Computer algebra independent integration tests

0-Independent-test-suites/Welz-Problems

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [116]. This is test number [11].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 86.21 (100)	% 13.79 (16)
Mathematica	% 81.90 (95)	% 18.10 (21)
Maple	% 66.38 (77)	% 33.62 (39)
Maxima	% 17.24 (20)	% 82.76 (96)
Fricas	% 76.72 (89)	% 23.28 (27)
Sympy	% 25.00 (29)	% 75.00 (87)
Giac	% 26.72 (31)	% 73.28 (85)
Mupad	% 31.90 (37)	% 68.10 (79)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

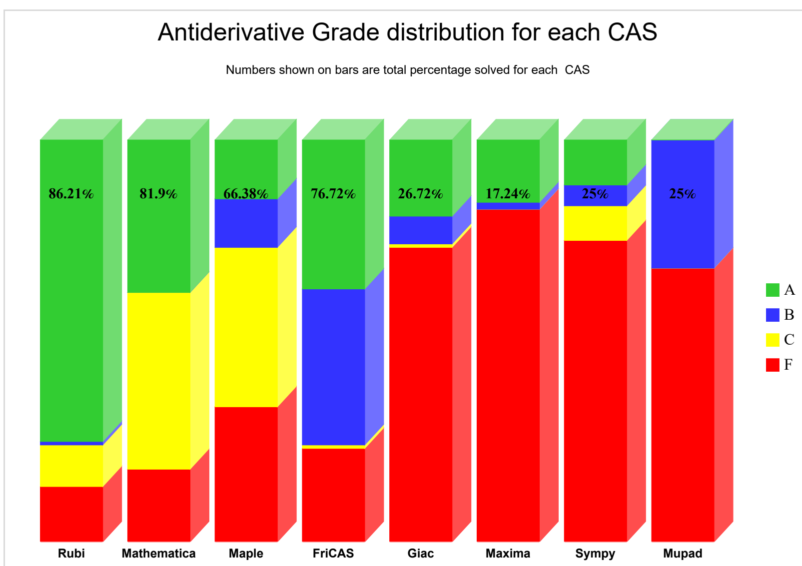
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

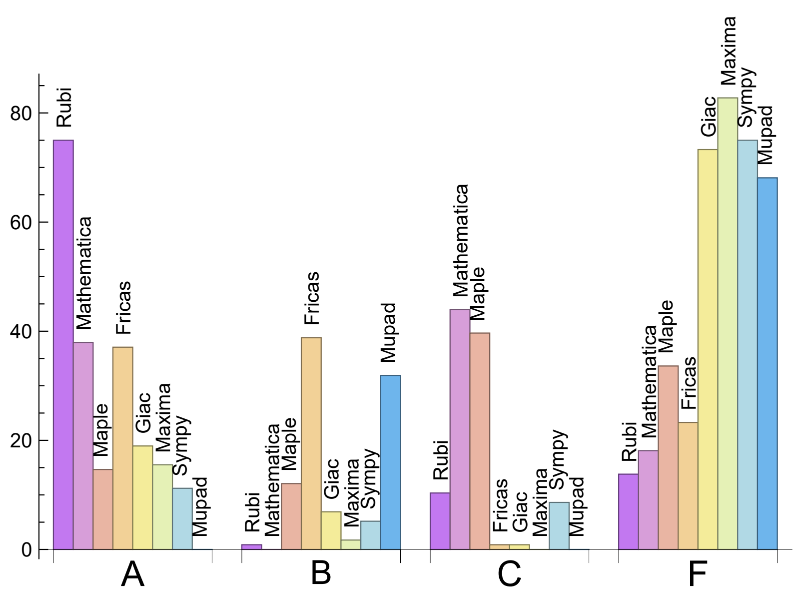
System	% A grade	% B grade	% C grade	% F grade
Rubi	75.00	0.86	10.34	13.79
Mathematica	37.93	0.00	43.97	18.10
Maple	14.66	12.07	39.66	33.62
Maxima	15.52	1.72	0.00	82.76
Fricas	37.07	38.79	0.86	23.28
Sympy	11.21	5.17	8.62	75.00
Giac	18.97	6.90	0.86	73.28
Mupad	0.00	31.90	0.00	68.10

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	16	100.00 %	0.00 %	0.00 %
Mathematica	21	100.00 %	0.00 %	0.00 %
Maple	39	92.31 %	5.13 %	2.56 %
Maxima	96	98.96 %	0.00 %	1.04 %
Fricas	27	37.04 %	33.33 %	29.63 %
Sympy	87	87.36 %	11.49 %	1.15 %
Giac	85	96.47 %	1.18 %	2.35 %
Mupad	79	98.73 %	1.27 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

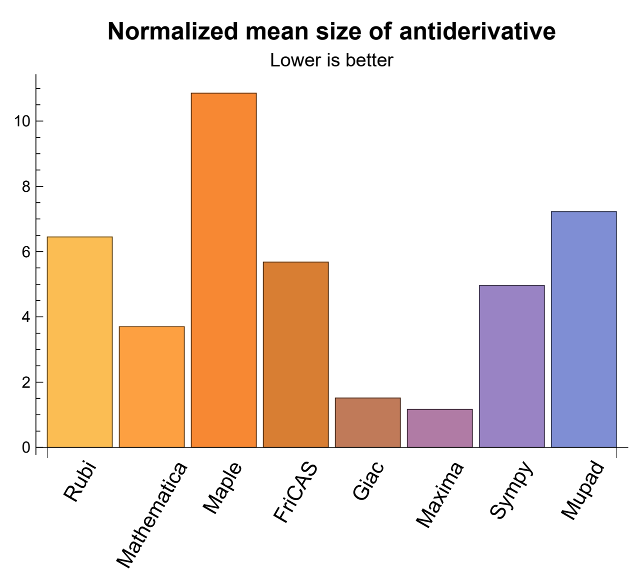
1.3 Performance

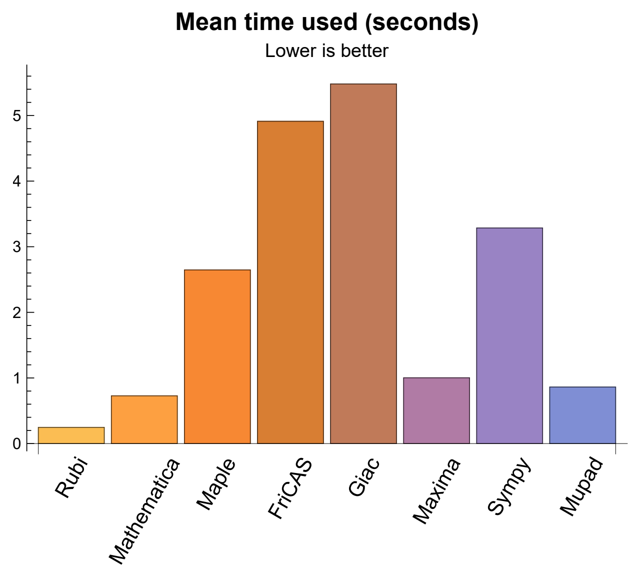
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.25	144.10	6.45	81.00	1.00
Mathematica	0.73	211.85	3.70	86.00	1.00
Maple	2.65	1851.86	10.85	317.00	2.31
Maxima	1.00	81.80	1.16	62.00	1.05
Fricas	4.91	802.17	5.68	171.00	1.86
Sympy	3.29	202.10	4.96	37.00	0.82
Giac	5.48	261.68	1.51	72.00	1.12
Mupad	0.86	151.70	7.22	76.00	1.09

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {9, 10, 82, 98, 113, 114, 116}

Mathematica {4, 9, 10, 39, 40, 51, 53, 54, 55, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 87, 88, 89, 90, 91, 98, 101, 106, 113, 114, 115, 116}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

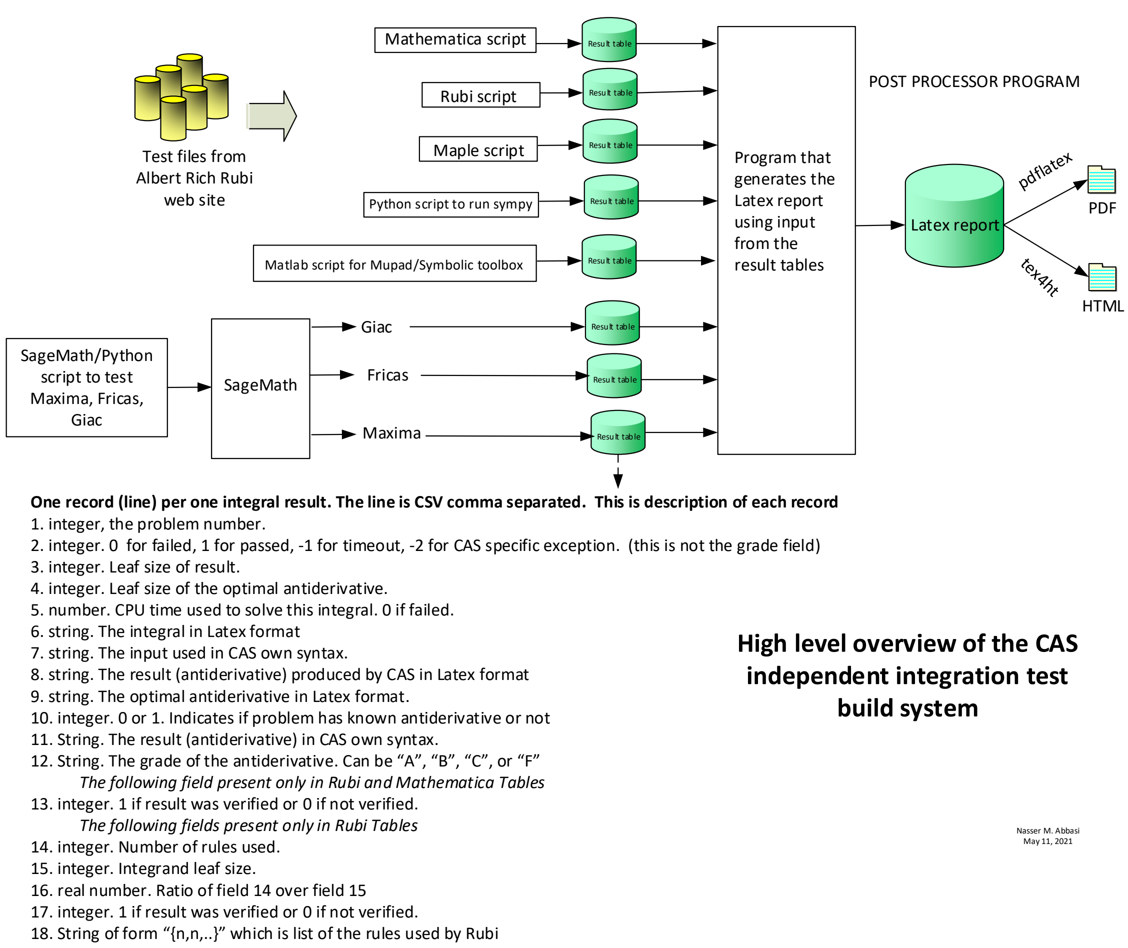
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 99, 105, 106, 107 }

B grade: { 10 }

C grade: { 2, 46, 52, 82, 83, 98, 100, 101, 102, 113, 114, 116 }

F grade: { 43, 44, 45, 58, 59, 60, 61, 95, 103, 104, 108, 109, 110, 111, 112, 115 }

2.1.2 Mathematica

A grade: { 1, 3, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 57, 62, 85, 94, 96, 97, 99, 103, 104, 105, 107 }

B grade: { }

C grade: { 2, 4, 24, 39, 40, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 98, 101, 106, 113, 114, 115, 116 }

F grade: { 12, 13, 37, 38, 44, 45, 46, 58, 59, 60, 61, 92, 93, 95, 100, 102, 108, 109, 110, 111, 112 }

2.1.3 Maple

A grade: { 1, 7, 8, 16, 21, 22, 23, 32, 47, 48, 49, 62, 63, 64, 103, 104, 105 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 17, 24, 31, 50, 51, 65, 68 }

C grade: { 2, 15, 28, 33, 34, 35, 36, 37, 39, 40, 52, 55, 56, 57, 58, 59, 66, 67, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 99, 100, 101, 102, 106, 107, 116 }

F grade: { 12, 13, 14, 18, 19, 20, 25, 26, 27, 29, 30, 38, 41, 42, 43, 44, 45, 46, 53, 54, 60, 61, 69, 70, 71, 72, 80, 94, 95, 96, 98, 108, 109, 110, 111, 112, 113, 114, 115 }

2.1.4 Maxima

A grade: { 1, 6, 16, 21, 22, 23, 29, 32, 33, 34, 35, 36, 56, 57, 62, 96, 99, 107 }

B grade: { 2, 106 }

C grade: { }

F grade: { 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 40, 52, 56, 57, 62, 63, 64, 65, 66, 67, 81, 83, 99, 106, 107, 113, 116 }

B grade: { 4, 7, 9, 10, 12, 13, 14, 24, 35, 37, 39, 41, 42, 43, 47, 48, 49, 50, 51, 55, 59, 68, 73, 74, 75, 76, 77, 78, 79, 80, 85, 86, 87, 88, 89, 90, 91, 93, 96, 97, 98, 100, 101, 102, 110 }

C grade: { 82 }

F grade: { 29, 38, 44, 45, 46, 53, 54, 58, 60, 61, 69, 70, 71, 72, 84, 92, 94, 95, 103, 104, 105, 108, 109, 111, 112, 114, 115 }

2.1.6 Sympy

A grade: { 1, 2, 14, 15, 20, 21, 22, 23, 25, 26, 62, 63, 64 }

B grade: { 8, 16, 17, 19, 30, 31 }

C grade: { 27, 28, 33, 34, 35, 36, 56, 57, 106, 107 }

F grade: { 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.7 Giac

A grade: { 1, 6, 8, 16, 21, 22, 23, 25, 26, 33, 34, 36, 41, 48, 49, 57, 62, 63, 64, 96, 99, 107 }

B grade: { 2, 3, 4, 7, 9, 10, 24, 47 }

C grade: { 50 }

F grade: { 5, 11, 12, 13, 14, 15, 17, 18, 19, 20, 27, 28, 29, 30, 31, 32, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 5, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 35, 36, 41, 47, 48, 49, 57, 62, 63, 64, 73, 74, 75, 81, 82, 84, 85, 96, 99, 106, 107 }

C grade: { }

F grade: { 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18, 24, 27, 28, 29, 31, 32, 37, 38, 39, 40, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.001	0.005	0.002	0.510	0.841	0.062	0.907	0.027

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	52	37	34	41	1	42	41	43
normalized size	1	3.47	2.47	2.27	2.73	0.07	2.80	2.73	2.87
time (sec)	N/A	0.056	0.034	0.004	0.466	0.853	59.577	1.057	0.515

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	99	370	0	100	0	177	204
normalized size	1	1.00	1.21	4.51	0.00	1.22	0.00	2.16	2.49
time (sec)	N/A	0.076	0.191	0.049	0.000	1.173	0.000	0.913	0.569

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	167	172	0	80	0	94	-1
normalized size	1	1.00	3.88	4.00	0.00	1.86	0.00	2.19	-0.02
time (sec)	N/A	0.012	3.763	0.053	0.000	1.529	0.000	0.938	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	126	115	0	105	0	0	82
normalized size	1	1.00	1.70	1.55	0.00	1.42	0.00	0.00	1.11
time (sec)	N/A	0.056	0.166	0.017	0.000	0.904	0.000	0.000	1.707

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	59	125	53	92	0	84	-1
normalized size	1	1.00	0.92	1.95	0.83	1.44	0.00	1.31	-0.02
time (sec)	N/A	0.026	0.086	0.039	1.452	0.978	0.000	1.068	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	75	45	0	83	0	101	-1
normalized size	1	1.00	1.56	0.94	0.00	1.73	0.00	2.10	-0.02
time (sec)	N/A	0.013	0.130	0.021	0.000	1.011	0.000	0.961	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	21	0	18	53	20	21
normalized size	1	1.00	1.00	0.70	0.00	0.60	1.77	0.67	0.70
time (sec)	N/A	0.082	0.058	0.004	0.000	0.886	0.753	0.949	0.381

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	365	340	902	0	424	0	367	-1
normalized size	1	1.66	1.55	4.10	0.00	1.93	0.00	1.67	-0.00
time (sec)	N/A	0.506	0.855	0.138	0.000	0.780	0.000	8.762	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F(-1)	B	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	541	311	1542	0	424	0	358	-1
normalized size	1	2.46	1.41	7.01	0.00	1.93	0.00	1.63	-0.00
time (sec)	N/A	0.751	0.713	0.023	0.000	0.612	0.000	9.218	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	125	278	0	161	0	0	-1
normalized size	1	1.00	0.91	2.01	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.203	0.032	0.000	1.061	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	394	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.240	0.098	0.000	4.546	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	0	0	0	369	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	4.56	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.173	0.088	0.000	10.157	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	60	15	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.94	0.48	0.00	-0.03
time (sec)	N/A	0.054	0.009	0.080	0.000	1.215	1.225	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	22	0	29	15	0	-1
normalized size	1	1.00	1.00	0.67	0.00	0.88	0.45	0.00	-0.03
time (sec)	N/A	0.063	0.011	0.100	0.000	1.621	0.860	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	28	56	15	15
normalized size	1	1.00	1.00	0.84	0.79	1.47	2.95	0.79	0.79
time (sec)	N/A	0.270	0.018	0.003	0.635	0.986	6.629	1.147	0.400

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	32	2147	0	-1
normalized size	1	1.00	0.83	2.31	0.00	0.62	41.29	0.00	-0.02
time (sec)	N/A	0.023	0.042	0.033	0.000	1.382	3.534	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	33	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.59	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.068	0.090	0.000	1.168	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	15	311	0	15
normalized size	1	1.00	1.00	0.00	0.00	0.88	18.29	0.00	0.88
time (sec)	N/A	0.054	0.007	0.082	0.000	1.089	2.986	0.000	0.297

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	18	36	0	18
normalized size	1	1.00	1.00	0.00	0.00	0.90	1.80	0.00	0.90
time (sec)	N/A	0.060	0.007	0.092	0.000	0.625	1.598	0.000	0.296

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	48	40	52	36	47	58
normalized size	1	1.00	0.86	1.14	0.95	1.24	0.86	1.12	1.38
time (sec)	N/A	0.033	0.042	0.013	0.588	1.108	0.147	0.986	0.413

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	19	20	19	24
normalized size	1	1.00	1.00	0.95	0.91	0.86	0.91	0.86	1.09
time (sec)	N/A	0.024	0.021	0.003	0.604	1.247	0.120	0.776	0.385

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	51	51	58	51	74	47
normalized size	1	1.00	0.79	0.82	0.82	0.94	0.82	1.19	0.76
time (sec)	N/A	0.091	0.066	0.014	0.588	0.740	0.185	1.211	0.406

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	961	455	0	358	0	304	-1
normalized size	1	1.00	11.17	5.29	0.00	4.16	0.00	3.53	-0.01
time (sec)	N/A	0.084	2.520	0.043	0.000	1.013	0.000	1.061	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	0	15	15	15	15
normalized size	1	1.00	1.00	0.00	0.00	0.79	0.79	0.79	0.79
time (sec)	N/A	0.065	0.012	0.104	0.000	1.020	0.230	0.787	0.423

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	22	27	22	22
normalized size	1	1.00	1.00	0.00	0.00	0.85	1.04	0.85	0.85
time (sec)	N/A	0.097	0.018	180.000	0.000	0.985	1.072	0.984	0.506

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	198	46	0	-1
normalized size	1	1.00	0.89	0.00	0.00	3.14	0.73	0.00	-0.02
time (sec)	N/A	0.219	0.203	0.085	0.000	1.087	1.469	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	127	25	0	216	51	0	-1
normalized size	1	1.00	1.55	0.30	0.00	2.63	0.62	0.00	-0.01
time (sec)	N/A	0.072	0.048	0.013	0.000	0.771	1.316	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	679	412	0	518	0	0	0	-1
normalized size	1	1.12	0.68	0.00	0.85	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.799	0.262	0.092	0.557	1.017	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	15	311	0	15
normalized size	1	1.00	1.00	0.00	0.00	0.88	18.29	0.00	0.88
time (sec)	N/A	0.055	0.011	0.091	0.000	0.884	2.577	0.000	0.314

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	120	0	32	2147	0	-1
normalized size	1	1.00	0.88	2.31	0.00	0.62	41.29	0.00	-0.02
time (sec)	N/A	0.024	0.061	0.027	0.000	1.028	2.715	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	44	48	48	0	0	-1
normalized size	1	1.00	0.97	1.29	1.41	1.41	0.00	0.00	-0.03
time (sec)	N/A	0.044	0.126	0.048	0.902	0.943	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	65	62	64	36	64	86
normalized size	1	1.00	0.98	1.12	1.07	1.10	0.62	1.10	1.48
time (sec)	N/A	0.036	0.015	0.105	1.092	0.948	0.944	0.916	0.543

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	81	48	62	64	37	64	76
normalized size	1	1.00	1.40	0.83	1.07	1.10	0.64	1.10	1.31
time (sec)	N/A	0.035	0.012	0.100	1.296	0.711	0.977	1.091	0.457

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	12	78	82	29	0	10
normalized size	1	1.00	1.76	0.24	1.59	1.67	0.59	0.00	0.20
time (sec)	N/A	0.005	0.040	0.092	1.361	0.896	0.893	0.000	0.334

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	65	62	64	32	63	80
normalized size	1	1.00	1.00	1.18	1.13	1.16	0.58	1.15	1.45
time (sec)	N/A	0.033	0.013	0.103	1.211	1.114	0.919	0.824	0.514

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	0	1143	0	301	0	0	-1
normalized size	1	1.00	0.00	11.78	0.00	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.080	8.055	0.000	7.448	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.080	0.616	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	176	59	1069	0	277	0	0	-1
normalized size	1	1.60	0.54	9.72	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.023	8.325	0.000	4.940	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	131	85	653	0	120	0	0	-1
normalized size	1	1.62	1.05	8.06	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.015	1.348	0.000	1.747	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	117	127	0	0	415	0	67	37
normalized size	1	1.77	1.92	0.00	0.00	6.29	0.00	1.02	0.56
time (sec)	N/A	0.056	0.079	0.141	0.000	3.859	0.000	1.049	0.394

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	145	140	0	0	665	0	0	-1
normalized size	1	1.84	1.77	0.00	0.00	8.42	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.151	0.037	0.000	3.454	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	F	F	B	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	0	55	0	0	1496	0	0	-1
normalized size	1	0.00	0.47	0.00	0.00	12.68	0.00	0.00	-0.01
time (sec)	N/A	21.937	0.195	0.047	0.000	52.735	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.611	1.638	0.126	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.462	0.666	0.097	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	493	576	0	0	0	0	0	0	-1
normalized size	1	1.17	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	0.396	0.276	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	198	584	0	957	0	767	343
normalized size	1	1.00	0.49	1.43	0.00	2.35	0.00	1.88	0.84
time (sec)	N/A	0.676	2.129	0.120	0.000	1.518	0.000	2.819	0.474

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	610	719	0	1563	0	972	567
normalized size	1	1.00	0.94	1.11	0.00	2.41	0.00	1.50	0.88
time (sec)	N/A	1.158	6.092	0.069	0.000	1.716	0.000	4.186	0.561

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1058	1058	1100	989	0	2763	0	1382	1017
normalized size	1	1.00	1.04	0.93	0.00	2.61	0.00	1.31	0.96
time (sec)	N/A	2.490	6.175	0.105	0.000	151.296	0.000	6.839	0.972

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	342	21028	0	1873	0	2509	-1
normalized size	1	1.00	0.90	55.63	0.00	4.96	0.00	6.64	-0.00
time (sec)	N/A	0.774	6.135	0.648	0.000	1.029	0.000	93.770	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	1431	86793	0	2775	0	0	-1
normalized size	1	1.00	2.24	136.04	0.00	4.35	0.00	0.00	-0.00
time (sec)	N/A	1.397	11.601	5.033	0.000	1.554	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	204	213	1275	0	546	0	0	-1
normalized size	1	3.09	3.23	19.32	0.00	8.27	0.00	0.00	-0.02
time (sec)	N/A	1.234	1.150	0.119	0.000	1.128	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	145	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.234	0.122	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	153	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.238	0.114	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	2378	0	171	0	0	-1
normalized size	1	1.00	1.14	24.52	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.055	14.133	0.000	4.153	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	107	20	69	105	96	32	0	-1
normalized size	1	1.47	0.27	0.95	1.44	1.32	0.44	0.00	-0.01
time (sec)	N/A	0.044	0.003	0.104	1.252	1.033	1.022	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	90	49	71	73	37	72	83
normalized size	1	1.00	1.34	0.73	1.06	1.09	0.55	1.07	1.24
time (sec)	N/A	0.037	0.021	0.107	1.297	0.877	0.986	1.116	0.366

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	C	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	482	0	0	2972	0	0	0	0	-1
normalized size	1	0.00	0.00	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.052	0.353	19.957	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	C	F	B	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	0	0	1404	0	3085	0	0	-1
normalized size	1	0.00	0.00	5.01	0.00	11.02	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.105	33.314	0.000	16.511	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.050	0.344	0.850	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.138	4.236	0.000	18.875	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	23	23
normalized size	1	1.00	1.00	0.96	0.92	0.92	0.88	0.92	0.92
time (sec)	N/A	0.023	0.007	0.004	0.540	0.921	0.106	0.989	0.074

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	86	52	0	43	73	40	53
normalized size	1	1.00	1.46	0.88	0.00	0.73	1.24	0.68	0.90
time (sec)	N/A	0.032	0.018	0.031	0.000	0.847	0.172	0.848	0.335

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	99	75	0	66	100	64	76
normalized size	1	1.00	1.27	0.96	0.00	0.85	1.28	0.82	0.97
time (sec)	N/A	0.077	0.021	0.026	0.000	0.883	0.184	0.969	0.090

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	110	100	0	56	0	0	-1
normalized size	1	1.00	2.24	2.04	0.00	1.14	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.095	0.026	0.000	0.676	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	108	365	0	61	0	0	-1
normalized size	1	1.00	2.04	6.89	0.00	1.15	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.091	0.038	0.000	0.908	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	5727	1512	0	359	0	0	-1
normalized size	1	1.00	76.36	20.16	0.00	4.79	0.00	0.00	-0.01
time (sec)	N/A	0.094	7.076	0.092	0.000	0.750	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	322	456	0	2667	0	0	-1
normalized size	1	1.00	1.88	2.67	0.00	15.60	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.421	0.111	0.000	5.831	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	157	0	0	0	0	0	-1
normalized size	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.256	0.135	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	162	0	0	0	0	0	-1
normalized size	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.264	0.120	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.215	0.139	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.186	0.119	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1191	0	0	653
normalized size	1	1.00	0.22	1.29	0.00	9.38	0.00	0.00	5.14
time (sec)	N/A	0.019	0.021	0.186	0.000	1.260	0.000	0.000	0.449

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	54	240	0	1666	0	0	331
normalized size	1	1.00	0.34	1.53	0.00	10.61	0.00	0.00	2.11
time (sec)	N/A	0.033	0.030	0.219	0.000	1.815	0.000	0.000	15.034

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	421	0	547	0	0	533
normalized size	1	1.00	0.65	5.69	0.00	7.39	0.00	0.00	7.20
time (sec)	N/A	0.156	0.021	0.200	0.000	1.101	0.000	0.000	0.211

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	32	383	0	497	0	0	-1
normalized size	1	1.00	0.31	3.72	0.00	4.83	0.00	0.00	-0.01
time (sec)	N/A	0.305	0.028	0.352	0.000	1.428	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	538	0	1792	0	0	-1
normalized size	1	1.00	1.56	6.64	0.00	22.12	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.098	10.056	0.000	3.816	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	126	444	0	345	0	0	-1
normalized size	1	1.00	1.56	5.48	0.00	4.26	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.095	2.458	0.000	2.760	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	704	0	1943	0	0	-1
normalized size	1	1.00	1.04	6.23	0.00	17.19	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.069	50.764	0.000	1.818	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	1685	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	15.46	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.101	180.000	0.000	1.731	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	159	206	0	85	0	0	217
normalized size	1	1.00	1.83	2.37	0.00	0.98	0.00	0.00	2.49
time (sec)	N/A	0.874	0.934	0.041	0.000	0.702	0.000	0.000	0.169

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	C	F	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	529	127	317	0	70	0	0	207
normalized size	1	529.00	127.00	317.00	0.00	70.00	0.00	0.00	207.00
time (sec)	N/A	1.669	0.602	0.049	0.000	1.209	0.000	0.000	0.478

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	180	133	536	0	63	0	0	-1
normalized size	1	3.91	2.89	11.65	0.00	1.37	0.00	0.00	-0.02
time (sec)	N/A	1.492	1.091	0.059	0.000	0.722	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	323	258	0	0	0	0	67
normalized size	1	1.00	10.09	8.06	0.00	0.00	0.00	0.00	2.09
time (sec)	N/A	0.096	0.463	0.122	0.000	0.000	0.000	0.000	1.686

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	46	240	0	44	0	0	204
normalized size	1	1.00	2.00	10.43	0.00	1.91	0.00	0.00	8.87
time (sec)	N/A	0.054	0.008	0.110	0.000	0.550	0.000	0.000	0.219

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	47	353	0	7739	0	0	-1
normalized size	1	1.00	0.22	1.62	0.00	35.50	0.00	0.00	-0.00
time (sec)	N/A	0.041	0.060	0.285	0.000	10.636	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	50	350	0	8237	0	0	-1
normalized size	1	1.00	0.24	1.67	0.00	39.22	0.00	0.00	-0.00
time (sec)	N/A	0.032	0.077	0.263	0.000	10.425	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	65	349	0	7910	0	0	-1
normalized size	1	1.00	0.29	1.57	0.00	35.63	0.00	0.00	-0.00
time (sec)	N/A	0.030	0.073	0.271	0.000	9.991	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	68	350	0	8105	0	0	-1
normalized size	1	1.00	0.32	1.64	0.00	37.87	0.00	0.00	-0.00
time (sec)	N/A	0.029	0.057	0.259	0.000	9.371	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	685	327	0	323	0	0	-1
normalized size	1	1.00	10.54	5.03	0.00	4.97	0.00	0.00	-0.02
time (sec)	N/A	0.129	3.211	0.161	0.000	0.710	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	876	311	0	112	0	0	-1
normalized size	1	1.00	13.90	4.94	0.00	1.78	0.00	0.00	-0.02
time (sec)	N/A	0.128	7.991	0.158	0.000	0.911	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	0	818	0	0	0	0	-1
normalized size	1	1.00	0.00	15.43	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.156	2.997	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	0	2134	0	267	0	0	-1
normalized size	1	1.00	0.00	19.76	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.036	5.820	0.000	2.696	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	135	120	0	0	0	0	0	-1
normalized size	1	1.38	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.087	0.134	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.199	0.206	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	80	0	110	387	0	113	157
normalized size	1	1.00	0.83	0.00	1.15	4.03	0.00	1.18	1.64
time (sec)	N/A	0.078	0.040	0.120	1.474	0.932	0.000	20.990	0.590

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	122	112	614	0	253	0	0	-1
normalized size	1	1.39	1.27	6.98	0.00	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.073	2.416	0.000	4.673	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	26	26	0	0	373	0	0	-1
normalized size	1	0.11	0.11	0.00	0.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.011	0.017	1.865	0.000	3.433	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	655	86	90	0	87	100
normalized size	1	1.00	0.89	7.99	1.05	1.10	0.00	1.06	1.22
time (sec)	N/A	0.058	0.028	3.427	1.512	0.663	0.000	0.884	0.548

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	409	0	720	0	318	0	0	-1
normalized size	1	3.03	0.00	5.33	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.337	0.138	7.804	0.000	12.832	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	409	150	676	0	318	0	0	-1
normalized size	1	3.03	1.11	5.01	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.301	0.308	7.503	0.000	12.883	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	399	0	652	0	268	0	0	-1
normalized size	1	3.35	0.00	5.48	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.299	0.128	7.938	0.000	13.536	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	43	34	0	0	0	0	-1
normalized size	1	0.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.425	0.154	0.096	0.000	0.578	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	43	34	0	0	0	0	-1
normalized size	1	0.00	1.00	0.79	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.220	0.085	0.092	0.000	0.603	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	34	0	0	0	0	-1
normalized size	1	1.00	1.10	0.87	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.019	0.095	0.000	0.725	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	A	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	101	12	105	94	31	0	10
normalized size	1	1.00	1.51	0.18	1.57	1.40	0.46	0.00	0.15
time (sec)	N/A	0.010	0.120	0.102	1.300	0.622	1.021	0.000	0.342

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	66	73	75	41	74	91
normalized size	1	1.00	0.93	0.94	1.04	1.07	0.59	1.06	1.30
time (sec)	N/A	0.038	0.027	0.104	1.410	0.871	1.017	0.889	0.403

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-1)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	384	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.068	0.282	0.127	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.415	0.352	0.194	0.000	4.434	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F(-1)	F	B	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	0	0	0	0	1827	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	9.18	0.00	0.00	-0.01
time (sec)	N/A	0.945	0.284	180.000	0.000	5.170	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.326	0.114	0.000	5.349	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.153	0.017	0.000	4.596	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	21	111	0	0	191	0	0	-1
normalized size	1	0.16	0.84	0.00	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.008	0.096	0.603	0.000	0.631	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	26	26	0	0	0	0	0	-1
normalized size	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.012	0.014	0.116	0.000	4.310	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	383	0	138	0	0	0	0	0	-1
normalized size	1	0.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.157	0.112	0.000	11.791	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	21	109	1214	0	341	0	0	-1
normalized size	1	0.08	0.40	4.46	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.008	0.093	6.549	0.000	3.530	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [2.143]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	10	0.100
2	C	5	2	3.47	37	0.054
3	A	9	6	1.00	15	0.400
4	A	3	3	1.00	19	0.158
5	A	8	7	1.00	17	0.412
6	A	6	6	1.00	17	0.353
7	A	3	3	1.00	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	4	2	1.00	23	0.087
9	A	18	12	1.66	27	0.444
10	B	25	13	2.46	39	0.333
11	A	7	3	1.00	45	0.067
12	A	7	4	1.00	32	0.125
13	A	5	3	1.00	32	0.094
14	A	2	2	1.00	27	0.074
15	A	2	2	1.00	29	0.069
16	A	2	1	1.00	30	0.033
17	A	3	2	1.00	13	0.154
18	A	3	2	1.00	15	0.133
19	A	2	2	1.00	23	0.087
20	A	2	2	1.00	25	0.080
21	A	3	2	1.00	11	0.182
22	A	2	2	1.00	18	0.111
23	A	6	6	1.00	20	0.300
24	A	6	5	1.00	31	0.161
25	A	2	2	1.00	29	0.069
26	A	2	2	1.00	35	0.057
27	A	5	5	1.00	32	0.156
28	A	6	6	1.00	21	0.286
29	A	359	30	1.12	14	2.143
30	A	2	2	1.00	23	0.087
31	A	3	2	1.00	13	0.154
32	A	2	2	1.00	33	0.061
33	A	5	5	1.00	15	0.333
34	A	5	5	1.00	15	0.333
35	A	1	1	1.00	11	0.091
36	A	5	5	1.00	15	0.333
37	A	1	1	1.00	17	0.059
38	A	3	3	1.00	18	0.167
39	A	2	2	1.60	16	0.125
40	A	5	5	1.62	17	0.294
41	A	5	5	1.77	13	0.385
42	A	5	5	1.84	16	0.312
43	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	F	0	0	N/A	0	N/A
45	F	0	0	N/A	0	N/A
46	C	7	3	1.17	32	0.094
47	A	19	9	1.00	20	0.450
48	A	29	9	1.00	20	0.450
49	A	49	9	1.00	20	0.450
50	A	14	6	1.00	23	0.261
51	A	24	6	1.00	23	0.261
52	C	9	8	3.09	48	0.167
53	A	7	7	1.00	24	0.292
54	A	7	7	1.00	24	0.292
55	A	1	1	1.00	18	0.056
56	A	8	8	1.47	13	0.615
57	A	6	6	1.00	15	0.400
58	F	0	0	N/A	0	N/A
59	F	0	0	N/A	0	N/A
60	F	0	0	N/A	0	N/A
61	F	0	0	N/A	0	N/A
62	A	1	1	1.00	38	0.026
63	A	1	1	1.00	33	0.030
64	A	2	2	1.00	39	0.051
65	A	1	1	1.00	19	0.053
66	A	4	4	1.00	19	0.210
67	A	4	4	1.00	24	0.167
68	A	1	1	1.00	24	0.042
69	A	7	7	1.00	24	0.292
70	A	7	7	1.00	24	0.292
71	A	3	3	1.00	26	0.115
72	A	3	3	1.00	22	0.136
73	A	1	1	1.00	22	0.045
74	A	1	1	1.00	23	0.043
75	A	8	8	1.00	18	0.444
76	A	8	7	1.00	23	0.304
77	A	1	1	1.00	21	0.048
78	A	1	1	1.00	19	0.053
79	A	1	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
80	A	1	1	1.00	19	0.053
81	A	4	4	1.00	34	0.118
82	C	5	5	529.00	40	0.125
83	C	7	7	3.91	51	0.137
84	A	2	2	1.00	29	0.069
85	A	2	2	1.00	18	0.111
86	A	1	1	1.00	25	0.040
87	A	1	1	1.00	25	0.040
88	A	1	1	1.00	25	0.040
89	A	1	1	1.00	25	0.040
90	A	2	2	1.00	40	0.050
91	A	2	2	1.00	40	0.050
92	A	1	1	1.00	18	0.056
93	A	3	3	1.00	15	0.200
94	A	7	7	1.38	21	0.333
95	F	0	0	N/A	0	N/A
96	A	5	5	1.00	24	0.208
97	A	7	7	1.39	19	0.368
98	C	1	1	0.11	20	0.050
99	A	5	5	1.00	22	0.227
100	C	4	2	3.03	25	0.080
101	C	5	3	3.03	24	0.125
102	C	4	2	3.35	23	0.087
103	F	0	0	N/A	0	N/A
104	F	0	0	N/A	0	N/A
105	A	3	3	1.00	19	0.158
106	A	2	2	1.00	11	0.182
107	A	6	6	1.00	15	0.400
108	F	0	0	N/A	0	N/A
109	F	0	0	N/A	0	N/A
110	F	0	0	N/A	0	N/A
111	F	0	0	N/A	0	N/A
112	F	0	0	N/A	0	N/A
113	C	1	1	0.16	19	0.053
114	C	1	1	0.10	20	0.050
115	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	C	1	1	0.08	19	0.053

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

```
[Out] -2*(-a*x+1)^(1/2)/a
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {32}

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - a*x],x]
```

```
[Out] (-2*Sqrt[1 - a*x])/a
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - a*x],x]
```

```
[Out] (-2*Sqrt[1 - a*x])/a
```

fricas [A] time = 0.84, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*x + 1)/a
```

giac [A] time = 0.91, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(-a*x + 1)/a
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/2),x)
[Out] -2*(-a*x+1)^(1/2)/a
```

maxima [A] time = 0.51, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-a*x + 1)/a
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{1-a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - a*x)^(1/2),x)
[Out] -(2*(1 - a*x)^(1/2))/a
```

sympy [A] time = 0.06, size = 12, normalized size = 0.80

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/2),x)
[Out] -2*sqrt(-a*x + 1)/a
```

3.2 $\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx$

Optimal. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

[Out] -2*(-a*x+1)^(1/2)/a

Rubi [C] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {12, 2295}

$$\frac{\sqrt{ax-1}\,\log(ax-1)}{\pi a}-\frac{2\sqrt{ax-1}\,\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Antiderivative was successfully verified.

[In] Int[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]

[Out] (-2*Sqrt[-1 + a*x]*Log[-Sqrt[-1 + a*x]])/(a*Pi) + (Sqrt[-1 + a*x]*Log[-1 + a*x])/(a*Pi)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{2\pi\sqrt{-1+ax}}\,dx &= \frac{\int \frac{-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)}{\sqrt{-1+ax}}\,dx}{2\pi}\\
&= \frac{\text{Subst}\left(\int \left(-2\log(-x)+\log\left(x^2\right)\right)\,dx,x,\sqrt{-1+ax}\right)}{a\pi}\\
&= \frac{\text{Subst}\left(\int \log\left(x^2\right)\,dx,x,\sqrt{-1+ax}\right)}{a\pi}-\frac{2\,\text{Subst}\left(\int \log(-x)\,dx,x,\right.}{a\pi}\\
&= -\frac{2\sqrt{-1+ax}\,\log\left(-\sqrt{-1+ax}\right)}{a\pi}+\frac{\sqrt{-1+ax}\,\log(-1+ax)}{a\pi}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 37, normalized size = 2.47

$$\frac{\sqrt{ax-1}\left(\log(ax-1)-2\log\left(-\sqrt{ax-1}\right)\right)}{\pi a}$$

Antiderivative was successfully verified.

[In] Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x
]

[Out] (Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)

fricas [A] time = 0.85, size = 1, normalized size = 0.07

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="fricas")
```

```
[Out] 0
```

giac [B] time = 1.06, size = 41, normalized size = 2.73

$$\frac{\sqrt{ax-1}\log\left(ax-1\right)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="giac")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

maple [C] time = 0.00, size = 34, normalized size = 2.27

$$\frac{\sqrt{ax-1}\,\left(-2\ln\left(-\sqrt{ax-1}\right)+\ln\left(ax-1\right)\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x)
```

```
[Out] (a*x-1)^(1/2)*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/a/Pi
```

maxima [B] time = 0.47, size = 41, normalized size = 2.73

$$\frac{\sqrt{ax-1}\log\left(ax-1\right)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="maxima")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

mupad [B] time = 0.51, size = 43, normalized size = 2.87

$$-\frac{2\,\ln\left(-\sqrt{a\,x-1}\right)\,\sqrt{a\,x-1}-\ln\left(a\,x-1\right)\,\sqrt{a\,x-1}}{\Pi\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x - 1)/2 - log(-(a*x - 1)^(1/2)))/(Pi*(a*x - 1)^(1/2)),x)
```

```
[Out] -(2*log(-(a*x - 1)^(1/2))*(a*x - 1)^(1/2) - log(a*x - 1)*(a*x - 1)^(1/2))/(
Pi*a)
```

sympy [A] time = 59.58, size = 42, normalized size = 2.80

$$\frac{\begin{cases}\frac{-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\log\left(ax-1\right)}{a} & \text{for}\ a\neq 0\\ \pi x & \text{otherwise}\end{cases}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
```

```
[Out] Piecewise(((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x -
1))/a, Ne(a, 0)), (pi*x, True))/pi
```

3.3 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{x^2+1}}{3\left(1-3x^2\right)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

```
[Out] 4/3*x/(-3*x^2+1)-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(1/2*3^(1/2)*(x^
2+1)^(1/2))*3^(1/2)-2/3*(x^2+1)^(1/2)/(-3*x^2+1)
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6742, 199, 207, 444, 47, 63}

$$\frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{x^2+1}}{3\left(1-3x^2\right)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]
*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx = \int \left(\frac{8}{3\left(-1+3x^2\right)^2}-\frac{4x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}+\frac{5}{3\left(-1+3x^2\right)}\right)dx$$

$$= \frac{5}{3}\int \frac{1}{-1+3x^2}\,dx+\frac{8}{3}\int \frac{1}{\left(-1+3x^2\right)^2}\,dx-4\int \frac{x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}\,dx$$

$$= \frac{4x}{3\left(1-3x^2\right)}-\frac{5\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{4}{3}\int \frac{1}{-1+3x^2}\,dx-2\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(-1+3x)^2}\,dx,x,x$$

$$= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(-1+3x)}\,dx,x,x^2$$

$$= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-4+3x^2}\,dx,x,\sqrt{1+x^2}\right)$$

$$= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{1+x^2}\right)}{3\sqrt{3}}$$

Mathematica [A] time = 0.19, size = 99, normalized size = 1.21

$$\frac{1}{9}\left(\frac{12x}{1-3x^2}-\frac{\frac{6x^2+6}{1-3x^2}+\sqrt{3}\sqrt{-x^2-1}\,\tan^{-1}\left(\frac{1}{2}\sqrt{3}\sqrt{-x^2-1}\right)}{\sqrt{x^2+1}}-\sqrt{3}\tanh^{-1}\left(\sqrt{3}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] ((12*x)/(1 - 3*x^2) - ((6 + 6*x^2)/(1 - 3*x^2) + Sqrt[3]*Sqrt[-1 - x^2]*Arc
Tan[(Sqrt[3]*Sqrt[-1 - x^2])/2])/Sqrt[1 + x^2] - Sqrt[3]*ArcTanh[Sqrt[3]*x]
)/9
```

fricas [A] time = 1.17, size = 100, normalized size = 1.22

$$\frac{\sqrt{3}\left(3\,x^2-1\right)\log\left(\frac{3\,x^2-2\,\sqrt{3}\,x+1}{3\,x^2-1}\right)+\sqrt{3}\left(3\,x^2-1\right)\log\left(\frac{3\,x^2+4\,\sqrt{3}\,\sqrt{x^2+1}+7}{3\,x^2-1}\right)-24\,x+12\,\sqrt{x^2+1}}{18\left(3\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(3)*(3*x^2 - 1)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + sqrt
(3)*(3*x^2 - 1)*log((3*x^2 + 4*sqrt(3)*sqrt(x^2 + 1) + 7)/(3*x^2 - 1)) - 24
*x + 12*sqrt(x^2 + 1))/(3*x^2 - 1)
```

giac [B] time = 0.91, size = 177, normalized size = 2.16

$$\frac{1}{18}\sqrt{3}\log\left(\frac{\left|6x-2\sqrt{3}\right|}{\left|6x+2\sqrt{3}\right|}\right)-\frac{1}{18}\sqrt{3}\log\left(-\frac{\left|-6x-8\sqrt{3}+6\sqrt{x^2+1}-\frac{6}{x-\sqrt{x^2+1}}\right|}{2\left(3x-4\sqrt{3}-3\sqrt{x^2+1}+\frac{3}{x-\sqrt{x^2+1}}\right)}\right)-\frac{4\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)}{3\left(3\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/(3
*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3*(x - sqrt(
x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 +
 1)))^2 - 16) - 4/3*x/(3*x^2 - 1)
```

maple [B] time = 0.05, size = 370, normalized size = 4.51

$$-\frac{x}{2\left(3x^2-1\right)}-\frac{5x}{18\left(x^2-\frac{1}{3}\right)}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\sqrt{3}\,x\right)}{9}-\sqrt{3}\left(\frac{\sqrt{\left(x-\frac{\sqrt{3}}{3}\right)^2+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}}\,x}{12}+\frac{\operatorname{arcsinh}(x)}{12}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+(x^2+1)^(1/2))^2,x)
```

```
[Out] -1/2*x/(3*x^2-1)-1/9*arctanh(3^(1/2)*x)*3^(1/2)-5/18*x/(x^2-1/3)-3^(1/2)*(-
1/12/(x-1/3*3^(1/2))*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(3
/2)+1/36*3^(1/2)*(1/3*(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1
/2)+1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3+2/3*3^(1/2)*(x-1/3*
3^(1/2)))*3^(1/2)/(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2))
)+1/12*x*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(1/2)+1/12*arc
sinh(x))+3^(1/2)*(-1/12/(x+1/3*3^(1/2))*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1
/3*3^(1/2))+4/3)^(3/2)-1/36*3^(1/2)*(1/3*(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+
1/3*3^(1/2))+12)^(1/2)-1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3-
2/3*3^(1/2)*(x+1/3*3^(1/2)))*3^(1/2)/(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*
3^(1/2))+12)^(1/2)))+1/12*x*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+
4/3)^(1/2)+1/12*arcsinh(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

mupad [B] time = 0.57, size = 204, normalized size = 2.49

$$\frac{\sqrt{3}\,\left(\ln\left(x-\frac{\sqrt{3}}{3}\right)-\ln\left(x+\sqrt{3}+2\,\sqrt{x^2+1}\right)\right)}{18}-\frac{4\,x}{9\,\left(x^2-\frac{1}{3}\right)}+\frac{\sqrt{3}\,\left(\ln\left(x+\frac{\sqrt{3}}{3}\right)-\ln\left(x-\sqrt{3}-2\,\sqrt{x^2+1}\right)\right)}{18}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + (x^2 + 1)^(1/2))^2,x)
```

```
[Out] (3^(1/2)*(log(x - 3^(1/2)/3) - log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/18 +
(3^(1/2)*atan(3^(1/2)*x*1i)*1i)/9 - (4*x)/(9*(x^2 - 1/3)) + (3^(1/2)*(log(x
 + 3^(1/2)/3) - log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/18 - (3^(1/2)*(6*log
(x - 3^(1/2)/3) - 6*log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/54 - (3^(1/2)*(6
*log(x + 3^(1/2)/3) - 6*log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/54 + (3^(1/2
)*(x^2 + 1)^(1/2))/(9*(x - 3^(1/2)/3)) - (3^(1/2)*(x^2 + 1)^(1/2))/(9*(x +
3^(1/2)/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

```
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

3.4 $\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx$

Optimal. Leaf size=43

$$\frac{3\sqrt{x^2-1}\,x}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

```
[Out] 5/16*arctanh(1/2*x/(x^2-1)^(1/2))+3/8*x*(x^2-1)^(1/2)/(-3*x^2+4)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {382, 377, 207}

$$\frac{3\sqrt{x^2-1}\,x}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)^2}\,dx &= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}-\frac{5}{8}\int \frac{1}{\sqrt{-1+x^2}\left(-4+3x^2\right)}\,dx\\
&= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}-\frac{5}{8}\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\
&= \frac{3x\sqrt{-1+x^2}}{8\left(4-3x^2\right)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 3.76, size = 167, normalized size = 3.88

$$-\frac{x\sqrt{x^2-1}\left(\frac{8x^2(x^2-1)\,{}_2F_1\left(2,3;\frac{7}{2};\frac{x^2}{4-3x^2}\right)}{45x^2-60}-\frac{x^2(2x^2-3)\sqrt{\frac{x^2-1}{3x^2-4}}\left(2\sqrt{\frac{x^2-x^4}{(4-3x^2)^2}}-\sin^{-1}\left(\sqrt{\frac{x^2}{4-3x^2}}\right)\right)}{4\left(\frac{x^2}{4-3x^2}\right)^{5/2}(x^2-1)}\right)}{16\left(1-\frac{3x^2}{4}\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] -1/16*(x*Sqrt[-1 + x^2]*(-1/4*(x^2*(-3 + 2*x^2)*Sqrt[(-1 + x^2)/(-4 + 3*x^2
)]*(2*Sqrt[(x^2 - x^4)/(4 - 3*x^2)^2] - ArcSin[Sqrt[x^2/(4 - 3*x^2)]]))/((x
^2/(4 - 3*x^2))^(5/2)*(-1 + x^2)) + (8*x^2*(-1 + x^2)*Hypergeometric2F1[2,
3, 7/2, x^2/(4 - 3*x^2)])/(-60 + 45*x^2)))/(1 - (3*x^2)/4)^2
```

fricas [B] time = 1.53, size = 80, normalized size = 1.86

$$-\frac{12\,x^2+5\left(3\,x^2-4\right)\log\left(3\,x^2-3\,\sqrt{x^2-1}\,x-2\right)-5\left(3\,x^2-4\right)\log\left(x^2-\sqrt{x^2-1}\,x-2\right)+12\,\sqrt{x^2-1}\,x-16}{32\left(3\,x^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(12*x^2 + 5*(3*x^2 - 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) - 5*(3*x^2
 - 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) + 12*sqrt(x^2 - 1)*x - 16)/(3*x^2 - 4)
```

giac [B] time = 0.94, size = 94, normalized size = 2.19

$$\frac{5\left(x-\sqrt{x^2-1}\right)^2-3}{4\left(3\left(x-\sqrt{x^2-1}\right)^4-10\left(x-\sqrt{x^2-1}\right)^2+3\right)}-\frac{5}{32}\log\left(\left|3\left(x-\sqrt{x^2-1}\right)^2-1\right|\right)+\frac{5}{32}\log\left(\left|\left(x-\sqrt{x^2-1}\right)^2-3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(x
^2 - 1))^2 + 3) - 5/32*log(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*log(abs
((x - sqrt(x^2 - 1))^2 - 3))
```

maple [B] time = 0.05, size = 172, normalized size = 4.00

$$\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x-\frac{2\sqrt{3}}{3}\right)^2+12\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^2-12\sqrt{3}\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{\sqrt{\left(x+\frac{2\sqrt{3}}{3}\right)^2-\frac{4\sqrt{3}\left(x+\frac{2}{\ldots}\right.}{3}}}{16\left(x+\frac{2\sqrt{3}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x)
```

```
[Out] -5/32*arctanh(3/2*(2/3-4/3*3^(1/2)*(x+2/3*3^(1/2)))*3^(1/2)/(9*(x+2/3*3^(1/
2))^2-12*3^(1/2)*(x+2/3*3^(1/2))+3)^(1/2))+5/32*arctanh(3/2*(2/3+4/3*3^(1/2
```

```
)*(x-2/3*3^(1/2)))*3^(1/2)/(9*(x-2/3*3^(1/2))^2+12*3^(1/2)*(x-2/3*3^(1/2))+
3)^(1/2))-1/16/(x+2/3*3^(1/2))*((x+2/3*3^(1/2))^2-4/3*3^(1/2)*(x+2/3*3^(1/2
))+1/3)^(1/2)-1/16/(x-2/3*3^(1/2))*((x-2/3*3^(1/2))^2+4/3*3^(1/2)*(x-2/3*3^
(1/2))+1/3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-4\right)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2-1}\,\left(3\,x^2-4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)\,(x+1)}\,\left(3x^2-4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

3.5 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

```
[Out] 8/9/(1-3*x)-8/9*arcsinh(x^(1/2))+10/9*arctanh(2*x^(1/2)/(1+x)^(1/2))+5/9*ln
(1-3*x)-4/3*x^(1/2)*(1+x)^(1/2)/(1-3*x)
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6742, 97, 157, 54, 215, 93, 207}

$$-\frac{4\sqrt{x}\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x
]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx &= \int \left(\frac{8}{3(-1+3x)^2}-\frac{4\sqrt{x}\sqrt{1+x}}{(-1+3x)^2}+\frac{5}{3(-1+3x)}\right)dx\\ &= \frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-4\int \frac{\sqrt{x}\sqrt{1+x}}{(-1+3x)^2}\,dx\\ &= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{4}{3}\int \frac{\frac{1}{2}+x}{\sqrt{x}\sqrt{1+x}\,(-1+3x)}\,dx\\ &= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{4}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx-\frac{10}{9}\int \frac{1}{\sqrt{x}\sqrt{1+x}\,(-\\ &= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)-\frac{20}{9}\text{Su}\\ &= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\sqrt{1+x}}{3(1-3x)}-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right)+\frac{5}{9}\log(1-3x)\end{aligned}$$

Mathematica [A] time = 0.17, size = 126, normalized size = 1.70

$$\frac{12x^{3/2}+12\sqrt{x}-8\sqrt{x+1}+15\sqrt{x+1}\,x\log(1-3x)-5\sqrt{x+1}\log(1-3x)+10\sqrt{-x-1}\,(3x-1)\tan^{-1}\left(\frac{2\sqrt{x}}{\sqrt{-x-1}}\right)-}{9\sqrt{x+1}\,(3x-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] (12*Sqrt[x] + 12*x^(3/2) - 8*Sqrt[1 + x] - 8*Sqrt[1 + x]*(-1 + 3*x)*ArcSinh
[Sqrt[x]] + 10*Sqrt[-1 - x]*(-1 + 3*x)*ArcTan[(2*Sqrt[x])/Sqrt[-1 - x]] - 5
*Sqrt[1 + x]*Log[1 - 3*x] + 15*x*Sqrt[1 + x]*Log[1 - 3*x])/(9*Sqrt[1 + x]*(
-1 + 3*x))
```

fricas [A] time = 0.90, size = 105, normalized size = 1.42

$$-\frac{5\,(3\,x-1)\log\left(3\,\sqrt{x+1}\,\sqrt{x}-3\,x-1\right)-4\,(3\,x-1)\log\left(2\,\sqrt{x+1}\,\sqrt{x}-2\,x-1\right)-5\,(3\,x-1)\log\left(\sqrt{x+1}\,\sqrt{x}-x\right.}{9\,(3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/9*(5*(3*x - 1)*log(3*sqrt(x + 1)*sqrt(x) - 3*x - 1) - 4*(3*x - 1)*log(2*
sqrt(x + 1)*sqrt(x) - 2*x - 1) - 5*(3*x - 1)*log(sqrt(x + 1)*sqrt(x) - x +
```

```
1) - 5*(3*x - 1)*log(3*x - 1) - 12*sqrt(x + 1)*sqrt(x) - 12*x + 12)/(3*x -
1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters
values [-89]Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,
1] at parameters values [63]2*((-5*(x+1)+4)/6/(3*(x+1)-4)+5/18*ln(abs(3*(x+
1)-4))+2*(1/9*ln((sqrt(x)-sqrt(x+1))^2)+5/36*ln(abs((sqrt(x)-sqrt(x+1))^2-3
))-5/36*ln(abs(3*(sqrt(x)-sqrt(x+1))^2-1))-(10*(sqrt(x)-sqrt(x+1))^2-6)/9/(
3*(sqrt(x)-sqrt(x+1))^4-10*(sqrt(x)-sqrt(x+1))^2+3)))
```

maple [B] time = 0.02, size = 115, normalized size = 1.55

$$\frac{5\ln\left(3x-1\right)}{9}-\frac{\sqrt{x+1}\,\left(-15x\,\mathrm{arctanh}\left(\frac{5x+1}{4\sqrt{(x+1)x}}\right)+12x\ln\left(x+\frac{1}{2}+\sqrt{(x+1)\,x}\right)+5\,\mathrm{arctanh}\left(\frac{5x+1}{4\sqrt{(x+1)x}}\right)-4\ln\left(\right.\right.}{9\sqrt{(x+1)\,x}\,\left(3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)+(x+1)^(1/2))^2,x)
```

```
[Out] -8/9/(3*x-1)+5/9*ln(3*x-1)-1/9*x^(1/2)*(x+1)^(1/2)*(12*ln(1/2+x+((x+1)*x)^(
1/2))*x-15*arctanh(1/4*(1+5*x)/((x+1)*x)^(1/2))*x-4*ln(1/2+x+((x+1)*x)^(1/2
))+5*arctanh(1/4*(1+5*x)/((x+1)*x)^(1/2))-12*((x+1)*x)^(1/2))/((x+1)*x)^(1/
2)/(3*x-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{x+1}+2\,\sqrt{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

mupad [B] time = 1.71, size = 82, normalized size = 1.11

$$\frac{10\,\mathrm{atanh}\left(\frac{2662400\,\sqrt{x}}{81\left(\frac{665600\,x}{81\,\left(\sqrt{x+1}-1\right)^2}+\frac{665600}{81}\right)\left(\sqrt{x+1}-1\right)}\right)}{9}+\frac{5\,\ln\left(x-\frac{1}{3}\right)}{9}-\frac{16\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)}{9}-\frac{8}{27\,\left(x-\frac{1}{3}\right)}+\frac{4\,\sqrt{x}\,\sqrt{x+1}}{3\,\left(3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + 2*x^(1/2))^2,x)
```

```
[Out] (10*atanh((2662400*x^(1/2))/(81*((665600*x)/(81*((x + 1)^(1/2) - 1)^2) + 66
5600/81)*((x + 1)^(1/2) - 1))))/9 + (5*log(x - 1/3))/9 - (16*atanh(x^(1/2)/
((x + 1)^(1/2) - 1)))/9 - 8/(27*(x - 1/3)) + (4*x^(1/2)*(x + 1)^(1/2))/(3*(
3*x - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\sqrt{x} + \sqrt{x+1}\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} \, dx$

Optimal. Leaf size=64

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] arctanh(x/(x^2-1)^(1/2))-1/2*I*arctan(1/2*(1-I*x)*2^(1/2)/(x^2-1)^(1/2))*2^
(1/2)+(x^2-1)^(1/2)/(I-x)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {733, 844, 217, 206, 725, 204}

$$\frac{\sqrt{x^2-1}}{-x+i} - \frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sqr
t[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx &= \frac{\sqrt{-1+x^2}}{i-x}+\int \frac{x}{(-i+x)\sqrt{-1+x^2}}\,dx\\
&= \frac{\sqrt{-1+x^2}}{i-x}+i\int \frac{1}{(-i+x)\sqrt{-1+x^2}}\,dx+\int \frac{1}{\sqrt{-1+x^2}}\,dx\\
&= \frac{\sqrt{-1+x^2}}{i-x}-i\,\text{Subst}\left(\int \frac{1}{-2-x^2}\,dx,x,\frac{-1+ix}{\sqrt{-1+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\
&= \frac{\sqrt{-1+x^2}}{i-x}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.09, size = 59, normalized size = 0.92

$$-\frac{\sqrt{x^2-1}}{x-i}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x^2]/(-I + x)) + ArcTanh[x/Sqrt[-1 + x^2]] - ArcTanh[(I + x)/(S
qrt[2]*Sqrt[-1 + x^2])]/Sqrt[2]
```

fricas [A] time = 0.98, size = 92, normalized size = 1.44

$$-\frac{\sqrt{2}\,(x-i)\log\left(-x+i\sqrt{2}+\sqrt{x^2-1}+i\right)-\sqrt{2}\,(x-i)\log\left(-x-i\sqrt{2}+\sqrt{x^2-1}+i\right)+(2\,x-2i)\log\left(-x+\sqrt{x^2}\right.}{2\,x-2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(x - I)*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)*(x - I)
*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) + (2*x - 2*I)*log(-x + sqrt(x^2 -
1)) + 2*x + 2*sqrt(x^2 - 1) - 2*I)/(2*x - 2*I)
```

giac [A] time = 1.07, size = 84, normalized size = 1.31

$$i\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2-1}-i\right)\right)+\frac{2\left(ix-i\sqrt{x^2-1}-1\right)}{\left(x-\sqrt{x^2-1}\right)^2-2ix+2i\sqrt{x^2-1}+1}-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="giac")
```

```
[Out] I*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - 1) - I)) + 2*(I*x - I*sqrt(x^
2 - 1) - 1)/((x - sqrt(x^2 - 1))^2 - 2*I*x + 2*I*sqrt(x^2 - 1) + 1) - log(a
bs(-x + sqrt(x^2 - 1)))
```

maple [B] time = 0.04, size = 125, normalized size = 1.95

$$-\frac{\sqrt{(x-i)^2-2+2i(x-i)}\;x}{2}+\frac{i\sqrt{2}\;\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4\sqrt{(x-i)^2-2+2i(x-i)}}\right)}{2}+\ln\left(x+\sqrt{(x-i)^2-2+2i(x-i)}\right)+\frac{\left((x-i)^2-\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x-I)^2,x)
```

```
[Out] 1/2/(x-I)*((x-I)^2-2+2*I*(x-I))^(3/2)+ln(x+((x-I)^2-2+2*I*(x-I))^(1/2))+1/2
*I*2^(1/2)*arctan(1/4*(-4+2*I*(x-I))*2^(1/2)/((x-I)^2-2+2*I*(x-I))^(1/2))-1
/2*I*((x-I)^2-2+2*I*(x-I))^(1/2)-1/2*x*((x-I)^2-2+2*I*(x-I))^(1/2)
```

maxima [A] time = 1.45, size = 53, normalized size = 0.83

$$\frac{1}{2}i\sqrt{2}\;\arcsin\left(\frac{ix}{|x-i|}-\frac{1}{|x-i|}\right)-\frac{\sqrt{x^2-1}}{x-i}+\log\left(2x+2\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I)
 + log(2*x + 2*sqrt(x^2 - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^2-1}}{(x-\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(1/2)/(x - 1i)^2,x)
```

```
[Out] int((x^2 - 1)^(1/2)/(x - 1i)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)}}{(x-i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

3.7 $\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=48

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

```
[Out] 3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))*2^(1/2)-1/4*x*(x^2-1)^(1/2)/(x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {382, 377, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[-1 + x^2])/(4*(1 + x^2)) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^2]])
/(4*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)^2}\,dx&=-\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)}+\frac{3}{4}\int \frac{1}{\sqrt{-1+x^2}\left(1+x^2\right)}\,dx\\&=-\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)\\&=-\frac{x\sqrt{-1+x^2}}{4\left(1+x^2\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 75, normalized size = 1.56

$$\frac{\sqrt{x^2-1}\left(3\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\left(x^2+1\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\right)-2x^2\right)}{8\left(x^3+x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] (Sqrt[-1 + x^2]*(-2*x^2 + 3*Sqrt[2]*Sqrt[x^2/(-1 + x^2)]*(1 + x^2)*ArcTanh[
Sqrt[2]*Sqrt[x^2/(-1 + x^2)]]))/(8*(x + x^3))
```

fricas [B] time = 1.01, size = 83, normalized size = 1.73

$$\frac{3\sqrt{2}\left(x^2+1\right)\log\left(\frac{9x^2+2\sqrt{2}\left(3x^2-1\right)+2\sqrt{x^2-1}\left(3\sqrt{2}x+4x\right)-3}{x^2+1}\right)-4x^2-4\sqrt{x^2-1}\,x-4}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(x^2 + 1)*log((9*x^2 + 2*sqrt(2)*(3*x^2 - 1) + 2*sqrt(x^2 -
 1)*(3*sqrt(2)*x + 4*x) - 3)/(x^2 + 1)) - 4*x^2 - 4*sqrt(x^2 - 1)*x - 4)/(x
^2 + 1)
```

giac [B] time = 0.96, size = 101, normalized size = 2.10

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2-1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2-1}\right)^2+2\sqrt{2}+3}\right)-\frac{3\left(x-\sqrt{x^2-1}\right)^2+1}{2\left(\left(x-\sqrt{x^2-1}\right)^4+6\left(x-\sqrt{x^2-1}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 -
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2
- 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

maple [A] time = 0.02, size = 45, normalized size = 0.94

$$-\frac{x}{8\sqrt{x^2-1}\left(\frac{x^2}{x^2-1}-\frac{1}{2}\right)}+\frac{3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] -1/8*x/(x^2-1)^(1/2)/(x^2/(x^2-1)-1/2)+3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))
*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(x^2+1\right)^2\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2-1}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)\,(x+1)}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

3.8 $\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}} \, dx$

Optimal. Leaf size=30

$$-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

```
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6689, 43}

$$-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}} \, dx &= \int \left(-\frac{1}{\sqrt{-1+x}}-2\sqrt{x}+\frac{2x}{\sqrt{-1+x}}\right) dx \\
&= -2\sqrt{-1+x}-\frac{4x^{3/2}}{3}+2\int \frac{x}{\sqrt{-1+x}} \, dx \\
&= -2\sqrt{-1+x}-\frac{4x^{3/2}}{3}+2\int \left(\frac{1}{\sqrt{-1+x}}+\sqrt{-1+x}\right) dx \\
&= 2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}-\frac{4x^{3/2}}{3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 30, normalized size = 1.00

$$-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

fricas [A] time = 0.89, size = 18, normalized size = 0.60

$$\frac{2}{3}(2x+1)\sqrt{x-1} - \frac{4}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="fricas")
[Out] 2/3*(2*x + 1)*sqrt(x - 1) - 4/3*x^(3/2)
```

giac [A] time = 0.95, size = 20, normalized size = 0.67

$$\frac{4}{3}(x-1)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="giac")
[Out] 4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

maple [A] time = 0.00, size = 21, normalized size = 0.70

$$-\frac{4x^{\frac{3}{2}}}{3} + \frac{4(x-1)^{\frac{3}{2}}}{3} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-1)^(1/2)/((x-1)^(1/2)+x^(1/2))^2,x)
[Out] 4/3*(x-1)^(3/2)-4/3*x^(3/2)+2*(x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\left(\sqrt{x-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)
```

mupad [B] time = 0.38, size = 21, normalized size = 0.70

$$\frac{4x\sqrt{x-1}}{3} + \frac{2\sqrt{x-1}}{3} - \frac{4x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)^(1/2) + x^(1/2))^2*(x - 1)^(1/2)),x)
[Out] (4*x*(x - 1)^(1/2))/3 + (2*(x - 1)^(1/2))/3 - (4*x^(3/2))/3
```

sympy [B] time = 0.75, size = 53, normalized size = 1.77

$$-\frac{4\sqrt{x}}{6\sqrt{x}\sqrt{x-1}+6x-3} - \frac{2\sqrt{x-1}}{6\sqrt{x}\sqrt{x-1}+6x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
[Out] -4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*sqrt(x)*sqr
t(x - 1) + 6*x - 3)
```

3.9 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x^2-1}+\sqrt{x}\right)}-\frac{1}{50}\sqrt{50\sqrt{5}-110}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2}\,\sqrt{x^2-1}}{2-\left(1-\sqrt{5}\right)x}\right)-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{x}}{-\sqrt{5}\,x-x+}\right.$$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6742, 736, 826, 1166, 207, 203, 1018, 1034, 725, 206, 204, 985}

$$-\frac{2\sqrt{x^2-1}\,(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}\,(1-2x)}{5\left(-x^2+x+1\right)}-\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\,\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\,\tan^{-1}\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(
1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sq
rt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(
5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt
[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sq
rt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int\left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2}+\frac{2x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{1}{\sqrt{-1+x^2}\left(-1-x\right.}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right)+2\int \frac{x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx+\int$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{2}{5}\int \frac{-\frac{1}{2}-x}{\sqrt{x}\left(-1-x+x^2\right)}\,dx+\frac{2}{5}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{4}{5}\,\text{Subst}\left(\int \frac{-\frac{1}{2}-x^2}{-1-x^2+x^4}\,dx,x,\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tan^{-1}\left(\frac{2-}{\sqrt{2\left(-1\right.}}\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\tan^{-1}\left(\sqrt{-}\right.$$

Mathematica [A] time = 0.85, size = 340, normalized size = 1.55

$$\frac{2}{5}\left(\frac{\sqrt{x}(1-2x)}{-x^2+x+1}+\frac{\sqrt{x^2-1}(1-2x)}{x^2-x-1}-\frac{1}{2}\sqrt{\frac{5}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+x-2}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\sqrt{\sqrt{5}-\frac{2}{5}}\tan^{-1}\left(\frac{(}{\sqrt{2}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(((1 - 2*x)*Sqrt[x])/(1 + x - x^2) + ((1 - 2*x)*Sqrt[-1 + x^2])/(-1 - x
+ x^2) + Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]]
- (Sqrt[(5*(1 + Sqrt[5]))/2]*ArcTan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1 + Sqrt
[5])]*Sqrt[-1 + x^2])])/2 - Sqrt[-2/5 + Sqrt[5]]*ArcTan[(2 + (-1 + Sqrt[5])
*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - Sqrt[(11 + 5*Sqrt[5])/10]*Ar
cTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]] - Sqrt[5/(2*(1 + Sqrt[5]))]*ArcTanh[(-
2 + x + Sqrt[5]*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])] - Sqrt[2/5 + Sqr
```

```
t[5]]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])]
))/5
```

fricas [B] time = 0.78, size = 424, normalized size = 1.93

$$\frac{4\sqrt{5}\left(x^2-x-1\right)\sqrt{10\sqrt{5}-22}\,\arctan\left(\frac{1}{2}\sqrt{2x^2-\sqrt{x^2-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

giac [B] time = 8.76, size = 367, normalized size = 1.67

$$\frac{2}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}-11}\,\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)+\frac{1}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}+11}\,\log\left(\left|-153040x+22956\sqrt{5}\sqrt{50}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/5*sqrt(1/10)*sqrt(5*sqrt(5) - 11)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1)
 - 1)/sqrt(2*sqrt(5) - 2)) + 1/5*sqrt(1/10)*sqrt(5*sqrt(5) + 11)*log(abs(-1
53040*x + 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 76520*sqrt(5) + 153040*sqr
t(x^2 - 1) - 38260*sqrt(50*sqrt(5) + 110) + 76520)) - 1/5*sqrt(1/10)*sqrt(5
*sqrt(5) + 11)*log(abs(-153040*x - 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 7
6520*sqrt(5) + 153040*sqrt(x^2 - 1) + 38260*sqrt(50*sqrt(5) + 110) + 76520)
) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*sqrt(5) - 1/2)) - 1
/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5) + 1/2)) + 1/50*sq
rt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1/2*sqrt(5) + 1/2))) + 4/5*((x
- sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2 + 3*x - 3*sqrt(x^2 - 1) - 2)/(
(x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))^3 - 2*(x - sqrt(x^2 - 1))^2 -
 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2) - sqrt(x))/(x^2 - x - 1)
```

maple [B] time = 0.14, size = 902, normalized size = 4.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x)
```

```
[Out] -1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)
+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+2/5/(1/2-1/2*5^(1/2))/(-2+2*
```

```
5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*
5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+
2-2*5^(1/2))^(1/2))*5^(1/2)-6/5/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arct
an(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(
x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))
+1/5*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(
-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-6/25*5^(1/2)/(2+2*5^
(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(
1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*
5^(1/2))^(1/2))-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1
/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+6/5/(1/2+1/2*5
^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)
-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^
(1/2)-1/2)+2+2*5^(1/2))^(1/2))+2/5/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*ar
ctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*
(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))
*5^(1/2)-1/5*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-
1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-6/25*5^(1/2)/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)+4/5/(-2+2*5^(1/2)
)^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))-8/25/(-2+2*5^(1/2))^(1/2)*ar
ctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2
)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*
5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\left(\sqrt{x}+\sqrt{x^2-1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)
```

3.10 $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2 \sqrt{-1+x^2}} \, dx$

Optimal. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x^2-1}+\sqrt{x}\right)}-\frac{1}{50}\sqrt{50\sqrt{5}-110}\,\tan^{-1}\left(\frac{\sqrt{2\sqrt{5}-2}\,\sqrt{x^2-1}}{2-\left(1-\sqrt{5}\right)x}\right)-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\tanh^{-1}\left(\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{x^2-}}{-\sqrt{5}x-x+2}\right.$$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

Rubi [B] time = 0.75, antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 25, number of rules used = 13, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 736, 826, 1166, 207, 203, 975, 1034, 725, 206, 204, 1018, 1065}

$$-\frac{\sqrt{x^2-1}\,(1-2x)}{5\left(-x^2+x+1\right)}+\frac{2\sqrt{x}\,(1-2x)}{5\left(-x^2+x+1\right)}-\frac{(3-x)\sqrt{x^2-1}}{5\left(-x^2+x+1\right)}+\frac{(x+2)\sqrt{x^2-1}}{5\left(-x^2+x+1\right)}+\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\tan^{-1}\left(\frac{2-\left(1-\sqrt{5}\right.}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^2])/(5*(1
+ x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + ((2 + x)*Sqrt[-1
 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(
-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1 - S
qrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5*Sqrt
[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 +
 Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt
[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 +
 Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2 + 5*Sqrt
[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1
 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^m*(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)
*(b^2 - 4*a*c)), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
```

```
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1065

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2
)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a
*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x)
)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f))))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx = \int\left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2}-\frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right)-\int \frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx+\int \frac{}{\sqrt{-1+x^2}\,(}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+$$

Mathematica [A] time = 0.71, size = 311, normalized size = 1.41

$$\frac{1}{25}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\left(5+2\sqrt{5}\right)\tanh^{-1}\left(\frac{\sqrt{5}x+x-2}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\frac{-20x^{3/2}+20\sqrt{x^2-1}\,x-10\sqrt{x^2-1}+\sqrt{50\sqrt{5}}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] ((10*Sqrt[x] - 20*x^(3/2) - 10*Sqrt[-1 + x^2] + 20*x*Sqrt[-1 + x^2] + Sqrt[
-110 + 50*Sqrt[5]]*(1 + x - x^2)*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[x]] + S
qrt[10*(1 + Sqrt[5])]*(1 + x - x^2)*ArcTan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1
 + Sqrt[5])]*Sqrt[-1 + x^2])] + 5*Sqrt[2/(-1 + Sqrt[5])]*(-1 - x + x^2)*Arc
Tan[(-2 + x - Sqrt[5]*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/(1 + x -
 x^2) - Sqrt[110 + 50*Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]] + Sqr
t[2/(1 + Sqrt[5])]*(5 + 2*Sqrt[5])*ArcTanh[(-2 + x + Sqrt[5]*x)/(Sqrt[2*(1
+ Sqrt[5])]*Sqrt[-1 + x^2])])/25
```

fricas [B] time = 0.61, size = 424, normalized size = 1.93

$$\frac{4\sqrt{5}\left(x^2-x-1\right)\sqrt{10\sqrt{5}-22}\,\arctan\left(\frac{1}{2}\sqrt{2x^2-\sqrt{x^2-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
```

```
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

giac [B] time = 9.22, size = 358, normalized size = 1.63

$$\frac{2}{5}\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)+\frac{1}{25}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{\sqrt{x}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{50}\sqrt{50\sqrt{5}+11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="giac")
```

```
[Out] 2/5*sqrt(1/2*sqrt(5) - 11/10)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1) - 1)/
sqrt(2*sqrt(5) - 2)) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*
sqrt(5) - 1/2)) - 1/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5
) + 1/2)) - 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-520*x - 78*sqrt(5)*sqrt(
50*sqrt(5) + 110) + 260*sqrt(5) + 520*sqrt(x^2 - 1) + 130*sqrt(50*sqrt(5) +
 110) + 260)) + 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-1040*x + 156*sqrt(5)
*sqrt(50*sqrt(5) + 110) + 520*sqrt(5) + 1040*sqrt(x^2 - 1) - 260*sqrt(50*sq
rt(5) + 110) + 520)) + 1/50*sqrt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1
/2*sqrt(5) + 1/2))) + 4/5*((x - sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2
+ 3*x - 3*sqrt(x^2 - 1) - 2)/((x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))
^3 - 2*(x - sqrt(x^2 - 1))^2 - 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2)
- sqrt(x))/(x^2 - x - 1)
```

maple [B] time = 0.02, size = 1542, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x)
```

```
[Out] -8/25/(-2+2*5^(1/2))^(1/2)*5^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x^(1/2))-8
/25/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5^(1/2))^(1/2)*x^(1/2))*5^(1/2)-1/10
/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*
(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+4/25*5^(1/2)/(2+2*5^(1/2))^(1/2)
*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/
(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/
2))-1/10/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1
/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)+4/25*5^(1/2)/(-2+2*5^(1/2
))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2
))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^
(1/2))^(1/2))+4/5/(-2+2*5^(1/2))^(1/2)*arctan(2/(-2+2*5^(1/2))^(1/2)*x^(1/2
))+2/5/(x-1/2*5^(1/2)-1/2)*x^(1/2)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2/(2+2*5
^(1/2))^(1/2)*x^(1/2))+1/20/(1/2-1/2*5^(1/2))*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-
5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)-2/5/(-2+2*5^(1/2))^(1/2)*
arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/
(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1
/2))+2/5/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2
)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5
```

```
^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+1/25*5^(1/2)*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5
^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)+1/25*ln(x+((x+1/2*5^(1/2)-
1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))*5^(1/2)-1/2
5*5^(1/2)*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^
(1/2))^(1/2)-1/25*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/
2)+1/2+1/2*5^(1/2))^(1/2))*5^(1/2)+1/10/(1/2+1/2*5^(1/2))*ln(x+((x-1/2*5^(1
/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))+1/20/(1/
2+1/2*5^(1/2))*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2
+2*5^(1/2))^(1/2)+1/10/(1/2-1/2*5^(1/2))*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1
/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))+2/5/(x+1/2*5^(1/2)-1/2)*
x^(1/2)+1/5/(1/2-1/2*5^(1/2))*x*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*
5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/5/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/
2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^
(3/2)-1/20/(1/2-1/2*5^(1/2))*5^(1/2)*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1
)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)-1/10/(1/2-1/2*5^(1/2))*ln(x+((x+1/
2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))*5
^(1/2)+1/20/(1/2+1/2*5^(1/2))*5^(1/2)*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1
)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/10/(1/2+1/2*5^(1/2))*ln(x+((x-1/
2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))*5^
(1/2)+1/5/(1/2+1/2*5^(1/2))*x*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(
1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)-1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*
((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(3
/2)-1/5*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/
2*5^(1/2))^(1/2))-1/5*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2
)-1/2)+1/2+1/2*5^(1/2))^(1/2))+1/10*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2
)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1
/2))^(1/2)-1/10*5^(1/2)/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/
2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\left(x^{\frac{5}{2}}-3\,x^{\frac{3}{2}}\right)}{5\left(x^2-x-1\right)}+\int\frac{x^{\frac{3}{2}}+\sqrt{x}}{5\left(x^2-x-1\right)}\,dx+\int\frac{x^2+x-1}{\left(x^4-2\,x^3-x^2+2\,x+1\right)\sqrt{x+1}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2) + sqrt(x)
)/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2*x^3 - x^2 + 2*x + 1
)*sqrt(x + 1)*sqrt(x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\sqrt{x^2-1}\,\left(-x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2),x)
```

```
[Out] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)

[Out] Timed out
```

3.11 $\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

```
[Out] 1/2*arctanh((I+x)/(1-I)^(1/2)/(-I+x^2)^(1/2))/(1-I)^(3/2)*2^(1/2)-1/2*arcta
nh((I-x)/(1+I)^(1/2)/(I+x^2)^(1/2))/(1+I)^(3/2)*2^(1/2)-(1/4+1/4*I)*(-I+x^2
)^(1/2)/(1+x)*2^(1/2)+(-1/4+1/4*I)*(I+x^2)^(1/2)/(1+x)*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {731, 725, 206}

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x^
2]),x]
```

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sqrt[I + x^2
])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])]/((1 -
I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + I)^(
3/2)*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}} + \frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right) dx = \frac{\int \frac{1}{(1+x)^2\sqrt{-i+x^2}} dx}{\sqrt{2}} + \frac{\int \frac{1}{(1+x)^2\sqrt{i+x^2}} dx}{\sqrt{2}}$$
$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{}}}{\sqrt{2}}$$
$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\text{Subst}}{}$$
$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)} - \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)} + \frac{\tanh^{-1}\left(\frac{i+x}{\sqrt{1-i}\sqrt{-}}\right)}{(1-i)^{3/2}\sqrt{2}}$$

Mathematica [A] time = 0.20, size = 125, normalized size = 0.91

$$\frac{i\left((1+i)\left(i\sqrt{x^2-i}+\sqrt{x^2+i}\right)+\sqrt{1-i}(x+1)\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\sqrt{x^2-i}}\right)+\sqrt{1+i}(x+1)\tanh^{-1}\left(\frac{(1+i)^{3/2}(1+ix)}{2\sqrt{x^2+i}}\right)\right)}{2\sqrt{2}(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[
I + x^2]),x]
```

```
[Out] ((I/2)*((1 + I)*(I*Sqrt[-I + x^2] + Sqrt[I + x^2]) + Sqrt[1 - I]*(1 + x)*Ar
cTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])] + Sqrt[1 + I]*(1 + x)*ArcTanh[(
(1 + I)^(3/2)*(1 + I*x))/(2*Sqrt[I + x^2])]))/(Sqrt[2]*(1 + x))
```

fricas [A] time = 1.06, size = 161, normalized size = 1.17

$$\frac{\sqrt{-\frac{1}{2}i+\frac{1}{2}}\left(-(i-1)x-i+1\right)\log\left(\sqrt{2}\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^2-i}-1\right)+\sqrt{-\frac{1}{2}i+\frac{1}{2}}\left((i-1)x+i-1\right)\log\left(-\sqrt{2}\sqrt{-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-1/2*I + 1/2)*(-(I - 1)*x - I + 1)*log(sqrt(2)*sqrt(-1/2*I + 1/2) - x
 + sqrt(x^2 - I) - 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x + I - 1)*log(-sqrt(2)
*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I - 1/2)*(-(I + 1)
*x - I - 1)*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + sqr
t(-1/2*I - 1/2)*((I + 1)*x + I + 1)*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2) - x +
 sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x - I - 1) - sqrt(2)*sqrt(x^2 + I)
- I*sqrt(2)*sqrt(x^2 - I))/((2*I + 2)*x + 2*I + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
```

```
variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.
```

maple [B] time = 0.03, size = 278, normalized size = 2.01

$$-\frac{\sqrt{2}\,\ln\left(\frac{-2x-2i+2\sqrt{1-i}\,\sqrt{-2x+(x+1)^{2}-1-i}}{x+1}\right)}{4\sqrt{1-i}}-\frac{i\sqrt{2}\,\ln\left(\frac{-2x-2i+2\sqrt{1-i}\,\sqrt{-2x+(x+1)^{2}-1-i}}{x+1}\right)}{4\sqrt{1-i}}-\frac{\sqrt{2}\,\ln\left(\frac{-2x+2i+2\sqrt{1+i}\,\sqrt{-2x+(x+1}{x+1}\right)}{4\sqrt{1+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(x+1)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(x+1)^2*2^(1/2)/(I+x^2)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)/(x+1)*((x+1)^2-2*x-1-I)^(1/2)-1/4*I*2^(1/2)/(x+1)*((x+1)^2-2*x
-1-I)^(1/2)-1/4*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((x+1)^2-2*x
-1-I)^(1/2))/(x+1))-1/4*I*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((
x+1)^2-2*x-1-I)^(1/2))/(x+1))-1/4*2^(1/2)/(x+1)*((x+1)^2-2*x-1+I)^(1/2)+1/4
*I*2^(1/2)/(x+1)*((x+1)^2-2*x-1+I)^(1/2)-1/4*2^(1/2)/(1+I)^(1/2)*ln((2*I-2*
x+2*(1+I)^(1/2)*((x+1)^2-2*x-1+I)^(1/2))/(x+1))+1/4*I*2^(1/2)/(1+I)^(1/2)*l
n((2*I-2*x+2*(1+I)^(1/2)*((x+1)^2-2*x-1+I)^(1/2))/(x+1))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  1which is not of the expected type LIST
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2}}{2\sqrt{x^2-\mathrm{i}}\,(x+1)^2}+\frac{\sqrt{2}}{2\sqrt{x^2+1\mathrm{i}}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2),x)
```

```
[Out] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x
**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -1/4*(1-I)^(3/2)*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))-1/4*(1+I)^(3/
2)*arctanh((1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))-1/2*(1-I*x^2)^(1/2)/(1+x)-1
/2*(1+I*x^2)^(1/2)/(1+x)
```

Rubi [A] time = 0.19, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2133, 731, 725, 206\}$

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 - I*x^2]/(2*(1 + x)) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^(3/2)
*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - ((1 + I)^(3/2)*ArcTa
nh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1-ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1+ix^2}}\,dx$$
$$= -\frac{\sqrt{1-ix^2}}{2(1+x)} - \frac{\sqrt{1+ix^2}}{2(1+x)} - \frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx + \frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx$$
$$= -\frac{\sqrt{1-ix^2}}{2(1+x)} - \frac{\sqrt{1+ix^2}}{2(1+x)} + \frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx, x, \frac{1+ix}{\sqrt{1-ix^2}}\right) - \frac{1}{2}i\,\text{Subst}\left(\int \right.$$
$$= -\frac{\sqrt{1-ix^2}}{2(1+x)} - \frac{\sqrt{1+ix^2}}{2(1+x)} - \frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right) - \frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\right.$$

Mathematica [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]), x]
```

fricas [B] time = 4.55, size = 394, normalized size = 3.15

$$\frac{4\,(x+1)\sqrt{\sqrt{2}+1}\,\arctan\left(\frac{2\left(x^3+x^2-\sqrt{2}\left(x^3+1\right)+\sqrt{x^4+1}\left(\sqrt{2}\,x-x-1\right)-x+1\right)\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{\sqrt{2}+1}+\left(2\,x^2-\sqrt{2}\left(x^2+1\right)+2\,\sqrt{x^4+1}\left(\sqrt{2}-1\right.\right.}{2\left(x^2-2\,x+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fri
cas")
[Out] 1/8*(4*(x + 1)*sqrt(sqrt(2) + 1)*arctan(1/2*(2*(x^3 + x^2 - sqrt(2)*(x^3 +
1) + sqrt(x^4 + 1)*(sqrt(2)*x - x - 1) - x + 1)*sqrt(x^2 + sqrt(x^4 + 1))*s
qrt(sqrt(2) + 1) + (2*x^2 - sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(sqrt(2) -
1) + 2)*sqrt(2*sqrt(2) + 2)*sqrt(sqrt(2) + 1))/(x^2 - 2*x + 1)) + (x + 1)*s
qrt(sqrt(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)
*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (sqrt(2)*(x^2 + 1
) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) - (x + 1)*sqrt(sqr
t(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(
2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (sqrt(2)*(x^2 + 1) + 2*s
qrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) + 4*sqrt(x^2 + sqrt(x^4 +
 1))*(x^2 - sqrt(x^4 + 1) - 1))/(x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)^2\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x+1)^2/(x^4+1)^(1/2),x)
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x+1)^2/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="max
ima")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)^2\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)
```

3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=81

$$-\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

```
[Out] -1/2*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))*(1-I)^(1/2)-1/2*arctanh((
1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))*(1+I)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2133, 725, 206}

$$-\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/2 - (Sqrt[1
 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2133

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx\\ &= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx,x,\frac{1-ix}{\sqrt{1+ix^2}}\right)+\left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\right.\\ &= -\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\sqrt{1+ix^2}}\right)\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]), x]
```

fricas [B] time = 10.16, size = 369, normalized size = 4.56

$$\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{\left(2x^2-\sqrt{2}\left(x^3-x^2+x+1\right)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2\right)-2x\right)\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{2\sqrt{2}-2}}{2\left(x^2-2x+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*((2*x^2 - sqrt(2)*(x^3 - x^2 + x + 1) +
sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2
*sqrt(2) - 2) + (x^2 + sqrt(2)*sqrt(x^4 + 1) + 1)*sqrt(2*sqrt(2) + 2)*sqrt(
2*sqrt(2) - 2))/(x^2 - 2*x + 1)) - 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - s
qrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqr
t(x^2 + sqrt(x^4 + 1)) + (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2)
- 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1)) + 1/8*sqrt(2*sqrt(2) + 2)*l
og(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1)
- 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4
 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x+1)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x+1)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{x^4+1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx = \text{Subst}\left(\int \frac{1}{1-2x^2} \, dx, x, \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

fricas [B] time = 1.22, size = 60, normalized size = 1.94

$$\frac{1}{4}\sqrt{2}\log\left(4x^4+4\sqrt{x^4+1}x^2+2\left(\sqrt{2}x^3+\sqrt{2}\sqrt{x^4+1}x\right)\sqrt{x^2+\sqrt{x^4+1}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt
(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

sympy [A] time = 1.22, size = 15, normalized size = 0.48

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1,1 & \frac{1}{2} \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

3.15 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctan(x*2^(1/2)/(-x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2132, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]

[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2132

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

fricas [A] time = 1.62, size = 29, normalized size = 0.88

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^4+1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^4 + 1))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

maple [C] time = 0.10, size = 22, normalized size = 0.67

$$-\frac{\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
[Out] -1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

sympy [**A**] time = 0.86, size = 15, normalized size = 0.45

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix} \frac{1}{2},1 & 1 \\ \frac{1}{4},\frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

3.16 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

```
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {6688}

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}}\,dx &= \int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{(1+x)^{3/2}}\right)dx\\ &= -\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x
]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

fricas [A] time = 0.99, size = 28, normalized size = 1.47

$$-\frac{2\left((x+1)\sqrt{x-1}+\sqrt{x+1}\,(x-1)\right)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2*((x + 1)*sqrt(x - 1) + sqrt(x + 1)*(x - 1))/(x^2 - 1)
```

giac [A] time = 1.15, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"giac")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2}{\sqrt{x-1}}-\frac{2}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-1)^(3/2)+(x+1)^(3/2))/(x-1)^(3/2)/(x+1)^(3/2),x)
```

```
[Out] -2/(x-1)^(1/2)-2/(x+1)^(1/2)
```

maxima [A] time = 0.63, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

mupad [B] time = 0.40, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x-1}}-\frac{2}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^(3/2) + (x + 1)^(3/2))/((x - 1)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] - 2/(x - 1)^(1/2) - 2/(x + 1)^(1/2)
```

sympy [B] time = 6.63, size = 56, normalized size = 2.95

$$-\frac{2x\sqrt{x-1}}{x^2-1}-\frac{2x\sqrt{x+1}}{x^2-1}-\frac{2\sqrt{x-1}}{x^2-1}+\frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt(x - 1)/(x
**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

3.17 $\int \left(x+\sqrt{a+x^2}\right)^b dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

```
[Out] -1/2*a*(x+(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x+(x^2+a)^(1/2))^(1+b)/(1+b)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2117, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] -(a*(x + Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x + Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(x+\sqrt{a+x^2}\right)^b dx &= \frac{1}{2}\text{Subst}\left(\int x^{-2+b}\left(a+x^2\right)dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right)dx, x, x+\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b-1}\left((b-1)x\left(\sqrt{a+x^2}+x\right)+ab\right)}{b^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-1 + b)*(a*b + (-1 + b)*x*(x + Sqrt[a + x^2])))/(-1 +
 b^2)
```

fricas [A] time = 1.38, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+a}\,b-x\right)\left(x+\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*b - x)*(x + sqrt(x^2 + a))^b/(b^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x+\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

maple [B] time = 0.03, size = 120, normalized size = 2.31

$$\frac{\left(\frac{8\sqrt{\pi}\left(b+\frac{ab}{x^2}-1\right)a^{-\frac{b}{2}-\frac{1}{2}}x^{b+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{b-1}}{(b+1)(2b-2)b}+\frac{4\sqrt{\pi}\,\sqrt{\frac{a}{x^2}+1}\,a^{-\frac{b}{2}-\frac{1}{2}}x^{b+1}\left(\sqrt{\frac{a}{x^2}+1}+1\right)^{b-1}}{(b+1)b}\right)b\,a^{\frac{b}{2}+\frac{1}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b,x)
```

```
[Out] 1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(1/
x^2*a*b+b-1)/(-2+2*b)*((1+1/x^2*a)^(1/2)+1)^(-1+b)+4*Pi^(1/2)/(1+b)/b*x^(1+
b)*a^(-1/2*b-1/2)*(1+1/x^2*a)^(1/2)*((1+1/x^2*a)^(1/2)+1)^(-1+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x+\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x+\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^b,x)
```

```
[Out] int((x + (a + x^2)^(1/2))^b, x)
```

<u>**sympy**</u> **[B]** time = 3.53, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(b/2)*b**2*x*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a
)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2)
 + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) +
a**(9/2)*a**(b/2)*b*x*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2
*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1
- b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - a**(7/2)*a**(b/2)*b**2*x**3*sqrt
(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1
- b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
2*a**(7/2)*x**2*gamma(1 - b/2)) + a**(7/2)*a**(b/2)*b*x**3*cosh(b*asinh(x/s
qrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 -
 b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2
)) + 2*a**5*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(
7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**5*a*
*(b/2)*b*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(
1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 -
b/2)) - 2*a**4*a**(b/2)*b*x**2*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2
)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*ga
mma(1 - b/2)) + 4*a**4*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1
 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b
/2)) - 2*a**4*a**(b/2)*b*x**2*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) - 2*a**4*a**(b/2)*x**2*sqrt(a/x**2 + 1)*sinh(b*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1
 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
 2*a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**4*a**(b/2)*x**2*cosh(b*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2
*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)
*x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*b*x**4*sqrt(a/x**2 + 1)*sinh(b*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 -
b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*
a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*x**4*cosh(b*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*
a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*
x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*x**4*sqrt(a/x**2 + 1)*sinh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*x**4*cosh(b*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9
/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*
gamma(1 - b/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(b/2)*b*x*sqrt(1 + x**2/
a)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b
**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) + a**(5/2)*a**(b/2)*b*x*cos
h(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5
/2)*gamma(1 - b/2)) - 2*a**(5/2)*a**(b/2)*x*sqrt(1 + x**2/a)*sinh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2
) - 2*a**(5/2)*gamma(1 - b/2)) - a**3*a**(b/2)*b**2*sqrt(1 + x**2/a)*sinh(b
*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)
*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 -
```

```
 b/2)) + 2*a**2*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))
*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)
) + 2*a**2*a**(b/2)*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)), True
))
```

3.18 $\int \left(x-\sqrt{a+x^2}\right)^b \, dx$

Optimal. Leaf size=56

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

```
[Out] -1/2*a*(x-(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x-(x^2+a)^(1/2))^(1+b)/(1+b)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2117, 14}

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] -(a*(x - Sqrt[a + x^2])^(-1 + b))/(2*(1 - b)) + (x - Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(x-\sqrt{a+x^2}\right)^b \, dx &= \frac{1}{2}\text{Subst}\left(\int x^{-2+b}\left(a+x^2\right) \, dx, x, x-\sqrt{a+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right) \, dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{b-1}\left(\frac{\left(x-\sqrt{a+x^2}\right)^2}{b+1}+\frac{a}{b-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b,x]
[Out] ((x - Sqrt[a + x^2])^(-1 + b)*(a/(-1 + b) + (x - Sqrt[a + x^2])^2/(1 + b)))
/2
```

fricas [A] time = 1.17, size = 33, normalized size = 0.59

$$-\frac{\left(\sqrt{x^2+a}\,b+x\right)\left(x-\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="fricas")
[Out] -(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b,x)
[Out] int((x-(x^2+a)^(1/2))^b,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x-\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^b,x)
[Out] int((x - (a + x^2)^(1/2))^b, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{a+x^2}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**b, x)
```

3.19 $\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

```
[Out] (x+(x^2+a)^(1/2))^b/b
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2122, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx &= \text{Subst}\left(\int x^{-1+b}\,dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{\left(x+\sqrt{a+x^2}\right)^b}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

[Out] (x + Sqrt[a + x^2])^b/b

fricas [A] time = 1.09, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")

[Out] (x + sqrt(x^2 + a))^b/b

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

[Out] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)

mupad [B] time = 0.30, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)

[Out] (x + (a + x^2)^(1/2))^b/b

sympy [B] time = 2.99, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{a}\,a^{\frac{b}{2}}\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{bx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}}x\cosh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,b} - \frac{a^{\frac{b}{2}}x\sinh\left(-b \cdots\right.}{\cdots} \\ -\frac{a^{\frac{b}{2}}\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{b}{2}}\cosh\left(b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} - \frac{a^{\frac{b}{2}}x^2\sinh\left(-b\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{ab\sqrt{1+\frac{x^2}{a}}} + \frac{a^{\frac{b}{2}}x\cosh\left(-b\operatorname{as}\cdots\right.}{\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(a)*a**(b/2)*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(
b*x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(1 - b/2)/
(b**2*gamma(-b/2)) + a**(b/2)*x*cosh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a))
)/(sqrt(a)*b) - a**(b/2)*x*sinh(-b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sq
rt(a)*b*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (-a**(b/2)*sinh(-b*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asinh(x/
sqrt(a)))*gamma(1 - b/2)/(b**2*gamma(-b/2)) - a**(b/2)*x**2*sinh(-b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh(-b*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*b), True))
```

3.20 $\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] -(x-(x^2+a)^(1/2))^b/b
```

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2122, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx &= -\text{Subst}\left(\int x^{-1+b}\,dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

fricas [A] time = 0.62, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] -(x - sqrt(x^2 + a))^b/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
[Out] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

mupad [B] time = 0.30, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
[Out] -(x - (a + x^2)^(1/2))^b/b
```

sympy [A] time = 1.60, size = 36, normalized size = 1.80

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b} & \text{for } b \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

3.21 $\int \frac{1}{(a+be^{px})^2} dx$

Optimal. Leaf size=42

$$-\frac{\log(a+be^{px})}{a^2p}+\frac{x}{a^2}+\frac{1}{ap(a+be^{px})}$$

```
[Out] 1/a/(a+b*exp(p*x))/p+x/a^2-ln(a+b*exp(p*x))/a^2/p
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 44}

$$-\frac{\log(a+be^{px})}{a^2p}+\frac{x}{a^2}+\frac{1}{ap(a+be^{px})}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(p*x))^(-2),x]
```

```
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+be^{px})^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} dx, x, e^{px}\right)}{p}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx, x, e^{px}\right)}{p}\\
&= \frac{1}{a(a+be^{px})p}+\frac{x}{a^2}-\frac{\log(a+be^{px})}{a^2p}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.86

$$\frac{\frac{a}{a+be^{px}}-\log(a+be^{px})+px}{a^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(p*x))^(-2),x]
```

```
[Out] (a/(a + b*E^(p*x)) + p*x - Log[a + b*E^(p*x)])/(a^2*p)
```

fricas [A] time = 1.11, size = 52, normalized size = 1.24

$$\frac{bpxe^{(px)} + apx - \left(be^{(px)} + a\right)\log\left(be^{(px)} + a\right) + a}{a^2bpe^{(px)} + a^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(p*x) + a*p*x - (b*e^(p*x) + a)*log(b*e^(p*x) + a) + a)/(a^2*b*p*e
^(p*x) + a^3*p)
```

giac [A] time = 0.99, size = 47, normalized size = 1.12

$$\frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(px)}+a}+1\right|\right)}{a^2b} + \frac{1}{\left(be^{(px)}+a\right)ab}\right)}{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*e^(p*x) + a) + 1))/(a^2*b) + 1/((b*e^(p*x) + a)*a*b))/p
```

maple [A] time = 0.01, size = 48, normalized size = 1.14

$$\frac{1}{\left(b\,e^{px}+a\right)ap} - \frac{\ln\left(b\,e^{px}+a\right)}{a^2p} + \frac{\ln\left(e^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*exp(p*x))^2,x)
```

```
[Out] -ln(a+b*exp(p*x))/a^2/p+1/a/(a+b*exp(p*x))/p+1/p/a^2*ln(exp(p*x))
```

maxima [A] time = 0.59, size = 40, normalized size = 0.95

$$\frac{x}{a^2} + \frac{1}{\left(abe^{(px)} + a^2\right)p} - \frac{\log\left(be^{(px)} + a\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - log(b*e^(p*x) + a)/(a^2*p)
```

mupad [B] time = 0.41, size = 58, normalized size = 1.38

$$\frac{\frac{x}{a} + \frac{b\,x\,e^{p\,x}}{a^2} - \frac{b\,e^{p\,x}}{a^2\,p}}{a + b\,e^{p\,x}} - \frac{\ln\left(a + b\,e^{p\,x}\right)}{a^2\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*exp(p*x))^2,x)
```

```
[Out] (x/a + (b*x*exp(p*x))/a^2 - (b*exp(p*x))/(a^2*p))/(a + b*exp(p*x)) - log(a
+ b*exp(p*x))/(a^2*p)
```

sympy [A] time = 0.15, size = 36, normalized size = 0.86

$$\frac{1}{a^2p + abpe^{px}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))**2,x)
```

```
[Out] 1/(a**2*p + a*b*p*exp(p*x)) + x/a**2 - log(a/b + exp(p*x))/(a**2*p)
```

3.22 $\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

```
[Out] -1/2/a/(b+a*exp(2*p*x))/p
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2282, 261}

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/(2*a*(b + a*E^(2*p*x))*p)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{\left(b+ax^2\right)^2}\,dx, x, e^{px}\right)}{p}\\
&= -\frac{1}{2a\left(b+ae^{2px}\right)p}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$-\frac{1}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/2*1/(a*(b + a*E^(2*p*x))*p)
```

fricas [**A**] time = 1.25, size = 19, normalized size = 0.86

$$-\frac{1}{2\left(a^2pe^{(2px)} + abp\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
[Out] -1/2/(a^2*p*e^(2*p*x) + a*b*p)
```

giac [**A**] time = 0.78, size = 19, normalized size = 0.86

$$-\frac{1}{2\left(ae^{(2px)} + b\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
[Out] -1/2/((a*e^(2*p*x) + b)*a*p)
```

maple [**A**] time = 0.00, size = 21, normalized size = 0.95

$$-\frac{1}{2\left(a\,e^{2px} + b\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(p*x)+a*exp(p*x))^2,x)
[Out] -1/2/p/a/(a*exp(p*x)^2+b)
```

maxima [**A**] time = 0.60, size = 20, normalized size = 0.91

$$\frac{1}{2\left(b^2e^{(-2px)} + ab\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
[Out] 1/2/((b^2*e^(-2*p*x) + a*b)*p)
```

mupad [**B**] time = 0.39, size = 24, normalized size = 1.09

$$\frac{e^{2\,p\,x}}{2\,b\,p\,\left(b + a\,e^{2\,p\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*exp(p*x) + b*exp(-p*x))^2,x)
[Out] exp(2*p*x)/(2*b*p*(b + a*exp(2*p*x)))
```

sympy [**A**] time = 0.12, size = 20, normalized size = 0.91

$$\frac{1}{2abp + 2b^2pe^{-2px}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)
[Out] 1/(2*a*b*p + 2*b**2*p*exp(-2*p*x))
```

3.23 $\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx$

Optimal. Leaf size=62

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

```
[Out] 1/2*x/a/b/p-1/2*x/a/(b+a*exp(2*p*x))/p-1/4*ln(b+a*exp(2*p*x))/a/b/p^2
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2283, 2191, 2282, 36, 29, 31}

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2*p*x)]/(4*a*b*p^2
)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2283

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
```

```
[n, 0] && LinearQ[{v, w}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(be^{-px}+ae^{px})^2}\,dx &= \int \frac{e^{2px}x}{\left(b+ae^{2px}\right)^2}\,dx\\ &= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\int \frac{1}{b+ae^{2px}}\,dx}{2ap}\\ &= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx,x,e^{2px}\right)}{4ap^2}\\ &= -\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\text{Subst}\left(\int \frac{1}{b+ax}\,dx,x,e^{2px}\right)}{4bp^2}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2px}\right)}{4abp^2}\\ &= \frac{x}{2abp}-\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\log\left(b+ae^{2px}\right)}{4abp^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.79

$$\frac{\frac{2pxe^{2px}}{ae^{2px}+b}-\frac{\log\left(ae^{2px}+b\right)}{a}}{4bp^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b/E^(p*x) + a*E^(p*x))^2,x]
[Out] ((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4*b*p^2)
```

fricas [A] time = 0.74, size = 58, normalized size = 0.94

$$\frac{2\,apxe^{(2\,px)}-\left(ae^{(2\,px)}+b\right)\log\left(ae^{(2\,px)}+b\right)}{4\left(a^2bp^2e^{(2\,px)}+ab^2p^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
[Out] 1/4*(2*a*p*x*e^(2*p*x) - (a*e^(2*p*x) + b)*log(a*e^(2*p*x) + b))/(a^2*b*p^2
*e^(2*p*x) + a*b^2*p^2)
```

giac [A] time = 1.21, size = 74, normalized size = 1.19

$$\frac{2\,apxe^{(2\,px)}-ae^{(2\,px)}\log\left(-ae^{(2\,px)}-b\right)-b\log\left(-ae^{(2\,px)}-b\right)}{4\left(a^2bp^2e^{(2\,px)}+ab^2p^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
[Out] 1/4*(2*a*p*x*e^(2*p*x) - a*e^(2*p*x)*log(-a*e^(2*p*x) - b) - b*log(-a*e^(2*
p*x) - b))/(a^2*b*p^2*e^(2*p*x) + a*b^2*p^2)
```

maple [A] time = 0.01, size = 51, normalized size = 0.82

$$\frac{x\,\mathrm{e}^{2px}}{2\left(a\,\mathrm{e}^{2px}+b\right)bp}-\frac{\ln\left(a\,\mathrm{e}^{2px}+b\right)}{4ab\,p^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b/exp(p*x)+a*exp(p*x))^2,x)
```

```
[Out] -1/4/p^2/b/a*ln(a*exp(p*x)^2+b)+1/2/p*x*exp(p*x)^2/b/(a*exp(p*x)^2+b)
```

maxima [A] time = 0.59, size = 51, normalized size = 0.82

$$\frac{xe^{(2\,px)}}{2\left(abpe^{(2\,px)}+b^{2}p\right)}-\frac{\log\left(\frac{ae^{(2\,px)}+b}{a}\right)}{4\,abp^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x) + b)/a)/(a
*b*p^2)
```

mupad [B] time = 0.41, size = 47, normalized size = 0.76

$$\frac{x\,\mathrm{e}^{2\,p\,x}}{2\,b\,p\,\left(b+a\,\mathrm{e}^{2\,p\,x}\right)}-\frac{\ln\left(b+a\,\mathrm{e}^{2\,p\,x}\right)}{4\,a\,b\,p^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

```
[Out] (x*exp(2*p*x))/(2*b*p*(b + a*exp(2*p*x))) - log(b + a*exp(2*p*x))/(4*a*b*p^
2)
```

sympy [A] time = 0.18, size = 51, normalized size = 0.82

$$\frac{x}{2abp+2b^{2}pe^{-2px}}-\frac{x}{2abp}-\frac{\log\left(\frac{a}{b}+e^{-2px}\right)}{4abp^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] x/(2*a*b*p + 2*b**2*p*exp(-2*p*x)) - x/(2*a*b*p) - log(a/b + exp(-2*p*x))/(
4*a*b*p**2)
```

3.24 $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2} dx$

Optimal. Leaf size=86

$$\frac{\sqrt{x^2-x+1}\,(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

```
[Out] arctan((1+x)*2^(1/2)/(x^2-x+1)^(1/2))*2^(1/2)-1/6*arctanh(1/3*(1-x)*6^(1/2)
/(x^2-x+1)^(1/2))*6^(1/2)+(1+x)*(x^2-x+1)^(1/2)/(x^2+x+1)
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{x^2-x+1}\,(x+1)}{x^2+x+1}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{x^2-x+1}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x)
)/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[
6]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
```

```
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\left(1+x+x^2\right)^2}\,dx = \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{12}\int \frac{18-6x}{\sqrt{1-x+x^2}\left(1+x+x^2\right)}\,dx$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{48}\int \frac{24+24x}{\sqrt{1-x+x^2}\left(1+x+x^2\right)}\,dx - \frac{1}{48}\int \frac{-48}{\sqrt{1-x+x^2}}$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + 24\,\mathrm{Subst}\left(\int \frac{1}{1728-2x^2}\,dx, x, \frac{-24+24x}{\sqrt{1-x+x^2}}\right) + 288\,\mathrm{Sul}$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}(1+x)}{\sqrt{1-x+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}}$$

Mathematica [C] time = 2.52, size = 961, normalized size = 11.17

$$\frac{\sqrt{x^2-x+1}\,(x+1)}{x^2+x+1} + \frac{\left(7-i\sqrt{3}\right)\tan^{-1}\left(\frac{3\left(\left(-21-4i\sqrt{3}\right)x^4+14\left(7-2i\sqrt{3}\right)x^3+\left(-103-36i\right.\right.}{\left(84i-113\sqrt{3}\right)x^4+2\left(52\sqrt{3-3i\sqrt{3}}\,\sqrt{x^2-x+1}+21\sqrt{3}+138i\right)x^3+\left(52\sqrt{3-3i\sqrt{3}}\,\sqrt{x^2-x+1}-59\sqrt{3}-18\right.}\right.}{4\sqrt{3-3i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + ((7 - I*Sqrt[3])*ArcTan[(3*(-17
 - (64*I)*Sqrt[3] + (94 + (32*I)*Sqrt[3])*x + (-103 - (36*I)*Sqrt[3])*x^2 +
```

```
14*(7 - (2*I)*Sqrt[3])*x^3 + (-21 - (4*I)*Sqrt[3])*x^4))/(96*I + 67*Sqrt[3
] + (84*I - 113*Sqrt[3])*x^4 - 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]
 + 2*x*(132*I - 69*Sqrt[3] + 26*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
+ x^2*(-180*I - 59*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
+ 2*x^3*(138*I + 21*Sqrt[3] + 52*Sqrt[3 - (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2])
)])/(4*Sqrt[3 - (3*I)*Sqrt[3]]) - ((I/4)*(-7*I + Sqrt[3])*ArcTan[(3*(-17 +
(64*I)*Sqrt[3] + (94 - (32*I)*Sqrt[3])*x + (-103 + (36*I)*Sqrt[3])*x^2 + 14
*(7 + (2*I)*Sqrt[3])*x^3 + (-21 + (4*I)*Sqrt[3])*x^4))/(96*I - 67*Sqrt[3] +
 (84*I + 113*Sqrt[3])*x^4 + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2] +
x^2*(-180*I + 59*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) +
x*(264*I + 138*Sqrt[3] - 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]) - 2*
x^3*(-138*I + 21*Sqrt[3] + 52*Sqrt[3 + (3*I)*Sqrt[3]]*Sqrt[1 - x + x^2]))])
/Sqrt[3 + (3*I)*Sqrt[3]] - ((-7*I + Sqrt[3])*Log[16*(1 + x + x^2)^2])/(8*Sq
rt[3 + (3*I)*Sqrt[3]]) - ((7*I + Sqrt[3])*Log[16*(1 + x + x^2)^2])/(8*Sqrt[
3 - (3*I)*Sqrt[3]]) + ((7*I + Sqrt[3])*Log[(1 + x + x^2)*(11*I + 4*Sqrt[3]
+ (11*I + 4*Sqrt[3])*x^2 + (10*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2] - x
*(17*I + 4*Sqrt[3] + (8*I)*Sqrt[1 - I*Sqrt[3]]*Sqrt[1 - x + x^2]))])/(8*Sqr
t[3 - (3*I)*Sqrt[3]]) + ((-7*I + Sqrt[3])*Log[(1 + x + x^2)*(-11*I + 4*Sqrt
[3] + (-11*I + 4*Sqrt[3])*x^2 - (10*I)*Sqrt[1 + I*Sqrt[3]]*Sqrt[1 - x + x^2
] + x*(17*I - 4*Sqrt[3] + (8*I)*Sqrt[1 + I*Sqrt[3]]*Sqrt[1 - x + x^2]))])/(
8*Sqrt[3 + (3*I)*Sqrt[3]])
```

fricas [B] time = 1.01, size = 358, normalized size = 4.16

$$-\frac{8\sqrt{6}\sqrt{3}\left(x^2+x+1\right)\arctan\left(\frac{2}{3}\sqrt{6}\sqrt{3}(x-1)+\frac{2}{3}\sqrt{2x^2-\sqrt{x^2-x+1}\left(2x-\sqrt{6}+1\right)-\sqrt{6}(x+1)+4}\left(\sqrt{6}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(2/3*sqrt(6)*sqrt(3)*(x - 1) +
 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - sqrt(6)*(x + 1) +
 4)*(sqrt(6)*sqrt(3) + 3*sqrt(3)) - 2/3*sqrt(x^2 - x + 1)*(sqrt(6)*sqrt(3)
+ 3*sqrt(3)) + sqrt(3)*(2*x - 1)) + 8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(
2/3*sqrt(6)*sqrt(3)*(x - 1) + 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x + sqr
t(6) + 1) + sqrt(6)*(x + 1) + 4)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - 2/3*sqrt(x
^2 - x + 1)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - sqrt(3)*(2*x - 1)) - sqrt(6)*(x
^2 + x + 1)*log(12168*x^2 - 6084*sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + 60
84*sqrt(6)*(x + 1) + 24336) + sqrt(6)*(x^2 + x + 1)*log(12168*x^2 - 6084*sq
rt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - 6084*sqrt(6)*(x + 1) + 24336) - 12*x^
2 - 12*sqrt(x^2 - x + 1)*(x + 1) - 12*x - 12)/(x^2 + x + 1)
```

giac [B] time = 1.06, size = 304, normalized size = 3.53

$$-\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2x+\sqrt{6}-2\sqrt{x^2-x+1}+1}{\sqrt{3}+\sqrt{2}}\right)+\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2x-\sqrt{6}-2\sqrt{x^2-x+1}+1}{\sqrt{3}-\sqrt{2}}\right)+\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(6)*sqrt(3)*arctan(-(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)/(sqr
t(3) + sqrt(2))) + 1/3*sqrt(6)*sqrt(3)*arctan(-(2*x - sqrt(6) - 2*sqrt(x^2
- x + 1) + 1)/(sqrt(3) - sqrt(2))) + 1/12*sqrt(6)*log(4*(sqrt(6)*sqrt(3) +
3*sqrt(3))^2 + 36*(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)^2) - 1/12*sqrt(
6)*log(4*(sqrt(6)*sqrt(3) - 3*sqrt(3))^2 + 36*(2*x - sqrt(6) - 2*sqrt(x^2 -
 x + 1) + 1)^2) + ((x - sqrt(x^2 - x + 1))^3 + 4*(x - sqrt(x^2 - x + 1))^2
```

```
- 10*x + 10*sqrt(x^2 - x + 1) + 5)/((x - sqrt(x^2 - x + 1))^4 + 2*(x - sqrt
(x^2 - x + 1))^3 + (x - sqrt(x^2 - x + 1))^2 - 6*x + 6*sqrt(x^2 - x + 1) +
3)
```

maple [B] time = 0.04, size = 455, normalized size = 5.29

$$-\frac{-\frac{6\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,\sqrt{6}\,(x+1)^2\operatorname{arctanh}\left(\frac{\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,\sqrt{6}}{4}\right)}{(-x+1)^2}-2\sqrt{6}\,\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,\sqrt{6}}{4}\right)+\frac{9\sqrt{2}\,\sqrt{\frac{(x+1)^2}{(-x+1)^2}+3}\,(x+1)^2\cdots}{(-x\cdots}}{6\sqrt{\frac{\frac{(x+1)^2}{(-x+1)^2}+3}{\left(\frac{x+1}{-x+1}+1\right)^2}}\left(\frac{x+1}{-x+1}+1\right)\left(\frac{3(x+\cdots}{(-x+\cdots}\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x)
```

```
[Out] -1/6*(9*2^(1/2)*((x+1)^2/(-x+1)^2+3)^(1/2)*arctan(2*2^(1/2)/((x+1)^2/(-x+1)
^2+3)^(1/2)*(x+1)/(-x+1))*(x+1)^2/(-x+1)^2-6*((x+1)^2/(-x+1)^2+3)^(1/2)*arc
tanh(1/4*((x+1)^2/(-x+1)^2+3)^(1/2)*6^(1/2))*6^(1/2)*(x+1)^2/(-x+1)^2+3*2^(
1/2)*arctan(2*2^(1/2)/((x+1)^2/(-x+1)^2+3)^(1/2)*(x+1)/(-x+1))*((x+1)^2/(-x
+1)^2+3)^(1/2)-2*6^(1/2)*arctanh(1/4*((x+1)^2/(-x+1)^2+3)^(1/2)*6^(1/2))*((
x+1)^2/(-x+1)^2+3)^(1/2)-12*(x+1)^3/(-x+1)^3-36*(x+1)/(-x+1))/(((x+1)^2/(-x
+1)^2+3)/((x+1)/(-x+1)+1)^2)^(1/2)/((x+1)/(-x+1)+1)/(3*(x+1)^2/(-x+1)^2+1)+
1/2*((x+1)^2/(-x+1)^2+3)^(1/2)*(3*2^(1/2)*arctan(2*2^(1/2)/((x+1)^2/(-x+1)^
2+3)^(1/2)*(x+1)/(-x+1))-6^(1/2)*arctanh(1/4*((x+1)^2/(-x+1)^2+3)^(1/2)*6^(
1/2)))/((x+1)/(-x+1)+1)/(((x+1)^2/(-x+1)^2+3)/((x+1)/(-x+1)+1)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\,x^2-x+1}{\left(x^2+x+1\right)^2\sqrt{x^2-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2-x+1}{\sqrt{x^2-x+1}\,\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2),x)
```

```
[Out] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2-x+1}{\sqrt{x^2-x+1}\,\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2), x)
```

3.25 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=19

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

```
[Out] 2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2122, 30}

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

fricas [A] time = 1.02, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

giac [A] time = 0.79, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
[Out] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

mupad [B] time = 0.42, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/(a^2 + x^2)^(1/2),x)
[Out] 2*(x + (a^2 + x^2)^(1/2))^(1/2)
```

sympy [A] time = 0.23, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

3.26 $\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx$

Optimal. Leaf size=26

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

```
[Out] 2*(b*x+(b^2*x^2+a)^(1/2))^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2122, 30}

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, bx+\sqrt{a+b^2x^2}\right)}{b}\\ &= \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

fricas [A] time = 0.99, size = 22, normalized size = 0.85

$$\frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

giac [A] time = 0.98, size = 22, normalized size = 0.85

$$\frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+\sqrt{b^2x^2+a}}}{\sqrt{b^2x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

```
[Out] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+\sqrt{b^2x^2+a}}}{\sqrt{b^2x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

mupad [B] time = 0.51, size = 22, normalized size = 0.85

$$\frac{2\sqrt{\sqrt{b^2\,x^2+a}+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^2)^(1/2) + b*x)^(1/2)/(a + b^2*x^2)^(1/2),x)
```

```
[Out] (2*((a + b^2*x^2)^(1/2) + b*x)^(1/2))/b
```

sympy [A] time = 1.07, size = 27, normalized size = 1.04

$$\begin{cases} \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**(1/4), Tru
e))
```

3.27 $\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))/a^(3/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

Rubi [A] time = 0.22, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2120, 329, 212, 206, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTanh[Sqrt[x
+ Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2120

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + p + 1)*e^(p +
 1)*f^(2*m)), Subst[Int[x^(n - 2*m - p - 2)*(-(a*f^2) + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
```

```
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a} - \frac{2\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \sqrt{x+\sqrt{}}\right.}{a}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.20, size = 56, normalized size = 0.89

$$-\frac{2\left(\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + ArcTanh[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]]))/a^(3/2)
```

fricas [A] time = 1.09, size = 198, normalized size = 3.14

$$\left[-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a}}}{a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-(2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) - sqrt(a)*log((a^2 +
 sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + s
qrt(a^2 + x^2)))/x))/a^2, (2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x
^2))/a) - sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a - (sqrt(-a)*(a + x) - sqrt
(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x))/a^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{x+\sqrt{a^2+x^2}}\,\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)), x)
```

sympy [C] time = 1.47, size = 46, normalized size = 0.73

$$-\frac{\Gamma^2\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_3F_2\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4}\end{matrix}\right|\frac{a^2e^{i\pi}}{x^2}\right)}{\pi x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2*exp_polar
(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

3.28 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$

Optimal. Leaf size=82

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2119, 459, 329, 212, 206, 203}

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx &= \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}+\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}+\left(4a^2\right)\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}-(2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)-(2a)\,\text{Subst}\left(\int \frac{1}{a+ \right.\\ &= 2\sqrt{x+\sqrt{a^2+x^2}}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 127, normalized size = 1.55

$$-\frac{2\sqrt{a^2+x^2}\left(\sqrt{a^2+x^2}+x\right)\left(-\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)\right)}{x\left(\sqrt{a^2+x^2}+x\right)+a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] (-2*Sqrt[a^2 + x^2]*(x + Sqrt[a^2 + x^2])*(-Sqrt[x + Sqrt[a^2 + x^2]] + Sqr
t[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + Sqrt[a]*ArcTanh[Sqrt[x + S
qrt[a^2 + x^2]]/Sqrt[a]]))/(a^2 + x*(x + Sqrt[a^2 + x^2]))
```

fricas [A] time = 0.77, size = 216, normalized size = 2.63

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\,\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
```

```
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

maple [C] time = 0.01, size = 25, normalized size = 0.30

$$2\sqrt{2}\,\sqrt{x}\,\operatorname{hypergeom}\left(\left[-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4}\right],-\frac{a^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
[Out] int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

sympy [C] time = 1.32, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\,\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

3.29 $\int x^3 \log^3(2+x) \log(3+x) \, dx$

Optimal. Leaf size=606

$$-\frac{5609}{96} \text{PolyLog}(2,-x-2)-\frac{563}{8} \text{PolyLog}(3,-x-2)-\frac{195}{2} \text{PolyLog}(4,-x-2)-\frac{195}{4} \log^2(x+2) \text{PolyLog}(2,-x-2)+$$

```
[Out] -302177/1152*x+3/256*x^4+8029/2304*x^2-763/3456*x^3+3891/128*ln(3+x)+2069/1
44*ln(2+x)-43/12*ln(2+x)^2+377/64*(2+x)^2-71/216*(2+x)^3+3/256*(2+x)^4-5609
/96*polylog(2,-2-x)-563/8*polylog(3,-2-x)-195/2*polylog(4,-2-x)+1/2*x^3*ln(
2+x)^2*ln(3+x)-3/16*x^4*ln(2+x)^2*ln(3+x)+1/4*x^4*ln(2+x)^3*ln(3+x)-25*x*ln
(2+x)*ln(3+x)+13/4*x^2*ln(2+x)*ln(3+x)-7/12*x^3*ln(2+x)*ln(3+x)+3/32*x^4*ln
(2+x)*ln(3+x)+6*x*ln(2+x)^2*ln(3+x)-3/2*x^2*ln(2+x)^2*ln(3+x)-81/4*ln(2+x)^
3*ln(3+x)+563/8*ln(2+x)*polylog(2,-2-x)-195/4*ln(2+x)^2*polylog(2,-2-x)+195
/2*ln(2+x)*polylog(3,-2-x)-187/64*x^2*ln(2+x)+83/288*x^3*ln(2+x)-3/128*x^4*
ln(2+x)+6733/32*(2+x)*ln(2+x)-377/32*(2+x)^2*ln(2+x)+71/72*(2+x)^3*ln(2+x)-
3/64*(2+x)^4*ln(2+x)-17/48*x^3*ln(2+x)^2+3/64*x^4*ln(2+x)^2-1251/16*(2+x)*l
n(2+x)^2+273/32*(2+x)^2*ln(2+x)^2-3/4*(2+x)^3*ln(2+x)^2+3/64*(2+x)^4*ln(2+x
)^2+65/4*(2+x)*ln(2+x)^3-33/8*(2+x)^2*ln(2+x)^3+3/4*(2+x)^3*ln(2+x)^3-1/16*
(2+x)^4*ln(2+x)^3-115/48*x^2*ln(3+x)+37/144*x^3*ln(3+x)-3/128*x^4*ln(3+x)+4
15/12*(3+x)*ln(3+x)-4083/32*ln(2+x)*ln(3+x)+963/16*ln(2+x)^2*ln(3+x)
```

Rubi [A] time = 4.80, antiderivative size = 679, normalized size of antiderivative = 1.12, number of steps used = 359, number of rules used = 30, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.143, Rules used = {2439, 2416, 2389, 2296, 2295, 2401, 2390, 2305, 2304, 2396, 2433, 2374, 2383, 6589, 2411, 2346, 2302, 30, 2330, 2319, 43, 2334, 2301, 6742, 2430, 2393, 2391, 2394, 2395, 2398}

$$-\frac{5609}{96} \text{PolyLog}(2,-x-2)-\frac{563}{8} \text{PolyLog}(3,-x-2)-\frac{195}{2} \text{PolyLog}(4,-x-2)-\frac{195}{4} \log^2(x+2) \text{PolyLog}(2,-x-2)+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2
 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 + (2069*Log[2 + x])/14
4 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 + x])/288 - (3*x^4*Log[2 + x])/
128 + (6365*(2 + x)*Log[2 + x])/32 - (273*(2 + x)^2*Log[2 + x])/32 + ((2 +
x)^3*Log[2 + x])/2 - (3*(2 + x)^4*Log[2 + x])/128 + ((384*(2 + x) - 144*(2
+ x)^2 + 32*(2 + x)^3 - 3*(2 + x)^4 - 192*Log[2 + x])*Log[2 + x])/128 + (17
*(36*(2 + x) - 9*(2 + x)^2 + (2 + x)^3 - 24*Log[2 + x])*Log[2 + x])/72 + (4
3*Log[2 + x]^2)/12 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2 + x]^2)/64 - (
1251*(2 + x)*Log[2 + x]^2)/16 + (273*(2 + x)^2*Log[2 + x]^2)/32 - (3*(2 + x
)^3*Log[2 + x]^2)/4 + (3*(2 + x)^4*Log[2 + x]^2)/64 + (65*(2 + x)*Log[2 + x
]^3)/4 - (33*(2 + x)^2*Log[2 + x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2
 + x)^4*Log[2 + x]^3)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48
+ (37*x^3*Log[3 + x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x
])/12 - (4083*Log[2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 + x] + (13*
x^2*Log[2 + x]*Log[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3 + x])/12 + (3*x^4*Lo
g[2 + x]*Log[3 + x])/32 + (963*Log[2 + x]^2*Log[3 + x])/16 + 6*x*Log[2 + x]
^2*Log[3 + x] - (3*x^2*Log[2 + x]^2*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3
 + x])/2 - (3*x^4*Log[2 + x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x]
)/4 + (x^4*Log[2 + x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563
*Log[2 + x]*PolyLog[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 - x])/4
 - (563*PolyLog[3, -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -2 - x])/2 - (19
5*PolyLog[4, -2 - x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

```
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 2346

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3\log^3(2+x)\log(3+x)\,dx = \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int\frac{x^4\log^3(2+x)}{3+x}\,dx - \frac{3}{4}\int\frac{x^4\log^2(2+x)\mathrm{lo}}{2+x}$$

$$= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int\Big(-27\log^3(2+x) + 9x\log^3(2+x) - 3x^2\mathrm{l}$$

$$= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int x^3\log^3(2+x)\,dx + \frac{3}{4}\int x^2\log^3(2+x)\,dx$$

$$= 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x) + \frac{1}{2}x^3\log^2(2+x)\mathrm{lo}$$

$$= \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+$$

$$= -\frac{81}{4}(2+x)\log^2(2+x) + \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x)$$

$$= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{81}{4}(2$$

$$= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{765}{16}($$

$$= -\frac{765x}{8} + \frac{27}{32}(2+x)^2 - \frac{1}{6}(2+x)^3 + \frac{3}{512}(2+x)^4 + \frac{765}{8}(2+x)\log(2+x) -$$

$$= -\frac{857x}{8} + \frac{79}{32}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) + \frac{83}{288}$$

$$= -\frac{16463x}{96} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) +$$

$$= -\frac{213473x}{1152} + \frac{6013x^2}{2304} - \frac{763x^3}{3456} + \frac{3x^4}{256} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2$$

Mathematica [A] time = 0.26, size = 412, normalized size = 0.68

$$-224640\,\mathrm{PolyLog}(4,-x-2) - 24\left(4680\log^2(x+2) - 6756\log(x+2) + 5609\right)\mathrm{PolyLog}(2,-x-2) + 288(78$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-195984 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[2 + x] + 4
00008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Log[2 + x] - 162*x^4*L
og[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Log[2 + x]^2 + 11880*x^2*Log[2 +
 x]^2 - 1680*x^3*Log[2 + x]^2 + 216*x^4*Log[2 + x]^2 + 48384*Log[2 + x]^3 +
 15552*x*Log[2 + x]^3 - 2592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 - 144*
x^4*Log[2 + x]^3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*Log[3
+ x] + 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Log[3 + x
] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[3 + x] - 1344*x
^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[3 + x] + 138672*Log[2 + x
]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Log[3 + x] - 3456*x^2*Log[2 + x]^2*Lo
g[3 + x] + 1152*x^3*Log[2 + x]^2*Log[3 + x] - 432*x^4*Log[2 + x]^2*Log[3 +
x] - 46656*Log[2 + x]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3 + x] - 24*(
5609 - 6756*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x] + 288*(-563
+ 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 - x])/2304
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^3\log\left(x+3\right)\log\left(x+2\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(x + 3)*log(x + 2)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \log(x+3) \log(x+2)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(x + 3)*log(x + 2)^3, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^3 \ln(x+2)^3 \ln(x+3) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(x+2)^3*ln(x+3),x)
```

```
[Out] int(x^3*ln(x+2)^3*ln(x+3),x)
```

maxima [A] time = 0.56, size = 518, normalized size = 0.85

$$\frac{3}{128}x^4+\frac{1}{16}\left(4x^4\log(x+3)-x^4+4x^3-18x^2+108x-324\log(x+3)\right)\log(x+2)^3-\frac{65}{4}\log(x+3)\log(x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="maxima")
```

```
[Out] 3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x - 324*log
(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 + 195/4*log(x + 3)*log
(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9*x^4 - 70*x^3 + 495*x^2 - 6*(3
*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x + 3) + 4680*log(x + 3)*log(-x - 2) - 49
50*x + 4680*dilog(x + 3) + 5778*log(x + 3) + 6048*log(x + 2))*log(x + 2)^2
+ 195/4*dilog(x + 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2 + 563/
16*log(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*(780*lo
g(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4 - 296*x^3 - 187
20*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 - 39840*x - 67308*log(x + 2
))*log(x + 3) - 1/1152*(81*x^4 - 1036*x^3 + 56160*log(x + 3)*log(x + 2)^2 +
 112320*log(x + 3)*log(x + 2)*log(-x - 2) + 11418*x^2 - 12*(9*x^4 - 56*x^3
+ 312*x^2 + 4680*log(x + 2)^2 - 2400*x - 6756*log(x + 2))*log(x + 3) + 1123
20*dilog(x + 3)*log(x + 2) + 112320*dilog(-x - 2)*log(x + 2) - 81072*log(x
+ 3)*log(x + 2) + 72576*log(x + 2)^2 - 200004*x - 81072*dilog(-x - 2) + 146
988*log(x + 3) + 302016*log(x + 2) - 112320*polylog(3, -x - 2))*log(x + 2)
+ 563/8*dilog(-x - 2)*log(x + 2) - 5609/96*log(x + 3)*log(x + 2) + 1573/12*
log(x + 2)^2 - 279145/1152*x - 5609/96*dilog(-x - 2) + 17171/128*log(x + 3)
 + 14227/36*log(x + 2) - 195/2*polylog(4, -x - 2) - 563/8*polylog(3, -x - 2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \ln(x+2)^3 \ln(x+3) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(x + 2)^3*log(x + 3),x)
```

```
[Out] int(x^3*log(x + 2)^3*log(x + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

```
[Out] (x+(x^2+b)^(1/2))^a/a
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 30}

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
```

```
[Out] (x + Sqrt[b + x^2])^a/a
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2122

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), S
ubst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx &= \text{Subst}\left(\int x^{-1+a}\,dx, x, x+\sqrt{b+x^2}\right)\\ &= \frac{\left(x+\sqrt{b+x^2}\right)^a}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
```

```
[Out] (x + Sqrt[b + x^2])^a/a
```

fricas [A] time = 0.88, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + b))^a/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
[Out] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

mupad [B] time = 0.31, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (b + x^2)^(1/2))^a/(b + x^2)^(1/2),x)
[Out] (x + (b + x^2)^(1/2))^a/a
```

sympy [**B**] time = 2.58, size = 311, normalized size = 18.29

$$\begin{cases} -\frac{\sqrt{b}\,b^{\frac{a}{2}}\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ax\sqrt{\frac{b}{x^2}+1}} + \frac{b^{\frac{a}{2}}x\cosh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} - \frac{b^{\frac{a}{2}}x\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}\sqrt{\frac{b}{x^2}+1}} - \frac{2b^{\frac{a}{2}}\cosh\left(\right.}{} \\ -\frac{b^{\frac{a}{2}}\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{1+\frac{x^2}{b}}} - \frac{b^{\frac{a}{2}}x^2\sinh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab\sqrt{1+\frac{x^2}{b}}} + \frac{b^{\frac{a}{2}}x\cosh\left(-a\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)+\operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} - \frac{2b^{\frac{a}{2}}\cosh\left(a\right.}{a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*b**(a/2)*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(
a*x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(
b)))/(a*sqrt(b)) - b**(a/2)*x*sinh(-a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/
(a*sqrt(b)*sqrt(b/x**2 + 1)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(1
- a/2)/(a**2*gamma(-a/2)), Abs(x**2/b) > 1), (-b**(a/2)*sinh(-a*asinh(x/sqr
t(b)) + asinh(x/sqrt(b)))/(a*sqrt(1 + x**2/b)) - b**(a/2)*x**2*sinh(-a*asin
h(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*cosh(-
a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(a/2)*cosh(a*asin
h(x/sqrt(b)))*gamma(1 - a/2)/(a**2*gamma(-a/2)), True))
```

3.31 $\int \left(x+\sqrt{b+x^2}\right)^a \, dx$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

```
[Out] -1/2*b*(x+(x^2+b)^(1/2))^(-1+a)/(1-a)+1/2*(x+(x^2+b)^(1/2))^(1+a)/(1+a)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2117, 14}

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] -(b*(x + Sqrt[b + x^2])^(-1 + a))/(2*(1 - a)) + (x + Sqrt[b + x^2])^(1 + a)
/(2*(1 + a))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(x+\sqrt{b+x^2}\right)^a \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^{-2+a}\left(b+x^2\right) \, dx,x,x+\sqrt{b+x^2}\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(bx^{-2+a}+x^a\right) \, dx,x,x+\sqrt{b+x^2}\right)\\
&= -\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)}+\frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 0.88

$$\frac{1}{2}\left(\sqrt{b+x^2}+x\right)^{a-1}\left(\frac{\left(\sqrt{b+x^2}+x\right)^2}{a+1}+\frac{b}{a-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] ((x + Sqrt[b + x^2])^(-1 + a)*(b/(-1 + a) + (x + Sqrt[b + x^2])^2/(1 + a)))
/2
```

fricas [A] time = 1.03, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+b}\,a-x\right)\left(x+\sqrt{x^2+b}\right)^a}{a^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x+\sqrt{x^2+b}\right)^a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

maple [B] time = 0.03, size = 120, normalized size = 2.31

$$\frac{\left(\frac{8\sqrt{\pi}\left(a+\frac{ab}{x^2}-1\right)b^{-\frac{a}{2}-\frac{1}{2}}x^{a+1}\left(\sqrt{\frac{b}{x^2}+1}+1\right)^{a-1}}{(a+1)(2a-2)a}+\frac{4\sqrt{\pi}\,\sqrt{\frac{b}{x^2}+1}\,b^{-\frac{a}{2}-\frac{1}{2}}x^{a+1}\left(\sqrt{\frac{b}{x^2}+1}+1\right)^{a-1}}{(a+1)a}\right)a\,b^{\frac{a}{2}+\frac{1}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a,x)
```

```
[Out] 1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(a+1)/a*x^(a+1)*b^(-1/2*a-1/2)*(a*
b/x^2+a-1)/(2*a-2)*((1+1/x^2*b)^(1/2)+1)^(a-1)+4*Pi^(1/2)/(a+1)/a*x^(a+1)*b
^(-1/2*a-1/2)*(1+1/x^2*b)^(1/2)*((1+1/x^2*b)^(1/2)+1)^(a-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x+\sqrt{x^2+b}\right)^a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x+\sqrt{x^2+b}\right)^a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (b + x^2)^(1/2))^a,x)
```

```
[Out] int((x + (b + x^2)^(1/2))^a, x)
```

<u>**sympy**</u> **[B]** time = 2.71, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a,x)
```

```
[Out] Piecewise((-a**2*b**(9/2)*b**(a/2)*x*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b
)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamm
a(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) -
a**2*b**(7/2)*b**(a/2)*x**3*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)))*gamma
(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(9/2
)*b**(a/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(7/2)*b**(a/2)*x**3*cosh(a*asinh(x/s
qrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2
*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2
)) + 2*a*b**5*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1
- a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**5*
b**(a/2)*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x
**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 -
a/2)) - 2*a*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + a
sinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b*
*(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*ga
mma(1 - a/2)) + 4*a*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sq
rt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x*
*2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a
/2)) - 2*a*b**4*b**(a/2)*x**2*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a
*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma
(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2)
 - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*a*b**3*b**(a/2)*x**4*cosh(a*asinh(x/
sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2)
 + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(
7/2)*x**2*gamma(1 - a/2)) - 2*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*as
inh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(
b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*
a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*
x**2*gamma(1 - a/2)) - 2*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) + 2*b**3*b**(a/2)*x**4*cosh(a*asinh(x/sqrt(b)) +
 asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*
b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*
gamma(1 - a/2)), Abs(x**2/b) > 1), (-a**2*b**3*b**(a/2)*sqrt(1 + x**2/b)*si
nh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(
5/2)*gamma(1 - a/2)) - 2*a*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asin
h(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 -
a/2) - 2*b**(5/2)*gamma(1 - a/2)) + a*b**(5/2)*b**(a/2)*x*cosh(a*asinh(x/sq
rt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 -
a/2)) + 2*a*b**3*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma
(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) + 2*
a*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 -
a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) - 2*b**(5
```

```
/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))
*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)
) + 2*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(
1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)), True
))
```

3.32 $\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx$

Optimal. Leaf size=34

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

```
[Out] 1/6*x^(1+a)*(6+3*x^a+2*x^(2*a))^(1+1/a)/(1+a)
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1594, 1747}

$$\frac{x^{a+1}\left(3x^a+2x^{2a}+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1747

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.)*
((d_) + (e_.)*(x_)^(n_.) + (f_.)*(x_)^(n2_.)), x_Symbol] :> Simp[(d*(g*x)^(
m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*g*(m + 1)), x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*e*(m + 1) - b*d*(m + n*
(p + 1) + 1), 0] && EqQ[a*f*(m + 1) - c*d*(m + 2*n*(p + 1) + 1), 0] && NeQ[
m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx &= \int x^a \left(1+x^a+x^{2a}\right)\left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} dx\\ &= \frac{x^{1+a}\left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6(1+a)}\end{aligned}$$

Mathematica [A] time = 0.13, size = 33, normalized size = 0.97

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6a+6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6 + 6*a)
```

fricas [A] time = 0.94, size = 48, normalized size = 1.41

$$\frac{\left(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a}\right)\left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)}\left(x^{3\,a}+x^{2\,a}+x^{a}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a), x)
```

maple [A] time = 0.05, size = 44, normalized size = 1.29

$$\frac{\left(3x^{a}+2x^{2a}+6\right)x\,x^{a}\left(3x^{a}+2x^{2a}+6\right)^{\frac{1}{a}}}{6a+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x)
```

```
[Out] 1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(a+1)*(6+3*x^a+2*(x^a)^2)^(1/a)
```

maxima [A] time = 0.90, size = 48, normalized size = 1.41

$$\frac{\left(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a}\right)\left(2\,x^{2\,a}+3\,x^{a}+6\right)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="max
ima")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(x^{a}+x^{2\,a}+x^{3\,a}\right)\,{\left(3\,x^{a}+2\,x^{2\,a}+6\right)}^{1/a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a),x)
```

```
[Out] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

```
[Out] Timed out
```

3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

Optimal. Leaf size=58

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))+1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 -
 (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^2\right)$$
$$= -\frac{\log(x)}{2} - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)$$
$$= -\frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)$$
$$= \frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

Mathematica [A] time = 0.01, size = 57, normalized size = 0.98

$$\frac{1}{2}\left(\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right) + \sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right) - \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - Log[x] + (3*Log[1 - (1 -
 x^2)^(1/3)])/2)/2
```

fricas [A] time = 0.95, size = 64, normalized size = 1.10

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}} + \left(-x^2+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((-
x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

giac [A] time = 0.92, size = 64, normalized size = 1.10

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}} + \left(-x^2+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

maple [C] time = 0.10, size = 65, normalized size = 1.12

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\pi\sqrt{3}\,x^2\,\text{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^2\right)}{9\Gamma\left(\frac{2}{3}\right)} + \frac{2\left(2\ln(x) - \frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3),x)
```

[Out] 1/4/Pi*3^(1/2)*GAMMA(2/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1,1,4/3
],[2,2],x^2)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(
2/3))

maxima [A] time = 1.09, size = 62, normalized size = 1.07

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="maxima")

[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)

mupad [B] time = 0.54, size = 86, normalized size = 1.48

$$\frac{\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-9\left(-\frac{1}{4}+\frac{\sqrt{3}\,1i}{4}\right)^2\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1i}{4}\right)-\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-9\left(\frac{1}{4}+\frac{\sqrt{3}\,1i}{4}\right)^2\right)\left(\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(1 - x^2)^(1/3)),x)

[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/4 - 9*((3^(1/2
)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4) - log((9*(1 - x^2)^(1/3))/4 - 9*((
3^(1/2)*1i)/4 + 1/4)^2)*((3^(1/2)*1i)/4 + 1/4)

sympy [C] time = 0.94, size = 36, normalized size = 0.62

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x**2+1)**(1/3),x)

[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-2))/(2*x**(2/3)*gam
ma(4/3))

3.34 $\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))-1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 57, 618, 204, 31}

$$\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1
- (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^2\right)$$
$$= -\frac{\log(x)}{2} - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)$$
$$= -\frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right) + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)$$
$$= -\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

Mathematica [A] time = 0.01, size = 81, normalized size = 1.40

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{1}{4}\log\left(\left(1-x^2\right)^{2/3} + \sqrt[3]{1-x^2} + 1\right) - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]) + Log[1 - (1 - x^2)^
(1/3)]/2 - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)]/4
```

fricas [A] time = 0.71, size = 64, normalized size = 1.10

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}} + \left(-x^2+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((
-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

giac [A] time = 1.09, size = 64, normalized size = 1.10

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}} + \left(-x^2+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

maple [C] time = 0.10, size = 48, normalized size = 0.83

$$\frac{\frac{2\Gamma\left(\frac{2}{3}\right)x^2\,\text{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^2\right)}{3} + \left(2\ln(x) + \frac{\pi\sqrt{3}}{6} - \frac{3\ln(3)}{2} + i\pi\right)\Gamma\left(\frac{2}{3}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(2/3),x)
```

[Out] 1/2/GAMMA(2/3)*(2/3*GAMMA(2/3)*x^2*hypergeom([1,1,5/3],[2,2],x^2)+(1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3))

maxima [A] time = 1.30, size = 62, normalized size = 1.07

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="maxima")

[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)

mupad [B] time = 0.46, size = 76, normalized size = 1.31

$$\frac{\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{2}+\frac{9}{4}-\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\left(1-x^2\right)^{1/3}}{2}+\frac{9}{4}+\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(1 - x^2)^(2/3)),x)

[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/2 - (3^(1/2)*9i)/4 + 9/4)*((3^(1/2)*1i)/4 - 1/4) - log((3^(1/2)*9i)/4 + (9*(1 - x^2)^(1/3))/2 + 9/4)*((3^(1/2)*1i)/4 + 1/4)

sympy [C] time = 0.98, size = 37, normalized size = 0.64

$$-\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x**2+1)**(2/3),x)

[Out] -exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**(-2))/(2*x**(4/3)*gamma(5/3))

3.35 $\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/2*ln(x+(-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(
1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {239}

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(-1/3),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^
(1/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [A] time = 0.04, size = 86, normalized size = 1.76

$$\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(-\frac{x}{\sqrt[3]{1-x^3}}+\frac{x^2}{\left(1-x^3\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(-1/3),x]
```

```
[Out] ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3
)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3
```

fricas [B] time = 0.90, size = 82, normalized size = 1.67

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{3}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{6}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*
log((x + (-x^3 + 1)^(1/3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 +
 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(-1/3), x)
```

maple [C] time = 0.09, size = 12, normalized size = 0.24

$$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3),x)
```

```
[Out] x*hypergeom([1/3,1/3],[4/3],x^3)
```

maxima [A] time = 1.36, size = 78, normalized size = 1.59

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)+\frac{1}{3}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{6}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3*log((-x^3
 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1
)
```

mupad [B] time = 0.33, size = 10, normalized size = 0.20

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^3)^(1/3),x)
```

```
[Out] x*hypergeom([1/3, 1/3], 4/3, x^3)
```

sympy [C] time = 0.89, size = 29, normalized size = 0.59

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3
))
```

3.36 $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(
1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 55, 618, 204, 31}

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)$$
$$= -\frac{\log(x)}{2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)$$
$$= -\frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)$$
$$= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

fricas [A] time = 1.11, size = 64, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

giac [A] time = 0.82, size = 63, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

maple [C] time = 0.10, size = 65, normalized size = 1.18

$$\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\pi\sqrt{3}\,x^3\,\text{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^3\right)}{9\Gamma\left(\frac{2}{3}\right)} + \frac{2\left(3\ln(x)-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3),x)
```

```
[Out] 1/6/Pi*3^(1/2)*GAMMA(2/3)*(2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3
],[2,2],x^3)+2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(
2/3))
```

maxima [A] time = 1.21, size = 62, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

mupad [B] time = 0.51, size = 80, normalized size = 1.45

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2\right)\left(\frac{1}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(1/3)),x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6)
```

sympy [C] time = 0.92, size = 32, normalized size = 0.58

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{1}{x^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-3))/(3*x*gamma(4/3)
)
```

3.37 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=97

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

```
[Out] -1/8*ln((1-x)*(1+x)^2)*2^(2/3)+3/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-
1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2148}

$$\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/
3)) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)])/(4*2^(1/3))
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((1 + x)*(1 - x^3)^(1/3)), x]
```

fricas [B] time = 7.45, size = 301, normalized size = 3.10

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(13\,x^6+2\,x^5+19\,x^4-4\,x^3+19\,x^2+2\,x+13\right)-4\,\sqrt{2}\left(5\,x^5-5\,x^4+6\,x^3-6\,x^2+\right.\right.}{6\left(3\,x^6-18\,x^5-3\,x^4-28\,x^3-3\,x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(13*x^6 + 2*x^5 +
19*x^4 - 4*x^3 + 19*x^2 + 2*x + 13) - 4*sqrt(2)*(5*x^5 - 5*x^4 + 6*x^3 - 6*
x^2 + 5*x - 5)*(-x^3 + 1)^(1/3) + 16*2^(1/6)*(x^4 + 2*x^3 + 2*x^2 + 2*x + 1
)*(-x^3 + 1)^(2/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) -
 1/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x^4 +
6*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2
 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)*(-x^3
+ 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

maple [C] time = 8.06, size = 1143, normalized size = 11.78

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] 1/4*RootOf(_Z^3-4)*ln(-(8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+8*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+26*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+9*(-x^3+1)^(1/3)*RootOf(_Z^3-4
)^2*x-26*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
*RootOf(_Z^3-4)-9*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+28*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-21*RootOf(_Z^3-4)*x^2+52*(-x^3+1)^(2/3)+2
4*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-18*RootOf(_Z^3-4)*x
+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-21*RootOf(_Z^3-4))/
(x+1)^2)-1/4*ln((8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)*RootOf(_Z^3-4)^2+20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)^2*RootOf(_Z^3-4)^2*x+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)*RootOf(_Z^3-4)^3*x-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*R
ootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x-13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x
+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*Root
Of(_Z^3-4)+13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2-70*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-21*RootOf(_Z^3-4)*x^2-36*(-x^3+1)^(2/3)-20*Ro
otOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-6*RootOf(_Z^3-4)*x-70*R
ootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-21*RootOf(_Z^3-4))/(x+1)
^2)*RootOf(_Z^3-4)-1/2*ln((8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
```

```
otOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x-13*(-x^3+1)^(1/3)*RootOf(_
Z^3-4)^2*x+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*
_Z^2)*RootOf(_Z^3-4)+13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2-70*RootOf(RootOf(_Z
^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-21*RootOf(_Z^3-4)*x^2-36*(-x^3+1)^(
2/3)-20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-6*RootOf(_Z^3
-4)*x-70*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-21*RootOf(_Z^3
-4))/(x+1)^2)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^3\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

3.38 $\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+\right.}{4\sqrt[3]{2}}$$

```
[Out] 1/8*ln((1-x)*(1+x)^2)*2^(2/3)+1/2*ln(x+(-x^3+1)^(1/3))-3/8*ln(-1+x+2^(2/3)*
(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1
/2)+1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3
)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2152, 239, 2148}

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(x+\right.}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3
)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 +
 x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(
1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2152

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]
[Out] Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x}{(x+1)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(-x^3+1)^(1/3),x)
[Out] int(x/(x+1)/(-x^3+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(1-x^3\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

3.39 $\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx$

Optimal. Leaf size=110

$$\frac{3\log\left(-2^{2/3}\sqrt[3]{x^2-3x+2}-x+2\right)}{4\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}(2-x)}{\sqrt{3}\sqrt[3]{x^2-3x+2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log(2-x)}{4\sqrt[3]{2}}-\frac{\log(x)}{2\sqrt[3]{2}}$$

```
[Out] -1/8*ln(2-x)*2^(2/3)-1/4*ln(x)*2^(2/3)+3/8*ln(2-x-2^(2/3)*(x^2-3*x+2)^(1/3)
)*2^(2/3)+1/4*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-x)/(x^2-3*x+2)^(1/3)*3^(1/
2))*3^(1/2)*2^(2/3)
```

Rubi [A] time = 0.02, antiderivative size = 176, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {755, 123}

$$\frac{3\sqrt[3]{x-2}\sqrt[3]{x-1}\log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{x-1}\right)}{4\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\log(x)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt{3}\sqrt[3]{x-2}\sqrt[3]{x-1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3}\sqrt[3]{x-1}}\right)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-2 + x
)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3)) + (3*
(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)*(-1 +
 x)^(1/3)])/(4*2^(1/3)*(2 - 3*x + x^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^(1
/3)*Log[x])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3))
```

Rule 123

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[(b*(b*e - a*f))/(b*c - a*d)^2, 3]}, -Sim
p[Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[(Sqrt[3]*ArcTan[1/Sqrt[3] + (
2*q*(c + d*x)^(2/3))/(Sqrt[3]*(e + f*x)^(1/3))])/(2*q*(b*c - a*d)), x] + Si
mp[(3*Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)])/(4*q*(b*c - a*d)), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rule 755

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[((b + q + 2*c*x)^(1/3)*(b - q
+ 2*c*x)^(1/3))/(a + b*x + c*x^2)^(1/3), Int[1/((d + e*x)*(b + q + 2*c*x)^(
1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[c^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx &= \frac{\left(\sqrt[3]{-4+2x}\sqrt[3]{-2+2x}\right)\int \frac{1}{x\sqrt[3]{-4+2x}\sqrt[3]{-2+2x}}\,dx}{\sqrt[3]{2-3x+x^2}}\\ &= -\frac{\sqrt{3}\sqrt[3]{-2+x}\sqrt[3]{-1+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(-2+x)^{2/3}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{2\sqrt[3]{2}\sqrt[3]{2-3x+x^2}}+\frac{3\sqrt[3]{-2+x}\sqrt[3]{-1+x}\log\left(-\frac{(-2+x)^{2/3}}{\sqrt[3]{2}}-\right.}{4\sqrt[3]{2}\sqrt[3]{2-3x+x^2}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.54

$$-\frac{3\sqrt[3]{1-\frac{2}{x}}\sqrt[3]{1-\frac{1}{x}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{1}{x},\frac{2}{x}\right)}{2\sqrt[3]{x^2-3x+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (-3*(1 - 2/x)^(1/3)*(1 - x^(-1))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x^(-1),
 2/x])/(2*(2 - 3*x + x^2)^(1/3))
```

fricas [B] time = 4.94, size = 277, normalized size = 2.52

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^6+36\,x^5-612\,x^4+2880\,x^3-5760\,x^2+5184\,x-1728\right)+12\sqrt{2}\left(x^5-38\,x^4\right.\right.}{6\left(x^6-108\,x^5+972\,x^4-3456\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 36*x^5 - 6
12*x^4 + 2880*x^3 - 5760*x^2 + 5184*x - 1728) + 12*sqrt(2)*(x^5 - 38*x^4 +
252*x^3 - 648*x^2 + 720*x - 288)*(x^2 - 3*x + 2)^(1/3) + 48*2^(1/6)*(x^4 -
6*x^3 + 6*x^2)*(x^2 - 3*x + 2)^(2/3))/(x^6 - 108*x^5 + 972*x^4 - 3456*x^3 +
 6048*x^2 - 5184*x + 1728)) + 1/12*2^(2/3)*log((2^(2/3)*x^2 + 6*2^(1/3)*(x^
2 - 3*x + 2)^(1/3)*(x - 2) + 12*(x^2 - 3*x + 2)^(2/3))/x^2) - 1/24*2^(2/3)*
log((12*2^(2/3)*(x^2 - 3*x + 2)^(2/3)*(x^2 - 6*x + 6) + 2^(1/3)*(x^4 - 36*x
^3 + 180*x^2 - 288*x + 144) - 6*(x^3 - 14*x^2 + 36*x - 24)*(x^2 - 3*x + 2)^
(1/3))/x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2-3\,x+2\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

maple [C] time = 8.32, size = 1069, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-3*x+2)^(1/3),x)
```

```
[Out] 1/4*RootOf(_Z^3-4)*ln(-(68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z
^2)*RootOf(_Z^3-4)^3*x^2+112*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*
_Z^2)^2*RootOf(_Z^3-4)^2*x^2-306*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)*RootOf(_Z^3-4)^3*x-504*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-216*(x^2-3*x+2)^(2/3)*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+306*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3+504*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-108*(x^2-3*x+2)^(1/3)*RootOf(_
```

```
Z^3-4)^2*x+258*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)*RootOf(_Z^3-4)*x+216*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2-516*(x^2-
3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^
3-4)+119*RootOf(_Z^3-4)*x^2+196*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+4*_Z^2)*x^2-1020*RootOf(_Z^3-4)*x-1680*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+4*_Z^2)*x-948*(x^2-3*x+2)^(2/3)+1020*RootOf(_Z^3-4)+1680*RootOf(Ro
otOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)+1/2*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln((28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2+68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2-126*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-306*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-108*(x^2-3*x+2)^(2/3)*RootOf(RootOf
(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+126*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3+306*RootOf(RootOf(_Z^
3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-54*(x^2-3*x+2)^(1/3)*
RootOf(_Z^3-4)^2*x-237*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+108*(x^2-3*x+2)^(1/3)*RootOf(_Z^3-4)^2+4
74*(x^2-3*x+2)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*Ro
otOf(_Z^3-4)+7*RootOf(_Z^3-4)*x^2+17*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*x^2+168*RootOf(_Z^3-4)*x+408*RootOf(RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+4*_Z^2)*x+258*(x^2-3*x+2)^(2/3)-168*RootOf(_Z^3-4)-408*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2 - 3\,x + 2\right)^{\frac{1}{3}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\left(x^2 - 3\,x + 2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 3*x + 2)^(1/3)),x)
[Out] int(1/(x*(x^2 - 3*x + 2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{(x-2)\,(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-3*x+2)**(1/3),x)
[Out] Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

3.40 $\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx$

Optimal. Leaf size=81

$$-\frac{3}{4}\log\left(\sqrt[3]{x^3-3x^2+7x-5}-x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{x^3-3x^2+7x-5}}+\frac{1}{\sqrt{3}}\right)+\frac{1}{4}\log(1-x)$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+(x^3-3*x^2+7*x-5)^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*
(-1+x)/(x^3-3*x^2+7*x-5)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2067, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{(x-1)^2+4}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3+4(x-1)}}-\frac{3\sqrt[3]{(x-1)^2+4}\sqrt[3]{x-1}\log\left((x-1)^{2/3}-\sqrt[3]{(x-1)^2+4}\right)}{4\sqrt[3]{(x-1)^3+4(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (Sqrt[3]*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)^(2/3
))/(4 + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(4*(-1 + x) + (-1 + x)^3)^(1/3)) -
(3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(4 + (-1 + x)^2)^(1/3) + (-1
+ x)^(2/3)])/(4*(4*(-1 + x) + (-1 + x)^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
```

```
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{4x+x^3}}\,dx, x, -1+x\right)$$
$$= \frac{\left(\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{4+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}$$
$$= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{4+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}$$
$$= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{4+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{4(-1+x)+(-1+x)^3}}$$
$$= \frac{\sqrt{3}\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{4+(-1+x)^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-4(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{4+(-1+x)^2}\sqrt[3]{-1+x}\log}{4\sqrt[3]{-4(1-x}}$$

Mathematica [C] time = 0.01, size = 85, normalized size = 1.05

$$\frac{3\sqrt[3]{ix+(2-i)}\sqrt[3]{i(x-1)}(x-(1-2i))F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{1}{4}i(x-(1-2i)),-\frac{1}{2}i(x-(1-2i))\right)}{4\sqrt[3]{x^3-3x^2+7x-5}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((2 - I) + I*x)^(1/3)*(I*(-1 + x))^(1/3)*((-1 + 2*I) + x)*AppellF1[2/3,
1/3, 1/3, 5/3, (-1/4*I)*((-1 + 2*I) + x), (-1/2*I)*((-1 + 2*I) + x)])/(4*(-
5 + 7*x - 3*x^2 + x^3)^(1/3))
```

fricas [A] time = 1.75, size = 120, normalized size = 1.48

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{22791076\sqrt{3}\left(x^3-3x^2+7x-5\right)^{\frac{1}{3}}(x-1)+\sqrt{3}\left(20389537x^2-40779074x+53222437\right)+179}{7204617x^2-14409234x-20666867}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan((22791076*sqrt(3)*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1)
 + sqrt(3)*(20389537*x^2 - 40779074*x + 53222437) + 17987998*sqrt(3)*(x^3 -
 3*x^2 + 7*x - 5)^(2/3))/(7204617*x^2 - 14409234*x - 20666867)) - 1/4*log(3
*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1) - 3*(x^3 - 3*x^2 + 7*x - 5)^(2/3) +
4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-3x^2+7x-5\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

maple [C] time = 1.35, size = 653, normalized size = 8.06

$$-\frac{\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(-304x^2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2-320x^2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)+608x\,\mathrm{RootO}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-3*x^2+7*x-5)^(1/3),x)
```

```
[Out] 1/2*RootOf(_Z^2-_Z+1)*ln(-304*RootOf(_Z^2-_Z+1)^2*x^2+624*RootOf(_Z^2-_Z+1)
*(x^3-3*x^2+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x+60
8*RootOf(_Z^2-_Z+1)^2*x+928*RootOf(_Z^2-_Z+1)*x^2+51*(x^3-3*x^2+7*x-5)^(2/3
)-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)+51*(x^3-3*x^2+7*x-5)^(1/3)*
x-1856*RootOf(_Z^2-_Z+1)*x-253*x^2-51*(x^3-3*x^2+7*x-5)^(1/3)+2356*RootOf(_
Z^2-_Z+1)+506*x-713)-1/2*ln(-304*RootOf(_Z^2-_Z+1)^2*x^2-624*RootOf(_Z^2-_Z
+1)*(x^3-3*x^2+7*x-5)^(2/3)-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x
+608*RootOf(_Z^2-_Z+1)^2*x-320*RootOf(_Z^2-_Z+1)*x^2+675*(x^3-3*x^2+7*x-5)^
(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)+675*(x^3-3*x^2+7*x-5)^(
1/3)*x+640*RootOf(_Z^2-_Z+1)*x+371*x^2-675*(x^3-3*x^2+7*x-5)^(1/3)-2356*Roo
tOf(_Z^2-_Z+1)-742*x+1643)*RootOf(_Z^2-_Z+1)+1/2*ln(-304*RootOf(_Z^2-_Z+1)^
2*x^2-624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(2/3)-624*RootOf(_Z^2-_Z+1)*(
x^3-3*x^2+7*x-5)^(1/3)*x+608*RootOf(_Z^2-_Z+1)^2*x-320*RootOf(_Z^2-_Z+1)*x^
2+675*(x^3-3*x^2+7*x-5)^(2/3)+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)
+675*(x^3-3*x^2+7*x-5)^(1/3)*x+640*RootOf(_Z^2-_Z+1)*x+371*x^2-675*(x^3-3*x
^2+7*x-5)^(1/3)-2356*RootOf(_Z^2-_Z+1)-742*x+1643)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3-3\,x^2+7\,x-5\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3-3\,x^2+7\,x-5\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3),x)
```

```
[Out] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^3-3x^2+7x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)
```

```
[Out] Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)
```

3.41 $\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx$

Optimal. Leaf size=66

$$-\frac{3}{4}\log\left(\sqrt[3]{x\left(x^2-q\right)}-x\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x}{\sqrt{3}\sqrt[3]{x\left(x^2-q\right)}}+\frac{1}{\sqrt{3}}\right)+\frac{\log(x)}{4}$$

```
[Out] 1/4*ln(x)-3/4*ln(-x+(x*(x^2-q))^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*x/(x*(x^2
-q))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1979, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{x^2-q}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}}-\frac{3\sqrt[3]{x}\sqrt[3]{x^2-q}\log\left(x^{2/3}-\sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-q + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/3))
/Sqrt[3]])/(2*(-(q*x) + x^3)^(1/3)) - (3*x^(1/3)*(-q + x^2)^(1/3)*Log[x^(2/
3) - (-q + x^2)^(1/3)])/(4*(-(q*x) + x^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx = \int \frac{1}{\sqrt[3]{-qx+x^3}}\,dx$$
$$= \frac{\left(\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\int \frac{1}{\sqrt[3]{x}\sqrt[3]{-q+x^2}}\,dx}{\sqrt[3]{-qx+x^3}}$$
$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-q+x^6}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{-qx+x^3}}$$
$$= \frac{\left(3\sqrt[3]{x}\sqrt[3]{-q+x^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-q+x^3}}\,dx, x, x^{2/3}\right)}{2\sqrt[3]{-qx+x^3}}$$
$$= \frac{\sqrt{3}\sqrt[3]{x}\sqrt[3]{-q+x^2}\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-q+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-qx+x^3}} - \frac{3\sqrt[3]{x}\sqrt[3]{-q+x^2}\log\left(x^{2/3}-\sqrt[3]{-q+x^2}\right)}{4\sqrt[3]{-qx+x^3}}$$

Mathematica [A] time = 0.08, size = 127, normalized size = 1.92

$$\frac{\sqrt[3]{x}\sqrt[3]{x^2-q}\left(-2\log\left(1-\frac{x^{2/3}}{\sqrt[3]{x^2-q}}\right)+\log\left(\frac{x^{4/3}}{\left(x^2-q\right)^{2/3}}+\frac{x^{2/3}}{\sqrt[3]{x^2-q}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (x^(1/3)*(-q + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/
3))/Sqrt[3]] - 2*Log[1 - x^(2/3)/(-q + x^2)^(1/3)] + Log[1 + x^(4/3)/(-q +
x^2)^(2/3) + x^(2/3)/(-q + x^2)^(1/3)]))/(4*(-(q*x) + x^3)^(1/3))
```

fricas [B] time = 3.86, size = 415, normalized size = 6.29

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(q^{12}-15q^{10}+90q^8-351q^6+810q^4-1215q^2+729\right)\left(x^3-qx\right)^{\frac{1}{3}}x-2\sqrt{3}\left(q^{12}+6q^{11}-\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((4*sqrt(3)*(q^12 - 15*q^10 + 90*q^8 - 351*q^6 + 810*q^4
- 1215*q^2 + 729)*(x^3 - q*x)^(1/3)*x - 2*sqrt(3)*(q^12 + 6*q^11 - 15*q^10
- 54*q^9 + 90*q^8 + 270*q^7 - 351*q^6 - 810*q^5 + 810*q^4 + 1458*q^3 - 1215
*q^2 - 1458*q + 729)*(x^3 - q*x)^(2/3) - sqrt(3)*(q^13 + 10*q^12 - 15*q^11
- 282*q^10 + 90*q^9 + 2178*q^8 - 351*q^7 - 6534*q^6 + 810*q^5 + 7614*q^4 -
1215*q^3 - (q^12 - 6*q^11 - 15*q^10 + 54*q^9 + 90*q^8 - 270*q^7 - 351*q^6 +
 810*q^5 + 810*q^4 - 1458*q^3 - 1215*q^2 + 1458*q + 729)*x^2 - 2430*q^2 + 7
29*q))/(q^13 + 18*q^12 + 81*q^11 - 162*q^10 - 1350*q^9 + 810*q^8 + 6561*q^7
```

```
 - 2430*q^6 - 12150*q^5 + 4374*q^4 + 6561*q^3 - 9*(q^12 + 2*q^11 - 15*q^10
- 18*q^9 + 90*q^8 + 90*q^7 - 351*q^6 - 270*q^5 + 810*q^4 + 486*q^3 - 1215*q
^2 - 486*q + 729)*x^2 - 4374*q^2 + 729*q)) - 1/4*log(-3*(x^3 - q*x)^(1/3)*x
 + q + 3*(x^3 - q*x)^(2/3))
```

giac [A] time = 1.05, size = 67, normalized size = 1.02

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{4}\log\left(\left(-\frac{q}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{2}\log\left(\left|\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-q/x^2 + 1)^(1/3) + 1)) + 1/4*log((-q/x
^2 + 1)^(2/3) + (-q/x^2 + 1)^(1/3) + 1) - 1/2*log(abs((-q/x^2 + 1)^(1/3) -
1))
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-q\right)x\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2-q))^(1/3),x)
```

```
[Out] int(1/(x*(x^2-q))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-q\right)x\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

mupad [B] time = 0.39, size = 37, normalized size = 0.56

$$\frac{3\,x\,{\left(1-\frac{x^2}{q}\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{x^2}{q}\right)}{2\,{\left(x^3-q\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(q - x^2))^(1/3),x)
```

```
[Out] (3*x*(1 - x^2/q)^(1/3)*hypergeom([1/3, 1/3], 4/3, x^2/q))/(2*(x^3 - q*x)^(1
/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x**2-q))**(1/3),x)
```

```
[Out] Integral((x*(-q + x**2))**(-1/3), x)
```

3.42 $\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx$

Optimal. Leaf size=79

$$-\frac{3}{4}\log\left(\sqrt[3]{(x-1)\left(q+x^2-2x\right)}-x+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2(x-1)}{\sqrt{3}\sqrt[3]{(x-1)\left(q+x^2-2x\right)}}+\frac{1}{\sqrt{3}}\right)+\frac{1}{4}\log(1-x)$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+((-1+x)*(x^2+q-2*x))^(1/3))+1/2*arctan(1/3*3^(1/2)+2
/3*(-1+x)/((-1+x)*(x^2+q-2*x))^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2067, 2011, 329, 275, 239}

$$\frac{\sqrt{3}\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-1)^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}}-\frac{3\sqrt[3]{x-1}\sqrt[3]{q+(x-1)^2-1}\log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2\cdots}\right.}{4\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)
^(2/3))/(-1 + q + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(-((1 - q)*(-1 + x)) + (-
1 + x)^3)^(1/3)) - (3*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(-1 +
 q + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4*(-((1 - q)*(-1 + x)) + (-1 + x
)^3)^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{-(1-q)x+x^3}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-1+q+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+q+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+q+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}$$

$$= \frac{\sqrt{3}\,\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{q-(2-x)x}}}{\sqrt{3}}\right)}{2\sqrt[3]{(1-q)(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{-1+q+(-1+x)^2}\ \cdots}{4\sqrt[3]{(1\cdots}}$$

Mathematica [A] time = 0.15, size = 140, normalized size = 1.77

$$\frac{\sqrt[3]{x-1}\,\sqrt[3]{q+(x-2)x}\left(-2\log\left(1-\frac{(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}\right)+\log\left(\frac{(x-1)^{4/3}}{(q+(x-2)x)^{2/3}}+\frac{(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-2)x}}+1}{\sqrt{3}}\right)\right)}{4\sqrt[3]{(x-1)(q+(x-2)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] ((-1 + x)^(1/3)*(q + (-2 + x)*x)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(-1 + x)^(
2/3))/(q + (-2 + x)*x)^(1/3))/Sqrt[3]] - 2*Log[1 - (-1 + x)^(2/3)/(q + (-2
+ x)*x)^(1/3)] + Log[1 + (-1 + x)^(4/3)/(q + (-2 + x)*x)^(2/3) + (-1 + x)^(
2/3)/(q + (-2 + x)*x)^(1/3)]))/(4*((-1 + x)*(q + (-2 + x)*x))^(1/3))
```

fricas [B] time = 3.45, size = 665, normalized size = 8.42

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(q^{12}-18q^{11}+117q^{10}-346q^9+414q^8-18q^7+69q^6-774q^5-234q^4+1058q^3+621q^2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((2*sqrt(3)*(q^12 - 18*q^11 + 117*q^10 - 346*q^9 + 414*q^
8 - 18*q^7 + 69*q^6 - 774*q^5 - 234*q^4 + 1058*q^3 + 621*q^2 + 378*q - 539)
*(x^3 + (q + 2)*x - 3*x^2 - q)^(2/3) + 4*sqrt(3)*(q^12 - 12*q^11 + 51*q^10
- 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5 - 810*q^4 + 320*q^3 + 291*q^2
```

```
 - (q^12 - 12*q^11 + 51*q^10 - 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5
- 810*q^4 + 320*q^3 + 291*q^2 + 714*q + 49)*x + 714*q + 49)*(x^3 + (q + 2)*
x - 3*x^2 - q)^(1/3) - sqrt(3)*(q^13 - 22*q^12 + 177*q^11 - 514*q^10 - 434*
q^9 + 5346*q^8 - 8247*q^7 - 4542*q^6 + 19638*q^5 - 8050*q^4 - 10343*q^3 + (
q^12 - 6*q^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 -
 1386*q^4 - 418*q^3 - 39*q^2 + 1050*q + 637)*x^2 + 6186*q^2 - 2*(q^12 - 6*q
^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 - 1386*q^4
- 418*q^3 - 39*q^2 + 1050*q + 637)*x + 1501*q + 32))/(q^13 - 22*q^12 + 249*
q^11 - 1546*q^10 + 4702*q^9 - 4230*q^8 - 10623*q^7 + 25338*q^6 - 3546*q^5 -
 31306*q^4 + 18817*q^3 + 9*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 1
86*q^7 - 15*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x^2
+ 9714*q^2 - 18*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 186*q^7 - 15
*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x - 995*q + 8))
 - 1/4*log(3*(x^3 + (q + 2)*x - 3*x^2 - q)^(1/3)*(x - 1) + q - 3*(x^3 + (q
+ 2)*x - 3*x^2 - q)^(2/3) - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2+q-2\,x\right)(x-1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="giac")
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x-1)\left(x^2+q-2x\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x-1)*(x^2+q-2*x))^(1/3),x)
[Out] int(1/((x-1)*(x^2+q-2*x))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2+q-2\,x\right)(x-1)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="maxima")
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left((x-1)\left(x^2-2\,x+q\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(q - 2*x + x^2))^(1/3),x)
[Out] int(1/((x - 1)*(q - 2*x + x^2))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)\left(q+x^2-2x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
```

```
[Out] Integral(((x - 1)*(q + x**2 - 2*x))**(-1/3), x)
```

3.43 $\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$

Optimal. Leaf size=118

$$-\frac{3\log\left(\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}-\sqrt[3]{q}\,(x-1)\right)}{4\sqrt[3]{q}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{q}\,(x-1)}{\sqrt{3}\sqrt[3]{(x-1)\left(-2qx+q+x^2\right)}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{q}}+\frac{\log(1-x)}{4\sqrt[3]{q}}+\frac{\log(x)}{2\sqrt[3]{q}}$$

```
[Out] 1/4*ln(1-x)/q^(1/3)+1/2*ln(x)/q^(1/3)-3/4*ln(-q^(1/3)*(-1+x)+((-1+x)*(-2*q*
x+x^2+q))^(1/3))/q^(1/3)+1/2*arctan(1/3*3^(1/2)+2/3*q^(1/3)*(-1+x)/((-1+x)*
(-2*q*x+x^2+q))^(1/3)*3^(1/2))*3^(1/2)/q^(1/3)
```

Rubi [F] time = 21.94, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[
-((-1 + q)^3*q)])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[-((-1 +
 q)^3*q)])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(
1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q -
15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (3*(1 - 5*q + 4*q^2 +
 (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*((-1 - 2*q
)/3 + x))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) +
9*((-1 - 2*q)/3 + x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*
(-(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q
])^(2/3))/(3*(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)
) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2
 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 +
 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 + ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*
q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 +
 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 - 2*
q)/3 + x])/(3*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(1+2q)+x\right)\sqrt[3]{-\frac{2}{27}(1-q)^2(1+8q)-\frac{1}{3}(1-4q)(1-q)x+x^3}}\right.$$

$$\left(\sqrt[3]{-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{2/3}}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}}}+3x}\ \sqrt[3]{-1+5q-4\right.$$

$$=\ \overline{\qquad\qquad}$$

Mathematica [C] time = 0.20, size = 55, normalized size = 0.47

$$\frac{3\left((x-1)\left(-2qx+q+x^2\right)\right)^{2/3}\ {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^2-2qx+q}{q(x-1)^2}\right)}{4q(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] (3*((-1 + x)*(q - 2*q*x + x^2))^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (q - 2
*q*x + x^2)/(q*(-1 + x)^2)])/(4*q*(-1 + x)^2)
```

fricas [B] time = 52.74, size = 1496, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(3)*q*sqrt((-q)^(1/3)/q)*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 + 5
4*(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 + 486*q^3*x + 54
0*(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 + 9*((2*q^2 - q - 1)*x
^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^
(2/3)*(-q)^(1/3) + 9*((q^2 + 7*q + 1)*x^5 - (19*q^2 + 25*q + 1)*x^4 + 9*(7*
q^2 + 3*q)*x^3 + 45*q^2*x - 9*(9*q^2 + q)*x^2 - 9*q^2)*(-(2*q + 1)*x^2 + x^
3 + 3*q*x - q)^(1/3)*(-q)^(2/3) + sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4 - 6*(7
*q^2 + 5*q)*x^3 - 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*(-(2*q + 1)*x^2
 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5 + 5*(q^3
+ 7*q^2 + q)*x^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 + 9*q^3 + 15*(5*q^3 + q^2)
*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) + ((q^3 + 24*q^2 + 3*q - 1)*
x^6 - 54*(q^3 + 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^
3 + q^2)*x^3 + 27*q^3 + 27*(14*q^3 + q^2)*x^2)*(-q)^(1/3))*sqrt((-q)^(1/3)/
q))/x^6) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*(-(2*q + 1)*x^2 + x
^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q + 1)*x^2 + x^3 + 3*q*
x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2 - 6*q*x + 3*q)*(-(2
*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^2 + 2*q)*x^3 + 9*q^
2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) -
 ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*(5*q^2 + q)*x^2 + 9
*q^2)*(-q)^(1/3))/x^4))/q, 1/12*(2*sqrt(3)*q*sqrt(-(-q)^(1/3)/q)*arctan(1/3
*sqrt(3)*(6*((2*q^2 - q - 1)*x^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*
q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) - 6*((q^3 + 7*q^2 + q)*x^5 -
 (19*q^3 + 25*q^2 + q)*x^4 + 45*q^3*x + 9*(7*q^3 + 3*q^2)*x^3 - 9*q^3 - 9*(
9*q^3 + q^2)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) - ((q^3 - 12*q^2
 - 15*q - 1)*x^6 + 18*(q^3 + 6*q^2 + 2*q)*x^5 - 9*(17*q^3 + 26*q^2 + 2*q)*x
^4 + 162*q^3*x + 180*(2*q^3 + q^2)*x^3 - 27*q^3 - 45*(8*q^3 + q^2)*x^2)*(-q
)^(1/3))*sqrt(-(-q)^(1/3)/q)/((q^3 + 24*q^2 + 3*q - 1)*x^6 - 54*(q^3 + 2*q^
2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^3 + q^2)*x^3 + 27*q^
3 + 27*(14*q^3 + q^2)*x^2)) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*
(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q +
1)*x^2 + x^3 + 3*q*x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2
- 6*q*x + 3*q)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^
2 + 2*q)*x^3 + 9*q^2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 +
 3*q*x - q)^(1/3) - ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*
(5*q^2 + q)*x^2 + 9*q^2)*(-q)^(1/3))/x^4))/q]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(2\,qx - x^2 - q\right)(x-1)\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="giac")
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x-1)\left(-2qx+x^{2}+q\right)\right)^{\frac{1}{3}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((x-1)*(-2*q*x+x^2+q))^(1/3),x)
[Out] int(1/x/((x-1)*(-2*q*x+x^2+q))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(2\,qx-x^{2}-q\right)(x-1)\right)^{\frac{1}{3}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left((x-1)\left(x^{2}-2\,q\,x+q\right)\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)),x)
[Out] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
[Out] Timed out
```

3.44 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$

Optimal. Leaf size=111

$$\frac{\log(x)}{2\sqrt[3]{k}}+\frac{\log(1-(k+1)x)}{2\sqrt[3]{k}}-\frac{3\log\left(\sqrt[3]{(1-x)x(1-kx)}-\sqrt[3]{k}\,x\right)}{2\sqrt[3]{k}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{k}\,x}{\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{\sqrt[3]{k}}$$

```
[Out] 1/2*ln(x)/k^(1/3)+1/2*ln(1-(1+k)*x)/k^(1/3)-3/2*ln(-k^(1/3)*x+((1-x)*x*(-k*
x+1))^(1/3))/k^(1/3)+arctan(1/3*(1+2*k^(1/3)*x/((1-x)*x*(-k*x+1))^(1/3))*3^
(1/2))*3^(1/2)/k^(1/3)
```

Rubi [F] time = 0.61, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x*(1 - k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x, k*x])/(2
*((1 - x)*x*(1 - k*x))^(1/3)) + ((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defe
r[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 + (-1 - k)*x)*(1 - k*x)^(1/3)), x])/((1
- x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\begin{aligned}\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx &= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{2-(1+k)x}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\,(1-(1+k)x)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}\\ &= \frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{1}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-}}}{\sqrt[3]{(1-x)x(1}}\\ &= \frac{3\sqrt[3]{1-x}\,x\sqrt[3]{1-kx}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};x,kx\right)}{2\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{}{\sqrt[3]{1-x}\,\sqrt[3]{}}}{\sqrt[3]{(1-x)x(1-}}\end{aligned}$$

Mathematica [F] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="gi
ac")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-(k+1)\,x+2}{((-x+1)\,(-kx+1)\,x)^{\frac{1}{3}}\,(-(k+1)\,x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(1+k)*x)/((-x+1)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

```
[Out] int((2-(1+k)*x)/((-x+1)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(k+1)x-2}{((kx-1)(x-1)x)^{\frac{1}{3}}((k+1)x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="ma
xima")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(k+1)-2}{(x\,(k+1)-1)\,{(x\,(k\,x-1)\,(x-1))}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)),x)
```

```
[Out] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx+x-2}{\sqrt[3]{x\,(x-1)\,(kx-1)}\,(kx+x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
```

```
[Out] Integral((k*x + x - 2)/((x*(x - 1)*(k*x - 1))**(1/3)*(k*x + x - 1)), x)
```

3.45 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

Optimal. Leaf size=176

$$\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-kx)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{3\log\left(kx+2^{2/3}\sqrt[3]{1-k}\,\sqrt[3]{(1-x)x(1-kx)}-1\right)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-kx)}{\sqrt[3]{1-k}\,\sqrt[3]{(1-x)x(1-kx)}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{1-k}}$$

```
[Out] 1/2*ln(1-(2-k)*x)*2^(1/3)/(1-k)^(1/3)+1/4*ln(-k*x+1)*2^(1/3)/(1-k)^(1/3)-3/
4*ln(-1+k*x+2^(2/3)*(1-k)^(1/3)*((1-x)*x*(-k*x+1))^(1/3))*2^(1/3)/(1-k)^(1/
3)-1/2*arctan(1/3*(1+2^(1/3)*(-k*x+1)/(1-k)^(1/3)/((1-x)*x*(-k*x+1))^(1/3))
*3^(1/2))*3^(1/2)*2^(1/3)/(1-k)^(1/3)
```

Rubi [F] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] ((1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^
(2/3)*x^(2/3)*(1 + (-2 + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx=\frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)}\,dx}{((1-x)x(1-kx))^{2/3}}$$

Mathematica [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fric
as")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{kx-1}{((kx-1)(x-1)x)^{\frac{2}{3}}((k-2)x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="giac
")
```

```
[Out] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{-kx+1}{\left(\left(k-2\right)x+1\right)\left(\left(-x+1\right)\left(-kx+1\right)x\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-k*x+1)/(1+(-2+k)*x)/((-x+1)*(-k*x+1)*x)^(2/3),x)
```

```
[Out] int((-k*x+1)/(1+(-2+k)*x)/((-x+1)*(-k*x+1)*x)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx-1}{\left((kx-1)(x-1)x\right)^{\frac{2}{3}}\left((k-2)x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="maxi
ma")
```

```
[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{k\,x-1}{\left(x\,\left(k-2\right)+1\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)),x)
```

```
[Out] -int((k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\,dx-\int\left(-\frac{}{kx\left(kx^3-kx^2-x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] -Integral(k*x/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x*
*2 - x**2 + x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x) - Integral
(-1/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x**2 - x**2
+ x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x)
```

3.46 $\int \frac{a+bx+cx^2}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=493

$$\frac{(a+b)\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{(a+b)\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{(a+b)\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{(a+b)\tan^{-1}}{\sqrt[3]{2}}$$

```
[Out] 1/24*(a+b)*ln((1-x)*(1+x)^2)*2^(2/3)-1/12*(a-c)*ln(x^3+1)*2^(2/3)-1/12*(b+c
)*ln(x^3+1)*2^(2/3)+1/12*(a+b)*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*
(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*(a+b)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
2^(2/3)+1/4*(b+c)*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/4*(a-c)*ln(-2^(1/3)*
x-(-x^3+1)^(1/3))*2^(2/3)+1/2*c*ln(x+(-x^3+1)^(1/3))-1/8*(a+b)*ln(-1+x+2^(2
/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*(a+b)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*(a+b)*arctan(1/3*(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/3*c*arctan(1/3*(1-2*x/(-x^3+1)^(1/
3))*3^(1/2))*3^(1/2)-1/6*(a-c)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*2^(2/3)*3^(1/2)+1/6*(b+c)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1
/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.85, antiderivative size = 576, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {6728, 239, 2148}

$$-\frac{\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)\left(3ib-\sqrt{3}\left(2a+b-i\sqrt{3}c-c\right)\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)\left(\sqrt{3}\right.}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]) - ((2*a + b - I*
Sqrt[3]*b - (1 + I*Sqrt[3])*c)*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/
(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((2*a + b + I*Sq
rt[3]*b - c + I*Sqrt[3]*c)*ArcTan[(2 - (2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 -
 x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I - Sqrt[3])) + (((3*I)*b - Sqrt[3]*
(2*a + b - c - I*Sqrt[3]*c))*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3] +
 2*x))])/(12*2^(1/3)*(I + Sqrt[3])) + (((3*I)*b + Sqrt[3]*(2*a + b - c + I*
Sqrt[3]*c))*Log[-((1 + I*Sqrt[3] - 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(12*2^(1
/3)*(I - Sqrt[3])) + (c*Log[x + (1 - x^3)^(1/3)])/2 - (((3*I)*b - Sqrt[3]*(
2*a + b - c - I*Sqrt[3]*c))*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I + Sqrt[3])) - (((3*I)*b + Sqrt[3]*(2*a + b - c + I*Sqr
t[3]*c))*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(
I - Sqrt[3]))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))]/Sq
```

```
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx &= \int \left(\frac{c}{\sqrt[3]{1-x^3}}+\frac{a-c+(b+c)x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\right)dx \\
&= c\int \frac{1}{\sqrt[3]{1-x^3}}\,dx+\int \frac{a-c+(b+c)x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx \\
&= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right)+\int \left(\frac{b-\frac{i(2a+b-c)}{\sqrt{3}}+c}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\right. \\
&= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right)+\frac{1}{3}\left(3b-i\sqrt{3}\,(2a+b-c)+3c\right)\int \\
&= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(2a+b-i\sqrt{3}\,b-c-i\sqrt{3}\,c\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}+\frac{(2a}{}
\end{aligned}$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
[Out] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 + bx + a}{\left(-x^3 + 1\right)^{\frac{1}{3}} \left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{c\,x^2 + bx + a}{\left(x^2 - x + 1\right) \left(-x^3 + 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
[Out] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 + bx + a}{\left(-x^3 + 1\right)^{\frac{1}{3}} \left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^2 + b\,x + a}{\left(1 - x^3\right)^{1/3}\,\left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
[Out] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{\sqrt[3]{-\left(x - 1\right) \left(x^2 + x + 1\right)} \left(x^2 - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)
[Out] Integral((a + b*x + c*x**2)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1
)), x)
```

3.47 $\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx$

Optimal. Leaf size=407

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73}{1176(3-2x)}$$

```
[Out] -19255/395136/(3-2*x)^(9/2)-462025/30118144/(3-2*x)^(7/2)-38491/8605184/(3-
2*x)^(5/2)-141045/120472576/(3-2*x)^(3/2)+1/28*x/(3-2*x)^(9/2)/(2*x^2+x+1)^
4+1/1176*(23+73*x)/(3-2*x)^(9/2)/(2*x^2+x+1)^3+1/32928*(1387+3049*x)/(3-2*x
)^(9/2)/(2*x^2+x+1)^2+5/153664*(3049+4377*x)/(3-2*x)^(9/2)/(2*x^2+x+1)-3822
5/240945152/(3-2*x)^(1/2)+5/13492928512*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+
2*14^(1/2))^(1/2))*(-298093007954+81630132224*14^(1/2))^(1/2)-5/13492928512
*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(-298093007954+81630
132224*14^(1/2))^(1/2)+5/6746464256*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))
^(1/2))/(-7+2*14^(1/2))^(1/2))*(298093007954+81630132224*14^(1/2))^(1/2)-5/
6746464256*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1
/2))*(298093007954+81630132224*14^(1/2))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{x}{28(3-2x)^{9/2}\left(2x^2+x+1\right)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}\left(2x^2+x+1\right)}+\frac{3049x+1387}{32928(3-2x)^{9/2}\left(2x^2+x+1\right)^2}+\frac{73}{1176(3-2x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2)) - 38491
/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x)^(3/2)) - 38225/(24
0945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + (23 + 7
3*x)/(1176*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 -
 2*x)^(9/2)*(1 + x + 2*x^2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2
)*(1 + x + 2*x^2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTa
n[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/33732321
28 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sqrt[7 + 2*Sq
rt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3373232128 + (5*Sqrt[(-1
49046503977 + 40815066112*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[1
4]]*Sqrt[3 - 2*x] - 2*x])/6746464256 - (5*Sqrt[(-149046503977 + 40815066112
*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])
/6746464256
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{1}{784}\int \frac{700-644x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^4}\,dx$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{\int \frac{325752-54}{(3-2x)^{11/2}(1+}}{4609}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{138}{32928(3-2}$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} + \frac{138}{32928(3-2}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+\right.}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} +$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} + \frac{}{28(3-2x}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14}{12047257}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14}{12047257}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14}{12047257}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14}{12047257}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14}{12047257}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14}{12047257}$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} - \frac{14}{12047257}$$

Mathematica [C] time = 2.13, size = 198, normalized size = 0.49

$$\frac{45i\sqrt{14-2i\sqrt{7}}\left(146319\sqrt{7}+115739i\right)\tanh^{-1}\left(\frac{\sqrt{6-4x}}{\sqrt{7-i\sqrt{7}}}\right)-45i\sqrt{14+2i\sqrt{7}}\left(146319\sqrt{7}-115739i\right)\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((56*(-40289347 + 429812744*x - 135202154*x^2 + 1073855156*x^3 - 1627773523
*x^4 + 1470758860*x^5 - 2888625656*x^6 + 3106712560*x^7 - 2343370048*x^8 +
2443779648*x^9 - 1873554048*x^10 + 677249280*x^11 - 88070400*x^12))/((3 - 2
*x)^(9/2)*(1 + x + 2*x^2)^4) + (45*I)*Sqrt[14 - (2*I)*Sqrt[7]]*(115739*I +
146319*Sqrt[7])*ArcTanh[Sqrt[6 - 4*x]/Sqrt[7 - I*Sqrt[7]]] - (45*I)*Sqrt[14
 + (2*I)*Sqrt[7]]*(-115739*I + 146319*Sqrt[7])*ArcTanh[Sqrt[6 - 4*x]/Sqrt[7
 + I*Sqrt[7]]])/121436356608
```

fricas [B] time = 1.52, size = 957, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/85228286570792313424725137817 6*(2263908918780*22241759018113166^(1/4)*sqr
t(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10
+ 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3
 - 1242*x^2 - 162*x - 243)*sqrt(21292357711*sqrt(14) + 81630132224)*arctan(
1/1005218715695186946952690868575343722872940181 5040*22241759018113166^(3/4
)*sqrt(12577271771)*sqrt(79716926)*sqrt(-2089731384934400*22241759018113166
^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 8163013222
4)*(7645*sqrt(14) - 115739) - 4190418993502514995568679111884800*x + 209520
9496751257497784339555942400*sqrt(14) + 6285628490253772493353018667827200)
*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357711*sqrt(14) + 816
30132224) - 1/1958184534851295802906658902*22241759018113166^(3/4)*sqrt(797
16926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2*x + 3)*sqrt(21292
357711*sqrt(14) + 81630132224) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 22639089
18780*22241759018113166^(1/4)*sqrt(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2
816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^
6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)*sqrt(2129235771
1*sqrt(14) + 81630132224)*arctan(1/2462861907259396838466870075605045544 2*2
2241759018113166^(3/4)*sqrt(12577271771)*sqrt(22241759018113166^(1/4)*sqrt(
79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)*(7645*sqr
t(14) - 115739) - 2005242886101391892*x + 1002621443050695946*sqrt(14) + 30
07864329152087838)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357
711*sqrt(14) + 81630132224) - 1/1958184534851295802906658902*22241759018113
166^(3/4)*sqrt(79716926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2
*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224) + 2/7*sqrt(14)*sqrt(7) + s
qrt(7)) + 315*22241759018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 -
 229870452342784*x^12 + 459740904685568*x^11 - 480638218534912*x^10 + 55900
3145469952*x^9 - 734018148958208*x^8 + 498923368153088*x^7 - 34611176062976
0*x^6 + 407660880326656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 10
1384624222208*x^2 - 21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11
 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x
^4 + 936*x^3 - 1242*x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)
*sqrt(21292357711*sqrt(14) + 81630132224)*log(2089731384934400/12577271771*
22241759018113166^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt
(14) + 81630132224)*(7645*sqrt(14) - 115739) - 3331739243453861593088 00*x +
 1665869621726930796544 00*sqrt(14) + 4997608865180792389632 00) - 315*222417
```

```
59018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 - 229870452342784*x^1
2 + 459740904685568*x^11 - 480638218534912*x^10 + 559003145469952*x^9 - 734
018148958208*x^8 + 498923368153088*x^7 - 346111760629760*x^6 + 407660880326
656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 101384624222208*x^2 -
21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x
^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*
x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)*sqrt(21292357711*sq
rt(14) + 81630132224)*log(-2089731384934400/12577271771*22241759018113166^(
1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)
*(7645*sqrt(14) - 115739) - 333173924345386159308800*x + 166586962172693079
654400*sqrt(14) + 499760886518079238963200) + 393027605675872810832*(880704
00*x^12 - 677249280*x^11 + 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^
8 - 3106712560*x^7 + 2888625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 107
3855156*x^3 + 135202154*x^2 - 429812744*x + 40289347)*sqrt(-2*x + 3))/(512*
x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 -
 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)
```

giac [B] time = 2.82, size = 767, normalized size = 1.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] -5/24179327893504*sqrt(7)*(856240*14^(3/4)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 2
568720*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 183480*14^(3/4)
*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) - 7645*14^(3/4)*sqrt(7)*(-8*
sqrt(14) + 32)^(3/2) + 103702144*14^(1/4)*sqrt(2)*sqrt(sqrt(14) + 4) + 7407
296*14^(1/4)*sqrt(7)*sqrt(-8*sqrt(14) + 32))*arctan(1/28*14^(3/4)*(14^(1/4)
*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)
) - 5/24179327893504*sqrt(7)*(856240*14^(3/4)*sqrt(2)*(sqrt(14) + 4)^(3/2)
+ 2568720*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 183480*14^(3
/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) - 7645*14^(3/4)*sqrt(7)*(
-8*sqrt(14) + 32)^(3/2) + 103702144*14^(1/4)*sqrt(2)*sqrt(sqrt(14) + 4) + 7
407296*14^(1/4)*sqrt(7)*sqrt(-8*sqrt(14) + 32))*arctan(-1/28*14^(3/4)*(14^(
1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) +
1/2)) - 5/48358655787008*sqrt(7)*(122320*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14)
 + 4)^(3/2) + 366960*14^(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14)
- 4) - 1284360*14^(3/4)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) + 53515*14^(3
/4)*(-8*sqrt(14) + 32)^(3/2) + 14814592*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(
14) + 4) - 51851072*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(14^(1/4)*sqrt(1/2)
*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 5/48358655787008
*sqrt(7)*(122320*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 366960*14^
(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) - 1284360*14^(3/4)*
(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) + 53515*14^(3/4)*(-8*sqrt(14) + 32)^(
3/2) + 14814592*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4) - 51851072*14^(
1/4)*sqrt(-8*sqrt(14) + 32))*log(-14^(1/4)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sq
rt(14) + 4) - 2*x + sqrt(14) + 3) + 1/5059848192*(1578405*(2*x - 3)^7*sqrt(
-2*x + 3) + 37939930*(2*x - 3)^6*sqrt(-2*x + 3) + 400127266*(2*x - 3)^5*sqr
t(-2*x + 3) + 2394090608*(2*x - 3)^4*sqrt(-2*x + 3) + 8763772549*(2*x - 3)^
3*sqrt(-2*x + 3) + 19602865030*(2*x - 3)^2*sqrt(-2*x + 3) - 24778425644*(-2
*x + 3)^(3/2) + 13623638952*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7)^4 + 1/
59295096*(9090*(2*x - 3)^4 - 3885*(2*x - 3)^3 + 2394*(2*x - 3)^2 - 2520*x +
 4172)/((2*x - 3)^4*sqrt(-2*x + 3))
```

maple [A] time = 0.12, size = 584, normalized size = 1.43

$$-\frac{731595\left(7+2\sqrt{14}\right)\sqrt{14}\ \arctan\left(\frac{2\sqrt{-2x+3}-\sqrt{7+2\sqrt{14}}}{\sqrt{-7+2\sqrt{14}}}\right)}{6746464256\sqrt{-7+2\sqrt{14}}}+\frac{1424965\left(7+2\sqrt{14}\right)\arctan\left(\frac{2\sqrt{-2x+3}-\sqrt{7+2\sqrt{14}}}{\sqrt{-7+2\sqrt{14}}}\right)}{3373232128\sqrt{-7+2\sqrt{14}}}-\frac{5786}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x)
```

```
[Out] 1/6588344*(567651623/32*(3-2*x)^(1/2)-6194606411/192*(3-2*x)^(3/2)+98014325
15/384*(3-2*x)^(5/2)-8763772549/768*(3-2*x)^(7/2)+149630663/48*(3-2*x)^(9/2
)-200063633/384*(3-2*x)^(11/2)+18969965/384*(3-2*x)^(13/2)-526135/256*(3-2*
x)^(15/2))/((3-2*x)^2-7+14*x)^4-731595/13492928512*ln(3-2*x+14^(1/2)-(3-2*x
)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)*14^(1/2)+1424965/6746464
256*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1
/2)-731595/6746464256/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14
^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))*14^(1/2)+1424965/33732
32128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(
-7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))-578695/3373232128/(-7+2*14^(1/2))^(1/2
)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(
1/2)+731595/13492928512*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2
))*(7+2*14^(1/2))^(1/2)*14^(1/2)-1424965/6746464256*ln(3-2*x+14^(1/2)+(3-2*
x)^(1/2)*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2)-731595/6746464256/(-7+2
*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/
2))^(1/2))*(7+2*14^(1/2))*14^(1/2)+1424965/3373232128/(-7+2*14^(1/2))^(1/2)
*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(7+2*
14^(1/2))-578695/3373232128/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(
7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*14^(1/2)+1/151263/(3-2*x)^(9/2)
+5/235298/(3-2*x)^(7/2)+19/470596/(3-2*x)^(5/2)+185/2823576/(3-2*x)^(3/2)+5
05/3294172/(3-2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^5(-2\,x+3)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

mupad [B] time = 0.47, size = 343, normalized size = 0.84

$$-\frac{\frac{272\,x}{441}-\frac{164\,(2\,x-3)^2}{441}+\frac{1966\,(2\,x-3)^3}{3087}-\frac{9091\,(2\,x-3)^4}{3087}-\frac{32070727\,(2\,x-3)^5}{5531904}-\frac{41014777\,(2\,x-3)^6}{11063808}-\frac{141921511\,(2\,x-3)^7}{154893312}+\frac{23262655\,(2\,x}{30978662}}{38416\,(3-2\,x)^{9/2}-76832\,(3-2\,x)^{11/2}+68600\,(3-2\,x)^{13/2}-35672\,(3-2\,x)^{15/2}+11809\,(3-2\,x)^{17/2}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(11/2)*(x + 2*x^2 + 1)^5),x)
```

```
[Out] (atan(((3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^(1/2)*15721589
71375i)/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/1958315
2812683802696692531 2 - 230036728532618625/27975932589548289566703616)) - (1
572158971375*7^(1/2)*(3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^
(1/2))/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/19583152
812683802696692531 2 - 230036728532618625/27975932589548289566703616)))*(7^(
1/2)*12577271771i - 149046503977)^(1/2)*5i)/3373232128 - ((272*x)/441 - (16
4*(2*x - 3)^2)/441 + (1966*(2*x - 3)^3)/3087 - (9091*(2*x - 3)^4)/3087 - (3
2070727*(2*x - 3)^5)/5531904 - (41014777*(2*x - 3)^6)/11063808 - (141921511
*(2*x - 3)^7)/154893312 + (23262655*(2*x - 3)^8)/309786624 + (1571659*(2*x
- 3)^9)/15059072 + (468427*(2*x - 3)^10)/17210368 + (394105*(2*x - 3)^11)/1
20472576 + (38225*(2*x - 3)^12)/240945152 - 520/441)/(38416*(3 - 2*x)^(9/2)
 - 76832*(3 - 2*x)^(11/2) + 68600*(3 - 2*x)^(13/2) - 35672*(3 - 2*x)^(15/2)
```

```
 + 11809*(3 - 2*x)^(17/2) - 2548*(3 - 2*x)^(19/2) + 350*(3 - 2*x)^(21/2) -
28*(3 - 2*x)^(23/2) + (3 - 2*x)^(25/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)
*12577271771i - 149046503977)^(1/2)*1572158971375i)/(3916630562536760539338
50624*((7^(1/2)*181960107187971125i)/195831528126838026966925312 + 23003672
8532618625/27975932589548289566703616)) + (1572158971375*7^(1/2)*(3 - 2*x)^
(1/2)*(- 7^(1/2)*12577271771i - 149046503977)^(1/2))/(391663056253676053933
850624*((7^(1/2)*181960107187971125i)/195831528126838026966925312 + 2300367
28532618625/27975932589548289566703616)))*(- 7^(1/2)*12577271771i - 1490465
03977)^(1/2)*5i)/3373232128
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Timed out
```

3.48 $\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

Optimal. Leaf size=648

$$\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(2x^2+x+1\right)}+\frac{451(14627273x+28962039)}{283351498752(3-2x)^{19/2}\left(2x^2+x+1\right)^2}+\frac{451(998691x+}{10119696384(3-2x)^{19}}$$

```
[Out] 4718120139975/351733660450816/(3-2*x)^(19/2)-815900548375/629418129227776/(
3-2*x)^(17/2)-3029508823715/1555033025150976/(3-2*x)^(15/2)-13515743021825/
13476952884641792/(3-2*x)^(13/2)-5846828446875/14513641568075776/(3-2*x)^(1
1/2)-37283626871975/261245548225363968/(3-2*x)^(9/2)-132355162272575/284467
3747342852096/(3-2*x)^(7/2)-11557581705725/812763927812243456/(3-2*x)^(5/2)
-46601678385075/11378694989371408384/(3-2*x)^(3/2)+1/63*x/(3-2*x)^(19/2)/(2
*x^2+x+1)^9+1/7056*(53+173*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^8+1/691488*(8477+2
1409*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^7+5/6453888*(21409+47471*x)/(3-2*x)^(19/
2)/(2*x^2+x+1)^6+41/90354432*(47471+92875*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^5+4
1/5059848192*(3436375+5677637*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^4+451/101196963
84*(811091+998691*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^3+451/283351498752*(2896203
9+14627273*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^2+11275/3966920982528*(14627273-35
058731*x)/(3-2*x)^(19/2)/(2*x^2+x+1)-24229218097975/22757389978742816768/(3
-2*x)^(1/2)+11275/127441383880959773900 8*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7
+2*14^(1/2))^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)
-11275/1274413838809597739008*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2)
)^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)+11275/6372
06919404798869504*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(
1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)-11275/6
37206919404798869504*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14
^(1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

$$\frac{11275(14627273-35058731x)}{3966920982528(3-2x)^{19/2}\left(2x^2+x+1\right)}+\frac{451(14627273x+28962039)}{283351498752(3-2x)^{19/2}\left(2x^2+x+1\right)^2}+\frac{451(998691x+}{10119696384(3-2x)^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
 - 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(22757389978742816768*Sqrt[3 -
 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
 2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
 + 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/
```

```
2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235
 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr
t[-7 + 2*Sqrt[14]]])/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2
]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[
3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(975658923
5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[
7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(97565
89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S
qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
```

```
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\int \frac{1680-1484x}{(3-2x)^{21/2}\left(1+x+2x^2\right)^9}\,dx}{1764}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\int \frac{2534672\ldots}{(3-2x)^{21/\ldots}}}{27\ldots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\ldots}{691488(3\ldots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\ldots}{691488(3\ldots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\ldots}{691488(3\ldots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\ldots}{691488(3\ldots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\ldots}{691488(3\ldots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\ldots}{691488(3\ldots}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{8\ldots}{691488(3\ldots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{5\ldots}{7056(3-2\ldots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} + \frac{\ldots}{63(3-2x\ldots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{302\ldots}{155503302\ldots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{302\ldots}{155503302\ldots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{302\ldots}{155503302\ldots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{302\ldots}{155503302\ldots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{302\ldots}{155503302\ldots}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{302\ldots}{155503302\ldots}$$

Mathematica [C] time = 6.09, size = 610, normalized size = 0.94

$$\frac{x}{63(3-2x)^{19/2}\left(2x^2+x+1\right)^9}+\frac{\frac{67816x+20776}{1568(3-2x)^{19/2}\left(2x^2+x+1\right)^8}+\frac{\frac{117492592x+46521776}{1372(3-2x)^{19/2}\left(2x^2+x+1\right)^7}+\frac{\frac{164128134240x+74020332960}{1176(3-2x)^{19/2}\left(2x^2+x+1\right)^6}+\frac{\frac{184316990760000x+94\cdots}{980(3-2x)^{19/2}\left(2\cdots\right.}}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + ((20776 + 67816*x)/(1568*(3 - 2
*x)^(19/2)*(1 + x + 2*x^2)^8) + ((46521776 + 117492592*x)/(1372*(3 - 2*x)^(
19/2)*(1 + x + 2*x^2)^7) + ((74020332960 + 164128134240*x)/(1176*(3 - 2*x)^
(19/2)*(1 + x + 2*x^2)^6) + ((94209549053760 + 184316990760000*x)/(980*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)^5) + ((95476201213680000 + 157747397367934080*
x)/(784*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^4) + ((72879297583985544960 + 8973
5798552133000960*x)/(588*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^3) + ((3643273421
2165998389760 + 18400346379541577848320*x)/(392*(3 - 2*x)^(19/2)*(1 + x + 2
*x^2)^2) + ((6440121232839552246912000 - 1543571914665913655846400 0*x)/(196
*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)) + (3947992688254522195411200 0/(19*(3 - 2
*x)^(19/2)) + (-90802166413848096693024000 0/(17*(3 - 2*x)^(17/2)) + (-19105
520493023248582746201600/(3 - 2*x)^(15/2) + (-268495574355372394658843107 20
000/(13*(3 - 2*x)^(13/2)) + (-1509944238585987965392741200 00000/(3 - 2*x)^(
11/2) + (-82377181135875141397849766198 40000/(3 - 2*x)^(9/2) + (-3383893120
36560466460044072847040000/(3 - 2*x)^(7/2) + (-101353055285765105508363945 1
5648960000/(3 - 2*x)^(5/2) + (-20433437573849564881280595679107 3600000/(3 -
 2*x)^(3/2) + (-22309948665198897968285610364062 28800000/Sqrt[3 - 2*x] + ((
Sqrt[(7 - I*Sqrt[7])/2]*(-31233928131278457155599854509687203200000 - (7175
059724092334984605434771301389120 0000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 -
 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) + (Sqrt[(7 + I*Sqrt[7])/
2]*(-31233928131278457155599854509687203200000 + (717505972409233498460543 4
7713013891200000*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqr
t[7]]])/(-14 - (2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)
/196)/392)/588)/784)/980)/1176)/1372)/1568)/1764
```

fricas [B] time = 1.72, size = 1563, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/109475537308620060324699564466344763160536147866564198767001 6*(4732002380
085251586622550100*478793617507582534294314731468 6^(1/4)*sqrt(1169607525756
986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 6468403
2*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21
+ 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 46771
2000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^
12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 2003648
4*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 +
 137781*x + 59049)*sqrt(3275718505284624031 99*sqrt(14) + 122642238092815735
1936)*arctan(1/3656217085193197024885534011338703535441745724187062686602 49
453794890088327253112192 52*4787936175075825342943147314686^(3/4)*sqrt(27763
87167632535361)*sqrt(12865682783326846)*sqrt(1169607525756986)*sqrt(4787936
```

1750758253429431473146863^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(3
275718505284624031 99*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14
) - 9756589235) - 7144023316491899220969682663120 2812*x + 28280279689505005
187146*sqrt(2233502127208610080 2556094) + 107160349747378488314545239946804
218)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3275718505284
62403199*sqrt(14) + 1226422380928157351936) - 1/102357367080615767666910014
42582284413274479000967 42*4787936175075825342943147314686^(3/4)*sqrt(116960
7525756986)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(-2*x +
3)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936) + 2/7*sqr
t(14)*sqrt(7) + sqrt(7)) + 4732002380085251586622550100*4787936175075825342
943147314686^(1/4)*sqrt(1169607525756986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5
505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^
23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 54
0503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720
*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 -
49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 32
76126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt(32757185052846
2403199*sqrt(14) + 1226422380928157351936)*arctan(1/392966702348163030765555
3305426032970833884806359730277975856974543991435989283703354643447808 00*47
87936175075825342943147314686^(3/4)*sqrt(2776387167632535361)*sqrt(11696075
25756986)*sqrt(-1486210744040984254522889076736 0000*4787936175075825342943 1
47314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(32757185052846240
3199*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235)
- 10617524208649565481090930614955423990381925855618094353584698163200 00*x
+ 42030455519026368931685279500166434110241662834835456 0000*sqrt(2233502127
2086100802556094) + 159262863129743482216363959224331359855728887834271415 3
037704724480000)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3
27571850528462403199*sqrt(14) + 1226422380928157351936) - 1/102357367080615
76766691001442582284413274479000967 42*4787936175075825342943147314686^(3/4)
sqrt(1169607525756986)(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))
*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14) + 122642238092815735193
6) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 271150425*4787936175075825342943147 3
14686^(1/4)*sqrt(1169607525756986)*(6429985372520617617318215 68*x^28 - 6751
484641146648498184126464*x^27 + 30381680885159918241828569088*x^26 - 793299
44533473119853663485952*x^25 + 146844790944939604835504750592*x^24 - 237989
833600419359560990457856*x^23 + 362048363881489025715123781632*x^22 - 47435
2077153419437787597242368*x^21 + 550984441886077267281495195648*x^20 - 6323
63154136437844718544486 40*x^19 + 662885025215707070319757885440*x^18 - 609
018199514371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 50
5075664975624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 3
04685469106942025132773736448*x^13 + 228722407218762404519491928064*x^12 -
129043951976611196927641387008*x^11 + 102555257051181053298083889152*x^10 -
61068067637283818105902989312*x^9 + 23430879305087206538965155840*x^8 - 24
573192412708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 274119
3833525857491515080704*x^5 + 4017914249140640432768679936*x^4 + 90121441102
2199723237834752*x^3 + 1013866212399974688642564096*x^2 - 32757185052846240
3199*sqrt(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 +
119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261
568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^1
6 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105
219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 +
5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*
x + 59049) + 168977702066662448107094016*x + 72419015171426763474468864)*sq
rt(327571850528462403199*sqrt(14) + 1226422380928157351936)*log(14862107440
40984254522889076736 0000/2776387167632535361*47879361750758253429431473146 8
6^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sq
rt(14) + 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 38242
231964006946013272086827269818478925709312 0000*x + 151385426388014656165701
481356328960000*sqrt(22335021272086100802556094) + 573633479460104190199081

```
302409047277183885639680000) - 271150425*4787936175075825342943147314686^(1
/4)*sqrt(1169607525756986)*(6429985372520617617318215 68*x^28 - 675148464114
6648498184126464*x^27 + 30381680885159918241828569088*x^26 - 79329944533473
119853663485952*x^25 + 146844790944939604835504750592*x^24 - 23798983360041
9359560990457856*x^23 + 362048363881489025715123781632*x^22 - 4743520771534
19437787597242368*x^21 + 550984441886077267281495195648*x^20 - 632336315413
643784471854448640*x^19 + 662885025215707070319757885440*x^18 - 60901819951
4371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 5050756649
75624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 304685469
106942025132773736448*x^13 + 228722407218762404519491928064*x^12 - 12904395
1976611196927641387008*x^11 + 102555257051181053298083889152*x^10 - 6106806
76372838181059029893312*x^9 + 23430879305087206538965155840*x^8 - 2457319241
2708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 27411938335258
57491515080704*x^5 + 4017914249140640432768679936*x^4 + 9012144110221997232
37834752*x^3 + 1013866212399974688642564096*x^2 - 327571850528462403199*sqr
t(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 1197342
72*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20
 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 4118
28480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x
^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*
x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 5904
9) + 168977702066662448107094016*x + 72419015171426763474468864)*sqrt(32757
1850528462403199*sqrt(14) + 1226422380928157351936)*log(-148621074404098425
45228890767360000/2776387167632535361*4787936175075825342943147314686^(1/4)
*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14)
+ 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235) - 382422319640
069460132720868272698184789257093120000*x + 1513854263880146561657014813563
28960000*sqrt(22335021272086100802556094) + 5736334794601041901990813024090
47277183885639680000) + 1272935063665829315736416183610522832*(240031204937
714427494400*x^27 - 2621948941596237063782400*x^26 + 1236504505589681110548
4800*x^25 - 33969890064381284111155200*x^24 + 65360120291258796757811200*x^
23 - 106701725825102321939251200*x^22 + 162290307223249502039654400*x^21 -
216634228326470609547509760*x^20 + 253788172995391086570485760*x^19 - 28727
9159180291305208156160*x^18 + 304010591010966811155955200*x^17 - 2826446645
39994827031006720*x^16 + 258819256815163249845447936*x^15 - 229408132984166
521977166336*x^14 + 172649692294614969274168896*x^13 - 13331254137724638611
5890240*x^12 + 102031573634317834547976132*x^11 - 5979110268149411757214917
6*x^10 + 41613884937255303086792337*x^9 - 27246604251076689552043953*x^8 +
10718131725916893151555068*x^7 - 8685973988079840377705700*x^6 + 3673303058
277822225386926*x^5 - 809990362095044210054958*x^4 + 1362587089603925431664
856*x^3 + 111926768697602999806116*x^2 + 205702452014540322797289*x - 48844
17100172357749737)*sqrt(-2*x + 3))/(524288*x^28 - 5505024*x^27 + 24772608*x
^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 38
6777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120
*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 +
 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*
x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 +
 826686*x^2 + 137781*x + 59049)
```

giac [A] time = 4.19, size = 972, normalized size = 1.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
```

```
[Out] 11275/2283749599146799148302336*sqrt(7)*(240680481328*14^(3/4)*sqrt(2)*(sqr
t(14) + 4)^(3/2) + 722041443984*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(1
4) - 4) - 51574388856*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32
) + 2148932869*14^(3/4)*sqrt(7)*(-8*sqrt(14) + 32)^(3/2) + 8741903954560*14
```

```
^(1/4)*sqrt(2)*sqrt(sqrt(14) + 4) - 624421711040*14^(1/4)*sqrt(7)*sqrt(-8*s
qrt(14) + 32))*arctan(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4)
+ 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) + 11275/22837495991467991483
02336*sqrt(7)*(240680481328*14^(3/4)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 7220414
43984*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) - 51574388856*14^(
3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) + 2148932869*14^(3/4)*sq
rt(7)*(-8*sqrt(14) + 32)^(3/2) + 8741903954560*14^(1/4)*sqrt(2)*sqrt(sqrt(1
4) + 4) - 624421711040*14^(1/4)*sqrt(7)*sqrt(-8*sqrt(14) + 32))*arctan(-1/2
8*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(
-1/8*sqrt(14) + 1/2)) - 11275/4567499198293598296604672*sqrt(7)*(3438292590
4*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 103148777712*14^(3/4)*sqr
t(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 361020721992*14^(3/4)*(sqr
t(14) + 4)*sqrt(-8*sqrt(14) + 32) - 15042530083*14^(3/4)*(-8*sqrt(14) + 32)
^(3/2) + 1248843422080*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4) + 437095
1977280*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(14^(1/4)*sqrt(1/2)*sqrt(-2*x +
 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 11275/456749919829359829660
4672*sqrt(7)*(34382925904*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 10
3148777712*14^(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 361
020721992*14^(3/4)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) - 15042530083*14^(
3/4)*(-8*sqrt(14) + 32)^(3/2) + 1248843422080*14^(1/4)*sqrt(7)*sqrt(2)*sqrt
(sqrt(14) + 4) + 4370951977280*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(-14^(1/
4)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/20
4816509808685350912*(232787883652335*(2*x - 3)^17*sqrt(-2*x + 3) + 13820106
668010555*(2*x - 3)^16*sqrt(-2*x + 3) + 389618236717151904*(2*x - 3)^15*sqr
t(-2*x + 3) + 6925854690067471092*(2*x - 3)^14*sqrt(-2*x + 3) + 86924717622
268515682*(2*x - 3)^13*sqrt(-2*x + 3) + 817308030405306394458*(2*x - 3)^12*
sqrt(-2*x + 3) + 5960699611609964201316*(2*x - 3)^11*sqrt(-2*x + 3) + 34438
539253455396724476*(2*x - 3)^10*sqrt(-2*x + 3) + 159569809573892673649239*(
2*x - 3)^9*sqrt(-2*x + 3) + 596312099501239401271299*(2*x - 3)^8*sqrt(-2*x
+ 3) + 1797250621001927736488676*(2*x - 3)^7*sqrt(-2*x + 3) + 4343978582610
098069631672*(2*x - 3)^6*sqrt(-2*x + 3) + 8317212692450176764092592*(2*x -
3)^5*sqrt(-2*x + 3) + 12350951282904546626644288*(2*x - 3)^4*sqrt(-2*x + 3)
 + 13738697725192288735303872*(2*x - 3)^3*sqrt(-2*x + 3) + 1078847966186370
2869789824*(2*x - 3)^2*sqrt(-2*x + 3) - 5340653236079401357791744*(-2*x + 3
)^(3/2) + 1255138952440667471476992*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7
)^9 + 1/3280733202692679552*(235862511885*(2*x - 3)^9 - 107316677325*(2*x -
 3)^8 + 80348352084*(2*x - 3)^7 - 64554208290*(2*x - 3)^6 + 49954696792*(2*
x - 3)^5 - 35035280280*(2*x - 3)^4 + 21058773120*(2*x - 3)^3 - 10093321056*
(2*x - 3)^2 + 6831901440*x - 10859127552)/((2*x - 3)^9*sqrt(-2*x + 3))
```

maple [A] time = 0.07, size = 719, normalized size = 1.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x+3)^(21/2)/(2*x^2+x+1)^10,x)
```

```
[Out] 1/86812553324672*(-1352841099712333/8192*(-2*x+3)^(31/2)+4606702222670185/7
86432*(-2*x+3)^(33/2)-25865320405815/262144*(-2*x+3)^(35/2)-347698778390586
0258979/1536*(-2*x+3)^(3/2)+9364999706478908741137/2048*(-2*x+3)^(5/2)-2385
1905772903279054347/4096*(-2*x+3)^(7/2)+192983613795383541041317/36864*(-2*
x+3)^(9/2)-57758421475348449750643/16384*(-2*x+3)^(11/2)+603330358695846954
11551/32768*(-2*x+3)^(13/2)-149770885083493978040723/196608*(-2*x+3)^(15/2)
+66256899944582155696811/262144*(-2*x+3)^(17/2)-17729978841543630405471/262
144*(-2*x+3)^(19/2)+2869878271121283060373/196608*(-2*x+3)^(21/2)-165574989
211387894481/65536*(-2*x+3)^(23/2)+454060016891836885 81/131072*(-2*x+3)^(25
/2)-43462358811134257841/1179648*(-2*x+3)^(27/2)+192384852501874197/65536*(
-2*x+3)^(29/2)+544765170330150812273/1024*(-2*x+3)^(1/2))/(14*x+(-2*x+3)^2-
7)^9-206922416016525/127441383880959773 9008*(7+2*14^(1/2))^(1/2)*14^(1/2)*l
n(-2*x+3+14^(1/2)-(-2*x+3)^(1/2)*(7+2*14^(1/2))^(1/2))+389615613935075/6372
```

```
06919404798869504*(7+2*14^(1/2))^(1/2)*ln(-2*x+3+14^(1/2)-(-2*x+3)^(1/2)*(7
+2*14^(1/2))^(1/2))-206922416016525/637206919404798869504/(-7+2*14^(1/2))^(
1/2)*(7+2*14^(1/2))*14^(1/2)*arctan((2*(-2*x+3)^(1/2)-(7+2*14^(1/2))^(1/2))
/(-7+2*14^(1/2))^(1/2))+389615613935075/318603459702399434752/(-7+2*14^(1/2
))^(1/2)*(7+2*14^(1/2))*arctan((2*(-2*x+3)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+
2*14^(1/2))^(1/2))-110005543624625/318603459702399434752/(-7+2*14^(1/2))^(1
/2)*14^(1/2)*arctan((2*(-2*x+3)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))
^(1/2))+206922416016525/1274413838809597739008*(7+2*14^(1/2))^(1/2)*14^(1/2
)*ln(-2*x+3+14^(1/2)+(-2*x+3)^(1/2)*(7+2*14^(1/2))^(1/2))-389615613935075/6
37206919404798869504*(7+2*14^(1/2))^(1/2)*ln(-2*x+3+14^(1/2)+(-2*x+3)^(1/2)
*(7+2*14^(1/2))^(1/2))-206922416016525/637206919404798869504/(-7+2*14^(1/2)
)^(1/2)*(7+2*14^(1/2))*14^(1/2)*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/
2))/(-7+2*14^(1/2))^(1/2))+389615613935075/318603459702399434752/(-7+2*14^(
1/2))^(1/2)*(7+2*14^(1/2))*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/2))/(
-7+2*14^(1/2))^(1/2))-110005543624625/318603459702399434752/(-7+2*14^(1/2))
^(1/2)*14^(1/2)*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/
2))^(1/2))+1/5367029731/(-2*x+3)^(19/2)+5/4802079233/(-2*x+3)^(17/2)+73/237
27920916/(-2*x+3)^(15/2)+165/25705247659/(-2*x+3)^(13/2)+2365/221460595216/
(-2*x+3)^(11/2)+30349/1993145356944/(-2*x+3)^(9/2)+854095/43406276662336/(-
2*x+3)^(7/2)+75933/3100448333024/(-2*x+3)^(5/2)+8519225/260437659974016/(-2
*x+3)^(3/2)+891605/12401793332096/(-2*x+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{10}\left(-2\,x+3\right)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)
```

mupad [B] time = 0.56, size = 567, normalized size = 0.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(21/2)*(x + 2*x^2 + 1)^10),x)
```

```
[Out] ((184192*(2*x - 3)^2)/47481 - (18944*x)/2261 - (15552*(2*x - 3)^3)/4199 + (
5666272*(2*x - 3)^4)/1440257 - (63490768*(2*x - 3)^5)/12962313 + (533495672
*(2*x - 3)^6)/70572593 - (1111521492*(2*x - 3)^7)/70572593 + (78007323158*(
2*x - 3)^8)/1482024453 - (250239440467*(2*x - 3)^9)/494008151 + (1118693654
785651073*(2*x - 3)^10)/453254454575104 + (1624300450152249301*(2*x - 3)^11
)/97125954551808 + (35048653520674948897*(2*x - 3)^12)/906508909150208 + (9
5527511967437577915*(2*x - 3)^13)/1813017818300416 + (564066299973141561054
7*(2*x - 3)^14)/114220122552926208 + (1737142288764447500149*(2*x - 3)^15)/
50764498912411648 + (12971210667229097601055*(2*x - 3)^16)/7107029847737630
72 + (32723441206946795665235*(2*x - 3)^17)/4264217908642578432 + (10264579
7034777710681325*(2*x - 3)^18)/39799367147330732032 + (14609317874302006653
15*(2*x - 3)^19)/2094703534070038528 + (687618468821894139745*(2*x - 3)^20)
/4528256169239642112 + (39968995676603847725*(2*x - 3)^21)/1509418723079880
704 + (5940132943613849875*(2*x - 3)^22)/1625527855624486912 + (57179785036
20010375*(2*x - 3)^23)/14629750700620382208 + (178056995818325525*(2*x - 3)
^24)/5689347494685704192 + (179665281323275*(2*x - 3)^25)/10159549097653043
2 + (1433237383402275*(2*x - 3)^26)/22757389978742816768 + (24229218097975*
(2*x - 3)^27)/22757389978742816768 + 37120/2261)/(20661046784*(3 - 2*x)^(19
/2) - 92974710528*(3 - 2*x)^(21/2) + 199231522560*(3 - 2*x)^(23/2) - 270069
397248*(3 - 2*x)^(25/2) + 259475340096*(3 - 2*x)^(27/2) - 187609683744*(3 -
```

```
 2*x)^(29/2) + 105782451264*(3 - 2*x)^(31/2) - 47554666992*(3 - 2*x)^(33/2)
 + 17278167438*(3 - 2*x)^(35/2) - 5111496103*(3 - 2*x)^(37/2) + 1234154817*
(3 - 2*x)^(39/2) - 242625852*(3 - 2*x)^(41/2) + 38550456*(3 - 2*x)^(43/2) -
 4883634*(3 - 2*x)^(45/2) + 482454*(3 - 2*x)^(47/2) - 35868*(3 - 2*x)^(49/2
) + 1890*(3 - 2*x)^(51/2) - 63*(3 - 2*x)^(53/2) + (3 - 2*x)^(55/2)) - (atan
(((- 7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*(3 - 2*x
)^(1/2)*43774618035829144330316520640625i)/(3300086980477615835608700826192
63806430093600589158123831296*((7^(1/2)*42709096709460747387242744942497717
8671875i)/165004349023880791780435041309631903215046800294579061915648 + 80
3365829195061345550676106938401175484375/2357204986055439882577643447280455
7602149542899225580273664)) + (43774618035829144330316520640625*7^(1/2)*(-
7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*(3 - 2*x)^(1/
2))/(330008698047761583560870082619263806430093600589158123831296*((7^(1/2)
*427090967094607473872427449424977178671875i)/16500434902388079178043504130
9631903215046800294579061915648 + 80336582919506134555067610693840117548437
5/23572049860554398825776434472804557602149542899225580273664)))*(- 7^(1/2)
*30540258843957888971i - 2293002953699236822393)^(1/2)*11275i)/318603459702
399434752 + (atan(((7^(1/2)*30540258843957888971i - 2293002953699236822393)
^(1/2)*(3 - 2*x)^(1/2)*43774618035829144330316520640625i)/(3300086980477615
83560870082619263806430093600589158123831296*((7^(1/2)*42709096709460747387
2427449424977178671875i)/16500434902388079178043504130963190321504680029457
9061915648 - 803365829195061345550676106938401175484375/2357204986055439882
5776434472804557602149542899225580273664)) - (43774618035829144330316520640
625*7^(1/2)*(7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*
(3 - 2*x)^(1/2))/(330008698047761583560870082619263806430093600589158123831
296*((7^(1/2)*427090967094607473872427449424977178671875i)/1650043490238807
91780435041309631903215046800294579061915648 - 8033658291950613455506761069
38401175484375/23572049860554398825776434472804557602149542899225580273664)
))*(7^(1/2)*30540258843957888971i - 2293002953699236822393)^(1/2)*11275i)/3
18603459702399434752
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

3.49 $\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\, dx$

Optimal. Leaf size=1058

result too large to display

```
[Out] 1/33516*(113+373*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^18+1/7976808*(40657+107329*x
)/(3-2*x)^(39/2)/(2*x^2+x+1)^17+5/595601664*(751303+1831285*x)/(3-2*x)^(39/
2)/(2*x^2+x+1)^16+1/25015269888*(184959785+429411497*x)/(3-2*x)^(39/2)/(2*x
^2+x+1)^15+1/4902992898048*(41652915209+92630823167*x)/(3-2*x)^(39/2)/(2*x^
2+x+1)^14+1/297448235814912*(2871555518177+6100156355517*x)/(3-2*x)^(39/2)/
(2*x^2+x+1)^13+1/7138757659557888*(77559130805859+156274047129113*x)/(3-2*x
)^(39/2)/(2*x^2+x+1)^12+5/1099368679571914752*(2656658801194921+50208801761
34289*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^11+1/3420258114223734784*(4518792158520
8601+78752911037377255*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^10+1/43095252239219058
2784*(6063974149878048635+9477172618423641847*x)/(3-2*x)^(39/2)/(2*x^2+x+1)
^9+1/4826668250792534527180 8*(6918336011449258548 31+9194981928740555812 21*x
)/(3-2*x)^(39/2)/(2*x^2+x+1)^8+23/15767116285922279455457 28*(91949819287405
5581221+9082871360924674685 17*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^7+115/101879828
30903626725064704*(908287136092467468517+298281884944522225747*x)/(3-2*x)^(
39/2)/(2*x^2+x+1)^6+23/20375965661807253450129408*(2599313568802265110081-1
042614244862318737918 7*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^5-23/20018492580021161
284337664*(10426142448623187379187+27513723463194262383705*x)/(3-2*x)^(39/2
)/(2*x^2+x+1)^4-115/764342443964444339947438 08*(265132244281690164788 43+306
73415406553789342019*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^3-115/125891696652967303
050166272*(88411609113007981044643-57122695362451521629 63*x)/(3-2*x)^(39/2)
/(2*x^2+x+1)^2+115/195831528126838026966925312*(28561347681225760814815+965
93481283901949034610 7*x)/(3-2*x)^(39/2)/(2*x^2+x+1)+1/133*x/(3-2*x)^(39/2)/
(2*x^2+x+1)^19+115/32482612650988307365321273688297313696481 28*ln(3-2*x+14^
(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(30297118912219360725028693061-80
611109111432760539830227 87*14^(1/2))*(-14+4*14^(1/2))^(1/2)-115/32482612650
98830736532127368829731369648128*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1
/2))^(1/2))*(30297118912219360725028693061-8061110911143276053983022787*14^
(1/2))*(-14+4*14^(1/2))^(1/2)+115/16241306325494153682660636844148656848240
64*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(3
0297118912219360725028693061+8061110911143276053983022787*14^(1/2))*(14+4*1
4^(1/2))^(1/2)-115/1624130632549415368266063684414865684824064*arctan((2*(3
-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(30297118912219360
725028693061+8061110911143276053983022787*14^(1/2))*(14+4*14^(1/2))^(1/2)-9
270277547814767462080476205 05/5800466544819340600950227444338806017228 8/(3-
2*x)^(1/2)+11155168222970774232376891145/1685166332532616560247354224017408
/(3-2*x)^(23/2)+14011818498091020272474956375/1011099799519569936148412534 4
104448/(3-2*x)^(21/2)-1305695962836335553428578 5425/10692401435725356272394
1220352/(3-2*x)^(39/2)-3948194343291401740321996415/202881463139404195937 73
4623232/(3-2*x)^(37/2)-3046882292626202227364808 11/537361713180043545997243
056128/(3-2*x)^(35/2)+21243158467565674556538629 25/168885109856585114456276
3890688/(3-2*x)^(33/2)+4765751507451411879609592953 5/666328524343253997036 5
8138959872/(3-2*x)^(31/2)+3491161999397471406217275198 5/1246679174577701026
71360389021696/(3-2*x)^(29/2)+1490663098087947608430174048 25/16249818206564
5168309566300173 1072/(3-2*x)^(27/2)+1584861396416906654373438017 1/601845118
7616487715169122228633 60/(3-2*x)^(25/2)-10119027441277961867857327524 5/3963
5112141167141497017771348889 43616/(3-2*x)^(15/2)-460503190416958283087439 33
7135/34350430522344855964082068502370844672/(3-2*x)^(13/2)-2211619588790911
794826342607495/40692048464931598603604911918193154457 6/(3-2*x)^(11/2)-4986
6814791877818534173165227 75/870069981722901090142534116650820902584 32/(3-2*
x)^(3/2)+17344136814980437866193586970 5/8965084889073520100515924471772610 5
6/(3-2*x)^(19/2)-2272409082346990515271351954 5/160427834857105096535548122 1
264572416/(3-2*x)^(17/2)-14340146755077724762794043702 5/7398554266351199746
109983985126028083 2/(3-2*x)^(9/2)-4611053278117143010907562317585/725058318
```

```
1024175751187784305423507521536/(3-2*x)^(7/2)-405965372440630510720926890227/2071595194578335928910795515835287863296/(3-2*x)^(5/2)
```

Rubi [A] time = 2.49, antiderivative size = 1058, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {740, 822, 828, 826, 1169, 634, 618, 204, 628}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] -13056959628363355534285785425/(106924014357253562723941220352*(3 - 2*x)^(39/2)) - 3948194343291401740321996415/(202881463139404195937734623232*(3 - 2*x)^(37/2)) - 304688229262620222736480811/(537361713180043545997243056128*(3 - 2*x)^(35/2)) + 2124315846756567455653862925/(1688851098565851144562763890688*(3 - 2*x)^(33/2)) + 47657515074514118796095929535/(66632852434325399703658138959872*(3 - 2*x)^(31/2)) + 34911619993974714062172751985/(124667917457770102671360389021696*(3 - 2*x)^(29/2)) + 149066309808794760843017404825/(1624981820656451683095663001731072*(3 - 2*x)^(27/2)) + 15848613964169066543734380171/(601845118761648771516912222863360*(3 - 2*x)^(25/2)) + 11155168222970774232376891145/(1685166332532616560247354224017408*(3 - 2*x)^(23/2)) + 14011818498091020272474956375/(10110997995195699361484125344104448*(3 - 2*x)^(21/2)) + 173441368149804378661935869705/(896508488907352010051592447177261056*(3 - 2*x)^(19/2)) - 22724090823469905152713519545/(1604278348571050965355481221264572416*(3 - 2*x)^(17/2)) - 101190274412779618678573275245/(3963511214116714149701777134888943616*(3 - 2*x)^(15/2)) - 460503190416958283087439337135/(34350430522344855964082068502370844672*(3 - 2*x)^(13/2)) - 2211619588790911794826342607495/(406920484649315986036049119181931544576*(3 - 2*x)^(11/2)) - 143401467550777247627940437025/(73985542663511997461099839851260280832*(3 - 2*x)^(9/2)) - 4611053278117143010907562317585/(7250583181024175751187784305423507521536*(3 - 2*x)^(7/2)) - 405965372440630510720926890227/(2071595194578335928910795515835287863296*(3 - 2*x)^(5/2)) - 4986681479187781853417316522775/(87006998172290109014253411665082090258432*(3 - 2*x)^(3/2)) - 927027754781476746208047620505/(58004665448193406009502274443388060172288*Sqrt[3 - 2*x]) + x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (113 + 373*x)/(33516*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x)/(7976808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (41652915209 + 92630823167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + (2871555518177 + 6100156355517*x)/(297448235814912*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13) + (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(1099368679571914752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 + 78752911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (691833601144925854831 + 919498192874055581221*x)/(48266682507925345271808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23*(919498192874055581221 + 908287136092467468517*x))/(1576711628592227945545728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298281884944522225747*x))/(10187982830903626725064704*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^6) + (23*(2599313568802265110081 - 10426142448623187379187*x))/(20375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(10426142448623187379187 + 27513723463194262383705*x))/(20018492580021161284337664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(26513224428169016478843 + 30673415406553789342019*x))/(76434244396444433994743808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^3) - (115*(88411609113007981044643 - 5712269536245152162963*x))/(125891696652967303050166272*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(28561347681225760814815 + 965934812839019490346107*x))/(19583152812683802696692
```

```
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302
97118912219360725028693061 + 806110911143276053983022787*Sqrt[14])*ArcTan[
(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162
74707684133031842207432842412032 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
12219360725028693061 + 8061110911143276053983022787*Sqrt[14])*ArcTan[(Sqrt[
7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162747076
84133031842207432842412032 + (115*(30297118912219360725028693061 - 80611109
11143276053983022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] -
 Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414
865684824064 - (115*(30297118912219360725028693061 - 8061110911143276053983
022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sq
rt[14]]*Sqrt[3 - 2*x] - 2*x])/1624130632549415368266063684414865684824064
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
```

```
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 828

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1))/((m + 1)*(
c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x
)^(m + 1)*Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x])/(a + b*x + c*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx = \frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}} + \frac{\int \frac{3640-3164x}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{19}}\,dx}{3724}$$

Mathematica [C] time = 6.18, size = 1100, normalized size = 1.04

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + ((44296 + 146216*x)/(3528*(3
- 2*x)^(39/2)*(1 + x + 2*x^2)^18) + ((223125616 + 589021552*x)/(3332*(3 - 2
*x)^(39/2)*(1 + x + 2*x^2)^17) + ((865861681440 + 2110519336800*x)/(3136*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + ((2984274342235200 + 6928434268875840*
x)/(2940*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + ((9408813737133390720 + 209
24013532366815360*x)/(2744*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + ((2724306
5619141593598720 + 57873497074462503141120*x)/(2548*(3 - 2*x)^(39/2)*(1 + x
 + 2*x^2)^13) + ((72110377354780278913835520 + 145295342948683106164016640*
x)/(2352*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + ((1729014581089328963351798
01600 + 326770416680301421681066214400*x)/(2156*(3 - 2*x)^(39/2)*(1 + x + 2
*x^2)^11) + ((370557652515461812186329087129600 + 6458029672318863068265404
24448000*x)/(1960*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + ((6961755986759734
38759010577554944000 + 1088028437838790621809440473088716800*x)/(1764*(3 -
2*x)^(39/2)*(1 + x + 2*x^2)^9) + ((1111965063471244015489248163496668569600
 + 1477884081820868038735185945420330393600*x)/(1568*(3 - 2*x)^(39/2)*(1 +
```

```
x + 2*x^2)^8) + ((1427636023038958525418189623276039160217600 + 14102294542
802935921085802172484323479552OO*x)/(1372*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
7) + ((1283308803395067168818807997696073436639232000 + 4214391612869991217
701355842462048362373120OO*x)/(1176*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^6) + (
(35990904373909724999169578894625893014666444800O - 14436361213243981948316
934609927589309137960960OO*x)/(980*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) + ((
-115202162481686975947569138187222162686920928460800O - 3040089329780519199
0311701662609533815702602547200OO*x)/(784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
4) + ((-22557462826971452456811282633656274091251331090022400OO - 260969551
132552925541038265166507347084573298900992OOOO*x)/(588*(3 - 2*x)^(39/2)*(1
+ x + 2*x^2)^3) + ((-179025112076931306921152249904224040100017283046080512
0000 + 115668033214143596894295804604678509924267822733393920000*x)/(392*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + ((7287086092491046604340635690094746125
228872832203816960000O + 24644670900872826929692130734587768100251906626103
43034880000*x)/(196*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (-53055056666589708
749302646546014801249192995757488046080000O/(3 - 2*x)^(39/2) + (-1708089006
242241264480481073293611769771298388785813753364480000/(37*(3 - 2*x)^(37/2)
) + (-696740950089909200017539783692427216704271188038402697920512000/(3 -
2*x)^(35/2) + (757366667762147355602446006474261151597409525795681824661504
000000/(3 - 2*x)^(33/2) + (616772664905423340350737254793402194192083509401
0816556282758758400000/(31*(3 - 2*x)^(31/2)) + (980445504127015992472138196
645778610361943940861637274650890661068800000/(29*(3 - 2*x)^(29/2)) + (4496
4233234365801798259356678072391756466292408034159102502223134720000O0/(3 -
2*x)^(27/2) + (487904184130260773926886832047572655461484781443782543411352
841560457216000/(3 - 2*x)^(25/2) + (429268867215238023064148871550918822599
025420880676981706228025454182400000O0/(3 - 2*x)^(23/2) + (2893692593980364
723231826294558630623656919099359688069727689450554368000000000/(3 - 2*x)^(
21/2) + (118767476492930264374166633243140666046068763101817907661320807641
190359040000000/(3 - 2*x)^(19/2) + (-23130641371662285970537372414163682847
2251691242315976748933281043780325376000000O/(3 - 2*x)^(17/2) + (-992239519
653790860422623948957964852355985846800936213338418761762097950023680000000
/(3 - 2*x)^(15/2) + (-10941518315154632243157241587901809625083601209973176
6901467841654602614755123200000000/(3 - 2*x)^(13/2) + (-8073268485314233063
84033793409543156006921653522584930074801894393063474562191360000000O/(3 -
2*x)^(11/2) + (-44337987226211231305207361494572283981715203938096393248399
666651183999754721382400000000O/(3 - 2*x)^(9/2) + (-18330190892216697744173
706790143700087358561576136178754174544727578117325359791923200000000/(3 -
2*x)^(7/2) + (-553541210002735957048844214716028245499086746401723523324780
66055766166841372505894912000000O/(3 - 2*x)^(5/2) + (-113323856633918397403
43974428370683887566771471384841151672642393999283182139266339840000000000/
(3 - 2*x)^(3/2) + (-1327220262908131487403839635355234271426655189754352930
6435677723641008864036246751334400000000O/Sqrt[3 - 2*x] + ((Sqrt[(7 - I*Sqr
t[7])/2]*(-1858108368071384082365375489497327979997317265656094102900994881
3097412409650745451868160000000000 - (38534140062781031467679876224014966993
36335555921865837542016885265897482833115690092544000000000*I)*Sqrt[7])*Arc
Tanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) +
(Sqrt[(7 + I*Sqrt[7])/2]*(-185810836807138408236537548949732797999731726565
6094102900994881309741240965074545186816000000000O + (3853414006278103146767
987622401496699336335555921865837542016885265897482833115690092544000000000
*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqrt[7]]])/(-14 - (
2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)/294)/322)/350)/
378)/406)/434)/462)/490)/518)/546)/196)/392)/588)/784)/980)/1176)/1372)/156
8)/1764)/1960)/2156)/2352)/2548)/2744)/2940)/3136)/3332)/3528)/3724
```

fricas [B] time = 151.30, size = 2763, normalized size = 2.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="fricas")
```

```
[Out] 1/392164866433134591452265700785383646021637848990793338705942972429050741O
8177191061306767655456730731382076063194779070544791852259532242520616485 25
72160*(616525316537858546962128448983043227187951381815778781478549978900*5
795904991921858556653045419515717067178458593845454142080244780765852057823
32794174344701326^(1/4)*sqrt(128684608824630489703584217174321785034500513 9
4)*sqrt(14)*sqrt(7)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754
688*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*
x^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112
*x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 517079841212727
296*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 14912870282
33404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 27862
74020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 +
 3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*
x^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 28615445794952
97024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 17127610
05459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 79
0511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 +
 282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^2
2 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^1
9 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16
- 2296849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 18
2213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 265002817
27302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3
755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2
+ 19758444939*x + 3486784401)*sqrt(378148402880167888800346812933915372766 2
345024772741260943*sqrt(14) + 141490223718487283855707890366841241012101616
40127797919744)*arctan(1/34885554762731597076008789349408244975617249636749
132425750095898949140452865810818124470791304767731061126710516699978714580
822916583226301682355823209315648798319267851525748818094906005095731630992
227838434460546889854820576222503959439208139217OO*579590499192185855665304
541951571706717845859384545414208024478076585205782332794174344701326^(3/4)
*sqrt(163485733532311285081249267709263950334945132741841731 1)*sqrt(6434230
441231524485179210858716089251725025697O)*sqrt(128684608824630489703584217 1
7432178503450051394)*sqrt(579590499192185855665304541951571706717845859384 5
454142080244780765852057823327941743447O1326^(1/4)*sqrt(128684608824630489 7
035842171743217850345005139 4)*sqrt(-2*x + 3)*sqrt(378148402880167888800346 8
12933915372766234502477274126O943*sqrt(14) + 14149022371848728385570789036 6
84124101210161640127797919744)*(8061110911143276053983022787*sqrt(14) - 302
971189122193607250286930 61) - 21038097668013253556956344328723682390547871 9
2594512041684573248748652161620808567413707456508928153 40*x + 9637320505996
21794425456308219340060829468062999882820661390*sqrt(166789371965963959581 0
98742817586289130679764812156476721038706576007991289033281726) + 315571465
020198803354345164930855235858218078888917680625268598731229782424312128511 2
056118476339223010)*(3029711891221936072502869306 1*sqrt(14)*sqrt(7) - 11285
555275600586475576231901 8*sqrt(7))*sqrt(378148402880167888800346812933915 37
27662345024772741260943*sqrt(14) + 1414902237184872838557078903668412410121
016164012779791974 4) - 1/3316417226807754157604240694473580354307118412805 7
805445740643992848947205475131833297639875732592434272266883677954804521721
584006729715127306903510*579590499192185855665304541951571706717845859384 54
541420802447807658520578233279417434470132 6^(3/4)*sqrt(128684608824630489 70
3584217174321785034500513 94)*(3029711891221936072502869306 1*sqrt(14)*sqrt(7
) - 1128555527560058647557623190 18*sqrt(7))*sqrt(-2*x + 3)*sqrt(37814840288
016788880034681293391537276623450247727412609 43*sqrt(14) + 1414902237184872
838557078903668412410121016164012779791974 4) + 2/7*sqrt(14)*sqrt(7) + sqrt(
7)) + 61652531653785854696212844898304322718795138181577878147854997890 0*57
959049919218585566530454195157170671784585938454541420802447807658520578233
2794174344701326^(1/4)*sqrt(12868460882463048970358421717432178503450051394
)*sqrt(14)*sqrt(7)*(549755813888*x^58 - 11269994184704*x^57 + 1070649447546
88*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x
```

```
^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112*
x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 5170798412127272
96*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 149128702823
3404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 278627
4020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 +
3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x
^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 286154457949529
7024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 171276100
5459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 790
511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 +
282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22
 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19
 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 -
 2296849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 182
213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 2650028172
7302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 37
55740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 +
 19758444939*x + 3486784401)*sqrt(37814840288016788880034681293391537276623
45024772741260943*sqrt(14) + 1414902237184872838557078903668412410121016164
0127797919744)*arctan(1/882212681369915578508303477421571883476414798304262
215955022242191758442824830361504884464751899654962438205380446610027450746
192342098621348232500464932063975676279800152639314467699279173080080434944
06341475991998227625530289790494302092900288913988891810201600*579590499192
185855665304541951571706717845859384545414208024478076585205782332794174344
701326^(3/4)*sqrt(1634857335323112850812492677092639503349451327418417311)*
sqrt(12868460882463048970358421717432178503450051394)*sqrt(-411483036686051
02441456509058170322829014409271501775935163876370158714880*579590499192185
855665304541951571706717845859384545414208024478076585205782332794174344701
326^(1/4)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(-2*x +
 3)*sqrt(3781484028801678888003468129339153727662345024772741260943*sqrt(14
) + 14149022371848728385570789036684124101210161640127797919744)*(806111091
1143276053983022787*sqrt(14) - 30297118912219360725028693061) - 86568203145
318221187283609975727790977789396744211474536443644223954841516195622280522
256097905125871171599697702873041301905017935839321574155334914762936684914
90879550259200*x + 39655939073240735699697464832307040228010334971057954913
097176452275244656121253105297648773136457461603590116807858434249634034483
200*sqrt(166789371965963959581098742817586289130679764812156476721038706576
007991289033281726) + 12985230471797733178092541496359168646668409511631721
180466546633593226227429343342078338414685768880675739954655430956195285752
690375898236123300237214440502737236319325388800)*(302971189122193607250286
93061*sqrt(14)*sqrt(7) - 112855552756005864755762319018*sqrt(7))*sqrt(37814
84028801678888003468129339153727662345024772741260943*sqrt(14) + 1414902237
1848728385570789036684124101210161640127797919744) - 1/33164172268077541576
042406944735803543071184128057805445740643992848947205475131833297639875732
592434272266883677954804521721584006729715127306903510*57959049919218585566
5304541951571706717845859384545414208024478076585205782332794174344701326^(
3/4)*sqrt(12868460882463048970358421717432178503450051394)*(302971189122193
60725028693061*sqrt(14)*sqrt(7) - 112855552756005864755762319018*sqrt(7))*s
qrt(-2*x + 3)*sqrt(37814840288016788880034681293391537276623450247727412609
43*sqrt(14) + 14149022371848728385570789036684124101210161640127797919744)
- 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 131989413465*57959049919218585566530454
1951571706717845859384545414208024478076585205782332794174344701326^(1/4)*s
qrt(12868460882463048970358421717432178503450051394)*(777850730975521785282
7317402300628134029188898204494505702056024604672*x^58 - 159459399849981965
982960006747162876747598372413192137366892148504395776*x^57 + 1514864298574
828676838120064098047329102184537925325304985475410791759872*x^56 - 8922920
197702954279424536475114108048248233314852830759853471017224634368*x^55 + 3
7049516473070962334132314494495535591639403546454145111565499223693590528*x
^54 - 118034716093527123457170227067542059725196738523651032986322545956112
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826368*x^53 + 309150088371501812670279456678545863084687431424928239641662378993516544000*x^52 - 70298132195777230673383983075115795266608470751142757
92779434714800806952966*x^51 + 143463306723712355468305112439226911663471236
034390984807425153131791305932 8*x^50 - 26682695055901722800490443671090900 0
228611047955821555812116193496004139417 6*x^49 + 45738412074465502626998211 9
71750106503531633604394327579162124902780779888 64*x^48 - 731617424135086661
98700167998340894258382768146404489814486698266055661322 24*x^47 + 110096075
22130108303720327150964714549103431151620256934238701471906607923200*x^46 -
 156437415843115561838300936832880604912543053580606458357805837278211714 58
048*x^45 + 21100253525322245323369387355308423247443333718020271970799243 11
9655863189504*x^44 - 271651277558601625195627765823180653075779937594404074
127310796642528589250 56*x^43 + 33434860614488797445569848022836846490704329
3071770126202950467443548865167 36*x^42 - 39423053452220154576417414767228 02
069850250206709428727323239502827040538624 0*x^41 + 44727121020483655457697 1
6368457363266929149987816260086889148108993794880307 2*x^40 - 48867241641804
491090588875438285611681021058445732456635375173947494808682496*x^39 + 5141
89405125347735489480063031695833076028700966156531804416445886706986516 48*x
^38 - 523215843733739214051023856561058527374728861876979686267810380133929
69793536*x^37 + 515116630975130382984180173022800410425552022016633269579 09
758403045704597504*x^36 - 489810603519174731161686619741905999571949810222 8
527313355726274372093503078 4*x^35 + 451996214903394043451534985348179466091
561636956251934795503873165785615564 80*x^34 - 404880582733214195930596097 36
2336238083826588763975168633651775570985940418 56*x^33 + 350560838720748004 8
7067567855249064178835868891722789291640219304625845567488*x^32 - 295492464
30146582583344598166261232974679787401051000932261220746996918255616*x^31 +
 242338937838739945110707883369250065804077743121445205880335495460844040 35
584*x^30 - 19178256794663007372927055945418126742331837591606363287110323 07
2444760326144*x^29 + 148414880649560666743967289911504439321719723677103997
2745088667423745245184 0*x^28 - 11184958165680426483017504655762227434727279
7340381879341496926504615810170 88*x^27 + 80897166364264609042152707250524 10
8677377533249635571542371192388023106928 64*x^26 - 577809432215066768352460 4
05565609178817603203484066718688992476626078715084 8*x^25 + 400198121753487 5
0941930164544740009926063843881646165362756970090082934456 32*x^24 - 2633329
69512268109981550996833745185555279818648244876840348760007175779123 2*x^23
+ 17542366897322253251089903529813657607145515074994145464959180389051321 54
880*x^22 - 110527023465119560828545083126526137277054522543239186566677257 1
865191809024*x^21 + 65776796070937473101898342886098749096522587138982913 91
668030557060721541 12*x^20 - 41868450855170421700463282020153827146482312561
5024056822995425518287847424*x^19 + 229225420425444294191147646176341699520
3235018284963744662787585752222924 80*x^18 - 1265190599528928078062208761554
970753195041029849816038292222519662220410 88*x^17 + 78935884826693368065254
82893190821189151939412773442470504902282581534310 4*x^16 - 3249817795278117
57715701023568508462292255883939165331194467267444295598 08*x^15 + 204918859
4701091333375687670878228916224957778925743317752942310286635827 2*x^14 - 10
709341936487878696123475460109781566666232126271592963297115430325321728*x^
13 + 25781421518498561820751339189884584471120079495619574973074655469350 42
048*x^12 - 340705360655172629909903757378974739082831055743924947119182311 8
374010880*x^11 + 36149668743642480393069390477870078076949048741584098494 16
22807133945856*x^10 - 374953079019890060800626107146254604879378550577160 50
19726591306896506 88*x^9 + 361092974099972372832558605269197722032357635208 0
49111095191908288299008*x^8 + 141002184747768997009392595202128779106364989
8404768703196210192082534 40*x^7 + 146999213652233656885845331514573869620 24
50401105478544813090771472220 16*x^6 + 531400537229235556119293924358341982 7
5650668430851979724323340159025152*x^5 + 2575996506690501628749016279663913
8798257488660077636512705521601478656*x^4 + 656150053590976829964372071235 1
478750499548998321662130594802672074752*x^3 + 197886524098866028084493164 34
0758882951573699949383461081430556776857 6*x^2 - 37814840288016788880034681 2
93391537276623450247727412609 43*sqrt(14)*(549755813888*x^58 - 1126999418470
4*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x^5
```

4 - 8342252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 + 3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 + 282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 + 19758444939*x + 3486784401) + 2795626794748522834434667972681081171892038420337550281205805649756 16*x + 4933459049556216766649414069437202068044773682948618143304362911 3344)*sqrt(3781484028801678888003468129339153727662345024772741260943*sqrt(14) + 14149022371848728385570789036684124101210161640127797919744)*log(41148303668605102441456509058170322829014409271501775935163876370158714880/16348573353231128508124926770926395033494513274184173 11*5795904991921858556653045419515717067178458593845454142080244780765852057823327941743447013 26^(1/4)*sqrt(1286846088246304897035842171743217850345005139 4)*sqrt(-2*x + 3)*sqrt(3781484028801678888003468129339153727662345024772741260943*sqrt(14) + 14149022371848728385570789036684124101210161640127797919744)*(8061110911143276053983022787*sqrt(14) - 3029711891221936072502869306 1) - 52951533613915553191922904161192663574869868505075814894398859634892573324132628969090213563223147240807581427299254272 00*x + 2425651352960602183821419752470082360447570412145758189601975348144486083461 1200*sqrt(16678937196596395958109874281758628913067976481215647672103870657600799128903328172 6) + 79427300420873329787884356241788995362304802757613722341598289452338859986198943453635320344834720861211372140948881408 00) - 131989413465*5795904991921858556653045419515717067178458593845454142080244780765852057823327941743447013 26^(1/4)*sqrt(1286846088246304897035842171743217850345005139 4)*(77785073097552178528273174023006281340291888982044945057020560246046 72*x^58 - 1594593998499819659829600067471628767475983724131921373668921485043957 76*x^57 + 151486429857482867683812006409804732910218453792532530498547541079175987 2*x^56 - 89229201977029542794245364751141080482482333148528307598534710172246343 68*x^55 + 37049516473070962334132314494495535591639403546454145111565499223693590528*x^54 - 118034716093527123457170227067542059725196738523651032986322545956112826368*x^53 + 30915008837150181267027945667854586308468743142492823964166237899351654400 0*x^52 - 702981321957772306733839830751157952666084707511427579277943471480080695296*x^51 + 143463306723712355468305112439226911663471236034390984807425153131791305932 8*x^50 - 266826950559017228004904436710909000228611047955821555812116193496004139417 6*x^49 + 45738412074465502626998211971750106503531633604394327579162124902780779888 64*x^48 - 73161742413508666198700167998340894258382768146404489814486698266055661322 24*x^47 + 11009607522130108303720327150964714549103431151620256934238701471906607923200*x^46 - 15643741584311556183830093683288060491254305358060645835780583727821171458048*x^45 + 21100253525322245323369387355308423247443333718020271970799243119655863189504*x^44 - 27165127755860162519562776582318065307577993759440407412731079664252858925056*x^43 + 3343486061448879744556984802283684649070432930717701262029504674435488651673 6*x^42 - 39423053452220154576417414767228020698502502067094287273232395028270405386240*x^41 + 44727121020483655457697163684573632669291499878162600868891481089937948803072*x^40 - 48867241641804491090588875438285611681021058445732456635375173947494808682496*x^39 + 5141894051253477354894800630316958330760287

0096615653180441644588670698651648*x^38 - 5232158437337392140510238565610 58
5273747288618769796862678103801339296979353 6*x^37 + 5151166309751303829841 8
0173022800410425552022016633269579097584030457045975 04*x^36 - 4898106035191
7473116168661974190599957194981022285273133557262743720935030784*x^35 + 451
99621490339404345153498534817946609156163695625193479550387316578561556480*
x^34 - 40488058273321419593059609736233623808382658876397516863365177557098
594041856*x^33 + 35056083872074800487067567855249064178835868891722789291 64
0219304625845567488*x^32 - 29549246430146582583344598166261232974679787401 0
5100093226122074699691825561 6*x^31 + 24233893783873994511070788336925006580
4077743121445205880335495460844040355 84*x^30 - 1917825679466300737292705594
5418126742331837591606363287110323072444760326144*x^29 + 148414880649560666
7439672899115044393217197236771039972745088667423745245184 0*x^28 - 11184958
1656804264830175046557622274347272797340381879341496926504615810170 88*x^27
+ 808971663642646090421527072505241086773775332496355715423711923880231069 2
864*x^26 - 577809432215066768352460405565609178817603203484066718688992476 6
260787150848*x^25 + 40019812175348750941930164544740009926063843881646165 36
27569700900829344563 2*x^24 - 263332969512268109981550996833745185555279818 6
4824487684034876000717577912 32*x^23 + 175423668973222532510899035298136576 0
714551507499414546495918038905132154880*x^22 - 110527023465119560828545083 1
2652613727705452254323918656667725718651918090 24*x^21 + 6577679607093747310
1898342886098749096522587138982913916680305570607215411 2*x^20 - 41868450855
1704217004632820201538271464823125615024056822995425518287847424*x^19 + 229
22542042544429419114764617634169952032350182849637446627875857522229248 0*x^
18 - 1265190599528928078062208761554970753195041029849816038292222519662220
41088*x^17 + 789358848266933680652548289319082118915193941277344247050490 22
825815343104*x^16 - 324981779527811757715701023568508462292255883939165331 1
944672674442955980 8*x^15 + 204918859470109133337568767087822891622495777892
574331775294231028663582 72*x^14 - 1070934193648787869612347546010978156666 6
2321262715929632971154303253217 28*x^13 + 25781421518498561820751339189884 58
4471120079495619574973074655469350420 48*x^12 - 3407053606551726299099037573
7897473908283105574392494711918231183740108 80*x^11 + 361496687436424803930 6
9390477870078076949048741584098494162280713394585 6*x^10 - 37495307901989006
0800626107146254604879378550577160501972659130689650688*x^9 + 3610929740999
7237283255860526919772203235763520804911109519190828829900 8*x^8 + 141002184
7477689970093925952021287791063649898404768703196210192082534 40*x^7 + 14699
9213652233656885845331514573869620245040110547854481309077147222016*x^6 + 5
3140053722923555611929392435834198275650668430851979724323340159025152*x^5
+ 257599650669050162874901627966391387982574886600776365127055216014786 56*x
^4 + 6561500535909768299643720712351478750499548998321662130594802672074752
*x^3 + 19788652409886602808449316434075888295157369994938346108143055677685
76*x^2 - 37814840288016788880034681293391537276623450247727412609 43*sqrt(14
)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754688*x^56 - 6306386
38006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x^53 + 21849572376
576000*x^52 - 49684091485814784*x^51 + 101394501297242112*x^50 - 1885833123
63618304*x^49 + 323261995581177856*x^48 - 517079841212727296*x^47 + 7781178
96260812800*x^46 - 1105641165387988992*x^45 + 1491287028233404416*x^44 - 19
19929663119949824*x^43 + 2363050939901804544*x^42 - 2786274020645928960*x^4
1 + 3161145685194047488*x^40 - 3453753931369283584*x^39 + 36340984671025233
92*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x^36 - 34617982122
47617536*x^35 + 3194540251789393920*x^34 - 2861544579495297024*x^33 + 24776
32938217930752*x^32 - 2088430257127768064*x^31 + 1712761005459316736*x^30 -
1355447485390974976*x^29 + 1048940886155151360*x^28 - 790511024135089152*x
^27 + 571750925528393856*x^26 - 408374103192240192*x^25 + 28284506959981372
8*x^24 - 186113897194906128*x^23 + 123982890381352520*x^22 - 78116367732251
996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^19 + 16200795673453
545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16 - 2296849711499532
*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 182213447974992*x^12
- 240797810407770*x^11 + 25549234281774*x^10 - 26500281727302*x^9 + 255207
01332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3755740313808*x^5 +

```
 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2 + 19758444939*x +
3486784401) + 2795626794748522834434667972681081171892038420337550281205805
64975616*x + 49334590495562167666494140694372020680447736829486181433043629
113344)*sqrt(3781484028801678888003468129339153727662345024772741260943*sqr
t(14) + 14149022371848728385570789036684124101210161640127797919744)*log(-4
1148303668605102441456509058170322829014409271501775935163876370158714880/1
634857335323112850812492677092639503349451327418417311*57959049919218585566
5304541951571706717845859384545414208024478076585205782332794174344701326^(
1/4)*sqrt(12868460882463048970358421717432178503450051394)*sqrt(-2*x + 3)*s
qrt(3781484028801678888003468129339153727662345024772741260943*sqrt(14) + 1
4149022371848728385570789036684124101210161640127797919744)*(80611109111432
76053983022787*sqrt(14) - 30297118912219360725028693061) - 5295153361391555
319192290416119266357486986850507581489439885963489257332413262896909021356
322314724080758142729925427200*x + 2425651352960602183821419752470082360447
5704121457581896019753481444860834611200*sqrt(16678937196596395958109874281
7586289130679764812156476721038706576007991289033281726) + 7942730042087332
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746225179791317800502510255796338156401518821079958557305776*(5285259508814
1665875251392948545451373376947250790400*x^57 - 109896779506627331516285609
3421299059440183747910041600*x^56 + 106072094893168533908962287996508349484
44579920210821120*x^55 - 63571167550234753994014104400074223346580880315719
352320*x^54 + 268751102085050752152483783816672599931031121283482910720*x^5
3 - 870946973219521114804962921504691759517713269107195904000*x^52 + 231375
8021932448312425321649336084981029506072497608458240*x^51 - 531660404716026
7290459856323292969345744886768161070776320*x^50 + 109354424880090472643664
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056465998405634456352194560*x^48 + 3530279423919880211160423903973594412746
2536376667298856960*x^47 - 567147089880685206131013139748919822977787771083
53803878400*x^46 + 85640241664030935730039797515882941408552267458802253561
856*x^45 - 122063250700174316553425220949165095613494323059071276548096*x^4
4 + 165018067996212231343716673011244333927488403644331103092736*x^43 - 212
762579742469905820226823821664465308559175943457404354560*x^42 + 2622073258
52831458520928585736224018299226513096563188826112*x^41 - 30944053790611241
1118620445892815079684504011563969741324288*x^40 + 351087306412578660000108
019219405351826065473130972707815424*x^39 - 3835545821005862463621676456708
92818138191443491318786949120*x^38 + 40349260752084990899888351465254740391
5763268860927101370368*x^37 - 410091833382540310980618746942733242840005307
528588546801664*x^36 + 4032324074419917922323480275120810038796848466261573
08542976*x^35 - 38299557981652752964191530266540999587508486258926597505024
0*x^34 + 352587259766861713156680120052199648639816399610100338851840*x^33
- 315079971582181801347294250924732868231627903206246048727040*x^32 + 27231
6634459399870536836933035003973818695505518285221314560*x^31 - 228671395190
671097020869564500875726797589816165421143277568*x^30 + 1868861116889859290
98566117844019918629526116042561389293568*x^29 - 14757502905599999483940628
7648843693901181887610273533861888*x^28 + 113537974641311616719165089124033
846938888435216187251000320*x^27 - 8519641562323339617019718851297502630839
3874494506050046976*x^26 + 614907175198867437939779042891506812095480715428
12762022208*x^25 - 43499929568624033785147670292431465440609985987022819309
056*x^24 + 30015307199183492418426115232917702261364741866517547318384*x^23
 - 19714530664252367893694794632442175393727220660187813722224*x^22 + 12908
687419060491715559483506875260114803121732707547895900*x^21 - 8152620728427
620176711248504306621849196751343566681977176*x^20 + 4826566229889649998651
082918574281667310767186073269174097*x^19 - 2980031288821257171626437270731
358463613690258748044875631*x^18 + 1674381797717888336240082619136481913141
447194739865411447*x^17 - 8938932115161338699060832431287058759588041285935
29339933*x^16 + 539470558336347193822687371553759571054898242285358894340*x
^15 - 242275403875001443743419975934494764357192021279244664252*x^14 + 1307
86287070310326986845647168054788265093887227255620788*x^13 - 73538381632205
```

```
950970872198730312615396368885742113789428*x^12 + 2033263055373138660211729
32490188746668950007879116154590*x^11 - 185841889627321318186553873625864802
12623851120277665058*x^10 + 45785290434797442432221248640851770216520645234
34159250*x^9 - 1589976397316459177542751340814719678836965386418728758*x^8
+ 2136884518140645208822032972708844209401147725933248644*x^7 + 52743183825
2429406648106098496733847843023830337908772*x^6 + 5912933716464809804680808
56862103952285194702447206232*x^5 + 153671770129689537528196360895808154174
885919188027188*x^4 + 77286799075459568078148376312494588624748077088337625
*x^3 + 13203155064763141960070155528810313105199695006969241*x^2 + 41100428
98499321701713055782797445718557813264221007*x + 14248811486313979718769861
8852924003909944526763627)*sqrt(-2*x + 3))/(549755813888*x^58 - 11269994184
704*x^57 + 107064944754688*x^56 - 630638638006272*x^55 + 2618521301286912*x
^54 - 8342252417974272*x^53 + 21849572376576000*x^52 - 49684091485814784*x^
51 + 101394501297242112*x^50 - 188583312363618304*x^49 + 323261995581177856
*x^48 - 517079841212727296*x^47 + 778117896260812800*x^46 - 110564116538798
8992*x^45 + 1491287028233404416*x^44 - 1919929663119949824*x^43 + 236305093
9901804544*x^42 - 2786274020645928960*x^41 + 3161145685194047488*x^40 - 345
3753931369283584*x^39 + 3634098467102523392*x^38 - 3697893960325791744*x^37
 + 3640651752731836416*x^36 - 3461798212247617536*x^35 + 319454025178939392
0*x^34 - 2861544579495297024*x^33 + 2477632938217930752*x^32 - 208843025712
7768064*x^31 + 1712761005459316736*x^30 - 1355447485390974976*x^29 + 104894
0886155151360*x^28 - 790511024135089152*x^27 + 571750925528393856*x^26 - 40
8374103192240192*x^25 + 282845069599813728*x^24 - 186113897194906128*x^23 +
 123982890381352520*x^22 - 78116367732251996*x^21 + 46488580159296898*x^20
- 29591055660829971*x^19 + 16200795673453545*x^18 - 8941894120163277*x^17 +
 5578893209169441*x^16 - 2296849711499532*x^15 + 1448289882400788*x^14 - 75
6896247319212*x^13 + 182213447974992*x^12 - 240797810407770*x^11 + 25549234
281774*x^10 - 26500281727302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 +
 10389354811164*x^6 + 3755740313808*x^5 + 1820618017974*x^4 + 463742325333*
x^3 + 139858796529*x^2 + 19758444939*x + 3486784401)
```

giac [**A**]   time = 6.84, size = 1382, normalized size = 1.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="giac")
```

```
[Out] 115/5820884187057104679865572244942878614409445376*sqrt(7)*(902844422048046
918046098552144*14^(3/4)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 2708533266144140754
138295656432*14^(3/4)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) - 193466661
867438625295592546888*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32
) + 8061110911143276053983022787*14^(3/4)*sqrt(7)*(-8*sqrt(14) + 32)^(3/2)
+ 27146218545348547209625708982656*14^(1/4)*sqrt(2)*sqrt(sqrt(14) + 4) - 19
39015610382039086401836355904*14^(1/4)*sqrt(7)*sqrt(-8*sqrt(14) + 32))*arct
an(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))
/sqrt(-1/8*sqrt(14) + 1/2)) + 115/58208841870571046798655722449428786144094
45376*sqrt(7)*(902844422048046918046098552144*14^(3/4)*sqrt(2)*(sqrt(14) +
4)^(3/2) + 2708533266144140754138295656432*14^(3/4)*sqrt(2)*sqrt(sqrt(14) +
 4)*(sqrt(14) - 4) - 193466661867438625295592546888*14^(3/4)*sqrt(7)*(sqrt(
14) + 4)*sqrt(-8*sqrt(14) + 32) + 8061110911143276053983022787*14^(3/4)*sqr
t(7)*(-8*sqrt(14) + 32)^(3/2) + 27146218545348547209625708982656*14^(1/4)*s
qrt(2)*sqrt(sqrt(14) + 4) - 1939015610382039086401836355904*14^(1/4)*sqrt(7
)*sqrt(-8*sqrt(14) + 32))*arctan(-1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sq
rt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 115/1164176837
4114209359731144489885757228818890752*sqrt(7)*(1289777745782924168637283645
92*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 386933323734877250591185
093776*14^(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 1354266
633072070377069147828216*14^(3/4)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32) - 5
6427776378002932377881159509*14^(3/4)*(-8*sqrt(14) + 32)^(3/2) + 3878031220
```

```
764078172803672711808*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4) + 1357310
9272674273604812854491328*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(14^(1/4)*sqr
t(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 115/116417
68374114209359731144489885757228818890752*sqrt(7)*(12897774578292416863728
364592*14^(3/4)*sqrt(7)*sqrt(2)*(sqrt(14) + 4)^(3/2) + 38693332373487725059
1185093776*14^(3/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4)*(sqrt(14) - 4) + 135
4266633072070377069147828216*14^(3/4)*(sqrt(14) + 4)*sqrt(-8*sqrt(14) + 32)
 - 56427776378002932377881159509*14^(3/4)*(-8*sqrt(14) + 32)^(3/2) + 387803
1220764078172803672711808*14^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(14) + 4) + 135
73109272674273604812854491328*14^(1/4)*sqrt(-8*sqrt(14) + 32))*log(-14^(1/4
)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/241
12597431479447071556104988390860001680293888*(38591279629413862313248614614
4809805*(2*x - 3)^37*sqrt(-2*x + 3) + 4994416662656937088431754278268478521
5*(2*x - 3)^36*sqrt(-2*x + 3) + 3157104325190190818790417015768672100251*(2
*x - 3)^35*sqrt(-2*x + 3) + 129862663539742829727010168448772257537793*(2*x
 - 3)^34*sqrt(-2*x + 3) + 3907056032933059027385185682832433217956200*(2*x
- 3)^33*sqrt(-2*x + 3) + 91626342308240062913659469031676941328847688*(2*x
- 3)^32*sqrt(-2*x + 3) + 1743051839783716654458570168808933730174627004*(2*
x - 3)^31*sqrt(-2*x + 3) + 27638544507622729125093621837291437830917462708*
(2*x - 3)^30*sqrt(-2*x + 3) + 372498510070445411629537388290851713705080145
718*(2*x - 3)^29*sqrt(-2*x + 3) + 43299535169306873423374720142726663639696
51587314*(2*x - 3)^28*sqrt(-2*x + 3) + 438994445601123086236053311571438967
25828415934650*(2*x - 3)^27*sqrt(-2*x + 3) + 391609357365773780316151578457
972453648367489837454*(2*x - 3)^26*sqrt(-2*x + 3) + 30950317017588495750406
26937399363198202032753884252*(2*x - 3)^25*sqrt(-2*x + 3) + 217907196222246
81379416567825910093368668334676797780*(2*x - 3)^24*sqrt(-2*x + 3) + 137261
402924198725794062163116053277099106968046586092*(2*x - 3)^23*sqrt(-2*x + 3
) + 776171183055652545384871388553173912691352168500951876*(2*x - 3)^22*sqr
t(-2*x + 3) + 3950095526376994607880784338655934603802167995433166405*(2*x
- 3)^21*sqrt(-2*x + 3) + 18125803816832861597832766873339882118924015183338
007655*(2*x - 3)^20*sqrt(-2*x + 3) + 75083414508694050144426639977685085540
038804754309758915*(2*x - 3)^19*sqrt(-2*x + 3) + 28093265207334834351777609
0631271895235611343284275820345*(2*x - 3)^18*sqrt(-2*x + 3) + 9494495163668
91514866641779309597536478490489987954462580*(2*x - 3)^17*sqrt(-2*x + 3) +
2896666953760570249650513456393600983703549509654469117900*(2*x - 3)^16*sqr
t(-2*x + 3) + 7968283692957988567650795129108295704483768260379820818752*(2
*x - 3)^15*sqrt(-2*x + 3) + 19727494578812277658606009712831861626922226523
266435734336*(2*x - 3)^14*sqrt(-2*x + 3) + 43844103379423695842480030320760
116666491172278035172870400*(2*x - 3)^13*sqrt(-2*x + 3) + 87180772449453719
112409715850861698835279004734515297162496*(2*x - 3)^12*sqrt(-2*x + 3) + 15
4427451620079851403012035013949923367197814895239131529728*(2*x - 3)^11*sqr
t(-2*x + 3) + 242351725944359254347670713000225450988365795247877220072960*
(2*x - 3)^10*sqrt(-2*x + 3) + 334646091432259174045261099248092390902126268
663782608549888*(2*x - 3)^9*sqrt(-2*x + 3) + 403034519668261986708991686890
381317841470126237337802123264*(2*x - 3)^8*sqrt(-2*x + 3) + 418646794645473
329714896095169087072615863373434634780753920*(2*x - 3)^7*sqrt(-2*x + 3) +
369621715112196031007775193340564258755874521674193323966464*(2*x - 3)^6*sq
rt(-2*x + 3) + 272008032423513780299697431707644217391623176190273099661312
*(2*x - 3)^5*sqrt(-2*x + 3) + 162377109720555022535973021706211388170650620
411678744248320*(2*x - 3)^4*sqrt(-2*x + 3) + 755566667488842917662208922971
66603376040200755275694800896*(2*x - 3)^3*sqrt(-2*x + 3) + 2571521747914715
6311480451271603595696519278112265697558528*(2*x - 3)^2*sqrt(-2*x + 3) - 56
95058898488457914056616763522088045930624578769252515840*(-2*x + 3)^(3/2) +
 616047393270423249767303997369406352855404127230297374720*sqrt(-2*x + 3))/
((2*x - 3)^2 + 14*x - 7)^19 + 1/43768013439874312895399492130064309616640*(
991856055479912729664933375*(2*x - 3)^19 - 465215115289202563341931875*(2*x
 - 3)^18 + 376870004361848629670138100*(2*x - 3)^17 - 347816399209073565143
694750*(2*x - 3)^16 + 333480450533749292133360000*(2*x - 3)^15 - 3197782482
61094005065228000*(2*x - 3)^14 + 300292311231869293365336000*(2*x - 3)^13 -
```

```
 2722255222799805295582980000*(2*x - 3)^12 + 2355088194765073024377120000*(2*
x - 3)^11 - 1924039146350363202166406400*(2*x - 3)^10 + 14687029154936715246
1094400*(2*x - 3)^9 - 1035449637189814847512512000*(2*x - 3)^8 + 66520770217
483444975816704*(2*x - 3)^7 - 383082228160329893651456000*(2*x - 3)^6 + 1936
45363104610494632755200*(2*x - 3)^5 - 83518859448878344178688000*(2*x - 3)^4
+ 29503969631711848046592000*(2*x - 3)^3 - 80039800340355395764224 0*(2*x - 3
)^2 + 2964997328805454086144000*x - 4588143302395106515353600)/((2*x - 3)^19*
sqrt(-2*x + 3))
```

maple [A] time = 0.10, size = 989, normalized size = 0.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x+3)^(41/2)/(2*x^2+x+1)^20,x)
```

```
[Out] -71922796940311334682104901840 35/16241306325494153682660636844148656848240 6
4/(-7+2*14^(1/2))^(1/2)*(7+2*14^(1/2))*14^(1/2)*arctan((2*(-2*x+3)^(1/2)-(7
+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))-7192279694031133468210490184035/
16241306325494153682660636844148656848240 64/(-7+2*14^(1/2))^(1/2)*(7+2*14^(
1/2))*14^(1/2)*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2
))^(1/2))-7192279694031133468210490184035/324826126509883073653212736882973
1369648128*(7+2*14^(1/2))^(1/2)*14^(1/2)*ln(-2*x+3+14^(1/2)-(-2*x+3)^(1/2)*
(7+2*14^(1/2))^(1/2))+134575316332807901902129327475 65/8120653162747076841 3
3031842207432842412032/(-7+2*14^(1/2))^(1/2)*(7+2*14^(1/2))*arctan((2*(-2*x
+3)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))-34841686749052264833
78299702015/81206531627470768413303184220743284241203 2/(-7+2*14^(1/2))^(1/2
)*14^(1/2)*arctan((2*(-2*x+3)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(
1/2))+7192279694031133468210490184035/324826126509883073653212736882973 1369
648128*(7+2*14^(1/2))^(1/2)*14^(1/2)*ln(-2*x+3+14^(1/2)+(-2*x+3)^(1/2)*(7+2
*14^(1/2))^(1/2))+13457531633280790190212932747565/812065316274707684133031
842207432842412032/(-7+2*14^(1/2))^(1/2)*(7+2*14^(1/2))*arctan((2*(-2*x+3)^
(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))-348416867490522648337829
9702015/812065316274707684133031842207432842412032/(-7+2*14^(1/2))^(1/2)*14
^(1/2)*arctan((2*(-2*x+3)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)
)+683151246370725/30145677658996078082575630336/(-2*x+3)^(1/2)+769045155125
/100934188590388654294338048/(-2*x+3)^(9/2)+838467657280275/105509871806486
273289014706176/(-2*x+3)^(7/2)+9270470094105/1076631344964145645806272512/(
-2*x+3)^(5/2)+320421783064625/30145677658996078082575630336/(-2*x+3)^(3/2)+
8192823353/1863702218870150079292928/(-2*x+3)^(19/2)+8972680075/16675230379
36450070946304/(-2*x+3)^(17/2)+102495360575/164790511984308007011164 16/(-2*
x+3)^(15/2)+122484655975/17852305464966700759542784/(-2*x+3)^(13/2)+1081587
8546425/148036809932570026298362 4704/(-2*x+3)^(11/2)+1/3111898385606868039/
(-2*x+3)^(39/2)+10/2952313853011644037/(-2*x+3)^(37/2)+143/781964209716597 6
098/(-2*x+3)^(35/2)+355/52662895756423920 66/(-2*x+3)^(33/2)+52865/277038748
585308867472/(-2*x+3)^(31/2)+14333/3239566011683047240 6/(-2*x+3)^(29/2)+147
8345/16890426929878508371 68/(-2*x+3)^(27/2)+475387/3127856838866390439 20/(-
2*x+3)^(25/2)+16575515/70063993190607145838 08/(-2*x+3)^(23/2)+246866015/735
671928501375031299 84/(-2*x+3)^(21/2)+13457531633280790190212932747565/16241
306325494153682660636844148656848240 64*(7+2*14^(1/2))^(1/2)*ln(-2*x+3+14^(1
/2)-(-2*x+3)^(1/2)*(7+2*14^(1/2))^(1/2))-13457531633280790190212932747565/1
6241306325494153682660636844148656848240 64*(7+2*14^(1/2))^(1/2)*ln(-2*x+3+1
4^(1/2)+(-2*x+3)^(1/2)*(7+2*14^(1/2))^(1/2))+1/30145677658996078082575630 33
6*(26722399847903378442920192943151823852165730773017 85/117922622078976*(-2
*x+3)^(41/2)+11863238464538262372125171963121938194527617640188225 45/391539
9561216*(-2*x+3)^(21/2)-1765094235896326267587117316622980931674493927114 3/
51904512*(-2*x+3)^(11/2)+80759773649264137894226893721799583535384946 5/1048
576*(-2*x+3)^(1/2)-55066091420817590167865401986871791412011888132876913/55
27622909952*(-2*x+3)^(31/2)+2737487528928439357869138774910126923363791747 1
41675/755914244096*(-2*x+3)^(33/2)-11664572170215876884203668230743495214 48
```

```
8310113371105/9826885173248*(-2*x+3)^(35/2)+126466293338272271690443076373
2665179119615389552413/25098715136*(-2*x+3)^(17/2)-259367320368504444169504
2001860835122939346700333136537/6199382638592*(-2*x+3)^(19/2)-7559301164046
82856570195190192032441294632160945523631/3915399561216*(-2*x+3)^(23/2)+853
508502207214511987093866021124087908041634697244059/7830799122432*(-2*x+3)^
(25/2)-6886173809894005543994516442461871486007042005189775/125627793408*(-
2*x+3)^(27/2)+136329987967245395141848253765147208279814148352958009/552762
2909952*(-2*x+3)^(29/2)+1808668971148992206490172102870787954874541181/3341
14095890432*(-2*x+3)^(57/2)-11968977253082880651292892111395530933265219/25
701084299264*(-2*x+3)^(59/2)+339556544641293541759958988614814460549873/982
6885173248*(-2*x+3)^(61/2)-64243396719140374998473027009027485263697/294806
55519744*(-2*x+3)^(63/2)+129886852748727110357425618672922324659/1133871366
144*(-2*x+3)^(65/2)-503502693505289734438057515605193725/103079215104*(-2*x
+3)^(67/2)+133883313322119397348791732981953297/824633720832*(-2*x+3)^(69/2
)-3254850748003483429666738850178379/824633720832*(-2*x+3)^(71/2)+360433340
020130123942335063779145/5772436045824*(-2*x+3)^(73/2)-92834223707457673455
7978321305/1924145348608*(-2*x+3)^(75/2)+1380572274182261258625859209942856
6280191230197271405/39307540692992*(-2*x+3)^(37/2)-100630472583456033324523
3940167063186576585913370455/10720238370816*(-2*x+3)^(39/2)-447963293570690
82297154473725670903546220392558695/9070970929152*(-2*x+3)^(43/2)+286072233
17693223698395672584150593863016075796143/29480655519744*(-2*x+3)^(45/2)-50
59022664167725408892162874688680417923742003781/29480655519744*(-2*x+3)^(47
/2)+73012476452577571533836489036461787385135079265/2680059592704*(-2*x+3)^
(49/2)-1939242920901534821454026903132433081580221023737/501171143835648*(-
2*x+3)^(51/2)+490738543064879423955077165987434152441563270473/100234228767
1296*(-2*x+3)^(53/2)-55011835288361289002011693179378316699033102675/100234
2287671296*(-2*x+3)^(55/2)-22397546321209486953062074374795737299957063565/
3145728*(-2*x+3)^(3/2)+404531566689883337048499233527781983599187634017/125
82912*(-2*x+3)^(5/2)-1188598027552254830082683218064697188605612952419/1258
2912*(-2*x+3)^(7/2)+3831583379166294091823572953989993625772471445345/18874
368*(-2*x+3)^(9/2)+9977850126168010187169130424774568330973123412551261/215
92276992*(-2*x+3)^(13/2)-12556967184995885809797263315720723203579692970777
45/2399141888*(-2*x+3)^(15/2))/(14*x+(-2*x+3)^2-7)^19
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2+x+1\right)^{20}(-2x+3)^{\frac{41}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

mupad [B] time = 0.97, size = 1017, normalized size = 0.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(41/2)*(x + 2*x^2 + 1)^20),x)
```

```
[Out] ((64356352*(2*x - 3)^2)/38073 - (5767168*x)/1443 - (7517962240*(2*x - 3)^3)
/5444439 + (1357449428992*(2*x - 3)^4)/1181443263 - (34130408095744*(2*x -
3)^5)/34261854627 + (1965832636456960*(2*x - 3)^6)/2158496841501 - (9552588
571922432*(2*x - 3)^7)/10792484207505 + (69571472879183872*(2*x - 3)^8)/755
47389452535 - (5204838729946112*(2*x - 3)^9)/5036492630169 + (3250820527817
55904*(2*x - 3)^10)/257635969158645 - (461538785202937088*(2*x - 3)^11)/272
428464995505 + (17726678744562203264*(2*x - 3)^12)/6992330601551295 - (1432
471149647610304*(2*x - 3)^13)/332968123883395 + (2043463601243388704*(2*x -
```

```
3)^14)/241114848329355 - (96972768477343976816*(2*x - 3)^15)/4840844262612
435 + (1083387067012254592765 6*(2*x - 3)^16)/181389282075536535 - (44340157
049832305729324*(2*x - 3)^17)/181389282075536535 + (691509778132186261807 28
2*(2*x - 3)^18)/423241658176251915 - (1357735833153708223970340 7*(2*x - 3)^
19)/423241658176251915 + (509495943858959939640753039465067261498 1*(2*x - 3
)^20)/2035946169792430536236 25646080 + (47475340273724148225749886260884632
526403*(2*x - 3)^21)/2035946169792430536236 25646080 + (54736240672766734586
8176230754600752341499*(2*x - 3)^22)/5182408432198914092237743718 40 + (1363
21739916884674180325053144349616764755 9*(2*x - 3)^23)/438511482724523500112
424468480 + (40035704814224807138997531075224002020138815 9*(2*x - 3)^24)/59
8568173918974577653459399475 20 + (16780353218671061875177851217431652450855
3291*(2*x - 3)^25)/1496420434797436444133648498688 0 + (51108771060698315319
124863093548144195799415067*(2*x - 3)^26)/3351981773946257634859372637061 12
0 + (39398708318720673508200388938122166434609005 3*(2*x - 3)^27)/2280259710
167522200584607236096 0 + (19450991951225490080928815092282978539677719528 1*
(2*x - 3)^28)/11688962083504898418996786631802880 + (39904941217415859849 67
8112809547525787872838871677*(2*x - 3)^29)/288717363462570990949220629805 53
11360 + (298202908298252068565416529654031351573999658954519*(2*x - 3)^30)/
297834753887704811716038123378337382 40 + (172783707178371264987902065794355
725529860298244 11*(2*x - 3)^31)/27075886717064073792367102125303398 40 + (13
65899091406231574832296169611108676090874691954 57*(2*x - 3)^32)/37906241403
889703309313942975424757760 + (12124448510282132213121066777925721516746772
830847*(2*x - 3)^33)/668933671833347705458481346625142784 0 + (5268103225464
00392428459875677051456589568282412 9*(2*x - 3)^34)/645866993494266750097844
0588104826880 + (6171761009286202626631300590201603951028773271141 3*(2*x -
3)^35)/18730142811333735752837477705503997952 0 + (236279120368028123256783 3
349111778791410563265773 87*(2*x - 3)^36)/197226802936437285876032243873341 2
433920 + (1006918289966448819369741773577875830109223667348001*(2*x - 3)^37
)/256394843817368471638841917035343616409 60 + (8343152514122341340412513706
840068954518337251868859*(2*x - 3)^38)/71790556268863172058875736769896212 5
946880 + (669016452611293431036170511813057767424944839195492 3*(2*x - 3)^39
)/215371668806589516176627210309688637784064 0 + (305581065207833944849384 01
543314040887488123057461 3*(2*x - 3)^40)/40745991395841259817199742491022174
7159040 + (7318673393711958469818414571768081348141036133 09*(2*x - 3)^41)/4
477581472070468111780191482529909309440 + (98156536112115492322904146290693
53244130713267641*(2*x - 3)^42)/305594935468809448628998068682666310369280
+ (111998015172594816786872871418593904041451326 17*(2*x - 3)^43)/1971580228
831028700832245604404298776576 + (136564747272427838170630719416707180545 54
74221*(2*x - 3)^44)/151422305976050793637586261153308213248 0 + (40305011659
049347862186541819167541945005655 01*(2*x - 3)^45)/314621902416905537891429 2
3150742928752640 + (142800962844555649098866729552205491584243363 1*(2*x - 3
)^46)/880941326767335506096001848220802005073 92 + (160089053926633694221849
8464088424576826036 21*(2*x - 3)^47)/8809413267673355060960018482208020050 73
92 + (21001998140967208924158271678544758006824603 89*(2*x - 3)^48)/11716519
6460055622310768245813366666674831 36 + (731521029491460764762993572365861 79
9703833*(2*x - 3)^49)/474353022105488349436308687503508771962 88 + (14527825
011424680881745287944067060548347 7*(2*x - 3)^50)/126959191210586587643247 32
5184762641907712 + (3054176246891199033401768204622054595917*(2*x - 3)^51)/
423197304035288625477491083949208806359 04 + (43226241215596960235839037876 4
52347793*(2*x - 3)^52)/113937735701808476090093753370940832481 28 + (1675721
4169421265746492710756597657 5*(2*x - 3)^53)/103579759728916796445539775791 7
643931648 + (935756145095208333386444273642906999*(2*x - 3)^54)/17401399634
458021802850682333016418051686 4 + (325001551972552320039960952878829 9*(2*x
- 3)^55)/248591423349400311469295461900234543595 52 + (359910711199433658030
176367535945*(2*x - 3)^56)/174013996344580218028506823330164180516864 + (92
70277547814767462080476205 05*(2*x - 3)^57)/580046654481934060095022744433 88
060172288 + 79953920/10101)/(597630395894891439718 4*(3 - 2*x)^(39/2) - 5677
488761001468677324 8*(3 - 2*x)^(41/2) + 263597692475068188590080*(3 - 2*x)^(
43/2) - 796876101097706139353088*(3 - 2*x)^(45/2) + 176320786164367039994 26
```

56*(3 - 2*x)^(47/2) - 30432498430143586695905 28*(3 - 2*x)^(49/2) + 42641375
22753475514499072*(3 - 2*x)^(51/2) - 49843240754085725297541 12*(3 - 2*x)^(5
3/2) + 49565680630574224014581 76*(3 - 2*x)^(55/2) - 42553157713737085185290
24*(3 - 2*x)^(57/2) + 31897796134848733452912 64*(3 - 2*x)^(59/2) - 21062355
39086912777861632*(3 - 2*x)^(61/2) + 12337084486097831501690 88*(3 - 2*x)^(6
3/2) - 6446157886660770294535 68*(3 - 2*x)^(65/2) + 3017871570807632507216 64
*(3 - 2*x)^(67/2) - 1270378343546601881504 64*(3 - 2*x)^(69/2) + 48214067552
985728953272*(3 - 2*x)^(71/2) - 1653094793600791863646 8*(3 - 2*x)^(73/2) +
512755062408649562651 8*(3 - 2*x)^(75/2) - 144001037979237504041 9*(3 - 2*x)^
(77/2) + 36625361600617825903 7*(3 - 2*x)^(79/2) - 8434157110208121753 3*(3 -
2*x)^(81/2) + 1757072432688984291 3*(3 - 2*x)^(83/2) - 330689906171022980 4*
(3 - 2*x)^(85/2) + 561126236614140036*(3 - 2*x)^(87/2) - 85611621840452988*
(3 - 2*x)^(89/2) + 11703514272799272*(3 - 2*x)^(91/2) - 1427192816292922*(3
- 2*x)^(93/2) + 154386157043846*(3 - 2*x)^(95/2) - 14711313018374*(3 - 2*x
)^(97/2) + 1223975378934*(3 - 2*x)^(99/2) - 87916389372*(3 - 2*x)^(101/2) +
5372380188*(3 - 2*x)^(103/2) - 273870408*(3 - 2*x)^(105/2) + 11333994*(3 -
2*x)^(107/2) - 365883*(3 - 2*x)^(109/2) + 8645*(3 - 2*x)^(111/2) - 133*(3
- 2*x)^(113/2) + (3 - 2*x)^(115/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)*817
42866766155642540624633854631975167472566370920865 55i - 2647038820161175221
60242769053740760936364151734091888266 01)^(1/2)*124320682492976962848972490
01366340523282983937937427139335625i)/(546445444973747744833043391094451536
03853101336983676390259568994646097049868196343130260955236337673144554235 5
7204931772416*((7^(1/2)*376655850073799072335964720186587398406296145585988
88628455806290315259752942013758759812 5i)/273222722486873872416521695547225
76801926550668491838195129784497323048524934098171565130477618168836572277 1
1778602465886208 + 777520974123765253499790235238946338576340593433713638 91
67367539445563937310492112511456 25/3903181749812483891664595650674653828846
65009549883402787568349961757836070487116736644721108830983379603958825514 6
37983744)) + (1243206824929769628489724900136634052328298393793742713933562
5*7^(1/2)*(3 - 2*x)^(1/2)*(- 7^(1/2)*8174286676615564254062463385463197516 7
47256637092086555i - 264703882016117522160242769053740760936364151734091888
26601)^(1/2))/(546445444973747744833043391094451536038531013369836763902595
68994646097049868196343130260955236337673144554235572049317724 16*((7^(1/2)*
376655850073799072335964720186587398406296145585988886284558062903152597529
420137587598125i)/273222722486873872416521695547225768019265506684918381951
29784497323048524934098171565130477618168836572277117786024658862 08 + 77752
097412376525349979023523894633857634059343371363891673675394455639373104921
1251145625/390318174981248389166459565067465382884665009549883402787568349 9
617578360704871167366447211088309833796039588255146379837 44)))*(- 7^(1/2)*8
17428667661556425406246338546319751674725663709208655 5i - 26470388201611752
21602427690537407609363641517340918882660 1)^(1/2)*115i)/812065316274707684 1
33031842207432842412032 + (atan(((3 - 2*x)^(1/2)*(7^(1/2)*81742866766155642
5406246338546319751674725663709208655 5i - 264703882016117522160242769053740
760936364151734091888266 01)^(1/2)*124320682492976962848972490013663405232 82
98393793742713933562 5i)/(546445444973747744833043391094451536038531013369 83
676390259568994646097049868196343130260955236337673144554235572049317724 16*
((7^(1/2)*376655850073799072335964720186587398406296145585988886284558062 90
315259752942013758759812 5i)/27322272248687387241652169554722576801926550668
49183819512978449732304852493409817156513047761816883657227711778602465886 2
08 - 777520974123765253499790235238946338576340593433713638916736753944556 3
93731049211251145625/39031817498124838916645956506746538288466500954988340 2
7875683499617578360704871167366447211088309833796039588255146379837 44)) - (
1243206824929769628489724900136634052328298393793742713933562 5*7^(1/2)*(3 -
2*x)^(1/2)*(7^(1/2)*8174286676615564254062463385463197516747256637092086 55
5i - 264703882016117522160242769053740760936364151734091888266 01)^(1/2))/(5
4644544497374774483304339109445153603853101336983676390259568994646097049 86
819634313026095523633767314455423557204931772416*((7^(1/2)*376655850073799 0
72335964720186587398406296145585988886284558062903152597529420137587598125i
)/27322272248687387241652169554722576801926550668491838195129784497323048 52

```
4934098171565130477618168836572277117786024658862O8 - 777520974123765253499
790235238946338576340593433713638916736753944556393731049211251145625/39031
817498124838916645956506746538288466500954988340278756834996175783607048711
6736644721108830983379603958825514637983744)))*(7^(1/2)*8174286676615564254
062463385463197516747256637092086555i - 264703882016117522160242769053740 76
093636415173409188826601)^(1/2)*115i)/812065316274707684133031842207432842 4
12032
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
```

```
[Out] Timed out
```

3.50 $\int \frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx$

Optimal. Leaf size=378

$$-\frac{63043297-29625922x}{41160000000\left(x^2-2x+3\right)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}+\frac{}{11760}$$

```
[Out] 1/123480000*(-3450497+2004270*x)/(x^2-2*x+3)^(9/2)+1/411600000*(-4878869+25
78034*x)/(x^2-2*x+3)^(7/2)+1/6860000000*(-30316369+15043110*x)/(x^2-2*x+3)^
(5/2)+1/41160000000*(-63043297+29625922*x)/(x^2-2*x+3)^(3/2)+1/280*(-1+10*x
)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^4+1/1050*(28+67*x)/(x^2-2*x+3)^(9/2)/(2*x^2
+x+1)^3+1/117600*(5485+8878*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^2+3/343000*(88
22+8233*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)-31/411600000000*(7434109-3088870*x
)/(x^2-2*x+3)^(1/2)-1/9604000000000*arctanh(1/7*(308108167+x*(932587773-620
347970*2^(1/2))-312239803*2^(1/2))*35^(1/2)/(-151363871237318045+1103204757
41093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-105954709866122631 50+772243330
1876572160*2^(1/2))^(1/2)+1/9604000000000*arctan(1/7*(308108167+312239803*2
^(1/2)+x*(932587773+620347970*2^(1/2)))*35^(1/2)/(151363871237318045+110320
475741093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(1059547098661226315 0+772243
3301876572160*2^(1/2))^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{63043297-29625922x}{41160000000\left(x^2-2x+3\right)^{3/2}}-\frac{31(7434109-3088870x)}{411600000000\sqrt{x^2-2x+3}}+\frac{3(8233x+8822)}{343000\left(x^2-2x+3\right)^{9/2}\left(2x^2+x+1\right)}+\frac{}{11760}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -(3450497 - 2004270*x)/(123480000*(3 - 2*x + x^2)^(9/2)) - (4878869 - 25780
34*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(68600000
00*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x +
 x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(411600000000*Sqrt[3 - 2*x + x^2]
) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/
(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3
 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2
*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045 + 110320475741
093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093
888*Sqrt[2]))]*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000 - (Sqrt[(-151363871237318045 +
110320475741093888*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 1
10320475741093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
```

```
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^5}\,dx = -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} - \frac{\int \frac{-1235+1335x-800x^2}{\left(3-2x+x^2\right)^{11/2}\left(1+x+2x^2\right)^4}\,dx}{1400}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x\right.}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x\right.}$$

$$= -\frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \frac{28+67x}{1050\left(3-2x+x^2\right)^{9/2}\left(1+x+2x\right.}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{1-10x}{280\left(3-2x+x^2\right)^{9/2}\left(1+x+2x^2\right)^4} + \ldots$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{\ldots}{280\left(3-\ldots\right.}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{303163\ldots}{68600000\ldots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{303163\ldots}{68600000\ldots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{303163\ldots}{68600000\ldots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{303163\ldots}{68600000\ldots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{303163\ldots}{68600000\ldots}$$

$$= -\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}} - \frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}} - \frac{303163\ldots}{68600000\ldots}$$

Mathematica [C] time = 6.13, size = 342, normalized size = 0.90

$$\frac{-9i\sqrt{50+10i\sqrt{7}}\left(932587773\sqrt{7}-299844895i\right)\sqrt{x^2-2x+3}\left(2x^4-3x^3+5x^2+x+3\right)^4\tanh^{-1}\left(\frac{\left(-5-i\sqrt{7}\right)x+i\ldots}{\sqrt{50+10i\sqrt{7}}\sqrt{x\ldots}}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] (560*(-53205422447 + 261702502714*x - 266966654968*x^2 + 1002897791524*x^3
- 1409335257371*x^4 + 2503427226914*x^5 - 3359813871472*x^6 + 4591320676952
*x^7 - 5134334619701*x^8 + 5380603084494*x^9 - 4915797913008*x^10 + 3999656
132532*x^11 - 2679143870481*x^12 + 1459208021718*x^13 - 606785954952*x^14 +
 188603773872*x^15 - 38639385552*x^16 + 4596238560*x^17) - (9*I)*Sqrt[50 +
(10*I)*Sqrt[7]]*(-299844895*I + 932587773*Sqrt[7])*Sqrt[3 - 2*x + x^2]*(3 +
 x + 5*x^2 - 3*x^3 + 2*x^4)^4*ArcTanh[(13 + I*Sqrt[7] + (-5 - I*Sqrt[7])*x)
/(Sqrt[50 + (10*I)*Sqrt[7]]*Sqrt[3 - 2*x + x^2])] + 9*Sqrt[50 - (10*I)*Sqrt
[7]]*(299844895 - (932587773*I)*Sqrt[7])*Sqrt[3 - 2*x + x^2]*(3 + x + 5*x^2
 - 3*x^3 + 2*x^4)^4*ArcTanh[(-13 + I*Sqrt[7] + (5 - I*Sqrt[7])*x)/(Sqrt[50
- (10*I)*Sqrt[7]]*Sqrt[3 - 2*x + x^2])])/(691488000000000*(3 - 2*x + x^2)^(
9/2)*(1 + x + 2*x^2)^4)
```

fricas [B] time = 1.03, size = 1873, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/7108652444723216758028075295024000000000*(264602060868765123019815591460
74412800*x^18 - 211681648695012098415852473168595302400*x^17 + 1018717934344
745723626290027123864892800*x^16 - 3214915039555496244690759436248041155200
*x^15 + 7688343631118056605744516779381246569200*x^14 - 1398091139115337718
7559506313807067863200*x^13 + 20977982138251784909414754860497120398000*x^1
2 - 25712705264922250829450580100197810638400*x^11 + 287572827277934795261 9
7333249442997761200*x^10 - 27283780001330543747380735174495978898400*x^9 +
25562212842803140665733059982554512415600*x^8 - 1804586055124978138995 14233
37622749529600*x^7 + 15206349685551845663545027271759639106000*x^6 - 726663
4096608462190931685680490685615200*x^5 - 3602042876982878244*33780221308347
3608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16
- 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10
- 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*
x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 22064095148
2187776)*arctan(1/964393622349963919677467835514205441102895152270484353118
304*sqrt(205487899)*(1207121086772200941513110092511294 0*sqrt(4167294734812
9)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(205487899)*(5*337802213083473608
^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876
830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(201502465*sq
rt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) +
 241424217354440188302622018502258 8*sqrt(41672947348129)*sqrt(7)*(125*sqrt(
2) + 172))*sqrt(16448360508869491318497096 8*x^2 + sqrt(205487899)*(33780221
3083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 4
2834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x -
 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 22
0640951482187776) - 411209012721737282962427 42*sqrt(x^2 - 2*x + 3)*(4*x + 1
) - 1233627038165211848887282 26*x + 205604506360868641481213710*sqrt(2) + 2
87846308905216098073699194) + 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10*x - 19) +
9*x - 29) + 1/11491792746071352963204808080707518 88*sqrt(205487899)*(5*3378
02213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349) - 573381349
*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(sqrt(2)*(20
1502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213083473608^(
3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*3378022130834736
08^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*x + 3))*sqr
t(151363871237318045*sqrt(2) + 220640951482187776) - 1/476*sqrt(x^2 - 2*x +
 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) + 172)) + 1/
476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) - 3602042876982878244*3378
02213083473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 +
```

616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 1
7389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*
x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 2
20640951482187776)*arctan(-1/9643936223499639196774678355142054411028951522
70484353118304*sqrt(205487899)*(12071210867722009415131100925112940*sqrt(41
672947348129)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) - sqrt(205487899)*(5*3378022
13083473608^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381
349) + 2876830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(2
01502465*sqrt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 2206409514
82187776) + 2414242173544401883026220185022588*sqrt(41672947348129)*sqrt(7)
*(125*sqrt(2) + 172))*sqrt(164483605088694913184970968*x^2 - sqrt(205487899
)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*
sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(8
9801606*x - 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*s
qrt(2) + 220640951482187776) - 41120901272173728296242742*sqrt(x^2 - 2*x +
3)*(4*x + 1) - 123362703816521184888728226*x + 205604506360868641481213710*
sqrt(2) + 287846308905216098073699194) - 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10
*x - 19) + 9*x - 29) + 1/1149179274607135296320480808070751888*sqrt(2054878
99)*(5*337802213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349)
- 573381349*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(
sqrt(2)*(201502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213
083473608^(3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*33780
2213083473608^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*
x + 3))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) + 1/476*sqrt(
x^2 - 2*x + 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) +
172)) - 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) + 9*33780221308
3473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(7)*(3530255223715004416*x^18 -
28242041789720035328*x^17 + 135914826113027670016*x^16 - 428926009681373036
544*x^15 + 1025759783440690970624*x^14 - 1865298603830415458304*x^13 + 2798
830469551551938560*x^12 - 3430525513645055541248*x^11 + 3836725505323763236
864*x^10 - 3640134417553133928448*x^9 + 3410447187060176453632*x^8 - 240763
4062573633011712*x^7 + 2028793548878716600320*x^6 - 969496340812733087744*x
^5 + 972364673182001528832*x^4 - 87373816786946359296*x^3 + 363395647091163
267072*x^2 - 151363871237318045*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16 - 19
44*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16
498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3
+ 1647*x^2 + 162*x + 243) + 35743834140114419712*x + 53615751210171629568)*
sqrt(151363871237318045*sqrt(2) + 220640951482187776)*log(19083512352618334
937598521302939860992*x^2 + 236911417693579806112743424/2041974420058321*sq
rt(205487899)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3
)*(89801606*sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)
(sqrt(2)(89801606*x - 132636591) - 42834985*x + 222438197))*sqrt(15136387
1237318045*sqrt(2) + 220640951482187776) - 47708780881545837343996303257349
65248*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 1431263426446375120319889097720489574
4*x + 23854390440772918671998151628674826240*sqrt(2) + 33396146617082086140
797412280144756736) - 9*337802213083473608^(1/4)*sqrt(205487899)*sqrt(35)*s
qrt(7)*(3530255223715004416*x^18 - 28242041789720035328*x^17 + 135914826113
027670016*x^16 - 428926009681373036544*x^15 + 1025759783440690970624*x^14 -
1865298603830415458304*x^13 + 2798830469551551938560*x^12 - 34305255136450
55541248*x^11 + 3836725505323763236864*x^10 - 3640134417553133928448*x^9 +
3410447187060176453632*x^8 - 2407634062573633011712*x^7 + 20287935488787166
00320*x^6 - 969496340812733087744*x^5 + 972364673182001528832*x^4 - 8737381
6786946359296*x^3 + 363395647091163267072*x^2 - 151363871237318045*sqrt(2)*
(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*
x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x
^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243) + 357438341401
14419712*x + 53615751210171629568)*sqrt(151363871237318045*sqrt(2) + 220640
951482187776)*log(19083512352618334937598521302939860992*x^2 - 236911417693
579806112743424/2041974420058321*sqrt(205487899)*(337802213083473608^(1/4)*

```
sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 42834985) - 337802
213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x - 132636591) - 428
34985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 220640951482187776)
 - 477087808815458373439963032573496524 8*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 14
3126342644637512031988909772048957 44*x + 238543904407729186719981516286748 2
6240*sqrt(2) + 33396146617082086140797412280144756736) + 728813301405404935
717704569729687107560 0*x^4 - 654890100650193679474043588865341716800*x^3 +
272374746406785098508522674459903486760 0*x^2 + 5756926178104321961473983880
*(4596238560*x^17 - 38639385552*x^16 + 188603773872*x^15 - 606785954952*x^1
4 + 1459208021718*x^13 - 2679143870481*x^12 + 3999656132532*x^11 - 49157979
13008*x^10 + 5380603084494*x^9 - 5134334619701*x^8 + 4591320676952*x^7 - 33
59813871472*x^6 + 2503427226914*x^5 - 1409335257371*x^4 + 1002897791524*x^3
 - 266966654968*x^2 + 261702502714*x - 53205422447)*sqrt(x^2 - 2*x + 3) + 2
679095866296246870575632863540034296 00*x + 401864379944437030586344929531 00
5144400)/(16*x^18 - 128*x^17 + 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13
 + 12685*x^12 - 15548*x^11 + 17389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7
 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)
```

<u>**giac**</u> **[C]** time = 93.77, size = 2509, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*l
og(3136*(24743015359883014513591422663809146512801777907125132721610123618 1
293485559300330785024470114864584026604284622700*sqrt(7)*sqrt(2)*sqrt(77224
33301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2)
- 151363871237318045)^2 + 1443342562659842513292832988722200213246770377915
632742093923877242119990959185962459760756700434068218583269657 50*sqrt(7)*
(110320475741093888*sqrt(2) - 151363871237318045)^3 + 28866851253196850265 8
566597744440042649354075583126548418784775544842399819183719249195215134008
6813643716653931500*sqrt(2)*(110320475741093888*sqrt(2) - 15136387123731804
5)^3 + 2061917946656917876132618555317428876066814825593761060134176968177 4
4571299416942320853725095720486688836903852250*sqrt(7722433301876572160*sqr
t(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318 0
45)^3 - 10491385411229696257352208072962304143673340426562259232108409328 92
592514150275756869331443550064380041514200248812294 81000*sqrt(7)*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 - 104913854112296962573 52
208072962304143673340426562259232108409328925925141502757568693314435500643
800415142002488122948100*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 1059547
0986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 209827
708224593925147044161459246082873466808531245184642168186578518502830055151
37386628871001287600830284004976245896200*sqrt(2)*sqrt(7722433301876572160*
sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237 3
18045)^2 - 122399496464346456335775760851226881676188971643226357707931442 1
70802459984198838301422001747507511004843323362361434394500*(11032047574109
3888*sqrt(2) - 151363871237318045)^3 + 724503996256958016683144110303659048
527226579099834730629600402015843465282345034154512602754539838193840458016
1329156260047220070 0*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 6339
409967248382662472854538412359683674181009662984901543522122388718802293934
794271550978055578969864402015298801946834493625741 25*sqrt(7)*(110320475741
093888*sqrt(2) - 151363871237318045)^2 + 126788199344967653208218731835108 8
361508312490869111205095341459358991548431951565218821053012281909331172952
868319415989224917443750*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237
318045)^2 + 1267881993449676535381256033002156963320502179376997406802245 18
030900924464663471430273419380295298646483255439984720301061537907975*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
```

t(2) - 151363871237318045)^2 - 21264528220686985082784156444749824400286141
322404339508073021441899306648522000622634130083780322911432744853704088500
185306368048860002880*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045) - 354408803678116418845746257791902502469699160932438355128709
627960208195159816911628734559796548815662417323599879097891861163466576081
8080*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 7088176073562328374916566029
889536476564991204751185360922127716049447858985212646112610216597117728735
334198568887900615291459333783931760*sqrt(2)*sqrt(7722433301876572160*sqrt(
2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045
) - 62021540643670373420405091577929394267973114014403593823713156536262791
860899401705831007948594881042345033377486353135919026969012248710900*(1103
20475741093888*sqrt(2) - 151363871237318045)^2 + 32899118880973852710380417
819638428242533122398715783525424459521797423442868355883980260844124987688
79366524177397299680463206375263895127106986700*sqrt(7)*sqrt(2)*sqrt(772243
3301876572160*sqrt(2) - 10595470986612263150) + 575734580417042425329673296
871504559468760385065959616403976214712902997702584706455877479060569410162
15068838775424495834136105150442722719839966690*sqrt(7)*(110320475741093888
*sqrt(2) - 151363871237318045) + 115146916083408484993484259748605110296131
995582491774181190935500623259371276266538520099917051160383062730419039816
825148360727220332176154717624880*sqrt(2)*(110320475741093888*sqrt(2) - 151
363871237318045) + 19191152680568080928847909459028587443440938343434315974
646074253344559590752575102628626876766807523046716011108365196910710221166
349350116972946360*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) - 4219083468134411357204
411829911575030935087599894152681923125521189411158761429808471398986722740
73729706037742726523339689588219472775435857599079592655320*sqrt(7)*sqrt(2)
 - 210954173406720568670297857045559135287403242478401319837374548657411401
676856640700014506909797905726420309651469993318236639490549988690120558780
166576108*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 4219083468134411371380763977036231747195786343738197611526042097212408620
850995875571065161946256176959320357812676471048958085730443394260704501501
50098280*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) -
 92058995214015314954086068104250082609842263587571418594372338376217809267
922249720111767473806366273424910555828722901509465001692379638157996477568
878185171912773*sqrt(7) - 8145397671270700392375835131235891700241076006749
872956875216413467916040501467073084588856054007249498459026692980650865954
42158392477166657536193932267555915756504095096650*sqrt(2) - 92058995214015
311115855531990633917126372266976678439922113048655699796659517564449092929
141850974541546613899009112709389970185858812421123596605881876967985540965
*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) + 115328821454793
719911555111506838054918092483693711591974348115163386041674344938146595482
926045968304684846611487161656745020914533707787852282815214568718566601574
0023320320)^2 + 3136*(34124314806601555041367954040995026009203193083759390
91863756572751913121135591082568720686452963161513000*sqrt(7)*sqrt(2)*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
t(2) - 151363871237318045)^2 + 19905850303850907107464639857247098505368529
298859644702538580007719493206624281314984204004308951775492500*sqrt(7)*(11
0320475741093888*sqrt(2) - 151363871237318045)^3 + 398117006077018142149292
797144941970107370585977192894050771600154389864132485626299684080086179035
50985000*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 2843
692900550129586780662836749585500766932756979949243219797143959927600946325
902140600572044135967927500*sqrt(7722433301876572160*sqrt(2) - 105954709866
12263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 76858147264
002002271476515530447061472149684267202070486664776785256332938718528406617
89826971901179214250*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 15136387
1237318045)^2 + 76858147264002002271476515530447061472149684267202070486664
7767852563329387185284066178982697190117921425*sqrt(7)*sqrt(772243330187657
2160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 15136387

```
1237318045)^2 + 15371629452800400454295303106089412294429936853440414097332
955357051266587743705681323579653943802358428 50*sqrt(2)*sqrt(77224333018765
72160*sqrt(2) - 105954709866122631 50)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 896678384746690026500559347855215717175079649784024155677 7
557291613238842850494980772088131467218042416625*(110320475741093888*sqrt(2
) - 151363871237318045)^3 + 976510829351336985318426092720643328053917556 99
90063034314276690919311524349854768153191859518751603398057755715947500140 0
0*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*
(110320475741093888*sqrt(2) - 151363871237318045) + 8544469756824198644285 7
71515706665814717081317737989161160454160393991529564500073813463684649514 8
700981051126045418060425 00*sqrt(7)*(110320475741093888*sqrt(2) - 1513638712
37318045)^2 + 17088939513648397282884157230313072455872836961976807320787 04
280682808457268940585970707216740637793591500908112081167623000 0*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 + 17088939513648397328383
24363911514584436344234997017533305796691850727246420616016306591378254759 2
37016461823382698716307000*sqrt(7722433301876572160*sqrt(2) - 1059547098661
2263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 161566291032
16429859182505251559538530528459085626839375307780956912834768081659431452 9
252834261093151623154745024213145434 0*sqrt(7)*sqrt(2)*(110320475741093888*s
qrt(2) - 151363871237318045) - 26927715172027383040078920360978657724993896
16713268545625606596270076614743912626365481228097225578689498425200383606 3
5590*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 538554303440547660947974878 1
98624587397928429114933826214930241667733552447191623911679736931289931750 5
05205553827219922880*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986
612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 471235015510
47920230470272157043648368683383506960627830940150456323925190242898045263 5
330631964978193453411263345326870620 0*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 41938592547058175637865452041109794172496152182758838080 83
69900699790874591792115928637572856409303229934928633430010503747776545857 5
2870920*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105954709866122 6
3150) + 73392536957351807756868872812476933721242879824990105229931845076 95
33849414676062708934630055035134045830109366690499889371007117918795898700*
sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045) + 146785073914703
61541608666268982016900814119268249174920123751742827764242323014015762064 1
963615809777688763882024614874428500622223231650140 0*sqrt(2)*(1103204757410
93888*sqrt(2) - 151363871237318045) + 2446417898578393603288255436181495977
78089793408164936529659929761653877816458460985919888559249587651218708898 8
513142432922439383397920921 00*sqrt(7722433301876572160*sqrt(2) - 1059547098
6612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 12920777533
12694019345893430149910326331364817398263414897431400481314306688204378121 5
921471858282975500842590978635984214415094963315999115 0*sqrt(7)*sqrt(2) + 7
58762916008240003409819101375410993692930390503123728189868562072536874773 1
98417845474059096092571231597614031305995383233253822344649520984786771597 2
44159013334688*x - 64603887665634701028843734757843820568795044140595103353
00445902481346769867158741187507320312111455790395720782521985923352484305 4
466661575*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 12920777533126940204230020370310056506867752862435766623805367556854879 26
2268547580075255076534844025016463869994363709342246652349944171058 2*sqrt(2
)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 36018223914820
16791468032806227754184457992608529203568071788577705554082231768803383499 5
139282160837267303073663773328283377733660710835980727723860519306 60*sqrt(7
) - 25665325155285930274575537408488222512257955813553854680296641781933443
84084826780557660928107456109583996394402379692154048400978571146920888612 6
0268773750000*sqrt(2) - 758762916008240003409819101375410993692930390503123
72818986856207253687477319841784547405909609257123159761403130599538323325 3
82234464952098478677159724415901333468 8*sqrt(x^2 - 2*x + 3) - 3601822391482
01674253967483465988239211591610327364049310125917227965268177456885468827 3
070958284684112917867030199254538909056956640775330160129944904845080*sqrt(
```

```
7722433301876572160*sqrt(2) - 10595470986612263150) + 189691081878191176772
044460297494206938487074845844976108591897448936648822154611593038166610328
9009899584124255840798697429302566249001909432732806568484301818509997)^2) -
 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*
log(3136*(247430153294512807701166330456641266086589954634290558518246802634
46391921433693240822436073126340210807888656257700*sqrt(7)*sqrt(2)*sqrt(7722
433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2)
 - 151363871237318045)^2 + 144334256088465804492347026099707405217177473536
6694924689773015368728620836321057146421042657031789637935049483250*sqrt(7)
*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 28866851217693160898
469405219941481043435494707333898493795460307374572416726421142928420853140
63579275870098966500*sqrt(2)*(110320475741093888*sqrt(2) - 1513638712373180
45)^3 + 2061917944120940064176386087138677217388249621952421320985390021955
32660119474436735203006093861684233990721354750*sqrt(7722433301876572160*sq
rt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318
045)^3 + 104913854900605463598088261225277871792243668954806319376337814136
029745746425269205752115419545432491274148221669945001000*sqrt(7)*sqrt(2)*(
110320475741093888*sqrt(2) - 151363871237318045)^2 + 1049138549006054635980
882612252778717922436689548063193763378141360297457464252692057521154195454
3249127414822166994500100*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 105954
70986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 + 20982
770980121092719617652245055574358448733790961263875267562827205949149285053
841150423083909086498254829644333989000200*sqrt(2)*sqrt(7722433301876572160
*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237
318045)^2 + 122399497384039707531102971429490850424284280447274039272394116
492034703370829480740044134656136337906486506258614935834500*(1103204757410
93888*sqrt(2) - 151363871237318045)^3 + 72450399556079180248329015146999235
208892617070047340278059551131392900835267393877060925502058563859855540246
511032336003698229700*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10
595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 633
940996115692828822413237832995359418919269073856001343620769961624939096901
713988544379098807927655360025907865006811958130347875*sqrt(7)*(11032047574
1093888*sqrt(2) - 151363871237318045)^2 + 126788199223138565723244288684180
270600256132071997655920959161553276561399672542358602343795866698184031403
9628006645155934817986250*sqrt(2)*(110320475741093888*sqrt(2) - 15136387123
7318045)^2 + 12678819922313856605315115974353068086847790601418601070307910
1065663972757334945871454599987025796960356213712979359289978635966225*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sq
rt(2) - 151363871237318045)^2 + 2126452792712690096116437713266837331182032
949571436835653998293704475706783435782714029336413768430799117811124620902
4321545469979809839680*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 151363
871237318045) + 35440879878544835015208327526337165880148748188307087674038
005051117505114076550006018390461315393241510718124111630271982879190286307
82880*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110
320475741093888*sqrt(2) - 151363871237318045) + 708817597570896700104330636
430351958368521132860850569114581683965576160612316116051739722090259733009
7341426354794006943734919549262613360*sqrt(2)*sqrt(7722433301876572160*sqrt
(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 15136387123731804
5) + 6202153978745346139901407055512974782136328075902825385385078928672967
3222028079002566886678131418912687891373331690882456544841615974534900*(110
320475741093888*sqrt(2) - 151363871237318045)^2 + 3289911884763015221101541
114405505474753270156481065387887051759144023849028573565153578955503100992
856636792237219317239230096498179223616314209700*sqrt(7)*sqrt(2)*sqrt(77224
33301876572160*sqrt(2) - 10595470986612263150) + 57573457983352766659078567
726413066650683718699283969000978205977352244855554840995810704156013886991
645243042141434050503060360394862431867078254790*sqrt(7)*(11032047574109388
8*sqrt(2) - 151363871237318045) + 11514691596670553324570673589674695312373
275176428664785053988930084036831493297504145438133911660943041218690890125
3978934468277888412836527277030080*sqrt(2)*(110320475741093888*sqrt(2) - 15
```

```
1363871237318045) + 19191152661117588970884988724230066021393869653903156391
55609757570957373975372338370836193986424378585490167158630684697779577032
6107328538508811760*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150
)*(110320475741093888*sqrt(2) - 151363871237318045) + 421908326717543641924
358820689861254773878411648930379381901809299202750782331001508374803517870
628151016355380370573185797674929411754956761480005114821080*sqrt(7)*sqrt(2
) + 210954163358771821772256664711669711256358377483192354207771689878568670
89453384049969627290987030043155180365618048585416083969716512306115822804
6039881708*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
 + 421908326717543643341994015831654651625296327690259460980119715812011991
659696962885724904699692807901543935675281424621198266774916716624735720821
529103720*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 920589964087560320916761581042015283905433780645215921762515407245365322
3479396020474916109653924902956542165396460831286616319128616458259997590278
2992478976911083*sqrt(7) + 814539727961527046694837833062145864584590634452
974565084319757683318515854195479301757363227685529077360827463499277712322
1082185639753988444491134597154930322061732374698 50*sqrt(2) - 9205899640875
602825344562588068375449965750892067890031749508881924444728233783716057581
267806102568826874405209859338973827430110551686881735838388090152238069483
5*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 11532881607348
687037747868321346713994558911627676219472353978719356293894710618931600085
322372043380298052663139565536675674564328460228800898840894611032414487006
75305025280)^2 + 3136*(341243148066015550413679540409950260092031930837593 9
091863756572751913121135591082568720686452963161513000*sqrt(7)*sqrt(2)*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sq
rt(2) - 151363871237318045)^2 + 19905850303850907107464639857247098505368 52
929885964470253858000771949320662428131498420400430895177549 2500*sqrt(7)*(1
10320475741093888*sqrt(2) - 151363871237318045)^3 + 398117006077018142149 29
279714494197010737058597719289405077160015438986413248562629968408008617903
550985000*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 284
369290055012958678066283674958550076693275697994924321979714395992760094632
590214060057204413596792 7500*sqrt(7722433301876572160*sqrt(2) - 10595470986
612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^3 + 7685814726
400200227147651553044706147214968426720207048666477678525633293871852840661
7898269719011792 14250*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 768581472640020022714765155304470614721496842672020704866 6
477678525633293871852840661789826971901179214 25*sqrt(7)*sqrt(77224333018765
72160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638
71237318045)^2 + 153716294528004004542953031060894122944299368534404140973 3
295535705126658774370568132357965394380235842850*sqrt(2)*sqrt(7722433301876
572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363
871237318045)^2 + 896678384746690026500559347855215717175079649784024155677
755729161323884285049498077208813146721804241 6625*(110320475741093888*sqrt(
2) - 151363871237318045)^3 + 976510829351336985318426092720643328053917556 9
990063034314276690919311524349854768153191859518751603398057755715947500140
00*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) + 854446975682419864428 5
771515706665814717081317737989161160454160393991529564500073813463684649514
8700981051126045418060425 00*sqrt(7)*(110320475741093888*sqrt(2) - 151363871
237318045)^2 + 170889395136483972828841572303130724558728369619768073207870
428068280845726894058597070721674063779359150090811208116762300 00*sqrt(2)*(
110320475741093888*sqrt(2) - 151363871237318045)^2 + 170889395136483973283 8
324363911514584436344234997017533305796691850727246420616016306591378254759
2370164618233826987163070 00*sqrt(7722433301876572160*sqrt(2) - 105954709866
12263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 16156629103
216429859182505251559538530528459085626839375307780956912834768081659431452
92528342610931516231547450242131454340*sqrt(7)*sqrt(2)*(110320475741093888*
sqrt(2) - 151363871237318045) - 269277151720273830400789203609786577249938 9
616713268545625606596270076614743912626365481228097225578689498425200383606
```

```
35590*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(110
320475741093888*sqrt(2) - 151363871237318045) - 53855430344054760947974878
198624587397928429114933826214930241667733552447191623911679736931289931750
505205553827219922880*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 1059547098
6612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 47123501551
047920230470272157043648368683383506960627830940150456323925190242898045263
53306319649781934534112633453268706200*(110320475741093888*sqrt(2) - 151363
871237318045)^2 + 419385925470581756378654520411097941724961521827588380808
369900699790874591792115928637572856409303229934928633430010503747776545857
52870920*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105954709866122
63150) + 733925369573518077568688728124769337212428798249901052299318450769
533849414676062708934630055035134045830109366690499889371007117918795898700
*sqrt(7)*(110320475741093888*sqrt(2) - 151363871237318045) + 14678507391470
361541608666268982016900814119268249174920123751742827764242323014015762064
19636158097776887638820246148744285006222232316501400*sqrt(2)*(110320475741
093888*sqrt(2) - 151363871237318045) + 244641789857839360328825543618149597
778089793408164936529659929761653877816458460985919888559249587651218708898
851314243292243938339792092100*sqrt(7722433301876572160*sqrt(2) - 105954709
86612263150)*(110320475741093888*sqrt(2) - 151363871237318045) - 1292077753
312694019345893430149910326331364817398263414897431400481314306688204378121
59214718582829755008425909786359842144150949633159991150*sqrt(7)*sqrt(2) -
758762916008240003409819101375410993692930390503123728189868562072536874773
198417845474059096092571231597614031305995383233253822344649520984786771597
244159013334688*x - 6460388766563470102884373475784382056879504414059510335
300445902481346769867158741187507320312111455790395720782521985923352484305
4466661575*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
 - 129207775331269402042300203703100565068677528624357666238053675568548792
622685475800752550765348440250164638699943637093422466523499441710582*sqrt(
2)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150) - 3601822391482
016791468032806227754184457992608529203568071788577705554082231768803383499
513928216083726730307366377332828337773366071083598072772386051930660*sqrt(
7) - 2566532515528593027457553740848822251225795581355385468029664178193344
384084826780557660928107456109583996394402379692154048400978571146920888612
60268773750000*sqrt(2) + 75876291600824000340981910137541099369293039050312
372818986856207253687477319841784547405909609257123159761403130599538323325
3822344649520984786771597244159013334688*sqrt(x^2 - 2*x + 3) - 360182239148
201674253967483465988239211591610327364049310125917227965268177456885468827
3070958284684112917867030199254538909056956640775330160129944904845080*sqrt
(7722433301876572160*sqrt(2) - 10595470986612263150) - 18969037612592882493
286509039021128990797812040571688798634238358733178856444459732969886293771
7384625840394590068917821873696654547424569549120105141773649324816347)^2)
+ 41672947348129/28000000000*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150)*arctan(-1/14*(424907232964614401909498696770230156468041
018681749287786326394760620649872991113993465473093811839889694663857531357
41461062214051300373175148059209445672904746742528*x - 42490723296461440190
949869677023015646804101868174928778632639476062064987299111399346547309381
183988969466385753135741461062214051300373175148059209445672904746742528*sq
rt(x^2 - 2*x + 3) - 1062268082411536004773746741925575391170102546704373219
465815986901551624682477784983663682734529599724236659643828393536526555351
2825093293787014802361418226186685632*I*sqrt(20*sqrt(2) - 25) + 10622680824
115360047737467419255753911701025467043732194658159869015516246824777849836
6368273452959972423665964382839353652655553512825093293787014802361418226186
685632)/(240456903400223659024212076421857594876596481118805954534597897163
5505204657923525607623616276623751473316663*(sqrt(7) + 2*sqrt(2) + sqrt(772
2433301876572160*sqrt(2) - 10595470986612263150))^7 - 142739937567750969487
785143849827267260861774511051146015080399033005784238132756036643733816335
2898015665578569812646*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt
(2) - 10595470986612263150))^6 + 103500570893851144987969410044573593528466
831057025915469516609398945671350435784286059468190138006696033871436116351
```

```
184730996386*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 1059
5470986612263150))^5 - 5062982909687377389957399074025778336103290716597321
5049765624930757608699456368874374542973258344666404687384663029077837715815
6904194414*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 105954
70986612263150))^4 + 109663729603246175183765204929539035364198260073648815
3626251774799452192758375873916572220994961819156070680914794516208869024576
339316465291922847*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2)
 - 10595470986612263150))^3 - 42190834681344113420154631008494428482305699
904760055166613689392979753170802688007006183501301022019174774824416270657
81217996707276477846427694382052541146*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301
876572160*sqrt(2) - 10595470986612263150))^2 - 3682359808560612938121004494
2331473355861294289510039804332924533313682318158002472147635428150206679235
686050868347855430430947554077103367216545381319104549548320 0*sqrt(7) - 73
64719617121225876242008988466294671172258857902007960866584906662736463631600
494429527085630041335847137210173669571086086189510815420673443309076263820
90990966400*sqrt(2) - 3682359808560612938121004494233147335586129428951003
98043329245333136823181580024721476354281502066792356860508683478554304309
4755407710336721654538131910454954832 00*sqrt(7722433301876572160*sqrt(2) -
10595470986612263150) + 14283521052038872882790385369323228930040295489364860
04185262156116702645830414445699158147702914990438794861101657539550235073
914317402544794766591661892781130702646113968 86))/(110320475741093888*sqrt
(2) - 151363871237318045) - 41672947348129/28000000000*sqrt(7)*sqrt(7722433
301876572160*sqrt(2) - 10595470986612263150)*arctan(-1/14*(4249072329646144
0190949869677023015646804101868174928778632639476062064987299111399346547309
381183988969466385753135741461062214051300373175148059209445672904746742528*x - 42490723296461440190949869677023015646804101868174928778632639476062
06498729911139934654730938118398896946638575313574146106221405130037317514805
92094456729047467425 28*sqrt(x^2 - 2*x + 3) - 1062268082411536004773746741925575391170102546704373219465815986901551624682477784983663682734529599724236659643828393536526555351282509329378701480236141822618668563 2*I*sqrt(20*s
qrt(2) - 25) + 106226808241153600477374674192557539117010254670437321946581
5986901551624682477784983663682734529599724236659643828393536526555351282509
32937870148023614182261866856 32)/(240456903104482806317945899374772853339737565242264877082844317429192606553322958291781931304794967036723873 3*(sqrt
(7) + 2*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))
^7 + 14273993864027954231032416493235084597584172646912424404943920290616291938289152272891444274768086053234577989342849 66*(sqrt(7) + 2*sqrt(2) + sqr
t(7722433301876572160*sqrt(2) - 10595470986612263150))^6 + 1035005707943988
2867370455909314910366457483004227041867621777239761173739910974366657765669338027073627974391069533080573093050 6*(sqrt(7) + 2*sqrt(2) + sqrt(77224333
01876572160*sqrt(2) - 10595470986612263150))^5 + 5062982839792119265762213443090551781921004176672345684321390261731100045776430693261660418696870154376935391839699513483958286559484145 4*(sqrt(7) + 2*sqrt(2) + sqrt(7722433301
876572160*sqrt(2) - 10595470986612263150))^4 + 109663729492100506852548516508189371952187779338119894553749726838300179564207424160536321027108218235029918594318831383181052429437744237854709567 7*(sqrt(7) + 2*sqrt(2) + sqrt(7
722433301876572160*sqrt(2) - 10595470986612263150))^3 + 4219083267175436404054639687522254737619823303456517594357863135556649401630968020316178357804829860461135845191147375397762170758509647257854327953473930 26*(sqrt(7) + 2
*sqrt(2) + sqrt(7722433301876572160*sqrt(2) - 10595470986612263150))^2 - 36
8235985635024162362460774967963334004243897793303578695550410507799195844567196904285900746309831059944379885387481474312223533832259969580105714665875528461138380*sqrt(7) - 73647197127004832472492154993592666800848779558660715739110082101559839168913439380857180149261966211988875977077496294862444706766451993916021142933175105692227676 0*sqrt(2) - 368235985635024162362460774967963334004243897793303578695550410507799195844567196904285900746309831059944379885387481474312223533832259969580105714665875528461138380*sqrt(7722
433301876572160*sqrt(2) - 10595470986612263150) - 142835208193613592915330036722260451533458578946048957790307351258262277239751604641916384506889981 1
```

```
400707486297236131450624307160336213112668130748323345181748400248288944 86)
)/(110320475741093888*sqrt(2) - 151363871237318045) + 1/205800000000*(10812
1281*(x - sqrt(x^2 - 2*x + 3))^15 + 135317265*(x - sqrt(x^2 - 2*x + 3))^14
- 2309618731*(x - sqrt(x^2 - 2*x + 3))^13 - 4089866767*(x - sqrt(x^2 - 2*x
+ 3))^12 + 23951599406*(x - sqrt(x^2 - 2*x + 3))^11 + 45641347654*(x - sqrt
(x^2 - 2*x + 3))^10 - 149568395690*(x - sqrt(x^2 - 2*x + 3))^9 - 2882154309
78*(x - sqrt(x^2 - 2*x + 3))^8 + 660704292769*(x - sqrt(x^2 - 2*x + 3))^7 +
 1062639157153*(x - sqrt(x^2 - 2*x + 3))^6 - 2094971437979*(x - sqrt(x^2 -
2*x + 3))^5 - 2301192104575*(x - sqrt(x^2 - 2*x + 3))^4 + 4977175786352*(x
- sqrt(x^2 - 2*x + 3))^3 + 1302994004424*(x - sqrt(x^2 - 2*x + 3))^2 - 6052
879270032*x + 6052879270032*sqrt(x^2 - 2*x + 3) + 2841437414928)/((x - sqrt
(x^2 - 2*x + 3))^4 + (x - sqrt(x^2 - 2*x + 3))^3 - 5*(x - sqrt(x^2 - 2*x +
3))^2 - 7*x + 7*sqrt(x^2 - 2*x + 3) + 14)^4 + 1/3150000000*(3*((((((((29420
*x - 332589)*x + 1860912)*x - 6743744)*x + 17167416)*x - 31960026)*x + 4336
2368)*x - 42014736)*x + 26516604)*x - 27199867)/(x^2 - 2*x + 3)^(9/2)
```

maple [B] time = 0.65, size = 21028, normalized size = 55.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^5\left(x^2-2\,x+3\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(2\,x^2+x+1\right)}^5\,{\left(x^2-2\,x+3\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)),x)
[Out] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
[Out] Timed out
```

3.51 $\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

Optimal. Leaf size=638

$$-\frac{12105495874518671061833-5117656435043679338190x}{1042737204880000000000000000\sqrt{x^2-2x+3}}-\frac{146548895467025x+37857197792117}{2421216420000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+\right.}$$

```
[Out] 1/18401244792000000000*(37358055634422583-14024622879097678*x)/(x^2-2*x+3)^(
19/2)+1/1042737204880000000000*(476849951294984711-125181871472148210*x)/(x^
2-2*x+3)^(17/2)+1/15641058073200000000000*(78517583754833335 11+194216499620
4584234*x)/(x^2-2*x+3)^(15/2)-11/4066675099032000000000000*(7502325106308201
089-7813986379726516886*x)/(x^2-2*x+3)^(13/2)-3/1147010925368000000000000*(
69053268515296359011-44840736195018286006*x)/(x^2-2*x+3)^(11/2)+1/938463484
39200000000000000*(-838519439380295335657+466189390555853643870*x)/(x^2-2*x+
3)^(9/2)+1/312821161464000000000000000*(-1117646664729238460189+568839749685
437871554*x)/(x^2-2*x+3)^(7/2)+1/521368602440000000000000000*(-655140551156
5449301689+3127298559983309301910*x)/(x^2-2*x+3)^(5/2)+1/1042737204880000000
00000000*(-4179039782398459850819+18869934455896524026 94*x)/(x^2-2*x+3)^(
3/2)+1/630*(-1+10*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^9+1/88200*(887+2218*x)/
(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^8+1/1080450*(14453+29371*x)/(x^2-2*x+3)^(19/
2)/(2*x^2+x+1)^7+1/605052000*(8837931+17459234*x)/(x^2-2*x+3)^(19/2)/(2*x^2
+x+1)^6+1/26471025000*(447940041+813432205*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1
)^5+1/29647548000000*(592729157441+911061463974*x)/(x^2-2*x+3)^(19/2)/(2*x^
2+x+1)^4+1/12353145000000*(277010166219+310705340015*x)/(x^2-2*x+3)^(19/2)/
(2*x^2+x+1)^3+1/276710448000000*(5488221294349+1384103301166*x)/(x^2-2*x+3)
^(19/2)/(2*x^2+x+1)^2+1/2421216420000000*(-37857197792117-146548895467025*x
)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)+1/1042737204880000000000000000*(-12105495
874518671061833+5117656435043679338190*x)/(x^2-2*x+3)^(1/2)-1/2259801992000
000000000000000*arctanh(1/7*(272944589523248381749+x*(656826642296538601431
-464885615909893491590*2^(1/2))-191941026386645109841*2^(1/2))*35^(1/2)/(-8
104222592127468960547894479780085484 6405+573059225230017071260263638786665 0
0308992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-5672955814489228272383526135846
059839248350+4011414576610119498821845471506655021629440*2^(1/2))^(1/2)+1/2
259801992000000000000000000*arctan(1/7*(272944589523248381749+1919410263866
45109841*2^(1/2)+x*(656826642296538601431+464885615909893491590*2^(1/2)))*3
5^(1/2)/(8104222592127468960547894479780085484 6405+57305922523001707126026 3
638786665003 08992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(5672955814489228272383
526135846059839248350+4011414576610119498821845471506655021629440*2^(1/2))^
(1/2)
```

Rubi [A] time = 1.40, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{12105495874518671061833-5117656435043679338190x}{1042737204880000000000000000\sqrt{x^2-2x+3}}-\frac{146548895467025x+37857197792117}{2421216420000000\left(x^2-2x+3\right)^{19/2}\left(2x^2+x+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] (37358055634422583 - 14024622879097678*x)/(18401244792000000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(1042737204880000
```

```
00000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 194216499620458423 4*
x)/(15641058073200000000000*(3 - 2*x + x^2)^(15/2)) - (11*(7502325106308201
089 - 7813986379726516886*x))/(4066750990320000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(6905326851529635901 1 - 4484073619501828600 6*x))/(114701092536800
0000000000*(3 - 2*x + x^2)^(11/2)) - (83851943938029533565 7 - 4661893905558
53643870*x)/(93846348439200000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(31282116146400000000000 0*(3 - 2*
x + x^2)^(7/2)) - (6551405511565449301689 - 3127298559983309301910*x)/(5213
686024400000000000000*(3 - 2*x + x^2)^(5/2)) - (41790397823984598508 19 -
18869934455896524026 94*x)/(10427372048800000000000 000*(3 - 2*x + x^2)^(3/
2)) - (12105495874518671061833 - 5117656435043679338190*x)/(104273720488000
00000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2
)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2)*(1 + x
+ 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*
x^2)^7) + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2)*(1 + x +
 2*x^2)^6) + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^2) -
(37857197792117 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2)*(1 + x + 2*x^2)) + (Sqrt[(8104222592127468960547894479780085484 6405 + 57
30592252300170712602636387866650030899 2*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(810
4222592127468960547894479780085484640 5 + 573059225230017071260263638786665 0
0308992*Sqrt[2]))]*(2729445895232483817 49 + 1919410263866451098 41*Sqrt[2] +
 (6568266422965386014 31 + 4648856159098934915 90*Sqrt[2])*x))/Sqrt[3 - 2*x +
 x^2]])/3228288560000000000000000 - (Sqrt[(-810422259212746896054789447978
00854846405 + 5730592252300170712602636387866650030899 2*Sqrt[2])/70]*ArcTan
h[(Sqrt[5/(7*(-8104222592127468960547894479780085484640 5 + 5730592252300170
712602636387866650030899 2*Sqrt[2]))]*(2729445895232483817 49 - 1919410263866
45109841*Sqrt[2] + (6568266422965386014 31 - 4648856159098934915 90*Sqrt[2])*
x))/Sqrt[3 - 2*x + x^2]])/3228288560000000000000000
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
```

```
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f))))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} - \frac{\int \frac{-2960+3060x-1800x^2}{\left(3-2x+x^2\right)^{21/2}\left(1+x+2x^2\right)^9}\,dx}{3150}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x \cdots\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x \cdots\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x \cdots\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x \cdots\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x \cdots\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x \cdots\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x \cdots\right.}$$

$$= -\frac{1-10x}{630\left(3-2x+x^2\right)^{19/2}\left(1+x+2x^2\right)^9} + \frac{887+2218x}{88200\left(3-2x+x^2\right)^{19/2}\left(1+x \cdots\right.}$$

$$= \frac{37358055634422583-14024622879097678x}{184012447920000000\left(3-2x+x^2\right)^{19/2}} - \frac{1-10x}{630\left(3-2x+x^2\right)^{19/2} \cdots}$$

$$= \frac{37358055634422583-14024622879097678x}{184012447920000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847\cdots}{10427372048800\cdots}$$

$$= \frac{37358055634422583-14024622879097678x}{184012447920000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847\cdots}{10427372048800\cdots}$$

$$= \frac{37358055634422583-14024622879097678x}{184012447920000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847\cdots}{10427372048800\cdots}$$

$$= \frac{37358055634422583-14024622879097678x}{184012447920000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847\cdots}{10427372048800\cdots}$$

$$= \frac{37358055634422583-14024622879097678x}{184012447920000000\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847\cdots}{10427372048800\cdots}$$

$$= \frac{37358055634422583-14024622879097678x}{\cdots\left(3-2x+x^2\right)^{19/2}} + \frac{4768499512949847\cdots}{\cdots}$$

Mathematica [C] time = 11.60, size = 1431, normalized size = 2.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] Sqrt[3 - 2*x + x^2]*((1 - x)/(11875000000*(3 - 2*x + x^2)^10) + (265 - 113*
x)/(403750000000*(3 - 2*x + x^2)^9) + (82361 - 4841*x)/(60562500000000*(3 -
 2*x + x^2)^8) + (1062937 + 1642511*x)/(1574625000000000*(3 - 2*x + x^2)^7)
 + (7*(-678331 + 833371*x))/(2220625000000000*(3 - 2*x + x^2)^6) + (7*(-731
61291 + 43964675*x))/(90843750000000000*(3 - 2*x + x^2)^5) + (-1340879383 +
 430593031*x)/(181687500000000000*(3 - 2*x + x^2)^4) - (11*(1626125723 + 11
2950205*x))/(3028125000000000000*(3 - 2*x + x^2)^3) - (11*(3311570647 + 152
86717673*x))/(36337500000000000000*(3 - 2*x + x^2)^2) - (11*(-411521923277
+ 484788625685*x))/(363375000000000000000*(3 - 2*x + x^2)) + (251943 + 2217
70*x)/(6300000000000*(1 + x + 2*x^2)^9) - (73*(-888423 + 1604678*x))/(88200
0000000000*(1 + x + 2*x^2)^8) + (-2596903794 - 4965311863*x)/(1080450000000
0000*(1 + x + 2*x^2)^7) + (-539608494637 - 334647150510*x)/(121010400000000
0000*(1 + x + 2*x^2)^6) + (-40800462989458 + 56711874696335*x)/(26471025000
0000000000*(1 + x + 2*x^2)^5) + (42018358198215561 + 129196597088670934*x)/
(2964754800000000000000*(1 + x + 2*x^2)^4) + (62819559864314747 + 1696303
89653846945*x)/(3705943500000000000000*(1 + x + 2*x^2)^3) + (108242210919
6374795 + 4797048907791526114*x)/(830131344000000000000000*(1 + x + 2*x^2)
^2) + (65571203144429922747 + 367152793968978953465*x)/(3631824630000000000
00000000*(1 + x + 2*x^2))) + ((23244280795494674579 5*I + 216341778311919248
41*Sqrt[7])*ArcTan[(-1350637388604350168995865589487332591135 15 + (18863089
4626466690216855285995045889396405*I)*Sqrt[7] - 150624136187268800855926877
6761430483700000*x - (1057115009374721927181156513503524479386 80*I)*Sqrt[7]
*x + 4911535405084435870258097898135419857073 60*x^2 - (46076406417713999339
9975100872663310399420*I)*Sqrt[7]*x^2 - 180084985147246689199448745264977 67
8818020*x^3 + (19786829637791387086383768095344600939686 0*I)*Sqrt[7]*x^3 -
1760048165007618809267744855998310477758 25*x^4 - (207342833228459577163557 0
43035558264835165*I)*Sqrt[7]*x^4 + (186244248199755548159585682605666126004
224*I)*Sqrt[10*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (1146118450460034142
52052727757333000617984*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2]
+ (300856093245758962411638410362999126622208*I)*Sqrt[10*(-5 + I*Sqrt[7])]*
x^2*Sqrt[3 - 2*x + x^2] - (143264806307504267815065909696666250772480*I)*Sq
rt[10*(-5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])/(236877329083883697986467 8
493023884746594823*I + 423642940259238735473942663180025956729505*Sqrt[7] +
 (189061348606562030176007421855674531164693 6*I)*x + 615057455931122825839 4
328777942059796320*Sqrt[7]*x + (251130025985582296234089302785223915766782 0
*I)*x^2 - 2027867550801106189867763431094227596320*Sqrt[7]*x^2 - (313421774
6230760357128318797499380812303788*I)*x^3 + 634304316027200432791928669683 6
9397935660*Sqrt[7]*x^3 + (94474906488662646732838536919046070366969 7*I)*x^4
 + 16381317765107264789462917221030750634835*Sqrt[7]*x^4)])/(16141442800000
000000000000*Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/16141442800000000000000000)*(
-232442807954946745795*I + 21634177831191924841*Sqrt[7])*ArcTan[(35*(436249
429066394667658518621821260762859 5*I + 121040840074068210135412189480007416
20843*Sqrt[7] - (409190315966173327071960945007832374050 00*I)*x + 175730701
694606521668409393655487422752*Sqrt[7]*x + (264872883292651275777339658533 6
4310310620*I)*x^2 - 5793907288003160542479324088840650275 2*Sqrt[7]*x^2 - (1
523889414975282568392481402100786307062 0*I)*x^3 + 18122980457920012365483 67
627667697083876*Sqrt[7]*x^3 - (7958372719599758089132442037656199635 95*I)*x
^4 + 468037650431636136841797634886592875281*Sqrt[7]*x^4))/(135063738860435
0168995865589487332591135 15 + (18863089462646669021685528599504588939640 5*I
)*Sqrt[7] + 150624136187268800855926877676143048370 0000*x - (10571150093747
2192718115651350352447938680*I)*Sqrt[7]*x - 491153540508443587025809789813 5
41985707360*x^2 - (460764064177139993399975100872663310399420*I)*Sqrt[7]*x^
```

```
2 + 1800849851472466891994487452649776788180020*x^3 + (197868296377913870863
837680953446009396860*I)*Sqrt[7]*x^3 + 176004816500761880926774485599831047
775825*x^4 - (207342833228459577163557043035558264835165*I)*Sqrt[7]*x^4 - (
143264806307504267815065909696666250772480*I)*Sqrt[70*(5 + I*Sqrt[7])]*Sqrt[
3 - 2*x + x^2] - (143264806307504267815065909696666250772480*I)*Sqrt[70*(5 +
 I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] + (286529612615008535630131819393332500
154496*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(5 +
I*Sqrt[7])] - ((-232442807954946745795*I + 21634177831191924841*Sqrt[7])*Lo
g[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/(32282885600000000
000000000*Sqrt[70*(5 + I*Sqrt[7])]) + ((I/322828856000000000000000000)*(2324
42807954946745795*I + 21634177831191924841*Sqrt[7])*Log[(-I + Sqrt[7] - (4*
I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/Sqrt[70*(-5 + I*Sqrt[7])] - ((I/3228288
5600000000000000000)*(232442807954946745795*I + 21634177831191924841*Sqrt[7
])*Log[(1 + x + 2*x^2)*(-13*I + 15*Sqrt[7] + (22*I)*x - 10*Sqrt[7]*x + (9*I
)*x^2 + 5*Sqrt[7]*x^2 + I*Sqrt[70*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] - I
*Sqrt[70*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(-5 + I*Sqrt[7]
)] + ((-232442807954946745795*I + 21634177831191924841*Sqrt[7])*Log[(1 + x
+ 2*x^2)*(-163*I + 15*Sqrt[7] + (122*I)*x - 10*Sqrt[7]*x - (41*I)*x^2 + 5*S
qrt[7]*x^2 - (13*I)*Sqrt[10*(5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (5*I)*Sq
rt[10*(5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2])])/(322828856000000000000000000
*Sqrt[70*(5 + I*Sqrt[7])])
```

fricas [B] time = 1.55, size = 2775, normalized size = 4.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/1434466423992337676564216536036272863645522031860857110126813332618112898
720320000000000000000000*(36045960776272236628083717974972055111190660172853
358396135728761934386631817942748278579200*x^38 - 5587123920322196677352976
28612066854223455232679227055140103795809982992793178112598317977600*x^37 +
 48121357636323435898491763496587693573439531330759233458841197897182406153
47695356895190323200*x^36 - 28710607758300836474268681367065241896063360827
677699962522107958880738952242991399003888332800*x^35 + 1315091821471322213
079849345669386715662902248081338714385016894218223678278587868742508613888
00*x^34 - 48615412586213217241775435956082154303807288316753404802358233229
5256693402493082467454965081600*x^33 + 150125118590038017914558770715112989
4284646412544039883821859865409233818038611634065993336166400*x^32 - 396012
076807241950819334539073291504431090617269457971847700920669696894988016137
7086262197766400*x^31 + 909142000002142860704234021157216421398787616081889
7418150894645435329015717575267031369642428000*x^30 - 184247649298721582706
989900442437618212098383039369354048250008452972140026730578162474910272628
00*x^29 + 33413073756673638925333625169011170445598811516221975115590200411
293389434416479555356860509015600*x^28 - 5481653256044929545971751700338269
9673242410936114304344629842103656622934490247108012261346586400*x^27 + 822
459830940635186677366276046635475885728402385815973257367014937498803836507
49401133206999014400*x^26 - 11372284806763969440259273586264909409387404544
3618754295078471234595964240139128161766283626302000*x^25 + 146086574413322
248286514192550522624098477614094095488624493581512454991258074867544318895
241990800*x^24 - 1750270940810010216829737529974120232517363052261271442728
11232619626419165679723993392477178363200*x^23 + 19688729160578415943345565
4443374481739030277196290989156609388218395099469530751149958413044135200*x
^22 - 208068683375682167383215047521697995267539026087882795784482813901791
360434798005710722616487282000*x^21 + 2081714449184784825196181653920157303
47012009814583465001141378703189206795143605224483243158516400*x^20 - 19622
755618454040835316742234157685550832000179582185155831117699557408106901596
9836642878534431200*x^19 + 176534941677723459681422280024952573032106299529
482816321219585323399086976471958310981405494523200*x^18 - 1491362557380113
```

```
8055695482939892925873700761520407473033056588722073078338292382261957134O7
37358000*x^17 + 12189081448358772438901196169673375625310538365442623433615
091379956996287788323526370448053414440O*x^16 - 919831860532221296355370692
78588580392985745730700928388526309371776740142438834607398588992195200*x^1
5 + 69317814132471559316390137037592557060398996838342232414889371690271398
0980987386433144021309544OO*x^14 - 4574307084113250024797073972709329687876
58973237085936599028628836672372493907006547585746109180OO*x^13 + 329969655
216763949298031215090491433294517890491697894556446151291991903089176733485
18481311574800*x^12 - 17770083757788737971933739892049927033484890029804651
9382701821617409378512807078348222722743544OO*x^11 + 1354422526745145970196
036923825637435189936268397849855148372985225665526414709333739259622802880
0*x^10 - 481375973272848865172866855106995818624092546697867179982576756873
2599092879797201593187475517200*x^9 + 5091181133639025216832620106123280320
347641869015804163342220634415255665812683873707564839486000*x^8 - 46421311
850305640075834899457188406077339946202653776901797199608409580382714283736
3184426478400*x^7 + 1771233883264782126042267141811413849986971398265032235
916172889879027134542752439323372429279200*x^6 + 23911503454316320991841103
2521665649750496447867853609069487786445410804754998849116452338787600*x^5
+ 79817891129994413353362937273464455099835468*1264938752804265123815574105
1177996081490572724 18^(1/4)*sqrt(1590558865810545927822094)*sqrt(35)*sqrt(2
)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376
*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 4
74602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 207502
6563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 295688552
9*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x
^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 -
252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7
 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 1
37781*x + 59049)*sqrt(810422259212746896054789447978008548464O5*sqrt(2) + 1
14611845046003414252052727757333000617984)*arctan(1/54206850781156887023310
518673090274966005685838243268724684064391985051350175945649154733957770247
43167351056637371274953501437271981836435236061968*sqrt(7952794329052729639
11047)*(9939513250523192816422116593216797292815016511001378462170679301990
*sqrt(11005224487862873621128239642490888848098)*sqrt(288886807671054271567
2947094311)*sqrt(7)*(10*sqrt(2) + 9) + sqrt(1590558865810545927822094)*(5*1
26493875280426512381557410511779960814905727241 8^(3/4)*sqrt(288886807671054
271567294709431 1)*sqrt(35)*(34061369711090637000O*sqrt(2) - 483753219647003
202703) + 556695603033691074737732 9*126493875280426512381557410511779960814
9057272418^(1/4)*sqrt(2888868076710542715672947094311)*sqrt(35)*(4373478266
4604992355*sqrt(2) - 66269826580994560232))*sqrt(81042225921274689605478944
797800854846405*sqrt(2) + 114611845046003414252052727757333000617984) + 147
461812540444568715696613114138557910359478676937042172325597372869522935182
724790786*sqrt(2888868076710542715672947094311)*sqrt(7)*(125*sqrt(2) + 172)
)*sqrt(5191798731734901573730421875012971256390643826285581511813805064*x^2
 + sqrt(1590558865810545927822094)*(126493875280426512381557410511779960814
9057272418^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43268355662383849682
*sqrt(2) - 62135959399493560795) - 1264938752804265123815574105117799608149
057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 1054043
15061877410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(810
422259212746896054789447978008548464O5*sqrt(2) + 11461184504600341425205272
7757333000617984) - 1297949682933725393432605468753242814097660956571395377
953451266*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 389384904880117618029781640625972
84422929828697141861338603537 98*x + 874869761179272589826814757400740628067
45190*sqrt(11005224487862873621128239642490888848098) + 9085647780536077754
028238281272699698683626695999767645674158862) + 5/35309486994022006419332*
sqrt(11005224487862873621128239642490888848098)*sqrt(7)*(sqrt(2)*(10*x - 19
) + 9*x - 29) + 1/701918227692516147086715878423299535653311118502220320740
2698434948589291714697700041413 6*sqrt(1590558865810545927822094)*(5*1264938
75280426512381557410511779960814905727241 8^(3/4)*sqrt(35)*(sqrt(2)*(3406136
```

```
97110906370000*x + 143139522536096832703) - 483753219647003202703*x - 19747
4174574809537297) + 556695603033691074737732 9*1264938752804265123815574105l
1779960814905727241 8^(1/4)*sqrt(35)*(sqrt(2)*(43734782664604992355*x + 2253
5043916389567877) - 66269826580994560232*x - 211997387482154244 78) - (5*126
4938752804265123815574105117799608149057272418^(3/4)*sqrt(35)*(340613697110
906370000*sqrt(2) - 483753219647003202703) + 5566956030336910747377329*1264
9387528042651238155741051177996081490572724 18^(1/4)*sqrt(35)*(4373478266460
4992355*sqrt(2) - 66269826580994560232))*sqrt(x^2 - 2*x + 3))*sqrt(81042225
9212746896054789447978008548464 05*sqrt(2) + 114611845046003414252052727757 3
33000617984) - 1/353094869940220064193 32*sqrt(x^2 - 2*x + 3)*(5*sqrt(110052
2448786287362112823964249088884809 8)*sqrt(7)*(10*sqrt(2) + 9) + 74179594525
256316007*sqrt(7)*(125*sqrt(2) + 172)) + 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1
) + 172*x - 82)) + 7981789112999441335336293727346445509983546 8*12649387528
0426512381557410511779960814905727241 8^(1/4)*sqrt(15905588658105459278220 94
)*sqrt(35)*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 18679
68*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 2
61706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329
345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895
*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^
17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 +
468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*
x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3
+ 1023516*x^2 + 137781*x + 59049)*sqrt(8104222592127468960547894479780085 48
46405*sqrt(2) + 11461184504600341425205272775733300061798 4)*arctan(-1/54206
8507811568870233105186730902749660056858382432687246840643919850513501759 45
6491547339577702474316735105663737127495350143727198183643523606196 8*sqrt(7
952794329052729639110 47)*(9939513250523192816422116593216797292815016511001
378462170679301990*sqrt(1100522448786287362112823964249088884809 8)*sqrt(288
88680767105427156729470943 11)*sqrt(7)*(10*sqrt(2) + 9) - sqrt(1590558865810
545927822094)*(5*1264938752804265123815574105117799608149057272418^(3/4)*sq
rt(288886807671054271567294709431 1)*sqrt(35)*(340613697110906370000*sqrt(2)
 - 483753219647003202703) + 5566956030336910747377329*12649387528042651238 1
5574105117799608149057272418^(1/4)*sqrt(28888680767105427156729470943 11)*sq
rt(35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(81042225
9212746896054789447978008548464 05*sqrt(2) + 114611845046003414252052727757 3
33000617984) + 147461812540444568715696613114138557910359478676937042172325
5973728695229351827247907 86*sqrt(288886807671054271567294709431 1)*sqrt(7)*(
125*sqrt(2) + 172))*sqrt(51917987317349015737304218750129712563906438262855
81511813805064*x^2 - sqrt(1590558865810545927822094)*(12649387528042651238 1
557410511779960814905727241 8^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43
268355662383849682*sqrt(2) - 62135959399493560795) - 126493875280426512381 5
574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383
849682*x - 105404315061877410477) - 62135959399493560795*x + 14867267072426
1260159))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 11461184
50460034142520527277573330006179 84) - 12979496829337253934326054687532428 14
097660956571395377953451266*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 389384904880117
61802978164062597284422929828697141861338603537 98*x + 874869761179272589826
81475740074062806745190*sqrt(1100522448786287362112823964249088884809 8) + 9
08564778053607775402823828127269969868362669599976764567415886 2) - 5/353094
86994022006419332*sqrt(1100522448786287362112823964249088884809 8)*sqrt(7)*(
sqrt(2)*(10*x - 19) + 9*x - 29) + 1/70191822769251614708671587842329953565 3
3111185022203207402698434948589291714697700041413 6*sqrt(1590558865810545927
822094)*(5*1264938752804265123815574105117799608149057272418^(3/4)*sqrt(35)
*(sqrt(2)*(340613697110906370000*x + 143139522536096832703) - 4837532196470
03202703*x - 197474174574809537297) + 5566956030336910747377329*12649387528
042651238155741051177996081490572724 18^(1/4)*sqrt(35)*(sqrt(2)*(43734782664
604992355*x + 22535043916389567877) - 66269826580994560232*x - 211997387482
15424478) - (5*1264938752804265123815574105117799608149057272418^(3/4)*sqrt
(35)*(340613697110906370000*sqrt(2) - 483753219647003202703) + 556695603033
```

6910747377329*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(
35)*(43734782664604992355*sqrt(2) - 66269826580994560232))*sqrt(x^2 - 2*x +
3))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 1146118450460
03414252052727757333000617984) + 1/35309486994022006419332*sqrt(x^2 - 2*x +
3)*(5*sqrt(11005224487862873621128239642490888848098)*sqrt(7)*(10*sqrt(2)
+ 9) + 74179594525256316007*sqrt(7)*(125*sqrt(2) + 172)) - 1/476*sqrt(7)*(2
5*sqrt(2)*(5*x - 1) + 172*x - 82)) + 24453*12649387528042651238155741051177
99608149057272418^(1/4)*sqrt(1590558865810545927822094)*sqrt(35)*sqrt(7)*(5
8681264663553748097050996611754496316407808*x^38 - 909559602285083095504290
447482194692904321024*x^37 + 7833948832584425370956308047669225258240442368
*x^36 - 46739627304520560359301118801262456316018819072*x^35 + 214091258966
892905713578429763409810498374336512*x^34 - 7914378840963908726941828569900
21126475411881984*x^33 + 2443971981023852389183004169635504201209831489536*
x^32 - 6446905281100567635350197739288116580793540673536*x^31 + 14800438431
924516080565532176343356954693567774720*x^30 - 2999472018305304975160392093
8291975718069728182272*x^29 + 543950385039779684976755634437931462765495913
02144*x^28 - 89238943444544755685020562033988611037555993870336*x^27 + 1338
92902214827011092889528472923281328218944045056*x^26 - 18513587658740219001
1531997633716034835132689940480*x^25 + 237822622904897041561702187373073394
318812215508992*x^24 - 284936536851054039764888677994792967684286178131968*
x^23 + 320523992669231941724388481023319612454782787125248*x^22 - 338726814
722685956738928688519407226164440916295680*x^21 + 3388941060685178348864870
69250634573161464946753536*x^20 - 31944997194601654648963735003466931034597
1696140288*x^19 + 287391247503511322489973442496808422958321912250368*x^18
- 242787372161112804792074580815007335314268010577920*x^17 + 19843297253643
7767771981576557768362169992275296256*x^16 - 149744647365292015359562891324
224536622995129499648*x^15 + 1128464024652710230240544677245701140256867808
70656*x^14 - 74467740316666419201365857719494322341993062072320*x^13 + 5371
7632299767958169950489423652550531043035185152*x^12 - 289289275588053521479
65359067020100302706002886656*x^11 + 22049412762644251773309104679542809356
433843290112*x^10 - 7836592584014101531712860147940403658565697404928*x^9 +
8288222622431088813634530458617273979494874480640*x^8 - 755718873364113816
635702120278992782166815932416*x^7 + 28834921318932789503548025890975347172
93712375808*x^6 + 389268931244541502203228657135011133961927655424*x^5 + 77
0266211020267891996472416855047787936254787584*x^4 + 1518975997000593369833
59025256804087045027790848*x^3 + 117307057194105230541603999703274443460516
511744*x^2 - 81042225921274689605478944797800854846405*sqrt(2)*(512*x^38 -
7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 - 6905376*x^33 + 213239
04*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*x^29 + 474602241*x^28
- 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25 + 2075026563*x^24 - 24
86100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2956885529*x^20 - 27872
33482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731347859*x^16 - 13065372
72*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803*x^12 - 252407834*x^11
+ 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^
6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516*x^2 + 137781*x + 5904
9) + 15791334622283396419062076883133098158146453504*x + 676771483812145560
8169461521342756353491337216)*sqrt(8104222592127468960547894479780085484640
5*sqrt(2) + 114611845046003414252052727757333000617984)*log(514926300974084
6168871608737947327093513510106682349523414420454231938660554455908352*x^2
+ 16517307604525632141069927349727551216675979497245715202048/1665374957748
9013357854121082231147111*sqrt(1590558865810545927822094)*(1264938752804265
123815574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3
)*(43268355662383849682*sqrt(2) - 62135959399493560795) - 12649387528042651
23815574105117799608149057272418^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(432683556
62383849682*x - 105404315061877410477) - 62135959399493560795*x + 148672670
724261260159))*sqrt(81042225921274689605478944797800854846405*sqrt(2) + 114
611845046003414252052727757333000617984) - 12873157524352115422179021844868
31773378377526670587380853605113557984665138613977088*sqrt(x^2 - 2*x + 3)*(
4*x + 1) - 3861947257305634626653706553460495320135132580011762142560815340

```
6739539954158419312 64*x + 86770206865778458070361238647050247531051756333308
5943213766737920*sqrt(110052244878628736211282396424908888848098) + 90112102
67046480795525315291407822413648642686694111665975235794905892655970297839б
16) - 24453*1264938752804265123815574105117799608149057272418^(1/4)*sqrt(15
90558865810545927822094)*sqrt(35)*sqrt(7)*(5868126466355374809705099661175 4
496316407808*x^38 - 9095596022850830955042904474821946929043210 24*x^37 + 78
339488325844253709563080476692252582404423 68*x^36 - 4673962730452056035930 1
11880126245631601881907 2*x^35 + 2140912589668929057135784297634098104983743
36512*x^34 - 791437884096390872694182856990021126475411881984*x^33 + 244397
198102385238918300416963550420120983148953 6*x^32 - 6446905281100567635350 19
77392881165807935406735 36*x^31 + 148004384319245160805655321763433569546935
67774720*x^30 - 299947201830530497516039209382919757180697281822 72*x^29 + 5
4395038503977968497675563443793146276549591302144*x^28 - 892389434445447556
850205620339886110375559938703 36*x^27 + 1338929022148270110928895284729232 8
1328218944045056*x^26 - 185135876587402190011531997633716034835132689940480
*x^25 + 2378226229048970415617021873730733943188122155 08992*x^24 - 28493653
6851054039764888677994792967684286178131968*x^23 + 320523992669231941724388
48102331961245478278712524 8*x^22 - 3387268147226859567389286885194072261644
40916295680*x^21 + 3388941060685178348864870692506345731614649467535 36*x^20
 - 31944997194601654648963735003466931034597169614028 8*x^19 + 2873912475035
113224899734424968084229583219122503 68*x^18 - 242787372161112804792074580 81
500733531426801057792 0*x^17 + 1984329725364377677719815765577683621699922 75
296256*x^16 - 149744647365292015359562891324224536622995129499648*x^15 + 11
2846402465271023024054467724570114025686780870656*x^14 - 744677403166664192
0136585771949432234199306207232 0*x^13 + 5371763229976795816995048942365255 0
531043035185152*x^12 - 289289275588053521479653590670201003027060028866 56*x
^11 + 220494127626442517733091046795428093564338432901 12*x^10 - 78365925840
1410153171286014794040365856569740492 8*x^9 + 82882226224310888136345304586 1
7273979494874480640*x^8 - 755718873364113816635702120278992782166815932416*
x^7 + 288349213189327895035480258909753471729371237580 8*x^6 + 3892689312445
41502203228657135011133961927655424*x^5 + 770266211020267891996472416855047
787936254787584*x^4 + 1518975997000593369833590252568040870450277908 48*x^3
+ 117307057194105230541603999703274443460516511744*x^2 - 810422259212746896
05478944797800854846405*sqrt(2)*(512*x^38 - 7936*x^37 + 68352*x^36 - 407808
*x^35 + 1867968*x^34 - 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 12913
5330*x^30 - 261706983*x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x
^26 - 1615329345*x^25 + 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22
 - 2955425895*x^21 + 2956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 -
2118344505*x^17 + 1731347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 6497
38605*x^13 + 468691803*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^
9 + 72315585*x^8 - 6593724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 +
 1325322*x^3 + 1023516*x^2 + 137781*x + 59049) + 157913346222833964190620 76
883133098158146453504*x + 676771483812145560816946152134275635349133721 6)*s
qrt(81042225921274689605478944797800854846405*sqrt(2) + 1146118450460034142
5205272775733300061798 4)*log(514926300974084616887160873794732709351351010 6
6823495234144204542319386605544559083 52*x^2 - 1651730760452563214106992734 9
7275512166759794972457152020 48/16653749577489013357854121082231147111*sqrt(
15905588658105459278220 94)*(126493875280426512381557410511779960814905727 24
18^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(43268355662383849682*sqrt(2)
 - 62135959399493560795) - 126493875280426512381557410511779960814905727241
8^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(43268355662383849682*x - 105404315061877
410477) - 62135959399493560795*x + 148672670724261260159))*sqrt(81042225921
27468960547894479780085484640 5*sqrt(2) + 114611845046003414252052727757333 0
00617984) - 128731575243521154221790218448683177337837752667058738085360511
35579846651386139770 88*sqrt(x^2 - 2*x + 3)*(4*x + 1) - 386194725730563462 66
537065534604953201351325800117621425608153406739539954158419312 64*x + 86770
206865778458070361238647050247531051756333308594321376673792 0*sqrt(110052244
878628736211282396424908888848098) + 901121026704648079552531529140782241364
86426866941116659752357949058926559702978396 16) + 473149067064481998763217 7
```

```
09555105306943512932580756046793648401639888862209988063963205432771600*x^4
 + 933056734920520960789163462383318633282684143000124192505535084262195413
27449647498113404575200*x^3 + 720578468552481534074799505602958944515340229
24762066351912610467773507163772995097552926305600*x^2 + 106889973888659738
28268515625026705527863090230587961936087245720*(33722490019334222378242713
60*x^37 - 53502205399640031394796147712*x^36 + 4691493940829897017294945758
72*x^35 - 2847499220912667753383035299072*x^34 + 13254252261100740556512388
253568*x^33 - 49770080058525077628064229832576*x^32 + 156010734937008739388
220889457760*x^31 - 417516398850754397130111919794336*x^30 + 97153817191336
5251873706873353652*x^29 - 1993653213575521837888601204380228*x^28 + 365555
3471852957606257345414140031*x^27 - 6054769996581738503753686155104785*x^26
 + 9155494158513869230271529746307221*x^25 - 127401066776850481786936051030
09787*x^24 + 16442770202470076313197215936814318*x^23 - 1977256973428874472
0189854470201506*x^22 + 22286437617621909921609206629636086*x^21 - 23584986
647560742443188031208946882*x^20 + 23579397211179175240196614296051673*x^19
 - 22218747553941794885903840542461607*x^18 + 19912295454080246583636391613
811979*x^17 - 16801760806053390242995145349148613*x^16 + 136134079650064752
88139078599341572*x^15 - 10279305650733178669223634020962076*x^14 + 7606288
378303449524327938977040824*x^13 - 5069838234992751929471190426115248*x^12
+ 3507425970596197680016078213030977*x^11 - 1974814483061344405275851094534
735*x^10 + 1357002388430055881833293557852283*x^9 - 56696901075916946161595
1049236597*x^8 + 458426000073846882432457044306894*x^7 - 947045576652534893
32536549937026*x^6 + 135183920426913231415208872303230*x^5 - 10230953189017
74638403186272874*x^4 + 29398041153524973343917601742151*x^3 + 193395719557
0062708781629134823*x^2 + 3397462350398947848063583843461*x - 8003871087155
5316861345369643)*sqrt(x^2 - 2*x + 3) + 97000947689757129586992241138859857
91552656932179508931988236024507972118200210878516740079600*x + 41571834724
181626965853817630939939106654243995055038279949582962177023363715189479357
45748400)/(512*x^38 - 7936*x^37 + 68352*x^36 - 407808*x^35 + 1867968*x^34 -
 6905376*x^33 + 21323904*x^32 - 56249904*x^31 + 129135330*x^30 - 261706983*
x^29 + 474602241*x^28 - 778618854*x^27 + 1168229184*x^26 - 1615329345*x^25
+ 2075026563*x^24 - 2486100252*x^23 + 2796604422*x^22 - 2955425895*x^21 + 2
956885529*x^20 - 2787233482*x^19 + 2507517852*x^18 - 2118344505*x^17 + 1731
347859*x^16 - 1306537272*x^15 + 984596334*x^14 - 649738605*x^13 + 468691803
*x^12 - 252407834*x^11 + 192383368*x^10 - 68375067*x^9 + 72315585*x^8 - 659
3724*x^7 + 25158762*x^6 + 3396411*x^5 + 6720651*x^4 + 1325322*x^3 + 1023516
*x^2 + 137781*x + 59049)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 5.03, size = 86793, normalized size = 136.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{10}\left(x^2-2\,x+3\right)^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^{10}\,\left(x^2-2\,x+3\right)^{21/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)),x)
[Out] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
[Out] Timed out
```

3.52 $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)(1+x^2)}}\,dx$

Optimal. Leaf size=66

$$-\sqrt{2}\sqrt{\sqrt{a^2+1}+a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,(x-a)}{\sqrt{(x^2+1)\,(x-a)}}\right)$$

```
[Out] -arctan((-a+x)*2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2)/((-a+x)*(x^2+1))^(1/2))*2^(
1/2)*(a+(a^2+1)^(1/2))^(1/2)
```

Rubi [C] time = 1.23, antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6719, 6742, 719, 419, 932, 168, 538, 537}

$$\frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,\Pi\left(\frac{2}{1-i\left(a-\sqrt{a^2+1}\right)};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\left(1-i\left(a-\sqrt{a^2+1}\right)\right)\sqrt{(x^2+1)\,(-(a-x))}}+\frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{(x^2+1)\,(-(a-x))}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*(1 + x
^2)]),x]
```

```
[Out] ((2*I)*Sqrt[(a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[1 - I*x]/S
qrt[2]], 2/(1 - I*a)])/Sqrt[-((a - x)*(1 + x^2))] + (4*Sqrt[1 + a^2]*Sqrt[(
a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticPi[2/(1 - I*(a - Sqrt[1 + a^2])), Arc
Sin[Sqrt[1 - I*x]/Sqrt[2]], 2/(1 - I*a)])/((1 - I*(a - Sqrt[1 + a^2]))*Sqrt
[-((a - x)*(1 + x^2))])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx = \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{-a-\sqrt{1+a^2}+x}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int\left(\frac{1}{\sqrt{-a+x}\sqrt{1+x^2}} - \frac{2\sqrt{1+a^2}}{\sqrt{-a+x}\left(-a+\sqrt{1+a^2}+x\right)\sqrt{1+\cdots}}\right.}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-a+x}\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}} - \frac{\left(2\sqrt{1+a^2}\sqrt{-a+x}\cdots\right.}{\cdots}$$

$$= -\frac{\left(2\sqrt{1+a^2}\sqrt{-a+x}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-ix}\sqrt{1+ix}\sqrt{-a+x}\left(-a+\sqrt{1+a^2}\cdots\right.}}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{\left(4\sqrt{1+a^2}\sqrt{-a+\cdots}\right.}{\cdots}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{\left(4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\cdots\right.}{\cdots}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1\cdots}}{\left(1-i\left(a\cdots\right.\right.}$$

Mathematica [C] time = 1.15, size = 213, normalized size = 3.23

$$\frac{2\sqrt{\frac{a-x}{a+i}}\left(2i\sqrt{a^2+1}\sqrt{1-ix}\sqrt{x^2+1}\,\Pi\left(\frac{2i}{a-\sqrt{a^2+1}+i};\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2i}{a+i}\right)-\left(\sqrt{a^2+1}-a-i\right)\sqrt{1+ix}\,(x+i)\,\text{Ellip}\cdots\right.}{\left(-\sqrt{a^2+1}+a+i\right)\sqrt{1-ix}\sqrt{\left(x^2+1\right)(x-a)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*
(1 + x^2)]),x]
```

```
[Out] (2*Sqrt[(a - x)/(I + a)]*(-((-I - a + Sqrt[1 + a^2])*Sqrt[1 + I*x]*(I + x)*
EllipticF[ArcSin[Sqrt[1 - I*x]/Sqrt[2]], (2*I)/(I + a)]) + (2*I)*Sqrt[1 + a
^2]*Sqrt[1 - I*x]*Sqrt[1 + x^2]*EllipticPi[(2*I)/(I + a - Sqrt[1 + a^2]), A
rcSin[Sqrt[1 - I*x]/Sqrt[2]], (2*I)/(I + a)]))/((I + a - Sqrt[1 + a^2])*Sqr
t[1 - I*x]*Sqrt[(-a + x)*(1 + x^2)])
```

fricas [A] time = 1.13, size = 546, normalized size = 8.27

$$\left[\frac{1}{4}\sqrt{-2\,a-2\sqrt{a^2+1}}\,\log\left(-\frac{8\,ax^7+x^8+4\left(2\,a^2+15\right)x^6-8\left(4\,a^3+15\,a\right)x^5+2\left(8\,a^4+80\,a^2+67\right)x^4+64\,a^{\cdots}}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log(-(8*a*x^7 + x^8 + 4*(2*a^2 + 15)*x^6
- 8*(4*a^3 + 15*a)*x^5 + 2*(8*a^4 + 80*a^2 + 67)*x^4 + 64*a^4 - 8*(20*a^3 +
 37*a)*x^3 + 4*(16*a^4 + 74*a^2 + 15)*x^2 + 48*a^2 - 4*(a*x^6 + 2*(2*a^2 +
3)*x^5 - (4*a^3 - a)*x^4 - 8*a^3 - (4*a^3 + 29*a)*x^2 + 20*x^3 + 2*(10*a^2
+ 3)*x - (4*a*x^5 + x^6 - (4*a^2 - 15)*x^4 - 16*a*x^3 + (4*a^2 + 15)*x^2 +
8*a^2 - 20*a*x + 1)*sqrt(a^2 + 1) - 5*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2
*a - 2*sqrt(a^2 + 1)) - 8*(24*a^3 + 13*a)*x + 16*(a*x^6 - x^7 + 15*a*x^4 -
7*x^5 - (12*a^2 + 7)*x^3 + 4*a^3 + (4*a^3 + 15*a)*x^2 - (12*a^2 + 1)*x + a)
*sqrt(a^2 + 1) + 1)/(8*a*x^7 - x^8 - 4*(6*a^2 - 1)*x^6 + 8*(4*a^3 - 3*a)*x^
5 - 2*(8*a^4 - 24*a^2 + 3)*x^4 - 8*(4*a^3 - 3*a)*x^3 - 4*(6*a^2 - 1)*x^2 -
8*a*x - 1)), -1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(-1/4*sqrt(-a*x^2 + x^3
 - a + x)*(2*a^2 - 2*a*x - x^2 - 2*sqrt(a^2 + 1)*(a - x) - 1)*sqrt(2*a + 2*
sqrt(a^2 + 1))/(a*x^2 - x^3 + a - x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-(x^2 + 1)(a - x)}\left(a - x - \sqrt{a^2 + 1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="giac")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

maple [C] time = 0.12, size = 1275, normalized size = 19.32

$$\frac{2(-a-i)\sqrt{\frac{-a+x}{-a-i}}\sqrt{\frac{x-i}{a-i}}\sqrt{\frac{x+i}{a+i}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-a+x}{-a-i}},\sqrt{\frac{a+i}{a-i}}\right)}{\sqrt{-a x^2 + x^3 - a + x}} - 2\sqrt{a^2+1}\left(-\frac{i\sqrt{-ix+1}\sqrt{-\frac{a}{-a-i}+\frac{x}{-a-i}}\sqrt{ix+1}\,a^2\,\mathrm{Ellip}}{\sqrt{a^2+1}\sqrt{-a^3x^2+a^2x^3-a^3+a^2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x)
```

```
[Out] 2*(-a-I)*((-a+x)/(-a-I))^(1/2)*((x-I)/(a-I))^(1/2)*((x+I)/(a+I))^(1/2)/(-a*
x^2+x^3-a+x)^(1/2)*EllipticF(((-a+x)/(-a-I))^(1/2),((a+I)/(a-I))^(1/2))-2*(
a^2+1)^(1/2)*(-I/(a^2+1)^(1/2)*(-I*x+1)^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2
)*(I*x+1)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2
+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)
),2^(1/2)*(-I/(-a-I))^(1/2))*a^2-I/(a^2+1)^(1/2)*(-I*x+1)^(1/2)*(-1/(-a-I)*
a+1/(-a-I)*x)^(1/2)*(I*x+1)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x
)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(
-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/2))+I/(a^2+1)^(1/2)*(-I*x+1)^(1/
2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(I*x+1)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2*x
-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I)
)^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/2))*a^2+I/(a^2+1)^
(1/2)*(-I*x+1)^(1/2)*(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(I*x+1)^(1/2)/(-a^3*x^2
+a^2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*EllipticPi(1/2
*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(-I/(-a-I))^(1/
2))+(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(
a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))
^(1/2),-(a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2))*a+I*(-1/(-a-I)*a+1/(-a-I)*
```

```
x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+
x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),-(a+I)/(a^2+1)^(1/2
),((a+I)/(a-I))^(1/2))-(-1/(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(
1/2)*(1/(a+I)*x+I/(a+I))^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*Ellipti
cPi(((-a+x)/(-a-I))^(1/2),(a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2))*a-I*(-1/
(-a-I)*a+1/(-a-I)*x)^(1/2)*(1/(a-I)*x-I/(a-I))^(1/2)*(1/(a+I)*x+I/(a+I))^(1
/2)/(-a*x^2+x^3-a+x)^(1/2)/(a^2+1)^(1/2)*EllipticPi(((-a+x)/(-a-I))^(1/2),(
a+I)/(a^2+1)^(1/2),((a+I)/(a-I))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-\left(x^2 + 1\right)(a - x)}\left(a - x - \sqrt{a^2 + 1}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="maxima")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{a - x + \sqrt{a^2 + 1}}{\sqrt{-\left(x^2 + 1\right)\,(a - x)}\,\left(x - a + \sqrt{a^2 + 1}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))),x)
```

```
[Out] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**
(1/2),x)
```

```
[Out] Timed out
```

3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\,2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\,2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{\cdot}\right.}{4\,2^{2/3}}$$

```
[Out] -1/12*a*arctanh(x)*2^(1/3)+1/4*a*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1
/3)-1/8*b*ln(x^2+3)*2^(1/3)+3/8*b*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+1/12*a
*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*a*arctan((1-2^(1/3)*(-x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)+1/4*b*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*
3^(1/2)*2^(1/3)
```

Rubi [A] time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 393, 444, 55, 617, 204, 31}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\,2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\,2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{\cdot}\right.}{4\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2
)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh
[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3
)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
```

```
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = a\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx + b\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}}$$

Mathematica [C] time = 0.23, size = 145, normalized size = 0.73

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)-\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2])/6 - (9*a*x*AppellF1[1/2, 1/3,
 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3,
1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] -
 AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(x^2 + 3\right)\left(-x^2 + 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(-x^2 + 1\right)^{\frac{1}{3}}\left(x^2 + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(x^2 + 3\right)\left(-x^2 + 1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,x}{\left(1 - x^2\right)^{1/3}\,\left(x^2 + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral((a + b*x)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.54 $\int \frac{a+bx}{\left(3-x^2\right) \sqrt[3]{1+x^2}} \, dx$

Optimal. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\,2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{\right.}}{4\,2^{2/3}}$$

```
[Out] -1/12*a*arctan(x)*2^(1/3)+1/4*a*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)
+1/8*b*ln(-x^2+3)*2^(1/3)-3/8*b*ln(2^(2/3)-(x^2+1)^(1/3))*2^(1/3)-1/12*a*ar
ctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*a*arctanh((1-2^(1/3)*(x^2+1)^(1/3))*3
^(1/2)/x)*2^(1/3)*3^(1/2)-1/4*b*arctan(1/3*(1+2^(1/3)*(x^2+1)^(1/3))*3^(1/2
))*3^(1/2)*2^(1/3)
```

Rubi [A] time = 0.09, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 392, 444, 55, 617, 204, 31}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{a\tan^{-1}(x)}{6\,2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{b\log\left(3-x^2\right)}{4\,2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{\right.}}{4\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -(a*ArcTan[x])/(6*2^(2/3)) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))])/(2
*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/(2*2^
(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[(Sqrt[3]*(
1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4
*2^(2/3)) - (3*b*Log[2^(2/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
```

```
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = a\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx + b\int \frac{x}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$$

$$= -\frac{a\tan^{-1}(x)}{6\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

$$= -\frac{a\tan^{-1}(x)}{6\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

$$= -\frac{a\tan^{-1}(x)}{6\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

$$= -\frac{a\tan^{-1}(x)}{6\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}} - \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} -$$

Mathematica [C] time = 0.24, size = 153, normalized size = 0.77

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;-x^2,\frac{x^2}{3}\right)-\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(x^2-3)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3])/6 - (9*a*x*AppellF1[1/2, 1/3, 1
, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3
/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF
1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx+a}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^2+3\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx+a}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{{\left(x^2+1\right)}^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
```

```
[Out] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx-\int \frac{bx}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) - Integral(b
*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

3.55 $\int \frac{1}{x \sqrt[3]{4-6x+3x^2}}\, dx$

Optimal. Leaf size=97

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}$$

```
[Out] -1/4*ln(x)*2^(1/3)+1/4*ln(6-3*x-3*2^(1/3)*(3*x^2-6*x+4)^(1/3))*2^(1/3)+1/6*
arctan(-1/3*3^(1/2)-1/3*2^(2/3)*(2-x)/(3*x^2-6*x+4)^(1/3)*3^(1/2))*2^(1/3)*
3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {750}

$$\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(
2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x +
 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rule 750

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, -Simp[(Sqrt[3]*c*e*ArcTan
[1/Sqrt[3] + (2*(c*d - b*e - c*e*x))/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3))])/
q^2, x] + (-Simp[(3*c*e*Log[d + e*x])/(2*q^2), x] + Simp[(3*c*e*Log[c*d - b
*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)])/(2*q^2), x])] /; FreeQ[{a, b, c, d
, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*c*
e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\, dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\,2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.05, size = 111, normalized size = 1.14

$$-\frac{\sqrt[3]{\frac{3x+i\sqrt{3}-3}{x}}\sqrt[3]{\frac{9x-3i\sqrt{3}-9}{x}}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{3-i\sqrt{3}}{3x},\frac{3+i\sqrt{3}}{3x}\right)}{2\sqrt[3]{3x^2-6x+4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -1/2*(((-3 + I*Sqrt[3] + 3*x)/x)^(1/3)*((-9 - (3*I)*Sqrt[3] + 9*x)/x)^(1/3)
*AppellF1[2/3, 1/3, 1/3, 5/3, (3 - I*Sqrt[3])/(3*x), (3 + I*Sqrt[3])/(3*x)]
)/(4 - 6*x + 3*x^2)^(1/3)
```

fricas [B] time = 4.15, size = 171, normalized size = 1.76

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x^3+2\cdot 4^{\frac{2}{3}}\left(3x^2-6x+4\right)^{\frac{2}{3}}(x-2)+4\left(3x^2-6x+4\right)^{\frac{1}{3}}\left(x^2-4x+4\right)\right)}{6\left(x^3-12x^2+24x-16\right)}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x^3 + 2*4^(2/3)*(3
*x^2 - 6*x + 4)^(2/3)*(x - 2) + 4*(3*x^2 - 6*x + 4)^(1/3)*(x^2 - 4*x + 4))/
(x^3 - 12*x^2 + 24*x - 16)) + 1/12*4^(2/3)*log((4^(1/3)*(x - 2) + 2*(3*x^2
- 6*x + 4)^(1/3))/x) - 1/24*4^(2/3)*log((4^(2/3)*(3*x^2 - 6*x + 4)^(2/3) +
4^(1/3)*(x^2 - 4*x + 4) - 2*(3*x^2 - 6*x + 4)^(1/3)*(x - 2))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-6x+4\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

maple [C] time = 14.13, size = 2378, normalized size = 24.52

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-6*x+4)^(1/3),x)
```

```
[Out] 1/6*RootOf(_Z^3-2)*ln((160*RootOf(_Z^3-2)-8*RootOf(RootOf(_Z^3-2)^2+2*_Z*Ro
otOf(_Z^3-2)+4*_Z^2)*x^3+192*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)*x^2-384*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-36*(3*x
^2-6*x+4)^(2/3)+256*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-5*R
ootOf(_Z^3-2)*x^3+120*RootOf(_Z^3-2)*x^2-240*RootOf(_Z^3-2)*x-48*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-30*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-64*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2-40*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3+48*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2
/3)*x+18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)
*(3*x^2-6*x+4)^(1/3)*x^2-72*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x-96*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)-15*RootOf(_Z^3-2
)^2*(3*x^2-6*x+4)^(1/3)*x^2+60*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x+72*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x
+4)^(1/3)+96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_
Z^3-2)^2*x+60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)^3*x-60*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)+18*(3*x^2-6*x+4)^(2/3)*x+
16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x
^3+10*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*
x^3)/x^3)-1/3*ln((-200*RootOf(_Z^3-2)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)*x^3-60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
*x^2+120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+60*(3*x^2-6*
```

```
x+4)^(2/3)-80*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+15*RootOf
(_Z^3-2)*x^3-150*RootOf(_Z^3-2)*x^2+300*RootOf(_Z^3-2)*x-12*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-30*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-16*RootOf(Root
Of(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2-40*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3-48*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)*x
-30*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x
^2-6*x+4)^(1/3)*x^2+120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x+96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)+9*RootOf(_Z^3-2)^2*(
3*x^2-6*x+4)^(1/3)*x^2-36*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x-120*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^
(1/3)+24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-
2)^2*x+60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2
)^3*x+36*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)-30*(3*x^2-6*x+4)^(2/3)*x+4*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+10
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3)/
x^3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-1/6*ln((-200*RootO
f(_Z^3-2)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-60*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+120*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+60*(3*x^2-6*x+4)^(2/3)-80*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+15*RootOf(_Z^3-2)*x^3-150*RootOf(_Z^3
-2)*x^2+300*RootOf(_Z^3-2)*x-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-30*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3
-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2-16*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2-40*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3
-2)+4*_Z^2)*RootOf(_Z^3-2)^3-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)*x-30*RootOf(RootOf(_Z^3-2)^2+
2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x^2+120*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4
)^(1/3)*x+96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^2*(3*x^2-6*x+4)^(2/3)+9*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x^2-36*Ro
otOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x-120*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)+24*RootOf(RootOf(_Z^3-
2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x+60*RootOf(RootOf(_Z^3
-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x+36*RootOf(_Z^3-2)^2*(3
*x^2-6*x+4)^(1/3)-30*(3*x^2-6*x+4)^(2/3)*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3+10*RootOf(RootOf(_Z^3-2)^2+2*_
Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3)/x^3)*RootOf(_Z^3-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-6\,x+4\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(3\,x^2-6\,x+4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)),x)
```

```
[Out] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{3x^2 - 6x + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

3.56 $\int x\sqrt[3]{1-x^3}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\sqrt[3]{1-x^3}\,x^2$$

```
[Out] 1/3*x^2*(-x^3+1)^(1/3)-1/6*ln(-x-(-x^3+1)^(1/3))-1/9*arctan(1/3*(1-2*x/(-x^
3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {279, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{3}\sqrt[3]{1-x^3}\,x^2+\frac{1}{18}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sq
rt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/18 - Log[1 + x/(1
 - x^3)^(1/3)]/9
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt[3]{1-x^3}\,dx &= \frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{3}\int\frac{x}{\left(1-x^3\right)^{2/3}}\,dx\\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{3}\text{Subst}\left(\int\frac{x}{1+x^3}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{1}{9}\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{9}\text{Subst}\left(\int\frac{1+x}{1-x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{18}\text{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{6}\text{Subst}\left(\right.\\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{3}\text{Subst}\left(\int\right.\\
&= \frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right.
\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.27

$$\frac{1}{2}x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^(1/3),x]
[Out] (x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, x^3])/2
```

fricas [A] time = 1.03, size = 96, normalized size = 1.32

$$\frac{1}{3}\left(-x^3+1\right)^{\frac{1}{3}}x^2-\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)-\frac{1}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{18}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(1/3)*x^2 - 1/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
-x^3 + 1)^(1/3))/x) - 1/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/18*log((x^2 - (
-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^3+1\right)^{\frac{1}{3}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)*x, x)
```

maple [C] time = 0.10, size = 69, normalized size = 0.95

$$\frac{\left(x^3-1\right)^{\frac{2}{3}} \left(-\mathrm{signum}\left(x^3-1\right)\right)^{\frac{2}{3}} x^2 \,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{6\left(-x^3+1\right)^{\frac{2}{3}} \mathrm{signum}\left(x^3-1\right)^{\frac{2}{3}}}-\frac{\left(x^3-1\right) x^2}{3\left(-x^3+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(1/3),x)
```

```
[Out] -1/3*x^2*(x^3-1)/(-x^3+1)^(2/3)+1/6*(x^3-1)^(2/3)/signum(x^3-1)^(2/3)*(-sig
num(x^3-1))^(2/3)*x^2*hypergeom([2/3,2/3],[5/3],x^3)/(-x^3+1)^(2/3)
```

maxima [A] time = 1.25, size = 105, normalized size = 1.44

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{3x\left(\frac{x^3-1}{x^3}-1\right)}-\frac{1}{9}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1/3)/x + 1) + 1/18*lo
g(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(1-x^3\right)^{1/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1 - x^3)^(1/3),x)
```

```
[Out] int(x*(1 - x^3)^(1/3), x)
```

sympy [C] time = 1.02, size = 32, normalized size = 0.44

$$\frac{x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3))
```

3.57 $\int \frac{\sqrt[3]{1-x^3}}{x} dx$

Optimal. Leaf size=67

$$\sqrt[3]{1-x^3} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

[Out] (-x^3+1)^(1/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 57, 618, 204, 31}

$$\sqrt[3]{1-x^3} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)^(1/3)/x,x]

[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x^3)^(1/3)]/2

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1-x^3}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{x}\,dx, x, x^3\right)\\ &= \sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right)\\ &= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-\right.\\ &= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\ &= \sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 90, normalized size = 1.34

$$\sqrt[3]{1-x^3} + \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{1}{6}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{1-x^3} + 1\right) - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 -
 (1 - x^3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

fricas [A] time = 0.88, size = 73, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) + \left(-x^3+1\right)^{\frac{1}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3}\log\left(\left(-x^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + (-x^3 + 1
)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3
+ 1)^(1/3) - 1)
```

giac [A] time = 1.12, size = 72, normalized size = 1.07

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}} + 1\right)\right) + \left(-x^3+1\right)^{\frac{1}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3}\log\left(\left|\left(-x^3+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 +
1)^(1/3) - 1))
```

maple [C] time = 0.11, size = 49, normalized size = 0.73

$$-\frac{\Gamma\left(\frac{2}{3}\right)x^3 \operatorname{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,2],x^3\right) - 3\left(3\ln(x)+3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+i\pi\right)\Gamma\left(\frac{2}{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/x,x)
```

```
[Out] -1/9/GAMMA(2/3)*(GAMMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],x^3)-3*(3+1/6*Pi*
3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3))
```

maxima [A] time = 1.30, size = 71, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(
1/3) - 1)
```

mupad [B] time = 0.37, size = 83, normalized size = 1.24

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3i}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3i}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log(3*(1 - x^3)^(1/3) - (3^(1/2)*3i)/2 + 3/2)*
((3^(1/2)*1i)/6 - 1/6) - log((3^(1/2)*3i)/2 + 3*(1 - x^3)^(1/3) + 3/2)*((3^
(1/2)*1i)/6 + 1/6) + (1 - x^3)^(1/3)
```

sympy [C] time = 0.99, size = 37, normalized size = 0.55

$$-\frac{xe^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/x,x)
```

```
[Out] -x*exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**(-3))/(3*gamma(2/
3))
```

3.58 $\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$

Optimal. Leaf size=482

$$\sqrt[3]{1-x^3}-\frac{1}{3}\sqrt[3]{2}\log\left(x^3+1\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-$$

```
[Out] (-x^3+1)^(1/3)-1/3*2^(1/3)*ln(x^3+1)+1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*
2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3
))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)
+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(2^(1/3
)-(-x^3+1)^(1/3))*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/2*ln(-2^(1/3)*x-(-x^3
+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))
*2^(1/3)*3^(1/2)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3
*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*2^(
1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(1/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx=\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

maple [C] time = 19.96, size = 2972, normalized size = 6.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x+1),x)
```

```
[Out] -(x^3-1)/(-x^3+1)^(2/3)+(1/2*RootOf(_Z^3-2)*ln((2*RootOf(RootOf(_Z^3-2)^2+_
Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*
RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+_Z*Root
Of(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf
(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^
3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)
+_Z^2)*RootOf(_Z^3-2)^3*x+5*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(Roo
tOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-2*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2-7*RootOf(RootOf(_Z^3-2)
^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^4+8*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2-1
4*RootOf(_Z^3-2)*x^4-2*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootO
f(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x-9*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2
)+_Z^2)*x^3+8*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x-18*RootOf(_Z^3-2)*x^3-
2*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootO
f(_Z^3-2)-16*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2+8*(x^6-2*x
^3+1)^(1/3)*RootOf(_Z^3-2)^2-32*RootOf(_Z^3-2)*x^2-9*RootOf(RootOf(_Z^3-2)^
2+_Z*RootOf(_Z^3-2)+_Z^2)*x-18*RootOf(_Z^3-2)*x+2*(x^6-2*x^3+1)^(2/3)-7*Roo
tOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-14*RootOf(_Z^3-2))/(x+1)^2/(x^
2+x+1))-1/2*ln((2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(
_Z^3-2)^2*x^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3
-2)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2
)^2*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*
x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x+
4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x-5*(x^6
-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+
_Z^2)+8*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)
*RootOf(_Z^3-2)*x^2+7*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^4-2
*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2+14*RootOf(_Z^3-2)*x^4+8*(x^6-2*x^
3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x
+13*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3-2*(x^6-2*x^3+1)^(1/
3)*RootOf(_Z^3-2)^2*x+26*RootOf(_Z^3-2)*x^3+8*(x^6-2*x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)+20*RootOf(RootOf(_Z^3
-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-2*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2+40
*RootOf(_Z^3-2)*x^2+13*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x+26
*RootOf(_Z^3-2)*x-8*(x^6-2*x^3+1)^(2/3)+7*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf
(_Z^3-2)+_Z^2)+14*RootOf(_Z^3-2))/(x+1)^2/(x^2+x+1))*RootOf(RootOf(_Z^3-2)^
2+_Z*RootOf(_Z^3-2)+_Z^2)-1/2*ln((2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-
2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)
```

```
+_Z^2)*RootOf(_Z^3-2)^3*x^3+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^
2)^2*RootOf(_Z^3-2)^2*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)
*RootOf(_Z^3-2)^3*x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*R
ootOf(_Z^3-2)^2*x+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(
_Z^3-2)^3*x-5*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+
_Z*RootOf(_Z^3-2)+_Z^2)+8*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*Ro
otOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*x^2+7*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_
Z^3-2)+_Z^2)*x^4-2*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2+14*RootOf(_Z^3-
2)*x^4+8*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2
)*RootOf(_Z^3-2)*x+13*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3-2
*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x+26*RootOf(_Z^3-2)*x^3+8*(x^6-2*x^3+
1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)+20*
RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-2*(x^6-2*x^3+1)^(1/3)*R
ootOf(_Z^3-2)^2+40*RootOf(_Z^3-2)*x^2+13*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)*x+26*RootOf(_Z^3-2)*x-8*(x^6-2*x^3+1)^(2/3)+7*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+14*RootOf(_Z^3-2))/(x+1)^2/(x^2+x+1))*Root
Of(_Z^3-2)-1/3*ln((RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf
(_Z^3-2)^4*x^6-RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_
Z^2)^2*x^3+8*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^
2)*x^6-6*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*Ro
otOf(_Z^3-2)+_Z^2)*x^2-10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*R
ootOf(_Z^3-2)^2*x^3+16*x^6-12*(x^6-2*x^3+1)^(1/3)*x^4+2*RootOf(_Z^3-2)^2*Ro
otOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-24*x^3+12*(x^6-2*x^3+1)^(1/3)
*x+8)/(x-1)/(x^2+x+1))+1/3*ln((RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z
^2)^2*RootOf(_Z^3-2)^4*x^6-RootOf(_Z^3-2)^4*RootOf(RootOf(_Z^3-2)^2+_Z*Root
Of(_Z^3-2)+_Z^2)^2*x^3+2*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf
(_Z^3-2)+_Z^2)*x^6-6*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^4+6*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*R
ootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-8*x^6+6*(x^6-2*x^3+1)^(1
/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x+12*(
x^6-2*x^3+1)^(2/3)*x^2-2*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf
(_Z^3-2)+_Z^2)+16*x^3-8)/(x-1)/(x^2+x+1))+1/6*ln((RootOf(RootOf(_Z^3-2)^2+_
Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^6-RootOf(_Z^3-2)^4*RootOf(RootO
f(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*x^3+2*RootOf(_Z^3-2)^2*RootOf(RootOf(
_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^6-6*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^4+6*(x^6-2*x^3+1)^(2/3)
*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-8*x^6
+6*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*Root
Of(_Z^3-2)^2*x+12*(x^6-2*x^3+1)^(2/3)*x^2-2*RootOf(_Z^3-2)^2*RootOf(RootOf(
_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+16*x^3-8)/(x-1)/(x^2+x+1))*RootOf(_Z^3-2)
^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2))/(-x^3+1)^(2/3)*((x^3-1)
^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x + 1),x)
```

```
[Out] int((1 - x^3)^(1/3)/(x + 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 1), x)
```

3.59 $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$

Optimal. Leaf size=280

$$\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}+\frac{3\log\left(\sqrt[3]{1-x^3}-\sqrt[3]{2}(x-1)\right)}{2\,2^{2/3}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\log\left(\sqrt[3]{1-x^3}+\sqrt[3]{2}x\right)}{2\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}}{}$$

```
[Out] -1/4*ln(-3*(-1+x)*(x^2-x+1))*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
+3/4*ln(-2^(1/3)*(-1+x)+(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(x+(-x^3+1)^(1/3))-1/
4*ln(2^(1/3)*x+(-x^3+1)^(1/3))*2^(1/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)+1/2*arctan(1/3*(1+2*2^(1/3)*(-1+x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*
2^(1/3)
```

Rubi [F] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] ((2*I)*Defer[Int][(1 - x^3)^(1/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] + ((2*
I)*Defer[Int][(1 - x^3)^(1/3)/(-1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx &= \int \left(\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(1+i\sqrt{3}-2x\right)}+\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(-1+i\sqrt{3}+2x\right)}\right)dx\\ &= \frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{1+i\sqrt{3}-2x}\,dx}{\sqrt{3}}+\frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{-1+i\sqrt{3}+2x}\,dx}{\sqrt{3}}\end{aligned}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]
```

fricas [B] time = 16.51, size = 3085, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*2^(1/3)*arctan(1/3*(26795748*sqrt(3)*2^(2/3)*(586745*x^11 - 70
6109*x^10 - 191742*x^9 - 43779*x^8 + 396304*x^7 + 323715*x^6 - 462255*x^5 +
 73568*x^4 + 24102*x^3 + 2372*x^2 - 2008*x)*(-x^3 + 1)^(1/3) + 26795748*sqr
t(3)*2^(1/3)*(340975*x^10 + 46080*x^9 - 970873*x^8 + 685704*x^7 - 289743*x^
6 + 397966*x^5 - 203166*x^4 - 21912*x^3 + 29756*x^2 - 4016*x)*(-x^3 + 1)^(2
/3) + 7*sqrt(273426)*2^(1/6)*(6*sqrt(3)*2^(2/3)*(338078915*x^10 - 459916473
*x^9 - 111133574*x^8 + 235674676*x^7 + 297312537*x^6 - 494815414*x^5 + 2448
15194*x^4 - 34383000*x^3 - 8933924*x^2 + 2566224*x)*(-x^3 + 1)^(2/3) + sqrt
(3)*2^(1/3)*(2332175065*x^12 - 3283524318*x^11 + 1882024851*x^10 - 39193009
70*x^9 + 2796090405*x^8 + 610770276*x^7 + 98233512*x^6 + 140867400*x^5 - 11
45424564*x^4 + 430987096*x^3 + 108889824*x^2 - 54987072*x + 4032064) - 6*sq
rt(3)*(493920245*x^11 - 452201839*x^10 - 276972599*x^9 - 661557480*x^8 + 13
75964914*x^7 - 191435014*x^6 - 333786162*x^5 - 47180632*x^4 + 107411572*x^3
 - 13096840*x^2 - 2566224*x)*(-x^3 + 1)^(1/3)) - 3*sqrt(3)*(2247079524645*x
^12 - 5276442179264*x^11 + 3816306322874*x^10 - 3280399521884*x^9 + 6278089
258290*x^8 - 6181108351032*x^7 + 2698150339136*x^6 + 1210170331680*x^5 - 25
58541243960*x^4 + 1136906331664*x^3 - 42652634816*x^2 - 54080708992*x + 515
2977792))/(18230538112975*x^12 - 14115716188440*x^11 - 20854883745366*x^10
+ 1856205891292*x^9 + 11854156958820*x^8 + 23868971173080*x^7 - 27900743059
560*x^6 + 8785124358048*x^5 - 2880050871456*x^4 + 1047429829408*x^3 + 24296
4112512*x^2 - 141331907328*x + 8096384512)) + 1/18*sqrt(3)*2^(1/3)*arctan(-
1/3*(13397874*sqrt(3)*2^(2/3)*(18803*x^11 - 25367*x^10 - 203754*x^9 + 40802
1*x^8 - 139829*x^7 + 7128*x^6 - 233871*x^5 + 225275*x^4 - 47094*x^3 - 10225
*x^2 + 2921*x)*(-x^3 + 1)^(1/3) + 26795748*sqrt(3)*2^(1/3)*(10589*x^10 - 73
935*x^9 + 63883*x^8 + 142959*x^7 - 173613*x^6 - 31588*x^5 + 79410*x^4 - 437
7*x^3 - 13328*x^2 + 2921*x)*(-x^3 + 1)^(2/3) - 7*sqrt(273426)*(6*sqrt(3)*2^
(2/3)*(309683372*x^10 - 328552599*x^9 - 24698630*x^8 - 422031122*x^7 + 7021
64163*x^6 - 95703451*x^5 - 206316094*x^4 + 60985482*x^3 + 11167816*x^2 - 37
33038*x)*(-x^3 + 1)^(2/3) + sqrt(3)*2^(1/3)*(2345654785*x^12 - 2502234618*x
^11 - 252041853*x^10 - 4416416426*x^9 + 6899968311*x^8 - 1680852528*x^7 + 1
576960038*x^6 - 2990585436*x^5 + 642930363*x^4 + 528479914*x^3 - 117963261*
x^2 - 38399466*x + 8532241) - 6*sqrt(3)*(491687266*x^11 - 516958230*x^10 -
69305552*x^9 - 808934094*x^8 + 1418391515*x^7 - 385704187*x^6 - 112721241*x
^5 - 69510422*x^4 + 47121139*x^3 + 11465929*x^2 - 4799203*x)*(-x^3 + 1)^(1/
3))*sqrt((6*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 4
8*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 -
201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3
 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^
6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11
 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^
3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(2995162579*x^12 + 315959718008*x^11 - 8
49682072424*x^10 + 177300060912*x^9 - 508006765899*x^8 + 3583876884636*x^7
- 3031033916540*x^6 - 1410763301208*x^5 + 2375077456341*x^4 - 546587071308*
x^3 - 175036021936*x^2 + 63861157012*x - 3114267965))/(367648430113*x^12 -
1408582980384*x^11 - 1269375810828*x^10 + 5714713216048*x^9 - 1087485936795
*x^8 - 126379999188*x^7 - 10319650860540*x^6 + 10854292018608*x^5 - 1383220
291365*x^4 - 1828745373668*x^3 + 426327416076*x^2 + 93479232396*x - 2492267
5961)) - 1/18*sqrt(3)*2^(1/3)*arctan(1/3*(13397874*sqrt(3)*2^(2/3)*(17344*x
^11 - 120304*x^10 + 110610*x^9 + 203214*x^8 - 213415*x^7 - 96387*x^6 + 3010
2*x^5 + 157561*x^4 - 101868*x^3 + 15151*x^2 + 913*x)*(-x^3 + 1)^(1/3) - 267
95748*sqrt(3)*2^(1/3)*(1277*x^10 + 57510*x^9 - 189677*x^8 + 108972*x^7 + 10
2426*x^6 - 47461*x^5 - 82155*x^4 + 56409*x^3 - 7301*x^2 - 913*x)*(-x^3 + 1)
^(2/3) + 7*sqrt(273426)*(6*sqrt(3)*2^(2/3)*(8733539*x^10 - 122586360*x^9 +
269810944*x^8 - 28009538*x^7 - 316185126*x^6 + 161786897*x^5 + 95479640*x^4
 - 80193978*x^3 + 11163982*x^2 + 1166814*x)*(-x^3 + 1)^(2/3) - sqrt(3)*2^(1
/3)*(1971824*x^12 - 78264612*x^11 + 705529692*x^10 - 1556393152*x^9 + 93384
9120*x^8 + 135726408*x^7 - 213906684*x^6 + 446158968*x^5 - 582881445*x^4 +
182390318*x^3 + 31120185*x^2 - 12999294*x - 833569) + 6*sqrt(3)*(12965988*x
^11 - 175265260*x^10 + 270273662*x^9 + 299814882*x^8 - 663644613*x^7 + 7755
```

```
3085*x^6 + 286893603*x^5 - 82332150*x^4 - 33723265*x^3 + 10863861*x^2 + 333
245*x)*(-x^3 + 1)^(1/3))*sqrt((6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x
^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1
/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x
^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^1
0 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x
)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 +
141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(67113
679084*x^12 - 61534090748*x^11 - 1006807736260*x^10 + 1996201310444*x^9 + 1
93806523788*x^8 - 2673973669800*x^7 + 775957356356*x^6 + 2110159119756*x^5
- 1821028473882*x^4 + 377014646048*x^3 + 67410900094*x^2 - 19835743048*x -
1369553867))/(168032067092*x^12 - 2318893136652*x^11 + 4401905935020*x^10 +
 1550444734940*x^9 - 6210007783092*x^8 - 1634341806144*x^7 + 6341768478444*
x^6 - 948091553244*x^5 - 2281774840272*x^4 + 1036207535072*x^3 - 5948022808
2*x^2 - 20085678624*x - 761048497)) + 1/3*sqrt(3)*arctan((4*sqrt(3)*(-x^3 +
 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3)*(x^3 - 1))/(9*x^3 -
1)) + 1/48*2^(1/3)*log(7717175424*(6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 -
54*x^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) +
2^(1/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 +
33*x^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281
*x^10 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2
 - x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^
7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/48*2^(1/3)
*log(1929293856*(6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x^7 + 141*x^6 -
 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(1081*x^12
 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x^6 - 630*x^5 +
 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^10 - 3*x^9 - 16
2*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x)*(-x^3 + 1)^(
1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*
x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) - 1/48*2^(1/3)*log(7717175424*(6
*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 48*x^4 - 6*x
^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 - 201*x^10 +
338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3 + 15*x^2 +
 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^6 - 99*x^5
+ 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10
- 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2
- 6*x + 1)) - 1/48*2^(1/3)*log(1929293856*(6*2^(2/3)*(4*x^10 - 27*x^9 + 32*
x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 48*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3
) - 2^(1/3)*(35*x^12 - 66*x^11 - 201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249
*x^6 - 18*x^5 + 306*x^4 - 166*x^3 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 -
 93*x^9 + 66*x^8 - 19*x^7 + 87*x^6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)
*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 1
41*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - 6*x + 1)) + 1/6*log(3*(-x^3 +
 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

maple [C] time = 33.31, size = 1404, normalized size = 5.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^2-x+1),x)
```

```
[Out] 1/3*RootOf(_Z^2+_Z+1)*ln(-x^3*RootOf(_Z^2+_Z+1)^2-x^3*RootOf(_Z^2+_Z+1)+3*x
*(-x^3+1)^(2/3)-3*x^2*(-x^3+1)^(1/3)+2*x^3-RootOf(_Z^2+_Z+1)-2)-1/3*ln(x^3*
RootOf(_Z^2+_Z+1)^2+3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(2/3)*x-3*(-x^3+1)^(1/3)*R
ootOf(_Z^2+_Z+1)*x^2+4*x^3*RootOf(_Z^2+_Z+1)+3*x*(-x^3+1)^(2/3)-3*x^2*(-x^3
+1)^(1/3)+4*x^3-RootOf(_Z^2+_Z+1)-2)*RootOf(_Z^2+_Z+1)-1/3*ln(x^3*RootOf(_Z
^2+_Z+1)^2+3*RootOf(_Z^2+_Z+1)*(-x^3+1)^(2/3)*x-3*(-x^3+1)^(1/3)*RootOf(_Z^
2+_Z+1)*x^2+4*x^3*RootOf(_Z^2+_Z+1)+3*x*(-x^3+1)^(2/3)-3*x^2*(-x^3+1)^(1/3)
+4*x^3-RootOf(_Z^2+_Z+1)-2)-1/18*ln(-(36*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+
162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^3-5*RootOf(_Z^3+324*RootOf(_Z^2+_Z+
1)+162)^2*x^4-12*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(
-x^3+1)^(1/3)*x^2+2*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^3+18*RootOf(
_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x^3-12*RootOf(_Z^3+324*RootO
f(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x+RootOf(_Z^3+324*RootOf
(_Z^2+_Z+1)+162)^2*x^2-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1
/3)*x^2+216*(-x^3+1)^(2/3)*x^2+2*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x
-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x-108*x*(-x^3+1)^(
2/3)-RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2)/(x^2-x+1)^2)*RootOf(_Z^2+_Z+
1)*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)-1/18*ln(-(36*RootOf(_Z^3+324*Root
Of(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^3-5*RootOf(_Z^3+324*R
ootOf(_Z^2+_Z+1)+162)^2*x^4-12*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*RootO
f(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2+2*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*
x^3+18*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x^3-12*RootOf(
_Z^3+324*RootOf(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x+RootOf(_
Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^2-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+16
2)*(-x^3+1)^(1/3)*x^2+216*(-x^3+1)^(2/3)*x^2+2*RootOf(_Z^3+324*RootOf(_Z^2+
_Z+1)+162)^2*x-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x-10
8*x*(-x^3+1)^(2/3)-RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2)/(x^2-x+1)^2)*R
ootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)+1/18*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1
)+162)*ln(-(RootOf(_Z^2+_Z+1)*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^4+
2*RootOf(_Z^2+_Z+1)*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^3+6*RootOf(_
Z^3+324*RootOf(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^3-RootOf(
_Z^2+_Z+1)*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)^2*x^2-18*RootOf(_Z^3+324*
RootOf(_Z^2+_Z+1)+162)*RootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x^2-6*RootOf(_Z^3+3
24*RootOf(_Z^2+_Z+1)+162)*(-x^3+1)^(1/3)*x^3-2*RootOf(_Z^2+_Z+1)*RootOf(_Z^
3+324*RootOf(_Z^2+_Z+1)+162)^2*x+6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*R
ootOf(_Z^2+_Z+1)*(-x^3+1)^(1/3)*x+18*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)
*(-x^3+1)^(1/3)*x^2+108*(-x^3+1)^(2/3)*x^2+RootOf(_Z^2+_Z+1)*RootOf(_Z^3+32
4*RootOf(_Z^2+_Z+1)+162)^2-6*RootOf(_Z^3+324*RootOf(_Z^2+_Z+1)+162)*(-x^3+1
)^(1/3)*x-108*x*(-x^3+1)^(2/3))/(x^2-x+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x^2 - x + 1),x)
```

```
[Out] int((1 - x^3)^(1/3)/(x^2 - x + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**2-x+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)
```

3.60 $\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$

Optimal. Leaf size=232

$$\frac{1}{2}xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-\frac{x^3}{8}\right)+\sqrt[3]{1-x^3}-\frac{\log\left(x^3+8\right)}{\sqrt[3]{3}}+\frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right)-\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{1}{2}3^{2/3}\log$$

```
[Out] (-x^3+1)^(1/3)+1/2*x*AppellF1(1/3,-1/3,1,4/3,x^3,-1/8*x^3)-3^(1/6)*arctan(2
/9*(-x^3+1)^(1/3)*3^(5/6)+1/3*3^(1/2))+3^(1/6)*arctan(1/3*(1-3^(2/3)*x/(-x^
3+1)^(1/3))*3^(1/2))-1/3*ln(x^3+8)*3^(2/3)+1/2*3^(2/3)*ln(3^(2/3)-(-x^3+1)^
(1/3))-ln(-x-(-x^3+1)^(1/3))+1/2*3^(2/3)*ln(-1/2*3^(2/3)*x-(-x^3+1)^(1/3))-
2/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(1/3)/(2 + x), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx = \int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$$

Mathematica [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{2+x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(2 + x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x+2),x)
[Out] int((-x^3+1)^(1/3)/(x+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x + 2),x)
[Out] int((1 - x^3)^(1/3)/(x + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(2+x),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 2), x)
```

3.61 $\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=168

$$-\frac{x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{\sqrt[3]{3}}+\frac{2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}+\text{tan}$$

```
[Out] -1/4*x^2*AppellF1(2/3,1,1/3,5/3,x^3,-1/2*x^3)*2^(2/3)+1/3*arctan(1/3*(3^(1/
3)+2*(x^3+2)^(1/3))*3^(1/6))*3^(1/6)+2/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2)^
(1/3))*3^(1/2))*3^(1/6)+1/18*ln(-x^3+1)*3^(2/3)+1/6*ln(3^(1/3)-(x^3+2)^(1/3
))*3^(2/3)-1/3*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

Rubi [F] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

```
[Out] (1 - I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(2 + x^3)^(1/3)), x] +
(1 + I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(2 + x^3)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx &= \int \left(\frac{1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}+\frac{1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\right)dx\\ &= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\,dx+\left(1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\end{aligned}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{2+x}{\left(1+x+x^2\right)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)), x]
```

fricas [F] time = 18.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^3+2\right)^{\frac{2}{3}}(x+2)}{x^5+x^4+x^3+2x^2+2x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 2)^(2/3)*(x + 2)/(x^5 + x^4 + x^3 + 2*x^2 + 2*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

maple [F] time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^2+x+1\right)\left(x^3+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+x+1)/(x^3+2)^(1/3),x)
[Out] int((x+2)/(x^2+x+1)/(x^3+2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{{\left(x^3+2\right)}^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)),x)
[Out] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt[3]{x^3+2}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x+1)/(x**3+2)**(1/3),x)
[Out] Integral((x + 2)/((x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

3.62 $\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1587}

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

fricas [A] time = 0.92, size = 23, normalized size = 0.92

$$\frac{1}{8}\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="fricas")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

giac [A] time = 0.99, size = 23, normalized size = 0.92

$$\frac{1}{8}\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="giac")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{\ln\left(320x^{4}+80x^{3}-12x^{2}+24x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

maxima [A] time = 0.54, size = 23, normalized size = 0.92

$$\frac{1}{8}\,\log\left(320\,x^{4}+80\,x^{3}-12\,x^{2}+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="maxima")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

mupad [B] time = 0.07, size = 23, normalized size = 0.92

$$\frac{\ln\left(320\,x^{4}+80\,x^{3}-12\,x^{2}+24\,x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^2 - 3*x + 160*x^3 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

```
[Out] log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9)/8
```

sympy [A] time = 0.11, size = 22, normalized size = 0.88

$$\frac{\log\left(320x^{4}+80x^{3}-12x^{2}+24x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

3.63 $\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

```
[Out] -1/22*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2)+1/22*arctan(1/66*(800*x^3-40*
x^2+30*x+57)*11^(1/2))*11^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2090}

$$\frac{\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] -ArcTan[(7 - 40*x)/(5*Sqrt[11])]/(2*Sqrt[11]) + ArcTan[(57 + 30*x - 40*x^2
+ 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rule 2090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-(C*(2*e*(B*d - 4*
A*e) + C*(d^2 - 4*c*e))), 2]}, Simp[(2*C^2*ArcTan[(C*d - B*e + 2*C*e*x)/q])
/q, x] - Simp[(2*C^2*ArcTan[(C*(4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e + 4*
C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3))/(q*(B^2 -
 4*A*C))])/q, x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rubi steps

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4}\,dx=-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}+\frac{\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

Mathematica [C] time = 0.02, size = 86, normalized size = 1.46

$$\frac{1}{8}\text{RootSum}\left[320\#1^4+80\#1^3-12\#1^2+24\#1+9\&,\frac{20\#1^2\log(x-\#1)+12\#1\log(x-\#1)+3\log(x-\#1)}{160\#1^3+30\#1^2-3\#1+3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #1] + 12*Lo
g[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/8
```

fricas [A] time = 0.85, size = 43, normalized size = 0.73

$$\frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)+\frac{1}{22}\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40x-7)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="fric
as")
```

```
[Out] 1/22*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + 1/22*s
qrt(11)*arctan(1/55*sqrt(11)*(40*x - 7))
```

giac [A] time = 0.85, size = 40, normalized size = 0.68

$$\frac{1}{22}\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}\,(40\,x-7)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="giac
")
```

```
[Out] 1/22*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arcta
n(-1/55*sqrt(11)*(40*x - 7)))
```

maple [A] time = 0.03, size = 52, normalized size = 0.88

$$\frac{\sqrt{11}\;\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\;\arctan\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 1/22*11^(1/2)*arctan(1/55*(40*x-7)*11^(1/2))+1/22*11^(1/2)*arctan(-20/33*11
^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2)+400/33*11^(1/2)*x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{20\,x^2+12\,x+3}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="maxi
ma")
```

```
[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)
```

mupad [B] time = 0.34, size = 53, normalized size = 0.90

$$\frac{\sqrt{11}\;\mathrm{atan}\left(\frac{8\,\sqrt{11}\,x}{11}-\frac{7\,\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\;\mathrm{atan}\left(\frac{400\,\sqrt{11}\,x^3}{33}-\frac{20\,\sqrt{11}\,x^2}{33}+\frac{5\,\sqrt{11}\,x}{11}+\frac{19\,\sqrt{11}}{22}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 20*x^2 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

```
[Out] (11^(1/2)*atan((8*11^(1/2)*x)/11 - (7*11^(1/2))/55))/22 + (11^(1/2)*atan((5
*11^(1/2)*x)/11 + (19*11^(1/2))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x
^3)/33))/22
```

sympy [A] time = 0.17, size = 73, normalized size = 1.24

$$\frac{\sqrt{11}\left(2\,\mathrm{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)+2\,\mathrm{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sqrt(11)*x**
3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22))/44
```

3.64 $\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

Optimal. Leaf size=78

$$-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

```
[Out] 2*ln(320*x^4+80*x^3-12*x^2+24*x+9)+2*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2
)-2*arctan(1/66*(800*x^3-40*x^2+30*x+57)*11^(1/2))*11^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2100, 2090}

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-2\sqrt{11}\tan^{-1}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-((84 + 576*x + 400*x^2 - 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4)),x]
```

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(57 + 30*x -
 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4]
```

Rule 2090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-(C*(2*e*(B*d - 4*
A*e) + C*(d^2 - 4*c*e))), 2]}, Simp[(2*C^2*ArcTan[(C*d - B*e + 2*C*e*x)/q])
/q, x] - Simp[(2*C^2*ArcTan[(C*(4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e + 4*
C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3))/(q*(B^2 -
 4*A*C))])/q, x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rule 2100

```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[(Coeff[Pm, x, m]*Log[Qn])/(n*Coeff[Qn, x, n]), x] + Dist[1/(n*Coeff[Qn,
x, n]), Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x]
/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Rubi steps

$$\int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\log\left(9+24x-12x^2+80x^3+320x^4\right)-\frac{\int \frac{168960+675840x+1126400x^2}{9+24x-12x^2+80x^3+320x^4}\,d}{1280}$$

$$= 2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)-2\sqrt{11}\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)+$$

Mathematica [C] time = 0.02, size = 99, normalized size = 1.27

$$\frac{1}{2}\text{RootSum}\left[320\#1^4+80\#1^3-12\#1^2+24\#1+9\&,\frac{640\#1^3\log(x-\#1)-100\#1^2\log(x-\#1)-144\#1\log(x-}{160\#1^3+30\#1^2-3\#1+3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 +
320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 144
*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1 +
30*#1^2 + 160*#1^3) & ]/2
```

fricas [A] time = 0.88, size = 66, normalized size = 0.85

$$-2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)-2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}\left(40x-7\right)\right)+2\log\left(320x^4+80x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="fricas")
```

```
[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(1
1)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*
x + 9)
```

giac [A] time = 0.97, size = 64, normalized size = 0.82

$$-2\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}\left(40x-7\right)\right)\right)+2\log\left(320x^4+80x^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="giac")
```

```
[Out] -2*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arctan(
-1/55*sqrt(11)*(40*x - 7))) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

maple [A] time = 0.03, size = 75, normalized size = 0.96

$$-2\sqrt{11}\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)-2\sqrt{11}\arctan\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)+2\ln\left(6400x^4+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

```
[Out] 2*ln(6400*x^4+1600*x^3-240*x^2+480*x+180)-2*11^(1/2)*arctan(400/33*11^(1/2)
*x^3-20/33*11^(1/2)*x^2+5/11*11^(1/2)*x+19/22*11^(1/2))-2*11^(1/2)*arctan(1
/55*(40*x-7)*11^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4\int\frac{640x^3-100x^2-144x-21}{320x^4+80x^3-12x^2+24x+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="maxima")
```

```
[Out] 4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9), x)
```

mupad [B] time = 0.09, size = 76, normalized size = 0.97

$$2\,\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)-2\,\sqrt{11}\,\operatorname{atan}\left(\frac{8\,\sqrt{11}\,x}{11}-\frac{7\,\sqrt{11}}{55}\right)-2\,\sqrt{11}\,\operatorname{atan}\left(\frac{400\,\sqrt{11}\,x^3}{33}-\frac{20\,\sqrt{11}}{33}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(576*x + 400*x^2 - 2560*x^3 + 84)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 +
9),x)
```

```
[Out] 2*log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9) - 2*11^(1/2)*atan((8*11^(1/2)*x
)/11 - (7*11^(1/2))/55) - 2*11^(1/2)*atan((5*11^(1/2)*x)/11 + (19*11^(1/2))
/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x^3)/33)
```

sympy [A] time = 0.18, size = 100, normalized size = 1.28

$$\sqrt{11}\left(-2\operatorname{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)-2\operatorname{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)\right)+2\log\left(x^4+\frac{x^3}{4}-\frac{3}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),x
)
```

```
[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x*
*3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(x*
*4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

3.65 $\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\tan^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

```
[Out] 1/2*arctan(x*(x^2+1)/(-x^4+1)^(1/2))+1/2*arctanh(x*(-x^2+1)/(-x^4+1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {405}

$$\frac{1}{2}\tan^{-1}\left(\frac{x\left(x^2+1\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqrt[1 - x^4]
]/2
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}}{1+x^4}\,dx=\frac{1}{2}\tan^{-1}\left(\frac{x\left(1+x^2\right)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x\left(1-x^2\right)}{\sqrt{1-x^4}}\right)$$

Mathematica [C] time = 0.10, size = 110, normalized size = 2.24

$$-\frac{5x\sqrt{1-x^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)}{\left(x^4+1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};x^4,-x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};x^4,-x^4\right)\right)-5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};x^4,-x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] (-5*x*Sqrt[1 - x^4]*AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4])/((1 + x^4)*(-5*
AppellF1[1/4, -1/2, 1, 5/4, x^4, -x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9/
4, x^4, -x^4] + AppellF1[5/4, 1/2, 1, 9/4, x^4, -x^4])))
```

fricas [A] time = 0.68, size = 56, normalized size = 1.14

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}\,x}{x^2-1}\right)+\frac{1}{4}\log\left(-\frac{x^4-2\,x^2-2\,\sqrt{-x^4+1}\,x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)*x/(x^2 - 1)) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-x
^4 + 1)*x - 1)/(x^4 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

maple [B] time = 0.03, size = 100, normalized size = 2.04

$$\frac{\arctan\left(-\frac{\sqrt{-x^4+1}}{x}+1\right)}{4}-\frac{\arctan\left(\frac{\sqrt{-x^4+1}}{x}+1\right)}{4}-\frac{\ln\left(\frac{-\frac{\sqrt{-x^4+1}}{x}+\frac{-x^4+1}{2x^2}+1}{\frac{\sqrt{-x^4+1}}{x}+\frac{-x^4+1}{2x^2}+1}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(x^4+1),x)
```

```
[Out] -1/4*arctan((-x^4+1)^(1/2)/x+1)+1/4*arctan(-(-x^4+1)^(1/2)/x+1)-1/8*ln((1/2
*(-x^4+1)/x^2-(-x^4+1)^(1/2)/x+1)/(1/2*(-x^4+1)/x^2+(-x^4+1)^(1/2)/x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{1-x^4}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(x^4 + 1),x)
```

```
[Out] int((1 - x^4)^(1/2)/(x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)\left(x^2+1\right)}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

3.66 $\int \frac{\sqrt{1+x^4}}{1-x^4} \, dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*arctanh(x*2^(1/2)/(x^4+1)^(
1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {404, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(2*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx = \text{Subst}\left(\int \frac{1}{1-4x^4}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.09, size = 108, normalized size = 2.04

$$-\frac{5x\sqrt{x^4+1}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)}{\left(x^4-1\right)\left(2x^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-x^4,x^4\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-x^4,x^4\right)\right)+5F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-x^4,x^4\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] (-5*x*Sqrt[1 + x^4]*AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4])/((-1 + x^4)*(5*
AppellF1[1/4, -1/2, 1, 5/4, -x^4, x^4] + 2*x^4*(2*AppellF1[5/4, -1/2, 2, 9/
4, -x^4, x^4] + AppellF1[5/4, 1/2, 1, 9/4, -x^4, x^4])))
```

fricas [A] time = 0.91, size = 61, normalized size = 1.15

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/8*sqrt(2)*log((x^4 + 2*sqrt
(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)
```

maple [C] time = 0.04, size = 365, normalized size = 6.89

$$-\frac{i\sqrt{-ix^2+1}\sqrt{ix^2+1}\ \text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} - \frac{\sqrt{-ix^2+1}\sqrt{ix^2+1}\ \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} + \frac{i\sqrt{-ix^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/(-x^4+1),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/
```

```
2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticE(x*(1/2*2^(1/2)+
1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*
EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))+1/2*I/(1/2*2^(1/2)+1/2*I*
2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(
1/2)+1/2*I*2^(1/2)),I)-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1
+I*x^2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-Ell
ipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I))-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^
2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 1)^(1/2)/(x^4 - 1),x)
```

```
[Out] -int((x^4 + 1)^(1/2)/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(x**4 + 1)/(x**4 - 1), x)
```

3.67 $\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx$

Optimal. Leaf size=75

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}\,x}{\sqrt{px^2+x^4+1}}\right)$$

```
[Out] 1/4*arctan(x*(2-p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2-p)^(1/2)+1/4*arctanh(x*(2+
p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2+p)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2071, 1093, 205, 208}

$$\frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\tanh^{-1}\left(\frac{\sqrt{p+2}\,x}{\sqrt{px^2+x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p
]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 2071

```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2*b*x^2
 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[a
*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{1-2px^2+\left(-4+p^2\right)x^4}\,dx,x,\frac{x}{\sqrt{1+px^2+x^4}}\right)\\ &= \frac{1}{4}\left(-4+p^2\right)\text{Subst}\left(\int \frac{1}{-2-p+\left(-4+p^2\right)x^2}\,dx,x,\frac{x}{\sqrt{1+px^2+x^4}}\right)-\frac{1}{4}\left(-4+p^2\right)\text{Su}\\ &= \frac{1}{4}\sqrt{2-p}\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{1+px^2+x^4}}\right)+\frac{1}{4}\sqrt{2+p}\tanh^{-1}\left(\frac{\sqrt{2+p}\,x}{\sqrt{1+px^2+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 7.08, size = 5727, normalized size = 76.36

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.75, size = 359, normalized size = 4.79

$$\left[\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2\left(p-1\right)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}\,x+1}{x^4+2\,x^2+1}\right)+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2\left(p+1\right)x^2+2\sqrt{x^4+px}}{x^4-2\,x^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p
- 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2 +
 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sqrt(-p
 + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x
^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^
2 + 1)), -1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*sqrt(-p - 2)/((p +
2)*x)) + 1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)
*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2
)*x/sqrt(x^4 + p*x^2 + 1)) - 1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*
sqrt(-p - 2)/((p + 2)*x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{x^4+px^2+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

maple [C] time = 0.09, size = 1512, normalized size = 20.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+p*x^2+1)^(1/2)/(-x^4+1),x)
```

```
[Out] 1/2*(-p-1)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^
(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*Elliptic
F(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2
))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*
(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/
2))*(EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)
^(1/2)))^(1/2))-EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-
1/2*(p^2-4)^(1/2)))^(1/2)))+1/4*(p+2)*(-1/2/(p+2)^(1/2)*arctanh(1/2*(p*x^2+
2*x^2+p+2)/(p+2)^(1/2)/(x^4+p*x^2+1)^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1
/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*
x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*
x,1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2
*(p^2-4)^(1/2))^(1/2)))-1/2*(p+1)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1
```

```
/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+
p*x^2+1)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1
/2*(p^2-4)^(1/2)))^(1/2))-1/4*(p+2)*(-1/2/(p+2)^(1/2)*arctanh(1/2*(p*x^2+2*
x^2+p+2)/(p+2)^(1/2)/(x^4+p*x^2+1)^(1/2))-1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2
)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^
2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,
1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(
p^2-4)^(1/2))^(1/2)))+1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(
p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)
^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-
4)^(1/2)))^(1/2))*p-1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^
2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(
1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)
^(1/2)))^(1/2))-2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)
^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)
/(p+(p^2-4)^(1/2))*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2
*p-1/2*(p^2-4)^(1/2)))^(1/2))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1
/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+
p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/
2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1
/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^
(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2
)*x,-1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+
1/2*(p^2-4)^(1/2))^(1/2))-1/2*p/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x
^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(
x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,-1/(-1/2*p
+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(1
/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^4 + px^2 + 1}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
[Out] -integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^4 + p\,x^2 + 1}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1),x)
[Out] -int((p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{px^2 + x^4 + 1}}{x^4 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

3.68 $\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx$

Optimal. Leaf size=171

$$\frac{\sqrt{\sqrt{p^2+4}-p}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}\,x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}\,x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctanh(1/4*x*(p-2*x^2+(p^2+4)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/
(-x^4+p*x^2+1)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)-1/4*arctan(1/4*x*(p-
2*x^2-(p^2+4)^(1/2))*(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/(-x^4+p*x^2+1)^(1/2))*
(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2072}

$$\frac{\sqrt{\sqrt{p^2+4}-p}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}\,x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}+p}\,x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] -(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4 + p
^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2]) + (Sqrt[-p +
Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x
^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])
```

Rule 2072

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, -Simp[(a*Sqrt[b + q]*ArcTan[(Sqrt[b + q
]*x*(b - q + 2*c*x^2))/(2*Sqrt[2]*Rt[-(a*c), 2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*Sqrt[2]*Rt[-(a*c), 2]*d), x] + Simp[(a*Sqrt[-b + q]*ArcTanh[(Sqrt[-b + q
]*x*(b + q + 2*c*x^2))/(2*Sqrt[2]*Rt[-(a*c), 2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*Sqrt[2]*Rt[-(a*c), 2]*d), x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d +
a*e, 0] && NegQ[a*c]
```

Rubi steps

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx=-\frac{\sqrt{p+\sqrt{4+p^2}}\,\tan^{-1}\left(\frac{\sqrt{p+\sqrt{4+p^2}}\,x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\,\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}}+\frac{\sqrt{-p+\sqrt{4+p^2}}\,\tanh^{-1}\left(\frac{\sqrt{-p+\sqrt{4+p^2}}}{2\sqrt{2}}\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.42, size = 322, normalized size = 1.88

$$\frac{\sqrt{\frac{4x^2}{\sqrt{p^2+4}-p}+2}\,\sqrt{1-\frac{2x^2}{\sqrt{p^2+4}+p}}\left(2i\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{\sqrt{p^2+4}-p}}\,x\right),\frac{p-\sqrt{p^2+4}}{\sqrt{p^2+4}+p}\right)-(p+2i)\Pi\left(\frac{1}{2}i\left(p-\sqrt{p^2+4}\right)\right.\right.}{4\sqrt{\frac{1}{\sqrt{p^2+4}-p}}\,\sqrt{p}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] (Sqrt[2 + (4*x^2)/(-p + Sqrt[4 + p^2])]*Sqrt[1 - (2*x^2)/(p + Sqrt[4 + p^2]
)]*((2*I)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (
p - Sqrt[4 + p^2])/(p + Sqrt[4 + p^2])] - (2*I + p)*EllipticPi[(I/2)*(p - S
qrt[4 + p^2]), I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (p - S
qrt[4 + p^2])/(p + Sqrt[4 + p^2])] + (-2*I + p)*EllipticPi[(I/2)*(-p + Sqrt
[4 + p^2]), I*ArcSinh[Sqrt[2]*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*x], (p - Sqrt
[4 + p^2])/(p + Sqrt[4 + p^2])]))/(4*Sqrt[(-p + Sqrt[4 + p^2])^(-1)]*Sqrt[1
 + p*x^2 - x^4])
```

fricas [B] time = 5.83, size = 2667, normalized size = 15.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*arctan(1/4
*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3 + 4*p)*x^8 + 2*(
3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)*x^2 - 2*((p^2 +
4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x^4 + (p*x^12 - (p
^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^2)*sqrt(p^2 + 4))
*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (
p^2 + 4)*x^4)*sqrt(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(2*(sqrt(2)*(
x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*(x^11 - 2
*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(
3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x)*sqrt(-x^4 + p*x
^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 + 4)*x^5 - 2*(p^3 +
 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(1/4)) - (2*((p^
3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^4 + p*x^2 + 1)*sq
rt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6 - (p^4 - 4*p^2 -
 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^2 + 1) - 2*((p*x^
10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt(-x^4 + p*x^2 + 1
)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p^2 + 4)*x^6 + (p^
3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4) - sqrt(
p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5 - x^3)*sqrt(p^2
+ 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p*x^7 - (p^2 - 6)*
x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 - 6)*x^9 - 10*p*x^7
 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^11
 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^2 + 4)^(1/4)))*sq
rt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*s
qrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 -
4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 + 4*p)*x^10 + (2*p^
4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28)*x^4 - 3*(p^3 +
4*p)*x^2 - p^2 - 4)) + 8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^
(3/4)*arctan(-1/4*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3
 + 4*p)*x^8 + 2*(3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)
*x^2 - 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x
^4 + (p*x^12 - (p^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^
2)*sqrt(p^2 + 4))*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p
^3 + 4*p)*x^6 - (p^2 + 4)*x^4)*sqrt(p^2 + 4) - sqrt(p^2 + sqrt(p^2 + 4)*p +
 4)*(2*(sqrt(2)*(x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) +
sqrt(2)*(x^11 - 2*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2
+ 1))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x
)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 +
4)*x^5 - 2*(p^3 + 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)
^(1/4)) - (2*((p^3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^
4 + p*x^2 + 1)*sqrt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6
 - (p^4 - 4*p^2 - 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^
```

```
2 + 1) - 2*((p*x^10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt
(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p
^2 + 4)*x^6 + (p^3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt
(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5
 - x^3)*sqrt(p^2 + 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p
*x^7 - (p^2 - 6)*x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 -
6)*x^9 - 10*p*x^7 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2
)*((p^2 + 4)*x^11 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^
2 + 4)^(1/4)))*sqrt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x
^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 +
4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 +
4*p)*x^10 + (2*p^4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28
)*x^4 - 3*(p^3 + 4*p)*x^2 - p^2 - 4)) - (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*
(p^2 + 4))*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*
x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(
p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x
^4 + p^2 + 4)) + (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*(p^2 + 4))*sqrt(p^2 + s
qrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x
^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 +
4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/(p^2 +
4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

maple [B] time = 0.11, size = 456, normalized size = 2.67

$$-\frac{\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}\;p\ln\left(\frac{\sqrt{-x^4+p\,x^2+1}\,\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}}{x}+\frac{-x^4+p\,x^2+1}{x^2}+\sqrt{p^2+4}\right)}{32}+\frac{\sqrt{2}\,\sqrt{p+\sqrt{p^2+4}}\;p\ln\left(\frac{\sqrt{-x^4+p\,x^2+}\cdots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x)
```

```
[Out] 1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*(p^2+4)^(1/2)*ln((-x^4+p*x^2+1)/x^2+(-
x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/4*2^(
1/2)/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x+
2*(p+(p^2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))-1/32*2^(1/2)*(p+(p^2+4
)^(1/2))^(1/2)*p*ln((-x^4+p*x^2+1)/x^2+(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p
^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*(p^2
+4)^(1/2)*ln((-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)-(-x^4+p
*x^2+1)/x^2-(p^2+4)^(1/2))+1/4*2^(1/2)/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*
(2*(p+(p^2+4)^(1/2))^(1/2)-2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x)/(-p+(p^2+4)^(1
/2))^(1/2))+1/32*2^(1/2)*(p+(p^2+4)^(1/2))^(1/2)*p*ln((-x^4+p*x^2+1)^(1/2)*
2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)-(-x^4+p*x^2+1)/x^2-(p^2+4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+px^2+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^4+p\,x^2+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1),x)
[Out] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^2-x^4+1}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
[Out] Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)
```

3.69 $\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=80

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

```
[Out] -b*arctan((x^2-1)^(1/4))+b*arctanh((x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(x^2-1
)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 398, 444, 63, 298, 203, 206}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1 + x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[
(-1 + x^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
```

```
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx &= a\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx + b\int \frac{x}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx\\
&= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{(2-x)\sqrt[4]{-1+x}}\,dx,\right.\\
&= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + (2b)\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt[4]{-1+\right.\\
&= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+x^2}\right.\\
&= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b\tan^{-1}\left(\sqrt[4]{-1+x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\tanh^{-1}\left(\sqrt[4]{}\right.
\end{aligned}$$

Mathematica [C] time = 0.26, size = 157, normalized size = 1.96

$$\frac{x\left(bx\sqrt[4]{1-x^2}\,(x^2-2)F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right)-\frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}\right)}{4(x^2-2)\sqrt[4]{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (x*(b*x*(1 - x^2)^(1/4)*(-2 + x^2)*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] - (24
*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1/2, 1/4, 1, 3/2, x^
2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5
/4, 1, 5/2, x^2, x^2/2]))))/(4*(-2 + x^2)*(-1 + x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx + a}{\left(x^2 - 1\right)^{\frac{1}{4}}\left(x^2 - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(-x^2 + 2\right)\left(x^2 - 1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
[Out] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx + a}{\left(x^2 - 1\right)^{\frac{1}{4}}\left(x^2 - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a + b\,x}{{\left(x^2 - 1\right)}^{1/4}\,\left(x^2 - 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
[Out] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[4]{x^2 - 1} - 2\sqrt[4]{x^2 - 1}}\,dx - \int \frac{bx}{x^2\sqrt[4]{x^2 - 1} - 2\sqrt[4]{x^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
[Out] -Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) - Integral(b
*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

3.70 $\int \frac{a+bx}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx$

Optimal. Leaf size=88

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

```
[Out] b*arctan((-x^2-1)^(1/4))-b*arctanh((-x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(-x^2
-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(-x^2-1)^(1/4)*2^(1/2))*2^(1
/2)
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1010, 398, 444, 63, 298, 203, 206}

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1 - x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[
(-1 - x^2)^(1/4)]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
```

```
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx = a\int \frac{1}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx + b\int \frac{x}{\sqrt[4]{-1-x^2}\left(2+x^2\right)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-1-x}\,(2+x)}\,dx, x, x$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - (2b)\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt[4]{-1-x^2}$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1-x^2}\right) +$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b\tan^{-1}\left(\sqrt[4]{-1-x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\tanh^{-1}\left(\sqrt[4]{-1\right.$$

Mathematica [C] time = 0.26, size = 162, normalized size = 1.84

$$\frac{x\left(bx\sqrt[4]{x^2+1}\,F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{\left(x^2+2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)\right)}\right)}{4\sqrt[4]{-x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (x*(b*x*(1 + x^2)^(1/4)*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2] - (24*a*Appe
llF1[1/2, 1/4, 1, 3/2, -x^2, -1/2*x^2])/((2 + x^2)*(-6*AppellF1[1/2, 1/4, 1
, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^2]
+ AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))))/(4*(-1 - x^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^2+2\right)\left(-x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^2-1\right)^{\frac{1}{4}}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
[Out] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^2+2\right)\left(-x^2-1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x}{\left(-x^2-1\right)^{1/4}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[4]{-x^2-1}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
[Out] Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)
```

3.71 $\int \frac{a+bx}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx$

Optimal. Leaf size=149

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*a*arctan((1-(-x^2+1)^(1/2))/x/(-x^2+1)^(1/4))+1/2*a*arctanh((1+(-x^2+1)
^(1/2))/x/(-x^2+1)^(1/4))+1/2*b*arctan(1/2*(1-(-x^2+1)^(1/2))/(-x^2+1)^(1/4
)*2^(1/2))*2^(1/2)+1/2*b*arctanh(1/2*(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/4)*2^(1
/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1010, 397, 439}

$$\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcT
an[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x
^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(
x*(1 - x^2)^(1/4))])/2
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx = a\int \frac{1}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx + b\int \frac{x}{\sqrt[4]{1-x^2}\left(2-x^2\right)}\,dx$$
$$= \frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{b\tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\tanh^{-1}\Bigg($$

Mathematica [C] time = 0.21, size = 144, normalized size = 0.97

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}\left(x^2-2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/4 - (6*a*x*AppellF1[1/2, 1/4, 1,
 3/2, x^2, x^2/2])/((1 - x^2)^(1/4)*(-2 + x^2)*(6*AppellF1[1/2, 1/4, 1, 3/2
, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/
2, 5/4, 1, 5/2, x^2, x^2/2])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx+a}{\left(x^2-2\right)\left(-x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^2+1\right)^{\frac{1}{4}}\left(-x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{bx+a}{\left(x^2-2\right)\left(-x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{{\left(1-x^2\right)}^{1/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)),x)
```

```
[Out] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx-\int \frac{bx}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

```
[Out] -Integral(a/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x) - Integral(b
*x/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x)
```

3.72 $\int \frac{a+bx}{\sqrt[4]{1+x^2}\left(2+x^2\right)}\,dx$

Optimal. Leaf size=135

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*a*arctan(((x^2+1)^(1/2)+1)/x/(x^2+1)^(1/4))-1/2*a*arctanh((1-(x^2+1)^(
1/2))/x/(x^2+1)^(1/4))-1/2*b*arctan(1/2*(1-(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(
1/2))*2^(1/2)-1/2*b*arctanh(1/2*((x^2+1)^(1/2)+1)/(x^2+1)^(1/4)*2^(1/2))*2^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1010, 397, 439}

$$-\frac{1}{2}a\tan^{-1}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\tan^{-1}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]) - (a*A
rcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*ArcTanh[(1 - Sqrt[1
+ x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[(1 + Sqrt[1 + x^2])/(Sqrt[2]*(
1 + x^2)^(1/4))])/Sqrt[2]
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\left(2+x^2\right)}\,dx = a\int \frac{1}{\sqrt[4]{1+x^2}\left(2+x^2\right)}\,dx + b\int \frac{x}{\sqrt[4]{1+x^2}\left(2+x^2\right)}\,dx$$
$$= -\frac{b\tan^{-1}\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}} - \frac{1}{2}a\tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{b\tanh^{-}}{}$$

Mathematica [C] time = 0.19, size = 152, normalized size = 1.13

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{\sqrt[4]{x^2+1}\left(x^2+2\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right) - 6F_1\left(\frac{1}{2}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2])/4 - (6*a*x*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2])/((1 + x^2)^(1/4)*(2 + x^2)*(-6*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^
2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^2+2\right)\left(x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^2+1\right)^{\frac{1}{4}}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(x^2+2\right)\left(x^2+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x}{\left(x^2+1\right)^{1/4}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[4]{x^2+1}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
[Out] Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

3.73 $\int \frac{x}{\sqrt{1-x^3}\left(4-x^3\right)}\,dx$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1
/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^(1
/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcT
an[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqr
t[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\left(4-x^3\right)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Mathematica [C] time = 0.02, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

fricas [B] time = 1.26, size = 1191, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

maple [C] time = 0.19, size = 164, normalized size = 1.29

$$i\sqrt{i\left(2x+1-i\sqrt{3}\right)}\,\sqrt{\frac{x-1}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(2x+1+i\sqrt{3}\right)}{2}}\,\left(-2\,\mathrm{RootOf}\left(\_Z^3-4\right)^2+\mathrm{RootOf}\left(\_Z^3-4\right)+1+i\sqrt{3}\,\left(-\mathrm{Root}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(2*x+1-I*3^(1/2)))^(1/2)*((x-1)/(I*3^(1/
2)-3))^(1/2)*(-1/2*I*(2*x+1+I*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2+_
alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/2
)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-
4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

mupad [B] time = 0.45, size = 653, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

```
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

3.74 $\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx$

Optimal. Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\,2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\,2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\,2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt{3}\,d^{2/3}}$$

```
[Out] -1/6*arctan((1+2^(1/3)*d^(1/3)*x)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arc
tan((d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arctanh((1-2^(1/3)*d^(1/3)*x)*3^(
1/2)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/18*arctanh(1/3*(d*x^3-1)^(1
/2)*3^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {485}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}x+1}{\sqrt{dx^3-1}}\right)}{3\,2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{dx^3-1}\right)}{9\,2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{dx^3-1}}\right)}{3\,2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt{3}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] -ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*d^(2/3)) - Arc
Tan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*d
^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-1
+ d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))
```

Rule 485

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, -Simp[(q*ArcTan[Sqrt[c + d*x^3]/Rt[-c, 2]])/(9*2^(2/3)
*b*Rt[-c, 2]), x] + (-Simp[(q*ArcTan[(Rt[-c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c +
 d*x^3]])/(3*2^(2/3)*b*Rt[-c, 2]), x] - Simp[(q*ArcTanh[Sqrt[c + d*x^3]/(Sq
rt[3]*Rt[-c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[-c, 2]), x] - Simp[(q*ArcTanh[(S
qrt[3]*Rt[-c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*
Rt[-c, 2]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*
c - a*d, 0] && NegQ[c]
```

Rubi steps

$$\int \frac{x}{\left(4-dx^3\right)\sqrt{-1+dx^3}}\,dx=-\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}x}{\sqrt{-1+dx^3}}\right)}{3\,2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{-1+dx^3}\right)}{9\,2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{-1+dx^3}}\right)}{3\,2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\tanh^{-1}\left(\right.}{3\,2^{2/3}}$$

Mathematica [C] time = 0.03, size = 54, normalized size = 0.34

$$\frac{x^2\sqrt{1-dx^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};dx^3,\frac{dx^3}{4}\right)}{8\sqrt{dx^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - d*x^3]*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3)/4])/(8*Sqrt[
-1 + d*x^3])
```

fricas [B] time = 1.81, size = 1666, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*
d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt
(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1) + (2*sq
rt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))^(1/3) + sqrt(3)*(d*x^4 - x) + 3*(s
qrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1
/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d
*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4
 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^
2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3
)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*
d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^
6 + 48*d*x^3 - 64)))/(d*x^4 - x)) - 1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6
)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(
1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(
-4))^(5/6))*sqrt(d*x^3 - 1) - (2*sqrt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))
^(1/3) + sqrt(3)*(d*x^4 - x) - 3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*
sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*
x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 -
24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1
/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6
)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^
(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*
x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)))/(d*x^4 - x)) + 1/18
*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/
2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4
*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d
^(-4))^(5/6)*x^5 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4))
 + 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^
3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/6)*(
d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^
7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5
 - 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5
 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/
6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^
3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*lo
g((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d
^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1
/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 1
6*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*
x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 -
 64)) + 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 - 60*d^2*x^6 - 24*(1
/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(
d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6)*d^5
*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4))
 - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 -
 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{dx^3-1}\left(dx^3-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

maple [C] time = 0.22, size = 240, normalized size = 1.53

$$-\frac{i\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\;\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\;\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\;\left(-2\,\mathrm{RootOf}\left(d\_Z^3-4\right)^2 d+i\sqrt{3}\;\mathrm{RootOf}\left(d\_Z^3-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x)
```

```
[Out] -1/9*I*2^(1/2)*sum(1/_alpha/d^(4/3)*(-1/2*I*(2*x+1/d^(1/3)+I*3^(1/2)/d^(1/3
))*d^(1/3))^(1/2)*((x-1/d^(1/3))/(-3/d^(1/3)-I*3^(1/2)/d^(1/3)))^(1/2)*(1/2
*I*(2*x+1/d^(1/3)-I*3^(1/2)/d^(1/3))*d^(1/3))^(1/2)/(d*x^3-1)^(1/2)*(-2*_al
pha^2*d+I*3^(1/2)*_alpha*d^(2/3)+_alpha*d^(2/3)-I*3^(1/2)*d^(1/3)+d^(1/3))*
EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/d^(1/3)+1/2*I*3^(1/2)/d^(1/3))*3^(1/2)*d^
(1/3))^(1/2),1/3*I*_alpha^2*d^(2/3)*3^(1/2)-1/6*I*_alpha*d^(1/3)*3^(1/2)+1/
2*_alpha*d^(1/3)-1/6*I*3^(1/2)-1/2,(-I*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)-1/2*I*
3^(1/2)/d^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{dx^3-1}\left(dx^3-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

mupad [B] time = 15.03, size = 331, normalized size = 2.11

$$\frac{\sqrt{3}\;314928^{1/3}\,\ln\left(\frac{\left(54\sqrt{d\,x^3-1}+54\sqrt{3}-54\,2^{1/3}\sqrt{3}\,d^{1/3}\,x\right)\left(\sqrt{d\,x^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}\,x\right)^3}{\left(2^{2/3}-d^{1/3}\,x\right)^6}\right)}{2916\,d^{2/3}}+\frac{\sqrt{3}\;314928^{1/3}\,\ln\left(\frac{\left(2\sqrt{d\,x^3-1}+2\sqrt{3}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((d*x^3 - 1)^(1/2)*(d*x^3 - 4)),x)
```

```
[Out] (3^(1/2)*314928^(1/3)*log(((54*(d*x^3 - 1)^(1/2) + 54*3^(1/2) - 54*2^(1/3)*
3^(1/2)*d^(1/3)*x)*((d*x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*d^(1/3)*x
)^3)/(2^(2/3) - d^(1/3)*x)^6))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*log((
(2*(d*x^3 - 1)^(1/2) + 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*d
^(1/3)*x)^3*(108*3^(1/2) - 108*(d*x^3 - 1)^(1/2) + 2^(1/3)*d^(1/3)*x*162i +
 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3) - 2^(2/3)*3^(1/2)*1i + 2*d^(1/3)*x
)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*l
```

```
og(((2*(d*x^3 - 1)^(1/2) - 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i - 2^(1/3)*3^(1/
2)*d^(1/3)*x)^3*(108*(d*x^3 - 1)^(1/2) + 108*3^(1/2) - 2^(1/3)*d^(1/3)*x*16
2i + 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3)*3^(1/2)*1i + 2^(2/3) + 2*d^(1/
3)*x)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916*d^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{dx^3\sqrt{dx^3-1}-4\sqrt{dx^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)
```

3.75 $\int \frac{x}{\sqrt{-1+x^3}\left(8+x^3\right)}\,dx$

Optimal. Leaf size=74

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

```
[Out] 1/18*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))+1/18*arctan(1/3*(x^3-1)^(1/2))-1/18*
arctanh((1-x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {486, 444, 63, 204, 2138, 203, 2145, 206}

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - Arc
Tanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-1+x^3}\left(8+x^3\right)}\,dx &= -\left(\frac{1}{12}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx\right)-\frac{1}{12}\int \frac{-2-2x+x^2}{\left(4-2x+x^2\right)\sqrt{-1+x^3}}\,dx-\frac{1}{4}\int \frac{x^{2}}{\left(-8-x^3\right)\cdots}\\
&= -\left(\frac{1}{12}\,\mathrm{Subst}\left(\int \frac{1}{(-8-x)\sqrt{-1+x}}\,dx,x,x^3\right)\right)+\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{9+x^2}\,dx,x,\frac{(1-x)^2}{\sqrt{-1+x^3}}\right)\cdots\\
&= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}-\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{-9-x^2}\,dx,x,\sqrt{-1+x^3}\right)\\
&= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{1}{3}\sqrt{-1+x^3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.65

$$\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{8}\right)}{16\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -1/8*x^3])/(16*Sqrt[-1 +
 x^3])
```

fricas [B] time = 1.10, size = 547, normalized size = 7.39

$$\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48x^5+186x^4-56x^3+6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64\right)}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}\right)-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 + 186*x^4 - 56*x^3 + 6*sqrt(3)*(x^4 + 12*
x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24
*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) - 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
 + 1/54*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x +
 1)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) + (3*sqrt(3)*(x^3 +
 x^2 - 2*x) - sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 + 186*x^4
- 56*x^3 + 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2
 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)))/(x^3 -
 3*x^2 + 2)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) - (3*sqrt(3
)*(x^3 + x^2 - 2*x) + sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
)/(x^3 - 3*x^2 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+8\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

maple [C] time = 0.20, size = 421, normalized size = 5.69

$$-\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\sqrt{3}}{6}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\sqrt{x^3-1}}-\frac{i\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{3}{2}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+8)/(x^3-1)^(1/2),x)
```

```
[Out] 1/9*I*(1/2-1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3
/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(1/2)*(1+I*3^(1/2))+1/3*I*3^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/9*I*(1/2+1/2*I*3^(1/2))*(-3/2-1/
2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x
^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*3^(
1/2)*(1-I*3^(1/2))-2/3*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2))-1/9*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+8\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

mupad [B] time = 0.21, size = 533, normalized size = 7.20

$$\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{9\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{\sqrt{3}\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\cdots}{9\,(-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(x^3 + 8)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/(9*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (3^(1/2)*((3^(1/2)
*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((
x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*1i)/3, asi
n((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/
2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)
^(1/2)) - (3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*((3^(1
/2)*1i)/2 + 3/2)*1i)/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i + 1)*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)\left(x^2-2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)), x)
```

3.76 $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}\,x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}\,d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}\,x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

```
[Out] 1/18*arctanh(1/3*(1+d^(1/3)*x)^2/(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctanh(1/3*
(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctan((1+d^(1/3)*x)*3^(1/2)/(d*x^3+1)^(1/2))
/d^(2/3)*3^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}\,x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}\,d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}\,x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(6*Sqrt[3]*d^(2/3)) + Ar
cTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[1
+ d*x^3]/3]/(18*d^(2/3))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
```

```
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\left(8-dx^3\right)\sqrt{1+dx^3}}\,dx = -\frac{\int \frac{2d^{2/3}-2dx-d^{4/3}x^2}{\left(4+2\sqrt[3]{d}x+d^{2/3}x^2\right)\sqrt{1+dx^3}}\,dx}{12d} + \frac{\int \frac{1+\sqrt[3]{d}x}{\left(2-\sqrt[3]{d}x\right)\sqrt{1+dx^3}}\,dx}{12\sqrt[3]{d}} - \frac{1}{4}\sqrt[3]{d}\int \frac{x^2}{\left(8-dx^3\right)\sqrt{1+dx^3}}\,d$$

$$= \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{\left(1+\sqrt[3]{d}x\right)^2}{\sqrt{1+dx^3}}\right)}{6d^{2/3}} - \frac{1}{12}\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8-dx)\sqrt{1+dx}}\,dx, x, x^3\right) + \frac{1}{3}d^{4/}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \sqrt{1+dx^3}\right)}{6d^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}$$

Mathematica [C] time = 0.03, size = 32, normalized size = 0.31

$$\frac{1}{16}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-dx^3,\frac{dx^3}{8}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/16
```

fricas [B] time = 1.43, size = 497, normalized size = 4.83

$$2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(-\frac{\left(9\sqrt{3}d^3x^5-\sqrt{3}\left(d^2x^6-40dx^3-32\right)\left(d^2\right)^{\frac{2}{3}}+3\sqrt{3}\left(5d^2x^4+8dx\right)\left(d^2\right)^{\frac{1}{3}}\right)\sqrt{dx^3+1}\left(d^2\right)^{\frac{1}{6}}}{9\left(d^4x^7-7d^3x^4-8d^2x\right)}\right)+2\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{d^4x^9+318d^3x^6\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/108*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(-1/9*(9*sqrt(3)*d^3*x^5 - sqrt(3)*(d^
2*x^6 - 40*d*x^3 - 32)*(d^2)^(2/3) + 3*sqrt(3)*(5*d^2*x^4 + 8*d*x)*(d^2)^(1
/3))*sqrt(d*x^3 + 1)*(d^2)^(1/6)/(d^4*x^7 - 7*d^3*x^4 - 8*d^2*x)) + 2*(d^2)
^(2/3)*log((d^4*x^9 + 318*d^3*x^6 + 1200*d^2*x^3 + 18*(5*d^2*x^7 + 64*d*x^4
 + 32*x)*(d^2)^(2/3) + 6*(7*d^3*x^6 + 152*d^2*x^3 + (d^2*x^7 + 80*d*x^4 + 1
60*x)*(d^2)^(2/3) + 6*(5*d^2*x^5 + 32*d*x^2)*(d^2)^(1/3) + 64*d)*sqrt(d*x^3
 + 1) + 18*(d^3*x^8 + 38*d^2*x^5 + 64*d*x^2)*(d^2)^(1/3) + 640*d)/(d^3*x^9
- 24*d^2*x^6 + 192*d*x^3 - 512)) - (d^2)^(2/3)*log((d^4*x^9 - 276*d^3*x^6 -
 1608*d^2*x^3 - 18*(d^2*x^7 - 52*d*x^4 - 80*x)*(d^2)^(2/3) - 6*(4*d^3*x^6 +
 164*d^2*x^3 + (d^2*x^7 - 28*d*x^4 - 272*x)*(d^2)^(2/3) - 24*(d^2*x^5 + d*x
^2)*(d^2)^(1/3) + 160*d)*sqrt(d*x^3 + 1) + 18*(d^3*x^8 + 20*d^2*x^5 - 8*d*x
^2)*(d^2)^(1/3) - 1088*d)/(d^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 512)))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{dx^3+1}\left(dx^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

maple [C] time = 0.35, size = 383, normalized size = 3.72

$$-\frac{i\left(-d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-d^2\right)^{\frac{1}{3}}+\left(-d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8\right)^2 d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x)
```

```
[Out] -1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)*(d*(x-1/d*(-d^2)^(1/3))/(-
3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)/(d*x^3+1)^(1/2)*(I*(-d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1/3)*_
alpha*d-(-d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^(1/2),-1/18/d*(2*I*3^(1/2)
*(-d^2)^(1/3)*_alpha^2*d-I*3^(1/2)*(-d^2)^(2/3)*_alpha+I*3^(1/2)*d-3*(-d^2)
^(2/3)*_alpha-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2/d*(-d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{dx^3+1}\left(dx^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\sqrt{d\,x^3+1}\,\left(d\,x^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)),x)
[Out] -int(x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)
```

3.77 $\int \frac{1}{\sqrt[3]{1-3x^2}\left(3-x^2\right)}\,dx$

Optimal. Leaf size=81

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] 1/4*arctan((1-(-3*x^2+1)^(1/3))/x)+1/12*arctanh(1/3*x*3^(1/2))*3^(1/2)-1/12
*arctanh(1/9*(1-(-3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {395}

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcT
anh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

Rule 395

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, -Simp[(q*ArcTanh[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Si
mp[(q*ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[
a, 3]*d), x] - Simp[(q*ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-3x^2}\left(3-x^2\right)}\,dx=\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}$$

Mathematica [C] time = 0.10, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\left(x^2-3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};3x^2,\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};3x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x^
2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))
```

fricas [B] time = 3.82, size = 1792, normalized size = 22.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11
 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(
3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x^8 + 94152*x^6 - 13581*x^4 + 432*x^2
 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x^10 - 107*x^8 - 7262*x^6 + 2322*x^4
- 243*x^2 + 9) - 48*sqrt(3)*(5*x^9 - 245*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 +
 1)^(2/3) - 12*sqrt(3)*(29*x^11 + 293*x^9 - 2670*x^7 + 4986*x^5 - 1215*x^3
+ 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(49*x^10 - 5043*x^8 + 3658*x^6 + 378*x
^4 - 171*x^2 + 9) - 2*sqrt(3)*(x^11 + 917*x^9 - 40566*x^7 + 15786*x^5 - 204
3*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt((x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(
x^5 + 13*x^3) - 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-
3*x^2 + 1)^(2/3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2
 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)
*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2106*x^5 - 396*x^3 + 27*x) - 3*sqrt(3
)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4 - 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) -
3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8 + 4260*x^6 - 14337*x^4 + 2862*x^2 -
 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^11 - 1591*x^9 + 42426*x^7 - 15102*x^5
 + 1269*x^3 - 27*x) - 6*sqrt(3)*(27*x^10 + 2307*x^8 + 4574*x^6 - 2538*x^4 +
 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^12 - 4986*x^10 + 327519*x^8 - 159660*
x^6 + 25839*x^4 - 2106*x^2 + 81)) + 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9
*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255
*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x
^8 + 94152*x^6 - 13581*x^4 + 432*x^2 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x
^10 - 107*x^8 - 7262*x^6 + 2322*x^4 - 243*x^2 + 9) + 48*sqrt(3)*(5*x^9 - 24
5*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 + 1)^(2/3) + 12*sqrt(3)*(29*x^11 + 293*x
^9 - 2670*x^7 + 4986*x^5 - 1215*x^3 + 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(4
9*x^10 - 5043*x^8 + 3658*x^6 + 378*x^4 - 171*x^2 + 9) + 2*sqrt(3)*(x^11 + 9
17*x^9 - 40566*x^7 + 15786*x^5 - 2043*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt
((x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)*
sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sqr
t(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9
*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2
106*x^5 - 396*x^3 + 27*x) + 3*sqrt(3)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4
- 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) + 3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8
 + 4260*x^6 - 14337*x^4 + 2862*x^2 - 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^1
1 - 1591*x^9 + 42426*x^7 - 15102*x^5 + 1269*x^3 - 27*x) + 6*sqrt(3)*(27*x^1
0 + 2307*x^8 + 4574*x^6 - 2538*x^4 + 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^1
2 - 4986*x^10 + 327519*x^8 - 159660*x^6 + 25839*x^4 - 2106*x^2 + 81)) - 1/2
88*sqrt(6)*sqrt(2)*log(12*(x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3)
- 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-3*x^2 + 1)^(2/
3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2
 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/288*sqrt(6)*sqrt(2)*log(1
2*(x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)
*sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sq
rt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 -
9*x^4 + 27*x^2 - 27)) + 1/72*sqrt(3)*log(-(x^12 + 2598*x^10 + 55143*x^8 + 1
14228*x^6 - 22113*x^4 - 7290*x^2 + 8*(3*x^10 + 576*x^8 + 5598*x^6 + 5832*x^
4 - 729*x^2 - sqrt(3)*(41*x^9 + 1368*x^7 + 4482*x^5 + 864*x^3 - 243*x))*(-3
*x^2 + 1)^(2/3) - 4*sqrt(3)*(25*x^11 + 2359*x^9 + 15426*x^7 + 6966*x^5 - 43
47*x^3 + 243*x) - 4*(84*x^10 + 4536*x^8 + 20880*x^6 + 5832*x^4 - 2916*x^2 -
```

```
sqrt(3)*(x^11 + 521*x^9 + 7362*x^7 + 10746*x^5 - 1971*x^3 - 243*x))*(-3*x^
2 + 1)^(1/3) + 729)/(x^12 - 18*x^10 + 135*x^8 - 540*x^6 + 1215*x^4 - 1458*x
^2 + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(x^2-3)(-3x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

maple [C] time = 10.06, size = 538, normalized size = 6.64

$$-\operatorname{RootOf}\left(48\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^2-3\right)+1\right)\ln\left(\frac{-3x^2+48\left(-3x^2+1\right)^{\frac{1}{3}}x\operatorname{RootOf}\left(48\_Z^2+4\_Z\operatorname{RootOf}\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x)
```

```
[Out] -1/12*RootOf(_Z^2-3)*ln((8*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)^2*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*x+192*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*RootOf(4*_Z*R
ootOf(_Z^2-3)+48*_Z^2+1)^2*x-16*RootOf(_Z^2-3)^2*RootOf(4*_Z*RootOf(_Z^2-3)
+48*_Z^2+1)*x-384*RootOf(_Z^2-3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)^2*x+
12*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)*x^2+24*(-3*x^2+1)^(
1/3)*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)+12*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*RootOf(_Z^2-3)-4*RootOf(_Z^2-3)*x-96*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*x+6*(-3*x^2+1)^(2/3)+3*x^2+3)/(x^2-3))-1/12*ln((2*(
-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*x+48*(-3*x^2+1)^(1/3)*RootOf(4*_Z*RootOf(_Z^
2-3)+48*_Z^2+1)*x+4*RootOf(_Z^2-3)*x+96*RootOf(4*_Z*RootOf(_Z^2-3)+48*_Z^2+
1)*x+6*(-3*x^2+1)^(2/3)-3*x^2+6*(-3*x^2+1)^(1/3)-3)/(x^2-3))*RootOf(_Z^2-3)
-ln((2*(-3*x^2+1)^(1/3)*RootOf(_Z^2-3)*x+48*(-3*x^2+1)^(1/3)*RootOf(4*_Z*Ro
otOf(_Z^2-3)+48*_Z^2+1)*x+4*RootOf(_Z^2-3)*x+96*RootOf(4*_Z*RootOf(_Z^2-3)+
48*_Z^2+1)*x+6*(-3*x^2+1)^(2/3)-3*x^2+6*(-3*x^2+1)^(1/3)-3)/(x^2-3))*RootOf
(4*_Z*RootOf(_Z^2-3)+48*_Z^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(x^2-3)(-3x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^2-3\right)\left(1-3x^2\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 - 3)*(1 - 3*x^2)^(1/3)),x)
```

```
[Out] -int(1/((x^2 - 3)*(1 - 3*x^2)^(1/3)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{1-3x^2}-3\sqrt[3]{1-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)
```

```
[Out] -Integral(1/(x**2*(1 - 3*x**2)**(1/3) - 3*(1 - 3*x**2)**(1/3)), x)
```

3.78 $\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -1/4*arctanh((1-(3*x^2+1)^(1/3))/x)+1/12*arctan(1/3*x*3^(1/2))*3^(1/2)+1/12
*arctan(1/9*(1-(3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {394}

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]
*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rule 394

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTan[(q*x)/3])/(12*Rt[a, 3]*d), x] + (Simp[(q
*ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)])/(12*Rt[a, 3]*
d), x] - Simp[(q*ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)])/(4*Sqrt[3]*Rt[a, 3]*d), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(3+x^2\right)\sqrt[3]{1+3x^2}}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

Mathematica [C] time = 0.10, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)}{\left(x^2+3\right)\sqrt[3]{3x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2])/((3 + x^2)*(1 + 3*x^2)^
(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/
```

```
2, 1/3, 2, 5/2, -3*x^2, -1/3*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2, -1
/3*x^2])))
```

fricas [B] time = 2.76, size = 345, normalized size = 4.26

$$\frac{1}{36}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\left(3x^4-10x^3-36x^2+18x+9\right)\left(3x^2+1\right)^{\frac{2}{3}}-4\sqrt{3}\left(x^5+15x^4-26x^3-54x^2+9x-9\right)\left(3x\right.}{x^6+126x^5-225x^4-828x^3-81x^2-162}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*arctan((4*sqrt(3)*(3*x^4 - 10*x^3 - 36*x^2 + 18*x + 9)*(3*x^2
+ 1)^(2/3) - 4*sqrt(3)*(x^5 + 15*x^4 - 26*x^3 - 54*x^2 + 9*x - 9)*(3*x^2 +
1)^(1/3) + sqrt(3)*(x^6 - 2*x^5 - 105*x^4 - 28*x^3 + 63*x^2 + 126*x + 9))/(
x^6 + 126*x^5 - 225*x^4 - 828*x^3 - 81*x^2 - 162*x + 81)) - 1/36*sqrt(3)*ar
ctan(2*(2*sqrt(3)*(23*x^3 + 9*x)*(3*x^2 + 1)^(2/3) + sqrt(3)*(x^5 - 80*x^3
- 9*x)*(3*x^2 + 1)^(1/3) + sqrt(3)*(11*x^5 + 10*x^3 - 9*x))/(x^6 - 657*x^4
- 189*x^2 - 27)) + 1/24*log((x^6 + 108*x^5 + 549*x^4 + 360*x^3 + 99*x^2 + 6
*(3*x^4 + 32*x^3 + 42*x^2 + 3)*(3*x^2 + 1)^(2/3) + 6*(x^5 + 27*x^4 + 70*x^3
 + 18*x^2 + 9*x + 3)*(3*x^2 + 1)^(1/3) + 108*x - 9)/(x^6 + 9*x^4 + 27*x^2 +
 27))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

maple [C] time = 2.46, size = 444, normalized size = 5.48

$$\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)\ln\left(\frac{-12x^2\,\mathrm{RootOf}\left(48\_Z^2+12\_Z+1\right)-x^2+24\left(3x^2+1\right)^{\frac{1}{3}}x\,\mathrm{RootOf}\left(48\_Z^2+12\_\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3)/(3*x^2+1)^(1/3),x)
```

```
[Out] -1/4*ln(-(12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)*x-6*RootOf(48*_Z^2+12*
_Z+1)*x^2+12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)-24*RootOf(48*_Z^2+12*_
Z+1)*x+(3*x^2+1)^(2/3)+(3*x^2+1)^(1/3)*x-x^2+6*RootOf(48*_Z^2+12*_Z+1)+(3*x
^2+1)^(1/3)-4*x+1)/(x^2+3))-ln(-(12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)
*x-6*RootOf(48*_Z^2+12*_Z+1)*x^2+12*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)
-24*RootOf(48*_Z^2+12*_Z+1)*x+(3*x^2+1)^(2/3)+(3*x^2+1)^(1/3)*x-x^2+6*RootO
f(48*_Z^2+12*_Z+1)+(3*x^2+1)^(1/3)-4*x+1)/(x^2+3))*RootOf(48*_Z^2+12*_Z+1)+
RootOf(48*_Z^2+12*_Z+1)*ln((24*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)*x-12
*RootOf(48*_Z^2+12*_Z+1)*x^2+24*RootOf(48*_Z^2+12*_Z+1)*(3*x^2+1)^(1/3)-48*
RootOf(48*_Z^2+12*_Z+1)*x-2*(3*x^2+1)^(2/3)+4*(3*x^2+1)^(1/3)*x-x^2+12*Root
Of(48*_Z^2+12*_Z+1)+4*(3*x^2+1)^(1/3)-4*x+1)/(x^2+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+3\right)\,\left(3\,x^2+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)),x)
[Out] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\sqrt[3]{3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
[Out] Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

3.79 $\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

```
[Out] -1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+
1/12*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\,2^{2/3}}$$

Mathematica [C] time = 0.07, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

fricas [B] time = 1.82, size = 1943, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
```

```
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [C] time = 50.76, size = 704, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] 1/144*ln((-2*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^2-RootOf(_Z^6+108)^4*x^3-3
*RootOf(_Z^6+108)^4*x-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+216*(-x^2+1)^(
2/3)*x-126*RootOf(_Z^6+108)*x^2+54*RootOf(_Z^6+108))/(RootOf(_Z^6+108)^3*x-
18)^2/(RootOf(_Z^6+108)^3*x+18))*RootOf(_Z^6+108)^4+1/24*ln((-2*RootOf(_Z^6
+108)^5*(-x^2+1)^(1/3)*x^2-RootOf(_Z^6+108)^4*x^3-3*RootOf(_Z^6+108)^4*x-36
*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+216*(-x^2+1)^(2/3)*x-126*RootOf(_Z^6+1
08)*x^2+54*RootOf(_Z^6+108))/(RootOf(_Z^6+108)^3*x-18)^2/(RootOf(_Z^6+108)^
3*x+18))*RootOf(_Z^6+108)-1/216*RootOf(_Z^6+108)^4*ln((-RootOf(_Z^6+108)^4*
x^6-225*RootOf(_Z^6+108)^4*x^4+72*x^5*RootOf(_Z^6+108)^4-1296*x^5*RootOf(_Z
^6+108)+486*RootOf(_Z^6+108)+189*RootOf(_Z^6+108)^4*x^2-3402*RootOf(_Z^6+10
8)*x^2+4050*RootOf(_Z^6+108)*x^4-72*RootOf(_Z^6+108)^4*x^3+1296*RootOf(_Z^6
+108)*x^3-27*RootOf(_Z^6+108)^4-108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^2-3
24*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x-6*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*
x^5+108*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^4-144*RootOf(_Z^6+108)^5*(-x^2+
1)^(1/3)*x^3+36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+54*RootOf(_Z^6+108)^5
*(-x^2+1)^(1/3)*x-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4+864*RootOf(_Z^6
+108)^2*(-x^2+1)^(1/3)*x^3+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+3888*(
-x^2+1)^(2/3)*x+1296*(-x^2+1)^(2/3)*x^4-9072*(-x^2+1)^(2/3)*x^3+3888*(-x^2+
1)^(2/3)*x^2+18*RootOf(_Z^6+108)*x^6)/(x^2+3)^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(1-x^2\right)}^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.80 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/1
2*arctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {392}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rule 392

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[(q*ArcTanh[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1
/3)*d), x] + (-Simp[(q*ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] + Simp[(q*ArcTan[q*x])/(6*2^(2/3)*a^(1/3
)*d), x] + Simp[(q*ArcTanh[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(3-x^2\right)\sqrt[3]{1+x^2}}\,dx=-\frac{\tan^{-1}(x)}{6\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\,2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}$$

Mathematica [C] time = 0.10, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{\left(x^2-3\right)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

fricas [B] time = 1.73, size = 1685, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2592*432^(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7
- 373248*x^5 - 69984*x^3 - sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 +
 6030*x^7 + 7182*x^5 + 2511*x^3 + 243*x) + 3*432^(1/6)*sqrt(3)*(x^12 + 924*
x^10 - 33363*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) + (432^(5/6)*sqr
t(3)*(x^10 - 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) + 432*sqrt(3)*2^(1/3)*(13
*x^9 - 177*x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) + 36*(96*x^10 - 4032*x^
8 - 2592*x^6 + sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 2
43*x))*(x^2 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^
2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 -
18*x^2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 +
 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8
+ 1206*x^6 + 756*x^4 + 81*x^2) + 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 207
0*x^5 - 756*x^3 - 81*x))*(x^2 + 1)^(2/3) + 18*sqrt(3)*(x^12 + 1422*x^10 + 2
1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
 7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
)*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
 - 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
 9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
*x^2 + 27) - 864*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^2+3\right)\left(x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(x^2+1\right)^{\frac{1}{3}}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^2+1\right)^{1/3}\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] -int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

3.81 $\int \frac{a+x}{(-a+x)\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\sqrt{x}\sqrt{-\left(a^2+1\right)x+a^2+x^2}\,\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-\left(a^2+1\right)x+a^2+x^2}}\right)}{(1-a)\sqrt{-\left(a^2+1\right)x^2+a^2x+x^3}}$$

```
[Out] -2*arctan((1-a)*x^(1/2)/(a^2-(a^2+1)*x+x^2)^(1/2))*x^(1/2)*(a^2-(a^2+1)*x+x
^2)^(1/2)/(1-a)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2056, 6733, 1698, 205}

$$-\frac{2\sqrt{x}\sqrt{-\left(a^2+1\right)x+a^2+x^2}\,\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-\left(a^2+1\right)x+a^2+x^2}}\right)}{(1-a)\sqrt{-\left(a^2+1\right)x^2+a^2x+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2
 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\int \frac{a+x}{\sqrt{x}(-a+x)\sqrt{a^2-\left(1+a^2\right)x+x^2}}\,dx}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\text{Subst}\left(\int \frac{a+x^2}{\left(-a+x^2\right)\sqrt{a^2+\left(-1-a^2\right)x^2+x^4}}\,dx, x, \sqrt{\cdots}\right.}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= \frac{\left(2a\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\right)\text{Subst}\left(\int \frac{1}{-a-\left(-2a^2-a\left(-1-a^2\right)\right)x^2}\,dx, x, \frac{\cdots}{\sqrt{a^2-\cdots}}\right.}{\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

$$= -\frac{2\sqrt{x}\sqrt{a^2-\left(1+a^2\right)x+x^2}\,\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-\left(1+a^2\right)x+x^2}}\right)}{(1-a)\sqrt{a^2x-\left(1+a^2\right)x^2+x^3}}$$

Mathematica [C] time = 0.93, size = 159, normalized size = 1.83

$$-\frac{2i\left(a^2-x\right)^{3/2}\sqrt{\frac{x-1}{x-a^2}}\sqrt{\frac{x}{x-a^2}}\left((a+1)\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right),1-\frac{1}{a^2}\right)-2\Pi\left(\frac{a-1}{a};i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)\right)}{(a-1)\sqrt{-a^2}\sqrt{(x-1)x\left(x-a^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] ((-2*I)*(a^2 - x)^(3/2)*Sqrt[(-1 + x)/(-a^2 + x)]*Sqrt[x/(-a^2 + x)]*((1 +
a)*EllipticF[I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)] - 2*EllipticP
i[(-1 + a)/a, I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)]))/((-1 + a)*
Sqrt[-a^2]*Sqrt[(-1 + x)*x*(-a^2 + x)])
```

fricas [A] time = 0.70, size = 85, normalized size = 0.98

$$\frac{\arctan\left(\frac{\sqrt{a^2x-\left(a^2+1\right)x^2+x^3}\left(a^2-2\left(a^2-a+1\right)x+x^2\right)}{2\left((a-1)x^3-\left(a^3-a^2+a-1\right)x^2+\left(a^3-a^2\right)x\right)}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a^2 - 2*(a^2 - a + 1)*x + x^2
)/((a - 1)*x^3 - (a^3 - a^2 + a - 1)*x^2 + (a^3 - a^2)*x))/(a - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a+x}{\sqrt{a^2x-\left(a^2+1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

maple [C] time = 0.04, size = 206, normalized size = 2.37

$$-\frac{4\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,a^3\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-a^2+x}{a^2}},\frac{a^2}{a^2-a},\sqrt{\frac{a^2}{a^2-1}}\right)}{\sqrt{-a^2x^2+a^2x+x^3-x^2}\,\left(a^2-a\right)}-\frac{2\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,a^2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-a^2+}{a^2}}\right.}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x)
```

```
[Out] -2*a^2*(-(-a^2+x)/a^2)^(1/2)*((x-1)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+
a^2*x+x^3-x^2)^(1/2)*EllipticF((-(-a^2+x)/a^2)^(1/2),(a^2/(a^2-1))^(1/2))-4
*a^3*(-(-a^2+x)/a^2)^(1/2)*((x-1)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+a^
2*x+x^3-x^2)^(1/2)/(a^2-a)*EllipticPi((-(-a^2+x)/a^2)^(1/2),a^2/(a^2-a),(a^
2/(a^2-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a+x}{\sqrt{a^2x-\left(a^2+1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

mupad [B] time = 0.17, size = 217, normalized size = 2.49

$$\frac{4\,a\,\left(a^2-1\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}\,\Pi\left(-\frac{a^2-1}{a-a^2};\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)}{\left(a-a^2\right)\,\sqrt{a^2\,x-x^2\,\left(a^2+1\right)+x^3}}-\frac{2\,\left(a^2-1\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{}}{\sqrt{a^2\,x-x^2\,\left(a^2+1\right)+x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + x)/((a - x)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)),x)
```

```
[Out] (4*a*(a^2 - 1)*(x/a^2)^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1
))^(1/2)*ellipticPi(-(a^2 - 1)/(a - a^2), asin((-(x - a^2)/(a^2 - 1))^(1/2)
), (a^2 - 1)/a^2))/((a - a^2)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)) - (2*(a^
2 - 1)*ellipticF(asin((-(x - a^2)/(a^2 - 1))^(1/2)), (a^2 - 1)/a^2)*(x/a^2)
^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1))^(1/2))/(a^2*x - x^2
*(a^2 + 1) + x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+x}{\sqrt{x\left(-a^2+x\right)(x-1)}\,(-a+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)
```

3.82 $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}\,dx$

Optimal. Leaf size=1

$$0$$

```
[Out] 0
```

Rubi [C] time = 1.67, antiderivative size = 529, normalized size of antiderivative = 529.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2056, 6733, 1708, 1103, 1706}

$$\frac{2(1-a)\sqrt{x}\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}\tan^{-1}\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-\left(-a^2+2a+1\right)x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-\left(-a^2+2a+1\right)x^2+(2-a)ax+x^3}}+\frac{((2-a)a)^{3/4}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\cdots}{a\sqrt{-\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]),
x]
```

```
[Out] (2*(1 - a)*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]*ArcTan[(Sqrt[-
1 + 2*a - a^2]*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]])/(a*Sqrt
[-1 + 2*a - a^2]*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + (((2 - a)
*a)^(3/4)*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)
*x + x^2)/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*ArcTan[Sqrt[x]
/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4])/(a*Sqrt[(2 -
 a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + ((2 - a)*(1 - Sqrt[(2 - a)*a])*Sqrt
[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2)/((2
- a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(Sqrt[2 - a] + Sqrt[a])^2/(4*
Sqrt[(2 - a)*a]), 2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)
/Sqrt[(2 - a)*a])/4])/(((2 - a)*a)^(3/4)*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)
*x^2 + x^3])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
```

```
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\int \frac{}{\sqrt{x}\,(-a+x)\sqrt{(2-}}}{\sqrt{(2-a)ax+\left(-1-2a+a^2\right)x^2+x}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\mathrm{Subst}\left(\int \frac{}{(-a}\right.}{\sqrt{(2-a)ax+\left(-1-2a+a^2\right.}}$$

$$= \frac{\left(2\sqrt{(2-a)a}\,\sqrt{x}\sqrt{(2-a)a+\left(-1-2a+a^2\right)x+x^2}\right)\mathrm{Su}}{a\sqrt{(2-a)ax+\left(-1-2a+\right.}}$$

$$= \frac{2(1-a)\sqrt{x}\sqrt{(2-a)a-\left(1+2a-a^2\right)x+x^2}\,\tan^{-1}\left(\frac{}{\sqrt{(2}}\right.}{a\sqrt{-1+2a-a^2}\sqrt{(2-a)ax-\left(1+2a-a^2\right)}}$$

Mathematica [C] time = 0.60, size = 127, normalized size = 127.00

$$\frac{2\sqrt{\frac{1}{x-1}+1}\,(x-1)^{3/2}\sqrt{\frac{(a-1)^2}{x-1}+1}\left(\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{-(a-1)^2}}{\sqrt{x-1}}\right),\frac{1}{(a-1)^2}\right)-2\Pi\left(\frac{1}{1-a};\sin^{-1}\left(\frac{\sqrt{-(a-1)^2}}{\sqrt{x-1}}\right)|\frac{1}{(a-1)^2}\right)\right)}{\sqrt{-(a-1)^2}\sqrt{(x-1)x\left(a^2-2a+x\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 +
x^3]),x]
```

```
[Out] (2*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (-1 + a)^2/(-1 + x)]*(-1 + x)^(3/2)*(El
lipticF[ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)] - 2*Elliptic
Pi[(1 - a)^(-1), ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)]))/(
Sqrt[-(-1 + a)^2]*Sqrt[(-1 + x)*x*(-2*a + a^2 + x)])
```

fricas [C] time = 1.21, size = 70, normalized size = 70.00

$$\frac{\log\left(-\frac{a^2-2\left(a^2-a\right)x-x^2+2\sqrt{\left(a^2-2a-1\right)x^2+x^3-\left(a^2-2a\right)x}\,a}{a^2-2ax+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="fricas")
```

```
[Out] log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^3 - (a^2 -
 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a+x-2}{\sqrt{-(a-2)ax+\left(a^2-2\,a-1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

maple [C] time = 0.05, size = 317, normalized size = 317.00

$$\frac{2\left(a^2-2a\right)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\frac{x-1}{-a^2+2a-1}}\,\sqrt{\frac{x}{-a^2+2a}}\,\operatorname{EllipticF}\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}}+\frac{2\,(2a-2)\left(a^2-2a\right)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\frac{x-1}{-a}}}{\sqrt{a^2x^2-a^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x)
```

```
[Out] 2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((x-1)/(-a^2+2*a-1))^(1/2)*(x/(-a
^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)*EllipticF(((a^2-
2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))+2*(2*a-2)*(a^2-2*a
)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((x-1)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a))^(
1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticPi(((a^2-
2*a+x)/(a^2-2*a))^(1/2),(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-1))^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a+x-2}{\sqrt{-(a-2)ax+\left(a^2-2\,a-1\right)x^2+x^3}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="maxima")
```

```
[Out] -integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

mupad [B] time = 0.48, size = 207, normalized size = 207.00

$$\frac{2\sqrt{\frac{x}{2\,a-a^2}}\,\sqrt{-\frac{x-1}{a^2-2\,a+1}}\,(a-1)^2\,\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\,\left(a\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\frac{a^2-2\,a+1}{2\,a-a^2}\right)-2\,\Pi\left(-\frac{a^2-2\,a+1}{a-a^2};\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\right.\right.}{a\sqrt{x^3+\left(a^2-2\,a-1\right)\,x^2+\left(2\,a-a^2\right)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + x - 2)/((a - x)*(x^3 - x^2*(2*a - a^2 + 1) - a*x*(a - 2))^(1/2)),
x)
```

```
[Out] (2*(x/(2*a - a^2))^(1/2)*(-(x - 1)/(a^2 - 2*a + 1))^(1/2)*(a - 1)^2*((x - 2
*a + a^2)/(a^2 - 2*a + 1))^(1/2)*(a*ellipticF(asin(((x - 2*a + a^2)/(a^2 -
2*a + 1))^(1/2)), -(a^2 - 2*a + 1)/(2*a - a^2)) - 2*ellipticPi(-(a^2 - 2*a
+ 1)/(a - a^2), asin(((x - 2*a + a^2)/(a^2 - 2*a + 1))^(1/2)), -(a^2 - 2*a
+ 1)/(2*a - a^2))))/(a*(x*(2*a - a^2) - x^2*(2*a - a^2 + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+x-2}{\sqrt{x\left(x-1\right)\left(a^{2}-2a+x\right)}\left(-a+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)), x)
```

3.83 $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)x^2+(-1+2a)x^3}}\,dx$

Optimal. Leaf size=46

$$\log\left(\frac{-2\left(\sqrt{(1-x)x\left(a^2-2ax+x\right)}+x\right)-a^2+2ax+x^2}{(a-x)^2}\right)$$

```
[Out] ln((-a^2+2*a*x+x^2-2*x-2*((1-x)*x*(a^2-2*a*x+x))^(1/2))/(a-x)^2)
```

Rubi [C] time = 1.49, antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.137, Rules used = {2056, 6733, 1710, 1104, 419, 1220, 537}

$$\frac{4(1-a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,\Pi\left(\frac{1}{a};\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-(1-2a)x^3}}-\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{\frac{(1-2a)x}{a^2}+1}\,F\left(\sin^{-1}\left(\sqrt{x}\right)|-\frac{1-}{a}\right.}{\sqrt{\left(-a^2-2a+1\right)x^2+a^2x-(1-2a)x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 +
 2*a)*x^3]),x]
```

```
[Out] (-2*(1 - 2*a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticF[Arc
Sin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*
a)*x^3] + (4*(1 - a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*Ellipt
icPi[a^(-1), ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^
2)*x^2 - (1 - 2*a)*x^3]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1104

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b +
q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/(Sqrt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (
2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1220

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[1 + (2*c*x^2)/(b - q)]*
Sqrt[1 + (2*c*x^2)/(b + q)])/Sqrt[a + b*x^2 + c*x^4], Int[1/((d + e*x^2)*Sq
rt[1 + (2*c*x^2)/(b - q)]*Sqrt[1 + (2*c*x^2)/(b + q)]), x], x]] /; FreeQ[{a
```

```
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1710

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 2056

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-1+2a)x^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\int \frac{\cdots}{\sqrt{x}(-\cdots}}{\sqrt{a^2x-\left(-1+2a+a^2\right)x^2+(-\cdots}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)x^2}\right)\text{Subs}\cdots}{\sqrt{a^2x-\left(-1+2a+a^2\right)\cdots}}$$

$$= -\frac{\left(4(1-a)a\sqrt{x}\sqrt{a^2-\left(-1+2a+a^2\right)x+(-1+2a)\cdots}\right.}{\sqrt{a^2x-\left(-1+2a\cdots}}$$

$$= -\frac{\left(4(1-a)a\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}\sqrt{a^2-\left(-1+2a\cdots}\right.}{\sqrt{a^2+\left(1-2a-a^2\right)\cdots}}$$

$$= -\frac{2(1-2a)\sqrt{1-x}\sqrt{x}\sqrt{1+\frac{(1-2a)x}{a^2}}F\left(\sin^{-1}\left(\sqrt{x}\right)|-\cdots\right.}{\sqrt{a^2x+\left(1-2a-a^2\right)x^2-(1-2a)x^3}}$$

Mathematica [C] time = 1.09, size = 133, normalized size = 2.89

$$\frac{2i(x-1)^{3/2}\sqrt{\frac{x}{x-1}}\sqrt{-\frac{a^2-2ax+x}{(2a-1)(x-1)}}\left(2a\Pi\left(1-a;i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right)|-\frac{(a-1)^2}{2a-1}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\frac{1}{\sqrt{x-1}}\right),-\frac{(a-1)^2}{2a-1}\right)\right)\cdots}{\sqrt{-\left((x-1)x\left(a^2-2ax+x\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 +
 (-1 + 2*a)*x^3]),x]
```

```
[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[-((a^2 + x - 2*a*x)/((-1 + 2*a)
*(-1 + x)))]*(-EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a)
)] + 2*a*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*
a))]))/Sqrt[-((-1 + x)*x*(a^2 + x - 2*a*x))]
```

fricas [A] time = 0.72, size = 63, normalized size = 1.37

$$\log\left(-\frac{a^{2}-2\,(a-1)x-x^{2}+2\,\sqrt{(2\,a-1)x^{3}+a^{2}x-\left(a^{2}+2\,a-1\right)x^{2}}}{a^{2}-2\,ax+x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="fricas")
```

```
[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a -
 1)*x^2))/(a^2 - 2*a*x + x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^{3}+a^{2}x-\left(a^{2}+2\,a-1\right)x^{2}}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

maple [C] time = 0.06, size = 536, normalized size = 11.65

$$-\frac{4\sqrt{-\frac{\left(-\frac{a^{2}}{2a-1}+x\right)(2a-1)}{a^{2}}}\,\sqrt{\frac{x-1}{\frac{a^{2}}{2a-1}-1}}\,\sqrt{\frac{(2a-1)x}{a^{2}}}\,a^{3}\,\mathrm{EllipticF}\left(\sqrt{-\frac{\left(-\frac{a^{2}}{2a-1}+x\right)(2a-1)}{a^{2}}},\sqrt{\frac{a^{2}}{(2a-1)\left(\frac{a^{2}}{2a-1}-1\right)}}\right)}{(2a-1)\sqrt{-a^{2}x^{2}+2a\,x^{3}+a^{2}x-2a\,x^{2}-x^{3}+x^{2}}}-\frac{4\,(a-1)\sqrt{-\frac{\left(-\frac{a^{2}}{2a-1}+x\right)}{a^{2}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x)
```

```
[Out] 2*a^2/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((x-1)/(a^2/(-1+2*a)-
1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(
1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/(-
1+2*a)-1))^(1/2))-4*a^3/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((x
-1)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-
2*a*x^2-x^3+x^2)^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^
2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*a))
/a^2*(-1+2*a))^(1/2)*((x-1)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/
(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*EllipticPi(
(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a^2/(
-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2\,a-1)x-a}{\sqrt{(2\,a-1)x^{3}+a^{2}x-\left(a^{2}+2\,a-1\right)x^{2}}\,(a-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - x*(2*a - 1))/((a - x)*(x^3*(2*a - 1) - x^2*(2*a + a^2 - 1) + a^2*x
)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax - a - x}{\sqrt{x\,(x-1)\left(-a^2 + 2ax - x\right)}\,(-a+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**
(1/2),x)
```

```
[Out] Integral((2*a*x - a - x)/(sqrt(x*(x - 1)*(-a**2 + 2*a*x - x))*(-a + x)), x)
```

3.84 $\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

[Out] 2/3*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)

Rubi [A] time = 0.10, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2137, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]

[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2137

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt[3]{2}x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{1+x^3}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.46, size = 323, normalized size = 10.09

$$-\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left(\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)\text{EllipticF}\left(\sin^{-1}\left(\right.\right.\right.}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-2ix+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*
x]*(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4
*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I +
Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2
/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*S
qrt[3])/(3*I + Sqrt[3])]))/((1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

maple [C] time = 0.12, size = 258, normalized size = 8.06

$$-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{-}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

```
[Out] -2*2^(1/3)*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*(3/2-1/2*I*3^(1/2))*((
x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{1}{3}}x-1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

mupad [B] time = 1.69, size = 67, normalized size = 2.09

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2^(1/3)*x - 1)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)*(3^(1/2
)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*1i)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt[3]{2}\,x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx-\int\left(-\frac{1}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Int
egral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

3.85 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

```
[Out] -2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2138, 206}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 2.00

$$\frac{1}{3}\log\left(3-\frac{(x+1)^2}{\sqrt{x^3+1}}\right)-\frac{1}{3}\log\left(\frac{(x+1)^2}{\sqrt{x^3+1}}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
[Out] Log[3 - (1 + x)^2/Sqrt[1 + x^3]]/3 - Log[3 + (1 + x)^2/Sqrt[1 + x^3]]/3
```

fricas [B] time = 0.55, size = 44, normalized size = 1.91

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2-6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

maple [C] time = 0.11, size = 240, normalized size = 10.43

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x-2)/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^3+1}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

mupad [B] time = 0.22, size = 204, normalized size = 8.87

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
```

```
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

3.86 $\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)}{}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2
))*3^(1/4)*2^(1/2)-1/18*arctan(1/6*(1-3^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2
))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2))*2
^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*
(1-2*x+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^
3])])/(2*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 + x
^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^(1
/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) - ((
2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3]
)])/(6*Sqrt[2]*3^(1/4))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx=-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right.}{}$$

Mathematica [C] time = 0.06, size = 47, normalized size = 0.22

$$\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{20+12\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])/(20 + 12*Sq
rt[3])
```

fricas [B] time = 10.64, size = 7739, normalized size = 35.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3)
 + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(-1/1296*(6
*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*
x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 8437780
8*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 1260979
2*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x
^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*
x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x
^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 - 537864*x^14 + 1586477*
x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x^9 - 25445
5812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645168*x^4 +
78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3538*x^15 - 310536*x^14
+ 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41002448*x^9
 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 + 77737424*
x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204736*x + 13
160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273216)*(-672*sqrt(3) + 11
64)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 - 205368*x^12 - 289200*x
^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992*x^7 + 1849440*x^6 + 15
58464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4*sqrt(3)*(142*x^15 - 44
19*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 + 128152*x^9 -
10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4 + 9632*x^3 +
108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x^14 + 738528*x^13 - 186
0046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 154441
44*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2
 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^
11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 -
 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536)
+ 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x - 235008)*(-672*
sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) +
 24)*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 - 5976*x^16 + 5544*x^15 + 299664
*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 - 4761216*x^10 - 1046592*
x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016832*x^5 - 3699072*x^4 -
 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 + 142
55*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^9 +
415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*x^3
+ 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x^16 - 26052*x^15 + 3328
44*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11 + 318696*x^10 + 126210
72*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 - 20168256*x^5 + 1093612
8*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^
15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610
*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154
784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17 - 96354*x^16 + 84798*x
^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 7660368
0*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 1132
69536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 + sqrt(3)*(1351*x^17
 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 3
```

3465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-672*sqrt(3) + 1164) + 32 61696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(97*x^17 - 523*x^16 - 217 1*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688*x^5 + 9113 44*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610*x ^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 15478 4*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(-672*sqrt(3) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 + 1)*((45 9*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^ 11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088* x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 + sqrt(3)*(2 65*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1 383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 450359*x^13 + 4370159*x^1 2 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 146877876*x^8 - 197107784 *x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 45545344*x^3 + 695175 36*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^14 + 260015*x^13 + 2523 113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x^9 + 84799980*x^8 - 113 800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864*x^4 - 26295616*x^3 + 4 0135968*x^2 - 7907648*x - 5562368) - 13696448*x - 9634304)*sqrt(-672*sqrt(3) + 1164) + 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 - 14712*x^14 - 53940*x^13 + 411732*x^12 - 280248*x^11 - 324624*x^10 + 18081 6*x^9 - 518544*x^8 + 974304*x^7 - 887136*x^6 - 1404096*x^5 + 1843584*x^4 + 135936*x^3 - 696192*x^2 + 4*sqrt(3)*(142*x^15 - 2124*x^14 - 7773*x^13 + 594 47*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 75276*x^8 + 140472*x^7 - 12 7784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 - 100512*x^2 + 62400*x + 248 32) + (4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12 + 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 + 768064*x^6 + 1040 5056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 + sqrt(3)*(2855*x^15 - 21 635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863472*x^10 + 337953 6*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368*x - 691712)*sqrt (-672*sqrt(3) + 1164) + 432384*x + 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqr t(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 6*(4680*x^16 - 60552*x^15 + 89856*x^14 + 278280*x^13 + 64440*x^12 - 128 5200*x^11 - 255600*x^10 + 3098880*x^9 - 1770336*x^8 - 3614400*x^7 + 3895488 *x^6 + 1199232*x^5 - 2905344*x^4 + 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(2 5*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12 - 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 6240*x^5 - 15552*x^4 + 37 12*x^3 + 3456*x^2 - 4096*x - 1280) + (1164*x^17 + 1248*x^16 - 246120*x^15 + 518172*x^14 + 2607528*x^13 - 8301144*x^12 + 7017600*x^11 + 6258120*x^10 - 21360336*x^9 + 16998960*x^8 + 966336*x^7 - 18216672*x^6 + 15860544*x^5 - 47 20704*x^4 - 6023424*x^3 + 5362176*x^2 + 48*sqrt(3)*(14*x^17 + 15*x^16 - 296 0*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404*x^11 + 75267*x^10 - 25 6916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190768*x^5 - 56784*x^4 - 7 2448*x^3 + 64496*x^2 - 24480*x - 13376) + (2340*x^17 - 35850*x^16 - 106410* x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293732*x^11 + 591615 24*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45222000*x^6 - 1005 98112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 + sqrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 - 987292*x^12 - 26 727704*x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8 + 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3 - 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*sqrt(-672*sqrt(3) + 1164) - 203 5200*x - 1112064)*sqrt(-672*sqrt(3) + 1164) + 24*(627*x^16 - 14286*x^15 + 3 9762*x^14 + 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x

^9 - 3294*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*
x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^1
3 - 62584*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^
7 - 155648*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x +
 11584) + 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) - 764928*x - 239616)*
sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x
^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3
)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 221
4*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x
^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3
 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3
) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^
3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*
x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x +
 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16
*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^17 + 13*x^16 - 5
22*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656*x^11 + 23944*x^10 - 42
336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*x^3
 - 1664*x^2 + 256*x)) - 1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) +
1164) + 24)*(56*sqrt(3) + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^
(3/4)*arctan(-1/1296*(6*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14
+ 950652*x^13 - 997302*x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9
- 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 -
 36433344*x^3 + 12609792*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 +
 548864*x^13 - 575818*x^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 -
4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 2
1034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 -
 537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10
 + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864
*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3
538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623
494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 14
7990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 759
8336) + 21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273
216)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 -
 205368*x^12 - 289200*x^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992
*x^7 + 1849440*x^6 + 1558464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4
sqrt(3)(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 264
54*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 3
57936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x
^14 + 738528*x^13 - 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9
 - 6903864*x^8 + 15444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 1
55968*x^3 - 3016704*x^2 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1
073898*x^12 - 452980*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916
720*x^7 - 2490016*x^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2
 + 1543936*x + 545536) + 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 66
5088*x - 235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt
(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 36*(144*x^17 - 5976*x^
16 + 5544*x^15 + 299664*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 -
4761216*x^10 - 1046592*x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016
832*x^5 - 3699072*x^4 - 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 28
6*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226
888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 -
177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x
^16 - 26052*x^15 + 332844*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11
 + 318696*x^10 + 12621072*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 -
 20168256*x^5 + 10936128*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x
^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^
11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176

*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17
 - 96354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 +
57963744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^
7 - 42453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x
^2 + sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*
x^13 + 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 8078428
0*x^8 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 1733536
0*x^3 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-6
72*sqrt(3) + 1164) + 3261696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(9
7*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 48
3752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x
^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^1
7 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11
 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x
^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 1
26592)*sqrt(-672*sqrt(3) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97
) - (sqrt(x^3 + 1)*((459*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 467
7912*x^12 + 12651948*x^11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 10
5313392*x^7 + 99605088*x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8
256960*x^2 + sqrt(3)*(265*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700
776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 6080
2736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767
168*x^2 + 1207168*x + 1383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 45
0359*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 14
6877876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4
 - 45545344*x^3 + 69517536*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^
14 + 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x
^9 + 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864
*x^4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 5562368) - 13696448*x - 96
34304)*sqrt(-672*sqrt(3) + 1164) + 2090880*x + 2396160)*(-672*sqrt(3) + 116
4)^(3/4) + 3*(984*x^15 - 14712*x^14 - 53940*x^13 + 411732*x^12 - 280248*x^1
1 - 324624*x^10 + 180816*x^9 - 518544*x^8 + 974304*x^7 - 887136*x^6 - 14040
96*x^5 + 1843584*x^4 + 135936*x^3 - 696192*x^2 + 4*sqrt(3)*(142*x^15 - 2124
*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 7527
6*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 - 100
512*x^2 + 62400*x + 24832) + (4945*x^15 - 37473*x^14 - 490698*x^13 + 224946
8*x^12 + 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*
x^7 + 768064*x^6 + 10405056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 +
sqrt(3)*(2855*x^15 - 21635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11
 - 4863472*x^10 + 3379536*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 600
7360*x^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1
738368*x - 691712)*sqrt(-672*sqrt(3) + 1164) + 432384*x + 172032)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24
)*sqrt(-56*sqrt(3) + 97) + 6*(4680*x^16 - 60552*x^15 + 89856*x^14 + 278280*
x^13 + 64440*x^12 - 1285200*x^11 - 255600*x^10 + 3098880*x^9 - 1770336*x^8
 - 3614400*x^7 + 3895488*x^6 + 1199232*x^5 - 2905344*x^4 + 681984*x^3 + 6497
28*x^2 + 108*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12
 - 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 62
40*x^5 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280) + (1164*x^17 + 12
48*x^16 - 246120*x^15 + 518172*x^14 + 2607528*x^13 - 8301144*x^12 + 7017600
*x^11 + 6258120*x^10 - 21360336*x^9 + 16998960*x^8 + 966336*x^7 - 18216672*
x^6 + 15860544*x^5 - 4720704*x^4 - 6023424*x^3 + 5362176*x^2 + 48*sqrt(3)*(
14*x^17 + 15*x^16 - 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404
*x^11 + 75267*x^10 - 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190
768*x^5 - 56784*x^4 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) + (2340*x^17
 - 35850*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 -
 46293732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^
7 + 45222000*x^6 - 100598112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x
^2 + sqrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*

```
x^13 - 987292*x^12 - 26727704*x^11 + 34156928*x^10 + 10669552*x^9 - 7064835
2*x^8 + 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 1709504
0*x^3 - 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*sqrt(-67
2*sqrt(3) + 1164) - 2035200*x - 1112064)*sqrt(-672*sqrt(3) + 1164) + 24*(62
7*x^16 - 14286*x^15 + 39762*x^14 + 50142*x^13 - 216816*x^12 + 112284*x^11 +
 325707*x^10 - 586326*x^9 - 3294*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5
 + 483816*x^4 - 115296*x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 - 4124*x^15 +
 11478*x^14 + 14474*x^13 - 62584*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^
9 - 954*x^8 + 182368*x^7 - 155648*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3
- 31344*x^2 + 37024*x + 11584) + 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164
) - 764928*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 + 72*x^7 + 1656
*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 324*x^5 + 576*x^4 + 696*x^
3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (1
23*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*
x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164)
 + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6
 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5
+ 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x +
2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16
)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656
*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5
 + 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) + 1/5184*((7*sqrt(3) + 12)*sqr
t(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1
164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 + 72*x^7 + 1656*x^6
 + 720*x^5 + 1440*x^4 + 2016*x^3 + (60*x^6 + 324*x^5 + 576*x^4 + 696*x^3 +
432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + (123*x
^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 + 1164*x^5
+ 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) + 1
44*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164)
 + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 + 144*sqrt(3)*(x^7 + 4*x^6 + 6
*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(26*x^7 + 38*x^6 + 42*x^5 + 46
*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^
3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*sqrt(3) + 1164) - 576*x + 2304
)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) -
 1/5184*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 12)*sqrt(-2*(7*sqrt(3
) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4)*log(1/3
6*(36*x^8 + 72*x^7 + 1656*x^6 + 720*x^5 + 1440*x^4 + 2016*x^3 - (60*x^6 + 3
24*x^5 + 576*x^4 + 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 1
0*x^3 + 8*x^2 + 4*x) + (123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2
+ sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*
sqrt(-672*sqrt(3) + 1164) + 144*x + 96)*sqrt(x^3 + 1)*sqrt(-2*(7*sqrt(3) +
12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 +
 144*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) + 72*(
26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 + sqrt(3)*(15*x^7 + 22*
x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(-672*s
qrt(3) + 1164) - 576*x + 2304)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32
*x^3 + 64*x^2 + 32*x + 16)) + 1/36*sqrt(14*sqrt(3) - 24)*arctan(1/12*(3*x^2
 + sqrt(3)*(x^2 - 10*x - 8) - 18*x - 12)*sqrt(14*sqrt(3) - 24)/sqrt(x^3 + 1
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3 + 6\sqrt{3} + 10\right)\sqrt{x^3 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

maple [C] time = 0.28, size = 353, normalized size = 1.62

$$\frac{\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\left(2+\sqrt{3}\right)\sqrt{x^3+1}}-\frac{\sqrt{2}\ (-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alpha-2)/(-3^(1/2)+2*_alpha-1)*(-I*3^(1
/2)+3)*((x+1)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I*3^(1/2))/(-I*3^(1/2)-3))^(1/2
)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha-3^(1/2
)*_alpha)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*
_alpha*3^(1/2)+1/2*3^(1/2)*_alpha-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(-1-3^(1/2))*_Z+2*3^(
1/2)+4))+1/9*(-1-3^(1/2))/(2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I
*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2
+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticP
i(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+6\sqrt{3}+10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x^3+6\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)),x)
```

```
[Out] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\,\left(x^3+10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3))), x)
```

3.87 $\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=210

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{(2+}{}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*(1-2*x-3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))
*3^(3/4)*2^(1/2)-1/36*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^
(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2
))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/6*(1+3
^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{(2+}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3]
)])/(3*Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 +
 x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTanh
[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(3/
4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/4)
)])/(3*Sqrt[2]*3^(3/4))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx=-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{(2+}{}$$

Mathematica [C] time = 0.08, size = 50, normalized size = 0.24

$$-\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)}{4\left(3\sqrt{3}-5\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/4*(x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])/(-5 +
3*Sqrt[3])
```

fricas [B] time = 10.43, size = 8237, normalized size = 39.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/108*sqrt(3)*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(1/324*(2
16*sqrt(3)*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345
761*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^
7 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sq
rt(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12
- 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x
^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
 271808*x + 126592)*(56*sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96
354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963
744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 4
2453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 -
sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13
+ 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8
 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3
 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3)
+ 97) + 6*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 3457
61*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7
 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqr
t(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 -
 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^
6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
271808*x + 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + 3*sqrt(sq
rt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 -
6128*x^15 - 537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84
218362*x^10 + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6
- 256326864*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(21
31*x^16 - 3538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x
^11 + 48623494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 693427
76*x^6 - 147990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242
560*x + 7598336) + 21204736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) +
 (459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972
*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 8842732
8*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)
*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x
^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*
x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x
- 1889792) - 4310784*x - 3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672
*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 -
 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15
444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704
*x^2 - sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 45298
0*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x
^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 5455
36) + 2674176*x + 944896)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7
653*x^14 + 41169*x^13 - 51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 -
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 17892*x^8 - 490248*x^7 + 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3
+ 187584*x^2 - sqrt(3)*(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41
940*x^11 + 26454*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 +
224784*x^5 - 357936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272
*x - 58752)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)
) + 108*(12*x^17 - 498*x^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^1
2 + 300000*x^11 - 396768*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 22012
8*x^6 + 584736*x^5 - 308256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17
 - 286*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11
- 226888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x
^5 - 177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 3148
8)*sqrt(56*sqrt(3) + 97) + (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14
 + 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294
*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108
576*x^2 - 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584
*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 15564
8*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) +
 128256*x + 40128)*(56*sqrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 358
50*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293
732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45
222000*x^6 - 100598112*x^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 - s
qrt(3)*(1351*x^17 - 20698*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 -
 987292*x^12 - 26727704*x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8
+ 46883072*x^7 + 26108944*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3
- 12966272*x^2 + 4724480*x + 2581504) + 8183040*x + 4471296)*(56*sqrt(3) +
97) + 6*(97*x^17 + 104*x^16 - 20510*x^15 + 43181*x^14 + 217294*x^13 - 69176
2*x^12 + 584800*x^11 + 521510*x^10 - 1780028*x^9 + 1416580*x^8 + 80528*x^7
- 1518056*x^6 + 1321712*x^5 - 393392*x^4 - 501952*x^3 + 446848*x^2 - 4*sqrt
(3)*(14*x^17 + 15*x^16 - 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 +
84404*x^11 + 75267*x^10 - 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6
+ 190768*x^5 - 56784*x^4 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) - 16960
0*x - 92672)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(sqrt(3)*sq
rt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 + 17731*x^
15 - 951114*x^14 + 450359*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^
10 + 9429316*x^9 + 146877876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776
*x^5 - 132260896*x^4 - 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 + 1
0237*x^15 - 549126*x^14 + 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 4508
9132*x^10 + 5444020*x^9 + 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106
882416*x^5 - 76360864*x^4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 55623
68) - 13696448*x - 9634304)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 1
557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^11 - 55684
800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088*x^6 - 1889
7984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 -
899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11 - 3214964
0*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 1091078
4*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1383424) +
2090880*x + 2396160)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 11
64)^(3/4) + 6*(sqrt(3)*(4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12
 + 474132*x^11 - 8423784*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 +
768064*x^6 + 10405056*x^5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 - sqrt(3
)*(2855*x^15 - 21635*x^14 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863
472*x^10 + 3379536*x^9 + 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x
^5 - 3826528*x^4 - 404416*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368
*x - 691712)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 3678*x^14 - 13
485*x^13 + 102933*x^12 - 70062*x^11 - 81156*x^10 + 45204*x^9 - 129636*x^8 +
 243576*x^7 - 221784*x^6 - 351024*x^5 + 460896*x^4 + 33984*x^3 - 174048*x^2
 - sqrt(3)*(142*x^15 - 2124*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46
860*x^10 + 26308*x^9 - 75276*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 2
66016*x^4 + 19712*x^3 - 100512*x^2 + 62400*x + 24832) + 108096*x + 43008)*s
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qrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(130*
x^16 - 1682*x^15 + 2496*x^14 + 7730*x^13 + 1790*x^12 - 35700*x^11 - 7100*x^
10 + 86080*x^9 - 49176*x^8 - 100400*x^7 + 108208*x^6 + 33312*x^5 - 80704*x^
4 + 18944*x^3 + 18048*x^2 - 3*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482
*x^13 + 316*x^12 - 6984*x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7
 + 20976*x^6 + 6240*x^5 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280)
- 21248*x - 6656)*sqrt(56*sqrt(3) + 97))*sqrt((9*x^8 + 18*x^7 + 414*x^6 + 1
80*x^5 + 360*x^4 + 504*x^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5
+ 46*x^4 + 46*x^3 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 2
6*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(12
3*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x
^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt
(3) + 97) + 6*(5*x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 +
 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4)
 - 36*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144
*x + 576)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x
+ 16)))/(x^17 + 13*x^16 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 1
1656*x^11 + 23944*x^10 - 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256
*x^5 + 13376*x^4 - 5760*x^3 - 1664*x^2 + 256*x)) - 1/108*sqrt(3)*sqrt(sqrt(
3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4)*(
56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/324*(216*sqrt(3)*(97*x^17 - 52
3*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 +
 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 168068
8*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqrt(3)*(28*x^17 - 151*x^1
6 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^1
0 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080
*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*(56*
sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96354*x^16 + 84798*x^15 +
4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 76603680*x^10
 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 113269536*
x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 - sqrt(3)*(1351*x^17 - 556
30*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 3346537
6*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 - 245103
68*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 - 757158
4*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3) + 97) + 6*(97*x^17 - 523
*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 +
26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688
*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqrt(3)*(28*x^17 - 151*x^16
 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10
 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*
x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(
56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - 3*sqrt(sqrt(3)*sqrt(56*sqrt(3) +
97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 - 6128*x^15 - 537864*x^14
+ 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x
^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645
168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(2131*x^16 - 3538*x^15 - 31
0536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41
002448*x^9 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 +
 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204
736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 13425*x^15
- 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972*x^11 + 47069892*x^10 -
49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 88427328*x^6 + 25613856*x^5 + 2
7458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)*(265*x^16 - 7751*x^15 -
 19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x^11 + 27175852*x^10 - 28
730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*x^6 + 14788128*x^5 + 158
53184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x - 1889792) - 4310784*x -
 3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) +
 6*(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 - 1860046*x^12 - 784596*x

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^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15444144*x^7 - 4312832*x^6
 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2 - sqrt(3)*(2855*x^1
5 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^11 + 4427548*x^10 -
3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 - 5519008*x^5 + 54039
04*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536) + 2674176*x + 944896
)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7653*x^14 + 41169*x^13 -
51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 - 17892*x^8 - 490248*x^7
+ 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3 + 187584*x^2 - sqrt(3)*(
142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 +
128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4
+ 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272*x - 58752)*sqrt(x^3 + 1
)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(12*x^17 - 498*x
^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^12 + 300000*x^11 - 396768
*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 220128*x^6 + 584736*x^5 - 308
256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 +
 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^
9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*
x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 31488)*sqrt(56*sqrt(3) + 97)
 + (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14 + 50142*x^13 - 216816*x
^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294*x^8 + 631752*x^7 - 5392
20*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(181
*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584*x^12 + 32412*x^11 + 940
21*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 155648*x^6 - 53232*x^5 + 1396
64*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) + 128256*x + 40128)*(56*s
qrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 35850*x^16 - 106410*x^15 -
2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293732*x^11 + 59161524*x^10
 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45222000*x^6 - 100598112*x
^5 + 42207168*x^4 + 29609472*x^3 - 22458240*x^2 - sqrt(3)*(1351*x^17 - 2069
8*x^16 - 61436*x^15 - 1192081*x^14 + 6896998*x^13 - 987292*x^12 - 26727704*
x^11 + 34156928*x^10 + 10669552*x^9 - 70648352*x^8 + 46883072*x^7 + 2610894
4*x^6 - 58080352*x^5 + 24368320*x^4 + 17095040*x^3 - 12966272*x^2 + 4724480
*x + 2581504) + 8183040*x + 4471296)*(56*sqrt(3) + 97) + 6*(97*x^17 + 104*x
^16 - 20510*x^15 + 43181*x^14 + 217294*x^13 - 691762*x^12 + 584800*x^11 + 5
21510*x^10 - 1780028*x^9 + 1416580*x^8 + 80528*x^7 - 1518056*x^6 + 1321712*
x^5 - 393392*x^4 - 501952*x^3 + 446848*x^2 - 4*sqrt(3)*(14*x^17 + 15*x^16 -
 2960*x^15 + 6232*x^14 + 31362*x^13 - 99844*x^12 + 84404*x^11 + 75267*x^10
- 256916*x^9 + 204458*x^8 + 11616*x^7 - 219104*x^6 + 190768*x^5 - 56784*x^4
 - 72448*x^3 + 64496*x^2 - 24480*x - 13376) - 169600*x - 92672)*sqrt(56*sqr
t(3) + 97))*sqrt(56*sqrt(3) + 97) + sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*s
qrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 + 17731*x^15 - 951114*x^14 + 45035
9*x^13 + 4370159*x^12 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 14687
7876*x^8 - 197107784*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 4
5545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 + 10237*x^15 - 549126*x^14
+ 260015*x^13 + 2523113*x^12 + 17504406*x^11 - 45089132*x^10 + 5444020*x^9
+ 84799980*x^8 - 113800232*x^7 - 17802104*x^6 + 106882416*x^5 - 76360864*x^
4 - 26295616*x^3 + 40135968*x^2 - 7907648*x - 5562368) - 13696448*x - 96343
04)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + (459*x^16 - 1557*x^15 - 26415*x^14 -
1449954*x^13 + 4677912*x^12 + 12651948*x^11 - 55684800*x^10 + 62834256*x^9
+ 8526168*x^8 - 105313392*x^7 + 99605088*x^6 - 18897984*x^5 - 42499296*x^4
+ 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 - 899*x^15 - 15249*x^14 -
837130*x^13 + 2700776*x^12 + 7304604*x^11 - 32149640*x^10 + 36277360*x^9 +
4922568*x^8 - 60802736*x^7 + 57507040*x^6 - 10910784*x^5 - 24536992*x^4 + 2
1568448*x^3 - 4767168*x^2 + 1207168*x + 1383424) + 2090880*x + 2396160)*sqr
t(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(
4945*x^15 - 37473*x^14 - 490698*x^13 + 2249468*x^12 + 474132*x^11 - 8423784
*x^10 + 5853520*x^9 + 8451720*x^8 - 15320016*x^7 + 768064*x^6 + 10405056*x^
5 - 6627744*x^4 - 700480*x^3 + 2799552*x^2 - sqrt(3)*(2855*x^15 - 21635*x^1
4 - 283306*x^13 + 1298732*x^12 + 273748*x^11 - 4863472*x^10 + 3379536*x^9 +
 4879608*x^8 - 8845008*x^7 + 443456*x^6 + 6007360*x^5 - 3826528*x^4 - 40441
```

```
6*x^3 + 1616320*x^2 - 1003648*x - 399360) - 1738368*x - 691712)*sqrt(x^3 +
1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 3678*x^14 - 13485*x^13 + 102933*x^12 -
 70062*x^11 - 81156*x^10 + 45204*x^9 - 129636*x^8 + 243576*x^7 - 221784*x^6
 - 351024*x^5 + 460896*x^4 + 33984*x^3 - 174048*x^2 - sqrt(3)*(142*x^15 - 2
124*x^14 - 7773*x^13 + 59447*x^12 - 40626*x^11 - 46860*x^10 + 26308*x^9 - 7
5276*x^8 + 140472*x^7 - 127784*x^6 - 202896*x^5 + 266016*x^4 + 19712*x^3 -
100512*x^2 + 62400*x + 24832) + 108096*x + 43008)*sqrt(x^3 + 1)*sqrt(56*sqr
t(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) + 108*(130*x^16 - 1682*x^15 + 2496*
x^14 + 7730*x^13 + 1790*x^12 - 35700*x^11 - 7100*x^10 + 86080*x^9 - 49176*x
^8 - 100400*x^7 + 108208*x^6 + 33312*x^5 - 80704*x^4 + 18944*x^3 + 18048*x^
2 - 3*sqrt(3)*(25*x^16 - 324*x^15 + 489*x^14 + 1482*x^13 + 316*x^12 - 6984*
x^11 - 1312*x^10 + 16624*x^9 - 9792*x^8 - 19328*x^7 + 20976*x^6 + 6240*x^5
 - 15552*x^4 + 3712*x^3 + 3456*x^2 - 4096*x - 1280) - 21248*x - 6656)*sqrt(5
6*sqrt(3) + 97))*sqrt((9*x^8 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4 + 504*x
^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x
^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x +
4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) - (sqrt(3)*(123*x^6 + 2016*x^5 + 2214*
x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3
 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*x^6 + 2
7*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3
 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 9
7)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^7 + 4*x
^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 - 4*x^7 +
 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)))/(x^17 + 13*x^16
 - 522*x^15 + 1742*x^14 + 3008*x^13 - 16884*x^12 + 11656*x^11 + 23944*x^10 -
 42336*x^9 + 9136*x^8 + 36256*x^7 - 27360*x^6 - 256*x^5 + 13376*x^4 - 5760*
x^3 - 1664*x^2 + 256*x)) - 1/1296*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqr
t(3) - 12) + 6)*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) - 6)*(672*s
qrt(3) + 1164)^(1/4)*log(1/9*(9*x^8 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4
+ 504*x^3 - 72*x^2 + 36*sqrt(3)*(26*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3
 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 +
12*x + 4) + 20*x + 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 + 2016*x^5
+ 2214*x^4 + 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1
192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*
x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 +
 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt
(3) + 97)*(7*sqrt(3) - 12) + 6)*(672*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^
7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 + 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 -
4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 + 32*x^3 + 64*x^2 + 32*x + 16)) + 1/1296*s
qrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(sqrt(3)*sqrt(56*sq
rt(3) + 97)*(7*sqrt(3) - 12) - 6)*(672*sqrt(3) + 1164)^(1/4)*log(1/9*(9*x^8
 + 18*x^7 + 414*x^6 + 180*x^5 + 360*x^4 + 504*x^3 - 72*x^2 + 36*sqrt(3)*(26
*x^7 + 38*x^6 + 42*x^5 + 46*x^4 + 46*x^3 + 42*x^2 - sqrt(3)*(15*x^7 + 22*x^
6 + 24*x^5 + 27*x^4 + 26*x^3 + 24*x^2 + 12*x + 4) + 20*x + 8)*sqrt(56*sqrt(
3) + 97) - (sqrt(3)*(123*x^6 + 2016*x^5 + 2214*x^4 + 2064*x^3 + 396*x^2 - s
qrt(3)*(71*x^6 + 1164*x^5 + 1278*x^4 + 1192*x^3 + 228*x^2 - 112) - 192)*sqr
t(x^3 + 1)*sqrt(56*sqrt(3) + 97) + 6*(5*x^6 + 27*x^5 + 48*x^4 + 58*x^3 + 36
*x^2 - 3*sqrt(3)*(x^6 + 5*x^5 + 10*x^4 + 10*x^3 + 8*x^2 + 4*x) + 12*x + 8)*
sqrt(x^3 + 1))*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4) - 36*sqrt(3)*(x^7 + 4*x^6 + 6*x^5 + 5*x^4 - 4*x^3 +
 6*x^2 + 4*x - 8) - 144*x + 576)/(x^8 - 4*x^7 + 16*x^6 - 16*x^5 + 28*x^4 +
32*x^3 + 64*x^2 + 32*x + 16)) + 1/72*sqrt(14*sqrt(3) + 24)*log((x^8 - 16*x^
7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 64*x^2 + 2*(5*x^6 - 54*x^5 + 96*
x^4 - 56*x^3 - 36*x^2 - 3*sqrt(3)*(x^6 - 10*x^5 + 20*x^4 - 8*x^3 - 4*x^2 +
8*x) + 24*x - 16)*sqrt(x^3 + 1)*sqrt(14*sqrt(3) + 24) + 16*sqrt(3)*(x^7 - 2
*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7
 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3 - 6\sqrt{3} + 10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

maple [C] time = 0.26, size = 350, normalized size = 1.67

$$\frac{\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\left(-2+\sqrt{3}\right)\sqrt{x^3+1}} - \sqrt{2}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x)
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alpha+2)/(-3^(1/2)-2*_alpha+1)*(-I*3^(1
/2)+3)*((x+1)/(-I*3^(1/2)+3))^(1/2)*((2*x-1-I*3^(1/2))/(-I*3^(1/2)-3))^(1/2
)*((2*x-1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha+3^(1/2
)*_alpha)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*3
^(1/2)*_alpha-1/2*3^(1/2)*_alpha-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(3^(1/2)-1)*_Z-2*3^(1/
2)+4))+1/9*(3^(1/2)-1)/(-2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3
^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1
/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(
((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3 - 6\sqrt{3} + 10\right)\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x^3 - 6\sqrt{3} + 10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)),x)
[Out] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\,\left(x^3 - 6\sqrt{3} + 10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10)), x)
```

3.88 $\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=222

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right.}{}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2)
)*3^(3/4)*2^(1/2)+1/18*arctan(1/2*3^(1/4)*(1+2*x+3^(1/2))*2^(1/2)/(x^3-1)^(
1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1-x)*(1+3^(1/2)
)*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/18*arctanh(1/6*(1-3^
(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^
3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2
*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan
h[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3
/4))])/(3*Sqrt[2]*3^(3/4))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx=\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{(2}{}$$

Mathematica [C] time = 0.07, size = 65, normalized size = 0.29

$$-\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{10+6\sqrt{3}}\right)}{\left(20+12\sqrt{3}\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])])
/((20 + 12*Sqrt[3])*Sqrt[-1 + x^3]))
```

fricas [B] time = 9.99, size = 7910, normalized size = 35.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3) +
 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(1/1296*(6*sq
rt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7
 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2
+ 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^1
3 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018320*x^9 - 25445581
2*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 784
64672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 9
15953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 -
146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4
 - 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160
704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-672*sqrt(3) + 1164)
^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*x^12 - 289200*x^11
 - 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1849440*x^6 + 15584
64*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)*(142*x^15 + 4419*
x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 106
92*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108
288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 738528*x^13 + 186004
6*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903864*x^8 + 15444144*
x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^3 + 3016704*x^2 +
sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x^12 - 452980*x^11
- 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7 + 2490016*x^6 - 55
19008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543936*x - 545536) + 2
674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x + 235008)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)
*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 + 5976*x^16 + 5544*x^15 - 299664*x^1
4 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x^10 - 1046592*x^9
- 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 + 3699072*x^4 - 186
1632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x
^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 4153
84*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78
784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 26052*x^15 - 332844*x
^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696*x^10 + 12621072*x
^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256*x^5 - 10936128*x^
4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 -
 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9
 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*
x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*x^16 + 84798*x^15
- 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*x^11 + 76603680*x^
10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453216*x^6 + 11326953
6*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt(3)*(1351*x^17 + 5
5630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 1121030*x^12 + 33465
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376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 61330384*x^7 + 2451
0368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 15297472*x^2 - 7571
584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3) + 1164) + 326169
6*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 + 523*x^16 - 2171*x^
15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 10
51756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x
^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8
006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 +
 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^
3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sqrt(-672*sqrt(3) +
1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 - 1)*((459*x^
16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 -
 55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6
+ 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^2 + sqrt(3)*(265*x
^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 3
2149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 1
0910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 13834
24) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13 + 4370159*x^12 -
30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x^8 + 197107784*x^7
 - 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 45545344*x^3 + 69517536*x
^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x^14 - 260015*x^13 + 2523113*
x^12 - 17504406*x^11 - 45089132*x^10 - 5444020*x^9 + 84799980*x^8 + 1138002
32*x^7 - 17802104*x^6 - 106882416*x^5 - 76360864*x^4 + 26295616*x^3 + 40135
968*x^2 + 7907648*x - 5562368) + 13696448*x - 9634304)*sqrt(-672*sqrt(3) +
1164) - 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 14
712*x^14 - 53940*x^13 - 411732*x^12 - 280248*x^11 + 324624*x^10 + 180816*x^
9 + 518544*x^8 + 974304*x^7 + 887136*x^6 - 1404096*x^5 - 1843584*x^4 + 1359
36*x^3 + 696192*x^2 + 4*sqrt(3)*(142*x^15 + 2124*x^14 - 7773*x^13 - 59447*x
^12 - 40626*x^11 + 46860*x^10 + 26308*x^9 + 75276*x^8 + 140472*x^7 + 127784
*x^6 - 202896*x^5 - 266016*x^4 + 19712*x^3 + 100512*x^2 + 62400*x - 24832)
- (4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8423
784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405056
*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 + sqrt(3)*(2855*x^15 + 21635*
x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536*x^
9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 - 40
4416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(-67
2*sqrt(3) + 1164) + 432384*x - 172032)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*
(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) + 6
*(4680*x^16 + 60552*x^15 + 89856*x^14 - 278280*x^13 + 64440*x^12 + 1285200*
x^11 - 255600*x^10 - 3098880*x^9 - 1770336*x^8 + 3614400*x^7 + 3895488*x^6
- 1199232*x^5 - 2905344*x^4 - 681984*x^3 + 649728*x^2 + 108*sqrt(3)*(25*x^1
6 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 + 6984*x^11 - 1312*x^10 - 16
624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 6240*x^5 - 15552*x^4 - 3712*x^
3 + 3456*x^2 + 4096*x - 1280) + (1164*x^17 - 1248*x^16 - 246120*x^15 - 5181
72*x^14 + 2607528*x^13 + 8301144*x^12 + 7017600*x^11 - 6258120*x^10 - 21360
336*x^9 - 16998960*x^8 + 966336*x^7 + 18216672*x^6 + 15860544*x^5 + 4720704
*x^4 - 6023424*x^3 - 5362176*x^2 + 48*sqrt(3)*(14*x^17 - 15*x^16 - 2960*x^1
5 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^10 - 256916*
x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x^4 - 72448*
x^3 - 64496*x^2 - 24480*x + 13376) - (2340*x^17 + 35850*x^16 - 106410*x^15
+ 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 + sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108
944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*sqrt(-672*sqrt(3) + 1164) - 2035200*
x + 1112064)*sqrt(-672*sqrt(3) + 1164) - 24*(627*x^16 + 14286*x^15 + 39762*
x^14 - 50142*x^13 - 216816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 -
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3294*x^8 - 631752*x^7 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 -
108576*x^2 + 2*sqrt(3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 6
2584*x^12 - 32412*x^11 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 1
55648*x^6 + 53232*x^5 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 1158
4) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928*x - 239616)*sqrt(
-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 -
2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^
6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4
- 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 +
228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)
sqrt(2(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*sqrt(3) + 11
64)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*
x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 +
sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) +
20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 +
16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 - 13*x^16 - 522*x^1
5 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^
9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 166
4*x^2 + 256*x)) + 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) +
24)*(56*sqrt(3) + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*a
rctan(1/1296*(6*sqrt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652
*x^13 - 997302*x^12 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 821253
6*x^8 - 84377808*x^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 3643334
4*x^3 + 12609792*x^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*
x^13 - 575818*x^12 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*
x^8 - 48715600*x^7 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*
x^3 + 7280256*x^2 + 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*
x^14 - 1586477*x^13 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018
320*x^9 - 254455812*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 1
34645168*x^4 - 78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15
- 310536*x^14 - 915953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10
- 41002448*x^9 - 146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*
x^5 + 77737424*x^4 - 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) -
21204736*x + 13160704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-6
72*sqrt(3) + 1164)^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*
x^12 - 289200*x^11 - 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1
849440*x^6 + 1558464*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)
*(142*x^15 + 4419*x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10
+ 128152*x^9 + 10692*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^
4 + 9632*x^3 - 108288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 73
8528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 69038
64*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^
3 + 3016704*x^2 + sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x
^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7
+ 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 15439
36*x - 545536) + 2674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x +
235008)*(-672*sqrt(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqr
t(3) + 1164) + 24)*sqrt(-56*sqrt(3) + 97) - 36*(144*x^17 + 5976*x^16 + 5544
*x^15 - 299664*x^14 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x
^10 - 1046592*x^9 - 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 +
3699072*x^4 - 1861632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 +
238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10
- 48920*x^9 - 415384*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^
4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 260
52*x^15 - 332844*x^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696
*x^10 + 12621072*x^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256
*x^5 - 10936128*x^4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151
*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661
*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 26

```
3080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*
x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*
x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453
216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt
(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 11
21030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 6
1330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 1
5297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3
) + 1164) + 3261696*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 +
523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11
 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680
688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x
^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x
^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 2630
80*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sq
rt(-672*sqrt(3) + 1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt
(x^3 - 1)*((459*x^16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12
 - 12651948*x^11 - 55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x
^7 + 99605088*x^6 + 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^
2 + sqrt(3)*(265*x^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12
- 7304604*x^11 - 32149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7
+ 57507040*x^6 + 10910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 -
 1207168*x + 1383424) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13
 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x
^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 4554534
4*x^3 + 69517536*x^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x^14 - 2600
15*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 5444020*x^9 + 8479
9980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360864*x^4 + 26
295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x - 9634304)*sq
rt(-672*sqrt(3) + 1164) - 2090880*x + 2396160)*(-672*sqrt(3) + 1164)^(3/4)
+ 3*(984*x^15 + 14712*x^14 - 53940*x^13 - 411732*x^12 - 280248*x^11 + 32462
4*x^10 + 180816*x^9 + 518544*x^8 + 974304*x^7 + 887136*x^6 - 1404096*x^5 -
1843584*x^4 + 135936*x^3 + 696192*x^2 + 4*sqrt(3)*(142*x^15 + 2124*x^14 - 7
773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26308*x^9 + 75276*x^8 + 1
40472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 19712*x^3 + 100512*x^2 +
 62400*x - 24832) - (4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 +
474132*x^11 + 8423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768
064*x^6 + 10405056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 + sqrt(3)*(
2855*x^15 + 21635*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472
*x^10 + 3379536*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5
+ 3826528*x^4 - 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x
+ 691712)*sqrt(-672*sqrt(3) + 1164) + 432384*x - 172032)*(-672*sqrt(3) + 11
64)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*sqrt(-56
*sqrt(3) + 97) - 6*(4680*x^16 + 60552*x^15 + 89856*x^14 - 278280*x^13 + 644
40*x^12 + 1285200*x^11 - 255600*x^10 - 3098880*x^9 - 1770336*x^8 + 3614400*
x^7 + 3895488*x^6 - 1199232*x^5 - 2905344*x^4 - 681984*x^3 + 649728*x^2 + 1
08*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 + 6984*x^1
1 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 6240*x^5 - 1
5552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + (1164*x^17 - 1248*x^16 -
246120*x^15 - 518172*x^14 + 2607528*x^13 + 8301144*x^12 + 7017600*x^11 - 62
58120*x^10 - 21360336*x^9 - 16998960*x^8 + 966336*x^7 + 18216672*x^6 + 1586
0544*x^5 + 4720704*x^4 - 6023424*x^3 - 5362176*x^2 + 48*sqrt(3)*(14*x^17 -
15*x^16 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75
267*x^10 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 +
56784*x^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - (2340*x^17 + 35850*x
^16 - 106410*x^15 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*
x^11 - 59161524*x^10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 452220
00*x^6 - 100598112*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 + sqrt(
3)*(1351*x^17 + 20698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987
```

```
292*x^12 - 26727704*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46
883072*x^7 - 26108944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12
966272*x^2 + 4724480*x - 2581504) + 8183040*x - 4471296)*sqrt(-672*sqrt(3)
+ 1164) - 2035200*x + 1112064)*sqrt(-672*sqrt(3) + 1164) - 24*(627*x^16 + 1
4286*x^15 + 39762*x^14 - 50142*x^13 - 216816*x^12 - 112284*x^11 + 325707*x^
10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 539220*x^6 + 184392*x^5 + 483816*
x^4 + 115296*x^3 - 108576*x^2 + 2*sqrt(3)*(181*x^16 + 4124*x^15 + 11478*x^1
4 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 94021*x^10 + 169244*x^9 - 954*x^
8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 139664*x^4 + 33280*x^3 - 31344*x^
2 - 37024*x + 11584) - 128256*x + 40128)*sqrt(-672*sqrt(3) + 1164) + 764928
*x - 239616)*sqrt(-56*sqrt(3) + 97))*sqrt((36*x^8 - 72*x^7 + 1656*x^6 - 720
*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^
2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2
016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278
*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x +
 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*
(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 -
5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 +
46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*
x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8
+ 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)))/(x^17 -
 13*x^16 - 522*x^15 - 1742*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 239
44*x^10 - 42336*x^9 - 9136*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^
4 - 5760*x^3 + 1664*x^2 + 256*x)) + 1/5184*sqrt(2*(7*sqrt(3) + 12)*sqrt(-67
2*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) - 12)*(
-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72*x^7 + 1656*x^6 - 720*x^5 +
 1440*x^4 - 2016*x^3 + (60*x^6 - 324*x^5 + 576*x^4 - 696*x^3 + 432*x^2 + 36
*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - (123*x^6 - 2016*x^
5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 -
 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt(3) + 1164) - 144*x + 96)*s
qrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(-672*
sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3
 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 +
12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164) + 576*x + 2304)/(x^8 + 4*x^
7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/5184*sqrt(
2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*((7*sqrt(3) + 12)*sqrt(-
672*sqrt(3) + 1164) - 12)*(-672*sqrt(3) + 1164)^(1/4)*log(1/36*(36*x^8 - 72
*x^7 + 1656*x^6 - 720*x^5 + 1440*x^4 - 2016*x^3 - (60*x^6 - 324*x^5 + 576*x
^4 - 696*x^3 + 432*x^2 + 36*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2
- 4*x) - (123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 + sqrt(3)*(71*
x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(-672*sqrt
(3) + 1164) - 144*x + 96)*sqrt(x^3 - 1)*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*s
qrt(3) + 1164) + 24)*(-672*sqrt(3) + 1164)^(1/4) - 288*x^2 - 144*sqrt(3)*(x
^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 72*(26*x^7 - 38*x^6
 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 + sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 -
27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(-672*sqrt(3) + 1164)
+ 576*x + 2304)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 -
 32*x + 16)) + 1/72*sqrt(14*sqrt(3) - 24)*log((x^8 + 16*x^7 + 112*x^6 + 16*
x^5 + 112*x^4 - 224*x^3 + 64*x^2 - 2*(5*x^6 + 54*x^5 + 96*x^4 + 56*x^3 - 36
*x^2 + 3*sqrt(3)*(x^6 + 10*x^5 + 20*x^4 + 8*x^3 - 4*x^2 - 8*x) - 24*x - 16)
*sqrt(x^3 - 1)*sqrt(14*sqrt(3) - 24) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*
x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^
5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3 - 6\sqrt{3} - 10\right)\sqrt{x^3 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

maple [C] time = 0.27, size = 349, normalized size = 1.57

$$\frac{\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\left(2+\sqrt{3}\right)\sqrt{x^{3}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] 1/9*(-1-3^(1/2))/(2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2
)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3
^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/
(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alpha
+2)/(-3^(1/2)-2*_alpha-1)*(-I*3^(1/2)-3)*((x-1)/(-I*3^(1/2)-3))^(1/2)*((2*x
+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)
/(x^3-1)^(1/2)*(1+2*_alpha-3^(1/2)*_alpha)*EllipticPi(((x-1)/(-3/2-1/2*I*3^
(1/2)))^(1/2),1/3*I*3^(1/2)*_alpha-1/2*3^(1/2)*_alpha-1/2*I*_alpha+_alpha+1
/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=Ro
otOf(_Z^2+(1+3^(1/2))*_Z+2*3^(1/2)+4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}-6\sqrt{3}-10\right)\sqrt{x^{3}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{x^{3}-1}\,\left(-x^{3}+6\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)),x)
```

```
[Out] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x^{3}-6\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10)), x)
```

3.89 $\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right.}{}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2
))*3^(1/4)*2^(1/2)+1/18*arctan(1/6*(1+3^(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2
))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/2*3^(1/4)*(1+2*x-3^(1/2))*2^(
1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/36*arctanh(1/2*3^(1/4)*(1
-x)*(1+3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {488}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x
^3])])/(2*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-1 +
 x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[(3^
(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4))
 + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1 +
x^3])])/(3*Sqrt[2]*3^(1/4))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx=-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{(2}{}$$

Mathematica [C] time = 0.06, size = 68, normalized size = 0.32

$$\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{-10+6\sqrt{3}}\right)}{4\left(3\sqrt{3}-5\right)\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -(x^3/(-10 + 6*Sqrt[3]))
])/(4*(-5 + 3*Sqrt[3])*Sqrt[-1 + x^3])
```

fricas [B] time = 9.37, size = 8105, normalized size = 37.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*
(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/64
8*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 80158
4*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 -
2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x
^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 3214
12*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 3654
4) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*x^17
+ 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 5
7963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7
 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^
2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x
^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280
*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360
*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56*sqrt
(3) + 97) - 6*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584
*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 - 2
*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^
12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 32141
2*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544
) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(
1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 216816
*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 53
9220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(1
81*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 9
4021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 13
9664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128)*(56
*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*x^15
+ 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108
944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17 - 104
*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x^11 -
 521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 132171
2*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15*x^16
 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^1
0 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x
^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt(56*s
qrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)
*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x^14 -
 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 +
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 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x
^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 549126
*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 544402
0*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360
864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x -
 9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26415*x
^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 6283425
6*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 4249929
6*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15249*x
^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 36277360*
x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536992*x
^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 239616
0)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqr
t(3)*(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8
423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405
056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15 + 216
35*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536
*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 -
 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(
x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 102933*
x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 129636*x^8 + 243576*x^7 + 2217
84*x^6 - 351024*x^5 - 460896*x^4 + 33984*x^3 + 174048*x^2 - sqrt(3)*(142*x^
15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26308*x
^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 19712*
x^3 + 100512*x^2 + 62400*x - 24832) + 108096*x - 43008)*sqrt(x^3 - 1)*sqrt(
56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(130*x^16 + 1682*x^15 +
 2496*x^14 - 7730*x^13 + 1790*x^12 + 35700*x^11 - 7100*x^10 - 86080*x^9 - 4
9176*x^8 + 100400*x^7 + 108208*x^6 - 33312*x^5 - 80704*x^4 - 18944*x^3 + 18
048*x^2 - 3*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x^12 +
 6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6 - 624
0*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + 21248*x - 6656)*
sqrt(56*sqrt(3) + 97))*sqrt((18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4
 - 1008*x^3 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x
^3 - 42*x^2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2
+ 12*x - 4) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^
5 + 2214*x^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 -
 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(
5*x^6 - 27*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4
 - 10*x^3 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56
*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(
3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/
(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 3
*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(
3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^13 + 16210952*x^12 + 771817
56*x^11 + 84218362*x^10 - 71018320*x^9 - 254455812*x^8 - 196076008*x^7 + 12
0105208*x^6 + 256326864*x^5 + 134645168*x^4 - 78464672*x^3 - 78514944*x^2 -
 sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 915953*x^13 + 9359398*x^12
+ 44560908*x^11 + 48623494*x^10 - 41002448*x^9 - 146910132*x^8 - 113204536*
x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4 - 45301600*x^3 - 45330624
*x^2 - 12242560*x + 7598336) - 21204736*x + 13160704)*sqrt(x^3 - 1)*(56*sqr
t(3) + 97) - (459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 - sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7
 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2
+ 2488832*x - 1889792) + 4310784*x - 3273216)*sqrt(x^3 - 1)*sqrt(56*sqrt(3)
 + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 + 88617*x^14 + 7
38528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903
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864*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x
^3 + 3016704*x^2 - sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*
x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7
 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543
936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - 2*(
246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^10 +
221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728*x^4
+ 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13 + 296
08*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^7 - 2
67064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x + 339
20) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) +
 1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 88530*x^1
3 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 549408
*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2 - sqr
t(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^12 +
171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 125600*x^
6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176) - 67
584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 1742*x^1
4 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 9136*x^8
 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 + 256*
x)) + 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12)
+ 24)*(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan
(1/648*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x
^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 +
801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x
^2 - 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99
812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 -
 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x -
 36544) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*
x^17 + 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^1
2 + 57963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 10622736
0*x^7 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 264960
00*x^2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845
510*x^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 807
84280*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 173
35360*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56
sqrt(3) + 97) - 6(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^
13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 8
01584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^
2 - 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 998
12*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 -
321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x -
36544) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) -
sqrt(1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 2
16816*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7
 - 539220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(
3)*(181*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^1
1 + 94021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5
 + 139664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128
)*(56*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*
x^15 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 591615
24*x^10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 1005
98112*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17
 + 20698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 26
727704*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 -
26108944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 +
4724480*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17
- 104*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x
^11 - 521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 1

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321712*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15
*x^16 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 7526
7*x^10 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56
784*x^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt
(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + sqrt(-4*sqrt(3)*sqrt(56*sqrt(3)
+ 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x
^14 - 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*
x^9 + 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260
896*x^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 5
49126*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 5
444020*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 -
76360864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 1369644
8*x - 9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26
415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 62
834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 42
499296*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15
249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 3627
7360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536
992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 2
396160)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6
*(sqrt(3)*(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^1
1 + 8423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 +
10405056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15
+ 21635*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 33
79536*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*
x^4 - 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*
sqrt(x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 10
2933*x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 129636*x^8 + 243576*x^7 +
 221784*x^6 - 351024*x^5 - 460896*x^4 + 33984*x^3 + 174048*x^2 - sqrt(3)*(1
42*x^15 + 2124*x^14 - 7773*x^13 - 59447*x^12 - 40626*x^11 + 46860*x^10 + 26
308*x^9 + 75276*x^8 + 140472*x^7 + 127784*x^6 - 202896*x^5 - 266016*x^4 + 1
9712*x^3 + 100512*x^2 + 62400*x - 24832) + 108096*x - 43008)*sqrt(x^3 - 1)*
sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)) - 216*(130*x^16 + 1682*x
^15 + 2496*x^14 - 7730*x^13 + 1790*x^12 + 35700*x^11 - 7100*x^10 - 86080*x^
9 - 49176*x^8 + 100400*x^7 + 108208*x^6 - 33312*x^5 - 80704*x^4 - 18944*x^3
 + 18048*x^2 - 3*sqrt(3)*(25*x^16 + 324*x^15 + 489*x^14 - 1482*x^13 + 316*x
^12 + 6984*x^11 - 1312*x^10 - 16624*x^9 - 9792*x^8 + 19328*x^7 + 20976*x^6
- 6240*x^5 - 15552*x^4 - 3712*x^3 + 3456*x^2 + 4096*x - 1280) + 21248*x - 6
656)*sqrt(56*sqrt(3) + 97))*sqrt((18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720
*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 +
 46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24
*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 20
16*x^5 + 2214*x^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*
x^4 - 1192*x^3 + 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97)
- 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 1
0*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sq
rt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*
sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1
152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)
) + 3*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*((2*sqrt
(3)*(3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^13 + 16210952*x^12 + 7
7181756*x^11 + 84218362*x^10 - 71018320*x^9 - 254455812*x^8 - 196076008*x^7
 + 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 78464672*x^3 - 78514944*
x^2 - sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 915953*x^13 + 9359398*
x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 - 146910132*x^8 - 11320
4536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4 - 45301600*x^3 - 453
30624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160704)*sqrt(x^3 - 1)*(5
6*sqrt(3) + 97) - (459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 99730
2*x^12 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377
```

```
808*x^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609
792*x^2 - sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818
*x^12 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 4871560
0*x^7 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256
*x^2 + 2488832*x - 1889792) + 4310784*x - 3273216)*sqrt(x^3 - 1)*sqrt(56*sq
rt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 + 88617*x^1
4 + 738528*x^13 + 1860046*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 +
 6903864*x^8 + 15444144*x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155
968*x^3 + 3016704*x^2 - sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 107
3898*x^12 - 452980*x^11 - 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 891672
0*x^7 + 2490016*x^6 - 5519008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 +
 1543936*x - 545536) + 2674176*x - 944896)*sqrt(x^3 - 1)*(56*sqrt(3) + 97)
- 2*(246*x^15 + 7653*x^14 + 41169*x^13 + 51342*x^12 - 72300*x^11 - 45930*x^
10 + 221688*x^9 + 17892*x^8 - 490248*x^7 - 462360*x^6 + 389616*x^5 + 619728
*x^4 + 16608*x^3 - 187584*x^2 - sqrt(3)*(142*x^15 + 4419*x^14 + 23781*x^13
+ 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 10692*x^8 - 283320*x^
7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108288*x^2 - 96000*x
+ 33920) - 166272*x + 58752)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt
(3) + 1164)^(1/4)) - 216*(12*x^17 + 498*x^16 + 462*x^15 - 24972*x^14 - 8853
0*x^13 - 9726*x^12 + 300000*x^11 + 396768*x^10 - 87216*x^9 - 723072*x^8 - 5
49408*x^7 + 220128*x^6 + 584736*x^5 + 308256*x^4 - 155136*x^3 - 136704*x^2
- sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x^14 - 50390*x^13 - 5942*x^
12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 415384*x^8 - 315088*x^7 + 1256
00*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78784*x^2 - 39040*x + 18176)
 - 67584*x + 31488)*sqrt(56*sqrt(3) + 97))/(x^17 - 13*x^16 - 522*x^15 - 174
2*x^14 + 3008*x^13 + 16884*x^12 + 11656*x^11 - 23944*x^10 - 42336*x^9 - 913
6*x^8 + 36256*x^7 + 27360*x^6 - 256*x^5 - 13376*x^4 - 5760*x^3 + 1664*x^2 +
 256*x)) + 1/2592*(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt
(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 116
4)^(1/4)*log(1/18*(18*x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3
 - 144*x^2 + 72*sqrt(3)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^
2 - sqrt(3)*(15*x^7 - 22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4
) + 20*x - 8)*sqrt(56*sqrt(3) + 97) + (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x
^4 - 2064*x^3 + 396*x^2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3
+ 228*x^2 - 112) - 192)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27
*x^5 + 48*x^4 - 58*x^3 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3
+ 8*x^2 - 4*x) - 12*x + 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) +
 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 -
4*x^6 + 6*x^5 - 5*x^4 - 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x
^7 + 16*x^6 + 16*x^5 + 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/2592*(sqr
t(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*sqrt(-4*sqrt(3)*sqrt(56*sq
rt(3) + 97)*(7*sqrt(3) - 12) + 24)*(672*sqrt(3) + 1164)^(1/4)*log(1/18*(18*
x^8 - 36*x^7 + 828*x^6 - 360*x^5 + 720*x^4 - 1008*x^3 - 144*x^2 + 72*sqrt(3
)*(26*x^7 - 38*x^6 + 42*x^5 - 46*x^4 + 46*x^3 - 42*x^2 - sqrt(3)*(15*x^7 -
22*x^6 + 24*x^5 - 27*x^4 + 26*x^3 - 24*x^2 + 12*x - 4) + 20*x - 8)*sqrt(56*
sqrt(3) + 97) - (sqrt(3)*(123*x^6 - 2016*x^5 + 2214*x^4 - 2064*x^3 + 396*x^
2 - sqrt(3)*(71*x^6 - 1164*x^5 + 1278*x^4 - 1192*x^3 + 228*x^2 - 112) - 192
)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97) - 6*(5*x^6 - 27*x^5 + 48*x^4 - 58*x^3
 + 36*x^2 - 3*sqrt(3)*(x^6 - 5*x^5 + 10*x^4 - 10*x^3 + 8*x^2 - 4*x) - 12*x
+ 8)*sqrt(x^3 - 1))*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12)
+ 24)*(672*sqrt(3) + 1164)^(1/4) + 72*sqrt(3)*(x^7 - 4*x^6 + 6*x^5 - 5*x^4
- 4*x^3 - 6*x^2 + 4*x + 8) + 288*x + 1152)/(x^8 + 4*x^7 + 16*x^6 + 16*x^5 +
 28*x^4 - 32*x^3 + 64*x^2 - 32*x + 16)) - 1/36*sqrt(14*sqrt(3) + 24)*arctan
(-1/12*(3*x^2 - sqrt(3)*(x^2 + 10*x - 8) + 18*x - 12)*sqrt(14*sqrt(3) + 24)
/sqrt(x^3 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

maple [C] time = 0.26, size = 350, normalized size = 1.64

$$\frac{\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\ \mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\left(-2+\sqrt{3}\right)\sqrt{x^3-1}}-\frac{\sqrt{2}\ (-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alpha-2)/(-3^(1/2)+2*_alpha+1)*(-I*3^(1
/2)-3)*((x-1)/(-I*3^(1/2)-3))^(1/2)*((2*x+1-I*3^(1/2))/(-I*3^(1/2)+3))^(1/2
)*((2*x+1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)/(x^3-1)^(1/2)*(1+2*_alpha+3^(1/2)
*_alpha)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2*I*_alpha+1/3*I*3
^(1/2)*_alpha+1/2*3^(1/2)*_alpha+_alpha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(1-3^(1/2))*_Z-2*3^(1/2)
+4))+1/9*(3^(1/2)-1)/(-2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/
2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((
x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+6\sqrt{3}-10\right)\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3-1}\ \left(x^3+6\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)),x)
```

```
[Out] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\ \left(x^3-10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3))), x)
```

3.90 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

```
[Out] 1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/
2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1740, 207}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}x^2+x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)^3\left(1+\sqrt{3}\right)+\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 3.21, size = 685, normalized size = 10.54

$$\frac{\left(x+\sqrt{3}-1\right)^2\sqrt{-x^3+\left(\sqrt{3}-1\right)x^2-2\left(2+\sqrt{3}\right)x+2\left(1+\sqrt{3}\right)}\sqrt{\frac{-\frac{4}{x+\sqrt{3}-1}+\sqrt{3}+1}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\left(\left(\frac{2\left(2i\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right.}\right.}{x+\sqrt{3}-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] ((-1 + Sqrt[3] + x)^2*Sqrt[2*(1 + Sqrt[3]) - 2*(2 + Sqrt[3])*x + (-1 + Sqrt
[3])*x^2 - x^3]*Sqrt[(1 + Sqrt[3] - 4/(-1 + Sqrt[3] + x))/(3 + Sqrt[3] + I*
Sqrt[2*(2 + Sqrt[3])])]*((I*(-1 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])]) + (2*(
(2*I)*Sqrt[3] - Sqrt[2*(2 + Sqrt[3])] + Sqrt[6*(2 + Sqrt[3])]))/(-1 + Sqrt[
3] + x))*Sqrt[Sqrt[2*(2 + Sqrt[3])] + I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x)
)]*EllipticF[ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8/(-1 + S
qrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(2 + Sqrt[3])])
/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])] + 2*Sqrt[6]*Sqrt[(4 + 2*Sqrt[3] +
 x^2)/(-1 + Sqrt[3] + x)^2]*Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[3] + 8
/(-1 + Sqrt[3] + x))]*EllipticPi[(2*Sqrt[2*(2 + Sqrt[3])])/(Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3])), ArcSin[Sqrt[Sqrt[2*(2 + Sqrt[3])] - I*(1 - Sqrt[
3] + 8/(-1 + Sqrt[3] + x))]/(2^(3/4)*(2 + Sqrt[3])^(1/4))], ((2*I)*Sqrt[2*(
2 + Sqrt[3])])/(3 + Sqrt[3] + I*Sqrt[2*(2 + Sqrt[3])])]))/((Sqrt[2*(2 + Sqr
t[3])] + I*(3 + Sqrt[3]))*Sqrt[1 + Sqrt[3] - (2 + Sqrt[3])*x + ((-1 + Sqrt[
3])*x^2)/2 - x^3/2]*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]*Sqrt[Sqrt[2*(2 + Sqrt[3]
)] - I*(1 - Sqrt[3] + 8/(-1 + Sqrt[3] + x))])
```

fricas [B] time = 0.71, size = 323, normalized size = 4.97

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37x^{12}-204x^{11}+804x^{10}-2408x^9+3708x^8-5472x^7+6432x^6+10944x^5+14832x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

maple [C] time = 0.16, size = 327, normalized size = 5.03

$$\frac{\sqrt{-\left(-1+\frac{\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(1+\frac{\sqrt{3}}{2}\right)x^2+1}\ \mathrm{EllipticF}\left(\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)x, i\sqrt{1+4\sqrt{3}\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}-2\sqrt{3}\left(-\frac{\sqrt{-\left(-1\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*
(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^(1
/2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*3^(1/
2)*x^2+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1
/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2))*(1-
(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^(1/2)
*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^(1/2))/(-1-3^(1
/2))^2,(1+1/2*3^(1/2))^(1/2)/(-1+1/2*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^4+4\sqrt{3}x^2-4}\left(x+\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),x
)
```

```
[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

3.91 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\ \tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

```
[Out] -1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2
))*(3+2*3^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1740, 203}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\ \tan^{-1}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{x^4-4\sqrt{3}x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3])]*S
qrt[-4 - 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1740

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> -Dist[(A^2*(B*d + A*e))/e, Subst[Int[1/(6*A^3*B*d
 + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /; F
reeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^4*
(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && EqQ
[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}x^2+x^4}}\,dx=-\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\left(1+\sqrt{3}\right)^4+\right.\right.\right.$$

$$=-\frac{1}{3}\sqrt{3+2\sqrt{3}}\ \tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}x^2+x^4}}\right)$$

Mathematica [C] time = 7.99, size = 876, normalized size = 13.90

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{3}-1-\frac{4}{-x+\sqrt{3}+1}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}\left(-x+\sqrt{3}+1\right)^2\left(\left(\frac{2\left(2i\sqrt{3}\sqrt{i\left(\sqrt{3}+1-\frac{8}{-x+\sqrt{3}+1}\right)+\sqrt{4-2\sqrt{3}}}+\sqrt{6}\sqrt{2\sqrt{4-2\sqrt{3}}-\sqrt{12-6\sqrt{3}}+i\sqrt{3}-i+\frac{8i\left(-2+\sqrt{3}\right)}{-x+\sqrt{3}+1}}\cdots\right.}{x-\sqrt{3}-1}\right.\right.\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[(-1 + Sqrt[3] - 4/(1 + Sqrt[3] - x))/(-3 + Sqrt[3] - I*Sqrt
[4 - 2*Sqrt[3]])]*(1 + Sqrt[3] - x)^2*((I*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 +
 Sqrt[3] - 8/(1 + Sqrt[3] - x))] + I*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(
1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))] + Sqrt[-2*Sqrt[12 - 6*Sqrt[3]] + 4*Sqrt
[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 + Sqrt[3])*x))/(1 + Sqrt[3]
- x)] + (2*((2*I)*Sqrt[3]*Sqrt[Sqrt[4 - 2*Sqrt[3]] + I*(1 + Sqrt[3] - 8/(1
+ Sqrt[3] - x))] + Sqrt[6]*Sqrt[-I + I*Sqrt[3] - Sqrt[12 - 6*Sqrt[3]] + 2*S
qrt[4 - 2*Sqrt[3]] + ((8*I)*(-2 + Sqrt[3]))/(1 + Sqrt[3] - x)] + Sqrt[-2*Sq
rt[12 - 6*Sqrt[3]] + 4*Sqrt[4 - 2*Sqrt[3]] - ((2*I)*(14 - 8*Sqrt[3] + (-1 +
 Sqrt[3])*x))/(1 + Sqrt[3] - x)]))/(-1 - Sqrt[3] + x))*EllipticF[ArcSin[Sqr
t[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2
- Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 +
Sqrt[3]))] + 2*Sqrt[6]*Sqrt[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + S
qrt[3] - x))]*Sqrt[(4 - 2*Sqrt[3] + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(2
*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3])), ArcSin[Sqrt
[Sqrt[4 - 2*Sqrt[3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]/(2^(3/4)*(2 -
 Sqrt[3])^(1/4))], (2*Sqrt[4 - 2*Sqrt[3]])/(Sqrt[4 - 2*Sqrt[3]] + I*(-3 + S
qrt[3]))]))/((Sqrt[4 - 2*Sqrt[3]] - I*(-3 + Sqrt[3]))*Sqrt[Sqrt[4 - 2*Sqrt[
3]] - I*(1 + Sqrt[3] - 8/(1 + Sqrt[3] - x))]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]
))
```

fricas [B] time = 0.91, size = 112, normalized size = 1.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}\left(2x^4-10x^3+3x^2-10x+2\right)+24\right)\sqrt{x^4-4\sqrt{3}x^2-4}\sqrt{2\cdots}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

maple [C] time = 0.16, size = 311, normalized size = 4.94

$$\frac{\sqrt{-\left(-1-\frac{\sqrt{3}}{2}\right)x^2+1}\,\sqrt{-\left(1-\frac{\sqrt{3}}{2}\right)x^2+1}\ \mathrm{EllipticF}\left(\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)x, i\sqrt{1-4\sqrt{3}\,\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x^4-4\sqrt{3}\,x^2-4}}+2\sqrt{3}\left(-\frac{\sqrt{-\left(-1\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I*
(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(1/
2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*3^(1/2)
*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)/(
-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/
2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^
(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)^2,
(1-1/2*3^(1/2))^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}x^2-4}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),x
)
```

```
[Out] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

3.92 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

```
[Out] ln(1+x)-3/2*ln(2+x-(x^3+2)^(1/3))+arctan(1/3*(1+2*(2+x)/(x^3+2)^(1/3))*3^(1
/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2151}

$$-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (3
*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx=\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

maple [C] time = 3.00, size = 818, normalized size = 15.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(x+1)/(x^3+2)^(1/3),x)
```

```
[Out] RootOf(_Z^2-_Z+1)*ln(-(1239*RootOf(_Z^2-_Z+1)^2*x^3-2478*RootOf(_Z^2-_Z+1)^
2*x^2+4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)*x+4504*RootOf(_Z^2-_Z+1)*(x^3+2)
^(1/3)*x^2+3265*RootOf(_Z^2-_Z+1)*x^3-4956*RootOf(_Z^2-_Z+1)^2*x+9008*RootO
f(_Z^2-_Z+1)*(x^3+2)^(2/3)+18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x+10816*Ro
otOf(_Z^2-_Z+1)*x^2+335*(x^3+2)^(2/3)*x+335*(x^3+2)^(1/3)*x^2+1574*x^3+1801
6*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)+21632*RootOf(_Z^2-_Z+1)*x+670*(x^3+2)^(2/
3)+1340*(x^3+2)^(1/3)*x+7870*x^2+17346*RootOf(_Z^2-_Z+1)+1340*(x^3+2)^(1/3)
+15740*x+11018)/(x+1)^2)-ln(-(1239*RootOf(_Z^2-_Z+1)^2*x^3-2478*RootOf(_Z^2
-_Z+1)^2*x^2-4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)*x-4504*RootOf(_Z^2-_Z+1)*
(x^3+2)^(1/3)*x^2-5743*RootOf(_Z^2-_Z+1)*x^3-4956*RootOf(_Z^2-_Z+1)^2*x-900
8*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)-18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x-5
860*RootOf(_Z^2-_Z+1)*x^2+4839*(x^3+2)^(2/3)*x+4839*(x^3+2)^(1/3)*x^2+6078*
x^3-18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)-11720*RootOf(_Z^2-_Z+1)*x+9678*(x
^3+2)^(2/3)+19356*(x^3+2)^(1/3)*x+16208*x^2-17346*RootOf(_Z^2-_Z+1)+19356*(
x^3+2)^(1/3)+32416*x+28364)/(x+1)^2)*RootOf(_Z^2-_Z+1)+ln(-(1239*RootOf(_Z^
2-_Z+1)^2*x^3-2478*RootOf(_Z^2-_Z+1)^2*x^2-4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(
2/3)*x-4504*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)*x^2-5743*RootOf(_Z^2-_Z+1)*x^3-
4956*RootOf(_Z^2-_Z+1)^2*x-9008*RootOf(_Z^2-_Z+1)*(x^3+2)^(2/3)-18016*RootO
f(_Z^2-_Z+1)*(x^3+2)^(1/3)*x-5860*RootOf(_Z^2-_Z+1)*x^2+4839*(x^3+2)^(2/3)*
x+4839*(x^3+2)^(1/3)*x^2+6078*x^3-18016*RootOf(_Z^2-_Z+1)*(x^3+2)^(1/3)-117
20*RootOf(_Z^2-_Z+1)*x+9678*(x^3+2)^(2/3)+19356*(x^3+2)^(1/3)*x+16208*x^2-1
7346*RootOf(_Z^2-_Z+1)+19356*(x^3+2)^(1/3)+32416*x+28364)/(x+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-1}{{\left(x^3+2\right)}^{1/3}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x)
[Out] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

3.93 $\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=108

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

```
[Out] -1/2*ln(1+x)+3/4*ln(2+x-(x^3+2)^(1/3))-1/4*ln(-x+(x^3+2)^(1/3))+1/6*arctan(
1/3*(1+2*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1/3*(1+2*(2+x)/(x^3+2
)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2149, 239, 2151}

$$\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(
1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x
- (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2149

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2151

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx + \frac{1}{2}\int \frac{1-x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) + \frac{3}{4}\log\left(2+x-\sqrt[3]{2+x^3}\right.$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 + x)*(2 + x^3)^(1/3)),x]
[Out] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]
```

fricas [B] time = 2.70, size = 267, normalized size = 2.47

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{13910019318573948542\sqrt{3}\left(7114781247x^4 + 13663058416x^3 - 46178206896x^2 - 126842559\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*(13910019318573948542*sqrt(3)*(7114781247*x^4 + 1366
3058416*x^3 - 46178206896*x^2 - 126842559344*x - 77084338088)*(x^3 + 2)^(2/
3) - 27820038637147897084*sqrt(3)*(1625757424*x^5 + 16302821713*x^4 + 26102
613730*x^3 - 26431113242*x^2 - 80188343316*x - 42779182428)*(x^3 + 2)^(1/3)
 + sqrt(3)*(932925708335594356631323018 85*x^6 + 382151535711085278859235047
618*x^5 + 673924074224408772959625384792*x^4 + 8894265631830874680155802900
48*x^3 + 888876515195959220955879945824*x^2 + 35126059825850824001997196488
0*x - 47674000995597211057816884304))/(78905434814564721745708464883*x^6 +
337746705836458222863347934450*x^5 + 15598952776058587894336070976*x^4 - 89
5430525315100108684787964824*x^3 + 361667862240477028869533375352*x^2 + 254
1802301011632510645972090336*x + 1554815286823334092314485968880)) + 1/12*l
og((22*x^6 + 6*x^5 - 48*x^4 + 44*x^3 + 24*x^2 + 3*(7*x^4 - 2*x^3 - 32*x^2 -
 20*x + 4)*(x^3 + 2)^(2/3) + 3*(7*x^5 - 16*x^3 + 34*x^2 + 76*x + 32)*(x^3 +
 2)^(1/3) - 192*x - 140)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*x^2 + 6*x + 1)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

maple [C] time = 5.82, size = 2134, normalized size = 19.76

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x^3+2)^(1/3),x)
```

```
[Out] 1/6*RootOf(_Z^2+_Z+1)*ln((23004340956706368*x+234055794617652*x^6+213993869
3647104*x^5+802477010117664*x^4-7356039259411920*x^3+16049540202353 28*x^2-9
38391398532049*RootOf(_Z^2+_Z+1)^2*x^6-3075400381743690*RootOf(_Z^2+_Z+1)^2
*x^5-1160717687406137*RootOf(_Z^2+_Z+1)*x^6-3217341937824168*RootOf(_Z^2+_Z
+1)^2*x^4-7959206999356368*RootOf(_Z^2+_Z+1)*x^2+10391133689698608*RootOf(_
Z^2+_Z+1)*x-6434683875648336*RootOf(_Z^2+_Z+1)^2*x^2-4163618978360688*RootO
f(_Z^2+_Z+1)^2*x-3532767618003008*x^3*RootOf(_Z^2+_Z+1)^2+12498127505504256
*(x^3+2)^(1/3)+11008356238241516*RootOf(_Z^2+_Z+1)+16295099853018372*(x^3+2
)^(2/3)*x-3979603499678184*RootOf(_Z^2+_Z+1)*x^4-10197714008127436*x^3*Root
Of(_Z^2+_Z+1)-2832707206612248*RootOf(_Z^2+_Z+1)*x^5+10107087250606332*(x^3
+2)^(2/3)+6413512798877184*(x^3+2)^(1/3)*x^2+20062783627256832*(x^3+2)^(1/3
)*x+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4+780469570084659*Ro
otOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^5-1306943427662820*RootOf(_Z^2+_Z+1)^2*(x
^3+2)^(2/3)*x^3+3482095004993094*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^4-6019
04942144643*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4+1609243886794551*RootOf(_Z^
2+_Z+1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3
)*x^2+3842311729647552*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^3-32359551765899
22*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3+5249311303832568*RootOf(_Z^2+_Z+1)*(
x^3+2)^(1/3)*x^4-2613886855325640*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x-84050
5690860402*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^2-1442113972435308*RootOf(_Z
^2+_Z+1)*(x^3+2)^(2/3)*x^2+4061647060486932*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)
*x^3-2161300347926748*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x+7759251414704196*
RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x+3531674097632562*RootOf(_Z^2+_Z+1)*(x^3+2
)^(1/3)*x^2+11688730639030284*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x-92820189036
1806*(x^3+2)^(2/3)*x^4+740020707562752*(x^3+2)^(1/3)*x^5-1959537324097146*(
x^3+2)^(2/3)*x^3+657796184500224*(x^3+2)^(1/3)*x^4+5569211342170836*(x^3+2)
^(2/3)*x^2-1644490461250560*(x^3+2)^(1/3)*x^3+7115580883942020*RootOf(_Z^2+
_Z+1)*(x^3+2)^(2/3)+9125490357912936*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)+155591
37585059152)/(x+1)^6)-1/6*ln((8449588288647072*x+456382083491740*x^6+189724
5518515662*x^5+1564738571971680*x^4-691092869287492*x^3+3129477143943360*x^
2-938391398532049*RootOf(_Z^2+_Z+1)^2*x^6-3075400381743690*RootOf(_Z^2+_Z+1
)^2*x^5-716065109657961*RootOf(_Z^2+_Z+1)*x^6-3217341937824168*RootOf(_Z^2+
_Z+1)^2*x^4-4910160751940304*RootOf(_Z^2+_Z+1)*x^2-18718371646419984*RootOf
(_Z^2+_Z+1)*x-6434683875648336*RootOf(_Z^2+_Z+1)^2*x^2-4163618978360688*Roo
tOf(_Z^2+_Z+1)^2*x-3532767618003008*x^3*RootOf(_Z^2+_Z+1)^2+337263714759132
0*(x^3+2)^(1/3)-11008356238241516*RootOf(_Z^2+_Z+1)+5921961582988536*(x^3+2
)^(2/3)*x-2455080375970152*RootOf(_Z^2+_Z+1)*x^4+3132178772121420*x^3*RootO
f(_Z^2+_Z+1)-3318093556875132*RootOf(_Z^2+_Z+1)*x^5+2991506366664312*(x^3+2
)^(2/3)+2041333010384220*(x^3+2)^(1/3)*x^2+6212752640299800*(x^3+2)^(1/3)*x
+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4+780469570084659*RootO
f(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^5-1306943427662820*RootOf(_Z^2+_Z+1)^2*(x^3+
2)^(2/3)*x^3+3482095004993094*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^4+6745129
10348133*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4-48304746625233*RootOf(_Z^2+_Z+
1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^2
+3842311729647552*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^3+622068321264282*Roo
tOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3+1714878706153620*RootOf(_Z^2+_Z+1)*(x^3+2)
^(1/3)*x^4-2613886855325640*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x-84050569086
0402*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^2-6109114720727652*RootOf(_Z^2+_Z+
1)*(x^3+2)^(2/3)*x^2+3622976398808172*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^3-2
161300347926748*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x-12987025125355476*RootO
f(_Z^2+_Z+1)*(x^3+2)^(2/3)*x-5212685479353366*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/
3)*x^2-16011331334883780*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x-289992964115418*
(x^3+2)^(2/3)*x^4-88753609147140*(x^3+2)^(1/3)*x^5-30525575170044*(x^3+2)^(
2/3)*x^3-1109420114339250*(x^3+2)^(1/3)*x^4+3235710968024664*(x^3+2)^(2/3)*
x^2-1863825792089940*(x^3+2)^(1/3)*x^3-7115580883942020*RootOf(_Z^2+_Z+1)*(
x^3+2)^(2/3)-9125490357912936*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)+4550781346817
```

```
636)/(x+1)^6)*RootOf(_Z^2+_Z+1)-1/6*ln((8449588288647072*x+456382083491740*
x^6+1897245518515662*x^5+1564738571971680*x^4-691092869287492*x^3+312947714
3943360*x^2-938391398532049*RootOf(_Z^2+_Z+1)^2*x^6-3075400381743690*RootOf
(_Z^2+_Z+1)^2*x^5-716065109657961*RootOf(_Z^2+_Z+1)*x^6-3217341937824168*Ro
otOf(_Z^2+_Z+1)^2*x^4-4910160751940304*RootOf(_Z^2+_Z+1)*x^2-18718371646419
984*RootOf(_Z^2+_Z+1)*x-6434683875648336*RootOf(_Z^2+_Z+1)^2*x^2-4163618978
360688*RootOf(_Z^2+_Z+1)^2*x-3532767618003008*x^3*RootOf(_Z^2+_Z+1)^2+33726
37147591320*(x^3+2)^(1/3)-1100835623824151 6*RootOf(_Z^2+_Z+1)+5921961582988
536*(x^3+2)^(2/3)*x-2455080375970152*RootOf(_Z^2+_Z+1)*x^4+3132178772121420
*x^3*RootOf(_Z^2+_Z+1)-3318093556875132*RootOf(_Z^2+_Z+1)*x^5+2991506366664
312*(x^3+2)^(2/3)+2041333010384220*(x^3+2)^(1/3)*x^2+6212752640299800*(x^3+
2)^(1/3)*x+36303984101745*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x^4+78046957008
4659*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^5-1306943427662820*RootOf(_Z^2+_Z+
1)^2*(x^3+2)^(2/3)*x^3+3482095004993094*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x
^4+674512910348133*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4-48304746625233*RootO
f(_Z^2+_Z+1)*(x^3+2)^(1/3)*x^5-3775614346581480*RootOf(_Z^2+_Z+1)^2*(x^3+2)
^(2/3)*x^2+3842311729647552*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^3+622068321
264282*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3+1714878706153620*RootOf(_Z^2+_Z+
1)*(x^3+2)^(1/3)*x^4-2613886855325640*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(2/3)*x-8
4050569086040 2*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x^2-6109114720727652*RootO
f(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^2+3622976398808172*RootOf(_Z^2+_Z+1)*(x^3+2)^(
1/3)*x^3-2161300347926748*RootOf(_Z^2+_Z+1)^2*(x^3+2)^(1/3)*x-1298702512535
5476*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x-5212685479353366*RootOf(_Z^2+_Z+1)*(
x^3+2)^(1/3)*x^2-1601133133488378 0*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)*x-289992
964115418*(x^3+2)^(2/3)*x^4-88753609147140*(x^3+2)^(1/3)*x^5-30525575170044
*(x^3+2)^(2/3)*x^3-1109420114339250*(x^3+2)^(1/3)*x^4+3235710968024664*(x^3
+2)^(2/3)*x^2-1863825792089940*(x^3+2)^(1/3)*x^3-7115580883942020*RootOf(_Z
^2+_Z+1)*(x^3+2)^(2/3)-9125490357912936*RootOf(_Z^2+_Z+1)*(x^3+2)^(1/3)+455
0781346817636)/(x+1)^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+2\right)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 2)^(1/3)*(x + 1)),x)
[Out] int(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**3+2)**(1/3),x)
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

3.94 $\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=98

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3+a)^(1/3))/(a+b)^(1/3
)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/(a+b)^(1/3)*3
^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1-\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}}+\frac{\log\left(\frac{x^2(a+b)^{2/3}}{(a+bx^3)^{2/3}}+\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1\right)}{6\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) - Log[1 - ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*(a + b)^(1/3)) +
 Log[1 + ((a + b)^(2/3)*x^2)/(a + b*x^3)^(2/3) + ((a + b)^(1/3)*x)/(a + b*x
^3)^(1/3)]/(6*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(1-x^3\right)\sqrt[3]{a+bx^3}}\,dx = \text{Subst}\left(\int \frac{1}{1-(a+b)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{a+b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2+\sqrt[3]{a+b}\,x}{1+\sqrt[3]{a+b}\,x+(a+b)^{2/}}\right.$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{a+b}\,x+(a+b)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) +$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{a+b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a}}{\sqrt[3]{a+}}\right.}{\sqrt[3]{a+b}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}} - \frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{a+b}}$$

Mathematica [A] time = 0.09, size = 120, normalized size = 1.22

$$\frac{-2\log\left(1-\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)+\log\left(\frac{x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+\frac{x^2(a+b)^{2/3}}{\left(a+bx^3\right)^{2/3}}+1\right)}{6\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*
Log[1 - ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + ((a + b)^(2/3)*x^2)/
(a + b*x^3)^(2/3) + ((a + b)^(1/3)*x)/(a + b*x^3)^(1/3)]])/(6*(a + b)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^3-1\right)\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
```

```
[Out] -int(1/((x^3 - 1)*(a + b*x^3)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] -Integral(1/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

3.95 $\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=154

$$\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] 1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3
+a)^(1/3))/(a+b)^(1/3)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))/(a+b)^(1/3)*3^(1/2)+1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3)
)*3^(1/2))/(a+b)^(1/3)*3^(1/2)
```

Rubi [F] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ((3 - I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(a + b*x^3)^(1/3)), x]
)/3 + ((3 + I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(a + b*x^3)^(1/3
)), x])/3
```

Rubi steps

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx=\int \left(\frac{1-\frac{i}{\sqrt{3}}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}+\frac{1+\frac{i}{\sqrt{3}}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\right)dx$$
$$=\frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\,dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(x^2+x+1\right)\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
[Out] int((x+1)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\left(b\,x^3+a\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)),x)
[Out] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{a+bx^3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+x+1)/(b*x**3+a)**(1/3),x)
[Out] Integral((x + 1)/((a + b*x**3)**(1/3)*(x**2 + x + 1)), x)
```

3.96 $\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=96

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-
1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3))*3^(1/2))/(a+b)^(1/3)*3^(1/
2)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 55, 617, 204, 31}

$$\frac{\log(1-x^3)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3))) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b*x^3
)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-x^3\right)\sqrt[3]{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b)^{2/3}+\sqrt[3]{a+b}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{\ldots}}\right.}{}$$

$$= \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}\right)}{\sqrt[3]{a+b}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}} + \frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.83

$$\frac{-3\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)+\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]] + Log
[1 - x^3] - 3*Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)])/(6*(a + b)^(1/3))
```

fricas [B] time = 0.93, size = 387, normalized size = 4.03

$$\left[\frac{3\sqrt{\frac{1}{3}}(a+b)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}\log\left(\frac{2bx^3+3\sqrt{\frac{1}{3}}\left(\left(bx^3+a\right)^{\frac{1}{3}}(a+b)-(a+b)(-a-b)^{\frac{1}{3}}-2\left(bx^3+a\right)^{\frac{2}{3}}(-a-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-a-b)^{\frac{1}{3}}}{a+b}}+3a-3\left(bx^3+a\right)^{\frac{1}{3}}(-a-b)^{\frac{2}{3}}+b}{x^3-1}\right)+\ldots}{6(a+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log((2*b*x^3 + 3*sqr
t(1/3)*((b*x^3 + a)^(1/3)*(a + b) - (a + b)*(-a - b)^(1/3) - 2*(b*x^3 + a)^
(2/3)*(-a - b)^(2/3))*sqrt((-a - b)^(1/3)/(a + b)) + 3*a - 3*(b*x^3 + a)^(1
/3)*(-a - b)^(2/3) + b)/(x^3 - 1)) + (-a - b)^(2/3)*log((b*x^3 + a)^(2/3) -
```

```
(b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) - 2*(-a - b)^(2/3)*log(
(b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b), -1/6*(6*sqrt(1/3)*(a + b)*sqr
t(-(-a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a - b)
^(1/3))*sqrt(-(-a - b)^(1/3)/(a + b))) - (-a - b)^(2/3)*log((b*x^3 + a)^(2/
3) - (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) + 2*(-a - b)^(2/3)*
log((b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b)]
```

giac [A] time = 20.99, size = 113, normalized size = 1.18

$$-\frac{(a+b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3\,(a+b)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a+\sqrt{3}\,b}+\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{6\,(a+b)^{\frac{1}{3}}}-\frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}\right.\right.}{3\,(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] -(a + b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/(a
+ b)^(1/3))/(sqrt(3)*a + sqrt(3)*b) + 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) - 1/3*log(abs((b*x^3
+ a)^(1/3) - (a + b)^(1/3)))/(a + b)^(1/3)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^3+1\right)\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

maxima [A] time = 1.47, size = 110, normalized size = 1.15

$$-\frac{\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3\,(a+b)^{\frac{1}{3}}}\right)}{(a+b)^{\frac{1}{3}}}-\frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{(a+b)^{\frac{1}{3}}}+\frac{2\,b\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right)}{(a+b)^{\frac{1}{3}}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/
(a + b)^(1/3))/(a + b)^(1/3) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) + 2*b*log((b*x^3 + a)^(1/3) -
(a + b)^(1/3))/(a + b)^(1/3))/b
```

mupad [B] time = 0.59, size = 157, normalized size = 1.64

$$\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-\frac{9\,a+9\,b}{9\,{\left(-a-b\right)}^{2/3}}\right)}{3\,{\left(-a-b\right)}^{1/3}}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-\frac{{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(9\,a+9\,b\right)}{36\,{\left(-a-b\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,{\left(-a-b\right)}^{1/3}}-\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-\frac{(1+\sqrt{3}\,1\mathrm{i}}{36\,(-}\right.}{6\,(-a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - (9*a + 9*b)/(9*(- a - b)^(2/3)))/(3*(- a - b)^(1/3)
) + (log((a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)^2*(9*a + 9*b))/(36*(- a - b)
^(2/3)))*(3^(1/2)*1i - 1))/(6*(- a - b)^(1/3)) - (log((a + b*x^3)^(1/3) - (
(3^(1/2)*1i + 1)^2*(9*a + 9*b))/(36*(- a - b)^(2/3)))*(3^(1/2)*1i + 1))/(6*
(- a - b)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] -Integral(x**2/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

3.97 $\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=88

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right.$$

$$= \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ldots}{1-\sqrt[3]{2}\,x+2\ldots}\ldots\right.}{6\ldots}$$

$$= -\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

Mathematica [A] time = 0.07, size = 112, normalized size = 1.27

$$\frac{2\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}+1\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + (
2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)] + 2*Log[1 + (2^
(1/3)*x)/(1 - x^3)^(1/3)])/(6*2^(1/3))
```

fricas [B] time = 4.67, size = 253, normalized size = 2.88

$$-\frac{1}{18}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-\ldots\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [C] time = 2.42, size = 614, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/6*RootOf(_Z^3-4)*ln((-27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)^2*RootOf(_Z^3-4)^2*x^3-3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)*RootOf(_Z^3-4)^3*x^3+6*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-3*(-x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+2*(-x^3+1)^(1/3)*R
ootOf(_Z^3-4)^2*x^2-27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
*x^3-3*RootOf(_Z^3-4)*x^3+5*(-x^3+1)^(2/3)*x+9*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)+RootOf(_Z^3-4))/(x+1)/(x^2-x+1))+RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln(-(-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-9*RootOf(RootOf(_Z^3-4)^2+6*_
Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+30*(-x^3+1)^(
1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^
2+4*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x^2+12*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*x^3+3*RootOf(_Z^3-4)*x^3-2*(-x^3+1)^(2/3)*x-12*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)-3*RootOf(_Z^3-4))/(x+1)/(x^2-
x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.98 $\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=233

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} \cdots}{\sqrt{\cdots}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2
^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx=\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.02, size = 26, normalized size = 0.11

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

fricas [B] time = 3.43, size = 373, normalized size = 1.60

$$-\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}\,2^{\frac{2}{3}}\,(-1)^{\frac{2}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(x^{16}-\cdots\right.\right.}{6\left(x^{18}-102x^{15}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [F] time = 1.86, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.99 $\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=82

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,\right.}{2\sqrt[3]{2}}$$
$$= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$
$$= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.89

$$\frac{-\log\left(x^3+1\right)+3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^3] + 3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(6*2^(1/3))
```

fricas [A] time = 0.66, size = 90, normalized size = 1.10

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

giac [A] time = 0.88, size = 87, normalized size = 1.06

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

maple [C] time = 3.43, size = 655, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-5*RootOf(_Z^
3-4)*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^
3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/3)+35*RootOf(_Z^3-4)+42*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/6*ln((-12*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+1
8*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x
^3-12*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42*(-x^3+1)^(2/3)+28*RootOf(_Z
^3-4)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x
+1))*RootOf(_Z^3-4)-ln((-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-
4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+21*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+42
*(-x^3+1)^(2/3)+28*RootOf(_Z^3-4)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z
^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)
```

maxima [A] time = 1.51, size = 86, normalized size = 1.05

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

mupad [B] time = 0.55, size = 100, normalized size = 1.22

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right.}{4}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.100 $\int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=135

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.34, antiderivative size = 409, normalized size of antiderivative = 3.03, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6728, 2148}

$$-\frac{3\left(-\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{3\left(\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}-\frac{\left(3-i\sqrt{3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/(1 - x^3)^(1/
3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((3 + I*Sqrt[3])*ArcTan[(2 -
(2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(
I - Sqrt[3])) + ((I - Sqrt[3])*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3]
 + 2*x))])/(4*2^(1/3)*(I + Sqrt[3])) + ((I + Sqrt[3])*Log[-((1 + I*Sqrt[3]
- 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(4*2^(1/3)*(I - Sqrt[3])) - (3*(I - Sqrt[
3])*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I + S
qrt[3])) - (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} + \frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx + \left(1+i\sqrt{3}\right)\int \frac{1}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} + \frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)} + \frac{\left(i-\sqrt{3}\right)}{}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1+x}{\left(1-x+x^2\right)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

fricas [B] time = 12.83, size = 318, normalized size = 2.36

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}\left(-1\right)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}-4\sqrt{2}\left(-1\right)^{\frac{1}{3}}\left(x^5-x^4-3x^3+\right.\right.}{6\left(3x^6-9x^5+6x^4-x^3+6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

maple [C] time = 7.80, size = 720, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*(-x^3+1)^(2/3)+RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*x-2*RootOf(RootOf(
_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*(-x^3+1)^(1/3)*x+2*RootOf(RootOf(_Z^3
+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*(-x^3+1)^(1/3)-x^2+x-1)/(x^2-x+1))-1/2*ln
(-((-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootO
f(_Z^3+4)^2+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3
+4)^3*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x+(-x^3+1)^(1/3)*RootOf(
_Z^3+4)^2+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)*RootOf(_Z^3+4)+RootOf(_Z^3+4)*x^2-2*(-x^3+1)^(2/3)-3*RootOf(_Z^3+4)*x+R
ootOf(_Z^3+4))/(x^2-x+1))*RootOf(_Z^3+4)-ln(-((-x^3+1)^(2/3)*RootOf(RootOf(
_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2+RootOf(RootOf(_Z^3+4
)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x-(-x^3+1)^(1/3)*RootOf(_Z
^3+4)^2*x-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z
^2)*RootOf(_Z^3+4)*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+2*(-x^3+1)^(1/3)*RootO
f(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+RootOf(_Z^3+4
)*x^2-2*(-x^3+1)^(2/3)-3*RootOf(_Z^3+4)*x+RootOf(_Z^3+4))/(x^2-x+1))*RootOf
(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\left(1-x^3\right)^{1/3}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
```

```
[Out] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

3.101 $\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=135

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{2\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{\sqrt[3]{2}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.30, antiderivative size = 409, normalized size of antiderivative = 3.03, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1586, 6728, 2148}

$$-\frac{3\left(-\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x-i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}-\frac{3\left(\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{1-x^3}+2x+i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}-\frac{\left(3-i\sqrt{3}\right)\text{t}}{2\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] + 2*x))/(1 - x^3)^(1/
3))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) + ((3 + I*Sqrt[3])*ArcTan[(2 -
(2^(1/3)*(1 + I*Sqrt[3] + 2*x))/(1 - x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(
I - Sqrt[3])) + ((I - Sqrt[3])*Log[-((1 - I*Sqrt[3] - 2*x)^2*(1 - I*Sqrt[3]
 + 2*x))])/(4*2^(1/3)*(I + Sqrt[3])) + ((I + Sqrt[3])*Log[-((1 + I*Sqrt[3]
- 2*x)^2*(1 + I*Sqrt[3] + 2*x))])/(4*2^(1/3)*(I - Sqrt[3])) - (3*(I - Sqrt[
3])*Log[1 - I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3)*(I + S
qrt[3])) - (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x + 2*2^(2/3)*(1 - x^3)^(
1/3)])/(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$
$$= \int \left(\frac{1-i\sqrt{3}}{(-1-i\sqrt{3}+2x)\sqrt[3]{1-x^3}} + \frac{1+i\sqrt{3}}{(-1+i\sqrt{3}+2x)\sqrt[3]{1-x^3}}\right)dx$$
$$= (1-i\sqrt{3})\int \frac{1}{(-1-i\sqrt{3}+2x)\sqrt[3]{1-x^3}}\,dx + (1+i\sqrt{3})\int \frac{1}{(-1+i\sqrt{3}+2x)\sqrt[3]{1-x^3}}$$
$$= -\frac{(3-i\sqrt{3})\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}(1-i\sqrt{3}+2x)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\,(i+\sqrt{3})} + \frac{(3+i\sqrt{3})\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}(1+i\sqrt{3}+2x)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\,(i-\sqrt{3})} + \frac{(i-\sqrt{3})\,\mathrm{lo}}{}$$

Mathematica [C] time = 0.31, size = 150, normalized size = 1.11

$$\frac{1}{3}x^3F_1\left(1;\frac{1}{3},1;2;x^3,-x^3\right)+x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-}}\right.}{6\sqrt[3]{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] + (x^3*AppellF1[1, 1/3, 1, 2, x^3
, -x^3])/3 + (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3
]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)]
 + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)])/(6*2^(1/3))
```

fricas [B] time = 12.88, size = 318, normalized size = 2.36

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)\left(-x^3+1\right)^{\frac{2}{3}}-4\sqrt{2}\,(-1)^{\frac{1}{3}}\left(x^5-x^4-3x^3\right.\right.}{6\left(3x^6-9x^5+6x^4-x^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)

maple [C] time = 7.50, size = 676, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)^2/(-x^3+1)^(1/3)/(x^3+1),x)

[Out] RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+RootOf(_Z^3+4)*x^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-3*RootOf(_Z^3+4)*x-6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+2*(-x^3+1)^(2/3)+RootOf(_Z^3+4)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))+1/2*RootOf(_Z^3+4)*ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-RootOf(_Z^3+4)*x^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+RootOf(_Z^3+4)*x+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-RootOf(_Z^3+4)-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^2}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^2}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)

[Out] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

3.102 $\int \frac{1-x}{\left(1+x+x^2\right)\sqrt[3]{1+x^3}}\,dx$

Optimal. Leaf size=119

$$-\frac{\log\left(\frac{2^{2/3}(x+1)^2}{\left(x^3+1\right)^{2/3}}-\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{2\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}$$

```
[Out] -1/4*ln(1+2^(2/3)*(1+x)^2/(x^3+1)^(2/3)-2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3
)+1/2*ln(1+2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3)-1/2*arctan(1/3*(1-2*2^(1/3)
*(1+x)/(x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [C] time = 0.30, antiderivative size = 399, normalized size of antiderivative = 3.35, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6728, 2148}

$$\frac{3\left(-\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{x^3+1}-2x-i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(\sqrt{3}+i\right)}+\frac{3\left(\sqrt{3}+i\right)\log\left(2\,2^{2/3}\sqrt[3]{x^3+1}-2x+i\sqrt{3}+1\right)}{4\sqrt[3]{2}\left(-\sqrt{3}+i\right)}+\frac{\left(3-i\sqrt{3}\right)\text{ta}}{2\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] ((3 - I*Sqrt[3])*ArcTan[(2 - (2^(1/3)*(1 - I*Sqrt[3] - 2*x))/(1 + x^3)^(1/3
))/(2*Sqrt[3])])/(2*2^(1/3)*(I + Sqrt[3])) - ((3 + I*Sqrt[3])*ArcTan[(2 - (
2^(1/3)*(1 + I*Sqrt[3] - 2*x))/(1 + x^3)^(1/3))/(2*Sqrt[3])])/(2*2^(1/3)*(I
 - Sqrt[3])) - ((I - Sqrt[3])*Log[(1 - I*Sqrt[3] - 2*x)*(1 - I*Sqrt[3] + 2*
x)^2])/(4*2^(1/3)*(I + Sqrt[3])) - ((I + Sqrt[3])*Log[(1 + I*Sqrt[3] - 2*x)
*(1 + I*Sqrt[3] + 2*x)^2])/(4*2^(1/3)*(I - Sqrt[3])) + (3*(I - Sqrt[3])*Log
[1 - I*Sqrt[3] - 2*x + 2*2^(2/3)*(1 + x^3)^(1/3)])/(4*2^(1/3)*(I + Sqrt[3])
) + (3*(I + Sqrt[3])*Log[1 + I*Sqrt[3] - 2*x + 2*2^(2/3)*(1 + x^3)^(1/3)])/
(4*2^(1/3)*(I - Sqrt[3]))
```

Rule 2148

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sq
rt[3]])/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1-x}{\left(1+x+x^2\right)\sqrt[3]{1+x^3}}\,dx = \int\left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}+\frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\right)dx$$

$$= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\,dx + \left(-1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1}}$$

$$= \frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}-2x\right)}{\sqrt[3]{1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} - \frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}-2x\right)}{\sqrt[3]{1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)} - \frac{\left(i-\sqrt{3}\right.}{}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1-x}{\left(1+x+x^2\right)\sqrt[3]{1+x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)), x]
```

fricas [B] time = 13.54, size = 268, normalized size = 2.25

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^6+7x^5+10x^4+7x^3+10x^2+7x+1\right)-4\sqrt{2}\left(x^5+x^4-3x^3-3x^2+x+1\right)\left(x\right.\right.}{6\left(3x^6+9x^5+6x^4+x^3+6x^2+9x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 7*x^5 + 10*x
^4 + 7*x^3 + 10*x^2 + 7*x + 1) - 4*sqrt(2)*(x^5 + x^4 - 3*x^3 - 3*x^2 + x +
 1)*(x^3 + 1)^(1/3) + 4*2^(1/6)*(x^4 + 4*x^3 + 5*x^2 + 4*x + 1)*(x^3 + 1)^(
2/3))/(3*x^6 + 9*x^5 + 6*x^4 + x^3 + 6*x^2 + 9*x + 3)) - 1/12*2^(2/3)*log((
2^(2/3)*(x^3 + 1)^(2/3)*(x^2 + 3*x + 1) - 2^(1/3)*(x^4 - 3*x^2 + 1) - 4*(x^
3 + 1)^(1/3)*(x^2 + x))/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 1/6*2^(2/3)*log(
(2^(2/3)*(x^2 + x + 1) + 2*2^(1/3)*(x^3 + 1)^(1/3)*(x + 1) + 2*(x^3 + 1)^(2
/3))/(x^2 + x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

maple [C] time = 7.94, size = 652, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)/(x^2+x+1)/(x^3+1)^(1/3),x)
```

```
[Out] 1/2*RootOf(_Z^3-4)*ln(-((x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+4*_Z^2)*RootOf(_Z^3-4)^3*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2)^2*RootOf(_Z^3-4)^2*x+(x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x+2*(x^3+1)^(1/3
)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*x+(x^3
+1)^(1/3)*RootOf(_Z^3-4)^2+2*(x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Roo
tOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)+RootOf(_Z^3-4)*x^2+2*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+RootOf(_Z^3-4)*x+2*RootOf(RootOf(_Z^3-
4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+RootOf(_Z^3-4)+2*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+x+1))+RootOf(RootOf(_Z^3-4)^2+2*_Z*RootO
f(_Z^3-4)+4*_Z^2)*ln(((x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4
*_Z^2)*RootOf(_Z^3-4)^3*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_
Z^2)^2*RootOf(_Z^3-4)^2*x+(x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x+(x^3+1)^(1/3)*Ro
otOf(_Z^3-4)^2-RootOf(_Z^3-4)*x^2-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^
3-4)+4*_Z^2)*x^2+2*(x^3+1)^(2/3)-3*RootOf(_Z^3-4)*x-6*RootOf(RootOf(_Z^3-4)
^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-RootOf(_Z^3-4)-2*RootOf(RootOf(_Z^3-4)^2+2
*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\left(x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x-1}{\left(x^3+1\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)),x)
```

```
[Out] -int((x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\,dx-\int \left(-\frac{1}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3 + 1)**(1/
3)), x) - Integral(-1/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3
 + 1)**(1/3)), x)
```

3.103 $\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+x+x^2\right)^2}\,dx$

Optimal. Leaf size=43

$$x^2\left(-{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+x+x^2\right)^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]

[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x)^2, x])/3 + (((4*I)/3)*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] - (4*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x)^2, x])/3 + (((4*I)/3)*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]

Rubi steps

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+x+x^2\right)^2}\,dx=\int \left(-\frac{4\left(1-x^3\right)^{2/3}}{3\left(-1+i\sqrt{3}-2x\right)^2}+\frac{4i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(-1+i\sqrt{3}-2x\right)}-\frac{4\left(1-x^3\right)^{2/3}}{3\left(1+i\sqrt{3}+2x\right)^2}+\frac{4i\left(1-x\right.}{3\sqrt{3}\left(1+i\sqrt{\right.}}\right.$$

$$=-\left(\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}-2x\right)^2}\,dx\right)-\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}+2x\right)^2}\,dx+\frac{(4i)\int \frac{\left(1-x^3\right)^{2/3}}{-1+i\sqrt{3}-2x}\,dx}{3\sqrt{3}}+\frac{(4i)}{}$$

Mathematica [A] time = 0.15, size = 43, normalized size = 1.00

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(2x+1)\left(1-x^3\right)^{2/3}}{x^2+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]

[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3]

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4+2\,x^3+3\,x^2+2\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

maple [A] time = 0.10, size = 34, normalized size = 0.79

$$x^2 \,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)-\frac{(x-1)\,(2x+1)}{\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2+x+1)^2,x)
[Out] -(x-1)*(2*x+1)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2,x)
[Out] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2+x+1)**2,x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 + x + 1)**2, x)
```

3.104 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$

Optimal. Leaf size=43

$$x^2\left(-{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

Rubi [F] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] -((1 + I*Sqrt[3])*Defer[Int][1/((1 - I*Sqrt[3] + 2*x)*(1 - x^3)^(1/3)), x])
 - (1 - I*Sqrt[3])*Defer[Int][1/((1 + I*Sqrt[3] + 2*x)*(1 - x^3)^(1/3)), x]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx &= \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}+\frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx\\ &= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx+\left(-1+i\sqrt{3}\right)\int \frac{1}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 1.00

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(2x+1)\left(1-x^3\right)^{2/3}}{x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4+2x^3+3x^2+2x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

maple [A] time = 0.09, size = 34, normalized size = 0.79

$$x^2 \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{2}{3}\right], \left[\frac{5}{3}\right], x^3\right) - \frac{(x-1)(2x+1)}{\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)/(x^2+x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] -(x-1)*(2*x+1)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^2+x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x-1}{\left(1-x^3\right)^{1/3}\left(x^2+x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)),x)
```

```
[Out] -int((x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\, dx - \int \left(-\frac{1}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1/
3)), x) - Integral(-1/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 -
x**3)**(1/3)), x)
```

3.105 $\int \frac{(1-x)^2}{\left(1-x^3\right)^{4/3}} \, dx$

Optimal. Leaf size=39

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

```
[Out] (1+(1-2*x)*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3, 2/3],[5/3],x^3)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1854, 12, 364}

$$x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 + (1 - 2*x)*x)/(1 - x^3)^(1/3) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^2}{\left(1-x^3\right)^{4/3}} \, dx &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}}-\int -\frac{2x}{\sqrt[3]{1-x^3}} \, dx\\
&= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}}+2\int \frac{x}{\sqrt[3]{1-x^3}} \, dx\\
&= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}}+x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.10

$$x^2\left(-{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^2/(1 - x^3)^(4/3),x]
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4+2x^3+3x^2+2x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^2}{\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="giac")
[Out] integrate((x - 1)^2/(-x^3 + 1)^(4/3), x)
```

maple [A] time = 0.10, size = 34, normalized size = 0.87

$$x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)-\frac{(x-1)(2x+1)}{\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^2/(-x^3+1)^(4/3),x)
[Out] -(x-1)*(2*x+1)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}}-\int \frac{x^2-2x}{\left(x^3-1\right)\left(x^2+x+1\right)^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="maxima")
[Out] x/(-x^3 + 1)^(1/3) - integrate((x^2 - 2*x)/((x^3 - 1)*(x^2 + x + 1)^(1/3)*(
-x + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(x-1)^2}{\left(1-x^3\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^2/(1 - x^3)^(4/3),x)
[Out] int((x - 1)^2/(1 - x^3)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^2}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**2/(-x**3+1)**(4/3),x)
[Out] Integral((x - 1)**2/(-(x - 1)*(x**2 + x + 1))**(4/3), x)
```

3.106 $\int \left(1-x^3\right)^{2/3} dx$

Optimal. Leaf size=67

$$\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/3*x*(-x^3+1)^(2/3)+1/3*ln(x+(-x^3+1)^(1/3))-2/9*arctan(1/3*(1-2*x/(-x^3+1
)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 239}

$$\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3),x]
```

```
[Out] (x*(1 - x^3)^(2/3))/3 - (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/3
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \left(1-x^3\right)^{2/3} dx &= \frac{1}{3}x\left(1-x^3\right)^{2/3}+\frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}}\, dx\\ &= \frac{1}{3}x\left(1-x^3\right)^{2/3}-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.12, size = 101, normalized size = 1.51

$$\frac{3(x-1)\left(1-x^3\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};-\frac{x-1}{1-(-1)^{2/3}},-\frac{x-1}{1+\sqrt[3]{-1}}\right)}{5\left(\frac{x-1}{1+\sqrt[3]{-1}}+1\right)^{2/3}\left(\frac{x-1}{1-(-1)^{2/3}}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(2/3),x]
```

```
[Out] (3*(-1 + x)*(1 - x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 -
(-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))])/(5*(1 + (-1 + x)/(1 + (-1)^(1/
3)))^(2/3)*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))
```

fricas [A] time = 0.62, size = 94, normalized size = 1.40

$$\frac{1}{3}\left(-x^{3}+1\right)^{\frac{2}{3}}x-\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{2}{9}\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{9}\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x}{x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(2/3)*x - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x
^3 + 1)^(1/3))/x) + 2/9*log((x + (-x^3 + 1)^(1/3))/x) - 1/9*log((x^2 - (-x^
3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^{3}+1\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3), x)
```

maple [C] time = 0.10, size = 12, normalized size = 0.18

$$x\,\mathrm{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3),x)
```

```
[Out] x*hypergeom([-2/3,1/3],[4/3],x^3)
```

maxima [B] time = 1.30, size = 105, normalized size = 1.57

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{3x^{2}\left(\frac{x^{3}-1}{x^{3}}-1\right)}+\frac{2}{9}\log\left(\frac{\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{9}\log\left(-\frac{\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 2/9*log((-x^3 + 1)^(1/3)/x + 1) - 1/9*l
og(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

mupad [B] time = 0.34, size = 10, normalized size = 0.15

$$x\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3),x)
```

```
[Out] x*hypergeom([-2/3, 1/3], 4/3, x^3)
```

sympy [C] time = 1.02, size = 31, normalized size = 0.46

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3),x)
```

```
[Out] x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3))
```

3.107 $\int \frac{\left(1-x^3\right)^{2/3}}{x} \, dx$

Optimal. Leaf size=70

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] 1/2*(-x^3+1)^(2/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 55, 618, 204, 31}

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/x,x]
```

```
[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x
]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-x^3\right)^{2/3}}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(1-x)^{2/3}}{x}\,dx, x, x^3\right)\\ &= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)\\ &= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(x)}{2} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{\right.\\ &= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\ &= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.93

$$\frac{1}{2}\left(\left(1-x^3\right)^{2/3} + \log\left(1-\sqrt[3]{1-x^3}\right) - \log(x)\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/x,x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + ((1 - x^3)^(2/3) - Log[x]
 + Log[1 - (1 - x^3)^(1/3)])/2
```

fricas [A] time = 0.87, size = 75, normalized size = 1.07

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3}\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3}\log\left(\left(-x^3+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/2*(-x^3
+ 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x
^3 + 1)^(1/3) - 1)
```

giac [A] time = 0.89, size = 74, normalized size = 1.06

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}} + 1\right)\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}} + \left(-x^3+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{3}\log\left(\left|\left(-x^3+1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3
 + 1)^(1/3) - 1))
```

maple [C] time = 0.10, size = 66, normalized size = 0.94

$$-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^3\right)}{3\Gamma\left(\frac{2}{3}\right)}-\frac{\left(3\ln(x)+\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/x,x)
```

```
[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*(2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1,
1],[2,2],x^3)-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(
2/3))
```

maxima [A] time = 1.41, size = 73, normalized size = 1.04

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1
)^(1/3) - 1)
```

mupad [B] time = 0.40, size = 91, normalized size = 1.30

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6) + (1 - x^3)^(2/3)/2
```

sympy [C] time = 1.02, size = 41, normalized size = 0.59

$$-\frac{x^2 e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/x,x)
```

```
[Out] -x**2*exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**(-3))/(3*gam
ma(1/3))
```

3.108 $\int \frac{(1-x^3)^{2/3}}{a+bx} \, dx$

Optimal. Leaf size=384

$$-\frac{(a^3+b^3)^{2/3}\log(a^3+b^3x^3)}{3b^3}+\frac{(a^3+b^3)^{2/3}\log\left(-\frac{x\sqrt[3]{a^3+b^3}}{a}-\sqrt[3]{1-x^3}\right)}{2b^3}+\frac{(a^3+b^3)^{2/3}\log\left(\sqrt[3]{a^3+b^3}-b\sqrt[3]{1-x^3}\right)}{2b^3}-$$

```
[Out] 1/2*(-x^3+1)^(2/3)/b-1/2*(a^3+b^3)*x^2*AppellF1(2/3,1/3,1,5/3,x^3,-b^3*x^3/
a^3)/a^2/b^2+1/2*a*x^2*hypergeom([1/3, 2/3],[5/3],x^3)/b^2-1/3*(a^3+b^3)^(2
/3)*ln(b^3*x^3+a^3)/b^3+1/2*(a^3+b^3)^(2/3)*ln(-(a^3+b^3)^(1/3)*x/a-(-x^3+1
)^(1/3))/b^3-1/2*a^2*ln(x+(-x^3+1)^(1/3))/b^3+1/2*(a^3+b^3)^(2/3)*ln((a^3+b
^3)^(1/3)-b*(-x^3+1)^(1/3))/b^3+1/3*a^2*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3
^(1/2))/b^3*3^(1/2)-1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1-2*(a^3+b^3)^(1/3)*x/a
/(-x^3+1)^(1/3))*3^(1/2))/b^3*3^(1/2)+1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1+2*b
*(-x^3+1)^(1/3)/(a^3+b^3)^(1/3))*3^(1/2))/b^3*3^(1/2)
```

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{a+bx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] Defer[Int][(1 - x^3)^(2/3)/(a + b*x), x]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{a+bx} \, dx = \int \frac{(1-x^3)^{2/3}}{a+bx} \, dx$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{a+bx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

maple **[F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(b*x+a),x)
[Out] int((-x^3+1)^(2/3)/(b*x+a),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(a + b*x),x)
[Out] int((1 - x^3)^(2/3)/(a + b*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(b*x+a),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(a + b*x), x)
```

3.109 $\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$

Optimal. Leaf size=234

$$\frac{1}{3}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-x^3)^{2/3}\,x}{3(x^3+1)}-\frac{(1-x^3)^{2/3}}{3(x^3+1)}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2}{\sqrt[3]{...}}}{\sqrt{...}}\right)}{3\sqrt{3}}$$

```
[Out] -1/3*(-x^3+1)^(2/3)/(x^3+1)+1/3*x*(-x^3+1)^(2/3)/(x^3+1)+2/3*x^2*(-x^3+1)^(
2/3)/(x^3+1)+1/3*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/6*ln(2^(1/3)-(-x^3+1
)^(1/3))*2^(2/3)+1/6*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/9*arctan(1/3*(
1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/9*arctan(1/3*(1+2^
(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [F] time = 0.41, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 + (((4*I)/3)*
Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x), x])/Sqrt[3] - (4*Defer[In
t][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3 + (((4*I)/3)*Defer[Int][
(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x), x])/Sqrt[3]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx &= \int\left(-\frac{4(1-x^3)^{2/3}}{3(1+i\sqrt{3}-2x)^2}+\frac{4i(1-x^3)^{2/3}}{3\sqrt{3}(1+i\sqrt{3}-2x)}-\frac{4(1-x^3)^{2/3}}{3(-1+i\sqrt{3}+2x)^2}+\frac{4i(1-x^3)^{2/3}}{3\sqrt{3}(-1+i\sqrt{3}...}\right.\\ &= -\left(\frac{4}{3}\int\frac{(1-x^3)^{2/3}}{(1+i\sqrt{3}-2x)^2}\,dx\right)-\frac{4}{3}\int\frac{(1-x^3)^{2/3}}{(-1+i\sqrt{3}+2x)^2}\,dx+\frac{(4i)\int\frac{(1-x^3)^{2/3}}{1+i\sqrt{3}-2x}\,dx}{3\sqrt{3}}+\frac{(4i)\int\frac{(...}{-1...}}{3...}\end{aligned}$$

Mathematica [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]
```

fricas [F] time = 4.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^4-2\,x^3+3\,x^2-2\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
[Out] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2,x)
[Out] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 - x + 1)**2, x)
```

3.110 $\int \frac{(1-2x)(1-x^3)^{2/3}}{(1-x+x^2)^2} dx$

Optimal. Leaf size=199

$$\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{\sqrt[3]{2}}+\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+$$

```
[Out] (-x^3+1)^(2/3)/(x^2-x+1)+1/2*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(-2^(
1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+ln(x+(-x^3+1)^(1/3))-2/3*arctan(1/3*(1-2*x/(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3
))*3^(1/2))*2^(2/3)*3^(1/2)+1/3*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)
```

Rubi [F] time = 0.95, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] (-4*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 + (4*(1 + I*S
qrt[3])*Defer[Int][(1 - x^3)^(2/3)/(1 + I*Sqrt[3] - 2*x)^2, x])/3 - (4*Defe
r[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3 + (4*(1 - I*Sqrt[3])
*Defer[Int][(1 - x^3)^(2/3)/(-1 + I*Sqrt[3] + 2*x)^2, x])/3
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx &= \int \left(\frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}-\frac{2x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\right) dx\\
&= -\left(2\int \frac{x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx\right)+\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx\\
&= -\left(2\int \left(-\frac{2\left(1+i\sqrt{3}\right)\left(1-x^3\right)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2}+\frac{2i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)}-\frac{2\left(1-i\sqrt{3}\right)\left(1-x^3\right)^{2/3}}{3\left(-1+i\sqrt{3}+2x\right)^2}\right.\right.\\
&= -\left(\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2} dx\right)-\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2} dx+\frac{1}{3}\left(4\left(1-i\sqrt{3}\right)\right)\int
\end{aligned}$$

Mathematica [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]
```

fricas [B] time = 5.17, size = 1827, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(-1/6*(3822*4^(2/3)*sqrt(3)*(5
0*x^4 - 74*x^3 - 207*x^2 + 143*x + 19)*(-x^3 + 1)^(2/3) + 7644*4^(1/3)*sqrt
(3)*(19*x^5 - 150*x^4 + 43*x^3 + 112*x^2 + 57*x - 50)*(-x^3 + 1)^(1/3) - 7*
sqrt(39)*(6*4^(1/3)*sqrt(3)*(1150*x^4 - 3974*x^3 - 1911*x^2 + 1522*x + 3898
)*(-x^3 + 1)^(2/3) - 4^(2/3)*sqrt(3)*(1778*x^6 - 6366*x^5 - 8412*x^4 + 1725
4*x^3 + 15117*x^2 - 4227*x - 16105) + 12*sqrt(3)*(437*x^5 - 1539*x^4 - 333*
x^3 - 2074*x^2 + 372*x + 3261)*(-x^3 + 1)^(1/3))*sqrt((6*4^(1/3)*(5*x^4 + 4
*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^
4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-
x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6*sqrt(3
)*(29494*x^6 - 17582*x^5 + 153824*x^4 - 266248*x^3 - 129950*x^2 + 238106*x
- 29747))/(138718*x^6 - 463746*x^5 - 296508*x^4 - 115072*x^3 + 1093704*x^2
- 70446*x - 256859)) + 8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(3822*4^(
2/3)*sqrt(3)*(19*x^4 - 181*x^3 + 36*x^2 + 169*x - 31)*(-x^3 + 1)^(2/3) - 76
44*4^(1/3)*sqrt(3)*(31*x^5 + 57*x^4 - 131*x^3 - 119*x^2 + 93*x + 19)*(-x^3
+ 1)^(1/3) + 7*sqrt(39)*(6*4^(1/3)*sqrt(3)*(3385*x^4 + 3574*x^3 - 1911*x^2
- 2948*x + 124)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(13027*x^6 + 16539*x^5 -
 8961*x^4 - 32644*x^3 - 2361*x^2 + 17139*x - 239) - 12*sqrt(3)*(2748*x^5 +
3450*x^4 - 4126*x^3 - 2385*x^2 + 1539*x - 76)*(-x^3 + 1)^(1/3))*sqrt((6*4^(
1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x
^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x
+ 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6
*sqrt(3)*(53953*x^6 - 12994*x^5 - 396521*x^4 + 169424*x^3 + 300029*x^2 - 62
294*x - 41597))/(52723*x^6 + 682854*x^5 - 325173*x^4 - 1353400*x^3 + 193623
*x^2 + 640446*x - 16073)) + 16*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(76
44*4^(2/3)*sqrt(3)*(5*x^4 - 107*x^3 - 243*x^2 + 26*x + 157)*(-x^3 + 1)^(2/3
) - 7644*4^(1/3)*sqrt(3)*(307*x^5 + 300*x^4 - 140*x^3 - 221*x^2 - 186*x - 9
8)*(-x^3 + 1)^(1/3) + 7*sqrt(39)*4^(1/3)*(6*4^(1/3)*sqrt(3)*(3109*x^4 + 400
*x^3 - 3822*x^2 + 1426*x + 3622)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(15505*
x^6 + 11493*x^5 - 22383*x^4 - 22720*x^3 - 5454*x^2 + 13032*x + 10888) - 12*
sqrt(3)*(2111*x^5 + 3450*x^4 - 941*x^3 - 1111*x^2 - 372*x - 2624)*(-x^3 + 1
)^(1/3)) + 6*sqrt(3)*(307479*x^6 + 239258*x^5 - 543668*x^4 - 607716*x^3 + 1
9112*x^2 + 232000*x + 343788))/(933353*x^6 + 1472754*x^5 + 285042*x^4 - 100
8596*x^3 - 1598208*x^2 - 560184*x + 468980)) + 48*sqrt(3)*(x^2 - x + 1)*arc
tan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3
)*(x^3 - 1))/(9*x^3 - 1)) - 3*4^(1/3)*(x^2 - x + 1)*log(39626496*(6*4^(1/3)
*(5*x^4 + 4*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*
x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*
x^2 + 1)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1))
 - 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(5*x^4 + 4*x^3 - 3*x^2 -
4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12
*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-x^3 + 1)^(1/3))
/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)
*log(39626496*(6*4^(1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) +
 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 -
5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6
*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(x^4 - 4*
x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 -
25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1
```

```
)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) - 24*(x^2 - x + 1
)*log(3*(-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1) - 72*(-x^3 + 1)^(2
/3))/(x^2 - x + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(2x-1)}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(-(-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(-2x+1)\left(-x^3+1\right)^{\frac{2}{3}}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x+1)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

```
[Out] int((-2*x+1)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}(2x-1)}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -integrate((-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(2x-1)\left(1-x^3\right)^{2/3}}{\left(x^2-x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2,x)
```

```
[Out] -int(((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\left(1-x^3\right)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1}\right)dx-\int \frac{2x\left(1-x^3\right)^{\frac{2}{3}}}{x^4-2x^3+3x^2-2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x) - Integ
ral(2*x*(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x)
```

3.111 $\int \frac{(1-x^3)^{2/3}}{1+x} dx$

Optimal. Leaf size=177

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\cdots$$

[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Verification is Not applicable to the result.

[In] Int[(1 - x^3)^(2/3)/(1 + x),x]

[Out] Defer[Int][(1 - x^3)^(2/3)/(1 + x), x]

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx = \int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - x^3)^(2/3)/(1 + x),x]

[Out] Integrate[(1 - x^3)^(2/3)/(1 + x), x]

fricas [F] time = 5.35, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-x^3+1)^{\frac{2}{3}}}{x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="fricas")

[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

maple **[F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x+1),x)
```

```
[Out] int((-x^3+1)^(2/3)/(x+1),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^3\right)^{2/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x + 1),x)
```

```
[Out] int((1 - x^3)^(2/3)/(x + 1), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

3.112 $\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} dx$

Optimal. Leaf size=177

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\cdots$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] Defer[Int][(1 - x^3)^(2/3)/(1 + x), x]
```

Rubi steps

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} dx = \int \frac{(1-x^3)^{2/3}}{1+x} dx$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3), x]
```

fricas [F] time = 4.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(-x^3+1)^{\frac{2}{3}}}{x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}\left(x^{2}-x+1\right)}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}-x+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{2}{3}}\left(x^{2}-x+1\right)}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^{3}\right)^{2/3}\left(x^{2}-x+1\right)}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1),x)
[Out] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)*(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

3.113 $\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx$

Optimal. Leaf size=132

$$-\frac{\log(x^3+1)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/6*ln(x^3+1)*2^(2/3)+1/2*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(
-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*a
rctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 0.16, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] x*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.10, size = 111, normalized size = 0.84

$$-\frac{4x(1-x^3)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] (-4*x*(1 - x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-
4*AppellF1[1/3, -2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/
3, x^3, -x^3] + 2*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

fricas [A] time = 0.63, size = 191, normalized size = 1.45

$$-\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{3}\cdot 4^{\frac{1}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\left(-\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*4^(1/3)*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(-x^3 + 1)^(1
/3))/x) + 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/
x) + 1/3*4^(1/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x) - 1/6*4^(1/3)*log(
(2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) - 1/
3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3
 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^3\right)^{2/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x^3 + 1),x)
```

```
[Out] int((1 - x^3)^(2/3)/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

3.114 $\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx$

Optimal. Leaf size=250

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{2^2}{}$$

```
[Out] -1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/12*ln((1-x)*(1+x)^2)*2^(2/3)+1/6
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-
1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/4*ln(-1+x+2^(2/3)*(-x^3+1)
^(1/3))*2^(2/3)+1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^
(1/2)*2^(2/3)
```

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 0.10, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx=\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.10

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

fricas [F] time = 4.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}} x}{x^3+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}} x}{x^3+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{2}{3}} x}{x^3+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(1-x^3\right)^{2/3}}{x^3+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^3)^(2/3))/(x^3 + 1),x)
```

```
[Out] int((x*(1 - x^3)^(2/3))/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{\left(x+1\right)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x*(-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

3.115 $\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3} dx$

Optimal. Leaf size=383

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(x^3+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\right.}{\sqrt[3]{2}}$$

```
[Out] 1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/12*ln((1-x)*(1+x)^2)*2^(2/3)-1/6*
ln(x^3+1)*2^(2/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x
^3+1)^(1/3))*2^(2/3)+1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/4*ln(-1+x+2^(
2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/
3))*3^(1/2))*2^(2/3)*3^(1/2)-1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)-1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/
2)
```

Rubi [F] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (-2*Defer[Int][(1 - x^3)^(2/3)/(-1 - x), x])/3 - ((1 + (-1)^(2/3))*Defer[In
t][(1 - x^3)^(2/3)/(-1 + (-1)^(1/3)*x), x])/3 - ((1 - (-1)^(1/3))*Defer[Int
][(1 - x^3)^(2/3)/(-1 - (-1)^(2/3)*x), x])/3
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx &= \int\left(-\frac{2\left(1-x^3\right)^{2/3}}{3(-1-x)}+\frac{\left(-1-(-1)^{2/3}\right)\left(1-x^3\right)^{2/3}}{3\left(-1+\sqrt[3]{-1}\,x\right)}+\frac{\left(-1+\sqrt[3]{-1}\right)\left(1-x^3\right)^{2/3}}{3\left(-1-(-1)^{2/3}x\right)}\right)dx\\ &= -\left(\frac{2}{3}\int\frac{\left(1-x^3\right)^{2/3}}{-1-x}\,dx\right)+\frac{1}{3}\left(-1+\sqrt[3]{-1}\right)\int\frac{\left(1-x^3\right)^{2/3}}{-1-(-1)^{2/3}x}\,dx+\frac{1}{3}\left(-1-(-1)^{2/3}\right)\int\frac{(1}{-1}\end{aligned}$$

Mathematica [C] time = 0.16, size = 138, normalized size = 0.36

$$-\frac{4\left(1-x^3\right)^{2/3}xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}-\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] -1/2*(x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]) - (4*x*(1 - x^3)^(2/3)*Ap
pellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -2/3, 1,
```

```
4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2*Appell
F1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

fricas [F] time = 11.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(-x+1)\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-x^3+1\right)^{\frac{2}{3}}(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{\left(1-x^3\right)^{2/3}\,(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1),x)
```

```
[Out] -int(((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{3}+1}\right) dx - \int \frac{x\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{3}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**3 + 1), x) - Integral(x*(1 - x**3)**(2/3)/
(x**3 + 1), x)
```

3.116 $\int \frac{\sqrt[3]{1-x^3}}{1+x^3} \, dx$

Optimal. Leaf size=272

$$\frac{\log \left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log \left(\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2} (1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3} \sqrt[3]{2} \log \left(\frac{\sqrt[3]{2} (1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log \left(\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\sqrt[3]{1-x^3}}+2 \sqrt[3]{2}\right)}{6\ 2^{2/3}}$$

```
[Out] 1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^
3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)
/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)
```

Rubi [C] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 0.08, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$x F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3} \, dx = x F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.09, size = 109, normalized size = 0.40

$$-\frac{4 x \sqrt[3]{1-x^3} F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right) \left(x^3 \left(3 F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4 F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] (-4*x*(1 - x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-
4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3, 2, 7/
3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))
```

fricas [A] time = 3.53, size = 341, normalized size = 1.25

$$\frac{1}{18}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{6\sqrt{3}\,2^{\frac{2}{3}}\left(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-24\sqrt{3}\,2^{\frac{1}{3}}\left(x^{14}-2\,x^{11}-6\,x^{8}\right.}{3\left(x^{18}-102\,x^{15}+447\,x^{12}-628\,x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(1/3)*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(x^16 - 33*x^13 + 110*x
^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 24*sqrt(3)*2^(1/3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - sqrt(3)*(x^18 + 42*x^15 - 4
17*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 62
8*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/18*2^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^
2 + 2^(2/3)*(x^6 + 2*x^3 + 1) - 6*2^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3))/(x^6
+ 2*x^3 + 1)) - 1/36*2^(1/3)*log((12*2^(2/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)
^(2/3) + 2^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 6*(x^10 - 11*x^7 +
 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

maple [C] time = 6.55, size = 1214, normalized size = 4.46

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/2*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*ln(-(-6*RootOf(Root
Of(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^4*x^3-36*RootOf(Roo
tOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+x^6*RootOf
(_Z^3-2)^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^
3-2)*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3
-2)^2*(-x^3+1)^(2/3)*x^2+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*Roo
tOf(_Z^3-2)+9*_Z^2)*x^4-6*RootOf(_Z^3-2)^2*x^3-36*RootOf(RootOf(_Z^3-2)^2+3
*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3-18*(-x^3+1)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x+RootOf(_Z^3-2)^2+6*RootOf(RootOf
(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2))/(x+1)^2/(x^2-x+1)^2)
-1/6*ln((6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-
2)^4*x^3+36*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z
^3-2)^3*x^3+x^6*RootOf(_Z^3-2)^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3
-2)+9*_Z^2)*RootOf(_Z^3-2)*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-
2)+9*_Z^2)*RootOf(_Z^3-2)^2*(-x^3+1)^(2/3)*x^2+6*RootOf(_Z^3-2)*(-x^3+1)^(1
/3)*x^4+18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)*x^4-2*RootOf(_Z^3-2)^2*x^3-12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2
)+9*_Z^2)*RootOf(_Z^3-2)*x^3+12*(-x^3+1)^(2/3)*x^2-6*RootOf(_Z^3-2)*(-x^3+1
)^(1/3)*x-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_
Z^2)*x+RootOf(_Z^3-2)^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)*RootOf(_Z^3-2))/(x+1)^2/(x^2-x+1)^2)*RootOf(_Z^3-2)-1/2*ln((6*RootOf(Roo
```

```
tOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^4*x^3+36*RootOf(Ro
otOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+x^6*RootO
f(_Z^3-2)^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z
^3-2)*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^
3-2)^2*(-x^3+1)^(2/3)*x^2+6*RootOf(_Z^3-2)*(-x^3+1)^(1/3)*x^4+18*(-x^3+1)^(
1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^4-2*RootOf(_Z^3-
2)^2*x^3-12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3
-2)*x^3+12*(-x^3+1)^(2/3)*x^2-6*RootOf(_Z^3-2)*(-x^3+1)^(1/3)*x-18*(-x^3+1)
^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x+RootOf(_Z^3-2)
^2+6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2))/(x
+1)^2/(x^2-x+1)^2)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x^3 + 1),x)
```

```
[Out] int((1 - x^3)^(1/3)/(x^3 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}}{(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/((x + 1)*(x**2 - x + 1)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```