\,x + \mathrm{root}\left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)^4 - 2),x)
```

```
[Out] symsum(log(- root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(56*x
 + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4
/16384 - z^2/1048576 - 1/1073741824, z, k)*(4096*x + root(z^8 - z^4/16384 -
 z^2/1048576 - 1/1073741824, z, k)^2*(262144*x - 67108864*root(z^8 - z^4/16
384 - z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/1
6384 - z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.23, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8 - 65536t^4 - 1024t^2 - 1, \left(t \mapsto t\log\left(-\frac{67108864t^7}{3} + \frac{262144t^5}{3} + \frac{40t}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2-(-x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x)))
```

3.389 $\int \frac{x^2}{2+\left(1+x^2\right)^4}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}i$$

```
[Out] 1/8*(-1)^(1/4)*arctan(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/4)-
1/8*(-1)^(3/4)*2^(1/4)*arctan(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4))^(1
/2)-1/8*(-1)^(1/4)*arctan(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/2)*2^(1
/4)+1/8*I*arctan(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2))*((1+I)
/(1+I+2^(3/4)))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6740, 204, 203, 1972, 205}

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{1}{8}i$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4)
) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1 + I*(-2)^(1/4)]])/(4
*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTan[x/Sqrt[1 + (-2)^(1/4)]]
)/(4*2^(3/4)) + (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4
))]*ArcTan[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2+\left(1+x^2\right)^4}\,dx = \int \left(\frac{-\sqrt[4]{-2}+i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}-x^2\right)}+\frac{-\sqrt[4]{-2}-i\sqrt{2}}{8\left(1+\sqrt[4]{-2}+x^2\right)}+\frac{-\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1+x^2\right)\right)}+\frac{-\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1+x\right.\right.}\right.$$

$$=\frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right)\int\frac{1}{\sqrt[4]{-2}+i\left(1+x^2\right)}\,dx+\frac{1}{8}\left(-\sqrt[4]{-2}-i\sqrt{2}\right)\int\frac{1}{1+\sqrt[4]{-2}+x^2}\,dx+\frac{1}{8}\left(\right.$$

$$=\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\;2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\;2^{3/4}}+\frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right.$$

$$=\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\;2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\;2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{}}}{}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[\#1^8+4\#1^6+6\#1^4+4\#1^2+3\&,\frac{\#1\log(x-\#1)}{\#1^6+3\#1^4+3\#1^2+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] RootSum[3 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 + 3*#1^
2 + 3*#1^4 + #1^6) & ]/8
```

fricas [B] time = 2.94, size = 2271, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
 128*sqrt(1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2))) - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 -
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12
288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*
sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(
2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) - sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(
-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/
8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2
)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I
*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-
I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqr
t(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))
- sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(sqrt(-
12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*
I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192
```

```
*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(
-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/819
2*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqr
t(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(
2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*
(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*
sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1
/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) +
128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*
sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I
*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2))
 + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/2
56*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/
2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))
)*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)
) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + 163
84*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(1/256*I*s
qrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/
8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))
^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
- sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I*sqrt(2) +
 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2
) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8
192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(
2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt
(-1/8192*I*sqrt(2))) + 2*x) - sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 3
2768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*s
qrt(1/8192*I*sqrt(2))) + 32768*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2*(-I*sqrt(2) - 128*sqrt(-1/8192*I*sqrt(2)) + 1) - 2*I*sqrt(2) - 256*sqr
t(-1/8192*I*sqrt(2)) + 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))
) + x) + sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(-8*(8388608
*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(
2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)
)) + 32768*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 1
28*sqrt(-1/8192*I*sqrt(2)) + 1) - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2))
```

```
 + 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))) + x) + sqrt(-1/256
*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2)
- 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/81
92*I*sqrt(2)))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/2
56*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-
1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)
))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/256*I*sqrt(2)
 - 1/2*sqrt(-1/8192*I*sqrt(2))) + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

maple [C] time = 0.01, size = 54, normalized size = 0.29

$$\frac{\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^8+4\right.\right.}{8\,\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)^7+24\,\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)^5+24\,\mathrm{RootOf}\left(\_Z^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+(x^2+1)^4),x)
```

```
[Out] 1/8*sum(1/(_R^7+3*_R^5+3*_R^3+_R)*_R^2*ln(-_R+x),_R=RootOf(_Z^8+4*_Z^6+6*_Z
^4+4*_Z^2+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

mupad [B] time = 2.78, size = 142, normalized size = 0.76

$$\sum_{k=1}^{8} \ln\left(\mathrm{root}\left(z^8 + \frac{z^4}{16384} + \frac{z^2}{1048576} + \frac{3}{1073741824}, z, k\right)\left(40\,x + \mathrm{root}\left(z^8 + \frac{z^4}{16384} + \frac{z^2}{1048576} + \frac{3}{1073741824},\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(40*x +
 root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 + z^2/1048576 + 3/1073741824, z, k)*(4096*x - root(z^8 + z^4/16384 + z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x - 67108864*root(z^8 + z^4/1638
4 + z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 + z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.21, size = 39, normalized size = 0.21

$$\mathrm{RootSum}\left(1073741824t^8 + 65536t^4 + 1024t^2 + 3, \left(t \mapsto t\log\left(67108864t^7 - 262144t^5 + 4096t^3 + 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)))
```

3.390 $\int \frac{x^2}{2+(1-x^2)^4} \, dx$

Optimal. Leaf size=188

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}$$

```
[Out] -1/8*(-1)^(1/4)*arctanh(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/4
)+1/8*(-1)^(3/4)*2^(1/4)*arctanh(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4))
^(1/2)+1/8*(-1)^(1/4)*arctanh(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/2)*
2^(1/4)-1/8*I*arctanh(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2))*(
(1+I)/(1+I+2^(3/4)))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6740, 206, 207, 1972, 208}

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] -(((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/
4)) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt[1 + I*(-2)^(1/4)]])
/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTanh[x/Sqrt[1 + (-2)^(1/
4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^
(3/4))]*ArcTanh[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2+\left(1-x^2\right)^4}\,dx = \int \left(\frac{\sqrt[4]{-2}+i\sqrt{2}}{8\left(1+\sqrt[4]{-2}-x^2\right)}+\frac{\sqrt[4]{-2}-i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}+x^2\right)}+\frac{\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1-x\right.\right.}$$

$$= \frac{1}{8}\left(\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}-i\left(1-x^2\right)}\,dx+\frac{1}{8}\left(\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{-1+\sqrt[4]{-2}+x^2}\,dx+\frac{1}{8}\left(\sqrt[4]{\cdot}\right.$$

$$= -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{1}{8}\left(\sqrt[4]{-2}-\right.$$

$$= -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1}}{}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^6+6\#1^4-4\#1^2+3\&,\frac{\#1\log(x-\#1)}{\#1^6-3\#1^4+3\#1^2-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] RootSum[3 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1 + 3*#1
^2 - 3*#1^4 + #1^6) & ]/8
```

fricas [B] time = 2.70, size = 2259, normalized size = 12.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2
)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 +
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2
) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1
/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/81
92*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) +
sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*
(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + s
qrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(sqrt(-1228
8*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqr
t(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*s
```

```
qrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*
sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqrt(2)
 - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/25
6*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sq
rt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt
(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*
sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*s
qrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32
*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(
1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sq
rt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2)
- 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sq
rt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/
8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2
))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sq
rt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sq
rt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*
sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192
*I*sqrt(2))) + 2*x) - sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*l
og(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^3 + 32768*(1/
256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 128*sqrt(1/819
2*I*sqrt(2)) - 1) - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2
*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 2*I*sqrt(2) - 256*sqrt(1/8192
*I*sqrt(2)) - 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) +
 sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^3 + 32768*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 128*sqrt(1/8192*I*sqrt(2)) - 1) -
32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2))) - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 3)*sqr
```

```
t(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) + sqrt(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^2 - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2)
 - 1/2*sqrt(1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/81
92*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 2*I*sqrt(
2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/819
2*I*sqrt(2))) + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.30

$$\frac{\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)^2\ln\left(-\mathrm{RootOf}\left(\_Z^8-4\_\right.\right.}{8\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)^7-24\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)^5+24\,\mathrm{RootOf}\left(\_Z^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+(-x^2+1)^4),x)
```

```
[Out] 1/8*sum(1/(_R^7-3*_R^5+3*_R^3-_R)*_R^2*ln(-_R+x),_R=RootOf(_Z^8-4*_Z^6+6*_Z
^4-4*_Z^2+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

mupad [B] time = 2.74, size = 142, normalized size = 0.76

$$\sum_{k=1}^{8} \ln\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x-\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(40*x -
 root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 - z^2/1048576 + 3/1073741824, z, k)*(4096*x + root(z^8 + z^4/16384 - z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x + 67108864*root(z^8 + z^4/1638
4 - z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 - z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.21, size = 39, normalized size = 0.21

$$\mathrm{RootSum}\left(1073741824t^8+65536t^4-1024t^2+3,\left(t\mapsto t\log\left(67108864t^7+262144t^5+4096t^3-40t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(-x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)))
```

3.391 $\int \frac{1-x^2}{a+b(1-x^2)^4} dx$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{}$$

```
[Out] -1/4*arctan(b^(1/8)*x/((-a)^(1/4)-b^(1/4))^(1/2))/b^(3/8)/(-a)^(1/2)/((-a)^
(1/4)-b^(1/4))^(1/2)+1/4*arctanh(b^(1/8)*x/((-a)^(1/4)+b^(1/4))^(1/2))/b^(3
/8)/(-a)^(1/2)/((-a)^(1/4)+b^(1/4))^(1/2)-1/8*arctan((-b^(1/8)*x*2^(1/2)+(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1
/2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a
)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/8*arctan((b^(1/8)*x*2^(1/2)+(b^(1/4)+(
(-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/
2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/
((-a)^(1/2)+b^(1/2))^(1/2)+1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)-b
^(1/8)*x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)
^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2)
)^(1/2)-1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)+b^(1/8)*x*2^(1/2)*(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/
2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4)
 - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^
```

```
(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]]
 - b^(1/4)]]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTa
nh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^
(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-
a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x
+ b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[
2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = \int \left(-\frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(1-x^2\right)^2\right)} - \frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(1-x^2\right)^2\right)}\right)dx$$

$$= -\frac{\sqrt{b}\int \frac{1-x^2}{\sqrt{-a}\sqrt{b}-b\left(1-x^2\right)^2}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int \frac{1-x^2}{\sqrt{-a}\sqrt{b}+b\left(1-x^2\right)^2}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\sqrt{b}\int \frac{1-x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int \frac{1-x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(1+\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+\left(1+\cdots\right.}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}\cdots}}{\sqrt[8]{b}}\cdots}}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{\cdots}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\cdots}{\cdots}}}{8\sqrt{-a}\sqrt{b}\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(\sqrt{\cdots}\right.}{8\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} + \cdots$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.10

$$-\frac{\text{RootSum}\left[\#1^8 b-4\#1^6 b+6\#1^4 b-4\#1^2 b+a+b\&,\frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1
]/(#1 - 2*#1^3 + #1^5) & ]/b
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 1}{\left(x^2 - 1\right)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

maple [C] time = 0.07, size = 69, normalized size = 0.10

$$\frac{\left(-\mathrm{RootOf}\left(b\_Z^8 - 4b\_Z^6 + 6b\_Z^4 - 4b\_Z^2 + a + b\right)^2 + 1\right)}{8b\left(\mathrm{RootOf}\left(b\_Z^8 - 4b\_Z^6 + 6b\_Z^4 - 4b\_Z^2 + a + b\right)^7 - 3\,\mathrm{RootOf}\left(b\_Z^8 - 4b\_Z^6 + 6b\_Z^4 - 4b\_Z^2 + a + b\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(a+b*(-x^2+1)^4),x)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8*b-4*_Z
^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{\left(x^2 - 1\right)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

mupad [B] time = 2.70, size = 328, normalized size = 0.49

$$\sum_{k=1}^{8} \ln\left(a\,b^5\,\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

```
[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 104
8576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4
096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6
+ 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*
a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^
4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 167772
16*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1,
 z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 +
 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)
```

sympy [A] time = 3.92, size = 133, normalized size = 0.20

$$-\operatorname{RootSum}\left(t^8\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\left(-6\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

3.392 $\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(-\sqrt{2}\,\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{}$$

```
[Out] -1/4*arctan(b^(1/8)*x/((-a)^(1/4)-b^(1/4))^(1/2))/b^(3/8)/(-a)^(1/2)/((-a)^
(1/4)-b^(1/4))^(1/2)+1/4*arctanh(b^(1/8)*x/((-a)^(1/4)+b^(1/4))^(1/2))/b^(3
/8)/(-a)^(1/2)/((-a)^(1/4)+b^(1/4))^(1/2)-1/8*arctan((-b^(1/8)*x*2^(1/2)+(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1
/2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a
)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/8*arctan((b^(1/8)*x*2^(1/2)+(b^(1/4)+(
(-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/
2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/
((-a)^(1/2)+b^(1/2))^(1/2)+1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)-b
^(1/8)*x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)
^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2)
)^(1/2)-1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)+b^(1/8)*x*2^(1/2)*(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/
2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(-\sqrt{2}\,\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4)
 - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^
```

```
(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]]
 - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTa
nh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^
(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-
a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x
+ b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[
2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = -\int \frac{-1+x^2}{a+b\left(-1+x^2\right)^4}\,dx$$

$$= -\int \left(-\frac{\sqrt{b}\left(-1+x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(-1+x^2\right)^2\right)} - \frac{\sqrt{b}\left(-1+x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(-1+x^2\right)^2\right)}\right) dx$$

$$= \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}-b\left(-1+x^2\right)^2}\,dx}{2\sqrt{-a}} + \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}+b\left(-1+x^2\right)^2}\,dx}{2\sqrt{-a}}$$

$$= \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} + \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(-1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+\left(-1-\right.}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}}}{\sqrt[8]{b}}}}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-}}{8\sqrt{-a}\sqrt{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(\right.}{8}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} +$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.10

$$-\frac{\text{RootSum}\left[\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+a+b\&,\frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1
]/(#1 - 2*#1^3 + #1^5) & ]/b
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{\left(x^2-1\right)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

maple [C] time = 0.00, size = 69, normalized size = 0.10

$$\frac{\left(-\operatorname{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^2+1\right)}{8b\left(\operatorname{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^7-3\operatorname{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(a+b*(x^2-1)^4),x)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8*b-4*_Z
^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{\left(x^2-1\right)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

mupad [B] time = 0.00, size = 328, normalized size = 0.49

$$\sum_{k=1}^{8} \ln\left(a\, b^5\, \left(\mathrm{root}\left(16777216\, a^5\, b^3\, z^8 + 16777216\, a^4\, b^4\, z^8 + 1048576\, a^3\, b^3\, z^6 + 24576\, a^2\, b^2\, z^4 + 256\, a\, b\, z^2 + 1, z,\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

```
[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 104
8576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4
096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6
+ 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*
a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^
4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 167772
16*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1,
 z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 +
 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)
```

sympy [A] time = 3.92, size = 133, normalized size = 0.20

$$-\operatorname{RootSum}\left(t^8\left(16777216a^5b^3 + 16777216a^4b^4\right) + 1048576t^6a^3b^3 + 24576t^4a^2b^2 + 256t^2ab + 1, \left(t \mapsto t\log\left(-6\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

3.393 $\int \frac{\left(1+x^2\right)^2}{ax^6+b\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}$$

```
[Out] 1/3*arctan(x*(a^(1/3)+b^(1/3))^(1/2)/b^(1/6))/b^(5/6)/(a^(1/3)+b^(1/3))^(1/
2)+1/3*arctan(x*(-(-1)^(1/3)*a^(1/3)+b^(1/3))^(1/2)/b^(1/6))/b^(5/6)/(-(-1)
^(1/3)*a^(1/3)+b^(1/3))^(1/2)+1/3*arctan(x*((-1)^(2/3)*a^(1/3)+b^(1/3))^(1/
2)/b^(1/6))/b^(5/6)/((-1)^(2/3)*a^(1/3)+b^(1/3))^(1/2)
```

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1+x^2\right)^2}{ax^6+b\left(1+x^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] Defer[Int][(a*x^6 + b*(1 + x^2)^3)^(-1), x] + 2*Defer[Int][x^2/(a*x^6 + b*(
1 + x^2)^3), x] + Defer[Int][x^4/(a*x^6 + b*(1 + x^2)^3), x]
```

Rubi steps

$$\int \frac{\left(1+x^2\right)^2}{ax^6+b\left(1+x^2\right)^3}\,dx = \int \left(\frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}+\frac{2x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}+\frac{x}{b+3bx^2+3bx}\right.$$

$$= 2\int \frac{x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx+\int \frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx+\int \frac{}{b+}$$

$$= 2\int \frac{x^2}{ax^6+b\left(1+x^2\right)^3}\,dx+\int \frac{1}{ax^6+b\left(1+x^2\right)^3}\,dx+\int \frac{x^4}{ax^6+b\left(1+x^2\right)^3}\,dx$$

Mathematica [C] time = 0.06, size = 95, normalized size = 0.57

$$\frac{1}{6}\text{RootSum}\left[\#1^6a+\#1^6b+3\#1^4b+3\#1^2b+b\&,\frac{\#1^4\log(x-\#1)+2\#1^2\log(x-\#1)+\log(x-\#1)}{\#1^5a+\#1^5b+2\#1^3b+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1] + 2*Log[
x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) & ]/6
```

fricas [C] time = 2.70, size = 5653, normalized size = 33.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-
1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b
+ b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9331
2/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)
^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*log(1/6*sqrt(1/2)*s
qrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 +
 b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/933
12*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6)
+ 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/(
(a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*b + x) - 1/36*sqrt(1/2)*sqrt((-I*sq
rt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/
31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a +
 b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/
((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*
b^5))^(1/3) - 72/(a*b + b^2))*log(-1/6*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(
a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
 b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/
3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*
(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 7
2/(a*b + b^2))*b + x) + 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3
 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)
*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) -
1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*
b + b^2)) + 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*s
qrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1
```

```
/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a
+ b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104
/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2
*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(
a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
 b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/
3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*
(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 7
2/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b +
 b^2))*log(1/12*b*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1
/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2
)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqr
t(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1
/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) +
 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)
*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*
b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5)
)^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3
) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4
) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(
I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)
) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^
2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2)) + x)
 - 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^
2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/466
56/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*
(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*
b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/
3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3
 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)
*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) -
1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*
b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b
 + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) -
1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3)
+ 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/4665
6/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 2073
6*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2))*log(-1/12*b*sq
rt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/
93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b +
b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/
(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3
```

```
 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/3)*(a*b +
b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) -
1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^
2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sq
rt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) -
1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^
2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)
/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a
*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9
3312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b
^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184
*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2)) + x) + 1/72*sqrt(-((a*b
+ b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b
^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1
/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(
-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b
^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46
656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)
*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a
*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a
*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312
/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^
3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^
5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/
93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b
^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2))*log(1/12*b*sqrt(-((a*b + b^2)*((-I
*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) +
 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((
a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/311
04/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)
^2*b^5))^(1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3
+ 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/
93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b +
b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/
(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3
 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)
*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) +
1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a
 + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^
4 + b^5)) + 216)/(a*b + b^2)) + x) - 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) +
 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/(
```

```
(a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b
^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3
 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(
1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4
+ b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b
^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1
/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt
(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31
104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b
)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((
a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^
5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5))
+ 216)/(a*b + b^2))*log(-1/12*b*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*
b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b
^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3)
 - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a
*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/
(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-
I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6)
+ 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/(
(a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31
104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b
)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(
1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^
3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^
(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^
4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3)
- 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*
b + b^2)) + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{a x^6+\left(x^2+1\right)^3 b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

maple [C] time = 0.28, size = 67, normalized size = 0.40

$$\frac{\left(\operatorname{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)^4+2\operatorname{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b\right.\right.}{6\operatorname{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)^5a+6\operatorname{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)^5b+12\operatorname{RootO}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(-_R+x),_R=RootOf((
a+b)*_Z^6+3*b*_Z^4+3*b*_Z^2+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{ax^6+\left(x^2+1\right)^3b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

mupad [B] time = 3.08, size = 504, normalized size = 3.00

$$\sum_{k=1}^{6}\ln\left(-a^3\ (a+b)\ \left(-\operatorname{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^2b^2\,60-\operatorname{root}\left(46656\,a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/(b*(x^2 + 1)^3 + a*x^6),x)
```

```
[Out] symsum(log(-3*a^3*(a + b)*(504*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*
b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*b^3*x - 864*root(46656*a*b^5*z^6 + 46656
*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*b^4 - 864*root(46656*a*b
^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*a*b^3 - 60
*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z,
k)^2*b^2 + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b
^2*z^2 + 1, z, k)^5*b^5*x + 2*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b
^4*z^4 + 108*b^2*z^2 + 1, z, k)*a*x + 8*root(46656*a*b^5*z^6 + 46656*b^6*z^
6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*b*x + 12*root(46656*a*b^5*z^6 + 4
6656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*a*b - 144*root(46656
*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*a*b^2*
x + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2
```

```
+ 1, z, k)^5*a*b^4*x - 1))*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*
z^4 + 108*b^2*z^2 + 1, z, k), k, 1, 6)
```

sympy [A] time = 1.87, size = 42, normalized size = 0.25

$$\text{RootSum}\left(t^6\left(46656ab^5+46656b^6\right)+3888t^4b^4+108t^2b^2+1,\left(t\mapsto t\log\left(6tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

```
[Out] RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b**
2 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

3.394 $\int \frac{(d+ex)^3}{a+cx^4}\,dx$

Optimal. Leaf size=320

$$-\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{d}{}$$

```
[Out] 1/4*e^3*ln(c*x^4+a)/c+3/2*d^2*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
-1/8*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e^2*a^(1/2)+d
^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*c^(1/2))*(-3*e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*d
*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c
^(3/4)*2^(1/2)+1/4*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e^2*a^(1/2)+d^2
*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d
^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a + c*x^
4])/(4*c)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \int \left(\frac{d^3+3de^2x^2}{a+cx^4}+\frac{x\left(3d^2e+e^3x^2\right)}{a+cx^4}\right)dx$$

$$= \int \frac{d^3+3de^2x^2}{a+cx^4}\,dx + \int \frac{x\left(3d^2e+e^3x^2\right)}{a+cx^4}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{3d^2e+e^3x}{a+cx^2}\,dx, x, x^2\right) + \frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-3e^2\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c} + \frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+3e^2\right)\right)\int \frac{\sqrt{a}\cdots}{\cdots}}{2c}$$

$$= \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right) + \frac{1}{2}e^3\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right) + \frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+3e^2\right)\right)\int \cdots}{4c\cdots}$$

$$= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}} - \frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\cdots\right.}{\cdots}$$

$$= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}} - \frac{d\left(\sqrt{c}\,d^2+3\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}\,d^2+3\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\cdots\right.}{2\sqrt{2}\,a^{3/4}c^{3/4}}$$

Mathematica [A] time = 0.25, size = 322, normalized size = 1.01

$$-\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}d^3-3a^{3/4}de^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}d^3-3a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt
[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1/4
)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d
^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4])/(8*a*c)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.36, size = 311, normalized size = 0.97

$$\frac{e^3\log\left(\left|cx^4+a\right|\right)}{4c}+\frac{\sqrt{2}\left(3\sqrt{2}\sqrt{ac}c^2d^2e+\left(ac^3\right)^{\frac{1}{4}}c^2d^3+3\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3}+\frac{\sqrt{2}\left(3\sqrt{2}\sqrt{ac}c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*e^3*log(abs(c*x^4 + a))/c + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c^2*d^2*e
+ (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c
)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3
)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d
*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

maple [A] time = 0.01, size = 314, normalized size = 0.98

$$\frac{3d^2e\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{2\sqrt{ac}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}}\right.}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a),x)
```

```
[Out] 1/8*d^3*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x
^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)+3/2*e*d^2/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+3/8*d*e^2/c/(a/c
)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)
*x*2^(1/2)+(a/c)^(1/2)))+3/4*d*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+3/4*d*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-
1)+1/4*e^3*ln(c*x^4+a)/c
```

maxima [A] time = 1.96, size = 310, normalized size = 0.97

$$\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^3+cd^3-3\,\sqrt{a}\,\sqrt{c}\,de^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^3-cd^3+3\,\sqrt{a}\,\sqrt{c}\,de^2\right)\log\left(\right.}{8\,a^{\frac{3}{4}}c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 + c*d^3 - 3*sqrt(a)*sqrt(c)*d*e^2)
*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) +
 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 - c*d^3 + 3*sqrt(a)*sqrt(c)*d*e^2
)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
+ 1/4*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 - 6*sq
rt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 1/4*(sqrt(2
)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 + 6*sqrt(a)*c*d^2*e
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))
```

mupad [B] time = 2.84, size = 894, normalized size = 2.79

$$\sum_{k=1}^{4}\ln\left(-c\,d^2\,\left(-3\,c\,d^5\,e^2+5\,a\,d\,e^6+3\,a\,e^7\,x+\mathrm{root}\left(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4),x)
```

```
[Out] symsum(log(-2*c*d^2*(5*a*d*e^6 - 3*c*d^5*e^2 + 3*a*e^7*x + 8*root(256*a^3*c
^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2
 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*
e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)^2*a*c^2*d + 2*root(256*a
^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6
*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*
d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*c^2*d^4*x - 5*c*d^4*
e^3*x - 24*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2
*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a
^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)
^2*a*c^2*e*x + 32*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*
d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z
 - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12
, z, k)*a*c*d*e^3 - 6*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*
c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8
*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*
e^12, z, k)*a*c*e^4*x))*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^
2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d
^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^
3*e^12, z, k), k, 1, 4)
```

sympy [A] time = 5.29, size = 384, normalized size = 1.20

$$\operatorname{RootSum}\left(256t^4a^3c^4 - 256t^3a^3c^3e^3 + t^2\left(96a^3c^2e^6 + 480a^2c^3d^4e^2\right) + t\left(-16a^3ce^9 + 192a^2c^2d^4e^5 - 48ac^3d^8e\right) + a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c**
2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d**
4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*d**
8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 + 960
*_t**3*a**3*c**4*d**4*e**2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a**3*c*
*3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716*_t*a*
*3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12 - 27*a*
*4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*c**3*d**
12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a*c**3*d**
11*e**4 + c**4*d**15))))
```

3.395 $\int \frac{(d+ex)^2}{a+cx^4}\,dx$

Optimal. Leaf size=291

$$-\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e^2\right.}{}$$

```
[Out] d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)-1/8*ln(-a^(1/4)*c^(1/4)*x*2
^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1
/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2
*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^
(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sqr
t[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/
4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^2}{a+cx^4}\,dx &= \int \left(\frac{2dex}{a+cx^4}+\frac{d^2+e^2x^2}{a+cx^4}\right)dx\\
&= (2de)\int \frac{x}{a+cx^4}\,dx+\int \frac{d^2+e^2x^2}{a+cx^4}\,dx\\
&= (de)\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)+\frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c}+\frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}\\
&= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}+\frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c}+\frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c}-\frac{\left(\sqrt{c}\,d^2\right.}{}\\
&= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}-\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{\right.}{4\sqrt{2}}\\
&= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}-\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 243, normalized size = 0.84

$$\frac{-\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)\right)-2\left(4\sqrt[4]{a}\,\sqrt[4]{c}\,de+\right.}{8a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)*c
^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
)/(8*a^(3/4)*c^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.34, size = 285, normalized size = 0.98

$$\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}\,c^2de+\left(ac^3\right)^{\frac{1}{4}}c^2d^2+\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}\,c^2de+\left(ac^3\right)^{\frac{1}{4}}c^2d^2+\left(ac^3\right.\right.}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^
(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c
^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a
*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*
c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^3)
```

maple [A] time = 0.00, size = 292, normalized size = 1.00

$$\frac{de\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{\sqrt{ac}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+}\right.}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a),x)
```

```
[Out] 1/8*d^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x
^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*d^2*(a/c)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*d^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)+d*e/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+1/8*e^2/c/(a/c)^(1/4
```

```
)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/
2)*x+(a/c)^(1/2)))+1/4*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)
*x+1)+1/4*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.05, size = 275, normalized size = 0.95

$$\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c
^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*
e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4
)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqr
t(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4)
)/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 1/4*(sqr
t(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*
d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))
```

mupad [B] time = 2.66, size = 556, normalized size = 1.91

$$\sum_{k=1}^{4}\ln\left(3\,c^2\,d^4\,e^2-a\,c\,e^6+4\,c^2\,d^3\,e^3\,x-\mathrm{root}\left(256\,a^3\,c^3\,z^4+192\,a^2\,c^2\,d^2\,e^2\,z^2+32\,a^2\,c\,d\,e^5\,z-32\,a\,c^2\,d^5\,e\,z+2\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4),x)
```

```
[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 + 4*c^2*d^3*e^3*x - 4*root(256*a^3*c^3*z
^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*
d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(256*a^3*c^3*z^4 + 19
2*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4
 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 4*root(256*a^3*c^3*z^4 + 192*a^2*
c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d
^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8
+ a^2*e^8, z, k)*a*c^2*d*e^3 + 32*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^
2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2
*e^8, z, k)^2*a*c^3*d*e*x)*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 +
```

```
 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z
, k), k, 1, 4)
```

sympy [A] time = 2.68, size = 277, normalized size = 0.95

$$\operatorname{RootSum}\left(256t^4a^3c^3 + 192t^2a^2c^2d^2e^2 + t\left(32a^2cde^5 - 32ac^2d^5e\right) + a^2e^8 + 2acd^4e^4 + c^2d^8, \left(t \mapsto t\log\left(x + \frac{64t^3a^4}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*c
*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e**2
- 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**3*c*
d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*e**11
 - 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c*d**4*
e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))
```

3.396 $\int \frac{d+ex}{a+cx^4}\,dx$

Optimal. Leaf size=219

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}}\cdots\right.}{2\sqrt{2}\,a^{3/4}\cdots}$$

```
[Out] 1/4*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*d*ar
ctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*d*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*d*ln(a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/2*e*
arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}}\cdots\right.}{2\sqrt{2}\,a^{3/4}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1
/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
```

```
0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{d+ex}{a+cx^4}\,dx = \int \left(\frac{d}{a+cx^4}+\frac{ex}{a+cx^4}\right)dx$$
$$= d\int \frac{1}{a+cx^4}\,dx + e\int \frac{x}{a+cx^4}\,dx$$
$$= \frac{d\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{d\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)$$
$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} + \frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{d\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \ldots$$
$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \ldots$$
$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\ldots\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.06, size = 184, normalized size = 0.84

$$\frac{-2\left(2\sqrt[4]{a}e+\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{c}d-2\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{2}\sqrt[4]{c}d\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\ldots\right.\right.}{8a^{3/4}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/
4)*Sqrt[c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.32, size = 215, normalized size = 0.98

$$\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}} d\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}} d\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ac}\,ce-\left(ac^{3}\right)^{\frac{1}{4}} cd\right)}{4\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2)
```

maple [A] time = 0.00, size = 151, normalized size = 0.69

$$\frac{e\arctan\left(\sqrt{\frac{c}{a}}\,x^{2}\right)}{2\sqrt{ac}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a),x)
```

```
[Out] 1/8*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)+1/2*e/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)
```

maxima [A] time = 2.67, size = 207, normalized size = 0.95

$$\frac{\sqrt{2}\,d\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,d\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d-2\,\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\,x\right.}{2\,\sqrt{}}\right.}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{c}}\,c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/
4)*c^(1/4)) - 1/8*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4)*d - 2*sqrt(a)*e)*a
rctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt
(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4
)*d + 2*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4))
```

mupad [B] time = 2.32, size = 160, normalized size = 0.73

$$\begin{cases} -\frac{2\,d+3\,e\,x}{6\,c\,x^3} & \text{if } a=0\\ \frac{\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,e+\sqrt{2}\,c^{1/4}\,d\right)}{4\,a^{3/4}\,\sqrt{c}}-\frac{\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,e-2\,\sqrt{2}\,c^{1/4}\,d\right)}{8\,a^{3/4}\,\sqrt{c}}+\frac{\sqrt{2}\,d\,\ln\left(\frac{\sqrt{a}+\sqrt{c}\,x^2+\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}{\sqrt{a}+\sqrt{c}\,x^2-\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}\right)}{8\,a^{3/4}\,c^{1/4}} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4),x)
```

```
[Out] piecewise(a == 0, -(2*d + 3*e*x)/(6*c*x^3), a ~= 0, (atan((2^(1/2)*c^(1/4)*
x)/a^(1/4) - 1)*(2*a^(1/4)*e + 2^(1/2)*c^(1/4)*d))/(4*a^(3/4)*c^(1/2)) - (a
tan((2^(1/2)*c^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*e - 2*2^(1/2)*c^(1/4)*d))/(
8*a^(3/4)*c^(1/2)) + (2^(1/2)*d*log((a^(1/2) + c^(1/2)*x^2 + 2^(1/2)*a^(1/4
)*c^(1/4)*x)/(a^(1/2) + c^(1/2)*x^2 - 2^(1/2)*a^(1/4)*c^(1/4)*x)))/(8*a^(3/
4)*c^(1/4)))
```

sympy [A] time = 0.82, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3c^2+32t^2a^2ce^2-16tacd^2e+ae^4+cd^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3ce^2-16t^2a^2cd^2e-8ta^2e^4-}{4ade^4-cd^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*e
**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a**2
*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d*e**4 - c
*d**5))))
```

3.397 $\int \frac{1}{a+cx^4}\,dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

```
[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+cx^4}\,dx = \frac{\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 -\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

fricas [A] time = 0.42, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}+\sqrt{a^2\sqrt{-\frac{1}{a^3c}}+x^2}\,a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}\left(-\frac{1}{a^3c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

giac [A] time = 0.33, size = 179, normalized size = 0.97

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

maple [A] time = 0.00, size = 128, normalized size = 0.69

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a),x)
```

```
[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.37, size = 169, normalized size = 0.91

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2
```

```
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

mupad [B] time = 0.08, size = 33, normalized size = 0.18

$$-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)+\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{2\,{\left(-a\right)}^{3/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4),x)
[Out] -(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*c^(1/4))
```

sympy [A] time = 0.16, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a),x)
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

3.398 $\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=416

$$-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right.}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}$$

```
[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)-1/4*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)-1/2*d^2*e*arc
tan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)/a^(1/2)-1/8*c^(1/4)*d*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(
3/4)/(a*e^4+c*d^4)*2^(1/2)+1/8*c^(1/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)+1/
4*c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/
a^(3/4)/(a*e^4+c*d^4)*2^(1/2)+1/4*c^(1/4)*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1
/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right.}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4))
- (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4
 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*
d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a*e^4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx = \int \left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)}\right)dx$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{a+cx^4}\,dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \left(\frac{d^3+de^2x^2}{a+cx^4} + \frac{x\left(-d^2e-e^3x^2\right)}{a+cx^4}\right)dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \frac{d^3+de^2x^2}{a+cx^4}\,dx}{cd^4+ae^4} + \frac{c\int \frac{x\left(-d^2e-e^3x^2\right)}{a+cx^4}\,dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\,\text{Subst}\left(\int \frac{-d^2e-e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} + \frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^4+ae^4\right)} + \frac{\left(d\left(\right.\right.}{}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} - \frac{\left(cd^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} - \frac{\left(ce^3\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} + \frac{\left(a\right.}{}$$

$$= -\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)} + \frac{e^3\log(d+ex)}{cd^4+ae^4} - \frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x\right.}{4\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)}$$

$$= -\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)} + \frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\right.}{2\sqrt{2}\,a^{3/4}}$$

Mathematica [A] time = 0.15, size = 404, normalized size = 0.97

$$\frac{-2a^{3/4}e^3\log\left(a+cx^4\right)+8a^{3/4}e^3\log(d+ex)-\sqrt{2}\,c^{3/4}d^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\sqrt{2}\,c^{3/4}d^3\log\left(\sqrt{2}\,\sqrt[4]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] (-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a
]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqrt[
c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c
```

```
^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^(1/4)
*d*e^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(
3/4)*d^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*S
qrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
 - 2*a^(3/4)*e^3*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.42, size = 371, normalized size = 0.89

$$\frac{\left(ac^{3}\right)^{\frac{1}{4}} cd \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^{2}d^{2}-2\left(ac^{3}\right)^{\frac{1}{4}}acde+\sqrt{2}\sqrt{ac}\,ace^{2}\right)}+\frac{\left(ac^{3}\right)^{\frac{1}{4}} cd \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^{2}d^{2}+2\left(ac^{3}\right)^{\frac{1}{4}}acde+\sqrt{2}\sqrt{ac}\,ace^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3}-\left(ac\right.\right.}{4\left(\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(sqrt(2)*a*c^2*d^2 - 2*(a*c^3)^(1/4)*a*c*d*e + sqrt(2)*sqrt(a*c)*a*c
*e^2) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + 2*(a*c^3)^(1/4)*a*c*d*e + sqrt(2)*sqrt(
a*c)*a*c*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 +
 sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^
4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*
(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^4) - 1/4*e^
3*log(abs(c*x^4 + a))/(c*d^4 + a*e^4) + e^4*log(abs(x*e + d))/(c*d^4*e + a*
e^5)
```

maple [A] time = 0.01, size = 433, normalized size = 1.04

$$-\frac{c\,d^{2}e\arctan\left(\sqrt{\frac{c}{a}}\,x^{2}\right)}{2\left(a\,e^{4}+c\,d^{4}\right)\sqrt{ac}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{4}+c\,d^{4}\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{4}+c\,d^{4}\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{3}\ln\left(\frac{x^{2}+}{x^{2}-}\right.}{8\left(a\,e^{4}+c\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a),x)
```

```
[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)+1/8*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*l
n((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^
(1/2)))+1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)+1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)-1/2*c/(a*e^4+c*d^4)*e*d^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^
2)+1/8/(a*e^4+c*d^4)*d*e^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4/(a*e^4+c*d^4)*d
*e^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4/(a*e^4+c*d^4)*
d*e^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/4*e^3*ln(c*x^4+
a)/(a*e^4+c*d^4)
```

maxima [A] time = 2.73, size = 345, normalized size = 0.83

$$\frac{e^3\log(ex+d)}{cd^4+ae^4} - \frac{c\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3-cd^3+\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3+cd^3-\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\cdots\right.}{a^{\frac{3}{4}}c^{\frac{5}{4}}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^3*log(e*x + d)/(c*d^4 + a*e^4) - 1/8*c*(sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*
e^3 - c*d^3 + sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1
/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 +
 c*d^3 - sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x
 + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^
(3/4)*c^(3/4)*d*e^2 + 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(
c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^(3/4)*c^(3/4)*d*e
^2 - 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c*
d^4 + a*e^4)
```

mupad [B] time = 0.42, size = 874, normalized size = 2.10

$$\left(\sum_{k=1}^{4}\ln\left(\mathrm{root}\left(256\,a^3\,c\,d^4\,z^4+256\,a^4\,e^4\,z^4+256\,a^3\,e^3\,z^3+96\,a^2\,e^2\,z^2+16\,a\,e\,z+1,z,k\right)\,c^4\,e\,\left(d\,e^2+5\,e^3\,x+\mathrm{root}\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)),x)
```

```
[Out] symsum(log(root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*c^4*e*(d*e^2 + 5*e^3*x + 240*root(256*a^3
*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z
+ 1, z, k)^2*a^2*e^5*x + 320*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256
*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*e^6*x + 32*root(2
56*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*
a*e*z + 1, z, k)*a*d*e^3 + 60*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 25
6*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*a*e^4*x - 4*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)*c*d^4*x - 16*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^
3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^5 + 208*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)^2*a^2*d*e^4 + 384*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 2
56*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*d*e^5 - 128*roo
t(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 +
16*a*e*z + 1, z, k)^3*a^2*c*d^5*e - 192*root(256*a^3*c*d^4*z^4 + 256*a^4*e^
4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^2*c*d^4*
e^2*x - 48*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^4*e*x))*root(256*a^3*c*d^4*z^4 +
256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k), k
, 1, 4) + (e^3*log(d + e*x))/(a*e^4 + c*d^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.399 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$

Optimal. Leaf size=552

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)-\sqrt{a}\,e^2\left(3cd^4-ae^4\right)\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)-\sqrt{a}\,e \cdots\right.}{4 \cdots}$$

```
[Out] -e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-c*d^3*e^3*
ln(c*x^4+a)/(a*e^4+c*d^4)^2-d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*
c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(
1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4
)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*arctan(-1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2)
)/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^
(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(3/4)/
(a*e^4+c*d^4)^2*2^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)-\sqrt{a}\,e^2\left(3cd^4-ae^4\right)\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)-\sqrt{a}\,e \cdots\right.}{4 \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] -(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(Sqrt[c]*d^2*(
c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d
^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*Lo
g[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - S
qrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d^2*(c
*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1
```

```
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^
3*e^3*Log[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)}\,dx = \int\Bigg(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)^2}+\frac{4cd^3e^4}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-a\cdots\right.\right.}{\left(cd^4+a\cdots\right.}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4\right)\cdots}{a+cx^4}}{\left(cd^4+ae^4\right)^2}\cdots$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\left(\frac{x\left(-2de\left(cd^4-ae^4\right)-4cd^3e^3x^2\right)}{a+cx^4}+\frac{d^2\left(cd^4-3a\cdots\right.}{\cdots}\right.}{\left(cd^4+ae^4\right)^2}\cdots$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{x\left(-2de\left(cd^4-ae^4\right)-4cd^3e^3x^2\right)}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{d^2\cdots}{\cdots}}{\cdots}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\,\mathrm{Subst}\left(\int\frac{-2de\left(cd^4-ae^4\right)-4cd^3e^3x}{a+cx^2}\,dx,x,x^{\cdots}\right.}{2\left(cd^4+ae^4\right)^2}\cdots$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}-\frac{\left(2c^2d^3e^3\right)\mathrm{Subst}\left(\int\frac{x}{a+cx^2}\,dx,x,x^2\right)}{\left(cd^4+ae^4\right)^2}-\cdots$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}-\frac{\sqrt{c}\,de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^2}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\cdots}{\cdots}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}-\frac{\sqrt{c}\,de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{\sqrt{c}\,d^2(\cdots}{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\,(\cdots}$$

Mathematica [A] time = 0.61, size = 524, normalized size = 0.95

$$\frac{-\frac{\sqrt{2}\sqrt[4]{c}\left(a^{3/2}e^6-3\sqrt{a}\,cd^4e^2-3a\sqrt{c}\,d^2e^4+c^{3/2}d^6\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{3/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(a^{3/2}e^6-3\sqrt{a}\,cd^4e^2-3a\sqrt{c}\,d^2e^4+c^{3/2}d^6\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\cdots\right.}{a^{3/4}}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] ((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2) + Sqrt[a]*
e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d
^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/a^(3/4) + (2*c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[2]*c
*d^4 + 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4*a^(3
/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/a^(3/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 - 3*Sq
rt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 -
 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) - 8*c*d^3*e^3*Log[a + c*x^4]
)/(8*(c*d^4 + a*e^4)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.78, size = 646, normalized size = 1.17

$$-\frac{cd^3e^3\log\left(\left|cx^4+a\right|\right)}{c^2d^8+2\,acd^4e^4+a^2e^8}+\frac{4\,cd^3e^4\log\left(\left|xe+d\right|\right)}{c^2d^8e+2\,acd^4e^5+a^2e^9}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{2\left(\sqrt{2}\,ac^3d^4-4\,\left(ac^3\right)^{\frac{1}{4}}ac^2d^3e+4\,\sqrt{2}\,\sqrt{ac}\,ac^2d^2e^2+\sqrt{2}\,a^2c^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -c*d^3*e^3*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 4*c*d^
3*e^4*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/2*((a*c^3
)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 - 4*(a*c^3)^(1/4)*a*c^2*d^3*e + 4*
sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + sqrt(2)*a^2*c^2*e^4 - 4*(a*c^3)^(3/4)*a*d
*e^3) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 4*(a*c^3)^(1/
4)*a*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + sqrt(2)*a^2*c^2*e^4 +
4*(a*c^3)^(3/4)*a*d*e^3) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3*sqrt(2)*(
```

```
a*c^3)^(3/4)*c*d^4*e^2 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 + sqrt(2)*(a
*c^3)^(3/4)*a*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^8
+ 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3
*sqrt(2)*(a*c^3)^(3/4)*c*d^4*e^2 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 +
sqrt(2)*(a*c^3)^(3/4)*a*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - (c*d^4*e^3 + a*e^7)/((c*d^4
+ a*e^4)^2*(x*e + d))
```

maple [A] time = 0.01, size = 866, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a),x)
```

```
[Out] -e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-3/4*c/(a*e
^4+c*d^4)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^4+1/4
*c^2/(a*e^4+c*d^4)^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
d^6-3/8*c/(a*e^4+c*d^4)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x
+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e^4+1/8*c^2/(a*e
^4+c*d^4)^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)
)/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^6-3/4*c/(a*e^4+c*d^4)^2*(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^4+1/4*c^2/(a*e^4+c*d^4)
^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^6+c/(a*e^4+c*d^4
)^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d*a-c^2/(a*e^4+c*d^4)^2/(a*c)
^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^5-1/8/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1
/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2)))*a*e^6+3/8*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^
(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^4*e
^2-1/4/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
a*e^6+3/4*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)*d^4*e^2-1/4/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)*a*e^6+3/4*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/c)^(1/4)*x+1)*d^4*e^2-c*d^3*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)^2
```

maxima [A] time = 2.11, size = 561, normalized size = 1.02

$$\frac{4\,cd^3e^3\log\left(ex+d\right)}{c^2d^8+2\,acd^4e^4+a^2e^8}-\frac{e^3}{cd^5+ade^4+\left(cd^4e+ae^5\right)x}-\frac{c\left(\frac{\sqrt{2}\left(4\,\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3-c^2d^6+3\,\sqrt{a}\,c^{\frac{3}{2}}d^4e^2+3\,acd^2e^4-a^{\frac{3}{2}}\,\sqrt{c}\,e^6\right)\log\left(\sqrt{c}\,x^2+\sqrt{\right.}{a^{\frac{3}{4}}c^{\frac{5}{4}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 4*c*d^3*e^3*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - e^3/(c*d^5 +
 a*d*e^4 + (c*d^4*e + a*e^5)*x) - 1/8*c*(sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)
*d^3*e^3 - c^2*d^6 + 3*sqrt(a)*c^(3/2)*d^4*e^2 + 3*a*c*d^2*e^4 - a^(3/2)*sq
rt(c)*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*
c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 + c^2*d^6 - 3*sqrt(a)
*c^(3/2)*d^4*e^2 - 3*a*c*d^2*e^4 + a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x^2 - s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*
c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)
*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 + 4*sqrt(a)*c^2*d^5*e - 4*a^(3/2)*c*
d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*
c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)
*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 - 4*sqrt(a)*c^2*d^5*e + 4*a^(3/2)*c*
d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^2*d^8 + 2*a*c*d^4
*e^4 + a^2*e^8)
```

mupad [B] time = 2.78, size = 2436, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)^2),x)
```

```
[Out] symsum(log((c^5*d*e^6 + c^5*e^7*x + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3
*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2
*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^4*c^4*e^13 + 256*root(512*a^4*c*d^4*e^4*
z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*
a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^3*e^8 + 496*root(51
2*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^
3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6*d^8*e
^5 + 528*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4
 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)
^3*a^3*c^5*d^4*e^9 - 128*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 +
 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*
d*e*z + c, z, k)^4*a^2*c^7*d^13*e^2 + 128*root(512*a^4*c*d^4*e^4*z^4 + 256*
a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*
e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^9*e^6 + 640*root(512*a^4*c*d^
4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3
+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d^5*e^10 + 32*r
oot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
```

```
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d^2
*e^7 - 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^
4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k
)^3*a*c^7*d^12*e + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 25
6*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e
*z + c, z, k)*c^6*d^5*e^4*x - 4*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^
8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 +
32*a*c*d*e*z + c, z, k)^2*c^7*d^10*e*x + 64*root(512*a^4*c*d^4*e^4*z^4 + 25
6*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^
2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a*c^6*d^7*e^4 + 384*root(512*a^4*c*d^
4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3
+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*d*e^14 + 320*ro
ot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3
*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*
e^15*x + 248*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8
*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z
, k)^2*a*c^6*d^6*e^5*x - 64*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^
4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a
*c*d*e*z + c, z, k)^3*a*c^7*d^11*e^2*x + 32*root(512*a^4*c*d^4*e^4*z^4 + 25
6*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^
2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d*e^8*x + 316*root(512*a^4*c*d^4*
e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 +
320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^2*e^9*x + 640*r
oot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6
*d^7*e^6*x + 704*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5
*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z +
c, z, k)^3*a^3*c^5*d^3*e^10*x - 192*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^
2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^
2 + 32*a*c*d*e*z + c, z, k)^4*a^2*c^7*d^12*e^3*x - 64*root(512*a^4*c*d^4*e^
4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 32
0*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^8*e^7*x + 448*roo
t(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*
c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d
^4*e^11*x)/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))*root(512*a^4*c*d^4*e^4*z^4
+ 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*
c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k), k, 1, 4) - e^3/(c*d^5 + a*d*e^4 +
a*e^5*x + c*d^4*e*x) + (4*c*d^3*e^3*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*a*
c*d^4*e^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a),x)

[Out] Timed out
```

3.400 $\int \frac{1}{(d+ex)^3\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=680

$$-\frac{\sqrt{c}\,e\left(a^2e^8-12acd^4e^4+3c^2d^8\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\,\sqrt{c}\,d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^3}$$

```
[Out] -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)+2*c*d^
2*e^3*(-3*a*e^4+5*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^3-1/2*c*d^2*e^3*(-3*a*e^4+
5*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/2*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^
8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(3/4)*
d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^8-12*a*c*d^4*e^
4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*
d^4)^3*2^(1/2)+1/8*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*
c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*arctan(-1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*
d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*arctan(
1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-
5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\,\sqrt{c}\,d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] -e^3/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*(d
+ e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(c^2*d^8 - 12*a*
c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3)
+ (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*
e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[d + e*x
```

```
])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2
*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)
*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[a +
c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)}\,dx = \int\left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)^3} + \frac{4cd^3e^4}{\left(cd^4+ae^4\right)^2(d+ex)^2} + \frac{2cd^2e^4\left(5cd^4-3ae^4\right)}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(d\left(c^2d\right.\right.}{}\right.$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} +$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} +$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} +$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} +$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} -$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} - \frac{\sqrt{c}\,e\left(3c^2d^8-12acd^4e^4+a^2e^8\right)\tan}{2\sqrt{a}\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} - \frac{\sqrt{c}\,e\left(3c^2d^8-12acd^4e^4+a^2e^8\right)\tan}{2\sqrt{a}\left(cd^4+ae^4\right)^3}$$

Mathematica [A] time = 0.93, size = 738, normalized size = 1.09

$$\frac{-4a^{3/4}e^3\left(ae^4+cd^4\right)^2 - 32a^{3/4}cd^3e^3(d+ex)\left(ae^4+cd^4\right) + 4a^{3/4}cd^2e^3(d+ex)^2\left(3ae^4-5cd^4\right)\log\left(a+cx^4\right) + 16a^{3/4}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] (-4*a^(3/4)*e^3*(c*d^4 + a*e^4)^2 - 32*a^(3/4)*c*d^3*e^3*(c*d^4 + a*e^4)*(d
 + e*x) - 2*Sqrt[c]*(Sqrt[2]*c^(9/4)*d^9 - 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*
Sqrt[a]*c^(7/4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 + 24*a^(5/4)*c*d^4*e
^5 - 10*Sqrt[2]*a^(3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 - 2*a
^(9/4)*e^9)*(d + e*x)^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[c]
*(Sqrt[2]*c^(9/4)*d^9 + 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/4)*d^7
*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 - 24*a^(5/4)*c*d^4*e^5 - 10*Sqrt[2]*a^(
3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 + 2*a^(9/4)*e^9)*(d + e*
x)^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 16*a^(3/4)*c*d^2*e^3*(5*c*d^
4 - 3*a*e^4)*(d + e*x)^2*Log[d + e*x] - Sqrt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt
[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e
^8)*(d + e*x)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sq
rt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*
a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2] + 4*a^(3/4)*c*d^2*e^3*(-5*c*d^4 + 3*a*e^4)*(d
+ e*x)^2*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4)^3*(d + e*x)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.89, size = 901, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 + 2*a*c^2*e^3 - 3*sqrt(2)*(a*c^3)^(3/4)*
d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d
^6 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e + 9*sqrt(a*c)*a*c^2*d^4*e^2 + 9*a^
2*c^2*d^2*e^4 - 8*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^3 - 3*sqrt(2)*(a*c^3)^(1/4)
*a^2*c*d*e^5 + sqrt(a*c)*a^2*c*e^6) + 1/4*(sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 -
2*a*c^2*e^3 - 3*sqrt(2)*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d^6 + 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^
5*e - 9*sqrt(a*c)*a*c^2*d^4*e^2 + 9*a^2*c^2*d^2*e^4 + 8*sqrt(2)*(a*c^3)^(3/
4)*a*d^3*e^3 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^5 + sqrt(a*c)*a^2*c*e^6) +
 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 6*sqrt(2)*(a*c^3)^(3/4)*c*d^7*e^2 - 1
2*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6
+ 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sq
```

```
rt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12)
- 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 6*sqrt(2)*(a*c^3)^(3/4)*c*d^7*e^2 -
12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6
 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12)
 - 1/2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*
c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^4 - 3*a*c*d^2*e^
8)*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3*
e^13) - 1/2*(9*c^2*d^8*e^3 + 10*a*c*d^4*e^7 + a^2*e^11 + 8*(c^2*d^7*e^4 + a
*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(x*e + d)^2)
```

maple [B] time = 0.01, size = 1201, normalized size = 1.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a),x)
```

```
[Out] -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-6*e^7*
d^2*c/(a*e^4+c*d^4)^3*ln(e*x+d)*a+10*e^3*d^6*c^2/(a*e^4+c*d^4)^3*ln(e*x+d)+
3/4*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
*d*e^8-3*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)
*x-1)*d^5*e^4+1/4*c^3/(a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d^9+3/8*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*ln((x^2+(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d
*e^8-3/2*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^4+1/8*c^3/(
a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^9+3/4*c/(a*e^4+c*d^4)^3*(a/
c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^8-3*c^2/(a*e^4+c*d^4
)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^4+1/4*c^3/(a*
e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9-1/2*
c/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*a^2+6*c^2/(a*e^
4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*d^4*e^5*a-3/2*c^3/(a*e^4+c
*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^8-5/4*c/(a*e^4+c*d^4)^3/(
a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1
/4)*2^(1/2)*x+(a/c)^(1/2)))*a*d^3*e^6+3/4*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2
^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2)))*d^7*e^2-5/2*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x-1)*a*d^3*e^6+3/2*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^7*e^2-5/2*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*d^3*e^6+3/2*c^2/(a*e^4+c*d^4)^3/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7*e^2+3/2*c/(a*e^4+c*
d^4)^3*ln(c*x^4+a)*e^7*d^2*a-5/2*c^2/(a*e^4+c*d^4)^3*ln(c*x^4+a)*e^3*d^6
```

maxima [A] time = 2.34, size = 817, normalized size = 1.20

$$-\frac{c\left(\frac{\sqrt{2}\left(10\,\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{9}{4}}d^6e^3-6\,\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{5}{4}}d^2e^7-c^3d^9+6\,\sqrt{a}\,c^{\frac{5}{2}}d^7e^2+12\,ac^2d^5e^4-10\,a^{\frac{3}{2}}c^{\frac{3}{2}}d^3e^6-3\,a^2cde^8\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(10\,\sqrt{2}\,a^{\frac{3}{4}}c\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*c*(sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(
5/4)*d^2*e^7 - c^3*d^9 + 6*sqrt(a)*c^(5/2)*d^7*e^2 + 12*a*c^2*d^5*e^4 - 10*
a^(3/2)*c^(3/2)*d^3*e^6 - 3*a^2*c*d*e^8)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4
)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(5/4)*d^2*e^7 + c^3*d^9 - 6*sqrt(a)*c^(5/2)
*d^7*e^2 - 12*a*c^2*d^5*e^4 + 10*a^(3/2)*c^(3/2)*d^3*e^6 + 3*a^2*c*d*e^8)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2
*(sqrt(2)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 - 12*sq
rt(2)*a^(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt
(2)*a^(9/4)*c^(5/4)*d*e^8 + 6*sqrt(a)*c^3*d^8*e - 24*a^(3/2)*c^2*d^4*e^5 +
2*a^(5/2)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))
/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2
)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 - 12*sqrt(2)*a^
(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt(2)*a^(9
/4)*c^(5/4)*d*e^8 - 6*sqrt(a)*c^3*d^8*e + 24*a^(3/2)*c^2*d^4*e^5 - 2*a^(5/2
)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sq
rt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^3*d^12 + 3*a*c^
2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)
*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) - 1
/2*(8*c*d^3*e^4*x + 9*c*d^4*e^3 + a*e^7)/(c^2*d^10 + 2*a*c*d^6*e^4 + a^2*d^
2*e^8 + (c^2*d^8*e^2 + 2*a*c*d^4*e^6 + a^2*e^10)*x^2 + 2*(c^2*d^9*e + 2*a*c
*d^5*e^5 + a^2*d*e^9)*x)
```

mupad [B] time = 3.67, size = 1955, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)^3),x)
```

```
[Out] symsum(log((c^7*d^5*e^6 + a*c^6*d*e^10)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12
*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 +
 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a
^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*
a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((208*a*c^7*d^7*e^7 - 48*a^2*
c^6*d^3*e^11)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 +
6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 +
 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^
3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d
^2*e*z + c^2, z, k)*((144*a*c^8*d^13*e^4 + 16*a^4*c^5*d*e^16 + 2608*a^2*c^7
*d^9*e^8 - 592*a^3*c^6*d^5*e^12)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 +
4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^
4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^
2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e
^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((896*a^4*c^6*d^7*e^13 - 1120*a^3*c^
7*d^11*e^9 - 1024*a^2*c^8*d^15*e^5 + 976*a^5*c^5*d^3*e^17 + 16*a*c^9*d^19*e
)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^
8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3
*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^
3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2
, z, k)*((384*a^7*c^4*d*e^22 - 128*a^2*c^9*d^21*e^2 - 128*a^3*c^8*d^17*e^6
+ 768*a^4*c^7*d^13*e^10 + 1792*a^5*c^6*d^9*e^14 + 1408*a^6*c^5*d^5*e^18)/(a
^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^
8) + (x*(320*a^7*c^4*e^23 - 192*a^2*c^9*d^20*e^3 - 448*a^3*c^8*d^16*e^7 + 1
28*a^4*c^7*d^12*e^11 + 1152*a^5*c^6*d^8*e^15 + 1088*a^6*c^5*d^4*e^19))/(a^4
*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)
) + (x*(80*a*c^9*d^18*e^2 - 1536*a^2*c^8*d^14*e^6 - 2016*a^3*c^7*d^10*e^10
+ 896*a^4*c^6*d^6*e^14 + 1296*a^5*c^5*d^2*e^18))/(a^4*e^16 + c^4*d^16 + 4*a
*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(36*a^4*c^5*e^1
7 - 4*c^9*d^16*e + 792*a*c^8*d^12*e^5 + 1632*a^2*c^7*d^8*e^9 - 152*a^3*c^6*
d^4*e^13))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a
^2*c^2*d^8*e^8)) + (x*(40*c^8*d^10*e^4 - 16*a*c^7*d^6*e^8 + 72*a^2*c^6*d^2*
e^12))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c
^2*d^8*e^8)) + (x*(a*c^6*e^11 + c^7*d^4*e^7))/(a^4*e^16 + c^4*d^16 + 4*a*c^
3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8))*root(768*a^5*c*d^4*e^8*
z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1
536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2
+ 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k), k, 1, 4) - ((a*e^7 + 9*
c*d^4*e^3)/(2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (4*c*d^3*e^4*x)/(a^2*e
^8 + c^2*d^8 + 2*a*c*d^4*e^4))/(d^2 + e^2*x^2 + 2*d*e*x) + (log(d + e*x)*(1
0*c^2*d^6*e^3 - 6*a*c*d^2*e^7))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*
a^2*c*d^4*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.401 $\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{3d}{}$$

```
[Out] 1/4*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)+3/4*d^2*e*arctan
(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)-3/32*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/3
2*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(
1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(
e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1854, 27, 12, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{3d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] -(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(4*a*c*(a + c*x^4)) + (3*d^2
*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) - (3*d*(Sqrt[c]*d^2 +
 Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c
^(3/4)) + (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4)) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^2}\,dx = -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} - \frac{\int \frac{-3d^3-6d^2ex-3de^2x^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} - \frac{\int -\frac{3d(d+ex)^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{(3d)\int \frac{(d+ex)^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{(3d)\int \left(\frac{2dex}{a+cx^4}+\frac{d^2+e^2x^2}{a+cx^4}\right)dx}{4a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{(3d)\int \frac{d^2+e^2x^2}{a+cx^4}\,dx}{4a} + \frac{\left(3d^2e\right)\int \frac{x}{a+cx^4}\,dx}{2a}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{4a} + \frac{\left(3d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}}{\cdots}}{8ac}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{\left(3d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16ac}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\cdots\right.}{16\sqrt{2}\,a^{7/4}c^{3/4}}$$

$$= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\cdots}{\sqrt[4]{a}}\right.}{8\sqrt{2}\,a^{7/4}c^{3/4}}$$

Mathematica [A] time = 0.37, size = 347, normalized size = 0.99

$$\frac{3\sqrt{2}\sqrt[4]{c}\left(a^{3/4}de^2-\sqrt[4]{a}\sqrt{c}\,d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+3\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}\,d^3-a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^2,x]
```

[Out] ((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(1/4)*(-(a^(1/4)*Sqrt[c]*d^3) + a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^2*c)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 342, normalized size = 0.98

$$\frac{3\,cdx^3e^2+3\,cd^2x^2e+cd^3x-ae^3}{4\left(cx^4+a\right)ac}+\frac{3\,\sqrt{2}\left(2\,\sqrt{2}\,\sqrt{ac}\,c^2d^2e+\left(ac^3\right)^{\frac{1}{4}}c^2d^3+\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}+\frac{3\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(3*c*d*x^3*e^2 + 3*c*d^2*x^2*e + c*d^3*x - a*e^3)/((c*x^4 + a)*a*c) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)

maple [A] time = 0.00, size = 390, normalized size = 1.12

$$\frac{e^3x^4}{4\left(c\,x^4+a\right)a}+\frac{3d\,e^2x^3}{4\left(c\,x^4+a\right)a}+\frac{3d^2e\,x^2}{4\left(c\,x^4+a\right)a}+\frac{d^3x}{4\left(c\,x^4+a\right)a}+\frac{3d^2e\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{4\sqrt{ac}\,a}+\frac{3\sqrt{2}\,d\,e^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3/(c*x^4+a)^2,x)

[Out] 1/4*d^3*x/a/(c*x^4+a)+3/32*d^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+3/4*e*d^2*x^2/a/(c*x^4+a)+3/4*e*d^2/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+3/4*d*e^2*x^3/a/(c*x^4+a)+3/32*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*e^3*x^4/a/(c*x^4+a)

maxima [A] time = 2.20, size = 332, normalized size = 0.95

$$\frac{3\,d\left(\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\,\sqrt{a}\,\sqrt{c}\,de\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{c}}}\right.}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")

[Out] 3/32*d*(sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a + 1/4*(3*c*d*e^2*x^3 + 3*c*d^2*e*x^2 + c*d^3*x - a*e^3)/(a*c^2*x^4 + a^2*c)

mupad [B] time = 0.43, size = 670, normalized size = 1.92

$$\left(\sum_{k=1}^{4}\ln\left(\frac{c\,d^2\,\left(27\,c\,d^5\,e^2-9\,a\,d\,e^6+36\,c\,d^4\,e^3\,x-\mathrm{root}\left(65536\,a^7\,c^3\,z^4+27648\,a^4\,c^2\,d^4\,e^2\,z^2+3456\,a^3\,c\,d^4\,e^5\,z-\right.\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((3*c*d^2*(27*c*d^5*e^2 - 9*a*d*e^6 + 36*c*d^4*e^3*x - 256*root(6
5536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*
a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a
^3*c^2*d - 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3
*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81
*c^2*d^12, z, k)*a*c^2*d^4*x + 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^
4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 +
 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*e^4*x + 512*root(65536*a^7*c^3*z
^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*
z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c^2*e*x - 1
92*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*
z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12,
z, k)*a^2*c*d*e^3))/(64*a^3))*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^
2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*
a^2*d^4*e^8 + 81*c^2*d^12, z, k), k, 1, 4) + ((d^3*x)/(4*a) - e^3/(4*c) + (
3*d^2*e*x^2)/(4*a) + (3*d*e^2*x^3)/(4*a))/(a + c*x^4)
```

sympy [A] time = 8.33, size = 350, normalized size = 1.00

$$\mathrm{RootSum}\left(65536t^4a^7c^3 + 27648t^2a^4c^2d^4e^2 + t\left(3456a^3cd^4e^5 - 3456a^2c^2d^8e\right) + 81a^2d^4e^8 + 162acd^8e^4 + 81c^2d^{12}, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 27648*_t**2*a**4*c**2*d**4*e**2 + _t*(3456*
a**3*c*d**4*e**5 - 3456*a**2*c**2*d**8*e) + 81*a**2*d**4*e**8 + 162*a*c*d**
8*e**4 + 81*c**2*d**12, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 +
28672*_t**3*a**6*c**3*d**4*e**2 - 7680*_t**2*a**5*c**2*d**4*e**5 + 1536*_t*
*2*a**4*c**3*d**8*e + 2160*_t*a**4*c*d**4*e**8 + 9216*_t*a**3*c**2*d**8*e**
4 + 144*_t*a**2*c**3*d**12 + 162*a**3*d**4*e**11 - 648*a**2*c*d**8*e**7 - 8
10*a*c**2*d**12*e**3)/(27*a**3*d**3*e**12 - 891*a**2*c*d**7*e**8 - 891*a*c*
*2*d**11*e**4 + 27*c**3*d**15)))) + (-a*e**3 + c*d**3*x + 3*c*d**2*e*x**2 +
 3*c*d*e**2*x**3)/(4*a**2*c + 4*a*c**2*x**4)
```

3.402 $\int \frac{(d+ex)^2}{\left(a+cx^4\right)^2} dx$

Optimal. Leaf size=322

$$-\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{(\sqrt{a}}{}$$

```
[Out] 1/4*x*(e*x+d)^2/a/(c*x^4+a)+1/2*d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(
1/2)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+
3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/1
6*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/
c^(3/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2
*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{(\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((
3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^2}\,dx = \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d^2-4dex-e^2x^2}{a+cx^4}\,dx}{4a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \left(-\frac{4dex}{a+cx^4}+\frac{-3d^2-e^2x^2}{a+cx^4}\right)dx}{4a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d^2-e^2x^2}{a+cx^4}\,dx}{4a} + \frac{(de)\int \frac{x}{a+cx^4}\,dx}{a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{(de)\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a} + \frac{\left(\frac{3\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \ldots$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{\left(\frac{3\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} + \ldots$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}} + \ldots$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}} + \ldots$$

Mathematica [A] time = 0.32, size = 321, normalized size = 1.00

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e^2-3\sqrt[4]{a}\sqrt{c}d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}+\frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{c}d^2-a^{3/4}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}-\frac{2\sqrt[4]{a}\left(8\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2+3\sqrt{\cdots}\right.}{c^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a^
(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 - 8*a^(1/4)*c^(1/4)*d*e
+ Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (
Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2 - a^(
3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/
(32*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.36, size = 323, normalized size = 1.00

$$\frac{x^3e^2+2\,dx^2e+d^2x}{4\left(cx^4+a\right)a}+\frac{\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ac}\,c^2de+3\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2+\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}+\frac{\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ac}\,c^2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e^2 + 2*d*x^2*e + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(4*sqrt(2)
*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)
*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1
```

```
/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d
^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
^3)
```

maple [A] time = 0.01, size = 362, normalized size = 1.12

$$\frac{e^{2} x^{3}}{4 \left(c\,x^{4}+a\right) a}+\frac{d e\,x^{2}}{2 \left(c\,x^{4}+a\right) a}+\frac{d^{2} x}{4 \left(c\,x^{4}+a\right) a}+\frac{d e \arctan \left(\sqrt{\frac{c}{a}}\;x^{2}\right)}{2 \sqrt{a c}\;a}+\frac{\sqrt{2}\,e^{2} \arctan \left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16 \left(\frac{a}{c}\right)^{\frac{1}{4}} a c}+\frac{\sqrt{2}\,e^{2} \arctan \left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{16 \left(\frac{a}{c}\right)^{\frac{1}{4}} a c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d^2*x/a/(c*x^4+a)+3/32*d^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d^2/a^
2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d^2/a^2*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/2*d*e*x^2/a/(c*x^4+a)+1/2*d*e
/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+1/4*e^2*x^3/a/(c*x^4+a)+1/32*e^2/a
/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/16*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+1/16*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)
^(1/4)*x-1)
```

maxima [A] time = 2.63, size = 318, normalized size = 0.99

$$\frac{e^{2} x^{3}+2\,d e x^{2}+d^{2} x}{4 \left(a c x^{4}+a^{2}\right)}+\frac{\frac{\sqrt{2} \left(3\,\sqrt{c}\,d^{2}-\sqrt{a}\,e^{2}\right) \log \left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}-\frac{\sqrt{2} \left(3\,\sqrt{c}\,d^{2}-\sqrt{a}\,e^{2}\right) \log \left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}+\frac{2 \left(3\,\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{3}{4}} \right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/(a*c*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(c
)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))
/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2
- sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(3*sqrt(2)*a^(
1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 8*sqrt(a)*sqrt(c)*d*e)*arc
tan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c
```

```
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(3/4)*
d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 8*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(
2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*
sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a
```

mupad [B] time = 2.48, size = 391, normalized size = 1.21

$$\frac{\frac{d^2\,x}{4\,a}+\frac{e^2\,x^3}{4\,a}+\frac{d\,e\,x^2}{2\,a}}{c\,x^4+a}+\left(\sum_{k=1}^{4}\ln\left(\frac{39\,c^2\,d^4\,e^2-a\,c\,e^6}{64\,a^3}-\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z+25\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^2,x)
```

```
[Out] ((d^2*x)/(4*a) + (e^2*x^3)/(4*a) + (d*e*x^2)/(2*a))/(a + c*x^4) + symsum(lo
g((39*c^2*d^4*e^2 - a*c*e^6)/(64*a^3) - root(65536*a^7*c^3*z^4 + 11264*a^4*
c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4
 + 81*c^2*d^8 + a^2*e^8, z, k)*(root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*
e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^
2*d^8 + a^2*e^8, z, k)*(12*c^3*d^2 - 16*c^3*d*e*x) + (x*(18*a*c^3*d^4 - 2*a
^2*c^2*e^4))/(8*a^3) + (2*c^2*d*e^3)/a) + (5*c^2*d^3*e^3*x)/(8*a^3))*root(6
5536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a
^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k), k, 1, 4)
```

sympy [A] time = 3.52, size = 318, normalized size = 0.99

$$\mathrm{RootSum}\left(65536t^4a^7c^3+11264t^2a^4c^2d^2e^2+t\left(256a^3cde^5-2304a^2c^2d^5e\right)+a^2e^8+82acd^4e^4+81c^2d^8,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*a
**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81*c
**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**3*a
**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4*c**
3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 3888*_t
*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c**2*d*
*9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4 + 729*c
**3*d**12)))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x**4)
```

3.403 $\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=241

$$-\frac{3d\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(\cdots\right.}{8\sqrt{2}\cdots}$$

```
[Out] 1/4*x*(e*x+d)/a/(c*x^4+a)+3/16*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/
4)/c^(1/4)*2^(1/2)+3/16*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/
4)*2^(1/2)-3/32*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4
)/c^(1/4)*2^(1/2)+3/32*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/
a^(7/4)/c^(1/4)*2^(1/2)+1/4*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{3d\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(\cdots\right.}{8\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
```

```
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx = \frac{x(d+ex)}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d-2ex}{a+cx^4}\,dx}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} - \frac{\int \left(-\frac{3d}{a+cx^4} - \frac{2ex}{a+cx^4}\right)dx}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{(3d)\int \frac{1}{a+cx^4}\,dx}{4a} + \frac{e\int \frac{x}{a+cx^4}\,dx}{2a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{(3d)\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{(3d)\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{(3d)\int \cdots}{\cdots}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\cdots\right.}{16\sqrt{2}\,a^{7/4}\cdots}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} - \frac{3d\log\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.19, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(d+ex)}{a+cx^4}-\frac{2\left(4\sqrt[4]{a}e+3\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}}+\frac{2\left(3\sqrt{2}\sqrt[4]{c}d-4\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}}-\frac{3\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}}{}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)*
e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/4)
*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (3*Sqr
t[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3
*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))
/(32*a^(7/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.34, size = 241, normalized size = 1.00

$$\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}-\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}+\frac{x^2e+dx}{4\left(cx^4+a\right)a}+\frac{\sqrt{2}\left(2\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqr
t(a/c))/(a^2*c) + 1/4*(x^2*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e
+ 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a^2*c^2)
```

maple [A] time = 0.00, size = 188, normalized size = 0.78

$$\frac{e\,x^{2}}{4\left(c\,x^{4}+a\right)a}+\frac{dx}{4\left(c\,x^{4}+a\right)a}+\frac{e\arctan\left(\sqrt{\frac{c}{a}}\;x^{2}\right)}{4\sqrt{ac}\;a}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;d\arctan\left(\frac{\sqrt{2}\;x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^{2}}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\;d\arctan\left(\frac{\sqrt{2}\;x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right.}{16a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d/a^2*(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d/a^2*(a/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*e*x^2/a/(c*x^4+a)+1/4*e/a/(a*c)^(1/
2)*arctan((1/a*c)^(1/2)*x^2)
```

maxima [A] time = 2.43, size = 238, normalized size = 0.99

$$\frac{ex^{2}+dx}{4\left(acx^{4}+a^{2}\right)}+\frac{\frac{3\sqrt{2}\,d\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{3\sqrt{2}\,d\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d-4\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}\right.}{2\sqrt{\sqrt{a}\sqrt{c}}}\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e*x^2 + d*x)/(a*c*x^4 + a^2) + 1/32*(3*sqrt(2)*d*log(sqrt(c)*x^2 + sqr
t(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 3*sqrt(2)*d*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(3*sqrt
(2)*a^(1/4)*c^(1/4)*d - 4*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*
c^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*d + 4*sqrt(a)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(c))*c^(1/4)))/a
```

mupad [B] time = 0.27, size = 282, normalized size = 1.17

$$\left(\sum_{k=1}^{4}\ln\left(\frac{c^{2}\left(3\,d\,e^{2}+2\,e^{3}\,x-\mathrm{root}\left(65536\,a^{7}\,c^{2}\,z^{4}+2048\,a^{4}\,c\,e^{2}\,z^{2}-1152\,a^{2}\,c\,d^{2}\,e\,z+81\,c\,d^{4}+16\,a\,e^{4},z,k\right)^{2}a^{3}\,c\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((c^2*(3*d*e^2 + 2*e^3*x - 192*root(65536*a^7*c^2*z^4 + 2048*a^4*
c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*d + 128
*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^
4 + 16*a*e^4, z, k)^2*a^3*c*e*x - 36*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^
2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)*a*c*d^2*x))/(16*a^3
))*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*
d^4 + 16*a*e^4, z, k), k, 1, 4) + ((e*x^2)/(4*a) + (d*x)/(4*a))/(a + c*x^4)
```

sympy [A] time = 1.09, size = 155, normalized size = 0.64

$$\operatorname{RootSum}\left(65536t^4a^7c^2 + 2048t^2a^4ce^2 - 1152ta^2cd^2e + 16ae^4 + 81cd^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6ce^2 - 4608t^2a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d**
2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e**
2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 + 360
*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 + 4*a
*c*x**4)
```

3.404 $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}}\cdots\right.}{8\sqrt{2}\,a^{7/4}\cdots}$$

```
[Out] 1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)
*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)-3
/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1
/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*
2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}}\cdots\right.}{8\sqrt{2}\,a^{7/4}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx = \frac{x}{4a\left(a+cx^4\right)} + \frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$= \frac{x}{4a\left(a+cx^4\right)} + \frac{3\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$= \frac{x}{4a\left(a+cx^4\right)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} - \ldots$$

$$= \frac{x}{4a\left(a+cx^4\right)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \ldots$$

$$= \frac{x}{4a\left(a+cx^4\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\ldots\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.11, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

fricas [A] time = 0.63, size = 173, normalized size = 0.86

$$\frac{12\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}\,a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right)+3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\ldots\right.}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3
/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x
^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 +
 a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 +
 a^2)
```

giac [A] time = 0.25, size = 194, normalized size = 0.96

$$\frac{x}{4\left(cx^4+a\right)a}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2\right.}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

maple [A] time = 0.00, size = 143, normalized size = 0.71

$$\frac{x}{4\left(cx^4+a\right)a}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16/a^2*(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.11, size = 189, normalized size = 0.94

$$\frac{x}{4\left(acx^4+a^2\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\right.}{}\right.}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a
```

mupad [B] time = 0.09, size = 58, normalized size = 0.29

$$\frac{x}{4a\left(cx^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{8(-a)^{7/4}c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{8(-a)^{7/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^2,x)
```

```
[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4
)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

sympy [A] time = 0.30, size = 39, normalized size = 0.19

$$\frac{x}{4a^2+4acx^4}+\mathrm{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

3.405 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=855

$$\frac{\log(d+ex)e^7}{\left(cd^4+ae^4\right)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}\,d^2\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2\right.}{2}$$

```
[Out] 1/4*(a*e^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)+e^7*ln(e*
x+d)/(a*e^4+c*d^4)^2-1/4*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)^2-1/4*d^2*e*arctan(x
^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)-1/2*d^2*e^5*arctan(x^2*c^
(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*d*e^4*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)
/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2
)+1/4*c^(1/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c
^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*d*e^4*arctan(1+c^(1/4)*
x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2
)-1/32*c^(1/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a
^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)+1/32*c^(1/4)*d*ln(a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7
/4)/(a*e^4+c*d^4)*2^(1/2)+1/16*c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)+1/16*c^(1/4)*d
*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a
*e^4+c*d^4)*2^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{\log(d+ex)e^7}{\left(cd^4+ae^4\right)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}\,d^2\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4))
 - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^
4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a
*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(
```

```
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqr
t[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2)
+ (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])/(c*d^4 +
a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) -
(c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4
*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2
 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16
*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)
^2)
```

<u>Rule 204</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

<u>Rule 205</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 260</u>

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

<u>Rule 275</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

<u>Rule 617</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
```

```
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \int\left(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} - \frac{ce^4\left(-d^3+d^2ex-de^2x^2+\right.}{\left(cd^4+ae^4\right)^2\left(a+cx^4\right.}\right.$$

$$= \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2} + \frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{\left(a+cx^4\right)^2}\,dx}{cd^4+ae^4}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int\left(\frac{-d^3-de^2x^2}{a+cx^4}+\frac{x\left(d^2e+e^3x^2\right)}{a+cx^4}\right)dx}{\left(cd^4+ae^4\right)^2} -$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \frac{-d^3-de^2x^2}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \frac{x\left(d^2e+e\right.}{a+cx}}{\left(cd^4+ae^4\right.}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\text{Subst}\left(\int \frac{d^2e+e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2} + \frac{(}{}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(cd^2e^5\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2} - \frac{(}{}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} + \frac{e^7\log}{\left(cd^4\right.}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}\,de^4}{}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}\,de^4}{}$$

Mathematica [A] time = 0.39, size = 558, normalized size = 0.65

$$\frac{\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}de^6+\sqrt{a}cd^5e^2-7a\sqrt{c}d^3e^4-3c^{3/2}d^7\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(-5a^{3/2}de^6-\sqrt{a}cd^5e^2+7a\sqrt{c}d^3e^4+3c^{3/2}d^7\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{\right.}}{a^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)
) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]
*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*e^5
 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4)
+ (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*S
qrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d*e^5 +
 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) +
32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*d^5*e^2
- 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 - Sqrt[a]*c
*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/(32*(c*d^4 +
 a*e^4)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.50, size = 771, normalized size = 0.90

$$\frac{\left(4\sqrt{2}\sqrt{ac}\,c^2d^2e+3\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^3d^4-4\left(ac^3\right)^{\frac{1}{4}}a^2c^2d^3e+4\sqrt{2}\sqrt{ac}\,a^2c^2d^2e^2+\sqrt{2}a^3c^2e^4-4\left(ac^3\right)^{\frac{3}{4}}a^2de^3\right)}+\frac{\left(4\sqrt{2}\sqrt{ac}\,c^2d^2e+3\right.}{8\left(\sqrt{2}a^2c^3d^4+4\left(ac^3\right)^{\frac{1}{4}}a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3
/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqr
t(2)*a^2*c^3*d^4 - 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*
c^2*d^2*e^2 + sqrt(2)*a^3*c^2*e^4 - 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/8*(4*sqr
t(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c
^3*d^4 + 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^
2 + sqrt(2)*a^3*c^2*e^4 + 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/32*(3*sqrt(2)*(a*c
^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sqrt(2)*(a*c^3)^(1/
4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/
c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/3
2*(3*sqrt(2)*(a*c^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sq
rt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^
2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a
^4*c^2*e^8) - 1/4*e^7*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^
8) + e^8*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/4*(a*c
*d^4*e^3 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 +
a^2*e^7 + (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(c*x^4 + a)*a)
```

maple [A] time = 0.02, size = 1122, normalized size = 1.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] e^7*ln(e*x+d)/(a*e^4+c*d^4)^2+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*d*e^6*x^3+1/4
*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^5*e^2/a*x^3-1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a
)*e^5*d^2*x^2-1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*e*d^6/a*x^2+1/4*c/(a*e^4+c*
d^4)^2/(c*x^4+a)*d^3*x*e^4+1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^7/a*x+1/4/(a
*e^4+c*d^4)^2/(c*x^4+a)*e^7*a+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*e^3*d^4+7/16*
c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3
*e^4+3/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x-1)*d^7+7/32*c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(
1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e^
4+3/32*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7+7/16*c/(a*e
^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^4+3
/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^7-3/4*c/(a*e^4+c*d^4)^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^
2-1/4*c^2/(a*e^4+c*d^4)^2/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^6+5/3
2/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(
1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^6+1/32*c/(a*e^4+c*d^4)^2
```

```
/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^2+5/16/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^6+1/16*c/(a*e^4+c*d^4)^2/a/(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^2+5/16/(a*e^4+c*d^4)^2/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^6+1/16*c/(a*e^4+c*d
^4)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^2-1/4*e^7
*ln(c*x^4+a)/(a*e^4+c*d^4)^2
```

maxima [A] time = 2.29, size = 601, normalized size = 0.70

$$\frac{e^7 \log(ex+d)}{c^2d^8+2\,acd^4e^4+a^2e^8} - \frac{c\left(\frac{\sqrt{2}\left(4\,\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{1}{4}}e^7-3\,c^2d^7+\sqrt{a}\,c^{\frac{3}{2}}d^5e^2-7\,acd^3e^4+5\,a^{\frac{3}{2}}\sqrt{c}\,de^6\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(4\,\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{1}{4}}e^7+\cdots\right)}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] e^7*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - 1/32*c*(sqrt(2)*(4*s
qrt(2)*a^(7/4)*c^(1/4)*e^7 - 3*c^2*d^7 + sqrt(a)*c^(3/2)*d^5*e^2 - 7*a*c*d^
3*e^4 + 5*a^(3/2)*sqrt(c)*d*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(7/4)*c^(1/4)*e^7 + 3
*c^2*d^7 - sqrt(a)*c^(3/2)*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^(3/2)*sqrt(c)*d*e^
6)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
 - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^2 + 7*s
qrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6 + 4*sqrt(a
)*c^2*d^6*e + 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^
2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6 - 4
*sqrt(a)*c^2*d^6*e - 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
- sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqr
t(c))*c^(5/4)))/(a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8) + 1/4*(c*d*e^2*x^3
- c*d^2*e*x^2 + c*d^3*x + a*e^3)/(a^2*c*d^4 + a^3*e^4 + (a*c^2*d^4 + a^2*c*
e^4)*x^4)
```

mupad [B] time = 2.99, size = 1591, normalized size = 1.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)),x)
```

```
[Out] e^3/(4*(a^2*e^4 + c^2*d^4*x^4 + a*c*d^4 + a*c*e^4*x^4)) + symsum(log((81*c^
5*d^5*e^6 + 64*a*c^4*d*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)
) + root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z
^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*
a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a
^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^
7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4
096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4
+ 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*
c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 8
1*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d
^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 2
4576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e
^4, z, k)*((98304*a^9*c^4*d*e^14 - 32768*a^6*c^7*d^13*e^2 + 32768*a^7*c^6*d
^9*e^6 + 163840*a^8*c^5*d^5*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4
*e^4)) + (x*(81920*a^9*c^4*e^15 - 49152*a^6*c^7*d^12*e^3 - 16384*a^7*c^6*d^
8*e^7 + 114688*a^8*c^5*d^4*e^11))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4
*e^4))) + (52224*a^7*c^4*d*e^13 - 3072*a^4*c^7*d^13*e + 13312*a^5*c^6*d^9*e
^5 + 68608*a^6*c^5*d^5*e^9)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))
 + (x*(61440*a^7*c^4*e^14 - 8192*a^4*c^7*d^12*e^2 - 4096*a^5*c^6*d^8*e^6 +
65536*a^6*c^5*d^4*e^10))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) +
 (8704*a^5*c^4*d*e^12 + 3584*a^3*c^6*d^9*e^4 + 15360*a^4*c^5*d^5*e^8)/(256*
(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(15360*a^5*c^4*e^13 - 576*a
^2*c^7*d^12*e + 1920*a^3*c^6*d^8*e^5 + 18880*a^4*c^5*d^4*e^9))/(256*(a^6*e^
8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (192*a*c^6*d^9*e^3 + 704*a^3*c^4*d*e
^11 + 1536*a^2*c^5*d^5*e^7)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))
 + (x*(1280*a^3*c^4*e^12 + 256*a*c^6*d^8*e^4 + 2240*a^2*c^5*d^4*e^8))/(256*
(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (81*c^5*d^4*e^7*x)/(256*(a^6*
e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)))*root(131072*a^8*c*d^4*e^4*z^4 + 6553
6*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*
e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^
4 + 256*a*e^4, z, k), k, 1, 4) + (e^7*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*
a*c*d^4*e^4) + (c*d^3*x)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^
4*x^4)) - (c*d^2*e*x^2)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4
*x^4)) + (c*d*e^2*x^3)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*
x^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.406 $\int \frac{1}{(d+ex)^2(a+cx^4)^2}\,dx$

Optimal. Leaf size=1141

$$\frac{8cd^3\log(d+ex)e^7}{(cd^4+ae^4)^3}-\frac{2cd^3\log\left(cx^4+a\right)e^7}{(cd^4+ae^4)^3}-\frac{e^7}{(cd^4+ae^4)^2(d+ex)}-\frac{\sqrt{c}\,d\left(3cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}\left(5c\right.\right.}{}$$

```
[Out] -e^7/(a*e^4+c*d^4)^2/(e*x+d)+1/4*c*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*d
*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a)+
8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)
^3-1/2*d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^
4+c*d^4)^2-d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^
4+c*d^4)^3/a^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c
^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(7
/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/
2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2
)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^
(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(7/4)/(
a*e^4+c*d^4)^2*2^(1/2)-1/8*c^(1/4)*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^(1/2
))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(1/4)*e^4*ln(a^(1/4)*c^(1/4)*x*2^(
1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*
d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*e^4*arctan(-1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4
)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*e^4*arctan(1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^
(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)
```

Rubi [A] time = 1.66, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{8cd^3\log(d+ex)e^7}{(cd^4+ae^4)^3}-\frac{2cd^3\log\left(cx^4+a\right)e^7}{(cd^4+ae^4)^3}-\frac{e^7}{(cd^4+ae^4)^2(d+ex)}-\frac{\sqrt{c}\,d\left(3cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}\left(5c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] -(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*
a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4
 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*
Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/
4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3
) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*
e^4)^2) + (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*
d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)
*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
+ a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(
7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a
*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(
Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a
*e^4)^3) - (2*c*d^3*e^7*Log[a + c*x^4])/(c*d^4 + a*e^4)^3
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx = \int\left(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)^2}+\frac{8cd^3e^8}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-\right.\right.}{\left(cd^4+a\right.}\right.$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{8cd^3e^7\log(d+ex)}{\left(cd^4+ae^4\right)^3}+\frac{\left(ce^4\right)\int\frac{d^2\left(5cd^4-3ae^4\right)-2de\left(3cd^4-ae^4\right)x+e^2\left(7c\right.}{a+cx^4}}{\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

Mathematica [A] time = 0.86, size = 807, normalized size = 0.71

$$\frac{256cd^3\log(d+ex)e^7-64cd^3\log\left(cx^4+a\right)e^7-\frac{32\left(cd^4+ae^4\right)e^7}{d+ex}+\frac{8c\left(cd^4+ae^4\right)\left(cx\left(d^2-2exd+3e^2x^2\right)d^4+ae^3\left(4d^3-3exd^2+2e^2x^2d-e^3x^3\right.\right.}{a\left(cx^4+a\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] ((-32*e^7*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c*d^4*x*(d^2 -
 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))
/(a*(a + c*x^4)) + (2*c^(1/4)*(-3*Sqrt[2]*c^(5/2)*d^10 + 8*a^(1/4)*c^(9/4)*
d^9*e - 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 - 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 48*a
^(5/4)*c^(5/4)*d^5*e^5 - 30*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 21*Sqrt[2]*a^2*Sqrt
[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 + 5*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*(3*Sqrt[2]*c^(5/2)*d^10
 + 8*a^(1/4)*c^(9/4)*d^9*e + 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 14*Sqrt[2]*a*c
^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 + 30*Sqrt[2]*a^(3/2)*c*d^4*e^6
- 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 - 5*Sqrt[2]*a^(
5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 256*c*d^3*e^7
*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 +
14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^
(5/2)*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4)
 + (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 + 14*a*c^(3/2)*
d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^(5/2)*e^10)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 64*c*d^3*e
^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 103.64, size = 1104, normalized size = 0.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -2*c*d^3*e^7*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*
e^8 + a^3*e^12) + 8*c*d^3*e^8*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e
^5 + 3*a^2*c*d^4*e^9 + a^3*e^13) + 1/8*(5*sqrt(2)*sqrt(a*c)*c^2*d^3*e + 3*(
a*c^3)^(1/4)*c^2*d^4 + 3*sqrt(2)*a*c^2*d*e^3 + 6*(a*c^3)^(3/4)*d^2*e^2 - 5*
(a*c^3)^(1/4)*a*c*e^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(sqrt(2)*a^2*c^3*d^6 - 6*(a*c^3)^(1/4)*a^2*c^2*d^5*e + 9*sqrt(2)*sq
rt(a*c)*a^2*c^2*d^4*e^2 + 9*sqrt(2)*a^3*c^2*d^2*e^4 - 16*(a*c^3)^(3/4)*a^2*
d^3*e^3 - 6*(a*c^3)^(1/4)*a^3*c*d*e^5 + sqrt(2)*sqrt(a*c)*a^3*c*e^6) + 1/8*
(5*sqrt(2)*sqrt(a*c)*c^2*d^3*e + 3*(a*c^3)^(1/4)*c^2*d^4 - 3*sqrt(2)*a*c^2*
d*e^3 + 6*(a*c^3)^(3/4)*d^2*e^2 - 5*(a*c^3)^(1/4)*a*c*e^4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^6 + 6*(a*c^3
)^(1/4)*a^2*c^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^2*c^2*d^4*e^2 + 9*sqrt(2)*a^3
*c^2*d^2*e^4 + 16*(a*c^3)^(3/4)*a^2*d^3*e^3 + 6*(a*c^3)^(1/4)*a^3*c*d*e^5 +
 sqrt(2)*sqrt(a*c)*a^3*c*e^6) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 - 3*
sqrt(2)*(a*c^3)^(3/4)*c^2*d^8*e^2 + 14*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4
- 30*sqrt(2)*(a*c^3)^(3/4)*a*c*d^4*e^6 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d
^2*e^8 + 5*sqrt(2)*(a*c^3)^(3/4)*a^2*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4)
+ sqrt(a/c))/(a^2*c^5*d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^
2*e^12) - 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 - 3*sqrt(2)*(a*c^3)^(3/4)*
c^2*d^8*e^2 + 14*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4 - 30*sqrt(2)*(a*c^3)^(
3/4)*a*c*d^4*e^6 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^2*e^8 + 5*sqrt(2)*(a*
c^3)^(3/4)*a^2*e^10)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*
d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2*e^12) + 1/4*(3*c^2*d
^4*x^4*e^3 + c^2*d^5*x^3*e^2 - c^2*d^6*x^2*e + c^2*d^7*x - 5*a*c*x^4*e^7 +
a*c*d*x^3*e^6 - a*c*d^2*x^2*e^5 + a*c*d^3*x*e^4 + 4*a*c*d^4*e^3 - 4*a^2*e^7
)/((a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8)*(c*x^5*e + c*d*x^4 + a*x*e + a*d
))
```

maple [A] time = 0.02, size = 1636, normalized size = 1.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)
^3-e^7/(a*e^4+c*d^4)^2/(e*x+d)-1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^6*x*e^4-
3*c^2/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^5-5/32/(a
*e^4+c*d^4)^3*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^10-5/16/(a*e^4+c*d^4)^3*a/(a
/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^10-5/16/(a*e^4+c*d^4)^3
*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^10+1/4*c^3/(a*e^4+
c*d^4)^3/(c*x^4+a)*d^10/a*x-1/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)*e^10*a*x^3+c/(a
*e^4+c*d^4)^3/(c*x^4+a)*e^7*d^3*a+1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*e^6*x^3
*d^4+c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^7*e^3-21/16*c/(a*e^4+c*d^4)^3*(a/c)^(1
```

```
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^4)^
3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^10-1/2*c^3/(a*e
^4+c*d^4)^3/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^9+15/16*c/(a*e^4+c*
d^4)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+
(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^4*e^6+15/8*c/(a*e^4+c*d^4)^3/(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4*e^6+15/8*c/(a*e^4+c*d^4)^3/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^4*e^6+3/2*c/(a*e^4+c*
d^4)^3*a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*d+3/4*c^3/(a*e^4+c*d^4)^
3/(c*x^4+a)*e^2/a*x^3*d^8-1/2*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*d^9*e/a*x^2+1/2
*c/(a*e^4+c*d^4)^3/(c*x^4+a)*d*e^9*a*x^2-3/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)*d^
2*a*x*e^8-21/16*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x-1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)*d^10-21/32*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*l
n((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^
(1/2)))*d^2*e^8+3/32*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d
^10+7/16*c^2/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^6*e^4+7/8*c^2
/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^6*
e^4+3/16*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
4)*x+1)*d^8*e^2+3/16*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/c)^(1/4)*x-1)*d^8*e^2+7/8*c^2/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^6*e^4+3/32*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/
4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2)))*d^8*e^2
```

maxima [A] time = 2.54, size = 961, normalized size = 0.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 8*c*d^3*e^7*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^
3*e^12) - 1/32*c*(sqrt(2)*(32*sqrt(2)*a^(7/4)*c^(5/4)*d^3*e^7 - 3*c^3*d^10
+ 3*sqrt(a)*c^(5/2)*d^8*e^2 - 14*a*c^2*d^6*e^4 + 30*a^(3/2)*c^(3/2)*d^4*e^6
 + 21*a^2*c*d^2*e^8 - 5*a^(5/2)*sqrt(c)*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(
1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(7/4)*c
^(5/4)*d^3*e^7 + 3*c^3*d^10 - 3*sqrt(a)*c^(5/2)*d^8*e^2 + 14*a*c^2*d^6*e^4
- 30*a^(3/2)*c^(3/2)*d^4*e^6 - 21*a^2*c*d^2*e^8 + 5*a^(5/2)*sqrt(c)*e^10)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2
*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14
*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*
sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^10 + 8*sqrt(
a)*c^3*d^9*e + 48*a^(3/2)*c^2*d^5*e^5 - 24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt
```

```
(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)
*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sq
rt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sq
rt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt
(2)*a^(11/4)*c^(3/4)*e^10 - 8*sqrt(a)*c^3*d^9*e - 48*a^(3/2)*c^2*d^5*e^5 +
24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/
4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a*c^3*
d^12 + 3*a^2*c^2*d^8*e^4 + 3*a^3*c*d^4*e^8 + a^4*e^12) + 1/4*(4*a*c*d^4*e^3
 - 4*a^2*e^7 + (3*c^2*d^4*e^3 - 5*a*c*e^7)*x^4 + (c^2*d^5*e^2 + a*c*d*e^6)*
x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 + (c^2*d^7 + a*c*d^3*e^4)*x)/(a^2*c^2*d
^9 + 2*a^3*c*d^5*e^4 + a^4*d*e^8 + (a*c^3*d^8*e + 2*a^2*c^2*d^4*e^5 + a^3*c
*e^9)*x^5 + (a*c^3*d^9 + 2*a^2*c^2*d^5*e^4 + a^3*c*d*e^8)*x^4 + (a^2*c^2*d^
8*e + 2*a^3*c*d^4*e^5 + a^4*e^9)*x)
```

mupad [B] time = 4.10, size = 2246, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)^2),x)
```

```
[Out] symsum(log(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 655
36*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 1812
48*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 1
9200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((120*a*c^8*d^14*e^3 +
 2664*a^2*c^7*d^10*e^7 - 10904*a^3*c^6*d^6*e^11 + 19320*a^4*c^5*d^2*e^15)/(
256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^
6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^
4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3
 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*
e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((4096*a^3*c^8*
d^15*e^4 + 54272*a^4*c^7*d^11*e^8 - 2048*a^5*c^6*d^7*e^12 + 144384*a^6*c^5*
d^3*e^16)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12
*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2
*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*
d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a
^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*(roo
t(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^1
2*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e
^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e
^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((98304*a^11*c^4*d*e^22 - 32768*a^6*
c^9*d^21*e^2 - 32768*a^7*c^8*d^17*e^6 + 196608*a^8*c^7*d^13*e^10 + 458752*a
^9*c^6*d^9*e^14 + 360448*a^10*c^5*d^5*e^18)/(256*(a^8*e^16 + a^4*c^4*d^16 +
 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(81920*a^
11*c^4*e^23 - 49152*a^6*c^9*d^20*e^3 - 114688*a^7*c^8*d^16*e^7 + 32768*a^8*
```

```
c^7*d^12*e^11 + 294912*a^9*c^6*d^8*e^15 + 278528*a^10*c^5*d^4*e^19))/(256*(
a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2
*d^8*e^8))) + (5120*a^9*c^4*e^21 - 3072*a^4*c^9*d^20*e + 17408*a^5*c^8*d^16
*e^5 + 337920*a^6*c^7*d^12*e^9 + 616448*a^7*c^6*d^8*e^13 + 304128*a^8*c^5*d
^4*e^17)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*
e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(356352*a^6*c^7*d^11*e^10 - 32768*a^5*c^8*d^
15*e^6 - 10240*a^4*c^9*d^19*e^2 + 770048*a^7*c^6*d^7*e^14 + 391168*a^8*c^5*
d^3*e^18))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^1
2*e^4 + 6*a^6*c^2*d^8*e^8))) + (x*(768*a^3*c^8*d^14*e^5 - 576*a^2*c^9*d^18*
e + 105088*a^4*c^7*d^10*e^9 + 221952*a^5*c^6*d^6*e^13 + 183744*a^6*c^5*d^2*
e^17))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^
4 + 6*a^6*c^2*d^8*e^8))) + (x*(200*a*c^8*d^13*e^4 + 19400*a^4*c^5*d*e^16 +
7512*a^2*c^7*d^9*e^8 + 2136*a^3*c^6*d^5*e^12))/(256*(a^8*e^16 + a^4*c^4*d^1
6 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8))) + (81*c^7*
d^9*e^6 - 254*a*c^6*d^5*e^10 + 625*a^2*c^5*d*e^14)/(256*(a^8*e^16 + a^4*c^4
*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(6
25*a^2*c^5*e^15 + 81*c^7*d^8*e^7 - 894*a*c^6*d^4*e^11))/(256*(a^8*e^16 + a^
4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)))*r
oot(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d
^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2
*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d
*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k), k, 1, 4) + ((x^4*(3*c^2*d^4*e^3 -
 5*a*c*e^7))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (a*e^7 - c*d^4*e^3
)/(a*e^4 + c*d^4)^2 + (c*d^3*x)/(4*a*(a*e^4 + c*d^4)) - (c*d^2*e*x^2)/(4*a*
(a*e^4 + c*d^4)) + (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)))/(a*d + a*e*x + c*d*
x^4 + c*e*x^5) + (8*c*d^3*e^7*log(d + e*x))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*
d^8*e^4 + 3*a^2*c*d^4*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.407 $\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=1384

$$\frac{12cd^2\left(3cd^4-ae^4\right)\log(d+ex)e^7}{\left(cd^4+ae^4\right)^4}-\frac{3cd^2\left(3cd^4-ae^4\right)\log\left(cx^4+a\right)e^7}{\left(cd^4+ae^4\right)^4}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex}$$

```
[Out] -1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)+1/4*
c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-e
*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2))/
a/(a*e^4+c*d^4)^3/(c*x^4+a)+12*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(e*x+d)/(a*e^4+
c*d^4)^4-3*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/4*e*(a^
2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)
/(a*e^4+c*d^4)^3-1/2*e^5*(a^2*e^8-26*a*c*d^4*e^4+21*c^2*d^8)*arctan(x^2*c^(
1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/32*c^(3/4)*d*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8-2*
d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)
+1/32*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^
8-36*a*c*d^4*e^4+9*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^
(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^
(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(
1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(1+c^(1/
4)*x*2^(1/2)/a^(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e
^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(3/4)*d*
e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*a^2*e^8+30*a*c*d
^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*
e^4+c*d^4)^4*2^(1/2)-1/8*c^(3/4)*d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(-3*a^2*e^8+30*a*c*d^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c
*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(3/4)*d*e^4*ar
ctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*a^2*e^8-30*a*c*d^4*e^4+15*c^2*d^8+4*d
^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+
1/4*c^(3/4)*d*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*a^2*e^8-30*a*c*d^4
*e^4+15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^
4+c*d^4)^4*2^(1/2)
```

Rubi [A] time = 1.96, antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{12cd^2\left(3cd^4-ae^4\right)\log(d+ex)e^7}{\left(cd^4+ae^4\right)^4}-\frac{3cd^2\left(3cd^4-ae^4\right)\log\left(cx^4+a\right)e^7}{\left(cd^4+ae^4\right)^4}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] -e^7/(2*(c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^3*(
d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^
4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e
^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c
]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 +
a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3) - (c^
(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2
*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a
^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*
d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d
*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5
*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(3*
c*d^4 - a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 - 36
*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5
*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c
^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^
2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt
[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[a + c*x^4])/(c*d^4
 + a*e^4)^4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx = \int\left(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)^3}+\frac{8cd^3e^8}{\left(cd^4+ae^4\right)^3(d+ex)^2}+\frac{12cd^2e^8\left(3cd^4-ae^4\right)}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{c\left(d\left(c\right.\right.}{}\right.$$

$$=-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{12cd^2e^7\left(3cd^4-ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^4}$$

$$=-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

$$=-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

$$=-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

$$=-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

$$=-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

$$=-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

$$=-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

Mathematica [A] time = 1.36, size = 996, normalized size = 0.72

$$\frac{384cd^2\left(3cd^4-ae^4\right)\log(d+ex)e^7-96cd^2\left(3cd^4-ae^4\right)\log\left(cx^4+a\right)e^7-\frac{256cd^3\left(cd^4+ae^4\right)e^7}{d+ex}-\frac{16\left(cd^4+ae^4\right)^2e^7}{(d+ex)^2}+\frac{8c\left(cd^4\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] ((-16*e^7*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (256*c*d^3*e^7*(c*d^4 + a*e^4))/
(d + e*x) + (8*c*(c*d^4 + a*e^4)*(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) +
c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x +
6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x^4)) - (6*Sqrt[c]*(Sqrt[2]*c^(13/4)*d
^13 - 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*Sqrt[2
]*a*c^(9/4)*d^9*e^4 - 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(7/4)*d
^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 + 84*a^(9/4)*c*d^4*e^9 - 30*Sqrt[2]
*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 - 4*a^(13/4)*e^13)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[c]*(Sqrt[2]*c^(
13/4)*d^13 + 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9
*Sqrt[2]*a*c^(9/4)*d^9*e^4 + 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^
(7/4)*d^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 - 84*a^(9/4)*c*d^4*e^9 - 30*
Sqrt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 + 4*a^(13/4
)*e^13)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 384*c*d^2*e^7*(3
*c*d^4 - a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*Sqrt[a]*c^(
5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 49*a^2*c*d^5
*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*
Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 4
9*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 96*c*d^2*e^7*(3*c*d^4
- a*e^4)*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.15, size = 1488, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/8*(2*sqrt(2)*sqrt(a*c)*c^3*d^4*e + (a*c^3)^(1/4)*c^3*d^5 + 4*sqrt(2)*a*c^
3*d^2*e^3 + 2*(a*c^3)^(3/4)*c*d^3*e^2 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2*sqr
t(2)*sqrt(a*c)*a*c^2*e^5)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(sqrt(2)*a^2*c^4*d^8 - 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 16*sqrt(2
)*sqrt(a*c)*a^2*c^3*d^6*e^2 + 34*sqrt(2)*a^3*c^3*d^4*e^4 - 40*(a*c^3)^(3/4)
*a^2*c*d^5*e^3 - 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^
3*c^2*d^2*e^6 + sqrt(2)*a^4*c^2*e^8 - 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/8*(2*s
qrt(2)*sqrt(a*c)*c^3*d^4*e + (a*c^3)^(1/4)*c^3*d^5 - 4*sqrt(2)*a*c^3*d^2*e^
3 + 2*(a*c^3)^(3/4)*c*d^3*e^2 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2*sqrt(2)*sqr
t(a*c)*a*c^2*e^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(sqrt(2)*a^2*c^4*d^8 + 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 16*sqrt(2)*sqrt(a
*c)*a^2*c^3*d^6*e^2 + 34*sqrt(2)*a^3*c^3*d^4*e^4 + 40*(a*c^3)^(3/4)*a^2*c*d
^5*e^3 + 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^3*c^2*d^
2*e^6 + sqrt(2)*a^4*c^2*e^8 + 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/32*(sqrt(2)*(a
*c^3)^(1/4)*c^4*d^13 - 2*sqrt(2)*(a*c^3)^(3/4)*c^2*d^11*e^2 + 9*sqrt(2)*(a*
c^3)^(1/4)*a*c^3*d^9*e^4 - 36*sqrt(2)*(a*c^3)^(3/4)*a*c*d^7*e^6 - 49*sqrt(2
)*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 30*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10 + 7
*sqrt(2)*(a*c^3)^(1/4)*a^3*c*d*e^12)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt
(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^12*e^4 + 6*a^4*c^3*d^8*e^8 + 4*a^5*c^2*d
^4*e^12 + a^6*c*e^16) - 3/32*(sqrt(2)*(a*c^3)^(1/4)*c^4*d^13 - 2*sqrt(2)*(a
*c^3)^(3/4)*c^2*d^11*e^2 + 9*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^9*e^4 - 36*sqrt(
2)*(a*c^3)^(3/4)*a*c*d^7*e^6 - 49*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 3
0*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10 + 7*sqrt(2)*(a*c^3)^(1/4)*a^3*c*d*e^12
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^
12*e^4 + 6*a^4*c^3*d^8*e^8 + 4*a^5*c^2*d^4*e^12 + a^6*c*e^16) - 3*(3*c^2*d^
6*e^7 - a*c*d^2*e^11)*log(abs(c*x^4 + a))/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*
a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) + 12*(3*c^2*d^6*e^8 - a*c*d^
2*e^12)*log(abs(x*e + d))/(c^4*d^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2*d^8*e^
9 + 4*a^3*c*d^4*e^13 + a^4*e^17) + 1/4*(10*a*c^3*d^12*e^3 - 30*a^2*c^2*d^8*
e^7 - 42*a^3*c*d^4*e^11 + 6*(c^4*d^11*e^4 - 6*a*c^3*d^7*e^8 - 7*a^2*c^2*d^3
*e^12)*x^5 + 3*(3*c^4*d^12*e^3 - 11*a*c^3*d^8*e^7 - 15*a^2*c^2*d^4*e^11 - a
^3*c*e^15)*x^4 - 2*a^4*e^15 + (c^4*d^13*e^2 + 3*a*c^3*d^9*e^6 + 3*a^2*c^2*d
^5*e^10 + a^3*c*d*e^14)*x^3 - (c^4*d^14*e + 3*a*c^3*d^10*e^5 + 3*a^2*c^2*d^
6*e^9 + a^3*c*d^2*e^13)*x^2 + (c^4*d^15 + 9*a*c^3*d^11*e^4 - 33*a^2*c^2*d^7
*e^8 - 41*a^3*c*d^3*e^12)*x)/((c*d^4 + a*e^4)^4*(c*x^4 + a)*(x*e + d)^2*a)
```

maple [A] time = 0.03, size = 2121, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^2,x)
```

```
[Out] -1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)-12*e
^11*d^2*c/(a*e^4+c*d^4)^4*ln(e*x+d)*a-1/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*e^13*
a^2*x^2-3/2*c/(a*e^4+c*d^4)^4/(c*x^4+a)*e^11*d^2*a^2-c^3/(a*e^4+c*d^4)^4/(c
*x^4+a)*d^7*e^6*x^3+9/4*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)*e^5*x^2*d^8-11/4*c^3/
(a*e^4+c*d^4)^4/(c*x^4+a)*d^9*x*e^4+1/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^13/
a*x+c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*a*d^6*e^7-33/4*c^3/(a*e^4+c*d^4)^4/(a*c)^
(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^8-3/4*c/(a*e^4+c*d^4)^4*a^2/(a*c)^(1/
2)*arctan((1/a*c)^(1/2)*x^2)*e^13+3*c/(a*e^4+c*d^4)^4*a*ln(c*x^4+a)*e^11*d^
2+36*e^7*d^6*c^2/(a*e^4+c*d^4)^4*ln(e*x+d)+5/2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a
)*d^10*e^3-9*c^2/(a*e^4+c*d^4)^4*ln(c*x^4+a)*e^7*d^6+27/16*c^3/(a*e^4+c*d^4
)^4/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9*e^4+21/32*c/(
a*e^4+c*d^4)^4*a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^12+21/16*c/(a*e^4+c*d^4)^
4*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^12-45/16*c/(a*e
^4+c*d^4)^4*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)
)/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e^10-45/8*c/(a*e^4+c*d^4)^4*
a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^10-45/8*c/(a*e^
4+c*d^4)^4*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^10-1
47/16*c^2/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)*d^5*e^8+27/8*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)
*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7*e^6+27
/4*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
d^7*e^6+27/4*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)*d^7*e^6+63/4*c^2/(a*e^4+c*d^4)^4*a/(a*c)^(1/2)*arctan((1/a*c)^(1/
2)*x^2)*e^9*d^4+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x-1)*d^13+3/32*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)
*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c
)^(1/2)))*d^13+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x+1)*d^13-3/4*c^4/(a*e^4+c*d^4)^4/a/(a*c)^(1/2)*arctan((1/a
*c)^(1/2)*x^2)*e*d^12-147/16*c^2/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^8-147/32*c^2/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^
(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x
+(a/c)^(1/2)))*d^5*e^8-5/2*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*d^3*e^10*a*x^3+3/2
*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^11*e^2/a*x^3+11/4*c^2/(a*e^4+c*d^4)^4/(c*x
^4+a)*e^9*a*x^2*d^4-3/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*e/a*x^2*d^12-9/4*c^2/
(a*e^4+c*d^4)^4/(c*x^4+a)*d^5*a*x*e^8+3/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*d*a^2
*x*e^12+3/16*c^3/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^11*e^2+3/
8*c^3/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
*d^11*e^2+3/8*c^3/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x+1)*d^11*e^2+27/16*c^3/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x-1)*d^9*e^4+27/32*c^3/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*
2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)
```

```
*x+(a/c)^(1/2)))*d^9*e^4+21/16*c/(a*e^4+c*d^4)^4*a*(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^12
```

maxima [A] time = 2.60, size = 1394, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -3/32*c*(sqrt(2)*(48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 16*sqrt(2)*a^(11/4)*
c^(5/4)*d^2*e^11 - c^4*d^13 + 2*sqrt(a)*c^(7/2)*d^11*e^2 - 9*a*c^3*d^9*e^4
+ 36*a^(3/2)*c^(5/2)*d^7*e^6 + 49*a^2*c^2*d^5*e^8 - 30*a^(5/2)*c^(3/2)*d^3*
e^10 - 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a
))/(a^(3/4)*c^(5/4)) + sqrt(2)*(48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 16*sqr
t(2)*a^(11/4)*c^(5/4)*d^2*e^11 + c^4*d^13 - 2*sqrt(a)*c^(7/2)*d^11*e^2 + 9*
a*c^3*d^9*e^4 - 36*a^(3/2)*c^(5/2)*d^7*e^6 - 49*a^2*c^2*d^5*e^8 + 30*a^(5/2
)*c^(3/2)*d^3*e^10 + 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1
/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(17/4)*d^13 + 2*s
qrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 + 9*sqrt(2)*a^(5/4)*c^(13/4)*d^9*e^4 + 36*
sqrt(2)*a^(7/4)*c^(11/4)*d^7*e^6 - 49*sqrt(2)*a^(9/4)*c^(9/4)*d^5*e^8 - 30*
sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(13/4)*c^(5/4)*d*e^12 + 4*s
qrt(a)*c^4*d^12*e + 44*a^(3/2)*c^3*d^8*e^5 - 84*a^(5/2)*c^2*d^4*e^9 + 4*a^(
7/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^
(1/4)*c^(17/4)*d^13 + 2*sqrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 + 9*sqrt(2)*a^(5/
4)*c^(13/4)*d^9*e^4 + 36*sqrt(2)*a^(7/4)*c^(11/4)*d^7*e^6 - 49*sqrt(2)*a^(9
/4)*c^(9/4)*d^5*e^8 - 30*sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(1
3/4)*c^(5/4)*d*e^12 - 4*sqrt(a)*c^4*d^12*e - 44*a^(3/2)*c^3*d^8*e^5 + 84*a^
(5/2)*c^2*d^4*e^9 - 4*a^(7/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqr
t(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))
*c^(5/4)))/(a*c^4*d^16 + 4*a^2*c^3*d^12*e^4 + 6*a^3*c^2*d^8*e^8 + 4*a^4*c*d
^4*e^12 + a^5*e^16) + 12*(3*c^2*d^6*e^7 - a*c*d^2*e^11)*log(e*x + d)/(c^4*d
^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) +
 1/4*(10*a*c^2*d^8*e^3 - 40*a^2*c*d^4*e^7 - 2*a^3*e^11 + 6*(c^3*d^7*e^4 - 7
*a*c^2*d^3*e^8)*x^5 + 3*(3*c^3*d^8*e^3 - 14*a*c^2*d^4*e^7 - a^2*c*e^11)*x^4
 + (c^3*d^9*e^2 + 2*a*c^2*d^5*e^6 + a^2*c*d*e^10)*x^3 - (c^3*d^10*e + 2*a*c
^2*d^6*e^5 + a^2*c*d^2*e^9)*x^2 + (c^3*d^11 + 8*a*c^2*d^7*e^4 - 41*a^2*c*d^
3*e^8)*x)/(a^2*c^3*d^14 + 3*a^3*c^2*d^10*e^4 + 3*a^4*c*d^6*e^8 + a^5*d^2*e^
12 + (a*c^4*d^12*e^2 + 3*a^2*c^3*d^8*e^6 + 3*a^3*c^2*d^4*e^10 + a^4*c*e^14)
*x^6 + 2*(a*c^4*d^13*e + 3*a^2*c^3*d^9*e^5 + 3*a^3*c^2*d^5*e^9 + a^4*c*d*e^
13)*x^5 + (a*c^4*d^14 + 3*a^2*c^3*d^10*e^4 + 3*a^3*c^2*d^6*e^8 + a^4*c*d^2*
e^12)*x^4 + (a^2*c^3*d^12*e^2 + 3*a^3*c^2*d^8*e^6 + 3*a^4*c*d^4*e^10 + a^5*
e^14)*x^2 + 2*(a^2*c^3*d^13*e + 3*a^3*c^2*d^9*e^5 + 3*a^4*c*d^5*e^9 + a^5*d
*e^13)*x)
```

mupad [B] time = 5.04, size = 3256, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)^3),x)
```

```
[Out] symsum(log(root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 2
62144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 -
 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d
^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c
^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((
108*a*c^10*d^19*e^3 + 3888*a^2*c^9*d^15*e^7 - 99576*a^3*c^8*d^11*e^11 + 591
408*a^4*c^7*d^7*e^15 - 79380*a^5*c^6*d^3*e^19)/(256*(a^10*e^24 + a^4*c^6*d^
24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c
^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 39
3216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16
*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^
6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*
a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^
2*e^4 + 81*c^3*d^4, z, k)*((6912*a^8*c^5*d*e^24 + 4608*a^3*c^10*d^21*e^4 +
154368*a^4*c^9*d^17*e^8 - 331776*a^5*c^8*d^13*e^12 + 5976576*a^6*c^7*d^9*e^
16 - 612864*a^7*c^6*d^5*e^20)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*
e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15
*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*
e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11
*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 75571
2*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 +
58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^
4, z, k)*((18432*a^5*c^10*d^23*e^5 - 3072*a^4*c^11*d^27*e + 1170432*a^6*c^9
*d^19*e^9 + 2863104*a^7*c^8*d^15*e^13 + 1797120*a^8*c^7*d^11*e^17 - 423936*
a^9*c^6*d^7*e^21 - 506880*a^10*c^5*d^3*e^25)/(256*(a^10*e^24 + a^4*c^6*d^24
 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3
*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 3932
16*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z
^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*
e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^
6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*
e^4 + 81*c^3*d^4, z, k)*((98304*a^13*c^4*d*e^30 - 32768*a^6*c^11*d^29*e^2 -
 98304*a^7*c^10*d^25*e^6 + 98304*a^8*c^9*d^21*e^10 + 819200*a^9*c^8*d^17*e^
14 + 1474560*a^10*c^7*d^13*e^18 + 1277952*a^11*c^6*d^9*e^22 + 557056*a^12*c
^5*d^5*e^26)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*
d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16
)) + (x*(81920*a^13*c^4*e^31 - 49152*a^6*c^11*d^28*e^3 - 212992*a^7*c^10*d^
```

```
24*e^7 - 245760*a^8*c^9*d^20*e^11 + 245760*a^9*c^8*d^16*e^15 + 901120*a^10*
c^7*d^12*e^19 + 933888*a^11*c^6*d^8*e^23 + 442368*a^12*c^5*d^4*e^27))/(256*
(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*
c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) - (x*(12288*a^
4*c^11*d^26*e^2 + 98304*a^5*c^10*d^22*e^6 - 1413120*a^6*c^9*d^18*e^10 - 403
0464*a^7*c^8*d^14*e^14 - 2813952*a^8*c^7*d^10*e^18 + 393216*a^9*c^6*d^6*e^2
2 + 675840*a^10*c^5*d^2*e^26))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4
*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 1
5*a^8*c^2*d^8*e^16))) + (x*(20736*a^8*c^5*e^25 - 576*a^2*c^11*d^24*e - 576*
a^3*c^10*d^20*e^5 + 484992*a^4*c^9*d^16*e^9 + 2468736*a^5*c^8*d^12*e^13 + 4
093632*a^6*c^7*d^8*e^17 - 228672*a^7*c^6*d^4*e^21))/(256*(a^10*e^24 + a^4*c
^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*
a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(216*a*c^10*d^18*e^4 + 2505
6*a^2*c^9*d^14*e^8 - 2160*a^3*c^8*d^10*e^12 + 59616*a^4*c^7*d^6*e^16 + 8661
6*a^5*c^6*d^2*e^20))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*
a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*
d^8*e^16))) + (81*c^9*d^13*e^6 - 2430*a*c^8*d^9*e^10 + 1296*a^3*c^6*d*e^18
+ 3969*a^2*c^7*d^5*e^14)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20
+ 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*
c^2*d^8*e^16)) + (x*(1296*a^3*c^6*e^19 + 81*c^9*d^12*e^7 - 6318*a*c^8*d^8*e
^11 + 5265*a^2*c^7*d^4*e^15))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*
e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15
*a^8*c^2*d^8*e^16)))*root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e
^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*
e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712
*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 5
8752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4
, z, k), k, 1, 4) - ((a^2*e^11 - 5*c^2*d^8*e^3 + 20*a*c*d^4*e^7)/(2*(a*e^4
+ c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^5*(c^3*d^7*e^4 - 7*a*c
^2*d^3*e^8))/(2*a*(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)
) + (3*x^4*(a^2*c*e^11 - 3*c^3*d^8*e^3 + 14*a*c^2*d^4*e^7))/(4*a*(a^3*e^12
+ c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)) + (c*d^2*e*x^2)/(4*a*(a*e^
4 + c*d^4)) - (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)) - (d*x*(c^3*d^10 + 8*a*c^
2*d^6*e^4 - 41*a^2*c*d^2*e^8))/(4*a*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*
a*c*d^4*e^4)))/(a*d^2 + a*e^2*x^2 + c*d^2*x^4 + c*e^2*x^6 + 2*a*d*e*x + 2*c
*d*e*x^5) + (log(d + e*x)*(36*c^2*d^6*e^7 - 12*a*c*d^2*e^11))/(a^4*e^16 + c
^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.408 $\int \frac{(d+ex)^3}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}$$

```
[Out] 1/32*x*(15*d*e^2*x^2+18*d^2*e*x+7*d^3)/a^2/(c*x^4+a)+1/8*(-a*e^3+c*x*(3*d*e
^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)^2+9/16*d^2*e*arctan(x^2*c^(1/2)/a^(1/2
))/a^(5/2)/c^(1/2)-3/256*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))*(-5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+3/256*d*ln(a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^
(11/4)/c^(3/4)*2^(1/2)+3/128*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*
a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+3/128*d*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a*e^3 - c*x
*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2) + (9*d^2*e*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[
a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/
4)) + (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(
11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^3}\,dx = -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}-\frac{\int \frac{-7d^3-18d^2ex-15de^2x^2}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$=\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}+\frac{\int \frac{21d^3+36d^2ex+15de^2x^2}{a+cx^4}\,dx}{32a^2}$$

$$=\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}+\frac{\int \left(\frac{36d^2ex}{a+cx^4}+\frac{21d^3+15de^2x^2}{a+cx^4}\right)dx}{32a^2}$$

$$=\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}+\frac{\int \frac{21d^3+15de^2x^2}{a+cx^4}\,dx}{32a^2}+\frac{\left(9d^2e\right)\ldots}{8\ldots}$$

$$=\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}+\frac{\left(9d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x\ldots\right.}{16a^2}$$

$$=\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}+\frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{\left(3d\left(7\ldots\right.\right.}{\ldots}$$

$$=\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}+\frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{3d\left(7\ldots\right.}{\ldots}$$

$$=\frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}+\frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{3d\left(7\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.35, size = 388, normalized size = 0.98

$$\frac{\frac{3\sqrt{2}\left(5a^{3/4}de^2-7\sqrt[4]{a}\sqrt{c}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}+\frac{3\sqrt{2}\left(7\sqrt[4]{a}\sqrt{c}d^3-5a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}-\frac{32a^2\left(ae^3-cdx\left(d^2+3dex\ldots\right.\right.}{c\left(a+cx^4\right)^2}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] ((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 - c
*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*Sqrt
[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d
^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a^(3/4)
*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (
3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 389, normalized size = 0.99

$$\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}\,c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3}+\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}\,c^2d^2e+7\left(ac^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5
*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3
)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^
3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*lo
g(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2*d*x^7*e
^2 + 18*c^2*d^2*x^6*e + 7*c^2*d^3*x^5 + 27*a*c*d*x^3*e^2 + 30*a*c*d^2*x^2*e
 + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)
```

maple [A] time = 0.01, size = 470, normalized size = 1.19

$$\frac{e^3x^4}{8\left(cx^4+a\right)^2 a}+\frac{3d\,e^2x^3}{8\left(cx^4+a\right)^2 a}+\frac{e^3x^4}{8\left(cx^4+a\right)a^2}+\frac{3d^2e\,x^2}{8\left(cx^4+a\right)^2 a}+\frac{15d\,e^2x^3}{32\left(cx^4+a\right)a^2}+\frac{d^3x}{8\left(cx^4+a\right)^2 a}+\frac{9d^2e\,x^2}{16\left(cx^4+a\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d^3*x/a/(c*x^4+a)^2+7/32*d^3/a^2*x/(c*x^4+a)+21/256*d^3/a^3*(a/c)^(1/4)
*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2)))+21/128*d^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+21/128*d^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)+3/8*e*d^2*x^2/a/(c*x^4+a)^2+9/16*e*d^2/a^2*x^2/(c*x^4+a)+9/16*e*d^2/a^2/(
a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+3/8*d*e^2*x^3/a/(c*x^4+a)^2+15/32*d*e^
2/a^2*x^3/(c*x^4+a)+15/256*d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1
/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+15/128*
d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+15/128*d*e^
2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*e^3*x^4/a/(
c*x^4+a)^2+1/8*e^3/a^2*x^4/(c*x^4+a)
```

maxima [A] time = 2.37, size = 392, normalized size = 0.99

$$\frac{15\,c^2de^2x^7+18\,c^2d^2ex^6+7\,c^2d^3x^5+27\,acde^2x^3+30\,acd^2ex^2+11\,acd^3x-4\,a^2e^3}{32\left(a^2c^3x^8+2\,a^3c^2x^4+a^4c\right)}+\frac{3\,d\left(\frac{\sqrt{2}\left(7\,\sqrt{c}\,d^2-5\,\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2\cdots\right.}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(15*c^2*d*e^2*x^7 + 18*c^2*d^2*e*x^6 + 7*c^2*d^3*x^5 + 27*a*c*d*e^2*x^
3 + 30*a*c*d^2*e*x^2 + 11*a*c*d^3*x - 4*a^2*e^3)/(a^2*c^3*x^8 + 2*a^3*c^2*x
^4 + a^4*c) + 3/256*d*(sqrt(2)*(7*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*
x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(7*s
qrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^
```

```
(3/4)*c^(1/4)*e^2 - 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
 + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(c))*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1
/4)*e^2 + 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)
*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(
3/4)))/a^2
```

mupad [B] time = 0.48, size = 721, normalized size = 1.83

$$\frac{\frac{11\,d^3\,x}{32\,a}-\frac{e^3}{8\,c}+\frac{7\,c\,d^3\,x^5}{32\,a^2}+\frac{15\,d^2\,e\,x^2}{16\,a}+\frac{27\,d\,e^2\,x^3}{32\,a}+\frac{9\,c\,d^2\,e\,x^6}{16\,a^2}+\frac{15\,c\,d\,e^2\,x^7}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\left(\sum_{k=1}^{4}\ln\left(\frac{c\,d^2\,\left(6867\,c\,d^5\,e^2-1125\,a\,d\,e^6+7992\,c\,d\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^3,x)
```

```
[Out] ((11*d^3*x)/(32*a) - e^3/(8*c) + (7*c*d^3*x^5)/(32*a^2) + (15*d^2*e*x^2)/(1
6*a) + (27*d*e^2*x^3)/(32*a) + (9*c*d^2*e*x^6)/(16*a^2) + (15*c*d*e^2*x^7)/
(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c*d^2*(6867*c*d^5*e^2
 - 1125*a*d*e^6 + 7992*c*d^4*e^3*x - 114688*root(268435456*a^11*c^3*z^4 + 6
3111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e
^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)^2*a^
5*c^2*d + 9600*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 -
 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 5
0625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*e^4*x - 18816*root(26843545
6*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4
147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^
2*d^12, z, k)*a^2*c^2*d^4*x + 196608*root(268435456*a^11*c^3*z^4 + 63111168
*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z +
245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)^2*a^5*c^2*e
*x - 46080*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 812
8512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625
*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*d*e^3))/(32768*a^6))*root(26843
5456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z
+ 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481
*c^2*d^12, z, k), k, 1, 4)
```

sympy [A] time = 8.01, size = 413, normalized size = 1.05

$$\operatorname{RootSum}\left(268435456t^4a^{11}c^3+63111168t^2a^6c^2d^4e^2+t\left(4147200a^4cd^4e^5-8128512a^3c^2d^8e\right)+50625a^2d^4e^8+24\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 + _
t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4*e
**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, Lambda(_t, _t*log(x + (2621
44000*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 539688
960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 77328000
*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t*a**3*
c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 60566940
*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*e**8 - 5
8012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15)))) + (-4*a**2*e**3 + 11*a*c
*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x**5 + 18*c
**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2*x**4 + 32*
a**2*c**3*x**8)
```

3.409 $\int \frac{(d+ex)^2}{\left(a+cx^4\right)^3} dx$

Optimal. Leaf size=360

$$-\frac{\left(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{\left(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}-\cdots$$

```
[Out] 1/8*x*(e*x+d)^2/a/(c*x^4+a)^2+1/32*x*(5*e^2*x^2+12*d*e*x+7*d^2)/a^2/(c*x^4+
a)+3/8*d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)-1/256*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/
4)/c^(3/4)*2^(1/2)+1/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*
(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+1/128*arctan(-1+c^
(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^
(1/2)+1/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/
2))/a^(11/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{\left(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(3
2*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*Sqrt[
c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*
Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(1
1/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^3}\,dx = \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} - \frac{\int \frac{-7d^2-12dex-5e^2x^2}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d^2+24dex+5e^2x^2}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{\int \left(\frac{24dex}{a+cx^4}+\frac{21d^2+5e^2x^2}{a+cx^4}\right)dx}{32a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d^2+5e^2x^2}{a+cx^4}\,dx}{32a^2} + \frac{(3de)\int \frac{x}{a+cx^4}\,dx}{4a^2}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{(3de)\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{8a^2} + \frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}}-5e^2\right)}{64a}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}}-5e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}}}{128\sqrt{2}\,a^{9/4}c^{3/4}}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}-\sqrt{}\right.}{128\sqrt{2}\,a^{9/4}c}$$

$$= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2} + \frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(21\sqrt{c}\,d^2+5\sqrt{a}\,e^2\right)\tan^{-1}\left(1\right.}{64\sqrt{2}\,a^{11/4}c^{3/4}}$$

Mathematica [A] time = 0.29, size = 358, normalized size = 0.99

$$\frac{\frac{\sqrt{2}\left(5a^{3/4}e^2-21\sqrt[4]{a}\sqrt{c}\,d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} + \frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{c}\,d^2-5a^{3/4}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} + \frac{32a^2x(d+ex)^2}{\left(a+cx^4\right)^2} - \frac{2\sqrt[4]{a}\left(4\right.}{}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x^
2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)*d
*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4
) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]
*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]*(
-21*a^(1/4)*Sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5*a^(3/4
)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(25
6*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.43, size = 356, normalized size = 0.99

$$\frac{5\,cx^7e^2+12\,cdx^6e+7\,cd^2x^5+9\,ax^3e^2+20\,adx^2e+11\,ad^2x}{32\,\left(cx^4+a\right)^2a^2}+\frac{\sqrt{2}\left(24\,\sqrt{2}\,\sqrt{ac}\,c^2de+21\,\left(ac^3\right)^{\frac{1}{4}}c^2d^2+5\,\left(ac^3\right)^{\frac{3}{4}}e^2\right.}{128\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*c*x^7*e^2 + 12*c*d*x^6*e + 7*c*d^2*x^5 + 9*a*x^3*e^2 + 20*a*d*x^2*e
 + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*c^
2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*sqrt
(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/256*
sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)*
x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*
d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3)
```

maple [A] time = 0.01, size = 419, normalized size = 1.16

$$\frac{e^{2}x^{3}}{8\left(c\,x^{4}+a\right)^{2}a}+\frac{de\,x^{2}}{4\left(c\,x^{4}+a\right)^{2}a}+\frac{5e^{2}x^{3}}{32\left(c\,x^{4}+a\right)a^{2}}+\frac{d^{2}x}{8\left(c\,x^{4}+a\right)^{2}a}+\frac{3de\,x^{2}}{8\left(c\,x^{4}+a\right)a^{2}}+\frac{7d^{2}x}{32\left(c\,x^{4}+a\right)a^{2}}+\frac{3de\arctan\left(\right.}{8\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d^2*x/a/(c*x^4+a)^2+7/32*d^2/a^2*x/(c*x^4+a)+21/256*d^2/a^3*(a/c)^(1/4)
*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2)))+21/128*d^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+21/128*d^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)+1/4*d*e*x^2/a/(c*x^4+a)^2+3/8*d*e/a^2*x^2/(c*x^4+a)+3/8*d*e/a^2/(a*c)^(1/
2)*arctan((1/a*c)^(1/2)*x^2)+1/8*e^2*x^3/a/(c*x^4+a)^2+5/32*e^2/a^2*x^3/(c*
x^4+a)+5/256*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+5/128*e^2/a^2/c/(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+5/128*e^2/a^2/c/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.18, size = 364, normalized size = 1.01

$$\frac{5\,ce^{2}x^{7}+12\,cdex^{6}+7\,cd^{2}x^{5}+9\,ae^{2}x^{3}+20\,adex^{2}+11\,ad^{2}x}{32\left(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4}\right)}+\frac{\frac{\sqrt{2}\left(21\,\sqrt{c}\,d^{2}-5\,\sqrt{a}\,e^{2}\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\,\sqrt{c}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^2
 + 11*a*d^2*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(
c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(
a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(21*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c
)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(21*sqrt
(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 48*sqrt(a)*sqrt(c
)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(21*sqrt(2)*a^(1/
4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 48*sqrt(a)*sqrt(c)*d*e)*ar
```

```
ctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a^2
```

mupad [B] time = 0.47, size = 676, normalized size = 1.88

$$\frac{\frac{11\,d^2\,x}{32\,a}+\frac{9\,e^2\,x^3}{32\,a}+\frac{7\,c\,d^2\,x^5}{32\,a^2}+\frac{5\,c\,e^2\,x^7}{32\,a^2}+\frac{5\,d\,e\,x^2}{8\,a}+\frac{3\,c\,d\,e\,x^6}{8\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\left(\sum_{k=1}^{4}\ln\left(-\frac{c\,\left(125\,a\,e^6-9891\,c\,d^4\,e^2+\mathrm{root}\left(268435456\,a^{11}\,c^3\right.\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^3,x)
```

```
[Out] ((11*d^2*x)/(32*a) + (9*e^2*x^3)/(32*a) + (7*c*d^2*x^5)/(32*a^2) + (5*c*e^2
*x^7)/(32*a^2) + (5*d*e*x^2)/(8*a) + (3*c*d*e*x^6)/(8*a^2))/(a^2 + c^2*x^8
+ 2*a*c*x^4) + symsum(log(-(c*(125*a*e^6 - 9891*c*d^4*e^2 + 344064*root(268
435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*
z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^
8, z, k)^2*a^5*c^2*d^2 - 8784*c*d^3*e^3*x - 3200*root(268435456*a^11*c^3*z^
4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d
*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*e^4
*x + 56448*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 541
9008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c
^2*d^8 + 625*a^2*e^8, z, k)*a^2*c^2*d^4*x + 30720*root(268435456*a^11*c^3*z
^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*
d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*d*
e^3 - 393216*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5
419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481
*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x))/(32768*a^6))*root(268435456
*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 30
7200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z,
k), k, 1, 4)
```

sympy [A] time = 4.88, size = 374, normalized size = 1.04

$$\mathrm{RootSum}\left(268435456t^4a^{11}c^3+25755648t^2a^6c^2d^2e^2+t\left(307200a^4cde^5-5419008a^3c^2d^5e\right)+625a^2e^8+111906a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 + _
t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 11190
6*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t, _t*log(x + (262144000*_t**3*
a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t**2*
```

```
a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*a**5*
c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c**3*d*
*10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*c**2*d*
*9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*a*c**2*d
**8*e**4 + 85766121*c**3*d**12)))) + (11*a*d**2*x + 20*a*d*e*x**2 + 9*a*e**
2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 + 5*c*e**2*x**7)/(32*a**4 + 64*a**3*
c*x**4 + 32*a**2*c**2*x**8)
```

3.410 $\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=266

$$-\frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\tan^{-1}}{64}$$

```
[Out] 1/8*x*(e*x+d)/a/(c*x^4+a)^2+1/32*x*(6*e*x+7*d)/a^2/(c*x^4+a)+21/128*d*arcta
n(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*d*arctan(1+
c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*d*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*d*ln(a
^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+3/16
*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)}-\frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a + c*x^4))
+ (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (21*d*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*d*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/
4)*c^(1/4)) + (21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(128*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx = \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} - \frac{\int \frac{-7d-6ex}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d+12ex}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{\int \left(\frac{21d}{a+cx^4} + \frac{12ex}{a+cx^4}\right)dx}{32a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{(21d)\int \frac{1}{a+cx^4}\,dx}{32a^2} + \frac{(3e)\int \frac{x}{a+cx^4}\,dx}{8a^2}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{(21d)\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(21d)\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(3e)\,\text{Subst}\Big(}{}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} + \frac{(21d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} + \frac{(21d)\int \frac{}{\frac{\sqrt{a}}{\sqrt{c}}}}{128a}$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}} +$$

$$= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}} + \frac{21d\tan^{-1}\Big(1}{64\sqrt{2}\,a^{1}}$$

Mathematica [A] time = 0.21, size = 249, normalized size = 0.94

$$\frac{\frac{32a^{7/4}x(d+ex)}{\left(a+cx^4\right)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6\left(8\sqrt[4]{a}\,e+7\sqrt{2}\sqrt[4]{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6\left(7\sqrt{2}\sqrt[4]{c}\,d-8\sqrt[4]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}} - \frac{21\sqrt{2}\,d\log\Big(-\sqrt{2}\sqrt[4]{a}\,\sqrt[4]{}}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e*x))/(a +
c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)*d - 8*a^(1/4)*e)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.32, size = 260, normalized size = 0.98

$$\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} d\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c}-\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}} d\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}\,ce+7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))
/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1
/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32*(6*c*
x^6*e + 7*c*d*x^5 + 10*a*x^2*e + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

maple [A] time = 0.01, size = 222, normalized size = 0.83

$$\frac{e\,x^2}{8\left(c\,x^4+a\right)^2 a}+\frac{dx}{8\left(c\,x^4+a\right)^2 a}+\frac{3e\,x^2}{16\left(c\,x^4+a\right)a^2}+\frac{7dx}{32\left(c\,x^4+a\right)a^2}+\frac{3e\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{16\sqrt{ac}\,a^2}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{c}\right)}\right.}{128a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d*x/a/(c*x^4+a)^2+7/32*d/a^2*x/(c*x^4+a)+21/256*d/a^3*(a/c)^(1/4)*2^(1/
2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2)))+21/128*d/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1
)+21/128*d/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*e*x^
2/a/(c*x^4+a)^2+3/16*e/a^2*x^2/(c*x^4+a)+3/16*e/a^2/(a*c)^(1/2)*arctan((1/a
*c)^(1/2)*x^2)
```

maxima [A] time = 2.20, size = 269, normalized size = 1.01

$$\frac{6\,cex^6 + 7\,cdx^5 + 10\,aex^2 + 11\,adx}{32\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)} + \frac{3\left(\frac{7\,\sqrt{2}\,d\log\left(\sqrt{c}\,x^2 + \sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{7\,\sqrt{2}\,d\log\left(\sqrt{c}\,x^2 - \sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{2\left(7\,\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d - 8 \cdots\right.}{\cdots}\right.}{256\,a \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/(a^2*c^2*x^8 + 2*a^3*c
*x^4 + a^4) + 3/256*(7*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(1/4)) - 7*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(1/4)*d
 - 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 2*(7*sqrt(
2)*a^(1/4)*c^(1/4)*d + 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(1/4)))/a^2
```

mupad [B] time = 0.30, size = 315, normalized size = 1.18

$$\frac{\frac{5\,e\,x^2}{16\,a} + \frac{11\,d\,x}{32\,a} + \frac{7\,c\,d\,x^5}{32\,a^2} + \frac{3\,c\,e\,x^6}{16\,a^2}}{a^2 + 2\,a\,c\,x^4 + c^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(\frac{c^2\,\left(63\,d\,e^2 + 36\,e^3\,x - \mathrm{root}\left(268435456\,a^{11}\,c^2\,z^4 + 4718592\,a^6\,c\,e^2\,z^2 - 2 \cdots\right.\right.}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^3,x)
```

```
[Out] ((5*e*x^2)/(16*a) + (11*d*x)/(32*a) + (7*c*d*x^5)/(32*a^2) + (3*c*e*x^6)/(1
6*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c^2*(63*d*e^2 + 36*e^3*
```

```
x - 7168*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*
c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)^2*a^5*c*d - 1176*root(2684354
56*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*
d^4 + 20736*a*e^4, z, k)*a^2*c*d^2*x + 4096*root(268435456*a^11*c^2*z^4 + 4
718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4,
z, k)^2*a^5*c*e*x))/(2048*a^6))*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c
*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k), k, 1,
 4)
```

sympy [A] time = 1.48, size = 192, normalized size = 0.72

$$\operatorname{RootSum}\left(268435456t^4a^{11}c^2 + 4718592t^2a^6ce^2 - 2709504ta^3cd^2e + 20736ae^4 + 194481cd^4, \left(t \mapsto t\log\left(x + \frac{-67}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_t
*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e**4
- 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 453789*c*
d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4 + 64
*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

3.411 $\int \frac{1}{(a+cx^4)^3}\,dx$

Optimal. Leaf size=219

$$-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\tan^{-1}\left(\right.}{64\sqrt{2}\,}$$

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/128*arctan(-1+c^(1/4)*x*2^(1/2)
/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4
))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{7x}{32a^2\left(a+cx^4\right)}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-3),x]
```

```
[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) +
(21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^
(11/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^4\right)^3}\,dx = \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7\int \frac{1}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} + \frac{21\int \frac{1}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} + \frac{21\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{21\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} - \frac{21\int \cdots}{128\cdots}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{\cdots}\right.}{128\sqrt{2}\,a^{1\cdots}}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}} - \frac{21\log\left(\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.08, size = 200, normalized size = 0.91

$$\frac{\frac{32a^{7/4}x}{\left(a+cx^4\right)^2} + \frac{56a^{3/4}x}{a+cx^4} - \frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} - \frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2}\tan\cdots}{\cdots}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-3),x]
```

```
[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

fricas [A] time = 0.64, size = 232, normalized size = 1.06

$$\frac{28\,cx^5+84\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\arctan\left(-a^8cx\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}+\sqrt{a^6\sqrt{-\frac{1}{a^{11}c}}+x^2}\,a^8c\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}\right)+21\left(a^2c^2\right.}{128\left(a^2c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(28*c*x^5 + 84*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*
arctan(-a^8*c*x*(-1/(a^11*c))^(3/4) + sqrt(a^6*sqrt(-1/(a^11*c)) + x^2)*a^8
*c*(-1/(a^11*c))^(3/4)) + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c)
)^(1/4)*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 +
a^4)*(-1/(a^11*c))^(1/4)*log(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c
^2*x^8 + 2*a^3*c*x^4 + a^4)
```

giac [A] time = 0.36, size = 204, normalized size = 0.93

$$\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c}+\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c}+\frac{21\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right.\right.}{256\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(1/4
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*
c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32*(7*c
*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

maple [A] time = 0.00, size = 158, normalized size = 0.72

$$\frac{x}{8\left(c\,x^4+a\right)^2a}+\frac{7x}{32\left(c\,x^4+a\right)a^2}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128a^3}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128a^3}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^3,x)
```

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/256/a^3*(a/c)^(1/4)*2^(1/2)*ln(
(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2)))+21/128/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+21/128
/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.70, size = 212, normalized size = 0.97

$$\frac{7\,cx^5+11\,ax}{32\,\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)}+\frac{21\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}}+\frac{\sqrt{2}\,\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x\right.}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{256\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a^2
```

mupad [B] time = 0.10, size = 80, normalized size = 0.37

$$\frac{\frac{11\,x}{32\,a}+\frac{7\,c\,x^5}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{64\,{(-a)}^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{64\,{(-a)}^{11/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^3,x)
```

```
[Out] ((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*atan
((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x)/
(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))
```

sympy [A] time = 0.46, size = 63, normalized size = 0.29

$$\frac{11ax+7cx^5}{32a^4+64a^3cx^4+32a^2c^2x^8}+\mathrm{RootSum}\left(268435456t^4a^{11}c+194481,\left(t\mapsto t\log\left(\frac{128ta^3}{21}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**3,x)
```

```
[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootSu
m(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)))
```

3.412 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=1352

$$\frac{\log(d+ex)e^{11}}{\left(cd^4+ae^4\right)^3}-\frac{\log\left(cx^4+a\right)e^{11}}{4\left(cd^4+ae^4\right)^3}-\frac{\sqrt{c}\,d^2\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^3}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\cdots\right.}{\cdots}$$

```
[Out] 1/32*c*x*(5*d*e^2*x^2-6*d^2*e*x+7*d^3)/a^2/(a*e^4+c*d^4)/(c*x^4+a)+1/8*(a*e
^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)^2+1/4*e^4*(a*e^3+
c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)^2/(c*x^4+a)+e^11*ln(e*x+d)/(a*
e^4+c*d^4)^3-1/4*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/4*d^2*e^5*arctan(x^2*c^
(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^2-3/16*d^2*e*arctan(x^2*c^(1/2
)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)-1/2*d^2*e^9*arctan(x^2*c^(1/2)/a^(
1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(1/4)*d*e^8*ln(-a^(1/4)*c^(1/4)
*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c
*d^4)^3*2^(1/2)+1/8*c^(1/4)*d*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(
1/4)*d*e^8*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a
^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*d*e^8*arctan(1+c^(1/4)*x*2^(1/2)
/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)-1/32*c^
(1/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2
)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/32*c^(1/4)*d*e^4*ln(a^(1
/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(
7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)
/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*
c^(1/4)*d*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2
))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/256*c^(1/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+
c*d^4)*2^(1/2)+1/256*c^(1/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)+1/128*
c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2
))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)+1/128*c^(1/4)*d*arctan(1+c^(1/4)*x*2^(1/2
)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)
```

Rubi [A] time = 1.41, antiderivative size = 1352, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^3),x]

[Out] (c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4)
) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c*x
^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a*e^
4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4
*a^(3/2)*(c*d^4 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(16*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (e^11*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (
c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e
^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*Sqrt[
c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(
c*d^4 + a*e^4)) - (e^11*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
```

```
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx = \int \left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)^3} - \frac{ce^4\left(-d^3+d^2ex-de^2x^2\right.}{\left(cd^4+ae^4\right)^2\left(a+cx\right.}\right.$$

$$= \frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^8\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^4\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{\left(a+cx^4\right)^2}\,dx}{\left(cd^4+ae^4\right)^2} + \frac{c\int}{}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\right)\right)}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)} + \frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a\right.}$$

Mathematica [A] time = 0.74, size = 835, normalized size = 0.62

$$256\log(d+ex)e^{11}-64\log\left(cx^4+a\right)e^{11}+\frac{32\left(cd^4+ae^4\right)^2\left(ae^3+cdx\left(d^2-exd+e^2x^2\right)\right)}{a\left(cx^4+a\right)^2}+\frac{8\left(cd^4+ae^4\right)\left(8a^2e^7+acdx\left(15d^2-14exd+13e^2x^2\right)e^4\cdots\right.}{a^2\left(cx^4+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] ((32*(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x
^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d^2 - 6*d*e*x + 5*e^2
*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2)))/(a^2*(a + c*x^4)) -
(2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 - 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2
]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 - 80*a^(5/4)*c^(5/4)*d
^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 12
0*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/a^(11/4) + (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 + 24*a^(1
/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^
6*e^4 + 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt
[2]*a^2*Sqrt[c]*d^2*e^8 + 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^
10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 256*e^11*Log[d + e*
x] + (Sqrt[2]*c^(1/4)*(-21*c^(5/2)*d^11 + 5*Sqrt[a]*c^2*d^9*e^2 - 66*a*c^(3
/2)*d^7*e^4 + 18*a^(3/2)*c*d^5*e^6 - 77*a^2*Sqrt[c]*d^3*e^8 + 45*a^(5/2)*d*
e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) + (S
qrt[2]*c^(1/4)*(21*c^(5/2)*d^11 - 5*Sqrt[a]*c^2*d^9*e^2 + 66*a*c^(3/2)*d^7*
e^4 - 18*a^(3/2)*c*d^5*e^6 + 77*a^2*Sqrt[c]*d^3*e^8 - 45*a^(5/2)*d*e^10)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 64*e^11*Lo
g[a + c*x^4])/(256*(c*d^4 + a*e^4)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.91, size = 1259, normalized size = 0.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(51*sqrt(2)*sqrt(a*c)*c^2*d^4*e + 21*(a*c^3)^(1/4)*c^2*d^5 - 75*sqrt(2
)*a*c^2*d^2*e^3 + 122*(a*c^3)^(3/4)*d^3*e^2 + 45*(a*c^3)^(1/4)*a*c*d*e^4)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3
*d^6 - 6*(a*c^3)^(1/4)*a^3*c^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2
+ 9*sqrt(2)*a^4*c^2*d^2*e^4 - 16*(a*c^3)^(3/4)*a^3*d^3*e^3 - 6*(a*c^3)^(1/4
)*a^4*c*d*e^5 + sqrt(2)*sqrt(a*c)*a^4*c*e^6) + 1/64*(51*sqrt(2)*sqrt(a*c)*c
^2*d^4*e + 21*(a*c^3)^(1/4)*c^2*d^5 + 75*sqrt(2)*a*c^2*d^2*e^3 + 122*(a*c^3
)^(3/4)*d^3*e^2 + 45*(a*c^3)^(1/4)*a*c*d*e^4)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^6 + 6*(a*c^3)^(1/4)*a^3*c
^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2 + 9*sqrt(2)*a^4*c^2*d^2*e^4
+ 16*(a*c^3)^(3/4)*a^3*d^3*e^3 + 6*(a*c^3)^(1/4)*a^4*c*d*e^5 + sqrt(2)*sqrt
(a*c)*a^4*c*e^6) + 1/256*(21*sqrt(2)*(a*c^3)^(1/4)*c^4*d^11 - 5*sqrt(2)*(a*
c^3)^(3/4)*c^2*d^9*e^2 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 - 18*sqrt(2
)*(a*c^3)^(3/4)*a*c*d^5*e^6 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 45
*sqrt(2)*(a*c^3)^(3/4)*a^2*d*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a
/c))/(a^3*c^5*d^12 + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^12)
- 1/256*(21*sqrt(2)*(a*c^3)^(1/4)*c^4*d^11 - 5*sqrt(2)*(a*c^3)^(3/4)*c^2*d^
9*e^2 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 - 18*sqrt(2)*(a*c^3)^(3/4)*a
*c*d^5*e^6 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 45*sqrt(2)*(a*c^3)^
(3/4)*a^2*d*e^10)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^5*d^1
2 + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^12) - 1/4*e^11*log(ab
s(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + e
^12*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3
*e^13) + 1/32*(4*a^2*c^2*d^8*e^3 + 16*a^3*c*d^4*e^7 + (5*c^4*d^9*e^2 + 18*a
*c^3*d^5*e^6 + 13*a^2*c^2*d*e^10)*x^7 - 2*(3*c^4*d^10*e + 10*a*c^3*d^6*e^5
+ 7*a^2*c^2*d^2*e^9)*x^6 + (7*c^4*d^11 + 22*a*c^3*d^7*e^4 + 15*a^2*c^2*d^3*
e^8)*x^5 + 8*(a^2*c^2*d^4*e^7 + a^3*c*e^11)*x^4 + 12*a^4*e^11 + (9*a*c^3*d^
9*e^2 + 26*a^2*c^2*d^5*e^6 + 17*a^3*c*d*e^10)*x^3 - 2*(5*a*c^3*d^10*e + 14*
a^2*c^2*d^6*e^5 + 9*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 30*a^2*c^2*d^7*e^
4 + 19*a^3*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(c*x^4 + a)^2*a^2)
```

maple [A] time = 0.02, size = 2098, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^3,x)
```

```
[Out] e^11*ln(e*x+d)/(a*e^4+c*d^4)^3-1/4*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^3+3/8/(a*
e^4+c*d^4)^3/(c*x^4+a)^2*e^11*a^2+33/128*c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4
)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/
2)*x+(a/c)^(1/2)))*d^7*e^4+33/64*c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7*e^4+77/128*c/(a*e^4+c*d^4)^3/a*(a/c)^
```

```
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^8+5/256*c^2/(a*e^4+c*d^
4)^3/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^
2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^9*e^2+5/128*c^2/(a*e^4+c*d^4)^3/a^2
/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^9*e^2+5/128*c^2/(a*e
^4+c*d^4)^3/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9*e^2
+9/64*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-
1)*d^5*e^6+9/64*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x+1)*d^5*e^6+77/128*c/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^8+77/256*c/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2)))*d^3*e^8+9/128*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*ln((x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
)*d^5*e^6+33/64*c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d^7*e^4+13/32*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d*e^10*x^7-7
/16*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^2*e^9*x^6+15/32*c^2/(a*e^4+c*d^4)^3/(
c*x^4+a)^2*d^3*x^5*e^8+7/32*c^4/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^11/a^2*x^5+1/
4*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*x^4*e^7*d^4+13/16*c^2/(a*e^4+c*d^4)^3/(c*
x^4+a)^2*d^5*e^6*x^3+1/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*x^4*e^11*a+1/2*c/(a*
e^4+c*d^4)^3/(c*x^4+a)^2*e^7*d^4*a-7/8*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^6*
e^5*x^2+15/16*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^7*x*e^4+11/32*c^3/(a*e^4+c*
d^4)^3/(c*x^4+a)^2*d^11/a*x-15/16*c/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a
*c)^(1/2)*x^2)*e^9*d^2+45/256/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*
d*e^10+45/128/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)*d*e^10+45/128/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)*d*e^10+9/32*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^9*e^2/a*x^3-5/1
6*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^10*e/a*x^2-9/16*c/(a*e^4+c*d^4)^3/(c*x^
4+a)^2*d^2*e^9*a*x^2+19/32*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^3*a*x*e^8+21/128
*c^3/(a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)*d^11+21/256*c^3/(a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/
4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^11+21/
128*c^3/(a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^11-5/8*c^2/(a*e^4+c*d^4)^3/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e
^5*d^6-3/16*c^3/(a*e^4+c*d^4)^3/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e
*d^10-3/16*c^4/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^10*e/a^2*x^6+11/16*c^3/(a*e^4+
c*d^4)^3/(c*x^4+a)^2*d^7/a*x^5*e^4+17/32*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d*e^
10*a*x^3+9/16*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^5*e^6/a*x^7+5/32*c^4/(a*e^4
+c*d^4)^3/(c*x^4+a)^2*d^9*e^2/a^2*x^7+1/8*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*e
^3*d^8-5/8*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^6*e^5/a*x^6
```

maxima [A] time = 2.41, size = 1015, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] e^11*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12)
 - 1/256*c*(sqrt(2)*(32*sqrt(2)*a^(11/4)*c^(1/4)*e^11 - 21*c^3*d^11 + 5*sqr
t(a)*c^(5/2)*d^9*e^2 - 66*a*c^2*d^7*e^4 + 18*a^(3/2)*c^(3/2)*d^5*e^6 - 77*a
^2*c*d^3*e^8 + 45*a^(5/2)*sqrt(c)*d*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(11/4)*c^(1
/4)*e^11 + 21*c^3*d^11 - 5*sqrt(a)*c^(5/2)*d^9*e^2 + 66*a*c^2*d^7*e^4 - 18*
a^(3/2)*c^(3/2)*d^5*e^6 + 77*a^2*c*d^3*e^8 - 45*a^(5/2)*sqrt(c)*d*e^10)*log
(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(
21*sqrt(2)*a^(1/4)*c^(13/4)*d^11 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*
sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*s
qrt(2)*a^(9/4)*c^(5/4)*d^3*e^8 + 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10 + 24*sq
rt(a)*c^3*d^10*e + 80*a^(3/2)*c^2*d^6*e^5 + 120*a^(5/2)*c*d^2*e^9)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(
a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(21*sqrt(2)*a^(1/4)*c^(13/4)*d^1
1 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4
 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*sqrt(2)*a^(9/4)*c^(5/4)*d^3*e^8
+ 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10 - 24*sqrt(a)*c^3*d^10*e - 80*a^(3/2)*c
^2*d^6*e^5 - 120*a^(5/2)*c*d^2*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(5/4)))/(a^2*c^3*d^12 + 3*a^3*c^2*d^8*e^4 + 3*a^4*c*d^4*e^8 + a^5*e^12) +
1/32*(8*a^2*c*e^7*x^4 + 4*a^2*c*d^4*e^3 + 12*a^3*e^7 + (5*c^3*d^5*e^2 + 13*
a*c^2*d*e^6)*x^7 - 2*(3*c^3*d^6*e + 7*a*c^2*d^2*e^5)*x^6 + (7*c^3*d^7 + 15*
a*c^2*d^3*e^4)*x^5 + (9*a*c^2*d^5*e^2 + 17*a^2*c*d*e^6)*x^3 - 2*(5*a*c^2*d^
6*e + 9*a^2*c*d^2*e^5)*x^2 + (11*a*c^2*d^7 + 19*a^2*c*d^3*e^4)*x)/(a^4*c^2*
d^8 + 2*a^5*c*d^4*e^4 + a^6*e^8 + (a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^
2*e^8)*x^8 + 2*(a^3*c^3*d^8 + 2*a^4*c^2*d^4*e^4 + a^5*c*e^8)*x^4)
```

mupad [B] time = 4.41, size = 2720, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)),x)
```

```
[Out] symsum(log((194481*c^7*d^13*e^6 + 871362*a*c^6*d^9*e^10 + 425984*a^3*c^4*d*
e^18 + 1148881*a^2*c^5*d^5*e^14)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^1
1*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + root(805306368*a
^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^1
2*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*
d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 96522
24*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*
a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*
c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^
```

4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*((402653184*a^15*c^4*d*e^22 - 134217728*a^10*c^9*d^21*e^2 - 134217728*a^11*c^8*d^17*e^6 + 805306368*a^12*c^7*d^13*e^10 + 1879048192*a^13*c^6*d^9*e^14 + 1476395008*a^14*c^5*d^5*e^18)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(335544320*a^15*c^4*e^23 - 201326592*a^10*c^9*d^20*e^3 - 469762048*a^11*c^8*d^16*e^7 + 134217728*a^12*c^7*d^12*e^11 + 1207959552*a^13*c^6*d^8*e^15 + 1140850688*a^14*c^5*d^4*e^19))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (211288064*a^12*c^4*d*e^21 - 11010048*a^7*c^9*d^21*e + 20447232*a^8*c^8*d^17*e^5 + 204472320*a^9*c^7*d^13*e^9 + 514850816*a^10*c^6*d^9*e^13 + 553123840*a^11*c^5*d^5*e^17)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(251658240*a^12*c^4*e^22 - 28311552*a^7*c^9*d^20*e^2 - 67108864*a^8*c^8*d^16*e^6 + 18874368*a^9*c^7*d^12*e^10 + 377487360*a^10*c^6*d^8*e^14 + 571473920*a^11*c^5*d^4*e^18))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (36962304*a^9*c^4*d*e^20 + 11010048*a^5*c^8*d^17*e^4 + 57999360*a^6*c^7*d^13*e^8 + 138805248*a^7*c^6*d^9*e^12 + 141361152*a^8*c^5*d^5*e^16)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(62914560*a^9*c^4*e^21 - 1806336*a^4*c^9*d^20*e + 2670592*a^5*c^8*d^16*e^5 + 43032576*a^6*c^7*d^12*e^9 + 143179776*a^7*c^6*d^8*e^13 + 171732992*a^8*c^5*d^4*e^17))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (4030464*a^6*c^4*d*e^19 + 576576*a^2*c^8*d^17*e^3 + 5061824*a^3*c^7*d^13*e^7 + 15959232*a^4*c^6*d^9*e^11 + 17863744*a^5*c^5*d^5*e^15)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(5242880*a^6*c^4*e^20 + 755136*a^2*c^8*d^16*e^4 + 6023488*a^3*c^7*d^12*e^8 + 19579200*a^4*c^6*d^8*e^12 + 22240704*a^5*c^5*d^4*e^16))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (x*(194481*c^7*d^12*e^7 + 871362*a*c^6*d^8*e^11 + 970321*a^2*c^5*d^4*e^15))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)))*root(805306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11

```
*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 4305715
2*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2
 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z +
 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k), k, 1, 4) + (
(3*a*e^7 + c*d^4*e^3)/(8*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^5*(7*c^3
*d^7 + 15*a*c^2*d^3*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x
^2*(5*c^2*d^6*e + 9*a*c*d^2*e^5))/(16*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)
) + (c*e^7*x^4)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2*d^7 +
19*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(3*c^3*d
^6*e + 7*a*c^2*d^2*e^5))/(16*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (e^
2*x^3*(9*c^2*d^5 + 17*a*c*d*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)
) + (e^2*x^7*(5*c^3*d^5 + 13*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a
*c*d^4*e^4)))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (e^11*log(d + e*x))/(a^3*e^12 +
 c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**3,x)
[Out] Timed out
```

3.413 $\int \frac{1}{(d+ex)^2 (a+cx^4)^3} dx$

Optimal. Leaf size=1830

result too large to display

```
[Out] -e^11/(a*e^4+c*d^4)^3/(e*x+d)+1/32*c*x*(7*d^2*(-3*a*e^4+c*d^4)-12*d*e*(-a*e
^4+c*d^4)*x+5*e^2*(-a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^2/(c*x^4+a)+1/8*c
*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*d*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*
c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a)^2+1/4*c*e^4*(8*a*d^3*e^3+x*(d^2*(-
3*a*e^4+5*c*d^4)-2*d*e*(-a*e^4+3*c*d^4)*x+e^2*(-a*e^4+7*c*d^4)*x^2))/a/(a*e
^4+c*d^4)^3/(c*x^4+a)+12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3*e^11*
ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/2*d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^(1/2)/a
^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^3-3/8*d*e*(-a*e^4+c*d^4)*arctan(x^2*c
^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)^2-d*e^9*(-a*e^4+5*c*d^4)*arct
an(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/8*c^(1/4)*e^8*ln(
-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)*d^6+a^(3/2)*e^6-
11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1
/8*c^(1/4)*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)
*d^6+a^(3/2)*e^6-11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c
*d^4)^4*2^(1/2)-1/256*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(
11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/256*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*c^(1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)
*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)*arctan(-1+c^(1/4)*
x*2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*
c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^
(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/32*c^(1/4)*e^4*ln(-a^(1/4)*c^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+3*d^2*(-3*a
*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(1/4)*e^4*ln(
a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/
2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c
^(1/4)*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/
2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c
^(1/4)*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2
)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(
1/4)*e^8*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+11*c*d^4)*a^(1/2
)+3*d^2*(-a*e^4+3*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(1/
4)*e^8*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+11*c*d^4)*a^(1/2)+3
*d^2*(-a*e^4+3*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)
```

Rubi [A] time = 2.78, antiderivative size = 1830, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17,

$\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12*
d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^
4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d
^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*
x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4
 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^
4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt
[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x
^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*Sqrt[c]*d*e*(c*d^4 - a*e^4
)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(
21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) -
 (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a
*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 +
 a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(1
1*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt
[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3
*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(3*Sqrt[c]*d^2*
(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^3*e^11*Lo
g[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*
d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*
(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*
d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]
*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(9*c^(3/2)*d^6 -
 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^
4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 -
a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2
]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*
e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
```

```
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^11*L
og[a + c*x^4])/(c*d^4 + a*e^4)^4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^3}\,dx = \int\left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)^2}+\frac{12cd^3e^{12}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-\right.\right.}{\left(cd^4+\right.}\right.$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{12cd^3e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^4}+\frac{\left(ce^8\right)\int\frac{3d^2\left(3cd^4-ae^4\right)-2de\left(5cd^4-ae^4\right)x+}{a+cx^4}}{\left(cd^4+ae^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3c\right.\right.\right.}{8a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)^2}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-ae\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

Mathematica [A] time = 1.55, size = 1115, normalized size = 0.61

$$\frac{3072cd^3\log(d+ex)e^{11}-768cd^3\log\left(cx^4+a\right)e^{11}-\frac{256\left(cd^4+ae^4\right)e^{11}}{d+ex}+\frac{8c\left(cd^4+ae^4\right)\left(c^2x\left(7d^2-12exd+15e^2x^2\right)d^8+2ace^4x\left(13d^2-24e\right.\right.}{a^2\left(cx^4+\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] ((-256*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c^2*d^8*x*(7
*d^2 - 12*d*e*x + 15*e^2*x^2) + 2*a*c*d^4*e^4*x*(13*d^2 - 24*d*e*x + 33*e^2
*x^2) + a^2*e^7*(64*d^3 - 45*d^2*e*x + 28*d*e^2*x^2 - 13*e^3*x^3)))/(a^2*(a
 + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(c*d^4*x*(d^2 - 2*d*e*x + 3*e^2*x^2) +
 a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)^2) - (6
*c^(1/4)*(7*Sqrt[2]*c^(7/2)*d^14 - 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*S
qrt[a]*c^3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 - 80*a^(5/4)*c^(9/4)*d^
9*e^5 + 27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 - 2
40*a^(9/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^
3*Sqrt[c]*d^2*e^12 + 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*c^(1/4)*(7*Sqrt[2]*c
^(7/2)*d^14 + 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a]*c^3*d^12*e^2 +
 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 + 80*a^(5/4)*c^(9/4)*d^9*e^5 + 27*Sqrt[2]*a^
(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 + 240*a^(9/4)*c^(5/4)*d^
5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^3*Sqrt[c]*d^2*e^12 -
80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/a^(11/4) + 3072*c*d^3*e^11*Log[d + e*x] - (3*Sqrt[2]*c^
(1/4)*(7*c^(7/2)*d^14 - 5*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27
*a^(3/2)*c^2*d^8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77
*a^3*Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(11/4) + (3*Sqrt[2]*c^(1/4)*(7*c^(7/2)*d^14 - 5*Sqr
t[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27*a^(3/2)*c^2*d^8*e^6 + 77*a^2
*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77*a^3*Sqrt[c]*d^2*e^12 + 15*a^
(7/2)*e^14)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4
) - 768*c*d^3*e^11*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 2769, normalized size = 1.51

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^3,x)
```

```
[Out] 12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3*e^11*ln(c*x^4+a)/(a*e^4+c*d
^4)^4-e^11/(a*e^4+c*d^4)^3/(e*x+d)+231/128*c^2/(a*e^4+c*d^4)^4/a*(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^6*e^8+99/128*c^3/(a*e^4+c*d^4)^
4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^10*e^4+231/256*
c^2/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/
c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^6*e^8+1/2*c^3/(a*e^4+c
*d^4)^4/(c*x^4+a)^2*e^3*d^11+99/256*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2)))*d^10*e^4+231/128*c^2/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^6*e^8+99/128*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^10*e^4+81/256*c^2/(a*e^4+c*
d^4)^4/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^
2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^8*e^6+81/128*c^2/(a*e^4+c*d^4)^4/a/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^8*e^6+15/128*c^3/(a*e
^4+c*d^4)^4/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^12*e^
2+15/128*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)*d^12*e^2+81/128*c^2/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x+1)*d^8*e^6+15/256*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^(1/4)
*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2)))*d^12*e^2+5/2*c/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^11*d^3*a^2-1
7/32*c/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^14*a^2*x^3+11/32*c^4/(a*e^4+c*d^4)^4/(
c*x^4+a)^2*d^14/a*x+2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*x^4*d^7*e^7-13/32*c^2
/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^14*a*x^7+7/32*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^
2*d^14/a^2*x^5+3*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^7*d^7*a+101/32*c^3/(a*e^
4+c*d^4)^4/(c*x^4+a)^2*e^6*x^3*d^8-17/8*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^9
*e^5*x^2+29/32*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^10*x*e^4+53/32*c^3/(a*e^4+
c*d^4)^4/(c*x^4+a)^2*e^10*x^7*d^4-5/8*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^5*e
^9*x^6-19/32*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^6*x^5*e^8-45/8*c^2/(a*e^4+c*
```

```
d^4)^4/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*d^5-45/256/(a*e^4+c*d^4)^4
*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^14-45/128/(a*e^4+c*d^4)^4*a/(a/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^14-45/128/(a*e^4+c*d^4)^4*a/(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^14+9/8*c/(a*e^4+c*d^4)^4/(c
*x^4+a)^2*d*e^13*a^2*x^2-57/32*c/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^2*a^2*x*e^12
-15/8*c^3/(a*e^4+c*d^4)^4/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^9-3
/8*c^4/(a*e^4+c*d^4)^4/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^13+405
/256*c/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^4*e^10+405/128*c/(a*e
^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4*e^10+40
5/128*c/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
*d^4*e^10+27/32*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^2/a*x^3*d^12-3/8*c^2/(a*e
^4+c*d^4)^4/(c*x^4+a)^2*d^5*e^9*a*x^2-5/8*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d
^13*e/a*x^2-39/32*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^6*a*x*e^8+2*c^2/(a*e^4+
c*d^4)^4/(c*x^4+a)^2*x^4*a*d^3*e^11+81/32*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e
^6/a*x^7*d^8+15/32*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^2/a^2*x^7*d^12+7/8*c^2
/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d*e^13*a*x^6-15/8*c^4/(a*e^4+c*d^4)^4/(c*x^4+a
)^2*d^9*e^5/a*x^6-3/8*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^13*e/a^2*x^6-45/32*
c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^2*a*x^5*e^12+33/32*c^4/(a*e^4+c*d^4)^4/(c
*x^4+a)^2*d^10/a*x^5*e^4+57/32*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^10*a*x^3*d
^4+15/8*c/(a*e^4+c*d^4)^4*a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^13*d-23
1/128*c/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
*d^2*e^12+21/128*c^4/(a*e^4+c*d^4)^4/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x-1)*d^14-231/256*c/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*ln((x^
2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)
))*d^2*e^12+21/256*c^4/(a*e^4+c*d^4)^4/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c
)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^1
4-231/128*c/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^2*e^12+21/128*c^4/(a*e^4+c*d^4)^4/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)*d^14
```

maxima [A] time = 3.10, size = 1564, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 12*c*d^3*e^11*log(e*x + d)/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8
 + 4*a^3*c*d^4*e^12 + a^4*e^16) - 3/256*c*(sqrt(2)*(128*sqrt(2)*a^(11/4)*c^
(5/4)*d^3*e^11 - 7*c^4*d^14 + 5*sqrt(a)*c^(7/2)*d^12*e^2 - 33*a*c^3*d^10*e^
4 + 27*a^(3/2)*c^(5/2)*d^8*e^6 - 77*a^2*c^2*d^6*e^8 + 135*a^(5/2)*c^(3/2)*d
^4*e^10 + 77*a^3*c*d^2*e^12 - 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt(c)*x^2 + sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(128*sqrt(2)
```

```
*a^(11/4)*c^(5/4)*d^3*e^11 + 7*c^4*d^14 - 5*sqrt(a)*c^(7/2)*d^12*e^2 + 33*a
*c^3*d^10*e^4 - 27*a^(3/2)*c^(5/2)*d^8*e^6 + 77*a^2*c^2*d^6*e^8 - 135*a^(5/
2)*c^(3/2)*d^4*e^10 - 77*a^3*c*d^2*e^12 + 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt
(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(7*sqr
t(2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^12*e^2 + 33*sqrt(
2)*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*e^6 + 77*sqr
t(2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^4*e^10 - 77*s
qrt(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 15*sqrt(2)*a^(15/4)*c^(3/4)*e^14 + 16*sq
rt(a)*c^4*d^13*e + 80*a^(3/2)*c^3*d^9*e^5 + 240*a^(5/2)*c^2*d^5*e^9 - 80*a^
(7/2)*c*d*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(7*sqrt(
2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^12*e^2 + 33*sqrt(2)
*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*e^6 + 77*sqrt(
2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^4*e^10 - 77*sqr
t(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 15*sqrt(2)*a^(15/4)*c^(3/4)*e^14 - 16*sqrt
(a)*c^4*d^13*e - 80*a^(3/2)*c^3*d^9*e^5 - 240*a^(5/2)*c^2*d^5*e^9 + 80*a^(7
/2)*c*d*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a^2*c^4*d^16
 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6*e^16) +
1/32*(16*a^2*c^2*d^8*e^3 + 80*a^3*c*d^4*e^7 - 32*a^4*e^11 + 3*(5*c^4*d^8*e^
3 + 22*a*c^3*d^4*e^7 - 15*a^2*c^2*e^11)*x^8 + 3*(c^4*d^9*e^2 + 6*a*c^3*d^5*
e^6 + 5*a^2*c^2*d*e^10)*x^7 - (5*c^4*d^10*e + 22*a*c^3*d^6*e^5 + 17*a^2*c^2
*d^2*e^9)*x^6 + (7*c^4*d^11 + 26*a*c^3*d^7*e^4 + 19*a^2*c^2*d^3*e^8)*x^5 +
3*(9*a*c^3*d^8*e^3 + 46*a^2*c^2*d^4*e^7 - 27*a^3*c*e^11)*x^4 + (7*a*c^3*d^9
*e^2 + 26*a^2*c^2*d^5*e^6 + 19*a^3*c*d*e^10)*x^3 - 3*(3*a*c^3*d^10*e + 10*a
^2*c^2*d^6*e^5 + 7*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 34*a^2*c^2*d^7*e^4
 + 23*a^3*c*d^3*e^8)*x)/(a^4*c^3*d^13 + 3*a^5*c^2*d^9*e^4 + 3*a^6*c*d^5*e^8
 + a^7*d*e^12 + (a^2*c^5*d^12*e + 3*a^3*c^4*d^8*e^5 + 3*a^4*c^3*d^4*e^9 + a
^5*c^2*e^13)*x^9 + (a^2*c^5*d^13 + 3*a^3*c^4*d^9*e^4 + 3*a^4*c^3*d^5*e^8 +
a^5*c^2*d*e^12)*x^8 + 2*(a^3*c^4*d^12*e + 3*a^4*c^3*d^8*e^5 + 3*a^5*c^2*d^4
*e^9 + a^6*c*e^13)*x^5 + 2*(a^3*c^4*d^13 + 3*a^4*c^3*d^9*e^4 + 3*a^5*c^2*d^
5*e^8 + a^6*c*d*e^12)*x^4 + (a^4*c^3*d^12*e + 3*a^5*c^2*d^8*e^5 + 3*a^6*c*d
^4*e^9 + a^7*e^13)*x)
```

mupad [B] time = 5.75, size = 3572, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)^2),x)
```

```
[Out] symsum(log((194481*c^9*d^17*e^6 + 1527012*a*c^8*d^13*e^10 + 4100625*a^4*c^5
*d*e^22 + 1926342*a^2*c^7*d^9*e^14 - 3102300*a^3*c^6*d^5*e^18)/(1048576*(a^
14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c
^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(1610612
```

736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*((23592960*a^14*c^4*e^29 - 11010048*a^7*c^11*d^28*e + 33030144*a^8*c^10*d^24*e^5 + 504889344*a^9*c^9*d^20*e^9 + 3103260672*a^10*c^8*d^16*e^13 + 6799491072*a^11*c^7*d^12*e^17 + 6101139456*a^12*c^6*d^8*e^21 + 1967652864*a^13*c^5*d^4*e^25)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(1610612736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*((402653184*a^17*c^4*d*e^30 - 134217728*a^10*c^11*d^29*e^2 - 402653184*a^11*c^10*d^25*e^6 + 402653184*a^12*c^9*d^21*e^10 + 3355443200*a^13*c^8*d^17*e^14 + 6039797760*a^14*c^7*d^13*e^18 + 5234491392*a^15*c^6*d^9*e^22 + 2281701376*a^16*c^5*d^5*e^26)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + (x*(335544320*a^17*c^4*e^31 - 201326592*a^10*c^11*d^28*e^3 - 872415232*a^11*c^10*d^24*e^7 - 1006632960*a^12*c^9*d^20*e^11 + 1006632960*a^13*c^8*d^16*e^15 + 3690987520*a^14*c^7*d^12*e^19 + 3825205248*a^15*c^6*d^8*e^23 + 1811939328*a^16*c^5*d^4*e^27))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16))) + (x*(2554331136*a^10*c^8*d^15*e^14 - 144703488*a^8*c^10*d^23*e^6 - 154140672*a^9*c^9*d^19*e^10 - 34603008*a^7*c^11*d^27*e^2 + 7659847680*a^11*c^7*d^11*e^18 + 7556038656*a^12*c^6*d^7*e^22 + 2494562304*a^13*c^5*d^3*e^26))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16))) + (12681216*a^5*c^10*d^23*e^4 + 127107072*a^6*c^9*d^19*e^8 + 674168832*a^7*

```
c^8*d^15*e^12 + 1018626048*a^8*c^7*d^11*e^16 - 446201856*a^9*c^6*d^7*e^20 +
 906854400*a^10*c^5*d^3*e^24)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c
*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^
12 + 15*a^12*c^2*d^8*e^16)) + (x*(516096*a^5*c^10*d^22*e^5 - 1806336*a^4*c^
11*d^26*e + 90427392*a^6*c^9*d^18*e^9 + 896090112*a^7*c^8*d^14*e^13 + 19609
06752*a^8*c^7*d^10*e^17 + 1732829184*a^9*c^6*d^6*e^21 + 1183887360*a^10*c^5
*d^2*e^25))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*
c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d
^8*e^16))) + (387072*a^2*c^10*d^22*e^3 + 8004096*a^3*c^9*d^18*e^7 + 4937932
8*a^4*c^8*d^14*e^11 + 49572864*a^5*c^7*d^10*e^15 - 156930048*a^6*c^6*d^6*e^
19 + 125452800*a^7*c^5*d^2*e^23)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^1
3*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12
*e^12 + 15*a^12*c^2*d^8*e^16)) + (x*(126360000*a^7*c^5*d*e^24 + 561600*a^2*
c^10*d^21*e^4 + 9609408*a^3*c^9*d^17*e^8 + 75731328*a^4*c^8*d^13*e^12 + 114
991488*a^5*c^7*d^9*e^16 - 80136000*a^6*c^6*d^5*e^20))/(1048576*(a^14*e^24 +
 a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e
^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16))) + (x*(4100625*a^4*c^5*
e^23 + 194481*c^9*d^16*e^7 + 1527012*a*c^8*d^12*e^11 - 167994*a^2*c^7*d^8*e
^15 - 13988700*a^3*c^6*d^4*e^19))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^
13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^1
2*e^12 + 15*a^12*c^2*d^8*e^16)))*root(1610612736*a^13*c^2*d^8*e^8*z^4 + 107
3741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 + 268435456*
a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d^3*e^11*z^
3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2 + 1153105
920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e
*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 +
194481*c^3*d^8, z, k), k, 1, 4) + ((c^2*d^8*e^3 - 2*a^2*e^11 + 5*a*c*d^4*e^
7)/(2*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (3*x^8*(5*c^4*
d^8*e^3 - 15*a^2*c^2*e^11 + 22*a*c^3*d^4*e^7))/(32*a^2*(a^3*e^12 + c^3*d^12
 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)) + (x^5*(7*c^3*d^7 + 19*a*c^2*d^3*e^4
))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^2*(3*c^2*d^6*e + 7*a
*c*d^2*e^5))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^3*(7*c^2*d^5*e
^2 + 19*a*c*d*e^6))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2
*d^7 + 23*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(
5*c^3*d^6*e + 17*a*c^2*d^2*e^5))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4
)) + (3*e^2*x^7*(c^3*d^5 + 5*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a
*c*d^4*e^4)) + (3*e^2*x^4*(9*c^3*d^8*e - 27*a^2*c*e^9 + 46*a*c^2*d^4*e^5))/
(32*a*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)))/(a^2*d + c^2*d*
x^8 + c^2*e*x^9 + a^2*e*x + 2*a*c*d*x^4 + 2*a*c*e*x^5) + (12*c*d^3*e^11*log
(d + e*x))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a
^2*c^2*d^8*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.414 $\int \frac{1}{(d+ex)^3 (a+cx^4)^3} dx$

Optimal. Leaf size=2204

result too large to display

```
[Out] -1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)+1
/32*c*x*(7*d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-6*e*(a^2*e^8-12*a*c*d^4*e^4
+3*c^2*d^8)*x+10*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^3/(c*x
^4+a)+1/8*c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+
c^2*d^8)-e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d
^4)*x^2))/a/(a*e^4+c*d^4)^3/(c*x^4+a)^2+1/4*c*e^4*(12*a*d^2*e^3*(-a*e^4+3*c
*d^4)+x*(3*d*(a^2*e^8-10*a*c*d^4*e^4+5*c^2*d^8)-e*(a^2*e^8-26*a*c*d^4*e^4+2
1*c^2*d^8)*x+4*c*d^3*e^2*(-5*a*e^4+7*c*d^4)*x^2))/a/(a*e^4+c*d^4)^4/(c*x^4+
a)+6*c*d^2*e^11*(-3*a*e^4+13*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^5-3/2*c*d^2*e^1
1*(-3*a*e^4+13*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^5-1/4*e^5*(a^2*e^8-26*a*c*d
^4*e^4+21*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4
)^4-3/16*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c
^(1/2)/a^(5/2)/(a*e^4+c*d^4)^3-1/2*e^9*(a^2*e^8-40*a*c*d^4*e^4+55*c^2*d^8)*
arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^5/a^(1/2)+1/256*c^(3/4)*d
*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d^
4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a
*e^4+c*d^4)^3*2^(1/2)-1/256*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+
x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d^4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3
*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/128*c^(3/4)*d*a
rctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(63*a^2*e^8-252*a*c*d^4*e^4+21*c^2*d^8+
10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(
1/2)+1/128*c^(3/4)*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(63*a^2*e^8-252*a*
c*d^4*e^4+21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4
)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(3/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-9*a^2*e^8+90*a*c*d^4*e^4-45*c^2*d^8+4*d^2*e^2*(-5*a*
e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)-1/32*c^(3/4)*
d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-9*a^2*e^8+90*a*c*
d^4*e^4-45*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a
*e^4+c*d^4)^4*2^(1/2)+1/16*c^(3/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(9*a^2*e^8-90*a*c*d^4*e^4+45*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2
)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/16*c^(3/4)*d*e^4*arctan(1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(9*a^2*e^8-90*a*c*d^4*e^4+45*c^2*d^8+4*d^2*e^2*(-5*a
*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)-3/8*c^(3/4)*
d*e^8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(15*c^2*d^8-16*a*c
*d^4*e^4+a^2*e^8-2*d^2*e^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*
e^4+c*d^4)^5*2^(1/2)+3/8*c^(3/4)*d*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(15*c^2*d^8-16*a*c*d^4*e^4+a^2*e^8-2*d^2*e^2*(-5*a*e^4+11*c*d
^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^5*2^(1/2)+3/4*c^(3/4)*d*e^8*arct
```

```
an(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(15*c^2*d^8-16*a*c*d^4*e^4+a^2*e^8+2*d^2*e
^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^5*2^(1/2)+3/4
*c^(3/4)*d*e^8*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(15*c^2*d^8-16*a*c*d^4*e
^4+a^2*e^8+2*d^2*e^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*
d^4)^5*2^(1/2)
```

Rubi [A] time = 3.16, antiderivative size = 2204, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] -e^11/(2*(c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4 + a*e^4)^
4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(3*c^
2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)
)/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e
^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d
^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4 +
a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(3*d*(
5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^
2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^4*(a
 + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^2*
d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(
c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c^(3/4)*d*e
^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c
*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 -
 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*
Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4
+ 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)
*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e
^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(
4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e
^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*
e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (6*c*d^2*e^11*(13
*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*c^
```

```
2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*
a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c
*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4
)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2
*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^
8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*
d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*
e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) -
 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3*c*d^
2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^5)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^3}\,dx=\int\left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)^3}+\frac{12cd^3e^{12}}{\left(cd^4+ae^4\right)^4(d+ex)^2}+\frac{6cd^2e^{12}\left(13cd^4-3ae^4\right)}{\left(cd^4+ae^4\right)^5(d+ex)}+\frac{c\left(d\right.}{}\right.$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{6cd^2e^{11}\left(13cd^4-3ae^4\right)\log(d+e}{\left(cd^4+ae^4\right)^5}$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)\right.}{}$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)\right.}{}$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)\right.}{}$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)\right.}{}$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)\right.}{}$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)\right.}{}$$

$$=-\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right)\right.}{}$$

Mathematica [A] time = 2.87, size = 1338, normalized size = 0.61

$$\frac{1536cd^2\left(13cd^4-3ae^4\right)\log(d+ex)e^{11}-384cd^2\left(13cd^4-3ae^4\right)\log\left(cx^4+a\right)e^{11}-\frac{3072cd^3\left(cd^4+ae^4\right)e^{11}}{d+ex}-\frac{128\left(cd^4+ae^4\right.}{(d+ex)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] ((-128*e^11*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (3072*c*d^3*e^11*(c*d^4 + a*e^
4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(a^3*e^11*(-96*d^2 + 45*d*e*x - 14*e^2
*x^2) + c^3*d^11*x*(7*d^2 - 18*d*e*x + 30*e^2*x^2) + a*c^2*d^7*e^4*x*(43*d^
2 - 114*d*e*x + 204*e^2*x^2) + a^2*c*d^3*e^7*(288*d^3 - 303*d^2*e*x + 274*d
*e^2*x^2 - 210*e^3*x^3)))/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(-(a^
2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2)
+ 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x
^4)^2) - (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 - 24*a^(1/4)*c^4*d^16*e + 10*S
qrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 - 176*a^(
5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*
c^(9/4)*d^9*e^8 - 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7
*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 + 1200*a^(13/4)*c*d^4*e^13 - 390*S
qrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 - 40*a^(17/
4)*e^17)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*Sqrt[c]*(7*
Sqrt[2]*c^(17/4)*d^17 + 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)
*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 + 176*a^(5/4)*c^3*d^12*e^5 + 78*
Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 + 960*a
^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3
*c^(5/4)*d^5*e^12 - 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*
d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 + 40*a^(17/4)*e^17)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 1536*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*
Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 -
702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3
*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(11/4) + (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 -
702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3
*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(11/4) - 384*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(256*(c*
d^4 + a*e^4)^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.53, size = 2119, normalized size = 0.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/64*(23*sqrt(2)*a*c^4*d^6*e - 65*(a*c^3)^(1/4)*a*c^3*d^5*e^2 + 30*sqrt(2)*
sqrt(a*c)*a*c^3*d^4*e^3 - 7*(a*c^3)^(3/4)*c^2*d^7 - 115*sqrt(2)*a^2*c^3*d^2
*e^5 + 65*(a*c^3)^(3/4)*a*c*d^3*e^4 + 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 + 20*
sqrt(2)*sqrt(a*c)*a^2*c^2*e^7)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 25*sqrt(2)*a^4*c^4*d^8*e^
2 + 10*(a*c^3)^(3/4)*a^3*c^2*d^9*e + 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 90*
sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 90*sqrt(2)*a^5*c^3*d^4*e^6 + 148*(a*c^3
)^(3/4)*a^4*c*d^5*e^5 + 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 25*sqrt(2)*sqrt(
a*c)*a^5*c^2*d^2*e^8 - sqrt(2)*a^6*c^2*e^10 + 10*(a*c^3)^(3/4)*a^5*d*e^9) -
 3/64*(23*sqrt(2)*a*c^4*d^6*e + 65*(a*c^3)^(1/4)*a*c^3*d^5*e^2 - 30*sqrt(2)
*sqrt(a*c)*a*c^3*d^4*e^3 + 7*(a*c^3)^(3/4)*c^2*d^7 - 115*sqrt(2)*a^2*c^3*d^
2*e^5 - 65*(a*c^3)^(3/4)*a*c*d^3*e^4 - 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 - 20
*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/
4))/(a/c)^(1/4))/(sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 25*sqrt(2)*a^4*c^4*d^8*e
^2 - 10*(a*c^3)^(3/4)*a^3*c^2*d^9*e - 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 90
*sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 90*sqrt(2)*a^5*c^3*d^4*e^6 - 148*(a*c^
3)^(3/4)*a^4*c*d^5*e^5 - 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 25*sqrt(2)*sqrt
(a*c)*a^5*c^2*d^2*e^8 - sqrt(2)*a^6*c^2*e^10 - 10*(a*c^3)^(3/4)*a^5*d*e^9)
+ 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^17 - 10*sqrt(2)*(a*c^3)^(3/4)*c^3*d^
15*e^2 + 50*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*sqrt(2)*(a*c^3)^(3/4)
*a*c^2*d^11*e^6 + 220*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 702*sqrt(2)*(
a*c^3)^(3/4)*a^2*c*d^7*e^10 - 770*sqrt(2)*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 +
390*sqrt(2)*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*sqrt(2)*(a*c^3)^(1/4)*a^4*c*d*e
^16)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^20 + 5*a^4*c^5
*d^16*e^4 + 10*a^5*c^4*d^12*e^8 + 10*a^6*c^3*d^8*e^12 + 5*a^7*c^2*d^4*e^16
+ a^8*c*e^20) - 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^17 - 10*sqrt(2)*(a*c^3
)^(3/4)*c^3*d^15*e^2 + 50*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*sqrt(2)
*(a*c^3)^(3/4)*a*c^2*d^11*e^6 + 220*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 -
 702*sqrt(2)*(a*c^3)^(3/4)*a^2*c*d^7*e^10 - 770*sqrt(2)*(a*c^3)^(1/4)*a^3*c
^2*d^5*e^12 + 390*sqrt(2)*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*sqrt(2)*(a*c^3)^(
1/4)*a^4*c*d*e^16)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^
20 + 5*a^4*c^5*d^16*e^4 + 10*a^5*c^4*d^12*e^8 + 10*a^6*c^3*d^8*e^12 + 5*a^7
```

```
*c^2*d^4*e^16 + a^8*c*e^20) - 3/2*(13*c^2*d^6*e^11 - 3*a*c*d^2*e^15)*log(ab
s(c*x^4 + a))/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^12*e^8 + 10*a^3*c
^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 6*(13*c^2*d^6*e^12 - 3*a*c*d^2
*e^16)*log(abs(x*e + d))/(c^5*d^20*e + 5*a*c^4*d^16*e^5 + 10*a^2*c^3*d^12*e
^9 + 10*a^3*c^2*d^8*e^13 + 5*a^4*c*d^4*e^17 + a^5*e^21) + 1/32*(30*c^5*d^11
*x^9*e^4 + 42*c^5*d^12*x^8*e^3 + c^5*d^13*x^7*e^2 - 4*c^5*d^14*x^6*e + 7*c^
5*d^15*x^5 + 204*a*c^4*d^7*x^9*e^8 + 294*a*c^4*d^8*x^8*e^7 + 19*a*c^4*d^9*x
^7*e^6 - 28*a*c^4*d^10*x^6*e^5 + 97*a*c^4*d^11*x^5*e^4 + 78*a*c^4*d^12*x^4*
e^3 + 5*a*c^4*d^13*x^3*e^2 - 8*a*c^4*d^14*x^2*e + 11*a*c^4*d^15*x - 594*a^2
*c^3*d^3*x^9*e^12 - 546*a^2*c^3*d^4*x^8*e^11 + 35*a^2*c^3*d^5*x^7*e^10 - 44
*a^2*c^3*d^6*x^6*e^9 + 461*a^2*c^3*d^7*x^5*e^8 + 586*a^2*c^3*d^8*x^4*e^7 +
31*a^2*c^3*d^9*x^3*e^6 - 40*a^2*c^3*d^10*x^2*e^5 + 79*a^2*c^3*d^11*x*e^4 +
40*a^2*c^3*d^12*e^3 - 30*a^3*c^2*x^8*e^15 + 17*a^3*c^2*d*x^7*e^14 - 20*a^3*
c^2*d^2*x^6*e^13 - 1165*a^3*c^2*d^3*x^5*e^12 - 1078*a^3*c^2*d^4*x^4*e^11 +
47*a^3*c^2*d^5*x^3*e^10 - 56*a^3*c^2*d^6*x^2*e^9 + 269*a^3*c^2*d^7*x*e^8 +
304*a^3*c^2*d^8*e^7 - 50*a^4*c*x^4*e^15 + 21*a^4*c*d*x^3*e^14 - 24*a^4*c*d^
2*x^2*e^13 - 567*a^4*c*d^3*x*e^12 - 520*a^4*c*d^4*e^11 - 16*a^5*e^15)/((a^2
*c^4*d^16 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6
*e^16)*(c*x^5*e + c*d*x^4 + a*x*e + a*d)^2)
```

maple [A] time = 0.04, size = 3334, normalized size = 1.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^3,x)
```

```
[Out] -1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)+1
65/32*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)*d^9*e^8+75/64*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/c)^(1/4)*x-1)*d^13*e^4+165/64*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/
2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2)))*d^9*e^8+75/128*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
2)))*d^13*e^4+165/32*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/c)^(1/4)*x+1)*d^9*e^8+78*e^11*d^6*c^2/(a*e^4+c*d^4)^5*ln(e*x+d)+5/4*c
^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^3*d^14-39/2*c^2/(a*e^4+c*d^4)^5*ln(c*x^4+a
)*e^11*d^6-129/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a*x^5*e^12+25/16*c^5/
(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^13/a*x^5*e^4-125/16*c^2/(a*e^4+c*d^4)^5/(c*x^
4+a)^2*d^3*e^14*a^2*x^3-31/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^7*e^10*a*x^
3+27/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^15*e^2/a*x^3+75/8*c^2/(a*e^4+c*d^
4)^5/(c*x^4+a)^2*e^13*a^2*x^2*d^4+15/2*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*
a*x^2*d^8-15/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a*x^2*d^16-141/16*c^2/(a*
e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a^2*x*e^12-85/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^
2*d^9*a*x*e^8-3*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*a^2*d^2*e^15+6*c^3/(a*e
```

^4+c*d^4)^5/(c*x^4+a)^2*x^4*a*d^6*e^11-105/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)
^2*d^3*e^14*a*x^7+117/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^11*e^6/a*x^7+15/
16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^15*e^2/a^2*x^7-1155/64*c^2/(a*e^4+c*d^
4)^5*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^12-1155/128*
c^2/(a*e^4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)
^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^12-1155/64*c^2/(a*e^
4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^12+105
3/64*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)*d^7*e^10+1053/128*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(
1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7*e^
10+1053/64*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
4)*x+1)*d^7*e^10-9/16*c^5/(a*e^4+c*d^4)^5/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1
/2)*x^2)*e*d^16+21/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/c)^(1/4)*x-1)*d^17+21/256*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2)))*d^17+21/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x+1)*d^17+225/8*c^2/(a*e^4+c*d^4)^5*a/(a*c)^(1/2)*arc
tan((1/a*c)^(1/2)*x^2)*e^13*d^4-33/8*c^4/(a*e^4+c*d^4)^5/a/(a*c)^(1/2)*arct
an((1/a*c)^(1/2)*x^2)*e^5*d^12+57/32*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^3*x*
e^16+65/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^13*a*x^6*d^4-33/8*c^5/(a*e^4+c*
d^4)^5/(c*x^4+a)^2*e^5/a*x^6*d^12-9/16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a^
2*x^6*d^16+45/32*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^2*x^5*e^16-18*e^15*d^2
*c/(a*e^4+c*d^4)^5*ln(e*x+d)*a+5*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*x^6*d^
8-65/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^9*x^5*e^8+121/16*c^4/(a*e^4+c*d^4)
^5/(c*x^4+a)^2*d^11*e^6*x^3-27/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^5*x^2*d^
12+5/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^13*x*e^4+9*c^4/(a*e^4+c*d^4)^5/(c
*x^4+a)^2*x^4*d^10*e^7-7/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^17*a^2*x^6+7/
32*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^17/a^2*x^5+11/32*c^5/(a*e^4+c*d^4)^5/(
c*x^4+a)^2*d^17/a*x+43/4*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^7*d^10*a+23/4*c^
2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^11*d^6*a^2-3/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+
a)^2*d^7*e^10*x^7-9/16*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^17*a^3*x^2-15/4*c/(a
*e^4+c*d^4)^5/(c*x^4+a)^2*e^15*d^2*a^3-15/16*c/(a*e^4+c*d^4)^5*a^2/(a*c)^(1
/2)*arctan((1/a*c)^(1/2)*x^2)*e^17+9/2*c/(a*e^4+c*d^4)^5*a*ln(c*x^4+a)*e^15
*d^2-45/2*c^3/(a*e^4+c*d^4)^5/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*d^8
+75/64*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
4)*x+1)*d^13*e^4+117/64*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x-1)*d^11*e^6+117/128*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2
^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2)))*d^11*e^6+117/64*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^11*e^6+15/64*c^4/(a*e^4+c*d^4)^5/a^2/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^15*e^2+15/128*c^4/(a*e^4+c
*d^4)^5/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/
(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^15*e^2+15/64*c^4/(a*e^4+c*d^4)^5
/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^15*e^2-585/64*c/
(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e

```
^14-585/128*c/(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(
1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e^14-585/6
4*c/(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d
^3*e^14+231/128*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)*d*e^16+231/256*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*ln((x^2
+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
)*d*e^16+231/128*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)*d*e^16
```

maxima [A] time = 3.99, size = 2198, normalized size = 1.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -3/256*c*(sqrt(2)*(832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 192*sqrt(2)*a^(1
5/4)*c^(5/4)*d^2*e^15 - 7*c^5*d^17 + 10*sqrt(a)*c^(9/2)*d^15*e^2 - 50*a*c^4
*d^13*e^4 + 78*a^(3/2)*c^(7/2)*d^11*e^6 - 220*a^2*c^3*d^9*e^8 + 702*a^(5/2)
*c^(5/2)*d^7*e^10 + 770*a^3*c^2*d^5*e^12 - 390*a^(7/2)*c^(3/2)*d^3*e^14 - 7
7*a^4*c*d*e^16)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(
3/4)*c^(5/4)) + sqrt(2)*(832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 192*sqrt(2
)*a^(15/4)*c^(5/4)*d^2*e^15 + 7*c^5*d^17 - 10*sqrt(a)*c^(9/2)*d^15*e^2 + 50
*a*c^4*d^13*e^4 - 78*a^(3/2)*c^(7/2)*d^11*e^6 + 220*a^2*c^3*d^9*e^8 - 702*a
^(5/2)*c^(5/2)*d^7*e^10 - 770*a^3*c^2*d^5*e^12 + 390*a^(7/2)*c^(3/2)*d^3*e^
14 + 77*a^4*c*d*e^16)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a)
)/(a^(3/4)*c^(5/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 + 10*sqrt(2)*a^(3/
4)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 + 78*sqrt(2)*a^
(7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8 + 702*sqrt(2
)*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d^5*e^12 - 390*
sqrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5/4)*d*e^16 + 24
*sqrt(a)*c^5*d^16*e + 176*a^(3/2)*c^4*d^12*e^5 + 960*a^(5/2)*c^3*d^8*e^9 -
1200*a^(7/2)*c^2*d^4*e^13 + 40*a^(9/2)*c*e^17)*arctan(1/2*sqrt(2)*(2*sqrt(c
)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)
*sqrt(c))*c^(5/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 + 10*sqrt(2)*a^(3/4
)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 + 78*sqrt(2)*a^(
7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8 + 702*sqrt(2)
*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d^5*e^12 - 390*s
qrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5/4)*d*e^16 - 24*
sqrt(a)*c^5*d^16*e - 176*a^(3/2)*c^4*d^12*e^5 - 960*a^(5/2)*c^3*d^8*e^9 + 1
200*a^(7/2)*c^2*d^4*e^13 - 40*a^(9/2)*c*e^17)*arctan(1/2*sqrt(2)*(2*sqrt(c)
*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*
sqrt(c))*c^(5/4)))/(a^2*c^5*d^20 + 5*a^3*c^4*d^16*e^4 + 10*a^4*c^3*d^12*e^8
 + 10*a^5*c^2*d^8*e^12 + 5*a^6*c*d^4*e^16 + a^7*e^20) + 6*(13*c^2*d^6*e^11
- 3*a*c*d^2*e^15)*log(e*x + d)/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^
```

```
12*e^8 + 10*a^3*c^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 1/32*(40*a^2*
c^3*d^12*e^3 + 304*a^3*c^2*d^8*e^7 - 520*a^4*c*d^4*e^11 - 16*a^5*e^15 + 6*(
5*c^5*d^11*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^12)*x^9 + 6*(7*c^5*d^1
2*e^3 + 49*a*c^4*d^8*e^7 - 91*a^2*c^3*d^4*e^11 - 5*a^3*c^2*e^15)*x^8 + (c^5
*d^13*e^2 + 19*a*c^4*d^9*e^6 + 35*a^2*c^3*d^5*e^10 + 17*a^3*c^2*d*e^14)*x^7
 - 4*(c^5*d^14*e + 7*a*c^4*d^10*e^5 + 11*a^2*c^3*d^6*e^9 + 5*a^3*c^2*d^2*e^
13)*x^6 + (7*c^5*d^15 + 97*a*c^4*d^11*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*
c^2*d^3*e^12)*x^5 + 2*(39*a*c^4*d^12*e^3 + 293*a^2*c^3*d^8*e^7 - 539*a^3*c^
2*d^4*e^11 - 25*a^4*c*e^15)*x^4 + (5*a*c^4*d^13*e^2 + 31*a^2*c^3*d^9*e^6 +
47*a^3*c^2*d^5*e^10 + 21*a^4*c*d*e^14)*x^3 - 8*(a*c^4*d^14*e + 5*a^2*c^3*d^
10*e^5 + 7*a^3*c^2*d^6*e^9 + 3*a^4*c*d^2*e^13)*x^2 + (11*a*c^4*d^15 + 79*a^
2*c^3*d^11*e^4 + 269*a^3*c^2*d^7*e^8 - 567*a^4*c*d^3*e^12)*x)/(a^4*c^4*d^18
 + 4*a^5*c^3*d^14*e^4 + 6*a^6*c^2*d^10*e^8 + 4*a^7*c*d^6*e^12 + a^8*d^2*e^1
6 + (a^2*c^6*d^16*e^2 + 4*a^3*c^5*d^12*e^6 + 6*a^4*c^4*d^8*e^10 + 4*a^5*c^3
*d^4*e^14 + a^6*c^2*e^18)*x^10 + 2*(a^2*c^6*d^17*e + 4*a^3*c^5*d^13*e^5 + 6
*a^4*c^4*d^9*e^9 + 4*a^5*c^3*d^5*e^13 + a^6*c^2*d*e^17)*x^9 + (a^2*c^6*d^18
 + 4*a^3*c^5*d^14*e^4 + 6*a^4*c^4*d^10*e^8 + 4*a^5*c^3*d^6*e^12 + a^6*c^2*d
^2*e^16)*x^8 + 2*(a^3*c^5*d^16*e^2 + 4*a^4*c^4*d^12*e^6 + 6*a^5*c^3*d^8*e^1
0 + 4*a^6*c^2*d^4*e^14 + a^7*c*e^18)*x^6 + 4*(a^3*c^5*d^17*e + 4*a^4*c^4*d^
13*e^5 + 6*a^5*c^3*d^9*e^9 + 4*a^6*c^2*d^5*e^13 + a^7*c*d*e^17)*x^5 + 2*(a^
3*c^5*d^18 + 4*a^4*c^4*d^14*e^4 + 6*a^5*c^3*d^10*e^8 + 4*a^6*c^2*d^6*e^12 +
 a^7*c*d^2*e^16)*x^4 + (a^4*c^4*d^16*e^2 + 4*a^5*c^3*d^12*e^6 + 6*a^6*c^2*d
^8*e^10 + 4*a^7*c*d^4*e^14 + a^8*e^18)*x^2 + 2*(a^4*c^4*d^17*e + 4*a^5*c^3*
d^13*e^5 + 6*a^6*c^2*d^9*e^9 + 4*a^7*c*d^5*e^13 + a^8*d*e^17)*x)
```

mupad [B] time = 7.78, size = 6280, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)^3),x)
```

```
[Out] symsum(log(root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^1
2*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^3
6*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16
*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^
5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268
435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13
*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8
*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^3
2*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e
^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 81
28512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*
a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8
*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^
```

6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462
847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^
5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z
+ 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^
23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*(root(268435
4560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 322122547
20*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^
17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^1
9*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c
^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 +
45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 79
1941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 155544
4924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 8156676096
0*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^
14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z
^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2
+ 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 559382
63040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*
a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^2
4*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548
810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z -
977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d
^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*(root(2684354560*a^12*c^9*d^36*e^4
*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*
z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 +
56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 1
2079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268
435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d
^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^
8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*
e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z
^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 278520
4224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c
^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d
^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^1
7*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2
819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c
^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^
12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*
e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^
6*d^14*e^11, z, k)*((44040192*a^8*c^12*d^31*e^5 - 11010048*a^7*c^13*d^35*e
+ 994050048*a^9*c^11*d^27*e^9 + 13683916800*a^10*c^10*d^23*e^13 + 429360414
72*a^11*c^9*d^19*e^17 + 52628029440*a^12*c^8*d^15*e^21 + 23429382144*a^13*c
^7*d^11*e^25 - 2132803584*a^14*c^6*d^7*e^29 - 3125280768*a^15*c^5*d^3*e^33)
/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^

4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*((402653184*a^19*c^4*d*e^38 - 134217728*a^10*c^13*d^37*e^2 - 671088640*a^11*c^12*d^33*e^6 - 536870912*a^12*c^11*d^29*e^10 + 3758096384*a^13*c^10*d^25*e^14 + 13153337344*a^14*c^9*d^21*e^18 + 20669530112*a^15*c^8*d^17*e^22 + 18790481920*a^16*c^7*d^13*e^26 + 10200547328*a^17*c^6*d^9*e^30 + 3087007744*a^18*c^5*d^5*e^34)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(335544320*a^19*c^4*e^39 - 201326592*a^10*c^13*d^36*e^3 - 1275068416*a^11*c^12*d^32*e^7 - 2952790016*a^12*c^11*d^28*e^11 - 1879048192*a^13*c^10*d^24*e^15 + 4697620480*a^14*c^9*d^20*e^19 + 12213813248*a^15*c^8*d^16*e^23 + 13153337344*a^16*c^7*d^12*e^27 + 7784628224*a^17*c^6*d^8*e^31 + 2483027968*a^18*c^5*d^4*e^35))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) - (x*(40894464*a^7*c^13*d^34*e^2 + 276824064*a^8*c^12*d^30*e^6 + 968884224*a^9*c^11*d^26*e^10 - 13010731008*a^10*c^10*d^22*e^14 - 53433335808*a^11*c^9*d^18*e^18 - 71647100928*a^12*c^8*d^14*e^22 - 34313601024*a^13*c^7*d^10*e^26 + 1837105152*a^14*c^6*d^6*e^30 + 4193255424*a^15*c^5*d^2*e^34))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (33914880*a^12*c^5*d*e^32 + 13713408*a^5*c^12*d^29*e^4 + 225902592*a^6*c^11*d^25*e^8 + 2352070656*a^7*c^10*d^21*e^12 + 2474606592*a^8*c^9*d^17*e^16 - 21361803264*a^9*c^8*d^13*e^20 + 88707170304*a^10*c^7*d^9*e^24 - 5526503424*a^11*c^6*d^5*e^28)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c

^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(132710400*a^12*c^5*e^33 - 18063
36*a^4*c^13*d^32*e - 2027520*a^5*c^12*d^28*e^5 + 162017280*a^6*c^11*d^24*e^
9 + 4635316224*a^7*c^10*d^20*e^13 + 15604273152*a^8*c^9*d^16*e^17 + 3931866
3168*a^9*c^8*d^12*e^21 + 64184389632*a^10*c^7*d^8*e^25 - 2525073408*a^11*c^
6*d^4*e^29))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9
*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*
d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (320544*a^2*c
^12*d^27*e^3 + 11448000*a^3*c^11*d^23*e^7 + 114031584*a^4*c^10*d^19*e^11 -
213750144*a^5*c^9*d^15*e^15 - 3499271712*a^6*c^8*d^11*e^19 + 9699804864*a^7
*c^7*d^7*e^23 - 933615072*a^8*c^6*d^3*e^27)/(1048576*(a^16*e^32 + a^8*c^8*d
^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^
11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*
c^2*d^8*e^24)) + (x*(514944*a^2*c^12*d^26*e^4 + 14314752*a^3*c^11*d^22*e^8
+ 266343552*a^4*c^10*d^18*e^12 + 297948672*a^5*c^9*d^14*e^16 - 2642613120*a
^6*c^8*d^10*e^20 + 1782459648*a^7*c^7*d^6*e^24 + 846599040*a^8*c^6*d^2*e^28
))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*
e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16
+ 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (194481*c^11*d^21*e^6 +
2430324*a*c^10*d^17*e^10 + 12960000*a^5*c^6*d*e^26 - 5918346*a^2*c^9*d^13*
e^14 - 83522988*a^3*c^8*d^9*e^18 + 71628705*a^4*c^7*d^5*e^22)/(1048576*(a^1
6*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^
6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^
12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(12960000*a^5*c^6*e^27 + 194481*c^11*
d^20*e^7 + 2430324*a*c^10*d^16*e^11 - 21081546*a^2*c^9*d^12*e^15 - 22781444
4*a^3*c^8*d^8*e^19 + 105734241*a^4*c^7*d^4*e^23))/(1048576*(a^16*e^32 + a^8
*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 +
56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28
*a^14*c^2*d^8*e^24)))*root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a
^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^2
0*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^
6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*
d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^4
0*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148
482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810
048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*
a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8
*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^
13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 54
3361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499
328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280
*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^
4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 435
09753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*
c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3
*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k),

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k, 1, 5) - ((2*a^3*e^15 - 5*c^3*d^12*e^3 - 38*a*c^2*d^8*e^7 + 65*a^2*c*d^4*
e^11)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) + (3*x^8*(5*a^3*c^2*e^15 -
7*c^5*d^12*e^3 - 49*a*c^4*d^8*e^7 + 91*a^2*c^3*d^4*e^11))/(16*a^2*(a^4*e^16
 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) - (
x^5*(7*c^5*d^15 + 97*a*c^4*d^11*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*c^2*d^
3*e^12))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) - (3*x^9*(5*c^5*d^1
1*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^12))/(16*a^2*(a^4*e^16 + c^4*d^
16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x^2*(c^2*
d^6*e + 3*a*c*d^2*e^5))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^3*(5
*c^2*d^5*e^2 + 21*a*c*d*e^6))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) -
(x*(11*c^4*d^15 + 79*a*c^3*d^11*e^4 - 567*a^3*c*d^3*e^12 + 269*a^2*c^2*d^7*
e^8))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) + (x^6*(c^3*d^6*e + 5*a*
c^2*d^2*e^5))/(8*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^4*(25*a^3*c*
e^15 - 39*c^4*d^12*e^3 - 293*a*c^3*d^8*e^7 + 539*a^2*c^2*d^4*e^11))/(16*a*(
a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) - (e^2*x^7*(c^3*d^5 + 17*a*c^2*d*e^4)
)/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)))/(a^2*d^2 + a^2*e^2*x^2 + c^
2*d^2*x^8 + c^2*e^2*x^10 + 2*a^2*d*e*x + 2*a*c*d^2*x^4 + 2*a*c*e^2*x^6 + 2*
c^2*d*e*x^9 + 4*a*c*d*e*x^5)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.415 $\int \frac{-1+x}{1-x+x^2} \, dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 - x + x^2),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{1-x+x^2}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 - x + x^2),x]
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

fricas [A] time = 0.98, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

giac [A] time = 0.29, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(x^2-x+1),x)
[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.53, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^2 - x + 1),x)
[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.11, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-x+1),x)
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.416 $\int \frac{-1+x^2}{1+x^3} \, dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1872, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^3),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1872

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^2}{1+x^3}\,dx &= \int \frac{-1+x}{1-x+x^2}\,dx\\
&= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^3),x]
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

fricas [A] time = 1.09, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

giac [A] time = 0.38, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^3+1),x)
[Out] -1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*ln(x^2-x+1)
```

maxima [A] time = 2.53, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

mupad [B] time = 0.03, size = 30, normalized size = 0.94

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3 + 1),x)
[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.12, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2 - x + 1\right)}{2} - \frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3+1),x)
```

```
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.417 $\int \frac{-4+3x}{4-2x+x^2}\,dx$

Optimal. Leaf size=32

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {634, 618, 204, 628}

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x}{4-2x+x^2}\,dx &= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx - \int \frac{1}{4-2x+x^2}\,dx\\ &= \frac{3}{2}\log\left(4-2x+x^2\right)+2\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, -2+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(4-2x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-2x+4\right)-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x)/(4 - 2*x + x^2),x]
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

fricas [A] time = 0.80, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

giac [A] time = 0.39, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}+\frac{3\ln\left(x^{2}-2x+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4+3*x)/(x^2-2*x+4),x)
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))
```

maxima [A] time = 2.16, size = 26, normalized size = 0.81

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(x-1)\right)+\frac{3}{2}\,\log\left(x^{2}-2\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{3\,\ln\left(x^{2}-2\,x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4)/(x^2 - 2*x + 4),x)
[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.12, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x**2-2*x+4),x)
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.418 $\int \frac{-8+2x+3x^2}{8+x^3}\,dx$

Optimal. Leaf size=32

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1872, 634, 618, 204, 628}

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1872

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-8+2x+3x^2}{8+x^3}\,dx &= \frac{1}{2}\int \frac{-8+6x}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx - \int \frac{1}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\log\left(4-2x+x^2\right) + 2\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, -2+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-2x+4\right) - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

fricas [A] time = 0.88, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(x-1)\right) + \frac{3}{2}\log\left(x^2-2\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

giac [A] time = 0.33, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}+\frac{3\ln\left(x^2-2x+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x-8)/(x^3+8),x)
[Out] -1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))+3/2*ln(x^2-2*x+4)
```

maxima [A] time = 2.33, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

mupad [B] time = 0.03, size = 30, normalized size = 0.94

$$\frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2 - 8)/(x^3 + 8),x)
[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.12, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x-8)/(x**3+8),x)
```

```
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.419 $\int \frac{2+x}{-1+2x+x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[Out] 1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{-1+2x+x^2}\,dx &= -\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx\\ &= \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

fricas [A] time = 0.90, size = 45, normalized size = 1.00

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

giac [A] time = 0.42, size = 44, normalized size = 0.98

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\log\left(\left|x^2+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}+\frac{\ln\left(x^2+2x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+2*x-1),x)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

maxima [A] time = 2.62, size = 35, normalized size = 0.78

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

mupad [B] time = 2.32, size = 34, normalized size = 0.76

$$\ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(2*x + x^2 - 1),x)
```

```
[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
1/2)
```

sympy [A] time = 0.11, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+2*x-1),x)
```

```
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

3.420 $\int \frac{-4+x^2}{2-5x+x^3}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

```
[Out] 1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2055, 632, 31}

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2055

```
Int[(u_.)*(P_)*(Q_)^(q_), x_Symbol] :> Module[{gcd = PolyGCD[P, Q, x]}, Int
[u*gcd^(q + 1)*PolynomialQuotient[P, gcd, x]*PolynomialQuotient[Q, gcd, x]^
q, x] /; NeQ[gcd, 1]] /; ILtQ[q, 0] && PolyQ[P, x] && PolyQ[Q, x]
```

Rubi steps

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx=\int \frac{2+x}{-1+2x+x^2}\,dx$$
$$=-\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx$$
$$=\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

fricas [A] time = 0.76, size = 45, normalized size = 1.00

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

giac [A] time = 0.31, size = 44, normalized size = 0.98

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\log\left(\left|x^2+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}+\frac{\ln\left(x^{2}+2x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)/(x^3-5*x+2),x)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

maxima [A] time = 2.33, size = 35, normalized size = 0.78

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^{2}+2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

mupad [B] time = 0.05, size = 34, normalized size = 0.76

$$\ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 4)/(x^3 - 5*x + 2),x)
```

```
[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
1/2)
```

sympy [A] time = 0.12, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(x**3-5*x+2),x)
```

```
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

3.421 $\int \frac{2}{-1+4x^2}\,dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(2x)$$

```
[Out] -arctanh(2*x)
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 207}

$$-\tanh^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[2/(-1 + 4*x^2),x]
```

```
[Out] -ArcTanh[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2}{-1+4x^2}\,dx &= 2\int \frac{1}{-1+4x^2}\,dx\\ &= -\tanh^{-1}(2x)\end{aligned}$$

Mathematica [B] time = 0.00, size = 23, normalized size = 3.83

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(-1 + 4*x^2),x]
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

fricas [B] time = 0.67, size = 17, normalized size = 2.83

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="fricas")
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

giac [B] time = 0.31, size = 15, normalized size = 2.50

$$-\frac{1}{2}\log\left(\left|x+\frac{1}{2}\right|\right)+\frac{1}{2}\log\left(\left|x-\frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="giac")
[Out] -1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))
```

maple [B] time = 0.00, size = 18, normalized size = 3.00

$$\frac{\ln(2x-1)}{2}-\frac{\ln(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*x^2-1),x)
[Out] 1/2*ln(2*x-1)-1/2*ln(2*x+1)
```

maxima [B] time = 1.06, size = 17, normalized size = 2.83

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="maxima")
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

mupad [B] time = 2.27, size = 6, normalized size = 1.00

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*x^2 - 1),x)
```

```
[Out] -atanh(2*x)
```

sympy [B] time = 0.09, size = 15, normalized size = 2.50

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x**2-1),x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

3.422 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

```
[Out] 1/2*ln(1-2*x)-1/2*ln(1+2*x)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

```
[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

Rubi steps

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx=\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.10

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

```
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

fricas [A] time = 0.52, size = 17, normalized size = 0.81

$$-\frac{1}{2}\log\left(2x+1\right)+\frac{1}{2}\log\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

giac [A] time = 0.42, size = 19, normalized size = 0.90

$$-\frac{1}{2}\log(|2x+1|)+\frac{1}{2}\log(|2x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="giac")
[Out] -1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln(2x-1)}{2}-\frac{\ln(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x-1)-1/(2*x+1),x)
[Out] 1/2*ln(2*x-1)-1/2*ln(2*x+1)
```

maxima [A] time = 0.94, size = 17, normalized size = 0.81

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

mupad [B] time = 0.15, size = 6, normalized size = 0.29

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x - 1) - 1/(2*x + 1),x)
[Out] -atanh(2*x)
```

sympy [A] time = 0.10, size = 15, normalized size = 0.71

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x)
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

3.423 $\int \frac{x}{\left(1-x^2\right)^5} \, dx$

Optimal. Leaf size=13

$$\frac{1}{8\left(1-x^2\right)^4}$$

```
[Out] 1/8/(-x^2+1)^4
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{8\left(1-x^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^2)^5,x]
```

```
[Out] 1/(8*(1 - x^2)^4)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(1-x^2\right)^5} \, dx = \frac{1}{8\left(1-x^2\right)^4}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^2)^5,x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

fricas [B] time = 0.59, size = 24, normalized size = 1.85

$$\frac{1}{8\left(x^8 - 4x^6 + 6x^4 - 4x^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="fricas")
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

giac [A] time = 0.33, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2 - 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="giac")
[Out] 1/8/(x^2 - 1)^4
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{1}{8\left(x^2 - 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^5,x)
[Out] 1/8/(x^2-1)^4
```

maxima [A] time = 0.88, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2 - 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="maxima")
[Out] 1/8/(x^2 - 1)^4
```

mupad [B] time = 2.32, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x^2 - 1)^5,x)
```

```
[Out] 1/(8*(x^2 - 1)^4)
```

sympy [B] time = 0.12, size = 22, normalized size = 1.69

$$\frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**5,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

3.424 $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}\right.$

Optimal. Leaf size=13

$$\frac{1}{8\left(1-x^2\right)^4}$$

```
[Out] 1/8/(-x^2+1)^4
```

Rubi [B] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 0, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(1-x)}+\frac{5}{256(1-x)^2}+\frac{1}{64(1-x)^3}+\frac{1}{128(1-x)^4}$$

Antiderivative was successfully verified.

```
[In] Int[-1/(32*(-1 + x)^5) + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-
1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(25
6*(1 + x)^2),x]
```

```
[Out] 1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x))
+ 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 + x)
)
```

Rubi steps

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256\right.$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(
256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) -
 5/(256*(1 + x)^2),x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

fricas [B] time = 0.64, size = 24, normalized size = 1.85

$$\frac{1}{8\left(x^8-4x^6+6x^4-4x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

giac [B] time = 0.31, size = 57, normalized size = 4.38

$$\frac{5}{256(x+1)}-\frac{5}{256(x-1)}+\frac{5}{256(x+1)^2}+\frac{5}{256(x-1)^2}+\frac{1}{64(x+1)^3}-\frac{1}{64(x-1)^3}+\frac{1}{128(x+1)^4}+\frac{1}{128(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

maple [B] time = 0.00, size = 58, normalized size = 4.46

$$\frac{1}{128(x-1)^4}-\frac{1}{64(x-1)^3}+\frac{5}{256(x-1)^2}-\frac{5}{256(x-1)}+\frac{1}{128(x+1)^4}+\frac{1}{64(x+1)^3}+\frac{5}{256(x+1)^2}+\frac{5}{256(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/32/(x-1)^5+3/64/(x-1)^4-5/128/(x-1)^3+5/256/(x-1)^2-1/32/(x+1)^5-3/6
4/(x+1)^4-5/128/(x+1)^3-5/256/(x+1)^2,x)
```

```
[Out] 1/128/(x-1)^4-1/64/(x-1)^3+5/256/(x-1)^2-5/256/(x-1)+1/128/(x+1)^4+1/64/(x+
1)^3+5/256/(x+1)^2+5/256/(x+1)
```

maxima [B] time = 0.94, size = 57, normalized size = 4.38

$$\frac{5}{256(x+1)}-\frac{5}{256(x-1)}+\frac{5}{256(x+1)^2}+\frac{5}{256(x-1)^2}+\frac{1}{64(x+1)^3}-\frac{1}{64(x-1)^3}+\frac{1}{128(x+1)^4}+\frac{1}{128(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5/(256*(x - 1)^2) - 5/(256*(x + 1)^2) - 5/(128*(x - 1)^3) - 5/(128*(x +
 1)^3) + 3/(64*(x - 1)^4) - 3/(64*(x + 1)^4) - 1/(32*(x - 1)^5) - 1/(32*(x
+ 1)^5),x)
```

```
[Out] 1/(8*(x^2 - 1)^4)
```

sympy [B] time = 0.31, size = 22, normalized size = 1.69

$$\frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1/
32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

3.425 $\int \frac{1+x^6}{-1+x^6} \, dx$

Optimal. Leaf size=69

$$\frac{1}{6} \log \left(x^2-x+1\right)-\frac{1}{6} \log \left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3} \tanh^{-1}(x)$$

```
[Out] x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6} \log \left(x^2-x+1\right)-\frac{1}{6} \log \left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
```

```
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^6}{-1+x^6}\,dx &= x+2\int \frac{1}{-1+x^6}\,dx\\
&= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+\cdots}\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\cdots\\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+6x+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2}{\ }\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

fricas [A] time = 0.59, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

giac [A] time = 0.29, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

maple [A] time = 0.01, size = 67, normalized size = 0.97

$$x-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\ln\left(x^2+x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^6-1),x)
```

[Out] x+1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/6*l
n(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3*ln(x+1)

maxima [A] time = 2.01, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)/(x^6-1),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
1) + 1/3*log(x - 1)

mupad [B] time = 0.10, size = 94, normalized size = 1.36

$$x+\frac{\mathrm{atan}\left(x\,1\mathrm{i}\right)\,2\mathrm{i}}{3}-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6 + 1)/(x^6 - 1),x)

[Out] x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3
^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (3
2*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)

sympy [A] time = 0.25, size = 85, normalized size = 1.23

$$x+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**6+1)/(x**6-1),x)

[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3

3.426 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

Optimal. Leaf size=69

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

```
[Out] x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1593, 1584, 388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
```

```
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx &= \int \frac{x^3\left(\frac{1}{x^3}+x^3\right)}{-1+x^6}\,dx\\ &= \int \frac{1+x^6}{-1+x^6}\,dx\\ &= x+2\int \frac{1}{-1+x^6}\,dx\\ &= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx\\ &= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x\cdots}\\ &= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\cdots\right.\\ &= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+6x+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

fricas [A] time = 0.61, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

giac [A] time = 0.31, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 67, normalized size = 0.97

$$x-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\ln\left(x^2+x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^3+x^3)/(-1/x^3+x^3),x)
```

```
[Out] x-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^
(1/2))+1/3*ln(x-1)-1/3*ln(x+1)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)
```

maxima [A] time = 1.99, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

mupad [B] time = 0.04, size = 94, normalized size = 1.36

$$x+\frac{\mathrm{atan}\,(x\,1\mathrm{i})\,2\mathrm{i}}{3}-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\,\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\,\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/x^3 + x^3)/(1/x^3 - x^3),x)
```

```
[Out] x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3
^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (3
2*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

sympy [A] time = 0.25, size = 85, normalized size = 1.23

$$x+\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

3.427 $\int \frac{-x+x^3}{6+2x}\,dx$

Optimal. Leaf size=24

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

```
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 772}

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/(6 + 2*x),x]
```

```
[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x+x^3}{6+2x}\,dx &= \int \frac{x\left(-1+x^2\right)}{6+2x}\,dx\\ &= \int \left(4-\frac{3x}{2}+\frac{x^2}{2}-\frac{12}{3+x}\right)dx\\ &= 4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.29

$$\frac{1}{2}\left(\frac{x^3}{3}-\frac{3x^2}{2}+8x-24\log(x+3)+\frac{93}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/(6 + 2*x),x]
[Out] (93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2
```

fricas [A] time = 0.55, size = 20, normalized size = 0.83

$$\frac{1}{6}x^3-\frac{3}{4}x^2+4x-12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="fricas")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

giac [A] time = 0.31, size = 21, normalized size = 0.88

$$\frac{1}{6}x^3-\frac{3}{4}x^2+4x-12\log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="giac")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))
```

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(6+2*x),x)
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(x+3)
```

maxima [A] time = 0.63, size = 20, normalized size = 0.83

$$\frac{1}{6}x^3-\frac{3}{4}x^2+4x-12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$4\,x-12\,\ln\left(x+3\right)-\frac{3\,x^2}{4}+\frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/(2*x + 6),x)
```

```
[Out] 4*x - 12*log(x + 3) - (3*x^2)/4 + x^3/6
```

sympy [A] time = 0.08, size = 20, normalized size = 0.83

$$\frac{x^{3}}{6}-\frac{3x^{2}}{4}+4x-12\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(6+2*x),x)
```

```
[Out] x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)
```

3.428 $\int \frac{x+x^3}{-1+x} \, dx$

Optimal. Leaf size=26

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

```
[Out] 2*x+1/2*x^2+1/3*x^3+2*ln(1-x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 772}

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x+x^3}{-1+x} \, dx &= \int \frac{x\left(1+x^2\right)}{-1+x} \, dx\\ &= \int \left(2+\frac{2}{-1+x}+x+x^2\right) \, dx\\ &= 2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{1}{6}\left(2x^3+3x^2+12x+12\log(x-1)-17\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)/(-1 + x),x]
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

fricas [A] time = 0.63, size = 20, normalized size = 0.77

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

giac [A] time = 0.30, size = 21, normalized size = 0.81

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)/(x-1),x)
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(x-1)
```

maxima [A] time = 0.69, size = 20, normalized size = 0.77

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.77

$$2\,x+2\,\ln\left(x-1\right)+\frac{x^2}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3)/(x - 1),x)
```

```
[Out] 2*x + 2*log(x - 1) + x^2/2 + x^3/3
```

sympy [A] time = 0.07, size = 19, normalized size = 0.73

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

3.429 $\int (ac + (bc + d)x) \, dx$

Optimal. Leaf size=17

$$acx + \frac{1}{2}x^2(bc + d)$$

```
[Out] a*c*x+1/2*(b*c+d)*x^2
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$acx + \frac{1}{2}x^2(bc + d)$$

Antiderivative was successfully verified.

```
[In] Int[a*c + (b*c + d)*x,x]
```

```
[Out] a*c*x + ((b*c + d)*x^2)/2
```

Rubi steps

$$\int (ac + (bc + d)x) \, dx = acx + \frac{1}{2}(bc + d)x^2$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.29

$$acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c + (b*c + d)*x,x]
```

```
[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

fricas [A] time = 0.53, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

giac [A] time = 0.39, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}\,(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="giac")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$acx + \frac{(bc + d)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x+1/2*(b*c+d)*x^2
```

maxima [A] time = 0.66, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}\,(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="maxima")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

mupad [B] time = 0.02, size = 17, normalized size = 1.00

$$\left(\frac{d}{2} + \frac{b\,c}{2}\right)\,x^2 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c + x*(d + b*c),x)
```

```
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

sympy [A] time = 0.06, size = 15, normalized size = 0.88

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

3.430 $\int (dx + c(a + bx))\, dx$

Optimal. Leaf size=24

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

```
[Out] 1/2*d*x^2+1/2*c*(b*x+a)^2/b
```

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d*x + c*(a + b*x),x]
```

```
[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

Rubi steps

$$\int (dx+c(a+bx))\, dx=\frac{dx^2}{2}+\frac{c(a+bx)^2}{2b}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$acx+\frac{1}{2}bcx^2+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d*x + c*(a + b*x),x]
```

```
[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

fricas [A] time = 0.45, size = 18, normalized size = 0.75

$$\frac{1}{2}x^2cb+\frac{1}{2}x^2d+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

giac [A] time = 0.37, size = 20, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + \frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="giac")
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

maple [A] time = 0.00, size = 20, normalized size = 0.83

$$\frac{d\,x^2}{2} + \left(\frac{1}{2}b\,x^2 + ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c*(b*x+a),x)
[Out] 1/2*d*x^2+c*(1/2*b*x^2+a*x)
```

maxima [A] time = 0.57, size = 20, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + \frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="maxima")
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

mupad [B] time = 0.02, size = 17, normalized size = 0.71

$$\left(\frac{d}{2} + \frac{b\,c}{2}\right)\,x^2 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x + c*(a + b*x),x)
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

sympy [A] time = 0.06, size = 15, normalized size = 0.62

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

3.431 $\int \frac{4+4x}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=22

$$-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)-4\tan^{-1}(x)$$

```
[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {801, 635, 203, 260}

$$-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)-4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

```
[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx &= \int \left(\frac{4}{x^2}+\frac{4}{x}-\frac{4(1+x)}{1+x^2}\right)dx\\
&= -\frac{4}{x}+4\log(x)-4\int \frac{1+x}{1+x^2}\,dx\\
&= -\frac{4}{x}+4\log(x)-4\int \frac{1}{1+x^2}\,dx-4\int \frac{x}{1+x^2}\,dx\\
&= -\frac{4}{x}-4\tan^{-1}(x)+4\log(x)-2\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.09

$$4\left(-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 4*x)/(x^2*(1 + x^2)),x]
[Out] 4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2]/2)
```

fricas [A] time = 0.62, size = 25, normalized size = 1.14

$$-\frac{2\left(2\,x\arctan(x)+x\log\left(x^2+1\right)-2\,x\log(x)+2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] -2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x
```

giac [A] time = 0.39, size = 23, normalized size = 1.05

$$-\frac{4}{x}-4\,\arctan(x)-2\,\log\left(x^2+1\right)+4\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="giac")
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(abs(x))
```

maple [A] time = 0.01, size = 23, normalized size = 1.05

$$-4\arctan(x) + 4\ln(x) - 2\ln\left(x^2+1\right) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+4*x)/x^2/(x^2+1),x)
[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

maxima [A] time = 1.38, size = 22, normalized size = 1.00

$$-\frac{4}{x} - 4\,\arctan(x) - 2\,\log\left(x^2+1\right) + 4\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)
```

mupad [B] time = 0.04, size = 28, normalized size = 1.27

$$4\,\ln(x) - \frac{4}{x} + \ln\left(x - \mathrm{i}\right)\,\left(-2+2\mathrm{i}\right) + \ln\left(x + 1\mathrm{i}\right)\,\left(-2-2\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4)/(x^2*(x^2 + 1)),x)
[Out] 4*log(x) - log(x + 1i)*(2 + 2i) - log(x - 1i)*(2 - 2i) - 4/x
```

sympy [A] time = 0.13, size = 20, normalized size = 0.91

$$4\log\left(x\right) - 2\log\left(x^2+1\right) - 4\,\mathrm{atan}\left(x\right) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x**2/(x**2+1),x)
[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

3.432 $\int \frac{24+8x}{x(-4+x^2)} dx$

Optimal. Leaf size=17

$$5\log(2-x)-6\log(x)+\log(x+2)$$

```
[Out] 5*ln(2-x)-6*ln(x)+ln(2+x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {801}

$$5\log(2-x)-6\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(24 + 8*x)/(x*(-4 + x^2)),x]
```

```
[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{24+8x}{x\left(-4+x^2\right)}\,dx &= \int \left(\frac{5}{-2+x}-\frac{6}{x}+\frac{1}{2+x}\right)dx\\ &= 5\log(2-x)-6\log(x)+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.59

$$8\left(\frac{5}{8}\log(2-x)-\frac{3\log(x)}{4}+\frac{1}{8}\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(24 + 8*x)/(x*(-4 + x^2)),x]
```

```
[Out] 8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)
```

fricas [A] time = 0.71, size = 15, normalized size = 0.88

$$\log(x+2) + 5\log(x-2) - 6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="fricas")
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

giac [A] time = 0.33, size = 18, normalized size = 1.06

$$\log(|x+2|) + 5\log(|x-2|) - 6\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="giac")
[Out] log(abs(x + 2)) + 5*log(abs(x - 2)) - 6*log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$-6\ln(x) + 5\ln(x-2) + \ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24+8*x)/x/(x^2-4),x)
[Out] 5*ln(x-2)+ln(x+2)-6*ln(x)
```

maxima [A] time = 0.75, size = 15, normalized size = 0.88

$$\log(x+2) + 5\log(x-2) - 6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="maxima")
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$5\ln(x-2) + \ln(x+2) - 6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x + 24)/(x*(x^2 - 4)),x)
[Out] 5*log(x - 2) + log(x + 2) - 6*log(x)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$-6\log(x) + 5\log(x-2) + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x**2-4),x)
[Out] -6*log(x) + 5*log(x - 2) + log(x + 2)
```

3.433 $\int \frac{-1+x^2}{-2x+x^3}\,dx$

Optimal. Leaf size=19

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

[Out] 1/2*ln(x)+1/4*ln(-x^2+2)

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 446, 72}

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(-2*x + x^3),x]

[Out] Log[x]/2 + Log[2 - x^2]/4

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^2}{-2x+x^3}\,dx &= \int \frac{-1+x^2}{x\left(-2+x^2\right)}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+x}{(-2+x)x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{2(-2+x)}+\frac{1}{2x}\right)dx, x, x^2\right) \\
&= \frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$
\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(-2*x + x^3),x]
[Out] Log[x]/2 + Log[2 - x^2]/4
```

fricas [A] time = 0.63, size = 13, normalized size = 0.68

$$
\frac{1}{4}\,\log\left(x^2-2\right)+\frac{1}{2}\,\log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="fricas")
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

giac [A] time = 0.31, size = 16, normalized size = 0.84

$$
\frac{1}{4}\,\log\left(x^2\right)+\frac{1}{4}\,\log\left(\left|x^2-2\right|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="giac")
[Out] 1/4*log(x^2) + 1/4*log(abs(x^2 - 2))
```

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$
\frac{\ln(x)}{2}+\frac{\ln\left(x^2-2\right)}{4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^3-2*x),x)
```

```
[Out] 1/4*ln(x^2-2)+1/2*ln(x)
```

maxima [A] time = 0.66, size = 13, normalized size = 0.68

$$\frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

mupad [B] time = 2.34, size = 13, normalized size = 0.68

$$\frac{\ln\left(x^2-2\right)}{4}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(2*x - x^3),x)
```

```
[Out] log(x^2 - 2)/4 + log(x)/2
```

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$\frac{\log\left(x\right)}{2}+\frac{\log\left(x^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3-2*x),x)
```

```
[Out] log(x)/2 + log(x**2 - 2)/4
```

3.434 $\int \frac{1+x^2}{3x+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{3}\log\left(x^3+3x\right)$$

```
[Out] 1/3*ln(x^3+3*x)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1587}

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[3*x + x^3]/3
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x^2}{3x+x^3} \, dx = \frac{1}{3}\log\left(3x+x^3\right)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.42

$$\frac{1}{3}\log\left(x^2+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[x]/3 + Log[3 + x^2]/3
```

fricas [A] time = 0.64, size = 10, normalized size = 0.83

$$\frac{1}{3}\,\log\left(x^{3}+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="fricas")
[Out] 1/3*log(x^3 + 3*x)
```

giac [A] time = 0.40, size = 13, normalized size = 1.08

$$\frac{1}{3}\,\log\left(3\left|\frac{1}{3}\,x^{3}+x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="giac")
[Out] 1/3*log(3*abs(1/3*x^3 + x))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{\ln\left(\left(x^{2}+3\right)x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^3+3*x),x)
[Out] 1/3*ln(x*(x^2+3))
```

maxima [A] time = 0.62, size = 10, normalized size = 0.83

$$\frac{1}{3}\,\log\left(x^{3}+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")
[Out] 1/3*log(x^3 + 3*x)
```

mupad [B] time = 2.27, size = 10, normalized size = 0.83

$$\frac{\ln\left(x^{3}+3\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(3*x + x^3),x)
```

```
[Out] log(3*x + x^3)/3
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$\frac{\log\left(x^3+3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+3*x),x)
```

```
[Out] log(x**3 + 3*x)/3
```

3.435 $\int \frac{a+3bx^2}{ax+bx^3} \, dx$

Optimal. Leaf size=10

$$\log \left(ax+bx^3\right)$$

```
[Out] ln(b*x^3+a*x)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1587}

$$\log \left(ax+bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

```
[Out] Log[a*x + b*x^3]
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{a+3bx^2}{ax+bx^3} \, dx = \log \left(ax+bx^3\right)$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.10

$$\log \left(a+bx^2\right)+\log (x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

```
[Out] Log[x] + Log[a + b*x^2]
```

fricas [A] time = 0.71, size = 10, normalized size = 1.00

$$\log \left(bx^3+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="fricas")
[Out] log(b*x^3 + a*x)
```

giac [A] time = 0.33, size = 11, normalized size = 1.10

$$\log\left(\left|bx^3 + ax\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="giac")
[Out] log(abs(b*x^3 + a*x))
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\ln\left(\left(b\,x^2 + a\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b*x^2+a)/(b*x^3+a*x),x)
[Out] ln(x*(b*x^2+a))
```

maxima [A] time = 0.61, size = 10, normalized size = 1.00

$$\log\left(bx^3 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="maxima")
[Out] log(b*x^3 + a*x)
```

mupad [B] time = 0.06, size = 10, normalized size = 1.00

$$\ln\left(b\,x^3 + a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 3*b*x^2)/(a*x + b*x^3),x)
[Out] log(a*x + b*x^3)
```

sympy [A] time = 0.13, size = 8, normalized size = 0.80

$$\log\left(ax + bx^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x**2+a)/(b*x**3+a*x),x)
```

```
[Out] log(a*x + b*x**3)
```

3.436 $\int \frac{-2+4x}{-x+x^3}\,dx$

Optimal. Leaf size=17

$$\log(1-x)+2\log(x)-3\log(x+1)$$

```
[Out] ln(1-x)+2*ln(x)-3*ln(1+x)
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 801}

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+4x}{-x+x^3}\,dx &= \int \frac{-2+4x}{x\left(-1+x^2\right)}\,dx\\ &= \int \left(\frac{1}{-1+x}+\frac{2}{x}-\frac{3}{1+x}\right)dx\\ &= \log(1-x)+2\log(x)-3\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 4*x)/(-x + x^3),x]
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

fricas [A] time = 0.69, size = 15, normalized size = 0.88

$$-3 \log(x+1) + \log(x-1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="fricas")
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

giac [A] time = 0.36, size = 18, normalized size = 1.06

$$-3 \log(|x+1|) + \log(|x-1|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="giac")
[Out] -3*log(abs(x + 1)) + log(abs(x - 1)) + 2*log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$2\ln(x) + \ln(x-1) - 3\ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+4*x)/(x^3-x),x)
[Out] ln(x-1)-3*ln(x+1)+2*ln(x)
```

maxima [A] time = 0.61, size = 15, normalized size = 0.88

$$-3 \log(x+1) + \log(x-1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$\ln(x-1) - 3 \ln(x+1) + 2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 2)/(x - x^3),x)
```

```
[Out] log(x - 1) - 3*log(x + 1) + 2*log(x)
```

sympy **[A]** time = 0.13, size = 15, normalized size = 0.88

$$2\log(x) + \log(x - 1) - 3\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x**3-x),x)
```

```
[Out] 2*log(x) + log(x - 1) - 3*log(x + 1)
```

3.437 $\int \frac{4+x}{4x+x^3}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 801, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/(4*x + x^3),x]
```

```
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x}{4x+x^3}\,dx &= \int \frac{4+x}{x\left(4+x^2\right)}\,dx\\
&= \int \left(\frac{1}{x}+\frac{1-x}{4+x^2}\right)dx\\
&= \log(x)+\int \frac{1-x}{4+x^2}\,dx\\
&= \log(x)+\int \frac{1}{4+x^2}\,dx-\int \frac{x}{4+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/(4*x + x^3),x]
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

fricas [A] time = 0.64, size = 17, normalized size = 0.74

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="fricas")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

giac [A] time = 0.38, size = 18, normalized size = 0.78

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="giac")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln(x)-\frac{\ln\left(x^{2}+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+4)/(x^3+4*x),x)
[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

maxima [A] time = 1.64, size = 17, normalized size = 0.74

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^{2}+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="maxima")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

mupad [B] time = 0.05, size = 21, normalized size = 0.91

$$\ln(x)+\ln\left(x-2\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)/(4*x + x^3),x)
[Out] log(x) - log(x + 2i)*(1/2 - 1i/4) - log(x - 2i)*(1/2 + 1i/4)
```

sympy [A] time = 0.13, size = 17, normalized size = 0.74

$$\log\left(x\right)-\frac{\log\left(x^{2}+4\right)}{2}+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x**3+4*x),x)
[Out] log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

3.438 $\int \frac{-x+2x^3}{1-x^2+x^4}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[Out] 1/2*ln(x^4-x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1587}

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2+x^4\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

fricas [A] time = 0.47, size = 13, normalized size = 0.87

$$\frac{1}{2}\,\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x^4 - x^2 + 1)
```

giac [A] time = 0.28, size = 13, normalized size = 0.87

$$\frac{1}{2}\,\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
[Out] 1/2*log(x^4 - x^2 + 1)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-x)/(x^4-x^2+1),x)
[Out] 1/2*ln(x^4-x^2+1)
```

maxima [A] time = 0.65, size = 13, normalized size = 0.87

$$\frac{1}{2}\,\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x^4 - x^2 + 1)
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)
```

```
[Out] log(x^4 - x^2 + 1)/2
```

sympy [A] time = 0.09, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-x)/(x**4-x**2+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/2
```

3.439 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1594, 800}

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+x}{2x+3x^2+x^3}\,dx &= \int \frac{-3+x}{x\left(2+3x+x^2\right)}\,dx\\
&= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)dx\\
&= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

fricas [A] time = 0.61, size = 17, normalized size = 0.81

$$-\frac{5}{2}\log\left(x+2\right)+4\log\left(x+1\right)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

giac [A] time = 0.37, size = 20, normalized size = 0.95

$$-\frac{5}{2}\log\left(|x+2|\right)+4\log\left(|x+1|\right)-\frac{3}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{3\ln(x)}{2}+4\ln\left(x+1\right)-\frac{5\ln\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3)/(x^3+3*x^2+2*x),x)
[Out] -3/2*ln(x)+4*ln(x+1)-5/2*ln(x+2)
```

maxima [A] time = 0.59, size = 17, normalized size = 0.81

$$-\frac{5}{2}\log\left(x+2\right)+4\log\left(x+1\right)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
```

```
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

mupad [B] time = 0.07, size = 17, normalized size = 0.81

$$4\ \ln(x+1)-\frac{5\ \ln(x+2)}{2}-\frac{3\ \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/(2*x + 3*x^2 + x^3),x)
```

```
[Out] 4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

sympy [A] time = 0.13, size = 20, normalized size = 0.95

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
```

```
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

3.440 $\int \frac{2+4x}{x^2+2x^3+x^4}\,dx$

Optimal. Leaf size=10

$$-\frac{2}{x(x+1)}$$

```
[Out] -2/x/(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1594, 27, 74}

$$-\frac{2}{x(x+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

```
[Out] -2/(x*(1 + x))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = \int \frac{2+4x}{x^2\left(1+2x+x^2\right)}\,dx$$
$$= \int \frac{2+4x}{x^2(1+x)^2}\,dx$$
$$= -\frac{2}{x(1+x)}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
[Out] -2/(x + x^2)
```

fricas [A] time = 0.70, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="fricas")
[Out] -2/(x^2 + x)
```

giac [A] time = 0.26, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="giac")
[Out] -2/(x^2 + x)
```

maple [A] time = 0.00, size = 14, normalized size = 1.40

$$-\frac{2}{x}+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+4*x)/(x^4+2*x^3+x^2),x)
```

```
[Out] -2/x+2/(x+1)
```

maxima [A] time = 0.78, size = 9, normalized size = 0.90

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="maxima")
```

```
[Out] -2/(x^2 + x)
```

mupad [B] time = 2.20, size = 10, normalized size = 1.00

$$-\frac{2}{x\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)/(x^2 + 2*x^3 + x^4),x)
```

```
[Out] -2/(x*(x + 1))
```

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x**4+2*x**3+x**2),x)
```

```
[Out] -2/(x**2 + x)
```

3.441 $\int \frac{1+x}{-6x+x^2+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

```
[Out] 3/10*ln(2-x)-1/6*ln(x)-2/15*ln(3+x)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1594, 800}

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-6*x + x^2 + x^3),x]
```

```
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{-6x+x^2+x^3} \, dx &= \int \frac{1+x}{x\left(-6+x+x^2\right)} \, dx \\
&= \int \left(\frac{3}{10(-2+x)}-\frac{1}{6x}-\frac{2}{15(3+x)}\right) dx \\
&= \frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(3+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-6*x + x^2 + x^3),x]
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

fricas [A] time = 0.76, size = 17, normalized size = 0.68

$$-\frac{2}{15}\log(x+3)+\frac{3}{10}\log(x-2)-\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="fricas")
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

giac [A] time = 0.36, size = 20, normalized size = 0.80

$$-\frac{2}{15}\log(|x+3|)+\frac{3}{10}\log(|x-2|)-\frac{1}{6}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="giac")
[Out] -2/15*log(abs(x + 3)) + 3/10*log(abs(x - 2)) - 1/6*log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.72

$$-\frac{\ln(x)}{6}+\frac{3\ln(x-2)}{10}-\frac{2\ln(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^3+x^2-6*x),x)
[Out] 3/10*ln(x-2)-2/15*ln(x+3)-1/6*ln(x)
```

maxima [A] time = 0.65, size = 17, normalized size = 0.68

$$-\frac{2}{15}\log(x+3)+\frac{3}{10}\log(x-2)-\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="maxima")
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

mupad [B] time = 0.11, size = 17, normalized size = 0.68

$$\frac{3\ \ln(x-2)}{10} - \frac{2\ \ln(x+3)}{15} - \frac{\ln(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^2 - 6*x + x^3),x)
[Out] (3*log(x - 2))/10 - (2*log(x + 3))/15 - log(x)/6
```

sympy [A] time = 0.13, size = 20, normalized size = 0.80

$$-\frac{\log(x)}{6} + \frac{3\log(x-2)}{10} - \frac{2\log(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3+x**2-6*x),x)
[Out] -log(x)/6 + 3*log(x - 2)/10 - 2*log(x + 3)/15
```

3.442 $\int \frac{4x^2+x^3}{x+x^3} \, dx$

Optimal. Leaf size=14

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

```
[Out] x-arctan(x)+2*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1593, 1584, 774, 635, 203, 260}

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 + x^3)/(x + x^3),x]
```

```
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{4x^2+x^3}{x+x^3}\,dx &= \int \frac{4x^2+x^3}{x\left(1+x^2\right)}\,dx\\ &= \int \frac{x(4+x)}{1+x^2}\,dx\\ &= x+\int \frac{-1+4x}{1+x^2}\,dx\\ &= x+4\int \frac{x}{1+x^2}\,dx-\int \frac{1}{1+x^2}\,dx\\ &= x-\tan^{-1}(x)+2\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + x^3)/(x + x^3),x]
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

fricas [A] time = 0.53, size = 14, normalized size = 1.00

$$x-\arctan(x)+2\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2\ \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$x - \arctan(x) + 2\ln\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4*x^2)/(x^3+x),x)
[Out] x-arctan(x)+2*ln(x^2+1)
```

maxima [A] time = 1.56, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2\ \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

mupad [B] time = 2.22, size = 14, normalized size = 1.00

$$x + 2\ \ln\left(x^2 + 1\right) - \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + x^3)/(x + x^3),x)
[Out] x + 2*log(x^2 + 1) - atan(x)
```

sympy [A] time = 0.10, size = 12, normalized size = 0.86

$$x + 2\log\left(x^2 + 1\right) - \mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4*x**2)/(x**3+x),x)
[Out] x + 2*log(x**2 + 1) - atan(x)
```

3.443 $\int \frac{x+2x^3}{\left(x^2+x^4\right)^3} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

```
[Out] -1/4/(x^4+x^2)^2
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

```
[Out] -1/(4*(x^2 + x^4)^2)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3} \, dx = -\frac{1}{4\left(x^2+x^4\right)^2}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.08

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]
[Out] -1/4*1/(x^4*(1 + x^2)^2)
```

fricas [A] time = 0.60, size = 16, normalized size = 1.23

$$-\frac{1}{4\left(x^{8}+2\,x^{6}+x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

giac [A] time = 0.37, size = 11, normalized size = 0.85

$$-\frac{1}{4\left(x^{4}+x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")
[Out] -1/4/(x^4 + x^2)^2
```

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{1}{2x^{2}}-\frac{1}{4x^{4}}-\frac{1}{4\left(x^{2}+1\right)^{2}}-\frac{1}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+x)/(x^4+x^2)^3,x)
[Out] -1/4/(x^2+1)^2-1/2/(x^2+1)-1/4/x^4+1/2/x^2
```

maxima [A] time = 0.58, size = 11, normalized size = 0.85

$$-\frac{1}{4\left(x^{4}+x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")
[Out] -1/4/(x^4 + x^2)^2
```

mupad [B] time = 2.24, size = 20, normalized size = 1.54

$$-\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*x^3)/(x^2 + x^4)^3,x)
```

```
[Out] -1/(4*x^4 + 8*x^6 + 4*x^8)
```

sympy [A] time = 0.13, size = 17, normalized size = 1.31

$$-\frac{1}{4x^8+8x^6+4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+x)/(x**4+x**2)**3,x)
```

```
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

3.444 $\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx$

Optimal. Leaf size=26

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

```
[Out] b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2
```

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1593, 1584, 43}

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx &= \int \frac{x^2(a+bx)}{cx^2+dx^3}\,dx \\
&= \int \frac{a+bx}{c+dx}\,dx \\
&= \int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx \\
&= \frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$\frac{(ad-bc)\log(c+dx)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

fricas [A] time = 0.61, size = 25, normalized size = 0.96

$$\frac{bdx-(bc-ad)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="fricas")
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

giac [A] time = 0.28, size = 27, normalized size = 1.04

$$\frac{bx}{d}-\frac{(bc-ad)\log(|dx+c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

maple [A] time = 0.00, size = 32, normalized size = 1.23

$$\frac{a\ln(dx+c)}{d}-\frac{bc\ln(dx+c)}{d^2}+\frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/(d*x^3+c*x^2),x)
```

```
[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c
```

maxima [A] time = 0.72, size = 26, normalized size = 1.00

$$\frac{bx}{d} - \frac{(bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

mupad [B] time = 2.23, size = 25, normalized size = 0.96

$$\frac{\ln(c + d\,x)\,(a\,d - b\,c)}{d^2} + \frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/(c*x^2 + d*x^3),x)
```

```
[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

sympy [A] time = 0.15, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

3.445 $\int \frac{x+x^2}{-2x-x^2+x^3} \, dx$

Optimal. Leaf size=6

$$\log(2-x)$$

```
[Out] ln(2-x)
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1586, 31}

$$\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[2 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x+x^2}{-2x-x^2+x^3} \, dx &= \int \frac{1}{-2+x} \, dx \\ &= \log(2-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 0.67

$$\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[-2 + x]
```

fricas [A] time = 0.67, size = 4, normalized size = 0.67

$$\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="fricas")
[Out] log(x - 2)
```

giac [A] time = 0.28, size = 5, normalized size = 0.83

$$\log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="giac")
[Out] log(abs(x - 2))
```

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(x^3-x^2-2*x),x)
[Out] ln(x-2)
```

maxima [A] time = 0.57, size = 4, normalized size = 0.67

$$\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="maxima")
[Out] log(x - 2)
```

mupad [B] time = 0.02, size = 4, normalized size = 0.67

$$\ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2)/(2*x + x^2 - x^3),x)
```

```
[Out] log(x - 2)
```

sympy [A] time = 0.06, size = 3, normalized size = 0.50

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(x**3-x**2-2*x),x)
[Out] log(x - 2)
```

3.446 $\int \frac{1-5x^2}{x^3(1+x^2)} \, dx$

Optimal. Leaf size=20

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 77}

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 5*x^2)/(x^3*(1 + x^2)),x]

[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1-5x}{x^2(1+x)}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{6}{x}+\frac{6}{1+x}\right)dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
[Out] -1/2*1/x^2 - 6*Log[x] + 3*Log[1 + x^2]
```

fricas [A] time = 0.67, size = 25, normalized size = 1.25

$$\frac{6\,x^2\log\left(x^2+1\right)-12\,x^2\log(x)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="fricas")
[Out] 1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2
```

giac [A] time = 0.38, size = 27, normalized size = 1.35

$$\frac{6\,x^2-1}{2\,x^2}+3\,\log\left(x^2+1\right)-3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="giac")
[Out] 1/2*(6*x^2 - 1)/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

maple [A] time = 0.01, size = 19, normalized size = 0.95

$$-6\ln(x)+3\ln\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5*x^2+1)/x^3/(x^2+1),x)
```

```
[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

maxima [A] time = 0.72, size = 20, normalized size = 1.00

$$-\frac{1}{2\,x^2} + 3\,\log\left(x^2+1\right) - 3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.90

$$3\,\ln\left(x^2+1\right) - 6\,\ln(x) - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x^2 - 1)/(x^3*(x^2 + 1)),x)
```

```
[Out] 3*log(x^2 + 1) - 6*log(x) - 1/(2*x^2)
```

sympy [A] time = 0.11, size = 19, normalized size = 0.95

$$-6\log\left(x\right) + 3\log\left(x^2+1\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x**2+1)/x**3/(x**2+1),x)
```

```
[Out] -6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)
```

3.447 $\int \frac{2x}{(-1+x)\left(5+x^2\right)}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)+\frac{1}{3}\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] 1/3*ln(1-x)-1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 801, 635, 203, 260}

$$-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)+\frac{1}{3}\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x)/((-1 + x)*(5 + x^2)),x]
```

```
[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{2x}{(-1+x)\left(5+x^2\right)}\,dx &= 2\int \frac{x}{(-1+x)\left(5+x^2\right)}\,dx\\ &= 2\int \left(\frac{1}{6(-1+x)}+\frac{5-x}{6\left(5+x^2\right)}\right)dx\\ &= \frac{1}{3}\log(1-x)+\frac{1}{3}\int \frac{5-x}{5+x^2}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{x}{5+x^2}\,dx+\frac{5}{3}\int \frac{1}{5+x^2}\,dx\\ &= \frac{1}{3}\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(5+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.05

$$2\left(-\frac{1}{12}\log\left(x^2+5\right)+\frac{1}{6}\log(1-x)+\frac{1}{6}\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x)/((-1 + x)*(5 + x^2)),x]
[Out] 2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12)
```

fricas [A] time = 0.66, size = 27, normalized size = 0.71

$$\frac{1}{3}\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="fricas")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

giac [A] time = 0.39, size = 28, normalized size = 0.74

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="giac")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 28, normalized size = 0.74

$$\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}\,x}{5}\right)}{3}+\frac{\ln\left(x-1\right)}{3}-\frac{\ln\left(x^2+5\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x/(x-1)/(x^2+5),x)
[Out] 1/3*ln(x-1)-1/6*ln(x^2+5)+1/3*5^(1/2)*arctan(1/5*5^(1/2)*x)
```

maxima [A] time = 1.97, size = 27, normalized size = 0.71

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="maxima")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

mupad [B] time = 0.16, size = 44, normalized size = 1.16

$$\frac{\ln\left(x-1\right)}{3}-\ln\left(x-\sqrt{5}\,1\mathrm{i}\right)\left(\frac{1}{6}+\frac{\sqrt{5}\,1\mathrm{i}}{6}\right)+\ln\left(x+\sqrt{5}\,1\mathrm{i}\right)\left(-\frac{1}{6}+\frac{\sqrt{5}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x)/((x^2 + 5)*(x - 1)),x)
[Out] log(x - 1)/3 - log(x - 5^(1/2)*1i)*((5^(1/2)*1i)/6 + 1/6) + log(x + 5^(1/2)
*1i)*((5^(1/2)*1i)/6 - 1/6)
```

sympy [A] time = 0.14, size = 31, normalized size = 0.82

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^{2}+5\right)}{6}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,x}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x**2+5),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3
```

3.448 $\int \frac{2+x^2}{2+x}\,dx$

Optimal. Leaf size=17

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

[Out] -2*x+1/2*x^2+6*ln(2+x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {697}

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2 + 6*Log[2 + x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{2+x}\,dx &= \int \left(-2+x+\frac{6}{2+x}\right)dx\\ &= -2x+\frac{x^2}{2}+6\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.06

$$\frac{x^2}{2} - 2x + 6\log(x+2) - 6$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/(2 + x),x]
```

```
[Out] -6 - 2*x + x^2/2 + 6*Log[2 + x]
```

fricas [A] time = 0.58, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

giac [A] time = 0.37, size = 16, normalized size = 0.94

$$\frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="giac")
[Out] 1/2*x^2 - 2*x + 6*log(abs(x + 2))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^2}{2} - 2x + 6\ln\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x+2),x)
[Out] -2*x+1/2*x^2+6*ln(x+2)
```

maxima [A] time = 0.44, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$6\,\ln\left(x + 2\right) - 2\,x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/(x + 2),x)
```

```
[Out] 6*log(x + 2) - 2*x + x^2/2
```

sympy [A]   time = 0.07, size = 14, normalized size = 0.82

$$\frac{x^2}{2} - 2x + 6\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(2+x),x)
```

```
[Out] x**2/2 - 2*x + 6*log(x + 2)
```

3.449 $\int \frac{1}{(-3+x)(4+x^2)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -3/26*arctan(1/2*x)+1/13*ln(3-x)-1/26*ln(x^2+4)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-3 + x)*(4 + x^2)),x]
```

```
[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx &= \frac{1}{13}\int \frac{1}{-3+x}\,dx+\frac{1}{13}\int \frac{-3-x}{4+x^2}\,dx\\ &= \frac{1}{13}\log(3-x)-\frac{1}{13}\int \frac{x}{4+x^2}\,dx-\frac{3}{13}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{13}\log(3-x)-\frac{1}{26}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.16

$$-\frac{1}{26}\log\left((x-3)^2+6(x-3)+13\right)+\frac{1}{13}\log(x-3)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-3 + x)*(4 + x^2)),x]
[Out] (-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/13
```

fricas [A] time = 0.47, size = 21, normalized size = 0.68

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="fricas")
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

giac [A] time = 0.27, size = 22, normalized size = 0.71

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="giac")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(abs(x - 3))
```

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$-\frac{3\arctan\left(\frac{x}{2}\right)}{26}+\frac{\ln\left(x-3\right)}{13}-\frac{\ln\left(x^{2}+4\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-3)/(x^2+4),x)
```

```
[Out] -1/26*ln(x^2+4)-3/26*arctan(1/2*x)+1/13*ln(x-3)
```

maxima [A] time = 0.99, size = 21, normalized size = 0.68

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^{2}+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln\left(x-3\right)}{13}+\ln\left(x-2\mathrm{i}\right)\left(-\frac{1}{26}+\frac{3}{52}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\left(-\frac{1}{26}-\frac{3}{52}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 4)*(x - 3)),x)
```

```
[Out] log(x - 3)/13 - log(x - 2i)*(1/26 - 3i/52) - log(x + 2i)*(1/26 + 3i/52)
```

sympy [A] time = 0.14, size = 22, normalized size = 0.71

$$\frac{\log\left(x-3\right)}{13}-\frac{\log\left(x^{2}+4\right)}{26}-\frac{3\operatorname{atan}\left(\frac{x}{2}\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x**2+4),x)
```

```
[Out] log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

3.450 $\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx$

Optimal. Leaf size=19

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

```
[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 72}

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{-2+3x}{x(5+2x)}\,dx,x,x^6\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \left(-\frac{2}{5x}+\frac{19}{5(5+2x)}\right)dx,x,x^6\right)\\
&= -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

fricas [A] time = 0.48, size = 15, normalized size = 0.79

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2}{5}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="fricas")
[Out] 19/60*log(2*x^6 + 5) - 2/5*log(x)
```

giac [A] time = 0.41, size = 17, normalized size = 0.89

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{1}{15}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="giac")
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6-2)/x/(2*x^6+5),x)
[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)
```

maxima [A] time = 0.47, size = 17, normalized size = 0.89

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{1}{15}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")
```

```
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

mupad [B] time = 0.09, size = 13, normalized size = 0.68

$$\frac{19\ \ln\left(x^6+\frac{5}{2}\right)}{60}-\frac{2\ \ln(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6 - 2)/(x*(2*x^6 + 5)),x)
```

```
[Out] (19*log(x^6 + 5/2))/60 - (2*log(x))/5
```

sympy [A] time = 0.11, size = 17, normalized size = 0.89

$$-\frac{2\log\left(x\right)}{5}+\frac{19\log\left(2x^6+5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**6-2)/x/(2*x**6+5),x)
```

```
[Out] -2*log(x)/5 + 19*log(2*x**6 + 5)/60
```

3.451 $\int \frac{3+2x}{(-2+x)(5+x)} \, dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(x+5)$$

```
[Out] ln(2-x)+ln(5+x)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {72}

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

```
[Out] Log[2 - x] + Log[5 + x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{3+2x}{(-2+x)(5+x)} \, dx &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right) dx \\ &= \log(2-x)+\log(5+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$\log(x-2)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

```
[Out] Log[-2 + x] + Log[5 + x]
```

fricas [A] time = 0.58, size = 9, normalized size = 0.82

$$\log\left(x^2+3x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
```

```
[Out] log(x^2 + 3*x - 10)
```

giac [A] time = 0.29, size = 11, normalized size = 1.00

$$\log\left(|x+5|\right)+\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
```

```
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

maple [A] time = 0.00, size = 9, normalized size = 0.82

$$\ln\left(\left(x-2\right)\left(x+5\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(x-2)/(x+5),x)
```

```
[Out] ln((x-2)*(x+5))
```

maxima [A] time = 0.45, size = 9, normalized size = 0.82

$$\log\left(x+5\right)+\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
```

```
[Out] log(x + 5) + log(x - 2)
```

mupad [B] time = 2.22, size = 9, normalized size = 0.82

$$\ln\left(x^2+3\,x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((x - 2)*(x + 5)),x)
```

```
[Out] log(3*x + x^2 - 10)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$\log\left(x^2+3x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
```

```
[Out] log(x**2 + 3*x - 10)
```

3.452 $\int \frac{x^4}{4+5x^2+x^4} \, dx$

Optimal. Leaf size=18

$$x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)$$

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1122, 1166, 203}

$$x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \int \frac{4+5x^2}{4+5x^2+x^4}\,dx$$
$$= x + \frac{1}{3}\int \frac{1}{1+x^2}\,dx - \frac{16}{3}\int \frac{1}{4+x^2}\,dx$$
$$= x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$x + \frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

fricas [A] time = 0.64, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

giac [A] time = 0.37, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.72

$$x + \frac{\arctan(x)}{3} - \frac{8\arctan\left(\frac{x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+5*x^2+4),x)
```

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

maxima [A] time = 1.01, size = 12, normalized size = 0.67

$$x-\frac{8}{3}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

mupad [B] time = 2.22, size = 12, normalized size = 0.67

$$x-\frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3}+\frac{\mathrm{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(5*x^2 + x^4 + 4),x)
```

```
[Out] x - (8*atan(x/2))/3 + atan(x)/3
```

sympy [A] time = 0.14, size = 14, normalized size = 0.78

$$x-\frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3}+\frac{\mathrm{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
```

```
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

3.453 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {88}

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx &= \int \left(\frac{1}{8(1+x)}-\frac{1}{(2+x)^2}+\frac{2}{2+x}-\frac{1}{2(3+x)^3}-\frac{5}{4(3+x)^2}-\frac{17}{8(3+x)}\right)dx\\ &= \frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{x+2}+\frac{10}{x+3}+\frac{2}{(x+3)^2}+\log(-x-1)+16\log(x+2)-17\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

fricas [B] time = 0.63, size = 83, normalized size = 1.80

$$\frac{18\,x^2 - 17\left(x^3 + 8\,x^2 + 21\,x + 18\right)\log\left(x + 3\right) + 16\left(x^3 + 8\,x^2 + 21\,x + 18\right)\log\left(x + 2\right) + \left(x^3 + 8\,x^2 + 21\,x + 18\right)\log}{8\left(x^3 + 8\,x^2 + 21\,x + 18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

giac [A] time = 0.36, size = 52, normalized size = 1.13

$$\frac{1}{x + 2} - \frac{\frac{7}{x+2} + 6}{4\left(\frac{1}{x+2} + 1\right)^2} + \frac{1}{8}\log\left(\left|-\frac{1}{x + 2} + 1\right|\right) - \frac{17}{8}\log\left(\left|-\frac{1}{x + 2} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

maple [A] time = 0.01, size = 39, normalized size = 0.85

$$\frac{\ln\left(x + 1\right)}{8} + 2\ln\left(x + 2\right) - \frac{17\ln\left(x + 3\right)}{8} + \frac{1}{x + 2} + \frac{1}{4\left(x + 3\right)^2} + \frac{5}{4\left(x + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x+2)^2/(x+3)^3,x)
```

```
[Out] 1/(x+2)+1/4/(x+3)^2+5/4/(x+3)+1/8*ln(x+1)+2*ln(x+2)-17/8*ln(x+3)
```

maxima [A] time = 0.45, size = 46, normalized size = 1.00

$$\frac{9\,x^2 + 50\,x + 68}{4\left(x^3 + 8\,x^2 + 21\,x + 18\right)} - \frac{17}{8}\log\left(x + 3\right) + 2\log\left(x + 2\right) + \frac{1}{8}\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

mupad [B] time = 0.04, size = 45, normalized size = 0.98

$$\frac{\ln(x+1)}{8}+2\,\ln(x+2)-\frac{17\,\ln(x+3)}{8}+\frac{\frac{9\,x^2}{4}+\frac{25\,x}{2}+17}{x^3+8\,x^2+21\,x+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 + 1
7)/(21*x + 8*x^2 + x^3 + 18)
```

sympy [A] time = 0.19, size = 46, normalized size = 1.00

$$\frac{9x^2+50x+68}{4x^3+32x^2+84x+72}+\frac{\log(x+1)}{8}+2\log(x+2)-\frac{17\log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

3.454 $\int \frac{x}{-1+x^2}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {260}

$$\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2),x]
```

```
[Out] Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2),x]
```

```
[Out] Log[-1 + x^2]/2
```

fricas [A] time = 0.65, size = 8, normalized size = 0.67

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1)
```

giac [A] time = 0.27, size = 9, normalized size = 0.75

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1),x)
```

```
[Out] 1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [A] time = 0.44, size = 8, normalized size = 0.67

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 - 1)
```

mupad [B] time = 0.04, size = 8, normalized size = 0.67

$$\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 1),x)
```

```
[Out] log(x^2 - 1)/2
```

sympy [A] time = 0.08, size = 7, normalized size = 0.58

$$\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1),x)
```

```
[Out] log(x**2 - 1)/2
```

3.455 $\int \frac{1}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=21

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*x/(-x^2+1)+1/2*arctanh(x)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {199, 207}

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-1+x^2\right)^2} \, dx &= \frac{x}{2\left(1-x^2\right)}-\frac{1}{2}\int \frac{1}{-1+x^2} \, dx\\
&= \frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(-2),x]
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

fricas [B] time = 0.72, size = 34, normalized size = 1.62

$$\frac{\left(x^2-1\right)\log\left(x+1\right)-\left(x^2-1\right)\log\left(x-1\right)-2\,x}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

giac [A] time = 0.27, size = 25, normalized size = 1.19

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 28, normalized size = 1.33

$$-\frac{\ln\left(x-1\right)}{4}+\frac{\ln\left(x+1\right)}{4}-\frac{1}{4\left(x-1\right)}-\frac{1}{4\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2,x)
[Out] -1/4/(x-1)-1/4*ln(x-1)-1/4/(x+1)+1/4*ln(x+1)
```

maxima [A] time = 0.45, size = 23, normalized size = 1.10

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [B] time = 2.22, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}(x)}{2} - \frac{x}{2\left(x^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^2,x)
```

```
[Out] atanh(x)/2 - x/(2*(x^2 - 1))
```

sympy [A] time = 0.11, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2 - 2} - \frac{\log(x - 1)}{4} + \frac{\log(x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2,x)
```

```
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

3.456 $\int \frac{x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 203}

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(1+x^2\right)^2} \, dx &= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2} \, dx\\ &= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x)-\frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^2)^2,x]
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

fricas [A] time = 0.55, size = 21, normalized size = 1.11

$$\frac{\left(x^2+1\right)\arctan(x)-x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

giac [A] time = 0.29, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{x}{2\left(x^2+1\right)}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^2,x)
[Out] -1/2/(x^2+1)*x+1/2*arctan(x)
```

maxima [A] time = 1.00, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + 1)^2,x)
```

```
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

3.457 $\int \frac{1}{2+3x}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\log(3x+2)$$

```
[Out] 1/3*ln(2+3*x)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\log(2+3x)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

fricas [A] time = 0.50, size = 8, normalized size = 0.80

$$\frac{1}{3}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

giac [A] time = 0.38, size = 9, normalized size = 0.90

$$\frac{1}{3}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\ln\left(3x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x+2),x)
```

```
[Out] 1/3*ln(3*x+2)
```

maxima [A] time = 0.45, size = 8, normalized size = 0.80

$$\frac{1}{3}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(3*x + 2)
```

mupad [B] time = 0.07, size = 6, normalized size = 0.60

$$\frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + 2),x)
```

```
[Out] log(x + 2/3)/3
```

sympy [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log(3x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

3.458 $\int \frac{1}{a^2+x^2} dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

```
[Out] arctan(x/a)/a
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a^2+x^2} dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

fricas [A] time = 0.58, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="fricas")
[Out] arctan(x/a)/a
```

giac [A] time = 0.26, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="giac")
[Out] arctan(x/a)/a
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x^2),x)
[Out] arctan(x/a)/a
```

maxima [A] time = 0.99, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="maxima")
[Out] arctan(x/a)/a
```

mupad [B] time = 2.25, size = 10, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + x^2),x)
```

```
[Out] atan(x/a)/a
```

sympy **[C]** time = 0.11, size = 20, normalized size = 2.00

$$\frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x**2),x)
```

```
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

3.459 $\int \frac{1}{a+bx^2} \, dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2} \, dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.78, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.38, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 0.98, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 2.24, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.13, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.460 $\int \frac{1}{2-x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-x+x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, -1+2x\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)^(-1),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

fricas [A] time = 0.60, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

giac [A] time = 0.38, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

maple [A] time = 0.00, size = 17, normalized size = 0.89

$$\frac{2\sqrt{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-x+2),x)
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

maxima [A] time = 0.95, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

mupad [B] time = 0.03, size = 16, normalized size = 0.84

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,(2\,x-1)}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x + 2),x)
[Out] (2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

sympy [A] time = 0.11, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-x+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

3.461 $\int x^2 \left(4-x^2\right)^2 dx$

Optimal. Leaf size=22

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

```
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(4 - x^2)^2,x]
```

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(4-x^2\right)^2 dx &= \int \left(16x^2-8x^4+x^6\right) dx\\ &= \frac{16x^3}{3}-\frac{8x^5}{5}+\frac{x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(4 - x^2)^2,x]
```

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

fricas [A] time = 0.41, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

giac [A] time = 0.31, size = 16, normalized size = 0.73

$$\frac{1}{7}\,x^7 - \frac{8}{5}\,x^5 + \frac{16}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="giac")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+4)^2,x)
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{1}{7}\,x^7 - \frac{8}{5}\,x^5 + \frac{16}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

mupad [B] time = 0.02, size = 17, normalized size = 0.77

$$\frac{x^3\,\left(15\,x^4-168\,x^2+560\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2 - 4)^2,x)
```

```
[Out] (x^3*(15*x^4 - 168*x^2 + 560))/105
```

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+4)**2,x)
```

```
[Out] x**7/7 - 8*x**5/5 + 16*x**3/3
```

3.462 $\int x\left(1-x^3\right)^2 \,dx$

Optimal. Leaf size=22

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

```
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {270}

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^2,x]
```

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(1-x^3\right)^2 \,dx &= \int \left(x-2x^4+x^7\right) dx\\ &= \frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^2,x]
```

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

fricas [A] time = 0.51, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="fricas")
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

giac [A] time = 0.29, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="giac")
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^2,x)
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="maxima")
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{x^2\left(5x^6 - 16x^3 + 20\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^3 - 1)^2,x)
```

```
[Out] (x^2*(5*x^6 - 16*x^3 + 20))/40
```

sympy [**A**] time = 0.06, size = 15, normalized size = 0.68

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**2,x)
```

```
[Out] x**8/8 - 2*x**5/5 + x**2/2
```

3.463 $\int \frac{-4+5x^2+x^3}{x^2}\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

```
[Out] 4/x+5*x+1/2*x^2
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {14}

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 5*x^2 + x^3)/x^2,x]
```

```
[Out] 4/x + 5*x + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+5x^2+x^3}{x^2}\,dx &= \int \left(5-\frac{4}{x^2}+x\right)dx\\ &= \frac{4}{x}+5x+\frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 5*x^2 + x^3)/x^2,x]
```

```
[Out] 4/x + 5*x + x^2/2
```

fricas [A] time = 0.72, size = 15, normalized size = 0.94

$$\frac{x^3 + 10\,x^2 + 8}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + 10*x^2 + 8)/x
```

giac [A] time = 0.31, size = 14, normalized size = 0.88

$$\frac{1}{2}\,x^2 + 5\,x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 5*x + 4/x
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5*x^2-4)/x^2,x)
```

```
[Out] 4/x+5*x+1/2*x^2
```

maxima [A] time = 0.46, size = 14, normalized size = 0.88

$$\frac{1}{2}\,x^2 + 5\,x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 5*x + 4/x
```

mupad [B] time = 0.03, size = 15, normalized size = 0.94

$$\frac{x^3 + 10\,x^2 + 8}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + x^3 - 4)/x^2,x)
```

```
[Out] (10*x^2 + x^3 + 8)/(2*x)
```

sympy [A] time = 0.07, size = 10, normalized size = 0.62

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5*x**2-4)/x**2,x)
```

```
[Out] x**2/2 + 5*x + 4/x
```

3.464 $\int \frac{-1+x}{3-4x+3x^2} \, dx$

Optimal. Leaf size=37

$$\frac{1}{6}\log\left(3x^2-4x+3\right)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

```
[Out] 1/6*ln(3*x^2-4*x+3)+1/15*arctan(1/5*(2-3*x)*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {634, 618, 204, 628}

$$\frac{1}{6}\log\left(3x^2-4x+3\right)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(3 - 4*x + 3*x^2),x]
```

```
[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{3-4x+3x^2}\,dx &= \frac{1}{6}\int \frac{-4+6x}{3-4x+3x^2}\,dx - \frac{1}{3}\int \frac{1}{3-4x+3x^2}\,dx\\ &= \frac{1}{6}\log\left(3-4x+3x^2\right)+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-20-x^2}\,dx,x,-4+6x\right)\\ &= \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}+\frac{1}{6}\log\left(3-4x+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{6}\log\left(3x^2-4x+3\right)-\frac{\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(3 - 4*x + 3*x^2),x]
[Out] -1/3*ArcTan[(-2 + 3*x)/Sqrt[5]]/Sqrt[5] + Log[3 - 4*x + 3*x^2]/6
```

fricas [A] time = 0.60, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right)+\frac{1}{6}\,\log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="fricas")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

giac [A] time = 0.27, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right)+\frac{1}{6}\,\log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{5}\ \arctan\left(\frac{(6x-4)\sqrt{5}}{10}\right)}{15}+\frac{\ln\left(3x^2-4x+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(3*x^2-4*x+3),x)
[Out] 1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))
```

maxima [A] time = 0.98, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\ \arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right)+\frac{1}{6}\ \log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="maxima")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

mupad [B] time = 2.22, size = 30, normalized size = 0.81

$$\frac{\ln\left(x^2-\frac{4\,x}{3}+1\right)}{6}-\frac{\sqrt{5}\ \mathrm{atan}\left(\frac{3\,\sqrt{5}\,x}{5}-\frac{2\,\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(3*x^2 - 4*x + 3),x)
[Out] log(x^2 - (4*x)/3 + 1)/6 - (5^(1/2)*atan((3*5^(1/2)*x)/5 - (2*5^(1/2))/5))/
15
```

sympy [A] time = 0.12, size = 39, normalized size = 1.05

$$\frac{\log\left(x^2-\frac{4x}{3}+1\right)}{6}-\frac{\sqrt{5}\ \mathrm{atan}\left(\frac{3\sqrt{5}x}{5}-\frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x**2-4*x+3),x)
[Out] log(x**2 - 4*x/3 + 1)/6 - sqrt(5)*atan(3*sqrt(5)*x/5 - 2*sqrt(5)/5)/15
```

3.465 $\int \left(2+x^3\right)^2 \, dx$

Optimal. Leaf size=14

$$\frac{x^7}{7}+x^4+4 x$$

[Out] 4*x+x^4+1/7*x^7

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {194}

$$\frac{x^7}{7}+x^4+4 x$$

Antiderivative was successfully verified.

[In] Int[(2 + x^3)^2,x]

[Out] 4*x + x^4 + x^7/7

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(2+x^3\right)^2 \, dx &= \int \left(4+4 x^3+x^6\right) \, dx\\ &= 4 x+x^4+\frac{x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x^7}{7}+x^4+4 x$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^3)^2,x]

[Out] 4*x + x^4 + x^7/7

fricas [A] time = 0.49, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="fricas")
[Out] 1/7*x^7 + x^4 + 4*x
```

giac [A] time = 0.36, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="giac")
[Out] 1/7*x^7 + x^4 + 4*x
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)^2,x)
[Out] 4*x+x^4+1/7*x^7
```

maxima [A] time = 0.47, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="maxima")
[Out] 1/7*x^7 + x^4 + 4*x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.93

$$\frac{x\left(x^6 + 7x^3 + 28\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 2)^2,x)
```

```
[Out] (x*(7*x^3 + x^6 + 28))/7
```

sympy [A] time = 0.05, size = 10, normalized size = 0.71

$$\frac{x^7}{7} + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)**2,x)
```

```
[Out] x**7/7 + x**4 + 4*x
```

3.466 $\int \frac{-4+x^2}{2+x} \, dx$

Optimal. Leaf size=11

$$\frac{x^2}{2} - 2x$$

```
[Out] -2*x+1/2*x^2
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627}

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\begin{aligned} \int \frac{-4+x^2}{2+x} \, dx &= \int (-2+x) \, dx \\ &= -2x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2
```

fricas [A]   time = 0.60, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2*x
```

giac [A]   time = 0.29, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x
```

maple [A]   time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)/(x+2),x)
```

```
[Out] -2*x+1/2*x^2
```

maxima [A]   time = 0.45, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x
```

mupad [B]   time = 0.02, size = 6, normalized size = 0.55

$$\frac{x\,\left(x-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 4)/(x + 2),x)
```

```
[Out] (x*(x - 4))/2
```

sympy [A] time = 0.06, size = 7, normalized size = 0.64

$$\frac{x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(2+x),x)
```

```
[Out] x**2/2 - 2*x
```

3.467 $\int \frac{1}{(2+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

```
[Out] 2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)*(1 + x^2)),x]
```

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+x)\left(1+x^2\right)}\,dx &= \frac{1}{5}\int \frac{1}{2+x}\,dx+\frac{1}{5}\int \frac{2-x}{1+x^2}\,dx\\ &= \frac{1}{5}\log(2+x)-\frac{1}{5}\int \frac{x}{1+x^2}\,dx+\frac{2}{5}\int \frac{1}{1+x^2}\,dx\\ &= \frac{2}{5}\tan^{-1}(x)+\frac{1}{5}\log(2+x)-\frac{1}{10}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + x)*(1 + x^2)),x]
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

fricas [A] time = 0.69, size = 19, normalized size = 0.76

$$\frac{2}{5}\arctan(x)-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="fricas")
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

giac [A] time = 0.37, size = 20, normalized size = 0.80

$$\frac{2}{5}\arctan(x)-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{2\arctan(x)}{5}+\frac{\ln(x+2)}{5}-\frac{\ln\left(x^{2}+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+2)/(x^2+1),x)
[Out] 2/5*arctan(x)+1/5*ln(x+2)-1/10*ln(x^2+1)
```

maxima [A] time = 0.99, size = 19, normalized size = 0.76

$$\frac{2}{5}\arctan(x)-\frac{1}{10}\log\left(x^{2}+1\right)+\frac{1}{5}\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="maxima")
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

mupad [B] time = 0.05, size = 25, normalized size = 1.00

$$\frac{\ln(x+2)}{5}+\ln(x-\mathrm{i})\left(-\frac{1}{10}-\frac{1}{5}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{1}{10}+\frac{1}{5}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x + 2)),x)
[Out] log(x + 2)/5 - log(x - 1i)*(1/10 + 1i/5) - log(x + 1i)*(1/10 - 1i/5)
```

sympy [A] time = 0.14, size = 20, normalized size = 0.80

$$\frac{\log(x+2)}{5}-\frac{\log\left(x^{2}+1\right)}{10}+\frac{2\operatorname{atan}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x**2+1),x)
[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

3.468 $\int \frac{1}{(1+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\left(1+x^2\right)}\,dx &= \frac{1}{2}\int \frac{1}{1+x}\,dx + \frac{1}{2}\int \frac{1-x}{1+x^2}\,dx\\ &= \frac{1}{2}\log(1+x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

fricas [A] time = 0.63, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

giac [A] time = 0.29, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x^2+1),x)
[Out] 1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)
```

maxima [A] time = 1.36, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

mupad [B] time = 2.23, size = 25, normalized size = 1.00

$$\frac{\ln(x+1)}{2}+\ln(x-\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x + 1)),x)
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

sympy [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2}-\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+1),x)
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.469 $\int \frac{x}{(1+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*arctan(x)-1/2*ln(1+x)+1/4*ln(x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {801, 635, 203, 260}

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{2(1+x)}+\frac{1+x}{2\left(1+x^2\right)}\right)dx\\ &= -\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx\\ &= -\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

fricas [A] time = 0.80, size = 19, normalized size = 0.76

$$\frac{1}{2}\,\arctan(x)+\frac{1}{4}\,\log\left(x^2+1\right)-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

giac [A] time = 0.25, size = 20, normalized size = 0.80

$$\frac{1}{2}\,\arctan(x)+\frac{1}{4}\,\log\left(x^2+1\right)-\frac{1}{2}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1))
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2}-\frac{\ln(x+1)}{2}+\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(x^2+1),x)
```

```
[Out] 1/2*arctan(x)-1/2*ln(x+1)+1/4*ln(x^2+1)
```

maxima [A] time = 1.43, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

mupad [B] time = 2.21, size = 25, normalized size = 1.00

$$-\frac{\ln(x+1)}{2}+\ln(x-\mathrm{i})\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x + 1)),x)
```

```
[Out] log(x - 1i)*(1/4 - 1i/4) - log(x + 1)/2 + log(x + 1i)*(1/4 + 1i/4)
```

sympy [A] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{\log(x+1)}{2}+\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x**2+1),x)
```

```
[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

3.470 $\int \frac{2x+x^2}{(1+x)^2}\,dx$

Optimal. Leaf size=9

$$\frac{x^2}{x+1}$$

```
[Out] x^2/(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {683}

$$x+\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{2x+x^2}{(1+x)^2}\,dx &= \int \left(1-\frac{1}{(1+x)^2}\right)dx\\ &= x+\frac{1}{1+x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 0.78

$$x+\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

fricas [A] time = 0.54, size = 12, normalized size = 1.33

$$\frac{x^2 + x + 1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] (x^2 + x + 1)/(x + 1)
```

giac [A] time = 0.30, size = 8, normalized size = 0.89

$$x + \frac{1}{x + 1} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] x + 1/(x + 1) + 1
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x+1)^2,x)
```

```
[Out] x+1/(x+1)
```

maxima [A] time = 0.95, size = 7, normalized size = 0.78

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] x + 1/(x + 1)
```

mupad [B] time = 0.02, size = 7, normalized size = 0.78

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2)/(x + 1)^2,x)
```

```
[Out] x + 1/(x + 1)
```

sympy [A] time = 0.07, size = 5, normalized size = 0.56

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(1+x)**2,x)
```

```
[Out] x + 1/(x + 1)
```

3.471 $\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] arctan(1/2*x)-3/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1166, 203}

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx &= -\left(3\int \frac{1}{1+2x^2}\,dx\right)+4\int \frac{1}{8+2x^2}\,dx\\ &= \tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

fricas [A] time = 0.48, size = 16, normalized size = 0.73

$$-\frac{3}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)+\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="fricas")
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

giac [A] time = 0.29, size = 16, normalized size = 0.73

$$-\frac{3}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)+\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="giac")
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

maple [A] time = 0.01, size = 17, normalized size = 0.77

$$-\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)}{2}+\arctan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-10)/(2*x^4+9*x^2+4),x)
[Out] arctan(1/2*x)-3/2*2^(1/2)*arctan(2^(1/2)*x)
```

maxima [A] time = 1.58, size = 16, normalized size = 0.73

$$-\frac{3}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)+\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="maxima")
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

mupad [B] time = 0.05, size = 16, normalized size = 0.73

$$\operatorname{atan}\left(\frac{x}{2}\right)-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 10)/(9*x^2 + 2*x^4 + 4),x)
[Out] atan(x/2) - (3*2^(1/2)*atan(2^(1/2)*x))/2
```

sympy [A] time = 0.15, size = 20, normalized size = 0.91

$$\operatorname{atan}\left(\frac{x}{2}\right)-\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-10)/(2*x**4+9*x**2+4),x)
[Out] atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2
```

3.472 $\int \frac{31+5x}{11-4x+3x^2}\,dx$

Optimal. Leaf size=37

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

```
[Out] 5/6*ln(3*x^2-4*x+11)-103/87*arctan(1/29*(2-3*x)*29^(1/2))*29^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {634, 618, 204, 628}

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In] Int[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

```
[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{31+5x}{11-4x+3x^2}\,dx &= \frac{5}{6}\int \frac{-4+6x}{11-4x+3x^2}\,dx+\frac{103}{3}\int \frac{1}{11-4x+3x^2}\,dx\\ &= \frac{5}{6}\log\left(11-4x+3x^2\right)-\frac{206}{3}\,\mathrm{Subst}\left(\int \frac{1}{-116-x^2}\,dx,x,-4+6x\right)\\ &= -\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}+\frac{5}{6}\log\left(11-4x+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{5}{6}\log\left(3x^2-4x+11\right)+\frac{103\tan^{-1}\left(\frac{3x-2}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
[Out] (103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

fricas [A] time = 0.68, size = 30, normalized size = 0.81

$$\frac{103}{87}\,\sqrt{29}\,\arctan\left(\frac{1}{29}\,\sqrt{29}\,(3\,x-2)\right)+\frac{5}{6}\,\log\left(3\,x^2-4\,x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="fricas")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

giac [A] time = 0.38, size = 30, normalized size = 0.81

$$\frac{103}{87}\,\sqrt{29}\,\arctan\left(\frac{1}{29}\,\sqrt{29}\,(3\,x-2)\right)+\frac{5}{6}\,\log\left(3\,x^2-4\,x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="giac")
```

```
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{103\sqrt{29}\ \arctan\left(\frac{(6x-4)\sqrt{29}}{58}\right)}{87}+\frac{5\ln\left(3x^2-4x+11\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((31+5*x)/(3*x^2-4*x+11),x)
```

```
[Out] 5/6*ln(3*x^2-4*x+11)+103/87*29^(1/2)*arctan(1/58*(6*x-4)*29^(1/2))
```

maxima [A] time = 1.39, size = 30, normalized size = 0.81

$$\frac{103}{87}\sqrt{29}\ \arctan\left(\frac{1}{29}\sqrt{29}\,(3\,x-2)\right)+\frac{5}{6}\ \log\left(3\,x^2-4\,x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="maxima")
```

```
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.81

$$\frac{5\ \ln\left(x^2-\frac{4\,x}{3}+\frac{11}{3}\right)}{6}+\frac{103\,\sqrt{29}\ \mathrm{atan}\left(\frac{3\,\sqrt{29}\,x}{29}-\frac{2\,\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 31)/(3*x^2 - 4*x + 11),x)
```

```
[Out] (5*log(x^2 - (4*x)/3 + 11/3))/6 + (103*29^(1/2)*atan((3*29^(1/2)*x)/29 - (2
*29^(1/2))/29))/87
```

sympy [A] time = 0.12, size = 44, normalized size = 1.19

$$\frac{5\log\left(x^2-\frac{4x}{3}+\frac{11}{3}\right)}{6}+\frac{103\sqrt{29}\ \mathrm{atan}\left(\frac{3\sqrt{29}\,x}{29}-\frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x**2-4*x+11),x)
```

```
[Out] 5*log(x**2 - 4*x/3 + 11/3)/6 + 103*sqrt(29)*atan(3*sqrt(29)*x/29 - 2*sqrt(2
9)/29)/87
```

3.473 $\int \frac{-2+x^2+x^3}{x^4} \, dx$

Optimal. Leaf size=15

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

```
[Out] 2/3/x^3-1/x+ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {14}

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2 + x^3)/x^4,x]
```

```
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+x^2+x^3}{x^4} \, dx &= \int \left(-\frac{2}{x^4} + \frac{1}{x^2} + \frac{1}{x}\right) dx \\ &= \frac{2}{3x^3} - \frac{1}{x} + \log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2 + x^3)/x^4,x]
```

```
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

fricas [A] time = 0.50, size = 19, normalized size = 1.27

$$\frac{3\,x^{3}\log(x)-3\,x^{2}+2}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3
```

giac [A] time = 0.27, size = 16, normalized size = 1.07

$$-\frac{3\,x^{2}-2}{3\,x^{3}}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="giac")
[Out] -1/3*(3*x^2 - 2)/x^3 + log(abs(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x)-\frac{1}{x}+\frac{2}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-2)/x^4,x)
[Out] 2/3/x^3-1/x+ln(x)
```

maxima [A] time = 0.97, size = 15, normalized size = 1.00

$$-\frac{3\,x^{2}-2}{3\,x^{3}}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")
[Out] -1/3*(3*x^2 - 2)/x^3 + log(x)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\ln(x)-\frac{x^{2}-\frac{2}{3}}{x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 - 2)/x^4,x)
```

```
[Out] log(x) - (x^2 - 2/3)/x^3
```

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\log(x) + \frac{2 - 3x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-2)/x**4,x)
```

```
[Out] log(x) + (2 - 3*x**2)/(3*x**3)
```

3.474 $\int \frac{1+x+x^3}{x^2} \, dx$

Optimal. Leaf size=15

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

```
[Out] -1/x+1/2*x^2+ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {14}

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/x^2,x]

[Out] -x^(-1) + x^2/2 + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^3}{x^2} \, dx &= \int \left(\frac{1}{x^2} + \frac{1}{x} + x\right) dx \\
&= -\frac{1}{x} + \frac{x^2}{2} + \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/x^2,x]

[Out] -x^(-1) + x^2/2 + Log[x]
```

fricas [A] time = 0.65, size = 15, normalized size = 1.00

$$\frac{x^3 + 2\,x\log(x) - 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="fricas")
[Out] 1/2*(x^3 + 2*x*log(x) - 2)/x
```

giac [A] time = 0.28, size = 14, normalized size = 0.93

$$\frac{1}{2}\,x^2 - \frac{1}{x} + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="giac")
[Out] 1/2*x^2 - 1/x + log(abs(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{x^2}{2} + \ln(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/x^2,x)
[Out] -1/x+1/2*x^2+ln(x)
```

maxima [A] time = 0.95, size = 13, normalized size = 0.87

$$\frac{1}{2}\,x^2 - \frac{1}{x} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="maxima")
[Out] 1/2*x^2 - 1/x + log(x)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\ln(x) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 1)/x^2,x)
```

```
[Out] log(x) - 1/x + x^2/2
```

sympy [A]   time = 0.08, size = 10, normalized size = 0.67

$$\frac{x^2}{2} + \log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/x**2,x)
```

```
[Out] x**2/2 + log(x) - 1/x
```

3.475 $\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx$

Optimal. Leaf size=11

$$\log\left(x^2+2\right)-\log(x)$$

```
[Out] -ln(x)+ln(x^2+2)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {446, 72}

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-2+x}{x(2+x)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{2+x}\right)dx,x,x^2\right)\\ &= -\log(x)+\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x^2+2)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
[Out] -Log[x] + Log[2 + x^2]
```

fricas [A] time = 0.45, size = 11, normalized size = 1.00

$$\log(x^2+2)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
[Out] log(x^2 + 2) - log(x)
```

giac [A] time = 0.29, size = 13, normalized size = 1.18

$$\log(x^2+2)-\frac{1}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$-\ln(x)+\ln(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/x/(x^2+2),x)
[Out] -ln(x)+ln(x^2+2)
```

maxima [A] time = 0.69, size = 13, normalized size = 1.18

$$\log(x^2+2)-\frac{1}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
```

```
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$\ln\left(x^2+2\right)-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 2)/(x*(x^2 + 2)),x)
```

```
[Out] log(x^2 + 2) - log(x)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\log(x)+\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/x/(x**2+2),x)
```

```
[Out] -log(x) + log(x**2 + 2)
```

3.476 $\int(-3+x)\left(-7+4x^2\right)\,dx$

Optimal. Leaf size=22

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

```
[Out] 21*x-4*x^3+1/16*(-4*x^2+7)^2
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {641}

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int(-3+x)\left(-7+4x^2\right)\,dx&=\frac{1}{16}\left(7-4x^2\right)^2-3\int\left(-7+4x^2\right)\,dx\\&=21x-4x^3+\frac{1}{16}\left(7-4x^2\right)^2\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.86

$$x^4-4x^3-\frac{7x^2}{2}+21x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

fricas [A]   time = 0.49, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="fricas")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

giac [A]   time = 0.36, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="giac")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

maple [A]   time = 0.00, size = 18, normalized size = 0.82

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3)*(4*x^2-7),x)
[Out] x^4-4*x^3-7/2*x^2+21*x
```

maxima [A]   time = 0.88, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

mupad [B]   time = 0.03, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7\,x^2}{2} + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 7)*(x - 3),x)
```

```
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x**2-7),x)
```

```
[Out] x**4 - 4*x**3 - 7*x**2/2 + 21*x
```

3.477 $\int (-2+7x)^3\,dx$

Optimal. Leaf size=11

$$\frac{1}{28}(2-7x)^4$$

```
[Out] 1/28*(2-7*x)^4
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{1}{28}(2-7x)^4$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 7*x)^3,x]
```

```
[Out] (2 - 7*x)^4/28
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (-2+7x)^3\,dx = \frac{1}{28}(2-7x)^4$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{28}(7x-2)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 7*x)^3,x]
```

```
[Out] (-2 + 7*x)^4/28
```

fricas [B] time = 0.60, size = 19, normalized size = 1.73

$$\frac{343}{4}x^4-98x^3+42x^2-8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="fricas")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

giac [A] time = 0.22, size = 9, normalized size = 0.82

$$\frac{1}{28}(7x-2)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="giac")
```

```
[Out] 1/28*(7*x - 2)^4
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{(7x-2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+7*x)^3,x)
```

```
[Out] 1/28*(-2+7*x)^4
```

maxima [B] time = 0.86, size = 19, normalized size = 1.73

$$\frac{343}{4}x^4-98x^3+42x^2-8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="maxima")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

mupad [B] time = 0.14, size = 9, normalized size = 0.82

$$\frac{(7x-2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x - 2)^3,x)
```

```
[Out] (7*x - 2)^4/28
```

sympy [B] time = 0.06, size = 19, normalized size = 1.73

$$\frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)**3,x)
```

```
[Out] 343*x**4/4 - 98*x**3 + 42*x**2 - 8*x
```

3.478 $\int \frac{-7+4x^2}{3+2x} dx$

Optimal. Leaf size=13

$$x^2 - 3x + \log(2x + 3)$$

[Out] -3*x+x^2+ln(3+2*x)

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {697}

$$x^2 - 3x + \log(2x + 3)$$

Antiderivative was successfully verified.

[In] Int[(-7 + 4*x^2)/(3 + 2*x),x]

[Out] -3*x + x^2 + Log[3 + 2*x]

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-7 + 4x^2}{3 + 2x} dx &= \int \left(-3 + 2x + \frac{2}{3 + 2x}\right) dx \\ &= -3x + x^2 + \log(3 + 2x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.23

$$x^2 - 3x + \log(2x + 3) - \frac{27}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(-7 + 4*x^2)/(3 + 2*x),x]

[Out] -27/4 - 3*x + x^2 + Log[3 + 2*x]

fricas [A] time = 0.84, size = 13, normalized size = 1.00

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="fricas")
[Out] x^2 - 3*x + log(2*x + 3)
```

giac [A] time = 0.36, size = 14, normalized size = 1.08

$$x^2 - 3\,x + \log\left(\left|2\,x + 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="giac")
[Out] x^2 - 3*x + log(abs(2*x + 3))
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$x^2 - 3x + \ln\left(2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-7)/(2*x+3),x)
[Out] -3*x+x^2+ln(2*x+3)
```

maxima [A] time = 0.64, size = 13, normalized size = 1.00

$$x^2 - 3\,x + \log\left(2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")
[Out] x^2 - 3*x + log(2*x + 3)
```

mupad [B] time = 2.21, size = 11, normalized size = 0.85

$$\ln\left(x + \frac{3}{2}\right) - 3\,x + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 7)/(2*x + 3),x)
[Out] log(x + 3/2) - 3*x + x^2
```

sympy [A] time = 0.08, size = 12, normalized size = 0.92

$$x^2 - 3x + \log\left(2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-7)/(3+2*x),x)
[Out] x**2 - 3*x + log(2*x + 3)
```

3.479 $\int \frac{1+x}{(-1+x)x^2} \, dx$

Optimal. Leaf size=16

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

```
[Out] 1/x+2*ln(1-x)-2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {77}

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*x^2),x]
```

```
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(-1+x)x^2} \, dx &= \int \left(\frac{2}{-1+x}-\frac{1}{x^2}-\frac{2}{x}\right) \, dx\\ &= \frac{1}{x}+2\log(1-x)-2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*x^2),x]
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

fricas [A] time = 0.62, size = 18, normalized size = 1.12

$$\frac{2\,x\log\left(x-1\right)-2\,x\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")
[Out] (2*x*log(x - 1) - 2*x*log(x) + 1)/x
```

giac [A] time = 0.36, size = 16, normalized size = 1.00

$$\frac{1}{x}+2\,\log\left(|x-1|\right)-2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")
[Out] 1/x + 2*log(abs(x - 1)) - 2*log(abs(x))
```

maple [A] time = 0.01, size = 15, normalized size = 0.94

$$-2\ln\left(x\right)+2\ln\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x-1)/x^2,x)
[Out] 2*ln(x-1)+1/x-2*ln(x)
```

maxima [A] time = 0.59, size = 14, normalized size = 0.88

$$\frac{1}{x}+2\,\log\left(x-1\right)-2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")
[Out] 1/x + 2*log(x - 1) - 2*log(x)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{1}{x}-4\,\mathrm{atanh}\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^2*(x - 1)),x)
```

```
[Out] 1/x - 4*atanh(2*x - 1)
```

sympy **[A]** time = 0.10, size = 14, normalized size = 0.88

$$-2\log(x) + 2\log(x-1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x**2,x)
```

```
[Out] -2*log(x) + 2*log(x - 1) + 1/x
```

3.480 $\int \frac{1}{4x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=27

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

```
[Out] 1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1106, 199, 206}

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{2\left(1-(1+x)^2\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{2\left(1-(1+x)^2\right)} + \frac{1}{2}\tanh^{-1}(1+x)$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(x+1)}{x(x+2)} - \log(x) + \log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + 4*x^3 + x^4)^(-1),x]
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

fricas [A] time = 0.57, size = 39, normalized size = 1.44

$$\frac{\left(x^2+2\,x\right)\log\left(x+2\right) - \left(x^2+2\,x\right)\log(x) - 2\,x - 2}{4\left(x^2+2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="fricas")
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

giac [A] time = 0.38, size = 27, normalized size = 1.00

$$-\frac{x+1}{2\left(x^2+2\,x\right)} + \frac{1}{4}\,\log\left(|x+2|\right) - \frac{1}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="giac")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}-\frac{1}{4x}-\frac{1}{4(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^3+4*x^2),x)
[Out] -1/4*ln(x)+1/4*ln(x+2)-1/4/x-1/4/(x+2)
```

maxima [A] time = 0.81, size = 25, normalized size = 0.93

$$-\frac{x+1}{2\left(x^2+2x\right)}+\frac{1}{4}\log(x+2)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="maxima")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

mupad [B] time = 2.23, size = 23, normalized size = 0.85

$$\frac{\operatorname{atanh}(x+1)}{2}-\frac{\frac{x}{2}+\frac{1}{2}}{x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 4*x^3 + x^4),x)
[Out] atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)
```

sympy [A] time = 0.11, size = 24, normalized size = 0.89

$$\frac{-x-1}{2x^2+4x}-\frac{\log(x)}{4}+\frac{\log(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**3+4*x**2),x)
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

3.481 $\int \frac{1+x^2}{1+x} \, dx$

Optimal. Leaf size=17

$$\frac{x^2}{2} - x + 2\log(x+1)$$

```
[Out] -x+1/2*x^2+2*ln(1+x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {697}

$$\frac{x^2}{2} - x + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x),x]
```

```
[Out] -x + x^2/2 + 2*Log[1 + x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+x} \, dx &= \int \left(-1+x+\frac{2}{1+x}\right) dx \\ &= -x+\frac{x^2}{2}+2\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.06

$$\frac{1}{2}\left(x^2 - 2x + 4\log(x+1) - 3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x),x]
```

```
[Out] (-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

fricas [A] time = 0.59, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="fricas")
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

giac [A] time = 0.36, size = 16, normalized size = 0.94

$$\frac{1}{2}x^2 - x + 2\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="giac")
[Out] 1/2*x^2 - x + 2*log(abs(x + 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^2}{2} - x + 2\ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x+1),x)
[Out] -x+1/2*x^2+2*ln(x+1)
```

maxima [A] time = 0.74, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="maxima")
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$2\ln(x+1) - x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + 1),x)
```

```
[Out] 2*log(x + 1) - x + x^2/2
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{x^2}{2} - x + 2\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(1+x),x)
```

```
[Out] x**2/2 - x + 2*log(x + 1)
```

3.482 $\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx$

Optimal. Leaf size=18

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

```
[Out] 1/x-3*x+1/2*x^2+3*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

```
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+3x-3x^2+x^3}{x^2} \, dx &= \int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right) dx \\ &= \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

```
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

fricas [A] time = 0.62, size = 20, normalized size = 1.11

$$\frac{x^3-6\,x^2+6\,x\log(x)+2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")
[Out] 1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x
```

giac [A] time = 0.37, size = 17, normalized size = 0.94

$$\frac{1}{2}\,x^2-3\,x+\frac{1}{x}+3\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(abs(x))
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{x^2}{2}-3x+3\ln(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/x^2,x)
[Out] 1/x-3*x+1/2*x^2+3*ln(x)
```

maxima [A] time = 0.73, size = 16, normalized size = 0.89

$$\frac{1}{2}\,x^2-3\,x+\frac{1}{x}+3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.89

$$3\,\ln(x)-3\,x+\frac{1}{x}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 3*x^2 + x^3 - 1)/x^2,x)
```

```
[Out] 3*log(x) - 3*x + 1/x + x^2/2
```

sympy [A] time = 0.08, size = 15, normalized size = 0.83

$$\frac{x^2}{2} - 3x + 3\log(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/x**2,x)
```

```
[Out] x**2/2 - 3*x + 3*log(x) + 1/x
```

3.483 $\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx$

Optimal. Leaf size=18

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

[Out] -7*x+3/2*x^2+1/3*x^3

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {43}

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

[In] Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]

[Out] -7*x + (3*x^2)/2 + x^3/3

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx &= \int \left(-7+3x+x^2\right)\,dx\\ &= -7x+\frac{3x^2}{2}+\frac{x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

[In] Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: SyntaxError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="fricas")
[Out] Exception raised: SyntaxError >> Malformed expression
```

giac [A] time = 0.35, size = 14, normalized size = 0.78

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="giac")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

maple [A] time = 0.00, size = 28, normalized size = 1.56

$$\frac{x^3}{3} + \frac{3x^2}{2} + \left(\frac{3}{2} - \frac{\sqrt{37}}{2}\right)\left(\frac{3}{2} + \frac{\sqrt{37}}{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x)
[Out] 1/3*x^3+3/2*x^2+(3/2-1/2*37^(1/2))*(3/2+1/2*37^(1/2))*x
```

maxima [A] time = 1.71, size = 14, normalized size = 0.78

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="maxima")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(2\,x^2+9\,x-42\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 37^(1/2)/2 + 3/2)*(x + 37^(1/2)/2 + 3/2),x)
```

```
[Out] (x*(9*x + 2*x^2 - 42))/6
```

sympy [A] time = 0.06, size = 14, normalized size = 0.78

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)
```

```
[Out] x**3/3 + 3*x**2/2 - 7*x
```

3.484 $\int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx$

Optimal. Leaf size=23

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

```
[Out] -5/3/(1+x)^3+3/(1+x)+2*ln(1+x)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1850}

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
```

```
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx &= \int \left(\frac{5}{(1+x)^4} - \frac{3}{(1+x)^2} + \frac{2}{1+x}\right) dx \\ &= -\frac{5}{3(1+x)^3} + \frac{3}{1+x} + 2\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
```

```
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

fricas [B] time = 0.56, size = 46, normalized size = 2.00

$$\frac{9\,x^2 + 6\left(x^3 + 3\,x^2 + 3\,x + 1\right)\log\left(x + 1\right) + 18\,x + 4}{3\left(x^3 + 3\,x^2 + 3\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 4)/(x^3 + 3*x^2
+ 3*x + 1)
```

giac [A] time = 0.37, size = 25, normalized size = 1.09

$$\frac{9\,x^2 + 18\,x + 4}{3\,(x + 1)^3} + 2\,\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="giac")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$2\ln\left(x + 1\right) - \frac{5}{3\,(x + 1)^3} + \frac{3}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+4)/(x+1)^4,x)
```

```
[Out] -5/3/(x+1)^3+3/(x+1)+2*ln(x+1)
```

maxima [A] time = 0.74, size = 34, normalized size = 1.48

$$\frac{9\,x^2 + 18\,x + 4}{3\left(x^3 + 3\,x^2 + 3\,x + 1\right)} + 2\,\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)
```

mupad [**B**] time = 0.03, size = 23, normalized size = 1.00

$$2\ \ln(x+1) + \frac{3\,x^2 + 6\,x + \frac{4}{3}}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2*x^3 + 4)/(x + 1)^4,x)
```

```
[Out] 2*log(x + 1) + (6*x + 3*x^2 + 4/3)/(x + 1)^3
```

sympy [**A**] time = 0.11, size = 31, normalized size = 1.35

$$\frac{9x^2 + 18x + 4}{3x^3 + 9x^2 + 9x + 3} + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+4)/(1+x)**4,x)
```

```
[Out] (9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)
```

3.485 $\int \frac{x}{(1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2/(1+x)+1/2*arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {801, 203}

$$\frac{1}{2(x+1)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTan[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)^2\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2\left(1+x^2\right)}\right)dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.75

$$\frac{1}{2}\left(\frac{1}{x+1}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^2*(1 + x^2)),x]
[Out] ((1 + x)^(-1) + ArcTan[x])/2
```

fricas [A] time = 0.73, size = 15, normalized size = 0.94

$$\frac{(x+1)\arctan(x)+1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] 1/2*((x + 1)*arctan(x) + 1)/(x + 1)
```

giac [B] time = 0.26, size = 32, normalized size = 2.00

$$-\frac{1}{8}\pi-\frac{1}{2}\pi\left\lfloor-\frac{\pi-4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{1}{2(x+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="giac")
[Out] -1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x + 1) + 1/
2*arctan(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\arctan(x)}{2}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)^2/(x^2+1),x)
[Out] 1/2/(x+1)+1/2*arctan(x)
```

maxima [A] time = 1.79, size = 12, normalized size = 0.75

$$\frac{1}{2(x+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/2*arctan(x)
```

mupad [B] time = 2.22, size = 12, normalized size = 0.75

$$\frac{\operatorname{atan}(x)}{2}+\frac{1}{2\,\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x + 1)^2),x)
```

```
[Out] atan(x)/2 + 1/(2*(x + 1))
```

sympy [A] time = 0.11, size = 10, normalized size = 0.62

$$\frac{\operatorname{atan}{\left(x \right)}}{2}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2/(x**2+1),x)
```

```
[Out] atan(x)/2 + 1/(2*x + 2)
```

3.486 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx$

Optimal. Leaf size=29

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

```
[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(2+x)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1850}

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

```
[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx &= \int \left(-20+9x-3x^2+x^3+\frac{47}{2+x}\right) dx \\ &= -20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4} + 47\log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.03

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2) - 70$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

```
[Out] -70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

fricas [A] time = 0.67, size = 25, normalized size = 0.86

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="fricas")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

giac [A] time = 0.28, size = 26, normalized size = 0.90

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="giac")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(abs(x + 2))
```

maple [A] time = 0.00, size = 26, normalized size = 0.90

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+3*x^2-2*x+7)/(x+2),x)
[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(x+2)
```

maxima [A] time = 0.76, size = 25, normalized size = 0.86

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="maxima")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

mupad [B] time = 0.03, size = 25, normalized size = 0.86

$$47\ln(x+2) - 20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 2*x - x^3 + x^4 + 7)/(x + 2),x)
```

```
[Out] 47*log(x + 2) - 20*x + (9*x^2)/2 - x^3 + x^4/4
```

sympy [A] time = 0.08, size = 24, normalized size = 0.83

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)
```

```
[Out] x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)
```

3.487 $\int \frac{-1+x^3}{-1+x} \, dx$

Optimal. Leaf size=16

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

```
[Out] x+1/2*x^2+1/3*x^3
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1586}

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(-1 + x),x]
```

```
[Out] x + x^2/2 + x^3/3
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{-1+x} \, dx &= \int \left(1+x+x^2\right) \, dx \\ &= x+\frac{x^2}{2}+\frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(-1 + x),x]
```

```
[Out] x + x^2/2 + x^3/3
```

fricas [A] time = 0.63, size = 12, normalized size = 0.75

$$\frac{1}{3}\,x^{3} + \frac{1}{2}\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

giac [A] time = 0.35, size = 12, normalized size = 0.75

$$\frac{1}{3}\,x^{3} + \frac{1}{2}\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{1}{3}x^{3} + \frac{1}{2}x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x-1),x)
[Out] x+1/2*x^2+1/3*x^3
```

maxima [A] time = 0.89, size = 12, normalized size = 0.75

$$\frac{1}{3}\,x^{3} + \frac{1}{2}\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.81

$$\frac{x\,\left(2\,x^2+3\,x+6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x - 1),x)
```

```
[Out] (x*(3*x + 2*x^2 + 6))/6
```

sympy [A] time = 0.06, size = 10, normalized size = 0.62

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + x
```

3.488 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$

Optimal. Leaf size=17

$$\frac{1}{x-1}-\frac{1}{(1-x)^2}+\tan^{-1}(x)$$

```
[Out] -1/(1-x)^2+1/(-1+x)+arctan(x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {801, 203}

$$\frac{1}{x-1}-\frac{1}{(1-x)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x}{(-1+x)^3\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{(-1+x)^3}-\frac{1}{(-1+x)^2}+\frac{1}{1+x^2}\right)dx\\ &= -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{x+(x-1)^2\tan^{-1}(x)-2}{(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
[Out] (-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2
```

fricas [A] time = 0.69, size = 25, normalized size = 1.47

$$\frac{\left(x^2-2\,x+1\right)\arctan(x)+x-2}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
[Out] ((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)
```

giac [A] time = 0.24, size = 12, normalized size = 0.71

$$\frac{x-2}{(x-1)^2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="giac")
[Out] (x - 2)/(x - 1)^2 + arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\arctan(x)-\frac{1}{(x-1)^2}+\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+2)/(x-1)^3/(x^2+1),x)
[Out] -1/(x-1)^2+1/(x-1)+arctan(x)
```

maxima [A] time = 1.74, size = 17, normalized size = 1.00

$$\frac{x-2}{x^2-2\,x+1}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] (x - 2)/(x^2 - 2*x + 1) + arctan(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{x-2}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 2)/((x^2 + 1)*(x - 1)^3),x)
```

```
[Out] atan(x) + (x - 2)/(x^2 - 2*x + 1)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{x-2}{x^2 - 2x + 1} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)**3/(x**2+1),x)
```

```
[Out] (x - 2)/(x**2 - 2*x + 1) + atan(x)
```

3.489 $\int \frac{1}{bx+c(d+ex)^2} dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

```
[Out] -2*arctanh((b+2*c*e*(e*x+d))/b^(1/2)/(4*c*d*e+b)^(1/2))/b^(1/2)/(4*c*d*e+b)
^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1981, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{bx+c(d+ex)^2}\,dx = \int \frac{1}{cd^2+(b+2cde)x+ce^2x^2}\,dx$$
$$= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{b(b+4cde)-x^2}\,dx, x, b+2cde+2ce^2x\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

fricas [A] time = 0.75, size = 190, normalized size = 4.04

$$\left[\frac{\log\left(\frac{2\,c^2e^4x^2+2\,c^2d^2e^2+4\,bcde+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x}\right)}{\sqrt{4\,bcde+b^2}}, \frac{2\sqrt{-4\,bcde-b^2}\,\arctan\left(\frac{\sqrt{-4\,bcde-b^2}\,(2\,ce^2x+2}{4\,bcde+b^2}\right.}{4\,bcde+b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b*
c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 + c*
d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), 2*sqrt(-4*b*c*d*e - b^2)*arc
tan(sqrt(-4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))/(4*
b*c*d*e + b^2)]
```

giac [A] time = 0.28, size = 48, normalized size = 1.02

$$\frac{2\arctan\left(\frac{2\,cxe^2+2\,cde+b}{\sqrt{-4\,bcde-b^2}}\right)}{\sqrt{-4\,bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-4*b*c*d*e
- b^2)
```

maple [A] time = 0.01, size = 43, normalized size = 0.91

$$-\frac{2\,\mathrm{arctanh}\left(\frac{2c\,e^{2}x+2cde+b}{\sqrt{4bcde+b^{2}}}\right)}{\sqrt{4bcde+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+c*(e*x+d)^2),x)
```

```
[Out] -2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e+b^2)^(1/2
))
```

maxima [A] time = 1.02, size = 68, normalized size = 1.45

$$\frac{\log\left(\frac{2\,ce^{2}x+2\,cde+b-\sqrt{(4\,cde+b)b}}{2\,ce^{2}x+2\,cde+b+\sqrt{(4\,cde+b)b}}\right)}{\sqrt{(4\,cde+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] log((2*c*e^2*x + 2*c*d*e + b - sqrt((4*c*d*e + b)*b))/(2*c*e^2*x + 2*c*d*e
+ b + sqrt((4*c*d*e + b)*b)))/sqrt((4*c*d*e + b)*b)
```

mupad [B] time = 0.10, size = 42, normalized size = 0.89

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x\,e^2+2\,c\,d\,e+b}{\sqrt{b}\,\sqrt{b+4\,c\,d\,e}}\right)}{\sqrt{b}\,\sqrt{b+4\,c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(d + e*x)^2 + b*x),x)
```

```
[Out] -(2*atanh((b + 2*c*d*e + 2*c*e^2*x)/(b^(1/2)*(b + 4*c*d*e)^(1/2))))/(b^(1/2
)*(b + 4*c*d*e)^(1/2))
```

sympy [B] time = 0.30, size = 151, normalized size = 3.21

$$\sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{-b^{2}\sqrt{\frac{1}{b(b+4cde)}}-4bcde\sqrt{\frac{1}{b(b+4cde)}}+b+2cde}{2ce^{2}}\right)-\sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{b^{2}\sqrt{\frac{1}{b(b+4cde)}}+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)**2),x)
```

```
[Out] sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))) - 4*b*c*
d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)) - sqrt(1/(b*(b + 4
*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*e))) + 4*b*c*d*e*sqrt(1/(b*(b
+ 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2))
```

3.490 $\int \frac{1}{a+bx+c(d+ex)^2} dx$

Optimal. Leaf size=57

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

```
[Out] -2*arctanh((b+2*c*e*(e*x+d))/(-4*a*c*e^2+4*b*c*d*e+b^2)^(1/2))/(-4*a*c*e^2+
4*b*c*d*e+b^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1981, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/Sqrt[
b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx = \int \frac{1}{a+cd^2+(b+2cde)x+ce^2x^2}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{b^2+4bcde-4ace^2-x^2}\,dx, x, b+2cde+2ce^2x\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{4ace^2-b^2-4bcde}}\right)}{\sqrt{4ace^2-b^2-4bcde}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[-
b^2 - 4*b*c*d*e + 4*a*c*e^2]
```

fricas [A] time = 0.63, size = 240, normalized size = 4.21

$$\left[\frac{\log\left(\frac{2c^2e^4x^2+4bcde+2\left(c^2d^2-ac\right)e^2+b^2+2\left(2c^2de^3+bce^2\right)x-\sqrt{4bcde-4ace^2+b^2}\left(2ce^2x+2cde+b\right)}{ce^2x^2+cd^2+(2cde+b)x+a}\right)}{\sqrt{4bcde-4ace^2+b^2}}, -\frac{2\sqrt{-4bcde+4ace^2-b^2}\,\text{arctar}}{4bcde-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d*
e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e +
 b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e^2
+ b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e + 4*a
*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))/(4*b
*c*d*e - 4*a*c*e^2 + b^2)]
```

giac [A] time = 0.36, size = 60, normalized size = 1.05

$$\frac{2\arctan\left(\frac{2cxe^2+2cde+b}{\sqrt{-4bcde+4ace^2-b^2}}\right)}{\sqrt{-4bcde+4ace^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqrt
(-4*b*c*d*e + 4*a*c*e^2 - b^2)
```

maple [A] time = 0.00, size = 61, normalized size = 1.07

$$\frac{2\arctan\left(\frac{2c\,e^{2}x+2cde+b}{\sqrt{4ac\,e^{2}-4bcde-b^{2}}}\right)}{\sqrt{4ac\,e^{2}-4bcde-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x+c*(e*x+d)^2),x)
```

```
[Out] 2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-4
*b*c*d*e-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c*e^2>0)', see `assume?` for mo
re details)Is 4*a*c*e^2    -4*b*c*d*e-b^2    positive or negative?
```

mupad [B] time = 2.23, size = 82, normalized size = 1.44

$$\frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,d\,e}{\sqrt{-b^{2}-4\,c\,d\,b\,e+4\,a\,c\,e^{2}}}+\frac{2\,c\,e^{2}\,x}{\sqrt{-b^{2}-4\,c\,d\,b\,e+4\,a\,c\,e^{2}}}\right)}{\sqrt{-b^{2}-4\,c\,d\,b\,e+4\,a\,c\,e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*(d + e*x)^2 + b*x),x)
```

```
[Out] (2*atan((b + 2*c*d*e)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2) + (2*c*e^2*x)/(4*
a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)))/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)
```

sympy [B] time = 0.35, size = 294, normalized size = 5.16

$$-\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{-4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b+\right.}{2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

```
[Out] -sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (-4*a*c*e**2*sqrt(-1/(4*a
*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)
) + 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*
e**2)) + sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (4*a*c*e**2*sqrt(
-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b
*c*d*e)) - 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e
)/(2*c*e**2))
```

3.491 $\int \frac{x^2}{1+(-1+x^2)^2} \, dx$

Optimal. Leaf size=188

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

```
[Out] -1/4*arctan((-2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^
(1/2)+1/4*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/
2))^(1/2)+1/4*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-1/4
*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1989, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1 + S
qrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/
Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*
Sqrt[2*(1 + Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int \frac{x^2}{1+\left(-1+x^2\right)^2}\,dx = \int \frac{x^2}{2-2x^2+x^4}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right) + \frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx + \frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}}{-\sqrt{2}-\sqrt{2\left(1+\right.}}}{4\sqrt{2\left(1+\right.}}$$

$$= \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{}{2\left(1-\right.}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log}{}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x^2)^2),x]
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

fricas [A] time = 0.52, size = 247, normalized size = 1.31

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2\,x^2+2\sqrt{2}\right) - \frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2^(3/4)*x*sqrt(2*sqrt(2)
 + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)
*log(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(3/4)*sqrt
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqr
t(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2
*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2
*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

giac [A] time = 1.49, size = 147, normalized size = 0.78

$$\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{2\sqrt{2}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2)
)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x - 2
^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

maple [B] time = 0.10, size = 308, normalized size = 1.64

$$\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(x^2-1)^2),x)
```

```
[Out] -1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))+1/4*
2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2
))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^
(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)
-x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*
ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2
)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-1\right)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)
```

mupad [B] time = 2.30, size = 101, normalized size = 0.54

$$\operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 1)^2 + 1),x)
```

```
[Out] atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*
(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*((-
 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32
- 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

sympy [A] time = 0.52, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(x**2-1)**2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

3.492 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$

Optimal. Leaf size=60

$$\frac{x^4}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

```
[Out] 2/(x^4+x+3)^3-3*x/(x^4+x+3)^3+5*x^2/(x^4+x+3)^3+x^4/(x^4+x+3)^3-5*x^6/(x^4+
x+3)^3
```

Rubi [A] time = 0.14, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2102, 1588}

$$-\frac{5x^6}{(x^4+x+3)^3}+\frac{x^4}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*
x^9)/(3 + x + x^4)^4),x]
```

```
[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)^3 + x^4/(
3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2102

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn
, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{\left(3+x+x^4\right)^4}\,dx = -\frac{5x^6}{\left(3+x+x^4\right)^3}+\frac{1}{6}\int \frac{-90+216x-30x^2 \ldots}{\ldots}$$

$$= \frac{x^4}{\left(3+x+x^4\right)^3}-\frac{5x^6}{\left(3+x+x^4\right)^3}-\frac{1}{48}\int \frac{720 \ldots}{\ldots}$$

$$= \frac{5x^2}{\left(3+x+x^4\right)^3}+\frac{x^4}{\left(3+x+x^4\right)^3}-\frac{5x^6}{\left(3+x+\ldots\right.}$$

$$= -\frac{3x}{\left(3+x+x^4\right)^3}+\frac{5x^2}{\left(3+x+x^4\right)^3}+\frac{x^4}{\left(3+x\ldots\right.}$$

$$= \frac{2}{\left(3+x+x^4\right)^3}-\frac{3x}{\left(3+x+x^4\right)^3}+\frac{5x^2}{\left(3+x+\ldots\right.}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.45

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7
 - 30*x^9)/(3 + x + x^4)^4),x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

fricas [A] time = 0.46, size = 65, normalized size = 1.08

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

giac [**A**] time = 0.40, size = 30, normalized size = 0.50

$$-\frac{5\,x^6 - x^4 - 5\,x^2 + 3\,x - 2}{\left(x^4 + x + 3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="giac")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3
```

maple [**A**] time = 0.01, size = 28, normalized size = 0.47

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{\left(x^4 + x + 3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x
)
```

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

maxima [**A**] time = 1.25, size = 65, normalized size = 1.08

$$-\frac{5\,x^6 - x^4 - 5\,x^2 + 3\,x - 2}{x^{12} + 3\,x^9 + 9\,x^8 + 3\,x^6 + 18\,x^5 + 27\,x^4 + x^3 + 9\,x^2 + 27\,x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

mupad [**B**] time = 2.34, size = 27, normalized size = 0.45

$$\frac{-5\,x^6 + x^4 + 5\,x^2 - 3\,x + 2}{\left(x^4 + x + 3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x^2 - 36*x + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9 +
15)/(x + x^4 + 3)^4,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

sympy [A] time = 0.23, size = 60, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-5*x**2+36*x-15)/
(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.493 $\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) d$

Optimal. Leaf size=27

$$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$$

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75
*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] -19/(4*(3 + x + x^4)^3) + (3 + x + x^4)^(-2) - (621*Defer[Int][(3 + x + x^4
)^(-4), x])/4 + 684*Defer[Int][x/(3 + x + x^4)^4, x] + 360*Defer[Int][x^2/(
3 + x + x^4)^4, x] + 44*Defer[Int][(3 + x + x^4)^(-3), x] - 320*Defer[Int][
x/(3 + x + x^4)^3, x] - 75*Defer[Int][x^2/(3 + x + x^4)^3, x] + 30*Defer[In
t][x/(3 + x + x^4)^2, x]
```

Rubi steps

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx = 3\int \frac{-47+228x+120x^{2}}{(3+x+x^4)}$$

$$= -\frac{19}{4(3+x+x^4)^3}+\frac{}{(3+}$$

$$= -\frac{19}{4(3+x+x^4)^3}+\frac{}{(3+}$$

$$= -\frac{19}{4(3+x+x^4)^3}+\frac{}{(3+}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*
x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

fricas [B] time = 0.68, size = 65, normalized size = 2.41

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

giac [B] time = 0.40, size = 197, normalized size = 7.30

$$\frac{1}{195075}x\left(\frac{377432\,x^2-2808656\,x+703551}{x^4+x+3}-\frac{255032\,x^2-1829456\,x+680601}{x^4+x+3}-\frac{7650\left(16\,x^2-128\,x+3\right)}{x^4+x+3}\right)-\frac{2(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")
```

```
[Out] 1/195075*x*((377432*x^2 - 2808656*x + 703551)/(x^4 + x + 3) - (255032*x^2 -
 1829456*x + 680601)/(x^4 + x + 3) - 7650*(16*x^2 - 128*x + 3)/(x^4 + x + 3
)) - 2/51*(16*x^3 - 64*x^2 + x + 12)/(x^4 + x + 3) + 1/390150*(754864*x^7 -
 2808656*x^6 + 469034*x^5 + 1321012*x^4 - 417584*x^3 - 13339729*x^2 + 26964
30*x + 2183454)/(x^4 + x + 3)^2 - 1/390150*(510064*x^11 - 1829456*x^10 + 45
3734*x^9 + 1402676*x^8 - 472048*x^7 - 13501313*x^6 + 4720744*x^5 + 3747556*
x^4 - 10935781*x^3 - 30736107*x^2 + 10203894*x + 4117662)/(x^4 + x + 3)^3
```

maple [C] time = 0.03, size = 250, normalized size = 9.26

$$\frac{\left(-255032\,\mathrm{RootOf}\left(\_Z^4+\_Z+3\right)^2+1829456\,\mathrm{RootOf}\left(\_Z^4+\_Z+3\right)-680601\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4+\_Z+3\right)\right.}{780300\,\mathrm{RootOf}\left(\_Z^4+\_Z+3\right)^3+195075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x
+3)^3+30*x/(x^4+x+3)^2,x)
```

```
[Out] (377432/195075*x^7-1404328/195075*x^6+234517/195075*x^5+660506/195075*x^4-2
08792/195075*x^3-13339729/390150*x^2+89881/13005*x+121303/21675)/(x^4+x+3)^
2+1/195075*sum((377432*_R^2-2808656*_R+703551)/(4*_R^3+1)*ln(-_R+x),_R=Root
Of(_Z^4+_Z+3))+30*(-16/765*x^3+64/765*x^2-1/765*x-4/255)/(x^4+x+3)+2/51*sum
((-16*_R^2+128*_R-3)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_Z^4+_Z+3))+3*(-255032/
585225*x^11+914728/585225*x^10-226867/585225*x^9-701338/585225*x^8+236024/5
85225*x^7+13501313/1170450*x^6-2360372/585225*x^5-1873778/585225*x^4+109357
81/1170450*x^3+3415123/130050*x^2-62987/7225*x-76253/21675)/(x^4+x+3)^3+1/1
95075*sum((-255032*_R^2+1829456*_R-680601)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_
Z^4+_Z+3))
```

maxima [B] time = 1.20, size = 65, normalized size = 2.41

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

mupad [B] time = 0.05, size = 27, normalized size = 1.00

$$\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{{\left(x^4+x+3\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((684*x + 360*x^2 + 57*x^3 - 141)/(x + x^4 + 3)^4 - (320*x + 75*x^2 + 8*
x^3 - 42)/(x + x^4 + 3)^3 + (30*x)/(x + x^4 + 3)^2,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

sympy [B] time = 0.32, size = 60, normalized size = 2.22

$$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{x^{12}+3x^{9}+9x^{8}+3x^{6}+18x^{5}+27x^{4}+x^{3}+9x^{2}+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320*
x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.494 $\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right) dx$

Optimal. Leaf size=27

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x
+ 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] 7/(2*(3 + x + x^4)^3) - (63*x)/(22*(3 + x + x^4)^3) - (12*x^2)/(3 + x + x^4
)^3 - (5*x^3)/(3 + x + x^4)^3 + (3*x^4)/(2*(3 + x + x^4)^3) - (10*x^6)/(3 +
 x + x^4)^3 - 1/(2*(3 + x + x^4)^2) + (5*x^2)/(3 + x + x^4)^2 + (144*Defer[
Int][(3 + x + x^4)^(-4), x])/11 + (828*Defer[Int][x/(3 + x + x^4)^4, x])/11
 + 18*Defer[Int][x^2/(3 + x + x^4)^4, x] - 4*Defer[Int][(3 + x + x^4)^(-3),
 x] - 20*Defer[Int][x/(3 + x + x^4)^3, x]
```

Rubi steps

$$\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3} - \frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4} \right) dx = -\left(3\int \frac{\left(1+4x^3\right)\left(2-3x+5x^2+x^4-\right.}{\left(3+x+x^4\right)^4}\right.$$

$$= -\frac{10x^6}{\left(3+x+x^4\right)^3} + \frac{5x^2}{\left(3+x+x^4\right)^2} - \frac{1}{6}\int$$

$$= \frac{3x^4}{2\left(3+x+x^4\right)^3} - \frac{10x^6}{\left(3+x+x^4\right)^3} - \frac{}{2(}$$

$$= -\frac{5x^3}{\left(3+x+x^4\right)^3} + \frac{3x^4}{2\left(3+x+x^4\right)^3} - \frac{}{(}$$

$$= -\frac{12x^2}{\left(3+x+x^4\right)^3} - \frac{5x^3}{\left(3+x+x^4\right)^3} + \frac{}{2(}$$

$$= -\frac{63x}{22\left(3+x+x^4\right)^3} - \frac{12x^2}{\left(3+x+x^4\right)^3} -$$

$$= \frac{7}{2\left(3+x+x^4\right)^3} - \frac{63x}{22\left(3+x+x^4\right)^3} -$$

$$= \frac{7}{2\left(3+x+x^4\right)^3} - \frac{63x}{22\left(3+x+x^4\right)^3} -$$

$$= \frac{7}{2\left(3+x+x^4\right)^3} - \frac{63x}{22\left(3+x+x^4\right)^3} -$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2
- 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

fricas [B] time = 0.66, size = 65, normalized size = 2.41

$$-\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

giac [B] time = 0.45, size = 111, normalized size = 4.11

$$\frac{69136x^7 - 147344x^6 - 30784x^5 + 120988x^4 + 137584x^3 + 1167854x^2 - 100680x + 18621}{390150\left(x^4 + x + 3\right)^2} - \frac{69136x^{11} - 1473}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="giac")
```

```
[Out] 1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 11
67854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^11 - 1473
44*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5 + 1
29019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3)^3
```

maple [B] time = 0.02, size = 112, normalized size = 4.15

$$-\frac{-\frac{34568}{195075}x^7 + \frac{73672}{195075}x^6 + \frac{15392}{195075}x^5 - \frac{60494}{195075}x^4 - \frac{68792}{195075}x^3 - \frac{583927}{195075}x^2 + \frac{3356}{13005}x - \frac{2069}{43350}}{\left(x^4 + x + 3\right)^2} + \frac{-\frac{34568}{195075}x^{11} + \frac{73672}{195075}x^{10} + \frac{1}{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)
/(x^4+x+3)^4,x)
```

```
[Out] -(-34568/195075*x^7+73672/195075*x^6+15392/195075*x^5-60494/195075*x^4-6879
2/195075*x^3-583927/195075*x^2+3356/13005*x-2069/43350)/(x^4+x+3)^2+3*(-345
68/585225*x^11+73672/585225*x^10+15392/585225*x^9-95062/585225*x^8-98824/58
5225*x^7-1322894/585225*x^6+36022/585225*x^5-129019/1170450*x^4-790303/5852
25*x^3-80674/65025*x^2-10951/14450*x+26831/43350)/(x^4+x+3)^3
```

maxima [B] time = 0.98, size = 65, normalized size = 2.41

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

mupad [B] time = 0.04, size = 27, normalized size = 1.00

$$\frac{-5\,x^{6}+x^{4}+5\,x^{2}-3\,x+2}{{\left(x^{4}+x+3\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*x + 4*x^3 - 30*x^5 - 3)/(x + x^4 + 3)^3 - (3*(4*x^3 + 1)*(5*x^2 - 3
*x + x^4 - 5*x^6 + 2))/(x + x^4 + 3)^4,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

sympy [B] time = 0.29, size = 60, normalized size = 2.22

$$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{x^{12}+3x^{9}+9x^{8}+3x^{6}+18x^{5}+27x^{4}+x^{3}+9x^{2}+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**4
+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```